The London School of Economics and Political Science

# Essays on Financial Externalities

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# Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it). The copyright of this thesis rests with the author. Quotation from it is permitted, provided that full acknowledgement is made.
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# Statement of co-authored work

I confirm that Chapter 3 was jointly co-authored with Andrew Gimber and Professor David Miles and I contributed 33% of this work.

# Abstract

This thesis explores some of the trade-offs faced by policymakers in trying to prevent or moderate the impact of financial externalities generating instability in the macroeconomy.

The first chapter explores the role of cashflow constraints combined with lower equilibrium interest rates in inducing less productive firms (*zombies*) to invest and produce. Zombie firms generate a negative spillover on the borrowing capacity of more productive firms: by demanding capital they contribute to raising wages, reducing the value of profits for all firms and further tightening the borrowing constraint of productive firms. If the interest rate hits the effective lower bound however, aggregate demand is low, fewer low-productivity firms invest and liquidating zombie firms can be counterproductive. At the lower bound, these firms are not zombies but make use of idle resources, boosting output and welfare.

The second chapter focuses on the key role played by collateral assets in determining the distribution of productive capital across heterogeneous producers, as well as in inducing business cycle amplifications. From a prudential point of view, moderating firms' access to credit is helpful in avoiding fire sale externalities in a financial crisis; however, this impairs the distribution of productive capital. From a normative perspective, the use of capital requirements can help implementing the constrained efficient allocation, provided that the regulator has the ability to commit to future policies.

The last chapter is a co-authored work that explores the link between aggregate demand externalities and housing tenure choices. Shocks that induce households to deleverage sharply can push the policy rate at its lower bound, at which point output is demand-determined. Restricting access to mortgages is therefore beneficial in preventing this externality; however, it distorts housing choices when households have a preference for owning as opposed to renting. Macroprudential interventions have important distributional consequences, which are explored in a version of the model calibrated to match the UK data.

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# Contents

# Chapter 1

# Financial Stabilisation Policies in a Credit Crunch

## *Zombie Firms and the Effective Lower Bound*

*This paper explores the role of financial stabilisation policy interventions during a crisis. In the model, firms are subject to a shock that restricts their debt to a fraction of their future profits, which results in lower investment. To compensate for this fall in aggregate demand, a reduction in the interest rate is needed, to induce unconstrained firms with lower productivity to start investing thus reestablishing an equilibrium. The constrained equilibrium however features too many low-productivity firms: zombies. They generate a negative spillover on the borrowing capacity of more productive firms, as they contribute to reducing the value of profits for all firms by inflating real labour costs. If the interest rate hits the effective lower bound, the opportunity cost of investment is relatively high, so aggregate demand is low. As fewer low-productivity firms invest, future aggregate productivity is improved, however current output is below potential and employment is low. While liquidating zombie firms away from the lower bound can improve the efficiency of the allocation, it can be counterproductive at the lower bound, as these firms are not zombies but make use of idle resources, boosting output and welfare.*

## 1.1 Introduction

A decade of low interest rates and a pandemic-induced recession raise a classic question on the extent to which policymakers should intervene to support or liquidate inefficient firms. On the one hand, the Schumpeterian viewpoint underscores the cleansing effect of recessions: bailing out businesses may create zombie firms and generate other supply-side inefficiencies. On the other hand, Keynesians argue that intervening to stimulate the economy in a crisis is especially beneficial, considering that aggregate demand may be insufficient. This paper presents a model of the tension between demand management concerns and efficiency of supply by offering a theory of zombie firms that accounts for the effective lower bound (ELB), and analyses its efficiency properties.

This is important because the monetary authority in charge of setting interest rates does not always work in close collaboration with the financial authority setting financial regulation.[1] As a result, the objectives established for each institution do not necessarily account for the potential effects on the other authority's targets. Should financial stabilisation policies take into account the state of monetary policy? And how do financial stabilisation policies affect the availability of monetary tools to combat shocks?

This paper addresses these questions within the context of a simple theoretical framework. It considers the potential for financial stabilisation policy interventions when credit conditions tighten, both when the interest rate is free to adjust in response to the shock as well as when it hits the lower bound. In the model, a credit crunch restricts the borrowing capacity of all firms to a fraction of their earnings.[2] As a result, constrained firms have to lower their investment relative to the first best. In equilibrium, the real interest rate falls so as to offset the direct impact of the shock, by inducing less productive firms, which would otherwise be inactive, to enter the market. Aggregate investment is preserved, but this comes at the expense of lower productivity, due to capital being operated by less efficient firms. As the efficiency of production falls, real wages are lower. This in turn raises firms' future profits and loosens their borrowing limit, further offsetting the initial shock.

This outcome is however not constrained efficient.[3] A policymaker that takes the financial friction as given, but internalises the effect of individual choices on prices, can intervene to improve on the allocation by liquidating some low-productivity firms. The allocation can be improved through policy interventions because of the interaction of two inefficiencies. Firstly, constrained entrepreneurs have a relatively higher marginal valuation of wealth than unconstrained ones. This implies that changes in prices that

[1]As examples, in the UK these two authorities are different branches of the same institution, the Bank of England, while in the Euro Area, the European Central Bank (ECB) is a formally distinct body from the European System of Financial Supervision. The ECB however provides input to the European Systemic Risk Board, as set out in EU regulations.

[2]The credit constraint considered is an earning-based constraint, as in Drechsel (2018).

[3]A formal definition of constrained (sub)optimality can be found in Geanakoplos and Polemarchakis (1986).

redistribute resources towards constrained agents can be beneficial in the aggregate. Secondly, there is a pecuniary externality associated with the borrowing limit, given that real wages determine profits and affect the borrowing capacity of firms. In the model, investment by low-productivity entrepreneurs creates a negative spillover on more efficient producers. At the margin, one extra unit of investment from a low-productivity firm increases the aggregate stock of investment, pushing up labour costs, which lowers the value of productive firms' profits and so reduces their borrowing ability. The financially-constrained allocation features too many inefficient firms in operation: *zombies*.

When the economy is at the effective lower bound, these conclusions are reversed. Now, because the interest rate is too high relative to what it would be without the bound, firms invest less in physical capital. As a result, demand is lower than the production potential, and there are productive resources left idle in the economy. While productivity is high, since low-productivity firms are not operating as much capital, aggregate investment is inefficiently low, and output and welfare are low both in the present and in the near future: in the present due to the weak demand; in the future because of the low capital stock. The lower bound on the interest rate induces an upper bound on future wages, given the optimal choice of capital and labour input mix of firms. As a result, the spillovers at play through changes in the wage rate are not active when the interest rate is at the ELB. Compared to an individual entrepreneur, a policymaker would internalise additional benefits from firms' investment, not just in the form of higher future output but also consisting of uninternalised effects of higher current consumption and production. However, given the constraint on interest rates, no intervention can increase the amount of investment in the economy compared to a laissez-faire allocation. The previous result is turned on its head: it is now in the policymaker's interest not to liquidate low-productivity firms. They are no longer zombies, but rather effective users of idle resources that boost output and welfare.

Next, I consider policies that aim to prevent or moderate the crisis before it hits the economy. I show that the unregulated economy features under-issuance of corporate bonds and that an increase in the amount of debt that firms can access before the credit crunch can help the economy during the crisis. In particular, if productive firms need to repay a larger stock of initial debt, they can invest less once the shock hits. This corresponds to lower future wages,[4] and higher future earnings, which alleviate the impact of the financial friction during the crisis. These policies are also helpful in preventing the ELB from binding as well as ameliorating the effects of the crisis at the lower bound. Indeed, the lower future wages induced by a larger stock of corporate debt contribute to increasing the current return to investment for all entrepreneurs. As the interest rate offered on financial markets corresponds to the physical return to investment of the marginal investor, this increases the equilibrium real interest rate, pushing it away from the lower bound. Policies that are meant to moderate the effects of pecuniary externalities are also helpful in preventing aggregate demand externalities: financial stabilisation policies improve the availability of monetary tools when the shock hits.

The paper further considers some extensions, which emphasise the importance of specific assumptions. First, I introduce another, less capital-intensive sector. Fighting zombie firms in the capital-intensive sector comes at a cost of further exacerbating resource misallocation across sectors:[5] the redistribution of resources away from the constrained sector during a credit crunch is intensified when fewer low-productivity firms are allowed to operate. The capital-intensive sector shrinks even more than in the absence of interventions to liquidate zombies. Second, I consider the possibility of another, unconstrained type of agents in the economy. More specifically, in the main model workers are assumed to be hand-to-mouth, so completely unable to smooth out their resources overtime. However, when they can access financial markets, the result on optimal liquidation of zombie firms in a credit crunch is only preserved under certain conditions.[6] Low-productivity firms' investment contributes to increasing future wages, and the higher cost of labour in the future allows for a redistribution of resources towards entrepreneurs with a higher valuation of wealth in the present. More generally, the presence of any other type of unconstrained agents in the economy is likely to affect the optimal policy for zombie firms.

Drechsel (2018) and Lian and Ma (2020) document how widespread earning-based

[4]The reasoning is as follows: in the aggregate, a larger amount of bonds issued allows indebted but productive firms to invest less and low-productivity entrepreneurs, who are savers, to invest more. Entrepreneurs do take this effect into account, but they do not internalise how a less efficient allocation of resources in the future reduces the future cost of labour.

[5]In defining misallocation across sectors, a first best allocation is used as reference for the optimal relative sector size.

[6]That is, when workers can borrow against their future wage earnings, but they are subject to a borrowing constraint that is no slacker than the entrepreneurs' constraint.

borrowing constraints are. However, the normative implications of this type of constraint have been studied very little, compared to the more popular collateral-type borrowing constraints. As a final step, this work compares the policy implications of a cashflow constraint to a collateral constraint and shows that the specific type of financial frictions constraining investment plays an important role in shaping the second-best distribution of active firms. If firms are constrained, not by future profits, but rather by the value of a collateral asset, then increased demand for physical capital by any type of firm will boost the price of collateral and so increase the borrowing ability of constrained firms. In this case, the beneficial effects of Schumpeterian liquidationist policies are reduced.

The rest of the paper is organised as follows: after relating this work to the relevant literature, Section 1.2 presents the main framework used for the analysis. In Section 1.3 a credit crunch is introduced, restricting firms' borrowing possibilities. After analysing the efficiency properties of the allocation, Section 1.4 analyses how the credit crunch is affected by an effective lower bound on the interest rate. It then considers the potential for policy interventions. Section 1.5 considers the possibility for interventions before a crisis can hit the economy; Section 1.6 analyses some extensions of the main model. Section 1.7 concludes.

### 1.1.1 Related Literature

This paper combines two main strands of the literature: one on misallocation induced by financial frictions, and another on the liquidity trap and demand shortages. The first underlines the supply-side cost of low-productivity firms operating in the economy; the second focuses on the beneficial effects of demand stimuli in a situation of low demand.

Among the papers that formalise how financial frictions can distort the allocation of productive factors generating lower aggregate productivity, this work is most closely related to Kiyotaki (1998), Aoki, Benigno, and Kiyotaki (2010) and Reis (2013). The setting in Kiyotaki (1998) is augmented with the introduction of workers supplying labour within the period. This is important as it introduces a price in firms' earnings, which depends on firms' choices and influences how much they can borrow. As a result of this modification, the constrained economy is constrained inefficient.[7] The two-sector

[7]On the contrary, the baseline model in Kiyotaki (1998) is constrained efficient because the only price, the interest rate, is a constant depending on the marginal investor's productivity.

framework of Aoki et al. (2010) and Reis (2013) is considered as an extension of the main model, in order to analyse the effects of misallocation not only within but also across sectors. The consequences of fluctuations in the price of a good on the sector's borrowing capacity have been extensively studied from the point of view of open economy models,[8] but they generate interesting effects in a closed economy too. Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017) use an analogous setting to analyse the effects of lower interest rates due to the euro convergence process on the distribution of productivity in southern European countries. The present work further analyses the efficiency of such allocation, stressing when there is scope for policy interventions and when the misallocation is in fact constrained efficient.

Concerning the demand rationing, positive papers on the liquidity trap, such as Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017), highlight the importance of debt accumulation in amplifying recessions induced by financial constraints. Farhi and Werning (2016) and Korinek and Simsek (2016) explore the normative implications of aggregate consumption demand externalities in the presence of constrained monetary policy. This work adds an analysis of intertemporal choices related to capital to their insights. This has two implications: first, borrowing is motivated by the need to finance capital investment; second, a different source of aggregate demand externalities is explored in connection to capital investment. Differently from consumption externalities, what starts out as a demand deficiency can turn into a supply-side problem in the following period in presence of investment externalities. Among the normative papers, Rognlie, Shleifer, and Simsek (2018) propose a model to study the investment overhang of the great recession, and analyse the effects of the liquidity trap on misallocation and unbalanced recovery across sectors. This paper underlines the potential for misallocation not only across sectors but also within a sector.

This paper is also related to the literature on pecuniary externalities and financial stabilization policies, as in Jeanne and Korinek (2020); Bianchi and Mendoza (2018); Dávila and Korinek (2018); Benigno et al. (2016); Bianchi (2011); Lorenzoni (2008), as it also features pecuniary externalities connected to a borrowing constraint and unequal marginal rates of substitution (MRSs). Here, this is considered in combination with a demand externality, with a focus on ex-post policies related to investment. Schmitt-Grohé

[8]See e.g. Benigno, Chen, Otrok, Rebucci, and Young (2016) or Bianchi (2011).

and Uribe (2016) and Wolf (2020), among others, analyse the effect of downwardly rigid nominal wages in combination with a fixed exchange rate and find that they generate a pecuniary externality that leads to overborrowing and excessive hiring before a deleveraging shock. This work differs from these papers as the wage rate is perfectly flexible.

In the setting proposed in this work, *zombie* firms are defined as low productivity firms that generate a negative spillover on productive firms. The empirical literature has provided mixed evidence of spillovers from zombie to non-zombie firms. Using firm-level data in Japan up to the early 2000s, Caballero, Hoshi, and Kashyap (2008) find that investment and employment growth for healthy firms relative to zombies falls as the percentage of zombies in their industry rises. More recently, Acharya, Eisert, Eufinger, and Hirsch (2019) documented similar effects in Europe. Both investment and employment growth of healthy firms are found to be significantly lower compared to non-zombie firms active in industries with less zombies. Schivardi, Sette, and Tabellini (2017) use a different identification strategy to show that zombie firms as induced by low bank capitalisation have a negligible effects on the growth rate of healthy firms. They point at general equilibrium effects such as aggregate demand externalities to explain this difference with the rest of the relevant literature. This paper encompasses a potential explanation both for the presence and absence of negative spillovers from zombie to healthy firms.

## 1.2 Framework

The economy features three dates: $t = 0, 1, 2$. Sections 1.3 and 1.4 will mainly focus on the last two periods; in Section 1.5 period 0 is taken in consideration more explicitly. The environment features no uncertainty. Two types of agents populate the economy: workers and entrepreneurs. Entrepreneurs have access to a Cobb-Douglas production technology that employs capital and labour, $y_i = a_i(k_i)^\alpha(n_i)^{1-\alpha}$. The productive sector is composed of different types of entrepreneurs, some with a high and some with a low idiosyncratic productivity: $i \in \{h, \ell\}$, $a_h > a_\ell$. They can consume, invest in productive capital and save or borrow on the financial market. Entrepreneurs can choose not to operate their technology and become financiers, as opposed to running firms. Workers, on the other hand, do not have access to a production technology and are excluded from financial markets. They supply labour with a certain disutility and enjoy consumption. In the following subsections, the problem of each agent operating in the economy is described in more details.

### 1.2.1 Entrepreneurs

High and low productivity entrepreneurs in the productive sector represent a share $\pi_h$ and $\pi_\ell$ of the population respectively.[9] To ease the exposition, their problem is split into intra-temporal and inter-temporal decisions.

#### Static Choices

When starting the period with a positive amount of capital $k_i > 0$, firms solve a static problem of choosing the optimal level of employment $n_i$ to maximise their earnings $d_i$:[10]

$$\begin{aligned} & d_i = \max_{n_i} \; y_i - \omega n_i \\ \text{s.t.} \qquad & y_i \leq a_i (k_i)^\alpha (n_i)^{1-\alpha} \end{aligned} \tag{1.2.1}$$

where $\omega$ represents the wage. In choosing the optimal level of employment, the production technology represents the limit to the profit maximization problem. The firms' optimal choice of labour is to employ workers up to the point where the marginal product of labour equals the real wage rate:

$$(1-\alpha)\frac{y_i}{n_i} = \omega$$

#### Dynamic Choices

Entrepreneurs choose the level of consumption $c_i$, debt (if positive) or savings (if negative) in financial instruments $b_i'$ and investment to start or continue running a firm in the

[9] In general, there can be switching across the two types of productivity, according to a transition matrix $P$, such that the share of both types of entrepreneurs in the population remains constant. However, the results presented here assume that entrepreneurs maintain their type throughout their lifetime.

[10] Throughout the paper, capital letters are used to indicate aggregate variables, while a prime superscript is used to indicate future variables.

following period $k_i'$. They solve the following problem:

$$V_{it}(z_i; S) = \max_{c_i, k_i', b_i'} \log c_i + \beta V_{it+1}(z_i'; S') \tag{1.2.2}$$

$$\begin{aligned} \text{subject to} \qquad c_i + k_i' - \frac{b_i'}{R} &= z_i, \qquad\quad k_i' \geq 0 \\ z_i' &= a_i (k_i')^{\alpha} (n_i')^{1-\alpha} - \omega' n_i' - b_i' \\ b_i' &\leq \frac{\theta}{1-\theta} z_i' \end{aligned} \tag{1.2.3}$$

with $V_{i3}(\cdot) = 0$ in the last time period. $S = \{K_h, K_\ell, B, \theta\}$ is a vector of aggregate state variables, $R$ is the gross real interest rate, while $z_i$ is the entrepreneurs' net worth, which is taken as given after having chosen the level of employment according to (1.2.1).[11] The limit on debt in (1.2.3) requires that borrowing be at most a fraction $\frac{\theta}{1-\theta}$ of entrepreneurs' future net worth. This constraint can also be rewritten as depending on firms' output after labour cost payments, as in e.g. Drechsel (2018): $b_i' \leq \theta d_i'$.[12,13]

### 1.2.2 Workers

Workers supply labour to the economy. They do not have access to borrowing or lending, that is, they are hand-to-mouth consumers: $B^{w\prime} = 0$ in all time periods. Every period, they solve the following problem:

$$W_t(S) = \max_{C,L} \; \log \left( C - \frac{L^{1+\psi}}{1+\psi} \right) + \beta W_{t+1}(S') \tag{1.2.4}$$

$$\text{subject to} \qquad C = \omega L$$

with $W_3 = 0$ in the last time period. The workers' preferences are as in Greenwood, Hercowitz, and Huffman (1988) (GHH) and they imply that labour supply features no

[11]After production, capital is assumed to fully depreciate.

[12]To see this, notice that $z_i' = d_i' - b_i'$. Plugging this expression in the initial borrowing constraint and rearranging shows the equivalence. See also the discussion in Drechsel (2018) on the irrelevance of stock vs. flow distinction for borrowing.

[13]Firms obtain credit based on expected earnings as opposed to the entire continuation value of the firm. One can think of this type of constraint as arising from the fact that it is not possible to continue to operate the production technology if the entrepreneur withdraws from the firm. Then, the only thing that can be recouped is the production net of labour costs at the time of repayment.

wealth effect: $L^{\psi} = \omega$.

### 1.2.3 Equilibrium

It is now possible to define an equilibrium within this framework.

**Definition 1.1.** *An equilibrium is a path of allocations* $\{c_{it}, C_t, n_{it}, k_{it+1}, b_{it+1}, L_t\}$*, prices and profits* $\{\omega_t, R_t, d_{it}\}$ *for all time periods* $t = 0, 1, 2$*, and all types of entrepreneurs* $i = h, \ell$*, with initial conditions for debt and capital* $\{b_{i0}, k_{i0}\}_{i=h,\ell}$ *given, such that entrepreneurs in the productive sector solve problems* (1.2.1) *and* (1.2.2)*, workers solve problems* (1.2.4)*, and markets clear:* $C_t + \sum_i (c_{it} + k_{it+1}) = Y_t, \quad \sum_i b_{it+1} = 0$.

### 1.2.4 Constrained Efficiency

In a first best allocation, only high-productivity entrepreneurs would engage in production, while low-productivity entrepreneurs would rather lend to more efficient firms, hence earning a higher return than they could from their own production technology. The return offered on financial markets would in fact correspond to the return on capital operated by high productivity entrepreneurs.[14] For the analysis that follows, however, it is useful to give details on two additional allocations: a laissez-faire allocation with a financial constraint, and a constrained efficient, second best allocation.

#### Financially constrained, laissez-faire allocation

Consider a constrained allocation where the debt limit (1.2.3) is binding and workers are constrained in their access to financial instruments to borrow or save. Due to the financial constraint, firms are not free to borrow as much as they wish from financiers. The marginal rate at which resources can be transferred intertemporally is not the same for all the agents in the economy:

$$\begin{aligned} R^{-1} = \mathrm{MRS}'(\ell) = &\mathrm{MRS}'(h) + \mu_h c_h > \mathrm{MRS}'(h) \\ = &\mathrm{MRS}'(w) + \mu_w C \neq \mathrm{MRS}'(w) \end{aligned}$$

[14] More details on the first best allocation can be found in Appendix 1.A.2.

where $\text{MRS}'(i) = \beta c_i / c_i'$ with $i = \{h, \ell, w\}$ is the marginal rate of substitution and $\mu_i$ is the Lagrangian multipliers associated with borrowing constraints (1.2.3) and $B^{w\prime} = 0$. The Lagrange multiplier $\mu_w$ can in principle be positive if workers wish to borrow, or negative if workers would like to save. Here, I consider a setting where workers have pre-existing debt and would like to borrow, so that the Lagrange multiplier $\mu_w$ is positive, and $\text{MRS}'(\ell) > \text{MRS}'(w)$.[15] As for the high-productivity entrepreneurs, their marginal rates of substitution is necessarily lower than that of low-productivity firms, given that they are subject to a restriction on their borrowing. This implies that their marginal valuation of wealth is high at time 1, and low at time 2.

The interest rate consistent with market clearing is lower than in a perfect allocation, so as to ensure that all of the goods that could potentially be produced can be consumed and invested. In particular, if the interest rate is sufficiently low, financiers start investing in productive capital. So long as the interest rate on loans is exactly equal to the return of the $\ell$ technology, these entrepreneurs are in fact indifferent between investing in productive capital or in financial markets. Provided that the initial share of net worth and idiosyncratic productivity of low-productivity entrepreneurs are not too low, they will do both in equilibrium.[16]

$$R = \text{MPK}'(\ell) \equiv \frac{\alpha a_\ell}{(\hat{K}_\ell')^{1-\alpha}}$$

With $\hat{K}_\ell' \equiv K_\ell' / N_\ell'$ the capital labour ratio of the low productivity entrepreneurs. A binding borrowing limit generates a redistribution of capital within the productive sector from high to low productivity firms. As a result, TFP is lower than in a first best allocation. The lower efficiency in aggregate production in turn results in a lower equilibrium wage level.

[15] Section 1.6.2 analyses the consequences of relaxing this assumption.

[16] Formally, the conditions $\hat{a}_\ell > \theta \hat{a}_h$ and $Z_{0\ell}/Z_0 > \theta \hat{a}_h / \hat{a}_\ell$ are necessary to ensure positive capital investment of low productivity firms, $K_{\ell 2} > 0$. The first condition requires the marginal productivity of less efficient firms to be sufficiently large. The second condition requires the low productivity entrepreneurs to initially own a sufficiently large share of total net worth.

**Financially constrained, second best allocation**

So long as financial frictions are present, no interventions can help achieve a first best allocation. Consider instead a planning problem where a social planner can choose the allocation subject to the same financial constraints as the decentralised economy. Differently from private individuals, the social planner takes into account how prices are formed. The planner intervenes in one period and lets the competitive equilibrium be realised thereafter.

**Definition 1.2.** *A constrained efficient or second best allocation is the solution to the following problem:*

$$V_t^P(K_h, K_\ell; \theta) = \max_{\tilde{C}, c_i, K_i', B_i'} \Bigg\{ \sum_{i \in h,l} \chi_i \pi_i \left[\log c_i + \beta V_{it+1}(z_i'; K_h', K_\ell', B')\right] + \\ + \log \tilde{C} + \beta W_{t+1}(K_h', K_\ell', B') \Bigg\} \tag{1.2.5}$$

$$\text{subject to} \qquad \tilde{C} + \sum_{i \in h,l} \pi_i \left(c_i + K_i'\right) = Y - (1+\psi)^{-1} L^{1+\psi};$$

$$B' \leq \theta \left(Y_h' - \omega' N_h'\right)$$

*with* $\tilde{C} \equiv C - (1+\psi)^{-1} L^{1+\psi}$, $Y = \sum_i a_i k_i^\alpha n_i^{1-\alpha}$, *and where the planner internalises how current choices affect prices.*

The principle of optimality applies here, so that any inefficiency internalised by the planner but not by private individuals is connected to prices either entering the borrowing limit, or the budget constraint of agents who don't all share the same marginal valuation of wealth. For example, there are pecuniary externalities connected to changes in wages hitting workers and producers in opposite ways. If there is a difference in how these agents value wealth at the margin, then this creates the opportunity for a Pareto improvement. These forces can act to either reinforce the effect of pecuniary externalities arising from the borrowing limit, or they can go in the opposite direction.

The following two assumptions describe the limits and possibilities of interventions that are implicit in the social planner's problem: 1) the social planner does not have sufficient instruments to completely undo the financial frictions; 2) the social planner can utilise enough instruments to perfectly implement the second best allocation.

The first point implies that the borrowing limit has to be satisfied in both the constrained efficient and the laissez-faire economy. For example, it may not be possible for policy makers to completely undo the moral hazard and limited commitment problems that generate the credit crunch. Because the only advantage of the social planner compared to individual agents is to internalise how prices depend on choice variables, if prices are a constant then the laissez-faire allocation is second best.[17]

The second point clarifies that, while a first order concern in practice, problems of imperfect implementation are abstracted from here. Rather than focusing on how best to use one particular policy instrument, this work looks at what are all the possible margins of interventions in the laissez-faire economy. In practice, the social planner might not have sufficient instruments or information to achieve the second best allocation. As an example, interventions after the crisis may consist of resolution policies, but there may not be a way to directly subsidise a firm's investment. While this work abstracts from these issues, it indicates the correct use of existing or new tools towards tackling specific margins of inefficiency.

The definition of these allocations will be useful in the analysis that follows, where a shock moves the economy from an unconstrained to a financially constrained setting. I will then compare the laissez-faire allocation to a second best allocation.

## 1.3 A Financial Crisis

Consider the effect of a credit crunch restricting firms' access to credit. In particular, assuming no frictions at time 0 and a perfect capital allocation at the beginning of date 1, assume that a low $\theta_1$ precludes efficient firms from borrowing as much as would be optimal for date 2.[18] In Appendix 1.B.1, I show that a financial crisis can only occur if $\theta < 1$, that is, if firms cannot use the entire value of their profits to obtain credit. If profits can fully be recovered by potential lenders, then there is no financial friction and the economy is first best.

[17] See Appendix 1.B.3 for an example where the financially constrained laissez-faire allocation is second best.

[18] As can be gathered from the characterization of the first best allocation in Appendix 1.A.2, even when only productive entrepreneurs engage in production and run all the capital, they still want to borrow from low productivity firms every period to finance capital investment for the following period.

### 1.3.1 Analysis of the Financial Crisis

**Date 0: *Before the crisis.***

The economy at date 0 is at an unconstrained, first best steady state, where only productive firms engage in production, while less productive entrepreneurs lend their funds to the more productive ones.[19]

**Date 1: *During the crisis.***

As a consequence of the perfect allocation in the previous period, all of the capital is owned by the more productive entrepreneurs at the beginning of period 1. The borrowing restriction however means that productive firms are no longer on their Euler Equation and can no longer invest as much as in period 0. The supply of savings in the economy exceeds the demand. A lower interest rate induces a lower demand for financial savings and an increased demand for investment from unconstrained agents. While the lower interest rate has the potential to induce a lower aggregate productivity, and impair the *quality* of investment projects, it also ensures that the *quantity* of capital invested in the aggregate is kept at the efficient level. This is possible as the low productivity entrepreneurs invest in setting up firms, and engage in production in the following period.

**Date 2: *After the crisis.***

After date 2, the world ends. Therefore, there can be no demand for debt, and no entrepreneur would want to take on any new investment: agents simply make their static consumption and labour choices. The financial friction of date 1 implies that TFP at date 2 is lower than optimal.

**Proposition 1.3.1.** *A financial crisis at date* 1 *induces no change in aggregate output within the period, but lower aggregate productivity and production at date* 2.

*Proof.* See Appendix 1.B.2 □

At time 1, labour demand is chosen to solve problem (1.2.1), and for given level of pre-installed capital, it is unchanged compared to the frictionless case. Additionally, the GHH preferences imply that the supply of labour at time 1 is also unchanged. Hence,

[19] See Section 1.5 for an analysis of interventions in this period, before the shock takes place.

compared to the first best, output remains the same at time 1. The lower interest rate is what allows the low productivity entrepreneurs to pick up the slack, by absorbing the extra resources produced that can no longer be demanded for investment by the high-productivity entrepreneurs. As a result however, realised TFP following the credit crunch is reduced, as high productivity entrepreneurs are no-longer the only active firms, and part of the investment is carried out by less efficient firms. The aggregate productivity-weighted investment in the economy is therefore lower at time 2, which reduces the equilibrium real wage and the aggregate level of employment.

### 1.3.2 Interventions during the Crisis

This section compares a constrained efficient allocation as defined in Subsection 1.2 to the laissez-faire constrained allocation. A binding borrowing constraint combined with a lower equilibrium interest rate induces the low-productivity entrepreneurs to enter the market and start production, which poses the question of whether the resulting distribution of active firms' productivity is constrained efficient, or whether a regulator might want to intervene to alter it.

**Definition 1.3.** *Zombie firms are low-productivity firms that produce in the constrained laissez-faire economy, but remain inactive in a constrained efficient allocation.*

In a first best allocation the number of active low-productivity firms is zero. In this sense, all low-productivity firms investing in the financially constrained laissez-faire economy could be considered *zombie* firms, if a first best allocation is chosen as reference. However, in presence of a borrowing limit, the first best allocation cannot be achieved. It is therefore more useful to refer to a constrained efficient allocation in defining zombie firms.

While the setting proposed in this paper is highly stylised, we can look at how this definition of zombie firms maps to the one most commonly used in the empirical literature on zombie firms. They are mainly identified as low quality firms that have low interest payments compared to the most profitable type of firms.[20] In the data, zombie firms are shown to have low profitability as measured by their EBITDA, high leverage, and low interest coverage ratios. Like in the empirical literature, in this setting zombie firms

[20] For any reference to the empirical literature and its findings in this subsection, see for example Caballero et al. (2008); Acharya et al. (2019).

generate relatively low profits per unit of capital operated compared to other firms. Also, intuitively, a constraint on borrowing of productive firms should be equivalent to the imposition of a higher interest rate on their debt compared to the interest rate perceived by less productive firms. Finally, in terms of aggregate consequences of zombie firms, it will be shown next how zombie firms can generate negative spillovers on the ability to invest and borrow of more productive firms, just as demonstrated in the data. However, in the interest of simplicity, some additional features that are usually attributed to zombie firms, such as high leverage or low interest coverage ratio, are absent here.[21]

The definition of zombie firms provided is useful to give an insight into why the laissez-faire allocations differs from a constrained efficient allocation.

**Proposition 1.3.2.** *The allocation in a financial crisis is not second best. Compared to a constrained efficient allocation the laissez-faire economy features zombie firms.*

*Proof.* See Appendix 1.B.4. □

In order to clarify where this proposition emerges from, I will first show that the choices of the constrained workers and entrepreneurs cannot be improved in a second best allocation. I then show that the choice of investment for the low productivity entrepreneurs is not constrained efficient, and in particular, that in a second best allocation, these firms optimally invest less.

The choice of the planner for debt at date 1 corresponds to the laissez-faire allocation, as it is always optimal for high productivity firms to borrow as much as possible. As a result:

$$\tilde{\mu} = \Delta\mathrm{MRS}'(\ell, h); \tag{1.3.1}$$

where $\Delta\mathrm{MRS}'(h, \ell) \equiv \mathrm{MRS}'(h) - \mathrm{MRS}'(\ell)$ is the distance in marginal rates of substitution of entrepreneurs with high and low productivity and with $\tilde{\mu}$ the Lagrangian multiplier that the social planner attaches to the borrowing limit.[22] Likewise, the choice of investment of the high-productivity firms is also constrained efficient: it is always optimal to let efficient

[21]Low productivity firms are not indebted at all in this model. Rather, they represent savers who decide to invest in a production project only if the interest rate offered on the financial market is sufficiently low.

[22]By equating the two first order conditions for debt and savings of productive and unproductive entrepreneurs, a similar optimality condition follows in the laissez-faire economy.

firms invest as much as possible. However, the social planner's choice of capital for the low-productivity entrepreneurs is pinned down by the following optimality condition:

$$1 - \mathrm{MRS}'(\ell)\mathrm{MPK}'(\ell) = -\left[\tilde{\mu}\theta N_h' - \underbrace{\Delta\mathrm{MRS}'(\ell,h)}_{>0} N_h' + \underbrace{\Delta\mathrm{MRS}'(\ell,w)}_{\geq 0} L'\right]\frac{\partial\omega'}{\partial K_\ell'} \quad (1.3.2)$$

where

$$\frac{\partial\omega'}{\partial K_\ell'} = \frac{\alpha\psi}{\alpha+\psi}\frac{\omega'\hat{a}_\ell}{\sum_i \hat{a}_i K_i'} > 0.$$

The left-hand-side of this expression coincides with the decentralised optimal choice. The choice of the planner differs from private individuals as the right-hand-side is in general not zero. In particular, one extra unit of investment by a low-productivity firm generates up to three different spillovers that individuals do not take into account, all connected to an increase in wages. A higher level of aggregate investment in the sector in fact contributes to raising labour demand for every wage level, resulting in a higher equilibrium wage.

Firstly, there is a pecuniary externality connected to the cashflow constraint: a higher wage increases the cost of production and contributes to reducing the profits of all firms. This reduces the value of pledgeable resources that high-productivity firms use to obtain borrowing, and tightens their credit constraint. Second, a change in wages affects the budget constraint of productive entrepreneurs, who have a lower marginal rate of substitution than low-productivity firms. The increased cost of production reduces the resources available to the more productive firms at time 2, when their valuation of resources is lower. This is beneficial in the aggregate, as it reduces the distance in MRSs. The two aforementioned effects go in opposite directions, but because $\theta < 1$, the latter one always dominates in the aggregate. Third, a higher wage increases resources available to workers at time 2, when they value resources less because they are constrained today, so this is not welfare improving.

By combining Equation (1.3.2) with (1.3.1) one can derive the sign of the aggregate effect of an increase in wages, in the context of the optimal investment choice for low productivity firms:

$$1 = \mathrm{MRS}'(\ell)\mathrm{MPK}'(\ell) - \tilde{\mu}\left[\underbrace{L' - (1-\theta)N_h'}_{>0}\right]\frac{\partial \omega'}{\partial K_\ell'}$$

In a second best allocation, the social planner optimally chooses a lower amount of investment of low productivity firms. In particular, the amount of overinvestment that takes place in the unregulated economy is increasing in: 1) the responsiveness of the wage rate to investment of the low productivity firms, $\frac{\partial \omega'}{\partial K_\ell'}$; 2) the pledgeability of future profits, $\theta$, as this amplifies the effect of a change in earnings on the borrowing capacity of productive firms; 3) the tightness of the borrowing constraint $\tilde{\mu}$, an indication of the benefit of relaxing such constraint.

It is important to remark that for this result, the assumption of a perfectly competitive, frictionless labour market plays a key role. In particular, it is crucial that the production decisions of less-productive firms have an impact on the wage rate paid by more-productive firms, in order for the borrowing externality to be at play. This implies that, for example, the results derived in this section are unlikely to be preserved if there is labour market segmentation across the two types of firms.[23]

Provided that there is one, perfectly competitive labour market, zombie firms arise during a financial crisis. This force will now be assessed against the need to increase demand during a deleveraging phase that is not accompanied by a strong enough reduction in the real interest rate.

