The London School of Economics and Political Science

Essays on Corporate Finance and Governance

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 39516 words.

Statement of inclusion of previous work

I confirm that Chapter 3 was the result of previous study for Master of Research in Finance at London School of Economics and Political Science during 2016 - 2018.

Abstract

This thesis explores topics on Corporate Finance and Governance.

In the first chapter, I develop a dynamic agency model to investigate optimal managerial authority and its interaction with managerial compensation. I find that when hiring a manager, the principal delegates authority that is unresponsive to either the manager's outside options or the firm's recruitment costs, in contrast to promised compensation, which increases in both. Over time, both the manager's authority and his compensation rise after good performances and decline after bad realizations. Authority-performance sensitivity decreases as the manager's authority grows, resembling entrenchment. In contrast, pay-performance sensitivity increases with the manager's authority. If managerial authority can be adjusted only infrequently, the optimal contract may allow for self-dealing. I find that in this case, early-career luck plays a disproportionate role in determining the manager's authority and lifetime utility.

In Chapter Two, I investigate the optimal financing and investment strategies of a platform enterprise featuring cross-group network effects. Networks are analogous to capital assets. A platform enterprise invests in the networks by making subsidies to users. I find that a platform enterprise, even as a monopoly, should make aggressive subsidies in the early stages of network growth. A platform with stronger network effects should make more subsidies at initial stages and enjoy a higher valuation. In terms of financing patterns, staged financing mitigates the limited enforcement problem, and ceteris paribus, the number of funding rounds decreases with the profitability of the platform and increases with required profits by financiers. I also find that the value of funds raised each round increases and the financing frequency decreases over time.

In Chapter Three, I study a public firm's choice of seasoned equity offering

methods and the subsequent stock performances. I document that small-size public firms which have conducted shelf SEOs tend to underperform with respect to expectations in the long run; the underperformance mitigates when the firm's size becomes larger. A three-date model is built to capture this long-run underperformance, proposing that heterogeneity in investment opportunities and information asymmetry are two key underlying factors. Empirical tests in this paper support the model and show that small firms have lower cumulative abnormal returns and a lower level of new investments after shelf SEOs.

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Chapter 1

Optimal Managerial Authority

Joanne Juan Chen[1]

1.1. Introduction

Properly exercising decision-making authority is crucial in the operation of firms. In a modern corporation, shareholders seldom make operational decisions. They instead delegate a majority of operational decision-making authority to professional managers who possess expertise and superior information (Dessein 2002). A manager's main duty is to properly exercise the authority delegated by corporate owners (Bolton and Dewatripont 2013). Therefore, the optimal

[1] I am deeply indebted to Daniel Ferreira, Martin Oehmke, and Ulf Axelson for their invaluable guidance and advice. I am grateful to Ricardo Alonso, Matthieu Bouvard, Mike Burkart, Michael Fishman, Sebastian Gryglewicz, Yunzhi Hu, Dirk Jenter, Peter Kondor, Jin Li, Kristian Miltersen, John Moore, Sebastian Pfeil, Walker Ray, Michael Rebello, Lucy White, Xiaoyun Yu, Hongda Zhong, and seminar participants at Queen Mary University of London, Northwestern Kellogg, UNC Kenan-Flagler, Erasmus University Rotterdam, BU Questrom, UT Dallas Naveen Jindal, Warwick Business School, Copenhagen Business School, Shanghai Advanced Institute of Finance, University of Amsterdam, European Winter Meeting of the Econometric Society, the Toulouse School of Economics, the European Finance Association Doctoral Tutorial, World Finance Conference, and the London School of Economics for helpful and insightful comments. All errors are my own. Contact information: Joanne Juan Chen, Department of Finance, London WC2A 2AE, UK. Email: j.chen64@lse.ac.uk.

allocation of authority is central in designing a managerial job. The following questions arise naturally: How much authority should be granted to newly hired managers? How should their level of authority evolve over time?[2] And how should their authority interact with compensation?

In this paper, I build a dynamic agency model that characterizes the optimal delegation of authority and its interaction with managerial compensation. This model explains several stylized facts that have not yet been well addressed in the literature. First, when recruiting for a managerial position, companies alter the manager's compensation level in response to varying labor market conditions, but not the level of delegated authority.[3] Second, a manager's authority is sensitive to his past performance and increases after good performance, but this sensitivity decreases as his authority grows, which resembles managerial entrenchment. In contrast, since a manager is granted more stock and options as authority grows, his pay-performance sensitivity increases with authority.[4] Furthermore, the model provides novel implications concerning the interactions between managerial authority, compensation, and career trajectories, which will be discussed in detail as the paper proceeds.

The study of optimal delegation was pioneered by Holmstrom (1977, 1984).

[2]Authority delegation is dynamic in firms. A well-performing middle-level manager will usually be assigned to lead a larger team; a CEO with good past performance can be granted a dual role as the board chairman or president. Or conversely, a manager can also be divested of part of his authority due to misconduct or poor performance. One recent example is the Volkswagen case. In June 2020, Volkswagen AG replaced the company CEO Herbert Diess's dual role as chief of namesake brand after vehicle delays and clashes with labor unions. (https://www.wsj.com/articles/volkswagen-board-considering-management-shake-up-for-vw-brand-11591632658)

[3]Empirical studies document that managerial compensation varies according to labor market conditions (see, e.g., Bizjak et al. [2008]; Brookman and Thistle [2013], among others). On the other hand, an employment contract usually states "the executive shall have the duties and responsibilities **typical for such position** and may otherwise be **assigned or modified** by the CEO or the Board of Directors." This verbiage demonstrates that (1) the initial delegated authority is associated with the managerial position only and (2) the dynamic and evolving nature of authority delegation over time.

[4]Edmans et al. [2017] comprehensively review executive compensation. Figures 6, 7, and 8 give examples of where the proportions of stocks and options increase in managerial authority (by a cross-sectional comparison between CEO and non-CEO executives).

Much of this literature focuses on delegation without monetary transfers, which limits its application to firms. In firms, performance-sensitive compensation is an important tool to align interests of the owners and managers and make delegation profitable. Understanding how optimal managerial authority and managerial compensation interact, especially in a dynamic world, is therefore of importance.[5]

To investigate optimal managerial authority and the corresponding managerial compensation in firms, I adopt a dynamic contracting approach[6] and study multi-task delegation problems in a dynamic environment, allowing for private savings and borrowing, as well as costly managerial turnover. The model is set up in discrete time to clarify the agency problems and is solved in continuous time for analytical tractability.

In the model, a risk-neutral principal ("she") has one project in each period. A project comprises a continuum of different tasks, each affecting the project's probability of success. These tasks can be understood as operational decisions, for example, about setting budgets or selecting suppliers. Each task requires a decision to be made among many different options. The principal cannot distinguish among the options. A qualified manager ("he") has expertise and can distinguish among all the options. Therefore, the principal may want to delegate some decision-making authority to the manager. Among the delegated tasks, the manager can make decisions that increase the project's probability of success; alternatively, he can pick the options that contain private benefits but decrease the project's probability of success. The principal incentivizes the manager to make

[5] Ottaviani (2000) considers a static uniform-quadratic case with full delegation and action-contingent transfers. Krishna and Morgan (2008) focus on a static case in which the principal can commit to a transfer rule but retains decision-making authority.

[6] I apply the contracting approach, because, on the one hand, contracting is a common approach to managerial compensation problems. An employer contracts on compensation to incentivize a manager to make profitable decisions. On the other hand, the formal authority of a manager is delegated ultimately by firm owners through explicit or implicit contracts (Aghion and Tirole 1997). Therefore, contracting is also a natural approach to study delegation problems.

good decisions by linking his current and future compensation to the project's output. She also optimally chooses the set of tasks to delegate for each period. Specifically, when hiring, the principal provides a full-commitment contract on output-contingent managerial authority and the compensation process. This contract is equivalent to a series of spot contracts provided at the beginning of each period, specifying the manager's authority and wage in the current period, and his output-contingent continuation value. The principal commits intertemporally to the manager's continuation value in the firm.[7] In contrast, the manager has limited commitment and can quit at any time. If the manager leaves, the principal can hire a qualified replacement with a constant cost. All managerial candidates have constant absolute risk aversion (CARA) preference and can save and borrow privately.

In the main model, managerial authority can be adjusted in each period. This setting gives rise to the following optimal mechanism: the promised continuation value increases after good performances and decreases after bad realizations; managerial authority monotonically increases in the manager's continuation value, as does his pay-performance sensitivity; and the relative magnitude of change in authority and compensation decreases with the manager's continuation value.

This mechanism demonstrates the dynamic misalignment effect on authority delegation. The intuition underlying this effect is as follows. To extract more good decisions from the manager, the principal needs to delegate more authority and make the manager's compensation more sensitive to the project's output. However, raising pay-performance sensitivity is costly, not only because of the manager's risk aversion but also due to potential managerial turnover. In this model, without a wealth effect and because the manager can smooth consumption by private savings

[7]Spear and Srivastava (1987) and Phelan (1995) prove that efficient contracts can be written recursively with commitment on the continuation value of the manager. In Section 1.4, I assume that opportunities to adjust managerial authority follow a Poisson distribution and can be noncontractible. Therefore, I adopt recursive contracts.

and borrowing,[8] the dynamics of misalignment is entirely driven by the manager's limited commitment and associated turnover costs: the misalignment problem becomes more severe when the manager is closer to departure, which results in less authority delegation and lower pay-performance sensitivity. In contrast, in the benchmark case in which the manager has full commitment and never leaves the firm, dynamic misalignment disappears, and the degree of misalignment is constant over time. Therefore, the optimal managerial authority and pay-performance sensitivity are also constant and have reached a pinnacle.

The mechanism explains the stylized fact that a long-serving manager who has a high level of authority seems entrenched, in the sense that the level of his authority becomes less sensitive to performance, while his compensation becomes more equity or option based (e.g., Edmans et al. 2017). The intuition behind this result is as follows: The dynamic misalignment problem fades when the manager's continuation value is sufficiently high. Hence, the manager gains more authority, the level of which is less sensitive to the manager's performance. Meanwhile, the principal selects a contract with higher pay-performance sensitivity, which is implemented by options or additional units of stocks, to provide incentives for good decisions. The opposite evolvement of the authority-performance sensitivity and the pay-performance sensitivity is one of the main findings in this paper.

I also show that the initial authority delegated to a newly hired manager is tied to the position and is independent of the labor market conditions. The compensation level, in contrast, varies according to the labor market conditions. In other words, changes in the manager's outside options or the firm's recruitment costs do not affect the authority initially allocated to a newly hired manager.

[8]If the manager cannot save or borrow privately, the principal is able to control the manager's consumption path and will usually distort payment timing to provide additional incentives. For example, Hoffmann and Pfeil (2021) discuss how deferring compensation increases the agent's (manager's) stake in the firm and provides incentives. Grochulski and Zhang (2021) study how temporarily suspending the agent (manager) given no consumption can rebuild his "skin in the game" and restore his incentives.

This result comes from the principal optimally offsetting the effects of labor market conditions on the severity of dynamic misalignment by adjusting the initial continuation value of the manager.

I then extend the model so as to examine the case in which authority is adjusted less frequently than is pay-performance sensitivity. In reality, managerial authority is adjusted infrequently due to various frictions.[9] In contrast, the pay-performance adjusts automatically with the firm's performance if stock options constitute part of the compensation package or if the number of stocks granted to the manager changes. The paper finds that if the opportunity to change the manager's authority arises only intermittently, while the pay-performance sensitivity can be adjusted frequently, the manager may engage in self-dealing (i.e., inefficient consumption of private benefits). Managerial self-dealing is tolerated by the principal, even though she could eliminate it by setting a sufficiently high pay-performance sensitivity. Therefore, the principal is in effect using private benefits as a cheaper alternative to compensation, at the cost of productive efficiency.

Importantly, the case of infrequent authority adjustment predicts that luck in one's early career is paramount in determining the manager's authority and lifetime utility. Moreover, with the analysis I delineate the career trajectories leading to managerial self-dealing. If a manager experiences a series of good realizations in the early stages of his career, he becomes better aligned with the firm and will be granted more authority in the future. Thereafter, the manager has more discretion in making decisions, generating more profits for the firm and gaining higher compensation for himself. Later in his career, if he suffers from negative shocks, he can take advantage of his high level of authority and engage in self-dealing, that is, acting in his own best interest, rather than in that of the

[9] For example, infrequent changes to the board composition (Adams and Ferreira 2007) or a deadlock on the board (Donaldson et al. 2020) may lead to a lag in the adjustment of a top executive's authority.

firm's, thereby keeping his lifetime utility high.

In contrast, if the manager first encounters negative shocks, he will be stripped of part of his authority since the misalignment becomes more severe. Thereafter, he gets stuck in a low-authority situation even if he later experiences positive shocks and becomes better aligned with the principal. He cannot well exploit his superior knowledge. Consequently, his lifetime utility is lower. This story depicts the career trajectory that leads to managerial self-dealing: a manager who experiences good luck in the early stages and bad luck in the later years of his career is more likely to engage in self-dealing.

Related Literature This paper bridges two strands of literature, the literature of optimal delegation and the dynamic contracting literature.

The optimal delegation literature was pioneered by Holmstrom (1977, 1984). This strand of literature studies the optimal allocation of decision-making authority between the principal and the agent when the agent has private information. Alonso and Matouschek (2008) investigate conditions for interval delegation to be optimal and provide explanations for the widespread use of threshold delegation (a particular type of interval delegation). Amador and Bagwell (2013) generalize the results by considering a general class of preferences and provide necessary and sufficient conditions for the optimality of interval delegation. Some related work studies full delegation and compares delegation with communication, for example, Dessein (2002) and Ottaviani (2000). Li et al. (2017) and Lipnowski and Ramos (2020) examine dynamic delegation without monetary transfer in a repeated games setup.[10] Most of the delegation literature considers the case in

[10]More generally, the delegation literature belongs to a set of theories of optimal rules: the relationship between the ultimate objective of the rule-setter and the optimal rule to commit to. Some relevant studies are Aghion and Tirole (1997), Burkart et al. (1997), Armstrong and Vickers (2010), Frankel (2014), but they consider scenarios different from the delegation literature. For example, Aghion and Tirole (1997) and Burkart et al. (1997) investigate the

which monetary transfers are unavailable. Relative to this literature, my model sheds light on the dynamic interaction between optimal multi-task delegation and optimal compensation.

Methodologically, the paper belongs to the continuous-time dynamic contracting literature. Optimal delegation is a seldom-visited topic in this strand of literature. One relevant study is done by Malenko (2019), who examines the capital allocation process in an organization when the manager has empire-building preferences. He finds that the threshold delegation of investment decisions is optimal, and the level of delegation decreases with the agent's continuation value. In my paper, the manager's operational decision-making authority increases with the manager's continuation value.

More broadly, this paper is related to the hidden-action models in the dynamic contracting literature. One strand of the literature analyzes the agent's hidden efforts, for example, Sannikov (2008), Zhu (2013), and Grochulski and Zhang (2021). My paper differs from their hidden effort models in that authority delegation is a choice variable of the principal. The principal can use authority delegation to restrict the action space of the manager.[11] Another major strand of the dynamic contract literature considers cash-flow diversion models, for example, DeMarzo and Sannikov (2006), Biais et al. (2007), Piskorski and Westerfield (2016), and Hoffmann and Pfeil (2021), among others. In these papers, cash flows are privately observed and can be diverted, thereby creating an *ex post* moral hazard problem. Besides, in these papers, the optimal aggregate incentives level is constant due to the constant diversion efficiency, and, consequently, moral hazard

impact of the manager's authority on the information structure, while the delegation literature takes the information structure as given.

[11]The predictions in their paper are also different from mine. In Zhu (2013), effort levels are binary, and the effort may increase or decrease with the agent's continuation value, depending on the parameters. Sannikov (2008) and Grochulski and Zhang (2021) consider a risk-averse agent with non-monetary effort costs. It is more difficult to incentivize efforts as the agent's continuation value increases. Consequently, there exists a high retirement point.

(i.e., diversion) is eliminated in equilibrium.[12] Noe and Rebello (2012) study managerial compensation and costly monitoring with dynamic learning of a latent firm characteristic. They find that monitoring intensity is negatively correlated with managerial compensation and the firm's fortune, and managerial private benefits may ameliorate agency conflicts.

The technical assumptions of this paper follow He (2011), who solves the double-deviation problem with private savings and borrowing by adopting the CARA preference.[13] My paper adds to that model by allowing the agent (manager) to have limited commitment.[14] Relative to the continuous-time dynamic contracting literature, I provide a discrete-time setup with an *ex ante* moral hazard problem and also disentangle the dynamic misalignment effect due to one-sided commitment from the wealth effect and the deferred compensation effect.

The job design aspect of this paper is related to Itoh (1994), Axelson and Bond (2015), Ke et al. (2018), and Ferreira and Nikolowa (2020), as well as to studies in personnel economics.[15] Axelson and Bond (2015) develop an incentive-based theory of finance jobs in an equilibrium framework. They focus on how to allocate pre-specified jobs to agents with hidden efforts, and find that the labor market conditions profoundly affect the jobs allocated to a young agent

[12] Piskorski and Westerfield (2016) study costly monitoring. Monitoring differs from delegation in that monitoring deters undesirable actions by threat of potential punishment, which is an incentive device and can substitute for pay-performance sensitivity. In contrast, delegation directly controls the agent's action space. Moreover, more intensive monitoring generally improves the firm's performance but less delegation usually leads to worse outcomes compared to the first-best case.

[13] When private savings and borrowing are allowed, the first-order approach may fail, that is, the first-order conditions may not guarantee full incentive compatibility. Generally speaking, the contracting problem becomes very difficult with private savings and borrowing. See Section 6 of Sannikov (2008) for more discussion.

[14] A majority of models that simultaneously consider the agent's limited commitment and private savings and borrowing in the continuous-time contracting literature assume a risk-neutral agent with limited liability and inefficient private savings technology, for example, Hoffmann and Pfeil (2010), DeMarzo et al. (2012), and Hoffmann and Pfeil (2021). In these models, the principal can manipulate payment timing and there exists deferred compensation. In my model, due to the efficient private savings and borrowing technology, the principal cannot distort the payment timing and, therefore, cannot use it as an additional tool to incentivize the agent.

[15] Lazear and Shaw (2007) and Lazear (2018) provide literature review on personnel economics.

as well as his subsequent career. The timing distortion in payment and the consequent performance bond effect is a major force driving their results. Ferreira and Nikolowa (2020) develop a theory of optimal job creation technology where employees gain utility from both consumption and job prestige. Relative to this literature, my paper investigates the dynamic job design for a managerial position, emphasizing on-the-job authority dynamics and the interaction between the manager's authority-performance sensitivity and pay-performance sensitivity.

1.2. The Model

The model is set up in discrete time to better clarify the agency problem. This discrete-time setting lays down a clear conceptual foundation for continuous-time modeling and allows me to derive a rich set of predictions.

1.2.1 Technology and authority delegation

The model considers a firm with an infinite life span. Time is partitioned into intervals with a length of $\delta > 0$, that is, $t = 0, \delta, 2\delta, \ldots$. The discount factor is $1/(1 + r\delta)$, where $r < 1$ is the common discount rate in this economy. A risk-neutral principal ("she") hires a manager ("he") to operate the firm. The manager has CARA preferences, and his instantaneous utility is represented by $u(c_t) = -e^{-\gamma c_t}$. The manager is allowed to privately save or borrow against his employment contract at the risk-free rate, r.

In each period of time, the firm undertakes a project that may succeed or fail. If the project succeeds, it generates net profits of $y_H = \sqrt{\delta}$ at the end of the period; if it fails, it generates $y_L = -\sqrt{\delta}$. In other words, $y_{t+\delta} \in \{-\sqrt{\delta}, \sqrt{\delta}\}$. A project comprises a mass-one continuum of tasks, all of which affect the project's probability of success. Take a manufacturing firm as an example. The tasks typically involve designing production lines, investing in machinery, choosing input materials, selecting suppliers, training employees, advertising, and marketing, etc.

A task is modeled as making one decision among infinitely many options. Of all the options in a task, only two are most relevant: the good one increases the project's probability of success; the bad one decreases the project's probability of success but carries private benefits; other options neither affect the probability nor contain private benefits. Both parties are aware of the task's impact on the project and the level of private benefits contained in the bad option. However, only the manager can distinguish among the options.[16] Therefore, both the good option and the bad option are picked with a probability of zero if the principal makes the decision. Moreover, if the principal retains full authority and makes decisions on all the tasks herself, the project will succeed or fail with equal probability; that is, the net present value (NPV) of the project is zero.[17]

Assumption 1 *Let $i \in [0, 1]$ represent the index of a task.*
The good option in task i increases the project's probability of success by $\frac{1}{2}\sqrt{\delta}di$; the bad option in task i decreases the project's probability of success by $\frac{1}{2}\sqrt{\delta}di$ but carries private benefits of $B_i\delta di$, where $B_i = i$.

As an illustration of Assumption 1, consider that if all tasks are delegated to the manager and all decisions made by him are good (i.e., he chooses all the good

[16]This setup is for elaboration simplicity. An equivalent and more general setup could be as follows. The manager has full information while the principal knows the distribution of the options: their effects on the project and private benefits contained. The distribution has the following characteristics: (1) the effects to the project are zero in expectation; (2) only finite many options contain private benefits; (3) the good option has the greatest positive effect on the project's probability of success but contains no private benefits; and (4) the bad option contains the highest private benefits but has an inverse effect on the project compared to the good one.

[17]The assumption is realistic because, in practice, firm owners are usually aware of the levels of potential private benefits associated wth different tasks, but they lack available attention/expertise to identify all the options to reach the optimal decision for each task. For instance, they know that the supplier choice may deliver more private benefits than employee training arrangements to the manager. However, they usually lack the information, expertise, and attention to select the most suitable supplier or to design the best employee training program.

options), the project's probability of success increases by $\int_0^1 \frac{1}{2}\sqrt{\delta}di = \frac{1}{2}\sqrt{\delta}$, and the project's expected profits increase by $\frac{1}{2}\sqrt{\delta}\cdot\sqrt{\delta} + (-\frac{1}{2}\sqrt{\delta})\cdot(-\sqrt{\delta}) = \delta$; if he makes all bad decisions on this continuum of tasks (i.e., he chooses all the bad options), the expected profits of the project decrease by δ, but he gains private benefits amounting to $\int_0^1 i\delta di = \frac{1}{2}\delta$.

Table 1.1 provides another illustration of Assumption 1. The table demonstrates how the options in Task i affect the project's expected profits. For one unit of task i, the good option increases the project's expected profits by δ, while the bad option decreases the project's expected profits by the same amount but delivers private benefits of $i\delta$. All the other options are neutral on the project and do not deliver private benefits.

Table 1.1: Options and Impacts of Task i (per unit)

Options in Task i:	Good Option	Bad Option	Other Options
Change in the project's expected profit	δ	$-\delta$	0
Private benefits	0	$i \cdot \delta$	0

Assumption 1 implies that each task has the same effect on the project's probability of success. They differ in the level of potential private benefits and are sorted according to the level of potential private benefits. The assumption $B_i = i \leqslant 1$ makes sure that the bad option minimizes total surplus.

Let $\alpha_t \in [0, 1]$ denote the measure of authority (i.e., the number of tasks) delegated to the manager at the beginning of period t. It is intuitive that the tasks delegated are those with the smallest private benefits, because it is easier to incentivize good decisions in these tasks. The principal retains the decision-making authority on tasks with high private benefits to prevent the manager from making bad decisions. Let $\theta_t \in [0, \alpha_t]$ denote the measure of good decisions made by the manager in period t. Figure 1.1 visualizes the allocation of authority for the period t.

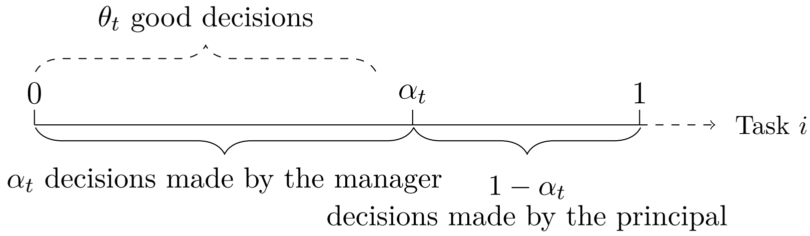


Figure 1.1: Authority delegation at time t

By simple calculations, the distribution of cash flows at the end of period t can be expressed in the manager's authority α_t, and the number of good decisions θ_t:

$$y_{t+\delta} = \begin{cases} y_H = \sqrt{\delta}, & Pr(y_H) = \dfrac{1}{2}[1 + (2\theta_t - \alpha_t)\sqrt{\delta}], \\ y_L = -\sqrt{\delta}, & Pr(y_L) = \dfrac{1}{2}[1 - (2\theta_t - \alpha_t)\sqrt{\delta}]. \end{cases} \tag{1.1}$$

REMARK 1.1 *The authority delegation considered in this paper can be viewed as an example of the general formulation of the delegation problem defined in Holmstrom (1977, 1984). I term the decision on a task i a "sub-decision", and the sequence of decisions on all the tasks $i \in [0,1]$ a "joint decision". Let $\mathbf{d}$ denote the joint decision. The decision space D is the set for all feasible joint decisions. The delegation process in this paper, where the principal makes decisions on tasks $i \in (\alpha_t, 1]$ and the manager makes decisions on tasks $i \in [0, \alpha_t]$, starts from the principal choosing the control set C, where $C \subseteq D$. The control set C contains all the joint decisions $\mathbf{d}$ where the sub-decisions on tasks $i \in (\alpha_t, 1]$ are fixed and predetermined by the principal. Then, the manager chooses a joint decision $\mathbf{d} \in C$, completing this delegation process.*

1.2.2 The contract

The contract provided by the principal specifies an authority delegation process $\alpha = \{\alpha_t\}_{t \geqslant 0}$ and an output-contingent wage process $w = \{w_t\}_{t \geqslant 0}$. Given the contract, the manager maximizes his expected discounted utility by choosing a decision process $\theta = \{\theta_t\}_{t \geqslant 0}$ and a consumption process $c = \{c_t\}_{t \geqslant 0}$.[18] Here, w_t and c_t are written as rates, so as to be consistent with the notations used for the continuous-time model that I will investigate later. That is, the wage for the period $[t, t + \delta)$ is $w_t \delta$, and the manager's consumption is $c_t \delta$. Without loss of generality, I assume both are end-of-period values.

The quadruple $(\alpha, w; \theta, c)$ is referred to as an **incentive-compatible contract**[19], where (θ, c) is the process of the manager's recommended decisions and consumption. The maximized expected discounted utility at time t is referred to as the manager's continuation value at t, denoted by V_t, which is a start-of-period value.

While the principal can commit to the above long-term contract, the manager will only stay in the relationship when his continuation value, V_t, derived from his future consumption in the firm, is greater than his outside option, $\underline{V}$. In other words, the model assumes a one-sided commitment by the principal. This assumption is consistent with the realities present in labor markets.[20] If the incumbent manager leaves, the principal can hire a replacement. Each time a manager is hired, the firm incurs a recruitment cost, $q \geqslant 0$, which can be understood as a search cost, like fees paid to headhunters, or an orientation cost.

[18]Given the wage process and the consumption process, the savings and borrowing process is pinned down. To simply the notation, I haven't written down that process explicitly.

[19]An incentive compatible contract is a contract including the agent's recommended strategies. For example, see DeMarzo and Sannikov (2006).

[20]Phelan (1995) points out that many long-term economic relationships are characterized by parties' differing abilities to commit to long-term contracts. In labor markets, while an employer could conceivably sign a contract that offers a worker a job for life, workers cannot promise to never quit or work for another firm.

Following Spear and Srivastava (1987) and Phelan (1995), the efficient contract can be written recursively. Since the incumbent manager's continuation value, V_t, is the only state variable in this model, the contract can be equivalently written in the following way. At the beginning of period t, the manager's continuation value V_t is given, and the principal writes an incentive-compatible spot contract $(\alpha_t, w_t, V_{t+\delta}(y_H), V_{t+\delta}(y_L); \theta_t, c_t)$, which keeps the principal's promise on V_t conditional on the manager behaving as suggested in the contract. This commitment on V_t can be expressed in the form of **the promise-keeping constraint** (Phelan 1995; Fernandes and Phelan 2000):

$$V_t = \frac{1}{1+r\delta} \cdot \Big\{ u(c_t)\delta + [Pr(y_H) \cdot V_{t+\delta}(y_H) + Pr(y_L) \cdot V_{t+\delta}(y_L)] \Big\}, \tag{1.2}$$

where $Pr(y_H)$ and $Pr(y_L)$ are defined in Equation (1.1).

He (2011) proves that if the manager can save or borrow privately, it is without loss of generality to focus on the incentive-compatible no-savings contracts.[21] In this paper, I follow He (2011) and focus on the contracts that lead to zero savings or borrowing in equilibrium.

REMARK 1.2 *Rewriting the contract in this fashion demonstrates a realistic way to implement the full commitment by the principal. Rather than providing an extremely complex state-contingent contract that covers the length of the employment relation, the principal only needs to write a spot contract* $(w_t, \alpha_t, V_{t+\delta}(y_H), V_{t+\delta}(y_L))$ *with recommendations on* (θ_t, c_t)*, which together satisfy* ***the promise-keeping constraint*** *(1.2), at each time* t.

[21] See He (2011) Lemma 2.

1.3. Model Solutions and Analysis

In this section, I derive the continuous-time limit of the model and solve for the optimal contract. The technical advantages of the continuous-time methods lead to a simpler computational procedure and make the optimal contracting tractable.

1.3.1 Manager's optimization problem

The manager's problem is to find the optimal choices of (θ, c) given the contract.

First, define

$$\beta_t = \frac{(V_{t+\delta}(y_H) - V_{t+\delta}(y_L))/u'(c_t)}{y_H - y_L}. \tag{1.3}$$

β_t measures the sensitivity of the manager's continuation value with respect to output normalized by his marginal utility. The incentive-compatible spot contract $(\alpha_t, w_t, V_{t+\delta}(y_H), V_{t+\delta}(y_L); \theta_t, c_t)$ with the promise-keeping constraint (1.2) can now be equivalently summarized as $(\alpha_t, w_t, \beta_t; \theta_t, c_t)$ with the same constraint. By the pay, the equilibrium β_t represents the sensitivity of the manager's certainty-equivalent pay with respect to the output when δ goes to zero in the model. Therefore, I term β_t as the "pay-performance sensitivity" hereafter. See Subsection 1.3.6 for the derivation and detailed discussion.

Following He (2011), the first-order approach applies, and the manager's consumption and operational decisions can be examined separately. Appendix A.1 adapts the proof of He (2011) to this model. Lemma 1.1 summarizes the manager's optimal decisions at each time t.

