The London School of Economics
& Political Science

Dynamic Group Decision Making

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A whale-ship was my Yale College and my Harvard.

Herman Melville, Moby Dick

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Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics & Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 33,940 words.

Statement of conjoint work

I confirm that Chapter 3 was jointly co-authored with Kateřina Šmídková and Roman Horváth and I contributed 33.3% of this work.

Abstract

A common theme running throughout the three chapters of this thesis is dynamic recurring group decision making. The first chapter sets up a model with endogenous status-quo (dynamic bargaining model) in which decision makers are uncertain about their own future preferences. The main focus of the chapter is on how different bargaining protocols influence equilibrium decisions. The two protocols considered are i) *implicit status-quo* bargaining protocol in which present period policy serves as the status-quo for the next period and ii) *explicit status-quo* bargaining protocol in which the current decision involves both current policy and a possibly different status-quo for the future. The main observation of the chapter is that the former bargaining protocol leads to decisions diverging from the preferences of the actors involved even in the periods in which their preferences coincide, this divergence being driven by the concerns to maintain a bargaining position for the future. The latter bargaining protocol, on the other hand, delivers decisions fully reflecting preferences of the actors involved in the periods when these coincide, but may lead to decisions reflecting only the proposer's preferences.

The second chapter shows how to construct equilibria in a class of dynamic bargaining models in which players have fixed preferences over all the dimensions of a policy space. The construction applies both to one-dimensional and multi-dimensional policy spaces and delivers equilibria with simple and intuitive structure. The chapter works out several examples to show i) the multiplicity of equilibria and ii) the non-monotonicity of the existence of the simple equilibria in the underlying model parameters.

The third paper is a collaborative work with Roman Horváth and Kateřina Šmídková from the Czech National Bank (currently published as a CNB working paper). The chapter analyses decision making in monetary policy committees, the decision making bodies of central banks. On the empirical

side, the chapter shows that voting records of monetary policy committees are informative about their own future decisions. On the theoretical side, the chapter shows that the voting records' predictive power can be generated through theoretical models used in the group decision making literature.

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Introduction

The common denominator of my thesis is recurrent group decision making. A group of decision makers is repeatedly making the same type of decision. Once a decision is made in a given bargaining round it determines the status-quo for the subsequent round. The resulting endogeneity of the status-quo makes the situation one of dynamic bargaining. The rationale for the explicit modelling of such situations comes from an attempt to understand the determination of policies that need to be continuously specified over time, such as central bank interest rates, tax rates or regulatory limits.

The dynamic bargaining literature, to which this thesis contributes, grew from the original bargaining model of Rubinstein (1982) and its political economy application by Baron and Ferejohn (1989). Where the dynamic bargaining literature diverges from these papers is in assuming that reaching a decision does not end the bargaining process, proceeding into a next period of typically infinite horizon interaction instead. What makes the interaction dynamic, as opposed to repeated, is the endogeneity of status-quo. A current decision is made against the status-quo determined by previous decisions and will in turn determine the status-quo in the future. Baron (1996) is among the first to analyse a model with these features.[1]

In this context, the first chapter focuses on the role of the bargaining protocol in the dynamic bargaining model with uncertainty regarding future preferences of the actors involved. The chapter compares two bargaining protocols: i) the *implicit status-quo* protocol under which present period policy serves as the status-quo for the next period and ii) the *explicit status-quo* protocol under which the decision in the current period involves both current policy and a (possibly different) status-quo for the ensuing period. While the former bargaining protocol is standard in the dynamic bargaining

[1] See chapter 2 for survey of the dynamic bargaining literature.

literature, the latter one and their comparison is the key novel feature.

The chapter shows that the two bargaining protocols lead to notably different policy outcomes, with the difference most marked in the periods of common interest. These are still characterized by disagreement under implicit status-quo bargaining, while under explicit status-quo bargaining lead to the policy decisions that fully reflect the congruent preferences of the committee members. This result arises due to the dual role of policy under the implicit status-quo protocol. Policy serves not only as policy but also determines the future status-quo and hence the bargaining position of committee members. The disagreement then arises even in periods of common interest due to the possibility of future conflict. The explicit status-quo protocol retains only the former role of policy with the policy decisions fully reflecting the common interests, when these arise.

A second difference arising from the two bargaining protocols is in the evolution of bargaining power as captured by the status-quo. The explicit status-quo protocol allows the proposer, by giving her more flexibility in crafting her proposals, to gain and retain the dominant position in the committee. Relative to implicit status-quo bargaining, this results in the policy outcomes that reflect, to a larger extent, the proposer's preferences, policy outcomes that are too extreme from the point of view of the committee as a whole.

The differences in the policy outcomes also determine the answer to a question on which of the two bargaining protocols would be chosen by a committee of decision makers who know it would be used in their subsequent interaction, or by a utilitarian central planner. The chapter shows that the explicit status-quo protocol is superior only if the initial status-quo gives little bargaining power to the proposer, such that the benefits of the common interests being reflected in the decisions outweigh the costs of the proposer's dominance.

The second chapter shows how to construct equilibria in the dynamic bargaining model, with fixed preferences of the players involved and, in the terminology of the previous chapter, with implicit status-quo bargaining protocol. For a similar model of bargaining over the share of a fixed-size budget, this has been done by Kalandrakis (2004), but for a model based on Baron (1996) where the bargaining proceeds over a one-dimensional space of policies, explicit characterization of the equilibria has so far been missing in the dynamic bargaining literature.

The bargaining proceeds over a space of policies with each player having quadratic preferences defined around a bliss point, where the policy space can be either one-dimensional as in Baron (1996) or multi-dimensional. In the absence of dynamic considerations the player recognized to be a proposer would propose her bliss point and, given sufficiently adverse status-quo, would have it approved by at least a minimum winning coalition of the remaining committee members. The chapter shows that in the same situation, the player recognized to be a proposer proposes her *strategic bliss point*, a policy between her and the median player's original bliss points.

The exact position of the strategic bliss point for a given player depends on the model parameters, such as the discount factor and the probability of being recognized as a proposer, but is shown to be the result of two opposing forces. One force pushes the player into proposing policies close to her original bliss point. The second and strategic force pushes the player in the opposite median player's direction, in an attempt to propose policies that constrain the future proposals of all the remaining players.

The chapter constructs an algorithm that calculates the strategic bliss points for a given model parametrization and shows how to use these to build conjectured equilibrium strategies. It further derives two conditions, one sufficient and one necessary, that the conjectured strategies need to satisfy in order to constitute an equilibrium. Both of these conditions are easy to check as they consider only a finite set of points in otherwise continuous policy space. Finally, the chapter works out several examples showing a number of interesting features that equilibria in the model may exhibit. These include a subset of players behaving in a way indistinguishable from the behaviour of the median player despite having different preferences, asymmetric equilibrium behaviour in otherwise symmetric environments and multiplicity of equilibria possibly complicating their computational approximation that typically relies on the uniqueness of the equilibrium being approximated.

The third chapter is an application of the dynamic bargaining framework to central bank decision making. In most modern central banks decisions to change or retain a given interest rate level is done by a group of decision makers, the monetary policy committee. The committee meets in a recurring manner with the interest rate from the previous meeting constituting the status-quo policy and the policy agreed upon in a given meeting becoming status-quo for the next one.

The chapter builds on the observation made by Gerlach-Kristen (2004) for the Bank of England that a voting pattern from a given monetary policy committee meeting is informative regarding interest rate changes in the subsequent meetings. The chapter finds similar result in voting and decision making data from several other central banks and, from the theoretical perspective, asks if it is possible to generate predictive power of the voting pattern in a formal group decision making model.

The model used posits a committee of decision makers making recurring decisions about policy, trying to match an unobserved time-varying optimal policy that follows $AR(1)$ process. Non-unanimous voting arises from the committee members having private signals about the optimal policy and hence voting either for the status-quo or for the policy proposed by the committee chairman. The alternative attracting (super-)majority of votes then becomes status-quo for the subsequent committee meeting.

The chapter investigates three variants of the model that differ in the degree of informational influence among the committee members. In the *democratic* version the influence is limited with committee members extracting no information from the chairman's proposal and the chairman having no information regarding signals of the remaining committee members. In the *consensual* version it is the chairman who has the informational influence, with the other committee members extracting information from her proposal, while in the *opportunistic* version the influence works in the opposite direction, with the chairman having information about private signals of the other committee members.

Using computer simulations, the chapter generates decision making and voting data from the three model versions and uses these in an econometric estimation akin to the one used with the real world data. The results suggest that in order to generate predictive power of the voting pattern for the future policy changes large degree of informational independence is needed, as in the democratic version of the model. The chapter further investigates the effect of the noise in private signals, of the variability in the optimal policy and of the size of the committee, showing that the size can be used as a substitute for quality of information in generating predictive power of the voting pattern.

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Chapter 1

Explicit and implicit status-quo determination in dynamic bargaining: Theory and application to FOMC directive

> *Several other members indicated that they would have preferred to tighten at that meeting. ... The asymmetric directive, which held prospect of near-term tightening, once again allowed FOMC to reach a consensus.*
>
> *Meyer (2004), page 83*

> *And we know from firsthand accounts that Greenspan was holding back an FOMC that was eager to raise rates.*
>
> *Blinder and Reis (2005), page 58*

1.1 Introduction

We study recurring group decision making problems with the preferences of the actors involved varying over time. As a leading example, consider periodic meetings of a monetary policy committee. In every period, the preferences of each committee member will be affected by host of factors, such as the state of the economy, his view of future economic development, his opinion about the strength of monetary policy transmission channels, or judgment about the suitable inflation monetary policy should aim for. Inevitably, most of those factors will change stochastically over time, opening the possibility for renegotiation of decisions reached at an earlier stage. Of course, there are many other examples of recurring decision making with varying preference, both in the economic and the political spheres.

With changing preferences of the involved parties, bargaining over a decision at any given point in time will proceed under varying degrees of disagreement. In the monetary policy committee case, ambiguity of information the committee holds can provoke disagreement over the most appropriate policy in some periods, but can lead to agreement in other periods when the information becomes more definite. Uncertainty over the future then implies uncertainty about the extent of future disagreement as well.

Recurring decisions naturally create dynamic linkages that need to be considered. First, strategic linkages arise due to the repeated nature of the interaction. The current action of any given decision maker will take into account its possible impact on the future behaviour of the remaining decision makers, with the repeated interaction allowing for cooperative outcomes unattainable in static settings. This is analysed in the folk theorems literature in general and in the political arena context in particular by Dixit, Grossman, and Gul (2000) and Maggi and Morelli (2006). We abstract away from these strategic linkages in recurring decisions by focusing on Stationary Markov Perfect equilibria.

Instead, we concentrate on the second type of linkages, procedural ones. These involve the role past decisions play during the determination of subsequent ones. These linkages stem from the need to ensure continuity in policy making. Protocols in place ensure that some policy is chosen even in the case of a decision not being reached.

The simplest of such protocols is the one under which a policy, once established, becomes the status-quo for the ensuing round of bargaining. Inaction, no change in a given policy or contract, leaves the previous decision in place. For example, in most countries personal income tax rates apply until changed. Labour unions negotiate agreements with firms regarding wage and employment levels which are effective until renegotiated. In effect, current policy *implicitly* determines status-quo.

However, there are several prominent examples of decision processes that enlarge the space of current decisions to include provisions for the future. These yield not only current policy but also *explicitly* determine a (potentially different) status-quo for the next round of negotiations. Legislative *sunset provisions* specify a time horizon for the statute or regulation in question, after which it automatically terminates. These are often found in tax laws or in laws impinging civil liberties, most prominently in the US Economic Growth and Tax Relief Reconciliation Act of 2001 and US Patriot Act of 2001. Regulatory *escape clauses* are another example of the present policy and the status-quo being distinct.[1]

In this paper, we investigate decisions reached in recurring negotiations with stochastically changing preferences of the actors involved. The status-

[1] Regulatory escape clauses can be viewed as temporary modifications of the regulatory rules in light of changed conditions with no change implied for the future. For example, the European Commission in the context of the Stability and Growth Pact initiates an excessive deficit procedure with a given country if its annual budget deficit exceeds 3% of its GDP, but can refrain from doing so if the breach of the limit is associated with, for example, a prolonged period of slow economic growth. This can be interpreted as the European Commission temporarily increasing the 3% limit during recessions but keeping it intact for the future, or in other words having different current and status-quo deficit limits.

quo for a given round of bargaining is determined endogenously during the previous negotiations. We call the distinction between the new status-quo being implicitly or explicitly decided upon *bargaining protocol* and ask how the bargaining protocol influences decisions reached.

We are motivated by normative, positive and theoretical questions. On the normative side we analyse how the different bargaining protocols influence the ability of the committee to respond to the changing preferences of the involved parties. The procedural linkages mentioned above imply that the behaviour of each decision maker will reflect both current and future incentives. This might imply that even in 'agreement' periods, characterized by similar current preferences of the decision makers, their behaviour might be driven by efforts to affect future decisions. We will show below that enlarging the space of current decisions to include provisions for the future via the explicit status-quo bargaining protocol delivers policies better tailored to changing circumstances. A potential downside of allowing for such provisions comes from the resulting increase in proposal power. Consequently, from the utilitarian perspective none of the bargaining protocols clearly dominates the other and we lay out conditions under which one or the other of them should be endorsed.

Our positive motivation builds on one of our motivating examples, monetary policy committees. Monetary policy in most central banks is decided upon by a committee composed of several members convening with regular frequency. The policy usually consists of the bank's operating target, its interest rate. In most central banks the interest rate decided in a given committee meeting serves also as a status-quo for the next meeting. Inaction leads to no change in monetary policy stance, hence the status-quo is implicitly determined by a given decision.

In contrast, the Federal Open Market Committee (FOMC), the decision body of the US Federal Reserve System, issues at the close of each meeting operating instructions for the Federal Reserve Bank of New York known as the *domestic policy directive*. The directive contains not only the decision about current policy but also a statement concerning the FOMC's expectation of future policy stance. Viewing the 'asymmetry', 'bias' or 'tilt' in the policy directive as explicitly specifying a status-quo policy possibly different from the current one allows us to gain deeper understanding of the FOMC decision making process.[2] Our model then suggests a novel rationale for the existence of the asymmetry.

The theoretical motivation is to advance growing dynamic bargaining literature. While acknowledging endogeneity of the status-quo in recurring decision making situations, this literature has invariably assumed that the status-quo is equal to the policy decision of the previous bargaining round. While this is a natural assumption in many environments, some environments might be more appropriately modelled as having an explicit status-quo bargaining protocol. Our analysis of explicit status-quo bargaining is not only, to our knowledge, novel in the literature, but also highlights differences the two bargaining protocols bring by analysing them in an otherwise identical model setup.

In the model, a committee composed of two members, one of whom possesses fixed proposal power, takes repeated decisions on a policy from a one-dimensional policy space over which each of the committee members has single peaked preferences represented by a bliss point. Every period is randomly selected to be either an agreement or disagreement one, with only the present period type being common knowledge. In agreement periods, the two members share a common bliss point whereas in the disagreement periods the bliss points of the two members differ. While certainly a crude simplification of the continuum on which conflict of preferences can take place, the agreement/disagreement dichotomy allows us to clearly illustrate the effect of the bargaining protocol on policy outcomes.

Besides the period type, every committee meeting is characterized by a one-dimensional status-quo. Under the implicit status-quo bargaining protocol, the status-quo is pitched against a proposed policy with the winning alternative being both the current policy outcome and the next period status-quo. Under the explicit status-quo bargaining protocol, the status-quo is pitched against a joint proposal for a policy and a new status-quo. If the committee selects the proposed pair, this proposal determines the current policy outcome and a possibly different future status-quo, otherwise, the status-quo becomes both the policy implemented today and the future status-quo.

We first show existence and uniqueness in a certain well defined sense

[2] See the opening part of section 1.5 for a discussion of why the asymmetry might constitute a status-quo.

of Stationary Markov Perfect equilibrium (S-MPE) under both bargaining protocols (propositions 1.1 and 1.4). The lack of general S-MPE existence results and typically ill behaved induced preferences over the 'state' variable in dynamic bargaining models (Baron, 1996; Baron and Herron, 2003; Kalandrakis, 2004; Duggan and Kalandrakis, 2012) make this a nontrivial exercise and we are forced to work with induced preferences that typically lack monotonicity, concavity and continuity. Adding further stochastic elements would allow us to use existing results on existence of S-MPE in dynamic bargaining context.[3] We refrain from doing so, limiting generality of our results to cases of sufficiently but not excessively strong conflict between the two players. On the other hand, this allows us to characterize equilibria of the model to a greater extent.

For the implicit status-quo bargaining protocol, we show that in equilibrium negotiations display inefficiency in agreement periods; the committee members are unable to agree on a policy corresponding to their common bliss point (proposition 1.2). The intuition for this result is the dual role of policy under the implicit status-quo bargaining protocol. Policy serves not only as policy but also determines the future status-quo. Moreover, we show that bargaining quickly reaches a point of gridlock, with the policy outcomes unresponsive to changing preferences (proposition 1.2). Once in gridlock, the two players have antithetic preference over policy even in agreement periods, as it determines the future status-quo and affects their future bargaining positions. Explicit status-quo bargaining reverses both of these results. In equilibrium, it leads to the policy outcomes corresponding to the common committee members' bliss point in the agreement periods (proposition 1.3) and does not lead to the gridlock as the policy outcomes remain responsive to the changing preferences of the committee members (proposition 1.5).

One possible side effect of explicit status-quo bargaining comes from the increase of proposal power relative to implicit status-quo bargaining. Allowing for proposals with different policy and status-quo creates room for the proposer to push through policies fully reflecting her preferences. Those are too extreme for the rest of the committee and the committee as a

[3] Duggan and Kalandrakis (2012) overcome the ill behaved induced preferences problem by adding noise elements to players' preferences and to the policy status-quo transition mechanism. This 'smooths out' the induced preferences and allows them to prove existence of S-MPE in a very general dynamic bargaining model.

whole might prefer different bargaining protocols in different environments (proposition 1.6).

Finally, we show that these results carry over to a committee composed of an odd number of members with preferences similar to those in the benchmark model (proposition 1.7). This allows us to shift attention to the FOMC decision making process and examine the role of the asymmetry in its directive. We focus mainly on its role as a predictor of future policy changes and as an instrument to achieve more consensual FOMC decisions. Our model delivers these two predictions and also suggests a novel explanation for the existence of the asymmetry as a tool which allows the FOMC chairman to maintain his dominant position in the committee.

The model we build belongs to the dynamic bargaining literature that assumes that the status-quo during a given round of bargaining is endogenously determined during previous bargaining rounds. Differently from most of the existing literature (Baron, 1996; Baron and Herron, 2003; Kalandrakis, 2004; Bernheim, Rangel, and Rayo, 2006; Battaglini and Coate, 2007; Baron, Diermeier, and Fong, 2012; Battaglini and Palfrey, 2012) we focus on an environment with stochastic preferences and abstract from distributional issues analysed in many of the mentioned papers.

Despite its obvious appeal, the dynamic bargaining literature with time-varying preferences is rather scarce. Battaglini and Coate (2008) build a dynamic model of legislative bargaining with general and targeted public spending. In their model, the status-quo is fixed but the intertemporal link is created by accumulated public debt while the time-varying preferences stem from a stochastic value of general public spending. Diermeier and Fong (2009) build a similar model. Riboni and Ruge-Murcia (2008) analyse a model similar to ours with the implicit status-quo bargaining protocol. They analytically solve the two period version of their model and resort to numerical simulation of the infinite period version. Dziuda and Loeper (2010) also analyse a model closely related to ours with the implicit status-quo bargaining protocol. In their model, a two member committee takes repeated decisions over a binary agenda with the preference parameter of each of the committee members being a continuous random variable distributed on the real line. In our model, it is the preference parameter that takes on two values with the policy being a continuous variable. However, none of the papers mentioned above analyses how expanding the space of current decisions to include provisions for the future changes policy outcomes and ability of the committee members to renegotiate in the changing environment, something our explicit status-quo bargaining protocol does. It is the comparison between the two bargaining protocols or institutions we are interested in.

Another strand of literature related to this paper is the literature investigating the effect of linking decisions. In Jackson and Sonnenschein (2007) agents are constrained to represent their preferences across decision problems such that the representation corresponds to the underlying distribution of their preferences. The main result of their paper is that linking large numbers of decisions leads to approximate ex ante Pareto efficiency. In Casella (2005) agents can store their votes and use them in future meetings when their preferences are more intense. This typically leads to ex ante welfare improvement over non-storable votes. Hortala-Vallve (2010) proves similar result in a setting where agents can distribute a given number of votes freely across a predetermined number of issues. The first mentioned paper improves efficiency by putting constraints on the misrepresentation of preferences allowed for, while the two latter papers improve efficiency by relaxing the one-person-one-vote constraint. In the context of this literature, our explicit status-quo bargaining protocol, by relaxing the 'policy equal to status-quo' constraint, can be viewed as relaxing constraint on the committee decision making but also as removing constraint on the proposal power.

We proceed as follows. The next section lays out the theoretical model. Section 1.3 solves for the equilibrium in a two period version. It is meant to build intuition for the infinite horizon version and to show that the key results are not sensitive to changes in the foresight horizon. Section 1.4 contains all the theoretical results. These describe equilibria for both of the bargaining protocols, discuss conditions under which one of them should be preferred and show that the model applies equally well to larger committees. Section 1.5 applies these results to the FOMC decision making, demonstrates that the model can replicate stylized facts about its decision patterns and suggests a novel interpretation of the asymmetric FOMC directive.

1.2 Model

We analyse the effect of bargaining protocol on dynamic policy making in a simple model. Policies in the model are set by a committee composed of two members. The first member is the chairman, who has policy proposal power and whom we denote by C (she). The second committee member is denoted by P (he) and has policy approval power. The voting rule used by the committee is simple majority with ties decided against C's proposal so that in order for C's proposal to pass, consent of both committee members is required.[4]

The committee sets policy p_t in each period t of an infinite horizon. The utility player $i \in \{C, P\}$ receives from the path of policies $\mathbf{p} = \{p_0, p_1, p_2, \ldots\}$ is given by

$$U_i(\mathbf{p}) = \sum_{t=0}^{\infty} \delta^t u_{i,t}(p_t)$$

where $\delta \in [0, 1)$ is common discount factor. Instantaneous utility $u_{i,t}(p_t)$ of each player is

$$u_{i,t}(p_t) = -(p_t - \pi^* - \varepsilon_{i,t})^2$$

where π^* is a common component in the committee members' preferences and $\varepsilon_{i,t}$ is a stochastic time-varying $i-$specific preference shock.

The timing of actions in period t is as follows. First, nature determines $\varepsilon_{i,t}$ according to the process specified below and the committee convenes with x_t being the default option. Both $\varepsilon_{i,t}$ and x_t are common knowledge. Second, the chairman C proposes a pair $\gamma_t = \{p_t, q_t\}$ against default option $\bar{\gamma}_t = \{x_t, x_t\}$. Third, voting takes place between γ_t and $\bar{\gamma}_t$. If P prefers γ_t it is implemented (C always votes for her proposal), players receive their payoffs from the offered policy p_t and the offered status-quo q_t becomes default option for the next period, i.e. $x_{t+1} = q_t$. If P prefers $\bar{\gamma}_t$, players receive their payoffs from the default policy x_t and the default status-quo x_t becomes default option for the next period, i.e. $x_{t+1} = x_t$. Finally, the committee adjourns and the game moves into period $t + 1$.

In the text we refer to the pair $\gamma_t = \{p_t, q_t\}$ C proposes as to (C's) *proposal* or *offer*, call its first element p_t *proposed (offered) policy* and its

[4] An alternative assumption that would not change any of the results is C making a take it or leave it offer to P. We opt for the voting rules specification as it naturally adapts once we expand the committee below.

second element q_t *proposed (offered) status-quo.* The pair $\bar{\gamma}_t = \{x_t, x_t\}$ is then *default option* or simply *default* and we abuse notation slightly in calling x_t by the same term.

Without loss of generality we assume that C whose utility maximizing offer γ_t coincides with the default option $\bar{\gamma}_t$ proposes $\bar{\gamma}_t$ instead of proposing a policy she knows would be rejected. It is also easy to see that in any equilibrium of the game it has to be the case that if P is indifferent between default $\bar{\gamma}_t$ and C's proposal γ_t he votes for γ_t. As a result C's offer γ_t is always accepted and we do not need to distinguish between proposed and accepted policies.

Up to this point the model generates dynamic policy making in that the proposed (and hence accepted) status-quo q_t from period t becomes the default option x_{t+1} for the $t+1$ period. To study how this feature interacts with the bargaining protocol used by the committee, we contrast two versions of the model. The first model version and bargaining protocol is with *implicit status-quo.* Under this bargaining protocol C's proposals are constrained to those that satisfy $p_t = q_t$ so that the t period status-quo q_t, and hence $t+1$ period default option x_{t+1}, is implicitly defined by the t period policy p_t. The second model version and bargaining protocol is with *explicit status-quo.* Under this bargaining protocol t period status-quo q_t, and hence $t+1$ period default option x_{t+1}, is explicitly determined during the committee bargaining.

To close the model we need to specify the distribution of the preference shocks $\varepsilon_{i,t}$. We assume those are generated according to

$$\varepsilon_{i,t} = \begin{cases} -\phi \text{ for } i = C \text{ and } \phi \text{ for } i = P & \text{with probability } r_d \\ \;\; 0 \text{ for } i \in \{C, P\} & \text{with probability } 1 - r_d \end{cases}$$

where $\phi > 0$ and $r_d \in [0, 1]$. In words, there are two types of periods. With probability r_d bliss points in the instantaneous utility functions of C and P are $\pi^* - \phi$ and $\pi^* + \phi$ respectively. We call those *disagreement* periods or D periods for short. The second type of period occurs with probability $1 - r_d$ and are called *agreement* periods or A periods for short. In these, bliss points in the instantaneous utility functions of both players are π^*.

Several comments regarding our modelling choices are in order. First, completely breaking the link between policy and status-quo and giving all the proposal power to C is motivated by our interest in the trade-off the explicit status-quo bargaining protocol creates. On the one hand, it should lead to more efficient policy outcomes, but it also opens the door to an abuse of proposal power. We want to see the full effect on both sides and thus opt for arguably strong assumptions.

Second, having A and D periods in the model reflects our belief that in recurrent decision making this is a natural assumption. We could have chosen either purely ideological or purely common preferences, which our model indeed includes as special cases with $r_d = 1$ or $r_d = 0$. However, it is easy to show that under both specifications the bargaining protocol plays no role. It is the interaction with the time varying preferences that creates an interesting problem to study.

1.3 Two period model

To build intuition for the results below, we first solve a two period version of the model. All the results are easily derived using backward induction and we state them without formal proofs.

Lemma 1.1 (Last period)**.** *For the last period default option x_1 and both bargaining protocols, equilibrium policy proposals $p_{A,1}(x_1)$ and $p_{D,1}(x_1)$, in A and D periods respectively, satisfy*

$$p_{A,1}(x_1) = \pi^*$$
$$p_{D,1}(x_1) = f(x_1, \phi)$$

where $f(x, \phi) = \max\{\min\{2(\pi^* + \phi) - x, x\}, \pi^* - \phi\}$.

In the last period there is no procedural link with the future via the status-quo and hence the bargaining protocol plays no role. It is thus easy to see why the two policy makers decide on π^* in A periods as it is a bliss point in their common utility function.

D period policy then reflects conflict in the committee. P's acceptance set consists of a symmetric interval around his bliss point $\pi^* + \phi$ with one boundary given by default option x_1, $[2(\pi^* + \phi) - x_1, x_1]$. C maximizes her utility with bliss point at $\pi^* - \phi$ by proposing minimum of P's acceptance (the min term) but only if she cannot propose her bliss point (the max term). The parameter ϕ captures the interval of disagreement, for $x_1 \in$

$[\pi^* - \phi, \pi^* + \phi]$, there is no other policy except for x_1 the two policy makers are willing to agree on. P would reject any policy below x_1 and C does not want to offer any policy above x_1 and thus $f(x_1, \phi) = x_1$.[5]

C's and P's expected utilities before nature determines the type of last period, as a function of x_1 (and hence as a function of the first period status-quo),

$$\mathbb{E}\left[U_{C,0}(x_1)\right] = -r_d(p_{D,1}(x_1) - \pi^* + \phi)^2 + (1 - r_d) \cdot 0$$
$$\mathbb{E}\left[U_{P,0}(x_1)\right] = -r_d(p_{D,1}(x_1) - \pi^* - \phi)^2 + (1 - r_d) \cdot 0,$$

reflect intertemporal preferences of the two policy makers and are interesting for several reasons. First, both are non-concave and non-monotone. $\mathbb{E}[U_{C,0}(x_1)]$ and $\mathbb{E}[U_{P,0}(x_1)]$ are non-increasing and non-decreasing respectively for $x_1 \leq \pi^* + \phi$ and vice-versa for $x_1 \geq \pi^* + \phi$. This is the reason why we cannot work with equilibria associated with well-behaved (concave, monotone) value functions as in, for example, Battaglini and Coate (2007, 2008), as the ill-behaved intertemporal preferences are an inherent feature of the model.

Second, potential future conflict spills over to the current period through the conflict in the intertemporal preferences. P prefers default option x_1 as close to $\pi^* + \phi$ as possible while C prefers it as far away from $\pi^* + \phi$ as possible. Thus the committee members have an incentive to manipulate x_1 in the first period as it determines their bargaining positions. Under implicit status-quo bargaining this is done via the enacted policy and under explicit status-quo bargaining this is done via the enacted status-quo.

Third, $\mathbb{E}[U_{C,0}(x_1)]$ and $\mathbb{E}[U_{P,0}(x_1)]$ are constant for $x_1 \notin (\pi^* - \phi, \pi^* + 3\phi)$. For the first period under explicit status-quo bargaining this means that whenever $z \notin (\pi^* - \phi, \pi^* + 3\phi)$ is an equilibrium status-quo proposal for some default option, so is $z' \notin (\pi^* - \phi, \pi^* + 3\phi)$. However, this multiplicity has no effect on the last period policy. No matter whether z or z' is proposed, last period P's acceptance set includes, on a policy dimension unique, unconstrained maximizer of C's overall utility.

[5] Notice also that the interval where $f(\cdot, \cdot)$ is not constant, the interval of default options for which C's proposal makes P indifferent between accepting and rejecting and interval of default options for which C cannot implement her bliss point, all coincide. This is a more general feature of the model, and it will hold for the first period irrespective of the type of period or bargaining protocol, and motivates our choice of equilibrium refinement (definition 1.3) for the infinite horizon model.

Lemma 1.2 (First period, D). *For the first period default option x_0, implicit status-quo protocol policy proposal $p^I_{D,0}(x_0)$ and explicit status-quo protocol policy and status-quo proposals $p^E_{D,0}(x_0)$ and $q^E_{D,0}(x_0)$ in D periods satisfy*

$$p^I_{D,0}(x_0) = p^E_{D,0}(x_0) = q^E_{D,0}(x_0) = f(x_0, \phi).$$

For the implicit status-quo protocol this reflects conflict in terms of both current and intertemporal preferences. The same holds for the explicit status-quo, but C can, in principle, offer a policy different from the status-quo. To see the nature of her trade-off, C can either concede on the policy dimension in order to gain a better bargaining position on the status-quo dimension, or vice versa. The strength of those two forces, to satisfy instantaneous or intertemporal utility, then determines her equilibrium proposal. As we will see below, the two forces exactly cancelling each other, which leads to $p^E_{D,0}(x_0) = q^E_{D,0}(x_0)$, is a result specific to the two period model.

