The London School of Economics and Political Science

Sequential Auctions and Resale

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work.

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 22,000 words.

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Abstract

In this thesis I study a market comprised of a sequence of auctions where buyers can choose to later resell any object they now buy. I develop a structural model of such a market and show how the possibility to resell shapes equilibrium strategies. I then estimate the model on data from classic car auctions. The model admits aggregate shocks to buyer and seller wealth and that way matches the positive empirical correlation between prices and the state of the economy.

Using a separate two-period model I show analytically that the resale option may increase average prices as compared to an otherwise identical market without resale. The same two-period model shows that with aggregate shocks resale may amplify price volatility. I then evaluate the quantitative importance of these effects in a number of counterfactual experiments on the estimated model. Resale raises prices moderately but does not lead to meaningfully more volatility. Allowing (counterfactually) for instantaneous resale increases average prices and their volatility substantially. A second set of counterfactuals reveals that centralizing trade lowers prices and increases the volume of trade, thereby increasing the efficiency of the market. Price volatility remains unchanged in this scenario, even with frequent resale opportunities.

An assumption in my model and several others in the literature is that bidders take a stationary distribution of rival bids as given and don't learn about that distribution from one auction to the next. This is different from the canonical model of sequential auctions in Weber (1983), where learning is present. I therefore compare the Weber model to a model where bidders face a stationary distribution of rival bids in each period. I show how equilibrium strategies differ in the two games and show that despite the differences, the two games yield the same expected prices and payoffs.

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Chapter 1

Introduction and Literature

In many markets trade takes place through a sequence of auctions. Examples include government procurement auctions, auctions for art, fine wine or classic cars, and a huge variety of goods on online auction platforms such as eBay. Sequential auctions are also a convenient way of modelling decentralized trade. For both of these reasons sequential auction markets have been studied extensively in both the theoretical and the empirical literature. This dissertation contributes to two aspects of sequential auctions that have as of yet received relatively little attention.

Sequential auctions are a dynamic setting in the first instance because buyers who lose an auction can bid again in future auctions. In many settings there is an additional dynamic aspect, namely that successful bidders can use future auctions to sell an object they just purchased. Chapter 2 studies this possibility of resale in sequential auction markets using the market for classic cars as an example, where resale is particularly important because classic cars have very low rates of depreciation.

Chapter 3 focusses on the information generated by the dynamic nature of sequential auction markets and its influence on bidder strategies. When bidders have to track the evolution of competition in the market, strategies in sequential auctions can become very complex, which is why many models make assumptions so that strategies of bidders are stationary. The chapter therefore compares the canonical model of sequential auctions, in which players have to track how the degree of competition changes over the course of the game, to a stationary environment.

The remainder of the introduction is devoted to a review of the related literature. Section 1.1 discusses the theory of sequential auctions. Section 1.2 begins with a review of the methods to estimate static models of auctions. Structural sequential auctions papers build on these methods to estimate their models and are discussed next. Section 1.3 first introduces the literature on the efficiency of static two sided trading mechanisms. Building on these papers a new literature studies the efficiency of sequential auction markets, both theoretically and empirically, and is reviewed next. Section 1.4 covers auctions with resale and related markets.

1.1 Sequential Auctions Theory

The first model of sequential auctions is in Weber (1983). The model has N players bidding on $M < N$ identical objects, which are sold one after the other in a sequence of M auctions. All N bidders are active in the first period and only exit the game by winning one of the auctions. This set-up means that continuing bidders are the ones who have not yet won an auction and so the distribution of values among the currently active bidders is declining over the course of the game. The famous result in the paper is that, in spite of this deteriorating distribution of bidder values, prices are in expectation the same in each auction and that the sequence of prices is a martingale.[1]

Said (2012) adds bidder entry to the Weber (1983) model. Entering bidders' values are drawn from a different distribution than values of continuing bidders since the latter have lost previous auctions and must therefore tend to have lower values. This creates asymmetric beliefs among the bidders about their respective competitors' values. For this reason there may be no symmetric equilibrium under a sealed-bid auction format and the auction may be inefficient. The paper shows that efficiency can be recovered with an open ascending auction format because this allows all bidders to learn about their competitors' value over the course of the stage game auction.

Many papers make assumptions to avoid the complications introduced by information release about bidders' values from one auction to the next. The complications arise because bidders' values are persistent over time. A different approach is to assume that bidders each period draw a value anew, just before any one auction takes place, and, importantly, do so independently across objects. Their value for one object is then not correlated with their value for other objects. Following Engelbrecht-Wiggans (1994), this is often referred to as stochastically equivalent objects when the values are all drawn from the same distribution. Since any one bidder's valuation for a given object is drawn independently from their valuation for the other objects, a bidder's past bids (or their staying in the game) are uninformative about their valuation for the objects at present and future auctions. A justification for this assumption may be that objects at each auction are not exactly identical and that bidders may learn the condition of the object just before the

[1] Surprisingly, given this result, Ashenfelter (1989) finds that prices in wine auctions have a statistically significant negative trend, although in most cases prices are constant. See also Ashenfelter and Genesove (1992), McAfee and Vincent (1993), Beggs and Graddy (1997), and Van den Berg et al. (2001) on the 'price decline anomaly'.

auction (e.g. for used items that were identical when new but have different conditions at the time of the auction). This also implies however that bidders have no persistent private information about their value for the object, which is likely to be violated in many settings. Engelbrecht-Wiggans (1994) shows that under the stochastic equivalence assumption, prices are not necessarily a martingale but may trend up or down, depending on the distribution of values. Said (2011) extends this setting to allow for a random number of bidders and random periods of time between auctions. Budish (2008) uses the assumption of stochastic equivalence in determining how best to combine several single-unit auctions to create a market for multiple units. The paper compares the efficiency of sequential to simultaneous separate auctions (in the latter case bidders have to choose only one auction to enter) and of providing public information about other auctions in the market to hiding it.

Zeithammer (2006) studies the amount of information about future auctions that bidders take into account when deciding their current bid on auction platforms such as eBay. Buyers in the model have perfectly persistent valuations but do not take the information generated by competitors staying in the game for more than one auction into account. Although not described by the author as such, this amounts to assuming a 'mean-field' equilibrium in which bidders do not fully optimise against the complex strategic environment but instead optimise against a stationary distribution of rival bids every period.[2] Buyers are interested in buying one unit of one specific product, i.e. there is no substitution between similar goods. On eBay the number of auctions due to end at any given time varies, which affects the bidders' near-term option value of losing one of these auctions. The author considers three different models of buyer behaviour that differ in the amount of information about future auctions they take into account. In the first bidders do not consider the number of auctions that will end soon. In the second case bidders consider the total number of auctions due to end soon and in the third case bidders consider the number of auctions only for the object they are interested in purchasing. If bidders take the number of all auctions or auctions for their desired object ending soon into account, they will shade their bid more the higher this number is. The author tests these three models using data from eBay auctions for MP3 players and movie DVDs and finds the strongest evidence for the third model, in which bidders take into account the number of auctions for their desired object due to end soon.

[2] This assumption was also adopted by Backus and Lewis (2019) and others in the structural literature on sequential auctions, which will be discussed below.

1.2 Structural Models

1.2.1 Static Models

The structural analysis of sequential auction models builds upon identification results and estimation methods developed first for static models of auctions. Guerre et al. (2000) show nonparametric identification for the first price auction with independent private values and suggest a nonparametric estimation method, which obtains the optimal rate of convergence to the true distribution. For the case without reserve price the approach is as follows. The objective of a bidder i with private value v_i in a first price auction where all other players play a monotone strategy $b(v)$ is:

$$\underset{B}{max} \quad F(b^{-1}(B))^{N-1}(v - B)$$

where $F(\cdot)$ is the distribution from which bidder values are drawn. The first order condition for i's optimal bid b_i is then:

$$v_i - b_i = \frac{1}{N-1}\frac{F(v_i)}{f(v_i)}b'(v_i)$$

Which uses the fact that in a symmetric increasing equilibrium $b_i = b(v_i)$. From this it also follows that $F(v_i) = F(b^{-1}(b_i)) = G(b_i)$, where $G(b)$ is now defined as the distribution of equilibrium bids. Moreover, $g(b_i) = \frac{d}{db_i}G(b_i) = \frac{d}{db_i}F(b^{-1}(b_i)) = f(b^{-1}(b_i))\frac{1}{b'(b^{-1}(b_i))} = f(v_i)/b'(v_i)$. Plugging this into the first order condition above yields a relationship between values and bids that depends on the distribution of bids, not the distribution of values:

$$v_i - b_i = \frac{1}{N-1}\frac{G(b_i)}{g(b_i)}$$

$G(\cdot)$ and $g(\cdot)$ can be estimated nonparametrically from observed bids. Each bid thus implies a private value through the equation above after plugging in the estimates for $G(\cdot)$, $g(\cdot)$, and the number of bidders N. This yields a sample of 'pseudo values', which can in a second step be used to estimate the distribution of values.

In the Milgrom and Weber (1982) model of English auctions bidders have a weakly dominant strategy to bid their value and the selling price will be the second highest valuation among the participating bidders. Athey and Haile (2002) show that the distribution of values is in that model identified from the selling price and the number of bidders alone. The distribution of equilibrium selling prices is equal to the distribution of the second order statistic from N draws, $F_{v_{(2:N)}}(v)$, and can be estimated nonparametrically. There is a unique relationship between a distribution and the distribution of order statistics associated with that distribution, which can be used to find the distribution of values $F(v)$, see the estimation strategy in Roberts (2013).

1.2.2 Dynamic Models

Most models of sequential auctions discussed below have an ascending auction as their stage game but the methods for estimating them are most similar to the approach developed in Guerre et al. (2000). The reason is that Guerre et al. (2000) showed how to account for the bid shading term stemming from the non-truthful first price auction mechanism while, similarly, estimators for sequential auction games have to account for an (additional) dynamic shading term that corrects equilibrium bids downwards by the continuation value associated with not winning the current auction. Estimation methods for sequential auction games express this continuation value in terms of the distribution of bids and plug estimates of this distribution into an inverted optimality condition to estimate unobservable values. Aguirregabiria and Mira (2007), Pesendorfer and Schmidt-Dengler (2008), and Arcidiacono and Miller (2011) develop estimators for dynamic games but focus on discrete action spaces.

Jofre-Bonet and Pesendorfer (2003) was the first paper to estimate a structural model of sequential auctions. The paper develops a model of repeated procurement auctions and estimates the model on data from highway paving contracts. Each period firms bid on a project for which they privately learn their cost just before the auction. Costs are, conditional on publicly known state variables, distributed independently. In the language of Engelbrecht-Wiggans (1994) projects are thus stochastically equivalent. Firms that win contracts accumulate 'backlog', i.e. projects that have not yet been completed at the time of further auctions. The model allows firms to take this effect into account and the estimates suggest that the cost of undertaking additional projects is increasing in backlog. The auction format is a first price auction with reserve price. The dynamic nature of the game means that different continuation values are associated with winning the auction and with losing it, which enters the equilibrium bids. To estimate the distribution of private costs the authors first estimate the distribution of bids nonparametrically. The value function can be expressed as a function of the distribution of bids. The estimates from the previous step can be plugged in to provide an estimate of the value function, and find costs using the inverted equilibrium bid function.

Recently, a new generation of structural models of sequential auctions has emerged, including Backus and Lewis (2019), Hendricks and Sorensen (2018), Bodoh-Creed et al. (2019), and Coey et al. (2019). All of these papers use data from auctions on the online marketplace eBay. Like Weber they also all assume perfectly persistent valuations but, rather than modelling information release from one auction to the next, make assumptions that lead to stationary equilibrium strategies. Hendricks and Sorensen (2018) and Coey et al. (2019) follow Backus and Lewis (2019) in adopting a 'mean-field' or 'oblivious' equilibrium assumption. Bidders form their bids by optimising against a stationary distribution of rival bids rather than a distribution of bids that changes over time and is in part determined by the bidder's own bid, as is the case in Weber (1983). This can be thought of as a 'large market approximation'. The idea being that in a large market, like one for popular products on eBay, newly arriving bidders are likely to dwarf those that a bidder may have encountered in a previous auction, and so learning from one auction to the next can safely be ignored. In Bodoh-Creed et al. (2019) stationarity follows from assuming a continuum of players, which will be discussed further below.

To illustrate the estimation approach given a stationary equilibrium adopted by these papers, consider the following simplified set-up. The cited papers extend this setting in various different directions. Bidders each period enter a second price auction without reserve price for identical objects. Bidder i has perfectly persistent value x_i. Values are drawn from a common distribution. Bidders believe that the highest rival bid they compete against follows a stationary distribution $G(\bar{b})$. The equilibrium bid function is then the following:

$$b_i = x_i - \eta V(x_i)$$

Where η is a discount factor that differs depending on the specifics of the model and the continuation value $V(x_i)$ is as follows:

$$V(x_i) = G(b_i)(x_i - \mathbf{E}[\bar{b}|\bar{b} < b]) + (1 - G(b_i))\eta V(x_i)$$

The first term is the probability of winning an auction given the distribution of the highest rival bid $G(\bar{b})$ times the payoff conditional on winning. The second term is the probability of not winning times the discounted continuation value. This recursion can be solved for the following expression:

$$V(x_i) = \frac{G(b_i)}{1 - \eta(1 - G(b_i))}(x_i - \mathbf{E}[\bar{b}|\bar{b} < b])$$

After plugging in the above expression for the continuation value, the bid function can be inverted to yield the following estimable equation:

$$x_i = b_i + \frac{\eta}{1 - \eta} G(b_i)(b_i - \mathbf{E}[\bar{b}|\bar{b} < b])$$

The distribution of values can thus be estimated nonparametrically given an estimate of the discount factor, a nonparametric estimate of $G(\bar{b})$ and of $\mathbf{E}[\bar{b}|\bar{b} < b]$.

Backus and Lewis (2019) set out to estimate a demand system for differentiated products from a sequential auctions market akin to the literature on demand estimation from fixed price markets. In a market like eBay, each auction is for one product only but consumers are choosing from a whole range of products available on the platform and therefore reduce their bids in any given auction by the amount at which they value the option of bidding in a future auction for a substitute product. Substitution thus has to take place across auctions, while traditional methods for estimating the distribution of taste parameters from auctions consider a static auction for one product in isolation.

A bidder who loses an auction and does not exit the game (which happens at an exogenous rate) gets to bid again on the same or a substitute product next period when that bidder is active (while in the game bidders are active with an exogenous probability each period). The continuation value thus depends on the matrix of transition probabilities between products, the probability of being active in a given period, and the stationary distribution of rival bids on all products. The simple set-up above is a simplification of this setting with only one product. The eBay proxy bidding system is modelled as a second price sealed-bid auction and so the equilibrium on each product is the bidder's value shaded by the continuation value of losing discounted by the probability of exit.

The authors show that the probability of being active while in the game and the probability of exit are identified in the data. The transition probabilities between products can be observed directly. To estimate the distribution of bids for all products one has to correct for the fact that higher types are more likely to bid on only one or a few products and exit early than lower types. Having done that, these estimates can again be plugged into the expression for the continuation values and be used to estimate values from bids using the equilibrium bid function. An additional complication is that the relationship between values and bids for a given product holds only for bidders who submit a positive bid on that product since some bidders may choose to wait for a more preferred product. This means that in the present context the inversion from bids to values provides only set identification of the distribution of values.

The authors extend their approach to allow for demand estimation in characteristics space (rather than product space) with random coefficients and estimate that model on data from eBay auctions for compact cameras focussing on resolution as the primary characteristic of a camera. Reduced form evidence suggests that buyers do substitute across camera resolutions: a substantial fraction bids on cameras with different resolutions when bidding in more than one auction. The estimated demand system allows for a random intercept and a random coefficient on taste for the resolution of a camera. The results suggest that on average resolution is the most important determinant of consumers' willingness to pay for a camera but that there is substantial heterogeneity in how much consumers value resolution. A counterfactual exercise using the estimated parameters shows that if a monopolist seller of cameras with one specific resolution on eBay were able to commit to future reserve prices (rather than commit only to a reserve price in the present auction), the seller could raise profits above and beyond what is possible with a reserve price without such commitment for future auctions.

Coey et al. (2019) introduce deadlines into consumers' decision problem to explain a number of phenomena observed on the eBay platform. In their model buyers have to obtain a certain product by a given idiosyncratic date. They arrive in the market randomly with different amounts of time left before that date. As long as the deadline remains in the future, bidders place relatively low bids in the hopes of buying at a discount. As the deadline comes closer, the remaining chances of buying the object at a low price are falling. Bidders therefore increase their bids over time. When the deadline arrives without the buyer having won an auction they will resort to a 'buy-it-now' option, which allows buyers to purchase a good at the full price without having to go through an auction. The model thus predicts that for bidders who participate in several auctions for the same product bids will tend to go up. This prediction is borne out in the eBay data. Providing both the auction and buy-it-now sales channels is in the model an endogenous response by sellers to the described buyer behaviour.

1.3 Efficiency of Decentralized Markets

1.3.1 Theory

Static mechanisms

The literature on efficiency in sequential auction markets as a way of modelling large decentralised markets follows a literature investigating whether static double auctions achieve an efficient outcome as the number of buyers and sellers grows large. Chatterjee and Samuelson (1983) show that the sealed-bid double auction with one seller and one buyer (so-called bilateral monopoly) who each have private information over their respective value for the item does not in general yield efficient trade, which would mean trade taking place whenever the buyer values the good more than the seller. Myerson and Satterthwaite (1983) show that in fact there is no mechanism that yields efficiency. It turns out however that this informational inefficiency is confined to small markets. Satterthwaite and Williams (1989) and Gresik and Satterthwaite (1989) show, respectively, that the double auction and the optimal mechanism both converge to the efficient outcome as the number of traders on both sides of the market grows.

Dynamic decentralized markets

Satterthwaite and Shneyerov (2007) show that decentralized markets can approximate a Walrasian market even under incomplete information as long as there is a large number of buyers and sellers and if there are very frequent trading opportunities. In the model there is a continuum of potential bidders and a continuum of potential sellers entering the game each period. They decide to enter based on their expected payoff of doing so. While in the game, they then incur a participation cost each unit of time. Bidders and sellers who have entered get matched randomly to trade through simultaneous single-unit first price auctions with a secret reserve price. This is repeated each period of the infinite horizon game. If there was a centralized trading mechanism each period, such as a double auction, the large number of traders on both sides of the market would guarantee an efficient allocation, even in a one-shot game, as shown in the literature cited above. The decentralized mechanism in the present setting on the other hand will in a single round of trading not achieve efficiency. Due to random matching, some auctions will be populated by many high-value bidders and some auctions will only have low-value bidders. To win, players may thus have to stay in the game for multiple periods and incur the associated participation cost. This will lead to inefficiently low entry of bidders. The novel result in Satterthwaite and Shneyerov (2007) is that in a dynamic game with many rounds of trading, also the outcome implemented by many separate single-unit auctions each period approximates the efficient outcome arbitrarily well as the time between periods becomes infinitesimal. The intuition is that as the waiting time (and hence cost) becomes very small between periods, price dispersion has to shrink as buyers would not find it optimal to bid anywhere above the lowest price they can expect to achieve by waiting. Sellers accept this low price because future bidders will bid just as low. As the distribution of equilibrium prices becomes degenerate it has to center on the Walrasian price or else the market would not clear and there would be profitable deviations.

In Satterthwaite and Shneyerov (2007) buyers and sellers play stationary strategies due to the assumed contiuum of buyers and sellers. Since buyers are matched to a contiuum of sellers each period, there are infinitely many simultaneous auctions each period and buyers will almost surely not encounter the same competitor twice. The continuum of traders also avoids the problem with entry, raised by Said (2012), that continuing and entering bidders have asymmetric beliefs. Since each bidder is infinitesimal, entering and continuing bidders have the same information. Both types face a continuum of competitors with a fixed proportion of entering to continuing ones. Beliefs are therefore symmetric.

Satterthwaite and Shneyerov (2008) replace the participation cost with an exogenous exit rate for bidders, which also has the effect of making them impatient, i.e. willing to pay a higher price in exchange for earlier trade. They show that convergence to Walrasian outcomes carries over to this setting also.

1.3.2 Structural Models

Hendricks and Sorensen (2018) take the theorem in Satterthwaite and Shneyerov (2007, 2008) to motivate a comparison of the outcome achieved by eBay to the limiting case of full efficiency. For popular products there are very frequent auctions on eBay and the platform attracts many traders. One may therefore think that it could come quite close to this benchmark, although, as in any real market, plenty of frictions are likely to remain. The model in Hendricks and Sorensen (2018) is similar to the one in Backus and Lewis (2019) restricted to one good. An important difference is that, instead of one sealed-bid auction per period, their model has overlapping ascending auctions that bidders sort into endogenously after observing the current bid. As in Backus and Lewis (2019) buyers enter at an exogenous rate and losing bidders exit at an exogenous rate. Bidders discount the future only at the rate with which they exit. Moreover there is no reserve price. Inefficiency arises only from the exogenous exit rate. Some high value bidders, who happen to bid against another high value bidder and lose, exit before winning a subsequent auction. Equilibrium prices fall when bidders exit at a lower rate, discount future auctions less, and shade more. However, the price level plays no role for efficiency. The inefficiency due to premature exogenous exit is present in Satterthwaite and Shneyerov (2008), too. In addition, due to the reserve price of the seller, the price does have an effect on efficiency in the models in Satterthwaite and Shneyerov (2007, 2008). When frictions are large, the reserve price means that the expected price can be high enough to deter entry by potential bidders with values above but close to the Walrasian price, who in an efficient market would be allocated an object.

The authors estimate their model on data from eBay auctions for ipads. Having estimated the distribution of buyers' values for an ipad, they calculate the Walrasian or market-clearing price. In a Walrasian market all buyers with a valuation above that price would receive the item. On eBay 59% of buyers with values above that level end up with an ipad. The market thus falls well short of the efficient benchmark. However, the dynamic nature of eBay does nevertheless substantially improve welfare: the authors estimate that holding all auctions in their data simultaneously, and thus giving each buyer the chance to bid only once, would result in only 31% of buyers with values above the Walrasian price receiving the object.

Bodoh-Creed et al. (2019) seek to understand whether a more efficient outcome could be achieved on eBay by changing the trading mechanism to a more centralised one. Instead of many single unit auctions within a given time frame, a number of them could be pooled into one (multi-unit) uniform price auction. They estimate a model very similar to the one in Satterthwaite and Shneyerov (2007). Each period a continuum of potential bidders arrive to the market and choose whether to enter or not. While in the market they incur a per-period participation cost and only exit by winning an auction. Auctions have a reserve price but these are taken as exogenous.

The authors show that this model is a good approximation for a model with a large but finite number of buyers and sellers. Specifically, if all players in a game with a finite number of traders play the equilibrium of the limiting game with a continuum of traders, for any $\epsilon > 0$ there is a number of players N such that the payoff a player can get by deviating from the limiting equilibrium is smaller than ϵ. An assumption behind this result is that as the number of players grows, there are increasingly many auctions per time period, of which buyers can enter only one. The probability of encountering the same competitor twice thus falls. In the limit this probability is zero as in Satterthwaite and Shneyerov (2007).

Estimation again proceeds by first using an estimate of the distribution of bids to obtain a plug-in estimate of the continuation value and then find values from the inverted equilibrium bid function as in Jofre-Bonet and Pesendorfer (2003) and Guerre et al. (2000). They extend these estimation methods in such a way that does not require observing the exact number of bidders but only those bids that resulted in an increase of the current price in the ascending eBay auction. They also argue that the eBay auction mechanism diverges from a second price sealed-bid auction and show that bid shading due to the non-truthful mechanism and due to the dynamic opportunity cost of winning can be identified and estimated separately.

They estimate their model on data from auctions for kindle fire tablets and find that 64% of buyers who would receive an object in a fully efficient market also receive one on eBay. This number is relatively close to the 59% found by Hendricks and Sorensen (2018). The authors then counterfactually allocate the same number of objects through fewer multi-unit uniform price auctions each period. They find that the majority of the efficiency gain obtainable by centralization can be achieved by implementing half as many two-unit auctions or a quarter as many four-unit auctions. Interestingly, this is reminiscent of the finding in Satterthwaite and Williams (1989), using numerical examples, that a small number of traders on both sides of the market (six in their case) is enough to realise almost all gains from trade in the double auction for bilateral monopoly.

Markets with search are an alternative to sequential auction markets of modelling decentralized markets with frictions. Gavazza (2016) studies the efficiency of a decentralized asset market in a bilateral search framework with intermediaries. Using data from the market for business aircraft he finds that frictions are substantial with almost 20 percent of aircraft misallocated. Intermediaries improve the allocation compared to a counterfactual without intermediaries but nevertheless, since they extract surplus, welfare would be higher without intermediaries.

1.4 Resale

1.4.1 Auction Models with Resale

Finally, chapter 2 of this dissertation is related to the literature on auctions with resale. Haile (2001) was the first empirical study of auctions with resale. The paper looks at auctions for the rights to harvest timber. The rights often get sold long before the time of the actual harvesting and the holders often end up subcontracting the job. This motivates the following model of two English auctions in sequence. Players have independent private values for the object. However, before the first auction they receive only a noisy signal about this value. The winner of the first auction becomes the seller of the second auction and the losers of the first auction get to bid again. In the first period players thus have to adjust their bids for the difference between the continuation values of being a seller and being a buyer. Between the first and the second auctions, all players observe their true values for the object. This realisation of uncertainty between the two periods provides scope for resale (the bidder with the highest signal may end up not having the highest value). Players place bids based on their signals in the first round and in the second round place bids based on their realised value. Signals are correlated with values, and so bids in the first and in the second auction will be correlated. This has two dynamic effects on first-period bidding: higher bids in the first period make it (i) less attractive to be a buyer again next period (since competing bids are expected to be higher) and make it (ii) more attractive to be a seller in the second period. Both effects mean that when a bidder's competitors bid more in the first period, the bidder wants to bid more too. The first-period auction thus has a common value element to it. This implies that bids should increase in the number of first-period bidders, which is testable. The paper finds empirical evidence for this effect in U.S. Forest Service timber auctions.

Garratt and Tröger (2006) and Hafalir and Krishna (2008) also present two-period models with resale. In these papers the reason for resale is not a realisation of uncertainty between periods but bidder asymmetry. In Garratt and Tröger (2006) a bidder with valuation for the object drawn from some distribution with positive support competes against a speculator who has a value of zero. Hafalir and Krishna (2008) is a generalisation of this set-up in that two bidders compete who have values drawn from different distributions. Such bidder asymmetry leads to inefficient equilibria in the first period, which in turn opens the possibility for trade in the second period.