## 1.4 A Financial Crisis at the Effective Lower Bound

Consider now a situation where the real interest rate is bounded from below:

$$R \geq \rho \tag{1.4.1}$$

[23] No formal characterization of this case is offered here, but further discussions on other possible relaxations of the assumption of frictionless labour markets can be found in Subsection 1.6.1.

This constraint is exogenous and taken as given here, but it is in general consistent with a lower bound on the nominal interest rate, combined with nominal rigidities.[24] Without loss of generality, in what follows, the lower bound is normalised to 1.

In normal times, the interest rate can adjust to ensure that the aggregate quantity of savings equals the total amount of investment:

$$Y - C = K'$$

Away from the lower bound, aggregate demand is sufficient to induce full capital utilisation, so labour demand is at the efficient level. That is, such that $\omega = \text{MPN}$, with MPN the marginal product of labour. If however the interest rate needed to clear the market is below the lower bound, then the real rate is constrained, aggregate demand for investment is too low, while demand for savings is too high. In this case, the pre-installed stock of capital cannot be fully utilised, so production is below the efficient level:

$$\begin{aligned} & d_i = \max_{n_i} \bar{y}_i - \omega n_i \\ \text{s.t.} \qquad & \bar{y}_i = \frac{1}{\pi_i}\left(C + K' - \sum_j y_j\right) \leq a_i (k_i)^{\alpha} (n_i)^{1-\alpha} \end{aligned} \tag{1.4.2}$$

Firms are capable of producing more, given their technology and previous capital investment. However, because the real interest rate is relatively too high to clear the market, aggregate demand is insufficient and cannot absorb the entire amount of potential output.[25] In this case, production is demand-determined, demand for labour is below the efficient level and the marginal product of labour is larger than the wage rate.

### 1.4.1 The Effects of a Binding ELB

An interest rate that is stuck at the lower bound, and is therefore inefficiently high, induces entrepreneurs to invest less than would be optimal. The level of investment of

[24] This would correspond to a situation where prices are fully rigid. Otherwise the real lower bound is not a constant but fluctuates over time with inflation expectations. Notice also that the real lower bound may emerge from other causes than a nominal lower bound, such as participation in a currency union.

[25] Equal rationing across all firms producing is assumed in this case.

low-productivity firms away from the lower bound would be such that the optimal level of aggregate investment is maintained, in spite of the cashflow constraint limiting how much productive firms can invest. When the interest rate is at the lower bound, low productivity firms invest less: investment is chosen so as to ensure that the return offered on the financial market exactly corresponds to the return on their investment technology. But as the return offered on bonds is too high at the ELB, they choose to invest less, maintaining a higher marginal product of capital.

$$\rho \equiv 1 = \frac{\alpha a_\ell}{(\hat{K}'_\ell)^{1-\alpha}} = \left[\frac{\hat{\alpha} a_\ell}{(\omega')^{1-\alpha}}\right]^{1/\alpha} \tag{1.4.3}$$

After the crisis, the economy exits the liquidity trap, so the choice of employment is optimal. This implies that the capital labour ratio $\hat{K}'_\ell$ depends on future wages.[26] This in turn means that a relationship between the current interest rate and the future wage level is maintained, even at the lower bound, through the future capital-labour ratio.

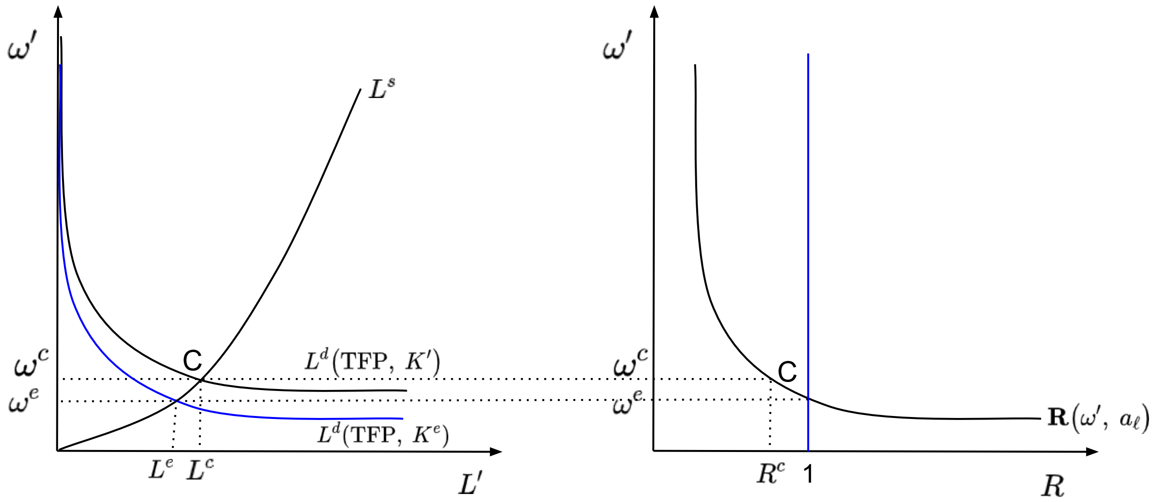


Figure 1.1: Labour market clearing and relationship between prices outside and at the ELB (blue line).

Figure 1.1 shows the labour market equilibrium and the relationship between future wages and current real rate at the lower bound. The effective lower bound implies that

[26]No intertemporal decision has to be made in the last period, so the capital available at date 2 is used to full capacity in order to maximise consumption. That is, the lower bound cannot be binding in the last time period.

the interest rate cannot fall below a threshold of 1. The corresponding wage is lower than in a financial crisis, as indicated by point C on the graph, where the lower bound is not binding. In turn, at the lower bound, a lower level of aggregate investment pushes the labour demand curve down resulting in lower real labour costs.

An interest rate above the optimal market clearing level has implications not just for aggregate investment but for consumption too. Unconstrained agents are on their Euler equation, but because the interest rate is too high, and future consumption is reduced due to lower future capital available, consumption demand of low-productivity entrepreneurs and financiers in the present is low. As aggregate demand for both investment and consumption is low, the pre-installed level of capital can no longer be fully utilised: the full-capacity level of production cannot be absorbed. As a result, a lower level of labour demand than full employment arises. The wage rate falls in the present period to ensure that the labour market is in equilibrium. This induces workers to also demand less consumption. As less output is produced at time 1, high productivity firms have access to less resources for investment. The weak demand at time 1 turns into a supply-side problem in the following period: production is low at date 2, even though the economy escapes the lower bound, due to the low level of capital that was invested at date 1.

**Proposition 1.4.1.** *At the effective lower bound, a financial crisis generates a recession featuring lower employment and output at date 1, as production is demand-determined. Date 2 features a supply-driven recession, where fewer low-productivity firms operate.*

*Proof.* See Appendix 1.C.1 □

The proposition clarifies that the low level of investment restricts production at time 1, but it also induces fewer low-productivity entrepreneurs to invest in their production technology. This implies that while the average *quality* of investment projects is higher, the *quantity* is inefficiently low, restricting production possibilities. In this sense, date 2 can see a higher level of productivity, yet it features a supply driven recession.

### 1.4.2 Interventions in a Financial Crisis at the Effective Lower Bound

When a financial crisis brings the economy to the effective lower bound, productive firms continue to be constrained in their choice of debt and investment. The less efficient entrepreneurs, on the other hand, invest less due to the high opportunity cost of operation, and consume less in the present due to lower future available resources. The social planner's problem now needs to take into account 1) the level of future investment limiting both future and current consumption demand of the low-productivity entrepreneurs (the Euler equation), as well as 2) the future wages - pinned down by the interest rate at the lower bound - determining the amount of investment of low-productivity entrepreneurs. The solution of this revised planner's problem, which can be found in Appendix 1.C.2, leads to the following proposition.

**Proposition 1.4.2.** *During a financial crisis where the economy is at the lower bound, the allocation at time 1 is constrained efficient: no planner's intervention can improve on the laissez-faire economy.*

*Proof.* See Appendix 1.C.2 □

When the interest rate is at the effective lower bound, the opportunity cost of investment is higher than what it should be. This has two consequences: first, fewer low-productivity entrepreneurs run firms, while more entrepreneurs keep their savings on the financial market. Second, the optimal future mix of capital and labour inputs for all active firms implies that the lower bound on the interest rate corresponds to an upper bound on the wage rate.[27] As a result of this, wages are fixed and equal to a constant, $\bar{\omega}' = (\hat{\alpha} a_\ell)^{\frac{1}{1-\alpha}}$. While it is normally the amount of capital invested that determines future wages, it is now the future wage implied by the interest rate at the ELB that determines the maximum possible level of investment of unproductive firms. The negative spillover that less efficient firms generate away from the lower bound, both on the borrowing capacity of productive entrepreneurs as well as on the distribution of resources, is therefore not active at the lower bound. The amount of investment demanded does not affect wages, as they are pinned down by the ELB on the interest rate.

At the same time, it is optimal to maintain the demand for capital of low productivity firms as large as possible. In fact, this not only directly contributes to boosting current demand, which is too low compared to the economy's production potential; it also positively impacts financiers' demand for consumption, further helping to relax the demand constraint. Consumption demand is inefficiently low due to the Euler equation

[27] See also Figure 1.1.

restricting current consumption of unconstrained agents to a multiple $1/\beta$ of their expected future consumption, which in turn depends on capital investment. So while the planner internalises additional benefits of investment of low productivity entrepreneurs compared to individual entrepreneurs, there is no way for the planner to intervene to ameliorate the efficiency of the allocation at the lower bound. Investment demand for both type of entrepreneurs is as large as can be, given the borrowing constraint and the constraint on the real interest rate.

## 1.5 Interventions Before the Crisis

In the analysis so far, period 0 has been ignored so as to focus on what happens when a credit crunch takes place at date 1. Much of the literature on financial regulation and financial stability however tends to focus on potential policy interventions before a crisis can take place. In this section, I analyse such ex-ante interventions, considering a setting where a financial crisis occurs with probability one, and where everyone anticipates it will happen.[28] I show that this economy features under-issuance of corporate bonds, which not only makes the impact of a credit crunch more dire, but it can also push the interest rate towards the lower bound. If this happens, a lower stock of debt makes demand even weaker and worsens the extent to which resources are left idle, as will be shown in more details below.

From Subsection 1.3.1, we know that before the shock takes place at time 1 ("*during the crisis*"), the allocation of capital at date 0 ("*before the crisis*") is perfect, the interest rate corresponds to the return to investment of the high-productivity producers and the wages depend only on capital and idiosyncratic productivity of the efficient entrepreneurs.

### 1.5.1 Interventions Before the Crisis without an ELB

While the borrowing constraint depends on labour costs at time 2, the planner has the ability to influence the tightness of the borrowing constraint through interventions at time 0. In particular, wages at date 2 depend on the choice of capital investment taking place one period before production, at date 1. The planner can influence this through

[28] Korinek and Simsek (2016) show in a setting without capital investment that adding aggregate uncertainty moderates some of the results, without changing the main intuition.

interventions at date 0. In net, the cost of distorting the laissez-faire allocation before the credit crunch is smaller than the benefit of partially undoing the borrowing limit when the shock takes place at date 1.

**Proposition 1.5.1.** *The allocation before a financial crisis is not constrained efficient: inactive entrepreneurs in the laissez-faire economy do not save sufficiently, while active firms issue too few bonds and over-invest.*

*Proof.* See Appendix 1.D.1. □

The choice of debt of the planner at time 0 to be repaid in time 1 can be summarised with the following optimality condition:

$$\left[1 - \frac{\mathrm{MRS}_\ell}{\mathrm{MRS}_h}\right]\left[1 + \frac{B'}{(R)^2}\underbrace{\frac{\partial R}{\partial B}}_{>0} - \frac{L' - N_h'}{R}\underbrace{\frac{\partial \omega'}{\partial B}}_{<0}\right] = -\mu C_h \left(L' - (1-\theta)N_h'\right)\underbrace{\frac{\partial \omega'}{\partial B}}_{<0} \quad (1.5.1)$$

In the decentralised economy, the distance in marginal rates of substitution of high and low productivity entrepreneurs is zero, as no financial friction affects the economy at date 0. There are two elements that the planner internalises which private individuals do not: 1) how a larger stock of initial debt $B_1$ increases the interest rate $R_1$, redistributing resources from the borrower to the lender; 2) how a larger stock of debt $B_1$ reduces future wages $\omega_2$, which both enter the budget constraint of all agents, but also contribute to relaxing the borrowing constraint by inducing higher profits.

I start by illustrating how pre-installed debt affects prices. The initial quantity of bonds outstanding $B_1$ affects the net worth of the two types of entrepreneurs' in opposite ways: borrowers face a lower net worth when they need to repay a larger amount of debt, while lenders have a higher initial net worth with more savings. Upon impact of the crisis, the net worth available to the productive entrepreneurs becomes key in determining the quantity of new capital investment that they can take on, given their limited ability to access new debt.[29] A marginal increase in the initial stock of bonds therefore restricts the net worth of productive entrepreneurs during the crisis and reduces their ability to

[29]Here, the assumption of full capital depreciation is important. With partial depreciation, the old undepreciated capital stock would reduce the need to invest in new capital, dampening the relevance of the initial stock of debt for determining investment.

invest for the future. Vice versa, financiers can invest more at date 1 for date 2 if they have a larger stock of initial savings. Entrepreneurs internalise the direct effect of changes in bonds issued or held at time 0 on their own investment possibility at time 1. They however do not internalise the effect this generates on future prices. A marginal increase in outstanding bonds corresponds to a redistribution of capital investment from high to low-productivity entrepreneurs, which lowers aggregate productivity at time 2, reducing equilibrium wages. In turn, lower wages increase the return to investment for all firms, raising the interest rate between time 1 and 2.

The crucial aspect is that labour costs enter the future borrowing constraint of productive firms negatively. The motive for policy interventions rests on the anticipation of such credit constraint: the redistribution of resources entailed by price changes would not matter in the aggregate without this, because markets are complete at time 0. The planner finds it optimal to induce a lower future wage rate, which helps relax the borrowing constraint, through a larger stock of bonds $B_1$. This in turn creates a wedge in the entrepreneurs' MRSs in the present. The distance between the marginal rates of substitution of high- and low-productivity firms should be positive, $\mathrm{MRS}_\ell < \mathrm{MRS}_h$, for the externality associated to future wages to be internalised in the present. Notice that, by making the future borrowing constraint less binding, ex-ante interventions can help reduce zombie investment and the negative externalities ex-post.

The choice of capital investment of the planner at time 0 is:

$$\begin{aligned}\mathrm{MRS}_h^{-1} - MPK_h = &\\ &\left(1 - \frac{\mathrm{MRS}_\ell}{\mathrm{MRS}_h}\right)\left(\frac{L' - N_h'}{R}\underbrace{\frac{\partial \omega'}{\partial K}}_{>0} - \frac{B'}{(R)^2}\underbrace{\frac{\partial R}{\partial K}}_{<0}\right) - \mu C_h\left(L' - (1-\theta_1)N_h'\right)\underbrace{\frac{\partial \omega'}{\partial K}}_{>0}\end{aligned} \tag{1.5.2}$$

Firstly, only high-productivity entrepreneurs invest in capital at time 0, given the absence of frictions in that time period. Secondly, an increase in investment of productive firms $K_1$ before the crisis leads to higher aggregate net worth $Z_1$ during the crisis. Investment of productive firms $K_{h2}$ depends positively on aggregate net worth, while investment of low productivity entrepreneurs $K_{\ell 2}$ is decreasing in aggregate net worth.[30] Therefore,

[30] From Equations (1.A.18) and (1.A.19) one can observe that investment of high-productivity entrepreneurs is increasing in initial aggregate net worth and decreasing in the aggregate stock of bonds,

similarly to what happens with an increase in pre-installed debt, a change in initial capital affects aggregate productivity at date 2, but in an opposite way, by redistributing resources from the low to the high productivity firms. In turn, a higher aggregate productivity corresponds to higher wages at date 2, which affect the borrowing constraint of productive firms negatively.

A larger initial capital stock also contributes to reducing the distance in MRSs that arises from Equation (1.5.1). This is seen as a benefit, but the effect is weaker than the direct impact of a larger wage on the borrowing constraint.[31] Overall, more investment in the present contributes to restricting the future borrowing capacity of productive firms, which they do not internalise, so the laissez-faire economy features over-investment.

These results on ex-ante interventions would not go through with a CRS production function that only employs capital. In that case, prices would be constants, and there would be little that a social planner could do to alter the efficiency of the allocation.[32] Proposition 1.5.1 would also not hold if production factors could be chosen in the same period that output is produced. Ottonello, Perez, and Varraso (2019) make the point that the timing of borrowing constraints is crucial for justifying macroprudential interventions, as only current-price constraints involve pecuniary externalities that a planner can intervene to internalise. While this is true in models where labour is employed within the period, the presence of an input that can be chosen one period before production affects the result. The proposition therefore demonstrates that there can be scope for macroprudential interventions even when the borrowing constraint emerges from borrowers' misbehaviour at the time of repayment. However, the particular type of required intervention crucially depends on the timing of the borrowing limit.

while the opposite holds for low-productivity entrepreneurs' investment.

[31] The appendix shows that combining (1.5.1) with (1.5.2), the right-hand-side of (1.5.2) is negative.

[32] The derivatives of prices with respect to choice variables would be zero if prices are a constant. See Appendix 1.B.3 for a proof that the allocation at time 1 is constrained efficient if only capital is used in production. Analogous arguments hold at time 0.

### 1.5.2 Conditions That Can Push the Interest Rate Towards the Lower Bound

The economy is at the ELB if the interest rate necessary to ensure that labour demand is optimal ($R^*$) is too low to satisfy the non-negativity constraint (1.4.1), so that $R = 1$.

$$R^* = \xi \hat{a}_\ell \left( \frac{1}{\hat{a}_h K'_h + \hat{a}_\ell K'_\ell} \right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}} < 1 \tag{1.5.3}$$

where $\xi$ is a function of parameters defined in Appendix 1.A.

**Lemma 1.5.1.** *There is a minimum level of initial aggregate debt $\underline{B}_1$ above which the effective lower bound is never binding.*

*Proof.* See Appendix 1.D.2 □

The previous subsection shows that the outstanding stock of initial debt influences the future efficiency of production in presence of a credit crunch. Productive firms, who borrow from low-efficiency entrepreneurs, can invest less for the future when they face larger amounts of old debt to repay, while low-productivity entrepreneurs can invest more when $B_1$ is large. Equation (1.5.3) shows that the interest rate consistent with efficient labour demand is a decreasing function of future productivity-weighted capital investments. Intuitively, a worse future level of aggregate productivity or a lower aggregate capital stock lowers labour demand for given labour supply, inducing lower future wages. This increases the return to investment in the present for both types of entrepreneurs and increases the equilibrium interest rate, pushing it away from the lower bound.[33] Hence, for sufficiently large levels of initial debt, the lower bound is never hit.

The macroprudential literature on aggregate demand externalities shed light on how larger stock of household debt is likely to push the economy towards the effective lower bound.[34] The equilibrium interest rate is in fact *decreasing* in the initial stock of debt in environments without capital. Intuitively, when debt only finances consumption, a larger amount of initial debt before a deleveraging shock requires a larger fall in the interest rate

[33]The interest rate corresponds to the return to investment of the low productivity entrepreneurs. It also positively affects the return to investment of productive entrepreneurs, which corresponds to $A_h \cdot R$ (where $A_h$ is a function of parameters defined in Appendix 1.A).

[34]See e.g. Korinek and Simsek (2016).

so as to induce savers to demand less bonds and consume more. In presence of corporate debt backing physical capital, however, a larger stock of initial debt affects aggregate productivity and labour costs in the following period. More borrowing induces a lower average productivity and a lower future wage rate, which boosts the return to investment, thus leading to a higher equilibrium interest rate.

From an empirical perspective, recent long run evidence by Jordà, Kornejew, Schularick, and Taylor (2020) finds that, contrarily to household debt, corporate debt accumulation does not seem to be associated with increased risk, nor with deeper and longer lasting crises, such as could be expected in the case of a demand driven recession.

### 1.5.3 Interventions Before the Crisis with an ELB

Lemma 1.5.1 shows that the choice of debt before the crisis can help keep the economy away from the effective lower bound, hence avoiding a deep recession at time 1, which the credit crunch combined with the ELB would generate. A social planner that can choose a sufficiently large level of debt in period 0 can ensure that the market clearing interest rate never becomes constrained. Moreover, Proposition 1.5.1 shows that increasing the stock of initial debt can be beneficial to partially undo the borrowing constraint. These observations lead to the following proposition.

**Proposition 1.5.2.** *For $B_1 > \underline{B}$, the effective lower bound is not binding and Proposition 1.5.1 applies. For $B_1 \leq \underline{B}$, the ELB on the real rate binds and the allocation at time 0 is not constrained efficient as it features under-borrowing.*

*Proof.* See Appendix 1.D.3. □

The planner's optimal choice of debt is:

$$1 - \frac{\text{MRS}_\ell}{\text{MRS}_h} = \underbrace{\frac{\partial \mathcal{D}(\cdot)}{\partial B}}_{>0} \tag{1.5.4}$$

Away from the lower bound, both the current interest rate and future wages depend on the level of bond demand: the planner internalises this and optimally chooses a larger quantity of bonds than individual agents. When the economy hits the lower bound, at

$t = 1$ the interest rate is stuck at one, and the future wage rate is also a constant.[35] These two prices, therefore, no longer respond to changes in choice variables at the lower bound. Nevertheless, the planner now internalises a different effect: larger quantities of debt directly affect the aggregate level of production. To see this, it is useful to inspect the aggregate resource constraint at time 1 when the economy is at the ELB:

$$
\begin{aligned}
C_h + C_\ell + C_w + K_h' + K_\ell' &= Y \\
(1-\hat{\beta})(Y - \omega L - B) + \frac{1}{\beta}\left(\theta \tilde{a} K_h' + K_\ell'\right) + K_h' + K_\ell' &= Y - \omega L \\
\implies \qquad \frac{\hat{a}_h - \hat{a}_\ell}{(1-\theta)\hat{a}_h} B + \frac{\mathrm{w}(\omega')}{\hat{\beta} A_h} &= Y - \omega L \qquad (1.5.5)
\end{aligned}
$$

where the Euler equation for the low-productivity entrepreneur was used, together with the constrained choice of capital of the productive entrepreneur $K_h' = \frac{\hat{\beta}\hat{a}_\ell(Y-\omega L - B)}{\hat{a}_\ell - \theta \hat{a}_h}$, the constrained future wage given by the lower bound on the interest rate $\omega' = (\hat{\alpha} a_\ell)^{\frac{1}{1-\alpha}}$, and where these two expressions now pin down the level of investment of the unconstrained entrepreneurs $\sum \hat{a}_i K_i' = \mathrm{w}(\omega')$. From Equation (1.5.5) it is apparent that $B$ affects aggregate demand positively, which in turn determines earnings and output at the lower bound.

In particular, a larger stock of debt directly contributes to reducing the resources available for investment to productive firms; this reduces aggregate demand at time 1 directly. The future wage rate is pinned down by the lower bound on the interest rate, but given the future optimal choices regarding labour, the wage must also depend on the sum of investment of the two entrepreneurs' types, weighted by their respective productivity type. As a result, for given wage, the lower investment of constrained firms must correspond to a larger quantity of investment of low-productivity firms. In fact, the increase in investment required from low productivity firms to maintain a constant future wage rate is larger than the initial reduction in investment of productive firms, because unconstrained firms' productivity is lower. This implies that in the aggregate, a larger stock of debt has an overall positive effect on boosting aggregate demand, and in particular, it contributes to increasing profits, so that $\partial \mathcal{D}(\cdot)/\partial B > 0$.

The social planner therefore consistently finds that the unregulated economy features

[35] See the right hand panel of Figure 1.1.

under-issuance of bonds. Away from the lower bound, this is because entrepreneurs do not internalise how more borrowing today induces a lower wage rate in the future, which relaxes the future borrowing conditions. At the lower bound, entrepreneurs do not consider how a larger stock of debt is associated with a higher level of aggregate demand, which in turn corresponds to increased output.

Concerning the planner's optimal choice of investment, future wages and the interest rate don't respond to changes in investment at the lower bound. Also, while the effects of changes in capital on the current wage rate are not taken into account by individual entrepreneurs, this affects entrepreneurs who pay the wage and workers who receive them in a symmetric way. As a result, this effect does not matter in the aggregate, analogously to what happens away from the lower bound. Therefore, the planner's choice of capital investment at the lower bound corresponds to the choice in the unregulated economy, and no intervention is needed.

Proposition 1.5.1 showed how even away from the lower bound, the allocation is not constrained efficient at date 0 as it features under-borrowing and over-investment connected to the future binding constraint. On the other hand, more debt can benefit the economy at the lower bound too, by reducing the demand rationing. As a result, the same interventions that are useful in counteracting pecuniary externalities are also helpful in reducing the impact of aggregate demand externalities.[36]

## 1.6 Extensions

Sections 1.3 and 1.4 presented the most minimal model suited to analyse the effect of zombie investment at the effective lower bound. Some of the distinctive features of the setting considered were the presence of only one sector, workers as hand-to-mouth consumers, and the type of financial friction considered linking firms' borrowing to their cashflows. In this section, each of these assumptions will be relaxed in turn. I show that the presence of zombie firms in a financial crisis is not altered by the presence of another, less capital-intensive sector. Nevertheless, the presence of a labour-intensive

[36]This result is consistent with findings in the literature (Farhi and Werning, 2016) that consider collateral externalities associated with a fixed asset like housing, in combination with aggregate demand externalities. Nevertheless, the direction of intervention is reversed when productive capital is considered and a collateral constraint is replaced by a net worth constraint.

sector introduces a further tradeoff between inefficient resource allocation within and across sectors. I then analyse the case where workers are free to borrow in proportion to their future labour income. I show that the extent to which they can collateralise their future earnings is an important aspect in driving the result of zombie firms in a crisis. Finally, the cashflow constraint is replaced with a collateral constraint to show that this would moderate some of the findings.

### 1.6.1 Introducing a Labour-Intensive Sector

Assume that the economy is composed not just of a capital-intensive sector with heterogeneous producers, but also of a labour intensive sector, where all producers have access to the same level of technology. I will call the capital intensive industry *manufacturing* ($m$) and the labour intensive one *service* ($s$). The production function in the service sector is linear: $Y_s = N_s$. Assume that workers supply labour to all sectors in the economy and are the owners of the service production technology. Perfect competition and full labour mobility imply that the nominal wage in both sectors equals the marginal product of labour in the service sector, $\omega = 1$.

Both workers and entrepreneurs demand manufacturing and service products according to a Cobb-Douglas composite function, $\hat{C} = (C_m)^\gamma (C_s)^{1-\gamma}$. This assumption implies that consumer demand is allocated to the two goods depending on relative prices. Normalising the price of service goods to 1, $p_m$ is the price of manufacturing goods in terms of services. As it is standard with this type of demand function, consumers devote a constant fraction of their overall consumption expenditure to each of the goods in the consumption bundle.

$$p_m c_{mi} = \gamma p \hat{c}_i, \qquad c_{si} = (1-\gamma) p \hat{c}_i \tag{1.6.1}$$

where $p$ is the aggregate price level of the consumption bundle: $p = \frac{(p_m)^\gamma}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$. The equilibrium price of manufacturing depends on the relative productivities in the two sectors. While productivity in the service sector is fixed at 1, the manufacturing sector level of productivity is affected by how capital is distributed among producers. In a first best allocation, only high-productivity firms operate and therefore the price is negatively related to $a_h K_h$.

With a cashflow-based borrowing limit, the price of manufacturing products contributes to determining the borrowing capacity of firms: $b'_{mi} \leq \theta \left( p'_m y'_{mi} - \omega' n'_{mi} \right)$. In a credit crunch, low-productivity firms start investing and producing, so TFP in manufacturing is lower than in a first best allocation. The lower efficiency in aggregate production in turn generates a higher price of manufacturing. Correspondingly, equilibrium real wages are lower and less labour is employed, causing both sectors to be smaller than in the first best. From consumers' preferences, it is clear that the reduction in manufacturing is always larger than in services, as manufacturing products become more expensive. A binding borrowing limit therefore generates a redistribution of capital within the sector from high to low productivity firms and a sectoral redistribution of output from production in the constrained manufacturing sector to the unconstrained service sector. Both of these effects contribute to reducing the efficiency of the economy.

**Lemma 1.6.1.** *In presence of two sectors the laissez-faire allocation in a financial crisis is not second best, as the economy features over-investment in manufacturing and zombie firms. In the aftermath of the crisis, the relative size of the manufacturing sector is too large.*

*Proof.* See Appendix 1.E.1. □

In a setting with only one sector, the wage played a crucial role as it introduced a pecuniary externality connected to the borrowing limit. But in presence of another, labour-intensive sector, the nominal wage rate is uniquely pinned down by productivity in that sector: it is fixed at one and it therefore no longer gives rise to any spillover. It is now the relative price of manufacturing goods that influences the borrowing ability of productive entrepreneurs, as well as entering the agents' budget constraints.

Just like in the case of a one sector economy, the borrowing constraint for productive firms can be relaxed by increasing the value of their cashflows, through an appropriate change in prices: in this case, an increase in the price of manufacturing. Less investment from low-productivity firms boosts the value of goods produced in the sector and helps alleviating the financial friction. At the same time, a higher price of manufacturing contributes to further shrinking the relative size of the manufacturing sector compared to the service sector: better TFP within the capital-intensive sector comes at the cost of a worse resource allocation across sectors. A higher price of manufacturing is however also helpful in redistributing resources away from workers, who face a higher aggregate consumption price level, at a time when they value resources less. These two effects are strong enough to overtake the benefit of a lower price of manufacturing, consisting in a redistribution of resources away from high productivity firms in a period when they value consumption less.

**Relaxing further assumptions: a discussion**

The previous subsection has shown how even in a setting with constant wages, an externality associated with zombie investment can be at play in a framework with multiple goods, via the relative price of output. In a more general setting, however, the wage rate need not be a constant. The second sector might rely on a production function which is not constant return to scale in the labour input, or alternatively also employ capital as an input. In both these cases, and similarly to the main setting, the equilibrium wage would be a function of aggregate investment. This implies that the externality associated with the wage rate, which is shut down here, would continue to be at play, reinforcing the one that is active through the output price. In general, externalities associated with an earning-based borrowing constraint can arise in connection to either input or output prices.

Similarly, imperfect labour mobility across the two sectors would also give rise to an additional externality through the wage rate, alongside the one generated by the output price. Indeed, with segmentation, aggregate investment in the manufacturing sector would be an important determinant of the wage rate in that industry, independently of the assumptions on labour demand in the service sector. Notice also that sectoral segmentation of the labour market is unlikely to overturn the result presented in this extension, to the extent that the shock hitting the manufacturing sector would continue to be transmitted to the other sector via changes in real wages.[37]

Throughout the paper, the only price rigidity considered is an effective lower bound on the real interest rate. But because the results presented depend on the response of prices to changes in investment demand, price rigidity has the potential to affect the findings when the ELB is not binding. So long as at least one price between the wage rate and

[37] On the contrary, labour market segmentation across types of firms within a sector would matter for the results, as discussed at the end of Subsection 1.3.2.

the relative output price is not fully rigid, and prices can to some degrees respond to current conditions, the borrowing externality would still play a role, although its impact would be reduced. In practice, the way in which the price rigidity is modelled would be important in determining the optimal intervention.

Finally, entrepreneurs may have market power and be able to set prices. If entrepreneurs can choose prices, they internalise the effect that changes in their price level can have on their borrowing ability. In general however, an externality would still be present, as entrepreneurs would not internalise the effects of their decisions on the economy-wide price level, which is relevant in the aggregate.[38]

### 1.6.2 Workers Can Borrow

Assume that workers are now free to save and borrow. They will choose between these two options depending on: 1) any outstanding level of old debt they may need to repay; 2) their expectations regarding future wage earnings. Their preferences imply that workers would like to smooth out their consumption. To do so, and ignoring outstanding debt, they would save part of their wages if they expect future falls in labour income, or borrow from the future if they expect increases in their labour income. When a credit crunch hits the economy, it is not just entrepreneurs that are subject to a borrowing limit, but also workers if they are borrowers. As they do not earn any income from production, assume that they can use their labour earnings in order to obtain credit:

$$B_w' \leq \theta_w \omega' L' \tag{1.6.2}$$

For constant or increasing labour earnings and $1 < R < \beta^{-1}$, workers would like to borrow and constraint (1.6.2) may hold with equality. Vice versa, if they expect negative wage growth and have no preinstalled debt, they optimally choose to save. Whether or not the workers are constrained is important, as the second-best level of investment depends on this aspect.

**Lemma 1.6.2.** *If workers are unconstrained, then the laissez-faire economy in a financial crisis does not feature zombie firms, but under-investment as low productivity firms*

[38] As Wolf (2020) points out, more market power is associated with stronger internalisation of the pecuniary externality and reduces the need for policy intervention, at the cost of larger monopolistic distortions.

*should invest more. If workers are no less constrained than entrepreneurs, then results in Proposition 1.3.2 continue to hold.*

*Proof.* See Appendix 1.E.2 □

If workers share the same marginal rate of substitution as unconstrained entrepreneurs, then changes in wages that affect workers are irrelevant in the aggregate - they do not constitute a reason for redistribution. With unconstrained workers, there are only two spillovers arising from a reduction in wages induced by lower investment of low-productivity firms: the positive pecuniary externality of a higher value of cashflows relaxing the borrowing constraint of productive firms, and the negative externality induced from increasing the amount of resources available to productive firms at time 2, when they value resources less at the margin. As explained in section 3, this latter effect on the budget constraint of productive entrepreneurs tends to dominate, as the impact on the borrowing constraint is linked to a borrowing parameter $\theta$ which is lower than 1.

This illustrates how the result of zombie firms can be overturned if there is an additional unconstrained party in the economy: when workers optimally choose not to demand any debt, it is no longer the case that a reduction in investment of low productivity firms helps the economy. On the contrary, increasing investment of low-productivity firms boosts the wage rate and helps redistributing resources away from high-productivity firms at time 2, when they have a lower valuation of consumption. In general, it is not the specific details of what workers do in the model that matters for this result, but rather the presence or absence of other constrained agents in the economy.

### 1.6.3 The Role of the Type of Financial Friction: A Collateral Constraint

Much of the literature on macroprudential policy and pecuniary externality is not based on a cashflow constraint as in (1.2.3), but rather on a collateral constraint,[39] such as:

$$b'_i \leq q'_h h'_i \tag{1.6.3}$$

[39] See e.g. Lorenzoni (2008), Bianchi and Mendoza (2018) etc.

where $q'_h$ is the price of a fixed asset that can be used as collateral, $h'_i$ the quantity of collateral available to entrepreneur $i$, and where the full value of collateral is assumed to be recouped by lenders in case of default.[40]

I now proceed to alter the main model to introduce a fixed asset. To this end, I also have to consider an additional time period, $t = 3$, so as to ensure that the borrowing limit at time 1 can feature a price of the asset at date 2 which is well defined.[41] Assume that every period entrepreneurs have to decide how to allocate their investment between two different types of capital: a fixed asset $h_i$ and physical capital $x_i$. The fixed asset can be thought of as land; it is available in positive fixed supply in the economy, and can be bought and sold at price $q_h$. The physical capital represents machines, which can be invested in one period before production, where conversion from output to machine occurs one-for-one. The physical capital is assumed to fully depreciate every period. Together, these form the stock of capital necessary to operate a firm: $k'_i \equiv (x'_i)^\delta (h'_i)^{1-\delta}$, with $\delta$ and $1-\delta$ the respective share of physical and fixed capital usage in the aggregate capital bundle. Similarly to the intratemporal allocation of consumption demand, the optimal demands for land and machines are the following:

$$x'_j = \delta q k'_j, \qquad uh'_j = (1-\delta) q k'_j \tag{1.6.4}$$

with $q$ the price of the aggregate investment bundle: $q = \frac{u^{1-\delta}}{\delta^\delta (1-\delta)^{1-\delta}}$. $u = q_h - \frac{q'_h}{R}$ represents the per-period user cost of the fixed asset, as the future resale value of the asset is netted from its purchasing price. Entrepreneurs choose to employ constant fractions of overall investment in the two forms of capital, where the fractions are pinned down by the respective shares in the capital stock bundle. Even in presence of a collateral constraint, the demands for investment remain the same, so long as the collateral parameter is set to one.