Lemma 1.1 *Given the contract (α_t, w_t, β_t) and the promised continuation value V_t:*

if $V_t > \underline{V}$*, the manager chooses* $\theta_t = \min\{2\beta_t,\ \alpha_t\}$*,* $c_t = -\frac{1}{\gamma}\ln(-rV_t)$*;*

if $V_t \leqslant \underline{V}$*, the manager quits.*

Lemma 1.1 shows that conditional on the manager staying in the firm, the number of good decisions he makes is determined by the pay-performance sensitivity. Given a certain level of authority, the manager makes more good decisions when his pay-performance sensitivity is higher. Intuitively, a higher pay-performance sensitivity makes bad decisions more costly for the manager and, therefore, can provide incentives for higher moral-hazard tasks. Allowing private savings and borrowing, the manager smooths consumption over time. The policies (θ_t, c_t) summarized in Lemma 1.1 satisfy the incentive compatibility constraint of the manager, and, therefore, are indeed the recommended decisions and consumption in the incentive-compatible spot contract at time t.

1.3.2 Principal's problem in recursive form

The principal's objective is to maximize her expected discounted profits by optimally designing the contract. Following the literature, I solve the principal's problem in a recursive way. Let $F(V_t)$ represent the principal's continuation value at the beginning of time t. The principal's problem at time t is summarized as follows:

$$F(V_t) = \max_{(\alpha_t, w_t, \beta_t)} \frac{1}{1+r\delta} \cdot \Big\{ -w_t\delta + E_t[y_{t+\delta}] + E_t[F(V_{t+\delta})] \Big\} \tag{1.4}$$

$$s.t. \quad V_t = \frac{1}{1+r\delta} \cdot \Big\{ u(c_t)\delta + [Pr(y_H) \cdot V_{t+\delta}(y_H) + Pr(y_L) \cdot V_{t+\delta}(y_L)] \Big\},$$

where

$$E_t[y_{t+\delta}] = (2\theta_t - \alpha_t)\delta,$$

$$E_t[F(V_{t+\delta})] = Pr(y_H) \cdot F(V_{t+\delta}(y_H)) + Pr(y_L) \cdot F(V_{t+\delta}(y_L)),$$

$$Pr(y_H) = \frac{1}{2}[1 + (2\theta_t - \alpha_t)\sqrt{\delta}], \;\; Pr(y_L) = \frac{1}{2}[1 - (2\theta_t - \alpha_t)\sqrt{\delta}],$$

$$\beta_t = \frac{(V_{t+\delta}(y_H) - V_{t+\delta}(y_L))/u'(c_t)}{y_H - y_L},$$

$$c_t = w_t + \int_{\theta_t}^{\alpha_t} i di.$$

That is, the principal maximizes the discounted profits by providing a spot contract (α_t, w_t, β_t) with recommendations on (c_t, θ_t), subject to the promise-keeping constraint. The last equation above comes from the fact that the principal provides a wage level at which there is no saving or borrowing in equilibrium, and the manager fully consumes the wage and private benefits at each time.

1.3.3 Continuous-time version of the problem

Taking the limit $\delta \to 0$, I derive the continuous-time version of the model (see Appendix A.2 for the detailed derivation), which is summarized in the following three points.

(i) The cumulative output Y_t evolves according to

$$dY_t = y_{t+\delta} = (2\theta_t - \alpha_t)dt + dZ_t, \tag{1.5}$$

where $\{Z_t\}_{t\geqslant 0}$ is a standard Brownian motion and dZ_t represents the limit of unexpected component of output $(y_{t+\delta} - E_t[y_{t+\delta}])$ when $\delta \to 0$.

(ii) The manager's continuation value evolves according to

$$dV_t = (rV_t - u(c_t))dt + \beta_t u'(c_t)dZ_t. \tag{1.6}$$

(iii) The principal's continuation value satisfies the Hamilton-Jacobian-Bellman (HJB) equation:

$$rF(V_t) = \max_{(\alpha_t, w_t, \beta_t)} \{-w_t + (2\theta_t - \alpha_t) + F'(V_t) \cdot [rV_t - u(c_t)] + \frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\}. \tag{1.7}$$

1.3.4 Optimal contract

I first solve for the relationship between the optimal authority level, α_t, and the pay-performance sensitivity, β_t.

Proposition 1.1 *If α_t and β_t can be freely chosen from the set $[0, 1]$, the principal would set $\alpha_t = 2\beta_t$.*

Proposition 1.1 demonstrates that more managerial authority should be accompanied by a higher pay-performance sensitivity. Combining Proposition 1.1 and Lemma 1.1, it's easy to find that $\alpha_t = \theta_t$. That is, there is no managerial self-dealing if the principal can freely choose the level of managerial authority and the pay-performance sensitivity in each period. Intuitively, bad decisions are inefficient and are dominated by the uninformed decisions made by the principal herself. Therefore, it is optimal for the principal to set the level of managerial authority and the corresponding pay-performance sensitivity so to eliminate self-dealing.

Corollary 1 *If α_t and β_t can be freely chosen by the principal at the beginning of time t, $\forall t \geqslant 0$, there is no managerial self-dealing. That is, all the decisions made by the manager are good and increase the project's probability of success.*

Applying the results in Lemma 1.1 and Proposition 1.1, the principal's Hamilton-Jacobi-Bellman (HJB) equation (1.7) simplifies to:

$$rF(V_t) = \max_{\alpha_t} \left\{ \frac{1}{\gamma} \ln(-rV_t) + \alpha_t + \frac{1}{8} F''(V_t)\alpha_t^2 \cdot (r\gamma V_t)^2 \right\}. \tag{1.8}$$

Taking the first-order condition with respect to α_t yields

$$\alpha_t = -\frac{4}{(r\gamma V_t)^2 F''(V_t)}. \tag{1.9}$$

Plugging the expression of α_t back into the HJB equation gives the ordinary differential equation (ODE):

$$rF(V_t) + \frac{2}{(r\gamma V_t)^2 F''(V_t)} - \frac{1}{\gamma} \ln(-rV_t) = 0. \tag{1.10}$$

It remains to find the boundary conditions to fully characterize the optimal contract. The first boundary condition is the "value-matching condition" at the manager's turnover point,

$$F(\underline{V}) = F(V_0) - q, \text{ with } V_0 \in \arg\max_V F(V). \tag{1.11}$$

This boundary condition reflects that, at the managerial turnover point in time, the principal hires a replacement with a recruitment cost q, and he optimally chooses the initial level of continuation value promised to the new manager, V_0.

The second boundary condition comes from the fact that if the manager's expected compensation level goes to infinity, or equivalently, his continuation value tends to the zero upper bound (since CARA utility is a negative exponential utility), he has no incentives to leave the firm, and the principal's continuation value converges to the level in the benchmark case without managerial turnover, denoted by $\bar{F}(V_t)$:

$$\lim_{V_t \to 0} \left[\bar{F}(V_t) - F(V_t) \right] = 0. \tag{1.12}$$

The optimal contract is fully characterized by the above equations and conditions.

1.3.5 Dynamics of managerial authority

Before investigating how the managerial authority evolves under the optimal contract, I first solve for the benchmark case, where the manager always stays with the firm, or in other words, the manager has full commitment. The proposition below summarizes the equilibrium results in this benchmark case.

Proposition 1.2 (Benchmark Case)
If the incumbent manager has full commitment (i.e., always stays with the firm), then for any level of his continuation value V_t,
1) the principal's policies are: $\alpha_t = \bar{\alpha} \equiv \min\{\frac{4}{r\gamma}, 1\}$, $\beta_t = \frac{\bar{\alpha}}{2}$, $w_t = -\frac{1}{\gamma}\ln(-rV_t)$;
2) the manager's policies are: $\theta_t = \bar{\alpha}$, $c_t = w_t$;
3) the principal's continuation value is: $\bar{F}(V_t) \equiv \frac{2}{r^2\gamma} + \frac{1}{r\gamma}\ln(-rV_t)$.

Proposition 1.2 shows that if the manager has full commitment, the principal will optimally delegate a constant authority level $\bar{\alpha}$, independent of the manager's continuation value. Pay-performance sensitivity would also be at a corresponding constant level.

Compared with the benchmark, the limited commitment by the manager creates an additional dynamic layer of misalignment. Furthermore, the severity of the dynamic misalignment grows as the manager's continuation value decreases, reducing optimal authority delegation. The following proposition confirms this intuition.

Proposition 1.3 (Dynamics of Managerial Authority)

1) Managerial authority α_t monotonically increases in the manager's continuation value V_t if there exists managerial turnover: $\frac{d\alpha_t}{dV_t} > 0$.

2) The upper limit of α_t is $\bar{\alpha}$, where $\bar{\alpha}$ is the authority level when the manager has full commitment (as shown in Proposition 1.2): $\lim_{V_t \to 0} \alpha_t = \bar{\alpha}$.

Part (1) of Proposition 1.3 states that, under the optimal contract, the principal delegates more authority to the manager when his continuation value is higher, i.e., the manager is dynamically better aligned with her. Part (2) shows that when the manager's continuation value tends toward the zero upper bound, or equivalently, his consumption level goes to infinity, his authority converges to the same level as in the case with full commitment (Proposition 1.2).

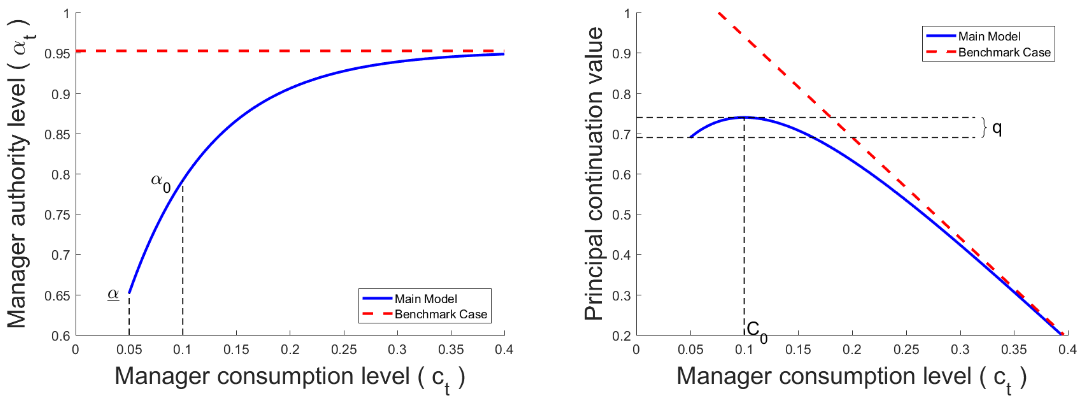


Figure 1.2: Dynamics of managerial authority

The left panel plots the manager's authority, α_t, as a function of the manager's consumption level c_t. According to Lemma 1.1, the manager's consumption is a monotonically increasing transformation of his continuation value: $c_t = -\frac{1}{\gamma}\ln(-rV_t)$. Additionally, the manager's consumption is equal to his compensation in the main model: $c_t = w_t$. Therefore, the horizontal axis also represents the manager's compensation. The right panel plots the principal's continuation value, $F(V_t)$, as a function of the manager's consumption level, c_t. The parameters are $r = 0.4$, $\gamma = 10.5$, $q = 0.05$, and $\underline{c} = 0.05$ ($\underline{V} = -1.4789$).

Figure 1.2 visualizes the optimal dynamic relationship between the manager's consumption level, the manager's authority, and the principal's continuation value. The manager's consumption level is equivalent to his wage level as there is no managerial self-dealing and no saving or borrowing in equilibrium. Moreover, according to Lemma 1.1, there is a one-to-one increasing relationship between the manager's consumption c_t and his continuation value V_t.

The left panel of Figure 1.2 provides an example of authority delegation as outlined in Proposition 1.2 and Proposition 1.3, respectively. The red dashed line represents the authority level without managerial turnover. Consistent with Proposition 1.2, the manager's authority is at a constant high level if he always stays with the firm. The blue curve illustrates how the manager's authority evolves when there is managerial turnover. It is obvious from the figure that $\bar{\alpha}$ is the highest level of authority the principal would ever delegate. This is precisely because the possibility of turnover creates an additional dynamic layer of misalignment and drives down optimal delegation. This result is consistent with the *Ally Principle* in the delegation literature, which states that the principal gives more discretion to a more aligned agent.[22]

The right panel of Figure 1.2 presents how the principal's continuation value evolves with the manager's consumption level. The principal's continuation value in the benchmark case, $\bar{F}(V_t)$, is higher than its counterpart, $F(V_t)$, in the main model, for any level of V_t. The intuition undergirding this result is simple: the principal is better off if the misalignment problem is less severe.

ANALYSIS: Degree of Misalignment

To confirm that it is indeed dynamics of misalignment that drives optimal delegation of authority, I now explicitly identify the magnitude of misalignment and analyze how the static and the dynamic components affect optimal managerial authority.

[22]See, for example, Holmstrom (1977) and Huber and Shipan (2006). Holmstrom (1977) shows that if the delegation set is a single interval, the Ally Principal holds under general conditions.

Let M_t denote the degree of misalignment at time t. It is defined as the loss in the principal's continuation value from one bad decision by the manager (i.e., one-step deviation of the manager). To obtain M_t, I take the partial derivative of the right-hand side of Equation (1.4) with respect to θ_t and normalize it by the time interval δ.

$$M_t \equiv \lim_{\delta \to 0} \frac{1}{\delta} \cdot \frac{\partial \Big[\frac{1}{1+r\delta} \cdot \Big\{ -w_t \delta + E_t[y_{t+\delta}] + E_t[F(V_{t+\delta})] \Big\} \Big]}{\partial \theta_t}$$

Simplifying this expression, we obtain

$$M_t = 2\Big[1 - \beta_t \cdot r\gamma V_t \cdot F'(V_t)\Big],$$

which demonstrates that, for any level of the pay-performance sensitivity, β_t, the degree of misalignment, M_t, monotonically decreases in the manager's continuation value, V_t.

Use M_t^{static} to denote the degree of misalignment in the benchmark case. The following equation shows that, indeed, only the static component of misalignment remains, and it is independent of V_t:

$$M_t^{static} = 2\Big[1 - \beta_t \cdot r\gamma V_t \cdot \bar{F}'(V_t)\Big] = 2[1 - \beta_t].$$

Hence, the degree of misalignment due to the manager's limited commitment is

$$M_t^{dynamic} = M_t - M_t^{static} = 2\beta_t\Big[1 - r\gamma V_t \cdot F'(V_t)\Big],$$

which monotonically decreases in the manager's continuation value, V_t.

The above analysis shows that a lower continuation value of the manager spells more severe dynamic misalignment and thus leads the principal to delegate less authority to the manager.

1.3.6 Authority-performance sensitivity

Extensive studies have been done on a manager's pay-performance sensitivity. However, the authority-performance sensitivity is much less investigated. One of the few related topics is managerial entrenchment. A long-serving manager who has moved up the corporate ladder maintains his authority or his authority may be slightly affected by the firm's bad performance, and this phenomenon is often explained by "managerial entrenchment". However, the entrenchment explanation makes it difficult to reconcile the fact that the compensation of a top manager is more equity based or option based (e.g., Edmans et al. 2017), which implies that a top manager's compensation is more sensitive to the firm's performance.

In this subsection, I apply the model to explain the puzzling phenomenon of simultaneous low authority-performance sensitivity and high pay-performance sensitivity for a manager with high authority. Moreover, I characterize the dynamics of the authority-performance sensitivity and his pay-performance sensitivity over the manager's tenure, and predict that near managerial turnover, the incumbent manager will be largely stripped of his authority after a bad performance, while his compensation, although being relatively low, is less affected.

First, let's revisit the definition of the pay-performance sensitivity, β_t, in this model. Applying Ito's lemma to the equilibrium wage expression, $w_t = -\frac{1}{\gamma}\ln(-rV_t)$, I obtain that

$$dw_t = \frac{1}{2}r^2\gamma\beta_t^2 dt + r\beta_t dZ_t. \tag{1.13}$$

Therefore,

$$\beta_t = \frac{1}{r}\cdot\frac{dw_t}{dY_t} = \frac{d(w_t/r)}{dY_t}. \tag{1.14}$$

β_t is the sensitivity of the discount-rate scaled compensation, w_t/r, to the output. A consumption stream of $\{w_t\}$ for all future periods delivers V_t to the manager.

Therefore, w_t/r is the present value of the certainty-equivalent wage that generates V_t. Hence, β_t is the sensitivity of the present value of the certainty-equivalent pay stream with respect to the firm's performance. I refer to β_t as the "pay-performance sensitivity" in this paper.

Similar to β_t, I define the authority-performance sensitivity, ψ_t, as

$$\psi_t = \frac{d\alpha_t}{dY_t}.$$

From the previous analysis, it's known that the pay-performance sensitivity decreases with misalignment. What about the authority-performance sensitivity? To better investigate this problem, I first decompose ψ_t into the product of two parts, the authority level α_t and the relative sensitivity of authority to compensation ψ_t/β_t:

$$\psi_t = \frac{1}{2} \cdot \alpha_t \cdot \frac{\psi_t}{\beta_t}, \tag{1.15}$$

and proves that ψ_t/β_t monotonically decreases in the manager's continuation value.

Proposition 1.4 *The ratio between authority-performance sensitivity and pay-performance sensitivity monotonically decreases in the manager's continuation value:* $\frac{d}{dV_t}(\frac{\psi_t}{\beta_t}) < 0$.

Proposition 1.4 predicts that the lower the manager's continuation value, the swifter are changes in the manager's authority compared to his compensation level when the firm's performance changes. That is, the principal primarily adjusts authority delegation to maximize her profits when the dynamic misalignment problem is severe, while he relies more on performance-sensitive compensation when the manager is dynamically better aligned with her. Figure 1.3 below provides two examples of this proposition. For both cases, the ratio of the manager's authority-performance sensitivity and pay-performance sensitivity quickly declines towards zero as the manager's consumption increases.

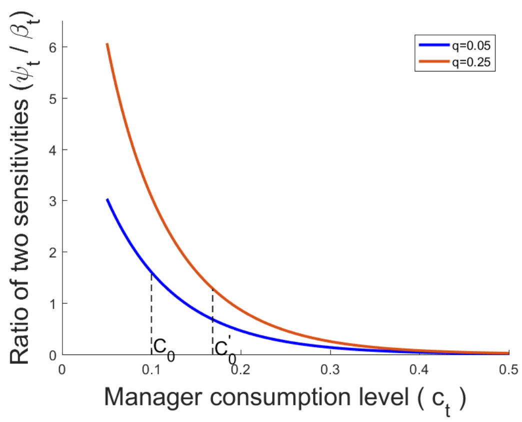


Figure 1.3: Ratio of the two sensitivities

This figure depicts the ratio of authority-performance sensitivity to pay-performance sensitivity, ψ_t/β_t. This ratio is monotonically decreasing in the manager's continuation value. According to Lemma 1.1, the manager's consumption is a monotonically increasing transformation of his continuation value: $c_t = -\frac{1}{\gamma}\ln(-rV_t)$. Additionally, the manager's consumption is equal to his compensation in the main model: $c_t = w_t$. Therefore, the horizontal axes also represent the manager's compensation. The parameters are $r = 0.4$, $\gamma = 10.5$, $\underline{c} = 0.05$ ($\underline{V} = -1.4789$), $q = 0.05$, and $q = 0.25$.

The result in Proposition 1.4 implies that when a manager is near departure, he should be stripped off a large fraction of this authority after bad performance, while his compensation level is less affected. In contrast, when the manager has a high continuation value and is less likely to leave, his authority is at a high level and should be less sensitive to the firm's performance, while at the same time, his compensation is also high but should become more sensitive to the firm's performance.

Proposition 1.3 shows that α_t increases in V_t. Proposition 1.4 states that ψ_t/β_t decreases in V_t. Therefore, the shape of ψ_t is determined by the relative strength of these two components. On the one hand, the manager's authority should be less sensitive to the firm's performance when his authority is lower, because the firm's performance is largely out of his control and has little to do with his decisions. Thus, authority-performance sensitivity decreases with misalignment. On the other hand, the principal tends to increase the relative change in authority delegation and managerial compensation when the misalignment problem deteriorates. This force drives authority-performance sensitivity to increase with misalignment. The resultant authority-performance sensitivity monotonically decreases in the manager's continuation value for a wide range of parameters. Proposition 1.5 provides a sufficient condition for authority-performance sensitivity ψ_t to be monotonically decreasing.

Proposition 1.5 *The range of recruitment costs q for the authority-performance sensitivity ψ_t to be monotonically decreasing in V_t takes a threshold form: $q \leqslant q^*$. A lower bound for the threshold q^* as a function of the parameters r and γ is given in the appendix.*

Proposition 1.5 shows that there exists a positive recruitment cost q^*, such that when $q \leqslant q^*$, the authority-performance sensitivity ψ_t monotonically decreases in the manager's continuation value, V_t. That means that if the authority-performance sensitivity monotonically decreases for a given level of recruitment cost q, it does so for any lower levels of recruitment costs $q' < q$.

The left panel of Figure 1.4 provides two examples of how the authority-performance sensitivity evolves with the manager's consumption or compensation levels. The blue curve depicts the case in which the recruitment cost is relatively small, $q = 0.05$. Then, the manager's authority-performance sensitivity is indeed monotonically decreasing with the manager's consumption levels, or equivalently the manager's continuation value. The orange curve depicts the case in which the recruitment cost is high, $q = 0.25$. In this case, the manager's authority-performance sensitivity mainly decreases in his continuation value, but increases for a narrow range of values near his departure. For both cases, the manager's authority-performance sensitivity quickly declines towards zero as his consumption increases, resembling the phenomenon of "managerial entrenchment". The right panel of Figure 1.4 depicts the principal's continuation values in these two cases.

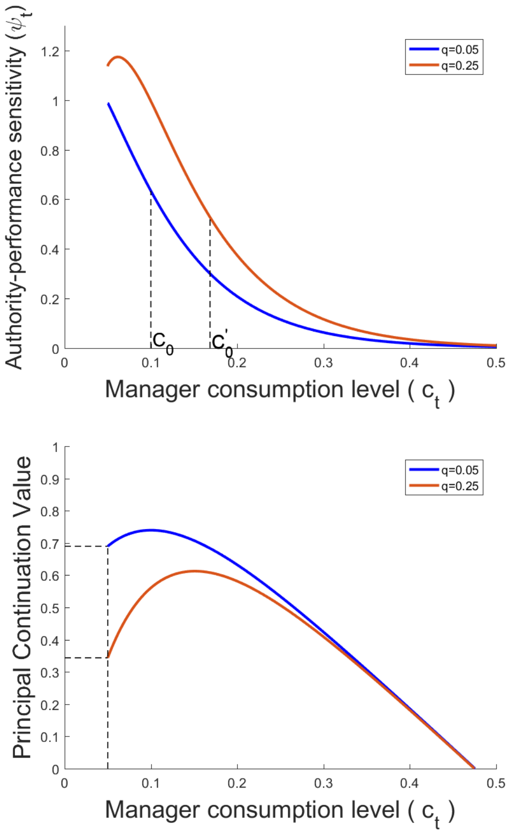


Figure 1.4: Authority-performance Sensitivity

The left panel depicts the authority-performance sensitivity as a function of the manager's consumption level, with the recruitment costs $q = 0.05$ and $q = 0.25$ respectively. The right panel depicts the principal's continuation value in these two cases. According to Lemma 1.1, the manager's consumption is a monotonically increasing transformation of his continuation value: $c_t = -\frac{1}{\gamma}\ln(-rV_t)$. Additionally, the manager's consumption is equal to his compensation in the main model: $c_t = w_t$. Therefore, the horizontal axes also represent the manager's compensation. The parameters are $r = 0.4$, $\gamma = 10.5$, $\underline{c} = 0.05$ ($\underline{V} = -1.4789$), $q = 0.05$, and $q = 0.25$.

1.3.7 Manager's initial authority

An old proverb goes "a new broom sweeps clean". Then, how much authority should be granted to a new manager? Moreover, is the manager's initial authority affected by managerial labor market conditions? And if so, how? The second question arises because empirical studies have documented that the managerial compensation outlined in executive contracts varies in accordance with labor market conditions (e.g., Bizjak et al. 2008; Brookman and Thistle 2013), while how his initial authority reacts to labor market conditions is not clear. Although people tend to focus on compensation more when signing an employment contract, the initial authority level is also an important aspect of the job and deserves attention. In this subsection, I apply the model to an analysis of the initial authority of a manager and an interaction of initial authority with managerial compensation to investigate how the managerial labor market influences both.

To begin with, the initially promised continuation value, V_0, should be within $[\underline{V}, 0)$. Hence, according to Proposition 1.3, the manager's initial authority, α_0, must satisfy $\underline{\alpha} \leqslant \alpha_0 < \bar{\alpha}$, where $\underline{\alpha}$ is an endogenously determined authority level at the point of managerial turnover, and $\bar{\alpha} = \min\{\frac{4}{r\gamma}, 1\}$, is the upper limit of the admissible managerial authority, as is defined in Proposition 1.2.

The managerial labor market influences authority and compensation through variations in the principal's recruitment costs and the manager's outside options. Before looking into its effect on the initial managerial authority, I first demonstrate how it affects the manager's expected compensation, or equivalently, the manager's initial continuation value, V_0. Lemma 1.2 states the results.

Lemma 1.2 *The manager's initial continuation value, V_0, increases in the recruitment cost, q, and the manager's outside option ,$\underline{V}$.*

The intuition behind this lemma is simple. *Ceteris paribus*, a higher recruitment cost, q, makes managerial turnover more costly to the principal and worsens the dynamic misalignment problem. Therefore, the principal is willing to give the manager more rents to align their interests and induce him to stay with the firm longer. Similarly, a better outside option also worsens the dynamic misalignment problem as the manager is more likely to leave. Hence, also in this case, the principal counteracts by promising a higher V_0.

Next, Proposition 1.6 summarizes the effects of the recruitment cost and the manager's outside option on the initial authority of a new manager.

Proposition 1.6 (Manager's Initial Authority)
The initial authority of a new manager, α_0, is independent of both the recruitment cost q, and the manager's outside option, $\underline{V}$.

Proposition 1.6 implies that α_0 is independent of managerial labor market conditions. This result is due to the canceling effect of the two opposite forces. On the one hand, a higher recruitment cost, q, or a better outside option, $\underline{V}$, aggravates the dynamic misalignment and makes the principal less willing to delegate authority. On the other hand, the principal counteracts these effects by promising a higher continuation value, V_0, to the incoming manager, and thus can delegate more authority compared to when the continuation value is at a nonreactive lower level of V_0. To put it another way, the principal optimally counteracts the impacts of the managerial labor market conditions to the extent that the initial delegated authority stays the same.

Results in this subsection help to explain the interesting phenomenon that compensation packages for managerial positions vary markedly with labor market conditions, while the initial authority granted to newly hired managers is usually unresponsive to labor market conditions.[23]

1.4. Managerial Self-dealing

Managerial self-dealing is socially inefficient and inimical to the firm's overall productivity. The previous model demonstrates that there is no self-dealing if both the manager's authority and his pay-performance sensitivity can be freely

[23] Refer to Footnote 2 in the introduction for evidence.

adjusted. However, in real-world situations, self-dealing takes place from time to time. A manager may use the company's aircrafts for private purposes, select a costly supplier to gain perks, hire employees on the grounds of friendship or kinship grounds, and so on. In this section, I analyze the reasons and occasions for managerial self-dealing to take place and delineate the managerial career trajectories more likely to lead to massive self-dealing.

1.4.1 Infrequent adjustment of authority

The previous model assumed authority delegation can be adjusted in every period. In reality, however, managerial authority generally changes less frequently than manager's pay-performance sensitivity. On the one hand, managerial authority is adjusted infrequently because of various frictions. For example, a deadlock on the board (Donaldson et al. 2020) may lead to a lag in the adjustment of a top manager's authority. Additionally, the board's composition also affects how authority is delegated to a top manager, and the board's composition changes once every several years (e.g., Adams and Ferreira 2007). Technological constraints could also limit the frequency of authority adjustments. For example, a manager often works on similar projects over some period of time, and authority adjustment is sensible only when the group of projects is completed. On the other hand, a manager's pay-performance sensitivity adjusts automatically with the firm's performance if stock options constitute part of the manager's compensation package. If the number of stocks granted to the manager changes, his pay-performance sensitivity also varies accordingly.

In this section, I assume the opportunity to change an incumbent manager's authority is exogenous and arrives at a rate of λ. Authority delegation is adjusted only when the opportunity to change the manager's authority arrives or when a new manager is hired. At all other times, managerial authority stays constant.

The principal still needs to keep her promise on the manager's continuation value each time when writing the contract. The other assumptions are the same as those made in the previous model.

I investigate the constrained-optimal behaviors of the principal and the manager in this setup. I let α_t represent the current level of managerial authority. Therefore, α_t stays constant when opportunities to adjust managerial authority have not arrived.

The principal's HJB equation becomes

$$rF(\alpha_t, V_t) = \max_{(w_t,\beta_t)} \Big\{-w_t+(2\theta_t-\alpha_t)+\frac{1}{2}\frac{\partial^2 F(\alpha_t, V_t)}{\partial V_t^2}\beta_t^2(u'(c_t))^2+\lambda[\max_{\alpha} F(\alpha, V_t)-F(\alpha_t, V_t)]\Big\}, \tag{1.16}$$

and the boundary conditions are

$$F(\alpha_t, \underline{V}) = \max_{(\alpha,V_0)} F(\alpha, V_0) - q \ ,$$
$$\lim_{V_t \to 0}[\bar{F}(\alpha_t, V_t) - F(\alpha_t, V_t)] = 0 \ .$$

The principal's HJB equation now has two state variables, α_t and V_t, and three choice variables, w_t, β_t, and α, where α represents the managerial authority level the principal would choose to delegate when the opportunity to change authority arrives. The first boundary condition suggests that the principal would reset the authority to the optimal level when hiring a new manager. The second boundary condition suggests that as V_t tends to the upper bound zero, the principal's value function approaches the level in the benchmark case without managerial turnover.

1.4.2 Results and analysis

In this section, I analyze constrained-optimal managerial authority, the corresponding pay-performance sensitivity, and the manager's behaviors in the above setting. With the results, I discover those managerial career trajectories that are more likely to lead to self-dealing. Moreover, I demonstrate that early-career luck is paramount in determining the manager's authority and lifetime utility.

Proposition 1.7 *For any $\lambda > 0$, the adjusted managerial authority level, α, increases in the manager's continuation value: $\frac{d\alpha}{dV_t} > 0$.*

Proposition 1.7 generalizes the results in Proposition 1.3 to any frequency of authority adjustment. It shows that when the manager's authority can be adjusted, the reset level is monotonically increasing with his continuation value. Figure 1.5 provides an example. The horizontal axis depicts the manager's consumption, which is a monotonically increasing function of the manager's continuation value, $c_t = -\frac{1}{\gamma}\ln(-rV_t)$. However, the manager's consumption level may be greater than his wage within that period if his authority is adjusted infrequently, because the manager may also engage in self-dealing and consume private benefits. Proposition 1.8 below provides the conditions by which managerial self-dealing ($\theta_t < \alpha_t$) occurs.