Lemma 1.3 (First period, A). *For the first period default option x_0, equilibrium policy proposal under the implicit and explicit status-quo protocol, $p^I_{A,0}(x_0)$ and $p^E_{A,0}(x_0)$ respectively, in A periods satisfy*

$$p^I_{A,0}(x_0) = f(x_0, \phi\kappa')$$
$$p^E_{A,0}(x_0) = \pi^*$$

where $\kappa' = \frac{\delta r_d}{1+\delta r_d} \leq \frac{1}{2}$.

First A periods reveal the key difference between the two bargaining protocols. Under the implicit status-quo bargaining, policy serves two roles. It is a policy in the standard sense but also determines future bargaining positions. Agreeing on π^* in A period would entail, at least for one of the players, giving up bargaining position relative to x_0. Combining current preferences favouring π^* and intertemporal preferences favouring $\pi^* - \phi$ ($\pi^* + \phi$) for C (P) makes A periods 'lesser disagreement' periods with the degree of conflict given by $\phi\kappa'$. The more probable the true D periods are and the more players care about future, the more of the conflict spills over to A periods.

Explicit status-quo bargaining on the other hand implies π^* is implemented in A periods. With the policy makers' preferences aligned on the policy dimension and, crucially, with the policy and status-quo possibly different, C does not compromise her bargaining position by proposing π^* policy. To the contrary, this allows C to propose status-quo that improves her bargaining position. She has a room to do so since proposing π^* on the policy dimension has made P better off compared to the default option x_0.

A key advantage of the two period model just discussed is that it delivers key predictions about the difference in policy outcomes under the two bargaining protocols in a relatively simple framework. On the other hand, with a fixed time horizon we are unable to discuss the evolution of policies in the long-run, and the fixed horizon also raises concerns about robustness of the results presented. For this reason we turn to the infinite horizon version of the model next.

1.4 Infinite horizon model

This section solves the infinite horizon dynamic bargaining model for the two bargaining protocols. For technical reasons we restrict the proposal space along any dimension to lie in a convex compact subset X of $\mathbb{R}$. Hence for both C's proposals and default options, we have $\gamma_t, \bar{\gamma}_t \in X^2 \subseteq \mathbb{R}^2$. However, it will become apparent from the model equilibria below that with X taken to be 'sufficiently large', this assumption is without loss of generality.

We focus on Stationary Markov Perfect Equilibria (S-MPE) where strategies in a given period depend only on the type of that period and on the default option for that period, i.e. only on payoff relevant variables. Focusing on the S-MPE we can drop all time subscripts. We denote by $x \in X$ the default option for a given period with the understanding that it is composed of a default policy status-quo pair $\bar{\gamma}(x) = \{x, x\} \in X^2$. Any policy is always denoted by (appropriately subscripted) $p \in X$ and any status-quo is always denoted by $q \in X$.

For this model, S-MPE will be a combination of several components. For C, we are looking for four functions, two of them mapping the space of default options X into proposed policies for each type of period, $p_D(x), p_A(x) : X^2 \to X$, and the remaining two mapping X into the proposed status-quo, $q_D(x), q_A(x) : X^2 \to X$. Formally, $\rho_C = \{p_D(x), p_A(x), q_D(x), q_A(x)\} : X^4 \to X^4$ denotes C's strategy and her proposal in period $i \in \{A, D\}$ given default option x is $\gamma_i(x) = \{p_i(x), q_i(x)\}$. For P, his strategy given period

$i \in \{A, D\}$ and default option x maps the combination of $\bar{\gamma}(x)$ and $\gamma_i(x)$ into his vote, hence it is a mapping $\rho_P : X^8 \to \{yes, no\}^2$.

It has to be acknowledged that our definition of ρ_C and ρ_P does not allow for mixed strategies. For P this is driven by the already mentioned fact that in any equilibrium P's voting strategy has to be to vote for C's proposal $\gamma_i(x)$ whenever indifferent between $\gamma_i(x)$ and $\bar{\gamma}(x)$ for $i \in \{A, D\}$. For C the reason behind focusing on pure strategies is twofold. First, we have assumed above that C whose utility maximizing proposal coincides with the default option $\bar{\gamma}(x)$ indeed proposes $\bar{\gamma}(x)$ instead of coming up with a proposal she knows would be rejected. Second, below we focus on a certain class of equilibria (see definition 1.3) for which it will be true that C's indifference among K proposals $\{\gamma_i^1(x), \ldots, \gamma_i^K(x)\}$ for some default option $x \in X$ and $i \in \{A, D\}$ will imply indifference by P among the same proposals. As a result, in case of C's indifference between two or more proposals we can pick one $\gamma_i^k(x)$ out of $\{\gamma_i^1(x), \ldots, \gamma_i^K(x)\}$ without changing the equilibrium (via changing the equilibrium value functions defined below) and hence we can think of ρ_C as a function instead of thinking of ρ_C as a distribution on X^4. With this qualification in mind, our definition of S-MPE is as follows.

Definition 1.1 (Stationary Markov Perfect Equilibrium)**.** *A pair of strategies $\rho^* = \{\rho_C^*, \rho_P^*\}$ constitutes S-MPE if it constitutes subgame perfect equilibrium.*

Notice that any given pair of strategies $\rho = \{\rho_C, \rho_P\}$ for given x and given path of A and D periods generates a unique path of implemented policies $\{p_0, p_1, \ldots\}$. Taking expectations over all possible paths gives a continuation value function for each policy maker who knows x but does not know whether the next period will be an A or D one,

$$V_C^\rho(x) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (p_t - \pi^* + \phi \mathbb{I}_D(t))^2\right]$$
$$V_P^\rho(x) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (p_t - \pi^* - \phi \mathbb{I}_D(t))^2\right]$$

where $\mathbb{I}_D(t)$ is D-period indicator function and the superscript ρ captures dependence on given ρ. Having the continuation value functions, we observe these can be equivalently derived as

$$V_C^\rho(x) = r_d\left[-(p_D(x) - \pi^* + \phi)^2 + \delta V_C^\rho(q_D(x))\right] + (1 - r_d)\left[-(p_A(x) - \pi^*)^2 + \delta V_C^\rho(q_A(x))\right]$$
$$V_P^\rho(x) = r_d\left[-(p_D(x) - \pi^* - \phi)^2 + \delta V_P^\rho(q_D(x))\right] + (1 - r_d)\left[-(p_A(x) - \pi^*)^2 + \delta V_P^\rho(q_A(x))\right].$$

Finally, we denote by $A_i^\rho(x)$ P's acceptance set in period $i \in \{A, D\}$ given default option x and strategies ρ. The acceptance sets are given by

$$A_D^\rho(x) = \{\{p, q\} \in X^2 | -(p - \pi^* - \phi)^2 + \delta V_P^\rho(q) \geq -(x - \pi^* - \phi)^2 + \delta V_P^\rho(x)\}$$
$$A_A^\rho(x) = \{\{p, q\} \in X^2 | -(p - \pi^*)^2 + \delta V_P^\rho(q) \geq -(x - \pi^*)^2 + \delta V_P^\rho(x)\}$$

and both are nonempty as $\bar{\gamma}(x) \in A_i(x)$ for $i \in \{A, D\}$.

With this notation, C's problem can be restated in terms of a pair of the usual Bellman functional equations

$$\begin{aligned} U_D^\rho(x) &= \max_{\{p,q\} \in A_D^\rho(x)} \{-(p - \pi^* + \phi)^2 + \delta r_d U_D^\rho(q) + \delta(1 - r_d)U_A^\rho(q)\} \\ U_A^\rho(x) &= \max_{\{p,q\} \in A_A^\rho(x)} \{-(p - \pi^*)^2 + \delta r_d U_D^\rho(q) + \delta(1 - r_d)U_A^\rho(q)\} \end{aligned} \tag{1.1}$$

where C's continuation value function V_C^ρ will be the probability-weighted sum of the value functions of the two optimization problems, i.e. $V_C^\rho = r_d U_D^\rho + (1 - r_d)U_A^\rho$. An alternative definition of S-MPE that exploits the recursive structure of the model and that we use is the following.

Definition 1.2 (Stationary Markov Perfect Equilibrium)**.** *A pair of strategies $\rho^* = \{\rho_C^*, \rho_P^*\}$ constitutes a S-MPE if for all $x \in X$ and any period $i \in \{A, D\}$*

1. *C's proposal strategy ρ_C^* solves (1.1)*
2. *P votes for C's proposal $\gamma_i(x)$ if and only if $\gamma_i(x) \in A_i^{\rho^*}(x)$.*

An equivalent way to express the requirement of the S-MPE is to say we are looking for ρ giving rise to V_C^ρ and V_P^ρ such that when C and P maximize their utility in the current period, their optimal behaviour is indeed expressed as ρ. If we can find such a ρ then by the one deviation principle we have an equilibrium.

Below, when we discuss S-MPE for the two bargaining protocols, it will become apparent that many of them satisfy an additional restriction in P being, for a given default option, indifferent between accepting and rejecting

C's offer, provided C's proposal differs from the unconstrained maximizer of her overall utility. Another way to view this is that as long as the default option x provides P with any real bargaining power, C's proposal will provide him with the minimum utility sufficient for her proposal to pass. We call S-MPE satisfying this feature Conflict S-MPE (CS-MPE). Denoting by γ_{CD}^{ρ} and γ_{CA}^{ρ} solutions to the two optimization problems in (1.1) when the restrictions on $\{p, q\}$ to lie in P's acceptance sets are removed, CS-MPE is defined as follows.

Definition 1.3 (Conflict Stationary Markov Perfect Equilibrium)**.** *A pair of strategies $\rho^* = \{\rho_C^*, \rho_P^*\}$ constitutes a CS-MPE if for all $x \in X$ and any period $i \in \{A, D\}$*

1. *ρ^* constitute a S-MPE*

2. *P is indifferent between $\gamma_i(x)$ and $\bar{\gamma}(x)$ provided $\gamma_{Ci} \notin A_i^{\rho}(x)$.*

Our focus on CS-MPE has another rationale as it can be viewed as a focus on equilibria with the minimum winning coalition property. Whenever C is constrained by the other committee member her proposal will make P indifferent between accepting and rejecting. Assuming P is a median member of some larger committee with C's proposal accepted if and only if P accepts, something we show in the context of larger committee in the proposition 1.7 below, CS-MPE will imply C establishes minimum winning coalitions supporting her proposals. This is reminiscent of the result by Duggan and Kalandrakis (2012) (see part 4 of their theorem 1) who show that minimum winning proposals are a natural feature of equilibria in dynamic bargaining models.[6]

From here on we focus on the equilibrium strategies and we drop the superscript ρ whenever the chance of confusion is minimal. Finally, for the bargaining protocol with implicit status-quo all results of this section additionally require any policy status-quo pair to have both of its elements equal.

[6] Formally the game just described can be viewed as a mapping from a pair of value functions, one for each player, into a new pair of value functions. The CS-MPE assumption makes sure this mapping is 'well behaved'. Without it, the way in which C reconciles indifference between two proposals has real consequences for P, inducing jumps in his value function.

Equilibrium with implicit status-quo

In this section we prove equilibrium existence and uniqueness result for the bargaining protocol with implicit status-quo. We then discuss predictions of the equilibrium about the evolution of policies under implicit status-quo bargaining.

Proposition 1.1 (S-MPE with implicit status-quo)**.** *Assume* $\delta^2 r_d(3-2r_d) \leq 1-\delta(1-r_d)$*. Then there exists unique CS-MPE. Equilibrium proposals satisfy*

$$p_D(x) = q_D(x) = max\{min\{z \in X | z \in A_D(x)\}, \gamma_{CD}\}$$
$$p_A(x) = q_A(x) = max\{min\{z \in X | z \in A_A(x)\}, \gamma_{CA}\}$$

for $\forall x \in X$*, where* $\gamma_{CD} = \pi^* - \phi$ *and* $\gamma_{CA} = \pi^* - \phi\delta r_d$*.*

Proof. See appendix 1.A1.

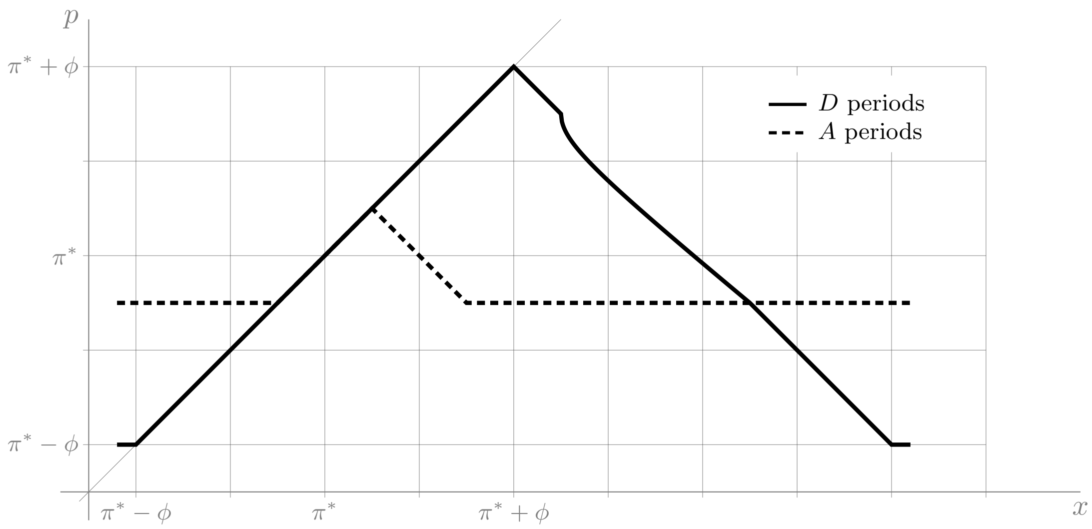
In words, for a given type of period $i \in \{A, D\}$ and default option x, C proposes the lowest policy out of P's acceptance set $A_i(x)$, provided the policy that is an unconstrained maximizer of her overall utility would be rejected, that is provided $\gamma_{Ci} \notin A_i(x)$.

The strategy of the proof follows. Existence follows by construction. We conjecture that the construction will give us a CS-MPE which allows us to derive P's continuation value function V_P and hence his acceptance sets A_A and A_D. Given the acceptance sets we conjecture that C's proposal strategy will be the one given in the proposition allowing us to derive her continuation value function V_C. Having the proposal strategy we note it indeed generates V_P and we confirm strategies generated by V_P and V_C satisfy definition 1.3 showing that the construction is CS-MPE.

To prove uniqueness of the CS-MPE, we note that it has to generate a unique V_P. What we then need to show is uniqueness of the solution to C's dynamic optimization program (1.1) given acceptance sets generated by V_P. We show this using an extended version, which we prove, of the theorem guaranteeing existence and uniqueness of solutions to Bellman functional equations from Stokey and Lucas (1989).

The assumption on $\{\delta, r_d\}$ in proposition 1.1 ensures existence of the CS-MPE equilibrium. The assumption can be alternatively expressed as $\delta \leq \varphi(r_d)$ where $\varphi(0) = \varphi(1) = 1$ and $\min_{r_d \in (0,1)} \varphi(r_d) = 7/9$ so that in

Figure 1.1: Equilibrium policy with implicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



effect we are ruling out cases where the 'future looms large' as δ approaches unity. When this happens the requirement on C's proposals under CS-MPE, to bring P to indifference between accepting and rejecting when unable to propose the unconstrained maximum of her overall utility, might fail in D periods. Intuitively, with δ large C focuses primarily on her bargaining position captured by the V_C function when determining which policy to propose. With the V_C function non-monotone, C might propose a policy strictly inside A_D, in effect disregarding her instantaneous utility. When this happen the equilibria become cumbersome to characterize due to non-continuity of V_P so that we rule those cases out by assumption.

To see how the equilibrium from proposition 1.1 looks in graphical form, figure 1.1 shows a particular parametrization for $\pi^* = 2, \phi = 1, \delta = 0.5, r_d = 0.5$. While proving proposition 1.1 we show that depending on the values of δ and r_d, the equilibrium falls into one of four (mutually exclusive) cases. For all four of those cases the A period proposed policy $p_A(x)$ has exactly the same shape as the one given in the figure, with the constant part given by γ_{CA} evaluated at particular values of $\{\delta, r_d\}$.

However, there are case dependent differences regarding the shape of the D period proposed policy $p_D(x)$. What is common to all of them is the constant and then linear increasing part for low values of x. Nevertheless, the default option x for which $p_D(x)$ reaches a maximum in general differs depending on the values of δ and r_d and the 'right' part of $p_D(x)$ (decreasing part in figure 1.1) is not necessarily monotone or even continuous. One common feature is that it eventually decreases to γ_{CD} where it becomes a constant function again.

Figure 1.1 (and proposition 1.1) shows that the equilibrium shares several features with the equilibrium in the two period version of the model. It is a CS-MPE, P is indifferent between accepting and rejecting unless C proposes the unconstrained maximizer of her utility. Furthermore, A periods are in effect lesser disagreement periods with degree of conflict captured by $\phi\delta r_d$ and the committee members are failing to agree on π^*, common bliss point in their instantaneous utility functions. Basic intuition for this result is again the dual role of policy under the implicit status-quo bargaining, it enters policy makers' instantaneous utility while at the same time determining their bargaining position. On the other hand it is the A periods during which C forgoes her bargaining position. By proposing $p_A(x)$ closer to π^* relative to the default option x, she compromises her intertemporal preferences in exchange for the current ones. D periods are then truly disagreement periods and C is fully using her proposal power to steer policy towards her most preferred one.

In order to discuss the long-term policy outcomes generated in equilibrium, we find it helpful to define a set of default options x which, when reached, implies a constant path of default options irrespective of the type of period. Constant default options then imply policies alternating between two (not necessarily) different values, one for A periods and the other for D periods. We call such a set a set of *stable default options* and define it along with two notions of efficiency in the following definition.

Definition 1.4 (Stable default options and efficiency)**.** *Set $S \subseteq X$ defined by*

$$S = \{x \in X | q_A(x) = q_D(x) = x\}$$

is called set of stable default options (stable set).

We say bargaining displays A-efficiency whenever

$$p_A(x) = \pi^*.$$

We say bargaining displays D-efficiency if

$$p_D(x) = p^*$$

for some $p^* \in [\pi^* - \phi, \pi^* + \phi]$ *across* D *periods.*

The rationale behind the definition of stable set is that once the bargaining reaches $x \in S$, resulting status-quo outcomes are constant forever for any path of A and D periods. If additionally we have $p_A(x) = p_D(x)$ for all $x \in S$ we can say that the bargaining outcomes are unresponsive to the changing preferences of the committee members.

Our notion of efficiency then comes from a static Pareto efficient mechanism implementing an infinite sequence of policies in the current environment. As we show in appendix 1.A2, such a mechanism implements π^* in A periods and $p^* \in [\pi^* - \phi, \pi^* + \phi]$ in D periods. The notion of A-inefficiency whenever $p_A(x) \neq \pi^*$ comes from the fact that the policy makers fail to agree on their current-period most preferred policy π^* due to their concerns about their bargaining position in the future. Given A period and default x such that $p_A(x) \neq \pi^*$, if they could sign a binding contract specifying that the next period default option will be x irrespective of today's policy (which they would set to π^*), both of them would be made better off. The notion of D-inefficiency on the other hand stresses the fact that both policy makers have a preference for policy smoothing. Finally, note that our notion of A-efficiency looks at each A period individually while D-efficiency compares policy decisions reached in different D periods.

Discussing equilibrium policy outcomes is further complicated by the fact that those will in general depend on the default x with which bargaining starts and on the path of A and D periods which is stochastic. Nevertheless, denoting by $x^t(x) \in X$ the default option after t periods of equilibrium play starting with default option x and some path of A and D periods, the following proposition captures the key features.

Proposition 1.2 (Policy outcomes with implicit status-quo)**.** *CS-MPE from proposition 1.1 generates policy and status-quo decisions satisfying following.*

1. *If* $x \in S$ *then the policy outcomes display D-efficiency in all subsequent periods*

2. *If* $x \in S$ *then* $p_A(x) = p_D(x) \in [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$

For initial default option x_0 *being continuous random variable with pdf* $f(x_0)$ *defined on* X*, for any* $t = 1, 2, \ldots$

3. $\int_{x_0 \in X} \mathbb{P}(x^t(x_0) \notin S) f(x_0) dx_0 \leq r_d^t$
4. $\int_{x_0 \in X} \mathbb{P}(p_A(x^t(x_0)) = \pi^*) f(x_0) dx_0 = 0$ *unless* $r_d = 0$.

Proof. See appendix 1.A1.

Recalling the equilibrium in figure 1.1 the intuition behind the result is straightforward. For any default option $x \in S$ we have policies constant not only in D periods (part one) but also in A periods (part two). For the third part, for any default option x in A period, policy and hence status-quo reaches S immediately and can stay out of S only for the path of D periods with the probability of t consecutive D periods being r_d^t. The last part then comes from the fact that the set of default options in X that can bring π^* as a policy outcome in the future for some combination of A and D periods has zero measure.

What proposition 1.2 says is that in CS-MPE from proposition 1.1 under the bargaining protocol with implicit status-quo, bargaining outcomes eventually become stable for any distribution of initial default option (part three). When this happens the policy outcomes display D-efficiency (part one) on the one hand but become unresponsive to the changing preferences of the two policy makers on the other (part two) with the policy constant henceforth. At the same time, unless $r_d = 0$ for any distribution of initial default option the chance that the bargaining satisfies A-efficiency is zero both on the path to S and once it is reached (part four). In other words, in the CS-MPE under the implicit status-quo bargaining there is no equilibrium force that would bring the bargaining outcome eventually to A-efficiency.

Equilibrium with explicit status-quo

We now show how policy outcomes change when C's proposals are not restricted to those with policy and status-quo equal. The first result we prove is that policy in A periods is equal to π^* for any default option. The logic behind the result is that since in the A periods the preferences of the two policy makers are aligned along the policy dimension, there is no reason they should not be able to reach an agreement on π^*, as doing so needs not compromise their bargaining position embodied in status-quo. The intuition is confirmed by the proposition.

Proposition 1.3 ($p_A(x)$ with explicit status-quo)**.** *In any S-MPE for any default option $x \in X$*

$$p_A(x) = \pi^*.$$

Proof. See appendix 1.A1.

A key strength of proposition 1.3 is that it applies to any S-MPE under the explicit status-quo bargaining protocol and shows that this bargaining protocol allows the committee members to reach consensus in A periods. What the proposition does not ensure is existence of such S-MPE, which is what the next proposition does.

Proposition 1.4 (S-MPE with explicit status-quo)**.** *Assume $\delta \geq \frac{1}{5r_d}$, $\delta \geq 1 - r_d^2$ and $\delta \leq 1 - \frac{(1-r_d)^2}{2}$. Then there exists a unique CS-MPE in terms of associated value functions V_C and V_P. Equilibrium proposals satisfy*

1. *$p_A(x) = \pi^*$ for $\forall x \in X$*
2. *$V_C(x) \leq V_C(q_A(x))$ for $\forall x \in X$*
3. *$V_C(q_D(x)) \leq V_C(q_D(x'))$ for $x, x' \in X$ satisfying $A_D(x) \subseteq A_D(x')$*
4. *C proposes γ_{CD} (γ_{CA}) for $\forall x \in X$ such that $\gamma_{CD} \in A_D(x)$ ($\gamma_{CA} \in A_A(x)$)*

where $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z'\}$ for some $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$.

Proof. See appendix 1.A1.

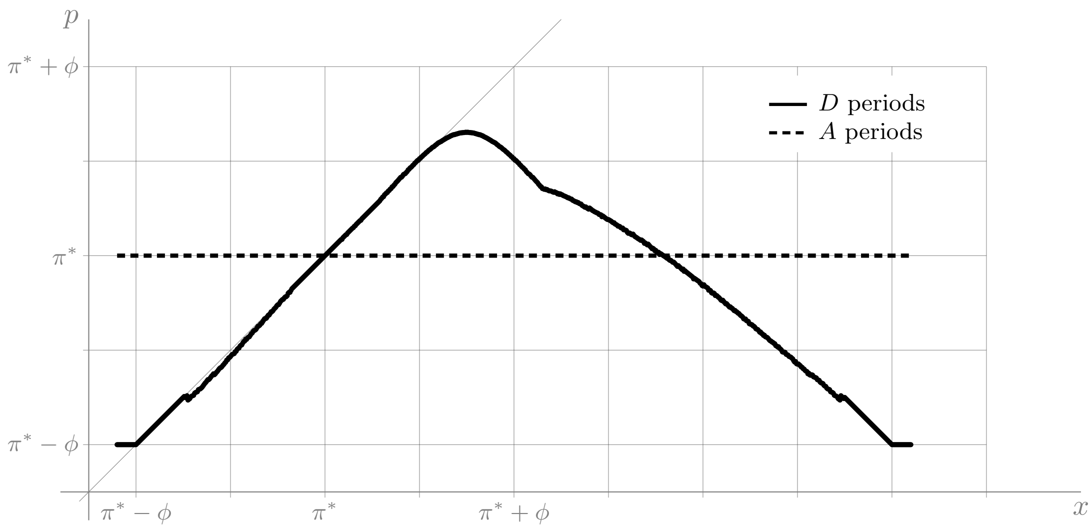
In words, equilibrium under explicit status-quo bargaining involves policy equal to π^* in A periods (part one) with C using A periods to improve her bargaining position (part two). Because C can improve her bargaining position in A periods, she is willing to surrender more of it in D periods in which P has more bargaining power (part three). Finally, C's unconstrained proposals are γ_{CA} and γ_{CD} in A and D periods respectively, implementing C's instantaneous utility bliss point and status-quo that maintains her bargaining position (part four).

The idea of the proof is similar to the proof of proposition 1.1. For the existence part we partially characterize the equilibrium, conjecturing first that we are characterizing CS-MPE. This gives us the V_P function and associated acceptance sets A_A and A_D. We prove these are well behaved, which allows us to prove existence of C's continuation value function V_C as a solution to C's dynamic optimization program (1.1). We then confirm that the proposal strategies generated by V_C indeed satisfy definition 1.3 of CS-MPE. Uniqueness in terms of associated value functions then follows from the uniqueness of V_P in any CS-MPE and resulting uniqueness of V_C.

The key difficulty in the proof of proposition 1.4 and the source of the assumptions on $\{\delta, r_d\}$ is confirming that proposal strategies associated with V_C indeed satisfy the definition of CS-MPE. What we need to ensure is that intertemporal incentives are strong enough (first two conditions) so that C is willing to use the status-quo dimension of her proposal space in A periods to bring P to indifference between accepting and rejecting as the definition of CS-MPE demands. On the other hand we need to make sure that the intertemporal incentives are not too strong (third condition). When this happens, in D periods P is willing to accept a wide range of policies when offered an even slightly more favourable status-quo compared to the default option. One of those policies is C's D period most preferred policy $\pi^* - \phi$. With proposals involving $\pi^* - \phi$ policy possibly violating requirements of CS-MPE, we need to make sure that C foregoes only little of her bargaining position exactly for those values of $\{\delta, r_d\}$, somewhat paradoxically, when the bargaining position is most valuable.

Figures 1.2 and 1.3 show equilibrium proposals from proposition 1.4 on the policy and status-quo dimension respectively for the same values of parameters used in figure 1.1. Even though we do not have an explicit expression for V_C we use computer simulation to estimate V_C and associated equilibrium proposal policies (see appendix 1.A3 for details of the numerical simulation). From proposition 1.4 we know that proposals on the status-quo dimension need not be unique and involve $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ for defaults such that $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$. When this happens figures 1.2 and 1.3 always use $z = z' = \pi^* - \phi$. Notice also that $\delta = 0.5$ and $r_d = 0.5$ used in the figures do not satisfy the assumption on $\{\delta, r_d\}$ from proposition 1.4. Nevertheless, given the simulated V_C and associated proposal strategies it is easy to confirm those satisfy the definition

Figure 1.2: Equilibrium policy with explicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$

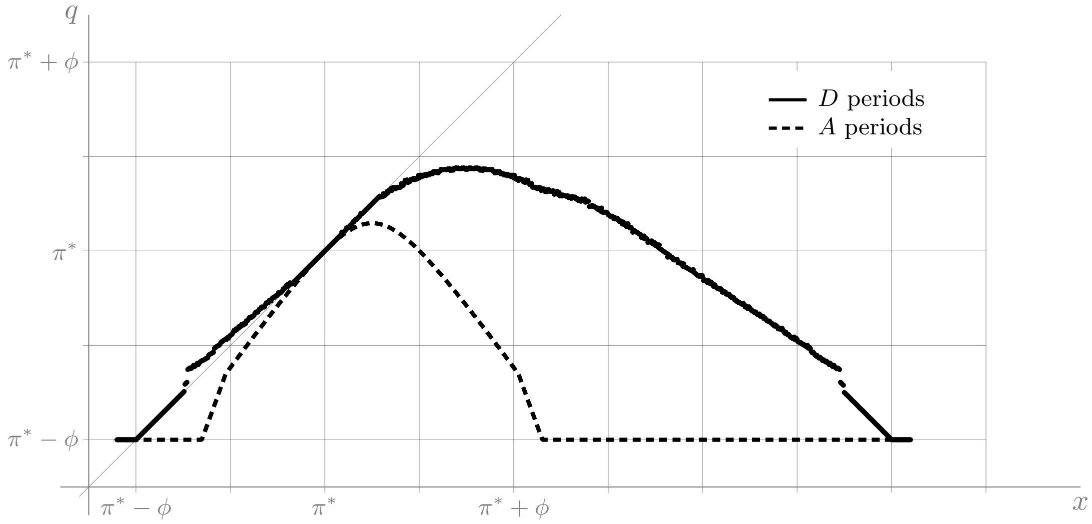


of CS-MPE and hence that the assumptions on $\{\delta, r_d\}$ in proposition 1.4 are sufficient but not necessary for existence of CS-MPE.

Figures 1.2 and 1.3 along with the proposition 1.4 show that under explicit status-quo bargaining, equilibrium in the infinite horizon again resembles equilibrium in the two period version of the model. It is a CS-MPE equilibrium, A period policy proposals are equal to π^* and C uses A periods to gain a better bargaining position. For any default option x, by offering π^* on the policy dimension P is made better off compared to $\bar{\gamma}(x) = \{x, x\}$, which allows C to gain a better bargaining position on the status-quo dimension in terms of proposing $q_A(x)$ providing her with higher intertemporal utility compared to x.

This in turn makes C willing to forego some of her bargaining position in D periods, a feature not present in the two period model. Intuitively, with C knowing she can gain bargaining position in future A periods without sacrificing on the policy dimension, she is willing to forego some of that bargaining position in D periods in exchange for a more favourable policy outcome. In the two period model any future A period is necessarily the last one with no bargaining position to be gained and hence with C not willing to trade-off policy for status-quo or vice versa in the first period,

Figure 1.3: Equilibrium status-quo with explicit status-quo
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



even though it might be a disagreement one.

The absence of an explicit expression for V_C and subsequently for C's proposal strategies further complicates characterization of the policy outcomes under explicit status-quo bargaining. Nevertheless, we are able to prove following.