One can obtain the price of the fixed asset at period 2 by combining the market clearing condition for output together with the terminal condition $q_3 = 0$, and the optimal relative ratio of physical capital to fixed asset, which can be derived from the two relationship in

[40] A collateral parameter $\phi$ could be introduced if the value of the collateral cannot be entirely recouped. Here, having a fully collateralisable asset is especially convenient. Contrarily to the case of an earning-based constraint, the efficiency of the economy is affected even when $\phi = 1$.

[41] In the absence of a date 3 there can be no trade in the asset at date 2, as every entrepreneur would want to sell, but no one would want to buy.

1.6.4. The price of land is a function of output at period 2:

$$q_2 = \frac{\hat{\beta}_2(1-\delta)}{1-\hat{\beta}_2(1-\delta)}\alpha Y_2 \tag{1.6.5}$$

In a similar way, using Equation 1.6.5 one can also derive an expression for the asset price at time 1, and so on recursively.

**Lemma 1.6.3.** *A credit crunch generated by a collateral constraint, away from the ELB, induces no change in aggregate output but changes in consumption and investment demand at time 1; aggregate productivity and production fall at date 2.*

*Proof.* See Appendix 1.E.3. □

Production only depends on pre-installed capital and employment, and the choice of employment is not altered by the presence of a fixed asset, hence production remains constant at time 1 when the credit crunch hits the economy. Nevertheless, the value of land changes to reflect 1) the future recession; and 2) the additional benefit of land holding, which now helps relaxing borrowing conditions in presence of a collateral constraint. As these two forces move the asset price in opposite directions, it is in principle not possible to state clearly what happens to it. In any case, the time varying nature of the asset price influences the net worth of entrepreneurs, thereby affecting equilibrium consumption and investment. In what follows, a financial crisis at time 1 is defined as a situation where the collateral constraint (1.6.3) becomes binding.

### Interventions During a Collateral-Constraint-Type of Financial Crisis

**Proposition 1.6.1.** *The laissez-faire allocation during a credit crunch induced by a collateral constraint is not constrained efficient, as the choice of investment of low-productivity firms can be too large or too small.*

*Proof.* See Appendix 1.E.4. □

A collateral constraint involves the price of land, which, as shown in Equation (1.6.5), depends on the aggregate level of output in the same period. The price $q_2^h$ is affected by the investment of low-productivity firms in differential ways, depending if physical or fixed capital is considered. Because land is in fixed supply, an increase in the fraction that is operated by low-productivity firms necessarily entails a reduction in the land operated by productive firms. This redistribution lowers the average productivity and hence the value of the land. On the contrary, a marginal increase in the physical capital demanded by low productivity firms does not necessitate any redistribution. It boosts the value of the land, allowing high-productivity firms to have access to more borrowing. In this sense, by demanding physical capital, less efficient entrepreneurs generate a positive externality on the borrowing capacity of productive firms, which the planner internalises.

To show this, consider the choice of physical investment of low productivity firms for the planner:[42]

$$1 - \mathrm{MRS}'(\ell)\mathrm{MPK}'(\ell) = \tilde{\mu} H_h' \frac{\partial q_h'}{\partial X_\ell'} + \Delta MRS'(\ell, h) \left( \Delta H_h'' \frac{\partial q_h'}{\partial X_\ell'} + N_h' \frac{\partial \omega'}{\partial X_\ell'} \right)$$

With $\frac{\partial q_h'}{\partial X_\ell'} > 0$ and $\Delta H_h'' \equiv H_{h3} - H_{h2}$. The left hand side corresponds to the optimal choice in the decentralised equilibrium. The planner, however, also internalises the effect of a larger aggregate investment from low-productivity firms on the collateral value, which relaxes the borrowing constraint. Moreover, there are effects connected to the distance in MRSs. The change in collateral price affects the net purchases or sales of land, which in turn depend on whether the financial friction eases, stays the same or becomes more stringent in the following period. Because this effect cannot be signed in general, it is hard to establish the direction of intervention in this respect. Just like with a cashflow constraint, larger investment by low-productivity firms also increases the wage rate, redistributing resources away from productive firms at time 2, which helps the economy at time 1.

Finally, whether or not workers are constrained may tilt the intervention in favour of less investment by low productivity firms, but it is not in general sufficient to establish the sign of the wedge between the planner and private entrepreneurs. This means that while the presence of a collateral constraint gives rise to the potential for intervention of a policy maker, it is not possible to conclude in general which direction such intervention should take. When workers are constrained and productive firms are future net sellers of capital, it is likely that the result on zombie firms may carry over to this case. Vice versa,

[42]See Appendix 1.E.4 for how the asset price depends on fixed asset investment of low-productivity entrepreneurs.

if workers are unconstrained and productive firms are buying or keeping their fixed asset, it is possible that an increase in investment by low productivity firms may improve the efficiency of the allocation.[43]

## 1.7 Conclusions

This paper presented a model of a credit crunch generating low aggregate productivity, where a lower bound on the interest rate can induce a demand rationing. This underlines the tension between the policy objectives of efficient demand and capital allocation. From an ex-ante perspective, larger amounts of corporate bonds help both ensuring financial stability in presence of a cashflow-based borrowing constraint, as well as moderating the impact of the effective lower bound on interest rate. While there may be other reasons why restricting debt in normal times may help moderate the impact of recessions, this work introduced one possible mechanism where larger corporate debt is helpful. In drafting financial policies, it is therefore important to keep in mind the desired target subject of such policies, and consider the potential interactions with the productive side of the economy. From an ex-post point of view, there can be a trade-off between boosting demand by letting less efficient but unconstrained firms operate and avoiding spillovers from these low-efficiency firms. Crucially, in a liquidity trap, the negative spillovers are dwarfed by demand-side concerns. For this result, two aspects are particularly important: the type of borrowing constraint, whether of a cashflow or collateral nature; and whether it is only one or multiple sectors of the economy that are subject to financial frictions.

There are various aspects of this work that could be further extended and explored, in order to look at other related questions of interest. As an example, the model could be altered to include nominal frictions, hence introducing a monetary authority facing a different objective function than a social planner. A conflict between these two authorities could potentially arise when their incentives are in contrast with each other. Coordination or lack thereof in a game between the two authorities would then play a role. Another interesting avenue of research could be the exploration of the long run consequences of an imperfect distribution of firms. In the long run, low-productivity firms are likely to generate additional and potentially larger costs for society, in the form of lower aggregate

[43] In a different environment, Lanteri and Rampini (2021) show that in a calibrated model, the distributive externality associated with fire-sales of capital tends to dominate the borrowing externality.

growth and stifled innovation. Finally, one could consider how to best implement the constrained efficient allocation in a decentralised economy. Both from a practical and political point of view, implementing ex-ante versus ex-post policies might have very different impacts, which should also be considered.

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# Appendix

## 1.A Main Model: Analytical Derivations

Throughout the paper and the appendix, the following definitions will be used:

$$\hat{\beta}_t \equiv 1 - \mathbb{1}_{t<T}\left(\frac{1}{\sum_{s=0}^{T-t}\beta^s}\right), \quad \hat{\alpha} \equiv \alpha^\alpha(1-\alpha)^{1-\alpha}, \quad \xi \equiv \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\psi}}$$

$$\hat{a}_i \equiv a_i^{1/\alpha}, \quad \tilde{a} \equiv \frac{\hat{a}_h}{\hat{a}_\ell}, \quad A_{i1} \equiv \frac{(1-\theta_1)\hat{a}_i}{\hat{a}_l - \theta_1\hat{a}_i}$$

### 1.A.1 Market clearing

$$\text{Aggregate employment:} \quad N_t = \pi_h n_{ht} + \pi_\ell n_{\ell t} = L_t \tag{1.A.1}$$

$$\text{Net worth:} \quad Z_t = \alpha Y_t \tag{1.A.2}$$

$$\text{Production goods:} \quad C_t + \pi_h c_{ht} + \pi_\ell c_{\ell t} + \pi_h K_{ht+1} + \pi_\ell K_{\ell t+1} = \pi_h y_{ht} + \pi_\ell y_{\ell t}$$

$$\iff \quad Z_t + \omega_t L_t = \frac{\omega_t N_t}{1-\alpha} \tag{1.A.3}$$

### 1.A.2 First Best Allocation

Optimal choices for workers and entrepreneurs imply:

$$L_t^\psi = \omega_t \tag{1.A.4}$$

$$C_t = \omega_t L_t \tag{1.A.5}$$

$$c_{it} = (1-\hat{\beta}_t)z_{it} \tag{1.A.6}$$

$$K_{t+1} = K_{ht+1} = \hat{\beta}_t Z_t \tag{1.A.7}$$

$$\frac{1}{R_t} = \frac{\beta c_{it}}{c_{jt+1}} \tag{1.A.8}$$

$$R_t = \hat{a}_h\left(\frac{\hat{\alpha}}{\omega_{t+1}^{1-\alpha}}\right)^{1/\alpha} \tag{1.A.9}$$

$$\omega_t = (1-\alpha)\left(\frac{\hat{a}_h k_{ht}}{n_{ht}}\right)^\alpha \tag{1.A.10}$$

$$Z_{t+1} = \hat{\beta}_t R_t Z_t \tag{1.A.11}$$

Plugging (1.A.10) and (1.A.4) in (1.A.1), one can solve for the equilibrium wage and solve for all prices and quantities:

$$\omega_t = \left[(1-\alpha)\left(\hat{a}_h K_{ht}\right)^{\alpha}\right]^{\frac{\psi}{\psi+\alpha}} \tag{1.A.12}$$

$$L_t = \left[(1-\alpha)(\hat{a}_h K_{ht})^{\alpha}\right]^{\frac{1}{\psi+\alpha}} \tag{1.A.13}$$

$$Z_t = \frac{\alpha}{1-\alpha}\left[(1-\alpha)(\hat{a}_h K_{ht})^{\alpha}\right]^{\frac{1+\psi}{\psi+\alpha}} \tag{1.A.14}$$

$$Z_{ht} = Z_t - B_t, \quad Z_{\ell t} = B_t \tag{1.A.15}$$

$$R_t = \alpha \hat{a}_h \left[\frac{1-\alpha}{(\hat{a}_h K_{ht+1})^{\psi}}\right]^{\frac{1-\alpha}{\psi+\alpha}} \tag{1.A.16}$$

With consumption of workers and entrepreneurs set according to (1.A.5), (1.A.6) and capital as in (1.A.7).

#### Description of first best allocation

The allocation is first best when no financial friction affects the economy: entrepreneurs are able to borrow as much as they wish, and workers can access financial markets. In this case, the low productivity entrepreneurs prefer to become financiers, extending loans to the high productivity entrepreneurs to run their firms. The high productivity technology provides a return proportional to $a_h$, and productive firms have to offer this return on any loans they take out, in order for the financial market to be in equilibrium.[44] Therefore, by extending loans to the high productivity entrepreneurs, the financiers have access to a higher return than they could achieve by operating their own technology. The constant return to scale technology implies that only high-productivity entrepreneurs run firms, while no low-productivity entrepreneur is active. Additionally, entrepreneurs and workers are able to transfer resources intertemporally at the same rate. This implies that their marginal rates of substitution (MRSs) are equated:

$$\beta \frac{c_h}{c_h'} \equiv \text{MRS}'(h) = \text{MRS}'(\ell) = \text{MRS}'(w) = R^{-1}$$

[44] A higher interest rate would lead to zero loan demand, as it would generate losses on each unit borrowed. A lower interest rate causes an infinite demand for borrowing as firms could make positive profits that way.

Employment demand is efficient, and the equilibrium wage rate ensures that labour demand and labour supply are equal. Production productivity is affected by how capital is distributed among entrepreneurs. Because only high-productivity firms operate, the TFP in this economy is high at $a_h$.

### 1.A.3 Financially Constrained Allocation

Conditions (1.A.4) to (1.A.6) remain valid. However, the other optimal choices are now replaced by the following conditions:

$$\omega_t = (1-\alpha)\left(\frac{\hat{a}_h k_{ht}}{n_{ht}}\right)^{\alpha} = (1-\alpha)\left(\frac{\hat{a}_\ell k_{\ell t}}{n_{\ell t}}\right)^{\alpha} \tag{1.A.17}$$

$$K_{ht+1} = \frac{\hat{\beta}_t\,(Z_t - B_t)}{1-\theta\tilde{a}} \tag{1.A.18}$$

$$K_{\ell t+1} = \hat{\beta}_t Z_t - K_{ht+1} = \frac{\hat{\beta}_t\,(B_t - \theta\tilde{a}Z_t)}{1-\theta\tilde{a}} \tag{1.A.19}$$

$$\frac{1}{R_1} = \frac{\beta A_i c_{i1}}{c_{i2}} \tag{1.A.20}$$

$$R_1 = \hat{a}_\ell \left(\frac{\hat{\alpha}}{\omega_2^{1-\alpha}}\right)^{1/\alpha} \tag{1.A.21}$$

$$Z_{t+1} = \hat{\beta}_t R_t\,(A_h Z_{ht} + Z_{\ell t}) \tag{1.A.22}$$

$$\omega_t = \left[(1-\alpha)\left(\sum \hat{a}_i K_{it}\right)^{\alpha}\right]^{\frac{\psi}{\psi+\alpha}} \qquad R_t = \alpha\hat{a}_l\left[\frac{1-\alpha}{(\sum \hat{a}_i K_{it+1})^{\psi}}\right]^{\frac{1-\alpha}{\alpha+\psi}} \tag{1.A.23}$$

$$Z_t = \frac{\alpha}{1-\alpha}\left[(1-\alpha)\left(\sum \hat{a}_i K_{it}\right)^{\alpha}\right]^{\frac{1+\psi}{\psi+\alpha}} \tag{1.A.24}$$

$$Z_{ht} = Z_t\left[\frac{(1-\theta_t)\hat{a}_h K_{ht}}{\sum \hat{a}_i K_{it}}\right] \qquad Z_{\ell t} = Z_t\left[\theta_t + \frac{(1-\theta_t)\hat{a}_\ell K_{\ell t}}{\sum \hat{a}_i K_{it}}\right] \tag{1.A.25}$$

## 1.B A Financial Crisis

**Lemma 1.B.1.** *If $\theta = 1$ the allocation is first best.*

### 1.B.1 Proof of lemma 1.B.1

Assume $\theta_1 = 1$. First order conditions for capital and savings for entrepreneurs are:

$$\frac{1}{R_1} = \beta\frac{c_{i1}}{c_{i2}} + c_{i1}\mu_{i1}$$
$$1 = \left(\frac{\beta c_{i1}}{c_{i2}} + c_{i1}\mu_{i1}\right)\frac{\partial y_{i2}}{\partial k_{i2}}$$

Assume by contradiction that the borrowing limit is binding and both agents engage in production. It must then be that:

$$R_1 = \alpha a_h\left(\frac{n_{h2}}{k_{h2}}\right)^{1-\alpha} = \alpha a_\ell\left(\frac{n_{\ell 2}}{k_{\ell 2}}\right)^{1-\alpha} \implies \hat{k}_{h2} = \left(\frac{a_h}{a_\ell}\right)^{\frac{1}{1-\alpha}}\hat{k}_{\ell 2}$$

But from the optimal choice of labour:

$$\omega_2 = (1-\alpha)a_h\left(\frac{k_{h2}}{n_{h2}}\right)^{\alpha} = (1-\alpha)a_\ell\left(\frac{k_{\ell 2}}{n_{\ell 2}}\right)^{\alpha} \implies \hat{k}_{h2} = \left(\frac{a_\ell}{a_h}\right)^{\frac{1}{\alpha}}\hat{k}_{\ell 2}$$

This leads to a contradiction as the capital-labour ratio implied by the optimal choice of labour differs from the one implied by the optimal choice of capital. It must be that the borrowing constraint is not binding when $\theta = 1$ and that high productivity entrepreneurs are the only active firms. □

### 1.B.2 Proof of Proposition 1.3.1

*A financial crisis induces no changes in aggregate output at date 1 but lower productivity and production at date 2.*

**Unchanged output at time 1.** Both demand and supply of labour when the shock hits the economy continue to be set according to (1.A.10) and (1.A.4). Labour market clearing then implies that both wages and aggregate employment are unchanged. For given level of capital, the level of output is therefore unchanged. This also implies that entrepreneurs' net worth in the period stays the same. Therefore, from consumption function (1.A.6), we know consumption not just for workers but also for entrepreneurs stays constant. The only effect of a borrowing limit at time 1 is then on capital demand, where (1.A.7) is replaced with (**??**), while total capital demanded stays constant.

**Lower TFP and output at time 2.** In a first best allocation, TFP is equal to $a_h$. Investment is entirely carried out by $h$ firms: $K_{ht} = K_t$. When a borrowing constraint binds, the economy's TFP is:[45]

$$\text{TFP}_2^c = \left( \frac{\hat{a}_h K_{h2}^c + \hat{a}_\ell K_{\ell 2}^c}{K_2^c} \right)^\alpha < a_h$$

given $K_{h2} \leq K_2$. Aggregate output can be rewritten as a function of just capital:

$$Y_t^* = \left[ (1-\alpha)^{1-\alpha} (\hat{a}_h K_t^*)^{\alpha(1+\psi)} \right]^\epsilon \qquad Y_t^c = \left[ (1-\alpha)^{1-\alpha} (\text{TFP}_t^c (K_t^c)^\alpha)^{1+\psi} \right]^\epsilon$$

with $\epsilon = \frac{1}{\psi+\alpha}$. Given $a_h > \text{TFP}_2^c$, a sufficient condition for $Y_2^* > Y_2^c$ is that $K_2^* \geq K_2^c$. As the aggregate quantity of investment in the constrained and unconstrained case is the same at $K_2^* = \hat{\beta}_1 Z_1^* = K_2^c$, this shows that output and TFP are both lower in the aftermath of the crisis. $\square$

### 1.B.3 Only capital used in production

*When physical capital is the only input in production, the allocation at time 1 is constrained efficient.*

$$\begin{aligned} & V_{i2}(z_{i2}; B_2, K_{i2}, K_{j2}) = \max \ \log c_{i2} \\ \text{s.to} \quad & c_{i2} = d_{i2} - b_{i2} \end{aligned}$$

where $d_{i2} = a_i k_{i2}$.

$$\frac{\partial V_{i2}}{\partial K_{i2}} = \lambda_{i2} \frac{\partial D_{i2}}{\partial K_{i2}}, \qquad \frac{\partial V_{i2}}{\partial K_{j2}} = 0, \qquad \frac{\partial V_{i2}}{\partial B_2} = -\lambda_{i2}$$

$$\text{with:} \qquad \lambda_{i2} = \frac{1}{C_{i2}}$$

[45] An asterisk superscript is used to indicate the first best allocation, while $c$ is used to indicate the constrained allocation.

**Planner's problem**

$$
\begin{aligned}
V_1^P(Z_1,S_1) = \max_{C_1,c_{i1}K_{i2},B_2} & \sum_{i\in h,l} \chi_i\pi_i \left[\log c_{i1} + \beta V_{i2}(z_{i2};B_2,K_{h2},K_{\ell 2})\right] \\
\text{s.to} \quad & \sum_{i\in h,l} \pi_i c_{i1} + K_{h2} + K_{\ell 2} = Y_1 && [\tilde{\lambda}_1] \\
& B_2 \leq \theta_1 D_{h2} && [\tilde{\mu}_1]
\end{aligned}
$$

$$
\begin{aligned}
c_{i1}: &\quad \frac{\chi_i}{c_{i1}} = \lambda_1 \\
B_2: &\quad \mu_1 = \beta \sum_i \chi_i\pi_i \frac{\partial V_{i2}}{\partial B_2} \\
K_{i2}: &\quad \lambda_1 = \beta \sum_j \chi_j\pi_j \frac{\partial V_{j2}}{\partial K_{i2}} + \theta_1\mu_1 \frac{\partial D_{h2}}{\partial K_{i2}}
\end{aligned}
$$

Combining these with the expressions obtained above, one can show the allocation chosen by the planner corresponds to the decentralised allocation. □

### 1.B.4 Proof of proposition 1.3.2

*Compared to a constrained efficient allocation, the laissez-faire economy features over-invesment an zombie firms; output at time 2 is too high.*

$$
\begin{aligned}
& V_{i2}(z_{i2};B_2,K_{i2},K_{j2}) = \max \ \log c_{i2} \\
\text{s.to} \quad & c_{i2} = d_{i2} - b_{i2}
\end{aligned}
$$

$$
\begin{aligned}
& W_2(B_2,K_{h2},K_{\ell 2}) = \max \ \log \tilde{C}_2 \\
\text{s.to} \quad & C_2 = \omega_2 L_2 - b_2^w, \qquad \tilde{C}_2 \equiv C_2 - v(L_2)
\end{aligned}
$$

where: $$d_{i2} = \hat{a}_i k_{i2}^{\alpha}(n_{i2})^{1-\alpha} - \omega_2 n_{i2}, \qquad \omega_2 = \left[(1-\alpha)\left(\sum \hat{a}_i K_{i2}\right)^{\alpha}\right]^{\frac{\psi}{\psi+\alpha}}.$$

$$
\frac{\partial V_{i2}}{\partial K_{j2}} = \lambda_{i2}\left[\mathbb{1}_{i=j}\left(\frac{\partial Y_{i2}}{\partial K_{j2}}\right) - N_{i2}\frac{\partial \omega_2}{\partial K_{j2}}\right], \qquad \frac{\partial V_{i2}}{\partial B_{j2}} = -\mathbb{1}_{i=j}\lambda_{i2}
$$

$$\frac{\partial W_2}{\partial K_{j2}} = \lambda_2^w L_2 \frac{\partial \omega_2}{\partial K_{j2}}, \qquad \frac{\partial W_2}{\partial B_2^w} = -\lambda_2^w$$

$$\text{with:} \quad \frac{\partial \omega_2}{\partial K_{j2}} = -\frac{\alpha\psi}{\psi+\alpha}\frac{\omega_2 \hat{a}_j}{\hat{a}_h K_{h2} + \hat{a}_\ell K_{\ell 2}}$$

**Planner's problem**

$$V_1^P(Z_1, S_1) = \max_{C_1, c_{i1} K_{i2}, B_2} \left\{ \log\left(\tilde{C}_1\right) + \beta W_2(K_{h2}, K_{\ell 2}) + \sum_{i \in h,l} \chi_i \pi_i \left[\log c_{i1} + \beta V_{i2}(z_{i2}; B_2, K_{h2}, K_{\ell 2})\right] \right\}$$

$$\begin{aligned}
\text{s.to} \quad & \sum_{i \in h,l} \pi_i (c_{i1} + k_{i2}) + \tilde{C}_1 = Y_1 - v(L_1) && [\tilde{\lambda}_1] \\
& B_{h2} + B_{\ell 2} + B_2^w = 0 \\
& B_{h2} + B_2^w \le \theta_1 D_{h2} && [\tilde{\lambda}_1 \tilde{\mu}_1] \\
& B_2^w = 0 && [\tilde{\lambda}_1 \tilde{\nu}_1]
\end{aligned}$$

$$\begin{aligned}
B_{i2}, B_2^w: \quad & \tilde{\mu}_1 = \beta\left(\frac{C_{\ell 1}}{C_{\ell 2}} - \frac{C_{h1}}{C_{h2}}\right), \qquad \tilde{\mu}_1 + \tilde{\nu}_1 = \beta\left(\frac{C_{\ell 1}}{C_{\ell 2}} - \frac{\tilde{C}_1}{\tilde{C}_2}\right) \\
K_{i2}: \quad & \tilde{\lambda}_1 = \beta\left(\sum_j \chi_j \pi_j \frac{\partial V_{j2}}{\partial K_{i2}} + \frac{\partial W_2}{\partial K_{i2}}\right) + \theta_1 \tilde{\mu}_1 \frac{\partial D_{h2}}{\partial K_{i2}}
\end{aligned}$$

Focusing on unconstrained choice of investment for low-productivity entrepreneurs:

$$\begin{aligned}
K_{\ell 2}: \quad 1 &= \frac{\beta C_{\ell 1}}{C_{\ell 2}}\frac{\partial Y_{\ell 2}}{\partial K_{\ell 2}} - \tilde{\mu}_1 \theta_1 N_{h2}\frac{\partial \omega_2}{\partial K_{\ell 2}} + \beta\left(\frac{C_{\ell 1}}{C_{\ell 2}} - \frac{C_{h1}}{C_{h2}}\right) N_{h2}\frac{\partial \omega_2}{\partial K_{\ell 2}} - \beta\left(\frac{C_{\ell 1}}{C_{\ell 2}} - \frac{\tilde{C}_1}{\tilde{C}_2}\right) L_2 \frac{\partial \omega_2}{\partial K_{\ell 2}} \\
1 &= \frac{\beta C_{\ell 1}}{C_{\ell 2}}\frac{\partial Y_{\ell 2}}{\partial K_{\ell 2}} - \tilde{\mu}_1\left[L_2 - (1-\theta_1)N_{h2}\right]\frac{\partial \omega_2}{\partial K_{\ell 2}} - \tilde{\nu}_1 L_2 \frac{\partial \omega_2}{\partial K_{\ell 2}}
\end{aligned}$$

One can show that $L_2 - (1-\theta_1)N_{h2} > 0$ by noticing that $L_2 - N_{h2} = N_{\ell 2} > 0$. Therefore:

$$\begin{aligned}
1 &= \frac{\beta C_{\ell 1}}{C_{\ell 2}}\frac{\partial Y_{\ell 2}}{\partial K_{\ell 2}} - \tau_{\ell 2} \\
(1+\tau_{\ell 2})\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}}\right)^{-1} &= \frac{\partial Y_{\ell 2}}{\partial K_{\ell 2}}
\end{aligned}$$

$$\text{where} \qquad \tau_{\ell 2} \equiv \left\{\tilde{\mu}_1 \left[L_2 - (1-\theta_1)N_{h2}\right] + \tilde{\nu}_1 L_2\right\} \frac{\partial \omega_2}{\partial K_{\ell 2}} > 0$$

This shows that $K_{\ell 2}^{\text{sp}} < K_{\ell 2}^{\text{c}}$, which implies a lower level of production at time 2. In turn, the lower capital investment reduces wages and allows productive firms to invest and produce more in the future. □

## 1.C The Effective Lower Bound

### 1.C.1 Proof of proposition 1.4.1

*A financial crisis where the interest rate is at the ELB generates lower employment and output at date 1.*

$$d_{h1} = \max_{n_{h1}} \; y_{h1} - w_1 n_{h1} \qquad \text{s.to} \quad y_{h1} \leq \frac{C_{h1} + C_{\ell 1} + C_1 + K_{h2} + K_{\ell 2}}{\pi_h} \qquad \text{if } R_1 = 1$$

At the lower bound, investment demand of unconstrained entrepreneurs is lower:

$$\begin{aligned} K_{\ell 2}: &\qquad \rho = \alpha a_\ell \left(\frac{1}{\hat{K}_{\ell 2}}\right)^{1-\alpha} \\ N_{\ell 2}: &\qquad a_\ell \left(\hat{K}_{\ell 2}\right)^{\alpha} = \frac{\omega_2}{1-\alpha} = a_h \left(\hat{K}_{h2}\right)^{\alpha} \end{aligned}$$

Combining the two one obtains $\omega_2 = \left(\frac{\hat{\alpha} a_\ell}{\rho}\right)^{\frac{1}{1-\alpha}}$. But from labour market clearing, we have:

$$\omega_2 = \left[(1-\alpha)\left(\sum_i \hat{a}_i K_{i2}\right)^{\alpha}\right]^{\frac{\psi}{\alpha+\psi}}$$

The choice of capital of the productive entrepreneurs continues to be constrained, and equal to $K_{h2} = \frac{\hat{\beta}_1 Z_{h1}}{1-\theta\tilde{a}}$. The level of investment of low productivity entrepreneurs can be found by combining the expressions above to obtain:

$$K_{\ell 2} = \Omega(\rho) - \frac{\hat{\beta}_1 \tilde{a}\left(Z_1 - B_1\right)}{1-\theta\tilde{a}}$$

with $\Omega(\rho) = \left[\frac{\hat{\alpha} a_\ell}{\rho}\right]^{\frac{\alpha+\psi}{\psi\alpha(1-\alpha)}} \frac{1}{\hat{a}_\ell(1-\alpha)^{1/\alpha}}$ a function of parameters decreasing in $\rho$, the ELB. If $\rho$ could fall to the equilibrium level of interest rate $R^*$, then the above expression for investment would be higher, such that aggregate capital investment corresponds to the efficient level. Because of the ELB, however, $K_{\ell 2}$ is lower. Furthermore, as a consequence of this lower investment demand, consumption of the low-productivity entrepreneurs is also lower:

$$C_{\ell 1} = \frac{C_{\ell 2}}{\beta\rho} = \frac{Z_{\ell 2}}{\beta\rho} = \frac{\alpha\left(Y_{\ell 2} + \theta Y_{h2}\right)}{\beta\rho} = \frac{K_{\ell 2} + \theta\tilde{a}K_{h2}}{\beta}$$

As a result of lower capital and lower consumption demanded, the level of employment at date 1 needs to be lower to ensure that all output produced corresponds to the aggregate amount of resources demanded:

$$\begin{aligned} & Y_h - \omega N_h = C_h + C_\ell + K'_h + K'_\ell \\ N_h: \qquad & \hat{a}_h K_h^\alpha N_h^{1-\alpha} - \omega N_h = \frac{\Omega(\rho)}{\hat{\beta} A_h} + \frac{\tilde{a}-1}{(1-\theta)\tilde{a}} B \end{aligned}$$

Due to $\rho > R^*$, the level of employment chosen is lower than away from the lower bound. The level of wage is also depressed, as it is set to ensure that labour supplied $L^\psi = \omega$ equals labour demanded in the expression above.

*A financial crisis where the interest rate is at the ELB generates better TFP but lower capital and production in manufacturing at* $t = 2$

First note that aggregate capital at the ELB is lower, due to both a lower $K'_\ell$, and the lower $Z_1$ inducing a lower $K'_h$. As for aggregate productivity:

$$\text{TFP}' = \left(\frac{\hat{a}_h K'_h + \hat{a}_\ell K'_\ell}{K'}\right)^\alpha = \left[\hat{a}_h - (\hat{a}_h - \hat{a}_\ell)\frac{K'_\ell}{K'}\right]^\alpha$$

One can show that $\frac{K'_\ell}{K'_h + K'_\ell}$ is lower in a liquidity trap due to the fall in $K'_\ell$ being larger than the fall in $K'_h$; as a result TFP increases. Finally, notice that in equilibrium output is a function of productivity and capital invested:

$$Y' = f(\text{TFP}' \cdot (K')^\alpha) = f(\hat{a}_h K'_h + \hat{a}_\ell K'_\ell)$$

As both $K'_h$ and $K'_\ell$ are lower, output at time 2 is lower although aggregate productivity is larger. □

### 1.C.2 Proof of proposition 1.4.2

*During a financial crisis where the economy is at the lower bound, the allocation at time 1 is constrained efficient and requires no planner's intervention.*

First note that derivatives with respect to $K_{i2}$ and $B_2$ of individuals' problem at time 2 are the same as in appendix 1.B.4. At time 1, the social planner's problem is:

$$V_1^P(K_h, K_\ell; \theta) = \max_{c_i, L, K'_i, B'_i} \left\{ \sum_{i \in h,l} \chi_i \pi_i \left[\log c_i + \beta V_{i2}(z'_i; K'_h, K'_\ell, B')\right] + \right. \\ \left. + \log \tilde{C} + \beta W_{t+1}(K'_h, K'_\ell, B') \right\}$$

$$\begin{aligned} \text{subject to} \qquad & \sum_{i \in h,l} \pi_i (c_i + K'_i) = a_h K^\alpha L^{1-\alpha} - L^{1+\psi} && (\lambda) \\ & B' \leq \theta (Y'_h - \bar{\omega}' N'_h) && (\lambda\tilde{\mu}) \\ & C_\ell \leq \frac{K'_\ell + \theta \tilde{a} K'_h}{\beta} && (\lambda\gamma) \qquad (1.C.1) \\ & \bar{\omega}' = (\hat{\alpha} a_\ell)^{\frac{1}{1-\alpha}} && \qquad (1.C.2) \end{aligned}$$

The optimal choice of debt is unchanged. As for capital:

$$K_{i2}: \quad \lambda_1 = \beta \left( \sum_j \chi_j \pi_j \frac{\partial V_{j2}}{\partial K_{i2}} + \frac{\partial W_2}{\partial K_{i2}} \right) + \theta_1 \lambda_1 \tilde{\mu}_1 \underbrace{\frac{\partial D_{h2}}{\partial K_{i2}}}_{=0} + [1 - \mathbb{1}_{i=h}(1 - \theta \tilde{a})] \frac{\lambda_1 \gamma_1}{\beta}$$

So long as $R$ is at the lower bound, it is not just the investment of the productive firms that is constrained, but also that of the low-productivity entrepreneurs. So while there are effects of a change in $K'_\ell$ that are not internalised, connected to constraint (1.C.1), the planner has no way of improving on the decentralised allocation, because the amount of capital demanded is pinned down by constraint (1.C.2). $\square$

# 1.D Interventions Before the Crisis

## 1.D.1 Proof of proposition 1.5.1

*The allocation before a financial crisis is not constrained efficient. Active firms in the decentralised economy issue too few bonds and they over-invest.*

$$\begin{aligned}
&W_1(B_1, K_1) = \max \ \log \tilde{C}_1 + \beta W_2(S_2) \\
&\text{s.to} \qquad \tilde{C}_1 = \omega_1 L_1 - v(L_1), \qquad \tilde{C}_1 \equiv C_1 - v(L_1)
\end{aligned}$$

$$\begin{aligned}
&V_{i1}(z_{i1}; B_1, K_1) = \max \ \log c_{i1} + \beta V_{i2}(z_{i2}; S_2) \\
\text{s.to} \qquad &c_{i1} + k_{i2} - \frac{b_{i2}}{R_1} = d_{i1} - b_{i1} \\
&b_{h2} \leq \theta_1 d_{h2}
\end{aligned}$$

$$\frac{\partial W_1}{\partial B_1} = \beta \frac{\partial W_2}{\partial B_1} + \lambda_1^w L_1 \frac{\partial \omega_1}{\partial B_1}, \tag{1.D.1}$$

$$\frac{\partial W_1}{\partial K_1} = \beta \frac{\partial W_2}{\partial K_1} + \lambda_1^w L_1 \frac{\partial \omega_1}{\partial K_1} \tag{1.D.2}$$

$$\frac{\partial V_{i1}}{\partial B_1} = \beta \frac{\partial V_{i2}}{\partial B_1} + \lambda_{i1} \left[ \frac{\partial D_{i1}}{\partial B_1} - \left( 1 + \frac{B_{i2}}{R_1^2} \frac{\partial R_1}{\partial B_1} + \frac{\partial K_{i2}}{\partial B_1} \right) \right] + \mathbb{1}_{i=h} \mu_{i1} \theta_1 \frac{\partial D_{h2}}{\partial B_1} \tag{1.D.3}$$

$$\frac{\partial V_{i1}}{\partial K_1} = \beta \frac{\partial V_{i2}}{\partial K_1} + \lambda_{i1} \left[ \frac{\partial D_{i1}}{\partial K_1} - \frac{B_{i2}}{R_1^2} \frac{\partial R_1}{\partial K_1} - \frac{\partial K_{i2}}{\partial K_1} \right] + \mathbb{1}_{i=h} \mu_{i1} \theta_1 \frac{\partial D_{h2}}{\partial K_1} \tag{1.D.4}$$

$$\text{with:} \qquad \lambda_{h1} = \frac{1}{C_{h1}}, \quad \lambda_{\ell 1} = \frac{1}{C_{\ell 1}}, \quad \lambda_1^w = \frac{1}{C_1}$$

$$V_0^P(Z_0, S_0) = \max_{\tilde{C}_0, c_{i0}, K_1, B_1} \left\{ \log \left( \tilde{C}_0 \right) + \beta W_1(B_1, K_1) + \sum_{i \in h,l} \chi_i \pi_i \left[ \log c_{i0} + \beta V_{i1}(z_{i1}; B_1, K_1) \right] \right\}$$

$$\text{s.to} \qquad \left(\sum_{i\in h,l} \pi_i c_{i0} + \tilde{C}_0\right) + K_1 = Y_0 - v(L_0)$$

$$\tilde{C}_0 : \qquad \frac{1}{\tilde{C}_0} = \lambda_0 \tag{1.D.5}$$

$$c_{i0} : \qquad \frac{\chi_i}{c_{i0}} = \lambda_0 \tag{1.D.6}$$

$$B_1 : \qquad \beta\left[\frac{\partial W_1}{\partial B_1} + \chi_h \pi_h \frac{\partial V_{h1}}{\partial B_1} + \chi_\ell \pi_\ell \frac{\partial V_{\ell 1}}{\partial B_1}\right] = 0 \tag{1.D.7}$$

$$K_1 : \qquad \lambda_0 = \beta\left[\frac{\partial W_1}{\partial K_1} + \chi_h \pi_h \frac{\partial V_{h1}}{\partial K_1} + \chi_\ell \pi_\ell \frac{\partial V_{\ell 1}}{\partial K_1}\right] \tag{1.D.8}$$