Lovo and Spaenjers (2018) study the auction market for art as a sequence of auctions with resale. Their market is populated by agents with heterogeneous tastes for a single art piece traded in the market. Buyers who win auctions become owners of the art piece. Owners of art are subject to a shock, which the authors term a 'liquidity shock' that has the effect of dropping the flow utility that owners derive from the art piece below zero for a random number of periods. Owners can choose to consign their art for an auction and set a reserve price for the auction. The reserve price leads to a fraction of 'buy-ins', i.e. auctions without successful sale. The authors argue that for this reason repeat sale indices that track the same art piece at different times it was sold, overestimate the expected return on art because the hypothetical selling price in the case of buy-ins is not observed.

Buyers anticipate that upon winning an auction they can later consign and sell the item. Willingness to pay is thus determined by the bidder's taste and the expected resale price. This generates the existence of 'flippers' who have a relatively low taste for art and buy only when the price is exceptionally low, not in order to keep it but in the expectation of selling it soon at a higher price. Agents with higher tastes for art sell only in response to having been hit by a liquidity shock.

The model also has an aggregate state with two possible values, contractions and expansions. All players are assumed to derive lower flow utility from owning art during contractions than during expansions. The existence of a state creates a further group of agents with values within an intermediate range who sell only when in distress or during expansions because during expansions the expected selling price increases by more than their value. The model thus generates a positive correlation between prices and volume (both are higher during expansions), which is observable in their data from art auctions.

A difference between this model of the auction market for art and the the sequential auctions literature discussed above is that buyers exit the market upon losing an auction so that the pool of buyers is replaced each period. This leaves out intertemporal substitution between periods. Moreover, as demonstrated in Satterthwaite and Shneyerov (2007) and Hendricks and Sorensen (2018), sequential auction markets in which bidders behave myopically, will not take advantage of their potential to increase efficiency as compared to static one-shot auctions and will thus remain far away from the efficient benchmark of a Walrasian market.

1.4.2 Related Markets

Resale and second hand markets have more generally been studied in a variety of other markets. Lewis (2011) studies information disclosure in auctions for used cars on eBay and finds that contractible information in the form of a detailed description and many photos of the car helps mitigate adverse selection in this market. The author finds that bidders take into account the information provided on the auction site when forming their bid and also that the amount of information provided responds to the cost of disclosure.

Schiraldi (2011) studies the demand for new and used cars. The author extends static models for demand estimation in fixed price markets to a dynamic model that captures the fact that cars depreciate over time and that trade is subject to transaction costs. Due to the presence of transaction costs consumers do not choose a level of quality to purchases statically for one moment in time. Rather, they buy a car anticipating its quality to deteriorate and to want to sell once the expected gain from doing so outweighs the transaction cost. The author finds that such a dynamic model fits the observed data from new and used car purchases in Italy much better than the static model. The estimation method allows the author to estimate unobservable transaction costs and they turn out to be substantial at 10 to 80 percent of the purchase price for different car models.

In the housing market transaction costs are also large and, while houses do not depreciate to the extent that new cars do, neighbourhoods change and housing needs change over time. Choosing a house to purchase is therefore a dynamic decision also. Bayer et al. (2016) study the housing market using a similar dynamic demand model. They find that a static model would yield wrong estimates of the willingness to pay of neighbourhood amenities that change stochastically or follow a trend over time. The possibility to resell a house is another reason why dynamic considerations are important in this market but is not modelled explicitly in the paper.

Chapter 2

Buying and Selling Classic Cars

Abstract: This chapter studies the interaction of resale, aggregate shocks, and decentralized trade in a dynamic structural model of a sequential auctions market with resale and aggregate shocks estimated on data from classic car auctions. The model matches the positive empirical correlation between prices and the state of the economy. Only half of the variation in prices over the business cycle stems from the direct effect of the aggregate wealth shock. The remaining half is due to the dynamic decisions of buyers and sellers. A series of counterfactual experiments shows that allowing for instantaneous resale substantially increases average car prices and their volatility over the business cycle compared to allowing no resale at all. A second set of counterfactuals reveals that centralizing trade lowers prices and increases the volume of trade. Price volatility remains unchanged in this scenario, even with frequent resale opportunities.

2.1 Introduction

Many goods can be purchased, used for some time, and then sold on. This is particularly relevant for products with low depreciation rates, such as real estate, art, and classic cars. Forward looking buyers value the possibility to resell for two reasons. First, it allows owners to respond to negative shocks. Following a drop in wealth, owners of a valuable good may want to sell it and rebalance consumption. Second, buyers may also purchase a resale good if they perceive its price to be low in order to sell at a profit. Prices for housing and for art have been found to be very volatile and correlated with the stock market (Arefeva, 2017; Goetzmann et al., 2011). I show that the same is true for classic car prices. Trade in these markets is often decentralized: Houses are sold through bargaining between each seller and several buyers (Arefeva, 2017). Art is often sold through single unit auctions, which are held at different times and in different locations. Classic cars are traded in the same way. Such sequential auction markets can be inefficient as compared to more centralized alternatives (Hendricks and Sorensen, 2018; Bodoh-Creed et al., 2019).

In this paper I study how resale, volatile prices, and decentralized trade are interrelated. I develop a structural model of a sequential auctions market with resale and estimate the model on data from classic car auctions. Classic car auctions provide an ideal setting to study these topics together. First, resale is important as classic cars are actively traded while there is no production. Second, the trading mechanism is known and trade is decentralized as each car is sold through an English auction and auctions are spread out over time and locations. Third, the state of the aggregate economy can be taken as exogenous to the market. While it is sizeable, the classic cars market is not large enough to have substantial spillovers to the rest of the economy. The last point in particular is important for estimating the structural model and may not be plausible in other markets for resale goods.

The model captures the main aspects of the market. Trade takes place in a sequence of single-unit auctions. Resale thus means that buyers who win an auction can, in the future, choose to sell it through a later auction. Agents in the market differ by wealth and are also subject to idiosyncratic wealth shocks, which generates a motive for trade. Random stock market returns (taken as exogenous) on monetary wealth create an opportunity cost of holding a car and aggregate fluctuations in wealth. Buyers submit bids in the auction and are forward looking in two respects. First, they take into account the possibility to later resell any car they may purchase now. Second, they know that if they are unsuccessful in the current auction, they will be able to bid on other cars in the future. Sellers set a reserve price and are also forward looking. They know that they will be able to try and sell again if they don't sell now and that if they do sell, they will be able to bid as buyers in future auctions. Supply is thus endogenous through the seller reserve price. The model is the first in the literature to combine a sequential auctions market, where losing bidders may return to bid again in future auctions, see Backus and Lewis (2019) and others cited below, with an endogenous seller continuation value through resale, see Lovo and Spaenjers (2018).

I collect data on prices from classic car auctions and estimate the parameters of the model using simulated method of moments. The estimated model shows that dynamic incentives over different states of the economy are important for explaining price volatility in the presence of aggregate uncertainty: Forcing buyers and sellers to use the same strategy in all states of the economy would remove more than half of the time series variation of prices in the estimated model. The remainder is due to the direct effect of changes in aggregate wealth.

The first set of counterfactuals illuminates the role of resale in this market. In a counterfactual market where resale is not possible, prices are 8% lower on average. In the data auctions are on average two months apart, which constrains the option value of resale and thus explains the moderate magnitude of this effect. When owners can sell close to instantaneously, average prices are 45% higher compared to no resale. In this scenario car prices are also 25% more volatile over the business cycle because the option value of being able to resell is higher in better states of the economy. This effect is mainly driven by the possibility to speculate, that is, the possibility to buy at a low price in order to resell at a higher price.

A second set of counterfactual simulations considers the changes that would result from centralized trade. Due to the model having an endogenous supply side and aggregate shocks, I can here study the seller response to centralizing trade and the effect on price volatility, in addition to the demand-side response and the effect on the average price. In a counterfactual market where buyers can bid on every available car, they will find it easier to substitute between cars, and prices consequently fall by 34%. Sellers become more than twice as likely to sell whenever the opportunity arises. With all bidders present at every auction, sellers are less likely to face a low draw of bids, which would induce them to hold out for the next possibility to sell, and hence the increased sales rate. Finally, in a centralized market frequent resale opportunities do not increase price volatility because there is little scope for speculation.

These results are significant for other markets where products are worth a substantial fraction of buyers' wealth and can be resold with little depreciation. A closely related and important market is housing, where prices have also been shown to fluctuate more than would be expected given the variation in fundamentals. According to my results, a big part of the explanation will be buyers adjusting their current willingness to pay according to expected prices in the near future. This will be exacerbated through the option value of selling if frictions in the market are low enough. Helping more potential buyers to bid on more available houses on the other hand would reduce the scope for speculation and help to reduce price volatility.

Having laid out the related literature in detail in chapter 1, the rest of the introduction focusses on connections and differences between this chapter and other papers. The rest of the chapter is organised as follows: Section 2.2 introduces the setting and describes the data. Section 2.3 presents the model. Section 2.4 describes the estimation method. The results are in section 2.5. Section 2.6 contains a number counterfactual exercises using the estimated parameters. Section 2.7 concludes.

Related Literature. Backus and Lewis (2019), Bodoh-Creed et al. (2019), Hendricks and Sorensen (2018), and Coey et al. (2019) model a market consisting of a sequence of auctions in which consumers substitute intertemporally between goods offered in different auctions. The demand side in the present model has bidders substitute across auctions in a very similar way. Importantly, the supply side is taken as exogenous in these papers. Bodoh-Creed et al. (2019) and Hendricks and Sorensen (2018) explain that the reserve price is not an important aspect of their market as reserves are set very low and most items sell. This is different in my application: In the classic car auction market a substantial fraction of auctions end without a sale.

Haile (2001) studies a two-period game in which the winner of a first auction can resell the object in the second period. Resale may occur because before the first auction bidders only receive a signal about their value and learn its true value before the second period. In my model resale is driven by the variation in the pool of bidders from one period to the next and by idiosyncratic wealth shocks. This makes the supply side closely related to Lovo and Spaenjers (2018), who study the market for art. Their model also incorporates resale at future auctions. However they have no intertemporal substitution between auctions as their market has one unique art piece only and unsuccessful buyers exit the market.

Other related papers include those on the housing market. In the literature on the housing market it is common to assume that once a house has been purchased, the new owner will keep it forever. Bayer et al. (2016) estimate dynamic demand in the housing market, where forward looking behaviour is in part motivated by a resale motive. Resale is however not modelled explicitly. Arefeva (2017) shows how modelling the auction-like way that houses are sold can explain the observed price volatility in the market better than the canonical model with Nash bargaining. The model does not account for resale and instead assumes that, upon purchase, houses are kept forever. I show below that depending on the frequency of resale opportunities, resale may amplify price volatility, which is likely to be similarly true in the housing market as in the data in this paper.

Satterthwaite and Shneyerov (2007, 2008), Hendricks and Sorensen (2018), and Bodoh-Creed et al. (2019) study the efficiency of sequential auction markets as compared to the Walrasian benchmark. Efficiency in their settings would be achieved if every buyer with a value above the Walrasian price receives an object. In my setting, where a durable good is traded repeatedly over time by agents who receive shocks to their willingness to pay, efficiency requires the ownership of the object to change quickly following a new shock. Efficiency would be achieved if at any given time the players with the currently highest value hold the object.

Classic cars are similar to other collectibles such as art, fine wine, stamps etc that have been studied extensively as investment objects. Pesando (1993) and Mei and Moses (2002) study the financial returns on art pieces. Burton and Jacobsen (1999) discuss measuring the returns on various collectibles and provide a list of studies that have done so. Goetzmann (1993) and Goetzmann et al. (2011) find that the level of the stock market strongly predicts prices at art auctions. Dimson et al. (2015) find the same for fine wine. These papers argue that the link between the two is buyer wealth and I follow the literature in focusing on aggregate wealth as the driver of price fluctuations in the model.

Dannefer (1980) studies the buying of classic cars from a sociological perspective and describes in detail the passion owners have for the cars and the activities they engage in (eg showing, touring, restoring). Although dated, the study's observations match mine described below.

2.2 Setting and Data

I collected a panel data set of classic car auctions held between 2003 and 2016. Worldwide, there are many events and shows dedicated to classic cars. At these events collectors buy and sell cars through auctions that are organised by specialised auction houses. Auctions are an important part of the market, although classic cars can also be traded in private or through dealers at fixed prices. The biggest auctions attract a lot of attention in specialised trade publications and are interpreted as indicators of current market conditions. The 'Classic Car Auction Yearbook' (Orsi and Gazzi, 2017) recorded 5600 classic car auctions in 2017/18. All of these (attempted) transactions are due to resale, since, by definition, there is no production in the market for classic cars.

The auction format is an English auction. The auctioneer incrementally raises the price. Any bidder who is willing to pay the current price can raise their hand. The process continues until no bidder is willing to put their hand up. After the auction ends, it is revealed whether the highest bid exceeds the seller's reserve. If yes, then the buyer has to pay and takes ownership of the car, which is always physically present at the auction. At some auctions there may also be phone bidders. Before the auction bidders can inspect the car but cannot test drive it.

From speaking to several of them, buyers are interested in classic cars for nostalgic reasons, for pleasure driving, to stand out, or because they enjoy doing their own maintenance. Buyers appear to be primarily middle-aged and older men. Dannefer (1980) provides a detailed description of classic car hobbyists.

2.2.1 Dataset

The data come from two auction houses which publish results of past auctions. The published results contain the model and year built of the car, the date of the auction, whether or not the auction was successful and, if so, the selling price (but no additional bids). The sample period is 2003 to 2016. There are three major auction locations where both auction houses hold auctions every year: Monterey (California), Phoenix (Arizona), and Amelia Island (Florida). Only one of the auction houses holds auctions in other locations also. Moreover, cars sold at these major auction events have to be in impeccable condition and represent the top of the market. This, together with reputational concern of auction houses also cited in Ashenfelter (1989), should alleviate possible adverse selection problems. I therefore focus on auctions in the three major locations, which yields a set of comparable cars in very good condition. I drop all brands that appear fewer than 10 times in the sample and focus on cars built between 1950 and 1999. The sample has 4941 auctions, of which 4560 ended with a sale. The median price of these cars is \$98200 and the mean is \$290600 with a standard deviation of \$748300. All prices are measured in constant 2002 US Dollars. As the standard deviation makes clear, there is a long tail of cars that are very expensive compared to the median.

I supplement this auction-level data with aggregate data from the Classic Car Auction Year Book (Orsi and Gazzi, 2017), which reports the annual average price of classic cars starting in 1993 until 2016.

2.2.2 Descriptive Statistics

Cars can be distinguished by brand, model, year built, and the time of the auction. There are 50 brands in the sample. The brand alone explains 11% of the observed variation in car prices. There are 2567 car models in the data. For most models there are only a few observations, in fact 1882 models appear only once. There are 63 models that appear at least 10 times. Limiting the data to the 1066 auctions that are for cars of one of these 63 models, the car's model explains 74% of the variation in the price. Adding an auction-year fixed effect to the regression, the explained price variation increases to 79%. The remaining price variation is due to the condition of the car, special features, famous previous owners, or the car having participated in big races. The car's model is thus the most important determinant of the price. Cross sectional variation in prices is larger than variation over time, i.e. differences between cars are bigger than yearly price changes for the same car. Among the car models with at least 10 observations, the highest average price (USD 1037000 for the Ferrari 275 GTB/4 Berlinetta) is 55 times larger than the lowest average price (USD 19045 for the MG TD Roadster). The most expensive auction year in the data is 2015 with an average price of USD 440700 and the cheapest year is 2004 with an average price of USD 82600, a factor of 5. This difference over time, while smaller than differences between models, is thus still very substantial, which the next section will explore in more detail.

For each car the auction houses publish a low and a high estimate. Ashenfelter (1989) finds that for art auctions the mid point of the low and the high estimate is a very good predictor for the selling price (conditional on sale), in line with theoretical results that it is in the auctioneer's interest to provide accurate information on the object being sold. This appears to be true for classic car auctions too, as the correlation between the price estimate and the selling price in my data is 0.97.

The secret reserve price is not observed in my data. For art auctions Ashenfelter and Graddy (2011) find that the secret reserve price is about 70% of the low estimate. In my data 95% of selling prices are above this level, and so it is plausible that the reserve price may be similar for classic car auctions. The correlations between the price estimates and the selling price are shown in the graphs in appendix B.

2.2.3 Correlation with the Stock Market

Figure 2.1a plots average deflated prices for classic cars and the S&P 500, both normalized to base year 1993 on a log scale. The underlying average prices for classic cars are taken from Orsi and Gazzi (2017). Prices for classic cars have more than quintupled since the 1990s and appear strongly correlated with the stock market. Figure 2.1b shows that also the return on classic cars (measured as the annual change in average prices) is strongly correlated with the average annual return on the S&P 500. See appendix B for the corresponding regression.

Average prices could go up in part due to a selection effect, for example if in years with a higher stock market owners of better cars are more likely to put them up for sale. To control for this, I run a regression of the log deflated sale price on a model fixed effect and an auction-year fixed effect on the sales in my auction-level data. The model fixed effect controls for selection of models into years sold. The year fixed effect is the expected log price difference between two classic cars of the same same model sold in an auction in a given year and in the baseline year 2003. Figure 2.1c plots the auction-year fixed effect, which shows a very similar pattern to the simple average: The expected price for a given model follows the S&P 500 closely and tripled between 2003 and 2015. See appendix B for regression results.

Akin to these results for the classic car market, Goetzmann (1993) and Goetzmann

Table 2.1: Main Brands in Sample

Brand	Price ($k)			No. Sales	No. Auctions
	Median	Mean	SD		
Ferrari	395	835	1508	674	772
Jaguar	86	198	917	480	556
Porsche	129	266	454	477	522
Mercedes	138	290	353	399	449
Chevrolet	65	84	82	340	391
Ford	48	187	589	183	183
Aston Martin	240	359	436	156	160
Austin-Healey	61	75	76	140	160
Shelby	353	471	416	140	140
Maserati	140	314	468	112	115
Other	67	134	210	1459	1649
Total	98	291	748	4560	5152

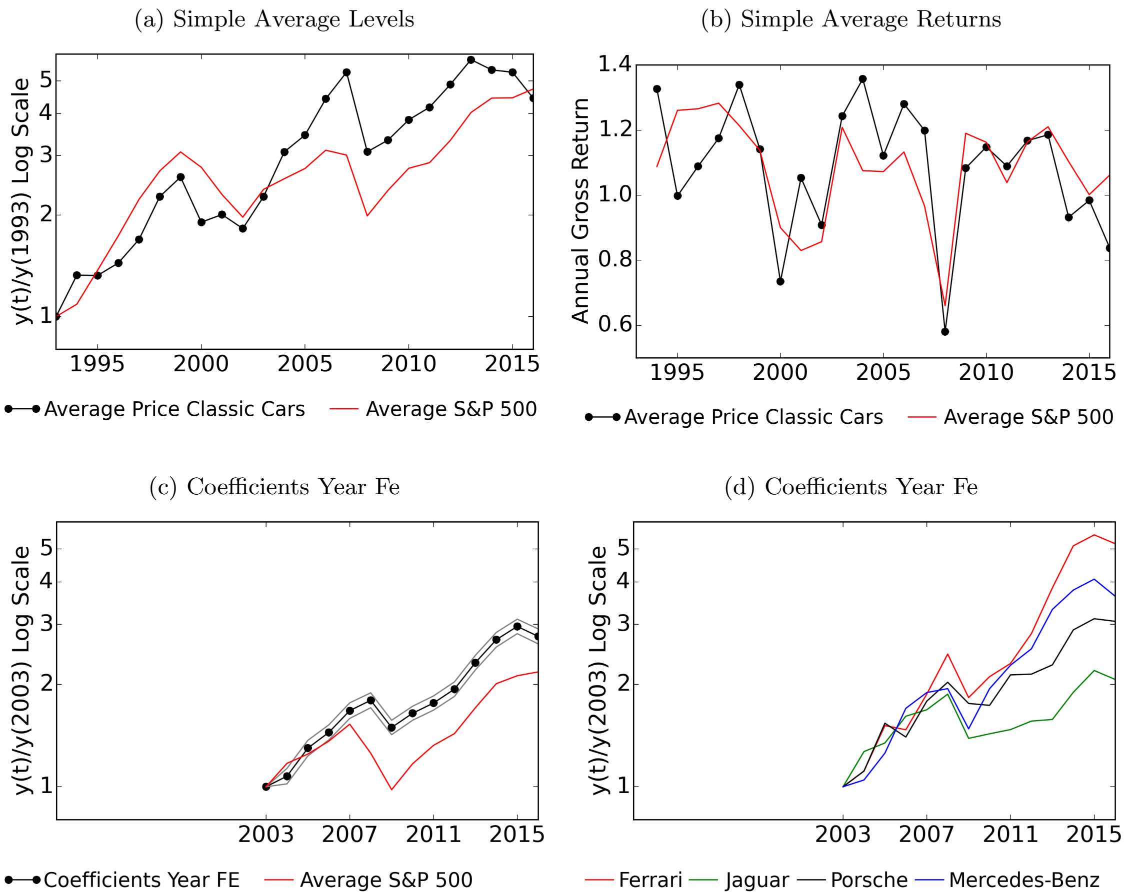
Notes: Top rows contain 4 biggest brands by number of sales, which are used for the estimation.

et al. (2011) use a repeat-sales index for the art market and find a strong positive correlation between the level of the stock market and prices for art. Lovo and Spaenjers (2018) argue that even a repeat-sales index of art may be biased due to selling decisions of the owners. Dimson et al. (2015) find the same correlation for wine using an index based on the sales of wines of the same chateau and vintage. My methodology is most similar to the latter paper as both control for characteristics of the object traded (respectively, the chateau plus vintage for wine and the model for cars). Repeat sales indices on the other hand treat the objects traded as unique and compare the prices of the very same object sold in different years. That way repeat sales indices control not only for selection by observable characteristics (e.g. the model) into years sold but also for selection by unobservable characteristics (e.g. condition and history of the car). As the model of a car explains most of the variation in prices, the results from a repeat-sales analysis would be expected to be very close.

2.2.4 Volume

The number of classic cars offered and sold, as covered by the Classic Car Auction Yearbook, has been trending up with some variation around that trend since the 1990s. The fraction of cars sold successfully has also been going from $60-70\%$ in the 1990s to $70-80\%$ in recent years. Total volume or the fraction sold appear not to be correlated with either prices or the stock market. Both time series are plotted in appendix B. Even in the years with the highest fraction of successful sales, more than 20% of auctions did not end in a sale. For classic cars the reserve price is thus important, in contrast to the eBay setting covered by for example Hendricks and Sorensen (2018) and Bodoh-Creed et al. (2019).

Figure 2.1: Price for Classic Cars over Time



Notes: Data for figures 2.1a and 2.1b come from the Classic Car Auction Year Book, which has data from 1993-2016. The definition in this book of a year is from September of that year until the following August. For that reason the annual average S&P 500 is plotted for the same definition of a year in figures 2.1a and 2.1b. Figures 2.1c and 2.1d and the rest of the paper use calendar years. Data for figures 2.1c and 2.1d is the auction-level data also used for the rest of the paper, which starts in 2003.

2.2.5 Data for Estimation

For the structural model I focus on the top 4 brands by number of sales. This ensures that there are enough observations per brand. These four brands are also more comparable to each other and are more likely to be substitutes for each other than some of the more unusual or less expensive brands in the market. Table 2.1 shows summary statistics for these and other common brands. Figure 2.1d shows the results from four separate regressions (one for each brand) of the price on a model and an auction-year fixed effect. The patterns are broadly similar across the brands. Expected prices do not change at the same rate in every year, but no brand consistently outperformed the others over the sample period. The results section will address how well the estimated model can match these different price paths.

2.3 Model

This section describes the model, its equilibrium, and the numerical solution method. The end of the section contains a discussion and illustration of the model that makes clear which trade offs agents face and how prices are determined in equilibrium.

2.3.1 Environment

There are N agents in the economy and $J < N$ cars. Time is discrete and the horizon is infinite. The future is discounted with discount factor β. Agents are either sellers if they own a car, or buyers if they do not own one.[1] By respectively buying and selling a car, agents move between being buyers and sellers and vice versa. The total pool of agents is stable (no entry or exit). Trade takes place through one auction each period. The auction is modelled as a second-price sealed bid auction.

2.3.2 Wealth

At the beginning of period t agent i has wealth $y_{i,t-1}$. Wealth delivers a random net return r_t, which is common to all agents and is drawn from a commonly known distribution, which will be estimated. The value of the return r_t is publicly observed at the beginning of the period. Agents consume the composite good $c_{i,t}$. Agents receive income m_i and privately observe the realisation of an idiosyncratic additive income shock $\epsilon_{i,t}$, which is drawn iid over i and t and independent of r_t with mean 0 and follows a commonly known distribution, which will be estimated. Car trading takes place after r_t and $\epsilon_{i,t}$ have been observed, after

[1] Agents can own at most one car. For tractability and because in the data I cannot identify owners, I will treat owners of more than one car as several separate agents.

m_i has been received, and after $c_{i,t}$ has been consumed. Let $A^j_{i,t} = 1$ if agent i holds car j at t and $A^j_{i,t} = 0$ otherwise. Assume that $\sum_j A^j_{i,t} \leq 1 \; \forall t$ (agents can hold at most one car simultaneously). The price of car j at time t is denoted by $P_{j,t}$. There is no borrowing, so $y_{i,t} \geq 0$. The intertemporal budget constraint is thus given by:

$$y_{i,t} = m_i + (1 + r_t)y_{i,t-1} - c_{i,t} + \epsilon_{i,t} + \sum_j P_{j,t}(A^j_{i,t-1} - A^j_{i,t}) \tag{2.1}$$

Period Utility

A seller who consumes c and holds car j in any given period has log-linear flow utility $ln(c) + w_j$. w_j is common to all agents but depends on the car j.

Composite Good Consumption

Consumption of the composite good is assumed to be linear in current monetary wealth: $c_{i,t} = \psi \cdot y_{i,t-1}$. Although this is not the exact solution to the problem facing agents in this economy, following results in Benhabib et al. (2015), I take it to be a good enough approximation to the optimal consumption path, see appendix C for details.

Given the log-linear utility function above, an agent with wealth $y_{i,t}$ enjoys utility $ln(\psi y_{i,t})$ from composite good consumption and individual wealth $y_{i,t}$ evolves according to:

$$y_{i,t} = m_i + (1 - \psi + r_t)y_{i,t-1} + \epsilon_{i,t} + \sum_j P_{j,t}(A^j_{i,t-1} - A^j_{i,t}) \tag{2.2}$$

Recall that $\epsilon_{i,t}$ is assumed to be drawn iid and independent of r_t with mean 0. Assume moreover that $\mathbf{E}[1 - \psi + r_t] < 1$. More details on the wealth process are contained in appendix C.