Proposition 1.5 (Policy outcomes with explicit status-quo). *CS-MPE from proposition 1.4 generates policy and status-quo decisions satisfying the following.*

1. *If $x \in S$ then the policy outcomes display D-efficiency in all subsequent periods*

2. *If $x \in S$ then $p_A(x) = \pi^*$ and $p_D(x) = \pi^* - \phi$ for almost all $x \in S$*

For an initial default option x_0 being a continuous random variable with pdf $f(x_0)$ defined on X, for any $t = 1, 2, \ldots$

3. $\int_{x_0 \in X} \mathbb{P}(x^t(x_0) \notin S) f(x_0) dx_0 \leq 1 - r_d \int_{x_0 \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)} f(x_0) dx_0$
$$-(1 - r_d) \int_{x_0 \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa)} f(x_0) dx_0$$

4. $\int_{x_0 \in X} \mathbb{P}(p_A(x^t(x_0)) = \pi^*) f(x_0) dx_0 = 1$

where $\kappa = \phi\sqrt{\delta r_d(3+\delta r_d)}$.

Proof. See appendix 1.A1.

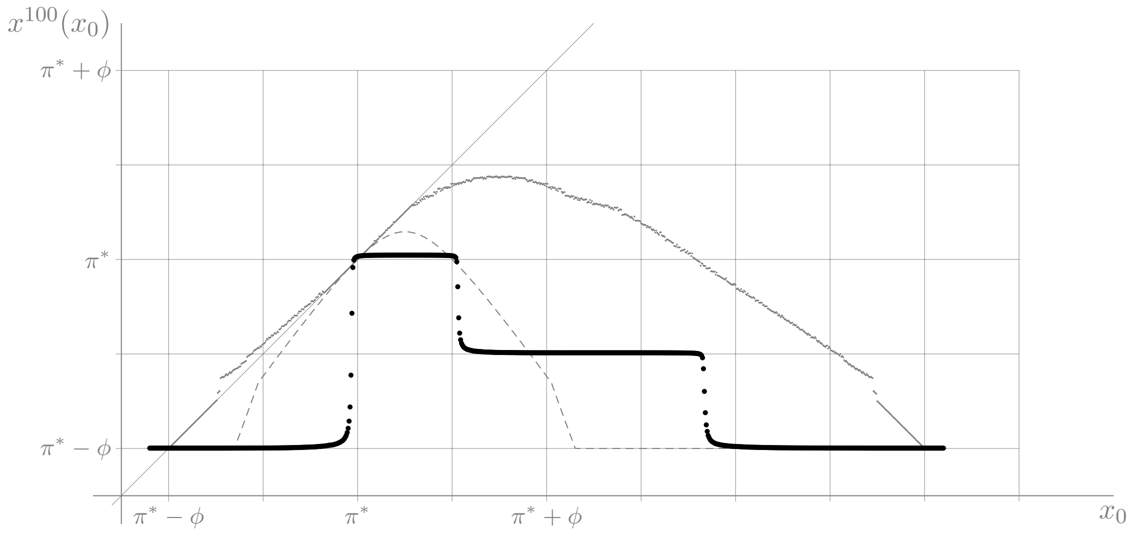
What the proposition 1.5 says is that in CS-MPE from proposition 1.4 under the bargaining protocol with explicit status-quo, when the bargaining outcomes become stable they display D-efficiency (part one). When this happens policy outcomes will be π^* in A periods and $\pi^* - \phi$ in D periods except for a finite set of discrete values of default options in S (part two). Indeed, in the proof of proposition 1.5 we show that the only other candidate default option for inclusion in the S set is $x = \pi^*$. With $\pi^* \in S$ we would have $p_A(\pi^*) = p_D(\pi^*) = \pi^*$. As a result under the explicit bargaining protocol, even when the bargaining outcomes become stable they almost always still reflect the changing preferences of the two policy makers unlike in proposition 1.2 for implicit status-quo bargaining and are both A-efficient and D-efficient, where the former holds no matter whether the bargaining has reached the stable set or not (part four).

Another difference explicit status-quo bargaining brings is that we cannot put an upper bound on the probability of the bargaining staying outside the stable set that would converge to zero over time (part three). We know from the proof of proposition 1.4 that for an initial period being an A (D) one and initial default option x_0 satisfying $x_0 \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa)$ ($X \setminus (\pi^* - \phi, \pi^* + 3\phi)$), we can set C's equilibrium proposal on the status-quo dimension equal to $q_A(x_0) = q_D(x_0) = \pi^* - \phi$ and $q_A(\pi^* - \phi) = q_D(\pi^* - \phi) = \pi^* - \phi$ such that the bargaining becomes stable in the initial period and remains so. When those conditions fail, convergence of the default option to the stable set S remains an open question.

To shed light on the convergence question we generated 10.000 one hundred period long random paths of A and D periods for the parameter values used in figures 1.2 and 1.3. For each path, we derived status-quo proposed in the last period $x^{100}(x_0)$ as a function of the initial default option x_0. Averaging over all the 10.000 paths gives figure 1.4, also depicting (thin lines) equilibrium status-quo offers $q_D(x)$ and $q_A(x)$.

Looking at figure 1.4, for default options x with $q_D(x) < \pi^*$ and $q_A(x) < \pi^*$, C proposes $q_A(x) < x$ in A periods improving her bargaining position by more than by how much it loses it in D periods by proposing $q_D(x) \geq x$. As a result status-quo in the long term converges to $\pi^* - \phi$, or more precisely to

Figure 1.4: Long-run status-quo with explicit status-quo
average over 10.000 random 100 period long paths
$\pi^* = 2, \ \phi = 1, \ \delta = 0.5, \ r_d = 0.5$



the $X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ set out of which figure 1.4 selects $\pi^* - \phi$. In terms of policy outcomes this implies convergence to $\pi^* - \phi$ in D periods and to π^* in A periods with C becoming effectively a dictator in the committee. We call such a committee *authoritarian* or the game being in *authoritarian regime*. However, C has to build up her dominant position gradually over time using A periods to improve her bargaining position and until the status-quo reaches $\pi^* - \phi$, she still has to take into account preferences of the other committee member when crafting her proposal.

For default options x with $q_D(x) > \pi^*$ and $q_A(x) > \pi^*$, status-quo in the long term converges to π^* with C never proposing status-quo that would start the convergent process to $\pi^* - \phi$ discussed above. Such a status-quo is not in P's acceptance set in A periods and would involve considerable loss on the policy dimension in D periods. With the status-quo converging to π^* policy outcomes converge to the same value in both types of periods with the committee becoming consensual and the D period policy outcomes midway in between the preferences of the committee members. We call such a committee *collegial* or the game being in *collegial regime*.

Finally, for default options x with $q_D(x) > \pi^*$ and $q_A(x) < \pi^*$, the long term outcome of the bargaining depends crucially on the nature of the first period. If the bargaining starts with an A period, C is able to start the convergent process towards $\pi^* - \phi$ and the committee eventually becomes authoritarian. Should the bargaining start with D period, C's proposal starts the convergence to π^* and the committee eventually becomes collegial. The line in figure 1.4 between π^* and $\pi^* - \phi$ then reflects the fact that a proportion r_d of the paths converges to $\pi^* - \phi$ whereas the remaining paths converge to π^*.

Notice the strong path dependency displayed by the model. For some default options the committee eventually becomes authoritarian, for some default options it eventually becomes collegial and for some default options the first period plays a crucial role in determining whether the committee becomes of the former or latter type.

Comparison of the bargaining protocols

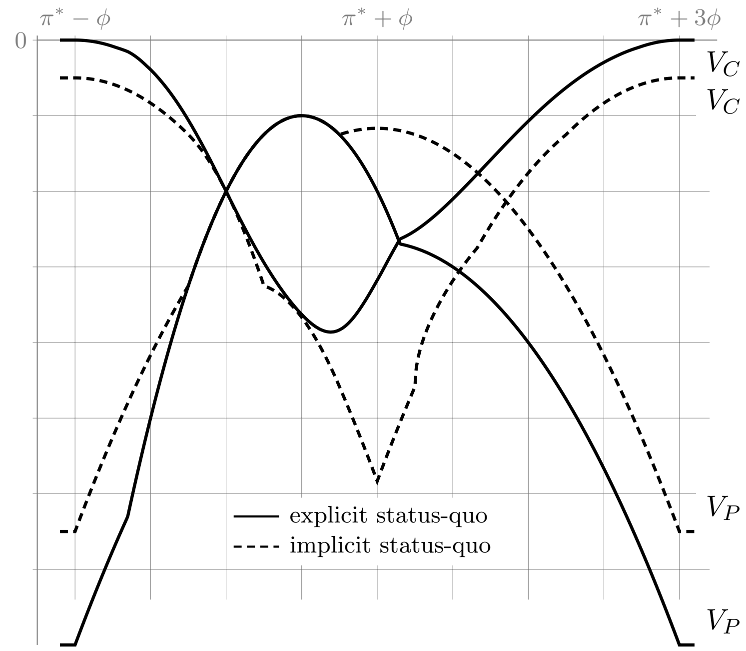
First, we want to provide an answer to the question of comparison between the bargaining protocols from the perspective of the two policy makers. Assume C and P before starting the game just analysed and before the first default option is known, have an option to choose between the bargaining protocols. Would they prefer either of the protocols and does it depend on their beliefs about the initial default option?

Figure 1.5 illustrates the answer to this question. It depicts the value functions of both policy makers for the two bargaining protocols. All the functions are based on the analytical results except for the V_C function in the model with explicit status-quo, which comes from the simulation exercise.

Note first that the intuition about C preferring the bargaining protocol with explicit status-quo as it relaxes the constraint on her optimization problem is misleading as it does not take into account changes in P's strategic behaviour. Nevertheless, figure 1.5 suggests C indeed prefers explicit status-quo bargaining protocol for any beliefs about the initial default option.

For P figure 1.5 suggests he is indifferent between the two bargaining protocols for intermediate values of initial default and strictly prefers bargaining under implicit status-quo otherwise. The intuition behind this result is that for the default options x for which P is indifferent between C's proposal $\gamma_i(x)$ and $\bar{\gamma}(x)$ for $i \in \{A, D\}$ under both bargaining protocols, his

Figure 1.5: Equilibrium value functions
$\pi^* = 2,\ \phi = 1,\ \delta = 0.5,\ r_d = 0.5$



continuation value is equal under the two protocols. On the other hand for the default options for which C is able to extract all the bargaining power in the long term under the bargaining with explicit status-quo, P prefers the other bargaining protocol as he retains some influence over the enacted policies, which then reflect, at least to some extent, his preferences.

Denoting by $V_i^j(x)$ the value function of player $i \in \{C, P\}$ under bargaining protocol $j \in \{E, I\}$ for default option x, the next proposition then shows that the situation depicted in figure 1.5 is a general feature of the model.

Proposition 1.6 (Policy makers' choice over bargaining protocol)**.**

1. $V_C^E(x) - V_C^I(x) \geq 0$ *for* $x \in X$
2. $V_P^E(x) - V_P^I(x) \leq 0$ *for* $x \in X$ *where the inequality is strict for* $x \in X \setminus [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d]$

3. $V_C^E(x) - V_C^I(x) + V_P^E(x) - V_P^I(x) = k$, *where*

$$k \geq 0 \qquad \textit{for} \quad x \in [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d]$$
$$k = -\frac{2\phi^2\delta r_d(1-r_d)}{1-\delta} \qquad \textit{for} \quad x \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$$

Proof. See appendix 1.A1.

The third part of the proposition shows choice over the bargaining protocol by players who are also uncertain over the role they will play in the game. Another interpretation is that it shows which bargaining protocol is preferred from the utilitarian perspective. For non-extreme values of the default option it is the explicit status-quo bargaining protocol. It allows for the A-efficient policy outcomes and the initial bargaining position of the P player prevents C from using her proposal power to determine D period policy fully according to her preferences.

On the other hand, for extreme values of the default option the implicit status-quo protocol dominates from the utilitarian perspective. Although it does not deliver A-efficiency it prevents C from fully using her proposal power. The explicit status-quo would allow C to hold on to her bargaining power, becoming a dictator in the committee.

The proposition also shows by how much the implicit status-quo protocol dominates for $x \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$, i.e. for default options generating the authoritarian regime under explicit status-quo bargaining. The difference increases with δ, is maximized for $r_d = \frac{1}{2}$ and equal to zero for $r_d \in \{0, 1\}$. The intuition for the effect of r_d comes from the benefits and costs of the explicit status-quo protocol. It delivers A-efficiency but creates too much proposal power, implying extreme policies viewed from the perspective of the committee as a whole. With $r_d = 1$ we need not be concerned either with A-efficiency, as there are no A periods, or with the excessive proposal power, as there are no A periods during which C gives up her bargaining position under the implicit status-quo protocol. At the other extreme, with $r_d = 0$ the model is a common preference one with no concerns over efficiency or excessive proposal power present as well.

Multi-member committee

Finally, to prepare for the next section, we want to show that the results presented above apply to any N member committee with fixed chairman, common preferences in A periods and D period preference shocks $-\phi$ and ϕ of the chairman and (not necessarily fixed) median committee member respectively. We call such a committee *essentially two-member* one and define it as follows.

Definition 1.5 (Essentially two-member committee)**.** *We say a committee composed of N (odd) members is an essentially two-member one if it possesses a fixed chairman with proposal power, has common preference for π^* in A periods, i.e. $\varepsilon_{i,t} = 0$ for $i \in \{1, \dots, N\}$, and its D period preference parameters satisfy either*

1. *$\varepsilon_{i,t} = \phi_i$ for $i \in \{1, \dots, N\}$ and $\phi_C = -\phi$, $\phi_m = \phi$ are chairman's and median member's preference parameters respectively,*

or

2. *$\varepsilon_{i,t} = -\phi$ for $i = C$ and $(N-1) \times 1$ vector of remaining preference parameters $\varepsilon_t = \{\varepsilon_{i,t}\}'_{i \in \{1,\dots,N\} \setminus \{C\}}$ satisfies $\varepsilon_t = \phi + \nu_t$ where ν_t is (possibly each D period specific) vector of random variables with number of negative, zero, positive elements equal to $\frac{N-3}{2}$, 2, $\frac{N-3}{2}$ respectively and $\mathbb{E}[\nu_{i,t}] = 0$ for $i \in \{1, \dots, N-1\}$ where $\nu_{i,t}$ is i-th element of ν_t.*

In words, any committee is essentially a two-member one if there is a fixed chairman with proposal power and D period preference shock equal to $-\phi$, the whole committee has common preferences in A periods and the D period preference parameters of the remaining committee members satisfy one of the conditions from the definition. The first condition requires the D period preference parameters to be fixed across periods for a given committee member and existence of a median member (among N members) with a preference shock equal to ϕ. The second condition allows for time varying D period preferences but requires those to be equal to ϕ on average and requires existence of two (each D period possibly different) median committee members (among $N-1$ members) with preference shock equal to ϕ. The reason for requiring two median members is that for the second condition we are now choosing among the $N-1$ non-chairman members, which is an even number, and need an equal number of those members with higher and lower preference shock $\phi + \nu_{i,t}$.

Next we need to rule out equilibria that possibly arise due to the committee members voting against their preferences as they realize they are not pivotal. Following Baron and Kalai (1993) we restrict attention to *stage-undominated* voting strategies that for all members $n \in \{1, \ldots, N\}$, all periods $i \in \{A, D\}$, all default options $x \in X$ and all proposals $\gamma(x) \in X^2$ satisfy

$$n \text{ votes for } \gamma(x) \text{ (against } \bar{\gamma}(x)) \Leftrightarrow \gamma(x) \in A_{i,n}(x).$$

where $A_{i,n}(x)$ is acceptance set of player n in period i and default option x.

With the preliminaries established, we are able to prove the following proposition asserting that the results presented above can be equally applied to any larger committee.

Proposition 1.7 (Committee with more than two members)**.** *Bargaining (policy, status-quo) outcomes under both bargaining protocols for any essentially two-member committee with its members using state-undominated strategies correspond to the bargaining outcomes of a game played between the committee chairman and player with median preference shock and thus to the results presented above.*

Proof. See appendix 1.A1.

1.5 Re-interpretation of asymmetric FOMC directive

In this section we interpret the asymmetry in FOMC directive in light of our model. We first discuss several reasons that make us believe that the FOMC decision making process is better viewed as proceeding under the explicit status-quo bargaining protocol. Adopting this perspective, we show that the model can replicate existing stylized facts about FOMC decision making. Finally, we discuss a novel interpretation of the asymmetry our model provides.

The structure of the model above is largely inspired by the decision making process in most modern central banks (see Mahadeva and Sterne, 2000, for further details). Typically a committee of several members with a well defined chairman is responsible for repeated decisions on a single monetary policy instrument with a goal to anchor inflation to some predetermined level. While having a single objective, the committee members do not always agree on the most appropriate stance of monetary policy, with differences driven both by personal preferences as well as by the external economic environment. Indeed, Chappell, McGregor, and Vermilyea (2005) show significant statistical differences in both intercepts and effects of economic variables in the 'individual reaction functions' of the FOMC members (see Blinder, 2007, for a discussion of possible causes of the preference heterogeneity in monetary policy committees). This opens up the possibility of time-varying disagreement which our model captures in the agreement/disagreement dichotomy. Finally, in most central banks the monetary policy instrument serves also as the status-quo for the next committee meeting.

FOMC, the decision body of the US Federal Reserve System, makes monetary policy decisions but also decides on the 'asymmetry', 'bias' or 'tilt' in its directive. What is formally known as the domestic policy directive is a set of operating instructions sent to the Open Market Trading Desk at the Federal Reserve Bank of New York. Every directive, in addition to current policy, specifies FOMC's expectations regarding future policy, specifying either asymmetry towards policy tightening or easing (asymmetric) or no change (symmetric). In its original form, the asymmetry has been used between 1983 and 1999 (see Thornton and Wheelock, 2000, for historal account), evolved endogenously, and FOMC has never clarified the meaning it has in its decision making. Additionally, the meaning seems to have evolved over time.[7]

We interpret asymmetry in the FOMC directive as a possible difference between the current policy and a status-quo for several reasons. First, its original intent has been to specify a contingency under which the Open Market Trading Desk would change the FOMC operating target before the next FOMC meeting. The transcript of the discussion during the first FOMC meeting to specify asymmetry in the directive reveals this intention. Chairman Volcker summarized that the whole proposed directive 'says we don't

[7] FOMC transcripts reveal a certain ambiguity regarding the meaning of the asymmetric directive. Chairman Greenspan, when asked this question by one of the new FOMC members, answers that FOMC does not have a 'specific formulation. Asymmetry merely means a general sense of the Committees's disposition or the direction' of its bias (Federal Reserve System, 2011, July 5-6, 1994 transcript, p. 69).

want to tighten right now but we do contemplate easing if the aggregates are noticeably, or quite visibly, soft' (Federal Reserve System, 2011, February 8-9, 1983 transcript, p. 83).

Second, it authorized the FOMC's 'chairman to notch up the Fed funds rate if necessary before the next regular meeting' (Greenspan, 2007, p. 102). Intermeeting change in the FOMC operating target, whether upon contingency or at the chairman's discretion, then implies change in the status-quo in between the committee meetings. At any given meeting the committee might find itself facing a default option different from the policy agreed upon at the previous meeting.

Third, the FOMC used the asymmetry to signal its future intentions over the intermediate horizon.[8] A difference between the policy and the status-quo then comes from credibility concerns.[9] Should the FOMC signal its intention to, say, tighten monetary policy without eventually doing so, its credibility would be compromised. Chairman Greenspan saying 'And I'm concerned about the credibility of the [FOMC] sitting with an asymmetric directive time and time again when the purpose of that is essentially to signal an intermediate trend' (Federal Reserve System, 2011, August 17, 1993 transcript, p. 36) lends itself to this explanation. On another occasion, after six consecutive meetings with no change in policy but asymmetry towards tightening, chairman Greenspan in his opening statement of the 'policy go-around' part of the FOMC meeting says that 'It is quite evident that we have come to a point, as we suggested we might at the last meeting, [...] We have to 'deliver'' (Federal Reserve System, 2011, March 25, 1997 transcript, p. 44). The three reasons taken together make us believe it is more appropriate to think of the FOMC as having the explicit status-quo bargaining protocol.

Notwithstanding the ambiguity regarding the meaning of the asymmetric directive, it generated several papers investigating its role in FOMC decision making. Three hypotheses have been put forward. First, the *authorizing intermeeting policy adjustments* hypothesis holds that the asymmetry gave

[8] Until early 1999 the directive has been published only as a part of FOMC minutes few days after its next meeting. Hence its immediate signalling role was rather limited. As Blinder (2007) notes, the long lag between the meeting and the publication of the minutes means that the 'minutes draw little press or market attention when they are published'.

[9] Recently, several central banks started publishing expected future policy paths along with their current monetary policy decision (see Kahn, 2007, for details). Present argument would apply to those central banks as well.

the FOMC chairman discretion to adjust policy stance in between regularly scheduled FOMC meetings. Chappell, McGregor, and Vermilyea (2007) confirm this hypothesis using data from the 1987 to 1992 period during which the intermeeting policy adjustments were common and refute it for the 1993 to 1999 period during which the intermeeting policy adjustments were rare. Thornton and Wheelock (2000) refute the hypothesis using data from the 1983 to 1999 period. However, with the intermeeting policy adjustments rare in the second half of their sample, they are effectively refuting it for the first half contradicting results of Chappell et al. (2007).

Second, the *predicting future policy changes* hypothesis holds that the asymmetry predicts the direction of future policy changes and increases their likelihood. Regarding the direction part of the hypothesis Thornton and Wheelock (2000), Lapp and Pearce (2000) and Pakko (2005) all confirm it using data from the 1983 to 1999, 1984 to 1998 and 1984 to 2003 periods respectively. Evidence on the likelihood part of the hypothesis is mixed with Thornton and Wheelock (2000) refuting it while Lapp and Pearce (2000) and Pakko (2005) reach an opposite conclusion.

Third, the *consensus building* hypothesis holds that the asymmetry allowed FOMC chairman to craft consensus among the FOMC members. Thornton and Wheelock (2000), Meade (2005) and Chappell et al. (2007) all confirm this hypothesis using data from the 1983 to 1999, 1989 to 1997 and 1987 to 1992 periods respectively, while the last paper refutes it for the 1993 to 1999 sample.

With our model silent on the intermeeting policy adjustment hypothesis, we focus on the latter two hypotheses and ask if our model is consistent with either of them.[10] In order to see how the two hypotheses are reflected in FOMC decision making, we use data about its decisions. For each of 48 meetings between February 4, 1994 and December 12, 1999 (inclusive) we record the change in the federal funds rate target and the adopted asymmetry in FOMC directive.[11] The reason for focusing on the period starting

[10] Intermeeting policy adjustments by FOMC chairman have become increasingly rare during the 1990's. For example, in the 1994 through 1999 period there have been only two intermeeting changes (see Thornton and Wheelock, 2000, for further details).

[11] While the federal funds rate target has not become FOMC's operating target until August 1997 with extent of restraint on commercial bank reserve positions being its operating target prior, there is considerable consensus that FOMC has been shifting its focus from the restraint on reserve positions to the federal funds rate as its operating target well before 1997 (see Thornton and Wheelock, 2000, for detailed discussion).

with the February 4, 1994 meeting is that it marks beginning of FOMC's practice of announcing target changes immediately upon making them and the beginning of the practice of making target changes almost exclusively at the regular FOMC meetings. The December 12, 1999 meeting is then the last meeting before the asymmetry in FOMC directive was replaced by a 'balance of risk assessment'.[12]

Existing hypotheses vs. simulated data

One way to generate theoretical predictions of our model is to simulate random path of equilibrium policy and status-quo proposals. To do so we took C's equilibrium proposal strategies depicted in figures 1.2 and 1.3 and generated 100.000 random 100 period long paths of policy and status-quo decisions, $\{p_1, \ldots, p_{100}\}$ and $\{q_1, \ldots, q_{100}\}$, with initial default option x_0 uniformly distributed on the $[\pi^* - \phi, \pi^* + \phi]$ interval.[13] We classify each meeting in period $t \in \{2, \ldots, 100\}$ of a given path as resulting in policy increase, no change or decrease depending on whether $p_t - p_{t-1} \geq \chi$, $|p_t - p_{t-1}| < \chi$ or $p_t - p_{t-1} \leq -\chi$ respectively. Each meeting also generates asymmetry towards increase, no change or decrease depending on whether $q_t - p_t \geq \chi$, $|q_t - p_t| < \chi$ or $q_t - p_t \leq -\chi$ respectively. We set $\chi = 0.075$ in order to match approximately the empirical ratio of the number of meetings resulting in no policy change to the number of meetings resulting in policy change (2.20). Finally, we rescale all data in the simulated sample to mach the number of meetings in the FOMC sample (48).

Table 1.1 shows data for the predicting future policy changes hypothesis, recording policy change during the given meeting and asymmetry adopted during the previous meeting. FOMC data clearly show support for the direction part of the hypothesis with FOMC never decreasing (increasing) the federal funds rate target with tightening (easing) asymmetry in its directive adopted previously. Similar holds for the simulated data with asymmetry towards policy increase (decrease) never followed by policy decrease (increase)

[12] Alternatively we could have focused on the period up to March 30, 1999 meeting after which FOMC began its practice of publishing statement immediately after each meeting that also included asymmetry contained in its directive (Farka, 2010), but none of the results would be substantially altered. Nor would the results change had we taken our data to start with the February 8-9, 1983 meeting, the very first one to specify the asymmetry in FOMC directive.

[13] We also experimented with either 2 period long paths or x_0 distributed uniformly on $[\pi^* - \phi, \pi^* + 3\phi]$ with little change in the results.

Table 1.1: Predicting future policy changes hypothesis
$t+1$ period policy change and t period asymmetry

t period asymmetry	$t+1$ period policy change					
	FOMC sample			Simulated sample		
	+	0	−	+	0	−
+	7	14	0	1	1	0*
0	3	18	4	7	25	0
−	0	1	1	0*	7	7

Note: Number of meetings in each cell. FOMC sample from February 4, 1994 to December 12, 1999. Simulated data rescaled and rounded to 48 meetings. * zero before rounding.

during the subsequent meeting.

For the likelihood part of the hypothesis, which holds that the asymmetric directive is associated with higher likelihood of policy change, results are mixed. FOMC meetings data in table 1.1 show that FOMC changed the federal funds rate target at 15 of its 48 meetings (31.3%) while conditional on asymmetric directive adopted at a previous meeting, FOMC changed the federal funds rate target at 8 of 23 meetings (34.8%). A simple proportions test of the hypothesis that 34.8% equals 31.3% (as opposed to the alternative of the former percentage being higher) yields insignificant test statistics (p-value 0.36).[14] Simulated data then show policy change at 15 out of 48 meetings (31.3%) and conditional on asymmetric directive at 8 out of 16 meetings (50.0%) with test statistics for the test of 50.0% being equal to 31.3% (with the same alternative as above) marginally significant (p-value 0.05).

In order to test the consensus building hypothesis we replicate the argument from Thornton and Wheelock (2000). They argue that the asymmetry in FOMC directive serves a consensus building role, with the asymmetric directives adopted more often during the meetings with no policy change as opposed to meetings with a policy change. Table 1.2 shows data for the consensus building hypothesis, recording policy change during given meeting and asymmetry adopted during the same meeting.

FOMC data in table 1.2 show that the asymmetric directive has been

[14] This test is based on normal approximation of binomial with the test statistic equal to $(r'-r)/\sqrt{r(1-r)/n}$ standard normal distributed. In this case $r'=0.348$, $r=0.313$ and $n=23$. We use similar test as Thornton and Wheelock (2000) for comparability.

Table 1.2: Consensus building hypothesis
t period policy change and t period asymmetry

t period asymmetry	t period policy change			
	FOMC sample		Simulated sample	
	+/−	0	+/−	0
+/−	3	20	8	8
0	12	13	7	25

Note: Number of meetings in each cell. FOMC sample from February 4, 1994 to December 12, 1999. Simulated data rescaled to 48 meetings.

adopted at 23 out of 48 meetings (47.9%) while conditional on no policy change at the same meeting the asymmetric directive has been adopted at 20 out of 33 meetings (60.6%). Using the same test as above to test the hypothesis that 60.6% equals 47.9% (as opposed to the alternative of the former percentage being higher) produces marginally significant test statistics (p-value 0.07). For the simulated data we obtain asymmetric directive adopted at 16 out of 48 meetings (33.3%) and conditional on no policy change asymmetric directive adopted at 8 out of 33 meetings (24.2%) with test statistic for the test of 24.2% being equal to 33.3% (with the same alternative as above) insignificant (p-value 0.87).

Existing hypotheses vs. authoritarian regime

Comparison of the simulated and FOMC decision data faces two possible objections. First, it is dependent on the choice of values for the model parameters. Second, empirical literature on FOMC decision making often notes dominance of chairman Greenspan (see for example Chappell et al., 2005). Hence comparison to the simulated data, which capture convergence to the authoritarian or the collegial regimes explained in the context of discussion of figure 1.4, might not be appropriate.

Table 1.3 shows the comparison the model generates assuming the bargaining has already converged to the authoritarian regime. For the policy, AA and DD paths generate no change while AD and DA paths generate policy decrease and increase respectively, as $p_D(x) = \pi^* - \phi$ and $p_A(x) = \pi^*$ in the authoritarian regime. For the asymmetry we have $q_D(x) = q_A(x) = \pi^* - \phi$ in the authoritarian regime and hence A peri-

Table 1.3: Authoritarian regime
predicting future policy/consensus building hypothesis

path	probability	asymmetry		policy change
		$t-1$	t	t
AA	$(1-r_d)^2$	–	–	0
AD	$r_d(1-r_d)$	–	0	–
DD	r_d^2	0	0	0
DA	$r_d(1-r_d)$	0	–	+

ods produce asymmetry towards policy decrease while D periods produce asymmetry towards no policy change.

It is apparent from table 1.3 that even in the authoritarian regime the asymmetry has an ability to predict the direction of future policy changes. Asymmetry at the $t-1$ period meeting towards lower policy predicts decrease or no change in the policy during the t period meeting while asymmetry towards no policy change predicts subsequent increase or no change in the policy.

For the increased likelihood of the policy change under the asymmetric directive hypothesis, the probability of the policy change is $2r_d(1-r_d)$ and conditional on asymmetric $t-1$ period asymmetry it is r_d, with the latter larger for $r_d \geq \frac{1}{2}$. For the consensus building hypothesis, the authoritarian regime predicts asymmetric directive adopted with probability $1-r_d$ and conditional on no policy change with probability $\frac{(1-r_d)^2}{(1-r_d)^2+r_d^2}$, with the latter larger for $r_d \leq \frac{1}{2}$. Finally, in the authoritarian regime the ratio of the number of meetings resulting in no policy change to the number of meetings resulting in a policy change is equal to $\frac{(1-r_d)^2+r_d^2}{2r_d(1-r_d)}$. This ratio is larger than 2, the approximate ratio in the FOMC data, either for $r_d \leq \frac{3-\sqrt{3}}{6} \doteq 0.21$ or for $r_d \geq \frac{3+\sqrt{3}}{6} \doteq 0.79$.

As a result, for the high degree of conflict in the committee ($r_d \geq \frac{1}{2}$) the authoritarian regime predicts increased likelihood of policy changes given asymmetric directive adopted during the previous meeting but no consensus building role of the asymmetry. On the other hand for the low degree of conflict in the committee ($r_d \leq \frac{1}{2}$) the authoritarian regime predicts a consensus building role of the asymmetry but not increased likelihood of policy changes under the asymmetric directive.

Adopting the view that the FOMC can be approximated by the authoritarian regime with low degree of conflict, our model then predicts a consensus building role of the asymmetry, its ability to predict direction of future policy changes and the majority of meetings resulting in no change in policy, but not that policy is changed more often under the asymmetric directive.