**Under-issuance of bonds and inefficient investment.**

$$D_{h1} = Z_1 = Y_1 - \omega_1 N_1 \qquad\qquad K_2 = \hat{\beta}_1 Z_1 \tag{1.D.9}$$

$$\omega_t = \left[(1-\alpha)\left(\sum_i \hat{a}_i K_{it}\right)^{\alpha}\right]^{\frac{\psi}{\alpha+\psi}} \qquad\qquad K_{h2} = \frac{K_2}{(1-\theta_1\tilde{a})}\left[1 - \frac{B_1}{Z_1}\right] \tag{1.D.10}$$

$$R_1^* = \hat{a}_\ell \left[\frac{\hat{\alpha}}{\omega_2^{1-\alpha}}\right]^{\frac{1}{\alpha}} \qquad\qquad K_{\ell 2} = K_2 \frac{B_1 - \theta_1 \tilde{a} Z_1}{(1-\theta_1\tilde{a})Z_1} \tag{1.D.11}$$

$$\frac{\partial D_{h1}}{\partial B_1} = 0, \qquad \frac{\partial D_{h1}}{\partial K_1} = \alpha\frac{\partial Y_1}{\partial K_1} - N_1 \frac{\partial \omega_1}{\partial K_1}$$

$$\frac{\partial \omega_1}{\partial B_1} = 0, \qquad \frac{\partial \omega_1}{\partial K_1} = \frac{\alpha\psi}{\psi+\alpha}\frac{\omega_1}{K_{h1}}$$

$$\frac{\partial R_1^*}{\partial B_1} = -\frac{\psi}{\psi+\alpha}\frac{R_1^*}{\sum \hat{a}_i K_{i2}}\sum \hat{a}_i \frac{\partial K_{i2}}{\partial B_1}, \qquad \frac{\partial R_1^*}{\partial K_{h1}} = -\frac{\psi}{\psi+\alpha}\frac{R_1^*}{\sum \hat{a}_i K_{i2}}\sum \hat{a}_i \frac{\partial K_{i2}}{\partial K_{h1}}$$

Using the fact that $\partial W_1/\partial B_1 = 0$ together with (1.D.6), and noticing that $D_{h2} = Y_{h2} - \omega_2 N_{h2}$ but the only effect of bonds choices that entrepreneurs do not internalises goes through the wage rate,[46] we can rewrite expression (1.D.7) as:

$$[\mathrm{MRS}_{h1} - \mathrm{MRS}_{\ell 1}]\left[\frac{B_2}{R_1^2}\frac{\partial R_1}{\partial B_1} + 1\right] = [(\mathrm{MRS}_{w1}\mathrm{MRS}_{w2} - \mathrm{MRS}_{\ell 1}\mathrm{MRS}_{\ell 2})\, L_2 -$$

[46]One can show that the derivative of $K_{h2}$ and $K_{\ell 2}$ with respect to $B_1$ and $K_1$ is pre-multiplied by an expression that corresponds to zero, given the entrepreneurs' choice of capital investment for time 2.

$$(\mathrm{MRS}_{h1}\mathrm{MRS}_{h2} - \mathrm{MRS}_{\ell 1}\mathrm{MRS}_{\ell 2})\, N_{h2} - \beta C_{h0}\theta_1\mu_1 N_{h2}]\, \frac{\partial \omega_2}{\partial B_1}$$

After dividing everything through by $\mathrm{MRS}_{h1}$, one can use the first order condition for low-productivity entrepreneurs' savings and the constrained debt choice of productive entrepreneurs at time 1, together with the derivative of the interest rate with respect to $B_1$. After rearranging, one obtains:

$$\left[1 - \frac{\mathrm{MRS}_{\ell 1}}{\mathrm{MRS}_{h1}}\right]\left[1 - (L_2 - (1-\theta_1)N_{h2})\,\frac{1}{R_1}\frac{\partial \omega_2}{\partial B_1}\right] = -\mu_1 C_{h1}\,(L_2 - (1-\theta_1)N_{h2})\,\frac{\partial \omega_2}{\partial B_1} \tag{1.D.12}$$

$$\text{where} \qquad \frac{\partial \omega_2}{\partial B_1} = -\frac{\alpha\psi}{\alpha+\psi}\frac{\hat{\beta}_1}{1-\theta_1\tilde{a}}\frac{(\hat{a}_h - \hat{a}_\ell)\omega_2}{\sum \hat{a}_i K_{i2}} < 0$$

The RHS of this expression is positive, and so is the second term in parenthesis on the LHS. Then, it must be that the first term in square brackets is also positive: $\mathrm{MRS}_{h1} > \mathrm{MRS}_{\ell 1}$ and the economy features undersaving. For capital, after substituting and rearranging:

$$\mathrm{MRS}_{h1}^{-1} = \frac{\partial Y_1}{\partial K_1} - \left[\mu_1 C_{h1} - \left(1 - \frac{\mathrm{MRS}_{\ell 1}}{\mathrm{MRS}_{h1}}\right)\frac{1}{R_1}\right](L_2 - (1-\theta_1)N_{h2})\,\frac{\partial \omega_2}{\partial K_1} \tag{1.D.13}$$

$$\text{where:} \qquad \frac{\partial \omega_2}{\partial K_1} = \frac{\alpha\psi}{\alpha+\psi}\frac{\hat{\beta}_1(1-\theta_1)\hat{a}_h}{1-\theta_1\tilde{a}}\frac{\omega_2}{\sum \hat{a}_i K_{i2}}\frac{\partial Z_1}{\partial K_1} > 0$$

The effect of a change in $K_1$ on the current wage cancels out when $\mathrm{MRS}_{h1} = \mathrm{MRS}_{w1}$ and the low productivity entrepreneurs are the only savers in the economy. Using (1.D.12) in (1.D.13), one can observe that the term in square brackets on the RHS of (1.D.13) is positive. Combined with the fact that the future wage rate is increasing in current capital stock, this means that there is over-investment in the economy at time 0. □

### 1.D.2 Proof of Lemma 1.5.1

*There is a minimum level of aggregate debt $\underline{B}$ above which the effective lower bound in never binding.*

$$\begin{aligned} R_1^* &= \xi \hat{a}_\ell \left( \frac{1}{\hat{a}_h K_{h2} + \hat{a}_\ell K_{\ell 2}} \right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}} \\ \text{with} \quad K_{h2} &= \hat{\beta}_1 \frac{Z_1 - B_1}{1 - \theta_1 \tilde{a}}, \quad K_{\ell 2} = \hat{\beta}_1 \frac{B_1 - \theta_1 \tilde{a} Z_1}{1 - \theta_1 \tilde{a}} \\ R_1^* &= \xi \hat{a}_\ell \left( \frac{1 - \theta_1 \tilde{a}}{\hat{\beta}_1 \left[ (1-\theta_1) \hat{a}_h Z_1 - (\hat{a}_h - \hat{a}_\ell) B_1 \right]} \right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}} \qquad (1.D.14) \\ \frac{\partial R_1^*}{\partial Z_1} &= -\frac{\psi(1-\alpha)}{\psi+\alpha} \frac{R_1 (1-\theta_1) \hat{a}_h}{(1-\theta_1) \hat{a}_h Z_1 - (\hat{a}_h - \hat{a}_\ell) B_1} < 0 \\ \frac{\partial R_1^*}{\partial B_1} &= \frac{\psi(1-\alpha)}{\psi+\alpha} \frac{R_1 (\hat{a}_h - \hat{a}_\ell)}{(1-\theta_1) \hat{a}_h Z_1 - (\hat{a}_h - \hat{a}_\ell) B_1} = -\frac{\partial R_1^*}{\partial Z_1} \frac{\hat{a}_h - \hat{a}_\ell}{(1-\theta_1) \hat{a}_h} > 0 \end{aligned}$$

We can define the lowest level of debt for which the economy is not in a liquidity trap, and $Z_1$ does not depend on $B_1$, because employment is still optimal at the margin. This is the level of debt for which $R_1^* = 1$. Define $\zeta_1 = (1-\theta_1 \tilde{a}) \hat{\beta}_1^{-1} (\xi \hat{a}_\ell)^{\frac{\alpha+\psi}{\psi(1-\alpha)}}$ :

$$\underline{B}_1 = \frac{(1-\theta_1) \hat{a}_h Z_1 - \zeta_1}{\hat{a}_h - \hat{a}_\ell}$$

With $Z_1$ defined in (1.A.14). A liquidity trap is triggered if:

$$R_1^* < 1 \quad \Longleftrightarrow \quad (1-\theta_1) \hat{a}_h Z_1 - (\hat{a}_h - \hat{a}_\ell) B_1 > \zeta_1 \quad \Longleftrightarrow \quad B_1 < \underline{B}_1$$

□

### 1.D.3 Proof of Proposition 1.5.2

*In presence of a lower bound on the real rate, the allocation before a financial crisis is not constrained efficient as it features under-borrowing.*

$$\max_{N_{h1}} D_{h1}^{lt} = Y_{h1} - w_1 N_{h1} \qquad \text{s.t.} \quad \sum_i (c_{i1} + K_{i2}) = Y_{h1}$$

Appendix 1.C.1 shows how the resource constraint in a liquidity trap corresponds to:

$$N^{lt}: \; \mathcal{D}(N^{lt}, K) = \left[a_h K^{\alpha}(N^{lt})^{1-\alpha} - \omega N^{lt}\right] = \frac{\Omega(\rho)}{\hat{\beta} A_h} + \frac{\tilde{a}-1}{(1-\theta)\tilde{a}} B \tag{1.D.15}$$
$$\implies \quad N^{lt} = \mathcal{N}\left(B, K\right)$$

The planners' derivatives of the agents value functions with respect to $B_1$ and $K_1$ are as in expressions (1.D.1) to (1.D.4). However, the relevant variables are determined by (1.D.15), as well as the following expressions:

$$D_{h2} = Y_{h2} - w_2 N_2, \qquad K_{h2} = \hat{\beta}_1 \frac{D_{h1} - B_1}{1 - \theta_1 \tilde{a}}, \qquad \omega_2 = (\hat{\alpha} a_\ell)^{1/(1-\alpha)}$$
$$\omega_1 = L_1^{\psi} = (N_1^{lt})^{\psi}, \qquad \omega_2 = \left[(1-\alpha)\left(\sum \hat{a}_i K_{i2}\right)^{\alpha}\right]^{\frac{\psi}{\psi+\alpha}}.$$

The planner's problem also leads to the same first order conditions as in (1.D.5) to (1.D.8). After collecting terms and simplifying, one obtains:

$$\left[1 - \frac{\text{MRS}_\ell}{\text{MRS}_h}\right]\left[1 + \frac{B'}{(R)^2}\underbrace{\frac{\partial R}{\partial B}}_{=0} - \frac{L' - N_h'}{R}\underbrace{\frac{\partial \omega'}{\partial B}}_{=0}\right] = -\mu C_h\left(L' - (1-\theta)N_h'\right)\underbrace{\frac{\partial \omega'}{\partial B}}_{=0} + \underbrace{\frac{\partial \mathcal{D}(\cdot)}{\partial B}}_{>0}$$
$$\text{MRS}_{h1}^{-1} = \frac{\partial Y_1}{\partial K_1} + \underbrace{\left(\frac{\partial Y_1}{\partial N_1} - w_1\right)}_{>0}\underbrace{\frac{\partial N_1}{\partial K_1}}_{<0} - \left(L_1 \underbrace{\frac{\text{MRS}_{w1}}{\text{MRS}_{h1}}}_{=1} - N_1\right)\frac{\partial \omega_1}{\partial K_1}$$

where:

$$\frac{\partial \mathcal{D}(\cdot)}{\partial B_1} = \frac{\tilde{a}-1}{(1-\theta_1)\tilde{a}} > 0 \tag{1.D.16}$$
$$\frac{\partial \omega_1}{\partial K_1} = \psi \frac{\omega_1}{N_1^{lt}} \frac{\partial N_1^{lt}}{\partial K_1} < 0 \tag{1.D.17}$$

The first two items on the right hand side are internalised by entrepreneurs. The third one cancels out when workers and productive firms share the same MRSs. □

## 1.E Extensions

### 1.E.1 Proof of Lemma 1.6.1

*In presence of two sectors the laissez-faire allocation in a financial crisis is not second best, as the economy features over-investment in manufacturing and zombie firms.*

$$
\begin{aligned}
& V_{i2}(z_{i2}; B_2, K_{i2}, K_{j2}) = \max\ \log \hat{c}_{i2} \\
\text{s.to} \quad & p_2 \hat{c}_{i2} = d^m_{i2} - b_{i2}
\end{aligned}
$$

$$
\begin{aligned}
& W_2(B_2, K_{h2}, K_{\ell 2}) = \max\ \log \tilde{C}_2 \\
\text{s.to} \quad & p_2 \hat{C}_2 = D^s_2 - B^s_2 + BL_2, \qquad \tilde{C}_2 \equiv C_2 - v(L_2)
\end{aligned}
$$

$$
\begin{aligned}
\text{where:} \quad & d^m_{i2} = p^m_2 \hat{a}_i k^\alpha_{i2} (n^m_{i2})^{1-\alpha} - B n^m_{i2} \\
& p^m_2 = \frac{B\Theta^\alpha_2}{1-\alpha} \frac{1}{(\hat{a}_h K_{h2} + \hat{a}_\ell K_{\ell 2})^{\frac{\alpha\psi}{\psi+\gamma\alpha}}}, \quad p_2 = \frac{(p^m_2)^\gamma}{\hat{\gamma}}
\end{aligned}
$$

with $\hat{\gamma} \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}$, $\Theta_t = \left[\frac{\hat{\gamma} B^{1-\gamma}(1-\alpha)^\gamma}{(1+\eta_t)^\psi}\right]^{\frac{1}{\psi+\alpha\gamma}}$ and $\eta_t = \frac{1-\gamma(1-\alpha\hat{\beta}_t)}{(1-\alpha)\gamma}$ time-dependent function of parameters.

$$
\begin{aligned}
\frac{\partial V_{i2}}{\partial K_{j2}} &= \lambda_{i2} \left[ \frac{\partial D^m_{i2}}{\partial K_{j2}} - \gamma p_2 \hat{C}_{i2} \frac{\partial p^m_2 / p^m_2}{\partial K_{j2}} \right] \\
\frac{\partial W_2}{\partial K_{j2}} &= -\tilde{\lambda}_2 \hat{C}_2 \frac{\partial p_2}{\partial K_{j2}}
\end{aligned}
$$

$$
\text{with:} \quad \frac{\partial D^m_{i2}}{\partial K_{j2}} = \mathbb{1}_{i=j} p^m_2 \frac{\partial Y^m_{i2}}{\partial K_{j2}} + Y^m_{i2} \frac{\partial p^m_2}{\partial K_{j2}}, \qquad \frac{\partial p^m_2}{\partial K_{j2}} = -\frac{\psi}{\psi+\gamma} \frac{p^m_2}{\hat{a}_h K_{h2} + \hat{a}_\ell K_{\ell 2}} \hat{a}_j
$$

**Planner's problem**

$$
V^P_1(Z_1, S_1) = \max_{\hat{C}_1, \hat{c}_{i1} K_{i2}, B_2} \left\{ \log\left(\tilde{C}_1\right) + \beta W_2(B^s_2, K_{h2}, K_{\ell 2}) + \sum_{i \in h,l} \chi_i \pi_i \left[\log \hat{c}_{i1} + \beta V_{i2}(z_{i2}; B_2, K_{h2}, K_{\ell 2})\right] \right\}
$$

$$
\begin{aligned}
\text{s.to} \quad & p_1 \left( \sum_{i \in h,l} \pi_i \hat{c}_{i1} + \tilde{C}_1 \right) + X_{h2} + X_{\ell 2} = p^m_1 Y^m_1 + Y^s_1 - p_1 v(L_1) && [\tilde{\lambda}_1] \\
& B_{h2} + B_{\ell 2} + B^s_2 = 0 && [\tilde{\nu}_1]
\end{aligned}
$$

$$B_2^m + B_2^s \leq \theta_1 D_{h2}^m \qquad [\tilde{\mu}_1]$$

Where the borrowing constraint of the service sector making zero profits is also considered and added to the manufacturing sector one.

$$B_2: \quad \frac{\mu_1}{\lambda_1} = \beta \left( \frac{p_1 \hat{C}_{\ell 1}}{p_2 \hat{C}_{\ell 2}} - \frac{p_1 \hat{C}_{h1}}{p_2 \hat{C}_{h2}} \right), \quad \frac{\mu_1}{\lambda_1} = \beta \left( \frac{p_1 \hat{C}_{\ell 1}}{p_2 \hat{C}_{\ell 2}} - \frac{p_1 \tilde{C}_1}{p_2 \tilde{C}_2} \right)$$

$$K_2: \quad 1 = \mathbb{1}_{i=h} \frac{\theta_1 \mu_1}{\lambda_1} p_2^m \frac{\partial Y_{i2}^m}{\partial K_{i2}} + \frac{\beta p_1 \hat{C}_{i1}}{p_2 \hat{C}_{i2}} p_2^m \frac{\partial Y_{i2}^m}{\partial K_{i2}} - \frac{\mu_1}{\lambda_1} \left[ (1-\theta_1) \alpha p_2^m Y_{h2}^m - \gamma p_2 \left( \hat{C}_{h2} + \hat{C}_2 \right) \right] \frac{\partial p_2^m / p_2^m}{\partial K_{i2}}$$

We know:

$$p_2 \hat{C}_{h2} = (1-\hat{\beta}_2) Z_{h2} = (1-\hat{\beta}_2)(1-\theta_1) \alpha p_2^m Y_{h2}^m;$$
$$p_2 \hat{C}_2 = w_2 L_2 = w_2 (1+\eta_2) N_2^m = (1-\alpha)(1+\eta_2) p_2^m Y_{h2}^m$$

Using these expressions in the last term in parenthesis in the FOC for capital we obtain:

$$(1-\theta_1) \alpha p_2^m Y_{h2}^m - \gamma p_2 \left( \hat{C}_{h2} + \hat{C}_2 \right) = -p_2^m \left\{ (1-\gamma\alpha) Y_{\ell 2}^m + \left[ 1 - \alpha \left( 1 - \theta_1 (1-\gamma) \right) \right] Y_{h2}^m \right\} < 0$$

This shows that the manufacturing sector features over-investment and zombie firms. □

### 1.E.2 Proof of Lemma 1.6.2

*If workers are unconstrained, then the laissez-faire economy in a financial crisis does not feature zombie firms, but under-investment as low productivity firms should invest more.*

When the borrowing constraint of workers is not binding, they share the same MRS with unconstrained low productivity entrepreneurs. Then, the choice of capital for the planner is:

$$1 = \mathbb{1}_{i=h} \frac{\theta_1 \mu_1}{\lambda_1} \frac{\partial Y_{i2}^m}{\partial K_{i2}} + \frac{\beta C_{i1}}{C_{i2}} \frac{\partial Y_{i2}^m}{\partial K_{i2}} + \frac{\mu_1}{\lambda_1} \underbrace{(1-\theta_1) N_{h2}}_{>0} \frac{\partial \omega_2}{\partial K_{i2}}$$

This means that more investment by low productivity firms is considered beneficial.

*The results in proposition 1.3.2 carry through provided that workers are sufficiently* *constrained.*

Assume $B_1^w = 0$.[47] Workers are constrained if

$$
\begin{aligned}
& \frac{1}{R_1} = \mathrm{MRS}_{1,2}(w) + \mu_1^w \tilde{C}_{w1} \\
\iff \quad & \frac{1}{R_1} > \beta \frac{\omega_1 L_1 - v(L_1) + \frac{B_2^w}{R_1}}{\omega_2 L_2 - v(L_2) - B_2^w}, \qquad \text{with } \; v(L_t) = \frac{L_t^{1+\psi}}{1+\psi} = \frac{\omega_t L_t}{1+\psi}, \quad B_2^w = \theta_1^w \omega_2 L_2; \\
\iff \quad & R_1 < \zeta(\theta_1^w) \frac{\omega_2 L_2}{\omega_1 L_1}.
\end{aligned}
$$

with $\zeta(\theta_1^w) = \beta^{-1}\left[1 - \theta_1^w \psi^{-1}(1+\psi)(1+\beta)\right]$ a function of parameters that is decreasing in $\theta_1^w$: a smaller pledgeability of future earnings induces workers constraint to be binding, so that the right-hand-side of the inequality above is larger than the left-hand side.

In turn, it is not sufficient for workers to be constrained: they need to be constrained sufficiently. Intuitively, the planner now internalises two effects associated with a change in wages in relation to the aggregate borrowing constraint:

$$
B_2 + B_2^w \leq \theta_1 D_{h2} + \theta_1^w \omega_2 L_2
$$

As in the baseline case, higher wages reduce firms' profits; however, they also relax the workers' borrowing constraints if they can borrow against their labour earnings. Provided that the share of employment in productive firms exceeds the ratio of collateral parameters, i.e. $\frac{N_h}{L} > \frac{\theta^w}{\theta}$, then the effect on firms profits remains dominant.

Finally, a sufficient but not necessary condition for the result in the proposition to go through is for workers to be at least as constrained as productive entrepreneurs. This can be seen in the proof 1.B.4 for the proposition, where $\nu \geq 0$.

□

### 1.E.3 Proof of Lemma 1.6.3

*A credit crunch generated by a collateral constraint induces no change in aggregate output but changes in consumption at time 1; aggregate productivity and production fall at date 2.*

[47] $B_1^w > 0$ would facilitate a binding constraint, $B_1^w < 0$ would require stricter conditions.

**Unchanged output at date 1.** Because the choice of employment is optimal when $R_1 = R_1^*$, the wage rate and level of employment continue to be set like in the unconstrained equilibrium, according to:

$$w_1 = \left[(1-\alpha)(\hat{a}_h K_{h1})^\alpha\right]^{\frac{\psi}{\alpha+\psi}}, \qquad L_1 = \left[(1-\alpha)(\hat{a}_h K_{h1})^\alpha\right]^{\frac{1}{\alpha+\psi}}$$

Output is then unchanged compared to an unconstrained setting, for given level of pre-installed capital. However, consumption depends directly on net worth, which is affected by the asset price. The price of land will be altered by the collateral constraint as it is now set according to:

$$q_1^h = \frac{(\widetilde{\beta\delta}_1 + \Lambda_1)\alpha Y_1 - \Lambda_1 B_1}{1 - \widetilde{\beta\delta}_1 - \Lambda_1}$$

as opposed to the unconstrained land value, $q_1^{h*} = \frac{\widetilde{\beta\delta}_1}{1-\widetilde{\beta\delta}_1}\alpha Y_1$. In the expressions above, the following definitions are used: $\widetilde{\beta\delta}_{T-1} \equiv (1-\delta)\hat{\beta}_{T-1}$, $\widetilde{\beta\delta}_t \equiv (1-\delta)\hat{\beta}_t \left(1 + \frac{\delta}{1-\delta}\widetilde{\beta\delta}_{t+1}\right)$ and $\Lambda_1 \equiv \left(\widetilde{\beta\delta}_1 - \hat{\beta}_1(1-\delta)\right)(\tilde{a}-1)$. As a result, consumption of the productive entrepreneurs, who own all of the land when the economy is unconstrained, is now:

$$C_{h1} = (1-\hat{\beta}_1)Z_{h1} = (1-\hat{\beta}_1)\left(\alpha Y_1 + q_1^h - B_1\right)$$

While output stays constant, the resources produced are used for different purposes after the credit crunch. If the collateral constraint negatively affects the demand for capital investment, then more will be demanded in consumption, thanks to the higher valuation of the fixed asset.

**Lower TFP and output at time 2.**
Because the unconstrained equilibrium is first best, consumption and output are maximised. The equilibrium in which the borrowing limit is binding therefore cannot feature a larger level of output then the first best equilibrium. To show that both output and productivity are in fact lower, the same procedure of Appendix 1.3.2 can be applied.

□

### 1.E.4 Proof of Proposition 1.6.1

*In a credit crunch generated by a collateral constraint the allocation is not constrained efficient.*

$$W_2(B_2, K_{h2}, K_{\ell 2}) = \max\ \log \tilde{C}_2 + \beta W_3(S_3)$$
$$\text{s.to} \qquad C_2 = w_2 L_2 - B_2^w, \qquad \tilde{C}_2 \equiv C_2 - v(L_2)$$
$$V_{i2}(z_{i2}; B_2, K_{i2}, K_{j2}) = \max\ \log c_{i2} + \beta V_{i3}(z_{i3}; S_3)$$
$$\text{s.to} \qquad c_{i2} + q_2^h h_{i3} + x_{i3} - \frac{b_{i3}}{R_2} = d_{i2} - b_{i2} + q_2^h h_{i2}$$
$$b_{h3} \leq 0$$
$$\frac{\partial W_2}{\partial K_{i2}} = \lambda_2^w \frac{\partial w_2}{\partial K_{i2}}$$
$$\frac{\partial V_{i2}}{\partial H_{j2}} = \lambda_{i2} \left[ \frac{\partial D_{i2}}{\partial H_{j2}} + \mathbb{1}_{i=j} q_2^h - \left( \Delta H_{i3} + \frac{B_{i3}}{R_2^2} \frac{\partial R_2}{\partial q_2^h} \right) \frac{\partial q_2^h}{\partial H_{j2}} \right]$$
$$\frac{\partial V_{j2}}{\partial X_{i2}} = \lambda_{j2} \left[ \frac{\partial D_{j2}}{\partial X_{i2}} - \left( \Delta H_{j3} + \frac{B_3}{R_2^2} \frac{\partial R_2}{\partial q_2^h} \right) \frac{\partial q_2^h}{\partial X_{i2}} \right]$$
$$\frac{\partial W_2}{\partial B_2^w} = -\lambda_2^w, \qquad \text{with:} \qquad \lambda_2^w = \frac{1}{\tilde{C}_2}$$
$$\frac{\partial V_{i2}}{\partial B_{i2}} = -\lambda_{i2}, \qquad \text{with:} \qquad \lambda_{i2} = \frac{1}{C_{i2}}$$

where:
$$d_{i2} = a_i k_{i2}^{\alpha} n_{i2}^{1-\alpha} - w_2 n_{i2}, \quad k_{i2} = x_{i2}^{\delta} h_{i2}^{1-\delta}$$
$$u_2 = q_2^h = \frac{\widetilde{\beta\delta}_2}{1 - \widetilde{\beta\delta}_2} \alpha Y_2, \qquad R_2 = \frac{\hat{a}_\ell}{q_2} \left( \frac{\hat{\alpha}}{w_3^{1-\alpha}} \right)^{1/\alpha}, \qquad w_t = \left[ (1-\alpha) \left( \sum \hat{a}_i K_{it} \right)^{\alpha} \right]^{\frac{\psi}{\psi+\alpha}}$$

$$\begin{aligned}
q_2^h &= \frac{\widetilde{\beta\delta}_2}{1 - \widetilde{\beta\delta}_2} \alpha \left( Y_{h2} + Y_{\ell 2} \right) \\
&= \frac{\widetilde{\beta\delta}_2}{1 - \widetilde{\beta\delta}_2} \alpha \left[ a_h \left( X_{h2}^{\delta} (1 - H_{\ell 2})^{1-\delta} \right)^{\alpha} N_{h2}^{1-\alpha} + a_\ell \left( X_{\ell 2}^{\delta} H_{\ell 2}^{1-\delta} \right)^{\alpha} N_{\ell 2}^{1-\alpha} \right] \\
&= \frac{\widetilde{\beta\delta}_2}{1 - \widetilde{\beta\delta}_2} \alpha \left\{ (1-\alpha)^{1-\alpha} \left[ (\hat{a}_h H - (\hat{a}_h - \hat{a}_\ell) H_{\ell 2}) \left( \frac{X_{h2} + X_{\ell 2}}{H} \right)^{1-\delta} \right]^{\alpha(1+\psi)} \right\}^{\frac{1}{\alpha+\psi}}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial q_2^h}{\partial X_{\ell 2}} &= \frac{\delta\widetilde{\beta\delta}_2}{1-\widetilde{\beta\delta}_2}\alpha^2\frac{Y_{\ell 2}}{X_{\ell 2}} > 0 \\
\frac{\partial q_2^h}{\partial H_{\ell 2}} &= \frac{(1-\delta)\widetilde{\beta\delta}_2}{1-\widetilde{\beta\delta}_2}\alpha^2\left(\frac{Y_{\ell 2}}{H_{\ell 2}} - \frac{Y_{h2}}{H_{h2}}\right) \\
&= -\frac{(1-\delta)\widetilde{\beta\delta}_2}{1-\widetilde{\beta\delta}_2}\alpha^2\left(\frac{1-\alpha}{\omega_2}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{\delta u_1}{1-\delta}\right)^{\delta}(\hat{a}_h - \hat{a}_\ell) < 0
\end{aligned}
$$

**Planner's problem**

$$
\begin{aligned}
V_1^P(Z_1, S_1) &= \max_{c_{i1}, X_{i2}, H_{i2}, B_2}\left\{\log\tilde{C}_1 + \beta W_2(B_2, K_{h2}, K_{\ell 2}) + \sum_{i\in h,l}\chi_{mi}\pi_i\left[\log c_{i1} + \beta V_{i2}(z_{i2}; B_2, K_{h2}, K_{\ell 2})\right]\right\} \\
\text{s.to} \quad & \sum_{i\in h,l}\pi_i c_{i1} + X_{h2} + X_{\ell 2} = Y_1 \\
& H_{h2} + H_{\ell 2} = 1 \\
& B_2 + B_2^w \leq q_2^h H_{h2}
\end{aligned}
$$

$$
\begin{aligned}
& c_{i1}: \quad \frac{\chi_i}{c_{i1}} = \lambda_1, \qquad B_2: \quad \mu_1 = \beta\sum_i \chi_i\pi_i\frac{\partial V_{i2}}{\partial B_2} \\
& \implies \quad \frac{\mu_1}{\lambda_1} = \beta\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta C_{h1}}{C_{h2}}\right) = \beta\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta\tilde{C}_1}{\tilde{C}_2}\right) \\
& H_{i2}: \quad \gamma_1 = \beta\frac{\partial W_2}{\partial H_{i2}} + \beta\sum_j \chi_j\pi_j\frac{\partial V_{j2}}{\partial H_{i2}} + \mu_1\left(H_{h2}\frac{\partial q_2^h}{\partial H_{i2}} + \mathbb{1}_{i=h}q_2^h\right) \\
& \implies \quad \frac{\gamma_1}{\lambda_1} = \frac{\beta C_{i1}}{C_{i2}}\left(\frac{\partial Y_{i2}}{\partial H_{i2}} + q_2^h\right) + \mathbb{1}_{i=h}\frac{\mu_1}{\lambda_1}q_2^h + \frac{\mu_1}{\lambda_1}H_{h2}\frac{\partial q_2^h}{\partial H_{i2}} - \beta\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta\tilde{C}_1}{\tilde{C}_2}\right)L_2\frac{\partial w_2}{\partial H_{i2}} + \\
& \qquad\qquad \beta\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta C_{h1}}{C_{h2}}\right)\left(\Delta H_{h3}\frac{\partial q_2^h}{\partial H_{i2}} + N_{h2}\frac{\partial w_2}{\partial H_{i2}}\right) \\
& X_{i2}: \quad \lambda_1 = \beta\frac{\partial W_2}{\partial X_{i2}} + \beta\sum_i \chi_i\pi_i\frac{\partial V_{i2}}{\partial X_{i2}} + \mu_1 H_{h2}\frac{\partial q_2^h}{\partial X_{i2}} \\
& \implies \quad 1 = \frac{\beta C_{i1}}{C_{i2}}\frac{\partial Y_{i2}}{\partial X_{i2}} + \frac{\mu_1}{\lambda_1}H_{h2}\frac{\partial q_2^h}{X_{i2}} + \beta\left(\frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta C_{h1}}{C_{h2}}\right)\left(\Delta H_{h3}\frac{\partial q_2^h}{\partial X_{i2}} + N_{h2}\frac{\partial w_2}{\partial X_{i2}}\right)
\end{aligned}
$$

$$- \beta \left( \frac{\beta C_{\ell 1}}{C_{\ell 2}} - \frac{\beta \tilde{C}_1}{\tilde{C}_2} \right) L_2 \frac{\partial w_2}{\partial X_{i2}}$$

where the fact that $b_{i3} = q_3^h h_{h3} = 0$ was used. Even if workers are constrained, it is not clear that the underinvestment result would be overturned. $\square$

# Chapter 2

# Financial Stabilization Policies and the Allocation of Capital

*Are financial stability and allocative efficiency compatible policy objectives? I answer this question in a model where producers exhibit heterogeneous productivity. An occasionally binding borrowing constraint dependent on the price of productive capital has the dual role of generating an externality justifying macroprudential intervention, as well as introducing misallocation. Compared to a constrained efficient allocation, the laissez-faire economy features inefficient wedges in both debt and capital markets. The optimal policy under commitment is in general not time consistent, due to both the forward looking nature of the price of capital as well as the non-linear stochastic discount factor used to price the asset. This prompts the regulator to make promises regarding the future so as to influence current prices, which are no longer optimal ex-post. The policy maker's incentive to deviate from past promises can however be low, if the promised direction of intervention remains the same ex post. Capital requirements can be adopted as policy instruments to implement the constrained efficient allocation in the decentralised economy. However, successful implementation relies on credible commitment to future policies.*

## 2.1 Introduction

Frictions and inefficiencies in credit markets have substantial consequences that go well beyond the realm of finance. If contracts cannot be perfectly enforced, lenders are subject to the possibility of borrowers failing to meet their obligations. Assets that can be pledged as collateral then become extremely valuable for protecting lenders, while ensuring access to credit to borrowers. Importantly, collateral ownership can ultimately determine the distribution of productive investment in the economy, hence affecting how efficiently a system can operate.[1]

The link between asset ownership and borrowing ability also plays a role in amplifying business cycle shocks. Whenever a shock drives asset prices down, this translates into a lower value of collateral, which forces constrained borrowers to deleverage. As they can no longer access liquidity through the debt market, they may be forced to sell assets at discounted prices. These fire sales, in turn, can trigger further rounds of asset prices falls and deleveraging. The problem is further exacerbated by the fact that no insurance contract can be written against certain states of the world. Because the ensuing crisis is systemic, no guarantees exist that the insurers could in turn survive it.

The 2008 financial meltdown has been a testament to the relevance of these mechanisms of financial shocks transmission to the real economy. This has brought the topic of prudential regulatory interventions to the forefront of policy debates. From a stability perspective, discouraging excessively leveraged positions during periods of relative financial tranquillity helps to prevent or reduce the likelihood of asset fire sales. However, the resulting allocation of resources will be impacted by such measures. In this paper, I underline the indissoluble link tying debt and capital markets together, so that interventions in either of the two markets have implications for the other. Starting from an environment featuring both financial instability and resource misallocation, I answer the following questions: 1) What is the optimal policy to address both stability and efficiency of the system? 2) How can it effectively be implemented in a competitive economy? 3) What are its time consistency properties?