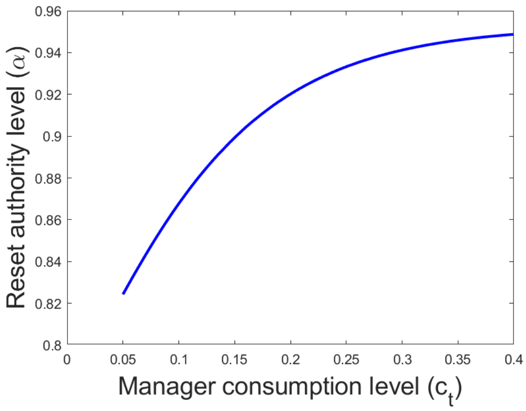


Figure 1.5: Reset levels of managerial authority, α

The figure plots the reset level of delegated authority, α, as a function of the manager's consumption level, c_t, when the opportunity to adjust managerial authority arises. According to Lemma 1.1, the manager's consumption is a monotonically increasing transformation of his continuation value: $c_t = -\frac{1}{\gamma}\ln(-rV_t)$. The parameters are $r = 0.4$, $\gamma = 10.5$, $q = 0.05$, $\underline{c} = 0.05$ ($\underline{V} = -1.4789$), and $\lambda = 2$.

Proposition 1.8 *There exists an authority level $\alpha^* \in [0, \bar{\alpha}]$, such that*

(1) if $\alpha_t \leqslant \alpha^*$, $\forall\ V_t \in [\underline{V}, 0)$, $\beta_t = \frac{\alpha_t}{2}$, and therefore, $\theta_t = \alpha_t$;

(2) if $\alpha_t > \alpha^*$, there exists a continuation value level $V^*(\alpha_t)$, such that

(2.1) if $V_t \geqslant V^*(\alpha_t)$, $\beta_t = \frac{\alpha_t}{2}$, and therefore, $\theta_t = \alpha_t$;

(2.2) if $V_t < V^*(\alpha_t)$, $\beta_t < \frac{\alpha_t}{2}$, and therefore, $\theta_t < \alpha_t$.

Proposition 1.8 demonstrates that a necessary condition for managerial self-dealing is that the manager possesses a relatively high level of authority ($\alpha_t > \alpha^*$). With a high level of authority, the manager engages in self-dealing when his continuation value is relatively low. One thing worth noting is that the principal can always eliminate managerial self-dealing by making the manager's compensation sufficiently sensitive to the firm's output, or more concretely, by setting $\beta_t = \alpha_t/2$. However, in certain situations, acquiescing to managerial self-dealing by providing a less-sensitive compensation is a superior strategy for the principal. This is because providing highly sensitive compensation may lead to frequent and costly managerial turnover. Therefore, by allowing managerial self-dealing, in the essence, the principal is making use of private benefits from the manager's self-dealing as a cheaper alternative to managerial compensation, even at the cost of overall firm efficiency.

Figure 1.6 provides a numerical example of Proposition 1.8. The flat surface represents the region of no managerial self-dealing in equilibrium, where $\alpha_t = 2\beta_t = \theta_t$. The curved surface is the region with self-dealing, where $\alpha_t > 2\beta_t = \theta_t$. The manager may engage in self-dealing when his authority level is higher than a certain level, α^*. At a particular authority level, $\alpha_t > \alpha^*$, self-dealing takes place if the manager's consumption level is relatively low, or equivalently if his continuation value is relatively low.

Together, Propositions 1.7 and 1.8 predict a time-series feature of a manager's authority and self-dealing. A manager who has performed well in the past will be

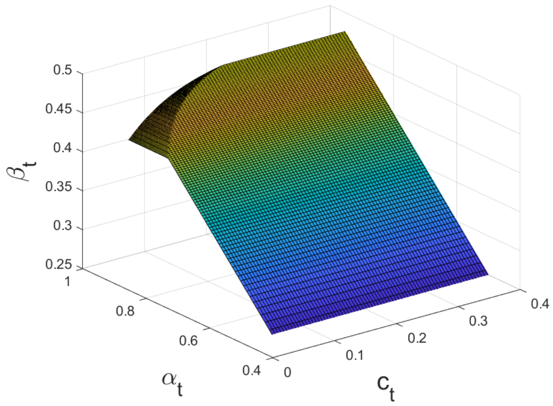


Figure 1.6: Pay-performance sensitivity and managerial self-dealing

The figure depicts the optimal choice of pay-performance sensitivity, β_t, as a function of the current authority, a_t, and consumption, c_t. The current authority, a_t, is a state variable and cannot be changed when the adjustment opportunity does not arise. Consumption, c_t, is a monotonically increasing transformation of the state variable, V_t: $c_t = -\frac{1}{\gamma}\ln(-rV_t)$, according to Lemma 1.1. The curved surface of this figure represents the region of managerial self-dealing, where $\beta_t = \theta_t/2 < \alpha_t/2$. The flat surface represents the region with no managerial self-dealing, where $\beta_t = \theta_t/2 = \alpha_t/2$. The parameters are $r = 0.4$, $\gamma = 10.5$, $q = 0.05$, $\underline{c} = 0.05$ ($\underline{V} = -1.4789$), and $\lambda = 2$.

conferred a high level of authority when there is an opportunity to do so. The manager will not abuse his authority if he continues to do well and is well-aligned with the principal. If, instead, the manager becomes unlucky and his performance trends downward, so that the misalignment problem becomes more severe and he is more likely to leave the firm, the manager may engage in self-dealing before his authority is adjusted downward. In summary, a manager who has a high level of authority but grim career prospects tends to engage in self-dealing.

1.4.3 Early luck and managerial self-dealing

With the above analysis, I can demonstrate that if managerial authority can be adjusted only infrequently, early-career luck plays a disproportionate role in determining the manager's lifetime authority and utility. Besides, I can delineate the career trajectories that lead to massive managerial self-dealing.

To illustrate the role of early-career luck, I consider the following two opposite career trajectories, which are summarized as "*two fates of a manager*". If a manager experiences a series of good realizations in the early stages of his career, he becomes better aligned with the firm and will be granted more authority when the opportunity to delegate more authority arrives. Thereafter, the manager will have more discretion in making decisions, generating more profits for the firm and gaining higher compensation for himself. Later in his career, if he suffers from negative shocks, he can take advantage of the high level of authority and engage in self-dealing, keeping a high lifetime utility. In contrast, if the manager encounters negative shocks in the early stage of his career, he will be stripped of part of his authority when there is an opportunity, since his continuation value becomes lower, and the misalignment problem becomes more severe. Thereafter, he gets stuck in a low-authority situation even if he later experiences positive shocks and becomes better aligned with the principal. He cannot well exploit his superior knowledge because his authority is restricted. Consequently, his lifetime utility is lower.

Figure 1.7 provides simulated career trajectories of two managers. The manager with early-career good luck is represented in red, and the manager with early-career bad luck is represented in blue. The top-left panel shows their authority levels over time. They start with the same level of authority. Then, the authority levels are adjusted in opposite directions after experiencing good luck and bad luck, respectively. The firm's cumulative output and the managerial consumption are higher for the manager with early luck, and the comparative advantage persists even after the aggregate shocks they experience converge ($t = 1$ in the figure). These results exhibit the disproportionate role of early-career luck on the firm's overall performance and the manager's lifetime utility.

The last panel of Figure 1.7 depicts the two manager's engagement with self-dealing and the levels of private benefits they obtain. Consistent with the analysis in the previous subsection, a manager who has a high level of authority

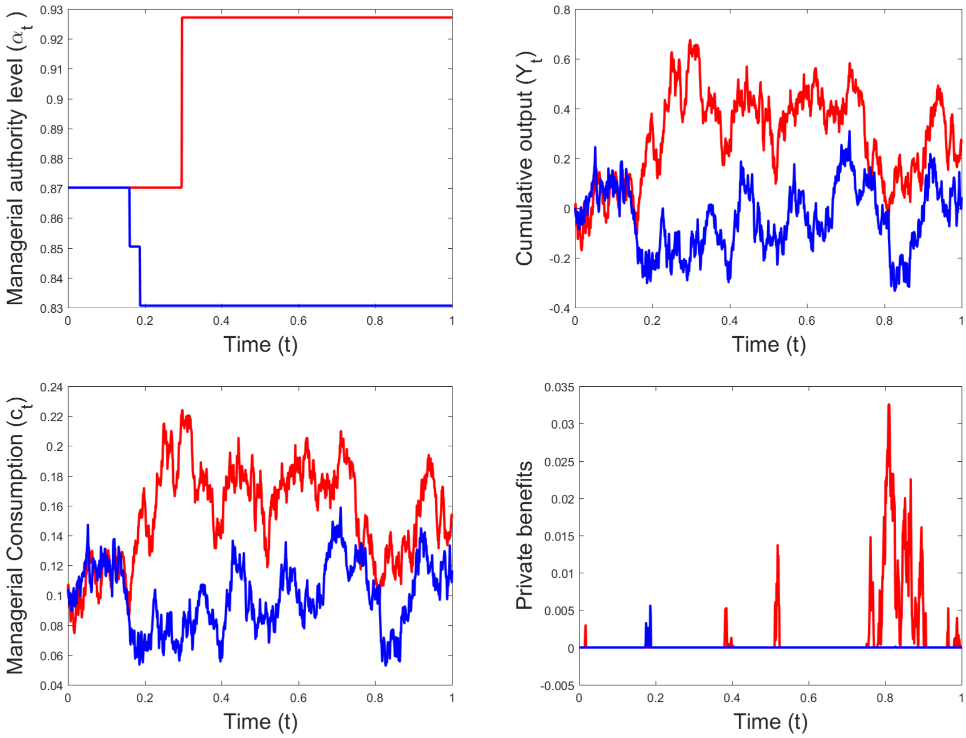


Figure 1.7: Two fates of a manager

The figure provides a simulated example of the "two fates of a manager" story. The red curves represent a manager with good luck in his early career; this manager encounters bad luck later. The blue curves represent a manager with bad luck in his early career; this manager experiences good luck later. To make a good comparison, I normalize the aggregate level of shocks for either of them to zero. The parameters for this simulation are $r = 0.4$, $\gamma = 10.5$, $q = 0.05$, $\underline{c} = 0.05$ ($\underline{V} = -1.4789$), and $\lambda = 2$.

but experiences bad luck and thus a grim prospect in the firm tends to engage in large-scale self-dealing. Combining this with the early-career experiences, I delineate the career trajectories that lead to massive self-dealing: a manager who experiences good luck in the early stages of his career and bad luck in the later years of his career is more likely to engage in massive self-dealing when he still holds a high level of authority but has a grim future with the firm.

1.5. Conclusion

In this paper, I develop a dynamic multi-task delegation model to analyze optimal managerial authority and its dynamic interaction with managerial compensation. With parsimonious assumptions, the model generates rich results and characterizes initial values and the dynamics of optimal managerial authority and consumption, which are consistent with real-world observations.

The model shows that when hiring a manager, the principle's delegation of authority is unresponsive to either the manager's outside options or the firm's recruitment costs, in contrast to promised compensation, which increases in both. Over time, both the manager's authority and his compensation rise after good performance and decline after bad realizations. Authority-performance sensitivity decreases as the manager's authority grows, resembling entrenchment. In contrast, pay-performance sensitivity increases with the manager's authority, consistent with the fact that firms grant more stocks or stock options to top managers. By exploiting the infrequent adjustment of authority, the model sheds light on managerial self-dealing and the impact of early-career luck: early luck plays a disproportionate role in the manager's career and lifetime utility, and a manager who experiences good luck in the early stages and bad luck in the later years of his career is more likely to engage in massive self-dealing.

The driving force of the model is the dynamic misalignment effect resulting from costly managerial turnover. Because of his limited commitment, the manager leaves the firm if past performances are bad and his continuation value to the firm drops below his outside options, as is consistent with empirical findings (e.g., Jenter and Lewellen 2020). Therefore, firm owners trade off among the projects' probabilities of success, the performance-sensitive compensation paid to the incumbent manager, and the cost from potential managerial turnover. Good past performance lowers the probability of managerial turnover, making the manager dynamically better aligned with the firm, and thus invites higher pay-performance sensitivity and more delegated authority. In the model, I have abstracted from unobservable skill differences and focused on the moral hazard problem. I also exclude the wealth effect by assuming CARA preference and the timing distortion of managerial compensation by allowing private savings and borrowing to isolate the dynamic misalignment effect. I explicitly identify the degree of misalignment and decompose it into dynamic and static components to aid the analysis.

The relatively simple structure of this model leaves several directions for future research. For instance, incorporating the search and matching model to provide a general equilibrium analysis on the effects of the managerial labor market would be desirable. Additionally, it might also be worth studying the dynamic trade-off between authority and costly efforts by considering a "power-hungry" manager with costly decision-making processes. Furthermore, the model can also be generalized to study the multiple hierarchical structure and the equilibrium authority distribution of a firm. I leave the full development of a richer model of this sort for future research.

1.6. Appendix A.1

CARA Preference and the First-order Approach

The analysis closely follows that of Lemma 3 and Section 2.3.3 in He (2011).

Consider a deviating manager with savings S who faces the contract $(\alpha_t,w_t,\beta_t;\theta_t,c_t)$. The principal is unaware of this deviation. Therefore, there exists a gap between the principal's promise V_t and the deviating manager's actual continuation value. Let $\widehat{V_t}(S,V_t)$ denote this actual continuation value. Then

$$\widehat{V_t}(S,V_t) = V_t \cdot e^{-r\gamma S}.$$

For a CARA agent without wealth effect,given the private savings S, his new optimal policy is to take the optimal decision-consumption policy without savings but to consume an extra rS more for all future dates $s \geqslant t$. $u(\theta_s, c_s + rS) = e^{-r\gamma S}u(\theta_s, c_s)$ explains the factor $e^{-r\gamma S}$.

Actual continuation value $\widehat{V_t}$ is hidden from the principal's view, and the authority and pay-performance sensitivity decisions only depend on the principal's promise V_t. Departure happens when the principal's promise V_t hits $\underline{V}$. I use τ to represent this departure time.

Let $\{\widehat{\theta_s}\}$, $s \in [t,\tau]$ represent the manager's optimal decision choice and $\{\widehat{c_s}\}$, $s \in [t,\infty)$ represent the optimal consumption choice with this private saving level S. $\{\theta_s\}$ and $\{c_s + rS\}$ also represent a feasible decision-consumption choice for this optimization problem and therefore,

$$\begin{aligned}
\widehat{V_t} &= \mathrm{E}_t\left[\sum_{n=0}^{\infty} \frac{1}{(1+r\delta)^{(n+1)}} u(\widehat{c_{t+n\delta}})\delta\right] \\
&\geqslant \mathrm{E}_t\left[\sum_{n=0}^{\infty} \frac{1}{(1+r\delta)^{(n+1)}} u(c_{t+n\delta} + rS)\delta\right] = V_t \cdot e^{-r\gamma S}.
\end{aligned}$$

Similarly, $\{\widehat{\theta}_s\}$ and $\{\widehat{c}_s - rS\}$ represent a feasible decision-consumption choice for the optimization problem without private saving. Thus,

$$
\begin{aligned}
V_t &= \mathrm{E}_t\left[\sum_{n=0}^{\infty} \frac{1}{(1+r\delta)^{(n+1)}} u(c_{t+n\delta})\delta\right] \\
&\geqslant \mathrm{E}_t\left[\sum_{n=0}^{\infty} \frac{1}{(1+r\delta)^{(n+1)}} u(\widehat{c_{t+n\delta}} - rS)\delta\right] = \widehat{V_t} \cdot e^{r\gamma S}
\end{aligned}
$$

The above two inequalities complete the proof $\widehat{V_t} = V_t \cdot e^{-r\gamma S}$ and show that $\widehat{\theta}_s = \theta_s$, $\widehat{c_s} = c_s + rS$. Optimal decision is not affected by the presence of private savings, while optimal consumption is higher by rS at all dates.

Now we return to the manager's consumption choice problem. The marginal utility from consumption must be equal to the marginal value of hidden wealth

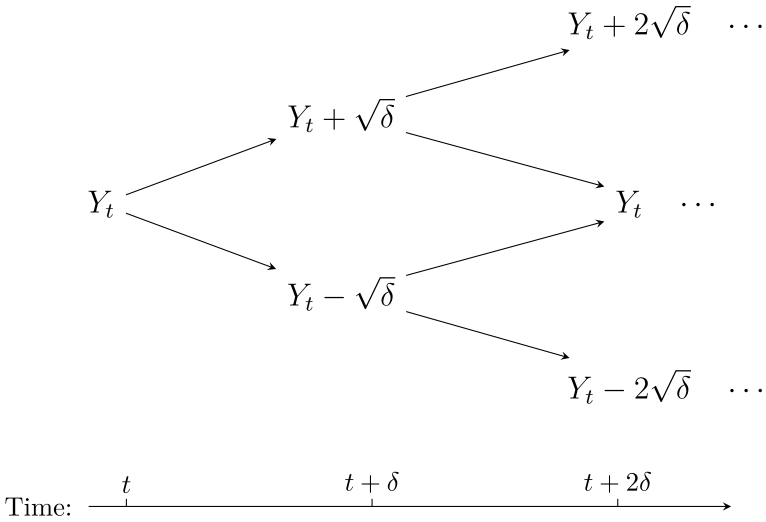
$$
\begin{aligned}
u'(c_t) &= \frac{\partial}{\partial S}\widehat{V_t}(S, V_t) = -r\gamma V_t \\
c_t &= -\frac{1}{\gamma}\ln(-rV_t)
\end{aligned}
$$

Hence, the first-order approach applies in this setup.

1.7. Appendix A.2

Continuous-time Limit of the Discrete-time Model

To begin with, we derive the continuous-time dynamics of the firm's cash flows. Let Y_t denote the cumulative cash flows till time t. From (1.1) we find that the dynamics of Y_t forms a binomial tree:



Use dY_t to denote the increment of the cumulative cash flows at time t: $dY_t = Y_{t+\delta} - Y_t$. dY_t follows a two-point distribution with mean $(2\theta_t - \alpha_t)\delta$ and variance $\delta + O(\delta^2)$, $\forall t \geqslant 0$.
Define $dZ_t = dY_t - E_t(dY_t) = dY_t - (2\theta_t - \alpha_t)\delta$. It has mean 0 and variance $\delta + O(\delta^2)$. Following Martingale central limit theorem (reference: Hall and Heyde (2014)), $\{Z_t\}_{t\geqslant 0}$ converges in distribution to the Wiener process (standard Brownian motion).

Therefore, the stochastic process of cash flows in continuous time becomes:

$$dY_t = (2\theta_t - \alpha_t)dt + dZ_t \ ,$$

where $\{Z_t\}_{t\geqslant 0}$ is a standard Brownian motion.

To get the continuous-time dynamics of the manager's continuation value, first combine Equation (1.2) and (1.3):

$$\begin{cases} V_t = \dfrac{1}{1+r\delta} \cdot \Big\{ u(c_t)\delta + [Pr(y_H) \cdot V_{t+\delta}(y_H) + Pr(y_L) \cdot V_{t+\delta}(y_L)] \Big\} \\ \\ \beta_t = \dfrac{(V_{t+\delta}(y_H) - V_{t+\delta}(y_L))/u'(c_t)}{y_H - y_L} \end{cases}$$

where

$$Pr(y_H) = \frac{1}{2}[1 + (2\theta_t - \alpha_t)\sqrt{\delta}] \ , \ Pr(y_L) = \frac{1}{2}[1 - (2\theta_t - \alpha_t)\sqrt{\delta}] \ , \ y_H = \sqrt{\delta} \ , \ y_L = -\sqrt{\delta}.$$

We get:

$$\begin{cases} V^H_{t+\delta} \equiv V_{t+\delta}(y_H) = (1+r\delta)V_t - u(c_t)\delta - (2\theta_t - \alpha_t)\beta_t u'(c_t)\delta + \beta_t u'(c_t)\sqrt{\delta} \\ \\ V^L_{t+\delta} \equiv V_{t+\delta}(y_L) = (1+r\delta)V_t - u(c_t)\delta - (2\theta_t - \alpha_t)\beta_t u'(c_t)\delta - \beta_t u'(c_t)\sqrt{\delta} \end{cases}$$

$$\Rightarrow \quad \begin{cases} V^H_{t+\delta} - V_t = [rV_t - u(c_t)]\delta + \beta_t u'(c_t)[y_H - (2\theta_t - \alpha_t)\delta] \\ \\ V^L_{t+\delta} - V_t = [rV_t - u(c_t)]\delta + \beta_t u'(c_t)[y_L - (2\theta_t - \alpha_t)\delta] \end{cases}$$

When $\delta \to 0$, the process for the manager's continuation value becomes:

$$dV_t = (rV_t - u(c_t))dt + \beta_t u'(c_t)dZ_t \ .$$

To derive the continuous-time version of the principal's problem, we first apply

Taylor-Young approximation to $F(V_{t+\delta}^H)$ and $F(V_{t+\delta}^L)$, and get

$$\begin{cases} F(V_{t+\delta}^H) = F(V_t) + F'(V_t)\cdot[rV_t - u(c_t) - (2\theta_t - \alpha_t)\beta_t u'(c_t)]\delta + \frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\delta \\ \qquad\qquad + F'(V_t)\cdot\beta_t u'(c_t)\sqrt{\delta} + o(\delta)\ ; \\ F(V_{t+\delta}^L) = F(V_t) + F'(V_t)\cdot[rV_t - u(c_t) - (2\theta_t - \alpha_t)\beta_t u'(c_t)]\delta + \frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\delta \\ \qquad\qquad - F'(V_t)\cdot\beta_t u'(c_t)\sqrt{\delta} + o(\delta)\ . \end{cases} \tag{A-1}$$

Plugging into the principal's problem expressed in Equation (1.4):

$$F(V_t) = \max_{(\alpha_t,w_t,\beta_t)} \frac{1}{1+r\delta}\cdot\{-w_t\delta+(2\theta_t-\alpha_t)\delta+F(V_t)+F'(V_t)[rV_t-u(c_t)]\delta+\frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\delta+o(\delta)\}$$

Let $\delta \to 0$, we get the principal's Hamilton-Jacobian-Bellman (HJB) equation:

$$rF(V_t) = \max_{(\alpha_t,w_t,\beta_t)} \{-w_t + (2\theta_t - \alpha_t) + F'(V_t)\cdot[rV_t - u(c_t)] + \frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\}\ .$$

1.8. Appendix B

Proof of Lemma 1.1

The manager maximizes his continuation value by choosing θ_t, taken the contract as given. As is proven in Appendix A.1, CARA preference prevents double deviation. Therefore, I do not need to consider the joint deviation of θ_t and c_t.

$$\begin{aligned} V_t =& \frac{1}{1+r\delta}\cdot\{u(c_t)\delta + [Pr(y_H)\cdot V_{t+\delta}(y_H) + Pr(y_L)\cdot V_{t+\delta}(y_L)]\} \\ =& \frac{1}{1+r\delta}\cdot\{u(w_t + \frac{\alpha_t^2-\theta_t^2}{2})\delta + \frac{1+(2\theta_t-\alpha_t)\sqrt{\delta}}{2}\cdot V_{t+\delta}(y_H) + \frac{1-(2\theta_t-\alpha_t)\sqrt{\delta}}{2}\cdot V_{t+\delta}(y_L)\}, \end{aligned}$$

where the first term of the second equality comes from the fact that the manager consumes his wage and private benefits in the no-saving equilibrium:

$$c_t = w_t + \int_{\theta_t}^{\alpha_t} i di = w_t + \frac{\alpha_t^2 - \theta_t^2}{2}.$$

Now, take F.O.C. of V_t with respect to θ_t and we get:

$$\begin{aligned}
& u'(c_t) \cdot (-\theta_t)\delta + \sqrt{\delta} \cdot [V_{t+\delta}(y_H) - V_{t+\delta}(y_L)] = 0 \\
\Rightarrow \quad & u'(c_t) \cdot (-\theta_t)\delta + \sqrt{\delta} \cdot \beta_t u'(c_t) \cdot 2\sqrt{\delta} = 0 \\
\Rightarrow \quad & \theta_t = 2\beta_t.
\end{aligned}$$

The above equation gives the interior solution of θ_t. If $2\beta_t > \alpha_t$, θ_t takes the upper bound value α_t. Therefore, $\theta_t = \min\{2\beta_t,\ \alpha_t\}$.

To find the manager's optimal consumption decision, we take a short cut and apply the Euler equation. (More through proof could be found in He (2011).)

With efficient private savings and borrowing technology, the manager can smooth his consumption intertemporally. Therefore, the Euler equation holds:

$$u'(c_t) = \mathrm{E}_t[u'(c_\tau)],\ \forall \tau > t$$

Apply the Euler equation to the manager's CARA utility, we find that

$$u(c_t) = \mathrm{E}_t[u(c_\tau)],\ \forall \tau > t.$$

Then,

$$\begin{aligned}
V_t &= \mathrm{E}_t\left[\sum_{n=0}^{\infty}\frac{1}{(1+r\delta)^{(n+1)}}u(c_{t+n\delta})\delta\right] \\
&= \sum_{n=0}^{\infty}\frac{1}{(1+r\delta)^{(n+1)}}u(c_t)\delta \\
&= \frac{u(c_t)}{r}.
\end{aligned}$$

Therefore, $c_t = -\frac{1}{\gamma}\ln(-rV_t),\ \forall t \geqslant 0.$ ■

Proof of Proposition 1.1

At time t, the principal's dynamic optimization problem is summarized in the HJB equation (1.7):

$$rF(V_t) = \max_{(\alpha_t, w_t, \beta_t)}\{-w_t + (2\theta_t - \alpha_t) + F'(V_t)\cdot[rV_t - u(c_t)] + \frac{1}{2}F''(V_t)\beta_t^2[u'(c_t)]^2\},$$

where the wage level w_t satisfies that there is no saving or borrowing in equilibrium by the manager:

$$w_t = c_t - \int_{\theta_t}^{\alpha_t} i di = c_t - \frac{\alpha_t^2 - \theta_t^2}{2},$$

and the manager's decision θ_t satisfies (according to Lemma 1.1)

$$\theta_t = \min\{2\beta_t,\ \alpha_t\}.$$

When $\alpha_t \in [0, 2\beta_t)$, the RHS of Equation (1.7) increases with α_t; when $\alpha_t \in [2\beta_t, 1]$, the RHS of Equation (1.7) decreases with α_t. Therefore, $\alpha_t = 2\beta_t$ maximizes Equation (1.7).

Together with Lemma 1.1, we get $\alpha_t = 2\beta_t = \theta_t$. ■

Proof of Proposition 1.2

Let $\bar{F}(V_t)$ represents the principal's continuation value at t if there is no potential manager departure. By definition,

$$\bar{F}(V_t) = \max_{\{\alpha_\tau, w_\tau, \beta_\tau\}_{\tau \geqslant t}} \mathrm{E}_t \left[\int_t^\infty e^{-r(\tau - t)} (2\theta_\tau - \alpha_\tau - w_\tau) d\tau \right].$$

According to Proposition 1.1, $\theta_t = \alpha_t$, and there is no private benefits in equilibrium. Thus, $w_t = c_t$, $\forall t \geqslant 0$, since the contract implies no saving.

Also, applying Ito's lemma to the equation $c_t = -\frac{1}{\gamma} \ln(-rV_t)$ in Lemma 1.1, we get $dc_t = \frac{1}{8} r^2 \gamma \alpha_t^2 dt + \frac{1}{2} r \alpha_t dZ_t$. Thus, $E_t[c_\tau] = c_t + E_t[\int_t^\tau \frac{1}{8} r^2 \gamma \alpha_s^2 ds]$.

Applying the above results into the expression of $\bar{F}(V_t)$, we find:

$$\begin{aligned}
\bar{F}(V_t) &= \max_{\{\alpha_\tau, c_\tau\}_{\tau \geqslant t}} \mathrm{E}_t [\int_t^\infty e^{-r(\tau - t)} (\alpha_\tau - c_\tau) d\tau] \\
&= \max_{\{\alpha_\tau, c_\tau\}_{\tau \geqslant t}} \mathrm{E}_t [\int_t^\infty e^{-r(\tau - t)} (\alpha_\tau - c_t - \int_t^\tau \frac{1}{8} r^2 \gamma \alpha_s^2 ds) d\tau] \\
&= \max_{\{\alpha_\tau, c_\tau\}_{\tau \geqslant t}} -\frac{c_t}{r} + \mathrm{E}_t [\int_t^\infty e^{-r(\tau - t)} (\alpha_\tau - \frac{1}{8} r \gamma \alpha_\tau^2) d\tau],
\end{aligned}$$

The problem now becomes an intra period maximization problem. It's obvious that we should set

$$\alpha_t \equiv \bar{\alpha} = \min\{\frac{4}{r\gamma}, 1\}.$$

Then,

$$\bar{F}(V_t) = \frac{2}{r^2 \gamma} - \frac{c_t}{r} = \frac{2}{r^2 \gamma} + \frac{1}{r\gamma} \ln(-rV_t), \ \forall t.$$

Also, the transversality condition $\lim_{\tau \to \infty} e^{-r(\tau - t)} \mathrm{E}_t [\bar{F}(V_\tau)] = 0$ is satisfied.■

Proof of Proposition 1.3

We've already got $\alpha_t = -\frac{4}{(r\gamma V_t)^2 F''(V_t)}$. So, the ODE could be rewritten as:

$$\alpha_t = 2rF(V_t) - \frac{2}{\gamma}\ln(-rV_t).$$

Differentiating with respect to V_t,

$$\frac{d\alpha_t}{dV_t} = 2rF'(V_t) - \frac{2}{\gamma V_t}$$

Remember that we have defined $\bar{F}(V_t)$ in Section 1.3.4. If we can prove that $F(V_t) < \bar{F}(V_t)$, $F'(V_t) > \bar{F}'(V_t)$, and $F''(V_t) < \bar{F}''(V_t)$, then

$$\frac{1}{2}\frac{d\alpha_t}{dV_t} > r\bar{F}'(V_t) - \frac{1}{\gamma V_t} = \frac{1}{\gamma V_t} - \frac{1}{\gamma V_t} = 0 \ ,$$

and

$$\alpha_t = -\frac{4}{(r\gamma V_t)^2 F''(V_t)} < -\frac{4}{(r\gamma V_t)^2 \bar{F}''(V_t)} = \bar{\alpha}_t \ .$$

We now prove that $F(V_t) < \bar{F}(V_t)$, $F'(V_t) > \bar{F}'(V_t)$, and $F''(V_t) < \bar{F}''(V_t)$.

Economic arguments dictate that $F(V_t) < \bar{F}(V_t)$. Intuitively, the principal's continuation value tend to be lower when the agent has an option to quit compared to the case where the agent does not have this option.