State Variable Mean Wealth

From the process for individual wealth in (2.2) follows a law of motion for the average wealth of economy, which is found by averaging (2.2) over i:

$$\begin{aligned}
\frac{1}{N}\sum_i y_{i,t} = \frac{1}{N}\sum_i m_i + (1 - \psi + s \cdot r_t)\frac{1}{N}\sum_i y_{i,t-1} + \frac{1}{N}\sum_i \epsilon_{i,t} \\
+ \sum_j P_{j,t}\frac{1}{N}\left(\sum_i A^j_{i,t-1} - \sum_i A^j_{i,t}\right) \\
\bar{y}_t = \bar{m} + (1 - \psi + r_t)\bar{y}_{t-1} \qquad (2.3)
\end{aligned}$$

Where the second line follows assuming that N is large enough for $\epsilon_{i,t}$ to obey a law or large numbers and because $\sum_i A_{i,t}^j = 1$ for all j and t, since every car is always held by exactly one agent. Put differently, trading of cars only moves money between agents and, while it does affect the *distribution* of wealth, it cancels out in the calculation of the mean. Hence $\bar{y}_t$ depends only on exogenous variables and is unaffected by the actions of agents in the model.

Mean wealth $\bar{y}_t$ is going to be the exogenous state variable of the economy. Agents are assumed to condition their beliefs (to be specified below) on the current value of y_t but not on higher moments of the wealth distribution. Under the assumptions on the individual budget constraint, in particular that $1 - \psi + r_t < 1$, the state variable will be stationary.

$\bar{y}_t$ is assumed to take values from a discrete grid $\mathcal{S}$ and the transition probabilities between values $\bar{y}_t$ and $\bar{y}_{t+1}$ are $\mathbf{Pr}(\bar{y}_t, \bar{y}_{t+1})$. For a given $\bar{m}$ and ψ, the transition probabilities will be determined by the distribution of returns r_t.

2.3.3 Auction Game

Bidders

Each potential buyer has probability σ of being an *active bidder* in the game. Active bidders privately observe their wealth at the beginning of the auction (equal to $m_i + (1 - \psi + r_t)y_{i,t-1} + \epsilon_{i,t}$) and observe the public state $\bar{y}_t$. Next one car is drawn for the auction. Car j has probability ρ_j of being picked. Active bidders observe the car for the auction and its value w_j, which is common to all agents. They also observe the number of active bidders, denoted N_A, but not their identities.[2] Active bidders then enter a second-price sealed bid auction.

Seller

Classic car auctions have a secret reserve price. After all bids have been received, the seller either accepts the selling price equal to the second-highest bid or does not. Unlike in the case of an announced reserve price, it is always optimal for sellers to accept a price above their value.[3] The seller of car j observes the same information as the active bidders and then chooses the reserve or lowest acceptable price a_j.

[2] In reality some participants in this market may know each other and learn something from the behaviour of particular bidders they know but I assume the market is large enough that this effect is negligible.

[3] Appendix A.2 shows in a two-period model the difference in equilibrium between sellers setting a reserve price equal to their valuation, as is optimal here when the reserve price is secret, and setting an optimal public reserve price.

Payoffs

Denote the expected value of leaving the current period as a potential buyer with wealth y conditional on the current state $\bar{y}$ by $B(y, \bar{y})$. Similarly, let $S_j(y, \bar{y})$ be the expected value of leaving the current period as a potential seller of car j with wealth y conditional on the current state $\bar{y}$.

For a bidder who is the last bidder in the auction at price P, the payoff from winning the auction is the immediate flow utility of the car w_j and the continuation value as seller evaluated at the current wealth minus the price paid for the car, i.e. $w_j + \beta S_j(y - P, \bar{y})$. The payoff of losing the auction is the continuation value associated with continuing into the next period as a buyer, namely $\beta B(y, \bar{y})$. The payoff for the seller is $\beta B(y + P, \bar{y})$ if the highest bid is above the reserve a_j and $w_j + S_j(y, \bar{y})$ otherwise.

Information Structure

The direct flow utility w_j is common to all agents and is common knowledge. Differences in the willingness to pay for a given car j in this model are generated by heterogeneous wealth y, which is private information. Buyers and the seller also consider the continuation values of being a buyer or seller next period when forming their bid, which will depend on future selling prices. In forming their beliefs over selling prices and hence the continuation values as buyers or sellers, agents are assumed to consider mean wealth and its transition probabilities only, which is exogenous and common knowledge. I thus follow Backus and Lewis (2019) in assuming a mean-field equilibrium, in which players do not attempt to update their beliefs on the value of continuing in the game from other bids. The implications of this assumption are explored further in chapter 3. Therefore incomplete information exists over private wealth only, which does not affect other bidders' valuations. The auction is an independent private values one.[4] I focus on the equilibrium in which both bidders and sellers play weakly dominant strategies.

Bidder Strategy

A bidder strategy is a mapping from the car j at auction, current private wealth y, and the current state variable $\bar{y}$ to a bid b_j. Define $b_j(y, \bar{y})$ as the weakly dominant strategy bid of a bidder with wealth y on car j in state $\bar{y}$. As discussed in the previous section, bidders have a dominant strategy to bid their willingness to pay net of continuation values. The

[4] In Haile (2001), bids are correlated between periods because bidders receive a signal about their value before the first period and learn their true value before the second period. This correlation introduces a common value element to the first auction. This does not happen in my model: Controlling for the state, bids are not correlated across periods.

weakly dominant strategy $b_j(y, \bar{y})$ therefore has to satisfy the following equation:

$$w_j + \beta S_j(y - b_j(y, \bar{y}), \bar{y}) = \beta B(y, \bar{y}) \tag{2.4}$$

In words, the bid $b_j(y, \bar{y})$ is such that the bidder is indifferent between winning and paying $b_j(y, \bar{y})$ and losing the auction.

Seller Strategy

A seller strategy is a mapping from the seller's car j, current private wealth y, and the current state variable $\bar{y}$ to a reserve price a_j. The weakly dominant strategy $a_j(y, \bar{y})$ has to satisfy the following equation:

$$\beta B(y + a_j(y, \bar{y}), \bar{y}) = w_j + \beta S_j(y, \bar{y}) \tag{2.5}$$

In words, the reserve price $a_j(y, \bar{y})$ is such that the seller is indifferent between selling and receiving $a_j(y, \bar{y})$ and not selling.

Beliefs

Let $\bar{b}_j$ be the second highest bid on car j. If all players play the equilibrium strategy as defined by (2.4) and (2.5), $\bar{b}_j$ and a_j follow stationary distributions conditional on the current state $\bar{y}$. In equilibrium players have consistent beliefs, i.e. their beliefs over the distribution of $\bar{b}_j$ and a_j are correct given the other players' strategies. Denote the stationary beliefs conditional on the state by $G_{\bar{b}_j|\bar{y}}(b)$ and $H_{a_j|\bar{y}}(a)$.

Equilibrium

A Markov Equilibrium in this game is a tuple

$$\left\{b_j(y, \bar{y}), a_j(y, \bar{y}), G_{\bar{b}_j|\bar{y}}, H_{a_j|\bar{y}}\right\}$$

such that $b_j(y, \bar{y})$ satisfies (2.4), $a_j(y, \bar{y})$ satisfies (2.5), $G_{\bar{b}_j|\bar{y}}(b) = \mathbf{Pr}(\bar{b}_j \leq b|\bar{y})$, and $H_{a_j|\bar{y}}(a) = \mathbf{Pr}(a_j \leq a|\bar{y})$.

2.3.4 Value Functions

A buyer's value function $B(y, \bar{y})$ equals the expected value of leaving the current period as a potential buyer with wealth y. A seller's value function $S_j(y, \bar{y})$ equals the expected value of leaving the current period as a seller of car j with wealth y.

Buyer

Define $\hat{B}_j(y, \bar{y})$ as the value of being drawn as an active bidder with private wealth y and public state $\bar{y}$ when car j is offered at the current auction. Since buyers are active with probability σ each period, the buyer value function can be expressed as follows:

$$B(y, \bar{y}) = \ln(\psi y) + (1 - \sigma)\beta\mathbf{E}[B(y', \bar{y}')|y, \bar{y}] + \sigma\mathbf{E}[\hat{B}_j(y', \bar{y}')|y, \bar{y}] \tag{2.6}$$

where

$$\begin{aligned}\hat{B}_j(y, \bar{y}) = \int\int \Big(&\mathbf{1}(b_j(y, \bar{y}) < \max(\bar{b}_j, a_j))\beta B(y, \bar{y}) \\ &+ \mathbf{1}(b_j(y, \bar{y}) > \max(\bar{b}_j, a_j))(w_j + \beta S_j(y - \bar{b}_j, \bar{y}))\Big) dG_{\bar{b}_j|\bar{y}} dH_{a_j|\bar{y}}\end{aligned}$$

$\hat{B}_j(y, \bar{y})$ is the sum of the value of winning the auction and paying price $\bar{b}_j$ and of losing the auction, weighted by the probability of each event, given the buyer's beliefs $G_{\bar{b}_j|\bar{y}}$ over the distribution of highest rival bids. See appendix C for the calculation of the expectations in equation (2.6). $B(y, \bar{y})$ is increasing in y but will be decreasing in $\bar{y}$. The latter relationship arises because expected prices for cars are higher for higher levels of $\bar{y}$. Intuitively, keeping y constant, it is better for buyers to bid against poorer competitors than against richer ones.

Seller

Let $\hat{S}_j(y, \bar{y})$ be the value of being drawn as the seller for the current auction with private wealth y and public state $\bar{y}$. Let $\tilde{\rho}$ be each seller's probability of being drawn for the next auction. The seller value function can then be expressed as follows:

$$S_j(y, \bar{y}) = \ln(\psi y) + (1 - \tilde{\rho})(w_j + \beta\mathbf{E}[S_j(y', \bar{y}')|y, \bar{y}]) + \tilde{\rho}\mathbf{E}[\hat{S}_j(y', \bar{y}')|y, \bar{y}] \tag{2.7}$$

where

$$\begin{aligned}\hat{S}_j(y, \bar{y}) = \int \Big(&\mathbf{1}(a_j(y, \bar{y}) > \bar{b}_j)(w_j + \beta S_j(y, \bar{y})) \\ &+ \mathbf{1}(a_j(y, \bar{y}) < \bar{b}_j)\beta B(y + \bar{b}_j, \bar{y})\Big) dG_{\bar{b}_j|\bar{y}}\end{aligned}$$

See appendix C for the calculation of the expectations in equation (2.7). $S_j(y, \bar{y})$ is increasing in both y and $\bar{y}$. The seller benefits from higher $\bar{y}$ because expected prices increase when bidders are rich on average.

2.3.5 Solution

A numerical solution of the model consists of a bid function $b_j(y, \bar{y})$, reserve-price function $a_j(y, \bar{y})$, and beliefs $G_{\bar{b}_j|\bar{y}}$ and $H_{a_j|\bar{y}}$. The bid function and reserve-price function maximise players' payoffs given beliefs while also generating a distribution of prices and highest rival bids. In equilibrium the distributions generated by the bid and reserve-price functions have to be equal to players's beliefs. A solution is thus found once a pair of bid and reserve-price functions generates a distribution of prices and highest rival bids, for which these same bid and reserve-price functions are also optimal.

To find the solution, initialise the buyer and seller value functions and calculate the corresponding bid and reserve-price functions. Simulate many rounds of trade from the bid and reserve-price functions and calculate the distribution of the second highest bid and the reserve price conditional on the state variable mean wealth. Variation in the second highest bid and the reserve price within a state will come from randomness of the bidder pool (both the number of bidders and their wealth are random). Now use these distributions of prices and highest rival bids as beliefs to calculate new buyer and seller value functions. Calculate new bid and reserve-price functions and simulate trade again. Repeat this until the value functions stop changing from one iteration to the next. More details on the algorithm for the numerical solution of the model are in appendix C.

2.3.6 Discussion of the Model

As the buyer and seller dominant strategies (2.4) and (2.5) make clear, both types of agents weigh continuing as buyers against continuing as sellers when choosing their bid. The relative attractiveness of each at different levels of wealth will depend on the parameters of the model, most importantly σ, the probability of buyers being active and $\tilde{\rho}$, the probability of sellers being drawn as the active seller. This is illustrated by the bid functions in figure 2.2.

As can be seen in figure 2.2a, the bid function and reserve-price functions have very similar shapes. For the same level of wealth the seller's reserve price will be above the buyer's bid because of the concavity of the utility function in composite good consumption: The seller gains extra wealth by selling, which at the same level of wealth and for the same price, has a smaller effect on utility than the buyer's reducing their wealth by buying.

For lower levels of wealth the bid function is first very steep before flattening off. This shape is due to 'flipping': In light of their ability to resell any car they may purchase, bidders with lower levels of wealth are willing to pay a large fraction of their wealth for a car, expecting to sell it soon after for a higher price. The bid function starts increasing again at higher levels of wealth where bidders are expecting to hold on to a car that they purchase, unless they get hit by a very negative shock.[5] Figure 2.2c shows bid functions for the same economy but with a higher probability for a seller of being drawn for any given auction. This reduces the expected waiting time before a new seller is able to resell after the initial purchase, and thereby increases the option value of resale. As the figure shows, the shape of the bid function of being very steep at lower levels of wealth becomes more pronounced, whereas at the highest levels of wealth the bid function is unaffected.

Increasing the probability for buyers of being active increases the value of waiting by losing the current auction, which means that bidders will shade their bids more. Hence in figure 2.2d the bid functions shift down. Moreover, the bid function for the car with the lower use value w_j is even decreasing for higher levels of wealth. Rich buyers prefer to wait for a better car to arrive at the next auction over buying the current car, which provides them with lower flow utility and, given the unit demand assumption, will preclude them from buying the better car until they sell the present one.

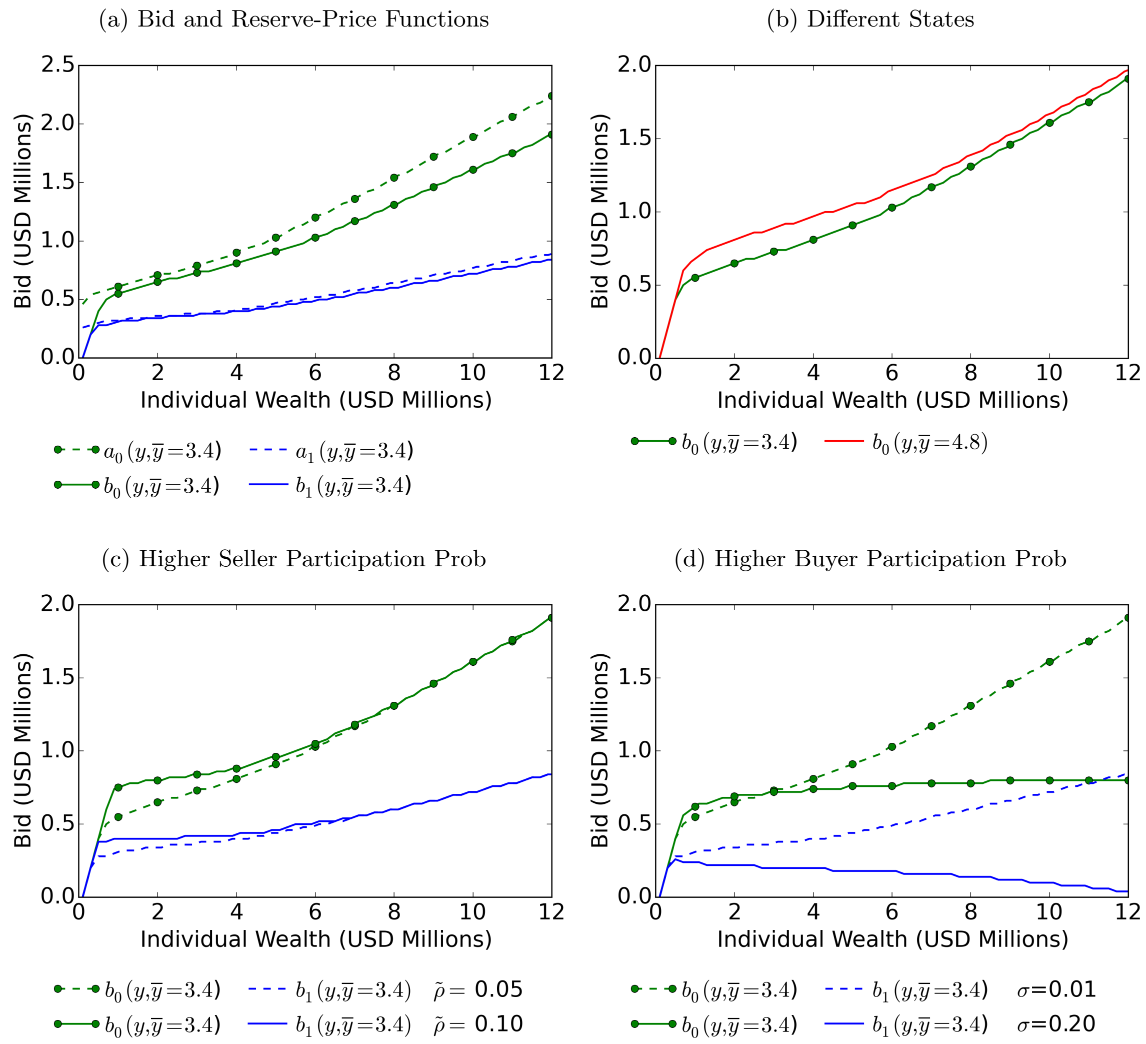
The buyer and seller value functions (2.6) and (2.7) are conditional expectations. The conditioning variables are private wealth and the economy's state variable mean wealth. When mean wealth is higher, the distribution of selling prices and of the highest rival bids is higher. The continuation value as a seller is therefore higher because selling is more attractive at higher prices, while the continuation value as a buyer is lower because buying becomes less attractive at higher prices. This means that bid functions differ by state: as can be seen in figure 2.2b, bids in the higher state are above bids in the lower state because the difference between the continuation values as seller and buyer increase, which determines the dominant-strategy bid, see equation (2.4). To consider price dynamics over states, the dynamic incentives of both the buyers and the seller are therefore important.

2.4 Estimation

Having described the data and the theoretical model, this section explains the estimation of the model. I first calibrate the parameters governing the wealth process from auxiliary data. Second, I use the auction data described in section 2.2 to calibrate the parameters governing auction participation. Third, I use the calibrated parameters together with the auction data from section 2.2 (again) to jointly estimate $w = (w_1, ..., w_J)$, the vector of flow utilities associated with each car, which I will refer to as demand parameters.

[5] Appendix A derives analytically the bid functions for two-period models of sequential auctions with and without resale and confirms this shape.

Figure 2.2: Bid Functions



Notes: Economy with two cars, $w_0 = 0.2$, $w_1 = 0.1$. Baseline parameters in figure 2.2a: $\sigma = 0.01$, $\tilde{\rho} = 0.05$, $\bar{y} =$ USD 3.8 Million

2.4.1 Wealth

The goal of estimating the intertemporal budget constraint (2.2) is a wealth distribution, which evolves according to (2.3) and is as close as possible to the empirical distribution of wealth among buyers and owners of classic cars over the sample period. Data on wealth comes from a separate data source, the Survey of Consumer Finances (SCF).[6]

The distribution of income m_i and the idiosyncratic shock $\epsilon_{i,t}$ are taken from the SCF, restricted to households in the 85th to the 99th percentiles in terms of both income and wealth. For the distribution of the common net return on wealth, r_t, I use one half of the average annual returns on the S&P 500 over the sample period. This corresponds to one half of agents' wealth not held in the form of a car being invested in the S&P 500 and the other half yielding a net return of 0. Empirically, most households invest less than one half of their wealth in the stock market but households have other investments, such as real estate, which are correlated with the stock market. The assumed fraction of 0.5 invested in the stock market is meant to capture this in a simple manner. Finally, the fraction of wealth spent on the composite good each period, ψ, matches simulated mean wealth to the empirical mean wealth over the sample period. This yields $\psi = 0.09$. See appendix D for more details.

2.4.2 Mean Wealth

Having parameterized the intertemporal budget constraint (2.2), one can simulate wealth for a panel of N agents by drawing repeatedly from the distribution of r_t and $\epsilon_{i,t}$. The state variable of the model in a given simulated period is then mean wealth in this period. The mean of simulated mean wealth is USD 3.99 Millions with a standard deviation of 0.8. Mean wealth has a very high autocorrelation at one lag of 0.94. (Returns are drawn iid.)

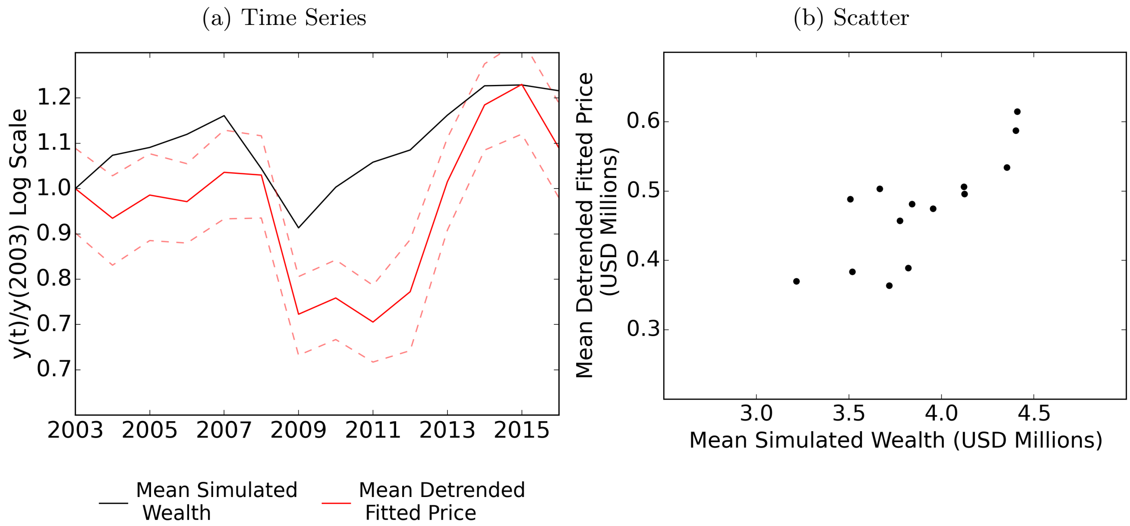
Finally, to assign a value of the state mean wealth to each year in the sample period, start with mean wealth in the 2001 SCF and calculate how mean wealth would be expected to evolve from there according to the mean wealth process (2.3) specified by the model and given the empirical average annual return on the S&P 500 in each of the years 2003-2016.

2.4.3 Prices

Bidders in the model condition their bid on the car j, their private wealth y, and the aggregate state $\bar{y}$. The model thus makes predictions about the distribution of prices by car and by state. Estimating the model therefore requires a measure of prices by car

[6]https://www.federalreserve.gov/econres/scfindex.htm

Figure 2.3: Mean Simulated Wealth vs Mean Fitted Prices



Notes: Scatter plots the two time series from the left against each other. Unit of observation is one year in both plots.

and by state. Having assigned a state to each year in the sample period (section 2.4.2), estimating expected prices for each brand and each year yields a measure of prices by car and state.

For the estimation I will use the top 4 brands in table 2.1 and estimate one taste parameter w_j per brand. There is heterogeneity within brands, which I will control for with a model fixed effect. Section 2.2.3 describes the regression of prices on a model and an auction-year fixed effect. The results from this regression for each of the 4 brands separately are plotted in figure 2.1d. While the model fixed effect controls for price variation within each brand, the fitted prices from this regression averaged by brand and by year yield a measure of the expected price for an exemplary car from each brand sold in each of the years in the sample period. Since the model assumes a stationary wealth process it will predict a stationary distribution of prices over time, while prices for classic cars display a clear upwards trend over the sample period, see figure 2.1a. I therefore detrend prices for the estimation of the model. For details see appendix D. Figure 2.3 shows the correlation between states and this measure of prices.

2.4.4 Trade Parameters

There are 4 brands in the estimated economy. Of those brands, there were 2100 sales over the sample period (see table 2.1), i.e. 150 sales per year over 14 years.

The total number of each brand sold relative to the total, also in table 2.1, yields the probability of each car being offered at any given auction, namely $\rho = (7/21, 5/21, 5/21, 4/21)$.

In the estimated model the sales rate (fraction of auctions that result in a sale) is about ten percent. I therefore assume that there are 1500 auctions per year.

In the model sellers are exogenously drawn for auctions. I assume that sellers get 6 chances per year to sell their car because there are 6 auction events in the data where sellers could enter a car. This means that the probability for a seller of being picked for an auction is $6/1500 = 0.004$. Since all sellers have the same probability of being active, the number of sellers is given by $1/0.004 = 250$. I assume the total number of agents is $N = 1000$, i.e. 3/4 of the market are buyers. Finally, I rescale these parameters by dividing the market size by 10 to estimate a smaller market for computational reasons. This yields Number of auctions: 150, $\tilde{\rho} = 0.04$, Number of sellers: 25, $N = 100$.

There are now 150 periods per year (since each period consists of one auction). Assume an annual discount rate of 0.99. This implies a per-period discount rate of $0.99^{1/150} = 0.999933$. Since the wealth process was calibrated to annual data, assume that wealth changes every period with probability 1/150 and stays the same with probability 149/150. More details in appendix D.1.

The probability of a buyer being active is $\sigma = 0.02$ since buyers will not be at most auctions. The low value of σ captures the decentralized nature of the market: Auctions in the data take place in only three locations in the US, which will be costly to get to for many potential buyers.

There is also a 5% transaction cost for the seller. The two auction houses in the data charge seller fees between 0% and 15%.[7] In the United States gains from selling a classic car are liable for capital gains tax, which is 15% or 20% (depending on the income of the seller). The model does not track the seller's gains so it is not possible to include the tax treatment of cars. Instead, all transactions costs are summarized into one figure of 5%. Buyers pay a fee to the auction house too but this fee is already included in the published prices that are used in this paper.[8]

2.4.5 Demand Parameters

I will estimate the vector of w, the flow utilities of each car, using method of simulated moments (McFadden, 1989). The estimator minimises the distance between the prices predicted by the model and the empirical prices. The model-predicted price is the expected second highest bid on each car, averaged over states. This yields four moments for four unknown parameters. The flow utility w_j associated with each brand is thus identified simply from the expected price of the average car of that brand over the sample period. Brands that had higher prices will be estimated to yield a higher flow utility.

[7] https://bringatrailer.com/bat-guide-how-classic-car-auction-fees-work/
[8] https://www.classicandsportsfinance.com/the-buyers-guide-to-classic-car-auctions/

2.5 Results

The estimation yields the estimates in table (2.2). Figure 2.4 shows the empirical and

Table 2.2: Estimates

	w_j
Ferrari	0.133
	(0.0236)
Jaguar	0.032
	(0.0131)
Porsche	0.028
	(0.0085)
Mercedes-Benz	0.038
	(0.0057)

Notes: Bootstrapped Standard Errors in Parentheses.

model moments for each state for two example brands. The estimation method matches the model-predicted prices for each brand averaged over states to the empirical counterpart, represented by the horizontal lines in figure 2.4. Prices by states were thus not used separately for the estimation. As the figure shows, the overall positive correlation between the state and prices is replicated by the model but there is additional variation in the empirical prices that the model does not match. Prices for cars in the data are very variable, which shows up in the wide standard deviation around the empirical prices in 2.4 and is also reflected in the standard error of the expected prices predicted by the model.