If markets are efficient, prices induce an optimal allocation of resources and no regulatory intervention can help improve on the optimality of the equilibrium. I introduce two very common inefficiencies in a model featuring risk averse, heterogeneous producers, which cause the laissez-faire economy to deviate from this first best outcome. First, there is a problem of limited enforcement linking how much debt borrowers can take on in each period to the value of productive capital they own. Second, contracts are incomplete in the sense that state-contingent financial assets are not available. The limited enforcement problem introduces financial instability concerns; the incomplete contracts problem simultaneously generates issues of allocative efficiency.

Collateralised debt generates a pecuniary externality as well causing misallocation.

[1] See e.g. Buera, Kaboski, and Shin (2011)

Ex-ante, producers are rational and anticipate the possibility of being constrained in their borrowing. However, they fail to internalise how, at an aggregate level, their decisions affect asset prices. As the value of collateral changes, this in turn impacts their ability to take on debt, giving rise to a collateral externality. An appropriate change in the net worth of the unconstrained agents would increase the price of capital and hence relax the constraint on borrowing. Ex-post, if the collateral constraint is binding, the real interest rate has to fall so as to induce savers to only save up to the point where constrained borrowers can borrow. As the return to savings through financial markets falls, producers who may be comparatively less productive engage in production activities. This generates capital misallocation. A redistribution of productive capital across producers would increase the aggregate production possibilities of the economy.

The lack of state-contingent contracts in presence of uncertainty implies that market participants cannot share risk efficiently. As a result, expected risk premia on risky production technologies available in the economy are not equated as they should be. Some investment technologies will carry a larger risk premium, if they are over-invested in. This, again, generates misallocation. Additionally, depending on the time and state of the world, certain agents have a higher marginal valuation of consumption than others. They would benefit by a favourable change in price more than the agent at the other end of the transaction would be hurt. Because producers do not internalise how their choices affect prices, they fail to take this in consideration when formulating their optimal plan. This represents another pecuniary externality.

A benevolent social planner, who cannot eliminate either of the distortions of the competitive economy, can nevertheless improve on its efficiency. As in a primal approach Ramsey problem, the planner maximizes producers' utilities subject to the same borrowing constraint that individuals face, but taking into account expressions for equilibrium prices as implementability constraints. This allows the social planner to internalize all externalities and achieve a constrained efficient allocation. Optimal policy requires interventions in both markets for bonds and for productive capital. Even if the idiosyncratic productivity of different producers cannot be identified by the regulator, it is still possible to implement the constrained efficient allocation by imposing a capital requirement on borrowers. The policy, however, requires contemporaneous announcement of current and future equity requirements, so as to shape expectations and induce optimal choices today.

The regulator's ability to credibly commit to future policies is therefore crucial, which leads to the analysis of the time consistency property of the allocation under commitment.

### 2.1.1 Related Literature

This paper builds on research on the themes of inefficient resource allocation and the macroeconomic impacts of collateral constraints. Kiyotaki (1998) and Kiyotaki and Moore (1997) explore the effects of a borrowing constraint on the productive capacity of the economy and how this can affect business cycle fluctuations. This paper builds on their contributions by focusing on the normative implications of their findings.

On the microfoundation of collateral constraints due to limited commitment and its connections to systemic risk, the work of Lorenzoni (2008) is of relevance. The paper considers risk-neutral agents subject to endogenous limits on both borrowing and savings. The unregulated economy features overborrowing (overinvestment) when the constraint on saving is binding. The paper revolves around a pecuniary externalities connected to unequal marginal rate of substitutions for borrowers and savers. The author finds that imposing a capital requirement is sufficient to implement the constrained efficient allocation for both capital and bond holding. Additionally, the allocation is time consistent. This is not the case here, and I show how interventions in both markets are necessary once a collateral externality is also considered.

A related strand of the literature focuses on macroprudential policy in presence of an occasionally binding collateral constraint in small open economy settings (Bianchi and Mendoza, 2018; Jeanne and Korinek, 2010; Benigno, Chen, Otrok, Rebucci, and Young, 2013). This paper pivots on their findings to apply them to a more simplified three-period setting, where entrepreneurs' productivity can be non-homogeneous.

The two papers that are closely related to this work are Dávila and Korinek (2018) and Iacoviello, Nunes, and Prestipino (2016). Both these papers consider optimal policy in a general equilibrium framework featuring collateral constraints and heterogeneous agents. This paper adds to their contribution by looking at different types of policy implementation, including with commitment, and considers the reasons for time inconsistency.

The rest of the paper is structured as follows. Section 2.2 presents the model and describes the efficient allocation. Section 2.3 analyses the laissez faire equilibrium. Section 2.4 presents the constrained efficient allocation under discretion and Section 2.5 the case of commitment. After reviewing how this differs from the discretionary allocation, I analyse the time consistency property of the allocation. Section 2.6 addresses what instruments can be used in the decentralised economy to implement the constrained efficient equilibrium. Section 2.7 concludes.

## 2.2 A model of Capital Allocation under Uncertainty

The economy lasts three periods, $t = \{0, 1, 2\}$. It is populated by entrepreneurs who enjoy consumption according to a concave utility function, which is time-separable and satisfies $U_c > 0$, $U_{cc} \leq 0$ plus standard Inada conditions. To simplify the analysis, consider a logarithmic utility function: $U(c_t) = \log c_t$. Agents have access to a production technology that yields output one period after investment:

$$y_{it} = z_t f_i \left(k_{it-1}\right), \qquad i \in [0, 1]$$

where $z_t$ is an exogenous aggregate productivity shock. There is heterogeneity in production possibilities, as not all entrepreneurs can access the same production technology. In particular, there are two groups of entrepreneurs: a fraction $n$ of producers has access to a technology exhibiting constant returns to scale, the remaining $1 - n$ have a decreasing returns to scale technology:

$$f_i(k_i) = \begin{cases} Ak_i, & \text{for } i \in (0, n), \text{ with } A > 1; \\ k_i^\alpha, & \text{for } i \in (n, 1), \text{ with } 0 < \alpha < 1. \end{cases}$$

The first class of agents can be thought of as a conglomeration of banks and high-productivity firms (CRS), while the second class of agents can be thought of as a less efficient productive sector (DRS).[2] Uncertainty is fully resolved in $t = 1$, with only one of two possible states realised:

[2]Interpreting more productive firms as a bank-firm conglomerate simplifies the analysis when implementing banking regulation. This setting is equivalent to having banks as separate agents, so long as there are no frictions between the financial and productive sectors (Stein, 2012).

$$z_1 = \begin{cases} z_g \geq 1 & \text{with prob } \; \pi \\ z_b \in (0,1) & \text{with prob } \; 1-\pi \end{cases}$$

where the $g$ and $b$ notation stands for good and bad state of the world. In all other time periods, $z_t = z \geq 1$ and $z$ is known with certainty. For simplicity, the total supply of capital used in production is assumed to be fixed at 1 over time, while depreciation is 0. Total supply of capital has to equate total demand: $n \cdot k^C + (1-n)k^D = 1$; initial conditions $k^C_{-1}, k^D_{-1}$ are given.[3]

### 2.2.1 Efficient Allocation

Consider a benevolent social planner who is able to choose the allocation for both type of producers subject to market clearing conditions. The social planner attaches a welfare weight equal to $\chi$ to the CRS-producer and correspondingly, $1-\chi$ to the DRS-producer. An efficient allocation is a sequence $\left\{c^C_t, c^D_t, k^C_t, k^D_t\right\}^2_{t=0}$ for $z_1 \in \{z_b, z_g\}$ such that, for given initial $k^C_{-1}$, $k^D_{-1}$, the social planner solves the following problem:

$$\max_{\left\{c^C_t, k^C_t, c^D_t, k^D_t\right\}^2_{t=0}} \sum_{t=0}^{2} \beta^t E_0 \left[n\chi U(c^C_t) + (1-n)(1-\chi)U(c^D_t)\right]$$

subject to

$$nc^C_t + (1-n)c^D_t = z_t \left[nf_C(k^C_{t-1}) + (1-n)f_D(k^D_{t-1})\right]; \qquad nk^C_t + (1-n)k^D_t = 1.$$

where $f_j$ with $j = C, D$ stands for the production function of the CRS and DRS agents respectively.[4]

**Proposition 2.2.1.** *An efficient allocation requires, for all possible states and time periods:*

1. *Perfect capital allocation*

$$f'_C(k^C_t) = f'_D(k^D_t). \tag{2.2.1}$$

[3] All variables relating to DRS-type agents are denoted with a $D$ superscript; a $C$ superscript is used to indicate the other type of producers.

[4] A prime symbol will be used to denote the first derivative of both the production function and the utility function with respect to the only input, $k$ and $c$ respectively.

*2. Full risk sharing*

$$\chi U'(c_t^C) = (1-\chi)U'(c_t^D). \tag{2.2.2}$$

*Proof.* Take first order conditions with respect to capital and debt of the two types of agents in the problem above. Noticing that the Lagrangian multipliers of the planner's resource constraint and asset market clearing are the same for both types of agents, the result can be obtained. □

Under the assumed functional forms, both agents should produce in equilibrium. In a first best allocation, the asset should be distributed according to:

$$k^{D*} = \left(\frac{\alpha}{A}\right)^{\frac{1}{1-\alpha}}; \qquad k^{C*} = \frac{1}{n} - \left(\frac{1-n}{n}\right)\left(\frac{\alpha}{A}\right)^{\frac{1}{1-\alpha}}.$$

## 2.3 The Laissez-Faire Economy

After describing the first best allocation, some frictions are introduced in the bond market, which affect the laissez-faire economy, causing the resulting equilibrium to deviate from first best. Before introducing such frictions, it is useful to consider specific initial conditions to clarify which type of agent will be selling bonds (borrowing) and which agent will be buying bonds (lending). Provided that at the beginning of time the CRS entrepreneurs are endowed with less capital than the optimal level, they will borrow positive amounts and roll this over in the following period, when the borrowing constraint can become binding. This is so that they can finance the gap between the level of capital they already own and the optimal level they wish to invest. More details on the exact condition on the initial distribution of capital can be found at the end of Appendix 2.A.2. This condition will be maintained in the rest of the paper.[5]

### 2.3.1 Two Sources of Inefficiency

Consider a competitive market economy where agents take prices as given and engage in trade based on these prices. Agents will have a motive to borrow or save on the basis of

[5]The term CRS-type entrepreneur and borrower will therefore be used interchangeably from now on.

1) their differential productivities, and 2) the initial distribution of capital. I make two crucial assumptions regarding the market for saving and borrowing:

**Assumption 2.1.** Incomplete contracts. *The only financial asset available in the decentralised economy is a one period risk-free bond in zero net supply.*

**Assumption 2.2.** Limited enforcement. *The maximal amount of bond holding of each agent is limited by a fraction of the value of productive capital that can be pledged as collateral:*

$$d_t^j \leq \theta_t q_t k_t^j.$$

*The value of $\theta_0$ is large enough, such that the collateral constraint is never binding in period 0. The value of $\theta_1$ is such that the borrowing constraint binds in the bad state and it is slack in the good state. See Appendix 2.A.3 for more details.*

Assumption 2.1 is going to be of relevance in period 0, before uncertainty is realised, as complete consumption smoothing in all states of the world will be precluded. In particular, the two agents' Marginal Rates of (intertemporal) Substitutions (MRSs) will only be equated on average, rather than state by state.

Assumption 2.2 will have consequences for the agents that borrow positive amounts. $d_t$ is the quantity of risk-free bonds underwritten in period $t$ to be repaid in period $t+1$, and $q_t$ the price of capital in period $t$.[6] $\theta_t$ represents the fraction of capital value that the lender can expect to recover in case the borrower does not pay back her loan. The presence of a collateral constraint will introduce a wedge between the two agents' Marginal Product of Capital (MPKs) as well as MRSs, when the bad state is realised.

The last part of Assumption 2.2 implies that the collateral constraint is slack in period 0. By definition, macroprudential policy is adopted in periods of financial tranquillity (i.e. when the collateral constraint is not binding), with the aim of addressing the potential build up of risk that could result in a crisis in the future. This assumption therefore allows one to focus on policies implemented before a potential crisis takes place, with the exclusive aim of preventing rather than managing the crisis.

[6] The current capital price is used as an approximation for the price in $t+1$ when debt becomes due, as the future price is uncertain.

### 2.3.2 Competitive equilibrium

For given $k^j_{-1}$, with $j = \{C, D\}$, each type of agent solves:

$$\max_{\{c^j_t, d^j_t, k^j_t\}_{t=0}^{2}} \quad U(c^j_0) + \beta E_0 \left[U(c^j_1) + \beta U(c^j_2)\right]$$

subject to

$$c^j_0 + q_0 \Delta k^j_0 = z f_j\left(k^j_{-1}\right) + \frac{d^j_0}{(1+r_0)} \qquad (\lambda^j_0)$$

$$c^j_1 + q_1 \Delta k^j_1 = z_1 f_j\left(k^j_0\right) - d^j_0 + \frac{d^j_1}{(1+r_1)}; \qquad z_1 \in \{z_b, z_g\} \qquad (\lambda^j_1)$$

$$c^j_2 = z f_j\left(k^j_1\right) - d^j_1 \qquad (\lambda^j_2)$$

$$d^j_1 \leq \theta_1 q_1 k^j_1; \qquad (\mu^j_1)$$

Where $\Delta k_t \equiv k_t - k_{t-1}$. Agents choose the sequences of consumption $c^j_t$, net purchases or sales of capital $\Delta k^j_t$ with $k^j_{t-1}$ given, and borrowing $d^j_t \geq 0$ or lending $d^j_t < 0$ with the objective of maximising their lifetime utility subject to the sequence of budget constraints for each period and for each state of the world, and subject to the collateral constraint. Each period, their past choice of capital and debt or savings determines their net worth, $\omega^j_t \equiv z_t f_j\left(k^j_{t-1}\right) - d^j_{t-1}$. This implies that the state realisation in $t = 1$ generates history dependence in $t = 2$.

In the last time period, the capital market shuts down, as everyone would like to sell their capital holdings but nobody would be willing to buy. No rational agent would leave any savings in the last time period, while borrowing is precluded. As a consequence, in period 2, entrepreneurs simply consume whatever is left of their production after repaying their debt or receiving the return on their savings and hold on to whatever amount of capital they had acquired in the previous period: $d^j_2 = 0,\ k^j_2 = k^j_1$.

**Definition 2.1.** *A competitive equilibrium in the unregulated economy is a set of allocations* $\left\{(c^C_t, k^C_t, d^C_t); (c^D_t, k^D_t, d^D_t)\right\}_{t=0}^{2}$, *prices* $\{q_t, r_t\}_{t=0}^{1}$ *and Lagrangian multipliers* $\left\{\mu^j_1\right\}_{j=C,D}$, *for* $z_1 \in \{z_b, z_g\}$ *such that:*

1. *Entrepreneurs solve the above problem taking prices and initial conditions* $k^j_{-1}$ *as given.*

*2. Markets for productive capital, risk-free bonds and consumption goods clear.*

The set of equations defining the equilibrium can be found in Appendix 2.A.1. I now describe the behaviour of the equilibrium and how it compares to the efficient allocation analysed in Subsection 2.2.1.

### Allocation in period 0

Assumption 2.2 ensures that the collateral constraint is not binding in period 0. However, due to the lack of state-contingent financial asset, bond repayments are the same regardless of what state is realised. Hence, ex-post, marginal utilities of consumption of the two producers cannot be equated. Define the stochastic discount factor: $m^j_{t,t+1s} \equiv \beta \frac{c^j_t}{c^j_{t+1s}}$ for $j = C, D$; $s = g, b$.[7]

**Lemma 2.3.1.** *In the laissez-faire economy:*

*i. The MRSs of the two agents are not equated state by state, but only on average.*

$$\begin{aligned} & E_0(m^C_{0,1}) = E_0(m^D_{0,1}) \\ \not\Rightarrow \qquad & m^C_{0,1s} = m^D_{0,1s}, \qquad \forall s \in \{g, b\}. \end{aligned}$$

*ii. The MPKs of the two agents are not equated state by state, but only on average.*

$$\begin{aligned} & E_0\left[m^C_{0,1}\left(q_1 + Az_1\right)\right] = E_0\left[m^D_{0,1}\left(q_1 + \alpha(k^D_0)^{\alpha-1}z_1\right)\right] \\ \not\Rightarrow \qquad & A = \alpha(k^D_0)^{\alpha-1}; \end{aligned}$$

*Proof.* Combine the first order conditions with respect to debt choices of the two types of agents to obtain point *i.*, the first order conditions with respect to capital to obtain point *ii.* □

The fact that MRSs are not equated state by state has implications for how the productive capital is allocated. While it is clear that the overall efficiency of production in period 1 is lower than in the efficient allocation, it is not obvious in which direction this can be corrected.

[7] Here, I adopt the convention that a variable indexed by $(s)$ with $s = b, g$ indicates the variable in a specific state of the world, where $b$ stands for bad and $g$ stands for the good state.

**Lemma 2.3.2.** *The difference in expected risk premia on the two production technologies provides an indication on the direction of investment misallocation.*

*Proof.* Consider the risk premium arising from investing in either of the two production technologies:

$$RP_0^j \equiv E_0(\widetilde{r}_1^j) - r_0 = -\frac{cov_0(m_{0,1}^j; \widetilde{r}_1^j)}{E_0(m_{0,1}^j)}$$

With $1 + \widetilde{r}_{t+1}^j \equiv \frac{q_{t+1}+z_{t+1}f_j'}{q_t}$, the gross return on capital of producers of type $j$. If MRSs were equated state-by-state, the risk premia on the two investment opportunities would be the same. This in turn would imply that the quantity of investment undertaken by each producer is efficient. What can be observed, instead, is the following:

$$\begin{aligned} RP_0^C - RP_0^D =& E_0\left(\widetilde{r}_1^C\right) - E_0\left(\widetilde{r}_1^D\right) \\ =& -\left[cov_0\left(m_{0,1}^C; \widetilde{r}_1^C\right) - cov_0\left(m_{0,1}^D; \widetilde{r}_1^D\right)\right](1+r_0) \\ =&(1+r_0)\left[cov_0\left(m_{0,1}^D - m_{0,1}^C; \widetilde{r}_1^C\right) - \left(\frac{A-\alpha(k_0^D)^{\alpha-1}}{q_0}\right)cov_0(m_{0,1}^D; z_1)\right] \neq 0 \end{aligned}$$

Because nothing ensures that the two stochastic discount factors are equated state-by-state, the expected risk premia on the two technologies are not equated. The technology exhibiting a larger risk premium will be over-invested in, and vice versa. □

**Allocation in period 1**

The quantity of bonds issued in $t = 0$ in the laissez-faire economy coincides with the amount that would implement the efficient allocation on average. In $t = 1$ however, repayments are not state-contingent. This will have an impact on agents' welfare.

**Lemma 2.3.3.** *Ex post, the risk-free interest rate is too low in the good state, too high in the bad state.*

*Proof.* If state contingent assets were available, the state-contingent price of debt would be:

$$(1+r_0^s)^{-1} = m_{0,1s}^j \text{ for } j \in \{C, D\},\ s \in \{b, g\}$$

In the good state, consumption is higher than in the low state. Hence $m^j_{0,1g} < E_0(m^j_{0,1}) < m^j_{0,1b}$, and therefore $r^b_0 < r_0 < r^g_0$. This generates the result above. $\square$

If the good state is realised, the productivity realisation is large and the price of the asset is high, so that the collateral constraint is not binding. Hence, given distortions inherited from the past, the allocation from $t = 1$ onwards is optimal. If the bad state is realised however, the bad productivity realisation depresses the asset price, causing the collateral constraint to bind and driving a wedge in the optimal allocation of investment.[8]

**Lemma 2.3.4.** *If the bad state is realised in $t = 1$:*

*i. The two agents' MRSs are not equated*

$$m^D_{1,2} - m^C_{1,2} = \mu^C_1 c^C_1 > 0.$$

*ii. Compared to the efficient allocation, the DRS-agent overinvest and the CRS-agent underinvest*

$$k^D_1 > \left(\frac{\alpha}{A}\right)^{\frac{1}{1-\alpha}} \equiv k^{D*}_1.$$

*Proof.* To obtain the first part of this lemma, use the first order conditions with respect to debt. To show part *ii*, combine the first order conditions for debt and asset choices for the two agents. $\square$

A financial crisis is defined as a period when the borrowing constraint is binding.

**Proposition 2.3.1.** *The fully efficient allocation cannot be implemented in the unregulated economy.*

*Proof.* See Appendix 2.A.4. $\square$

### 2.3.3 Alternative Market Structures

So as to illustrate the role of the two key financial frictions introduced in the previous section, I now present alternative market structures, where either assumptions of non-state-contingent contracts and limited enforcement have been removed.

[8]Technically, for the collateral constraint to hold with equality, a further assumption is needed, ensuring that the discounted value of borrowed funds when the constraint binds do not exceed the cost of financing new investment: $\frac{\theta q_1 k^C_1}{1+r_1} < q_1 k^C_1$. This corresponds to condition (2.A.26).

### Fully Enforcible, State-Contingent Contracts

Let's first consider an economy where state-contingent financial assets are traded, and no problem of limited enforcement is present. If there exist constant planner's welfare weights such that the social planner allocation coincides with the equilibrium in the laissez faire economy, then the decentralised economy is Pareto efficient.

**Lemma 2.3.5.** *The laissez-faire allocation in an economy without financial frictions is first best.*

*Proof.* See Appendix 2.A.5 □

From first order conditions in the laissez-faire economy, it is easy to see the equivalence with the first best allocation by setting welfare weight $1 - \chi = \frac{c_t^D}{c_t^C}$.

### Fully Enforcible, Non-State-Contingent Contracts

Consider the case where only risk-free bonds are available. The quantity of borrowing, however, is not constrained by problems of limited enforcement. In period 0, it will not be possible to undertake state contingent debt, hence Lemmas 2.3.1 to 2.3.3 regarding period 0 allocation still apply. Because after the initial period there is no uncertainty, the impact of the inefficiency is fully realised in $t = 0$ in this economy. In particular, this setting features no financial instability. This makes this case not very relevant if the main objective is to study interventions aimed at macroeconomic stabilisation.

### State-Contingent Contracts with Limited Enforcement

If state-contingent contracts are available, but can only finance up to a fraction of an agent's collateral asset, then whether or not the allocation is efficient depends on the realisation of the aggregate state When the good state is realised, the laissez-faire economy coincide with the first best allocation. When the constraint is binding, instead, the allocation is sub-optimal and interventions in the laissez-faire economy could be Pareto improving. Lemma 2.3.4, describing the allocation in period 1, would still hold.

If, as in the baseline model, the collateral constraint is not binding in $t = 0$, this setting allows for the analysis of prudential policies. In particular, the regulator would be facing a trade-off between an unrestricted efficient allocation in the present and improving the allocation in the crisis state in the future. Reducing bond holding in period 0 can make the constraint less binding in period 1, but at the cost of compromising what could be a fully efficient allocation in $t = 0$.

The availability of state contingent bonds, however, has an impact on the analysis of time consistency properties of the interventions. In period 1 a planner who can commit to future policies would exploit its ability to promise lower future consumption to prop up the current asset price and relax the collateral constraint, while a discretionary time-consistent planner does not have access to such a policy. Here, the assumption of non-linearity of the stochastic discount factor used to price assets is crucial.[9] This makes the overall plan non time consistent. However, at time 0, the choices of a discretionary planner and one with commitment would coincide. In both cases the only aim of any intervention is purely to tackle the potentially binding constraint in the future period, without any temptation to alter the allocation in the present. Therefore, the planner under commitment has no incentive to deviate from promised policies when implementing macroprudential policy in presence of state-contingent bonds, but the discretionary and commitment allocation differ ex-post, when policies are devoted to crisis management.

The lack of state contingent contract is not just a realistic assumption; it is also very frequently combined with a collateral constraint in the literature. While it complicates the analysis somewhat, it gives rise to some interesting interactions between the objectives of preventing a credit crunch and managing inefficiencies that are always present in the economy, even during tranquil times. The rest of the paper is therefore focused on optimal interventions when both problems of limited commitment and incomplete contracts are present, while a more detailed description of the allocation and policy intervention in the case of a collateral constraint with state contingent bonds can be found in Appendix 2.B.

## 2.4 Discretionary Constrained Efficient Allocation

After having illustrated how the laissez-faire economy deviates from first best, I will consider possible interventions aimed at improving on the efficiency of the equilibrium. A benevolent social planner is not able to remove either market failures affecting the competitive equilibrium: the collateral constraint and the availability of risk-free debt

[9] See Lorenzoni (2008) for an example with linear utility where this result is overturned.

only. Differently from the individual producers in the decentralised economy, however, the social planner internalizes the impact of real choices on prices, and how this in turn affects each producer's budget constraint and borrowing constraint.

I will start by considering the allocation resulting from a discretionary optimization of a planner without any commitment ability, who re-optimizes period by period, before analysing the allocation under commitment and its time consistency properties.

### 2.4.1 Constrained Markovian Planner's Problem

Define $k^C = k$, $k^D = \frac{1-nk}{1-n}$ and $d^C = d$, $d^D = -\frac{nd}{1-n}$. In each time period, the social planner takes past choices as given.[10] The problem faced by the planner is:

$$V_t(k, d; z) = \max_{\{c^C, c^D, k', d', q, r\}} \left[ n\chi \log c^C + (1-n)(1-\chi) \log c^D \right] + \beta E_{z'|z} \left[ V_{t+1}(k', d'; z') \right]$$

subject to

$$c^C + q\Delta k' - \frac{d'}{(1+r)} \leq Azk - d \qquad \lambda^C \geq 0$$

$$c^D - \frac{n}{1-n} \left[ q\Delta k' - \frac{d'}{(1+r)} \right] \leq z \left( \frac{1-nk}{1-n} \right)^{\alpha} + \frac{n}{1-n} d \qquad \lambda^D \geq 0$$

$$d' \leq \theta q k' \qquad \mu^P \geq 0$$

$$q = \beta E_{z'|z} \left[ \frac{c^D}{c^{D\prime}(k', d'; z')} \left( q'(k', d'; z') + \alpha z' \left( \frac{1-nk'}{1-n} \right)^{\alpha-1} \right) \right] \qquad \gamma^q$$

$$\frac{1}{1+r} = \beta E_{z'|z} \left[ \frac{c^D}{c^{D\prime}(k', d'; z')} \right] \qquad \gamma^r$$

For $t = 0, 1, 2$, with $V_3 = 0$. The set of endogenous state variables is $(k, d)$, while $z$ is exogenous.[11] Because the CRS producers can be constrained, and hence not on their Euler equation, the first order conditions of the unconstrained agents with DRS technology for both debt and capital are used as constraints in the planner's maximization. A Markovian planner cannot commit to future policies. However, the impact of current capital and

[10] Prime symbols are used here to indicate next period's variables. Given the notation used thus far, however, $k' = k_t$ and $d' = d_t$, the choices made at time $t$ for $t + 1$.

[11] Given that total asset supply is 1 and bond supply is 0, accounting for the choices of only one of the agents' type is sufficient.

borrowing and saving choices on the state variables that future regulators will face is internalised.

**Definition 2.2.** *A Markovian constrained efficient equilibrium is a set of allocations* $\{\hat{c}_t^C, \hat{c}_t^D, \hat{d}_t, \hat{k}_t\}$*, prices* $\{\hat{q}_t, \hat{r}_t\}$*, and Lagrangian multipliers* $\{\hat{\mu}_t\}$ *for* $t = 0, 1, 2$*, such that:*

1. *In each time period, the planner solves the above problem taking the policy rules of future planners as given.*
2. *Time consistency: the current planner's conjecture of future policy rules are consistent with those optimally chosen by the current planner.*

If there exists welfare weights such that the planner's FOCs coincide with the decentralised FOCs, then the laissez faire equilibrium is constrained efficient. Given the frictions considered and the additional information that the planner internalizes, however, the regulator's intervention can improve on the efficiency of the equilibrium. In general, without intervention, no welfare weight can ensure that the decentralised FOCs of the two groups of agents satisfy the planner's optimal choices.

### 2.4.2 Constrained Markovian Planner's Optimal Plan

Before describing the optimal plan chosen by the planner, it is useful to define some concepts that will be used later. In particular, the planner discounts future revenues using a different stochastic discount factor (SDF) than the agents' SDF:

$$\hat{m}_{t,t+1} = \frac{X_{t+1}}{X_t} m_{t,t+1}^C,$$

where $X_t$ accounts for all frictions driving a wedge between the two agents' marginal rates of substitutions in period $t$. Furthermore, the planner takes into account how changes in choice variables affect prices. $\frac{\mathrm{d}x}{\mathrm{d}y}$ represents the total differential of price $x$ with respect to choice variable $y$.

#### Adjusted SDF

The planner can discount future streams of income more or less than private individuals, depending on the ratio between current and future wedges between the two agents' MUCs (weighted by planner's welfare weights) as well as current and future binding collateral constraints:

$$\frac{X_t}{c_t^C} \equiv \frac{c_t^D}{\omega_t^D}\left[\frac{\chi}{c_t^C} - \frac{1-\chi}{c_t^D} - \frac{\theta q_t k_t \mu_t^P}{1-n}\right].$$

$X_t$ is non-zero in the laissez faire economy, reflecting the two main inefficiencies:

1. The lack of state-contingent bonds implies that no constant planner welfare weight can equate the two agents' marginal utilities of consumption for all time periods: $\frac{\chi}{c_t^C} \neq \frac{n(1-\chi)}{(1-n)c_t^D}$.
2. The presence of limited enforcement generates a positive multiplier on the collateral constraint, $\mu_t^P > 0$.

$\frac{c_t^D}{\omega_t^D}$ is the consumption share out of net worth of the DRS agent (or saver), which is an important variable in the determination of the planner's SDF. If constant over time, it does not affect how the planner discounts future revenue. Changes over time in this share will induce higher or lower discounting on the side of the planner.

### Price Derivatives

Individual agents take prices as given when taking decisions. The planner, however, internalises how their choices affect prices, and how these in turn affect the budget constraint of both agents. It is therefore useful to consider the following definitions:

$$\delta_{t+1}^{qd} \equiv \frac{\partial q_t}{\partial q_{t+1}}\frac{\partial q_{t+1}}{\partial d_t} + \frac{\partial q_t}{\partial c_{t+1}^D}\frac{\partial c_{t+1}^D}{\partial d_t} < 0; \qquad \delta_{t+1}^{Rd} \equiv \frac{\partial r_t^{-1}}{\partial c_{t+1}^D}\frac{\partial c_{t+1}^D}{\partial d_t} < 0$$

$$\delta_{t+1}^{qk} \equiv \frac{\partial q_t}{\partial q_{t+1}}\frac{\partial q_{t+1}}{\partial k_t} + \frac{\partial q_t}{\partial c_{t+1}^D}\frac{\partial c_{t+1}^D}{\partial k_t} > 0; \qquad \delta_{t+1}^{Rk} \equiv \frac{\partial r_t^{-1}}{\partial c_{t+1}^D}\frac{\partial c_{t+1}^D}{\partial k_t} > 0.$$

Prices depend positively on the saver's current net worth and negatively on their future net worth. When choosing an optimal plan, the planner does not have the ability to commit to future policies. Nevertheless, they internalise how their current choices affect future state variables, and how this in turn affects current prices. In a general equilibrium with complete markets, the planner's decisions would still involve these derivatives, but they would cancel out in the aggregate as all agents have the same marginal utility. For example, a price increase could hurt the agent at the buying end of a transaction, but it would also benefit the agent at the selling end by an equal amount. This transfer would not affect the efficiency of the allocation in the aggregate. Because only risk-free debt is available, and the collateral constraint can potentially bind, however, marginal utilities are not equated, and a change in price could benefit one agent more than it hurts another. The planner takes this into account when choosing their optimal plan, and will find it optimal to encourage price changes that go in favour of the agent with highest marginal utility.

The saver's net worth is affected by choices of the CRS-type through market clearing conditions. Increased debt holding for the CRS-type (increased savings for the DRS-type) raises the net worth of the saver in the following period, which depresses both price of debt and of the asset in the current period. Increased asset holding for the CRS-agent (reduced asset holding for the DRS-agent) decreases the saver's net worth in the following period, hence boosting current price of debt and of the asset.

### Constrained Planner's FOC's

The planner's optimal choices with respect to capital and bonds of the CRS-agent can be summarised as follow:[12]

$$\frac{1}{1+\hat{r}_t} = E_t\left(\hat{m}_{t,t+1}\right) + \frac{\hat{\mu}_t \hat{c}_t}{X_t}\left[1 + \theta \hat{k}_t E_t\left(\delta^{qd}_{t+1}\right)\right] + E_t\left(\delta^{rd}_{t+1}\hat{d}_t - \delta^{qd}_{t+1}\Delta\hat{k}_t\right) \tag{2.4.1}$$

$$\begin{aligned}\hat{q}_t = E_t\left[\hat{m}_{t,t+1}\left(\hat{q}_{t+1} + \alpha\left(\hat{k}'_t\right)^{\alpha-1} z_{t+1}\right)\right] + \frac{\theta\hat{\mu}_t\hat{c}_t}{X_t}\left[\hat{q}_t + \hat{k}_t E_t\left(\delta^{qk}_{t+1}\right)\right] + \\ E_t\left[\delta^{rk}_{t+1}\hat{d}_t - \delta^{qk}_{t+1}\Delta\hat{k}_t\right] + E_t\left[\frac{\hat{m}_{t,t+1}z_{t+1}}{X_{t+1}}\left(A - \alpha\left(\hat{k}'_t\right)^{\alpha-1}\right)\right]\end{aligned} \tag{2.4.2}$$

Equation (2.4.1) describes the optimal allocation of debt from the point of view of the planner. The aggregate benefit of a marginal increase in debt is a larger utility at the margin for the borrower and a lower marginal utility for the saver, by an amount equal to $\frac{1}{1+r_t}$. The marginal cost of increasing debt is realised in the following period in the form of a reduction in marginal utility of the CRS-agent net of the increase in marginal utility of the DRS-agent. Additionally, as savings for the DRS-agent increase, their net worth in the following period also increases. The planner internalises how changes in the net worth of the unconstrained agent affect prices, and this is important both because the

[12] More details on the derivation can be found in Appendix 2.C.

asset price enters the collateral constraint and hence influences the borrowing capacity directly, but also because prices affect the budget constraints of both agents in different ways. An increase in future net worth of the saver affects future prices positively. The increased future asset price boosts expected future borrowing capacity, therefore reducing the extent to which the collateral constraint is expected to bind in the following period. This represents an externality, which is not taken into account by individual producers. On the other hand, an expected increase in savers' future net worth depresses both the current price of debt as well as the price of the asset. The fall in the current asset price makes the current borrowing constraint more binding, hence increasing the cost of higher debt. There is therefore a trade-off between relaxing the current collateral constraint and acting prudentially to ensure that the collateral constraint next period is less binding in expectation.[13] At the same time, a reduced asset price makes asset purchases less expensive for the constrained agent. Finally, the reduction in the price of debt corresponds to an increase in the interest rate, which again represents an additional cost for the borrower and a benefit for the saver. Notice that if bonds were state-contingent, these price changes would still be present but they would be cancelling out in the aggregate, so the planner would not want to intervene on this margin. It is because one agent has a higher marginal utility that changing prices in favour of such agent generates a benefit that outweighs the cost imposed on the agent at the other end of the transaction.

Consider then Equation (2.4.2), defining the optimal asset allocation from the point of view of the planner. Increased asset holding of the CRS-agent entails a marginal cost in terms of reduced period $t$ consumption for CRS and increased consumption for DRS, who must be at opposite ends of the transaction. The lower asset availability for the DRS-agent translates into lower returns from asset holding in the following period. Hence, the saver will be consuming marginally less and the borrower marginally more. An increase in asset holding of the constrained agent affects the future net worth of the saver negatively, hence depressing asset prices and borrowing capacity in $t+1$. This is clearly a cost from the point of view of the planner. The fall in savers' net worth additionally boosts the current asset price, hence relaxing the collateral constraint, if binding, but increasing the cost of asset purchases. The increase in the price of debt/fall in the interest rate benefits

[13]Note that here I am describing FOCs for generic time t, when the constraint can be both currently binding and have a probability of being binding in the future. In the specific model I consider however, this is never the case and only one of these two things can be true at any given time.

the borrower and hurts the saver. Finally, part of the benefits of increasing the asset allocated to the CRS-agent are not related to prices but rather to quantities. Whenever the distance between the marginal productivity of the two agents is positive, the planner sees a benefit from re-allocating the asset from the DRS- to the CRS-type. A larger level of aggregate production can thus be achieved.