Mathematically, for any path of consumptions, the principal's obligation c_τ jumps upwards from $\underline{c} = -\ln(-r\underline{V})/\gamma$ to $c_0 = -\ln(-rV_0)/\gamma$ when the quit happens. As a result,

$$\mathrm{E}_t[c_\tau] > c_t + \int_t^\tau \frac{1}{8} r^2 \gamma \alpha_s^2 ds \ .$$

Thus,

$$
\begin{aligned}
F(V_t) &= \mathrm{E}_t\left[\int_t^\infty e^{-r(\tau-t)}(\alpha_\tau - c_\tau)d\tau\right] \\
&< \mathrm{E}_t\left[\int_t^\infty e^{-r(\tau-t)}(\alpha_\tau - c_t - \int_t^\tau \frac{1}{8}r^2\gamma\alpha_s^2 ds)d\tau\right] \\
&= -\frac{c_t}{r} + \mathrm{E}_t\left[\int_t^\infty e^{-r(\tau-t)}\left(\alpha_\tau - \frac{1}{8}r\gamma\alpha_\tau^2\right)d\tau\right] \\
&\leqslant -\frac{c_t}{r} + \frac{2}{r^2\gamma} \\
&= \bar{F}(V_t)
\end{aligned}
$$

On the other hand, $F(V_t) > \bar{F}(V_t) - 2/r^2\gamma$. The principal achieves the latter continuation value by setting all future consumption levels constant $c \equiv -\ln(-rV_t)/\gamma$ and zero power $\alpha \equiv 0$, which is obviously sub-optimal.

$$
\begin{aligned}
F(V_t) &= \mathrm{E}_t\left[\int_t^\infty e^{-r(\tau-t)}(\alpha_\tau - c_\tau)d\tau\right] \\
&> \mathrm{E}_t\left[\int_t^\infty e^{-r(\tau-t)}\ln(-rV_t)/\gamma d\tau\right] \\
&= \frac{1}{r\gamma}\ln(-rV_t) \\
&= \bar{F}(V_t) - 2/r^2\gamma
\end{aligned}
$$

The ODE could be rewritten as

$$
F''(V_t) = -\frac{2}{(r\gamma V_t)^2}\cdot\frac{1}{rF(V_t) - \ln(-rV_t)/\gamma}\ .
$$

Define $G(V_t) = \bar{F}(V_t) - F(V_t)$. From the above analysis, $0 < G(V_t) < 2/r^2\gamma$, $\forall V_t$.

$$
\begin{aligned}
G''(V_t) &= \bar{F}''(V_t) - F''(V_t) \\
&= -\frac{1}{r\gamma V_t^2} + \frac{2}{(r\gamma V_t)^2}\cdot\frac{1}{2/r\gamma - rG(V_t)} \\
&= \frac{1}{V_t^2}\cdot\frac{rG(V_t)}{2 - r^2\gamma G(V_t)} \\
&> 0\ .
\end{aligned}
$$

The limit of $G(V_t)$ as $V_t \to 0$ must be equal to 0. If this limit is not equal to zero, because $G''(V_t)$ is proportional to V_t^{-2}, we would have $G(V_t)$ unbounded after integration, which is contradictory to the previous analysis.

Since $G(V_t) > 0$ for any $V_t < 0$,

$$\lim_{V_t \to 0} G'(V_t) \leqslant 0 \ .$$

And because $G(V_t)$ is convex, $G'(V_t)$ is non-decreasing. Therefore, $G'(V_t) < 0$, $\forall V_t$. This completes the proof.■

Proof of Proposition 1.4

According to (1.10), $\alpha_t = 2rF(V_t) - \frac{2}{\gamma}\ln(-rV_t) = 2rF(V_t) + 2c_t$. Then, $\frac{d\alpha_t}{dw_t} = \frac{d\alpha_t}{dc_t} = 2rF'(V_t)\frac{dV_t}{dc_t} + 2 = 2F'(V_t)u'(c_t) + 2 = 2F'(V_t)(-r\gamma V_t) + 2$, since we already know that $w_t = c_t$, $V_t = \frac{u(c_t)}{r}$, and $u'(c_t) = -r\gamma V_t$. Then,

$$\frac{d}{dV_t}(\frac{d\alpha_t}{dw_t}) = 2F''(V_t)(-r\gamma V_t) - 2r\gamma F'(V_t).$$

Besides, we know that

$$\frac{d}{dV_t}(\frac{d\bar{\alpha}}{dw_t}) = 2\bar{F}''(V_t)(-r\gamma V_t) - 2r\gamma\bar{F}'(V_t) = 0.$$

Therefore,

$$\frac{d}{dV_t}(\frac{d\alpha_t}{dw_t}) = 2(-r\gamma V_t)[F''(V_t) - \bar{F}''(V_t)] - 2r\gamma[F'(V_t) - \bar{F}'(V_t)] < 0,$$

because we have shown that $F''(V_t) < \bar{F}''(V_t)$ and $F'(V_t) > \bar{F}'(V_t)$ in the proof of Proposition 1.3. ■

Proof of Proposition 1.5

The authority-performance sensitivity is given by

$$\begin{aligned}\psi_t &= \frac{1}{2}\alpha_t \cdot \frac{\psi_t}{\beta_t} \\ &= \alpha_t \cdot (1 - r\gamma V_t F'(V_t))\end{aligned}$$

Differentiating with respect to V_t

$$\begin{aligned}\frac{d\psi_t}{dV_t} &= \frac{d\alpha_t}{dV_t}(1 - r\gamma V_t F'(V_t)) + \alpha_t \cdot (-r\gamma F'(V_t) - r\gamma V_t F''(V_t)) \\ &= (2rF'(V_t) - \frac{2}{\gamma V_t})(1 - r\gamma V_t F'(V_t)) - r\gamma\alpha_t F'(V_t) - r\gamma V_t \alpha_t F''(V_t) \\ &= \frac{1}{-\gamma V_t}\Big[2(1 - r\gamma V_t F'(V_t))^2 - \gamma\alpha_t(-r\gamma V_t F'(V_t)) - r\gamma^2 V_t^2 \alpha_t F''(V_t)\Big]\end{aligned}$$

Substituting in $\alpha_t = -\dfrac{4}{r^2\gamma^2 V_t^2 F''(V_t)}$,

$$\frac{d\psi_t}{dV_t} = \frac{1}{-\gamma V_t}\Big[2(1 - r\gamma V_t F'(V_t))^2 - \gamma\alpha_t(-r\gamma V_t F'(V_t)) - \frac{4}{r}\Big]$$

ψ_t is decreasing in V_t if and only if

$$2(1 - r\gamma V_t F'(V_t))^2 - \gamma\alpha_t(-r\gamma V_t F'(V_t)) - \frac{4}{r} < 0 \tag{A-2}$$

The expression is a quadratic function of $-r\gamma V_t F'(V_t)$ and is easy to see that a sufficient condition for $d\psi_t/d\alpha_t < 0$ is

$$-1 < -r\gamma V_t F'(V_t) < \sqrt{\frac{2}{r}} - 1$$

If the authority-performance sensitivity monotonically decreased for a given level of recruitment cost q_A, so does it for any lower levels of recruitment costs $q_B < q_A$. The proof of this statement uses equation (A-6) in the proof of Proposition 1.6. Differentiating

(A-6) with respect to V_t,

$$F'_B(V_t) = \frac{V_0^A}{V_0^B} F'_A(V_t)$$
$$-r\gamma V_t F'_B(V_t) = -r\gamma \frac{V_0^A}{V_0^B} V_t \cdot F'_A\left(\frac{V_0^A}{V_0^B} V_t\right)$$

$-r\gamma V_t F'_A(V_t)$ satisfies equation (A-2) for $V_t \in [\underline{V}, 0)$ and in particular for the sub-interval $V_t \in [V_0^A/V_0^B \cdot \underline{V}, 0)$. Therefore $-r\gamma V_t F'_B(V_t)$ satisfies equation (A-2) for the entire range of V_t, $[\underline{V}, 0)$, and ψ_t is monotonically decreasing in V_t for the same range.

The previous section proves that the range of q for ψ_t to be monotonically decreasing in V_t takes a threshold form: $q \leqslant q^*$. Next we provide a lower bound for the threshold q^* as a function of parameters r and γ.

First observe that $F''(V_t) < \overline{F}''(V_t)$ and for $V_t < V_0$,

$$\begin{aligned} F'(V_t) &= F'(V_0) - \int_{V_t}^{V_0} F''(V)dV \\ &> F'(V_0) - \int_{V_t}^{V_0} \overline{F}''(V)dV \\ &= 0 - (\overline{F}'(V_0) - \overline{F}'(V_t)) \\ &= \frac{1}{r\gamma V_t} - \frac{1}{r\gamma V_0} \end{aligned}$$

Therefore,

$$\begin{aligned} q = F(V_0) - F(\underline{V}) &= \int_{\underline{V}}^{V_0} F'(V)dV \\ &> \int_{\underline{V}}^{V_0} \left(\frac{1}{r\gamma V} - \frac{1}{r\gamma V_0}\right) dV \\ &= \frac{1}{r\gamma}\left(-\ln\left(\frac{\underline{V}}{V_0}\right) + \frac{\underline{V} - V_0}{V_0}\right) \end{aligned}$$

This inequality gives us an upper bound for $\underline{V}/V_0$:

$$\frac{\underline{V}}{V_0} < -W_{-1}(-e^{-1-r\gamma q}) \tag{A-3}$$

where W_{-1} is the Lambert W function.

Next we establish lower bounds for α_0 and $\underline{\alpha}$. $G(V_t) = \overline{F}(V_t) - F(V_t)$ is a convex function. Because $F'(V_0) = 0$, $G'(V_0) = \overline{F}'(V_0) - F'(V_0) = -1/r\gamma V_0$.

$$G(V_0) = -\int_{V_0}^{0} G'(V)dV < -G'(V_0) \cdot (0 - V_0) = \frac{1}{r\gamma}$$
$$\alpha_0 = 2rF(V_0) + 2c_0 = 2rF(V_0) - 2r\overline{F}(V_0) + \frac{4}{r\gamma} = \frac{4}{r\gamma} - 2rG(V_0) > \frac{2}{r\gamma}$$

where the last inequality used the assumption $r < 1$. Therefore $\underline{\alpha}$ is bounded below by,

$$\begin{aligned}\underline{\alpha} &= \alpha_0 - 2r(F(V_0) - F(\underline{V})) - 2(c_0 - \underline{c}) \\ &\geqslant \frac{2}{r\gamma} - 2rq - \frac{2}{\gamma}\ln(\frac{\underline{V}}{V_0})\end{aligned}$$

The upper bound for $-r\gamma V_t F'(V_t)$ follows from the above inequalities,

$$\begin{aligned}F'(V_t) &= F'(V_0) - \int_{V_t}^{V_0} F''(V)dV \\ &= 0 + \int_{V_t}^{V_0} \frac{4}{r^2\gamma^2V^2\alpha(V)}dV \\ &< \int_{V_t}^{V_0} \frac{4}{r^2\gamma^2V^2\underline{\alpha}}dV \\ &= \frac{4}{r^2\gamma^2\underline{\alpha}}\left(\frac{1}{V_t} - \frac{1}{V_0}\right)\end{aligned}$$

Therefore,

$$-r\gamma V_t F'(V_t) < \frac{4}{r\gamma\underline{\alpha}}\left(\frac{V_t}{V_0} - 1\right) \leqslant \frac{2\left(-W_{-1}(-e^{-1-r\gamma q}) - 1\right)}{1 - r^2\gamma q + rW_{-1}(-e^{-1-r\gamma q})}$$

The right hand side of the above inequality is an increasing function of q and is equal to 0 when $q = 0$. Therefore q^* is higher than the solution to the following equation

$$\frac{2\left(-W_{-1}(-e^{-1-r\gamma q}) - 1\right)}{1 - r^2\gamma q + rW_{-1}(-e^{-1-r\gamma q})} = \sqrt{\frac{2}{r}} - 1 \tag{A-4}$$

and this completes the proof. ■

Proof of Lemma 1.2

First, we prove that V_0 increases with q.

Consider two firms, Firm A and Firm B. They have different recruitment costs, $0 \leqslant q_A < q_B$, but are otherwise identical. Then, the corresponding continuation values of the principal must satisfy $F_A(V_t) > F_B(V_t)$. This is because Firm A can at least always use the same strategy as Firm B, and saves the recruitment costs. Therefore, $F''_A(V_t) > F''_B(V_t)$, according to Equation (1.10).

Similar to the logic in Proof of Proposition 1.3, we define $g(V_t) = F_A(V_t) - F_B(V_t)$. So, $g(V_t) > 0$ and $g''(V_t) > 0$, $\forall\ V_t < 0$. Since $\lim_{V_t \to 0} g(V_t) = 0$ and $g(V_t) > 0$, $\lim_{V_t \to 0} g'(V_t) \leqslant 0$. $g(V_t)$ is a convex function, $g'(V_t)$ is non-decreasing. Therefore, $g'(V_t) < 0$, $\forall\ V_t < 0$.

From the proof of Proposition 1.3, we know that $0 > \bar{F}''(V_t) > F''(V_t)$, since $G''(V_t) > 0$. Therefore, $F_A(V_t)$ and $F_B(V_t)$ are concave. Besides, we know that $F'_A(V_0^A) = 0$ and $F'_B(V_0^B) = 0$. Therefore, $F'_A(V_0^B) < F'_B(V_0^B) < 0$, since we've got that $g'(V_t) < 0$. This leads to the result that $F'_A(V_0^B) < F'_A(V_0^A)$, and thus, $V_0^A < V_0^B$, i.e., V_0 increases with q.

To prove that V_0 increases with $\underline{V}$, consider two firms, Firm C and Firm D. They are in two different industries where the managers' outside options are different, $\underline{V_C} < \underline{V_D}$. Following the same steps as above, we reach the conclusion that V_0 increases with $\underline{V}$. ■

Proof of Proposition 1.6

To prove that α_0 is independent of q, adopt the same setting as in the proof of Lemma 1.2, when proving that V_0 increases with q.

According to Equation (1.9), we only need to prove that

$$(V_0^A)^2 F''_A(V_0^A) = (V_0^B)^2 F''_B(V_0^B) \tag{A-5}$$

If we could prove that

$$F_B(V_t) = F_A\left(\frac{V_0^A}{V_0^B}\cdot V_t\right) + \frac{1}{r\gamma}[\ln(-rV_0^B) - \ln(-rV_0^A)], \tag{A-6}$$

the above equality (A-5) is satisfied. We then prove that the relationship between $F_A(V_t)$ and $F_B(V_t)$ satisfies Equation (A-6). Therefore, it boils down to prove that if $F_A(V_t)$ is the solution to Firm A's problem, $F_B(V_t)$ as expressed in Equation (A-6) is the solution to Firm B's problem.

From the previous proof for the relationship between V_0 and q, we know that there is one-to-one mapping between V_0 and q. Therefore, the boundary conditions $F'(V_0) = 0$ and $\lim_{V_t\to 0}[\bar{F}(V_t) - F(V_t)] = 0$ together with the ODE (1.10) pin down the solution $F(V_t)$.

Suppose $F_A(V_t)$ satisfies the above boundary conditions and the ODE. We prove that $F_B(V_t)$ as expressed in Equation (A-6) also satisfies the boundary conditions and the ODE.

First,

$$F_B'(V)\Big|_{V=V_0^B} = \frac{V_0^A}{V_0^B}\cdot F_A'(V)\Big|_{V=V_0^A} = 0$$

Second,

$$\begin{aligned}
&\lim_{V\to 0}[F_B(V) - \bar{F}(V)] \\
=&\lim_{V\to 0}\left[F_A\left(\frac{V_0^A}{V_0^B}\cdot V\right) - \bar{F}(\frac{V_0^A}{V_0^B}\cdot V)\right] + \lim_{V\to 0}\left[\bar{F}\left(\frac{V_0^A}{V_0^B}\cdot V\right) - \bar{F}(V)\right] + \frac{1}{r\gamma}\left(\ln(-rV_0^B) - \ln(-rV_0^A)\right) \\
=&0 + \frac{1}{r\gamma}\ln\left(\frac{V_0^A}{V_0^B}\right) + \frac{1}{r\gamma}\ln\left(\frac{V_0^B}{V_0^A}\right) \\
=&0
\end{aligned}$$

Third,

$$
\begin{aligned}
\frac{2}{r\gamma V^2 F_B''(V)} &= \frac{2}{r\gamma\left(\frac{V_0^A}{V_0^B}\cdot V\right)^2 F_A''\left(\frac{V_0^A}{V_0^B}\cdot V\right)} \\
&= \frac{1}{\gamma}\ln\left(-r\cdot\frac{V_0^A}{V_0^B}\cdot V\right) - rF_A\left(\frac{V_0^A}{V_0^B}\cdot V\right) \\
&= \frac{1}{\gamma}\ln(-rV) - rF_B(V)
\end{aligned}
$$

Therefore, if $F_A(V_t)$ is the solution to Firm A's problem, $F_B(V_t)$ as expressed in Equation (A-6) is the solution to Firm B's problem. We have proven that that α_0 is independent of q.

To prove α_0 is independent of $\underline{V}$, again, as in the proof of Lemma 1.2, consider two firms, Firm C and Firm D. They are in two different industries where the managers' outside options are different, $\underline{V_C} < \underline{V_D}$. Following the same steps as in the above proof, we derive that α_0 is independent of $\underline{V}$. ■

Proof of Proposition 1.7

Suppose that the opportunity to change authority arrives at a rate λ. In this notes I use α_t to represent the current level of authority.

The principal's HJB equation is

$$
rF(\alpha_t, V_t) = \max_{\beta_t}\left\{-w_t + (2\theta_t - \alpha_t) + \frac{1}{2}\frac{\partial^2 F(\alpha_t, V_t)}{\partial V_t^2}\left[u'(c_t)\right]^2\beta_t^2 + \lambda\left(\max_{\alpha} F(\alpha, V_t) - F(\alpha_t, V_t)\right)\right\}
$$

where

$$
\begin{aligned}
\theta_t &= \min\{2\beta_t, \alpha_t\} \\
w_t &= c_t - \left(\frac{\alpha_t^2}{2} - \frac{\theta_t^2}{2}\right)
\end{aligned}
$$

Therefore

$$\beta_t = \min\left\{\frac{4}{4 - F_{VV}\left[u'(c_t)\right]^2}, \frac{\alpha_t}{2}\right\} \tag{A-7}$$

$$rF(\alpha_t, V_t) = -c_t + \lambda\left(\max_{\alpha} F(\alpha, V_t) - F(\alpha_t, V_t)\right) \tag{A-8}$$

$$+\begin{cases} \dfrac{\alpha_t^2}{2} - \alpha_t + \dfrac{8}{4 - F_{VV}\left[u'(c_t)\right]^2}, & \text{for } \beta_t < \dfrac{\alpha_t}{2} \\ \alpha_t + \dfrac{\alpha_t^2}{8} F_{VV}\left[u'(c_t)\right]^2, & \text{for } \beta_t = \dfrac{\alpha_t}{2} \end{cases} \tag{A-9}$$

The principal gets a chance to reset the authority after turnover. This gives rise to one boundary condition

$$F(\alpha_t, \underline{V}) = \max_{(\alpha, V_0)} F(\alpha, V_0) - q$$

Now let $\bar{F}(\alpha_t, V_t)$ represent the discounted profit of the principal if the current authority is α and there are no exits opportunities ($\underline{V} \to -\infty$). The principal would set $\beta_t = \alpha_t/2$ before the opportunity to change authority arrives. When the opportunity arrives, the principal would set the authority to $\bar{\alpha} = 4/r\gamma$, and pay-performance sensitivity to $\bar{\alpha}/2$.

$$\begin{aligned} \bar{F}(\bar{\alpha}, V_t) &= -\frac{c_t}{r} + \frac{2}{r^2\gamma} \\ \bar{F}(\alpha_t, V_t) &= -\frac{c_t}{r} + \frac{1}{r+\lambda}\left(\alpha_t - \frac{1}{8} r\gamma\alpha_t^2\right) + \frac{\lambda}{r+\lambda}\frac{2}{r^2\gamma} \end{aligned}$$

$\bar{F}(\bar{\alpha}, V_t)$ is equal to the flexible authority no-exit value function. $\bar{F}(\alpha_t, V_t)$ consists of 2 components: expected profits before and after the change of authority.

As V_t tends to 0, the with-exit value function approaches the no-exit value function. Therefore the other boundary condition is given by

$$\lim_{V_t \to 0} (\bar{F}(\alpha_t, V_t) - F(\alpha_t, V_t)) = 0$$

Define $G(\alpha_t, V_t) = \bar{F}(\bar{\alpha}, V_t) - F(\alpha_t, V_t)$,

$$G_{VV} = -\frac{1}{r\gamma V_t^2} - F_{VV} \tag{A-10}$$

Substituting in $F(\alpha_t, V_t) = \bar{F}(\bar{\alpha}, V_t) - G(\alpha_t, V_t)$ and using $u'(c_t) = -r\gamma V_t$

$$\begin{aligned}(r+\lambda)G(\alpha_t, V_t) =& \frac{2}{r\gamma} + \lambda \min_\alpha G(\alpha, V_t) \\ & - \begin{cases} \dfrac{\alpha_t^2}{2} - \alpha_t + \dfrac{8}{4 + r\gamma + (r\gamma V_t)^2 G_{VV}}, & \text{for } \beta_t < \dfrac{\alpha_t}{2} \\ \alpha_t - \dfrac{\alpha_t^2}{8}\left(r\gamma + (r\gamma V_t)^2 G_{VV}\right), & \text{for } \beta_t = \dfrac{\alpha_t}{2} \end{cases}\end{aligned} \tag{A-11}$$

with boundary conditions

$$\begin{aligned} G(\alpha_t, \underline{V}) &= \min_{(\alpha, V_0)} \left[G(\alpha, V_0) + \frac{1}{r}(c_0 - \underline{c})\right] + q \\ \lim_{V_t \to 0} G(\alpha_t, V_t) &= \frac{1}{r+\lambda} \frac{(\alpha_t - \bar{\alpha})^2}{2\bar{\alpha}} \end{aligned}$$

Solving for G_{VV},

$$G_{VV} = \begin{cases} \left((r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t) - \dfrac{2}{r\gamma} + \alpha_t - \dfrac{r\gamma}{8}\alpha_t^2\right) \Big/ \dfrac{\alpha_t^2}{8} r^2\gamma^2 V_t^2\,, \\ \text{if } (r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t) < \dfrac{2}{r\gamma} - \dfrac{\alpha_t^2}{2} \\ \dfrac{1}{r^2\gamma^2 V_t^2}\left[8 \Big/ \left(\lambda \min_\alpha G(\alpha, V_t) - (r+\lambda)G + \dfrac{2}{r\gamma} + \alpha_t - \dfrac{\alpha_t^2}{2}\right) - 4 - r\gamma\right], \\ \text{if } (r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t) > \dfrac{2}{r\gamma} - \dfrac{\alpha_t^2}{2} \end{cases} \tag{A-12}$$

G_{VV} is an increasing function of $(r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t)$. G_{VV} is increasing in α_t for $(r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t) < 2/r\gamma - \alpha_t/2$ and decreasing in α_t for $(r+\lambda)G - \lambda \min_\alpha G(\alpha, V_t) > 2/r\gamma - \alpha_t/2$.

Lemma A1. *Consider 2 levels of authority $\alpha_1 < \alpha_2$. Suppose $G(\alpha_1, V_t)$ and $G(\alpha_2, V_t)$ intersects at $V_t = \tilde{V}$. Then $G(\alpha_2, V_t)$ could only cross $G(\alpha_1, V_t)$ from above, not below*

$$G_V(\alpha_1, \tilde{V}) \geqslant G_V(\alpha_2, \tilde{V})$$

From Lemma A1. to Proposition 1.7

Because $G(\alpha_t, V_t)$ is inversely related to the principal's continuation value $F(\alpha_t, V_t)$, the value functions $F(\alpha_1, V_t)$ and $F(\alpha_2, V_t)$ intersects only once, and

$$\begin{aligned} F(\alpha_1, V_t) &> F(\alpha_2, V_t), \quad \text{for } V_t \in (\underline{V}, \tilde{V}) \\ F(\alpha_1, V_t) &< F(\alpha_2, V_t), \quad \text{for } V_t \in (\tilde{V}, 0) \end{aligned}$$

If α_2 is reset authority level when $V_t = \hat{V} \in (\tilde{V}, 0)$, then $F(\alpha_2, \hat{V}) > F(\alpha, \hat{V})$ for any other authority level α. Therefore for all $V_t \in (\hat{V}, 0)$ and any authority level α_1 that is below α_2,

$$F(\alpha_2, V_t) > F(\alpha_1, V_t) \tag{A-13}$$

As a consequence the reset authority level for all $V_t \in (\hat{V}, 0)$ is above α_2. ■

Proof of Lemma A1: Suppose the opposite is true and

$$G_V(\alpha_1, \tilde{V}) < G_V(\alpha_2, \tilde{V}) \tag{A-14}$$

We consider two cases $G_{VV}(\alpha_1, \tilde{V}) < G_{VV}(\alpha_2, \tilde{V})$ and $G_{VV}(\alpha_1, \tilde{V}) \geqslant G_{VV}(\alpha_2, \tilde{V})$. If $G_{VV}(\alpha_1, \tilde{V}) < G_{VV}(\alpha_2, \tilde{V})$ then from equation (A-12),

$$(r+\lambda)G(\alpha_1, \tilde{V}) - \lambda \min_{\alpha} G(\alpha, \tilde{V}) = (r+\lambda)G(\alpha_2, \tilde{V}) - \lambda \min_{\alpha} G(\alpha, \tilde{V}) < \frac{2}{r\gamma} - \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$$

Similar to the proof of $G'(V_t) < 0$ in Proposition 1.3, in this setup with infrequent adjustment of authority, $(r+\lambda)G(\alpha_t, V_t) - \lambda \min_\alpha G(\alpha, V_t)$ is decreasing in V_t,

$$(r+\lambda)\frac{\partial G(\alpha_t, V_t)}{\partial V_t} - \lambda \frac{d}{dV_t}\min_\alpha G(\alpha, V_t) < 0 \tag{A-15}$$

for any α_t, V_t.

Assumption (A-14) guarantees that $G(\alpha_1, V_t) < G(\alpha_2, V_t)$ for V_t between $\tilde{V}$ and the next intersection of 2 functions to the right of $\tilde{V}$. In this range,

$$(r+\lambda)G(\alpha_1, V_t) - \lambda \min_\alpha G(\alpha, V_t) < (r+\lambda)G(\alpha_2, V_t) - \lambda \min_\alpha G(\alpha, V_t) < \frac{2}{r\gamma} - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

From equation (A-12),

$$G_{VV}(\alpha_1, V_t) < G_{VV}(\alpha_2, V_t) \tag{A-16}$$

Combining (A-14) and (A-16), we find that

$$G_V(\alpha_1, V_t) < G_V(\alpha_2, V_t) \tag{A-17}$$

for any V_t between $\tilde{V}$ and the next intersection. However, this inequality indicates that $G_V(\alpha_1, \cdot)$ and $G_V(\alpha_2, \cdot)$ will diverge and $G(\alpha_1, V_t)$ will always be below $G_V(\alpha_2, V_t)$, contradicting

$$\begin{aligned}\lim_{V_t\to 0} G(\alpha_1, V_t) &= \frac{1}{r+\lambda}\frac{(\alpha_1-\hat{\alpha})^2}{2\hat{\alpha}} \\ &> \frac{1}{r+\lambda}\frac{(\alpha_1-\hat{\alpha})^2}{2\hat{\alpha}} = \lim_{V_t\to 0} G(\alpha_2, V_t)\end{aligned}$$

If $G_{VV}(\alpha_1, \tilde{V}) \geqslant G_{VV}(\alpha_2, \tilde{V})$, then from equation (A-12),

$$(r+\lambda)G(\alpha_1, \tilde{V}) - \lambda \min_\alpha G(\alpha, \tilde{V}) = (r+\lambda)G(\alpha_2, \tilde{V}) - \lambda \min_\alpha G(\alpha, \tilde{V}) > \frac{2}{r\gamma} - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

Between $\tilde{V}$ and the next intersection of 2 functions to the left of $\tilde{V}$,

$$(r+\lambda)G(\alpha_1, V_t) - \lambda \min_{\alpha} G(\alpha, V_t) > (r+\lambda)G(\alpha_2, V_t) - \lambda \min_{\alpha} G(\alpha, V_t) > \frac{2}{r\gamma} - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

$$G_{VV}(\alpha_1, V_t) > G_{VV}(\alpha_2, V_t)$$

Therefore,

$$G_V(\alpha_1, V_t) < G_V(\alpha_2, V_t)$$

$G_V(\alpha_1, \cdot)$ and $G_V(\alpha_2, \cdot)$ will diverge and $G(\alpha_1, V_t)$ will always be above $G_V(\alpha_2, V_t)$, contradicting

$$G(\alpha_1, \underline{V}) = \min_{(\alpha, V_0)} \left[G(\alpha, V_0) + \frac{1}{r}(c_0 - \underline{c})\right] + q = G(\alpha_2, \underline{V})$$

Neither $G_{VV}(\alpha_1, \tilde{V}) < G_{VV}(\alpha_2, \tilde{V})$ nor $G_{VV}(\alpha_1, \tilde{V}) \geqslant G_{VV}(\alpha_2, \tilde{V})$ are consistent with assumption (A-14). ■

Proof of Proposition 1.8

Substituting (A-10) into (A-7), we obtain

$$\beta_t(\alpha_t, V_t) = \min\left\{\frac{4}{4 + r\gamma + (r\gamma V_t)^2 G_{VV}}, \frac{\alpha_t}{2}\right\}$$

$\beta_t = \alpha_t/2$ if and only if

$$G_{VV} \leqslant \frac{1}{r\gamma V_t^2}\left[\frac{8}{\alpha_t} - 4 - r\gamma\right]$$

or equivalently from (A-12),

$$(r+\lambda)G(\alpha_t, V_t) - \lambda \min_{\alpha} G(\alpha, V_t) \leqslant \frac{2}{r\gamma} - \frac{\alpha_t^2}{2} \qquad \text{(A-18)}$$

At $V_t = \underline{V}$, $G(\alpha_t, V_t)$ is the same across α_t and equal to

$$\min_{(\alpha,V_0)} [G(\alpha, V_0) + \frac{c_0 - \underline{c}}{r}] + q.$$

Let

$$\alpha^* = \sqrt{\frac{4}{r\gamma} - 2r \min_{(\alpha,V_0)} [G(\alpha, V_0) - 2(c_0 - \underline{c})] - 2rq}$$

From the monotonicity of $(r+\lambda)G(\alpha_t, V_t) - \lambda \min_\alpha G(\alpha, V_t)$ in equation (A-15), for any $\alpha_t \in [0, \alpha^*]$ and $V_t \in [\underline{V}, 0)$,

$$\begin{aligned}(r+\lambda)G(\alpha_t, V_t) - \lambda \min_\alpha G(\alpha, V_t) &\leqslant (r+\lambda)G(\alpha_t, \underline{V}) - \lambda \min_\alpha G(\alpha, \underline{V}) \\ &= \frac{2}{r\gamma} - \frac{(\alpha^*)^2}{2} \\ &\leqslant \frac{2}{r\gamma} - \frac{\alpha_t^2}{2}\end{aligned}$$

Similarly, for $\alpha_t \in (\alpha^*, \hat{\alpha}]$, there exists a continuation level $V^*(\alpha_t)$ such that

$$(r+\lambda)G(\alpha_t, V_t) - \lambda \min_\alpha G(\alpha, V_t) \leqslant \frac{2}{r\gamma} - \frac{\alpha_t^2}{2}$$

if and only if $V_t \in [V^*(\alpha_t), 0)$. ■

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Chapter 2

Platform Enterprises: Financing, Investment, and Network Growth

Joanne Juan Chen[1]

2.1. Introduction

Enterprises that leverage the power of platform business models have grown dramatically in size and scale over the past few years. Prominent examples are Uber, Airbnb, Upwork, etc., through which services are traded; and Amazon, eBay, Taobao, etc., where commodities are traded. A platform enterprise doesn't produce goods by itself. Instead, it creates networks of users who interact and transact through the platform and establishes a platform market. Compared with traditional markets, the platform market is ultimately driven by network effects, which enable the users to benefit more from trading through the platform when the network size of the other group is larger.