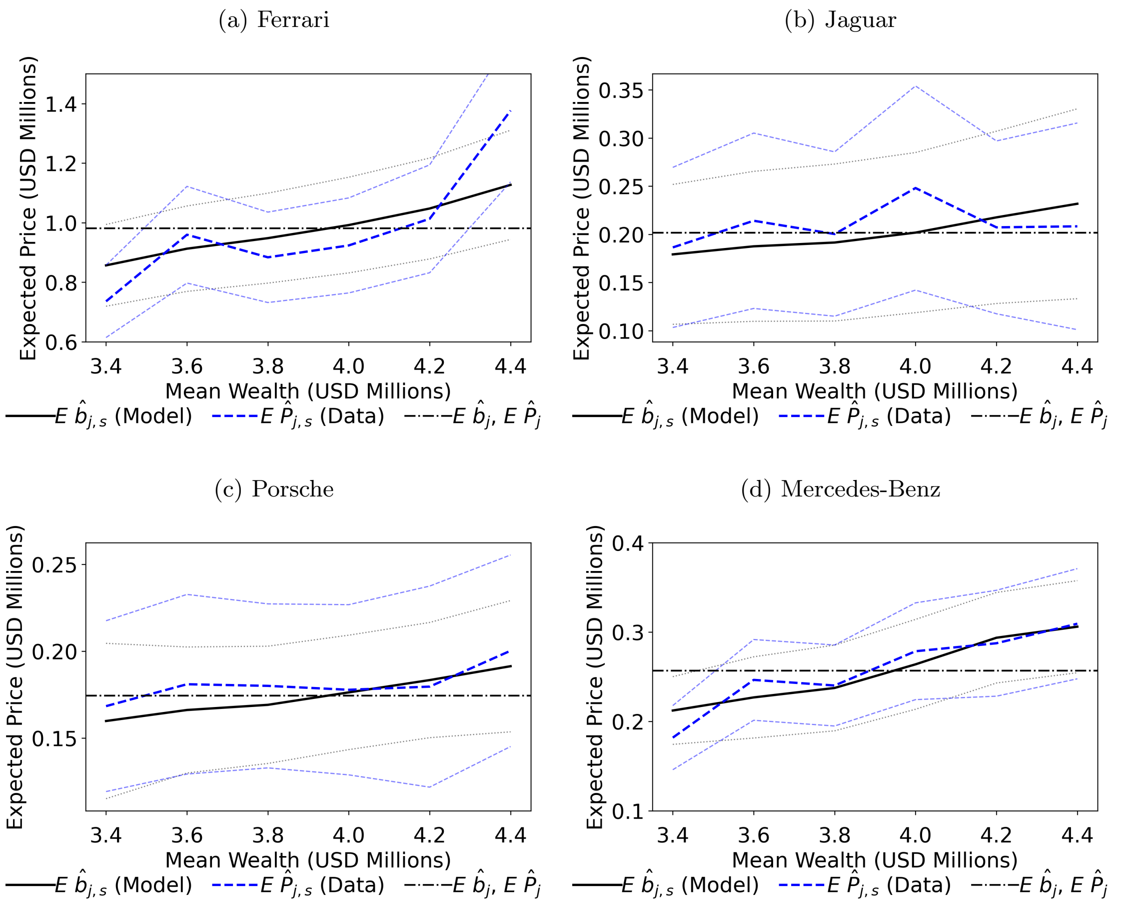
Prices over the sample period as predicted by the model compared to the data are shown in figure 2.5a. Again, empirical prices display more variance than the model predicts. In the model, expected prices change over time only due to changes in mean wealth (the exogenous state variable). In the data prices will have differed over time due to additional reasons, for example tastes changing over time.

Figure (2.6) plots the equilibrium bid functions $b_j(y, \bar{y})$ predicted by the model for the estimated parameters. The possibility to resell implies that even relatively poor bidders are willing to submit high bids so that at the lower end of the bid function, the slope is very steep. The poorest bidders are willing to almost exhaust their budget constraint and would, upon winning, sell the car again as soon as possible.

2.5.1 Price Volatility

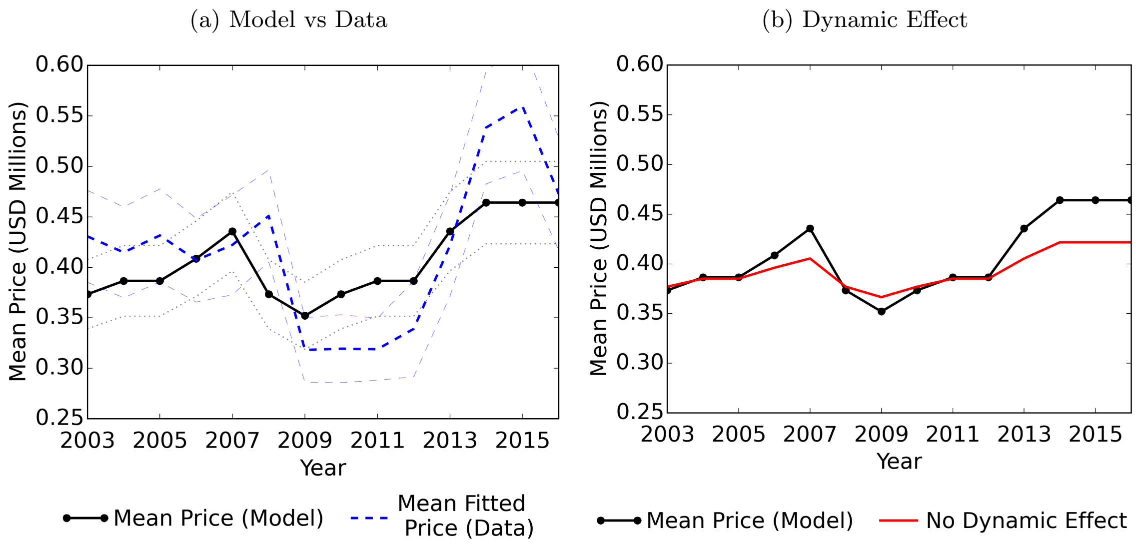
Variation in the price for a given car within a state in the model comes from uncertainty in the bidder pool. Since each buyer has probability σ of being active in any one auction, both the number of active bidders and their wealth is random for each auction. The simulation of the model draws different bidder pools, which yields an expected price for

Figure 2.4: Model and Empirical Moments



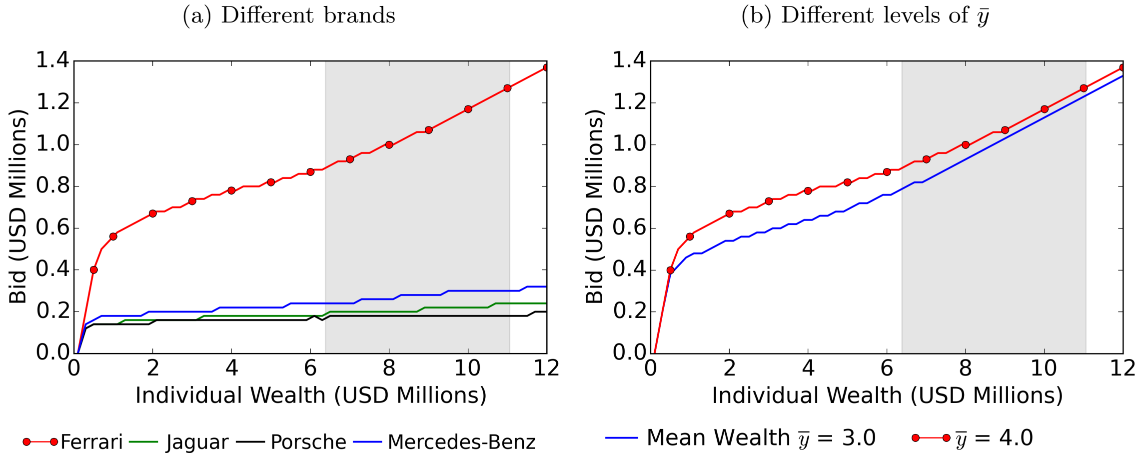
Notes: Expected price by brand and by state in black against empirical average price by brand and by state. Average over states for each brand is matched exactly by estimation (black horizontal line). Bootstrapped error bands in gray and light blue.

Figure 2.5: Expected Prices over Sample Period



Notes: Prices averaged over all brands. Left: Dotted lines are mean +/- one standard deviation. Right: Red line plots predicted average prices if bid function fixed at mean state over sample period but wealth allowed to vary.

Figure 2.6: Bid Functions



Notes: Bid functions $b_j(y, \bar{y})$ predicted by the model for the estimated parameters plotted as a function of private wealth y. Left: for each car j in the economy and mean income $\bar{y}$ fixed at USD 4 Million. Right: For Ferrari at two different values of the state variable mean wealth, $\bar{y}$ = USD 3 Million and $\bar{y}$ = USD 4 Million. Shaded areas: Mean +/- one standard deviation of wealth of the second richest active bidder when $\bar{y}$ = USD 4 Million. Given the bid functions turn out to be increasing for the estimated parameters, the bid of the second richest bidder will determine the selling price.

each car and state and a variance around it.

Moreover, there is variation in the expected price for a car across states of the economy. In higher states, buyers are richer on average. Since the bid function is increasing in wealth, one would expect bids and average prices to increase. This is a shift along the bid function. In addition, figure (2.6b) shows that the bid function shifts up for higher states. The bid function shifts out because in higher states future prices are expected to be high, which makes it more attractive to be a seller next period and less attractive to be a buyer next period. Both effects increase bidders' current willingness to pay at every level of wealth. As the figure shows the two bid functions converge for higher levels of wealth because bidders with high wealth expect that after purchase they are unlikely to sell, and hence care less about future expected prices. The shift in the bid function from a lower to a higher state amplifies volatility across states, as compared the direct wealth effect alone.

The red line in figure 2.5b plots the time series of prices that would results from shutting down this 'dynamic' effect by keeping the bid function fixed at the average state over the sample period and changing only the wealth of bidders. The standard deviation of prices without the dynamic effect is 0.018, or 48% of the standard deviation of prices predicted by the unrestricted model, which is 0.037. In other words, 52% of the standard deviation of prices over time as predicted by the model comes from the dynamic effect of agents changing their strategies according to the state of the economy and only 48% is generated by the direct wealth effect of stock market swings.

The counterfactuals below will explore the role of resale in generating price variation both within and across states of the economy.

2.5.2 Discussion

Before moving onto the counterfactuals, I briefly discuss some limitations imposed by the data and reflected in my estimation approach.

My data from classic cars has the advantage of covering a relatively long horizon and being from a small enough market to warrant the assumption of being exogenous to the state of the economy. These are both important for exploring price volatility over the business cycle, as I do here. A downside to the data is that I only observe transaction prices but not the number of bidders or bidder identities. This means I have to calibrate the number of bidders N and the probabilities of being active for sellers $\tilde{\rho}$ and for bidders σ.

Moreover, there are large amounts of unobserved heterogeneity between cars (unobserved by the econometrician but observed by market participants). This shows up in a high bootstrapped standard deviation of empirical and predicted prices in figure 2.4.

The presence of unobserved heterogeneity and the lack of data on the same bidder bidding on different cars, makes it infeasible to flexibly estimate a substitution matrix between cars as Backus and Lewis (2019) do in their setting. Cross price elasticities here come instead from the assumed utility function, estimated wealth process, and the estimated w vector.

2.6 Counterfactuals

2.6.1 No Resale

The fact that classic cars are actively traded and can be resold in future auctions is an important aspect of this market, as discussed above, and makes them asset-like rather than pure consumption goods. This first counterfactual exercise will rule out resale. Buyers who buy afterwards stay sellers forever. The seller value function then becomes the following:

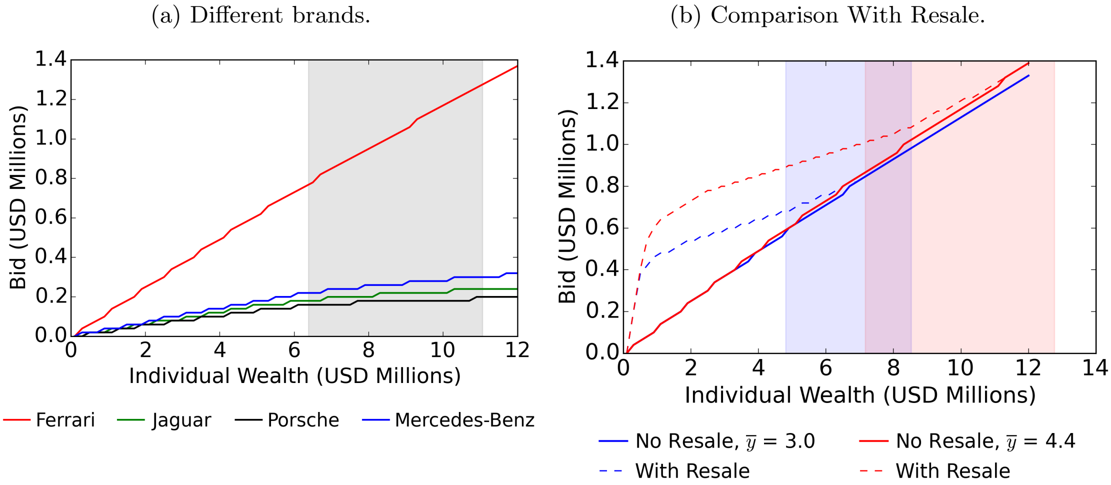
$$S_j^{\text{keep}}(y, \bar{y}) = \ln(\psi y) + w_j + \mathbf{E}[S_j^{\text{keep}}(y', \bar{y}')|y, \bar{y}] \tag{2.8}$$

The rest of the model remains the same.

Bid Functions

Solving this version of the model with the parameters estimated above yields the bid functions in figure (2.7a). Figure (2.7b) shows a comparison between one bid function in

Figure 2.7: Bid Functions No Resale



Notes: Bid functions $b_j(y, \bar{y})$ predicted by the model without resale for the estimated parameters plotted as a function of private wealth y. Shaded areas: Mean +/- one standard deviation of wealth of the second richest active bidder.

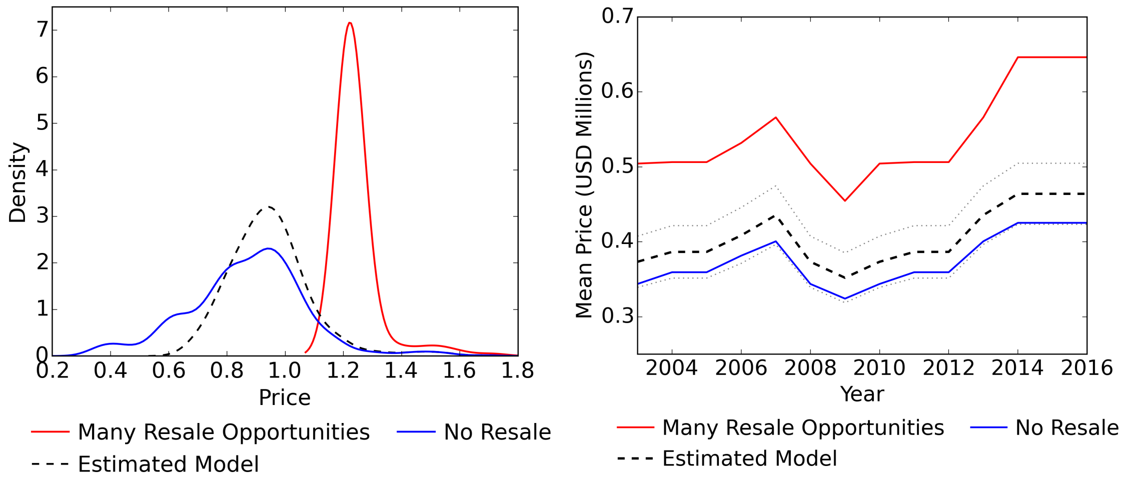
the original model with resale and the counterfactual one without resale. Since without resale there is no option value of selling, the bid function is strictly below the one with resale. Moreover the option to sell is worth more to bidders with less wealth, so that the two bid functions converge towards the upper end of the wealth distribution.

This has two implications for prices. First, average prices will be lower without resale because of the loss of the option value. Second, the variance around the expected price per car and state will be higher without resale because the bid function is steeper, i.e. random changes in the pool of active bidders will have a larger effect on the price. Intuitively, with resale there will be 'flipping': If there happen to be relatively poor bidders, someone will buy the car in the expectation to sell it again soon at a higher price. These flippers help to smooth out fluctuations in the price due to randomness of the pool of active bidders. Without resale this is not possible. Figure 2.8a accordingly shows that the density of prices within one state is more narrow with resale than without. Moreover, the table in appendix E shows that without resale the average price is lower and the standard deviation is higher in each state than with resale.

Figure (2.7b) also shows the difference in bid functions between states. With resale the difference is larger because, as explained above, in higher states the expected resale value is higher, which bidders thus pay extra for in higher states. When this resale value is not present, the difference in bids between states narrows. Note that the two curves again converge at the top of the wealth distribution. The richest bidders expect to never want to sell and therefore do not take the resale value into account much, even when it is possible to resell. This smaller gap between bids in different states implies that average

Figure 2.8: Prices with no and many resale opportunities

(a) Density within state. (b) Prices over sample period.



Notes: Left: Density of prices for Ferrari car in state $\bar{y} = 3.8$ Million. Right: Predicted Prices averaged over all brands. Bootstrapped error band of one standard deviation in gray.

prices will fluctuate less across states than with resale.

There are thus two effects of resale on the volatility of prices: *Within* a state resale helps to smooth out random fluctuations in the price but at the same time resale tends to amplify fluctuations in the average price *across* states. Appendix A.3 shows an analytical solution to a two-period model with aggregate shocks with and without resale. The analytical model confirms that with resale prices will differ more between states than without resale.

Prices over the Sample Period

Figure (2.8b) shows that indeed the predicted price averaged across brands over the sample period is lower in every year in the case without resale than with resale. The difference is about one (bootstrapped) standard deviation of the predicted average prices by state. Across the sample period, prices would have been 8% lower on average without resale than with resale. As the figure also shows, the volatility of prices over the sample period would have been very similar without resale. For the estimated parameters, resale does not seem to amplify price volatility.

More Resale Opportunities for Sellers

This conclusion changes if sellers have more frequent opportunities to resell their car. Suppose that, in the extreme, sellers could resell their car every period, i.e. $\tilde{\rho} = 1$. This would mean holding as many auctions as there are sellers every period. Alternatively, sellers might have other outside markets where they can try to sell their car, e.g. through private sellers. Figure 2.8a shows that with many resale opportunities the density moves to the right and becomes more narrow than in the baseline case, i.e. the exact opposite to the change without resale. A price due to a bad draw of active bidders is now very unlikely to be realised because speculators can buy the car and immediately (in the next period) try to resell at a higher price.

In this counterfactual with $\tilde{\rho} = 1$, prices would therefore have been 31% higher over the sample period. The difference in volatility is also substantial: The standard deviation normalised by the mean over the sample period increases from 0.09 to 0.11 , an increase of 21%. Very frequent resale opportunities thus increase the option value of resale and increase the fluctuations therein across states, which leads to more volatile prices.

Demand parameter estimates without resale

Given that predicted prices without resale are lower with the estimated parameters, one may expect that estimating the model while ignoring the possibility to resell a car will bias parameter estimates of w, the flow utilities of each car, upwards. Indeed, estimating the model without resale yields larger estimates for every element of w as shown in E. The difference is relatively small however, at about one standard error of the original parameter estimate.

2.6.2 Centralized Trade

Auctions for classic cars take place in various locations and at various times over the year. Therefore, at any given auction only a small fraction of potential buyers will be present. The estimation captures this with a low probability of being active for buyers of $\sigma = 0.02$. This section considers a counterfactual experiment where trade is centralized, which is modelled as letting all buyers be active at every auction, i.e. $\sigma = 1$. Bodoh-Creed et al. (2019) consider the effects of centralizing auctions on eBay by combining several auctions on the same day into fewer multi-unit auctions. Due to having a model with an endogenous supply side, I can here additionally consider the seller response to centralizing trade. The aggregate shocks in the model allow me to also look at the effect on price volatility, in addition to the effect on the average price.

A higher value of σ implies that more bidders will be present at each auction and hence the wealth of the richest bidders will be higher in expectation. This will tend to increase selling prices. On the other hand, a higher probability of being active again next period increases the option value of waiting by losing the auction. Hence bidders will shade their bids more, which will tend to decrease prices. Which effect dominates will determine the net effect on prices of centralizing trade. A larger bidder pool at each auction also means that the variation in the wealth of bidders from one auction to the next goes down.

It turns out that the overall effect of centralizing trade on prices is negative, i.e. the shading effect dominates the wealth effect on the active bidder pool. Prices by brand predicted by the baseline version of the model and in the centralized version are shown in appendix E. The average price across models falls very substantially: the difference is 34%, which is statistically significant as the table shows.

Centralizing trade also has an effect on the behaviour of sellers: With more active bidders at each auction, sellers are more likely to sell whenever they are given the chance to do so. In the baseline model 15% of auctions end with a sale. With a centralized buyer side the number of selling opportunities stays the same for sellers but the sales rate is now 35%. This is because with the same active bidders in every auction there is a lower incentive for sellers to wait and try to sell again to a richer bidder pool. A higher sales rate implies that cars are more often owned by the richest agents, i.e. the allocation improves.

More Resale Opportunities under Centralized Trading

Finally, consider what happens when sellers again get to sell every period ($\tilde{\rho} = 1$) but under the current scenario of centralized trade ($\sigma = 1$). Now the average price increases compared to centralized trade with the baseline frequency of selling opportunities, as one would expect, but is still below the baseline price level (decentralized trade and infrequent selling opportunities). Compared to the baseline, the average price is 19% lower.

More interestingly, price volatility over the sample period remains the same: The standard deviation divided by the mean price over the sample period is approximately the same as in the baseline model. This is in stark contrast to the counterfactual with frequent selling opportunities under decentralized trade, where price volatility across states of the economy was substantially above the baseline case. The explanation for this difference is that under centralized trade, the bidder pool does not change from one period to the next, eliminating the scope for buying cars at a low price to resell them at a higher one because the same bidders that are active in the current auction will be active in the next one.[9]

2.7 Conclusion

This paper has argued that taking into account resale opportunities is important whenever they are present, especially when market participants are subject to shocks, for example to their wealth. I have shown that resale increases average selling prices due to the option value of resale. When sellers have frequent opportunities to resell, the option value of

[9] Bidders' wealth will still change from one auction to the next due to the additive wealth shock $\epsilon_{i,t}$ but this effect is small compared to the changing composition of the active bidder pool in the decetralized case.

resale may also increase price volatility across states of the economy because the option value is higher in better states.

Centralizing trade in markets with resale not only benefits buyers by allowing them to substitute more easily to their most preferred good but also allows owners to sell at less variable prices and therefore leads to more frequent transactions. Centralized trade also reduces the scope for speculation and therefore more frequent resale opportunities do not lead to more volatile prices when trade is centralized. It can thus help guard against price volatility that the possibility to speculate introduces in markets where resale is very common.

While the model was estimated using data from auctions for classic cars, which has the advantage of having a known trading mechanism and being plausibly too small to affect the aggregate economy, these insights carry over to important markets, such as the housing market, where used goods can be sold on also.

Chapter 3

Information in Sequential Auctions

Abstract: This chapter compares two models of sequential auctions. The first model has all bidders enter in the first period, as in the canonical model in Weber (1983). The second model has bidders enter one by one. This makes the game stationary from the perspective of a bidder who anticipates bidding again next period, as in several structural models in the empirical literature on sequential auctions (for example Backus and Lewis, 2019). In the first model bidders bid as if they are going to win the next auction with certainty, even if their value is low, because only then are deviations in the present auction payoff relevant. In the second auction bidders account for the probability of winning next period conditional on their value, which implies that higher bidders shade more. Nevertheless, the two models turn out to be revenue equivalent. Moreover, in both models prices are in expectation the same in every period.

3.1 Introduction

While many markets can be modelled as a sequences of auctions, not all of these markets function identically, in particular with respect to how bidders arrive to these markets. Christie's and Sotheby's organise auctions for art that take place in a sequence over one or several days. The online marketplace eBay at any point in time has many auctions taking place one after the other. An important difference between these two examples is that at the beginning of a wine or art auction all potential buyers are typically present and can bid in all of the individual auctions taking place. On eBay bidders arrive at different times and so bidders do not get the opportunity to bid on auctions that took place before their arrival. Bidder entry also changes the expected outside option of bidding in the next auction whenever a bidder loses the current auction. This paper therefore investigates in a stylized setting how the timing of bidders' arrival to the game affects the optimal bidding strategies and auction outcomes including expected prices and efficiency of the allocation.

In the canonical model of sequential auctions, presented in Weber (1983), the game consists of a finite sequence of auctions for one of a pre-specified number of identical objects each. Bidders have unit demand and are all present in the first auction. After each auction the winner exits and the losing bidders continue into the next round. I will contrast this with an alternative model in which only two randomly selected bidders enter the first auction and one new player enters every subsequent auction. The rest of the games will be identical. Winning bidders again exit after each round and losing bidders continue into the next round. The important difference between the two models is that only in the Weber model can continuing bidders learn new information about their competitors' valuations from the first auction to the next. In the second model continuing bidders face a new entrant as competitor whom they have not learned anything about. This makes the second game stationary from the point of view of the continuing bidder, while the Weber model is not stationary.

How, if at at all, is the difference between these models reflected in equilibrium strategies, player payoffs, and seller revenue? I show in this chapter that the equilibrium bid functions differ in an interesting way: In the stationary model bidders shade their bid by the value of continuing into the next auction, which depends on their expected probability of winning that auction. In the Weber model, bidders shade their bid by the value of continuing into the next period *assuming they are going to win with certainty*, even if their value of the object is low. Bidders changing their bid marginally only affect their payoff in the auction if they are tied with the highest competing bidder. Since bidders in the Weber model will face the same competitors again next period, this implies winning next period and so bidders condition their bid on this scenario. In the stationary model on the other hand, being tied with their competitor this period says nothing about bidders' probability of winning next period because they will then face a new competitor.

I also show that the two models are payoff and revenue equivalent. The expected price in both models is the same and is moreover the same in all periods of each game.

A growing empirical literature on sequential auctions commonly models sequential auctions as stationary. Relating this class of stationary models to the canonical model in Weber (1983) is therefore interesting from this perspective, too.