**Proposition 2.4.1.** *The constrained social planner allocation differs from the laissez-faire allocation due to:*

*(i)* Imperfect risk sharing. *Current and expected distance in the agents' MUCs.*

*(ii)* Pecuniary externalities.

  *(ii.a) Changes in prices disproportionately affecting the agent with highest marginal utility.*

  *(ii.b) Changes in the price of capital affecting borrowing capacity of the constrained agent.*

*(iii)* Allocative inefficiency. *There exist benefits from redistributing capital.*

*Proof.* See First Order Conditions (2.4.1) and (2.4.2) and compare to decentralised First Order Conditions (2.A.1)-(2.A.2) and (2.A.3)-(2.A.4). □

Consider the case where state contingent contracts are available in $t = 0$. If the collateral constraint does not bind in that period, the distance between the two agents' marginal utilities is zero, $X_0 = 0$. Then, prices are efficient, and any change in price generated by increased debt or capital holding would also be efficient: the planner does not want to intervene to alter these price effects. Additionally, perfect capital allocation is possible in $t = 0$. Hence, the two agents' MPKs are equated and the planner does not see a benefit from re-allocating the asset to the CRS-agent. However, there would still be scope for improvement on the laissez-faire equilibrium due to the possibility of a binding collateral constraint in $t = 1$, and the externality that this generates. Interventions in this case would be purely prudential, as there would be no trade-off between current efficiency of the allocation and future stability of the system. In particular, in this case, intervention in the debt market exclusively would ensure a constrained efficient allocation in both debt and capital markets.

Consider now the case where there is no collateral constraint but only risk free debt. In general, $X_t \neq 0$ due to marginal utilities of the two agents not being equated state by state. For the same reason, the two agents' MPKs are also not equated. The planner's interventions in this case are not prudential but linked exclusively to an efficient resource distribution, as no risk of a crisis exists in $t = 1$. The planner's interventions are aimed at improving the period 0 allocation, by tackling inefficient asset allocation and the lack of risk sharing, taking also into account inefficient price movements. This clarifies the role of each inefficiency further.

### 2.4.3 Comparison with Laissez-Faire Equilibrium

To have a measure of how the optimal decisions of the planner differ from the decentralised choices of the agents, I define wedges between the two sets of optimal conditions. I will use notation $\tau^C_{t,d}, \tau^C_{t,k}$ to denote deviations of the first order conditions of the CRS-agent (the borrower) from the planner's choices, and $\tau^D_{t,d}, \tau^D_{t,k}$ for wedges on optimal savings and asset sales of the DRS-agent (the saver). These wedges will enter the first order conditions of the agents and alter their optimal choices in the following way:

$$\frac{1}{\tau^C_{t,d}(1+r_t)} = E_t(m^C_{t,t+1}) + \mu^C_t c^C_t; \quad q_t(1+\tau^C_{t,k}) = E_t\left[m^C_{t,t+1}\left(q_{t+1} + f'_C z_{t+1}\right)\right] + \theta q_t \mu^C_t c^C_t \tag{2.4.3}$$

$$\frac{1}{\tau^D_{t,d}(1+r_t)} = E_t(m^D_{t,t+1}); \qquad q_t(1+\tau^D_{t,k}) = E_t\left[m^D_{t,t+1}\left(q_{t+1} + f'_D z_{t+1}\right)\right]. \tag{2.4.4}$$

With both $\tau_i$'s taken as given by agents.[14]

#### Macroprudential Wedges

While it is not possible for regulators to manipulate competitive prices directly, the planner's choices differ from the agents', due to both a prudential and a re-allocative

[14]To ensure that the wealth of individuals is not altered, lump sum transfers or taxes can be used, exactly rebating the consequences of the wedges to each consumer:

$$T^j_t = q_t \tau^j_{t,k} k^j_t + \frac{\tau^j_{t,d} d^j_t}{(1+r_t)(1+\tau^j_{t,d})}.$$

.

objective.

**Lemma 2.4.1.** *Define* $\tilde{\theta} \equiv \theta/(1-\chi)$*;* $\zeta^j_{t+1} = \frac{z_{t+1} f'_j}{q_t}$ *for* $j = C, D$ *the dividend yield;* $\xi_t \equiv \frac{c^D_t}{\omega^D_t}$ *the consumption share out of net worth of the DRS agents;* $D^i_t \equiv \delta^{ri}_t d_{t-1} - \delta^{qi}_t \Delta k_{t-1}$ *for* $i = d, k$ *the combined effect of marginal changes in the price of debt and capital on the agents' budget constraints. Wedges on debt and saving choices are:*

$$\tau^C_{0,d} = \left[E_0(m^C_{0,1})\right]^{-1} \left\{ E_0 \left( \frac{\xi_1}{\xi_0} m^C_{0,1} \right) + E_0 \left[ D^d_1 \right] \right\}$$
$$\tau^D_{0,d} = \left[E_0(m^D_{0,1})\right]^{-1} \left\{ E_0 \left[ \frac{\xi_1}{\xi_0} \left( 1 + \tilde{\theta} q_1 k_1 \mu^P_1 \right) m^D_{0,1} \right] + E_0 \left[ D^d_1 \right] \right\}$$

*Wedges on investment decisions are:*

$$\tau^C_{0,k} = -E_0 \left[ \frac{\xi_1}{\xi_0} m^C_{0,1} (1 + \widetilde{r}^D_1) \right] - \frac{E_0 \left[ D^k_1 \right]}{q_0} - E_0 \left[ \frac{m^C_{0,1}}{\xi_0} \left( \zeta^C_1 - \zeta^D_1 \right) \right]$$
$$\tau^D_{0,k} = -E_0 \left[ \frac{\xi_1}{\xi_0} \left( 1 + \tilde{\theta} q_1 k_1 \mu^P_1 \right) m^D_{0,1} (1 + \widetilde{r}^D_1) \right] - \frac{E_0 \left[ D^k_1 \right]}{q_0}$$

*Proof.* Compare the decentralised First Order Conditions (2.4.3) and (2.4.4) to the planner's First Order Conditions (2.4.1) and (2.4.2) to obtain these expressions. See Appendix 2.C.1 for more details. □

Mapping into what was mentioned in the previous section, there are four margins of interventions:

1. *Consumption Smoothing*: an increasing consumption share of the saver over time motivates a discouragement of debt and an encouragement of capital holdings of the borrower. The opposite is true for the saver. If the consumption share is constant over time $\left(\frac{\xi_1}{\xi_0} = 1\right)$ then this margin does not require any intervention.

2. *Pecuniary Externalities*:

   2a. There are benefits from intervening in both debt and capital market whenever prices are not state-contingent. With state-contingent prices, the $D^j$ elements would disappear in the aggregate, and agents would be sharing risk perfectly between themselves.

2b. Expectations of a binding collateral constraint in the future requires higher savings and lower capital sales of the unconstrained agent. Encouraging both more savings and capital holding of the saver can help relax the constraint in the following period thanks to an increased asset price through larger net worth of the DRS-type.

3. *Allocative Inefficiency*: A positive gap between the return to the asset between constrained and unconstrained agents motivates a subsidy for capital holdings of the borrower.

Both the consumption smoothing and the first of the two pecuniary externalities hit the two producers in a symmetric way. On net, these can entail either subsidies or taxes depending on whether the consumption share of the saver is increasing over time and on whether effects of price changes on the budget constraint are positive or negative. It is not possible to sign these effects in general. The second pecuniary externality connected to the collateral constraint, on the other hand, impacts the policy instrument in a way that can be signed clearly. The regulator tries to increase the net worth of the saver so as to help relax the constraint via an increased capital price. This externality implies a subsidy for savings and a tax on capital sales of the unconstrained agent. The allocative efficiency motive only enters the choice of asset of the CRS-agent and implies a subsidy for asset purchases of the constrained agent.

**Proposition 2.4.2.** *The laissez faire economy is not constrained efficient. Both decisions of debt and capital holdings of the constrained social planner differ from the decentralised allocation.*

*Proof.* The wedges defined in Lemma 2.4.1 are non-zero for both capital and debt choices, for whatever choice of planner's weights, which proves that the laissez-faire allocation is not constrained efficient. See Appendix 2.C.1 for more details. □

Notice that, to the extent that the wedges between private individuals' optimal choices and the regulator's choices are implemented in the decentralised economy, it is not important which agent the policy is targeted to. This is because the two agents are always on opposite sides of each transaction, and markets have to clear. Obviously, the exact form of the wedges would change if the optimal policy was implemented targeting only one of the agents. However, using an anonymous system where every agent is subject to the same wedge is not possible. Given that the two agents' state-by-state MRSs are different, it remains crucial that the two agents are influenced in differential ways to achieve constrained efficiency.[15]

In a representative agent, small open economy model, restricting debt is sufficient to achieve the goal of the regulator. The price of capital is not relevant for how the asset is allocated: because there is only one type of agent, capital is necessarily allocated efficiently. Furthermore, the interest rate is exogenous and not affected by agents' choices. Here, however, interventions in the capital market are crucial. Intuitively, the increased resources available in period 1 in the form of larger quantities of borrowing will be spent on both consumption and capital, as it is not just the timing of consumption that is inefficient, but also the allocation of capital. Due to the lack of state contingent contracts, only the *expected* stochastic discount factor can be affected. Given that the planner has access to the same market structure as private individual, there is no way for them to alter the SDF state by state. As a result, even after intervention in the debt market to ensure an efficient debt/savings allocation, further action is necessary in the capital market to directly correct any additional capital misallocation. Both the price of capital and the interest rate are distorted and require interventions. Fixing inefficiencies affecting the debt market is not sufficient to ensure an efficient capital market allocation[16].

### Ex-post Wedges

Ex-post in period 1, no intervention is needed in the good state, $\tau^C_{1,i}(g) = \tau^D_{1,i}(g) = 0$ for $i = d, k$. In the bad state, interventions are aimed at improving the capital allocation so that production in period 2 is not too low. No further prudential concern is present for the planner at that point. Ex-post wedges on debt and capital in the bad state are:

$$\tau^C_{1,d} = \left(m^C_{1,2} + \mu^C_1 c^C_1\right)^{-1} \left\{ \frac{1}{\xi_1} \left[ m^C_{1,2} + \mu^P_1 c^C_1 \left(1 + \theta k_1 \delta^{qk}_2\right)\right] + D^d_2 \right\}$$

[15] See also Dávila and Korinek (2018) on this point.

[16] Notice that this result would not hold in presence of state-contingent bonds. In that case, intervening with a tax to alter the stochastic discount factor used to discount profits from state-contingent bonds would automatically imply that the SDF used to discount profits from real capital holding is also different, and this is enough to ensure that the allocation is constrained efficient. See also Lorenzoni (2008) and Appendix 2.B for more details.

$$\tau_{1,d}^D = \left(m_{1,2}^D\right)^{-1} \left[\frac{m_{1,2}^D}{\xi_1}\left(\frac{1}{1+\tilde{\theta} q_1 k_1 \mu_1^P}\right) + D_2^d\right]$$

$$\tau_{1,k}^C = -\frac{1}{\xi_1} m_{1,2}^C \zeta_2^C + \theta c_1^C \left[\mu_1^C - \frac{\mu_1^P}{\xi_1}\left(1 + k_1 \delta_2^{qk}\right)\right] - \frac{D_2^k}{q_1}$$

$$\tau_{1,k}^D = -\frac{1}{\xi_1}\left(\frac{1}{1+\tilde{\theta} q_1 k_1 \mu_1^P}\right) m_{1,2}^D \zeta_2^D - \frac{D_2^k}{q_1}$$

In the high productivity realisation, the Lagrangian multiplier on both the borrowing constraints of the borrower and the planner is zero and, with equal MRSs for the two agents, both taxes on capital and borrowing are zero. Aggregate production will not be at full potential, given the misallocation of the previous period, but there is nothing that a regulator can do to improve this.

In the low productivity realisation, discouraging savings can be beneficial in a moment when the constraint is binding, as this increases the current net worth of the saver and boosts prices, hence relaxing the constraint. Notice that, conditional on interventions in the previous period, the constraint for the private agents will be as binding as for the planner, $\mu_1^P = \mu_1$.

Interventions of the planner are aimed at tackling two inefficiencies: 1) when the constraint is binding, increasing the price of capital through increased asset holding of the constrained agent or through reduced savings helps relaxing the collateral constraint; 2) increasing the price of capital in a moment when the borrower is fire-selling capital is beneficial as the Lagrangian multiplier on the collateral constraint drives a positive wedge between the marginal utility of consumption of the borrower and the saver.

## 2.5 Constrained Efficient Allocation under Commitment

The problem of a social planner who has the ability to choose the allocation once and for all is now considered. Differently from the discretionary equilibrium, the planner will not take the policy rules of future regulators as given, but will rather be able to directly choose them. The efficiency of the allocation under commitment will be no lower than in the discretionary equilibrium, given the wider choice set of the planner in the former case.

### 2.5.1 Constrained Planner's Allocation

The problem faced is analogous to 2.4.1, with the only difference that the entire allocation sequence is chosen once and for all.

$$\max_{\{c_t^C, c_t^D, d_t, k_t, q_t, r_t\}_{t=0}^2} \beta^t \left[ n\chi \left(\log c_t^C\right) + (1-n)(1-\chi) \left(\log c_t^D\right) \right]$$

s.to

$$c_t^C + q_t \Delta k_t \leq A z_t k_{t-1} - d_{t-1} + \frac{d_t}{(1+r_t)} \qquad \lambda_t^C \geq 0$$

$$c_t^D - \frac{n}{1-n} q_t \Delta k_t \leq z_t \left(\frac{1-nk_{t-1}}{1-n}\right)^\alpha + \frac{n}{1-n}\left[d_{t-1} - \frac{d_t}{(1+r_t)}\right] \qquad \lambda_t^D \geq 0$$

$$d_t \leq \theta q_t k_t \qquad \mu_t^P \geq 0$$

$$q_t = \beta E_t \left[\frac{c_t^D}{c_{t+1}^D}\left(q_{t+1} + \alpha z_{t+1}\left(\frac{1-nk_t}{1-n}\right)^{\alpha-1}\right)\right] \qquad \gamma_t^q$$

$$\frac{1}{1+r_t} = \beta E_t \left[\frac{c_t^D}{c_{t+1}^D}\right] \qquad \gamma_t^r$$

**Definition 2.3.** *A constrained efficient equilibrium under commitment is a set of allocations* $\{c_t^C, c_t^D, d_t, k_t\}_{t=0}^2$*, prices* $\{q_t, r_t\}_{t=0}^2$*, and Lagrangian multipliers* $\{\mu_1^P\}$ *such that at time* $t = 0$ *the planner solves the above problem.*

The optimal choices of the planner under commitment differ from discretion, due to the planner's ability to make credible promises concerning future policy. In particular, the regulator will exploit her commitment ability to affect current prices by promising changes in:

(i) Future capital prices. This matters for current capital prices because the price is forward looking.

(ii) Future consumption of the saver. This matters for current prices because the stochastic discount factor is non-linear and depends on the saver's future consumption.

**Constrained Planner's FOCs under Commitment, $t$=0**

Optimal choices of the planner in period zero can be summarised as:

$$\frac{X_0}{(1+r_0)} = E_0\left(X_1 m^C_{0,1}\right) + \beta X_0 E_0\left[\xi_1 D^d_1\right] \tag{2.5.1}$$

$$X_0 q_0 = E_0\left[X_1 m^C_{0,1}\left(q_1 + z_1 f'_D\right)\right] - \beta X_0 E_0\left[D^k_1 - \xi_1 D^d_1\left(q_1 + z_1 f'_D\right)\right] + E_0\left[m^C_{0,1} z_1\left(f'_C - f'_D\right)\right] \tag{2.5.2}$$

In period 0, the planner faces no previous commitment, and is therefore in a similar position as a planner acting under discretion. Nevertheless, there are differences between the discretionary and commitment case. Firstly, the consumption share of savers is now used to weigh the indirect effect of asset and debt choices on prices. As a result of this effect, the pecuniary externality connected to imperfect risk sharing will have a smaller impact on the allocation than in the case of discretion, given that savers consume a share of their net worth which is smaller than one. Looking at Equation (2.5.2), the appearance of risk sharing elements connected to the price of debt can be observed. A no arbitrage condition links the return to capital to the return on bonds. Therefore, any promise made regarding future interest rate will have consequences on the asset price, and is therefore taken into account by the planner. Analogously to the debt market, interventions in the capital market will be smaller than the discretionary case, given again a share of consumption for the saver which is less than one, $\xi_1 < 1$. Hence, in period 0 the difference between choices of a discretionary planner and one operating under commitment is only linked to the risk sharing motive, and the planner will intervene less in the case of commitment than in the case of a Markovian equilibrium.

Note that if state-contingent bonds were available, then the elements connected to imperfect risk sharing would disappear, and the optimal decisions of the planner with commitment in period 0 would coincide with those of the Markovian planner. The next subsection focuses on policy choices in period 1.

**Constrained Planner's FOCs under Commitment, $t$=1**

In period 1, the optimal plan is:

$$\frac{1}{(1+r_1)}\left[\frac{X_1}{c_1^C}-\frac{X_0}{c_0^C}\xi_1 D_1^d\right]=\mu_1^P\left(1+\theta k_1\delta_2^{qd}\right)+\beta\left\{\frac{X_2}{c_2^C}+D_2^d\left[\frac{X_1}{c_1^C}-\frac{X_0}{c_0^C}\xi_1 D_1^d\right]+g(\delta_2^{qd})\right\}$$

$$\begin{aligned}q_1\left[\frac{X_1}{c_1^C}-\frac{X_0}{c_0^C}\xi_1 D_1^d\right]=\theta\mu_1^P\left(q_1+k_1\delta_2^{qk}\right)+\beta\Bigg\{&\frac{X_2 z_2 f_D'}{c_2^C}+\frac{z_2\left(f_C'-f_D'\right)}{c_2^C}\\&+D_2^k\left[\frac{X_1}{c_1^C}-\frac{X_0}{c_0^C}\xi_1 D_1^d\right]+g(\delta_2^{qk})\Bigg\}\end{aligned}$$

with g($\delta_2^{qi}$) $=\left[\left(\frac{X_1}{c_1}-\frac{X_0}{c_0}\xi_1 D_1^d\right)\theta\mu_1^P k_1-\frac{X_0}{c_0}m_{0,1}^D\Delta k_0\right]\delta_2^{qj}$ for $i=d,k$.

Looking at the left hand side of these two equations, it is apparent that promises made in period 0 are carried over and influence the allocation in period 1, changing the marginal cost of saving and of asset holding. Analogously, the risk sharing terms are also carrying forward the effect of past promises. Period 0 promises enter with an opposite sign as current period's promises, an indication of potential time inconsistency issues. Moreover, the presence of the functions $g(\delta_2^{qi})$, $i=d,k$, makes evident that one of the main reasons why the discretionary allocation differs from commitment is the forward looking nature of the asset price. Changes in asset and debt holding affect the price in period 2, which has consequences on the allocation in period 1 through changes in borrowing capacity, and in period 0 through changes in the (forward-looking) price of asset. These effects are absent for the interest rate, which does not share the same recursive nature of the asset price. Moreover, intervention under commitment in period 1 are aimed at tackling the binding constraint in period 1 by making promises of lower future consumption for the saver in period 2, which boosts the current asset price.

### 2.5.2 Time Consistency of Equilibrium Under Commitment

In the previous description of the optimal allocation, I hinted that the commitment case might be subject to problems of time inconsistency. Let's now focus our attention on this aspect of the equilibrium.

**Definition 2.4.** *An optimal plan is time consistent if, given a chance to re-optimize at any later date, the planner would make the same choices as in the original plan.*

A sufficient condition to ensure time consistency is that no promises are made under commitment which would not be carried forward by a discretionary planner. So long as the optimal allocation with commitment and of the Markov planner coincide, then the allocation of the planner with commitment is time consistent. Here, the two plans do not coincide and therefore, time consistency does not hold.

Already in period 0, the choices of the Markovian planner and the one with commitment ability do not coincide. What drives this divergence is the lack of state contingent asset. If the only reason for intervention in period 0 was purely prudential and linked to ensuring the collateral constraint is not too binding in the future, then the Markovian and commitment allocation would coincide at time 0[17]. With state contingent assets, current prices are state dependent: so long as the future net worth can be affected in an appropriate way, the relevant price today will change accordingly, so discretionary and commitment allocation do not differ.

In period 1, the Markovian allocation differs from the allocation with commitment for two reasons: 1) because of promises made in period 0; 2) because of promises regarding period 2. Notice that the crucial assumptions for the result of time inconsistency here is the concavity of the utility function and the forward looking capital price. With linear utility, the stochastic discount factor would not feature future consumption. The planner would not want to make promises in $t$ regarding consumption of the saver in $t+1$, as that would not affect current prices. Without forward looking asset prices, the planner would not make promises in period 0 regarding the asset price in 1 so as to affect the allocation in 0. Removing both these two assumptions would therefore bring about a time consistent plan. This is the case, for example, in Lorenzoni (2008).

Even with risk averse agents, the plan could still be time consistent if the price was not forward looking and, crucially, the planner attached a zero welfare weight to savers. Intuitively, if the planner does not care about the savers, then carrying forward a plan of lower consumption for that class of agent has no cost, but brings about a benefit in the period that the promise is made. This is the case in some of the open economy literature (e.g. Bianchi (2011)), where the asset price is static and the country is a borrower, so

[17]They would not coincide at time 1. See Appendix 2.B.

that zero weight is attached to savers.

To be more specific about the problem of time inconsistency, it is useful to rewrite here some of the planner's first order conditions under commitment for general time $t$:

$$\lambda_t^C = \frac{\chi}{c_t^C} \tag{2.5.3}$$

$$\lambda_t^D = \left\{ 1 - \chi + \gamma_t^q q_t + \frac{\gamma_t^r}{1 + r_t} - m_{t-1,t}^D \left[ \gamma_{t-1}^q \left( q_t + z_t f_D' \right) + \gamma_{t-1}^r \right] \right\} \frac{1}{c_t^D} \tag{2.5.4}$$

$$\gamma_t^q = \sum_{s=0}^{t} (\pi)^s m_{t-s,t}^D \left[ \theta k_{t-s} \mu_{t-s}^P - \left( \lambda_{t-s}^C - \frac{n}{1-n} \lambda_{t-s}^D \right) \Delta k_{t-s} \right], \quad m_{t,t} = 1 \tag{2.5.5}$$

$$\gamma_t^r = \left( \lambda_t^C - \frac{n}{1-n} \lambda_t^D \right) d_t \tag{2.5.6}$$

Equation (2.5.4) expresses the shadow value of wealth of savers as equal to their marginal utility of consumption plus any impact that changes in savers' wealth has on both current as well as past prices. Current Lagrangian multipliers $\gamma_t^q$ and $\gamma_t^r$ enter with an opposite sign with respect to past multipliers $\gamma_{t-1}^q$ and $\gamma_{t-1}^r$.

The sign of $\gamma_t^q$ and $\gamma_t^r$ cannot be determined uniquely. Laissez-faire equilibrium prices represent implementability constraints in the planner's problem, and because they have to hold with equality, the sign of the multipliers associated to these can be either positive or negative. Equations (2.5.5) and (2.5.6) show that depending on the distance in the shadow value of wealth of the two agents, and on the trading position of the CRS-agent, both $\gamma_t^q$ and $\gamma_t^r$ can be either positive or negative. The recursive nature of the asset price, in particular, implies that past wedges between the two agent's shadow value of wealth matter in determining the current value of the multiplier. Depending on the persistence of the process describing this wedge, it might or might not be possible for the multiplier on the asset price's implementability constraint to change sign from one period to the next.

If the sign of the multiplier on competitive prices remains the same over time, time consistency cannot be achieved[18]. For the allocation to be time consistent, it is necessary that the sign of the multipliers on prices alternates over time. Intuitively, a promised

[18] Bianchi and Mendoza (2018) show that time consistency is necessarily ruled out in their setting, as the Lagrangian multiplier on the price of debt is zero and the one on the asset price depends on the multiplier associated to the collateral constraint exclusively, which is always non-negative. Therefore, $\gamma_t^q$ is an always increasing sequence and time inconsistency follows.

decrease in future net worth affects current prices positively and future prices negatively. A necessary (although not sufficient) condition for time consistency is that the planner finds this sequence of price changes optimal over time, so that when time $t + 1$ comes, maintaining the promise of lower prices will not be too costly.

But in general, having multipliers with opposing sign is still not sufficient to guarantee time consistency: not only the planner needs to find the direction of interventions optimal ex-post, but their precise size too. While time inconsistency therefore holds in general, there can be cases where, conditional on a certain realisation of aggregate productivity, the planner's incentive to renege on past promises is low, because at least the sign of promised interventions remains optimal. As an example, if reducing the asset price is considered optimal at time 0 to re-allocate resources to the agent with the highest marginal utility, the planner can promise higher net worth of the saver in period 1, which reduces $q_0$. If the bad state is realised in period 1, a higher net worth, and hence a higher price $q_1$, is beneficial to help relax the collateral externality. Assuming this motive for intervention dominates in the planner's choices, a higher asset price might still be optimal, hence not violating the necessary condition of alternating signs of Lagrangian multipliers for time consistency.

Exact time consistency would require a very specific, knife-edge relation across the planner's margins of interventions, which is unlikely to hold in practice. In general, the planner's temptation to deviate from past plans is increasing in:

- *The amount of risk sharing.* The larger the distance between the agents' state contingent marginal rate of substitution at time 0 and 1, the more the planner could benefit from an ex-post deviation.

- *The size of the transaction.* The larger the trade in capital and bonds, the larger the effect of the pecuniary externality in period 0 and hence the temptation to deviate in period 1.

- *The price sensitivity.* For large changes in prices resulting from relatively small changes in quantities, the effect of the pecuniary externality on the agents' budget constraint is increased, hence the planner faces a bigger incentive to deviate ex-post.

Provided that risk sharing is close to perfect, capital and bonds are transacted in small quantities and their price changes modestly in response to the quantity changes, then the cost of commitment ex-post might not be so high. If the temptation to renege on past promises is relatively low, then it would not be as hard for the private sector to believe the regulator's promises.

The laissez-faire allocation differs from a constrained efficient equilibrium both in period 0 and in period 1. This implies that there is scope for policy interventions in both periods. In period 0, policies are partly prudential and partly aimed at addressing the problem of imperfect risk sharing. In period 1, in the event of a financial crisis, policies are aimed at its management. While the issue of how a constrained efficient allocation could be implemented in the decentralised economy has been left to the side so far, I now turn to this separate but related question.

## 2.6 The Regulated Economy

Many of the macroprudential and financial stabilisation policies adopted in developed countries are carried out in the form of interventions in the banking sector, as opposed to taxes and subsidies directly imposed on the debt and capital choices of firms. While a banking sector is absent from this model, any intervention in the banking sector will be passed on to firms if other frictions that might affect the relationship between banks and firms are abstracted away. Therefore, regulation will have an effect on borrowers' choices, here the CRS agents. I consider an instrument aimed at implementing macroprudential policies, such as a prudential leverage ratio, and an instrument to help the allocation ex-post, like bailout programs, quantitative easing, asset price supports.[19]

To derive the optimal instruments, I follow the primal approach: given the instruments available, I compare the first order conditions of the planner to the ones of agents in the decentralised economy.

### 2.6.1 Optimal Instruments in a Discretionary Allocation

To start with, consider a discretionary planner that cannot commit to future policies and optimally chooses the leverage ratio for ex-ante interventions and the capital price subsidy for ex-post interventions to maximize overall welfare, subject to individuals' optimal choices.

[19]Policies like the Troubled Assets Relief Program in the US could fall within this category.

Define $e_t = z_t f_C(k_{t-1}) - d_{t-1} + q_t k_{t-1}$, the level of CRS firms' equity. In this model, equity corresponds to total net worth. There is no way for a firm to raise new equity at the beginning of a period: it is a backward looking variable that is entirely determined by past choices and current shocks. While this assumption does not perfectly reflect reality, it is a fact that issuing new equity can be particularly difficult, especially during periods of financial distress. Implicitly, I am precluding this option completely by raising the cost to infinity, which simplifies the analysis. Let's be more precise about the policy instruments considered.

**Definition 2.5.** *Regulation of borrowers.*

- *Ex-ante interventions: firms are subject to:*

  *a.* A lower bound on their equity-to-asset ratio, $\frac{e_t}{q_t k_t} \geq \phi_t$. *(Leverage ratio requirements)*

- *Ex-post interventions: in case of a financial crisis, the planner can intervene by:*

  *a.* Providing a subsidy to the price of capital, $q_t k_t (1 + s_{t,k})$. *(Asset price support)*

  *b.* Lowering the interest rate on debt, $\frac{d_t}{(1+r_t)(1-s_{t,d})}$. *(Quantitative easing)*

  *Both these policies are financed through lump sum taxation of borrowers.*[20]

$\phi_t$ is the time- and state-varying equity requirement, while $s_{t,k}$, $s_{t,d}$ the ex-post subsidies.

Borrowers can use their equity and any amount of new borrowing undertaken to pay out dividends (consumption), and to invest in productive capital that provides a positive, risky return in the following period. Firms' choices today therefore affect the future value of their equity holding. This is important if they are aware of regulation that will be in place in the following period. Larger quantities of bond holdings allow the firm to satisfy their liquidity needs and buy more of the productive capital in the current period. In the future period, more debt will have to be repaid; at the same time, the increase in capital holding boosts their net worth as well as prices.

[20] One could argue that ultimately these kind of policy interventions are financed through taxation of the whole population and not just of borrowers. To abstract away from re-distributional aspects of such a policy, I ignore this aspect here. I additionally ignore the distortionary effects of proportional income taxation by considering lump-sum taxes only.

Starting from the last period, consider how the FOCs of private individual would be altered by the presence of subsidies. For simplicity, assume it is possible to target the subsidies to borrowers exclusively.[21]

$$
\begin{aligned}
\frac{1}{(1+r_1)(1-s_{1d})} &= m^C_{1,2} + \mu^C_1 c^C_1 \\
q_1(1+s_{1k}) &= A m^C_{1,2} z_2 + \theta \mu^C_1 c^C_1 q_1 (1+s_{1k})
\end{aligned}
$$

However, to implement the discretionary constrained efficient allocation in period 0, regulation has to also be imposed in period 1, so as to influence the choice of capital in an appropriate way. There is a chance that while the regulation is optimal from the point of view of period 0, it is no longer optimal from the point of view of period 1. Hence, even though the allocation is time consistent, the policy tools are not.

**Proposition 2.6.1.** *The period* t *constrained efficient allocation cannot be implemented by simply imposing an equity ratio requirement in either period* t *or* t+1 *only. It can however be implemented with a system of time-varying equity requirements imposed over consecutive periods.*

*Proof.* See Appendix 2.C.2. □

The result obtained in the previous section, showing that interventions in both bond and capital markets are necessary to implement the constrained efficient allocation remains valid. In particular, imposing equity requirements in period $t$ can only alter the agent's choice of capital investment, but not of bonds holding. A stringent equity ratio will induce lower capital holding for given level of equity. The desirability of this depends on whether, due to incomplete contracts, the CRS-type producer underinvests or overinvests in the first period. Additionally, nothing can be done to induce the correct level of bond holding in that period.

If an equity requirement was imposed in period $t+1$ only, both choices of capital and bond in period $t$ would be altered. The anticipation of a future equity requirement induces agents to take on less debt and/or purchase more capital in the current period, as this will boost equity in the future. While the direction of these incentives may be

[21]One might argue that QE is instead a policy that affects both borrowers and lenders. Here, this aspect of policy is not considered.

correct,the appendix shows that this is still not sufficient to implement the constrained efficient allocation. Inefficiencies in bond and capital markets involve interventions that cannot be reconciled with the use of just one instrument. In particular, the difference in MPK's can only be addressed by specific interventions in the capital market and cannot be implemented with one instrument without undesirably distorting the debt market.

Imposing equity requirements over consecutive periods, however, can implement the constrained efficient allocation. As mentioned before, anticipation of a capital requirement in $t+1$ is sufficient to alter both bond and capital choices in period $t$. Because an equity ratio has to be satisfied in period $t$ as well though, the choice of asset holding in $t$ will further be influenced by this margin. As a result, capital and debt decisions can be affected in differential ways, which allows implementation of the constrained efficient allocation.

Notice that for this result to hold, it is crucial that equity requirements be changed by the regulator in each period, so as to effectively implement the optimal equity ratios that the planner chooses in the constrained efficient allocation. These ratios will in general not be constant, as the economic stance changes. The ratio in time 0 depends on initial conditions which cannot be altered by the planner. On the other hand, the ratio in time 1 is fully the result of (present and past) optimal choices. Hence, the two optimal ratios will in general not be the same. Although the direction of intervention could be correct, imposing a constant equity requirement impacts on the optimality of the allocation.

It is important to highlight that the two equity requirements are both necessary to implement macroprudential policy in period 0, and do not take ex-post policy in period 1 into consideration. If ex-post interventions are needed (i.e. in the event of a crisis), then a further equity requirement in period 2 would have to be imposed[22]. However, a tension could arise between the implementation of optimal ex-ante and ex-post policy: the level of equity ratio requirement in period 2 would have to be chosen subject to the constraint of what was imposed in period 1 to implement period 0 policy.

From the discussion above, it is evident that the regulator needs to have commitment ability in order to implement the optimal policy, and that problems of time consistency may arise. In particular, after inducing borrowers to behave optimally in period 0, the regulator will be subject to the temptation of changing the equity ratio requirements

[22]In this setting, such a policy could not be credibly implemented, given that both bond and capital markets shut down in the last time period.

in period 1, so as to achieve her objectives for that period without constraining her instrument set in period 2.

It is also worth mentioning that while here only equity-asset ratios were considered, in practice more instruments are available to policy makers. This is crucial because employing more than one instrument at a time affords the policy maker not only to affect capital and bond markets differentially, but also to avoid problems of time inconsistency. This provides an additional argument in favour of simultaneously monitoring more than one ratio for borrowers.

### 2.6.2 Policy under Commitment

Similarly to the discretionary case, the optimal allocation under commitment can also be implemented in the competitive market. In general, however, because the allocation chosen by the planner is not time consistent, the instruments used to implement such allocation are also not time consistent.

**Proposition 2.6.1.** *The social planner's allocation under commitment can be implemented in the decentralised economy either through taxation or through capital requirements.*

*Proof.* See Appendix 2.D.1. □

Instruments in period 0 are not substantially different from the discretionary case, especially if the lack of state-contingent contracts does not impact risk sharing possibilities too badly (i.e. the $D_t^i,\ i = k, d$ terms are not too large). In period 1, there are more differences between policy under discretion and under commitment. These too are directly proportional to the size of $D_t^i$, and of $\delta^{qi}$ in particular. It is therefore evident that the source of time inconsistency is related to the price effects that the planner tries to tackle, and to the price of the asset especially.

## 2.7 Conclusions

This paper highlights the importance for a regulator to keep the connection between borrowing capacity and investment opportunity in mind. Limited enforcement problems generate uncertainty concerning the future stability of the system, motivating macroprudential interventions. However, the absence of state-contingent assets drives an inefficient resource allocation, which cannot be ignored by policy makers when drafting prudential policy. Interventions targeting financial stability without considering allocative efficiency can bring about undesired results. If this is to be avoided, it is important to keep track of both the funding of banks as well as the type of investment they engage in. Capital-asset ratio requirements imposed on banks can be useful in altering their debt and asset choices in a desirable way. The policy instrument can however only be used to correct the same type of inefficiencies in the two markets, hence the trade-off between stabilisation and efficiency of the system would still be present. Using more than one capital requirement over consecutive periods improves this, but maintaining the requirement constant over time equally induces a suboptimal allocation. Ideally, the ratio should be both state and time dependent, and announcements should be made concerning future state-contingent capital requirements, so that the private sector's expectations and choices are shaped by these announcements. The policy makers' ability to commit to future policies is therefore crucial. Regulators will likely be facing problems of time inconsistency, but it is possible that these are not too large. If policy makers' optimal decisions ex-post only differ from their ex-ante promises in terms of size, and not in terms of sign of intervention, then sticking to previously outlined plans could be not so costly.