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Usually, the platform enterprise charges spreads between the selling and purchasing prices, and it makes profits from these fees.[2]

The business model of platforms has attracted attention from both industry and academia. While there is copious academic work focusing on competition and fee structure of platforms, studies on the financing and investment of platform enterprises are sparse. Some interesting questions remain unanswered: is it rational for platforms to make highly aggressive subsidies to users at early stages, especially when the platform enterprise is a monopoly with no entry threat? Why does the number of financing rounds vary greatly for different platform enterprises?[3] How would the capital market conditions affect a platform enterprise's financing and investment decisions and its valuation? This paper develops a dynamic theory of platform financing and investment to study these questions.

The platform financing and investment questions are unique in the following aspects. First, a platform is an enterprise but it builds up a market. Therefore, it possesses properties of both a firm and a market. Besides, a platform tends to allocate a large portion of funds for making subsidies to users, even there is no competitor or entry threat. So, subsidies cannot be simply regarded as predatory pricing strategy of the platform enterprise. Instead, subsidies are investment into the networks, and the networks are analogous to capital-assets, which the platform enterprise first invests in and later generates income from. The model in this paper captures these distinctive features of a platform and solves for optimal financing and investment decisions of a monopolistic platform enterprise under various capital market conditions.

In the model, there is a massive fully-competitive traditional market and a newly-launched platform market where the same type of good is traded. Network effects in

[2]Fees charged by the platforms can be in different forms. Some platforms directly charge a per-transaction commission from sellers and buyers, e.g. Upwork and Amazon; some set selling and pricing prices for the users and make profits from the price differences, e.g. Uber. Here, the spread includes both of these forms. Some platforms charge membership fees in addition to per-transaction fees. In the model of this paper, the fixed membership fee is set to zero. This simplification assumption does not affect qualitative results.

[3]For example, Amazon raised two rounds of funding in total, while Uber has raised twenty-three rounds till the end of October 2019, according to data from Crunchbase.

the platform market generate additional surpluses, which can be shared by the platform enterprise and the agents. This is the ultimate reason why the platform enterprise can charge fees on transactions. Once a new agent gets to know the platform market, he will try it out and decide whether to switch to it from the traditional market, with an option to switch back. The traditional market is assumed to be immense and unresponsive to the platform market. The entrepreneurial platform enterprise has the incentive to make subsidies to users to boost network growth and maximize its value. With no internal funds, the entrepreneur has to raise external capital to invest in the networks. Therefore, he needs to decide jointly on the amount and timing of financing and investment in a world with financial frictions and financing costs.

The first key finding of the model is that in face of financial constraints, it is constrained-optimal for the platform to make highly aggressive subsidies by using up available funds early on. This result highlights the importance of networks as intangible assets of the platform enterprise. Timely and sufficient investment in the networks is crucial for the success of a platform.[4] Financial constraints are endogenized in this paper: the entrepreneur may rationally choose not to invest up to the unconstrained-optimal level because of financing costs. When the financial constraint is binding, it's suboptimal to use the funds gradually and smoothly. On the contrary, it is rational to make heavy subsidies early on and charge zero spread[5] subsequently for a certain period.

The model explicitly solves for the (constrained) optimal dynamic pricing strategies of the platform enterprise and the corresponding network growth paths. It finds that the spread charged by the platform is non-decreasing over time. That is, the per-transaction subsidies decrease over time and are followed by weakly increasing fees. The model also

[4]A prominent example of failure due to inadequate subsidies and slow network growth is SideCar, a ride-sharing company similar to Uber. According to an article on Havard Business Review: *it deliberately pursued innovation and a conservative slow-growth strategy in order to be financially responsible. The fatal flaw was not recognizing the importance of attracting both sides of the platform. Sidecar also raised much less venture capital than Uber and Lyft, and was unable to attract enough drivers and riders to survive much beyond the startup phase.* (Yoffie, Gawer and Cusumano, 2019).

[5]The model assumes a zero maintenance cost for simplicity. Intuitively, the platform may retain some funds and charge a maintenance-level commission if the maintenance costs are positive.

predicts that *ceteris paribus*, stronger network effects, defined as more user benefits from the same network size, lead to more aggressive subsidies early on and hence a higher platform valuation and faster network growth.

In terms of financing patterns, this paper argues that staged financing can be a natural choice to mitigate incentive problems. Intuitively, staging allows the financiers to abandon the project in case of misconduct, and fewer available funds in hand reduce the entrepreneur's incentives of misconduct. The type of potential misconduct this paper considers is limited enforcement – the entrepreneur could abscond with funds in hand. This is an extreme case of fund embezzlement, where the entrepreneur could embezzle all the funds he had just raised. In practice, fund embezzlement is indeed a severe problem for startups, especially for platform enterprises which need enormous investment before earning profits.[6] This paper finds that, with potential embezzlement, each round the entrepreneur cannot raise more funds than his expected profits from successfully managing the platform enterprise. This leads to an increased value of funds raised each round over time and decreased financing frequency. The model simultaneously endogenizes the number of rounds, the financing frequency, and the value of funds raised.

The paper also analyzes how the number of financing rounds is affected by the profitability of the platform as well as the capital market conditions. All things equal, higher profitability leads to fewer rounds of financing; a more competitive capital market leads to fewer rounds of financing. When the capital market is not fully competitive, the more profits the financiers require, the less investment the entrepreneur tends to make, and the more rounds of financing he has to raise. However, an excessively high cost of financing would lead to no financing and no investment in the networks. This paper characterizes the interaction between discrete financing choices and continuous investing decisions under different scenarios.

[6] Many failed platform enterprises have been accused of entrepreneur fund embezzlement. For example, Kongkonghu, a platform trading second-hand goods in China, was reported the CEO embezzlement of funds for private usage. In this paper, I assume that fund embezzlement can be detected before the next round of financing, because of investors' monitoring efforts or due diligence undertaken before each new round of financing.

Related Literature

Rochet and Tirole (2003, 2006) and Armstrong (2006) provide pioneering work on platform markets. Their work involves the type of platforms where price non-neutrality of the two sides is a key feature. They define the platform markets with this non-neutral price structure as two-sided markets. Rochet and Tirole (2006) summarize that factors making a market two-sided are: absence or limits on the bilateral pricing setting, platform-imposed constraints on pricing and membership fixed costs or fixed fees. Examples of those markets are credit card markets, newspapers, Videogame platforms, etc. Rochet and Tirole (2003, 2006) and Armstrong (2006) develop static models on the two-sided markets to discuss competition and price structure of platforms in various cases. This paper, in contrast, involves platforms which either allow bilateral pricing by end-users (e.g. Amazon, Deliveroo, Upwork, etc.) or charge no membership fees and optimally set prices satisfying the market clearing condition to maximize profits (e.g. Uber). Thus, in this paper, equilibrium transaction amounts and prices are endogenously determined by market clearing, and only the level of fees charged by the platform matters in equilibrium. This simple price structure allows me to explore the dynamics of the platform and find closed-form expressions. To my knowledge, this paper provides the first attempt to describe the optimal dynamic pricing strategy and network growth path of the platform market based on a fully micro-founded model.

Another strand of literature studies competition with same-side network effects, following the work by Katz and Shapiro (1985), and Farrell and Saloner (1985, 1986). Some more recent work of this literature addresses the issue of dynamic competition and growth. Mitchell and Skrzypacz (2006) derive the Markov perfect equilibrium of an infinite-period game with network effects. They assume the consumer's utility to be an increasing function of the network size at the time of trade. I make similar assumptions in this paper. That is, the agents are assumed to be myopic and not forward looking. Cabral (2011) considers dynamic pricing competition between two proprietary networks with forward-looking agents. My work is distinguished from this strand of literature in the following aspects. First, I model the cross-group network effects of the platform market. Second, I focus on the effect of network itself on the dynamic pricing decisions without competition or entry threat. Third, I introduce financial frictions in the model and study the joint decisions of dynamic financing and pricing. The topic on dynamic platform competition and its interaction with financing issues is a good direction for future research.

For the financing part, this paper is related to the literature on venture capital (VC) staging. Admati and Pfleiderer (1994) model the staging as a way to mitigate agency problems such as asymmetric information and overinvestment. Wang and Zhou (2004) investigate the cases with moral hazard and uncertainty and find that staged financing can control risk and mitigate moral hazard. Most work on VC staging has assumed either fixed investment levels or a fixed number of stages. This paper endogenizes the financing and investment levels, the number of stages, and frequency of financing simultaneously, without resort to uncertainty.

This paper is also related to work on continuous-time models of the firm's financing and investment decisions and the impact of external financing costs on investment. Decamps et al. (2011) explore a model where a firm has a fixed-size investment project and generate random cash flows, and they study the impact of external financing costs on equity issuance and stock prices. Bolton, Chen, and Wang (2011) study the case of flexible firm size and the dynamic patterns of corporate investment. Demarzo et al. (2012) study the dynamic investment theory with dynamic optimal incentive contracting, and endogenize financial constraints. Section 2.3 of this paper also endogenizes financial constraints, but this paper involves no uncertainty. Limited enforcement along with financing costs leads to endogenous financing level, staging, and pricing decisions in this paper. Besides, the literature consider either fixed-size investment or AK production technology, while this paper endogenizes the cash flows by modelling the unique business patterns of the platform enterprise.

The remainder of this paper proceeds as follows. Section 2.2 sets up and solves for the framework of the dynamics of the platform market, and compares it with the traditional markets and firms. Section 2.3 discusses the financing and investment issues with financial frictions, and characterizes the constrained-optimal financing and investment patterns. Section 2.4 concludes. Proofs appear in the Appendix.

2.2. Framework of the Platform Market

To understand the strategies of the platform enterprise, we first need to know how the platform market works. In this section, I build a framework to analyze the dynamics of the platform market, and the corresponding pricing strategies of the platform enterprise.

The investment strategy of the entrepreneur who runs the platform enterprise is highlighted in this section, assuming no financial frictions or financing costs, or equivalently, the entrepreneur has sufficient internal funds to make the investment. Discussion for financial frictions and the interaction between financing and investment decisions are deferred until next section.

2.2.1 The Model

Time is continuous and the discount rate in this economy is $r > 0$.

In this economy, there is a massive fully-competitive traditional market trading one good, with no network effects. All agents originally trade in this traditional market. At time $t = 0$, a monopolistic platform is launched by an entrepreneur to trade the same good. The platform enterprise creates networks of users, which in turn generate network effects. The initial network size is normalized to be one on each side of the platform market. An agent can switch frictionlessly between the traditional market and the platform market once he gets to know both. Hence, a user of the platform always compares the utility gained from trading in the platform market with his reservation utility when trading in the traditional market, and decides whether to stay in or exit the platform market. There is a fixed cost F_0 to launch the platform, which can be ignored in this section, since it is a sunk cost and does not affect the entrepreneur's investment decisions thereafter.

The information of the platform market is disseminated in the economy by “word of mouth”: current users of the platform will constantly tell his or her friends of it. An agent who has just heard of the platform market will transact one unit of the good and compare the utility with his corresponding reservation utility to decide whether to switch to the platform market. To simplify the analysis, I assume there is no cost to commence, keep, or terminate the platform membership. The platform generates profits by charging spreads between the purchasing and selling prices. The platform’s objective is to maximize all discounted future profits.

The remainder of Subsection 2.2.1 set up the model in details. Part A describes the instantaneous utilities of platform users; Part B then solves for the instantaneous supply and demand functions of the platform market and the market-clearing equilibrium; Part C introduces the law of motion of the platform market; Part D defines the problem of the platform.

A. Instantaneous Utilities of Platform Users

The good is indivisible. On each side of the market, agents are homogeneous. Their utilities are assumed to be quasilinear. A buyer has the standard decreasing marginal utility and a seller experiences an increasing marginal cost. A key setup factor in this model is the cross-group network effects in agents’ utilities when trading through the platform. That is, the net utility of an agent not only depends on the amount of goods/services he consumes or sells, but is also related to the network size of the group on the other side of the market.

The utility U_D of a buyer on the demand side and the utility U_S of a seller on the supply side are, respectively,

$$U_D(x_D) = \int_0^{x_D} (b_1 + \epsilon_i - N_S^{-\eta} x) dx - p_D x_D, \tag{1}$$

$$U_S(x_S) = p_S x_S - \int_0^{x_S} (b_2 + \epsilon_i + N_D^{-\eta} x) dx. \tag{2}$$

For a buyer, the term $(b_1 + \epsilon_i - N_S^{-\eta}x)$ is the utility he gains from consuming an infinitesimal unit of the good at consumption level x. b_1 denotes the average product quality. The zero-mean random variable ϵ_i is an identical and mutually independent shock component in the quality of each infinitesimal unit of the good, where i is an index for the ordinal value of the units. $-N_S^{-\eta}x$ exhibits a decreasing marginal utility. The component $N_S^{-\eta}$ represents the cross-group network effect, where η is a non-negative parameter representing the strength of the network effect.[7] p_D is the per-unit purchasing price of the good, and x_D is the level of consumption.

The above setting assumes that the cross-group network size directly affects the pace of decline in the marginal utility, but not the quality of the good. This is a novel way of specifying network effects. It is a reasonable assumption because the benefits of cross-group network effects usually hail from product variety, lower searching costs or better matching results, which slow down the decrease in the agent's marginal utility rather than affecting the quality of the good. For example, a buyer on Amazon enjoys slower utility decrease if there are more sellers and thus product differentiation; the utility of a Uber-rider also decreases more slowly if there are more drivers and hence higher matching frequency. More broadly, network effects can be understood as benefits of convenience generated by the platform market, which do not exist in the traditional market – to purchase more differentiated goods, the potential buyer has to go to different stores in the traditional market; to take more rides, the traveller has to book taxis several more times and wait longer in the traditional market; while with platforms like Amazon and Uber, they don't have to. Thus, users of the platform tend to experience a slower decrease in the marginal utility and potentially consume more.

A seller's utility is the profits he gets from selling x_S units of the good, since his utility is also quasilinear. p_S is the selling price of the good. p_S may be different from p_D because the platform can charge a spread between these two prices. This spread can be either positive or negative. The term $(b_2 + \epsilon_i + N_D^{-\eta}x)$ is the cost of

[7]When $\eta = 0$, there is no network effect. As will be shown later, the platform can never charge a positive commission in this case, so the optimal choice is not to launch the platform if there is no network effect.

providing an infinitesimal unit of the good at level x. Similar to the demand side, ϵ_i is a shock in the cost of each infinitesimal unit of good produced, $N_D^{-\eta}x$ demonstrates the increasing marginal cost, and $N_D^{-\eta}$ measures the cross-group network effect. The larger the network size of the demand side, the more slowly the marginal cost increases. This setting is most suitable for the suppliers in the "sharing economy" or "gig economy", where the economies of scales generally do not apply to an individual supplier, and they encounter increasing marginal costs. For example, a freelancer of Upwork or a room-provider of Airbnb usually experience increasing marginal costs from more work or longer waiting time in between, but the larger network size of the demand side can mitigate the increase.

In this model, I assume $\epsilon_i \sim U[-\frac{b}{2}, \frac{b}{2}]$, *i.i.d.*, where $b = b_1 - b_2$. More explanations of this assumption will be given in Section 2.2.1.C.

Integrate (1) and (2), and by the law of large numbers, the agents' utilities are in the quadratic quasilinear form:

$$U_D(x_D) = (b_1 x_D - \frac{1}{2}N_S^{-\eta}x_D^2) - p_D x_D, \tag{3}$$

$$U_S(x_S) = p_S x_S - (b_2 x_S + \frac{1}{2}N_D^{-\eta}x_S^2). \tag{4}$$

B. Demand, Supply, and Platform Market Equilibrium

Knowing agents' utilities, we can solve for the demand and supply functions as well as the market equilibrium, taken the spread charged by the platform as given.[8]

Lemma 2.1 ***(Market equilibrium with Spread*** m***)***
Denote $m = p_D - p_S$*,* $b = b_1 - b_2$*, and assume* $N_S = N_D = N$*, then the instantaneous*

[8] In the model, I assume the platform will impose the market-clearing equilibrium instead of rationing equilibrium, because the former maximizes the platform's profits and also generates the maximum transaction amounts. In practice, platforms do apply dynamic pricing strategies and follow a market-clearing equilibrium. For example, Uber uses a dynamic pricing model that matches fares to the rider-to-driver ratio. They call it "surge pricing". For platforms where end-users can set the prices, e.g. Upwork, Amazon, market-clearing equilibrium is even more prevailing because the sellers can adjust the prices according to the demand.

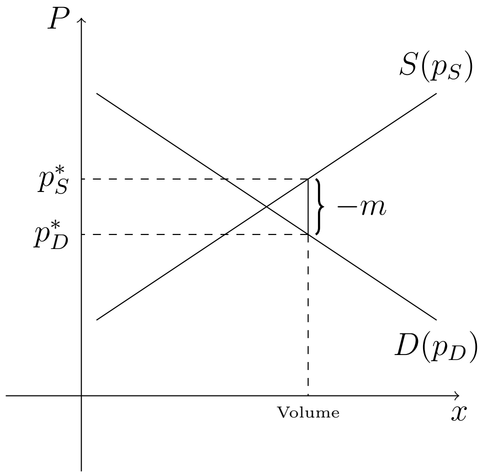


Figure 2.1: Example
The platform market equilibrium with a negative spread ($m < 0$).

equilibrium results of the platform market are: $p_S^* = \frac{b_1+b_2-m}{2}$, $p_D^* = \frac{b_1+b_2+m}{2}$, $x_S^* = \frac{b-m}{2}N^\eta$, $x_D^* = \frac{b-m}{2}N^\eta$, $x_S^* N_S^* = x_D^* N_D^* = \frac{b-m}{2}N^{1+\eta}$, $U_S^* = \frac{(b-m)^2}{8}N^\eta$, $U_D^* = \frac{(b-m)^2}{8}N^\eta$.

m is the spread charged by the platform. For simplicity, here I solve for a symmetric equilibrium where $N_S = N_D = N$. In this paper, the initial network sizes are assumed to be symmetric on the supply and demand sides, and the growth speeds are also symmetric, as will been shown in Section 2.2.1.C. Thus, $N_S = N_D = N$ throughout time in the model. The model best suits the normative "sharing economy" or "gig economy", where the participants are both goods/services providers and consumers. In this case, the supply side and the demand side are composed of the same group of people and thus symmetric.

Proof of Lemma 2.1 is direct. By utility maximization, we derive the individual demand functions: $x_D = (b_1 - p_D)N_S^\eta$, $x_S = (p_S - b_2)N_D^\eta$. Apply the market-clearing condition $N_D x_D = N_S x_S$, we can solve for the equilibrium prices and quantities, as well as agents' utilities.

Figure 2.1 depicts the market demand function, $D(p_D) = (b_1 - p_D)N_S^\eta N_D$, the market supply function, $S(p_S) = (p_S - b_2)N_D^\eta N_S$, and the equilibrium prices and trading volumes under a negative spread m. With a negative m, the trading volume is stimulated and agents trade more because of the subsidies. If instead m is positive, the platform makes profits and the equilibrium trading volume is lower than the zero-spread volume.

C. Law of Motion of the Platform Market

The law of motion guarantees how this platform market dynamically evolves. It consists of three parts - dissemination of news, the joining decision of a newcomer, and exit decision of an existing user.

To understand the joining and exit decisions of the agents, we need to know their outside opportunities. In this economy, there is a traditional competitive market trading the same good with the same quality, but with no network effects. Agents can always trade in this traditional market. The selling and purchasing prices are equal in the traditional market. Thus, utility parameters b_1 and b_2 remain the same in the traditional market; with no network effects, $\eta = 0$; with no price difference, $p_S = p_D = p$. Namely, agents' instantaneous utilities in the traditional market are:

$$U_D(x_D) = (b_1 x_D - \frac{1}{2}x_D^2) - p \cdot x_D, \quad U_S(x_S) = p \cdot x_S - (b_2 x_S + \frac{1}{2}x_S^2). \tag{5}$$

Therefore, the equilibrium price of the traditional market is $p^* = \frac{b_1+b_2}{2}$, and the equilibrium utility of a buyer or seller is $U_R^* = \frac{b^2}{8}$.

For the dissemination of news on the platform market, assume a member of the platform tells λ fraction of his friends about the platform each unit of time. That means, the arrival rate of newcomers on each side is λN.

A newly arrived agent will choose to "have a taste" of the network to gather information. To be specific, his decision rule is as follows: trade an infinitesimal unit of the good through this platform and compare the utility gained with the corresponding reservation utility in the traditional market. The latter is $b/2$. [9] If he gains a higher

[9] It is actually the marginal utility at $x = 0$ in the traditional market, which can be derived

utility than the corresponding reservation utility, he will join the network and switch from the traditional market to the platform market. With some abuse of notation, I denote the utility from trading an infinitesimal unit in the platform market by the marginal utility, MU:

$$MU_D|_{x=0} = (b_1 + \epsilon_i) - p_D = \frac{b-m}{2} + \epsilon_i, \tag{6}$$

$$MU_S|_{x=0} = p_S - (b_2 - \epsilon_i) = \frac{b-m}{2} + \epsilon_i. \tag{7}$$

Therefore, the probability of a newcomer to join the network is

$$Pr(\frac{b-m}{2} + \epsilon_i \geqslant \frac{b}{2}) = Pr(\epsilon_i \geqslant \frac{m}{2}) = \frac{b-m}{2b}, \tag{8}$$

because we have assumed that $\epsilon_i \sim U[-\frac{b}{2}, \frac{b}{2}]$, *i.i.d.* in Section 2.2.1.A.

Once joining the platform market, the user will optimally choose to trade the equilibrium amount as shown in Lemma 1. If the utility he gets is lower than the equilibrium reservation utility in the traditional market, $U_R^* = \frac{b^2}{8}$, he will immediately exit the platform. Because agents are homogeneous, the platform would collapse in this case. So, the platform will always guarantee $U_S^* = U_D^* \geqslant U_R^*$, i.e. $(b-m)^2 N^\eta \geqslant b^2$. I name this constraint "no-exit condition".

Combining all the three parts above, and assuming the potential size of the platform

---

from (5). There is no shock to the quality in the traditional market, because the buyers and sellers usually meet and examine the goods before the transaction is made. In the platform market, however, transactions are usually made before or even without personal contact between a buyer and a seller, which leads to shocks in the realization of the costs and the good quality.

market is $\bar{N}$, the law of motion for each side of the platform market is [10]

$$\dot{N} = \begin{cases} \dfrac{\lambda}{2b}(b-m)N, & if \quad N < \bar{N}, \\ 0, & if \quad N = \bar{N}, \end{cases} \tag{9}$$

where m and N must always satisfy the no-exit condition: $(b-m)^2 N^\eta \geqslant b^2$.

D. Problem of the Entrepreneur

In Section 2.2, I assume the entrepreneur who runs the platform enterprise is financially unconstrained. Therefore, his objective is to maximize the discounted cash flows generated by the platform enterprise, subject to the law of motion and the no-exit condition:

$$\begin{aligned} & \max_{m(t)} \int_0^\infty e^{-rt} \frac{m(t)[b-m(t)]}{2} N(t)^{(1+\eta)}\, dt \\ & s.t. \qquad \dot{N}(t) = \begin{cases} \dfrac{\lambda}{2b}[b-m(t)]N(t), & if\ N(t) < \bar{N}, \\ 0, & if\ N(t) = \bar{N}, \end{cases} \\ & \& \qquad [b-m(t)]^2 N(t)^\eta \geqslant b^2,\ \forall t. \end{aligned} \tag{P1}$$

2.2.2 Model Solutions and Analysis

In this subsection, I present the closed-form solutions for the entrepreneur's problem (P1). Problem (P1) is a standard dynamic optimization problem, with $m(t)$ as the choice variable and $N(t)$ as the state variable. Denote the optimal policy function by $m^*(t)$ and the corresponding optimal state function by $N^*(t)$. Denote value function by

[10]For the law of motion in (9) to hold, $m \in [-b, b]$, because the probability $\frac{b-m}{2b} \in [-1, 1]$. The upper bound never binds since the static monopolistic price is $\frac{b}{2}$; the lower bound may bind. In this paper, I only consider the set of parameters $\{\eta, \lambda, r, \bar{N}\}$ where the lower bound of m does not bind, which is realistic, since platforms in general will not make extreme subsidies. Even if this lower bound binds, it won't affect the qualitative results of this paper, but only generates a segment of $(-b)$ level of m at initial stages of a platform's dynamic pricing strategy.

$\Pi^{m^*}(t)$. [11] Then the optimized objective function in Problem (P1) is $\Pi^{m^*}(0)$.

We know that the static monopolistic price is $\frac{b}{2}$. Here, I first show that if the platform can indeed charge this monopolistic price when the network reaches the maximum size $\bar{N}$, and the optimal strategy is indeed to make subsidies at the very beginning ($m^*(0) < 0$), then the no-exit constraint will never bind all along the optimal path. Put differently, if the no-exit constraint does not bind on the initial point and the endpoint, then it does not bind on the whole path.

Lemma 2.2 *If $m^*(0) < 0$ and $\bar{N} \geqslant 4^{\frac{1}{\eta}}$, the no-exit constraint $[b - m(t)]^2 N(t)^\eta \geqslant b^2$ will not bind on the optimal path.*

Proof of Lemma 2.2 is in Appendix. In this paper, I will only consider the cases where $m^*(0) < 0$, because this paper focuses on the discussion of the optimal investment strategy and its interaction with financing decisions. Besides, as will be shown later, the cases where the platform optimally makes no investment at the very beginning ($m^*(0) > 0$) are less profitable cases. With a high launch cost F_0, these cases are naturally ruled out because the profits of the platform enterprise are not enough to cover the launch costs.

The closed-form solutions of the entrepreneur's problem (P1) is presented in Proposition 2.1. Please see Appendix for detailed derivation and proof.

Proposition 2.1 ***(Solutions to P1)***
Let $\frac{\lambda(1+\eta)}{2r} = a > 1$, and $\bar{N} \geqslant 4^{\frac{1}{\eta}}$. The optimal policy function $m^*(t)$, network growth $N^*(t)$, and discounted future profits $\Pi^{m^*}(t)$, are as follows:

[11] The value function can also be expressed as a function of the state variable, i.e. $\Pi^{m^*}(t) = \Pi(N^*(t))$.

$$m^*(t) = \begin{cases} \frac{b}{a}\sqrt{a-1}\tan(\frac{r}{2}\sqrt{a-1}\cdot t + c_1) + \frac{b}{a}(a-1) & , \ if \ t < T^* \\ b/2 & , \ if \ t \geqslant T^* \end{cases}$$

$$N^*(t) = \begin{cases} e^{\frac{r}{1+\eta}t+c_2} \cdot [\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)]^{\frac{2}{1+\eta}} & , \ if \ t < T^* \\ \bar{N} & , \ if \ t \geqslant T^* \end{cases}$$

$$\Pi^{m^*}(t) = \frac{1}{2r}N^*(t)^{(1+\eta)}[b - m^*(t)]^2,$$

if $c_1 < \arctan(-\sqrt{a-1})$*. The exogenous parameters are* $(b, r, \eta, \lambda, \bar{N})$*. Constants* c_1 *and* c_2*, and the market growth termination time* T^* *are determined by the end points:* $N^*(0) = 1,\ N^*(T^*) = \bar{N},\ m^*(T^*) = \frac{b}{2}$.

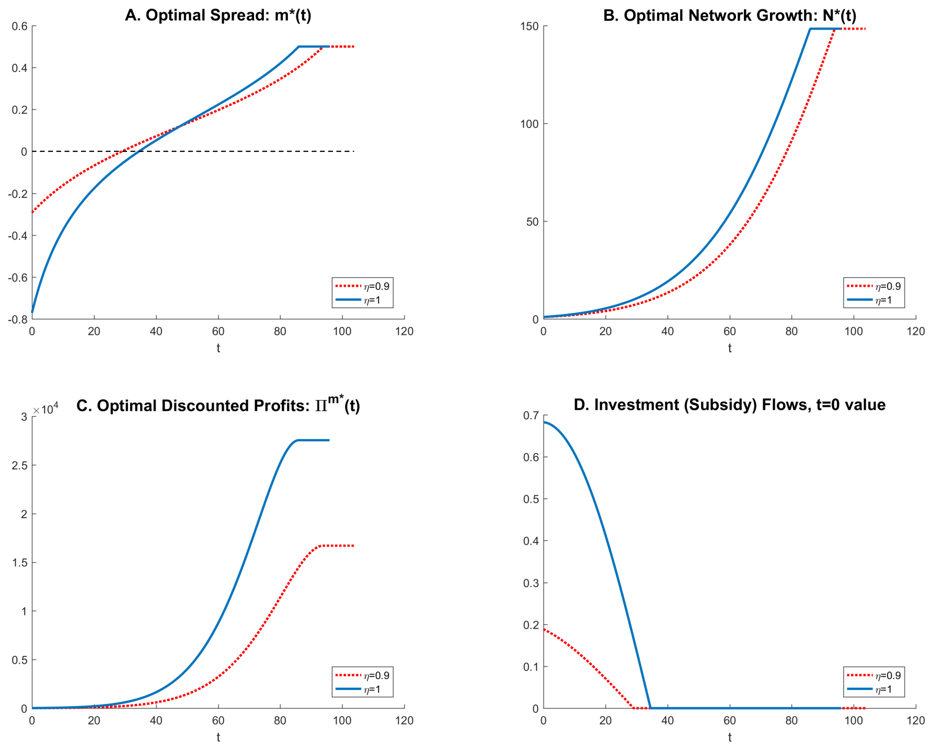
The exogenous parameters b and r are properties of the economy, and η, λ, and $\bar{N}$ are specific for the platform enterprise. Here, the inequality $c_1 < \arctan(-\sqrt{a-1})$ guarantees that the optimal strategy of the platform is indeed to make investment (subsidies) at initial stages; Restriction on the potential market size, $\bar{N} \geqslant 4^{\frac{1}{\eta}}$, guarantees the platform can charge the monopolistic price $\frac{b}{2}$, when the market reaches its maximum size. For the range of a, I will show in Appendix that if $a \leqslant 1$, $m^*(0) > 0$. Therefore, this paper focuses on $a > 1$. A visual example of Proposition 2.1 is given in Figure 2.2.