Novel role of asymmetric directive

Besides capturing major stylized facts about the FOMC decision outcomes, the model suggest a novel role of the asymmetry in its directive. One of the predictions for the explicit status-quo protocol, relative to the implicit status-quo one, is that it allows the chairman to gain or retain a dominant position in the committee. When this happens, the chairman is able to press for policy outcomes fully reflecting her preferences. We call this view of the asymmetric directive the *preservation of supremacy* hypothesis.

Dominance of chairman Greenspan in FOMC is not new. Chappell et al. (2005), Blinder (2007) and Meade (2005) all acknowledge it. Blinder (2007) even goes as far as claiming that it is 'quite possible for the Fed to adopt one policy even though the (unweighted) majority favoured another' and ranks the Federal Reserve System very low in terms of democracy in making monetary policy decisions.[15] Interestingly, the original inclusion of the asymmetry in the directive was made upon the suggestion of then chairman Volcker.

While we cannot rigorously test the preservation of supremacy hypothesis because we lack appropriate counterfactuals, the following anecdotal evidence is at least suggestive of its validity. For six consecutive meetings since the July 2-3, 1996 meeting, FOMC has kept the federal funds rate unchanged, adopting asymmetric directive towards tightening in all those meetings. The series was interrupted by the 25 basis point increase at the March 25, 1997 meeting (with symmetric directive) and followed by another 5 meetings with no change in the federal funds rate and asymmetric directive towards tightening, until the November 12, 1997 meeting.

During the whole period FOMC was receiving signals which would, un-

[15] The ranking includes central banks of (from the least to the most democratic) New Zealand, Canada, Australia, USA, Japan, Switzerland, Euro zone, Sweden and UK. In the first three central banks it is the governor responsible for the policy (see Maier, 2010, for details).

der normal circumstances, call for tighter monetary policy. But, as chairman Greenspan argued, the US economy was not operating under normal circumstances. His explanation for declining unemployment and non-increasing inflation was higher productivity growth, at that time not yet apparent from the economic data. But 'his insight played to an unresponsive audience' (Meyer, 2004, p. 80) with 'many committee members [...] leaning [...] toward an increase' (Greenspan, 2007, p. 171).

During the whole period chairman Greenspan tried to persuade the FOMC members out of tightening monetary policy move. The pattern started with the July 2-3, 1996 meeting with chairman Greenspan arguing that his 'judgment is that in all likelihood, if the Committee does not move at [that] meeting or during the intermeeting period, [it] will do so at the August meeting or later' (Federal Reserve System, 2011, July 2-3, 1996 transcript, p. 89). He made similar argument professing to believe that 'the probability of our having to move [...] is still above 50 percent', and that FOMC confronts 'far greater likelihood that the next move will be up rather than down' (Federal Reserve System, 2011, September 24, 1996 and December 17, 1996 transcripts, p. 29 and 36 respectively).

Chairman Greenspan did not use only the probability of near future policy tightening as his argument. When proposing yet another no change in the federal funds rate, he used asymmetry in the FOMC directive proposing an 'asymmetry that is unlike that at the previous couple of meetings. [...] a real asymmetry' (Federal Reserve System, 2011, February 4-5, 1997 meeting stranscript, p. 104).[16] Meyer (2004, p. 83) summarizes chairman Greenspan's behaviour during the periods as 'speaking like a hawk and walking like a dove'.

Combining chairman Greenspan's dominance in FOMC and his disagreement with many of the FOMC members, we can interpret the episode in light of our model as a series of D periods in the authoritative regime. The model then predicts series of $\pi^* - \phi$ policy choices with the status-quo set at the same level, i.e. with symmetric directive. Discrepancy with the asymmetric directives in FOMC decisions is nevertheless only apparent. Future no change or increase in the policy is in the model associated with symmetric di-

[16] FOMC transcripts reveal some of the committee members becoming increasingly uneasy with the continuing discrepancy between the unchanging policy and the asymmetric directive, such as when Ms. Rivlin remarks that she finds 'meaning of these asymmetries a little mysterious' (Federal Reserve System, 2011, December 17, 1996 transcript, p. 36).

rectives but with asymmetric (tightening) directives in the FOMC decisions. Crucially, it is the explicit status-quo protocol that allows the chairman to preserve his dominance in the committee.

Concluding remarks

We have shown that our model, with the explicit status-quo bargaining protocol, can well represent data generated by the FOMC decision making process. It can replicate existing stylized facts and, additionally, gives us an alternative perspective from which the FOMC decision making can be approached and discussed.

While hardly conclusive, we believe the asymmetry in FOMC directive allowed its chairman to influence US monetary policy, at least to some extent. The two opening quotes of the paper then capture the basic trade-off our model creates under the explicit status-quo, increased efficiency at the potential cost of disproportionate proposal power. The former quote, taken at face value, pertains to the efficiency part of the trade-off. But its meaning pertains to the disproportionate proposal power part. The quote can be taken to mean that the asymmetry allowed the FOMC chairman to carry out policy more to his liking than he would be allowed otherwise. The latter quote then shows that this is a real, not only hypothetical, possibility.[17]

Interestingly, the two quotes refer to the same episode, the 1996-1997 event described above. With hindsight, chairman Greenspan turned out to be correct and among the first to identifying a change in the productivity trend. Indeed, the second opening quote immediately goes on to say 'We give him enormous credit for doing so.' The 'we' does not include everybody (see The Economist, 2006, for an alternative view), but that is another story.

1.A1 Proofs

1.A1.1 Proof of proposition 1.1

Preliminaries

To prove the existence part of proposition 1.1 we construct CS-MPE in a model with implicit status-quo. We are forced to split the equilibria of

[17] The former quote is interesting from another perspective. It comes from Laurence H. Meyer who is often viewed as Greenspan's fiercest opponent in the productivity debate.

the model into four distinct cases depending on the parameters δ and r_d. However, the logic of the proof is always the same. For each case we first state C's equilibrium proposal strategies. These are generated conjecturing a CS-MPE equilibrium giving the V_P function with acceptance sets A_A and A_D of which proposal strategies, we conjecture, are minima (unless C's unconstrained optima lie in the appropriate acceptance sets). Next, we specify V_C and V_P generated by the proposal strategies and derive the shape of associated acceptance sets A_A and A_D. In the next step we characterize the shape of C's overall utility in A and D periods deriving her unconstrained maxima in the two periods to be $\gamma_{CA} = \pi^* - \phi\delta r_d$ and $\gamma_{CD} = \pi^* - \phi$ respectively. With P's acceptance sets and C's overall utility, we confirm the proposal strategy originally given is indeed optimal for C and can be written in the form given in the proposition.

Having established existence of CS-MPE by constructing it we next turn to the uniqueness part of proposition 1.1 by showing that the CS-MPE constructed is the unique one. Here we note that in any CS-MPE, V_P has to be the one derived in the existence part and we establish uniqueness of the solution to C's dynamic optimization problem (1.1).

Throughout the whole proof we maintain the assumption on $\{\delta, r_d\}$ expressed in the proposition, that is we maintain

Assumption 1.1. *For any pair $\{\delta, r_d\}$ with $\delta \in [0, 1)$ and $r_d \in [0, 1]$ assume $\delta^2 r_d(3 - 2r_d) \leq 1 - \delta(1 - r_d)$.*

Despite the logic of the proof being rather straightforward, the proof itself is rather lengthy and algebra intensive. Striving to keep its length to a minimum, we sometimes omit proofs of purely algebraic results but always indicate how those can be shown.

Throughout the proof, we often refer to C in D periods as to CD and similarly for P (PD) and by analogy in A periods to CA and PA respectively. To save on notation we denote instantaneous utility of the policy makers by

$$f_{CD}(x) = -(x - \pi^* + \phi)^2 \qquad f_{PD}(x) = -(x - \pi^* - \phi)^2$$
$$f_{CA}(x) = -(x - \pi^*)^2 \qquad f_{PA}(x) = -(x - \pi^*)^2$$

and the overall utility by

$$U_{CD}(x) = f_{CD}(x) + \delta V_C(x) \qquad U_{PD}(x) = f_{PD}(x) + \delta V_P(x)$$
$$U_{CA}(x) = f_{CA}(x) + \delta V_C(x) \qquad U_{PA}(x) = f_{PA}(x) + \delta V_P(x).$$

Throughout the proof we are forced to work with a series of intervals in the default option space X. Those are always denoted by I_i and are always closed (except where explicitly indicated) and convex subsets of X. The upper boundary of I_i is denoted by I_i^U and lower boundary by I_i^L.

Many of the functions in the proof are defined piecewise. If this is the case then we use the notation $f^{I_i}(x)$ for function $f(x)$ constrained to the appropriate interval. Derivatives are often denoted by primes when no confusion as to with respect to which variable the derivative is being taken is imminent.

It will become apparent that many of the functions we work with are differentiable only in the interior of the intervals but not at the point where the two intervals meet. Taking general $f(x)$, $f'(I_i^U)$ will often fail to exist as $f(x)$ has a kink at I_i^U. If this is the case then $f'^{I_i}(I_i^U)$ will always denote left derivative, i.e. derivative as $x \to I_i^U$ from below, and $f'^{I_i}(I_i^L)$ will denote right derivative, i.e. derivative as $x \to I_i^L$ from above.

It is helpful first to establish following lemmas.

Lemma 1.4.

$$U'_{CD}(x) \geq 0 \Rightarrow U'_{CA}(x) \geq 0 \qquad U'_{PD}(x) \geq 0 \Leftarrow U'_{PA}(x) \geq 0$$
$$U'_{CD}(x) \leq 0 \Leftarrow U'_{CA}(x) \leq 0 \qquad U'_{PD}(x) \leq 0 \Rightarrow U'_{PA}(x) \leq 0$$

Proof. The lemma follows from the readily verifiable facts that $f'_{CA}(x) > f'_{CD}(x)$ and $f'_{PA}(x) < f'_{PD}(x)$ that naturally assumes differentiability of the V_C and V_P functions. A similar result for V_C and V_P non-differentiable at some specific x but possessing left and right derivatives at x follows by analogy. □

Lemma 1.5. *Let $h(x)$ and $k(x)$ be two real valued continuously differentiable functions defined on $[t-r, t]$ and $[t, t+r]$ respectively, for some $t, r \in \mathbb{R}$ and $r > 0$. Assume $k(t) = h(t)$ and that the first derivative of the functions satisfies $k'(t+x) \leq -h'(t-x)$ for all positive $x \leq r$. Then $k(t+r) \leq h(t-r)$.*

Proof. Integrating the derivative inequality in the lemma with respect to x from 0 to r gives

$$\int_0^r k'(t+z)dz \leq -\int_0^r h'(t-z)dz$$
$$k(t+r) - k(t) \leq h(t-r) - h(t)$$
$$k(t+r) \leq h(t-r)$$

□

Lemma 1.6. *Define*

$$z(x) = \pi^* + \phi(1-\delta(1-r_d)) - \sqrt{\frac{1-\delta}{1-\delta r_d}(x-\pi^*-\phi)^2 + \phi^2\delta(1-r_d)\left(\frac{4\delta^2 r_d^2}{1-\delta r_d} - (1-\delta)\right)}.$$

Then

$$\mathrm{sgn}[z(x)'] = \mathrm{sgn}[\pi^* + \phi - x]$$
$$\mathrm{sgn}[z(x)''] = \mathrm{sgn}[-(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d))].$$

Proof. Denote the term in the square root of $z(x)$ by $T(x)$. Then

$$z(x)' = -\frac{1}{\sqrt{T(x)}}\frac{1-\delta}{1-\delta r_d}(x-\pi^*-\phi)$$
$$z(x)'' = -\frac{1}{T(x)^{3/2}}\frac{1-\delta}{(1-\delta r_d)^2}\phi^2\delta(1-r_d)(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d)).$$

□

Next we give explicit formulas for the continuation value functions of the two policy makers used throughout the proof. As already mentioned, both of the functions are defined piecewise on the different I_i intervals, but we leave the specific definition of the intervals for later when we will show that in the equilibrium the induced continuation value function of C can be pasted together from the following.

$$
\begin{aligned}
V_C^{I_1}(x) =& V_C^{I_{12}}(x) = -\frac{1-r_d}{1-\delta}\phi^2\delta r_d \\
V_C^{I_2}(x) =& V_C^{I_5}(x) = -\frac{r_d}{1-\delta r_d}\left[(x-\pi^*+\phi)^2+\phi^2\frac{\delta(1-r_d)(1-\delta r_d)}{1-\delta}\right] \\
V_C^{I_3}(x) =& -\frac{1}{1-\delta}\left[(x-\pi^*+\phi r_d)^2+\phi^2 r_d(1-r_d)\right] \\
V_C^{I_4}(x) =& V_C^{I_3}(x)+\frac{8(1-r_d)\delta r_d}{(1-\delta)(1-\delta r_d)}[\phi(x-\pi^*)-\phi^2\delta r_d] \\
V_C^{I_6}(x) =& V_C^{I_{11}}(x) = -\frac{r_d}{1-\delta r_d}\left[(\pi^*+3\phi-x)^2+\phi^2\frac{\delta(1-r_d)(1-\delta r_d)}{1-\delta}\right] \\
V_C^{I_7}(x) =& r_d\left[(2(\pi^*+\phi(1-\delta(1-r_d)))-x-\pi^*+\phi)^2+\delta V_C^{I_4}(2(\pi^*+\phi(1-\delta(1-r_d)))-x)\right] \\
& (1-r_d)\left[(2(\pi^*+\phi\delta r_d)-x-\pi^*)^2+\delta V_C^{I_3}(2(\pi^*+\phi\delta r_d)-x)\right] \\
V_C^{I_8}(x) =& r_d\left[(2(\pi^*+\phi(1-\delta(1-r_d)))-x-\pi^*+\phi)^2+\delta V_C^{I_3}(2(\pi^*+\phi(1-\delta(1-r_d)))-x)\right] \\
& (1-r_d)\left[(2(\pi^*+\phi\delta r_d)-x-\pi^*)^2+\delta V_C^{I_3}(2(\pi^*+\phi\delta r_d)-x)\right] \\
V_C^{I_9}(x) =& r_d\left[-(z(x)-\pi^*+\phi)^2+\delta V_C^{I_4}(z(x))\right]+(1-r_d)\left[-(-\phi\delta r_d)^2+\delta V_C^{I_3}(\pi^*-\phi\delta r_d)\right] \\
V_C^{I_{10}}(x) =& r_d\left[-(z(x)-\pi^*+\phi)^2+\delta V_C^{I_3}(z(x))\right]+(1-r_d)\left[-(-\phi\delta r_d)^2+\delta V_C^{I_3}(\pi^*-\phi\delta r_d)\right]
\end{aligned}
$$

Likewise, P's continuation value function in the equilibrium will be pasted together from the following functions.

$$
\begin{aligned}
V_P^{I_3}(x) =& -\frac{1}{1-\delta}\left[(x-\pi^*-\phi r_d)^2+\phi^2 r_d(1-r_d)\right] \\
=& V_P^{I_4}(x) = V_P^{I_7}(x) = V_P^{I_8}(x) \\
V_P^{I_2}(x) =& -\frac{r_d}{1-\delta r_d}\left[(x-\pi^*-\phi)^2+\phi^2\frac{\delta(1-r_d)(1+3\delta r_d)}{1-\delta}\right] \\
=& V_P^{I_5}(x) = V_P^{I_6}(x) = V_P^{I_9}(x) = V_P^{I_{10}}(x) = V_P^{I_{11}}(x) \\
V_P^{I_1}(x) =& V_P^{I_{12}}(x) = -\frac{\phi^2 r_d}{1-\delta}(4-3\delta(1-r_d))
\end{aligned}
$$

At the time being, use of 12 different I_i's might seem redundant, but as will become apparent the fact that the value functions are identical on some intervals is a coincidence. Indeed, they will be induced by parts of the equilibrium that are different in nature.

Having the V_P function we can explain the rationale behind the $z(x)$

function from lemma 1.6. Looking at V_P it consists of two quadratic terms that apply on different I_i intervals, a property the U_{PD} function will inherit. The $z(x)$ function then allows us to compare U_{PD} across intervals where it is given by different quadratic terms, or formally, $z(x)$ solves $U_{PD}(x) = U_{PD}(z(x))$ for $x \in I_3 \cup I_4$ and $z(x) \in I_9 \cup I_{10}$. More specifically, as the proposition claims that C implements the policy corresponding to the minimal accepted one, $z(x)$ gives us a lower boundary of A_D for default options in the $I_9 \cup I_{10}$ interval. We do not need similar functions for other intervals as those lower boundaries will be linear functions of the default option x.

We sometimes need to use an inverse of $z(x)$ as well. Formally speaking, as $z(x)$ is not monotone, $z^{-1}(x)$ is not well defined. However, it is apparent there are exactly two solutions x to the equation $k = z(x)$ for a given constant k. Taking the larger of the two, we can define the inverse of the function $z(x)$ as $z^{-1}(x) = \{\max\{y : x = z(y)\}\}$.

Existence

Case 1: Equilibrium for $\delta \leq \frac{1}{1+2r_d}$

For $\delta \leq \frac{1}{1+2r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for } x \in I_1 \cup I_2 \cup I_5 \cup I_6 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for } x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for } x \in I_4 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for } x \in I_1 \cup I_{12} \\ x & \text{for } x \in I_2 \cup I_3 \cup I_4 \cup I_5 \\ 2(\pi^* + \phi) - x & \text{for } x \in I_6 \cup I_{11} \\ z(x) & \text{for } x \in I_9 \cup I_{10} \end{cases}$$

where

$$\begin{aligned}
I_1 &= [x^-, \pi^* - \phi] & I_6 &= [\pi^* + \phi, \pi^* + \phi(2 - 3\delta r_d)] \\
I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_9 &= [\pi^* + \phi(2 - 3\delta r_d), \tau^+] \\
I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{10} &= [\tau^+, \pi^* + \phi(2 + \delta r_d)] \\
I_4 &= [\pi^* + \phi\delta r_d, \pi^* + 3\phi\delta r_d] & I_{11} &= [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi] \\
I_5 &= [\pi^* + 3\phi\delta r_d, \pi^* + \phi] & I_{12} &= [\pi^* + 3\phi, x^+]
\end{aligned}$$

where $\tau^+ = \pi^* + \phi + \phi\sqrt{(1 - \delta r_d)^2 - \frac{4\delta^3 r_d^2(1-r_d)}{1-\delta}}$ (τ^- to be used later is defined analogously with the term in the square root subtracted) and x^- and x^+ are respectively lower and upper boundaries of the policy space X.

To see the term in the square root of τ^+ is always positive, substitute in $\delta = 1/(1 + 2r_d)$ which gives a positive expression. Then, differentiating the term in the square root with respect to δ gives an expression that can be regarded as a cubic equation in δ. It has one real root and the derivative is negative below the root. As the root is always higher than unity, it follows that the original expression has to be positive.

It is straightforward to show that the equilibrium offers induce the continuation value functions given above on the appropriate I_i intervals and that both V_C and V_P are continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. Next we need to describe the shape of the U_{PA} and U_{PD} functions.

claim 1.1 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, U_{PD} has a global maximum at $\pi^* + \phi$ and both functions are quasi-concave.*

Proof. It is straightforward to show that U_{PA} is increasing (and hence U_{PD} as well by lemma 1.4) on $I_1 \cup I_2 \cup I_3$. Similarly U_{PD} is decreasing (and hence U_{PA} by the same lemma) on $I_6 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$. The remaining two intervals, I_4 and I_5, are easy to show as well. It follows U_{PA} has to have a global maximum at $\pi^* + \phi\delta r_d$, which is the boundary of I_3 with I_4 and U_{PD} has to have a global maximum at $\pi^* + \phi$, which is the boundary of I_5 with I_6. Quasi-concavity then follows. □

The next two claims outline the shape of P's acceptance sets.

claim 1.2 (Shape of $A_A(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_3$ then $A_A(x) = \{p : x \leq p \wedge p \leq x'\}$ with $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4$*

2. *if $x \in I_4$ then $A_A(x) = \{p : x' \leq p \wedge p \leq x\}$ with $x' = 2(\pi^* + \phi\delta r_d) - x \in I_3$*

3. *if $x \notin I_3 \cup I_4$ then $\pi^* - \phi\delta r_d \in A_A(x)$.*

Proof. Notice U_{PA} is symmetric around $\pi^* + \phi\delta r_d$, which is its global maximum on $I_3 \cup I_4$. Moreover, for any $x \in I_3$, U_{PA} is increasing up to x and for any $x \in I_4$, U_{PA} is decreasing from x on. Hence the first part follows. A similar argument proves the second part.

To see the third part, notice $U_{PA}(I_3^L) = U_{PA}(I_4^U)$ and $I_3^L = \pi^* - \phi\delta r_d$. The third part then follows by the same argument as in the preceding paragraph about the increasing and decreasing parts of U_{PA}. □

claim 1.3 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*

2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*

3. *if $x \in I_3 \cup I_4$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = z^{-1}(x) \in I_9 \cup I_{10}$*

4. *if $x \in I_5$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_6$*

5. *if $x \in I_6$ then $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ where $x' = 2(\pi^* + \phi) - x \in I_5$*

6. *if $x \in I_9 \cup I_{10}$ then $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ where $x' = z(x) \in I_3 \cup I_4$*

7. *if $x \in I_{11}$ then $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ where $x' = 2(\pi^* + \phi) - x \in I_2$.*

Proof. All the parts below use the fact that for $x \leq \pi^* + \phi$, U_{PD} is increasing up to x and for $x \geq \pi^* + \phi$, U_{PD} is decreasing from x on. Also convexity of $A_D(x)$ for given x follows from quasi-concavity of U_{PD}.

For part one, notice $U_{PD}(I_1^U) = U_{PD}(I_{12}^L)$ and $I_1^U = \pi^* - \phi$ which along with the argument in the preceding paragraph gives the result.

For part two, notice U_{PD} is symmetric around $\pi^* + \phi$ for $x \in I_2 \cup I_{11}$. This also proves part seven.

For part three, by quasi-concavity of U_{PD} and the fact that U_{PD} has a global maximum at $\pi^* + \phi$ there must exist an upper boundary of the acceptance set that satisfies $x' \geq \pi^* + \phi$. It is easy to confirm $x' \in I_9 \cup I_{10}$ and that x' has to solve $x = z(x')$, i.e. $x' = z^{-1}(x)$.

For part four, notice U_{PD} is symmetric around $\pi^* + \phi$ for $x \in I_5 \cup I_6$. Hence the fourth part follows. This also proves part five.

For part six, we are looking for x' that solves $U_{PD}(x) = U_{PD}(x')$ with $x \in I_9 \cup I_{10}$. It is easy to confirm $x' = z(x) \in I_3 \cup I_4$ is the solution to this equation. □

The following claim gives the shape of the U_{CD} and U_{CA} functions.

claim 1.4 (Shape of U_{CA} and U_{CD})**.**

1. *U_{CA} is increasing on $I_1 \cup I_2$ and decreasing on $I_3 \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$*

2. *U_{CD} is increasing on I_1 and decreasing on $I_2 \cup I_3 \cup I_4 \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$*

3. *$U_{CA}(x) \geq U_{CA}(x')$ where $x \in I_3$ and $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4$*

4. *$U_{CA}(\pi^* - \phi\delta r_d) \geq \max_{x \in I_9} U_{CA}(x)$*

5. *$U_{CD}(z(x)) \geq U_{CD}(x')$ $\forall x' \in [I_9^L, x]$ given $x \in I_9$*

6. *U_{CA} has a global maximum at $\pi^* - \phi\delta r_d$ and U_{CD} at $\pi^* - \phi$.*

Proof. The first part is straightforward given the continuation value functions above, except for I_{10}. To establish $U_{CA}^{I_{10}}$ is decreasing, first note

$$V_C''^{I_{10}}(x) = r_d z(x)'' \left[U_{CD}'^{I_3}(z(x)) \right] - r_d \frac{2}{1-\delta} [z(x)']^2.$$

The sign of $z(x)''$ by lemma 1.6 depends on the sign of $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d)$, which is negative for $\delta \leq 1/(1+2r_d)$, and hence $z(x)''$ is positive. The sign of $U_{CD}'^{I_3}(z(x))$ is negative by part two of this claim and the last term is negative so $V_C''^{I_{10}}(x)$ is negative. It follows $U_{CA}''^{I_{10}}$ is concave so if we can establish that $U_{CA}'^{I_{10}}(I_{10}^L)$ is negative the claim follows.

Evaluating $U'^{I_{10}}_{CA}(x)$ at $I^L_{10} = \tau^+$ gives

$$U'^{I_{10}}_{CA}(\tau^+) = -2\phi\left[1 + \left(\frac{\tau^+ - \pi^* - \phi}{\phi}\right)\left(\frac{1-\delta-2\delta r_d(1-\delta(1-r_d))}{(1-\delta)(1-\delta r_d)}\right)\right]$$

where the term in the brackets is positive. To see this, note that the last term in the equation $1-\delta-2\delta r_d(1-\delta(1-r_d)) > 0$. This can be seen regarding the expression as a quadratic equation in δ. It is negative between the roots. One of the roots is higher than unity and the second one is higher than $1/(1+2r_d)$. This establishes the first part.

For the second part, it is again straightforward to establish most of the results. For I_{10} the claim follows from part one of this claim and lemma 1.4 and for I_4 the claim follows by assumption 1.1.

The third part follows readily from the derivatives of U_{CA} on I_3 and I_4 using lemma 1.5 that can be used as I_3 and I_4 have the same width.

To establish the fourth part where we cannot use the derivative argument as U_{CA} may have local maximum on I_9. First note

$$V'^{I_9}_{C}(x) = r_d z(x)'\left[U'^{I_4}_{CD}(z(x))\right],$$

which by lemma 1.6 and part two of this claim is positive. Furthermore f_{CA} is decreasing on I_9. Using the inequality $\max_x f(x) + \max_x g(x) \geq \max_x f(x) + g(x)$ we can derive the upper bound on $U^{I_9}_{CA}$ as we know the maxima of the $f^{I_9}_{CA}$ and $V^{I_9}_{C}$ functions.

The upper bound is given by

$$f_{CA}(I^L_9) + \delta V^{I_9}_{C}(I^U_9) \geq \max_{x\in I_9} U^{I_9}_{CA}(x)$$

and we need to show it is lower than $U_{CA}(\pi^* - \phi\delta r_d)$. Some algebra gives

$$1 - 3\delta r_d + 3\delta^2 r_d^2 + \frac{\delta^3 r_d^3}{1-\delta} \geq 0,$$

which holds. To see this, we can disregard the last term in the expression that is positive. Regarding the remaining as a quadratic equation in δ gives a pair of roots both of which are complex and it is easy to confirm the expression has to be positive.

The fifth part is complicated by the fact that U_{CD} may have local maxima on I_9. First note that if we prove $U_{CD}(z(x)) \geq U_{CD}(x)\ \forall x \in I_9$ then we are done by the fact that U_{CD} is decreasing on I_4 and $z(x) \in I_4\ \forall x \in I_9$.

To start, we note the relevant parts of the V_C function can be alternatively expressed as

$$\begin{aligned} V_C^{I_9}(x) =& r_d[f_{CD}(z(x)) + \delta V_C^{I_4}(z(x))] \\ &+ (1-r_d)[f_{CA}(\pi^* - \phi\delta r_d) + \delta V_C^{I_3}(\pi^* - \phi\delta r_d)] \\ V_C^{I_4}(x) =& r_d[f_{CD}(x) + \delta V_C^{I_4}(x)] \\ &+ (1-r_d)[f_{CA}(2(\pi^* + \phi\delta r_d) - x) + \delta V_C^{I_3}(2(\pi^* + \phi\delta r_d) - x)], \end{aligned}$$

which upon substitution into $U_{CD}(z(x)) - U_{CD}(x)$ simplifies the algebra as the first square brackets disappear. Nevertheless, some lengthy and uninstructive algebra remains and gives

$$\begin{aligned} &U_{CD}(z(x)) - U_{CD}(x) = \\ &\qquad 4\phi\left[(x-\pi^*) - \frac{1-\delta-\delta^2 r_d + \delta^2 r_d^2}{1-\delta}(z(x)-\pi^*) - \frac{3\phi\delta^3 r_d^2(1-r_d)}{1-\delta}\right]. \end{aligned}$$

with the derivation using

$$(z(x)-\pi^*)^2 = \phi^2(1-\delta(1-r_d))^2 + T(x) + 2\phi(1-\delta(1-r_d))(z(x)-\pi^*).$$

It is easy to confirm this expression is positive for $x = I_9^L$. Taking the derivative with respect to x then gives

$$[U_{CD}(z(x)) - U_{CD}(x)]' = 4\phi\left[1 - \frac{1-\delta-\delta^2 r_d + \delta^2 r_d^2}{1-\delta} z(x)'\right],$$

which is positive. To see this notice $1-\delta-\delta^2 r_d+\delta^2 r_d^2 > 0$ for $\delta \leq 1/(1+2r_d)$ and $z(x)'$ is negative by lemma 1.6. This proves the fifth part. The sixth part is then a direct consequence of the above. □

It is now easy to confirm the specified offers are indeed an equilibrium and can be written in the way used in proposition 1.1. By claim 1.4, CA either implements her unconstrained maximum $\pi^* - \phi\delta r_d$ or minimum of $A_A(x)$. This follows from the shape of A_A given in claim 1.2, which implies that if $\pi^* - \phi\delta r_d \notin A_A(x)$ for some x then $A_A(x) \in I_3 \cup I_4$.

For CD, the best option is when the unconstrained maximum $\pi^* - \phi$ is available. If she cannot implement $\pi^* - \phi$, then the lowest possible policy is implemented. This follows directly from claim 1.4 where the only problematic interval is I_9. But in claim 1.3 we have shown that for $x \in I_4$ the acceptance set takes the form $[x, z^{-1}(x)]$ and for $x \in I_9$ the acceptance set takes the form $[z(x), x]$. But then by part five of claim 1.4, CD implements as low a policy as possible. This concludes proof of case 1.

Case 2: Equilibrium for $\delta \geq \frac{1}{1+2r_d}$ and $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$

For $\delta \geq \frac{1}{1+2r_d}$ and $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for} \quad x \in I_1 \cup I_2 \cup I_5 \cup I_6 \cup I_{9-} \cup I_{9+} \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for} \quad x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for} \quad x \in I_4 \cup I_7 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for} \quad x \in I_1 \cup I_{12} \\ x & \text{for} \quad x \in I_2 \cup I_3 \cup I_4 \cup I_5 \\ 2(\pi^* + \phi(1-\delta(1-r_d))) - x & \text{for} \quad x \in I_7 \\ 2(\pi^* + \phi) - x & \text{for} \quad x \in I_6 \cup I_{11} \\ z(x) & \text{for} \quad x \in I_{9-} \cup I_{9+} \cup I_{10} \end{cases}$$

where

$$\begin{aligned} I_1 &= [x^-, \pi^* - \phi] & I_5 &= (\tau_1^-, \pi^* + \phi] \\ I_2 &= [\pi^* - \phi, \pi^* - \phi\delta r_d] & I_6 &= [\pi^* + \phi, \tau_1^+) \\ I_3 &= [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] & I_{9+} &= [\tau_1^+, \tau^+] \\ I_4 &= [\pi^* + \phi\delta r_d, \pi^* + \phi(1-\delta(1-r_d))] & I_{10} &= [\tau^+, \pi^* + \phi(2+\delta r_d)] \\ I_7 &= [\pi^* + \phi(1-\delta(1-r_d)), \pi^* + 3\phi\delta r_d] & I_{11} &= [\pi^* + \phi(2+\delta r_d), \pi^* + 3\phi] \\ I_{9-} &= [\pi^* + 3\phi\delta r_d, \tau_1^-] & I_{12} &= [\pi^* + 3\phi, x^+] \end{aligned}$$

where as before $\tau^+ = \pi^* + \phi + \phi\sqrt{(1-\delta r_d)^2 - \frac{4\delta^3 r_d^2(1-r_d)}{1-\delta}}$ and $\tau_1^{\pm}$ are defined as $\tau_1^- = \pi^* + \phi - \phi\sqrt{\frac{\delta(1-r_d)}{1-\delta}((1-\delta)(1-\delta r_d) - 4\delta^2 r_d^2)}$ and τ_1^+ analogously with the term involving the square root being added.