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# Appendix

## 2.A Laissez-Faire Economy

### 2.A.1 Constrained Equilibrium in the Laissez-Faire Economy

The set of equilibrium conditions is:

for $t = 0, 1$ :

$$m^j_{t,t+1} \equiv \frac{\beta c^j_t}{c^j_{t+1}} \quad \text{for } j = C, D$$

$$q_t = E_t\left[m^C_{t,t+1}\left(q_{t+1} + z_{t+1} f'_C(k^C_t)\right)\right] + \theta q_t \mu^C_t c^C_t \tag{2.A.1}$$

$$q_t = E_t\left[m^D_{t,t+1}\left(q_{t+1} + z_{t+1} f'_D(k^D_t)\right)\right] \tag{2.A.2}$$

$$\frac{1}{(1+r_t)} = E_t\left(m^C_{t,t+1}\right) + \mu^C_t c^C_t \tag{2.A.3}$$

$$\frac{1}{(1+r_t)} = E_t\left(m^D_{t,t+1}\right) \tag{2.A.4}$$

$$q_2 = \mu^j_0 = 0, \quad \text{for } j = C, D \tag{2.A.5}$$

$$\mu^C_1\left[d^C_1 - \theta q_1 k^C_1\right] = 0, \quad \text{for} \quad z_1 \in \{z_b, z_g\} \quad \text{with} \tag{2.A.6}$$

$$d^C_1 < \theta q_1 k^C_1, \quad \mu^C_1 = 0 \qquad \text{for} \quad z_1 = z_g$$

$$d^C_1 = \theta q_1 k^C_1, \quad \mu^C_1 > 0 \qquad \text{for} \quad z_1 = z_b$$

for $t = 0, 1, 2$ :

$$c^C_t + q_t \Delta k^C_t = z_t f_C\left(k^C_{t-1}\right) - d^C_{t-1} + \frac{d^C_t}{(1+r_t)} \tag{2.A.7}$$

$$c^D_t + q_t \Delta k^D_t = z_t f_D\left(k^D_{t-1}\right) - d^D_{t-1} + \frac{d^D_t}{(1+r_t)} \tag{2.A.8}$$

$$n d^C_t + (1-n) d^D_t = 0 \tag{2.A.9}$$

$$n k^C_t + (1-n) k^D_t = 1 \tag{2.A.10}$$

### 2.A.2 Efficient Allocation in the Laissez-Faire Economy

Assume that at the beginning of time, entrepreneurs receive initial capital endowments. The notation $\bar{E}_0 \equiv n\bar{e}_0^C + (1-n)\bar{e}_0^D = nA\bar{k}^C + (1-n)(\bar{k}^D)^\alpha$ is used to indicate the initial aggregate endowment, where $\bar{k}^j \equiv k_{-1}^j$ and with $n\bar{k}^C + (1-n)\bar{k}^D = 1$. In a frictionless, decentralised allocation, $\mu_t^i = 0$ and state contingent debt exists. Using this information and equating (2.A.2) to (2.A.1), the optimal capital investment of each type of entrepreneurs is:

$$k_t^D = k_D^* = \left(\frac{\alpha}{A}\right)^{\frac{1}{1-\alpha}} \qquad \Longrightarrow \quad y_t^{*D} = z_t(K_D^*)^\alpha \tag{2.A.11}$$

$$k_t^C = k_C^* = \frac{1}{n} - \frac{1-n}{n}\left(\frac{\alpha}{A}\right)^{\frac{1}{1-\alpha}} \qquad \Longrightarrow \quad y_t^{*C} = z_t A K_C^* \tag{2.A.12}$$

The efficient level of output when capital is optimally allocated is

$$Y_t^* = \left[nA\left(\frac{1}{n} - \frac{1-n}{n}k_D^*\right) + (1-n)(k_D^*)^\alpha\right] z_t = \left[A + (1-n)(1-\alpha)\left(\frac{\alpha}{A}\right)^{\frac{\alpha}{1-\alpha}}\right] z_t \equiv \tilde{A} z_t$$

Combine Equations (2.A.3) and (2.A.4) with Market Clearing Condition (2.A.10) and the sum of the budget constraints of the two types (2.A.7) and (2.A.8), weighted by their respective share in the population. We obtain:

$$1 + r_{0s} = \frac{Y_{1s}}{\beta Y_0} = \frac{\tilde{A} z_{1s}}{\beta \bar{E}_0} \tag{2.A.13}$$

$$1 + r_1 = \frac{Y_2}{\beta Y_1} = \frac{z_2}{\beta z_1} \tag{2.A.14}$$

$$q_0 = \beta(1+\beta)\frac{A}{\tilde{A}}\bar{E}_0 \tag{2.A.15}$$

$$q_1 = \beta A z_1 \tag{2.A.16}$$

Using this information in the budget constraint of individuals (2.A.8) and (2.A.7) one can solve for the optimal levels of consumption, and the related quantity of borrowing or saving that is implied:

$$c_2^j = z_2 f_j(k_j^*) - d_1^j \equiv \omega_2^j \tag{2.A.17}$$

$$c^j_{1s} = z_{1s} f_j(k^*_j) - \frac{d^j_{0s}}{1+\beta} \equiv \frac{\omega^j_{1s} + \beta z_{1s} f_j(k^*_j)}{1+\beta} \tag{2.A.18}$$

$$d^j_{1s} = \frac{z_2}{z_{1s}} \frac{d^j_{0s}}{1+\beta} \tag{2.A.19}$$

$$c^C_0 = \frac{\bar{e}^C_0 + (\beta+\beta^2)(\bar{E}_0/\tilde{A}) A \bar{k}^C}{1+\beta+\beta^2} \tag{2.A.20}$$

$$c^D_0 = \frac{\bar{e}^D_0 + (\beta+\beta^2)(\bar{E}_0/\tilde{A}) \left[(k^*_D)^\alpha - A(k^*_D - \bar{k}^D)\right]}{1+\beta+\beta^2} \tag{2.A.21}$$

$$d^j_{0s} = \frac{(1+\beta)Y^*_{1s}}{1+\beta+\beta^2} \left[\frac{y^{*j}_{1s}}{Y^*_{1s}} - \frac{\bar{e}^j_0}{\bar{E}_0} + (\beta+\beta^2)\frac{A}{\tilde{A}}(k^*_j - \bar{k}^j)\right] \tag{2.A.22}$$

To ensure CRS agents are borrowers and DRS agents are lenders, we need to impose that the right-hand-side of Equation (2.A.22) be positive for CRS agents and negative for DRS. In either case, we obtain that $\bar{k}^D$ needs to be sufficiently large, such that the following condition is satisfied:

$$(\beta+\beta^2)A\left(\bar{k}^D - k^*_D\right) > -\left[\frac{\tilde{A}}{\bar{E}_0}(\bar{k}^D)^\alpha - (k^*_D)^\alpha\right] \tag{2.A.23}$$

A sufficient condition is therefore that the DRS agent starts the period with more capital than the efficient level:

$$\bar{k}^D > k^*_D \tag{2.A.24}$$

### 2.A.3 More details on Assumption 2.2

A necessary condition for the borrowing constraint to be slack at $t=0$ is that:

$$\begin{aligned} & d^C_{0s} < \theta_0 q_0 k^C_0 \\ \iff \quad & \theta_0 > \frac{\tilde{A} z_{1s}}{\beta \bar{E}_0}\left(1 - \frac{\tilde{A}/\bar{E}_0 + \beta + \beta^2}{1+\beta+\beta^2}\frac{\bar{k}^C}{k^*_C}\right) \end{aligned}$$

A necessary condition for the borrowing constraint to be binding at $t = 1$ in the bad state is that, in the unconstrained economy:

$$
\begin{aligned}
& d_1^C > \theta_1 q_{1b} k_1^C \\
& A z_2 \left( k_C^* - \frac{\tilde{A}/\bar{E}_0 + \beta + \beta^2}{1 + \beta + \beta^2} \bar{k}^C \right) > \theta_1 \beta A z_{1b} k_C^* \\
\iff \qquad & \theta_1 z_{1b} < \frac{z_2}{\beta} \left( 1 - \frac{\tilde{A}/\bar{E}_0 + \beta + \beta^2}{1 + \beta + \beta^2} \frac{\bar{k}^C}{k_C^*} \right)
\end{aligned}
\tag{2.A.25}
$$

Given $\theta > 0$ and $z_{1b} > 0$, for this to be possible the following condition, stronger than (2.A.24), needs to be satisfied:

$$
k_C^* > \left[ 1 + \frac{\tilde{A}/\bar{E}_0 - 1}{1 + \beta + \beta^2} \right] \bar{k}_C
$$

Additionally, notice that for the right-hand-side of (2.A.25) to exceed the left-hand-side, a further additional condition needs to be satisfied:

$$
\theta_1 < \frac{z_2}{\beta z_{1b}} = 1 + r_{1b} \tag{2.A.26}
$$

If the good state is realised, the following condition should hold:

$$
\theta_1 z_{1g} \geq \frac{z_2}{\beta} \left( 1 - \frac{\tilde{A}/\bar{E}_0 + \beta + \beta^2}{1 + \beta + \beta^2} \frac{\bar{k}^C}{k_C^*} \right)
$$

### 2.A.4 Proposition 2.3.1

*Proof.* Combining Equations (2.A.3) and (2.A.4) it is clear that condition (2.2.2) does not hold state by state but only on average. Additionally, combining (2.A.1) and (2.A.2), it is apparent that condition (2.2.1) fails too. □

### 2.A.5 Lemma 2.3.3

*Proof.* Assume state contingent assets are available and no borrowing constraint limits debt in $t = 1$. Then first order conditions would be:

$$\frac{1}{1+r_{ts}} = \beta \frac{c_t^C}{c_{t+1s}^C} = \beta \frac{c_t^D}{c_{t+1s}^D}$$
$$q_t = \beta E_t \left[ \frac{c_t^C}{c_{t+1}^C} \left( q_{t+1} + z_{t+1} f_C'(k_t^C) \right) \right] = \beta E_t \left[ \frac{c_t^D}{c_{t+1}^D} \left( q_{t+1} + z_{t+1} f_D'(k_t^D) \right) \right]$$

Which imply Equations (2.2.1) and (2.2.2). □

## 2.B State-Contingent Contracts with Limited Enforcement

The economy's only inefficiency is an occasionally binding collateral constraint in period 1. From the laissez faire economy, first order conditions are:

$$q_t = E_t \left[ m_{t,t+1}^C \left( q_{t+1} + z_{t+1} f_C'(k_t^C) \right) \right] + \theta \mu_t^C c_t^C$$
$$q_t = E_t \left[ m_{t,t+1}^D \left( q_{t+1} + z_{t+1} f_D'(k_t^D) \right) \right]$$
$$\frac{1}{(1+r_t^s)} = m_{t,t+1s}^C + \mu_t^C c_t^C$$
$$\frac{1}{(1+r_t^s)} = m_{t,t+1s}^D$$

### 2.B.1 Constrained Efficient Allocation, Commitment

Define $k_t^C = k_t$, $k_t^D = \frac{1-nk_t}{1-n}$ and $d_t^C = d_t$, $d_t^D = -\frac{nd_t}{1-n}$. The constrained planner, who has commitment ability, solves the following problem:

$$\max_{\{c_t^C, c_t^D, d_t, k_t, q_t, r_t\}_{t=0}^2} \sum_{t=0}^{2} \beta^t \left[ n\chi E_0 \left( \log c_t^C \right) + (1-n)(1-\chi) E_0 \left( \log c_t^D \right) \right]$$

s.to

$$c_t^C + q_t \Delta k_t \leq z_t f_C(k_{t-1}) - d_{t-1} + \sum_{s'|s} \frac{d_{ts'}}{1+r_{ts'}} \qquad \forall t, s$$

$$c_t^D - \frac{n}{1-n} q_t \Delta k_t \leq z_t f_D\left(\frac{1-nk_{t-1}}{1-n}\right) + \frac{n}{1-n}\left[d_{t-1} - \sum_{s'|s} \frac{d_{ts'}}{1+r_{ts'}}\right] \qquad \forall t, s$$

$$d_t \leq \theta q_t k_t \qquad t = 1, \forall s$$

$$q_t = \beta E_t\left[\frac{c_t^D}{c_{t+1}^D}\left(q_{t+1} + z_{t+1} f_D'\left(\frac{1-nk_t}{1-n}\right)\right)\right] \qquad \forall t, s$$

$$\frac{1}{1+r_{ts'}} = \beta \frac{c_t^D}{c_{t+1s'}^D} \qquad \forall t, s$$

**Period 0.**

In period 0, the economy features no inefficiencies. However, the planner internalises how choices in $t = 0$ affect the price of capital in period 1, which in turn determined the borrowing capacity in that period. The planner anticipates the *collateral externality* that affects the economy in period 1 with non-zero probability. Interventions are aimed at (1) increasing savings and (2) reducing investment of the CRS-agent, to implement the constrained efficient allocation. In particular, thanks to the presence of state-contingent contracts, interventions that modify the stochastic discount factor will naturally also affect the price of capital thanks to a no-arbitrage equation linking the asset price to the returns on bonds. Therefore, intervening in the bond market exclusively is sufficient to implement the constrained efficient allocation.

**Period 1.**

Once the uncertainty is realised in period 1, one of two scenarios are possible. If the realisation of the state is good, then the constraint is not binding and the allocation is efficient. Any policy intervention in the previous period generated a suboptimal resource allocation but no benefit ex-post. If, on the other hand, the bad state of the world is realised, then the constraint is less binding then it would have been without interventions. However, the planner has further ways to intervene ex-post. In particular, because the planner has commitment ability, she can exploit this to boost the current price of capital, by promising lower consumption of the unconstrained agent in the future.

Notice also that more than one externality is present in period 1. In fact, on top of the collateral externality, there is now also a *terms-of-trade externality*, driven by the multiplier on the collateral constraint opening up a wedge between MRSs of the two classes of producers. Due to this wedge, any policy that can bring about a higher asset price in a state where the CRS-producer has to fire-sell capital improves on the efficiency of the allocation, as the cost to the DRS-agent at the other end of the transaction, who is unconstrained, is not as high as the benefit to the constrained agent, who has a higher marginal utility of consumption. Here, both collateral externalities and terms-of-trade externalities require the same direction of intervention: increase capital holding of the more agent so as to increase the price of capital.

### 2.B.2 Constrained Efficient Allocation, Markov Equilibrium

In each time period, a constrained efficient, Markovian planner maximizes:

$$V_t(k, d; z) = \max_{c^C, c^D, d', k', q, r} [n\chi \log(c_t) + (1-n)(1-\chi)\log(c'_t)] + \beta E_{z'|z} V_{t+1}(k', d'; z')$$

subject to

$$\begin{aligned}
c^C + q\Delta k^{C\prime} &\leq zf_C(k) - d + \sum_s \frac{d'_s}{1+r_s} \\
c^D - \frac{n}{1-n} q\Delta k &\leq zf_D\left(\frac{1-nk}{1-n}\right) + \frac{n}{1-n}\left[d - \sum_s \frac{d'_s}{1+r_s}\right] \\
d' &\leq \theta q k' \\
q &= \beta E_{z'|z}\left[\frac{c^D}{c^{D\prime}(k', d'; z')}\left(q'(k', d'; z') + z' f'_D\left(\frac{1-nk'}{1-n}\right)\right)\right] \\
\frac{1}{1+r_s} &= \beta \frac{c^D}{c_s^{D\prime}(k', d'; z')}
\end{aligned}$$

taking both past and future variables as given, but internalising how current choices affect future variables.

**Period 0** In period 0, interventions are purely prudential and coincide with the choices of the planner under commitment.

**Period 1** In period 1, a Markovian planner cannot make promises regarding the future to ameliorate the binding constraint. Instead, this can only be done indirectly, by influencing tomorrow's state variable with today's choices. As a result, a Markovian planner is not as effective as a planner with commitment ability in the second period.

### 2.B.3 Time Consistency

Because the two allocations under commitment and discretion do not coincide, the allocation is not time consistent. The time inconsistency is generated by policy in period 1 in the bad state, however, while there is no difference in macroprudential policy choices with or without commitment ability. In this spacial case, even without commitment ability, a Markovian planner can be just as successful as a planner with commitment ability in implementing macroprudential policies. It is only ex-post that a planner that can commit to future policies wishes to exploit her advantage to promise lower future consumption of the unconstrained agent to increase the current asset price. In period 2 however, the benefits are in the past, and there is only a cost in carrying forward with this plan, in the form of a lower consumption value for the unconstrained agents. This is therefore no longer optimal once in period 2. Notice, however, that if a planner assigns a zero welfare weight to savers (as is always the case in small open economy models) then the policy is time consistent because there is no cost imposed in period 2 in terms of welfare. Also, if the agent pricing the asset had a linear utility (cfr. Lorenzoni (2008)) then the policy would be time consistent too, because the planner would have no way of intervening in period 1 to improve on the allocation. In that case, the laissez-faire allocation in period 1 would be constrained efficient.

## 2.C Constrained Efficient Allocation under Discretion

The set of equilibrium conditions comprises the aggregate resource constraint, which can be obtained by combining the two agents' budget constraints (2.A.7) and (2.A.8) with (2.A.9) and (2.A.10); the complementary slackness conditions associated with prices (2.A.4), (2.A.2) and the collateral constraint (2.A.6), together with the following equations:

$$c^C: \quad \lambda^C = \frac{\chi}{c^C} \tag{2.C.1}$$

$$c^D: \quad \lambda^D = \left(1 - \chi + \gamma^q q + \frac{\gamma^r}{1+r}\right)\frac{1}{c^D} \tag{2.C.2}$$

$$q: \quad \gamma^q = \theta k' \mu^P - \left(\lambda^C - \frac{n}{1-n}\lambda^D\right)\Delta k' \tag{2.C.3}$$

$$r: \quad \gamma^r = \left(\lambda^C - \frac{n}{1-n}\lambda^D\right) d' \tag{2.C.4}$$

$$\begin{aligned} d': \quad \frac{1}{(1+r)}\left(\lambda^C - \frac{n}{1-n}\lambda^D\right) &= \beta E_{z'|z}\left(\lambda^{C\prime} - \frac{n}{1-n}\lambda^{D\prime}\right) + \mu^P \\ &\quad - \frac{n}{1-n}E_{z'|z}\left[\frac{m^{D\prime}}{c^{D\prime}}\left(z' f_D'(\cdot)\gamma^q + \gamma^r\right)\right] \end{aligned} \tag{2.C.5}$$

$$\begin{aligned} k': \quad q\left(\lambda^C - \frac{n}{1-n}\lambda^D\right) &= \beta E_{z'|z}\left[\left(\lambda^{C\prime} - \frac{n}{1-n}\lambda^{D\prime}\right)q' + \left(\lambda^{C\prime} f_C'(\cdot) - \frac{n}{1-n}\lambda^{D\prime} f_D'(\cdot)\right)z'\right] \\ &\quad + \theta q \mu^P + \frac{n}{1-n}E_{z'|z}\left\{m^{D\prime}\left[\frac{f_D'}{c^{D\prime}}\left(z' f_D' \gamma^q + \gamma^r\right) - f_D'' \gamma^q\right] z'\right\} \end{aligned} \tag{2.C.6}$$

### 2.C.1 Implementation with Taxation

**Proposition 2.4.2**:

*Proof.* Consider a decentralised economy where taxes (if positive) or subsidies (if negative) on debt and asset purchases are imposed. The individual's problem is:

$$\max_{\{c_t^j, k_t^j, d_t^j\}_{t=0}^2} \quad \sum_{t=0}^{2}\sum_s \beta^t \pi_s log(c_{ts}^j)$$

s.to

$$c_t^j + q_t\left[k_t^j(1+\tau_{t,k}^j) - k_{t-1}^j\right] \leq z_t f_j(k_{t-1}^j) - d_{t-1}^j + \frac{d_t^j}{(1+r_t)(1+\tau_{t,d}^j)} + T_t^j$$

$$d_1^j \leq \theta q_1 k_1^j$$

First order conditions in this case are:

$$\begin{aligned}
\frac{1}{(1+\tau^C_{t,d})(1+r_t)} &= E_t(m^C_{t,t+1}) + \mu^C_t c^C_t \\
\frac{1}{(1+\tau^D_{t,d})(1+r_t)} &= E_t(m^D_{t,t+1}) \\
q_t(1+\tau^C_{t,k}) &= E_t\left[m^C_{t,t+1}\left(q_{t+1}+f'_C z_{t+1}\right)\right] + \theta q_t \mu^C_t c^C_t \\
q_t(1+\tau^D_{t,k}) &= E_t\left[m^D_{t,t+1}\left(q_{t+1}+f'_D z_{t+1}\right)\right]
\end{aligned}$$

Comparing these to the first order conditions of the planner, one can observe that by setting taxes appropriately, the social planner's allocation can be replicated perfectly. Define $\xi_t \equiv \frac{c^D_t}{\omega^D_t}$, $\zeta^i_{t+1} \equiv \frac{z_{t+1}f'_i(k^i_t)}{q_t}$ and $1+\widetilde{r}^i_{t+1} \equiv \frac{q_{t+1}+z_{t+1}f^i(k^i_t)}{q_t}$.

$$\begin{aligned}
1+\tau^C_{t,d} = \left[E_t(m^C_{t,t+1}) + \mu^C_t c^C_t\right]^{-1} \Bigg\{ E_t\left(\frac{\xi_{t+1}}{\xi_t} m^C_{t,t+1}\right) + E_t\left[\delta^{rd}_{t+1} d_t - \delta^{qd}_{t+1}\left(\Delta k_t - \theta k_t \mu^P_t c^C_t\right)\right] \\
+ \frac{\mu^P_t c^C_t}{\xi_t} \Bigg\}
\end{aligned} \tag{2.C.7}$$

$$\begin{aligned}
1+\tau^D_{t,d} = \left[E_t(m^D_{t,t+1})\right]^{-1} \Bigg\{ E_t\left[\frac{\xi_{t+1}}{\xi_t}\left(\frac{1-\chi+\theta q_{t+1}k_{t+1}\mu^P_{t+1}}{1-\chi+\theta q_t k_t \mu^P_t}\right) m^D_{t,t+1}\right] + E_t\Big[\delta^{rd}_{t+1} d_t + \\
\delta^{qd}_{t+1}\left(\Delta k_t - \theta k_t \mu^P_t c^C_t\right)\Big]\Bigg\}
\end{aligned} \tag{2.C.8}$$

$$\begin{aligned}
\tau^C_{t,k} = E_t\left[\left(1-\frac{\xi_{t+1}}{\xi_t}\right) m_{t,t+1}(1+\widetilde{r}^C_{t+1})\right] + \theta c^C_t\left(\mu_t - \frac{\mu^P_t}{\xi_t}\right) - E_t\left[\delta^{rk}_{t+1} d_t - \delta^{qk}_{t+1}\left(\Delta k_t - \theta k_t \mu^P_t c^C_t\right)\right] \\
- E_t\left[\frac{1-\xi_{t+1}}{\xi_t} m^C_{t,t+1}\left(\zeta^C_{t+1} - \zeta^D_{t+1}\right)\right]
\end{aligned} \tag{2.C.9}$$

$$\begin{aligned}
\tau^D_{t,k} = E_t\left[\left(1-\frac{\xi_{t+1}}{\xi_t}\left(\frac{1-\chi+\theta q_{t+1}k_{t+1}\mu^P_{t+1}}{1-\chi+\theta q_t k_t \mu^P_t}\right)\right) m^D_{t+1}(1+\widetilde{r}^D_{t+1})\right] - \\
E_t\left[\delta^{rk}_{t+1} d_t - \delta^{qk}_{t+1}\left(\Delta k_t - \theta k_t \mu^P_t c^C_t\right)\right]
\end{aligned} \tag{2.C.10}$$

$\square$

### 2.C.2 Implementation with Capital Requirements

**Proposition 2.6.1:**

*Proof.* Consider the problem of financial intermediaries that know that capital requirements are going to be in place in $t = 0$. At time 0, however, due to the regulator being unable to commit to future policies, they do not anticipate any regulation will be implemented in time 1. They will solve the following problem:

$$\max_{\{c_t^C, c_t^D, kt, d_t\}_{t=0}^2} \sum_{t=0}^{2} \sum_j \beta^t E_0 log(c_t^j)$$

$$\text{s.to} \quad c_t^j + q_t \Delta k_t \leq z_t f_j(k_{t-1}) - d_{t-1} + \frac{d_t}{(1+r_t)} \qquad \forall t, j \qquad (\lambda_t^j) \qquad (2.C.11)$$

$$d_1 \leq \theta q_1 k_1 \qquad (\mu_1) \qquad (2.C.12)$$

$$e_0 \geq \phi_0 q_0 k_0, \quad e_0 = z_0 f_c(k_{-1}) + q_0 k_{-1} \qquad (\kappa_0) \qquad (2.C.13)$$

Clearly, the presence of a capital requirement in period 0 only alters the optimal choice of asset in period 0. Debt choices in period 0 do not influence the net worth of banks in period 0, and as a consequence, it is not possible to implement the constrained efficient allocation.
Consider now the possibility of a regulator that credibly announces in period 0 that a capital requirement will be in place in period 1. In this case, Constraint (2.C.13) will be replaced by:

$$e_1 \geq \phi_1 q_1 k_1, \quad e_1 = z_1 f_C(k_0) - d_0 + q_1 k_0$$

Both first order conditions for debt and asset in period 0 will be distorted as a result of this announcement, and in the following way:

$$\frac{1}{1+r_0} = E_0(m_{0,1}^C) + E_0(\widetilde{\kappa}_1) \qquad (2.C.14)$$

$$q_0 = E_0\left[m_{0,1}^C\left(q_1 + z_1 f_C'(k_0)\right)\right] + E_0\left(\widetilde{\kappa}_1 z_1\right) f_C(k_0) \qquad (2.C.15)$$

Where $\widetilde{\kappa}_1$ is just a rescaling of the Lagrangian multiplier on the capital requirement:

$\widetilde{\kappa}_t = \kappa_t c_0^C$. By comparing Equation (2.C.14) to (2.4.1) one obtains this expression for $\widetilde{\kappa}_1$, measuring the extent to which the capital requirement constraint is binding:

$$\widetilde{\kappa}_1 = \frac{X_1 - X_0}{X_0} m_{0,1}^C + \delta_1^{rd} d_0 + \delta_1^{rk} \Delta k_0$$

But if this is plugged into Equation (2.C.15), we observe it is not possible to replicate Equation (2.4.2). Hence, imposing capital requirements over one period only is not sufficient to replicate the planner's allocation.

We now want to show that if capital requirements are imposed in period $t$ and $t+1$, it is then possible to replicate the planner's allocation in period $t$.
First order conditions become:

$$\frac{1}{(1+r_t)} = E_t\left(m_{t,t+1}^C\right) + \mu_t^C c_t^C + E_t\left(\widetilde{\kappa}_{t+1}\right)$$
$$q_t\left[1 + \phi_t \widetilde{\kappa}_t - \theta \mu_t^C c_t^C\right] = E_t\left[\left(m_{t,t+1}^C + \widetilde{\kappa}_{t+1}\right)\left(z_{t+1} f_C'(k_t) + q_{t+1}\right)\right]$$

We can compare these two to the planner's optimal choices (2.4.1) and (2.4.2). By imposing capital requirement such that the following relationships are satisfied, the constrained efficient allocation can be implemented in the decentralised economy:

$$\widetilde{\kappa}_{t+1} = \frac{X_{t+1} - X_t}{X_t} m_{t,t+1}^C + \left[\delta_{t+1}^{rd} d_t + \delta_{t+1}^{rk}\left(\Delta k_t - \theta k_t \mu_t^P c_t^C\right)\right] + c_t^C\left(\frac{\mu_t^P}{X_t} - \mu_t^C\right)$$
$$\phi_t\, \widetilde{\kappa}_t = E_t\left[\left(\widetilde{\kappa}_{t+1} - \frac{X_{t+1} - X_t}{X_t} m_{t,t+1}^C\right)\left(1 + \widetilde{r}_{t+1}^C\right)\right] + \frac{1}{q_t} E_t\left[\delta_{t+1}^{qd} d_t + \delta_{t+1}^{qk}\left(\Delta k_t - \theta k_t \mu_t^P c_t^C\right)\right] - \theta c_t^C\left(\frac{\mu_t^P}{X_t} - \mu_t^C\right) - E_t\left[\frac{1 - X_{t+1}}{X_t} m_{t,t+1}^C\left(\zeta_{t+1}^C - \zeta_{t+1}^D\right)\right]$$

Where time $t$ capital requirement helps correcting for the asset allocation while time $t+1$ capital requirement is used to meet the condition for debt. □

## 2.D Constrained Efficient Allocation under Commitment

First order conditions of the problem under commitment are:

$$
\begin{aligned}
c_t^C:&\quad \lambda_t^C = \frac{\chi}{c_t^C}\\
c_t^D:&\quad \lambda_t^D = \left\{1-\chi+\gamma_t^q q_t + \frac{\gamma_t^r}{(1+r_t)} - m_{t-1,t}^D\left[\gamma_{t-1}^q\left(q_t + z_t f_D'\right) + \gamma_{t-1}^r\right]\right\}\frac{1}{c_t^D}\\
q_t:&\quad \gamma_t^q = \theta k_t \mu_t^P - \left(\lambda_t^C - \frac{n}{1-n}\lambda_t^D\right)\Delta k_t + m_{t-1,t}^D \gamma_{t-1}^q\\
r_t:&\quad \gamma_t^r = \left(\lambda_t^C - \frac{n}{1-n}\lambda_t^D\right) d_t\\
d_t:&\quad \frac{1}{(1+r_t)}\left(\lambda_t^C - \frac{n}{1-n}\lambda_t^D\right) = \beta E_t\left(\lambda_{t+1}^C - \frac{n}{1-n}\lambda_{t+1}^D\right) + \mu_t^P\\
k_t:&\quad q_t\left(\lambda_t^C - \frac{n}{1-n}\lambda_t^D\right) = \beta E_t\left[\left(\lambda_{t+1}^C - \frac{n}{1-n}\lambda_{t+1}^D\right) q_{t+1} + \left(\lambda_{t+1}^C f_C' - \frac{n}{1-n}\lambda_{t+1}^D f_D'\right) z_{t+1}\right]\\
&\qquad\qquad + \theta q_t \mu_t^P - \frac{n}{1-n} E_t\left[m_{t,t+1}^D z_{t+1}\right] f_D'' \gamma_t^q
\end{aligned}
$$

where terms in blue are what sets these conditions apart from the discretionary case.

### 2.D.1 Commitment Implementation

Define $\tilde{\theta} = \frac{\theta}{1-\chi}$. Macroprudential policy:

$$1+\tau_{0,d}^C = \left[E_0(m_{0,1}^C)\right]^{-1}\left[E_0\left(\frac{\xi_1}{\xi_0} m_{0,1}^C\right) + \beta E_0\left(\xi_1 D_1^r\right)\right] \tag{2.D.1}$$

$$1+\tau_{0,d}^D = \left[E_0(m_{0,1}^D)\right]^{-1}\left\{E_0\left[\frac{\xi_1}{\xi_0} m_{0,1}^D\left(1+\tilde{\theta} q_1 k_1 \mu_1^P\right)\right] + \beta E_0\left(\xi_1 D_1^r\right)\right\} \tag{2.D.2}$$

$$
\begin{aligned}
\tau_{0,k}^C = {}& E_0\left[\left(1-\frac{\xi_1}{\xi_0}\right) m_{0,1}^C (1+\tilde{r}_1^C)\right] - E_0\left[\frac{1-\xi_1}{\xi_0} m_{0,1}^C\left(\zeta_1^C - \zeta_1^D\right)\right] + \beta E_0\left[\xi_1 D_1^r (1+\tilde{r}_1^D)\right]\\
& + \frac{n}{1-n} E_0\left[m_{0,1}^D z_1 f_D'' \frac{\Delta k_0}{q_0}\right]
\end{aligned} \tag{2.D.3}
$$

$$\tau^D_{0,k} = E_0\left[\left(1-\frac{\xi_1}{\xi_0}\left(1+\tilde{\theta}q_1k_1\mu^P_1\right)\right)m^D_{0,1}(1+\widetilde{r}^D_1)\right] + \beta E_0\left[\xi_1 D^r_1(1+\widetilde{r}^D_1)\right] + \frac{n}{1-n}E_0\left[m^D_{0,1}z_1 f''_D\frac{\Delta k_0}{q_0}\right] \tag{2.D.4}$$

Ex-post policy:

$$1+\tau^C_{1,d} = \left(m^C_{1,2}+\mu^C_1 c^C_1\right)^{-1}\left[\frac{(m^C_{1,2}+\mu^P_1 c^D_1)}{\xi_1}\left(1-\frac{D^r_1}{\left(m^C_{0,1}/(\beta\xi_0)+D^r_1\right)}\right) - \frac{\beta m^D_{0,1}\Delta k_0 \delta^{Rk}_2}{\xi_1\left(m_{0,1}/\beta\xi_0+D^r_1\right)} + \beta D^r_2\right] \tag{2.D.5}$$

$$1+\tau^D_{1,d} = \frac{1}{\xi_1}\left[\frac{m^D_{0,1}}{m^D_{0,1}(1+\tilde{\theta}q_1k_1\mu^P_1)+\beta\xi_0 D^r_1}\right] + \frac{\beta D^r_2}{m^D_{1,2}} - \frac{\beta m^D_{0,1}\Delta k_0\delta^{rk}_2}{m^D_{1,2}\left[m^D_{0,1}/\beta\xi_0(1+\tilde{\theta}q_1k_1\mu^P_1)+D^r_1\right]} \tag{2.D.6}$$

$$\tau^C_{1,k} = \left[1-\frac{1}{\xi_1}\left(1-\frac{D^r_1}{m^C_{0,1}/(\beta\xi_0)+D^r_1}\right)\right]\frac{m^C_{1,2}f'_C z_2}{q_1} + \frac{\beta}{q_1}\left(D^q_2+\delta^{qk}_2\theta\mu^P_1 k_1 c^C_1\right) + \theta c^C_1\left[\mu^C_1 - \frac{\mu^P_1}{\xi_1}\left(1-\frac{D^r_1}{m_{0,1}/(\beta\xi_0)+D^r_1}\right)\right] - \frac{\beta\delta^{qk}_2}{q_1\xi_1}\left(\frac{m^C_{0,1}\theta\mu^P_1 k_1 c^C_1+\beta\xi_0 m^D_{0,1}\Delta k_0}{m^C_{0,1}+\beta\xi_0 D^r_1}\right) \tag{2.D.7}$$

$$\tau^D_{1,k} = \left[1-\frac{1}{\xi_1}\left(\frac{m^D_{0,1}}{m_{0,1}D(1+\tilde{\theta}q_1k_1\mu^P_1)+\beta\xi_0 D^r_1}\right)\right]\frac{m^D_{1,2}f'_D z_2}{q_1} + \frac{\beta}{q_1}\left(D^q_2+\delta^{qk}_2\theta\mu^P_1 k_1 c^C_1\right) - \frac{\beta\delta^{qk}_2}{q_1\xi_1}\left(\frac{\xi_0 m^D_{0,1}\Delta k_0}{m^D_{0,1}/\beta(1+\tilde{\theta}q_1k_1\mu^P_1)+\xi_0 D^r_1}\right) \tag{2.D.8}$$

These taxes implement the constrained efficient allocation in case of commitment, which proves the first part of Proposition 2.6.1.