The remainder of the introduction reviews the literature related to this chapter with a focus on information release across auctions. The rest of the chapter is organised as follows: Section 3.2 explains the set-up of the two models. Section 3.3 derives the equilibria of the two games with two periods and explores the difference between them. Section 3.4

extends the game to more than two periods and establishes that the two games result in the same allocation and that in both games expected prices are the same in every period. Section 3.7 concludes.

Related Literature. The famous result in Weber (1983) is that in his model of sequential auctions prices are the same in expectation in each period, although the average valuation among bidders left in the game is falling. The paper shows that it does not affect the outcome of the game whether the selling price is announced after each auction or not. Said (2012) demostrates that in this setting, bidder entry may lead to the non-existence of a symmetric equilibrium with sealed bid auctions.

Another strand of literature, starting with Engelbrecht-Wiggans (1994), assumes that values are drawn not before the beginning of the game for the duration of the game but are drawn each period independently from other periods' values of the same bidder. Since values are not correlated across periods, there is no scope for learning as the game progresses. This leads to a stationary equilibrium but implies that bidders have no persistent private information about their values, which is not suitable for many applications.

Starting with Backus and Lewis (2019), a structural literature on sequential auction markets including Hendricks and Sorensen (2018) and Coey et al. (2019), assumes a mean-field equilibrium in which bidders do not take the full strategic environment into account but optimise their bid against a stationary distribution of rival bids. This approach maintains persistent differences between bidders but assumes that bidders do not take the information generated by one auction about the next into account.

Satterthwaite and Shneyerov (2007) and Bodoh-Creed et al. (2019) assume a continuum of players, which also leads to stationary strategies because the actions of any one player do not affect the aggregate outcome.

3.2 Model

There are M periods and $N = M+1$ bidders. In each period one of M identical objects is sold through a second price sealed-bid auction without reserve price. Each bidder's value is an independent random variable X with distribution $F(x)$ that takes values between 0 and $\bar{x}$. Bidders privately observe the realisation x of their value before the start of the game. Bidders have unit demand. There is no discounting between the two periods.

This section presents two models that have this set-up in common. In the first model all N bidders enter the first auction. After the first and all subsequent auctions, the winner exits while all losing bidders continue into the next auction. The second model has only two bidders enter the first auction and has one new bidder enter each subsequent auction to bid against the continuing bidder from the previous round. From the perspective of the continuing bidder the second model is stationary in that the continuing bidder expects to face a competitor in the next round with the same distribution of values as in the present one. I will refer to this model therefore as the stationary model.[1]

3.2.1 Weber Model

In the first period all $M+1$ bidders submit a bid on the first object. The winner of the first auction exits. The M remaining bidders move into the second period and bid again on an identical object. The winner of the second auction exits and the remaining bidders continue into the third auction, and so on. A bidder's value x for the object remains the same for all periods of the game. After each auction it will be revealed who won but the selling price will be kept hidden from the losing bidders. Weber (1983) shows that announcing the selling price after each auction does not change the outcome of the game.

3.2.2 Stationary Model

In the first period two out of the $M+1$ bidders are chosen at random to submit a bid on the first object. The winner exits while the remaining bidder moves into the second period. One of the $M-1$ bidders who did not participate in the first auction is chosen at random and enters the second auction. After the second auction the winner exits and the loser continues into the third auction. A new randomly picked bidder will enter the third auction, and so on. In each auction there are thus two bidders. After each auction it will be revealed who won but not what the selling price was.

3.3 Equilibrium for Two Periods

This section restricts both games to $M = 2$ and derives the equilibria for both games und this assumption. Section 3.4 will consider the case $M \geq 3$.

A player's strategy in both of these games with two periods consists of two bid functions, which, for each period, map the range of possible values $[0, \bar{x}]$ to a nonnegative bid. Both games end after the second period and so the second auction amounts to a static second price auction, in which players have a dominant strategy to bid their value. In both games the second-period bid function of any symmetric subgame perfect equilibrium

[1] The model is not fully stationary because after the first round the valuation of the continuing bidder does not follow the same distribution as that of the new entrant. But the important aspect is that it is stationary from the perspective of the continuing bidder.

will therefore be $b_2(x) = x$. We are then interested in the symmetric subgame perfect equilibrium bids at $t = 1$ in each of the two models.

3.3.1 Equilibrium in the Weber Model

There are $N = 3$ players in this version of the game with $M = 2$ and so each bidder faces two competitors in the first round of the Weber game. Throughout, I will be using the notation $X_{(i:n)}$ for the ith highest order statistic from n draws. $X_{(1:2)}$ is thus the larger of the two competing values and $X_{(2:2)}$ is the lower one. They have a joint distribution $f_{X_{(1:2)}X_{(2:2)}}(\cdot,\cdot)$. Conjecture that the other bidders are playing according to an increasing bid function $b_W(\cdot)$. A bidder with valuation x who submits bid B will then win the first auction if $B > b_W(X_{(1:2)})$. The bidder will lose the first and then win the second auction if $B < b_W(X_{(1:2)})$ and if, given that in the second round all bidders will bid their own value, $x > X_{(2:2)}$. If $B < b_W(X_{(1:2)})$ and $x < X_{(2:2)}$ the bidder will lose both auctions and have a payoff of zero. The objective in the first period is then as follows:

$$\max_B \int_{x_{(1:2)}} \int_{x_{(2:2)}} \Big(1_{B>b_W(x_{(1:2)})}(x - b_W(x_{(1:2)})) + 1_{B<b_W(x_{(1:2)})}1_{x>x_{(2:2)}}(x - x_{(2:2)})\Big) f_{X_{(1:2)}X_{(2:2)}}(x_{(1:2)}, x_{(2:2)})dx_{(2:2)}dx_{(1:2)} \quad (3.1)$$

Solving the problem in (3.1) yields the equilibrium in the Weber model.

Proposition 1. *The Weber Model with $M = 2$ periods has a unique symmetric subgame perfect equilibrium in which all players in period 1 bid according to the bid function $b_W(x)$, which satisfies:*

$$b_W(x) = \boldsymbol{E}[X|X < x] \quad (3.2)$$

Proof. See Appendix.

To build intuition for this result it helps to compare it to the static second price auction, where the optimal bid is equal to x. There bidding less than x exposes the bidder to the risk of losing the auction when the object could have been bought for an amount below x (when the highest competing bid is also below x). In that case the bidder would have preferred to buy. Similarly in this model, bidding any less than $\mathbf{E}[X|X < x]$ means to possibly forgo buying in the first period below the price at which the bidder can expect to buy in period two.

3.3.2 Equilibrium in the Stationary Model

This section derives the unique symmetric subgame perfect equilibrium of the stationary Model. In the stationary model each bidder faces one competitor in the first auction. Conjecture that this competitor in the first auction is playing a monotone bid function $b_S(\cdot)$. Let X_1 be the value of the competitor that a bidder faces in the first period and let X_2 be the value of the competitor that the bidder will face in the second round. A bidder with value x who submits bid B in the first period then wins the first auction if $B > b_S(X_1)$. The bidder will lose the first and then win the second auction if $B < b_S(X_1)$ and if, given that in the second round all bidders will bid their own value, $x > X_2$. If $B < b_S(X_1)$ and $x < X_2$ the bidder will lose both auctions and have a payoff of zero. The key difference to the Weber model is that X_1 and X_2 are two independent random variables with the same distribution as X. Their joint pdf is $f_{X_1X_2}(x_1, x_2) = f(x_1) \cdot f(x_2)$. The order statistics $X_{(1:2)}$ and $X_{(2:2)}$, which are relevant to the Weber model above, are not distributed independently. The objective in the stationary model can be written in similar form to above:

$$\max_B \int_{x_2} \int_{x_1} \left(1_{B>b_S(x_1)}(x - b_S(x_1)) + 1_{B<b_S(x_1)} 1_{x>x_2}(x - x_2)\right) f_{X_1X_2}(x_1, x_2) dx_1 dx_2 \tag{3.3}$$

Due to the independence of X_1 and X_2, this can then be simplified as follows:

$$\max_B \int_0^{b_S^{-1}(B)} (x - b_S(x_1)) f(x_1) dx_1 + \left(1 - F(b_S^{-1}(B))\right) \int_0^x (x - x_2) f(x_2) dx_2 \tag{3.4}$$

See appendix for details. Solving (3.4) yields the equilibrium in the stationary Model.

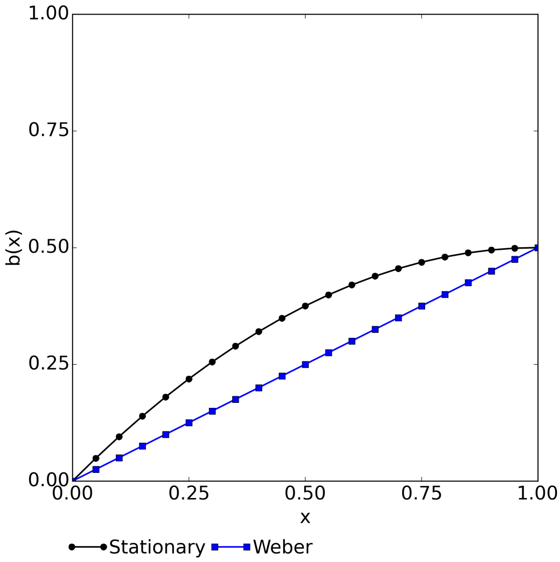
Proposition 2. *The stationary Model has a unique symmetric subgame perfect equilibrium in which all players at $t = 1$ bid according to the bid function $b_S(x)$, which satisfies:*

$$b_S(x) = x - F(x)(x - \boldsymbol{E}[X|X < x]) \tag{3.5}$$

Proof. See Appendix.

This equilibrium bid function can be interpreted as the value of the object less the value of continuing into the next period. The value of moving into the next period is the probability of winning the next auction, $\mathbf{Pr}[x > X] = F(x)$, times the expected payoff of winning the auction, $(x - \mathbf{E}[X|X < x])$.

Figure 3.1: Bid Functions with Uniformly Distributed Values



Notes: Equilibrium bid functions $b_W(x)$ and $b_S(x)$ as per equations (3.2) and (3.5) for the case of $X \sim \mathcal{U}[0,1]$. $b_W(x) = \frac{1}{2}x$ and $b_S(x) = x - \frac{1}{2}x^2$, see Appendix F.5.

3.3.3 Comparison

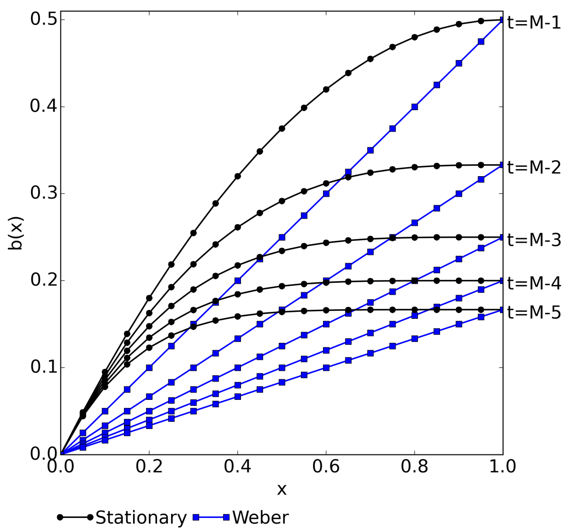
This section looks at how the equilibrium bid functions of the two models differ and why. Rewrite the bid function in the Weber model (3.2) as follows:

$$b_W(x) = x - (x - \mathbf{E}[X|X < x]) \tag{3.6}$$

Written this way, both equilibrium strategies in the Weber model in (3.6) and in the stationary model in (3.5) can be interpreted as the bidder's valuation x shaded by an opportunity cost of moving into the second auction. In the Weber model the amount of shading equals $(x - \mathbf{E}[X|X < x])$, the expected payoff of winning the second auction. In the stationary model the amount of shading equals $F(x)(x - \mathbf{E}[X|X < x])$, the expected payoff of winning the second auction discounted by $F(x)$, the probability of winning the next auction.

In both models, deviating to a lower first-period bid affects payoffs only if the deviating bidder has the highest valuation among the first-period bidders. In the Weber model the highest first-period bidder will also be the highest bidder in the second auction, should that bidder deviate and continue into that auction, because all players submit bids in the first round. In the stationary model this is not the case. The continuing bidder in the stationary model will face a new entrant who may have a higher valuation than even the highest first-period bidder because that new entrant had not been active in the first round. This explains why the equilibrium strategy in the Weber model conditions on winning the second period auction while in the stationary model the equilibrium strategy accounts for

Figure 3.2: Bid Functions in Different Periods with Uniformly Distributed Values



Notes: Equilibrium bid functions $b_W(x)$ and $b_S(x)$ as per equations (3.7) and (3.8) for the case of $X \sim \mathcal{U}[0,1]$. See appendix F.4.

the probability of winning the next auction: In the Weber model deviating in the first round affects payoffs only for bidder types that will win the second auction but in the stationary model a bidder who would have won the first auction may deviate to a low bid, continue, and then lose the second auction.

Since $F(x) \leq 1$, bidders in the Weber model shade more than in the stationary model. One way to think about this is that since bidders in the Weber model bid as if they are going to win the next auction with certainty, they assign a higher value to the outside option of moving into the second auction. One can also notice that the expected valuation of the competitor that the continuing bidder will face in the stationary model is higher than in the Weber model, and therefore the continuation value in the Weber model is indeed higher and so bidders should shade more.

In the stationary model bidders with higher values shade more since they have a higher probability of winning in period two. Backus and Lewis (2016) discuss this effect and explain that it leads to a concave bid function in their model (p.11). This effect depends on stationarity and is absent in the Weber model, where all bidders condition their bid on winning the second period. The two bid functions therefore converge as x becomes larger. (Since $F(x) \to 1$ as $x \to \bar{x}$.)

3.4 Equilibrium for $M \geq 3$ Periods

This section extends the analysis to M objects and $N = M + 1$ bidders. The assumption of $N = M + 1$ is necessary for the stationary model to retain a symmetric equilibrium, as will become clear below.

Proposition 3. *The Weber model with M objects has a unique symmetric subgame perfect equilibrium in which all players bid according to the following bid function in period $t = 1, ..., M$:*

$$b_t(x) = \boldsymbol{E}[X_{(M:M)}|X_{(t:M)} = x] \tag{3.7}$$

Proof. See Appendix.

Note that for $M = 2$, this reduces to the equilibrium in proposition 1, since $\mathbf{E}[X_{(2:2)}|X_{(1:2)} = x] = \mathbf{E}[X|X < x]$ (see equation F.14 in the appendix).

Proposition 4. *In the stationary model, let $V_t(x)$ be the continuation value in period $t = 1, ..., M$, i.e. the value of not winning at t and instead continuing into the auction $t + 1$. The stationary game has a unique symmetric subgame perfect equilibrium in which all players bid according to the following bid function:*

$$b_t(x) = x - V_t(x) \tag{3.8}$$

and the continuation value is as follows:

$$V_t(x) = F(x)(x - \boldsymbol{E}[b_{t+1}(x)|X < x]) + (1 - F(x))V_{t+1}(x) \tag{3.9}$$

Proof. See Appendix.

Note that $V_{M-1}(x) = F(x)(x - \mathbf{E}[X|X < x])$ and so for $M = 2$ this reduces to the equilibrium in proposition 2.

In the stationary model, the newly entering bidder and the bidder continuing from the previous period always have asymmetric beliefs about each others' values. The entering bidder's value has distribution $F(x)$ but in any increasing equilibrium the continuing bidder's valuation has distribution $F_{X_{(t:t)}}(x)$ in period t. Despite this asymmetry of beliefs however, a symmetric equilibrium exits. Due to the second price format of the auction, beliefs about competitor values in the current auction do not affect the optimal bid. Beliefs about competitor values in subsequent auctions are important because they determine the continuation value $V_t(x)$. Conditional on continuing, however, both bidders have the same beliefs about next period's competition because next period's competition always consists of a new entrant. The game thus remains stationary in this respect. This would break down with more than two bidders in any one period, which would mean there being more than one continuing bidder each period and next period's competition consisting of a new entrant as well as other continuing bidders from the present auction. In that case the asymmetry of beliefs would yield asymmetric bidding functions and no efficient equilibrium, as pointed out in Said (2012). Therefore, in addition to not accounting for the possibility to bid against the same competitor again in future auctions, both the meanfield equilibrium assumption and the assumption of a continuum of players avoid the asymmetry in optimal strategies between continuing and newly entering bidders. The former by assuming that continuing and entering bidders optimise against the same stationary distribution of rival bids. The latter because with a continuum of bidders, the asymmetry in beliefs, caused by one bidder being either a continuing bidder or an entrant, vanishes.

3.5 Payoffs and Prices

We have seen that both games have a unique symmetric equilibrium in increasing bid functions. In the Weber model the highest-value bidder then wins the first auction, the second-highest value bidder wins the second auction, and so on. The M highest-value bidders all win an item in equilibrium. In the stationary model the higher of the two active bidders will win in each auction. The losing bidder continues and bids again. All of the M highest-value bidders will eventually win an auction. The bidder with the lowest (i.e. the $(M+1)$th value) will lose every auction after entering the game and is the only bidder who does not receive an item in equilibrium. Both games therefore efficiently allocate the M objects to the M highest-value bidders. Since both games have the same allocation in equilibrium, it follows from the envelope theorem of Milgrom and Segal (2002) that expected payments must be the same also.

Another mechanism that allocates M items efficiently to N bidders with unit demand is the uniform price auction, in which the M highest bidders receive the object and all pay the $(M+1)$th highest bid.[2] Again, since the allocation is the same, equilibrium prices must be the same in the Weber and in the stationary games as in the uniform price auction. The symmetric equilibrium in the uniform price auction is to bid truthfully (Krishna, 2009). The expected price thus equals $\mathbf{E}[X_{(M+1:N)}]$. These results are summarised in the following proposition:

Proposition 5. *In both the Weber and the stationary model for M objects and $N = M+1$ bidders the M highest-value bidders receive the M items in equilibrium. They all pay the same expected price, namely the expected valuation of the $(M+1)$th highest bidder:*

$$\boldsymbol{E}[P_t] = \boldsymbol{E}[X_{(M+1:M+1)}] \quad \forall t = 1, ..., M$$

The two games are thus payoff equivalent, and moreover prices in both games are in ex-

[2] The uniform price auction is inefficient in general but efficient if bidders have unit demand (Krishna, 2009: p.196).

pectation the same in every period.

Note that since players enter the game one by one in the stationary model, the winner of auction t is going to have a value distributed according to the tth order statistic from $(t+1)$ draws, i.e. $X_{(t:t+1)}$. The price at time t is determined by the second highest bid, i.e. by valuation $X_{(t+1:t+1)}$. In the Weber model the winner has valuation $X_{(t:M+1)}$ and the price is determined by the bidder with valuation $X_{(t+1:M+1)}$. The price is thus determined by a bidder with a higher value in expectation in the Weber model than in the stationary model (since the distribution of $X_{(t+1:M+1)}$ first-order dominates $X_{(t+1:t+1)}$). On the other hand, bid functions in the stationary model are above the bid functions in the Weber model. These two effects turn out to exactly cancel each other, so that expected prices are the same. For illustration, a direct derivation of expected prices for the two-period case is in Appendix F.3.

3.6 Discounting

So far players were assumed to not discount the future (as in Weber, 1983). When players discount the future with factor β, the equilibrium bid functions change in an intuitive way, as follows:

Proposition 6. *The Weber model with discounting has a unique symmetric subgame perfect equilibrium in which all players bid according to the following bid function:*

$$b_t(x) = x - \beta(x - \boldsymbol{E}[X_{(M:M)}|X_{(t:M)} = x]) \tag{3.10}$$

Proof. See Appendix.

Proposition 7. *The stationary game with discounting has a unique symmetric subgame perfect equilibrium in which all players bid according to the following bid function:*

$$b_t(x) = x - \beta V_t(x) \tag{3.11}$$

Proof. See Appendix.

With discounting the two games are not payoff equivalent. In the Weber game bidders with relatively low values will always have to wait to receive the object because the highest-value bidders will receive the object first. In the stationary model lower-value players may receive the object early if the higher-value bidders happen to enter later on. With discounting this difference in timing matters and so expected payoffs are not the same in the two games. The envelope theorem does not apply anymore here. In the two period game with uniformly distributed values between 0 and 1 for example, the expected price in the first period of the Weber game is $\frac{1}{2} - \frac{\beta}{4}$ while in the stationary game it is $\frac{1}{3} - \frac{\beta}{12}$.

3.7 Conclusion

This chapter has compared two stylized models of sequential auctions. The difference between the models was such that one was stationary from the perspective of the bidders in the first period and the other wasn't. The stationary model is comparable to several models in the recent literature that assume that bidders play against a stationary distribution of rival bids rather than updating their beliefs from one auction to the next.

The analysis showed that the way bidders shade their bids below their value differs between the two games and that the amount of shading increases in values only in the stationary case. It also showed that the differences in the equilibrium strategies exactly offset the differences in the way bidders arrive at the auctions to make the two games revenue and payoff equivalent. This revenue equivalence fails when players discount the future.

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Appendix

A Illustrative Models of Sequential Auctions

This section presents a number of two period models of sequential auctions that can be solved analytically to shed more light on the mechanisms operating in the main model, which only has a numerical solution.

A.1 Continuation Values and Resale

There are two bidders and one seller active in each period of the game. The game consists of two periods t. Valuations are a random variable X which is distributed uniformly between 0 and 1: $X \sim \mathcal{U}[0, 1]$. Players observe the realisation of their value x before the start of the game. Valuations stay the same throughout the game in this version of the game. Players do not discount the future. Each period consists of a second-price auction with reserve price r. The reserve price is set by a seller with valuation X_S. Assume at first that each auction is efficient, i.e. $r(X_S) = X_S$. This will be the optimal secret reserve price if the seller has to accept or reject the selling price after all bids have been received. Section A.2 will look at the same game but with an optimal public reserve price.

Players in the game have stationary beliefs about other players' valuations, i.e. a continuing bidder will believe his competitor's value will again be drawn from $\mathcal{U}[0, 1]$.

Assume also a stationary distribution for X_S, namely that it is distributed according to the first order statistic among three draws, i.e. $X_{(1:3)}$, which will result if among two bidders and one seller, the object is in expectation owned by the agent with the highest valuation. Values X have cdf $F(x) = x$ and so $F_{X_S}(x) = x^3$.

We are interested in bidding strategies $t = 1$ under different assumptions about what bidders perceive to happen at $t = 2$.

Bidder Continuation Value

Upon losing the first auction, a bidder will continue into the second period to bid again. The game ends after the second period, so the equilibria in the second auction will be that of a static second price auction. We will focus on the unique symmetric equilibrium in which all players bid their own value. In the second period the bidder with value x will then win if $x > \max(X_S, \bar{X})$ where $\bar{X}$ is the competitor's value with cdf $F(\bar{x}) = \bar{x}$. With an efficient aucion in the second period, $\mathbf{Pr}[x > \max(X_S, X')] = \mathbf{Pr}[x > X' \cap x > X_S] = F(x)F_{X_S}(x) = x^4$. The continuation of losing is then:

$$\begin{aligned}
V(x) &= \mathbf{Pr}[x > \max(X_S, \bar{X})](x - \mathbf{E}[\max(X_S, \bar{X})|\max(X_S, \bar{X}) < x]) \\
&= F(x)F_{X_S}(x)\left(x - \int_0^x a\frac{\frac{d}{da}(F(a)F_{X_S}(a))}{F(x)F_{X_S}(x)}da\right) \\
&= x^4(x - \int_0^x \frac{4a^3}{x^4}da) \\
&= x^4(x - \frac{4}{5}x) \\
&= \frac{1}{5}x^5
\end{aligned}$$

Seller Continuation Value

Suppose bidders anticipate being able to sell their object in an efficient second price auction in the second period. In an efficient auction the reserve price will be equal to the seller's value, which is the revenue-maximising reserve price if the seller's action is to accept or reject the selling price after all bids have been received, as in the main model. There are two bidders. If $X_{(1:2)} < x$ there will be no sale, if $X_{(2:2)} < x < X_{(1:2)}$ there will be a sale at price x and if $x < X_{(2:2)}$ there will a sale at price $X_{(2:2)}$. Only in the last case will the seller gain from the sale. The continuation value upon buying in the first period is then as follows:

$$\begin{aligned}
S(x) &= \mathbf{Pr}[x < X_{(2:2)}](\mathbf{E}[X_{(2:2)}|x < X_{(2:2)}] - x) \\
&= (1-x)^2(\frac{1}{3} + \frac{2}{3}x - x) \\
&= \frac{1}{3} - x + x^2 - \frac{1}{3}x^3
\end{aligned}$$

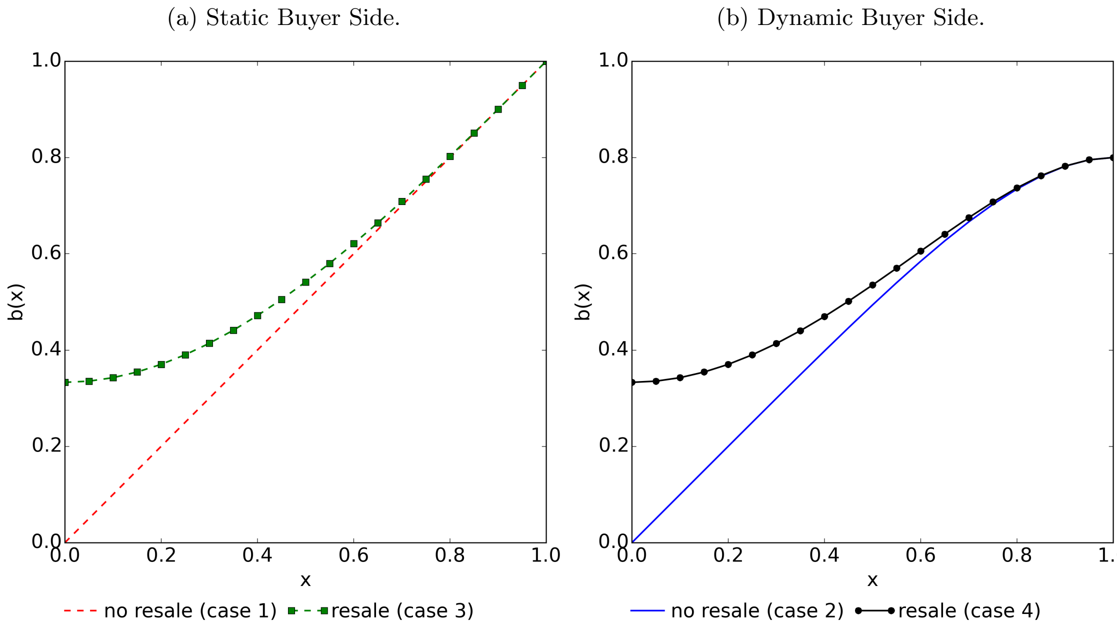
Bid Functions

We will now look at four different models, partly motivated by the existing literature, and compare the resulting bid functions in period one.