REMARK 2.1 *The optimal pricing strategy* $m^*(t)$ *is monotonically increasing.*

Remark 2.1 emphasizes that it is efficient for the platform to make subsidies early on and charge a commission later. It is never optimal to undulate this spread, switching between subsidies and commission. As will be shown in Section 2.3, this monotone-increasing property of the pricing strategy holds in more general cases with financial frictions.

REMARK 2.2 *Let the economy has constant* r *and* b*. Then for the platform enterprise, a stronger network effect* η*, a faster information dissemination speed* λ*, or a large maximum potential network size* $\bar{N}$*, will result in more subsidies at initial stages, and meanwhile, a higher valuation of the platform.*

Figure 2.2: Numerical examples: no financial constraints



Parameters: $b = 1$, $r = 0.1$, $\lambda = 0.12$, $\bar{N} = e^5$, $\eta = 0.9$ or 1.

Remark 2.2 predicts that with constant r and b, if we observe that a platform makes heavier subsidies at initial stages, then it tends to enjoy a higher valuation. This result can be got directly from Proposition 1 by plugging $t = 0$ into $\Pi^{m^*}(t)$: $\Pi^{m^*}(0) = \frac{[b-m^*(0)]^2}{2r}$. That is, a more negative $m^*(0)$ leads to a higher $\Pi^{m^*}(0)$. Besides, since a higher η, λ, or $\bar{N}$ leads to a higher valuation of the platform, each of them must lead to more subsidies at the beginning.

Figure 2.2 depicts how $m^*(t)$, $N^*(t)$ and $\Pi^{m^*}(t)$ evolve over time, and also flows of investment (subsidies) in the time-zero value. This example demonstrates that a stronger network effect leads to a steeper optimal pricing path, faster network growth, and a higher platform value. Here, $\Pi^{m^*}(0) = 15.68$ if $\eta = 1$, and $\Pi^{m^*}(0) = 8.35$ if $\eta = 0.9$. Figure 2.2(D) shows that the time-zero value of investment flows keep decreasing over time in this example.

A. Comparison with the Traditional Market

The main distinction between the platform market and the traditional market is the existence of cross-group network effects in the former.

Take Deliveroo as an example. It is a British online food ordering and delivering platform linking the restaurants and the consumers. Consumers can order online from a wide range of restaurants which may not be within walking distance, and wait at home for the food to be delivered. The traditional market counterpart is the decentralized local restaurant market. Through Deliveroo, consumers have access to a large group of restaurants. Hence, they tend to consume more because of the richer product variety brought by the platform, compared to limited choices of local restaurants near home. Deliveroo charges a commission from the restaurant which in turn affects the prices of food listed on the platform by the restaurants. The story is similar for Amazon. Uber is also similar if we regard the traditional market as the taxi-booking market. [12]

The network effects are indispensable for a platform enterprise to exist and make profits. If on the contrary, the platform contains no network effects, i.e. $\eta = 0$, then the platform is exactly the same as the traditional market and it can never charge a commission. Once the platform charges a positive spread, the no-exit condition $[b - m(t)]^2 N(t)^\eta \geqslant b^2$ immediately breaks and all agents exit. Intuitively, when the platform brings no additional surplus to agents compared with the traditional market, there is no room for it to charge a commission. Thus, *ex ante* the entrepreneur has no incentive to launch the platform if there are no network effects in this market. If the network effects are weak, the commission that can be charged by the platform is low and it leads to low profitability of the platform enterprise. With some fixed launch costs or flow maintenance costs, the platform enterprise cannot survive either.

[12] In many cities' taxi market, customers must book in advance for a taxi, which exhibits no network effect.

B. Comparison with the Traditional Production Firms

Perhaps surprisingly, the optimal behavior of a platform enterprise exhibits similarities to a traditional production firm. They both invest first and profit later. The networks of the platform are analogous to the capital assets of a production firm. The platform first invests in the networks and then extract profits from the networks. Therefore, the platform generates negative cash flows early on and positive cash flows later, which has a similar pattern to a traditional production firm.

Some characters of the platform enterprise's investment this paper would like to emphasize and which may be different from a traditional production firm are as follows. 1) The optimal investment-stage and production-stage of the platform enterprise are clearly separated. The entrepreneur running the platform enterprise has the choice to switch between investment (subsidy) and production (commission) as frequently as he wishes, but he will never do so because it's suboptimal. 2) The investment for a platform enterprise takes place dynamically and gradually and a perfect timing is extremely important. Once the platform cannot follow the optimal timing, it will suffer from slower growth and a lower valuation. 3) There is no depreciation in the network, unlike most capital assets in production firms. Once the network is built, it lasts forever unless the agents exit.

If the entrepreneur does not have adequate internal funds to make investment, he has to raise external funds. Section 2.3 discusses the entrepreneur's financing issues in detail.

2.3. Financing Under Limited Enforcement

Section 2.2 builds up a dynamic model for the platform business. The optimal investment and pricing strategies of the entrepreneur are studied, assuming he is financially unconstrained. What if the entrepreneur needs to raise external financing and there exist financial frictions and financing costs, as are common to start-up firms?

In this section, I analyze the financing and investment strategies of the entrepreneur when there exist financial frictions and financing costs. Moreover, to analyze the influences of financial market conditions, I relax the assumption of a fully-competitive capital market. Financiers may require positive profits for their investment. I examine how the required profits affect the incentives of the entrepreneur and thus his financing level, staging, and timing choices.

2.3.1 The Model

The type of financial friction I consider in this paper is limited enforcement, which constrains the entrepreneur's ability to make credible promises.[13] I assume the enforcement of contracts is limited as follows: the entrepreneur can abscond with funds in hand, instead of investing the funds in the networks. The entrepreneur has the incentive to abscond with funds once he has more funds in hand than what he expects to get from successfully managing the platform enterprise. The profits generated by the platform are assumed to be verifiable and paid out as dividends straight away. Thus, the entrepreneur can potentially embezzle the funds but not the operational cash inflows.

Specifically, the setting of the model is as follows: The platform enterprise is launched by an entrepreneur, who has skills but no money. There is a fixed cost F_0 to launch the platform enterprise, and any other costs for operating the platform are set to zero. The fixed cost has already been covered through the "angel investment" (or initial rounds of financing), and the entrepreneur is obliged to pay back F (time-zero value) to the angel investors. To invest in the networks, the entrepreneur has to raise additional financing. Assume that there exists a fixed financing cost C (time-zero value) for each round of financing. This fixed financing cost is a deadweight loss for raising a new round of capital, which can be understood as fees paid to the third parties for project valuation, endorsement, etc. So, each round the entrepreneur raises $(I_j + C)$ from financier j, where I_j (time-zero value) is invested into the networks. Financier j requires W_j (time-zero

[13] Some previous papers consider limited enforcement as a type of financial frictions are Chien and Lustig (2009), Rampini and Viswanathan (2010).

value) as investment profits. That is, he requires a time-zero value $(I_j + C + W_j)$ back for this investment, where $W_j \geqslant 0$.[14]

Denote the total number of financing rounds by n. Let $W = \sum_{j=1}^{n} W_j$, representing the aggregate time-zero value of profits required by all financiers, and $I = \sum_{j=1}^{n} I_j$, representing the aggregate time-zero value of funds invested in the networks. Besides, the model allows the entrepreneur to choose the timing of each new round of financing. Therefore, he is able to allocate the funds optimally through time into the networks. That is to say, only the aggregate fund level I, but not any individual level I_j, affects the optimal investment strategy and the valuation of the platform.

Let $\Pi_0(I)$ denote the optimized time-zero value of all discounted profits as a function of I. $\Pi_0(I)$ can be regarded as the solution to a lump-sum constrained optimization problem when the lump-sum investment level is I. Each $\Pi_0(I)$ corresponds to a unique pricing strategy and network growth trajectory, as will be demonstrated in Proposition 2 below. Intuitively, $\Pi_0(I)$ increases in I when $I \leqslant I^*$, where I^* is the optimal time-zero value of investment derived in Section 2.2,[15] and $\Pi_0(I^*) = \Pi^{m^*}(0)$.

In this model, the aggregate investment level I is endogenously chosen by the entrepreneur. To be exact, the amount of funds raised each round I_j, and the number of rounds n, are all endogenously determined by the entrepreneur's profit-maximization objective, subject to the incentive compatibility constraint for him not to embezzle funds and abscond, as well as the individual rationality constraint for him to launch the platform enterprise and raise external financing to invest in the networks.

[14] For simplicity of the problem, all the values in this section are denoted using the time-zero value.

[15] $I^* = \int_0^{\tau^*} e^{-rt} \frac{m^*(t)[m^*(t)-b]}{2} N^*(t)^{(1+\eta)} dt$, where τ^* is the time point when the platform charges zero spread, or $m^*(\tau^*) = 0$. The entrepreneur will rationally raise $I \leqslant I^*$.

The problem of the entrepreneur is summarized as follows:

$$
\begin{aligned}
& \max_{n, I_1, I_2, \ldots, I_n} \quad \Pi_0(I) - F - nC - W \\
s.t. \quad & \Pi_0(I) - F - nC - W \geqslant I_1 && (IC_1) \\
& \ldots \\
& \Pi_0(I) - F - nC - W \geqslant I_n && (IC_n) \\
& \Pi_0(I) - F - nC - W \geqslant \max\{\Pi_0(0) - F,\ 0\} && (IR) \\
& \text{where} \quad I = \sum_{j=1}^{n} I_j,\ W = \sum_{j=1}^{n} W_j,
\end{aligned}
\tag{P2}
$$

and $\Pi_0(I)$ is the solution to the following problem:

$$
\begin{aligned}
& \max_{m(t)} \int_0^{\infty} e^{-rt} \, \frac{m(t)[b - m(t)]}{2} \, N(t)^{(1+\eta)} \, dt \\
s.t. \quad & \dot{N}(t) = \begin{cases} \dfrac{\lambda}{2b}[b - m(t)]N(t), & if \ N(t) < \bar{N}, \\ 0, & if \ N(t) = \bar{N}, \end{cases} \\
\& \quad & [b - m(t)]^2 N(t)^{\eta} \geqslant b^2, \ \forall t, \\
\& \quad & \int_0^{t} e^{-rx} \, \frac{m(x)[m(x) - b]}{2} \, N(x)^{(1+\eta)} \, dx \leqslant I, \ \forall t.
\end{aligned}
$$

The entrepreneur maximizes the time-zero value of his expected profits from optimally and successfully managing the platform enterprise, subject to the incentive compatibility (IC) constraints and the individual rationality (IR) constraint. The incentive compatibility constraints say that the funds the entrepreneur receives each round should be no more than what he expects to gain from managing the platform enterprise. Otherwise, he would abscond with funds. The individual rationality constraint says that if the entrepreneur expects to get too little from raising funds and optimally investing in the networks, he would instead choose not to make the investment or not to launch the platform enterprise, *ex ante*.

2.3.2 Model Solutions and Analysis

To solve Problem (P2), we need to solve for $\Pi_0(I)$ first. As is defined, $\Pi_0(I)$ is the optimized time-zero value of all discounted future profits of the platform, with the constraint that the investment level is I. We can solve for the constraint-optimal pricing strategy $m(t)$ and the network growth function $N(t)$ to get $\Pi_0(I)$. A detailed derivation of $\Pi_0(I)$ is given in Proof of Proposition 2 in Appendix.

Lemma 2.3 *The constrained optimal pricing strategy $m(t)$ is non-decreasing.*

Lemma 2.3 shows that the platform's subsidy stages and commission stages are clearly divided. It is inefficient to charge a commission for a period and use the profits as internal funds to make subsidies. Therefore, the platform will never swing between subsidies and commission. The proof of Lemma 2.3 may be found in Appendix. Equipped with Lemma 2.3, we can solve for $m(t)$, $N(t)$, and $\Pi_0(I)$. The results are provided in Proposition 2.2.

Proposition 2.2 *Let $a > 1$ and $\bar{N} \geqslant 4^{\frac{1}{\eta}}$. Define τ^* as the time point when the unconstrained optimal spread is zero, or $m^*(\tau^*) = 0$; define I^* as the optimal aggregate subsidy amount, or $I^* = \int_0^{\tau^*} e^{-rt} \frac{m^*(t)[m^*(t)-b]}{2} N^*(t)^{(1+\eta)} dt$. When the available aggregate subsidy amount $I \leqslant I^*$, the constraint-optimal policy function $m(t)$, network growth $N(t)$, and the time-zero value of discounted future profits $\Pi_0(I)$, are as follows:*

$$m(t) = \begin{cases} \frac{b}{a}\sqrt{a-1}\tan(\frac{r}{2}\sqrt{a-1}\cdot t + c_3) + \frac{b}{a}(a-1) & , \ t \leqslant \underline{\tau} \\ 0 & , \ \underline{\tau} < t \leqslant \tilde{\tau} \\ m^*(t - \tilde{\tau} + \tau^*) & , \ t > \tilde{\tau} \end{cases}$$

$$N(t) = \begin{cases} e^{\frac{r}{1+\eta}t + c_4} \cdot [\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_3)]^{\frac{2}{1+\eta}} & , \ t \leqslant \underline{\tau} \\ e^{\frac{\lambda}{2}(t-\underline{\tau})} \cdot N(\underline{\tau}) & , \ \underline{\tau} < t \leqslant \tilde{\tau} \\ N^*(t - \tilde{\tau} + \tau^*) & , \ t > \tilde{\tau} \end{cases}$$

$$\Pi_0(I) = e^{-r\tilde{\tau}} \cdot \Pi^{m^*}(\tau^*) - I,$$

where c_3, c_4, $\underline{\tau}$, $\tilde{\tau}$ are determined by terminal conditions:
$N(0) = 1$, $N(\tilde{\tau}) = N^*(\tau^*)$, $m(\underline{\tau}) = 0$ *and* $\int_0^{\underline{\tau}} e^{-rt} \frac{m(t)[m(t)-b]}{2} N(t)^{(1+\eta)} dt = I$.

REMARK 2.3 *When the aggregate amount of available funds is less than the optimal amount, it is constraint-optimal for the platform to make highly aggressive subsidies early on and use up the available funds, and then wait with zero spread until it's optimal to start to charge a commission.*

The results highlight the importance of building up the networks at early stages. Timely and sufficient investment in the networks is crucial for the success and high valuation of a platform enterprise. If the entrepreneur misses the best timing to make investment, the platform enterprise will suffer from a lower valuation or even failure to survive. A prominent example is the failure of SideCar, as mentioned in the introduction of this paper. Therefore, it is rational for the entrepreneur running the platform to use up available funds to make heavy subsidies early on.

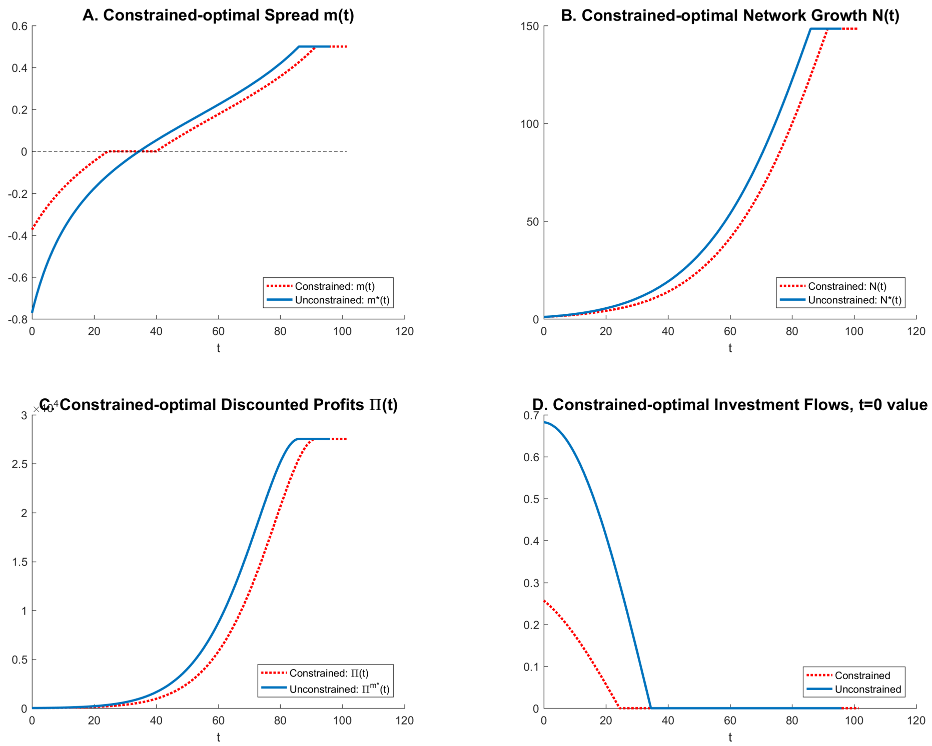
Figure 2.3 plots an example of constrained-optimal pricing strategy, network growth pattern, the value of discounted profits, and investment flows, compared with the unconstrained benchmarks the same as in Figure 2.2.

Having got $\Pi_0(I)$, I now set out to characterize the solutions for the entrepreneur's problem (P2). First, the entrepreneur's optimal financing patterns are summarized in Proposition 2.3.

Proposition 2.3 *Each round of financing has the same time-zero value. Thus, the face value (time-t value) of the funds raised each round keeps increasing. The frequency of new rounds keeps decreasing in most of the economically meaningful cases, where a sufficient condition is* $a > \frac{5}{4}$, *and* $\bar{N} < \bar{N}^*$ *such that* $m(0) > -b$.

In the entrepreneur's problem stated above, it is obvious that setting $I_1 = I_2 = ... = I_n = \frac{I}{n}$ weakly dominates all other strategies, because it is the strategy where the incentive compatibility constraint is easiest to be satisfied. That is, the face value (time-t value) of each round of financing is increasing. Besides, with the decreasing flows

Figure 2.3: Numerical examples: with financial constraints



Parameters: $b = 1$, $r = 0.1$, $\lambda = 0.12$, $\bar{N} = e^5$, $\eta = 1$.
The constrained case has $I = 3.5$ and $\Pi_0(I) = 14.16$.

of investment, as shown in Figure 2.2(D) and Figure 2.3(D), the time intervals between two rounds of financing are inclined to increase. A through proof of Proposition 2.3 is in Appendix.

How do the financing and investment decisions vary under different scenarios? In this paper, I analyze the effects of profitability and capital market conditions.

A. Profitability and Rounds of Financing

Proposition 2.4 *Ceteris Paribus, a platform enterprise with higher profitability* ($\frac{\Pi_0(I)-F}{I}$) *raises fewer rounds of financing.*

This proposition seems counter-intuitive, but it is actually a general result when the major financial friction is limited enforcement. All things equal, the more profitable the enterprise, the more profits left to the entrepreneur. Hence, he can credibly raise more funds within each round, which leads to fewer rounds of financing.

Proof of Proposition 2.4 is simple and direct. Using results in Proposition 2.3, we can rewrite the incentive compatibility constraints as: $\frac{\Pi_0(I)-F}{I} \geqslant n\frac{C}{I} + \frac{1}{n} + \frac{W}{I}$. Higher profitability makes the (IC) constraints easier to be satisfied, and thus leads to a smaller n required, so as to maximize the entrepreneur's profits.

B. Influences of Capital Market Conditions

In practice, the capital market is usually not fully competitive, and financiers may require positive profits to their investments (i.e. $W > 0$). If the capital market is abundant with money, financiers tend to require lower profits. On the other hand, if a lot of good projects are waiting to be financed in the capital market, the opportunity costs are higher and the required profits by financiers increase. The level of required profits by the financiers affects the entrepreneur's financing and investment decisions.

Proposition 2.5 *Ceteris Paribus, the financing and investment levels (I) weakly decrease with the required profits by the financiers (W), while the number of financing rounds (n) weakly increases with the required profits by the financiers (W) as long as the deadweight-loss financing cost (C) is not too high, where a sufficient condition is* $\frac{C}{I/n} < \frac{1}{n}$.

The underlying logic of Proposition 2.5 is similar to that of Proposition 2.4. The more profits required by the financiers, the fewer profits left to the entrepreneur, and thus the less funding the entrepreneur can credibly raise within one round. This results in a trade-off between raising lower aggregate level of funds to make less investment, and raising more rounds of financing to make adequate investment, depending on the cost of financing and relative profitability of the investment. Or mathematically, because I is a continuous choice variable while n is a discrete variable, the entrepreneur will optimally choose to decrease I and increase n by turns when W continues increasing. When W becomes excessively large, the participation constraint will bind and the entrepreneur will choose not to finance. Proof of Proposition 5 may be found in Appendix. Below is an example demonstrating the relationship between the required profits of financiers and the entrepreneur's financing decisions.

An Example. *The parameters for the platform are the same as in Figure 2.2 and 2.3:* $b = 1$, $r = 0.1$, $\lambda = 0.12$, $\eta = 1$, $\bar{N} = e^5$. $F = 10$, $C = 0.1$. *The relationship between the required profits of financiers and the entrepreneur's financing decisions are shown in the table:*

Required Profits: W	**Financing Rounds:** n	**Aggregate Financing Level:** I
$0 - 0.43$	3 rounds	$I = I^*$
$0.43 - 1.48$	3 rounds	$I < I^*$, and decreases with W
$1.48 - 1.57$	4 rounds	$I = I^*$
$1.57 - 2.34$	4 rounds	$I < I^*$, and decreases with W
$2.34 - 2.84$	5 rounds	$I < I^*$, and decreases with W
$2.84 - 3.15$	6 rounds	$I < I^*$, and decreases with W
$3.15 - 3.32$	7 rounds	$I < I^*$, and decreases with W
$3.32 - 3.43$	8 rounds	$I < I^*$, and decreases with W
$3.43 - 3.47$	9 rounds	$I < I^*$, and decreases with W
> 3.47	Not to finance	$I = 0$

Table 2.1: An Example

2.4. Conclusion

This paper develops a tractable micro-founded dynamic platform model. The defining property of a platform market is the existence of cross-group network effects. Networks are analogous to capital-assets of the platform enterprise. The platform enterprise invests in the networks first and generates income from the networks later on. With inadequate internal funds, the enterprise has to raise external capital. The paper solves for the optimal financing and investment strategies of the entrepreneur in a world with financial frictions and financing costs. Meanwhile, the paper depicts the platform market growth patterns.

The key findings of this paper are that: 1) in face of financial constraints, a monopolistic platform should make aggressive subsidies by using up available funds to boost network growth early on; 2) the optimal spread charged by the platform is non-decreasing over time, i.e. per-transaction subsidies decrease over time and are followed by increasing fees; 3) with stronger network effects, the platform has a propensity to make more subsidies early one and enjoys a higher valuation and faster network growth; 4) staging is a natural choice to mitigate financial frictions and *ceteris paribus*, the number of financing rounds decrease with profitability of the platform and increases with required profits by financiers; 5) the value of funds raised each round increases and the financing frequency decrease for a platform enterprise.

There are some other interesting questions this paper has not discussed, which might be directions for future research. First, what if there exists platform competition or potential entry of new platforms? How will financial frictions and costs interact with competition and entry threat? Whether it will be a winner-takes-all equilibrium and whether the deep pocket matters most are still not that clear. Second, what if there exists uncertainty in this market, say, uncertainty on the network growth path (the law of motion)? How will the uncertainty affect optimal financing and investment decisions? It may also be interesting to study other forms of incentive problems, such as asymmetric information between the platform and financiers. I leave studies on these questions to future research.

2.5. Appendix

Proof of Lemma 2.2

Assume the no-exit constraint does not bind, then $m^*(t)$ and $N^*(t)$ are as given in Proposition 1. Therefore, I only need to prove $m^*(t)$ and $N^*(t)$ given in Proposition 1 satisfy $[b - m^*(t)]^2 N^*(t)^\eta \geqslant b^2$ when $m^*(0) < 0$ and $\bar{N} \geqslant 4^{\frac{1}{\eta}}$. Since $m^*(t)$ will never be greater than the monopolistic price $\frac{b}{2}$, so we always have $b - m^*(t) > 0$. Thus, the no-exit constraint is equivalent to $[b - m^*(t)]N^*(t)^{\frac{\eta}{2}} \geqslant b$. Below is the proof.

Define $\theta_t = \frac{r}{2}\sqrt{a-1} \cdot t + c_1, \in (-\frac{\pi}{2}, \frac{\pi}{2})$. θ_t is increasing in t. Then, $\log\{[b - m^*(t)]N^*(t)^{\frac{\eta}{2}}\} = \log(1 - \sqrt{a-1}\tan\theta_t) + \frac{\eta}{1+\eta}[\log(\cos\theta_t)] + \mathit{constant}$. Denote it by $f(\theta_t)$. We then have:

$f(\theta_0) = \log[b - m^*(0)] > \log(b)$, and $f(\theta_{T^*}) = \log(\frac{b}{2}\bar{N}^{\frac{\eta}{2}}) \geqslant \log(b)$.

$$\begin{aligned} f'(\theta_t) &= \frac{-\sqrt{a-1}}{1 - \sqrt{a-1}\tan\theta_t}(1 + \tan^2\theta_t) + \frac{\eta}{1+\eta}[-\tan\theta_t + \frac{1}{\sqrt{a-1}}] \\ &= \underbrace{\frac{1}{\sqrt{a-1} - (a-1)\tan\theta_t}}_{>0} \underbrace{[-(a-1)(1+\tan^2\theta_t) + \frac{\eta}{1+\eta}(1 - \sqrt{a-1}\tan\theta_t)^2]}_{g(\theta_t)}. \end{aligned}$$

Since $b - m^*(\theta_t) > 0$, we have $\sqrt{a-1} - (a-1)\tan\theta_t > 0$.

Define $g(\theta_t) = -(a-1)(1+\tan^2\theta_t) + \frac{\eta}{1+\eta}(1 - \sqrt{a-1}\tan\theta_t)^2$. We get:

$$g(\theta_t) = -\frac{1}{1+\eta}(\sqrt{a-1}\tan\theta_t + \eta)^2 + (1 + \eta - a).$$

Case 1. $a \geqslant 1 + \eta$**:**

then we have $g(\theta_t) < 0$, thus $f'(\theta_t) < 0$, $f(\theta_t)$ is monotone decreasing. Since $f(\theta_{T^*}) \geqslant \log(b)$, we get $f(\theta_t) \geqslant \log(b)$, $\forall \theta_t \in [\theta_0, \theta_{T^*}]$. Thus, $\forall t \in [0, T^*]$, the no-exit constraint is satisfied.

Case 2. $a < 1+\eta$**:**

Since $m^*(0) < 0$, we get $\tan\theta_0 < -\sqrt{a-1}$. And from $m^*(T^*) = \frac{b}{2}$, we get $\theta_{T^*} = \frac{2-a}{2\sqrt{a-1}}$. Thus, $\exists\ \hat{t} \in (0, T^*)$ where $m^*(\hat{t}) = 0$, $\tan\theta_{\hat{t}} = -\sqrt{a-1}$.

For $t \in [0, \hat{t}]$, $m^*(t) \leqslant 0$, the no-exist constraint is satisfied. We now want to prove that the no-exist constraint is always satisfied when $t \in (\hat{t}, T^*]$, i.e., when $\tan\theta_t \in (-\sqrt{a-1}, \frac{2-a}{2\sqrt{a-1}}]$.

We take derivative of $g(\theta_t)$:

$$g'(\theta_t) = -\frac{2\sqrt{a-1}}{1+\eta}(\eta + \sqrt{a-1}\tan\theta_t).$$

$g'(\theta_t)$ decreases in $\tan\theta_t$. So $g'(\theta_t) < g'(\theta_{\hat{t}}) = -\frac{2\sqrt{a-1}}{1+\eta}(\eta + 1 - a) < 0$, since we are in Case 2. That is, $g(\theta_t)$ is monotone decreasing for $\theta_t \in (\theta_{\hat{t}}, \theta_{T^*}]$.

$g(\theta_{\hat{t}}) = -\frac{1}{1+\eta}(1+\eta-a)^2 + (1+\eta-a) = \frac{a}{1+\eta}(1+\eta-a) > 0.$

$g(\theta_{T^*}) = -\frac{1}{1+\eta}(1+\eta-\frac{a}{2})^2 + (1+\eta-a) = -\frac{a^2}{4(1+\eta)} < 0.$

So, $f(\theta_t)$ first increases and then decreases. That is, $f(\theta_t)$ is quasi-concave on $(\theta_{\hat{t}}, \theta_{T^*}]$. Since $f(\theta_{\hat{t}}) \geqslant \log(b)$, $f(\theta_{T^*}) \geqslant \log(b)$, $f(\theta_t) \geqslant \log(b)$ is always satisfied on $(\theta_{\hat{t}}, \theta_{T^*}]$.

To summarize Case 1 and Case 2, the no-exit constraint does not bind for on the optimal path if $m^*(0) < 0$ and $\bar{N} \geqslant 4^{\frac{1}{\eta}}$. ■

Proof of Proposition 2.1

Let T^* denote the time when the platform reaches its maximum size, i.e., $N(T^*) = \bar{N}$. When $t > T^*$, the network size of each side is constant $\bar{N}$. So the optimal pricing strategy is the monopoly pricing: $m^*(t) = \frac{b}{2}$. Thus, $\Pi^{m^*}(t) = \frac{b^2}{8r}\bar{N}^{(1+\eta)}$, $\forall t \geqslant T^*$. When $t \leqslant T^*$, it's a optimal control problem with fixed end points. We can apply techniques in calculus of variations to solve it.

First, define $h(t) = \log N(t)$, so $\dot{h}(t) = \frac{\dot{N}(t)}{N(t)} = \frac{\lambda}{2b}[b - m(t)]$. We can rewrite the problem as:

$$\max_{h(t)} \int_0^{T^*} (\frac{b}{\lambda})^2 \, e^{-rt+(1+\eta)h(t)} \, \dot{h}(t)[\lambda - 2\dot{h}(t)] \, dt$$

$$s.t. \quad h(0) = 0, \;\; h(T^*) = \log \bar{N}, \;\; \Pi(T^*) = \frac{b^2}{8r} \, \bar{N}^{(1+\eta)}.$$

Apply the Euler-Lagrange Equation, $\frac{\partial g}{\partial h} = \frac{d}{dt}(\frac{\partial g}{\partial \dot{h}})$, where

$$g(\; h(t), \;\; \dot{h}(t), \;\; t) = (\frac{b}{\lambda})^2 \, e^{-rt+(1+\eta)h(t)} \, \dot{h}(t)[\lambda - 2\dot{h}(t)],$$

and we get

$$(1+\eta)\dot{h}^2(t) - 2r\dot{h}(t) + 2\ddot{h}(t) + \frac{r\lambda}{2} = 0. \tag{A-1}$$

Since $\dot{h}(t) = \frac{\lambda}{2b}[b - m(t)]$, $\ddot{h}(t) = -\frac{\lambda}{2b}\dot{m}(t)$. Plug them into (A-1):

$$\dot{m}(t) = \frac{(1+\eta)\lambda}{4b} m^2(t) + [r - \frac{(1+\eta)\lambda}{2}] \, m(t) + \frac{(1+\eta)\lambda b}{4} - \frac{br}{2}, \tag{A-2}$$

$$\Rightarrow \qquad \frac{(1+\eta)\lambda}{4b} \, dt = \frac{dm(t)}{m^2(t) + 2b[\frac{2r}{(1+\eta)\lambda} - 1] \, m(t) + b^2 - \frac{2rb^2}{(1+\eta)\lambda}}. \tag{A-3}$$

The denominator on the right-hand side of (A-3) is a quadratic form, whose discriminant is $\Delta = 4b^2[\frac{2r}{(1+\eta)\lambda} - 1]^2 - 4[b^2 - \frac{2rb^2}{(1+\eta)\lambda}] = \frac{4b^2}{a^2}(1-a)$, where $a \equiv \frac{\lambda(1+\eta)}{2r}$. So we can divide the situation into three cases according to the range of Δ.