## Chapter 3

# Limiting Mortgage Debt

### *Aggregate demand externalities and housing market distortions*

*Many households prefer homeownership to renting but cannot afford to buy without borrowing, so mortgages can improve allocative efficiency in housing markets. However, highly indebted households may impose aggregate demand externalities when there are nominal rigidities and monetary policy is constrained. Optimal macroprudential limits on mortgage borrowing would trade off housing market distortions against reductions in aggregate demand externalities. In a model calibrated to match features of UK data, we find that debt limits affect interest rates, house prices and rents. Depending on the size and incidence of these general equilibrium effects, macroprudential policy can have different distributional consequences.*

## 3.1 Introduction

Mortgage debt played a prominent role in the 2007–2008 financial crisis and the Great Recession. Countries and regions that saw a greater build-up of household debt prior to the crisis tended to suffer the greatest falls in output and employment during the recession.[1] This motivated many central banks and financial regulators around the world to impose limits on mortgage debt. Mortgages, however, can improve the efficiency of the housing market by ensuring that households with low savings, like many first-time buyers, are able to purchase a home. Homeownership provides security of tenure and insures against rent increases when housing derivatives markets are incomplete.[2]

This paper examines the consequences of macroprudential debt restrictions in a model where debt causes a macroeconomic externality, there are deadweight costs in the housing rental market, and households have heterogeneous wealth. It shows that while a limit on borrowing helps to avoid costly output losses associated with excessive debt, it also

[1] Zabai (2017); Mian and Sufi (2018); Aikman, Bridges, Kashyap, and Siegert (2019).
[2] Sinai and Souleles (2005).

has important distributional consequences through general equilibrium effects on interest rates, house prices and rents.

Mortgage debt can exacerbate recessions if highly indebted households deleverage sharply in response to shocks. If other households could be induced to borrow more, deleveraging by one group of households should not lead to a reduction in aggregate demand. However, in the presence of nominal rigidities and an effective lower bound on nominal interest rates, monetary policy may be unable to achieve the reduction in the real interest rate that would be necessary to keep output at potential (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017). Before the deleveraging shock, individual borrowers do not internalize how their demand for debt contributes to pushing the interest rate against the lower bound, causing a demand-driven recession. This externality can be corrected by introducing limits on borrowing (Farhi and Werning, 2016; Korinek and Simsek, 2016).[3]

However, if households have a preference for owning housing as opposed to renting, a borrowing limit may prevent poor households from purchasing a home of the desired size, pushing them into the rental sector. This represents a cost not just for renters, who have to face moving and search costs if required to leave at the end of their contracts, and have to accept restrictions on modifications to the property; but for landlords as well, who must perform costly inspections to overcome moral hazard problems.[4] There is therefore a trade-off between staving off demand externalities connected to household debt and ensuring an efficient allocation in the housing market.

Importantly, unconstrained landlords and borrowing-constrained tenants do not have the same marginal valuation of wealth, as renters' binding borrowing limit increases their marginal propensity to spend. This implies that in the aggregate, there can be welfare gains not just from avoiding or moderating the aggregate demand externality connected to excessive debt accumulation, but also from price changes that help credit-constrained households.

In a stylized two-period framework, we model households who can inherit large or small amounts of wealth. Renting housing involves deadweight financial costs, which

[3]In this paper we model this aggregate demand externality from debt in reduced form, as in Mian and Sufi (2017).

[4]We model these costs associated with rental housing in reduced form as a deadweight cost. This deadweight cost causes households to prefer owner-occupied housing to rental housing.

can be avoided if the housing is owner occupied. In the absence of binding debt limits, households with low wealth would borrow from richer households to be able to purchase housing and avoid these deadweight rental costs. The rental market is therefore inactive in a laissez-faire equilibrium with no debt limits. If there were no aggregate demand externalities from debt accumulation, such a laissez-faire equilibrium would be efficient. However, we assume that, when borrowing and lending, households do not internalize that the accumulation of debt may in fact reduce future aggregate output. In reduced form, this externality approximates a demand-driven recession triggered by a liquidity trap, and calls for a prudential limit on debt.

We show that even when renters and landlords face symmetric deadweight costs upon signing a tenancy contract, they disagree regarding their preferred settings of macro-prudential policy. A no-arbitrage condition links the rates of return in the housing and financial markets. This implies that landlords need to be offered a higher rental rate, to be induced to rent out their properties and pay the deadweight cost associated with rental agreements. Due to this, landlords prefer a tighter borrowing limit compared to renters, as a strict constraint on debt helps to avoid the recession, while the extra cost of tenancy is borne by renters.

As the price of housing is influenced by the value of housing services, the increased rental rate pushes up house prices. On the other hand, a tighter debt limit causes the aggregate demand for housing to subside, as less wealthy households are forced to rent instead of buying. This pushes the house price down and hurts homeowners. From this point of view, not just tenants but landlords too would prefer a borrowing limit that is not too strict. Which of these two forces prevail depends on parameter conditions. We illustrate this point with alternative calibrations based on UK data.

### 3.1.1 Related literature

In the 2007–2008 financial crisis, US subprime mortgage losses contributed to the downfall of many financial institutions. Although press and policymaker attention focused mainly on these financial failures at the start of the crisis, there is now a widespread view that household indebtedness played an independent role in triggering and exacerbating the recession (Zabai, 2017; Mian and Sufi, 2018; Aikman et al., 2019).

The build-up of household debt has been associated with financial crises and recessions

(Schularick and Taylor, 2012; Zabai, 2017). Jordà, Schularick, and Taylor (2016) find that recessions following mortgage lending booms are particularly severe. Mian, Sufi, and Verner (2017) and Alter, Feng, and Valckx (2018) find that growth in household debt relative to GDP ratio is associated in the medium run with lower GDP growth and higher unemployment. US postal code areas where households suffered greater leveraged housing losses saw greater declines in economic activity (Mian, Rao, and Sufi, 2013). Central banks now monitor the proportion of highly indebted households as a potential source of economic and financial instability (Cateau, Roberts, and Zhou, 2015; Reserve Bank of Australia, 2017; Bank of Canada, 2017; Bank of England, 2017).

In spite of our choice of modelling demand externalities in reduced form, we contribute to the literature on the topic (Korinek and Simsek, 2016; Farhi and Werning, 2016) by introducing property ownership as a motive for borrowing. In one of the applications in Farhi and Werning (2016) the interaction between aggregate demand and pecuniary externalities is analysed by introducing a collateral constraint that depends on the house price. However, while collateral externalities tend to reinforce the effect of aggregate demand externalities, we show that unequal marginal rates of substitution can work against them. We leverage the theoretical findings of other research (Dávila and Korinek, 2018) to show this point in connection to the housing market.

While very stylized, our model can be related to the life-cycle literature on housing tenure choices, as in Iacoviello and Pavan (2013) or Kiyotaki, Michaelides, and Nikolov (2011). Both papers model a preference for owning over renting as embedded in the households' utility function. This paper, instead, assumes a financial cost that affects households' budget constraint. The result we obtain of a higher rental rate in the presence of rental costs are however confirmed when a utility premium is assumed (Kiyotaki et al., 2011), which seems to suggest that the two formulations may not have substantially different implications. Besides, one can make arguments for both psychological as well as practical costs associated with renting. Differently from Iacoviello and Pavan (2013), who consider lumpy housing with infrequent and costly housing adjustment, we model housing as a continuous rather than discrete choice. This simplifies tractability greatly and allows us to consider a general equilibrium model of housing prices, but at the cost of counterfactual situations where households partly own and partly rent properties.

Ortalo-Magné and Rady (2006) find that marginal first-time buyers who are credit constrained can play a crucial role in determining housing market volatility, confirming the importance of considering distributional aspects connected to debt limits, as not all players in the housing market contribute to house price movements to the same extent. In the same spirit, Kaplan, Mitman, and Violante (2017) model a further advantage of owning housing, as it gives access to additional opportunities for borrowing in the form of Home Equity Line of Credit.

## 3.2 Two-period, two-agent model with housing and debt

The economy lasts for two periods: $t \in \{1, 2\}$. All households are born at the beginning of the first period and die at the end of the second. In each period there are two goods: a non-durable consumption good produced by households, who all supply one unit of labour inelastically in each period, and durable housing, which is in fixed supply. Consumption goods and housing are both infinitely divisible. There are two types of households: rich and poor, who differ only in their initial endowment of housing: $h_0^R > h_0^P$. In each period there are spot markets for non-durable consumption goods $c_t$ and housing occupancy $s_t$. In the first period only, there is a market for housing ownership $h_1$ and a market in riskless one-period debt $d_2$, where $d_2 > 0$ denotes net borrowing.[5] Housing transactions take place at the start of the period, so it is the purchaser of a unit of housing who has the right to live in it or rent it out.

### 3.2.1 Preferences and budget constraints

A household's lifetime utility is

$$u(c_1) + \eta v(s_1) + \beta(u(c_2) + \eta v(s_2)),$$

where $\eta > 0$ captures the intensity of the preference for housing and $\beta > 0$ is the intertemporal discount factor. The utility functions $u(c_t)$ and $v(s_t)$ have the usual properties: they

[5] As the second period is the final period, there is no scope for any new borrowing to be repaid, and no meaningful distinction between housing occupancy and ownership.

are strictly increasing, concave, and continuously differentiable. For simplicity we focus on the special case with $u(\cdot) = v(\cdot) = \log(\cdot)$.

The period 1 and period 2 budget constraints are

$$c_1 + P_1(h_1 - h_0) + \rho_1(s_1 - h_1) + \mu m(s_1 - h_1) \leq y_1 + \frac{d_2}{1 + r_1} \text{ and}$$
$$c_2 + P_2(h_2 - h_1) + \rho_2(s_2 - h_2) + \mu m(s_2 - h_2) \leq y_2 - d_2,$$

where $P_t$ and $\rho_t$ are the purchase price of housing and the housing rental rate in period $t$.

### 3.2.2 Deadweight cost of rental housing

The function $m(s_t - h_t)$ denotes a deadweight cost of rental housing. This cost is incurred by both tenants and landlords, so $m(s_t - h_t) > 0$ whenever $s_t \neq h_t$. It is increasing in the amount of housing rented or rented out, so $m'(s_t - h_t) > 0$ for net tenants, for whom $s_t > h_t$, and $m'(s_t - h_t) < 0$ for net landlords, for whom $s_t < h_t$. The marginal deadweight cost of rental housing is increasing, so $m''(s_t - h_t) > 0$. Owner-occupancy imposes no deadweight costs, so $m(0) = 0$. The time-invariant parameter $\mu > 0$ captures the intensity of the rental market friction.

Each household supplies a fixed amount of labour and can produce $y_t$ units of the non-durable consumption good in period $t$. For concreteness, we can interpret the deadweight cost of rental housing as landlords and tenants having to divert some of their effort towards monitoring and maintenance activities, or having to employ property management agents to do so at the prevailing wage. The assumptions on the curvature of the cost function $m(s_t - h_t)$ imply that these activities are subject to increasing marginal costs.

### 3.2.3 Debt and the aggregate demand externality

Households can borrow or lend between the two periods at the risk-free interest rate $r_1$, subject to the borrowing constraint

$$d_2 \leq \bar{d}_2.$$

The debt limit $\bar{d}_2$ is an exogenous parameter of the model, which we think of as being set in advance by a macroprudential authority. The focus of this paper is on comparative statics exercises in which we solve the model for different values of the debt limit and examine the implications for equilibrium outcomes and the welfare of the two types of households.

In the second period only, output can be affected by a reduced-form aggregate demand externality from debt as in Mian and Sufi (2017):

$$y_2 = \begin{cases} \bar{y}_2 & \text{if } D_2 \leq \bar{D} \\ \bar{y}_2 - f\left(\frac{D_2}{\bar{D}}, \phi\right) & \text{otherwise,} \end{cases}$$

where $\bar{y}_2$ is production capacity, $D_2$ is aggregate gross debt, $\bar{D}$ is the threshold debt level above which output is constrained, and $\phi$ captures the severity of macroeconomic frictions. The penalty function $f(\cdot)$ has the properties $f(1,\cdot) = 0$, $f_1 > 0$ and $f_{1,2} > 0$. As Mian and Sufi (2017) point out, this functional form can be motivated by a model with nominal rigidities and an effective lower bound on the nominal interest rate, as in Farhi and Werning (2016) and Korinek and Simsek (2016).

### 3.2.4 Market clearing and equilibrium

Markets are competitive in the sense that households take prices and aggregate quantities as given. Using the superscripts $R$ and $P$ to distinguish between rich and poor households, normalizing the total number of households to one, and using $\pi$ to denote the fraction of poor households, the market-clearing conditions for goods, housing ownership and housing occupancy are:

$$\begin{aligned} (1-\pi)\left(c_t^R + \mu m(s_t^R - h_t^R)\right) + \pi\left(c_t^P + \mu(s_t^P - h_t^P)\right) &= y_t, \\ (1-\pi)h_t^R + \pi h_t^P &= H, \\ (1-\pi)s_t^R + \pi s_t^P &= (1-\pi)h_t^R + \pi h_t^P, \qquad t \in \{1,2\} \end{aligned}$$

An equilibrium in this economy is a set of prices and a set of choices for each type of household such that all choices are individually optimal and all markets clear. As there are no shocks, the rational expectations equilibrium will feature perfect foresight.

## 3.3 Household optimization

Households of type $i \in \{P, R\}$ solve the following problem:

$$\max_{c_1^i, c_2^i, s_1^i, s_2^i, d_2^i, h_1^i, h_2^i} u(c_1^i) + \eta v(s_1^i) + \beta\left((u(c_2^i) + \eta v(s_2^i)\right)$$

subject to the budget constraints

$$c_1^i + P_1(h_1^i - h_0^i) + \rho_1(s_1^i - h_1^i) + \mu m(s_1^i - h_1^i) \le y_1 + \frac{d_2^i}{1+r_1}, \tag{3.3.1}$$

$$c_2^i + P_2(h_2^i - h_1^i) + \rho_2(s_2^i - h_2^i) + \mu m(s_2^i - h_2^i) \le y_2 - d_2, \tag{3.3.2}$$

and the borrowing constraint

$$d_2^i \le \bar{d}_2.$$

Differentiating with respect to the household's choice variables, we have

$$c_1^i : \quad u'(c_1^i) = \lambda_1^i \tag{3.3.3}$$

$$c_2^i : \quad \beta u'(c_2^i) = \lambda_2^i \tag{3.3.4}$$

$$s_1^i : \quad \frac{\eta v'(s_1^i)}{\rho_1 + \mu m'(s_1^i - h_1^i)} = \lambda_1^i \tag{3.3.5}$$

$$s_2^i : \quad \frac{\beta\eta v'(s_2^i)}{\rho_2 + \mu m'(s_2^i - h_2^i)} = \lambda_2^i \tag{3.3.6}$$

$$d_2^i : \quad (1+r_1)\left(1 + \frac{\tilde{\lambda}_2^i}{\lambda_2^i}\right) = \frac{\lambda_1^i}{\lambda_2^i} \tag{3.3.7}$$

$$h_1^i : \quad \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)} = \frac{\lambda_1^i}{\lambda_2^i} \tag{3.3.8}$$

$$h_2^i : \quad \mu m'(s_2^i - h_2^i) = \rho_2 - P_2 \tag{3.3.9}$$

Combining (3.3.3) with (3.3.5) and (3.3.4) with (3.3.6) yields an intratemporal optimality condition for each period:

$$\frac{\eta v'(s_1^i)}{u'(c_1^i)} = \rho_1 + \mu m'(s_1^i - h_1^i), \tag{3.3.10}$$

$$\frac{\eta v'(s_2^i)}{u'(c_2^i)} = \rho_2 + \mu m'(s_2^i - h_2^i). \tag{3.3.11}$$

Within each period the ratio of a household's marginal utility of housing occupancy to the marginal utility of its non-durables consumption must equal the relative price it faces between these two goods. The price of non-durables is normalized to one, so for a given level of housing ownership the relative price of housing occupancy is given by the housing rental rate $\rho_t$ plus the marginal rental friction associated with an increase in net renting. Note that for landlords, with $s_t^i < h_t^i$, this marginal friction is negative because occupying more of the housing they own and renting less of it out reduces the deadweight rental cost they must pay.

Combining first-order conditions (3.3.3), (3.3.4), and (3.3.8) gives us an intertemporal optimality condition:

$$\frac{u'(c_1^i)}{\beta u'(c_2^i)} = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{3.3.12}$$

The ratio of the discounted marginal utilities of a household's consumption across the two periods must equal the effective rate of return it faces on housing ownership. The rate of return on owning a unit of housing is its future sale price $P_2$ divided by the net resources that must be forgone today to aquire it. As well as the purchase price $P_1$, we must take into account the rental rate $\rho_1$ that a landlord earns or a tenant saves, and the marginal deadweight rental cost of their housing purchase. For a given level of housing occupancy, purchasing an additional unit of housing worsens the rental friction for a landlord but alleviates it for a tenant.

By combining first-order conditions (3.3.7) and (3.3.8), we obtain an equation relating the effective rates of return on bonds and housing :

$$(1 + r_1)\left(1 + \frac{\tilde{\lambda}_2^i}{\lambda_2^i}\right) = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{3.3.13}$$

Households for whom the borrowing constraint is binding, and for whom the Lagrange multiplier on the borrowing constraint, $\tilde{\lambda}_2^i$, is therefore positive, will face a higher effective rate of return on housing than on bonds. For households who are unconstrained by the debt limit and for whom the Lagrange multiplier on the borrowing constraint is therefore zero, condition (3.3.13) simplifies to the following no-arbitrage condition:

$$1 + r_1 = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{3.3.14}$$

Households who can borrow or save as much as they like must be indifferent between using bonds and housing as financial assets to transfer resources between the two periods.

## 3.4 Analytical results

Using the household optimality conditions and market-clearing conditions above, we can show the following about the equilibrium of our model.

First, there will be no renting in the second period. Intuitively, since the second period is the final period, housing ceases to be a financial asset and so the distinction between owning and renting breaks down. Due to the deadweight cost of renting, the rental rate would need to be lower than the purchase price of housing in order to induce households to become tenants. Since landlords must also pay the deadweight cost, households would need to earn a rental rate in excess of the purchase price of housing in order to induce them to become landlords. These two requirements are contradictory, so the only equilibrium in the second period is one in which all housing is owner-occupied and the purchase price of housing is equal to its rental rate.

**Proposition 3.4.1.** *In the second period all housing is owner-occupied and the purchase price of housing is equal to its rental rate:* $s_2^R = h_2^R$, $s_2^P = h_2^P$ *and* $P_2 = \rho_2$.

*Proof.* The first-order condition (3.3.9) must be satisfied for both rich and poor households. All households face the same rental rate $\rho_2$ and the same purchase price of housing $P_2$, so the right-hand side of this equation is identical for all households. For equation (3.3.9) to be satisfied for all households, the left-hand side must also be the same for all households. The second derivative of the deadweight rental cost function is positive for all levels of net renting, so in order for the first derivative $m'(s_2^i - h_2^i)$ to be equal across households we must have $s_2^R - h_2^R = s_2^P - h_2^P$. For the housing ownership and occupancy markets to clear in the second period, we must have $(1-\pi)(s_2^R - h_2^R) + \pi(s_2^P - h_2^P) = 0$. For these two conditions to both be satisfied we must have $s_2^R = h_2^R$ and $s_2^P = h_2^P$. This in turn implies that the left-hand side of the first-order condition (3.3.9) is equal to zero, which means we must have $P_2 = \rho_2$ in order for the right-hand side to equal zero. $\square$

Second, if neither rich nor poor households are constrained by the debt limit, then there will be no renting in the first period, either. Similar to the previous result, the intuition is that the deadweight cost of renting drives a wedge between the rates of return on housing for landlords and tenants. If households face no constraints on their asset positions, they must all be indifferent between holding bonds and housing in equilibrium. They all face the same rate of return on bonds, but due to the rental friction they will only face the same effective return on housing if their net rental positions are the same. This can only be the case when all housing is owner-occupied.

**Proposition 3.4.2.** *If the borrowing constraint is slack for both rich and poor households, $\tilde{\lambda}_2^R = \tilde{\lambda}_2^P = 0$, then in the first period all housing is owner-occupied: $s_1^R = h_1^R$ and $s_1^P = h_1^P$.*

*Proof.* If the borrowing constraint is slack for both rich and poor, then the no-arbitrage condition (3.3.14) must be satisfied for both types of households. All households face the same interest rate $r_1$, house prices $P_1$ and $P_2$ and rental rate $\rho_1$. For equation (3.3.14) to be satisfied for both types of households, the marginal deadweight rental friction $m'(s_1^i - h_1^i)$ must therefore be equal across households. The second derivative of the deadweight rental cost function is positive for all levels of net renting, so equality of marginal frictions can only be achieved if net renting is equal across households: $s_1^R - h_1^R = s_1^P - h_1^P$. Market clearing in the housing ownership and occupancy markets requires $(1-\pi)(s_1^R - h_1^R) + \pi(s_1^P - h_1^P) = 0$. For these two conditions to both be satisfied we must have $s_1^R = h_1^R$ and $s_1^P = h_1^P$. $\square$

We have shown that in order for the rental market to be active, one of the household types must find itself constrained by the debt limit. If this is the case, we can also show that the constrained households will be the tenants and the unconstrained households will be the landlords. Intuitively, households constrained by the debt limit are unable to purchase as much housing as they would like to.

**Proposition 3.4.3.** *Suppose that the debt limit binds for one of the household types but not the other: $d_2^U < d_2^C = \bar{d}_2$. Then in the first period the constrained household type are* *tenants and the unconstrained type are landlords:*

$$s_1^C - h_1^C > 0 > s_1^U - h_1^U.$$

*Proof.* Let the superscripts $C$ and $U$ denote the constrained and unconstrained types of households. By assumption the borrowing constraint is binding for the constrained and slack for the unconstrained: $\tilde{\lambda}_2^C > \tilde{\lambda}_2^U = 0$. Non-satiation ensures $\lambda_2^C > 0$. From the household optimality condition (3.3.13) we have

$$\frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^C - h_1^C)} = (1 + r_1)\left(1 + \frac{\tilde{\lambda}_2^C}{\lambda_2^C}\right) > 1 + r_1 = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^U - h_1^U)}.$$

Rearranging and simplifying yields $m'(s_1^C - h_1^C) > m'(s_1^U - h_1^U)$, and because $m''(s_t^i - h_t^i) > 0$ we must have $s_1^C - h_1^C > s_1^U - h_1^U$. Letting $\kappa > 0$ denote the share of the constrained type of household in the population, market clearing requires that

$$\kappa(s_1^C - h_1^C) = -(1 - \kappa)(s_1^U - h_1^U).$$

Combining these gives us

$$s_1^C - h_1^C = -\frac{1 - \kappa}{\kappa}(s_1^U - h_1^U) > 0 > s_1^U - h_1^U.$$

□

**Proposition 3.4.4.** *Suppose that the poor households start the period with a lower housing stock than the rich, $h_0^P < h_0^R$. Then the poor households will be borrowers and the rich households will be lenders.*

*Proof.* The allocation for consumption and housing services will be the same as in the previous section:

$$\begin{aligned}
c_1^i =& \frac{1}{(1 + \beta)(1 + \eta)}\left[y_1^i + \beta\frac{y_1}{y_2}y_2^i + \eta(1 + \beta)\frac{h_0^i}{H}y_1\right] \\
c_2^i =& \frac{y_2}{y_1}c_1^i \\
h_t^i =& s_t^i = \frac{H}{y_t}c_t
\end{aligned}$$

$$d_2^i = y_2^i - c_2^i - P_2(h_2^i - h_1^i)$$

We now show that if $y_t^R = y_t^P = y_t \ \forall t, \quad h_0^P < h_0^R$, then $d_2^P > d_2^R$.

$$c_t^i = y_t \left( \frac{1}{1+\eta} + \frac{\eta}{1+\eta} \frac{h_0^i}{H} \right)$$
$$s_t^i = h_t^i = \frac{H + \eta h_0^i}{1+\eta}$$
$$d_2^i = \frac{\eta}{1+\eta} \left( 1 - \frac{h_0^i}{H} \right) y_2$$

This shows that

$$d_2^P = \frac{\eta}{1+\eta} \left( 1 - \frac{h_0^P}{H} \right) y_2 > \frac{\eta}{1+\eta} \left( 1 - \frac{h_0^R}{H} \right) y_2 = d_2^R$$

as long as $h_0^P < h_0^R$. □

### 3.4.1 First best allocation with log utility

As shown in Propositions 3.4.1 and 3.4.2 above, if the borrowing constraint is slack for both rich and poor households then all housing will be owner-occupied in both periods. With $s_t^i = h_t^i$ for $t \in \{1, 2\}$ and $i \in \{R, P\}$, the household optimality conditions simplify to

$$\frac{\eta v'(h_t^i)}{u'(c_t^i)} = \rho_t, \tag{3.4.1}$$
$$\frac{u'(c_1^i)}{\beta u'(c_2^i)} = 1 + r_1 = \frac{P_2}{P_1 - \rho_1}. \tag{3.4.2}$$

With log utility, $u(\cdot) = v(\cdot) = \log(\cdot)$, these become

$$\eta c_t^i = \rho_t h_t^i, \tag{3.4.3}$$
$$\frac{c_2^i}{\beta c_1^i} = 1 + r_1 = \frac{P_2}{P_1 - \rho_1}. \tag{3.4.4}$$

By combining these with the household budget constraints and market clearing conditions, we can derive the following closed-form solution of our model when the debt limit is not binding for either household type:

$$
\begin{aligned}
c_1^i &= \left(1+\eta\frac{h_0^i}{H}\right)\frac{y_1}{1+\eta},\\
c_2^i &= (1+r_1)\beta c_1^i = \left(1+\eta\frac{h_0^i}{H}\right)\frac{y_2}{1+\eta},\\
s_1^i &= s_2^i = h_1^i = \frac{\eta c_1^i}{\rho_1} = \frac{H+\eta h_0^i}{1+\eta},\\
d_2^i &= y_2 - c_2^i = \left(1-\frac{h_0^i}{H}\right)\frac{\eta y_2}{1+\eta},\\
1+r_1 &= \frac{y_2}{\beta y_1}, \qquad \rho_1 = \eta\frac{y_1}{H}, \qquad P_2 = \rho_2 = \eta\frac{y_2}{H},\\
P_1 &= \rho_1 + \frac{P_2}{1+r_1} = \frac{\eta}{H}\left(y_1 + \frac{y_2}{1+r_1}\right) = \eta(1+\beta)\frac{y_1}{H}.
\end{aligned}
$$

In line with our interpretation of the debt limit $\bar{d}_2$ as being set by a macroprudential authority, we can interpret the equilibrium of our model with a non-binding debt limit as a laissez-faire equilibrium. When households choose how much to borrow, they take into account the impact their borrowing will have on their own future consumption. However, individual households do not internalize the fact that their contribution to aggregate debt may exacerbate the aggregate demand externality.

We derive restrictions on the values of the model parameters such that the laissez-faire equilibrium will suffer from an aggregate demand externality.

**Proposition 3.4.5.** *Provided the parameter restriction*

$$
h_0^P \leq \left(1 - \frac{1+\eta}{\eta}\frac{\bar{D}}{\pi\bar{y}_2}\right) H
$$

*is satisfied, second-period output is reduced by an aggregate demand externality in the laissez-faire equilibrium with no binding debt limit.*

*Proof.* Equilibrium conditions for aggregate debt and output are:

$$
D_2 = \pi d_2^p
$$

$$d_2^i = \frac{\eta}{1+\eta}\left(1 - \frac{h_0^i}{H}\right) y_2$$
$$y_2 = \bar{y}_2 - \phi\left(\frac{D_2}{\bar{D}} - 1\right), \quad \text{for} \quad D_2 > \bar{D}$$

A fixed point for debt is therefore

$$D_2 = \frac{\pi\eta\bar{D}(H - h_0^P)(\bar{y}_2 + \phi)}{H\bar{D}(1+\eta) + \phi\pi\eta(H - h_0^p)}$$

and

$$\frac{\pi\eta\bar{D}(H - h_0^P)(\bar{y}_2 + \phi)}{H\bar{D}(1+\eta) + \phi\pi\eta(H - h_0^p)} > \bar{D}$$

when

$$h_0^P \leq \left(1 - \frac{1+\eta}{\eta}\frac{\bar{D}}{\pi\bar{y}_2}\right) H.$$

□

The condition in Proposition 3.4.5 says that the aggregate demand externality will be active in the laissez-faire equilibrium if poor households' inherited share of the aggregate stock of housing is low. The more unequal the inherited stock of housing, the more poor households will want to borrow. Similarly, the right-hand side of the condition is increasing in the housing preference parameter $\eta$ because poor households' desire for borrowing is greater the stronger is their preference for housing occupancy over consumption of non-durables.

## 3.5 General equilibrium effects of the debt limit

The presence of an aggregate demand externality in the laissez-faire equilibrium of our model could motivate a macroprudential policymaker to impose a binding debt limit. However, as well as alleviating the aggregate demand externality, the debt limit has general equilibrium effects on relative prices in our model.

### 3.5.1 Rent

We have seen above that a binding debt limit means poor households purchase less housing from the rich than in the laissez-faire equilibrium, and the rental market becomes active. As we consider tighter debt limits, poor households purchase less housing and rent more of it from the rich. Rich households are unconstrained by the debt limit, and so must be indifferent between holding housing and bonds. All else equal, the rental rate in the first period must rise in order to compensate rich landlords for the increased deadweight rental cost.

### 3.5.2 Interest rate

There are two channels through which the debt limit can affect the equilibrium interest rate. Both channels operate through the market-clearing condition for bonds, which are in zero net supply.

The first is that when the debt limit tightens, the interest rate must fall so that rich households' desire to lend falls to match poor households' reduced ability to borrow. The second channel operates via the aggregate demand externality. If a tighter debt limit alleviates the aggregate demand externality, output in the second period increases relative to the first period. With income now relatively more abundant in the second period and scarce in the first period, all else equal the interest rate must rise to induce households to defer their consumption.

These two channels have opposite effects, so for debt limits that do not completely eliminate the aggregate demand externality, the net effect on the interest rate of a tighter debt limit is ambiguous. For debt limits tight enough to completely eliminate the aggregate demand externality, only the first channel will operate and so the interest rate will fall as the debt limit tightens.

### 3.5.3 House price

As with the interest rate, there are two channels with opposite effects through which the debt limit can affect the equilibrium purchase price of housing.

The first of these works directly through the tightening of poor households' affordability constraints. As they can no longer afford to purchase as much housing, all else equal the price of housing must fall to induce rich households to hold more of it. The second channel operates via the deadweight cost of rental housing. A tighter debt limit means poor households rent more housing. This means the marginal deadweight cost they incur is higher, so all else equal the marginal value of housing ownership is higher for them.

### 3.5.4 Incidence on rich and poor households

Although the signs of the impacts of a tighter debt limit on the interest rate and the house price are ambiguous, the incidence of all the general equilibrium effects on rich and poor households are clear. In equilibrium, rich households are landlords, lenders, and net sellers of housing, whereas poor households are tenants, borrowers and net buyers of housing. It is therefore in the interests of rich households for the rental rate on housing, the interest rate, and the purchase price of housing to be high, and vice versa for poor households. A tighter debt limit also affects poor households' welfare directly by constraining their choices. Together, these effects mean that the tightness of the debt limit may have distributional consequences as well as alleviating the aggregate demand externality. Since the sign of the general equilibrium effects on the interest rate and the house price are ambiguous, the net consequences of a tighter debt for the welfare of rich and poor households will depend on the calibration of the parameters.

## 3.6 Calibration

We interpret each period in the model as lasting for 25 years and calibrate the model accordingly. We solve the model in such a way as to match the following targets, which are based on UK data, as closely as possible:

- An annualized growth rate of 3%.
- An annualized interest rate of 4%.
- The value of the housing stock is 3.5 times annual GDP.
- An owner-occupancy rate of 66%.
- A share of imputed rent in GDP of 10%.

| | | Calibration (a) | Calibration (b) |
|---|---|---|---|
| Discount factor | $\beta$ | 0.77 | 0.78 |
| Housing preference | $\eta$ | 0.1 | 0.11 |
| Rental friction | $\mu$ | 0.01 | 0.01 |
| Poor population share | $\pi$ | 0.52 | 0.86 |
| Total housing stock per capita | $H$ | 1 | 1 |
| Housing endowment of rich | $h_0^R$ | 2.11 | 7.3 |
| Housing endowment of poor | $h_0^P$ | 0 | 0 |
| Output in period 1 | $y_1$ | 1 | 1 |
| Potential output in period 2 | $\bar{y}_2$ | 1.67 | 1.67 |
| AD externality debt threshold | $\bar{D}$ | 0.056 | 0.0947 |
| AD externality severity | $\phi$ | 0.09 | 0.043 |
| Debt limit | $\bar{d}_2$ | 0.05 | 0.0522 |

Table 3.6.1: 25-year calibration. Targets met in first period (a) or on average across both periods (b)

- A private debt-to-GDP ratio of 150%.

As Table 1 shows, depending on whether we calibrate the model to match these targets in the first period only, or on average across the two periods (to capture the life-cycle dimension), the calibrated values of some of the parameters differ. In particular, the latter calibration features a higher degree of inequality, with the same aggregate housing endowment concentrated in a smaller share of the population.

As discussed above, the calibrated values of the parameters affect the relative strength of the general equilibrium effects of tighter debt limits. We find in both calibrations that the interest rate is lower for tighter debt limits, which implies that the first channel identified above dominates. We also find in both calibrations that the affordability effect on the house price dominates, so that the equilibrium house price is lower for tighter debt limits. This latter finding is in line with those of Gete and Reher (2018), who find that a reduction in mortgage availability since the Great Recession has led to higher rents.

These general equilibrium effects in turn determine the distributional impact of macroprudential policy. As shown in Figure 1, under calibration (a) the interests of rich and poor households are aligned. The welfare of both types of household is maximized around a debt limit just tight enough to eliminate the aggregate demand externality, but no tighter.

By contrast, under calibration (b) the interests of rich and poor households diverge. Poor households again prefer that the macroprudential authority sets a debt limit in the region of the point where the aggregate demand externality is eliminated. Rich households, however, prefer the laissez-faire equilibrium. Having observed that house prices and the interest rate both increase with less restrictive debt limits under this calibration, we can infer that this latter effect is what drives rich households to prefer the laissez-faire equilibrium. This is because all the other effects of a looser debt limit, namely a worse aggregate demand externality, lower rents, and a lower house price, are detrimental to rich households' welfare.

## 3.7 Conclusion

In this paper we have solved a simple two-period, two-agent model with housing and debt, and used it to study the impact of macroprudential debt limits motivated by an aggregate demand externality. We have shown that in our model such limits can have subtle general equilibrium effects on prices, which have distributional effects. In future, we aim to extend our model by introducing nominal rigidities and an effective lower bound on the nominal interest rate.

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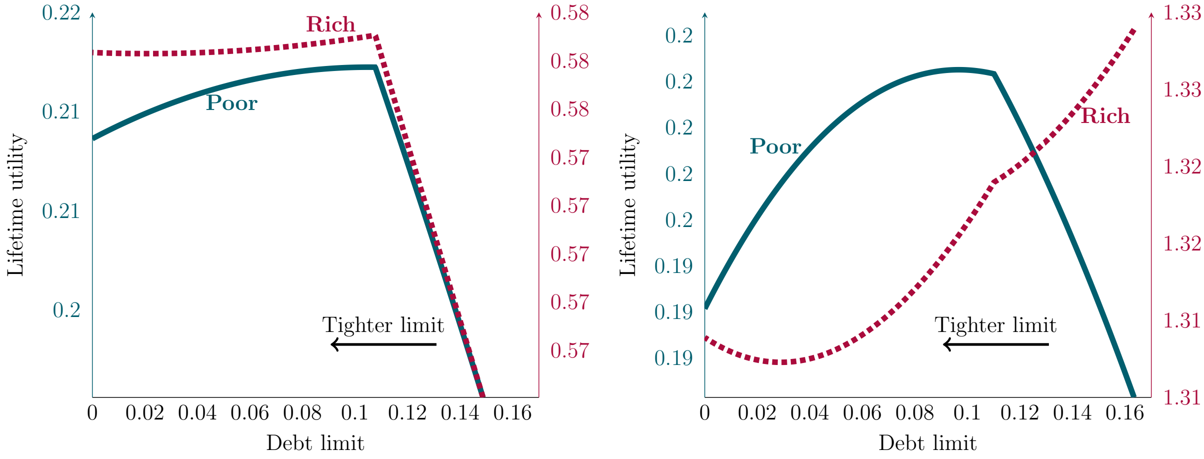


(a) Calibration with $\pi = 0.52$.



(b) Calibration with $\pi = 0.86$.

Figure 3.8.1: Distributional impact of macroprudential policy depends on strength and incidence of general equilibrium effects.