1. Static: Bidders perceive the first period to be a one-shot game.
   $b(x) = x$.

2. Dynamic buyers, no resale: Bidders take the continuation value of losing into account but there is no resale, i.e. no continuation value upon buying. This corresponds to

Figure A.3: Bid Functions



several models in the literature, including Backus and Lewis (2019), Hendricks and Sorensen (2018), and Bodoh-Creed et al. (2019).

$b(x) = x - V(x) = x - \frac{1}{5}x^5$.

3. Static buyers, with resale: Bidders ignore the continuation value of losing, e.g. because all losing bidders are assumed to exit but there is resale and hence a continuation value of winning the first auction. This corresponds to the assumptions in Lovo and Spaenjers (2018).

   $b(x) = x + S(x) = \frac{1}{3} + x^2 - \frac{1}{3}x^3$.

4. Both: Dynamic buyers, with resale. Bidders bid taking the continuation value of losing and winning into account.

   $b(x) = x + S(x) - V(x) = \frac{1}{3} + x^2 - \frac{1}{3}x^3 - \frac{1}{5}x^5$.

Figure A.3 plots the resulting bid function for each of the four cases. We can note the following about the bid functions:

- Resale affects the bottom of the bid function: bid functions with and without resale have different intercepts but converge at the top. Intuitively, in the absence of shocks, this is the result of the possibility to 'flip' and object, i.e. to sell it to a bidder with a higher valuation next period.

- Buyer continuation value affects the top of the bid function: dynamic and static cases have the same intercept but diverge at the top. This is because bidders shade down their bid by the option value of bidding again next period.

- Case 4 with buyer continuation value and resale has the least steep bid function, i.e. the price distribution in this case will be the narrowest.

A.2 Optimal Public Reserve Price

So far I have assumed that the seller sets an optimal secret reserve price. This is the assumption in the main model because in the empirical setting reserve prices are secret. If the seller can only accept or reject the selling price after bidding has ended, the optimal cutoff below which to reject the price, is equal to the seller's value, which is also the efficient reserve price. The reserve price that maximises the seller's revenue if that reserve is public is above the seller's value as demonstrated in Riley and Samuelson (1981). This section lays out the same model as above but with this optimal public reserve price.

In the second auction the seller with valuation x maximises their expected payoff by setting a reserve price $r(x)$ that solves $r = x + \frac{1-F(r)}{f(r)}$ (Riley and Samuelson, 1981: Prop 3). For $x \sim \mathcal{U}[0,1]$, $r(x) = \frac{1}{2} + \frac{x}{2}$. Given reserve price r, the expected payoff for the seller is as follows (see Riley and Samuelson, 1981: equ 11):

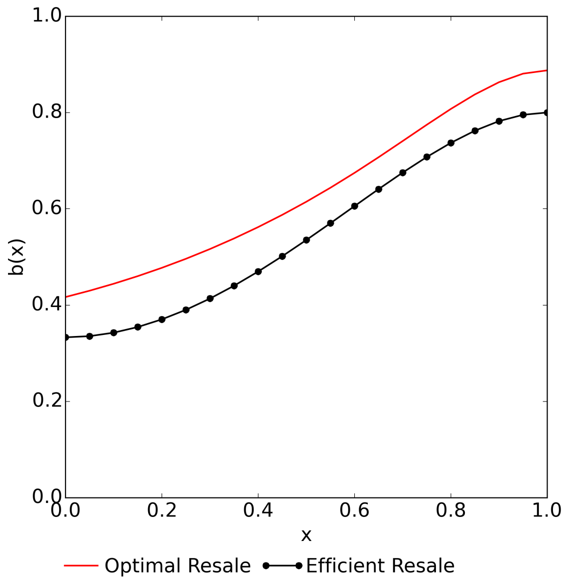
$$S(x) = n \int_{r}^{\bar{x}} (af(a) + F(a) - 1)F(a)^{n-1}da - x(1 - F(r)^n)$$

Where n is the number of bidders. Using the uniform distribution of valuations, $n = 2$, and the optimal reserve price $r(x)$:

$$\begin{aligned} S(x) &= 2 \int_{\frac{1}{2}+\frac{x}{2}}^{\bar{x}} (a + a - 1)a \ da - x(1 - r^2) \\ &= \frac{5}{12} - \frac{3}{4}x + \frac{1}{4}x^2 + \frac{1}{12}x^3 \end{aligned}$$

The reserve price also affects the buyer continuation value. As per Riley and Samuelson (1981): equ 4 and 8b, for all $x > r$, the expected payoff from an auction to a bidder is equal to $\int_r^x F^{(n-1)}(a)da$. Since in the first period, the reserve price in the second period is unknown to the bidders and has the following distribution: $F_r(a) = \mathbf{Pr}[r < a] = \mathbf{Pr}[\frac{1}{2} + \frac{x_S}{2} < a] = F_{x_S}(2x - 1) = (2x - 1)^3$. The value of continuing into the next auction

Figure A.4: Bid Functions for Efficient and Optimal Resale



as a bidder becomes:

$$\begin{aligned} V(x) &= \mathbf{Pr}[x > r]\mathbf{E}_r \left[\int_r^x F^{(n-1)}(a)da \mid x > r \right] \\ &= F_r(x) \int_{1/2}^x \int_r^x F^{(n-1)}(a)da \frac{f_r(r)}{F_r(x)} dr \\ &= \int_{1/2}^x (\frac{1}{2}x^2 - \frac{1}{2}r^2) 6(2r-1)^2 dr \\ &= \frac{1}{80} - \frac{1}{2}x^2 + 2x^3 - 3x^4 + \frac{8}{5}x^5 \quad \forall x \geq \frac{1}{2} \end{aligned}$$

If bidders in the first period take the continuation value of both losing and winning into account, with optimal resale the bid function becomes the following:

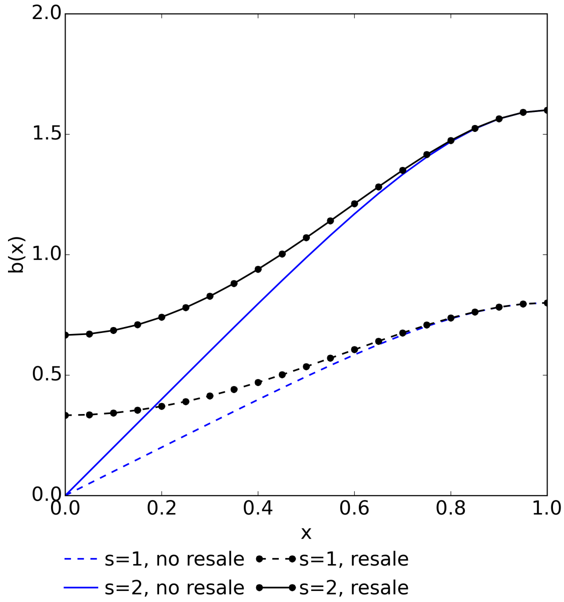
$$b(x) = \begin{cases} \frac{5}{12} + \frac{1}{4}x + \frac{1}{4}x^2 + \frac{1}{12}x^3, & x < \frac{1}{2} \\ \frac{97}{240} + \frac{1}{4}x + \frac{3}{4}x^2 - \frac{23}{12}x^3 + 3x^4 - \frac{8}{5}x^5, & x \geq \frac{1}{2} \end{cases}$$

Figure A.4 plots the bid functions for the case of resale with an efficient reserve price (which is the optimal secret reserve) and with an optimal public reserve.

A.3 Aggregate Shocks

Now suppose that each player has valuation sx, where again $x \sim \mathcal{U}[0,1]$ and s is the same for all players. x stays the same over time but s is subject to shocks. Assume specifically that s follows a random walk, i.e. $\mathbf{E}[s'|s] = s$. The value function of a bidder in the second

Figure A.5: Bid Functions with Aggregate Shock



period becomes:

$$\begin{aligned}
V(x,s) &= \mathbf{E}_{s'}\left[\mathbf{Pr}[x > \max(x_S, x')](s'x - \mathbf{E}[\max(s'x_S, s'x')|\max(x_S, x') < x])\right] \\
&= \mathbf{E}_{s'}\left[s'V(x)|s\right] \\
&= sV(x)
\end{aligned}$$

Similarly, $S(x,s) = sS(x)$ and $b(x,s) = sb(x)$. Hence, in case 2 without resale:

$$b(x,s) = s(x - \frac{1}{5}x^5)$$

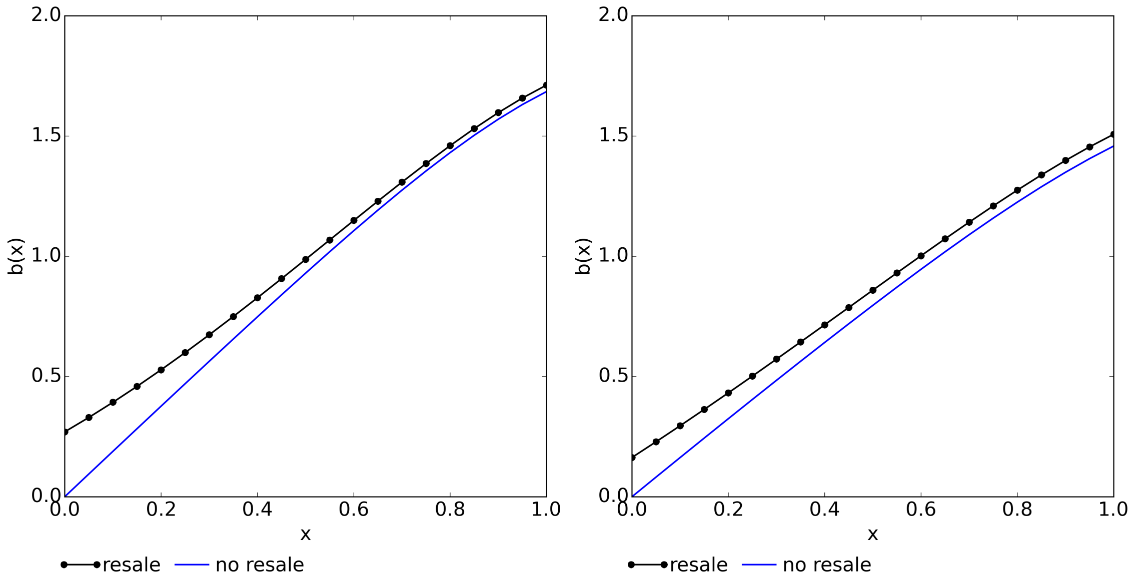
And in case 4 with resale:

$$b(x,s) = s(\frac{1}{3} + x^2 - \frac{1}{3}x^3 - \frac{1}{5}x^5)$$

As the bid functions make clear, the expected price will depend more strongly on s with resale than without. The expected price in the first period is $\int_0^1 b(x) f_{X_{(2:3)}} dx$, which for the case without resale equals $0.48s$ and for the case with resale equals $0.55s$. This demonstrates analytically in this simplified two-period model why we should expect more volatile prices in a setting with resale than without, as investigated in the counterfactual exercise on the estimated model.

Figure A.6: Bid Functions with Idiosyncratic Shock

(a) Probability shock: $p = 0.1$ (b) Probability shock: $p = 0.3$



Notes: Bid Functions with and without resale under idiosyncratic shock, with probability p each bidder's valuation drop to 0. Left panel: $p = 0.1$, right panel: $p = 0.5$.

A.4 Idiosyncratic Shocks

Now consider a slightly different model where players get payoff $\tilde{x}$ in each period they hold the object and $\tilde{x}$ equals x with probability $(1-p)$ and equals 0 with probability p. A buyer with value x who buys at $t = 1$ and does not resell at $t = 2$ thus has expected payoff equal to $x + (1-p)x$. The continuation value as buyer, $\tilde{V}(x)$ depends on whether the bidder gets shocked, the seller gets shocked, and whether the competitor gets shocked:

$$\begin{aligned}\tilde{V}(x) = (1-p)^3 V(x) + (1-p)\Big(p^2 x + (1-p)p\mathbf{Pr}[x > \bar{X}](x - \mathbf{E}[\bar{X}|\bar{X} < x]) \\ + (1-p)p\mathbf{Pr}[x > X_S](x - \mathbf{E}[X_S|X_S < x]\Big)\end{aligned}$$

Where the three terms represent the cases where nobody gets shocked, the bidder does not get shocked but the competitor and the seller do, only the seller does, and only the competitor gets shocked. This becomes:

$$\tilde{V}(x) = (1-p)^3 V(x) + (1-p)\left(p^2 x + (1-p)p\frac{1}{2}x^2 + (1-p)p\frac{1}{4}x^4\right)$$

The continuation value as seller, $\tilde{S}(x)$, in this model with idiosyncratic shocks is as follows:

$$\begin{aligned}\tilde{S}(x) = (1-p)^3\left(\mathbf{Pr}[x < X_{(2:2)}]\mathbf{E}[X_{(2:2)}|x < X_{(2:2)}] + \mathbf{Pr}[x > X_{(2:2)}]x\right) \\ + (1-p)\left(p^2 + 2p(1-p)\right)x + p(1-p)^2\mathbf{E}[X_{(2:2)}]\end{aligned}$$

Where the first term represents the case where nobody gets shocked, the second term represents the term where the seller does not get shocked and at least one bidder gets shocked, and the case where the bidder gets shocked but the two bidders do not.

The bid functions without resale and with are then as follows:

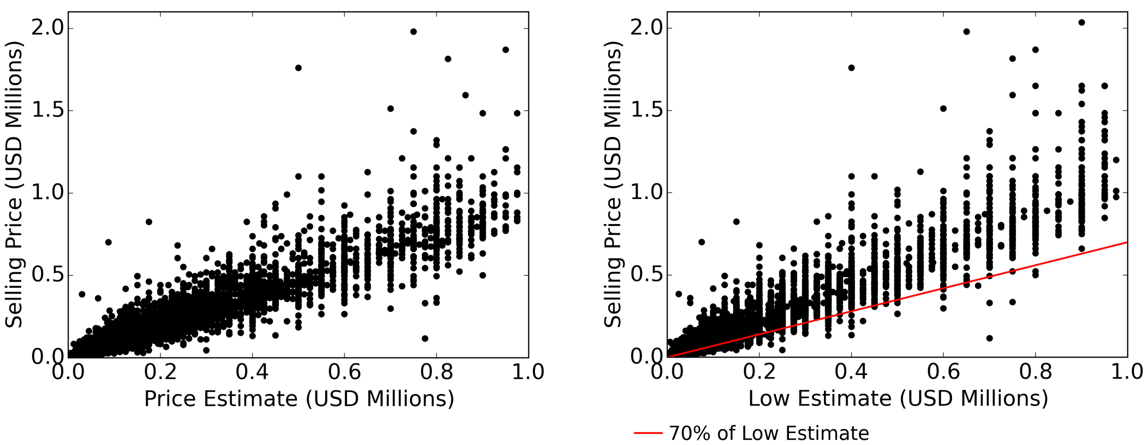
$$b_k(x) = x + (1-p)x - \tilde{V}(x)$$
$$b(x) = x + \tilde{S}(x) - \tilde{V}(x)$$

Figure A.6 plots these two bid functions. As can be seen from the figure there is now a gap between the two bid functions that persists even for high values of x because also high-value bidders are subject to shocks and value the possibility of selling after getting shocked. The larger gap between the bid functions at lower valuations accounts for the possibility of selling to a bidder with a higher valuation next period, which, without shocks, is not relevant for bidders with the highest valuations.

B Data

Price Estimate vs Selling Price

Ashenfelter (1989) shows that in his data from wine auctions the correlation between the price estimate published by the auction house and the selling price is very high. This is in line with theoretical results in Milgrom and Weber (1982) that it is in the auctioneer's interest to provide accurate information about the object at auction. The same result is true in my data from classic car auctions.



Following Ashenfelter (1989), the price estimate on the X axis of the left graph is the mid-point of the low and the high estimate published by the auction houses for each auction. The graph is restricted to cars with a predicted price below USD 1 Million. For these cars on the graph the correlation between the price estimate and the selling price is 0.95. For all cars the correlation is 0.97. The right graph is restricted to cars with a low estimate between USD 1 Million.

Regression Classic Car Returns on S&P 500 Returns

The following table shows the regression output corresponding to figure 2.1b. The dependent variable is the return on classic cars calculated as the change in the deflated annual average price on classic cars in Orsi and Gazzi (2017). The definition in the book of a year is from September of that year until the following August. The single explanatory variable is the annual return of the S&P 500 according to the book's definition of a year. The sample is years 1994-2016 (year 1993 being lost in calculating the return).

	Dependent Variable: Return on Classic Cars
Intercept	0.247
	(0.23)
S&P Average Annual Return	0.776
	(0.211)
N	23

(Standard errors in parantheses)

Year Fixed Effects Regression

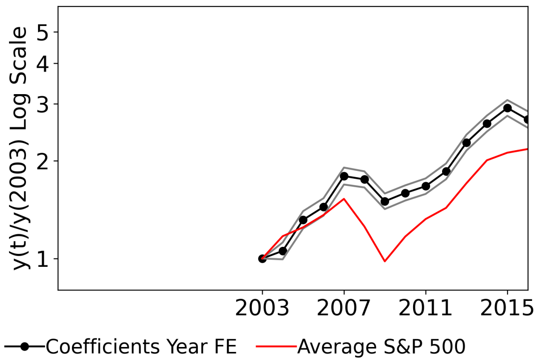
The following table shows the regression output corresponding to figure 2.1c (left column) and 2.1d (columns 2-5). The dependent variable is the log deflated sale price from the auction level data. The explanatory variables are fixed effects for years 2004-2016 and a fixed effect for each model in the data. The sample is all auctions in years 2003-2016 for the left column and all auctions for cars of the respective brand in years 2003-2016 for columns 2-5.

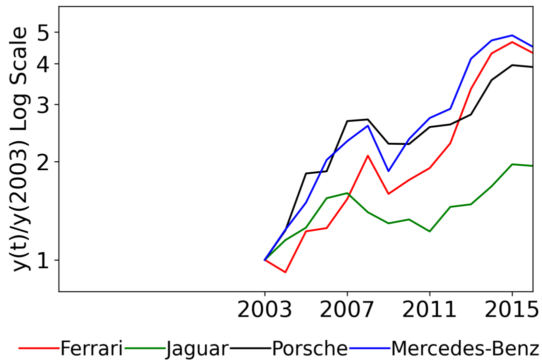
	Dependent Variable: Log Price				
	All	Ferrari	Jaguar	Porsche	Mercedes
Intercept	12.569	9.154	10.159	11.559	10.357
	(0.346)	(0.278)	(0.348)	(0.323)	(0.127)
Auction Year 2004	0.072	0.106	0.238	0.106	0.046
	(0.052)	(0.141)	(0.125)	(0.157)	(0.124)
Auction Year 2005	0.262	0.414	0.296	0.43	0.228
	(0.053)	(0.154)	(0.127)	(0.189)	(0.127)
Auction Year 2006	0.368	0.386	0.477	0.336	0.531
	(0.051)	(0.134)	(0.137)	(0.143)	(0.128)
Auction Year 2007	0.515	0.627	0.521	0.579	0.637
	(0.053)	(0.144)	(0.138)	(0.166)	(0.147)
Auction Year 2008	0.585	0.898	0.627	0.707	0.664
	(0.05)	(0.13)	(0.145)	(0.163)	(0.132)
Auction Year 2009	0.401	0.603	0.328	0.564	0.392
	(0.049)	(0.13)	(0.117)	(0.16)	(0.134)
Auction Year 2010	0.497	0.745	0.358	0.551	0.664
	(0.047)	(0.13)	(0.112)	(0.155)	(0.123)
Auction Year 2011	0.566	0.834	0.387	0.757	0.822
	(0.048)	(0.127)	(0.118)	(0.158)	(0.122)
Auction Year 2012	0.661	1.037	0.444	0.762	0.934
	(0.049)	(0.129)	(0.127)	(0.161)	(0.124)
Auction Year 2013	0.84	1.346	0.455	0.825	1.2
	(0.049)	(0.127)	(0.128)	(0.157)	(0.119)
Auction Year 2014	0.995	1.631	0.639	1.061	1.331
	(0.049)	(0.129)	(0.126)	(0.157)	(0.119)
Auction Year 2015	1.085	1.705	0.787	1.137	1.405
	(0.049)	(0.126)	(0.123)	(0.15)	(0.121)
Auction Year 2016	1.018	1.645	0.726	1.12	1.29
	(0.051)	(0.128)	(0.135)	(0.154)	(0.125)
Model FE	Yes	Yes	Yes	Yes	Yes
N	4554	671	480	477	399

(Standard errors in parantheses)

Regression with model-year-built fixed effect.

The definition of a model in the regressions above is the model of car regardless of the year built. In the below graphs and table are the results for the same regressions but controlling for a model-year-built fixed effect, i.e. defining the model as the model built in a specific year. The results are very similar for all brands together. For the brands separately, for Porsche and Mercedes-Benz the rate of growth changes but the pattern remains very similar.



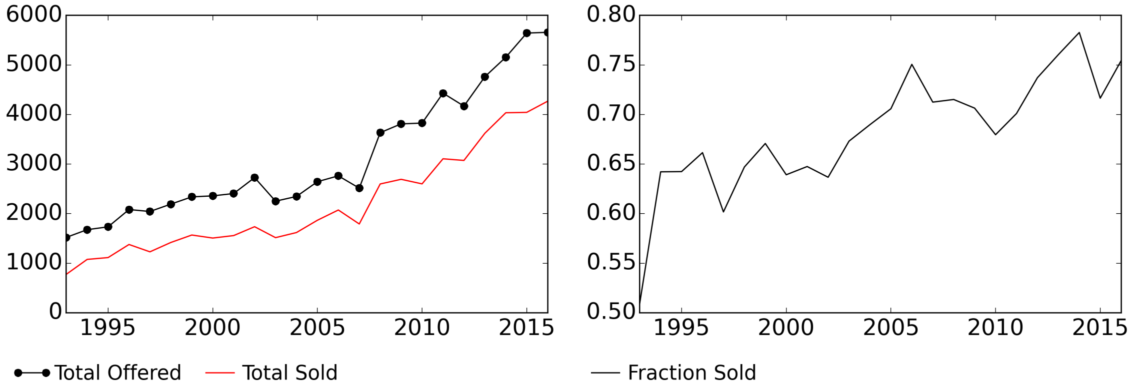


	Dependent Variable: Log Price				
	All	Ferrari	Jaguar	Porsche	Mercedes
Intercept	12.499	9.366	10.228	11.357	10.112
	(0.319)	(0.305)	(0.346)	(0.353)	(0.194)
Auction Year 2004	0.054	-0.088	0.14	0.208	0.213
	(0.06)	(0.167)	(0.149)	(0.188)	(0.147)
Auction Year 2005	0.274	0.202	0.228	0.61	0.405
	(0.062)	(0.202)	(0.157)	(0.243)	(0.145)
Auction Year 2006	0.366	0.224	0.436	0.625	0.705
	(0.062)	(0.175)	(0.18)	(0.206)	(0.158)
Auction Year 2007	0.585	0.43	0.471	0.98	0.839
	(0.06)	(0.18)	(0.166)	(0.222)	(0.17)
Auction Year 2008	0.562	0.736	0.336	0.992	0.948
	(0.057)	(0.165)	(0.181)	(0.216)	(0.165)
Auction Year 2009	0.406	0.466	0.258	0.821	0.627
	(0.055)	(0.164)	(0.134)	(0.218)	(0.171)
Auction Year 2010	0.465	0.565	0.285	0.818	0.854
	(0.054)	(0.165)	(0.127)	(0.2)	(0.154)
Auction Year 2011	0.513	0.649	0.201	0.938	1.002
	(0.055)	(0.16)	(0.139)	(0.201)	(0.157)
Auction Year 2012	0.619	0.824	0.374	0.957	1.067
	(0.056)	(0.164)	(0.145)	(0.208)	(0.155)
Auction Year 2013	0.823	1.208	0.392	1.027	1.421
	(0.056)	(0.159)	(0.147)	(0.202)	(0.157)
Auction Year 2014	0.958	1.459	0.519	1.27	1.55
	(0.057)	(0.162)	(0.148)	(0.207)	(0.157)
Auction Year 2015	1.069	1.539	0.675	1.376	1.586
	(0.056)	(0.16)	(0.14)	(0.201)	(0.158)
Auction Year 2016	0.987	1.462	0.664	1.362	1.506
	(0.059)	(0.162)	(0.156)	(0.203)	(0.161)
Model-Year-built FE	Yes	Yes	Yes	Yes	Yes
N	4554	671	480	477	399

(Standard errors in parantheses)

Total Volume

The following two graphs show the annual number of classic cars offered and sold through auctions, taken from the aggregate data in the Classic Car Auction Yearbook (Orsi and Gazzi, 2017).



C Model

Linear Composite Good Consumption Function

The intertemporal budget constraint in the economy is given by equation (2.2). Ignoring car trade and focusing on composite good consumption $c_{i,t}$, in the special case without additive income ($m_i + \epsilon_{i,t} = 0$), it is straightforward to show that the optimal consumption function is linear, namely $c_{i,t} = \psi y_{i,t-1}$. In this case the intertemporal budget constraint becomes

$$y_{i,t} = (1 + r_t)y_{i,t-1} - c_{i,t}$$
$$y_{i,t} \geq 0$$

Define a new variable $w_{i,t-1} = (1+r_t)y_{i,t-1}$, the agent's problem can then be characterized as

$$V(w_{i,t-1}) = \max_{c_{i,t}, y_{i,t}, w_{i,t}} U(c_{i,t}) + \beta \mathbf{E}[V(w_{i,t})] \text{ s.t. } \begin{aligned} c_{i,t} &= w_{i,t-1} - y_{i,t} \\ w_{i,t-1} &= (1 + r_t)y_{i,t-1} \\ w_{i,0} &\text{ given} \end{aligned}$$

Rewrite:

$$V(w_{i,t-1}) = \max_{y_{i,t}} U(w_{i,t-1} - y_{i,t}) + \beta \mathbf{E}[V((1 + r_{i,t+1})y_{i,t})]$$

This problem admits the following Euler equation:

$$U'(c_{i,t}) = \beta \mathbf{E}[U'(c_{i,t+1})(1 + r_{i,t+1})]$$

Now use $U(c_{i,t}) = \log(c_{i,t})$ and guess $c_{i,t} = \psi w_{i,t-1}$.

This guess implies that $w_{i,t} = (1 + r_{i,t+1})y_{i,t} = (1 + r_{i,t+1})(1 - \psi)w_{i,t-1}$.

The Euler equation becomes:

$$\begin{aligned}
\frac{1}{c_{i,t}} &= \beta\mathbf{E}\left[\frac{1}{c_{i,t+1}}(1 + r_t)\right] \\
\frac{1}{\psi w_{i,t-1}} &= \beta\mathbf{E}\left[\frac{1}{\psi w_{i,t}}(1 + r_t)\right] \\
\frac{1}{\psi w_{i,t-1}} &= \beta\mathbf{E}\left[\frac{(1 + r_t)}{\psi(1 + r_t)(1 - \psi)w_{i,t-1}}\right] \\
\psi &= 1 - \beta
\end{aligned}$$

Hence the guess is verified and we have a linear consumption function.