Case 1. $a > 1$, that is $\Delta < 0$.

Rewrite (A-3) as

$$\frac{ar}{2b} \, dt = \frac{dm(t)}{[m(t) - \frac{b}{a}(a-1)]^2 + \frac{b^2}{a^2}(a-1)} \; ,$$

and integrate each side of the equation. [1] Then we get when $t < T^*$,

$$m^*(t) = \frac{b}{a}\sqrt{a-1} \tan(\frac{r}{2}\sqrt{a-1} \cdot t + c_1) + \frac{b}{a}(a-1), \tag{A-4}$$

[1] Formula: $\int \frac{1}{x^2+v^2} dx = \frac{1}{v} \; \tan^{-1} \; (\frac{x}{v})$.

where c_1 is the constant of integration to be determined by terminal conditions.

To find $h^*(t)$ when $t < T^*$:

$$\begin{aligned}
h^*(t) &= \int_0^t \dot{h}^*(t)\, dt \\
&= \int_0^t \frac{\lambda}{2b}[b - m^*(t)]\, dt \\
&= \int_0^t \frac{\lambda}{2a}[1 - \sqrt{a-1}\tan(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)]\, dt \\
&= \frac{\lambda}{2a}t - \frac{\lambda}{ar}\log[\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)] + c_2 \\
&= \frac{r}{1+\eta}t + \frac{2}{1+\eta}\log[\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)] + c_2
\end{aligned}$$

where c_2 is also a constant to be determined by terminal conditions. [2]

Thus when $t < T^*$,

$$N^*(t) = e^{\frac{r}{1+\eta}t + c_2} \cdot [\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)]^{\frac{2}{1+\eta}} \; . \tag{A-5}$$

A quick way to derive the value function is to use the HJB equation:

$$r\Pi^{m^*}(h(t)) = \max_{m(t)} \{\frac{m(t)[b - m(t)]}{2} e^{(1+\eta)h(t)} + \frac{d\Pi}{dh}\dot{h}(t)\} \; . \tag{A-6}$$

Take F.O.C. with respect to $m(t)$:

$$\frac{d\Pi}{dh}(t) = \frac{2b}{\lambda}[\frac{b}{2} - m^*(t)]\, e^{(1+\eta)h^*(t)} \; .$$

Plug it into (A-6), and we get the value function:

$$\Pi^{m^*}(t) = \frac{1}{2r}N^*(t)^{(1+\eta)}[b - m^*(t)]^2 \; . \tag{A-7}$$

Since the value function is continuous at T^*, and $\Pi^{m^*}(T^*) = \frac{b^2}{8r}\bar{N}^{(1+\eta)}$, by applying

[2]Through out this paper, I define the domain of the trigonometric functions to be $(-\frac{\pi}{2}, \frac{\pi}{2})$, without loss of generality. Thus, $\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_1)$ is always positive.

(A-7), we get $m^*(T^*) = \frac{b}{2}$. Now we have three end points, $h(0) = 0$, $h(T^*) = \log \bar{N}$ and $m^*(T^*) = \frac{b}{2}$ to determine c_1, c_2 and T^*:

$$\begin{cases} \dfrac{2}{1+\eta}\log[\cos(c_1)+1] + c_2 = 0 \\ \dfrac{r}{1+\eta}T^* + \dfrac{2}{1+\eta}\log[\cos(\dfrac{r}{2}\sqrt{a-1}\cdot T^* + c_1)] + c_2 = \log \bar{N} \\ \dfrac{1}{a}\sqrt{a-1}\tan(\dfrac{r}{2}\sqrt{a-1}\cdot T^* + c_1) + \dfrac{1}{a}(a-1) = \dfrac{1}{2} \end{cases} \tag{A-8}$$

The existence is demonstrated in Figure 2.2.

Case 2. $a = 1$, that is $\Delta = 0$.

Rewrite (A-3) as

$$\frac{ar}{2b}\,dt = \frac{dm(t)}{m^2(t)} = -\,dm^{-1}(t) \ ,$$

$$\Rightarrow \qquad m(t) = -\frac{1}{\frac{ar}{2b}t - c_3} \ .$$

The constant c_3 must be positive, so that we can get a positive $m(t)$ when $t > 0$, as required by the problem. This leads to $m(0) = \frac{1}{c_3} > 0$. Since $\dot{m}(t) = \frac{ar}{2b}m^2(t) \geqslant 0$, we always have $m(t) > 0$. That is there is no investment in network in this case, or the platform always charges a positive commission on both sides.

Case 3. $0 < a < 1$, that is $\Delta > 0$.

Rewrite (A-3) as

$$\frac{ar}{2b}dt = \frac{dm(t)}{[m(t)-\alpha][m(t)-\beta]} \ ,$$

where

$$\alpha + \beta = \frac{2b}{a}(a-1) < 0, \quad \alpha\beta = \frac{b^2}{a}(a-1) < 0 \ .$$

Let $\beta > 0$ and $\alpha < 0$, then:

$$\beta - \frac{b}{2} = \frac{b}{a}(\frac{a}{2} - 1\sqrt{1-a}) < 0, \qquad \forall\ 0 < a < 1.$$

Since $\dot{m}(t) = \frac{ar}{2b}[m(t) - \alpha][m(t) - \beta]$, we must have $m(0) > \beta > 0$. Otherwise, $m(t)$ can never grow to $\frac{b}{2}$ and will never satisfy the terminal conditions. To conclude, there is no investment in network in this case, or the platform always charges a positive commission on both sides. ■

Proof of Lemma 2.3

When $a \leqslant 1$, $m^*(0) > 0$ and there is no subsidy stage. Thus the constraint never binds and the constraint-optimal pricing strategy is still $m^*(t)$, which is increasing.

When $a > 1$, let's prove by contradiction.

Suppose there is a segment of $m(t)$ that is decreasing. $m_{t_0-\Delta t}$ and m_{t_0} are two adjacent points along that segment, $m_{t_0-\Delta t} > m_{t_0}$. Denote the corresponding network sizes by $N_{t_0-\Delta t}$ and N_{t_0}, and normalize $N_{t_0-\Delta t}$ to be 1. So, $N_{t_0} = N_{t_0-\Delta t} \cdot [1 + \frac{\lambda}{2b}(b - m_{t_0-\Delta t})\Delta t] = 1 + \frac{\lambda}{2b}(b - m_{t_0-\Delta t})\Delta t$, and the network size after these two periods is: $N_{t_0+\Delta t} = [1 + \frac{\lambda}{2b}(b - m_{t_0-\Delta t})\Delta t][1 + \frac{\lambda}{2b}(b - m_{t_0})\Delta t]$. Note that in this proof, we use discrete-time approximations. When $\Delta t \to 0$, they converge to the continuous-time results.

Define $\hat{m} = \frac{m_{t_0-\Delta t}+m_{t_0}}{2}$. Thus, $m_{t_0-\Delta t} > \hat{m} > m_{t_0}$. We would like to prove that, whenever there is a decreasing pricing strategy $(m_{t_0-\Delta t}, m_{t_0})$, we can find a non-decreasing strategy $(\hat{m}, \hat{m})$ that weakly dominates it. That is, we need to prove that, 1) $(\hat{m}, \hat{m})$ is within the entrepreneur's action space, 2) it leads to weakly faster network growth, and 3) it leads to weakly higher profits.

If $\hat{m} \geqslant 0$, it's obvious that the lump-sum subsidy level constraint won't bind after choosing the strategy $(\hat{m}, \hat{m})$. If $\hat{m} < 0$, since the strategy $(\hat{m}, \hat{m})$ generates weakly higher profits (i.e. make weakly lower subsidies) than the original strategy $(m_{t_0-\Delta t}, m_{t_0})$, as will be proven later, the lump-sum subsidy level constraint won't bind after these two periods, since the original strategy satisfy the constraint. And by backward induction, the constraint will also not bind in the intermediate period. In this way, we have proven that $(\hat{m}, \hat{m})$ is indeed within the entrepreneur's action space.

As has been shown, with strategy $(m_{t_0-\Delta t}, m_{t_0})$, the network size after these two periods is $N_{t_0+\Delta t} = [1 + \frac{\lambda}{2b}(b - m_{t_0-\Delta t})\Delta t][1 + \frac{\lambda}{2b}(b - m_{t_0})\Delta t]$. If we instead choose $(\hat{m}, \hat{m})$, the network size after two periods becomes $\hat{N}_{t_0+\Delta t} = [1 + \frac{\lambda}{2b}(b - \hat{m})\Delta t]^2$. $\hat{N}_{t_0+\Delta t} - N_{t_0+\Delta t} = \frac{\lambda^2 \Delta t^2}{4b^2}(\hat{m}^2 - m_{t_0-\Delta t} m_{t_0}) = \frac{\lambda^2 \Delta t^2}{16b^2}(m_{t_0-\Delta t} - m_{t_0})^2 > 0$. Thus, $(\hat{m}, \hat{m})$ indeed leads to faster network growth.

The profits generated by the original strategy $(m_{t_0-\Delta t}, m_{t_0})$ during these two periods are $\Pi = \frac{1}{2} m_{t_0-\Delta t}(b - m_{t_0-\Delta t}) + \frac{1}{2} m_{t_0}(b - m_{t_0}) e^{-r\Delta t} N_{t_0}^{1+\eta}$, while the profits generated by the strategy $(\hat{m}, \hat{m})$ are $\hat{\Pi} = \frac{1}{2}\hat{m}(b - \hat{m})(1 + e^{-r\Delta t}\hat{N}_{t_0}^{1+\eta})$. We would like to prove that $\Pi - \hat{\Pi} < 0$.

For notational simplicity, denote $m_{t_0-\Delta t}$ and m_{t_0} by m_1 and m_2, respectively. So, $-b \leqslant m_2 < \hat{m} < m_1 \leqslant b/2$, and $\hat{m} = (m_1 + m_2)/2$.

$$
\begin{aligned}
& \Pi - \hat{\Pi} < 0 \\
\Leftrightarrow \quad & m_1(b - m_1)e^{r\Delta t} + m_2(b - m_2)[1 + \frac{\lambda}{2b}(b - m_1)\Delta t]^{1+\eta} \\
& - \hat{m}(b - \hat{m})e^{r\Delta t} - \hat{m}(b - \hat{m})[1 + \frac{\lambda}{2b}(b - \hat{m})\Delta t]^{1+\eta} < 0 \ , \\
\Leftrightarrow \quad & m_1(b - m_1)(1 + r\Delta t) + m_2(b - m_2)[1 + \frac{\lambda(1+\eta)}{2b}(b - m_1)\Delta t] \\
& - \hat{m}(b - \hat{m})(1 + r\Delta t) - \hat{m}(b - \hat{m})[1 + \frac{\lambda(1+\eta)}{2b}(b - \hat{m})\Delta t] < 0 \ ,
\end{aligned}
$$

where we apply $\lim\limits_{\Delta t \to 0} e^{r\Delta t} = 1 + r\Delta t$, $\lim\limits_{x \to 0}(1 + x)^{1+\eta} = 1 + (1 + \eta)x$,

$$\Leftrightarrow \quad (2\hat{m}^2 - m_1^2 - m_2^2) +$$
$$\Delta t \left\{ r[m_1(b-m_1) - \hat{m}(b-\hat{m})] + \frac{\lambda(1+\eta)}{2b}[m_2(b-m_2)(b-m_1) - \hat{m}(b-\hat{m})^2] \right\} < 0 \ ,$$
$$\Leftrightarrow \quad -\frac{(m_1-m_2)^2}{2} +$$
$$\Delta t \frac{\lambda(1+\eta)}{2b}[bm_1(b-m_1) - b\hat{m}(b-\hat{m}) + m_2(b-m_2)(b-m_1) - \hat{m}(b-\hat{m})^2] < 0 \ ,$$

since $[m_1(b-m_1) - \hat{m}(b-\hat{m})] > 0$ and $a = \frac{\lambda(1+\eta)}{2r} > 1$.

With some algebra, we finally get that the above inequality is equivalent to:

$$-\frac{(m_1-m_2)^2}{2} - \Delta t \frac{\lambda(1+\eta)}{2b}(m_1-\hat{m})[(b+m_2)(m_1-\hat{m}) + \hat{m}^2] < 0 \ ,$$

which is true. So we have proven that $\Pi - \hat{\Pi} < 0$.

Combining 1), 2), 3), we have proven Lemma 3. ■

Proof of Proposition 2.2

The terminal conditions $N(T) = \bar{N}$ and $m(T) = \frac{b}{2}$ are the same as in the non-constrained problem, but with $T > T^*$. According to Lemma 3, $m(t)$ is non-decreasing. So, there exists $\tilde{\tau}$ such that $m(t) \leqslant 0$ when $t \leqslant \tilde{\tau}$ and $m(t) > 0$ when $t > \tilde{\tau}$.

According to the Principle of Optimality, if the terminal values are determined and there are no additional constraints, we can find the same optimal path by backward induction. That is to say, for $t \in [\tilde{\tau}, T]$, the optimal path will be exactly the same as in the non-constrained problem when $t \in [\tau^*, T^*]$, where τ^* satisfies $m^*(\tau^*) = 0$. Thus, we must have $m(\tilde{\tau}) = 0$, $N(\tilde{\tau}) = N^*(\tau^*)$, $\Pi(\tilde{\tau}) = \Pi^{m^*}(\tau^*)$.

Then, $\Pi_0(I) = e^{-r\tilde{\tau}}\Pi(\tilde{\tau}) - I = e^{-r\tilde{\tau}}\Pi^{m^*}(\tau^*) - I$.

The problem turns into finding minimum $\tilde{\tau}$ given aggregate subsidy level I. And its dual problem is to find minimum I given $\tilde{\tau}$. Let's solve the dual problem:

$$\min_{m(t)} \int_0^{\tilde{\tau}} e^{-rt} \frac{m(t)[m(t)-b]}{2} N(t)^{1+\eta} dt$$

$$s.t. \qquad m(t) \leqslant 0, \ \forall t,$$

$$\dot{N}(t) = \frac{\lambda}{2b}[b - m(t)]N(t),$$

$$N(0) = 1, \ N(\tilde{\tau}) = N^*(\tau^*).$$

Again, define $h(t) = \log N(t)$, so $\dot{h}(t) = \frac{\dot{N}(t)}{N(t)} = \frac{\lambda}{2b}[b - m(t)]$. This problem is similar to the non-constrained problem, but with additional constraints, $m(t) \leqslant 0$, and different terminal conditions.

Because $m(t)$ is weakly increasing, let's denote the time $m(t)$ first touches zero by $\underline{\tau}$, $\underline{\tau} \leqslant \tilde{\tau}$. So, $m(t) < 0$ when $t \in [0, \underline{\tau})$ and $m(t) = 0$ when $t \in [\underline{\tau}, \tilde{\tau}]$. Thus, when $t \in [0, \underline{\tau})$, the additional constraint does not bind, and the solution to this problem follows the same Euler-Lagrange equation as in the non-constrained problem, only with different terminal conditions; when $t \in [\underline{\tau}, \tilde{\tau}]$, $m(t) = 0$, $\dot{h}(t) = \frac{\lambda}{2}$.

Since we have already known the terminal conditions, the constraint-optimal path is then determined:

$$m(t) = \begin{cases} \frac{b}{a}\sqrt{a-1}\tan(\frac{r}{2}\sqrt{a-1}\cdot t + c_3) + \frac{b}{a}(a-1) & , \ t \leqslant \underline{\tau} \\ 0 & , \ \underline{\tau} < t \leqslant \tilde{\tau} \\ m^*(t - \tilde{\tau} + \tau^*) & , \ t > \tilde{\tau} \end{cases}$$

$$N(t) = \begin{cases} e^{\frac{r}{1+\eta}t + c_4} \cdot [\cos(\frac{r}{2}\sqrt{a-1}\cdot t + c_3)]^{\frac{2}{1+\eta}} & , \ t \leqslant \underline{\tau} \\ e^{\frac{\lambda}{2}(t-\underline{\tau})} \cdot N(\underline{\tau}) & , \ \underline{\tau} < t \leqslant \tilde{\tau} \\ N^*(t - \tilde{\tau} + \tau^*) & , \ t > \tilde{\tau} \end{cases}$$

$$\Pi_0(I) = e^{-r\tilde{\tau}} \cdot \Pi^{m^*}(\tau^*) - I,$$

where c_3, c_4, $\underline{\tau}$, $\tilde{\tau}$ are determined by terminal conditions:
$N(0) = 1$, $N(\tilde{\tau}) = N^*(\tau^*)$, $m(\underline{\tau}) = 0$ and $\int_0^{\underline{\tau}} e^{-rt} \frac{m(t)[m(t)-b]}{2} N(t)^{(1+\eta)} dt = I$. ■

Proof of Proposition 2.3

The first part of Proposition 3 is already proven. Let's prove the second part.

The constrained-optimal time-t subsidy flows at time-zero value is:

$$
\begin{aligned}
& e^{-rt}\frac{m(t)[m(t)-b]}{2}N(t)^{(1+\eta)} \\
= & \frac{1}{2}e^{(1+\eta)c_4}(\frac{b}{a})^2[\sqrt{a-1}\tan\theta_t + a - 1][\sqrt{a-1}\tan\theta_t - 1]\cos^2\theta_t,
\end{aligned}
$$

where $\theta_t = \frac{r}{2}\sqrt{a-1}\cdot t + c_3$, $\theta_t \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Let $f(\theta_t) = [\sqrt{a-1}\tan\theta_t + a - 1][\sqrt{a-1}\tan\theta_t - 1]\cos^2\theta_t$, then

$$
\frac{df(\theta_t)}{d\theta_t} = 2(a-1)\sin(2\theta_t) + (a-2)\sqrt{a-1}\cos(2\theta_t).
$$

Since we are considering the subsidy stage, $m(t) < 0$. Thus, $\tan(\theta_t) < -\sqrt{a-1}$. We can restrict the range of θ_t to be $(-\frac{\pi}{2}, 0)$, and then $\cos(2\theta_t) < 0$.

$$
\frac{df(\theta_t)}{d\theta_t} < 0 \;\Leftrightarrow\; (a-2)\cot(2\theta_t) > -2\sqrt{a-1}. \tag{A-9}
$$

Case 1. $a > 2$:

$$
\frac{df(\theta_t)}{d\theta_t} < 0 \;\Leftrightarrow\; \cot(2\theta_t) > \frac{-2\sqrt{a-1}}{a-2}.
$$

Since $\cot(2\theta_t) = \frac{1}{2}[\frac{1}{\tan(\theta_t)} - \tan(\theta_t)]$ is decreasing in $\tan(\theta_t)$, we get $\cot(2\theta_t) > \frac{1}{2}(\sqrt{a-1} - \frac{1}{\sqrt{a-1}}) > 0$, the above inequality indeed holds.

Thus, when $a > 2$, the constrained-optimal subsidy flows (time-zero value) is decreasing with t.

Case 2. $1 < a < 2$**:**

$$\frac{df(\theta_t)}{d\theta_t} < 0 \Leftrightarrow \cot(2\theta_t) < \frac{2\sqrt{a-1}}{2-a}.$$

If we consider the cases where $m(0) > -b$,[3] then we must have $\tan\theta_t > \frac{1-2a}{\sqrt{a-1}}$.

$$\cot(2\theta_t) = \frac{1}{2}[\frac{1}{\tan\theta_t} - \tan\theta_t] > \frac{1}{2}(\frac{2a-1}{\sqrt{a-1}} - \frac{\sqrt{a-1}}{2a-1}).$$

Then, a sufficient condition for Inequality (A-9) to hold is that:

$$\frac{1}{2}(\frac{2a-1}{\sqrt{a-1}} - \frac{\sqrt{a-1}}{2a-1}) < \frac{2\sqrt{a-1}}{2-a}. \tag{A-10}$$

Let $y = a - 1$, then $y \in (0, 1)$. After some algebra, (A-10) is equivalent to

$$(4y-1)(y+1)^2 > 0$$

Thus, the sufficient condition for (A-9) to hold is $1 < a < 2$ and $m(0) > -b$ in Case 2. When $\bar{N}$ is not too large, or $\exists\ \bar{N}^*$ such that when $\bar{N} < \bar{N}^*$, we have $m(0) > -b$.

When $a = 2$, Inequality (A-9) indeed holds.

To summarize, a sufficient condition for the constrained-optimal subsidy flows to be decreasing in t is that: $a > \frac{5}{4}$, and $\bar{N} < \bar{N}^*$ such that $m(0) > -b$. ■

[3]In practice, platforms seldom give extremely large subsidies. For example, p_D is always positive.

Proof of Proposition 2.5

According to Proposition 3, we can rewrite the problem (P2) as:

$$\begin{aligned} &\max_{n,I}\ \Pi_0(I) - F - nC - W \\ s.t.\quad &\Pi_0(I) - F - nC - W \geqslant \frac{I}{n} \qquad (IC) \\ &\Pi_0(I) - F - nC - W \geqslant \max\{\Pi_0(0) - F, 0\} \qquad (IR) \end{aligned}$$

As we consider the case the entrepreneur participate in financing and subsidizing, let's omit the individual rationality constraint (IR) for a while. Then the Lagrangian function of this problem is:

$$\mathcal{L} = \Pi_0(I) - F - nC - W + \mu[\Pi_0(I) - F - nC - W - \frac{I}{n}].$$

Take F.O.C. with respect to I:

$$\Pi_0'(I) + \mu[\Pi_0'(I) - \frac{1}{n}] = 0$$

Since $\Pi_0(I)$ is weakly increasing in I, $\Pi_0'(I) \geqslant 0$. By definition, the Lagrangian multiplier $\mu \geqslant 0$. So we get $\Pi_0'(I) - \frac{1}{n} \leqslant 0$. That is, $\Pi_0(I) - \frac{I}{n}$ weakly decreases in I.

Let's rearrange the (IC) constraint as

$$\Pi_0(I) - \frac{I}{n} \geqslant F + nC + W.$$

Keep F and n constant and increases W. When the constraint does not bind, W does not affect I. When the constraint start to bind, increasing W must lead to an increase on the left hand side, and thus a decrease in I.

For the number of financing rounds n, *Ceteris Paribus*, increase in W leads to weakly increase in n as long as the financing cost C is not to large. A sufficient condition is $\frac{C}{I/n} < \frac{1}{n}$, which can be got directly from $(\frac{I}{n} + nC)$ decreasing in n.

Because I is a continuous choice variable while n is a discrete one, the entrepreneur will decrease I and increase n by turns when W is increasing, so as to maximize his expected profits. When W is too large, the participation constraint will bind and he will choose not to finance and make no subsidies. ■

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Chapter 3

Firms' Public SEO Method Choices and Subsequent Stock Performances

Joanne Juan Chen[1]

3.1. Introduction

Firms' seasoned equity offerings (SEOs) can be categorized into three major types by their offer methods: fully marketed offers (non-shelf SEOs), accelerated offers (shelf-SEOs), and rights offers. Among these three methods, the first two are public offering methods. The academic literature on the issue methods for SEOs has focused on rights offers versus public offers, and less attention is given to the comparisons between accelerated offers and fully marketed offers, also known as traditional bookbuilt offers.

Compared to traditional bookbuilt offering, shelf offering is a fast and low-cost

[1]This chapter is the thesis for Master of Research Degree in Finance at London School of Economics and Political Science during 2016 - 2018.

process for raising capital. It is introduced by the Securities and Exchange Commission (SEC) in 1982. Over time, the SEC looses rules of shelf registration to facilitate firms to raise capital. From 2008, the SEC began allowing firms with less than $75 million public floats to conduct shelf SEOs. [2] This financial deregulation, however, has raised concerns.

On the one hand, compared to traditional fully marketed seasoned equity offerings (non-shelf SEOs), shelf SEOs provide much less information about the firm and about the potential new investments: individual takedowns from the shelf (i.e. individual offering actions) are not subject to prior selective SEC staff review, and the firm is not required to provide more information about an offering beforehand either to the SEC or to the public. On the other hand, small firms *per se* are firms with high information asymmetry . The public know much less of a small firm than the insiders. This fact is also mentioned by the 2008 Amendment itself :

"It has been observed that the securities of smaller public companies are comparatively more vulnerable to price manipulation than the securities of larger public companies, and may also be more prone to financial reporting error and abuses...

A recent analysis of the reporting by public companies in response to SEC Staff Accounting Bulletin 108 found that (1) reporting errors at smaller public companies tend to be more significant than those of larger companies; and (2) smaller public companies are more likely to sit on errors that decrease earning than big companies."

So some small firms may take advantage of the information asymmetry by shelf SEOs and issue overpriced seasoned equities. However, the investors seem to be unaware of these concerns. According to Gustafson & Iliev (2017), this "removement of barriers" to small firms' equity issuance has resulted in a 49% increase in the annual probability of raising equity for small firms, and small firms' issuance discounts are on average similar to large firms. Besides, Bethel & Krigman(2005) document that between 1992

[2]In 2008, the SEC released *REVISIONS TO THE ELIGIBILITY REQUIREMENTS FOR PRIMARY SECURITIES OFFERINGS ON FORMS S-3 AND F-3* to make amendments to the shelf registration rule which was first introduced in 1982.
https://www.sec.gov/rules/final/2007/33-8878.pdf

and 2003, the average shelf SEO discount is 2.1%, compared to 2.6% for non-shelf SEOs.

I make the following inferences from the above facts: (1) a firm may take advantage of the information asymmetry and sell overpriced shelf SEOs when they do not have good investment opportunities, and thus result in the stocks' long-run underperformance; while a firm which has a better investment opportunity than that is expected by outside investors tends to choose non-shelf SEOs to provide more information about the new investment to the market, endorsed by the SEC and the underwriters; (2) this divergence is more severe for small firms, since the smaller the firm is, the nosier information the market has about it.

From some preliminary empirical work, I find that stocks of small shelf issued firms indeed have more severe negative abnormal returns than small non-shelf issued firms, and the firms also invest less; while there are no prominent differences among large firms.

I construct a three-date model to capture these long-run stock performance differences. The underlying variable in the model is the investment opportunity. Firms make two decisions: to choose the optimal level of new investments according to the investment opportunity, and to strategically choose the equity issuance techniques. The basic mechanism is that: a firm with investment opportunities worse than investors' expectations would like to conceal information and prefer shelf SEOs, while a firm with better investment opportunities than investors' expectations would like to reveal more information and it will trade off between more information revelation of non-shelf SEOs and low costs of shelf SEOs. The key assumption for this mechanism to work is that investors are of bounded rationality and don't regard shelf SEO itself as a negative signal. This assumption is reasonable because: (1) investors tend to update believes when they receive new information from the firm while tend to ignore the "signal " of no information provided by the firm; (2) firms of different sizes choose shelf SEO out of different reasons, some to conceal information and some only to reduce costs, so it is hard for investors to fully analyze and distinguish between those different purposes. This assumption represents the reality, as mentioned above: small firms' issuance discounts are on average similar to large firms (Gustafson and Ilieve, 2017) and the shelf SEO discount is even smaller than non-shelf SEO discount during 1992-2003 (Bethel and Krigman, 2005). I propose two propositions and three testable hypotheses from the model.

Then I empirically test the three hypotheses and find that the empirical evidence generally supports the hypotheses: (1) the stocks of small firms which have conducted shelf SEOs underperform with respect to expectations in the long run, while the stocks of small firms which have conducted non-shelf SEOs don't; the relative performance gap becomes less prominent when the firm size becomes larger; (2) small firms which have conducted shelf SEOs have less increases in investments than small firms which have conducted non-shelf SEOs; this difference becomes less prominent when the firm size becomes larger; (3) the difference in relative changes of investment levels is a major explanation of the performance gap between the two groups of small firms which have conducted shelf and non-shelf SEOs.

This paper makes contributions to the literature in the following aspects. (1) This is the first paper I know to compare different SEO methods from the perspective of information asymmetry. (2) This paper relates a firm's SEO method choice to its underlying investment opportunities and future investments, and thus explains the fundamental economic logic behind a firm's financing choice. (3) Instead of directly using long-run stock returns (or abnormal returns) to represent the firms' long-run performance, this paper analyzes the interaction between the firm and investors and explains the mechanism of long-run abnormal returns in a way that is consistent with the asset pricing literature.

The remainder of this paper is organized as follows. Section 3.2 summarizes relevant literature. Section 3.3 presents the theoretical model. Section 3.4 empirically tests the hypotheses and Section 3.5 concludes.

3.2. Literature Review

There are two recent studies focusing on the choice of shelf and non-shelf SEOs. One is Gao and Ritter's (2010). The authors examine the issuer's choice from the perspective of demand and supply. They argue that many firms choose a higher cost fully marketed offer because the marketing effort flattens the issuer's short-run demand curve, and helps to achieve a higher price after the offer. The other is Gustafson and Iliev (2017), studying the consequences of the 2008 SEC deregulation. They find that post-deregulation, small firms double their reliance on public equity and transition away from private investments in public equity.

From the perspective of issuing costs, due diligence is generally shorter and fees are lower for shelf SEOs. Bhagat, Marr, and Thompson (1985), and Blackwell, Marr, and Spivey (1990) document lower issuance costs for common equity offerings from allocated shelves in the 1980s. Autore, Kumar and Shome (2008) also document that shelf offerings result in no larger market penalties (short-run price declines) and significantly lower underwriter fees relative to non-shelf offerings in the 1990s.

From the perspective of market timing, Bethel and Krigman (2010) document that firms using shelf SEOs access the market faster than similar firms that use the slower traditional procedure that requires detailed advance disclosure. And in general, mangers are taking advantage of asymmetric information and timing the market when issuing SEOs. Graham and Harvey (2001), in a survey of CFOs, document that 67 percent of managers admit that ”the amount by which our stock is undervalued or overvalued by the market” is an important factor in their decision to issue common equity. Earlier, Loughran and Ritter (1997) document that operating performance (profit margin, ROA, operating income) peaks at approximately the time of the offering and then deteriorates. They argued that the investors are too optimistic about the prospects of issuing firms and expect the recent improvement of these firms to be permanent.

Another branch of literature related to this paper is on legality of long-run abnormal returns. The most serious problem with inference in studies of long-run abnormal stock returns is the reliance on a model of asset pricing. Kothari and Warner (1997) examine a variety of abnormal return models and find the degree of misspecification is not highly sensitive to the model employed. Their results support my usage of CAPM as the benchmark model to compute abnormal returns in this paper. For the time horizon, Speiss and Affleck-Graves (1995) document that there exists delayed stock price reaction to seasoned equity offerings, with abnormal performance apparently persisting for years following event. Following their study, I examine one and two years' cumulative abnormal returns after SEOs in Section 3.4 Empirical Tests.