By the condition on this case, the term under the square root in $\tau_1^{\pm}$ is positive. To see the term in the square root of τ^+ is positive, follow the same procedure as for case 1 but instead of substituting $\delta = 1/(1+2r_d)$ substitute condition $\delta = 1/(1+r_d)$ that is indeed a weaker condition than the condition defining case 2, $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \leq 0$.

It is a matter of simple algebra to confirm that the equilibrium offers induce the continuation value functions specified above where I_{9+} and I_{9-} correspond to I_9. For V_P it is easy to show that the function is continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. For V_C it can be shown that it is differentiable everywhere except at the boundaries of the I_i intervals. Regarding continuity, V_C is continuous everywhere except at I_5^L and I_6^U where it jumps in a discrete manner. This is a direct consequence of the equilibrium offers not being continuous at the same points with respect to the default x. We first describe the shape of U_{PA} and U_{PD}.

claim 1.5 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5 \cup I_{9-}$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$ and is quasi-concave. U_{PD} has two local maxima at $\pi^* + \phi(1-\delta(1-r_d))$ and $\pi^* + \phi$ the latter of which is also a global maximum. U_{PD} has one local minimum at $\pi^* + 3\phi\delta r_d$.*

Proof. It is easy to show U_{PA} is increasing (and hence U_{PD} as well by lemma 1.4) on $I_1 \cup I_2 \cup I_3$. Similarly U_{PD} is decreasing (and hence U_{PA} by the same lemma) on $I_7 \cup I_6 \cup I_{9+} \cup I_{10} \cup I_{11} \cup I_{12}$. The remaining three intervals, I_4, I_{9-} and I_5, are equally easy. It follows U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, which is a boundary of I_3 with I_4 and its quasi-concavity follows. Similarly, U_{PD} has two local maxima. One at the boundary of I_4 and I_7 and the second at the boundary of I_5 and I_6. Also, it follows that a local minimum has to be at the boundary of I_7 and I_{9-}. It is easy to show $\pi^* + \phi$ is the global maximum. □

Next we wish to characterize the acceptance sets. As the shape of the A_A is exactly the same as in claim 1.2 we do not repeat it here. For the A_D we have the following.

claim 1.6 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*
2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*

3. *if* $x \in I_3 \cup [I_4^L, \pi^* + 2\phi(1-\delta(1+r_d/2))]$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = z^{-1}(x) \in I_9 \cup I_{10}$

4. *if* $x \in [\pi^* + 2\phi(1-\delta(1+r_d/2)), I_4^U]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x \leq p \wedge p \leq x'\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x'' + x''' = 2(\pi^* + \phi)$, $x = z(x'') = z(x''')$, $x' \in I_7$, $x'' \in I_{9-}$ *and* $x''' \in I_{9+}$

5. *if* $x \in I_7$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x' \leq p \wedge p \leq x\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x'' + x''' = 2(\pi^* + \phi)$, $x' = z(x'') = z(x''')$, $x' \in I_4$, $x'' \in I_{9-}$ *and* $x''' \in I_{9+}$

6. *if* $x \in I_{9-}$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x \leq p \wedge p \leq x'\}$, $x'' + x''' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_4$, $x''' \in I_7$ *and* $x' \in I_{9+}$

7. *if* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x' \leq p \wedge p \leq x\}$, $x'' + x''' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_4$, $x''' \in I_7$ *and* $x' \in I_{9-}$

8. *if* $x \in I_5$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_6$

9. *if* $x \in I_6$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_5$

10. *if* $x \in [\pi^* + \phi(2 - 3\delta r_d), I_{9+}^U] \cup I_{10}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

11. *if* $x \in I_{11}$ *then* $A_D(x) = \{x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_2$.

Proof. Parts one through three and eight through eleven are very similar to the relevant parts in claim 1.3. What we cannot use is the quasi-concavity of U_{PD}. However, it is easy to confirm that the acceptance sets are convex.

Parts four through seven present the key difference compared to claim 1.3. To see these, first notice for the default options specified, U_{PD} has two peaks. One peak is symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$ and the second one around $\pi^* + \phi$. It then follows $U_{PD}(x) = U_{PD}(x')$ gives four solutions. One pair symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$ and the second pair symmetric around $\pi^* + \phi$. It is then a matter of straightforward algebra to work out the appropriate intervals. □

Following claim gives the shape of U_{CA} and U_{CD} functions.

claim 1.7 (Shape of U_{CA} and U_{CD})**.**

1. *U_{CA} is increasing on $I_1 \cup I_2$ and decreasing on $I_3 \cup I_{9-} \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$*
2. *U_{CD} is increasing on I_1 and decreasing on $I_2 \cup I_3 \cup I_4 \cup I_7 \cup I_{9-} \cup I_5 \cup I_6 \cup I_{10} \cup I_{11} \cup I_{12}$*
3. *$U_{CA}(x) \geq U_{CA}(x')$ where $x \in I_3$ and $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7$*
4. *$U_{CA}(x'') \geq U_{CA}(x')$ and $U_{CD}(x'') \geq U_{CD}(x')$ for every $x' \in [I^L_{9+}, x]$ given $x \in [I^L_{9+}, \pi^* + \phi(2 - 3\delta r_d)]$ with $x'' = 2(\pi^* + \phi) - x \in I_{9-}$.*
5. *$U_{CA}(\pi^* - \phi\delta r_d) \geq \max_{x \in [\pi^* + \phi(2 - 3\delta r_d), I^U_{9+}]} U_{CA}(x)$*
6. *$U_{CD}(z(x)) \geq U_{CD}(x') \, \forall x' \in [\pi^* + \phi(2 - 3\delta r_d), x]$ given $x \in [\pi^* + \phi(2 - 3\delta r_d), I^U_{9+}]$*
7. *U_{CA} has a global maximum at $\pi^* - \phi\delta r_d$ and U_{CD} at $\pi^* - \phi$.*

Proof. The first part is straightforward given the continuation value functions except for I_{10}. As in claim 1.4 we have V_C concave on this interval so if we can establish that $U'^{I_{10}}_{CA}(I^L_{10})$ is negative the claim follows. In claim 1.4 this gave us equation

$$U'^{I_{10}}_{CA}(\tau^+) = -2\phi \left[1 + \left(\frac{\tau^+ - \pi^* - \phi}{\phi} \right) \left(\frac{1 - \delta - 2\delta r_d(1 - \delta(1 - r_d))}{(1 - \delta)(1 - \delta r_d)} \right) \right]$$

where we could establish negativity by the fact that $1 - \delta - 2\delta r_d(1 - \delta(1 - r_d)) > 0$. For the current case we need to do more work as this inequality might not be satisfied.

Note that $\frac{\tau^+ - \pi^* - \phi}{\phi} < 1 + \delta r_d$, which can be seen consulting the definition of the I_i intervals. Hence if we can prove the derivative is negative when $\frac{\tau^+ - \pi^* - \phi}{\phi}$ is replaced by $1 + \delta r_d$ the claim follows. Doing that gives

$$U'^{I_{10}}_{CA}(\tau^+) = -4\phi \left[\frac{1 - \delta - \delta r_d(1 - \delta(1 - r_d))(1 + \delta r_d)}{(1 - \delta)(1 - \delta r_d)} \right],$$

which is negative as the term in the square brackets is positive. To see that, take the nominator and substitute $\delta = (1 + r_d - \sqrt{1 - 2r_d + 17r_d^2})/(2r_d(1 - 4r_d))$, which is the solution to the condition defining case 2, and confirm the expression is positive. Next, taking the derivative of the nominator with respect to δ gives a quadratic equation in δ with the derivative being negative between the roots. One of the roots is negative and the second one is higher than unity. This shows the $U'^{I_{10}}_{CA}(\tau^+)$ is negative and hence proves the first part of the claim.

The second part of the claim is straightforward using the similar argument as part two of claim 1.4. Likewise, the third part can be established using the same argument as part three of claim 1.4 noting that the width of I_3 is the same as the width of $I_4 \cup I_7$.

To see the fourth part, notice that if we show $U_{CA}(x') \geq U_{CA}(x)$ and $U_{CD}(x') \geq U_{CD}(x)$ where $x' = 2(\pi^* + \phi) - x \in I_{9-}$ for every default option $x \in [I^L_{9+}, \pi^* + \phi(2 - 3\delta r_d)]$ then we are done. However, it is easy to confirm $V_C(x') = V_C(x)$ for x, x' just defined. Hence the claim follows.

The fifth part can be established using a similar argument as in part 4 of claim 1.4 where the derivation of the upper bound on U^{9+}_{CA} is done using exactly the same values.

To prove the sixth part, again the same argument as in part five of claim 1.4 can be used. However, the conditions on δ defining case 2 alone are not sufficient to ensure $1 - \delta - \delta^2 r_d + \delta^2 r_d^2 > 0$. However, the inequality still holds by virtue of assumption 1.1. Finally, the last part is a direct consequence of the above. □

Again, putting claims 1.2, 1.6 and 1.7 together proves the specified offers are indeed an equilibrium. CA can either implement her unconstrained optimum $\pi^* - \phi\delta r_d$ and when this policy is not available, she offers as low a policy as possible.

The same logic applies for CD. Using claim 1.7, CD either offers her unconstrained maximizer $\pi^* - \phi$ and if this is not available she offers as low a policy as possible. This can be seen from the fact that U_{CD} is decreasing over the majority of I_i intervals for policies above $\pi^* - \phi$. When we cannot establish decreasing U_{CD}, claims 1.7 and 1.6 imply that whenever any policy from such an interval is available, there is also available another policy that gives CD higher utility, with this policy in turn rejected in favour of the lowest policy available. This concludes the proof of case 2.

Case 3: Equilibrium for $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \geq 0$ and $\delta \leq \frac{1}{3r_d}$

For $4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d) \geq 0$ and $\delta \leq \frac{1}{3r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for} \quad x \in I_1 \cup I_2 \cup I_{10-} \cup I_{9-} \cup I_{9+} \cup I_{10+} \cup I_{11} \cup I_{12} \\ x & \text{for} \quad x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for} \quad x \in I_4 \cup I_7 \cup I_8 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for} \quad x \in I_1 \cup I_{12} \\ x & \text{for} \quad x \in I_2 \cup I_3 \cup I_4 \\ 2(\pi^* + \phi(1-\delta(1-r_d))) - x & \text{for} \quad x \in I_7 \cup I_8 \\ z(x) & \text{for} \quad x \in I_{10-} \cup I_{9-} \cup I_{9+} \cup I_{10} \\ 2(\pi^* + \phi) - x & \text{for} \quad x \in I_{11} \end{cases}$$

where

$$I_1 = [x^-, \pi^* - \phi] \qquad I_{10-} = [\pi^* + 3\phi\delta r_d, \tau^-]$$
$$I_2 = [\pi^* - \phi, \pi^* - \phi\delta r_d] \qquad I_{9-} = [\tau^-, \pi^* + \phi]$$
$$I_3 = [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] \qquad I_{9+} = [\pi^* + \phi, \tau^+]$$
$$I_4 = [\pi^* + \phi\delta r_d, \pi^* + \phi(1-\delta(1-r_d))] \qquad I_{10+} = [\tau^+, \pi^* + \phi(2+\delta r_d)]$$
$$I_7 = [\pi^* + \phi(1-\delta(1-r_d)), \pi^* + 2\phi(1-\delta(1-r_d/2))] \qquad I_{11} = [\pi^* + \phi(2+\delta r_d), \pi^* + 3\phi]$$
$$I_8 = [\pi^* + 2\phi(1-\delta(1-r_d/2)), \pi^* + 3\phi\delta r_d] \qquad I_{12} = [\pi^* + 3\phi, x^+].$$

Case 3 indeed subsumes two important subcases depending on whether $\delta \leq 1/(1+r_d)$ holds and one of the subcases can even be split further. However, to economize on space and avoid extensive repetition of similar arguments we have decided to treat all the subcases at once.

We stress that some of the I_i intervals above might not be properly defined. For $\delta \geq 1/(1+r_d)$ the intervals are exactly as those just given with the qualification that I_{9-} and I_{9+} might not exist if τ^- and τ^+ become complex. If this happens, then I_{10-} and I_{10+} naturally extend all the way to $\pi^*+\phi$. If below we need to distinguish those two cases, we refer to case 3.1 if $\delta \geq 1/(1+r_d)$ with $\tau^\pm$ real and to case 3.2 if $\delta \geq 1/(1+r_d)$ with $\tau^\pm$ complex.

The remaining possibility, referred to as case 3.3, is when $\delta \leq 1/(1+r_d)$ in which case I_8 ceases to exist and I_7 extends all the way to $\pi^* + 3\phi\delta r_d$. If this happens, I_{10-} also ceases to exist and I_{9-} starts immediately at $\pi^* + 3\phi\delta r_d$.

As before, the equilibrium offers induce the continuation value functions given above where I_{9-} and I_{9+} map into I_9 and analogously for $I_{10\pm}$. Both V_C and V_P are continuous everywhere and differentiable everywhere except at the boundaries of the I_i intervals. Proceeding similarly, we first describe the shape of U_{PA} and U_{PD}.

claim 1.8 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_{10-} \cup I_{9-}$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$ and is quasi-concave. U_{PD} has two local maxima at $\pi^* + \phi(1-\delta(1-r_d))$ and $\pi^* + \phi$ the former of which is also a global maximum. U_{PD} has one local minimum at $\pi^* + 3\phi\delta r_d$.*

Proof. The argument is essentially as in claim 1.5 adjusting for different intervals. The key difference is that the global maximum is at $\pi^* + \phi(1 - \delta(1-r_d))$ and not at $\pi^* + \phi$, something that can be readily verified. □

To characterize the shape of the acceptance sets, A_A described in claim 1.2 applies for the current case as well and we do not repeat it here. Before we describe A_D let us define another pair of constants $\tau_2^{\pm}$ given by the expression $\tau_2^- = \pi^* + \phi(1-\delta(1-r_d)) - \phi\sqrt{\frac{\delta(1-r_d)}{1-\delta r_d}(4\delta^2 r_d^2 - (1-\delta)(1-\delta r_d))}$ and analogously for τ_2^+. Notice that by one of the conditions defining case 3, the term in the square root is positive. With this definition we have the following.

claim 1.9 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*
2. *if $x \in I_2$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = 2(\pi^* + \phi) - x \in I_{11}$*
3. *if $x \in [I_3^L, \pi^* + 2\phi(1-\delta(1+r_d/2))]$ then $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ where $x' = z^{-1}(x) \in I_{9+} \cup I_{10+}$*
4. *if $x \in [\pi^* + 2\phi(1-\delta(1+r_d/2)), \tau_2^-]$ then $A_D(x) = A_D^1(x) \cup A_D^2(x)$ where $A_D^1 = \{p : x \leq p \wedge p \leq x'\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$,*

$x+x' = 2(\pi^*+\phi(1-\delta(1-r_d)),\ x''+x''' = 2(\pi^*+\phi),\ x = z(x'') = z(x''')$, $x' \in I_7 \cup I_8$, $x'' \in I_{10-} \cup I_{9-}$ *and* $x''' \in I_{9+} \cup I_{10+}$

5. *if* $x \in [\tau_2^-, \pi^* + \phi(1-\delta(1-r_d))]$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi(1-\delta(1-r_d))) - x \in I_7 \cup I_8$

6. *if* $x \in [\pi^* + \phi(1-\delta(1-r_d)), \tau_2^+]$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi(1-\delta(1-r_d))) - x \in I_3 \cup I_4$

7. *if* $x \in [\tau_2^+, \pi^* + 3\phi\delta r_d]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x' \leq p \wedge p \leq x\}$, $A_D^2 = \{p : x'' \leq p \wedge p \leq x'''\}$, $x + x' = 2(\pi^* + \phi(1-\delta(1-r_d)),\ x'' + x''' = 2(\pi^* + \phi),\ x' = z(x'') = z(x''')$, $x' \in I_3 \cup I_4$, $x'' \in I_{10-} \cup I_{9-}$ *and* $x''' \in I_{9+} \cup I_{10+}$

8. *if* $x \in I_{10-} \cup I_{9-}$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x \leq p \wedge p \leq x'\}$, $x''+x''' = 2(\pi^*+\phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_3 \cup I_4$, $x''' \in I_7 \cup I_8$ *and* $x' \in I_{9+} \cup I_{10+}$

9. *if* $x \in [I_{9+}^L, \pi^* + \phi(2 - 3\delta r_d)]$ *then* $A_D(x) = A_D^1(x) \cup A_D^2(x)$ *where* $A_D^1 = \{p : x'' \leq p \wedge p \leq x'''\}$, $A_D^2 = \{p : x' \leq p \wedge p \leq x\}$, $x'' + x''' = 2(\pi^* + \phi(1-\delta(1-r_d))$, $x + x' = 2(\pi^* + \phi)$, $x'' = z(x) = z(x')$, $x'' \in I_3 \cup I_4$, $x''' \in I_7 \cup I_8$ *and* $x' \in I_{10-} \cup I_{9-}$

10. *if* $x \in [\pi^* + \phi(2-3\delta r_d), I_{10+}^U]$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

11. *if* $x \in I_{11}$ *then* $A_D(x) = \{x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^*+\phi) - x \in I_2$.

Proof. The proof is very similar to the proof of claim 1.6 where the key difference arises due to the fact that the higher of the peaks is the one symmetric around $\pi^* + \phi(1-\delta(1-r_d))$. □

To finish the proof of case 3, we need to show C indeed wants to implement as low a policy as possible. The next claim proves that.

claim 1.10 (Shape of U_{CA} and U_{CD})**.**

1. *U_{CA} is increasing on $I_1 \cup I_2$ and decreasing on $I_3 \cup I_{10-} \cup I_{9-} \cup I_{11} \cup I_{12}$*

2. *U_{CD} is increasing on I_1 and decreasing on $I_2 \cup I_3 \cup I_4 \cup I_{10-} \cup I_{9-} \cup I_{11} \cup I_{12}$*

3. *$U_{CA}(x) \geq U_{CA}(x')$ where $x \in I_3$ and $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7 \cup I_8$*

4. *$U_{CD}(x) \geq U_{CD}(x')$ where $x \in I_3 \cup I_4$ and $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_7 \cup I_8$*

5. *$U_{CA}(x'') \geq U_{CA}(x')$ and $U_{CD}(x'') \geq U_{CD}(x')$ for every $x' \in [I^L_{9+}, x]$ given $x \in [I^L_{9+}, \pi^* + \phi(2 - 3\delta r_d)]$ with $x'' = 2(\pi^* + \phi) - x \in I_{10-} \cup I_{9-}$.*

6. *U_{CA} and U_{CD} are decreasing on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$*

7. *U_{CA} has a global maximum at $\pi^* - \phi\delta r_d$ and U_{CD} at $\pi^* - \phi$.*

Proof. The first and second parts of the claim can be readily verified using expressions for the continuation value function V_C.

Part three can be established using lemma 1.6 where we note that we are allowed to use it given that the width of I_3 is the same as width of $I_4 \cup I_7 \cup I_8$. The same argument gives part four as the width of $I_3 \cup I_4$ is larger than the width of $I_7 \cup I_8$.

To see the fifth part, notice that if we show that $U_{CA}(x') \geq U_{CA}(x)$ and $U_{CD}(x') \geq U_{CD}(x)$ with $x' = 2(\pi^* + \phi) - x \in I_{10-} \cup I_{9-}$ for every default policy $x \in [I^L_{9+}, \pi^* + \phi(2 - 3\delta r_d)$ then we are done. However, it is easy to confirm $V_C(x') = V_C(x)$ for x, x' just defined and the claim follows.

Part six is the key difficulty. Note that by lemma 1.4 it suffices to show U_{CA} decreasing. However, we cannot rely on concavity of V_C as in claims 1.4 and 1.7. Instead we will use the following strategy. Writing $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ we replace $V'_C(x)$ by the upper bound on its maximum on the appropriate interval and show the resulting expression is negative, which also proves that U_{CA} is decreasing.

Here we are forced to split the proof according to different cases. For cases 3.1 and 3.2 the interval $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$ falls into I_{10+} and we can write

$$V'^{I_{10+}}_C(x) = r_d z(x)' \left[U'^{I_3}_{CD}(z(x))\right]$$

where we want to find an upper bound on the maximum of $V'^{I_{10+}}_C$ on the interval $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$. To do so notice both of the terms are negative and hence if we can find minima of the two terms treated separately this will give us something that has to be higher than the maximum of $V'^{I_{10+}}_C$.

It is easy to establish $z(x)'$ is decreasing on I_{10+} while the term in the square brackets is increasing on I_{10+}. It follows that if we evaluate $z(x)'$ at

I^U_{10+} and $U'^{I_3}_{CD}(z(x))$ at $\pi^* + \phi(2 - 3\delta r_d)$ the resulting expression will give us an upper bound on the maximum of $V'_C(x)$ on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$. Doing so gives

$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I^U_{10+}]} z(x)' \geq -1$$
$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I^U_{10+}]} U'_{CD}(z(x)) = -6\phi,$$

which gives us a maximum for V'_C. It is then a matter of straightforward algebra to substitute the maximum into $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ and confirm the resulting expression is negative on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{10+}]$.

For case 3.3, $\pi^* + \phi(2 - 3\delta r_d) \in I_{9+}$ so that we need to use a similar argument but separately on $[\pi^* + \phi(2 - 3\delta r_d), I^U_{9+}]$ and I_{10+}. We can still use

$$V'^{I_{9+}}_C(x) = r_d z(x)' \left[U'^{I_4}_{CD}(z(x))\right] \qquad V'^{I_{10+}}_C(x) = r_d z(x)' \left[U'^{I_3}_{CD}(z(x))\right]$$

and the fact that $z(x)'$ is decreasing on $I_{9+} \cup I_{10+}$ and $U'^{I_4}_{CD}(z(x))$ with $U'^{I_3}_{CD}(z(x))$ are increasing on I_{9+} and I_{10+} respectively. It follows we need to evaluate $z(x)'$ at I^U_{9+} and I^U_{10+}, $U'^{I_4}_{CD}(z(x))$ at $\pi^* + \phi(2-3\delta r_d)$ and $U'^{I_3}_{CD}(z(x))$ at I^L_{10+}.

The evaluation gives

$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I^U_{9+}] \cup I_{10+}} z(x)' \geq -1$$
$$\min_{x \in [\pi^* + \phi(2-3\delta r_d), I^U_{9+}]} U'_{CD}(z(x)) = -\frac{2\phi}{(1-\delta)(1-\delta r_d)}(3(1 - \delta - \delta r_d + \delta^2 r_d^2) - \delta^2 r_d(1 - r_d))$$
$$\min_{x \in I_{10+}} U'_{CD}(z(x)) = -\frac{2\phi}{1-\delta}(1 - \delta + 2\delta r_d).$$

Upon substitution of the maximum of V'_C into $U'_{CA}(x) = f'_{CA}(x) + \delta V'_C(x)$ the condition for U_{CA} decreasing on I_{10+} becomes

$$\frac{\delta r_d}{1-\delta}(1 - \delta + 2\delta r_d) - 1 - \sqrt{(1 - \delta r_d)^2 - \frac{4\delta^3 r_d^2(1 - r_d)}{1-\delta}} \leq 0,$$

which holds. To see this notice that for $r_d \leq 1/2$ we are done. Otherwise, substituting $\delta = 1/(1 + r_d)$ confirms the condition holds for maximum δ allowed for case 3.3. The derivative of the condition with respect to δ is positive and hence the condition must hold. Therefore U_{CA} (and hence U_{CD} by lemma 1.4) is decreasing on I_{10+}.

For $[\pi^* + \phi(2-3\delta r_d), I^U_{9+}]$, upon substitution the corresponding condition is $(1-\delta)(4\delta r_d - 1) - 3\delta^2 r_d^2(1-\delta r_d) + \delta^3 r_d^2(1-r_d) \leq 0$, which holds for case 3.3. To see this regard it as a cubic equation in δ. Solving for the roots, noticing that the condition holds for δ below the lowest root and showing that the lowest root is higher than $1/3r_d$ proves the claim. Finally the last part of the claim follows from all the above. □

Combining the information provided by claims 1.2, 1.9 and 1.10 proves the equilibrium for case 3. CA either offers her unconstrained maximizer $\pi^* - \phi\delta r_d$ and when this policy is not available, then she offers as low a policy as possible. This follows from the information about the intervals over which U_{CA} is decreasing provided by claim 1.10 and where we cannot use this argument the same claim implies that the minimum policy available gives CA the highest utility among the policies available. The same argument applies for CD and concludes the proof for case 3.

Case 4: Equilibrium for $\delta \geq \frac{1}{3r_d}$

For $\delta \geq \frac{1}{3r_d}$ the equilibrium offers are

$$p_A(x) = \begin{cases} \pi^* - \phi\delta r_d & \text{for } \; x \in I_1 \cup I_2 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12} \\ x & \text{for } \; x \in I_3 \\ 2(\pi^* + \phi\delta r_d) - x & \text{for } \; x \in I_4 \cup I_7 \cup I_8 \end{cases}$$

$$p_D(x) = \begin{cases} \pi^* - \phi & \text{for } \; x \in I_1 \cup I_{12} \\ x & \text{for } \; x \in I_2 \cup I_3 \cup I_4 \\ 2(\pi^* + \phi(1-\delta(1-r_d))) - x & \text{for } \; x \in I_7 \cup I_8 \\ z(x) & \text{for } \; x \in I_9 \cup I_{10} \\ 2(\pi^* + \phi) - x & \text{for } \; x \in I_{11} \end{cases}$$

where

$$I_1 = [x^-, \pi^* - \phi] \qquad I_9 = [\pi^* + 3\phi\delta r_d, \tau^+]$$
$$I_2 = [\pi^* - \phi, \pi^* - \phi\delta r_d] \qquad I_{10} = [\tau^+, \pi^* + \phi(2 + \delta r_d)]$$
$$I_3 = [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d] \qquad I_{11} = [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi]$$
$$I_4 = [\pi^* + \phi\delta r_d, \pi^* + \phi(1 - \delta(1 - r_d))] \qquad I_{12} = [\pi^* + 3\phi, x^+].$$
$$I_7 = [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 2\phi(1 - \delta(1 - r_d/2))]$$
$$I_8 = [\pi^* + 2\phi(1 - \delta(1 - r_d/2)), \pi^* + 3\phi\delta r_d]$$

As in the previous case we have subsumed two subcases and prove the equilibrium for those jointly. The first subcase, referred to as case 4.1, is for $\delta \geq 1/(1 + r_d)$. If this condition holds all the intervals are as those given except for I_9 that does not exist and I_{10} starts at $\pi^* + 3\phi\delta r_d$. For $\delta \leq 1/(1 + r_d)$, referred to as case 4.2, the interval I_8 does not exist and I_7 extends all the way to $\pi^* + 3\phi\delta r_d$.

Once again it is easy to confirm that the strategies given induce continuation value functions on the corresponding intervals. For the current case both V_C and V_P are continuous everywhere and differentiable everywhere except for points where the different I_i intervals meet. Proceeding similarly, we first give the properties of U_{PA} and U_{PD}.

claim 1.11 (Shape of U_{PA} and U_{PD})**.** *U_{PA} is increasing on $I_1 \cup I_2 \cup I_3$ and decreasing otherwise. U_{PD} is increasing on $I_1 \cup I_2 \cup I_3 \cup I_4$ and decreasing otherwise. U_{PA} has a global maximum at $\pi^* + \phi\delta r_d$, U_{PD} has global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$ and both functions are quasi-concave.*

Proof. The argument is very similar to the one used to prove claim 1.1 with minor adjustments for the fact that U_{PD} has a global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$, which is immediately apparent upon realizing that $\pi^* + \phi(1 - \delta(1 - r_d))$ is a boundary of I_4 and I_7. □

Proceeding to outline the shape of the acceptance sets, for A_A the claim 1.2 applies for the current case as well and we do not repeat it here. For A_D we have following.

claim 1.12 (Shape of $A_D(x)$)**.** *Let x be the default option. Then*

1. *if $x \in I_1 \cup I_{12}$ then $\pi^* - \phi \in A_D(x)$*

2. *if* $x \in I_2$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_{11}$

3. *if* $x \in [I_3^L, \pi^* + 2\phi(1 - \delta(1 + r_d/2))]$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = z^{-1}(x) \in I_9 \cup I_{10}$

4. *if* $x \in [\pi^* + 2\phi(1 - \delta(1 + r_d/2)), \pi^* + \phi(1 - \delta(1 - r_d))]$ *then* $A_D(x) = \{p : x \leq p \wedge p \leq x'\}$ *where* $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_7 \cup I_8$

5. *if* $x \in I_7 \cup I_8$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_3 \cup I_4$

6. *if* $x \in I_9 \cup I_{10}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = z(x) \in I_3 \cup I_4$

7. *if* $x \in I_{11}$ *then* $A_D(x) = \{p : x' \leq p \wedge p \leq x\}$ *where* $x' = 2(\pi^* + \phi) - x \in I_2$.

Proof. The proof is very similar to the proof of claim 1.3 where only minor adjustments have to be made for the current case due to the fact that U_{PD} is symmetric around its global maximum at $\pi^* + \phi(1 - \delta(1 - r_d))$ and hence some of the acceptance sets have to be made symmetric around $\pi^* + \phi(1 - \delta(1 - r_d))$. □

Having the acceptance sets the last thing we need to do is to describe the shape of U_{CA} and U_{CD}. The next claim does that.

claim 1.13 (Shape of U_{CA} and U_{CD})**.**

1. U_{CA} *is increasing on* $I_1 \cup I_2$ *and decreasing on* $I_3 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$

2. U_{CD} *is increasing on* I_1 *and decreasing on* $I_2 \cup I_3 \cup I_4 \cup I_9 \cup I_{10} \cup I_{11} \cup I_{12}$

3. $U_{CA}(x) \geq U_{CA}(x')$ *where* $x \in I_3$ *and* $x' = 2(\pi^* + \phi\delta r_d) - x \in I_4 \cup I_7 \cup I_8$

4. $U_{CD}(x) \geq U_{CD}(x')$ *where* $x \in I_3 \cup I_4$ *and* $x' = 2(\pi^* + \phi(1 - \delta(1 - r_d))) - x \in I_7 \cup I_8$

5. U_{CA} *has a global maximum at* $\pi^* - \phi\delta r_d$ *and* U_{CD} *at* $\pi^* - \phi$.

Proof. The first and second parts of the claim follow readily using the continuation value function, except for intervals I_9 and I_{10}.

For case 4.1 we do not have to worry about I_9 as it is empty. To show U_{CA} is decreasing on I_{10} we use the same argument as in claim 1.10. The only difference arises from the fact that in case 3.1 the relevant part of the claim 1.10 $U'^{I_3}_{CD}(z(x))$ has been evaluated at $\pi^* + \phi(2 - 3\delta r_d)$ whereas for case 4.1 we need to evaluate $U'^{I_3}_{CD}(z(x))$ at $\pi^* + 3\phi\delta r_d$. However, it is easy to confirm that $U'^{I_3}_{CD}(z(\pi^* + 3\phi\delta r_d)) = U'^{I_3}_{CD}(z(\pi^* + \phi(2 - 3\delta r_d)))$ and the argument is essentially the same.