For the case with additive income, i.e.

$$\begin{aligned}
&y_{i,t} = m_i + (1 + r_t)y_{i,t-1} - c_{i,t} + \epsilon_{i,t} \\
&y_{i,t} \geq 0
\end{aligned}$$

Benhabib et al. (2015) show that the consumption function is *asymptotically* linear (Proposition 5), namely:

$$\lim_{y \to \infty} \frac{c(y)}{y} = \psi$$

I rely on this result and use $c(y) = \psi y$ as a good enough approximation to the exact solution for the model.

Wealth Process

Ignoring car trade, the wealth process 2.2 is a so-called *Kesten Process*, see Benhabib and Bisin (2018): p.17. This is a stationary process with the following stationary distribution:

$$y_i = m_i \cdot \Big(\sum_{t=0}^{\infty}\prod_{k=0}^{t}(1 + r_{t-k} - \psi)\Big) + \sum_{t=0}^{\infty}\Big(\epsilon_{i,t}\prod_{k=0}^{t}(1 + r_{t-k} - \psi)\Big)$$

The first two moments of the aggregate wealth distribution will be as follows:

$$
\begin{aligned}
\mathbf{E}(y_i) &= \mathbf{E}(m_i) \cdot \Big(\sum_{t=0}^{\infty} \prod_{k=0}^{t} \mathbf{E}(r_{t-k}) \Big) + \sum_{t=0}^{\infty} \Big(\mathbf{E}(\epsilon_{i,t}) \prod_{k=0}^{t} \mathbf{E}(r_{t-k}) \Big) \\
&= \mathbf{E}(m_i) \cdot \Big(\sum_{t=0}^{\infty} (\mathbf{E}(r))^t \Big) \\
&= \frac{1}{1-\mathbf{E}(r_t)} \mathbf{E}(m_i) \\
\mathbf{Var}(y_i) &= \mathbf{Var}\Big(m_i \Big(\sum_{t=0}^{\infty} \prod_{k=0}^{t} r_{t-k} \Big) \Big) + \sum_{t=0}^{\infty} \mathbf{Var}\Big(\epsilon_{i,t} \prod_{k=0}^{t} r_{t-k} \Big) \\
&= \mathbf{Var}(m_i) \cdot \Big(\sum_{t=0}^{\infty} \prod_{k=0}^{t} r_{t-k} \Big)^2 + \sum_{t=0}^{\infty} \Big(\mathbf{Var}(\epsilon_{i,t}) \mathbf{E}\Big((\prod_{k=0}^{t} r_{t-k})^2 \Big) \Big) \\
&= \Big(\frac{1}{1-\mathbf{E}(r_t)} \Big)^2 \mathbf{Var}(m_i) + \sum_{t=0}^{\infty} \mathbf{Var}(\epsilon_{i,t}) (\mathbf{E}(r_t^2))^t \\
&= \Big(\frac{1}{1-\mathbf{E}(r_t)} \Big)^2 \mathbf{Var}(m_i) + \frac{1}{1-\mathbf{E}(r_t^2)} \sigma_\epsilon^2
\end{aligned}
$$

The second line of the variance derivation, follows because (i) r_t is constant over i and (ii) because $\mathbf{Var}(XY) = \mathbf{E}(X^2Y^2) - (\mathbf{E}XY)^2 = \mathbf{E}X^2\mathbf{E}Y^2 - (\mathbf{E}X\mathbf{E}Y)^2 = \mathbf{Var}(X)\mathbf{E}(Y^2)$ if X, Y are independent and $\mathbf{E}(X) = 0$.

Calculation Value Functions

Given the intertemporal budget constraint (2.2) and the state transition determined by $r(\bar{y}_t, \bar{y}_{t+1})$, the expectations in (2.6) are calculated as follows:

$$
\begin{aligned}
&\mathbf{E}[B(y', \bar{y}')|y, \bar{y}] \\
&\quad = \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_\epsilon B(m + (1 - \psi + r')y + \epsilon', \bar{y}') dF_\epsilon \\
&\mathbf{E}[\hat{B}_j(y', \bar{y}')|y, \bar{y}] \\
&\quad = \sum_{j=1}^{J} \rho_j \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_\epsilon \hat{B}_j(m + (1 - \psi + r')y + \epsilon', \bar{y}') dF_\epsilon
\end{aligned}
$$

And the expectations in (2.7):

$$
\begin{aligned}
&\mathbf{E}[S_j(y', \bar{y}')|y, \bar{y}]) \\
&\quad = \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_\epsilon S_j(m + (1 - \psi + r')y + \epsilon', \bar{y}') dF_\epsilon \\
&\mathbf{E}[\hat{S}_j(y', \bar{y}')|y, \bar{y}] \\
&\quad = \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_\epsilon \hat{S}_j(m + (1 - \psi + r')y + \epsilon', \bar{y}') dF_\epsilon
\end{aligned}
$$

Solution Algorithm

Initialise $B^{(1)}_{j,s,y}$ and $S^{(1)}_{j,s,y}$. Superscript (1) indicates the first iteration. j, s, y indicate grid-points in the 3-dimensional tensors. The dimensions are the car types, the state mean wealth, and private wealth. I use the following grid for mean wealth:
$(2.6, 3, 3.4, 3.6, 3.8, 4, 4.2, 4.4, 4.8, 5.2, 5.6)$ in USD millions. Private wealth is on a grid from 0.1 to 20.1 in increments of 0.1, all in USD millions.

Next, interpolate over y grid and obtain $B^{(1)}_{j,s}(y)$ and $S^{(1)}_{j,s}(y)$. Initialise beliefs $G^{(1)}_{\bar{b}_j|s}$ and $H^{(1)}_{a_j|s}$.

Then, for $p \in \{1, 2, ..., P\}$:

- Calculate the bid function as follows: $b^{(p)}_{j,s,y} = y - (S^{(p)}_{j,s}(B^{(p)}_{j,s}(y) - w_j/\beta))^{-1}$

- Calculate the reserve price function as follows: $a^{(p)}_{j,s,y} = (B^{(p)}_{j,s}(S^{(p)}_{j,s} + w_j/\beta))^{-1} - y$

- Calculate $\hat{B}_{j,s,y}$ as follows (here and below $\int$ indicates Monte Carlo integration):

$$\hat{B}^{(p)}_{j,s,y} = \int\int 1(b^{(p)}_{j,s,y} > max(\bar{b}, a_j))(w_j + \beta S^{(p)}_{j,s}(y - \bar{b})) \\ + 1(b^{(p)}_{j,s,y} < max(\bar{b}, a_j))\beta B^{(p)}_{j,s}(y) dG^{(p)}_{\bar{b}_j|s} dH^{(p)}_{a_j|s}$$

- Calculate $\hat{S}_{j,s,y}$ as follows:

$$\hat{S}^{(p)}_{j,s,y} = \int 1(a^{(p)}_{j,s,y} < \bar{b})(\beta B^{(p)}_{j,s}(y + \bar{b})) + 1(a^{(p)}_{j,s,y} < \bar{b})(w_j + \beta S^{(p)}_{j,s}(y)) dG^{(p)}_{\bar{b}_j|s}$$

- Interpolate $\hat{B}^{(p)}_{j,s,y}$ and $\hat{S}^{(p)}_{j,s,y}$ to get $\hat{B}^{(p)}_{j,s}(y)$ and $\hat{S}^{(p)}_{j,s,y}(y)$

- Calculate $B^{(p+1)}_{j,s,y}$ as follows:

$$B^{(p+1)}_{j,s,y} = \ln(\psi y) + (1 - \sigma)\beta \sum_{s'} \mathbf{Pr}(r' = r(s, s')) \int_{\epsilon} B^{(p)}_{j,s}(m + (1 - \psi + r')y + \epsilon) dF_{\epsilon} \\ + \sigma \sum_{j} \rho_j \sum_{s'} \mathbf{Pr}(r' = r(s, s')) \int_{\epsilon} \hat{B}^{(p)}_{j,s}(m + (1 - \psi + r')y + \epsilon) dF_{\epsilon}$$

- Calculate $S^{(p+1)}_{j,s,y}$ as follows:

$$S^{(p+1)}_{j,s,y} = \ln(\psi y) + (1 - \rho_j)\Big(w_j + \beta \sum_{s'} \mathbf{Pr}(r' = r(s, s')) \int_{\epsilon} S^{(p)}_{j,s}(m + (1 - \psi + r')y + \epsilon) dF_{\epsilon}\Big) \\ + \rho_j \sum_{s'} \mathbf{Pr}(r' = r(s, s')) \int_{\epsilon} \hat{S}^{(p)}_{j,s}(m + (1 - \psi + r')y + \epsilon) dF_{\epsilon}$$

- Interpolate $B_{j,s,y}^{(p+1)}$ and $S_{j,s,y}^{(p+1)}$ to get $B_{j,s}^{(p+1)}(y)$ and $S_{j,s}^{(p+1)}(y)$.

- Update beliefs:

  Initialise wealth distribution $y^1 = \{y_i^t\}_{i=1}^N$ to empirical distribution. Randomly pick sellers among initially richest agents. Then for $t \in \{1, 2, ..., T\}$:

  - Determine state as mean of y_t
  - Randomly pick active bidders from buyers (all agents who are not sellers)
  - Calculate bids according to $b_{j,s,y}^{(p)}$ and wealth of active bidders
  - Store second highest bid $\bar{b}_j^t$ for each car
  - Pick one random seller for each car
  - Calculate reserve price a_j^t for each car according to $a_{j,s,y}^{(p)}$ and store
  - Find preliminary wealth distribution y^{t+1} by drawing r_t and $\{\epsilon_{i,t}\}_{i=1}^N$
  - If $\bar{b}_j^t > a_j^t$, adjust list of sellers and adjust y^{t+1} of winners and previous sellers by selling prices

  This yields $\{\bar{b}_j^t\}_{t=1}^T$ and $\{a_j^t\}_{t=1}^T$. Calculate state for each t and make list of $\bar{b}_j^t$ and a_j^t for each state. Draw randomly from that list for each state to get $G_{\bar{b}_j|s}^{(p+1)}$ and $H_{a_j|s}^{(p+1)}$.

D Estimation

Wealth

- Sample: Households in the 85th to 99th percentiles in terms of both income and wealth in the SCF (Survey of Consumer Finances)

- m_i: Income in 2001 because wealth in 2001 is closest to average income in the sample period 2003-2016.

- r_t: Average annual return on the S&P500.

- $\epsilon_{i,t}$: For the idiosyncratic shock $\epsilon_{i,t}$ I need an individual mean-zero change over one year. I therefore use the de-meaned change in income from the SCF panel 2007-09 (the only panel there is) and divide by two to calibrate $\epsilon_{i,t}$.

- s: The share of wealth invested in the stock market is chosen to be $s = 0.5$.

- ψ: For the share of wealth spent on the composite good each period, simulate (2.2) for a grid of possible values of ψ and choose this parameter so that it minimises the distance between the average wealth over households and time in the simulation and over the sample period. This yields $\psi = 0.09$.

Prices by State

1. Calculate year fixed effects.

Time subscript t, model subscript i, brand subscript j.

$$P_{i,j,t} = \beta^0_j + \beta^1_{i,j} + \beta^2_{t,j} + \epsilon_{i,j,t}$$

2. Predict prices by year and brand using year-fe.

$$\hat{P}_{i,j,t} = \hat{\beta}^0_j + \hat{\beta}^1_{i,j} + \hat{\beta}^2_{t,j}$$

3. Average by year and brand.

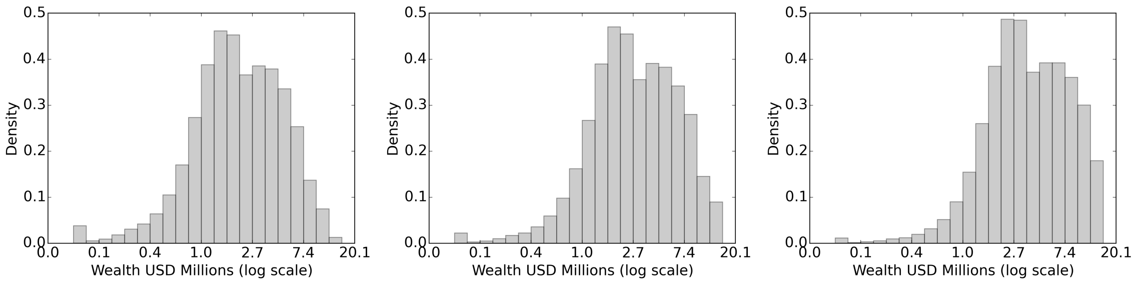
$$\hat{P}^{\text{trend}}_{j,t} = \frac{1}{N_j} \sum_j \hat{P}_{i,j,t}$$

4. De-trend.

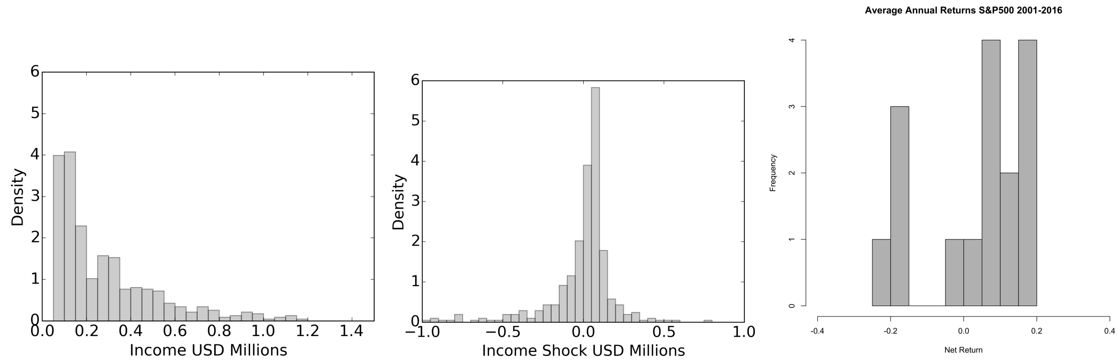
$$\hat{P}^{\text{trend}}_{j,t} - \hat{P}^{\text{trend}}_{j,0} = \beta^{\text{trend}}_j \cdot t + \epsilon_{j,t}$$
$$\hat{P}_{j,t} = \hat{P}^{\text{trend}}_{j,t} - \hat{\beta}^{\text{trend}}_j \cdot t + \frac{1}{T} \sum_t (\hat{\beta}^{\text{trend}}_j \cdot t)$$

5. Match to state by simulated mean wealth over sample period.

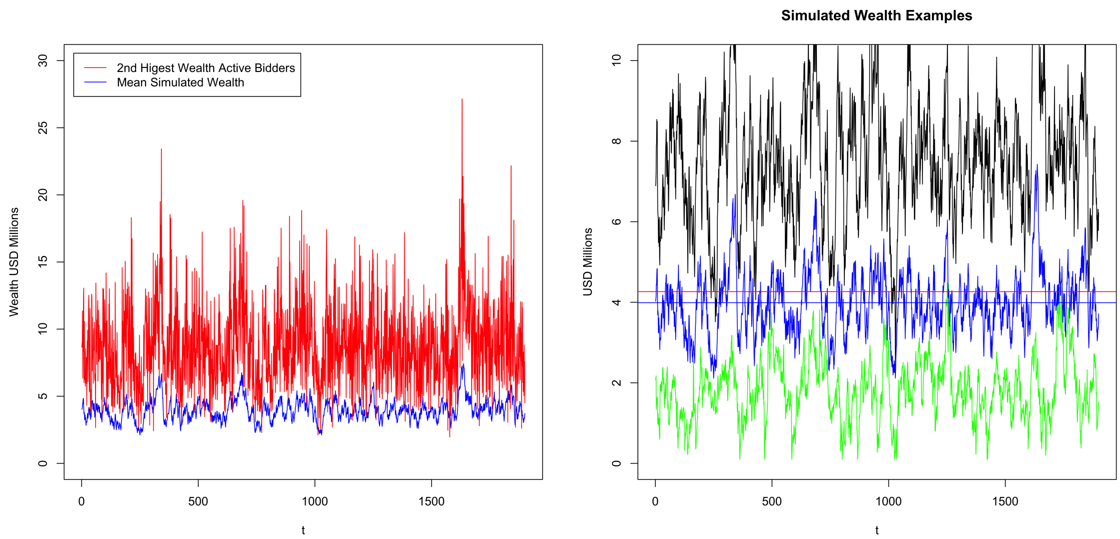
The following graphs show the estimated wealth distribution for mean wealth (state) 3.0, 3.8, and 4.8:



The following graphs graphs plot the income distribution, the distribution of the idiosyncratic shock, and the distribution of the net return:



The following graphs show the output of the wealth simulation, on the left the simulated mean and the second-highest wealth among active bidders, and on the right three examples of simulated individual wealth paths.



D.1 Rescaled Model for Estimation

Let A be the number of auctions per year. Then:

Per-period discount rate $\tilde{\beta} = \beta^{1/A}$.

$\tilde{\psi} = 1 - (1 - \psi)^{1/A}$.

Calculating expectations has to account for probability of wealth staying the same.

$$
\begin{aligned}
&\mathbf{E}[B(y', \bar{y}')|y, \bar{y}] \\
&\quad = \frac{A-1}{A} \cdot B(y, \bar{y}) \\
&\qquad + \frac{1}{A} \cdot \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_{\epsilon} B(m + (1 - \psi + sr')y + \epsilon', \bar{y}') dF_{\epsilon} \\
&\mathbf{E}[\hat{B}_j(y', \bar{y}')|y, \bar{y}] \\
&\quad = \frac{A-1}{A} \cdot \sum_{j=1}^{J} \rho_j \hat{B}_j(y, \bar{y}) \\
&\qquad + \frac{1}{A} \cdot \sum_{j=1}^{J} \rho_j \sum_{\bar{y}' \in \mathcal{S}} \mathbf{Pr}(r' = r(\bar{y}, \bar{y}')) \int_{\epsilon} \hat{B}_j(m + (1 - \psi + sr')y + \epsilon', \bar{y}') dF_{\epsilon}
\end{aligned}
$$

E Counterfactuals

Summary Statistics Predicted Prices

Prices for Ferrari:

State	With Resale		No Resale	
	Mean	SD	Mean	SD
3.0	0.77	0.12	0.69	0.17
3.4	0.86	0.13	0.78	0.19
3.6	0.91	0.15	0.83	0.21
3.8	0.95	0.14	0.87	0.2
4.0	0.99	0.13	0.92	0.21
4.2	1.05	0.16	0.96	0.23
4.4	1.13	0.17	1.03	0.25
4.8	1.21	0.18	1.1	0.25

Estimates assuming no resale

	w_j baseline	w_j no resale
Ferrari	0.133	0.148
	(0.0236)	
Jaguar	0.032	0.037
	(0.0131)	
Porsche	0.028	0.033
	(0.0085)	
Mercedes-Benz	0.038	0.043
	(0.0057)	

(Bootstrapped standard errors in parantheses)

Predicted Prices baseline and centralized trade

	Predicted Price	Predicted Price Centralized Trade
Ferrari	0.99	0.609
	(0.1585)	
Jaguar	0.203	0.162
	(0.0849)	
Porsche	0.175	0.128
	(0.0345)	
Mercedes-Benz	0.258	0.168
	(0.0476)	
Average	0.406	0.267
	(0.0365)	

(Bootstrapped standard errors in parantheses)

F Appendix to Information in Sequential Auctions

F.1 Distributions of Order Statistics

For reference, below are the distributions of the order statistics used below. Let X be a random variable with CDF $F(\cdot)$. Let $X_{(i:n)}$ be the ith largest among n draws from X. Let X_1, X_2, and X_3 be independent draws from X. The distribution of the order statistics used here is then as follows:

$$
\begin{aligned}
F_{X_{(1:2)}}(x) &= \mathbf{Pr}[X_{(1:2)} < x] \\
&= \mathbf{Pr}[X_1 < x \cap X_2 < x] \\
&= F(x)^2 \\
f_{X_{(1:2)}}(x) &= 2F(x)f(x) \\
F_{X_{(2:2)}}(x) &= \mathbf{Pr}[X_{(2:2)} < x] \\
&= \mathbf{Pr}[(X_1 < x \cap X_2 < x) \cup (X_1 < x \cap X_2 > x) \cup (X_1 > x \cap X_2 < x))] \\
&= F_{X_{(1:2)}}(x) + 2\mathbf{Pr}[(X_1 < x \cap X_2 > x)] \\
&= F(x)^2 + 2F(x)(1 - F(x)) \\
f_{X_{(2:2)}}(x) &= 2f(x)(1 - F(x)) \\
F_{X_{(1:2)}X_{(2:2)}}(x, y) &= \mathbf{Pr}[X_{(1:2)} < x \cap X_{(2:2)} < y] \quad x > y \\
&= \mathbf{Pr}[X_{(1:2)} < y] + \mathbf{Pr}[y < X_{(1:2)} < x \cap X_{(2:2)} < y] \\
&= \mathbf{Pr}[X_{(1:2)} < y] + 2\mathbf{Pr}[y < X_1 < x \cap X_2 < y] \\
&= F_{X_{(1:2)}}(y) + 2F(y)(F(x) - F(y)) \\
f_{X_{(1:2)}X_{(2:2)}}(x, y) &= 2f(x)f(y)
\end{aligned}
$$

$$\begin{aligned}
F_{X_{(2:3)}}(x) &= \mathbf{Pr}[X_1 < x \cap X_2 < x \cap X_3 < x] + 3\mathbf{Pr}[X_1 < x \cap X_2 < x \cap X_3 > x] \\
&= F(x)^3 + 3F(x)^2(1 - F(x)) \\
f_{X_{(2:3)}}(x) &= 6F(x)f(x)(1 - F(x)) \\
F_{X_{(3:3)}}(x) &= F_{X_{(2:3)}}(x) + 3\mathbf{Pr}[X_1 < x \cap X_2 > x \cap X_3 > x] \\
&= F(x)^3 + 3F(x)^2(1 - F(x)) + 3F(x)(1 - F(x))^2 \\
f_{X_{(3:3)}}(x) &= 3f(x)(1 - F(x))^2 \\
F_{X_{(2:3)}X_{(3:3)}}(x, y) &= F_{X_{(2:3)}}(y) + \mathbf{Pr}[X_{(3:3)} < y \cap y < x_{(2:3)} < x] \\
&= F_{X_{(2:3)}}(y) + 3\mathbf{Pr}[X_1 < y \cap y < X_{(2:2)} < x] \\
&= F_{X_{(2:3)}}(y) + 3F(y)(F_{X_{(2:2)}}(x) - F_{X_{(2:2)}}(y)) \\
f_{X_{(2:3)}X_{(3:3)}}(x, y) &= 3f(y)f_{X_{(2:2)}}(x) \\
&= 6f(x)f(y)(1 - F(x))
\end{aligned}$$

F.2 Proofs

Proof of Proposition 1

The objective (3.1) can be rewritten as follows:

$$\begin{aligned}
\max_B \int_0^{b_W^{-1}(B)} \int_{x_{(2:2)}} &(x - b_W(x_{(1:2)}))f_{X_{(1:2)}X_{(2:2)}}(x_{(1:2)}, x_{(2:2)})dx_{(2:2)}dx_{(1:2)} \\
&+ \int_{b_W^{-1}(B)}^{\bar{x}} \int_0^x (x - x_{(2:2)})f_{X_{(1:2)}X_{(2:2)}}(x_{(1:2)}, x_{(2:2)})dx_{(2:2)}dx_{(1:2)} \quad \text{(F.12)}
\end{aligned}$$

First order condition:

$$\begin{aligned}
\int_{x_{(2:2)}} &\left(x - b_W(b_W^{-1}(B))\right) f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = b_W^{-1}(B), x_{(2:2)})dx_{(2:2)}\frac{1}{b'_W(b_W^{-1}(B))} \\
&- \int_0^x \left(x - x_{(2:2)}\right) f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = b_W^{-1}(B), x_{(2:2)})dx_{(2:2)}\frac{1}{b'_W(b_W^{-1}(B))} = 0
\end{aligned}$$

Using that in equilibrium $B = b_W(x)$,

$$\begin{aligned}
(x - b_W(x)) \int_{x_{(2:2)}} &f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = x, x_{(2:2)})dx_{(2:2)} \\
&- \int_0^x \left(x - x_{(2:2)}\right) f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = x, x_{(2:2)})dx_{(2:2)} = 0
\end{aligned}$$

Then:

$$(x - b_W(x)) f_{X_{(1:2)}}(X_{(1:2)} = x) = \int_0^x \left(x - x_{(2:2)}\right) f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = x, x_{(2:2)}) dx_{(2:2)}$$

Solve for $b_W(x)$:

$$\begin{aligned} b_W(x) &= x - \int_0^x \left(x - x_{(2:2)}\right) \frac{f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = x, x_{(2:2)})}{f_{X_{(1:2)}}(X_{(1:2)} = x)} dx_{(2:2)} \\ &= \mathbf{E}[X_{(2:2)} | X_{(1:2)} = x] \end{aligned} \tag{F.13}$$

Finally:

$$\begin{aligned} b_W(x) &= \mathbf{E}[X_{(2:2)} | X_{(1:2)} = x] \\ &= \int_0^x x' \frac{f_{X_{(1:2)}X_{(2:2)}}(X_{(1:2)} = x, x')}{f_{X_{(1:2)}}(X_{(1:2)} = x)} dx' \\ &= \int_0^x x' \frac{2f(x)f(x')}{2F(x)f(x)} dx' \\ &= \int_0^x x' \frac{f(x')}{F(x)} dx' \\ &= \mathbf{E}[X | X < x] \end{aligned} \tag{F.14}$$

Monotonicity of $b_W(x)$ is thus confirmed. Anticipating the equilibrium in which all bidders bid $b_2(x) = x$ in the second period, the first period thus has a unique equilibrium in which all bidders play $b_W(x)$. Since $b_2(x)$ is the unique symmetric equilibrium in the second period, $(b_W(x), b_2(x))$ is the unique symmetric subgame perfect equilibrium of this game.