3.3. Theoretical Model

3.3.1 Model Setup

Consider three dates, t=0, 1, 2.

At t=0:

(1) a public firm with current value V and number of shares N finds a new investment opportunity. The production function for the new investment is $Y = A^{1-\alpha} I^{\alpha}$, where $A = e^a K$, and $a \sim N(\mu_a, \sigma_a^2)$. A represents the investment opportunity and μ_a represents the quality of the new investment opportunity. μ_a belongs to the same distribution for all the firms. K denotes the current capital level of the firm and I denotes the investment in the new project. Including the existing capital K into A means a larger firm tends to have a larger scale of investment opportunity.

(2) μ_a is observed by the firm, while the potential investors ("investors" for short) don't observe μ_a. Investors have a prior belief about μ_a: $\mu_a \sim N(\mu_0, \tau_0^{-1})$.

(3) There is public information about the firm's new investment opportunity, which comes out in different forms such as news coverage and third-party analysis. This information is denoted as a public signal: S_p. $S_p = \mu_a + \epsilon_p$, $\epsilon_p \sim N(0, \tau_p^{-1})$. ϵ_p is the noise term and the reciprocal of its variance τ_p denotes the precision of the signal.

An important assumption in this model is that: τ_p increases in the firm size. This assumption is reasonable as explained in the Introduction part of this paper. The public signal is observed by both the firm and the investors.

(4) Knowing the public signal, the firm makes the issuing technique decision.

At t=1,

(1) The firm announces issuance of ΔN number of new shares and the offering techniques. If the firm announces shelf SEO, then no more information about this offering is provided and the firm benefits from a lower cost of issuance. If the firm announces non-shelf SEO, it provides more information about the issuance, often in the form of a road show or detailed description of the new investments in the non-shelf SEO prospectus. This is summarized as an additional signal for the new investment: S_{ns}. $S_{ns} = \mu_a + \epsilon_{ns}$, $\epsilon_{ns} \sim N(0, \tau_{ns}^{-1})$. In the meantime, the firm has to pay more underwriting and marketing costs for this issuance.

Here, an important implicit assumption is that the investors are of bounded rationality: they are not aware of the differences between shelf and non-shelf SEOs *per se*. That is, they update their believes with the additional information provided by the non-shelf SEOs, but they don't update with the no-information-providing behavior of the shelf SEOs. To put it in another way, the shelf SEO is not working as a negative signal to investors.

(2) The offer price P_1 is determined by the firm, taking the investors' posterior believes into consideration. This paper assumes that the investors breaks even in their expectations. So P_1 is the after-issuance equilibrium price, or "fair price" from the investors' perspectives at t=1:

$$(N + \Delta N)P_1 = V + E_1[Y - I|Investors] + \Delta N \cdot P_1(1 - c), \tag{1}$$

where c is the proportional cost for issuance, $c_{ns} > c_s$.

At t=2,

(1) After seasoned equity issuance, the firm starts to make investments and produce. Without loss of generality, this paper assumes that after the finance, the firm has enough money to make optimal level of investments. So the firm's investment decision is independent of offering technique choice.

(2) The firm produces and the results of the new investment realize. The new price of the firm's shares become P_2. For simplicity, this paper assumes no time discount.

So we have:

$$(N + \Delta N)P_2 = V + (Y - I) + \Delta N \cdot P_1(1 - c) \tag{2}$$

In this three-date model, the firm makes decisions at t=0 and t=2, while the investors make decisions at t=1. The firm is fully rational and the investors are of bounded rationality.

3.3.2 The Investor's Problem

As mentioned above, the investors break even in their expectations. This is guaranteed by Equation (1) & (2),

At t=1, the investors posterior information about μ_a after receiving all the signals is:

$\mu_a \sim N(\frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p}, (\tau_0 + \tau_p)^{-1})$, if the firm announces shelf SEO;

$\mu_a \sim N(\frac{\tau_0\mu_0+\tau_p S_p+\tau_{ns} S_{ns}}{\tau_0+\tau_p+\tau_{ns}}, (\tau_0 + \tau_p + \tau_{ns})^{-1})$, if the firm announces non-shelf SEO.

After some algebra, we have:

$$E_1^s[Y - I|Investor] = \alpha^{\frac{\alpha}{1-\alpha}}(1 - \alpha)e^{\frac{1}{2}\sigma_a^2}e^{\frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p}+\frac{1}{2(\tau_0+\tau_p)}}K \tag{3}$$

$$E_1^{ns}[Y - I|Investor] = \alpha^{\frac{\alpha}{1-\alpha}}(1 - \alpha)e^{\frac{1}{2}\sigma_a^2}e^{\frac{\tau_0\mu_0+\tau_p S_p+\tau_{ns} S_{ns}}{\tau_0+\tau_p+\tau_{ns}}+\frac{1}{2(\tau_0+\tau_p+\tau_{ns})}}K \tag{4}$$

3.3.3 The Firm's Problem

The firm's ultimate objective is to maximize P_2, because P_2 is proportional to the firm's value at date 2. [3] So the firm's problem consists of 1) optimal offering technique choice and 2) optimal investment decision after SEO issuance. Backward induction is used to solve the firm's problem.

a. Investment Decision

The firm solve the following problem to maximize the expected profits from the new investment opportunity at t=2.

$$\max_{I} E_2[Y - I|Firm]$$

Taking first order conditions with respect to I:

$$\Rightarrow E_2[A^{1-\alpha}|Firm]\alpha I^{\alpha-1} - 1 = 0$$

$$\Rightarrow I^* = (\alpha E_2[A^{1-\alpha}|Firm])^{\frac{1}{1-\alpha}} = \alpha^{\frac{1}{1-\alpha}} e^{\mu_a + \frac{1}{2}\sigma_a^2} K \tag{5}$$

$$\Rightarrow \frac{I^*}{K} = \alpha^{\frac{1}{1-\alpha}} e^{\mu_a + \frac{1}{2}\sigma_a^2} \tag{6}$$

$$\Rightarrow NPV_{(t=2)} = E_2[Y - I|Firm] = \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) e^{\mu_a + \frac{1}{2}\sigma_a^2} K \tag{7}$$

b. Offering Technique Choice

At t=0, the firm chooses among non-shelf SEO and shelf SEO so as to:

$$\max_{\{s,ns\}} \{E_0^{ns}[P_2|Firm], E_0^{s}[P_2|Firm]\}$$

Solving this problem, we get Proposition 3.1.

[3] We assume that N and ΔN are pre-determined.

Proposition 3.1 *Firm chooses non-shelf SEO over shelf SEO if and only if*

$$\mu_a - E_0[\mu_a|S_p] > A + C, \tag{8}$$

where

$A \equiv \frac{\tau_{ns}}{2(\tau_0+\tau_p)(\tau_0+\tau_p+\tau_{ns})}$, *represents relative adjustments,*

$C \equiv \frac{\tau_0+\tau_p+\tau_{ns}}{\tau_{ns}} \cdot \frac{V+\bar{E}}{\bar{E}} \left[ln(\frac{1-c_s}{N+\Delta N\cdot c_s}) - ln(\frac{1-c_{ns}}{N+\Delta N\cdot c_{ns}})\right]$, *represents cost differences.*

μ_a *denotes the true investment opportunity.*

$E_0[\mu_a|S_p]$ *denotes the public expectation of the investment opportunity.*

τ_p *denotes the precision of the public signal and increases with the firm's size.*

$\bar{E} \equiv e^{\mu_0+\frac{1}{2\tau_0}}$ *is the ex-ante expectation of* μ_a*.*

From Proposition 3.1, we can find that a firm chooses between the two issuing techniques according to the difference between its true investment opportunities and public expectations about its investment opportunities.

Since $E_0[\mu_a|S_p] = \frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p}$, Inequality (8) becomes:

$$\mu_a - \frac{\tau_0\mu_0 + \tau_p S_p}{\tau_0 + \tau_p} > A(\tau_p) + C(\tau_p).$$

(1) When $\tau_p \to 0$, the firm will choose non-shelf SEO iff $\mu_a > A(\tau_p = 0) + C(\tau_p = 0)$. The right hand side is some constant. That is to say, if the firm is extremely small, its offering technique decision will exclusively depend on the quality of investment opportunities.

(2) When $\tau_p \to +\infty$, the firm will choose non-shelf SEO iff $\mu_a - S_p > A(\tau_p \to +\infty) + C(\tau_p \to +\infty)$. Since $A(\tau_p \to +\infty) = 0$, $C(\tau_p \to +\infty) \to +\infty$, an extremely large firm will always choose shelf SEO.

(3) When τ_p is some positive number, the firm will choose an issuing technique according to the difference between its true investment opportunities and public expectation about its investment opportunities, balancing between providing more information and issuing with lower costs. To be specific, if $\mu_a < E_0[\mu_a|S_p]$, the firm will always

choose shelf SEO, so as to provide the public with less information and pool with other firms that have better investment opportunities. On the other hand, if $\mu_a > E_0[\mu_a|S_p]$, the firm will balance between providing more information about its good investment opportunities and higher costs of providing the additional information.

Thus, it is implied from this model that the stocks of small firms which have conducted shelf SEOs tend to underperform with respect to expectations in the long run after issuances, with more information comes out gradually. That is, the stocks of these firms tend to have negative abnormal returns in the long run after shelf SEOs.

Proposition 3.2 *The firm's investment ratio for the new project* $\frac{I}{K}$ *is proportional to the exponential form of the quality of the investment opportunity* e^{μ_a}*. The firm's expected profits from the new project* $E_0[Y - I|Firm]$ *is proportional to the investment opportunity* $e^{\mu_a}K$*.*

It's easy to derive from Proposition 3.2 that:

(1) the two groups of small firms which choose non-shelf SEO and shelf SEOs have prominent differences in investment levels in the new project; while

(2) the two groups of large firms which choose non-shelf SEO and shelf SEOs don't have significant differences investment levels in the new project.

3.4. Empirical Tests

3.4.1 Hypotheses

Based on the model's implications, a firm's SEO behavior is consistent with the following hypotheses:

H1: the stocks of small firms which have conducted shelf SEOs underperform with respect to expectations in the long run, while the stocks of small firms which have conducted non-shelf SEOs don't; the relative performance gap becomes less prominent when the firm size becomes larger.

H2: small firms which have conducted shelf SEOs have less increases in investments than small firms which have conducted non-shelf SEOs; this difference becomes less prominent when the firm size becomes larger.

H3: the difference in relative changes of investment levels is a major explanation of the performance gap between the two groups of small firms which have conducted shelf and non-shelf SEOs.

3.4.2 Data

The initial sample start with 2097 shelf SEOs and 1000 non-shelf SEOs obtained from SDC Global New Issues database and CRSP monthly stock file. To be included in the sample, a firm must satisfy that: (1) it has conducted primary SEOs during 1999-2015, and the offerings are not rights offers, ADRs or utilities; (3) it has available data for each month on monthly returns and number of shares outstanding , starting from one year before the SEO to two years after the SEO; (4) it is not a financial company with SIC code 6000-6999, or regulated utilities with SIC code 4900-4950.

Then the sample is merged with data from COMPUSTAT Fundamentals Annual database.I require at least three years data on assets, CAPEX, book capital and long-term investments. So I'm left with 819 shelf SEOs and 538 non-shelf SEOs. [4]

Finally, the sample is merged with calculated ex-ante CAPM betas, and 688 shelf SEOs and 177 non-shelf SEOs are left. [5]

[4]This sample represents about 44% of the initial sample. In general, more than half of the data got lost when mixing the CRSP data with Compustat annual financial data. Lee and Masulis (2009) end up with 32% of the initial sample; Liu and Wysocki (2007) end up with 34% of the initial sample; Dechow and Dichev (2002) retain less than 26% of their initial sample.

[5]The firm must listed on the NYSE, AMEX or NASDAQ for at least two years before the SEO in order to generate ex-ante CAPM beta for the firm. I also require the firm to survive for at least two years after the SEO, so as to compare the accounting data among different firms. This implies that survival bias is not considered in this paper.

In Appendix.B, I show the descriptive statistics and figures of the sample.

3.4.3 Estimation and Results

Hypothesis 1

Cumulative abnormal returns measure the underperformance of a stock with respect to expectations. In this paper, the asset pricing model used to find abnormal returns is CAPM. CAPM is used because (1) it is the simplest model and is widely applied in literature to calculate abnormal returns; (2) according to Kothari and Warner (1997), the degree of misspecification is not highly sensitive to the model employed. Also, I take log transformation of the cumulative abnormal returns following the literature, to smooth out extreme values.

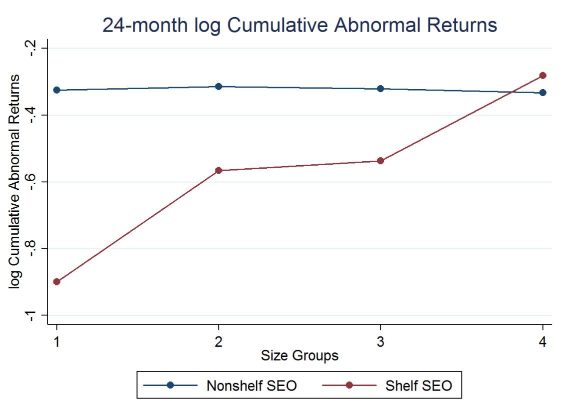
Figure 3.1 and Table 3.1 show the preliminary results. Figure 3.1 visually shows the average cumulative abnormal returns of different size groups, using the 12-month and 24-month log cumulative abnormal returns after issuances. Table 3.1 shows the preliminary regression results of the two groups separately, using book value of the firm ($logAsset$) to measure the firm size.

Table 3.1: SEO Methods, Firm Sizes, and Abnormal Returns

	Nonshelf SEO logCumABR	Shelf SEO logCumABR
logAsset	-0.00337	**0.102**
	(-0.10)	(6.28)
Intercept	-0.0932	**-0.788**
	(-0.48)	(-9.31)
N	177	688
adj. R^2	0.00	0.05

The table shows the results of the regressions: $logCumABR_i^{12m} = \alpha + \beta * logAsset_i + \epsilon_i$, where i denotes non-shelf-SEO firms and shelf-SEO firms, repectively. These are two cross-sectional regressions in Month 12 after equity issuances. Coefficient in bold if p-value<0.01. T statistics in parentheses.

Figure 3.1: 12-month and 24-month log Cumulative Abnormal Returns



(a)

The 4 size groups are defined using the market value (Market Cap) of the firm at the time of issuance. Group 1: Market Cap<\$75m; Group 2: \$75m <Market Cap<\$150m ; Group 3: \$150m< Market Cap<\$1000m ; Group 4: Market Cap>\$1000m. Within each group, the firms are equally weighted.

Table H.1 formally tests Hypothesis 1. The empirical model is :

$$logCumABR_i^{12m} = \alpha + \beta_1 logAsset_i + \beta_2 Shelf_i + \beta_3 logAsset_i * Shelf_i + X_i\theta + \epsilon_i \quad (9)$$

where $Shelf_i$ is a dummy which equals 1 if the issuance i is a shelf SEO. In this specification, its coefficient β_2 represents the gap of $logCumABR_i^{12m}$ between shelf and non-shelf issued firms when $logAsset$ goes to zero. The term $logAsset_i * Shelf_i$ is the intersection that represents the gap in slope for the two different groups of firms. X_i is a matrix of controls.

As shown in Table H.1, the dummy $Shelf$ is negatively significant and the intersection $logAsset * Shelf$ is positively significant. This means the stocks of small shelf issued firms underperform more severely in the long run than stocks of small non-shelf issued firms, and the performance gap becomes less prominent when the firm size is larger. In Table H.1, the performance gap vanishes when $logAsset \approx 7$, that is, when the size of the firm is around 1 billion U.S. dollars. The results are robust with different control variables. So, results in Table H.1 support Hypothesis 1.

Table 3.2: H.1

	logCumABR	logCumABR	logCumABR	logCumABR
logAsset	-0.00621	-0.00913	0.00159	0.0169
	(-0.18)	(-0.26)	(0.04)	(0.46)
Shelf	**-0.693**	**-0.708**	**-0.735**	**-0.683**
	(-3.27)	(-3.30)	(-3.37)	(-3.12)
logA*S	**0.107**	**0.110**	**0.114**	0.0959
	(2.76)	(2.79)	(2.84)	(2.39)
Intercept	-0.0951	-0.0642	0.0778	0.108
	(-0.49)	(-0.31)	(0.21)	(0.29)
IssueRatio		-0.108	-0.0806	-0.202
		(-0.45)	(-0.33)	(-0.82)
Number			Yes	Yes
Industry			Yes	Yes
Year				Yes
N	865	865	865	865
adj. R^2	0.05	0.05	0.04	0.07

$IssueRatio \equiv \frac{Amount\ Issued}{Market\ Cap}$, denotes the relative amount of equity issuance. $Number$ denotes the number of times a firm has issued seasoned equity in the sample. $Industry$ denotes different industry groups defined using SIC codes. $Year$ denotes the year dummies. Coefficient in bold if p-value<0.01. T statistics in parentheses.

Hypothesis 2

In this paper, I measure the increase in investments (or $\frac{I^*}{K}$ in the model) using the firm's relative change in capital expenditure to its lagged net "Property Plant and Equipment": $\frac{\hat{I^*}}{K} \equiv \frac{CAPX_{AfterSEO} - CAPX_{BeforeSEO}}{PPENT_{BeforeSEO}}$. Specifying in this way, I implicitly assume that without the SEO, the trend of capital expenditure of a firm would not change. This specification works as a noisy measure of investments in the new project. I winsorize the data to limit the domain between [-2, 2], in order to reduce the effect of spurious outliers.

The empirical model of Table H.2 is:

$$(\frac{\hat{I^*}}{K})_i = \alpha + \beta_1 logAsset_i + \beta_2 Shelf_i + \beta_3 logAsset_i * Shelf_i + X_i\theta + \epsilon_i \tag{10}$$

The results in Table H.2 show that small shelf issued firms increase less in investments than small non-shelf issued firms, and as the firm size becomes larger the difference becomes less prominent. This is because the dummy $Shelf$ is negatively significant and the intersection $logAsset * Shelf$ is positively significant, and the difference between shelf and non-shelf issued firms vanishes when $logAsset \approx 7$. The results are robust adding different control variables. Table H.2 supports H.2.

Table 3.3: H.2

	$\frac{\hat{I}^*}{K}$	$\frac{\hat{I}^*}{K}$	$\frac{\hat{I}^*}{K}$	$\frac{\hat{I}^*}{K}$
logAsset	-0.0738	**-0.0847**	-0.0776	**-0.0832**
	(-2.50)	(-2.83)	(-2.57)	(-2.67)
Shelf	**-0.701**	**-0.757**	**-0.714**	**-0.721**
	(-3.95)	(-4.22)	(-3.93)	(-3.91)
logA*S	**0.100**	**0.109**	**0.101**	**0.110**
	(3.06)	(3.31)	(3.03)	(3.25)
Intercept	**0.728**	**0.843**	**1.496**	**1.449**
	(4.50)	(4.92)	(4.78)	(4.57)
IssueRatio		-0.402	-0.336	-0.327
		(-2.00)	(-1.65)	(-1.57)
Number			Yes	Yes
Industry			Yes	Yes
Year				Yes
N	865	865	865	865
adj. R^2	0.03	0.03	0.04	0.04

Coefficient in bold if p-value<0.01. T statistics in parentheses.

Hypothesis 3

To test Hypothesis 3, I add the relative change of investment levels $\frac{\hat{I}^*}{K}$ as a control variable in Equation (9).

The results are shown in Table H.3. On the one hand, relative change of investment levels $\frac{\hat{I}^*}{K}$ explains the log cumulative returns after 12 months of the SEOs, in an economically and statistically significant way. On the other hand, the difference between shelf and non-shelf issued firms mitigates after controlling $\frac{\hat{I}^*}{K}$. Together, the results in Table H.3 show that the difference in relative changes of investment levels is a major explanation of the stock performance gap between the two groups of small firms.

Table 3.4: H.3

	logCumABR	logCumABR	logCumABR	logCumABR
logAsset	0.00626	0.00514	0.0144	0.0315
	(0.18)	(0.14)	(0.40)	(0.85)
Shelf	**-0.575**	**-0.581**	**-0.617**	-0.557
	(-2.72)	(-2.70)	(-2.83)	(-2.55)
logA*S	0.0905	0.0914	0.0969	0.0768
	(2.33)	(2.33)	(2.43)	(1.92)
$\frac{\hat{I}^*}{K}$	**0.169**	**0.168**	**0.165**	**0.174**
	(4.20)	(4.17)	(4.03)	(4.27)
Intercept	-0.218	-0.206	-0.169	-0.145
	(-1.13)	(-1.00)	(-0.45)	(-0.38)
IssueRatio		-0.0403	-0.0252	-0.145
		(-0.17)	(-0.10)	(-0.59)
Num(Issuance)			Yes	Yes
Industry Dummy			Yes	Yes
Year Dummy				Yes
N	865	865	865	865
adj. R^2	0.06	0.06	0.06	0.09

Coefficient in bold if p-value<0.01. T statistics in parentheses.

3.5. Conclusions

This paper documents that stocks of small public firms which have conducted shelf SEOs tend to underperform with respect to expectations (measured by cumulative abnormal returns) in the long run. The underperformance mitigates when the firm size becomes larger. I use a three-date model to capture the long-run performance differences of small and large firms after shelf and non-shelf SEOs. I propose in the model that a key underlying variable to create heterogeneity is the investment opportunity and a key assumption for abnormal returns is that investors are of bounded rationality so that they don't regard self SEO itself as a bad signal. Firms strategically choose the equity issuance techniques to reveal or conceal information, and after issuance, make optimal level of new investments according to different investment opportunities

This model is supported by the empirical evidences.

The empirical work in this paper show that: (1) the stocks of small firms which have conducted shelf SEOs underperform more severely in the long run than the stocks of small firms which have conducted non-shelf SEOs; the performance gap becomes less prominent when the firm size is larger; (2) increase in investments of small firms which have conducted shelf SEOs is less than that of small firms which have conducted non-shelf SEOs; this difference becomes less prominent when the firm size is larger; (3) the difference in relative changes of investment levels is a major explanation of the performance gap between the two groups of small firms considered.

Overall, this paper contributes to the literature by interacting the firm's investments, SEO method choices and the market's short-run and long-run reactions from the perspective of information asymmetry.

3.6. Appendix A

Proof of Proposition 3.1:

The firm chooses non-shelf SEO over shelf SEO if and only if:

$$E_0^{ns}[P_2|Firm] > E_0^{s}[P_2|Firm]$$

Plugging in $P_2 = \frac{V+(Y-I)+\Delta N P_1(1-c)}{N+\Delta N}$ from Equation (2), $P_1 = \frac{V+E_1[Y-I|Investors]}{N+\Delta N\cdot c}$ from Equation (1), and using the facts that V, N, Y, I and ΔN does not depend on the offering technique decision, we can get:

$$E_0^{ns}[\frac{V+E_1^{ns}[Y-I|Investors]}{N+\Delta N\cdot c_{ns}}(1-c_{ns})|Firm] > E_0^{s}[\frac{V+E_1^{s}[Y-I|Investors]}{N+\Delta N\cdot c_{s}}(1-c_{s})|Firm]$$

$$\Rightarrow \qquad \frac{V+E_0^{ns}}{N+\Delta N\cdot c_{ns}}(1-c_{ns}) > \frac{V+E_0^{s}}{N+\Delta N\cdot c_{s}}(1-c_{s}) \tag{3.6.1}$$

where $E_0^{ns} \equiv E_0^{ns}[E_1^{ns}[Y-I|Investor]|Firm]$, $E_0^{s} \equiv E_0^{s}[E_1^{s}[Y-I|Investor]|Firm]$.

Take natural logarithm transformation on both sides:

$$ln(V+E_0^{ns}) - ln(V+E_0^{s}) > ln(\frac{1-c_s}{N+\Delta N\cdot c_s}) - ln(\frac{1-c_{ns}}{N+\Delta N\cdot c_{ns}}) \tag{3.6.2}$$

Using Equation (3) & (4), we can get:

$$E_0^{ns} \equiv E_0^{ns}[E_1^{ns}[Y-I|Investor]|Firm] = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)e^{\frac{1}{2}\sigma_a^2}e^{\frac{\tau_0\mu_0+\tau_p S_p+\tau_{ns}\mu_a}{\tau_0+\tau_p+\tau_{ns}}+\frac{1}{2(\tau_0+\tau_p+\tau_{ns})}}e^{\frac{\tau_{ns}}{2(\tau_0+\tau_p+\tau_{ns})^2}}K$$

$$E_0^{s} \equiv E_0^{s}[E_1^{s}[Y-I|Investor]|Firm] = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)e^{\frac{1}{2}\sigma_a^2}e^{\frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p}+\frac{1}{2(\tau_0+\tau_p)}}K$$

And thus:

$$ln(E_0^{ns})-ln(E_0^s) = \frac{\tau_{ns}}{\tau_0+\tau_p+\tau_{ns}}(\mu_a - \frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p}) + \frac{1}{2(\tau_0+\tau_p+\tau_{ns})} + \frac{\tau_{ns}}{2(\tau_0+\tau_p+\tau_{ns})^2} - \frac{1}{2(\tau_0+\tau_p)} \tag{3.6.3}$$

To analytically solve Inequality (A.2), I take log linearization as an approximation:

Consider function: $f(x) = ln(V + e^x)$.

Expand it around $\bar{x} = ln(E[e^x])$. So $e^{\bar{x}} = E[e^x] \equiv \bar{E}$,

$\Rightarrow f(x) \approx f(\bar{x}) + f'(\bar{x})(x-\bar{x}) \approx f(\bar{x}) + \frac{e^{\bar{x}}}{V+e^{\bar{x}}}(x-\bar{x})$

Let $x_1 = ln(E_0^{ns}), x_2 = ln(E_0^s)$, then $\bar{E} = E[E_0^{ns}] = E[E_0^s] = e^{\mu_0+\frac{1}{2\tau_0}}$

$f(x_1) - f(x_2) \approx \frac{e^{\bar{x}}}{V+e^{\bar{x}}}(x_1 - x_2)$.

So, Inequality (A.2) is approximate to:

$$\frac{\bar{E}}{V+\bar{E}}(ln(E_0^{ns}) - ln(E_0^s)) > ln(\frac{1-c_s}{N+\Delta N \cdot c_s}) - ln(\frac{1-c_{ns}}{N+\Delta N \cdot c_{ns}} \tag{3.6.4}$$

Plugging (A.3) into (A.4) and after some algebra, we can finally get:

$\mu_a - \frac{\tau_0\mu_0+\tau_p S_p}{\tau_0+\tau_p} > \frac{\tau_{ns}}{2(\tau_0+\tau_p)(\tau_0+\tau_p+\tau_{ns})} + \frac{\tau_0+\tau_p+\tau_{ns}}{\tau_{ns}} \cdot \frac{V+\bar{E}}{\bar{E}} \left[ln(\frac{1-c_s}{N+\Delta N \cdot c_s}) - ln(\frac{1-c_{ns}}{N+\Delta N \cdot c_{ns}}) \right]$.

■

Proof of Proposition 3.2:

Directly from Equation (6) and Equation (7). ■

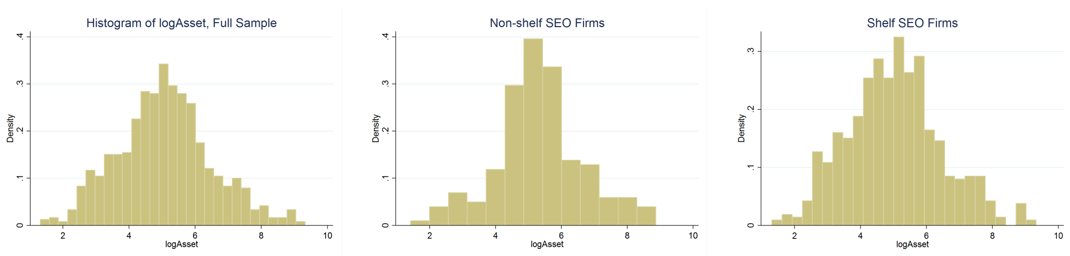
3.7. Appendix B

Tables and Figures

Table 3.5: Descriptive Statistics

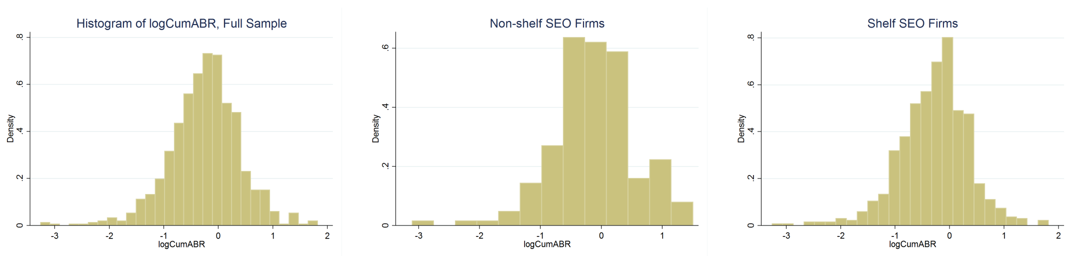
	Mean	StdD	P25	P50	P75
logCumABR	-0.25	0.64	-0.61	-0.22	0.14
logAsset	5.08	1.41	4.18	5.10	5.87
$\frac{\hat{I}^*}{K}$	0.19	0.53	-0.05	0.07	0.34

Figure 3.2: Histogram of LogAsset



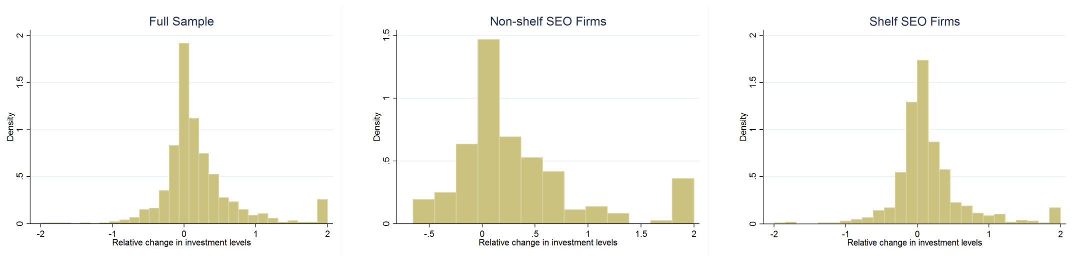
(a) (b) (c)

Figure 3.3: Histogram of logCumABR, 12 months after SEOs



(a) (b) (c)

Figure 3.4: Histogram of Relative Changes in Investment Levels($\frac{\hat{I^*}}{K}$)



(a) (b) (c)

Firure 4: the data are winsorized between [-2, 2] to aviod extreme values.

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