For case 4.2 we need to show the claim for both I_9 as well as I_{10}. Nevertheless, the resulting expressions for the maximum of V'_C on the appropriate intervals are the same as in case 3.3 of the relevant part of claim 1.10. This is due to the fact that the only change is that I_9 starts at $\pi^* + 3\phi\delta r_d$ not at $\pi^* + \phi(2 - 3\delta r_d)$ but $z(x)$ evaluated at those values is the same. Therefore for the I_{10} interval the claim follows by a similar argument as in claim 1.10. For I_9 the condition for U_{CA} to be decreasing becomes (note this change is due to the fact that the I_9^L now is different than in claim 1.10) $-\frac{4\delta^3 r_d^2(1-r_d)}{(1-\delta)(1-\delta r_d)} \leq 0$, which holds.

Finally, parts three and four follow by the use of lemma 1.6 where we note that we can use it as the width of I_3 is the same as $I_4 \cup I_7 \cup I_8$ (part three) and the width of $I_3 \cup I_4$ is larger than the width of $I_7 \cup I_8$ (part four). Part five then follows from the previous parts. □

By a now familiar argument we do not repeat here we have an equilibrium for case 4.

Uniqueness

First notice any distinct CS-MPE has to give rise to P's continuation value function V_P constructed in the previous part of the proof. It then suffices to show that given V_P, C's dynamic optimization program (1.1) has a unique solution. In order to do so we first need to establish properties of P's acceptance sets.

claim 1.14. *For any $x \in X$ the acceptance correspondences $A_D(x)$ and $A_A(x)$ are nonempty, compact valued and upper hemicontinuous.*

Proof. The nonempty part follows from the definition and the compact valued part follows from continuity of V_P along with compactness of X. To prove upper hemicontinuity of the acceptance correspondence

$$A_D(x) = \{p \in X | U_{PD}(p) \geq U_{PD}(x)\}$$

pick two sequences $\{x_\alpha\} \to x$ and $\{p_\alpha\} \to p$ such that $p_\alpha \in A_D(x_\alpha)\ \forall \alpha$. Note that by non-emptiness of A_D this can be done. We need to show $p \in A_D(x)$.

Suppose $p \notin A_D(x)$. Then

$$U_{PD}(x_\alpha) \leq U_{PD}(p_\alpha)\ \forall \alpha$$
$$U_{PD}(x) > U_{PD}(p).$$

Summing the two inequalities gives

$$U_{PD}(x_\alpha) - U_{PD}(x) < U_{PD}(p_\alpha) - U_{PD}(p)\ \forall \alpha.$$

Taking the limit for $\alpha \to \infty$ on both sides gives a contradiction to continuity of $U_{PD}(\cdot)$. For A_A the proof is analogous and hence omitted. □

We note that although we have proven upper hemicontinuity of the acceptance correspondences, for some of the cases above a stronger result, continuity, holds as well. More specifically, for all cases A_A can be proven continuous and for cases 1 and 4, A_D is continuous as well. Failure of lower hemicontinuity of A_D in cases 2 and 3 is then a consequence of the double peakedness of U_{PD} shown in claims 1.5 and 1.8. We can always find a sequence of policies approaching the higher peak as A_D is nonempty. On the other hand it is impossible to find a sequence of policies approaching the lower peak 'from above'. Given that we do not need this stronger result, we state it without proving.

Returning to our main argument, to prove the uniqueness of the CS-MPE we need to show uniqueness of the solution to C's optimization problem (1.1). The optimization problem can be rewritten as a Bellman type functional equation

$$V_C(x) = r_d \max_{p \in A_D(x)} \{f_{CD}(p) + \delta V_C(p)\} + (1 - r_d) \max_{p \in A_A(x)} \{f_{CA}(p) + \delta V_C(p)\}$$

and we already know the acceptance correspondences are upper hemicontinuous. If we could prove their continuity we would be able to use theorem 4.6

in Stokey and Lucas (1989) to prove uniqueness of V_C solving the functional equation above. It turns out a similar result holds for upper hemicontinuous correspondences as well (with associated value functions upper semicontinuous, not continuous as in Stokey and Lucas, 1989). The following theorem states the result formally.

Theorem 1.1. *Let X be a convex subset of $\mathbb{R}^n$, $\Gamma : X \twoheadrightarrow X$ nonempty, compact valued and upper hemicontinuous correspondence, $F : A \to \mathbb{R}$ on $A = \{(x, y) \in X \times X |\, y \in \Gamma(x)\}$ bounded and upper semicontinuous function, $SC(X)$ space of bounded upper semicontinuous functions $f : X \to R$ with the sup norm $\|f\| = \sup_{x\in X} |f(x)|$ and $\beta < 1$. Then, the T operator, defined by*

$$(Tf)(x) = \max_{y\in\Gamma(x)} [F(x, y) + \beta f(y)] \tag{1.2}$$

maps $SC(X)$ into itself and has a unique fixed point $v = Tv$.

Proof. The strategy of the proof is in the following. First, we make sure that a maximum in (1.2) exists, next we show that Tf is upper semicontinuous (u.s.c.) and, hence, T maps $SC(X)$ into itself. Next, we observe that T is a contraction and, hence, has a unique fixed point, provided that $SC(X)$ is complete. As is customary, we view the normed vector space $(X, \|\cdot\|)$ as a metric space on X with the uniform metric $d(f, g) = \|f - g\|$.

Since the notion of upper semicontinuity is not well known in the economic literature, we provide its definition.

Definition 1.6 (upper semicontinuous function)**.** *A function $f : X \to \bar{\mathbb{R}}$ on a topological space X is upper semicontinuous at $x \in X$ if, for each $\epsilon > 0$, there exists a neighbourhood U of x such that $f(y) \leq f(x) + \epsilon$ for all y in U. It is upper semicontinuous if it is upper semicontinuous $\forall x \in X$.*

An alternative definition, sometimes used, takes a sequence $\{x_n\}$ and defines u.s.c. as a function that satisfies $x_n \to x \Rightarrow \limsup_n f(x_n) \leq f(x)$ which is, indeed, the same requirement (Bourbaki, 2007, Chapter IV.6, Proposition 4). Yet, another definition requires the set $\{x \in X|f(x) < c\}$ to be open for any $c \in \mathbb{R}$, which is equal to the previous definition (Aliprantis and Border, 2006, Lemma 2.42).

Intuitively, u.s.c. functions are allowed to jump but, when they do so, the value of the function at the jump is 'the higher of the two'. The advantage of the u.s.c. functions is that they possess maxima on compact intervals.

Coming back to the proof, first observe that, for any $x \in X$, the function $F(x, \cdot) + \beta f(\cdot)$ is u.s.c. and is maximized on a compact, non-empty set $\Gamma(x)$, hence, the maximum exists (Aliprantis and Border, 2006, Theorem 2.43).

Furthermore, as Γ is upper hemicontinuous, T is u.s.c. (Aliprantis and Border, 2006, Lemma 17.30) and it is clearly bounded. Hence, $T : SC(X) \to SC(X)$.

Next, we need to make sure that T satisfies conditions under which Blackwell's Theorem (Aliprantis and Border, 2006, Theorem 3.53) holds. Denoting by $B(X)$ the space of bounded functions defined on X, we need T to map a closed linear subspace of $B(X)$ that includes constant functions into itself. Furthermore, we need T to satisfy *monotonicity* and *discounting*.

That $SC(X)$ is a linear subspace of $B(X)$ that includes constant functions follows trivially. To establish that $SC(X)$ is closed, we observe that $B(X)$ is complete and that any complete subset of a complete metric space is closed (Berberian, 1999, Chapter III.4, Theorem 1). Hence, if we can establish that $SC(X)$ is complete, then closedness follows.

To establish that $SC(X)$, with the uniform metric, is a complete metric space, we adopt the approach of the proof of theorem 3.1 in Stokey and Lucas (1989), with appropriate modifications. We find a function f to which a Cauchy sequence of functions $\{f_n\}$ converges, we show the sequence converges in the uniform metric and, finally, that $f \in SC(X)$.

First, fix $x \in X$ and take a sequence $\{f_n(x)\}$, which satisfies

$$|f_n(x) - f_m(x)| \leq \sup_{y \in X} |f_n(y) - f_m(y)| = \|f_n - f_m\|$$

and which satisfies the Cauchy criterion and, hence, converges to a limit $f(x)$.

Second, we need to show that $\{f_n\}$ converges in the uniform metric. Pick $\epsilon > 0$ and $N := N(\epsilon)$, such that $n, m \geq M \Rightarrow \|f_n - f_m\| \leq \epsilon/2$ (which can be done). For any $x \in X$ and all $n, m \geq N$

$$\begin{aligned} |f_n(x) - f(x)| &\leq |f_n(x) - f_m(m)| + |f_m(x) - f(x)| \\ &\leq \|f_n - f_m\| + |f_m(x) - f(x)| \\ &\leq \epsilon/2 + |f_m(x) - f(x)|. \end{aligned}$$

As $f_m(x) \to f(x)$, choose $m(x)$ for each $x \in X$ such that $|f_m(x) - f(x)| \leq$

$\epsilon/2$. As x was arbitrary, it follows that $\|f_n - f\| \le \epsilon$ for $\forall n \ge N$ and, as ϵ was arbitrary, we have convergence in the uniform metric.

Third, we need to show that f is bounded and u.s.c., the first of which follows readily. To show the u.s.c. part, pick $\epsilon > 0$ and k such that $\|f_k - f\| \le \epsilon/3$. As $f_n \to f$, this can be done. Then, choose δ such that $\|x - y\|_E < \delta \Rightarrow f_k(y) < f_k(x) + \epsilon/3$ where $\|\cdot\|_E$ is a usual Euclidean distance and it can be done by u.s.c. of f_k. Finally,

$$\begin{aligned} f(y) - f(x) &= f(y) - f_k(y) + f_k(y) - f_k(x) + f_k(x) - f(x) \\ &\le |f(y) - f_k(y)| + f_k(y) - f_k(x) + |f_k(x) - f(x)| \\ &\le 2\|f - f_k\| + f_k(y) - f_k(x) \\ &\le \epsilon. \end{aligned}$$

Furthermore, it is easy to confirm that $g \le f$ implies $Tg \le Tf$ (monotonicity) and that there exists $\beta \in (0,1)$, such that $T(f + c) \le Tf + \beta c$ for any constant function c (discounting). Hence, by Blackwell's Theorem, T is a contraction and it has a unique fixed point, which concludes the proof. □

With C's optimization problem we can define operator T similarly as in theorem 1.1 by

$$Tv(x) = r_d \max_{p \in A_D(x)} \{f_{CD}(p) + \delta v(p)\} + (1 - r_d) \max_{p \in A_A(x)} \{f_{CA}(p) + \delta v(p)\}.$$

It is easy to see T satisfies monotonicity and discounting, and existence of a fixed point $Tv = v$ can be proven in a similar way as in theorem 1.1. The fixed point of T is then C's continuation value function V_C derived in the existence part of the proof, which defines a unique equilibrium proposal strategy for C, proving uniqueness of the CS-MPE.

Notice that instead of using T, we could have used the original formulation of C's optimization problem given in (1.1) and worked with a pair of mappings defined by

$$\begin{aligned} T_D u_D(x) &= \max_{p \in A_D(x)} \{f_{CD}(p) + \delta r_d u_D(p) + \delta(1 - r_d) u_A(p)\} \\ T_A u_A(x) &= \max_{p \in A_A(x)} \{f_{CA}(p\ ) + \delta r_d u_D(p) + \delta(1 - r_d) u_A(p)\} \end{aligned}$$

using theorem 1.1 to prove existence of a unique fixed point of T_D for each

u_a and existence of a unique fixed point of T_A for each u_d. To complete the proof we would then need to show existence of coincidence solution u_D^*, u_A^* such that u_D^* is a fixed point of T_D for u_A^* and u_A^* is a fixed point of T_A for u_D^*. Using an approach similar to Liu, Agarwal, and Kang (2004) this is possible, but would not give us the uniqueness result that is the focus of this part of the proof.

1.A1.2 Proof of proposition 1.2

Using definition 1.4 of S and the results from the proof of proposition 1.1 it is easy to see $S = [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$ and hence parts one and two. For part three notice $p_A(x) \in S$ for any $x \in X$ and hence $x^t(x) \notin S$ for some $x \in X$ implies that all the t periods generating $x^t(x)$ need to be D periods. As a result we have $\mathbb{P}(x^t(x) \notin S) \leq r_d^t$ for any $x \in X$. Part four follows from the fact that $\mathbb{P}(p_A(x^t(x)) = \pi^*) = 0$ for almost all $x \in X$ except for a finite set of discrete values of zero measure.

1.A1.3 Proof of proposition 1.3

Assume there exists S-MPE with $p_A(x) = \pi^* + \varepsilon$ for some $x \in X$ and (not necessarily positive) $\varepsilon \neq 0$. Let $\gamma = \{p_A(x) = \pi^* + \varepsilon, q_D(x)\}$ be C's equilibrium proposal and $\gamma' = \{\pi^* + \varepsilon/2, q_D(x)\}$. By the definition of S-MPE it must be that γ solves C's optimization problem, that is it is a solution to

$$\max_{\{p,q\}\in X^2} \left\{-(p-\pi^*)^2 + \delta V_C(q)\right\}$$
$$\text{s.t. } -(p-\pi^*)^2 + \delta V_P(q) \geq -(x-\pi^*)^2 + \delta V_P(x).$$

By continuity of the constraint in p proposal $\gamma' \in A_A(x)$. C's utility from γ' is $-\varepsilon^2/4 + \delta V_C(q_D(x))$ and from γ it is $-\varepsilon^2 + \delta V_C(q_D(x))$. By assumption γ is an equilibrium hence

$$-\varepsilon^2 + \delta V_C(q_D(x)) \geq -\varepsilon^2/4 + \delta V_C(q_D(x)),$$

which implies $\varepsilon^2 \leq \varepsilon^2/4$, a contradiction.

1.A1.4 Proof of proposition 1.4

To prove existence of CS-MPE in the model with explicit status-quo bargaining protocol, we proceed as follows. First we give formal meaning to the term C's unconstrained proposals by deriving a global maximum of the V_C function. Then we conjecture that any equilibrium offer $\gamma(x)$ that C makes to P for default option $\bar{\gamma}(x)$ has to make P indifferent between the two options unless C can propose the unconstrained maximizer of her overall utility. This allows us to derive explicit expressions for the continuation value function V_P of the P player and hence the shape of his acceptance sets. Given the acceptance sets we show that those are well behaved and hence that the C's dynamic optimization program has a solution. We then go back and make sure that the equilibrium policies indeed satisfy the original conjecture of making P indifferent between $\gamma(x)$ and $\bar{\gamma}(x)$.

As before we refer to C in D period as to CD and analogously for P and A periods. We keep the notation

$$f_{CD}(x) = -(x - \pi^* + \phi)^2 \qquad f_{PD}(x) = -(x - \pi^* - \phi)^2$$
$$f_{CA}(x) = -(x - \pi^*)^2 \qquad f_{PA}(x) = -(x - \pi^*)^2$$

and denote the overall utility by

$$U_{CD}(p, q) = f_{CD}(p) + \delta V_C(q) \qquad U_{PD}(p, q) = f_{PD}(p) + \delta V_P(q)$$
$$U_{CA}(p, q) = f_{CA}(p) + \delta V_C(q) \qquad U_{PA}(p, q) = f_{PA}(p) + \delta V_P(q).$$

To prove existence of CS-MPE we need to constrain values of δ and r_d. The following assumption lists all the constraints we need.

Assumption 1.2. *For any pair $\{\delta, r_d\}$ with $\delta \in [0, 1)$ and $r_d \in [0, 1]$ assume*

1. $\delta \geq \frac{1}{5r_d}$
2. $\delta \geq 1 - r_d^2$
3. $\delta \leq 1 - \frac{(1-r_d)^2}{2}$.

Notice that the three requirements are mutually compatible and in general allow for values of δ and r_d with 'enough discounting and conflict'. Denoting the space of possible values for $\{\delta, r_d\}$ by $\mathbb{P} = [0, 1) \times [0, 1]$ (with the convention that its graphical representation has r_d on the horizontal axis) assumption 1.2 isolates the north-eastern part of $\mathbb{P}$. Notice also that we are not selecting a measure-zero set out of the $\mathbb{P}$ and hence our existence result will be generic in the sense that S-MPE will exist on some neighbourhood of $\{\delta, r_d\}$ strictly satisfying assumption 1.2.

claim 1.15. *Let $X^- = X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ and $z, z' \in X^-$. For any $x \in X^-$ the equilibrium is given by*

$$q_A(x) = z \qquad p_A(x) = \pi^*$$
$$q_D(x) = z' \qquad p_D(x) = \pi^* - \phi.$$

where the policy strategies are unique. Moreover, for any $x \in X^-$, $V_C(x) = 0$ and $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$.

Proof. We first show $\rho = \{q_D(x) = q_A(x) = x, p_D(x) = \pi^* - \phi, p_A(x) = \pi^*\}$ is an equilibrium for any $x \in X^-$. Fix $x \in X^-$. Note that $\{p_D(x) = \pi^* - \phi, x\} \in A_D(x)$ and $\{p_A(x) = \pi^*, x\} \in A_A(x)$ and both increase C's utility compared to $\{x, x\}$. It also follows ρ induces $V_C(x) = 0$ hence C clearly cannot do better. Therefore ρ is an equilibrium.

Having the equilibrium for given x, notice it induces the same path of policy decisions for a fixed path of A and D periods as any $x' \in X^-$. It follows $V_C(x)$ and $V_P(x)$ must be constant on X^-. Therefore the first part of the claim follows.

To show uniqueness of the policy offers notice C's utility strictly decreases by offering anything other than policy specified in the claim.

The fact that $V_C(x) = 0 \; \forall x \in X^-$ follows from the two previous remarks. To show $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$ using the constancy of $V_P(x)$ we can write

$$V_P(x) = r_d[-4\phi^2 + \delta V_P(x)] + (1 - r_d)[\delta V_P(x)],$$

which after rearranging gives $V_P(x)$ in the claim. □

claim 1.16. *Let $X^+ = (\pi^* - \phi, \pi^* + 3\phi)$. Then for all $x \in X^+$, $V_C(x) < 0$.*

Proof. Assume there exists an equilibrium such that $V_C(x) = 0$ for some $x \in X^+$. It follows $V_P(x) = -\frac{4\phi^2 r_d}{1-\delta}$. Take D period, if P rejects today and follows the equilibrium strategy from then on his utility is $f_{PD}(x) - \frac{4\phi^2 \delta r_d}{1-\delta}$ whereas if he accepts (as equilibrium demands) his utility is $f_{PD}(\pi^* - \phi) -$

$\frac{4\phi^2\delta r_d}{1-\delta}$. For this to be an equilibrium it must be that

$$f_{PD}(x) - \frac{4\phi^2\delta r_d}{1-\delta} \leq f_{PD}(\pi^* - \phi) - \frac{4\phi^2\delta r_d}{1-\delta},$$

which rewrites as $(x - \pi - \phi)^2 \geq 4\phi^2$ and holds for $x \notin (\pi^* - \phi, \pi^* + 3\phi)$, a contradiction to $x \in X^+$. □

Claims 1.15 and 1.16 give precise meaning to the term C's unconstrained maximizer as they imply that $\{\pi^* - \phi, z\}$ maximizes $U_{CD}(p, q)$ and $\{\pi^*, z\}$ maximizes $U_{CA}(p, q)$ for any $z \in X^-$. Denote those by $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z\}$. Notice that if $\gamma_{CD} \in A_D(x)$ for some default x then γ_{CD} has to be part of C's equilibrium strategy. Similar holds for $\gamma_{CA} \in A_A(x)$.

Next we wish to characterize P's continuation value function V_P conjecturing that if for some default option x we have $\gamma_{CD} \notin A_D(x)$, C's offer $\gamma(x)$ will make P indifferent between $\gamma(x)$ and default option $\bar{\gamma}(x) = \{x, x\}$ and similarly for A periods. The next claim helps in translating the conjecture into V_P.

claim 1.17. *For any $x \in X^+$ if P is brought to indifference in A periods for default option x, then he is brought to indifference in D periods for the same default option.*

Proof. We prove the converse, i.e. if P is not brought to indifference in D periods, then he is not brought to indifference in A periods.

Note that if P is not made indifferent in D periods, then C's proposal has to be $\{\pi^* - \phi, z\}$ for some $z \in X$. This implies

$$f_{PD}(x) + \delta V_P(x) \leq f_{PD}(\pi^* - \phi) + \delta V_P(z),$$

which after rearranging gives

$$f_{PA}(x) + \delta V_P(x) \leq \delta V_P(z) - [3\phi^2 + 2\phi(x - \pi^*)],$$

where the term in the square brackets is positive for any $x \in X^+$. It then follows that $\{\pi^*, z\} \in A_A(x)$. □

With the help of claim 1.17 we conjecture that for default options x close to P's D period bliss point $\pi^* + \phi$ he will be made indifferent for both A

and D periods and for x further away he will be made indifferent only in D periods. This gives rise to V_P of the following form.

$$V_P(x) = \begin{cases} -\frac{1}{1-\delta}\left[(x-\pi^*-\phi r_d)^2+\phi^2 r_d(1-r_d)\right] \\ \qquad \text{for } \; x \in [\pi^*+\phi\delta r_d-\kappa, \pi^*+\phi\delta r_d+\kappa] \\ -\frac{r_d}{1-\delta r_d}\left[(x-\pi^*-\phi)^2+\phi^2\frac{4\delta(1-r_d)}{1-\delta}\right] \\ \qquad \text{for } \; x \in [\pi^*-\phi, \pi^*+\phi\delta r_d-\kappa] \cup [\pi^*+\phi\delta r_d+\kappa, \pi^*+3\phi] \\ -\frac{4\phi^2 r_d}{1-\delta} \\ \qquad \text{otherwise} \end{cases}$$

with $\kappa = \phi\sqrt{\delta r_d(3+\delta r_d)}$ where the last constant part applies to x for which $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$. For future reference denote $\kappa^- = \pi^*+\phi\delta r_d-\kappa$ and $\kappa^+ = \pi^*+\phi\delta r_d+\kappa$. It is easy to confirm V_P is continuous and (strictly) piece-wise concave for $x \in X$ ($x \in X^+$). In the next claim we establish upper hemicontinuity of the acceptance correspondences generated by V_P.

claim 1.18. *For any $x \in X$ the acceptance correspondences $A_D(x)$ and $A_A(x)$ are nonempty, compact valued and upper hemicontinuous.*

Proof. The nonempty part follows by definition and the compact valued part follows from continuity and the fact that X is compact. To prove upper hemicontinuity of the acceptance correspondence

$$A_D(x) = \{(p,q) \in X^2 | f_{PD}(p) + \delta V_P(q) \geq f_{PD}(x) + \delta V_P(x)\}$$

denote $\mathbf{x} = (x,x)$, $\mathbf{p} = (p,q)$ and $f(\mathbf{p}) = f_{PD}(p) + \delta V_P(q)$.

Pick two sequences $\{\mathbf{x}_\alpha\} \to \mathbf{x}$ and $\{\mathbf{p}_\alpha\} \to \mathbf{p}$ such that $\mathbf{p}_\alpha \in A_D(x_\alpha)\ \forall\alpha$. Note that by non-emptiness of A_D this can be done. We need to show $\mathbf{p} \in A_D(x)$.

Suppose $\mathbf{p} \notin A_D(x)$. Then

$$f(\mathbf{x}_\alpha) \leq f(\mathbf{p}_\alpha)\ \forall\alpha$$
$$f(\mathbf{x}) > f(\mathbf{p}).$$

Summing the two inequalities gives

$$f(\mathbf{x}_\alpha) - f(\mathbf{x}) < f(\mathbf{p}_\alpha) - f(\mathbf{p}) \ \forall\alpha.$$

Taking the limit for $\alpha \to \infty$ on both sides gives a contradiction to continuity of $f(\cdot)$. For A_A the proof is analogous and hence omitted. □

Next we want to show C's dynamic optimization problem has a solution. Precise statement of the dynamic program is

$$U_D(x) = \max_{\{p,q\}\in A_D(x)} \{f_{CD}(p) + \delta(r_d U_D(q) + (1 - r_d)U_A(q))\}$$
$$U_A(x) = \max_{\{p,q\}\in A_A(x)} \{f_{CA}(p) + \delta(r_d U_D(q) + (1 - r_d)U_A(q))\},$$

which can alternatively be written as

$$V_P(x) = r_d \max_{\{p,q\}\in A_D(x)} \{f_{CD}(p)+\delta V_C(q)\}+(1-r_d) \max_{\{p,q\}\in A_A(x)} \{f_{CA}(p)+\delta V_C(q)\}.$$

With the acceptance correspondences possessing properties given in claim 1.18, existence and uniqueness of the solution to the dynamic program above follow using a similar argument as in proposition 1.2 for the no directive model. C's equilibrium proposal strategy for default x is then given by $\{p_D(x), q_D(x)\}$ in D periods and by $\{p_A(x), q_A(x)\}$ in A periods, where

$$\{p_D(x), q_D(x)\} \in \arg\max_{\{p,q\}\in A_D(x)} \{f_{CD}(p) + \delta V_C(q)\}$$
$$\{p_A(x), q_A(x)\} \in \arg\max_{\{p,q\}\in A_A(x)} \{f_{CA}(p) + \delta V_C(q)\}.$$

Notice that even though $p_D(x)$, $q_D(x)$, $p_A(x)$ and $q_A(x)$ are correspondences we can always take a unique selection out of each of them. For this reason below we treat those as functions. In the next claim we establish properties of the value functions that solve the dynamic optimization program above.

claim 1.19. *Under assumption 1.2 (namely its part one)*

1. $U_A(x)$, $U_D(x)$ *and* $V_C(x)$ *are all u.s.c.*

2. $U_A(x) = 0$ *for* $\forall x \in [x^-, \kappa^-] \cup [\kappa^+, x^+]$, $U_A(x)$ *is non-increasing for* $\forall x \in [\kappa^-, \pi^* + \phi\delta r_d]$ *and non-decreasing for* $\forall x \in [\pi^* + \phi\delta r_d, \kappa^+]$

3. $U_A(x) = U_A(x')$ *with* $\frac{x+x'}{2} = \pi^* + \phi\delta r_d$ *for* $\forall x \in [\kappa^-, \kappa^+]$

4. *$U_A(x) \geq U_A(x')$ for $\forall x' \in [x, 2(\pi^* + \phi\delta r_d) - x]$ with $x \in [\kappa^-, \pi^* + \phi\delta r_d]$*

5. *$U_D(x) = 0$ for $\forall x \in [x^-, \pi^* - \phi] \cup [\pi^* + 3\phi, x^+]$, $U_D(x)$ is non-increasing for $\forall x \in [\pi^* - \phi, \pi^* + \phi(1 - \delta(1 - r_d))]$ and non-decreasing for $\forall x \in [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 3\phi]$*

6. *$U_D(x) = U_D(x')$ with $\frac{x+x'}{2} = \pi^* + \phi$ for $\forall x \in [x^-, \kappa^-] \cup [2(\pi^* + \phi) - \kappa^-, x^+]$, with $x = z(x')$ where $z(x')$ is a uniquely defined decreasing function mapping the range $[\kappa^+, 2(\pi^* + \phi) - \kappa^-]$ into $[\kappa^-, 2(\pi^* + \phi(1 - \delta(1 - r_d))) - \kappa^+]$ and with $\frac{x+x'}{2} = \pi^* + \phi(1 - \delta(1 - r_d))$ for $\forall x \in [2(\pi^* + \phi(1 - \delta(1 - r_d))) - \kappa^+, \kappa^+]$*

7. *$U_D(x) \geq U_D(x')$ for $\forall x' \in [2(\pi^* + \phi(1 - \delta(1 - r_d))) - x, x]$ with $x \in [\pi^* + \phi(1 - \delta(1 - r_d)), \kappa^+]$ and for $\forall x' \in [z(x), x]$ with $x \in [\kappa^+, 2(\pi^* + \phi) - \kappa^-]$*

8. *$V_C(x)$ is non-increasing $\forall x \in [x^-, \pi^* + \phi\delta r_d]$ and non-decreasing $\forall x \in [\pi^* + \phi(1 - \delta(1 - r_d)), x^+]$.*

Proof. The first part follows immediately from the fact that the value functions are solutions to C's dynamic optimization program and theorem 1.1.

The second part follows from the fact that $\gamma_{CA} \in A_A(x)$ whenever $x \in [x^-, \kappa^-] \cup [\kappa^+, x^+]$. The non-increasing and non-decreasing parts follow from the fact that $U_{PA}(x, x)$, which defines $A_A(x)$, is under part one of assumption 1.2 increasing on $[\kappa^-, \pi^* + \phi\delta r_d]$ and decreasing on $[\pi^* + \phi\delta r_d, \kappa^+]$. With the default option x entering C's optimization only as a constraint in the form of $A_A(x)$, it follows $U_A(x)$ has to be non-increasing and non-decreasing on the two intervals respectively.

The third part follows from the fact that $U_{PA}(x, x) = U_{PA}(x', x')$ for x and x' satisfying the condition given in the claim, which implies $A_A(x) = A_A(x')$. Part four then follows from parts two and three.

Part five can be shown in a similar manner as part two, investigating properties of the $U_{PD}(x, x)$ function defining acceptance set $A_D(x)$, using again part one of assumption 1.2. The sixth part is analogous to part three using the fact that $U_{PD}(x, x) = U_{PD}(x', x')$ for the x and x' defined. Part seven is an implication of parts five and six. Part eight is a direct consequence of parts two and five upon observing that $V_C(x) = r_d U_D(x) + (1 - r_d) U_A(x)$. □

The next claim establishes a certain monotonicity property of the equilibrium status-quo offers $q_D(x)$ (as evaluated under the V_C function) that will become useful later. We denote by $A_D^\partial(x)$ the boundary of $A_D(x)$ and similarly by $A_A^\partial(x)$ the boundary of $A_A(x)$.

claim 1.20. *Let x be the default option with its associated equilibrium status-quo offer $q_D(x)$. Then for any $x' \in \{q : A_D(q) \subseteq A_D(x)\}$ with its associated equilibrium status-quo offer $q_D(x')$, $V_C(q_D(x)) \geq V_C(q_D(x'))$.*

Proof. Fix x and x' with $x' \in \{q : A_D(q) \subseteq A_D(x)\}$ where the interpretation of x' is that it is a default option with strictly smaller associated P's D period acceptance set. It is immediate that the claim holds for $q_D(x)$ and its associated policy offer $p_D(x)$ with $\{p_D(x), q_D(x)\} \in A_D(x) \setminus A_D^\partial(x)$ and any x' such that $\{p_D(x), q_D(x)\} \in A_D(x')$ for then $q_D(x) = q_D(x')$. So assume that $\{p_D(x), q_D(x)\} \in A_D^\partial(x)$.