Proof of Proposition 2

Using independence of X_1 and X_2, the objective (3.3) can be rewritten as follows:

$$\begin{aligned} &\max_B \int_0^{b_S^{-1}(B)} (x - b_S(x_1)) f(x_1) dx_1 \cdot \int_{x_2} f(x_2) dx_2 + \int_{b_S^{-1}(B)}^{\bar{x}} f(x_1) dx_1 \cdot \int_0^x (x - x_2) f(x_2) dx_2 \\ =&\max_B \int_0^{b_S^{-1}(B)} (x - b_S(x_1)) f(x_1) dx_1 + \left(1 - F(b_S^{-1}(B))\right) \int_0^x (x - x_2) f(x_2) dx_2 \end{aligned}$$

First order condition:

$$(x - b_S(b_S^{-1}(B))) f(b_S^{-1}(B)) \frac{1}{b_S'(b_S^{-1}(B))} - f(b_S^{-1}(B)) \frac{1}{b_S'(b_S^{-1}(B))} \int_0^x (x - x_2) f(x_2) dx_2 = 0$$

Simplify:

$$x - B = \int_0^x (x - x_2) f(x_2) dx_2$$

In equilibrium, $B = b_S(x)$, and so:

$$\begin{aligned}
b_S(x) &= x - \int_0^x (x - x_2) f(x_2) dx_2 \\
&= x - F(x) \int_0^x (x - x_2) \frac{f(x_2)}{F(x)} dx_2 \\
&= x - F(x)(x - \mathbf{E}[X|X < x]) \qquad \text{(F.15)}
\end{aligned}$$

Monotonicity of $b_S(x)$ is thus confirmed. Anticipating the equilibrium in which all bidders bid $b_2(x) = x$ in the second period, the first period thus has a unique equilibrium in which all bidders play $b_S(x)$. Since $b_2(x)$ is the unique symmetric equilibrium in the second period, $(b_S(x), b_2(x))$ is the unique symmetric subgame perfect equilibrium of this game.

Proof of Proposition 3

Suppose the equilibrium holds at $t + 1$ and that there is an increasing equilibrium in all periods. The objective in the Weber model at t is then as follows:

$$\begin{aligned}
&\max_B \int_{x_{(t:M)}} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \\
&\quad \Big(1_{B > b_t(x_{(t:M)})} (x - b_t(x_{(t:M)})) + 1_{B < b_t(x_{(t:M)})} 1_{x > x_{(t+1:M)}} (x - b_{t+1}(x_{(t+1:M)})) + ... \\
&\qquad + 1_{B < b_t(x_{(t:M)})} 1_{x < x_{(t+1:M)}} ... 1_{x > x_{(M:M)}} (x - x_{(M:M)}) \Big) \\
&\qquad f_{X_{(t:M)} X_{(t+1:M)} ... X_{(M:M)}} (x_{(t:M)} x_{(t+1:M)} ... x_{(M:M)}) dx_{(t:M)} x_{(t+1:M)} ... x_{(M:M)}
\end{aligned}$$

Rewrite:

$$\begin{aligned}
&\max_B \int_0^{b^{-1}(B)} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} (x - b_t(x_{(t:M)})) \\
&\qquad f_{X_{(t:M)} X_{(t+1:M)} ... X_{(M:M)}} (x_{(t:M)} x_{(t+1:M)} ... x_{(M:M)}) dx_{(t:M)} x_{(t+1:M)} ... x_{(M:M)} \\
&\qquad + \int_{b^{-1}(B)}^{\bar{x}} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x > x_{(t+1:M)}} (x - b_{t+1}(x_{(t+1:M)})) + ... \\
&\qquad\qquad + 1_{x < x_{(t+1:M)}} ... 1_{x > x_{(M:M)}} (x - x_{(M:M)}) \Big) \\
&\qquad f_{X_{(t:M)} X_{(t+1:M)} ... X_{(M:M)}} (x_{(t:M)} x_{(t+1:M)} ... x_{(M:M)}) dx_{(t:M)} x_{(t+1:M)} ... x_{(M:M)}
\end{aligned}$$

FOC:

$$0 = \frac{1}{b_W'(b_t^{-1}(B))} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} (x - b_t(b_t^{-1}(B)))$$

$$f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = b_t^{-1}(B), x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

$$- \frac{1}{b_W'(b_W^{-1}(B))} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x>x_{(t+1:M)}}(x - b_{t+1}(x_{(t+1:M)})) + ...$$

$$+ 1_{x<x_{(t+1:M)}}...1_{x>x_{(M:M)}}(x - x_{(M:M)})\Big)$$

$$f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = b_t^{-1}(B), x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

In equilibrium $B = b_t(x)$:

$$(x - b_t(x))\cdot$$

$$\int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = x, x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

$$= \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x>x_{(t+1:M)}}(x - b_{t+1}(x_{(t+1:M)})) + ...$$

$$+ 1_{x<x_{(t+1:M)}}...1_{x>x_{(M:M)}}(x - x_{(M:M)})\Big)$$

$$f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = x, x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

Then:

$$(x - b_t(x))f_{X_{(t:M)}}(x) = \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x>x_{(t+1:M)}}(x - b_{t+1}(x_{(t+1:M)})) + ...$$

$$+ 1_{x<x_{(t+1:M)}}...1_{x>x_{(M:M)}}(x - x_{(M:M)})\Big)$$

$$f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = x, x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

Note that

$$\frac{f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = x, x_{(t+1:M)}, ..., x_{(M:M)})}{f_{X_{(t:M)}}(x)}$$

$$= f_{X_{(t+1:M)}...X_{(M:M)}|X_{(t:M)}=x}(x_{(t+1:M)}, ..., x_{(M:M)})$$

Conditional on $X_{(t:M)} = x$, $1_{x<x_{(t+1:M)}} = 0$ and so inside the integral above all summation

terms but the first one drop out:

$$x - b_t(x) = \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(x - b_{t+1}(x_{(t+1:M)})\Big)$$

$$f_{X_{(t+1:M)}...X_{(M:M)}|X_{(t:M)}=x}(x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}$$

$$= \mathbf{E}[x - b_{t+1}(X_{(t+1:M)})|X_{(t:M)} = x]$$

$$= x - \mathbf{E}[\mathbf{E}[X_{M:M}|X_{t+1:M}]|X_{(t:M)} = x]$$

$$= x - \mathbf{E}[X_{M:M}|X_{(t:M)} = x]$$

Monotonicity is thus verified. The unique symmetric subgame perfect equilibrium is for all players to play $\mathbf{E}[X_{M:M}|X_{(t:M)} = x]$ in all periods $t = 1, ..., M$.

Proof of Proposition 4

Conjecture that all bidders play an increasing bid function in every period (verify later). A continuing bidder will face an entering bidder next period whose value is random variable X. Suppose the equilibrium holds at $t = M - s + 1$ (proof by backward induction) and that the entering bidder at $t = M - s$ plays the bid function $b_t(\cdot)$. The value of continuing from period $t = M - s - 1$ into period $t = M - s$ is then as follows:

$$V_{M-s-1}(x) = \max_B \int_0^{b^{-1}_{M-s}(B)} (x - b_{M-s}(x_1))f(x_1)dx_1 + (1 - F(b^{-1}_{M-s}(B)))V_{M-s}(x) \quad \text{(F.16)}$$

First order condition for this problem:

$$(x - b_{M-s}(b^{-1}_{M-s}(B)))f(b^{-1}_{M-s}(B))\frac{1}{b'_{M-s}(b^{-1}_{M-s}(B))} - f(b^{-1}_{M-s}(B))\frac{1}{b'_{M-s}(b^{-1}_{M-s}(B))}V_{M-s}(x) = 0 \quad \text{(F.17)}$$

Solving and setting $B = b_{M-s}(x)$:

$$b_{M-s}(x) = x - V_{M-s}(x) \quad \text{(F.18)}$$

The entering bidder solves a similar but not identical objective. The entering bidder will face a continuing player coming into auction t. Denote the value of the continuing bidder at t by X^t_c, which follows the distribution of the order statistic $X_{(t:t)}$. The problem therefore becomes:

$$\max_B \int_0^{b^{-1}_{M-s}(B)} (x - b_{M-s}(x^t_c))f_{X_{(t:t)}}(x^t_c)dx^t_c + (1 - F_{X_{(t:t)}}(b^{-1}_{M-s}(B)))V_{M-s}(x) \quad \text{(F.19)}$$

As can be seen from the first order condition (F.17), (F.18) is going to solve (F.19) also. Since we know from Proposition 2 that the proposed bid function is the equilibrium strategy at $t = M - 1$, it now follows by backward induction that it is also going to be the equilibrium strategy for all $t = M-1, M-2, ..., 1$. Plugging (F.18) into (F.16) yields (3.9). Since $V_t(x) < x$, the bid function is increasing in every period. The unique symmetric subgame perfect equilibrium is for all players to play $x - V_t(x)$ in all periods $t = 1, ..., M$.

Proof of proposition 6:

Weber Objective with discount factor β:

$$\begin{aligned}
\underset{B}{max} \int_0^{b^{-1}(B)} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} & (x - b_t(x_{(t:M)})) \\
& f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)}x_{(t+1:M)}...x_{(M:M)})dx_{(t:M)}x_{(t+1:M)}...x_{(M:M)} \\
& + \int_{b^{-1}(B)}^{\bar{x}} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x > x_{(t+1:M)}} \beta(x - b_{t+1}(x_{(t+1:M)})) + ... \\
& \qquad + 1_{x < x_{(t+1:M)}} ... 1_{x > x_{(M:M)}} \beta^{M-t}(x - x_{(M:M)})\Big) \\
& \qquad f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)}x_{(t+1:M)}...x_{(M:M)})dx_{(t:M)}x_{(t+1:M)}...x_{(M:M)}
\end{aligned}$$

FOC:

$$\begin{aligned}
0 = \frac{1}{b'_W(b_t^{-1}(B))} & \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} (x - b_t(b_t^{-1}(B))) \\
& f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = b_t^{-1}(B), x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)} \\
& - \frac{1}{b'_W(b_W^{-1}(B))} \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \Big(1_{x > x_{(t+1:M)}} \beta(x - b_{t+1}(x_{(t+1:M)})) + ... \\
& \qquad + 1_{x < x_{(t+1:M)}} ... 1_{x > x_{(M:M)}} \beta^{M-t}(x - x_{(M:M)})\Big) \\
& f_{X_{(t:M)}X_{(t+1:M)}...X_{(M:M)}}(x_{(t:M)} = b_t^{-1}(B), x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)}
\end{aligned}$$

For the same reasons as above this reduces to the following:

$$\begin{aligned}
x - b_t(x) = & \int_{x_{(t+1:M)}} ... \int_{x_{(M:M)}} \beta\Big(x - b_{t+1}(x_{(t+1:M)})\Big) \\
& f_{X_{(t+1:M)}...X_{(M:M)}|X_{(t:M)}=x}(x_{(t+1:M)}, ..., x_{(M:M)})dx_{(t+1:M)}...x_{(M:M)} \\
=& \mathbf{E}[\beta(x - b_{t+1}(X_{(t+1:M)}))|X_{(t:M)} = x] \\
=& \beta(x - \mathbf{E}[\mathbf{E}[X_{M:M}|X_{t+1:M}]|X_{(t:M)} = x]) \\
=& \beta(x - \mathbf{E}[X_{M:M}|X_{(t:M)} = x])
\end{aligned}$$

Proof of Proposition 7

As before, the value of the continuing player coming into auction t, denoted by X_c^t, follows the distribution of the order statistic $X_{(t:t)}$. Suppose the equilibrium holds at $t = M-s+1$ (proof by backward induction) and that the continuing bidder at $t = M-s$ plays the bid function $b_t(\cdot)$. The entering bidder then solves the following objective:

$$\max_B \int_0^{b_{M-s}^{-1}(B)} (x - b_{M-s}(x_c^t)) f_{X_{(t:t)}}(x_c^t) dx_c^t + \beta(1 - F_{X_{(t:t)}}(b_{M-s}^{-1}(B))) V_{M-s}(x)$$

The continuing bidder solves the following objective:

$$\max_B \int_0^{b_{M-s}^{-1}(B)} (x - b_{M-s}(x_1)) f(x_1) dx_1 + \beta(1 - F(b_{M-s}^{-1}(B))) V_{M-s}(x)$$

Following the same steps as above, both of these are maximised at:

$$b_{M-s}(x) = x - \beta V_{M-s}(x)$$

F.3 Prices in the two-period case

Second-period price

Proposition 8. *In both the Weber and the stationary model the expected second-period price P_2 equals the expected value of $X_{(3:3)}$, the lowest among three draws from X:*

$$\boldsymbol{E}[P_2] = \boldsymbol{E}[X_{(3:3)}] \tag{F.20}$$

Proof. In both models the unique symmetric equilibrium in the second period is for both active players to submit a bid equal to $b_2(x) = x$. The bidder with valuation $X_{(3:3)}$, the lowest among all three players' valuations, will be present in the second auction in both models and will submit the lower of the two second-period bids. The price in the second price auction will therefore be determined by this bidder. Therefore, $\mathbf{E}[P_2] = \mathbf{E}[b_2(X_{(3:3)})]$, and since $b_2(x) = x$, the proposition follows.

First-period price in the Weber model

Proposition 9. *In the Weber model the expected first-period price P_W equals the expected value of $X_{(3:3)}$, the lowest among three draws from X:*

$$\boldsymbol{E}[P_W] = \boldsymbol{E}[X_{(3:3)}] \tag{F.21}$$

Proof. In the Weber model all three bidders are active in the first auction. Therefore the first period price P_W is determined by the bidder with the second highest valuation from three draws, i.e. by $X_{(2:3)}$. Using the equilibrium bid function $b_W(x)$, the proposition follows:

$$\begin{aligned}
\mathbf{E}[P_W] &= \mathbf{E}[b_W(X_{(2:3)})] \\
&= \mathbf{E}_{X_{(2:3)}}[\mathbf{E}[X|X < X_{(2:3)}]] \\
&= \mathbf{E}[X_{(3:3)}]
\end{aligned}$$

First-Period Price in the Stationary Model

Proposition 10. *In the stationary model the expected first-period price P_S equals the expected value of $X_{(3:3)}$, the lowest among three draws from X:*

$$\boldsymbol{E}[P_S] = \boldsymbol{E}[X_{(3:3)}] \tag{F.22}$$

Proof. In the stationary model two randomly drawn bidders are present in the first auction and so the first-period price is determined by the bidder with lower valuation among two draws, i.e. by $X_{(2:2)}$. Using the equilibrium bid function $b_S(x)$ and the distributions of the order statistic $X_{(2:2)}$, the proposition follows:

$$\begin{aligned}
\mathbf{E}[P_S] &= \mathbf{E}[b_S(X_{(2:2)})] \\
&= \int_0^{\bar{x}} (x - F(x)(x - \mathbf{E}[X|X < x])) f_{X_{(2:2)}}(x)dx \\
&= \int_0^{\bar{x}} \mathbf{E}[X|X < x]F(x) f_{X_{(2:2)}}(x)dx + \int_0^{\bar{x}} x(1 - F(x)) f_{X_{(2:2)}}(x)dx \\
&= \frac{1}{3}\int_0^{\bar{x}} \mathbf{E}[X|X < x] f_{X_{(2:3)}}(x)dx + \frac{2}{3}\int_0^{\bar{x}} x f_{X_{(3:3)}}(x)dx \\
&= \frac{1}{3}\mathbf{E}_{X_{(2:3)}}[\mathbf{E}[X|X < X_{(2:3)}]] + \frac{2}{3}\mathbf{E}[X_{(3:3)}] \\
&= \mathbf{E}[X_{(3:3)}]
\end{aligned}$$

Where the fourth line follows from the fact that $F(x)f_{X_{(2:2)}}(x) = \frac{1}{3}f_{X_{(2:3)}}(x)$ and $(1 - F(x))f_{X_{(2:2)}}(x) = \frac{2}{3}f_{X_{(3:3)}}(x)$, see distributions in F.1.

F.4 Bid Functions for Uniformly Distributed Values

For $X \sim \mathcal{U}[0, 1]$, the bid functions in the Weber model are as follows:

$b_M = x$,

$b_{M-1} = \frac{1}{2}x,$
$b_{M-2} = \frac{1}{3}x,$
$b_{M-3} = \frac{1}{4}x,$
$b_{M-4} = \frac{1}{5}x,$
$b_{M-5} = \frac{1}{6}x.$

For the stationary model they are:

$b_M = x,$
$b_{M-1} = x - \frac{1}{2}x^2,$
$b_{M-2} = x - x^2 + \frac{1}{3}x^3,$
$b_{M-3} = x - \frac{3}{2}x^2 + x^3 - \frac{1}{4}x^4,$
$b_{M-4} = x - 2x^2 + 2x^3 - x^4 + \frac{1}{5}x^5,$
$b_{M-5} = x - \frac{5}{2}x^2 = \frac{10}{3}x^3 - \frac{5}{2}x^4 + x^5 - \frac{1}{6}x^6.$

See figure 3.2.

F.5 Alternative Derivation of Equilibrium

Weber Model

The equilibrium stated in Weber (1983) theorem 3b is the following:

$$b_W(x) = \mathbf{E}[X_{(3:3)}|X_{(2:3)} = x]$$

Note first of all that this is equal to the bid function derived above, since similarly to the second equality in (3.2), it turns out that:

$$\begin{aligned}
\mathbf{E}[X_{(3:3)}|X_{(2:3)} = x] &= \int_0^x x' \frac{f_{X_{(3:3)}X_{(2:3)}}(X_{(1:2)} = x, x')}{f_{X_{(2:3)}}(X_{(2:3)} = x)} dx' \\
&= \int_0^x x' \frac{6f(x)f(x')(1 - F(x))}{6F(x)f(x)(1 - F(x))} dx' \\
&= \int_0^x x' \frac{f(x')}{F(x)} dx' \\
&= \mathbf{E}[X|X < x]
\end{aligned}$$

The following derives this expression of the equilibrium assuming values are uniformly distributed on $[0, 1]$.

Conjecture a monotone equilibrium bid function $b_W(x)$ at $t = 1$. At $t = 2$ the remaining active bidders will then have values $X_{(2:3)}$ and $X_{(3:3)}$. The bidder with value $X_{(2:3)}$ will win the auction and pay $X_{(3:3)}$ (since both players bid their respective value at $t = 2$). If the equilibrium winner of round two were instead to buy at $t = 1$, the value of doing so would be not having to pay the expected price at $t = 2$, i.e. $\mathbf{E}[X_{(3:3)}|X_{(2:3)} = x]$. The equilibrium bid at $t = 1$ is equal to this value:

$$\begin{aligned} b_W(x) &= \mathbf{E}[X_{(3:3)}|X_{(2:3)} = x] \\ &= \frac{1}{2}x \end{aligned}$$

Where the second line follows from assuming a uniform distribution of values on $[0, 1]$.

Consider a bid $b'(x) < b_W(x)$. This deviation from the equilibrium bid will make a difference to payoffs only if x is the largest among three draws from X, i.e. if $x = X_{(1:3)}$, and if $b'(x) < b_W(X_{(2:3)})$. In equilibrium the highest-value bidder wins the first auction and receives payoff x minus the bid of the second-highest value bidder:

$$\begin{aligned} EU(x = X_{(1:3)}) &= x - \mathbf{E}_{X_{(2:3)}}[b_1(x_{(2:3)})|X_{(1:3)} = x] \\ &= x - \mathbf{E}_{X_{(2:3)}}[\mathbf{E}[X_{(3:3)}|X_{(2:3)}]|X_{(1:3)} = x] \\ &= x - \mathbf{E}[X_{(3:3)}|X_{(1:3)} = x] \end{aligned}$$

Where the third equality follows from the law of iterated expectation. Deviating to a low enough bid results in the bidder with $x = X_{(1:3)}$ winning the second period and paying the bid of the bidder with the third largest valuation:

$$EU'(x = X_{(1:3)}) = x - \mathbf{E}[X_{(3:3)}|X_{(1:3)} = x]$$

Hence the expected payoff from deviating to any bid $b'(x) < b_1(x)$ is the same as the equilibrium payoff in all scenarios.

Consider a bid $b''(x) > b_W(x)$. This will make a difference to payoffs only if $x = X_{(2:3)}$ or $x = X_{(3:3)}$ and if $b''(x) > b_W(X_{(1:3)})$. In equilibrium the bidder with $x = X_{(2:3)}$ wins at $t = 2$ and their expected payoff is:

$$\begin{aligned} EU(x = X_{(2:3)}) &= x - \mathbf{E}[X_{(3:3)}|X_{(2:3)} = x] \\ &= \frac{1}{2}x \end{aligned}$$

The bidder with $x = X_{(3:3)}$ on the other hand in equilibrium wins neither auction in equilibrium:

$$EU(x = X_{(3:3)}) = 0$$

Now consider the payoffs from instead outbidding the highest-value bidder in the first period.

$$\begin{aligned} EU''(x = X_{(2:3)}) &= x - \mathbf{E}[b_1(x_{(1:3)})|X_{(2:3)} = x] \\ &= x - \mathbf{E}[\frac{1}{2}x_{(1:3)}|X_{(2:3)} = x] \\ &= x - \frac{1}{2}(\frac{1}{2} + \frac{1}{2}x) \\ &= \frac{3}{4}x - \frac{1}{4} \end{aligned}$$

Note that $EU''(x = X_{(2:3)}) < EU(x = X_{(2:3)})\ \forall x < 1$. The lowest-value bidder could however sometimes gain from deviating:

$$\begin{aligned} EU''(x = X_{(3:3)}) &= x - \mathbf{E}[b_1(X_{(1:3)})|X_{(3:3)} = x] \\ &= x - \frac{1}{2}(\frac{2}{3} + \frac{1}{3}x) \\ &= \frac{5}{6}x - \frac{1}{3} \end{aligned}$$

The relevant tradeoff is whether a deviation has higher payoff *in expectation*:

$$\begin{aligned} EU''(x) - EU(x) &= \mathbf{Pr}[x = X_{(2:3)}|x](\frac{3}{4}x - \frac{1}{4} - \frac{1}{2}x) + \mathbf{Pr}[x = X_{(3:3)}|x](\frac{5}{6}x - \frac{1}{3}) \\ &= 2x(1-x)\frac{1}{4}(x-1) + (1-x)^2(\frac{5}{6}x - \frac{1}{3}) \\ &= \frac{1}{3}(x-1)^3 \\ &< 0\ \ \forall x < 1 \end{aligned}$$

Hence deviating to a bid $b''(x) > b_W(x)$ is never profitable in expectation. $b_W(x) = \frac{1}{2}x$ and $b_2(x) = x$ is an SPNE of this game (and monotonicity is confirmed).

Stationary Model

In the stationary model the loser from the first round faces a new bidder in the second round who has not been active before. The valuation of the new entrant is drawn from X. Assume again that $X \sim \mathcal{U}[0, 1]$. The realised value of the entrant may be above or below the valuation of the continuing bidder. When trading off buying at $t = 1$

against continuing, bidders have therefore to take into account the probability of facing a competitor with a higher valuation at $t = 2$. The value of buying at $t = 1$ is then the valuation of the object minus the expected value of continuing into the second period. Bidders bid this value:

$$\begin{aligned} b_S(x) &= x - Pr[X < x](x - \mathbf{E}[X|X < x]) \\ &= x - x \cdot (x - \frac{1}{2}x) \\ &= x - \frac{1}{2}x^2 \end{aligned}$$

Consider a deviation to a bid $b'(x) < b_S(x)$ when other bidders are playing the equilibrium strategy. This will make a difference to payoffs only if the bidder has the higher value among the two bidders active in the first auction, i.e. if $x = X_{(1:2)}$. The equilibrium payoff to the highest value bidder is:

$$\begin{aligned} EU(x = X_{(1:2)}) &= x - \mathbf{E}[b_1(X_{2:2})|X_{(1:2)} = x] \\ &= x - \mathbf{E}[X_{(2:2)} - \frac{1}{2}X^2_{(2:2)}|X_{(1:2)} = x] \\ &= x - \mathbf{E}[X_{(2:2)}|X_{(1:2)} = x] + \frac{1}{2}\mathbf{E}[X^2_{(2:2)}|X_{(1:2)} = x] \\ &= x - \frac{1}{2}x + \frac{1}{2}\int_0^x z^2 f_{X_{(2:2)}|X_{(1:2)}=x}(z)dz \\ &= \frac{1}{2}x + \frac{1}{2}\int_0^x z^2 \frac{1}{x} dz = \frac{1}{2}x + \frac{1}{2}\frac{1}{x}\frac{1}{3}x^3 \\ &= \frac{1}{2}x + \frac{1}{6}x^2 \end{aligned}$$

If by deviating to a lower bid the highest-value bidder from $t = 1$ continues into the second period, the new entrant may have a higher or a lower value. The expected payoff from deviating is thus:

$$\begin{aligned} EU'(x = X_{(1:2)}) &= Pr[X < x](x - \mathbf{E}[X|X < x]) \\ &= \frac{1}{2}x^2 \end{aligned}$$

Note that having had the highest draw of valuations in the first period does not provide the bidder with any information about their probability of winning in the second period. Note also that for all $x < 1$, $EU(x = X_{(1:2)}) > EU'(x = X_{(1:2)})$. Hence this is never a profitable deviation.

Consider a deviation to a bid $b''(x) > b_S(x)$ when the other bidders are playing the strategy $b_S(x)$. Such a deviation will change payoffs only if $x = X_{(2:2)}$. This bidder will in equilibrium continue to the second period and have the following expected payoff:

$$\begin{aligned} EU(x = X_{(2:2)}) &= Pr[X < x](x - \mathbf{E}[X|X < x]) \\ &= \frac{1}{2}x^2 \end{aligned}$$

If deviating to outbid the bidder with the higher valuation, the payoff will be:

$$\begin{aligned} EU''(x = X_{(2:2)}) &= x - \mathbf{E}[b_1(X_{(1:2)})|X_{(2:2)} = x] \\ &= x - \mathbf{E}[X_{(2:2)} - \frac{1}{2}X^2_{(2:2)}|X_{(2:2)} = x] \\ &= x - \mathbf{E}[X_{(2:2)}|X_{(2:2)} = x] + \frac{1}{2}\mathbf{E}[X^2_{(2:2)}|X_{(2:2)} = x] \\ &= x - (\frac{1}{2} + \frac{1}{2}x) + \frac{1}{2}\int_x^1 z^2 f_{X_{(1:2)}|X_{(2:2)}=x}(z)dz \\ &= \frac{1}{2}x - \frac{1}{2} + \frac{1}{2}\int_x^1 z^2 \frac{1}{1-x}dz \\ &= \frac{1}{2}x - \frac{1}{2} + \frac{1}{2}\frac{1}{1-x}(\frac{1}{3} - \frac{1}{3x^3}) \\ &= \frac{2}{3}x - \frac{1}{3} + \frac{1}{6}x^2 \end{aligned}$$

Note:

$$\begin{aligned} EU''(x = X_{(2:2)}) - EU(x = X_{(2:2)}) &= \frac{2}{3}x - \frac{1}{3} + \frac{1}{6}x^2 - \frac{1}{2}x^2 \\ &= \frac{2}{3}x - \frac{1}{3} - \frac{1}{3}x^2 \\ &= (-\frac{1}{3})(1 - 2x + x^2) \\ &= (-\frac{1}{3})(1 - x)^2 < 0 \end{aligned}$$

Hence $b_S(x) = x - \frac{1}{2}x^2$ and $b_2(x) = x$ is an SPNE of this game (and monotonicity is confirmed).