Easy argument shows that for p to be C's equilibrium policy offer for some default option it has to be that $p \in [\pi^* - \phi, \pi^* + \phi]$. We need to construct a set of offers C can be expected to choose from in equilibrium, i.e. those where the policy offer falls into the $[\pi^* - \phi, \pi^* + \phi]$ interval. This will be given as a set $A'_D(x) = \{\{\max\{p, \pi^* - \phi\}, q\} | \{p, q\} \in A_D^\partial(x) \wedge p \leq \pi^* + \phi\} \subseteq A_D(x)$ or in words as a subset of $A_D^\partial(x)$ for which the policy is smaller than $\pi^* + \phi$ and for which, if the policy falls below $\pi^* - \phi$, it is replaced by $\pi^* - \phi$. It is easy to see that for any default option $x'' \in X$, C's D period equilibrium offer satisfies $\{p_D(x''), q_D(x'')\} \in A'_D(x'')$.

Now with $\{p_D(x), q_D(x)\} \in A_D^\partial(x)$ and any $\{p, q\} \in A'_D(x)$ for which $p \leq p_D(x)$ it has to be the case that $V_C(q_D(x)) \geq V_C(q)$. To see this note that

$$\begin{aligned} f_{CD}(p_D(x)) + \delta V_C(q_D(x)) &\geq f_{CD}(p) + \delta V_C(q) \\ f_{CD}(p_D(x)) - f_{CD}(p) &\leq 0 \end{aligned}$$

where the first line follows from the fact that $\{p_D(x), q_D(x)\}$ is C's equilibrium offer and $\{p, q\} \in A_D(x)$ and the second line follows from the fact that $\pi^* - \phi \leq p \leq p_D(x)$.

Next we want to show that for any $\{p, q\} \in A'_D(x)$ for which $p > p_D(x)$, the associated q cannot be part of C's equilibrium offer for x'. To see this note that $\{p, q\} \in A'_D(x)$ with $p > p_D(x)$ and $\{p_D(x), q_D(x)\} \in A_D^\partial(x)$ has to satisfy

$$\begin{aligned} f_{PD}(x) + \delta V_P(x) &= f_{PD}(p) + \delta V_P(q) \\ &= f_{PD}(p_D(x)) + \delta V_P(q_D(x)). \end{aligned}$$

Keeping q and $q_D(x)$ the same, changing x to x' such that $A_D(x') \subseteq A_D(x)$, p and $p_D(x)$ will change to p' and $p'_D(x)$ with $\{p', q\} \in A'_D(x')$ and $\{p'_D(x), q_D(x)\} \in A'_D(x')$ that satisfy

$$\begin{aligned} f_{PD}(x') + \delta V_P(x') &= f_{PD}(p') + \delta V_P(q) \\ &= f_{PD}(p'_D(x)) + \delta V_P(q_D(x)) \end{aligned}$$

(if such a p' does not exist we are done as q cannot be part of equilibrium for x', if such p' does exist, $p'_D(x)$ has to exist as well). Taking the difference of the two systems of equations gives $f_{PD}(p) - f_{PD}(p') = f_{PD}(p_D(x)) - f_{PD}(p'_D(x))$ which rewrites as

$$(p' - p) = (p'_D(x) - p_D(x)) \left[\frac{p_D(x) + p'_D(x) - 2(\pi^* + \phi)}{p + p' - 2(\pi^* + \phi)} \right]$$

where the term in the square brackets is positive and strictly larger than unity (all p, p', $p_D(x)$ and $p'_D(x)$ are in the $[\pi^* - \phi, \pi^* + \phi]$ interval and $p_D(x) < p$ and $p_D(x)' < p'$). This in turn implies that $p' - p > p'_D(x) - p_D(x)$ and along with the fact that

$$f_{CD}(p) + \delta V_C(q) \leq f_{CD}(p_D(x)) + \delta V_C(q_D(x))$$

gives

$$f_{CD}(p') + \delta V_C(q) < f_{CD}(p'_D(x)) + \delta V_C(q_D(x))$$

so that q cannot be part of C's equilibrium offer for default option x'.

Combining the two results, if $\{p, q\} \in A'_D(x)$ with $p > p_D(x)$ then q cannot be part of C's equilibrium proposal for x'. If $\{p, q\} \in A'_D(x)$ with $p \leq p_D(x)$ then $V_C(q_D(x)) \geq V_C(q)$ so that if $q = q_D(x')$ it has to be the case that $V_C(q_D(x)) \geq V_C(q_D(x'))$. □

Next we want to confirm our original conjecture that in equilibrium for a given default option in a given type of period P is indifferent between accepting and rejecting C's offer given that the unconstrained maximizer of

C's overall utility is not in P's acceptance set. Formally, we want to show that for default option x if $\gamma_{CD} \notin A_D(x)$ then $\{p_D(x), q_D(x)\} \in A_D^{\partial}(x)$ and similarly if $\gamma_{CA} \notin A_A(x)$ then $\{p_A(x), q_A(x)\} \in A_A^{\partial}(x)$. A key complication is the fact that the V_C function can possess local maxima in the $[\pi^* + \phi\delta r_d, \pi^* + \phi(1-\delta(1-r_d))]$ interval and hence C's utility maximizing offer can lie in the interior of P's acceptance set, even though her unconstrained optimizer is outside of it. Denoting the problematic interval by $Z = [\pi^* + \phi\delta r_d, \pi^* + \phi(1-\delta(1-r_d))]$ we deal with A and D periods in the following two claims respectively. The two claims then deliver the conditions on δ specified in parts two and three of assumption 1.2.

claim 1.21. *Let x be the default policy and assumption 1.2 (namely its part one and two) holds. Then C's equilibrium proposal in A periods, provided $\gamma_{CA} \notin A_A(x)$, satisfies $\{p_A(x), q_A(A)\} \in A_A^{\partial}(x)$.*

Proof. We know by proposition 1.3 that for any $x \in X$, $p_A(x) = \pi^*$. Denoting by $A_A(x, y = z)$ a 'slice' through $A_A(x)$ when variable y (either p or q) is equal to z, C's optimization problem in A period for default option x can be rewritten as $\max_{q \in A_A(x, p=\pi^*)}\{\delta V_C(q)\}$ and we want to show that whenever $\gamma_{CA} \notin A_A(x)$ then $q_A(x) \in \{\min\{A_A(x, p = \pi^*)\}, \max\{A_A(x, p = \pi^*)\}\}$. It is easy to confirm that under part one of assumption 1.2 for any x, $A_A(x, p = \pi^*)$ is a non-empty, compact and convex subset of X.

Next note that by part eight of claim 1.19 if $\max\{V_C(x), V_C(y)\} \geq \max_{z \in (x,y)} V_C(z)$ for some $x \leq \pi^* + \phi\delta r_d$ and some $y \geq \pi^* + \phi(1-\delta(1-r_d))$ then $\max\{V_C(x'), V_C(y')\} \geq \max_{z \in (x',y')} V_C(z)$ for any $x' \leq x$ and any $y' \geq y$. Hence if we can show that the claim is true for $x = \pi^* + \phi\delta r_d$, which maximizes $U_{PA}(x, x)$ and hence delivers the smallest $A_A(x)$, we are done as $Z \in A_A(\pi^* + \phi\delta r_d, p = \pi^*)$.

Now minima and maxima of $A_A(\pi^* + \phi\delta r_d, p = \pi^*)$ are given respectively by $q_A^- = \pi^* + \phi r_d - \phi r_d\sqrt{1-\delta}$ and $q_A^+ = \pi^* + \phi r_d + \phi r_d\sqrt{1-\delta}$ for $\{\delta, r_d\} \in \mathbb{P}$ for which $q_A^+ \leq \kappa^+$ (it is easy to confirm $\kappa^- \leq q_A^-$). Then we can use part four (along with part five) of claim 1.19 to conclude that $V_C(q_A^-) \geq V_C(x)$ for any $x \in [q_A^-, 2(\pi^* + \phi\delta r_d) - q_A^-]$ and part seven (along with part two) of the same claim to conclude that $V_C(q_A^+) \geq V_C(x)$ for any $x \in [2(\pi^* + \phi(1-\delta(1-r_d))) - q_A^+, q_A^+]$. The condition for $2(\pi^* + \phi\delta r_d) - q_A^- \geq 2(\pi^* + \phi(1-\delta(1-r_d))) - q_A^+$ that rewrites as $\delta \geq 1 - r_d^2$ then delivers the claim. For values of $\{\delta, r_d\}$ for which $q_A^+ \geq \kappa^+$ the argument is similar if somewhat complicated by use of the function $z(x)$ mentioned in part six of claim 1.19. We do not repeat the purely algebraic argument here as it delivers a condition on $\{\delta, r_d\}$ that is strictly weaker than the condition $\delta \geq 1 - r_d^2$ just derived. □

claim 1.22. *Let x be the default policy and assumption 1.2 (namely its part one and three) holds. Then C's equilibrium proposal in D periods, provided $\gamma_{CD} \notin A_D(x)$, satisfies $\{p_D(x), q_D(x)\} \in A_D^{\partial}(x)$.*

Proof. First note that for default option x if $\gamma_{CD} \notin A_D(x)$ then if $\{p_D(x), q_D(x)\}$ is strictly inside $A_D(x)$ then it has to be the case that $p_D(x) = \pi^* - \phi$. If not and $p_D(x) = p \neq \pi^* - \phi$ then there exists (not necessarily positive) ε such that C can offer $\{p - \varepsilon, q_D(x)\} \in A_D(x)$ with $U_{CD}(p - \varepsilon, q_D(x)) > U_{CD}(p, q_D(x))$. Also it has to be the case that $q_D(x) = z$ for some $z \in Z$. If not, then by part eight of claim 1.19, there has to exist q such that $\{p_D(x), q\} \in A_D^{\partial}(x)$ and satisfies $U_{CD}(p_D(x), q) \geq U_{CD}(p_D(x), q_D(x))$ and we can specify $\{p_D(x), q\}$ to be C's equilibrium offer satisfying the claim.

Next $\gamma_{CD} \notin A_D(x)$ implies that $x \in X^+ = (\pi^* - \phi, \pi^* + 3\phi)$ and, under assumption 1.2, for any $x \in (\pi^* - \phi, \pi^* + \phi(1 - \delta(1 - r_d))]$ there exists a unique $x' \in [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 3\phi)$ such that $U_{PD}(x, x) = U_{PD}(x', x')$, which implies $A_D(x) = A_D(x')$. Denoting by $X^c = (\pi^* - \phi, \pi^* + \phi(1 - \delta(1 - r_d))]$ we focus on $x \in X^c$ since if the claim holds for any such x it has to hold for any $x' \in [\pi^* + \phi(1 - \delta(1 - r_d)), \pi^* + 3\phi) = X^+ \setminus X^c$.

Next assume that $p_D(x) = \pi^* - \phi$ and $q_D(x) = z$ for some $z \in Z$ are part of the equilibrium for some $x \in X^c$ such that $\{\pi^* - \phi, z\} \in A_D(x) \setminus A_D^{\partial}(x)$. We show that this leads to contradiction.

Observe that if $p_D(x) = \pi^* - \phi$ and $q_D(x) = z$ for some $z \in Z$ is C's equilibrium offer for $x \in X^c$ with $\{\pi^* - \phi, z\} \in A_D(x) \setminus A_D^{\partial}(x)$, it has to be the case that $p_D(x^c) = \pi^* - \phi$ and $q_D(x^c) = z$ is C's equilibrium offer for $x^c \in X^c$ such that $\{\pi^* - \phi, z\} \in A_D^{\partial}(x^c)$. Such x^c is implicitly defined by $U_{PD}(x^c, x^c) = U_{PD}(\pi^* - \phi, z)$. We denote x^c as a function of z by $x^c(z)$ for $z \in Z$. It is easy to show that $x^c(z)$ is increasing on Z^- and decreasing on Z^+ where $Z^- = [\pi^* + \phi\delta r_d, \pi^* + \phi r_d]$ and $Z^+ = [\pi^* + \phi r_d, \pi^* + \phi(1 - \delta(1 - r_d))]$ respectively with $Z = Z^- \cup Z^+$ and that $x^c(z) \leq z$.

Now from the fact that $p_D(x^c(z)) = \pi^* - \phi$ and $q_D(x^c(z)) = z$ is C's equilibrium offer it follows

$$f_{CD}(x^c(z)) + \delta V_C(x^c(z)) \leq f_{CD}(\pi^* - \phi) + \delta V_C(z),$$

which rewrites as

$$\begin{aligned}&-(x^c(z)-\pi^*+\phi)^2+\delta^2 r_d V_C(q_D(x^c(z)))+\delta^2(1-r_d)V_C(q_A(x^c(z)))\\ \le &-\delta r_d(p_D(z)-\pi^*+\phi)^2+\delta^2 r_d V_C(q_D(z))+\delta^2(1-r_d)V_C(q_A(z)).\end{aligned}$$

We show that this inequality fails under assumption 1.2.

First we show that $V_C(q_A(x^c(z))) \geq V_C(q_A(z))$ using part four of claim 1.19. Evaluating $x^c(z)$ at its maximum, that is for $z = \pi^* + \phi r_d$, gives $x^c(\pi^*+\phi r_d) = \pi^* + \phi(1-\delta(1-r_d)) - \phi\sqrt{(1-\delta)(4-\delta+2\delta r_d-\delta r_d^2)}$ and in order to use claim 1.19 we need this to be smaller than $\pi^* + \phi\delta r_d - \phi(1-\delta)$ (as $\phi(1-\delta)$ is size of the Z interval). This condition rewrites as $0 \leq \delta(1-\delta)(3-r_d)(1+r_d)$, which clearly holds for any $\{\delta, r_d\} \in \mathbb{P}$.

Next we show that $V_C(q_D(x^c(z))) \geq V_C(q_D(z))$ that follows from claim 1.20 along with the fact that $x^c(z) \leq z$, $x^c(z) \in X^c$ and $z \in X^c$, which implies $A_D(z) \subseteq A_D(x^c(z))$.

Finally we show that $-(x^c(z)-\pi^*+\phi)^2 \geq -\delta r_d(p_D(z)-\pi^*+\phi)^2$. As we do not know the exact value of $p_D(z)$ we replace it by the minimum value of policy in the $A_D(z)$ set. We denote this policy, as a function of z, by $p_m(z)$ and note it solves $f_{PD}(z)+\delta V_P(z) = f_{PD}(p_m(z)) + \delta V_P(\pi^*+\phi r_d)$ as $\pi^*+\phi r_d$ maximizes the V_P function under assumption 1.2. Similarly, $x^c(z)$ defined above by $\{\pi^*-\phi, z\} \in A_D^{\partial}(x^c(z))$ for some $z \in Z$ solves $f_{PD}(x^c(z)) + \delta V_P(x^c(z)) = f_{PD}(\pi^*-\phi) + \delta V_P(z)$.

In what follows we need to focus only on $z = \pi^* + \phi\delta r_d$. To see this note that using the implicit function theorem (non-differentiability of V_P poses no problem here as even at the point where V_P is not differentiable, it possesses left and right derivatives)

$$\begin{aligned}\frac{\partial p_m(z)}{\partial z} &= \frac{2(\pi^*+\phi-z)+\delta\frac{\partial V_P(z)}{\partial z}}{2(\pi^*+\phi-p_m(z))}\\ \frac{\partial x^c(z)}{\partial z} &= \frac{\delta\frac{\partial V_P(z)}{\partial z}}{2(\pi^*+\phi-x^c(z))+\delta\frac{\partial V_P(x^c(z))}{\partial x^c(z)}}.\end{aligned}$$

If we can prove that $\frac{\partial x^c(z)}{\partial z} \leq \sqrt{\delta r_d}\frac{\partial p_m(z)}{\partial z}$ for any $z \in Z$ then if the inequality $-(x^c(z)-\pi^*+\phi)^2 \geq -\delta r_d(p_m(z)-\pi^*+\phi)^2$ holds for $z = \pi^*+\phi\delta r_d$ it has to hold for any $z \in Z$.

For $z \in Z^+$ we have $\frac{\partial x^c(z)}{\partial z} \leq 0 \leq \sqrt{\delta r_d}\frac{\partial p_m(z)}{\partial z}$ (denominators in $\frac{\partial x^c(z)}{\partial z}$ and $\frac{\partial p_m(z)}{\partial z}$ are positive as $x^c(z) \in X^c$ and $p_m(z) \in X^c$ while nominators are positive and negative respectively). For $z \in Z^-$, $\frac{\partial x^c(z)}{\partial z} \leq \sqrt{\delta r_d}\frac{\partial p_m(z)}{\partial z}$ rewrites as (using only nominators as denominator in $\frac{\partial x^c(z)}{\partial z}$ is larger than denominator in $\frac{\partial p_m(z)}{\partial z}$ and using the fact that under assumption 1.2, $\kappa^- \leq z \leq \kappa^+$ for any $z \in Z$)

$$\frac{\delta - \sqrt{\delta r_d}}{1-\delta}(\pi^* + \phi - z) - \phi\frac{\delta(1-r_d)(1-\sqrt{\delta r_d})}{1-\delta} \leq 0$$

which, as straightforward algebra shows, holds for $z \in Z^-$.

We now focus on the inequality $-(x^c(z)-\pi^*+\phi)^2 \geq -\delta r_d(p_m(z)-\pi^*+\phi)^2$ evaluated at $z = \pi^* + \phi\delta r_d$. This gives us $x^c_m = x^c(\pi^* + \phi\delta r_d)$ and $p^c_m = p_m(\pi^* + \phi\delta r_d)$ that read as

$$\begin{aligned} x^c_m &= \pi^* + \phi(1-\delta(1-r_d)) - \phi\sqrt{(1-\delta)(4-\delta+2\delta r_d - \delta^2 r_d^2)} \\ p^c_m &= \pi^* + \phi - \phi\sqrt{1-2\delta r_d + \delta r_d^2} \end{aligned}$$

where the expression for x^c_m applies only as long as $\kappa^- \leq x^c_m$. We do not need to focus on the case when $\kappa^- > x^c_m$ as then $\kappa^- > x^c_m > x^c(\pi^* + \phi\delta r_d)$ (which is a direct consequence of the V_P function being the upper envelope of two quadratic functions).

At this point it is helpful to replace δr_d in expressions for x^c_m and p^c_m by k which gives

$$\begin{aligned} x^k_m &= \pi^* + \phi\left(1 - \frac{k}{r_d} + k\right) - \phi\sqrt{\left(1-\frac{k}{r_d}\right)\left(4 - \frac{k}{r_d} + 2k - k^2\right)} \\ p^k_m &= \pi^* + \phi - \phi\sqrt{1-2k+kr_d} \end{aligned}$$

where $k \in [\frac{1}{5}, 1]$ and $r_d \in [k, 1]$ under part one of assumption 1.2.

As a next step we prove that $\frac{\partial x^k_m}{\partial r_d} \leq k\frac{\partial p^k_m}{\partial r_d}$ for any $k \in [\frac{1}{5}, 1]$ and any $r_d \in [k, 1]$, which implies that if $-(x^k_m - \pi^* + \phi)^2 \geq -k(p^k_m - \pi^* + \phi)^2$ holds for some $k \in [\frac{1}{5}, 1]$ and $r_d \in [k, 1]$, then it has to hold for the same k and any $r'_d \geq r_d$. To confirm $\frac{\partial x^k_m}{\partial r_d} \leq k\frac{\partial p^k_m}{\partial r_d}$ for any $k \in [\frac{1}{5}, 1]$ and any $r_d \in [k, 1]$ we rewrite the inequality $\frac{\partial x^k_m}{\partial r_d} \leq k\frac{\partial p^k_m}{\partial r_d}$ into the form $\mathbb{P}_k(r_d) \leq 0$ where $\mathbb{P}_k(r_d)$ is a complicated expression of k and r_d that we view as a polynomial in r_d with coefficients given by k. We need to confirm $\mathbb{P}_k(r_d)$ does not have a root in $[k, 1]$ for any $k \in [\frac{1}{5}, 1]$. To do so we use the Descartes rule with a substitution $r_d = \frac{\alpha+\beta y}{1+y}$ with $\alpha = k$ and $\beta = 1$ (see Prasolov, 2004, corollary to theorem 1.4.1). This gives us polynomial $\mathbb{P}_k(y)$ in y with coefficients given by k. We use Sturm's theorem (Prasolov, 2004, theorem 1.4.3) to check that all the coefficients in $\mathbb{P}_k(y)$ are negative for $k \in [\frac{1}{5}, 1]$ which, by Descartes rule, implies that $\mathbb{P}_k(y)$ does not have a positive root for any $k \in [\frac{1}{5}, 1]$, which in turn implies that $\mathbb{P}_k(r_d)$ does not have a root in $[k, 1]$ for any $k \in [\frac{1}{5}, 1]$.

The last thing we need it to find is a line through the $\{\delta, r_d\}$ space $\mathbb{P}$, expressed as $\delta = f(r_d)$, for which $-(x_m^c - \pi^* + \phi)^2 \geq -\delta r_d(p_m^c - \pi^* + \phi)^2$ holds. This will imply that the inequality holds for any $\{\delta', r_d'\}$ such that $k = \delta r_d = \delta' r_d'$ and $r_d' \geq r_d$. Combined with part one of assumption 1.2 $\delta \geq \frac{1}{5r_d}$, if we can show that the inequality holds for any $\{\delta = f(r_d), r_d\}$ where $\delta \geq \frac{1}{5r_d}$, it has to hold for any $\{\delta', r_d\}$ such that $f(r_d) \geq \delta' \geq \frac{1}{5r_d}$. One such $f(r_d)$ is given by $f(r_d) = 1 - \frac{(1-r_d)^2}{2}$. To see this we substitute the expressions for x_m^c and p_m^c into $-(x_m^c - \pi^* + \phi)^2 \geq -\delta r_d(p_m^c - \pi^* + \phi)^2$ along with $\delta = 1 - \frac{(1-r_d)^2}{2}$, getting polynomial $\mathbb{P}(r_d)$ in r_d and we confirm that it has no root in the $[\frac{1}{5}, 1]$ interval using Sturm's theorem again. This delivers part three of assumption 1.2 and proves the claim. □

Claims 1.21 and 1.22 confirm our original conjecture that C's offers bring P to indifference between accepting and rejecting given the unconstrained maximizer of C's overall utility is not available. Hence C's strategy as a solution to her dynamic optimization program indeed generates P's acceptance sets conjectured in that optimization program. Therefore C's proposal strategies $\rho_C = \{p_D(x), p_A(x), q_D(x), q_A(x)\}$ generated by C's dynamic optimization problem under acceptance sets generated by V_P and P's voting strategies ρ_P generated by V_P constitute CS-MPE.

The rest of the proposition follows easily. Uniqueness of the CS-MPE in terms of associated value functions follows from the uniqueness of V_P in any CS-MPE and uniqueness of the solution to C's optimization program. Part one of the proposition follows from proposition 1.3, part two is trivial to establish, part three follows from claim 1.20 and part four follows from claims 1.15 and 1.16.

1.A1.5 Proof of proposition 1.5

Using definition 1.4 of S, $x \in S$ implies $q_A(x) = q_D(x) = x$ and hence stable D period policy decisions $p_D(x) = p^*$. D-efficiency and hence part one then follows from the fact that $p^* \in [\pi^* - \phi, \pi^* + \phi]$, which is easy to see. For part two denote as in the proof of proposition 1.4 by $X^- = X \setminus (\pi^* - \phi, \pi^* + 3\phi)$ and by $X^+ = (\pi^* - \phi, \pi^* + 3\phi)$. We know by claim 1.15 from the proof of proposition 1.4 that $p_D(x) = \pi^* - \phi$ and $p_A(x) = \pi^*$ for any $x \in X^-$. For $x \in X^+$ we know $q_A(x) \in \{\min\{A_A(x, p = \pi^*)\}, \max\{A_A(x, p = \pi^*)\}\}$ by claim 1.21 from the proof of proposition 1.4 and it is easy to show $q_A(x) = x$ possibly only for $x = \pi^*$ as $\min\{A_A(\pi^*, p = \pi^*)\} = \pi^*$. This opens the possibility that $q_A(\pi^*) = q_D(\pi^*) = \pi^*$ and hence possibly $\pi^* \in S$. This in turn would imply $p_A(\pi^*) = p_D(\pi^*) = \pi^*$, which is easy to show as well. In any case, π^* has zero measure.

For part three, from part four of proposition 1.4, C proposes unconstrained maximizers of her overall utility $\gamma_{CD} = \{\pi^* - \phi, z\}$ and $\gamma_{CA} = \{\pi^*, z'\}$ whenever $\gamma_{CD} \in A_D(x)$ and $\gamma_{CA} \in A_A(x)$ for some $z, z' \in X \setminus (\pi^* - \phi, \pi^* + 3\phi)$. As a result whenever x is such that C can propose γ_{CA} (γ_{CD}) in A (D) period, we can specify proposal strategies such that the bargaining reaches S immediately. Integration intervals in the proposition are then a translation of the conditions $\gamma_{CA} \in A_A(x)$ and $\gamma_{CD} \in A_D(x)$ that can be derived easily using V_P from proof of proposition 1.4. The fourth part is then a direct consequence of proposition 1.3.

1.A1.6 Proof of proposition 1.6

To prove the proposition we prove that the policy C proposes for a given default option x under the implicit status-quo bargaining is in P's acceptance set for the same default option under the explicit status-quo bargaining. This, along with the fact that the explicit status-quo bargaining relaxes the constraint on C's optimization problem, will imply the first part. We superscript all variables from the implicit status-quo bargaining by I and all variables from the explicit status-quo bargaining by E and use the notation from the proofs of propositions 1.1 and 1.4.

For D periods notice that by feasibility of equilibrium proposals under implicit status-quo bargaining

$$f_{PD}(p_D^I(x)) + \delta V_P^I(p_D^I(x)) \geq f_{PD}(x) + \delta V_P^I(x)$$

for any $x \in X$. Adding $\pm\delta V_P^E(p_D^I(x))$ and $\pm\delta V_P^E(x)$ to the left and right hand sides, the inequality after rearrangement becomes

$$\begin{aligned} f_{PD}(p_D^I(x)) + \delta V_P^E(p_D^I(x)) \geq & f_{PD}(x) + \delta V_P^E(x) \\ & +\delta[(V_P^I(x) - V_P^E(x)) - (V_P^I(p_D^I(x)) - V_P^E(p_D^I(x)))] \end{aligned}$$

so that if we can prove that the term in the square brackets is positive for any $x \in X$, $\{p_D^I(x), p_D^I(x)\} \in A_D^E(x)$ will follow.

The difference of P's value functions under the two bargaining protocols from the proofs of propositions 1.1 and 1.4 is

$$V_P^I(x) - V_P^E(x) = \begin{cases} \phi^2 \dfrac{3\delta r_d(1-r_d)}{1-\delta} \\ \qquad \text{for} \quad x \in X \setminus (\pi^* + \phi\delta r_d - \kappa, \pi^* + \phi\delta r_d + \kappa) \\ \dfrac{1-r_d}{(1-\delta)(1-\delta r_d)}(x - \pi^* + \phi\delta r_d)(x - \pi^* - 3\phi\delta r_d) \\ \qquad \text{for} \quad x \in [\pi^* + \phi\delta r_d - \kappa, \pi^* - \phi\delta r_d] \cup [\pi^* + 3\phi\delta r_d, \pi^* + \phi\delta r_d + \kappa] \\ 0 \\ \qquad \text{for} \quad x \in [\pi^* - \phi\delta r_d, \pi^* + 3\phi\delta r_d] \end{cases}$$

where $\kappa = \phi\sqrt{\delta r_d(3 + \delta r_d)}$ as before. Also note that $V_P^I(x) - V_P^E(x)$ is non-negative for $\forall x \in X$, which proves the second part of the proposition.

To prove

$$V_P^I(x) - V_P^E(x) - (V_P^I(p_D^I(x)) - V_P^E(p_D^I(x))) \geq 0$$

for $\forall x \in X$, first take $x \in X^-$. Then $p_D^I(x) = \pi^* - \phi$ and $V_P^I(x) - V_P^E(x) = V_P^I(\pi^* - \phi) - V_P^E(\pi^* - \phi)$ so that the inequality holds. For default options $x \in \{z | p_D^I(z) \geq \pi^* - \phi\delta r_d\}$ it is easy to show $x \in [\pi^* - \phi\delta r_d, \pi^* + \phi(2 + \delta r_d)]$ and $p_D^I(x) \leq x$ so that $\pi^* - \phi\delta r_d \leq p_D^I(x) \leq x$ and the inequality follows from the fact that $V_P^I(x) - V_P^E(x)$ is non-decreasing for $x \geq \pi^* - \phi\delta r_d$. For default options $x \in [\pi^* - \phi, \pi^* - \phi\delta r_d]$ the inequality holds as $p_D^I(x) = x$. Finally, for $x \in [\pi^* + \phi(2 + \delta r_d), \pi^* + 3\phi]$, $p_D^I(x) = 2(\pi^* + \phi) - x \in [\pi^* - \phi, \pi^* - \phi\delta r_d]$ and as $\pi^* + \phi\delta r_d + \kappa \leq \pi^* + \phi(2 + \delta r_d)$, we have $V_P^I(x) - V_P^E(x) = \phi^2 \frac{3\delta r_d(1-r_d)}{1-\delta}$, whereas $V_P^I(p_D^I(x)) - V_P^E(p_D^I(x)) \leq \phi^2 \frac{3\delta r_d(1-r_d)}{1-\delta}$, so that the inequality holds.

For A periods a similar argument shows that it suffices to show

$$V_P^I(x) - V_P^E(x) - (V_P^I(p_A^I(x)) - V_P^E(p_A^I(x))) \geq 0$$

for $\forall x \in X$ in order to show $\{p_A^I(x), p_A^I(x)\} \in A_A^E(x)$. The inequality then follows from the fact that $p_A^I(x) \in [\pi^* - \phi\delta r_d, \pi^* + \phi\delta r_d]$ so that the second term in the inequality is always equal to zero, whereas the first term is always positive. The third part of the proposition then follows using straightforward algebra and results from the proofs of propositions 1.1 and 1.4.

1.A1.7 Proof of proposition 1.7

We prove two claims that together prove the proposition. The strategy of the proof borrows heavily from Riboni and Ruge-Murcia (2008).

claim 1.23. *The difference in utilities associated with two sequences of policy decisions is linear in ϕ (for the first condition in definition 1.5) and in $\nu_{i,0}$ (for the second condition in definition 1.5).*

Proof. For the first condition in definition 1.5 of an essentially two-member committee, take two general sequences of policy decisions $\mathbf{p} = \{p_0, p_1, \ldots\}$ and $\mathbf{p}' = \{p_0', p_1', \ldots\}$. The utility associated with these policy sequences for a committee member with preference parameter ϕ is

$$U(\mathbf{p}, \phi) = -\sum_{t=0}^{\infty} \delta^t (p_t - \pi^* - \phi \mathbb{I}_D(t))^2$$

where $\mathbb{I}_D(t)$ is D period indicator function. Taking the derivative of the difference $U(\mathbf{p}, \phi) - U(\mathbf{p}', \phi)$ with respect to ϕ gives

$$\frac{\partial [U(\mathbf{p}, \phi) - U(\mathbf{p}', \phi)]}{\partial \phi} = \sum_{t=0}^{\infty} 2\delta^t \mathbb{I}_D(t)(p_t - p_t'),$$

which does not depend on ϕ. It follows that the difference in utility between $\mathbf{p}$ and $\mathbf{p}'$ is linear in ϕ.

For the second condition in definition 1.5, the utility associated with the sequence of policy decisions for a member with preference shock $\nu_{i,0}$ in the current period that is already realized and hence common knowledge is

$$U(\mathbf{p}, \nu_{i,0}) = -(p_0 - \pi^* - \phi - \nu_{i,0})^2 - \sum_{t=1}^{\infty} \delta^t \left[(p_t - \pi^* - \phi \mathbb{I}_D(t))^2 + r_d \mathbb{E}[\nu_{i,t}^2] \right]$$

with derivative of the difference $U(\mathbf{p}, \nu_{i,0}) - U(\mathbf{p}', \nu_{i,0})$ with respect to $\nu_{i,0}$

equal to

$$\frac{\partial [U(\mathbf{p}, \nu_{i,0}) - U(\mathbf{p}', \nu_{i,0})]}{\partial \nu_{i,0}} = 2(p_0 - p_0'),$$

which again does not depend on $\nu_{i,0}$. □

The next claim shows that the proposal is passed if and only if it is accepted by the median member. Formally, for the first condition in definition 1.5 for the committee of N (odd) members denote their preference parameters $\{\phi_1, \ldots, \phi_N\}$ such that $\phi_i < \phi_j$ for every pair $1 \leq i < j \leq N$. Then the median member has the preference shock ϕ_m that satisfies $|\{i|\phi_i > \phi_m\}| = |\{i|\phi_i < \phi_m\}|$. For the second condition in definition 1.5 for the $N-1$ (even) members denote their preference parameters $\{\phi + \nu_{1,0}, \ldots, \phi + \nu_{N-1,0}\}$ such that $\phi + \nu_{i,0} < \phi + \nu_{j,0}$ for every pair $1 \leq i < j \leq N-1$. Then the two median members have preference shocks $\phi + \nu_{m,0}$ where $\nu_{m,0} = 0$ and $|\{i|\nu_{i,0} > \nu_{m,0}\}| = |\{i|\nu_{i,0} < \nu_{m,0}\}|$.

claim 1.24. *Assuming stage-undominated voting strategies, for a committee with N members with N odd, C's proposal γ is passed if and only if it is accepted by the median committee member.*

Proof. For sufficiency, assume the median member accepts, then by the preceding claim either all committee members with $\phi_i > \phi_m$ ($\nu_{i,0} > \nu_{m,0}$) accept or all committee members with $\phi_i < \phi_m$ ($\nu_{i,0} < \nu_{m,0}$) accept. In either case, γ passes.

For necessity, assume the median member does not vote for γ. Then either all members with $\phi_i > \phi_m$ ($\nu_{i,0} > \nu_{m,0}$) do not vote for γ or all members with $\phi_i < \phi_m$ ($\nu_{i,0} < \nu_{m,0}$) do not vote for γ. In either case γ is not approved. □

Using claim 1.24 C's proposal strategy when faced with an essentially two-member committee will take into account only median member(s) of the committee. In the A periods for the first condition in definition 1.5 this is a player with D period preference shock ϕ and for the second condition of the same definition those are all the remaining committee members who in D periods have preference shocks equal to ϕ on average. In the D periods we have either one or two players with preference shock equal to ϕ being median ones, depending on the exact condition used in definition 1.5. As a result, C's proposal strategy in the dynamic bargaining game played by any essentially two-member committee will be equal to the proposal strategy in the dynamic bargaining game played by C with only one other player with the two players having D period preference shocks $-\phi$ and ϕ respectively. The proposition then follows from the fact that C's proposal is always approved in equilibrium by the median player and hence by the whole essentially two-member committee if its members use stage-undominated strategies.

1.A2 Static mechanism implementation

We restrict attention to static transfer-free direct mechanisms in which the policy in period t is independent of history. In mechanism $M : \{m_C, m_P\} \to \Delta(X)$ player $i \in \{C, P\}$ submits message $m_i \in \{A, D\}$ and M implements a policy from X chosen according to some distribution, so that $\Delta(X)$ denotes the set of all distributions on X.

Because player types are perfectly correlated we can restrict attention to mechanisms that learn the type of period with certainty. Each such mechanism will be characterized by a pair of distributions, one for A periods with *cdf* F_A and one for D periods with *cdf* F_D.

It is immediate that F_A implements π^* with certainty in any Pareto efficient mechanism. For D periods, C's expected utility is equal to

$$\int_X -(x - \pi^* + \phi)^2 dF_D(x) = -\operatorname{var}(x) - (\mathbb{E}(x) - \pi^* + \phi)^2$$

and P's expected utility is equal to

$$\int_X -(x - \pi^* - \phi)^2 dF_D(x) = -\operatorname{var}(x) - (\mathbb{E}(x) - \pi^* - \phi)^2$$

so that $\operatorname{var}(x) = 0$ in any Pareto efficient mechanism.

As a result, any Pareto efficient static transfer-free direct mechanism has to involve $M(A, A) = \pi^*$ and $M(D, D) = p^*$ where $p^* \in [\pi^* - \phi, \pi^* + \phi]$. Moreover, $p^* = \pi^*$ for utilitarian (maximizing sum of expected utilities) mechanism.

1.A3 Numerical simulation of equilibrium under explicit status-quo bargaining

This section describes the procedure to obtain numerical estimates of the equilibrium C's value function V_C and her proposals in the model with the explicit status-quo bargaining protocol. We use the standard value function iteration method along with several results proven earlier. First of all recall that by proposition 1.3 we know $p_A(x) = \pi^*$. Furthermore, from the proof of proposition 1.4 we know the shape of P's acceptance sets A_A and A_D and equilibrium proposals for $x \in X^-$. Finally by the same proposition we know C's value function V_C is unique.

To estimate the remaining part of the equilibrium, we restrict the proposal space along each dimension to $X' = [\pi^* - 1.1\phi, \pi^* + 3.1\phi]$ and specify a grid of discrete nodes $\{d_1, \ldots, d_N\} \in X'$. Call this grid G. We use $\pi^* = 2$, $\phi = 1$ which, with the distance of the neighbouring nodes equal to 0.001, gives $N = 4201$. With the proposal space specified, we implement the following iterative procedure. At the iteration step t we solve C's optimization problem for A and D periods for each default option in G. Denote by $V_C^t(G)$ the $N \times 1$ vector of C's continuation values, each of them associated with a distinct node (default option) $d_i \in G$ at the t-th step of the iteration.

For D periods we solve for each $d_i \in G$

$$\max_{\{p,q\} \in A_D(d_i) \subseteq G^2} -(p - \pi^* + \phi)^2 + \delta V_C^t(q)$$

by searching the discretized feasible proposal space $A_D(d_i) \subseteq G^2$. This gives us two $N \times 1$ vectors of proposals for the D period, one along the policy dimension, $\mathbf{p}_D^t$, and the second along the status-quo dimension, $\mathbf{q}_D^t$, with the i-th element of each being C's proposed policy and status-quo for default option d_i.

For A periods we already know $p_A(x) = \pi^*$ hence for each $d_i \in G$ we solve

$$\max_{\{\pi^*,q\} \in A_A(d_i) \subseteq G^2} V_C^t(q)$$

again by searching the feasible proposal grid $A_A(d_i) \subseteq G^2$. This gives us one $N \times 1$ vector of status-quo proposals for the A period, $\mathbf{q}_A^t$, with the i-th element being the proposed status-quo for default option d_i.

Finally we compute $N \times 1$ vector of C's continuation values as

$$V_C^{t+1}(G) = r_d \left[-(\mathbf{p}_D^t - \pi^* + \phi)^2 + \delta V_C^t(\mathbf{q}_D^t)\right] + (1 - r_d)\left[\delta V_C^t(\mathbf{q}_A^t)\right]$$

and proceed to the iteration step $t+1$. We stop the iteration procedure when $\max_{i \in \{1,\ldots,N\}} |V_C^{t+1}(d_i) - V_C^t(d_i)| \leq 1.0 \times 10^{-6}$. As usual, for the first step of the iteration we use $V_C^1(G) = \mathbf{0}$. We experience no problems with convergence and for the simulations shown the procedure converges in about 70 iterations.

The reason why we use this rather rudimentary numerical procedure instead of some more involved one (e.g. a better optimization algorithm and functional approximation for V_C) is twofold. First, we suspect the V_C to be ill-behaved with a number of local maxima and we do not want the optimization algorithm to pick a wrong one especially as the acceptance sets are in general not convex. Second, we suspect the resulting equilibrium to involve several discontinuities and we do not want the functional approximation to 'smooth out' the problem.

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Chapter 2

Simple equilibria in dynamic bargaining games over policies

2.1 Introduction

In recent years, the political science and the political economy literature has, among other things, been studying dynamic bargaining models. On the most general level, these models fully acknowledge the fact that real world policies remain in place until changed and hence the last period's policy serves as a status-quo during today's round of policy determination. Similarly, policy determined today will serve as a status-quo during the next round of policy making.

Despite considerable progress having been made, the more widespread use of dynamic bargaining models is complicated by a lack of general results that ensure the existence and uniqueness of (Markov perfect) equilibria, and by the complexity of equilibria in models where these have been derived analytically.

Faced with this complexity, a part of the literature has turned to computer simulations. What the simulations reveal is indeed the ill-behaved nature of key components of equilibria in dynamic bargaining models. The equilibrium strategies and induced preferences often lack convenient mathematical properties such as differentiability or continuity.

We hope to contribute to the growing literature on dynamic bargaining by constructing equilibria in games in which players care about all the dimensions of a policy space. All the equilibria that we derive here have very simple and arguably intuitive shape, and as such might be more useful in applied work.

We then use these equilibria to illustrate several potentially interesting features. Besides a standard comparative static exercise, we show the multiplicity of equilibria in certain environments. This multiplicity can prove challenging, especially in work where researchers use computer simulations, as the typically used value function iteration method relies on an assumption of uniqueness of the equilibrium being approximated.

Our construction can also prove beneficial in more complicated environments that are 'close' to the simple setup we use, as it readily produces value functions that can be used in the first step of the value function iteration method, in a hope to speed up convergence in the more complicated model.

The paper proceeds in the following way. We start with a brief survey of the dynamic bargaining literature in section 2.2 followed by a description of the model in section 2.3. Section 2.4 and 2.5 derives the corresponding equilibria when the policy space is assumed to be one-dimensional and multi-dimensional, respectively. Section 2.6 discusses our results and some possible extensions of the model in which our approach to the construction of the equilibria might still apply. Section 2.7 concludes. Proofs of all the propositions from the main text are included in the appendix.

2.2 Literature survey

We start with a description of a typical dynamic bargaining model. A set of N players interacts in an infinite horizon with discounting. In every period t, one of the players is randomly chosen to propose a policy $x_t \in \mathbb{R}^n$. This policy is then pitched against a status-quo policy q_t with a winning alternative becoming the new status-quo q_{t+1}. Players collect their utilities, given by an utility function $u_i(\cdot)$, and the bargaining moves to period $t+1$.

We distinguish the *dynamic bargaining* model just described from the closely related, but nevertheless different model with *evolving default*, in which the endogenous nature of the default policy, i.e. the alternative to the proposal, is preserved, but in which utilities are collected only once the game ends.

Both of the mentioned versions can be found in papers investigating divide-the-dollar problems, where policy space is usually $N-1$ dimensional simplex and the players only care about their share of the dollar, i.e. only about one dimension in the policy space. Kalandrakis (2004) derives the equilibrium analytically in a three player version of the dynamic bargaining model with linear utilities. Epple and Riordan (1987) on the other hand investigate a model with general utilities, but where players take fixed turns in proposing.

Diermeier and Fong (2008a,b) investigate a divide-the-dollar game with evolving default where the former model is complicated by the players having to decide about both, the size of the budget to share (with quadratic costs motivated by distortionary taxation) and about how to share it. Diermeier and Fong (2007) and Diermeier and Fong (2009) combine dynamic bargaining across periods with evolving default over individual rounds of bargaining within each period. The latter paper further adds a decision about the size of the budget with quadratic and stochastic costs. Nevertheless, all four papers just mentioned assume players derive linear utility from their share of a budget.

It is the role of concavity over players' share in a utility functions that motivates Battaglini and Palfrey (2012). They simulate equilibria in a standard divide-the-dollar three-player game with concave utilities and contrast these to the equilibria in a model with linear utilities, both theoretically and experimentally.

As opposed to divide-the-dollar games in which players only care about a single dimension of a policy space, several papers have considered models where players are interested in all the dimensions of a policy space X. We call these dynamic bargaining models over *policies*.

Among the first to investigate such a model is Baron (1996). In his paper, N players bargain over policies with $X = \mathbb{R}$ in a framework that is very similar to the one considered in (the one-dimensional part of) this paper. Fong (2005) and Baron and Herron (2003) try to expand the model by assuming $X = \mathbb{R}^2$. Both papers assume only three players with the most preferred policies on an equilateral triangle. Nevertheless, in order to derive some results they have to either put strong restrictions on X (Fong, 2005) or resort to a computer simulation of the equilibrium (Baron and Herron, 2003).

Several extensions of these models have been made. Duggan, Kalandrakis, and Manjunath (2008) make the institutional structure of their model richer by including legislature and a president with a policy veto. Alternatively, Cho (2004) includes elections in a model with three parties where voters' preferences are defined over $X \in \mathbb{R}$ and parties are interested also in the spoils of holding the office. Baron, Diermeier, and Fong (2012) have a similar model, except that $X \in \mathbb{R}^2$. In both models that include elections, it is the policy $x \in X$ that evolves endogenously, whereas spoils of the office are set to zero in case the parties do not reach an agreement.

Two papers take the dynamic bargaining to a monetary policy setting, modelling interest rate making decisions as a game with an endogenous status-quo. Riboni (2010) investigates model with N decision making players plus a public forming expectations about future monetary policy. His equilibrium then requires mutual consistency of both the decision makers' and the citizens' strategies. Riboni and Ruge-Murcia (2008) consider a model without public expectations but with the players' preferences changing stochastically from period to period. What is common to both models is that the proposal-making authority is assumed to be with a single fixed player, emulating the chairman-led nature of a typical monetary policy committee.

Faced with rather complicated equilibria that often defy attempts for analytical derivation, a series of papers have turned to computer simulations. Baron and Herron (2003) illustrate equilibrium in their model derived from simulations, along with the discontinuous property of corresponding value functions. Duggan et al. (2008) perform a similar exercise but use their simulations to investigate the welfare effects of several constitutional changes. Finally, Duggan and Kalandrakis (2011) propose a new method for the computer simulation of equilibria in dynamic bargaining games and compare its speed and robustness to several other methods. As an illustrative example they simulate equilibrium in a model with $N = 9$ and $X \in \mathbb{R}^2$.

Several other papers investigate related models that do not fit into any of the categories just mentioned. Bernheim, Rangel, and Rayo (2006) focus on a model with evolving default, where the policy space consists of a finite set of alternatives and a fixed horizon set for bargaining. They prove that if the policy space includes a Condorcet winner it will, under some conditions, be the final outcome of the bargaining independent of the original default. On the other hand, the independence of the final outcome and starting position arises without a Condorcet winner.

Battaglini and Coate (2007, 2008) analyse two related models with a tax-financed public spending used to finance public good and pork-barrel programs. The intertemporal link in their models arises due to the investment nature of the public good in the former model and possibility of debt finance in the latter.

Finally, Duggan and Kalandrakis (2012) prove the existence of a Markov perfect equilibrium in a general dynamic bargaining game. In order to smooth out the above mentioned discontinuities in the equilibrium value functions, the framework uses (possibly negligible) shocks in the utilities of the players and also a stochastic relationship between the agreed-on policy and the future status-quo. Besides the existence result, the paper proves the upper hemicontinuity of the equilibrium correspondence, suggesting a relative robustness of the equilibria to small changes in the model's parameters. As an illustrative example working paper version of the paper (Duggan and Kalandrakis, 2007) also numerically simulates equilibrium in a dynamic bargaining game with $N = 5$ and $X \in \mathbb{R}$.

2.3 Model

In this section we lay out our model and in the next one, we explain in detail the construction which leads to a conjectured equilibrium and derive the conditions under which the conjecture is indeed an equilibrium. Throughout, we include several examples for specific parameter values of the model.

The model of this section is simple. There is a set of N (odd) players choosing policies from $X = \mathbb{R}$ in an infinite sequence of periods $t = 0, 1, \ldots$ with discounting $\delta \in [0, 1)$. In each period t one of the players is randomly chosen to make a proposal r_t which is then pitched against the status-quo q_t policy and an alternative obtaining majority of votes is implemented and becomes a new status-quo q_{t+1}. Then players collect their utilities and bargaining moves into next period $t + 1$. Probabilities of recognition $p \in \{p_1, \ldots, p_N\}$ are fixed. The single period utility of each player i is taken to be $u_i(x) = -(x - x_i)^2$. We call the most preferred policies of players x_i *original* bliss points and order the players such that $x_1 < \ldots < x_i < x_{i+1} < \ldots < x_N$ denoting the whole vector by $x = \{x_1, \ldots, x_N\}$. We denote by $i = m$ the median player with i satisfying $|\{j|x_j < x_i\}| = |\{j|x_j > x_i\}|$ with the corresponding bliss point of $x_i = x_m$.

We focus solely on Stationary Markov Perfect equilibria (SMPE) of Maskin and Tirole (2001), in which pure strategies are measurable only with respect to the payoff relevant histories. In our model, this will imply the dependence of proposal and voting strategies on a state given by the status-quo q_t policy, not on a specific period t. For this reason we omit the time subscript from thereon.

SMPE will consist of two strategies for each player. The first one is a proposal strategy of player i when recognized in a period with a status-quo q, which we denote by $r_i(q)$. For convenience, and without the loss of generality, we assume that a proposer whose most preferred policy, out of the set of policies that would be accepted, is the current status-quo q, will indeed offer this policy, instead of offering a different policy knowing that it would be rejected.

The second strategy of each player is a voting strategy determining a player's vote when faced with status-quo q and offer r. Following Baron and Kalai (1993) we focus on *stage undominated voting* strategies, which assume that each player votes as if being pivotal and in effect for an alternative offering higher expected utility. This wipes out equilibria in which players vote against their most preferred alternative knowing their vote cannot change the resulting policy. Notice also that in any equilibrium it has to be true that the player who is indifferent between q and r votes for r.

With this structure, it is easy to see that any policy offered will always be accepted. As a result we do not have to distinguish between proposed and accepted policies and can focus solely on $r_i(q)$.

Two sets of equilibrium strategies just described give rise to a continuation value function of each player $V_i(q)$. These functions measure the expected utility from continuing the game at the beginning of each period with a status-quo q before a proposer for that given period is recognized. More formally, these can be written as

$$V_i(q) = \sum_{j=1}^{N} p_j[-(r_j(q) - x_i)^2 + \delta V_i(r_j(q))].$$

With the value functions defined, the proposal strategy of player i for a status-quo q solves

$$\max_{r \text{ accepted}|q} u_i(r) + \delta V_i(r)$$

and denoting overall expected utility by $U_i(r) = u_i(r) + \delta V_i(r)$, voting strategy of player i faced with status-quo q and alternative r dictates voting for r if and only if

$$U_i(r) \geq U_i(q).$$

2.4 Equilibria with $X \in \mathbf{R}$

The first result we prove greatly simplifies the derivation of decisive coalitions needed to approve any given proposal r. More specifically, we prove that, for a general set of (possibly non-Markov) equilibrium proposal and voting strategies, r is accepted, as opposed to q, if and only if the voter with a median bliss point prefers r to q. More formally we have

Proposition 2.1 (Dynamic median voter theorem for $X = \mathbb{R}$)**.** *For any set of proposal and voting strategies and any status-quo policy, a proposal is* *accepted if and only if it is accepted by a player with a median bliss point* x_m.

Proof. See appendix 2.A1.

An implication of this result is that the acceptance sets for each player, when recognized to make a proposal, will be given by the shape of a median player expected utility function and will be the same for all players. A second implication is that the median player will for any status-quo offer his original bliss point. Finally, when constructing the equilibrium, we do not need to specify voting strategies of all the players relying on proposition 2.1 and hence focus only on the voting strategy of the median player.

Conjectured proposal strategies will have the following shape

$$r_i(q) = \begin{cases} \max\{\min\{2x_m - q, q\}, \hat{x}_i\} & \text{for } i < m \\ \min\{\max\{2x_m - q, q\}, \hat{x}_i\} & \text{for } i > m \\ x_m & \text{for } i = m \end{cases} \tag{2.1}$$

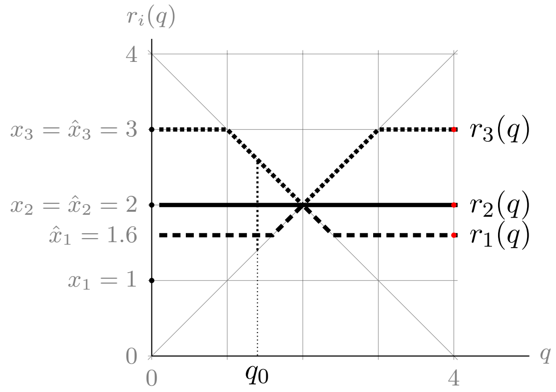
where the $\hat{x}_i$s are what we call *induced* or *strategic* bliss points as those will be offered by a player i, given a large enough acceptance set, even though the original bliss point policy would be accepted as well. The vector of these can be denoted by $\hat{x} = \{\hat{x}_1, \ldots, \hat{x}_N\}$.

Example 2.1. *To illustrate the construction in (2.1) consider a simple model with* $N = 3$, $x_i = i$, $p_i = \frac{1}{N}$ *and* $\delta = 0.9$. *As will be explained below, a set of induced bliss points that qualify as an equilibrium are* $\hat{x} = \{1.6, 2, 3\}$. *Figure 2.1 illustrates the construction of (2.1) graphically.*

An intuition behind the shape of construction (2.1) and figure 2.1 follows. Given the proposal strategies, the typical acceptance set with status-quo q will be an interval between q and the policy on the other side of the median bliss point with the same distance from x_m, i.e. $2x_m - q$. One such acceptance set is indicated in the figure for status-quo q_0. This general shape arises as the overall utility of the median player decreases with a distance of a policy from x_m and hence the median player rejects any policy further from x_m compared to the status-quo.

Faced with this constraint, the decision of player i regarding her proposed policy is driven by two forces. The first force drives the proposal as close as

Figure 2.1: Conjectured equilibrium in Example 2.1



possible to the player's original bliss point x_i increasing her current utility. The second force arises due to strategic considerations. Notice that by offering a policy that is closer to x_m player i sacrifices current utility, but potentially gains by constraining future policies to stay closer to the x_m. For a given player, this will be especially important if the probability of recognition of a player with the original bliss point on the other side of x_m is high.

The interplay of these two forces determines the shape of the construction in (2.1). For status-quo policies close to x_m the acceptance set is a narrow interval around x_m, the first force dominates, and both player 1 and 3 offer policies as close as possible to their bliss points. With increasing status-quo, the acceptance set widens, the second force gains force and player 1 switches to offering policy $\hat{x}_1$, for which the two forces even out, in an attempt to prevent player 3 from offering extreme policies in the future. The same logic holds for player 3 but for this player the second force is absent as player 1 does not offer extreme policies close to x_1 and hence player 3 will offer as high policies as possible given that her original bliss point x_3 is not in the acceptance set.

Finally, we need to specify a way to derive the strategic bliss points in the construction. These will be derived using an algorithm explained below, but first, we need additional piece of notation. Let us denote the general set of players in t-th step of the algorithm by $\mathbb{P}_t$, and define $p_t^+ = \sum_{i \in \mathbb{P}_t | x_i > x_m} p_i$ and $p_t^- = \sum_{i \in \mathbb{P}_t | x_i < x_m} p_i$. In words, p_t^+ is a sum of probabilities of recognition of players in set $\mathbb{P}_t$ with original bliss points above the median and analogously for p_t^-. With this notation the algorithm proceeds as follows.

Algorithm 2.1 (Strategic bliss points with $X = \mathbb{R}$)**.**

step 0 *Set* $\hat{x}_m = x_m$ *and* $\mathbb{P}_1 = \{1, \dots, N\} \setminus \{m\}$

step t *For* $i \in \mathbb{P}_t$ *compute*

$$\hat{x}_{i,t} = \begin{cases} x_i + 2\delta p_t^+(x_m - x_i) & i < m \\ x_i + 2\delta p_t^-(x_m - x_i) & i > m \end{cases}$$

and define $\mathbb{R}_t = \{i | (x_i - x_m)(\hat{x}_{i,t} - x_m) < 0\}$.

If $\mathbb{R}_t = \emptyset$ *pick a player with* $\hat{x}_{i,t}$ *closest to* x_m *out of* $\mathbb{P}_t$. *If more than one player is chosen, pick one of them in an arbitrary way. Denote the chosen player by* j. *Then* $\hat{x}_j = \hat{x}_{j,t}$ *and* $\mathbb{P}_{t+1} = \mathbb{P}_t \setminus \{j\}$. *If* $\mathbb{P}_{t+1} \neq \emptyset$, *proceed to the next step. If* $\mathbb{R}_t \neq \emptyset$, *proceed similarly except for picking player* j *out of* $\mathbb{R}_t$ *and setting* $\hat{x}_j = x_m$.

In words, the algorithm starts from a set of all players apart from the median, and assumes that all these players follow proposal strategies resembling the proposal strategies from figure 2.1 when status-quo q is close to x_m, i.e. assuming that players with $i > m$ ($i < m$) offer as high (low) policy as they can given the acceptance set of the form $[q, 2x_m - q]$.

Given these strategies, the algorithm computes the policy offering the maximum overall utility, $\hat{x}_{i,t}$, for each player and drops the player with $\hat{x}_{i,t}$ closest to x_m as this is the player first to switch into offering her strategic bliss point, i.e. the first to switch to the constant part of the equilibrium. The algorithm then proceeds similarly with a smaller set of players.

There are two possible complications. The first arises when the set $\mathbb{R}_t$, capturing the players with $\hat{x}_{i,t}$ on the other side of x_m compared to their x_i, is not empty. It is easy to confirm that this happens if and only if $2\delta p_1^+ > 1$ or $2\delta p_1^- > 1$. If this is the case, then the strategic bliss point of the chosen player is set to x_m and this player behaves in the same way as the median. Intuitively, this happens when the second force mentioned above is strong enough, which happens either when the future is important as captured by high δ, or the probability of recognition of players on the other side of x_m is high.

The second complication arises when the algorithm computes two $\hat{x}_{i,t}$s with the same distance from x_m. If this is the case the choice of which player to drop is arbitrary. This also implies that there will be two (or more) candidates for equilibria. If the algorithm at some step arrives at two players with an equal distance of $\hat{x}_{i,t}$ from x_m, eliminating one of them and proceeding, will give the first candidate equilibrium while eliminating the other will give the second candidate equilibrium.

Example 2.1 (continued)**.** *In the 0 step the algorithm drops the median player and sets* $\hat{x}_2 = x_2 = 2$*. In the first step, the algorithm computes* $\hat{x}_{1,1} = 1.6$ *and* $\hat{x}_{3,1} = 2.4$*, and by dropping the first player, finally gives* $\hat{x}_3 = \hat{x}_{3,2} = 3$ *as already anticipated and indeed drawn in figure 2.1. Notice that dropping player 3 in the first step of the algorithm would produce a symmetric around* x_m *but distinct set of strategic bliss points* $\hat{x} = \{1, 2, 2.4\}$*.*

The next example illustrates the first complication mentioned above, when either a high δ or a high probability of players on one side of the median (or both) induces a player on the other side of the median to behave as median.

Example 2.2 (Players behaving as median)**.** *Consider model with* $N = 5$*,* $x_i = i$*,* $\delta = 0.9$ *and* $p = \{0.4, 0.4, 0.1, 0.05, 0.05\}$*. It is easy to confirm that* $\mathbb{R}_1 = \{4, 5\}$ *with the algorithm dropping player 4 and* $\mathbb{R}_2 = \{5\}$ *with the algorithm dropping player 5. After two more steps, the algorithm produces* $\hat{x} = \{1, 2, 3, 3, 3\}$*. It is also easy to see that if the algorithm in the first step produces a nonempty* $\mathbb{R}$*, eliminating player, say, above the median, then all the remaining players on the same side of the median will be eliminated in the subsequent steps, and all the players on the opposite side of the median will have the strategic bliss points set to the original ones.*

Finally, we use the algorithm from above for parametrization of the model from Duggan and Kalandrakis (2007), who simulate equilibrium in a similar model. Our setup naturally lacks the utility and status-quo transition shocks their model has, but the values should be interesting for comparative purposes.

Example 2.3 (Duggan and Kalandrakis (2007) parametrization)**.** *Consider model with* $N = 5$*,* $x = \{1, 1.5, 2, 2.8, 3\}$*,* $\delta = 0.9$ *and* $p_i = \frac{1}{N}$*. The algorithm proceeds by eliminating players 2, 1, 4, and 5 in steps 1 through 4 respectively and produces a unique vector of strategic bliss points* $\hat{x} = \{1.72, 1.86, 2, 2.8, 3\}$*.*

We now proceed to specify the conditions under which the conjectured equilibrium constructed above is indeed an equilibrium. In order to do so we first need to construct several objects that will allow us to write the conditions in a concise way.

First, notice that the set of induced bliss points given by algorithm 2.1 induces a finite set of kinks in the proposal strategies. Combine all the (unique) values of q for which such kinks occur into vector $B = \{b_1, \ldots, b_k\}$ where k is the number of the kink-inducing values of q. Assume that B is ordered in such a way that $b_{j-1} < b_j$ for $j = 2, \ldots, k$.

Next, for a given status-quo q, it is helpful to split the set of players N into two subsets. Those who are on the constant part of the proposal strategies $C(q)$ and those who are not $N \setminus C(q)$. While well-defined for $X \setminus B$, $C(q)$ is not well-defined for any of the break points in B as it is not clear whether the player i for whom the $r_i(q)$ kinks at a specific q should be included in $C(q)$ or not. For this reason, let us define $C(b_j)$ for $j = 1, \ldots, k$ as a set of two sets, one including the player i (along with the rest of the non-problematic players) and one that does not (again including the non-problematic players). We regard $C(q)$ as a correspondence mapping X into sets, which is single valued for $q \in X \setminus B$ and double valued for $q \in B$.

Next, we define $p^+(q) = \sum_{i \in N \setminus C(q) | x_i > x_m} p_i$ and $p^-(q) = \sum_{i \in N \setminus C(q) | x_i < x_m} p_i$. In words, for a specific value of q, $p^+(q)$ gives the sum of probabilities of recognition of players above the median who are not on the constant part of the equilibrium. This is analogous for $p^-(q)$. Given that we view $C(q)$ as a correspondence, both $p^+(q)$ and $p^-(q)$ will be correspondences as well, mapping X into a single value for $q \in X \setminus B$ and into two values for $q \in B$. Notice also that both $p^+(q)$ and $p^-(q)$ are constant on every interval into which policy space X is divided by B, if we disregard the correspondence nature at the breaks in B. Finally, for any $b \in B$, denote by $p^+(\uparrow b)$ one of the values of $p^+(b)$, namely the one which is equal to $p^+(b - \epsilon)$ for small positive values of ϵ. Similarly, denote by $p^+(\downarrow b)$ the value of $p^+(b)$ which is equal to $p^+(b + \epsilon)$ for small positive values of ϵ. For $p^-(q)$ things are defined analogously.

With the notation in place, we are ready to state the following condition, under which the construction from (2.1) along with the strategic bliss points given by algorithm 2.1 gives an equilibrium.

Proposition 2.2 (Sufficient condition for equilibrium with $X = \mathbb{R}$). *For*

$i \in N$ denote

$$X_i^c = \begin{cases} B \cap (x_i, \hat{x}_i) & \text{for } i < m \\ B \cap (\hat{x}_i, x_i) & \text{for } i > m \end{cases}$$

with a typical element $x_{i,j}^c$. The construction from (2.1), with the strategic bliss points given by algorithm 2.1, is an equilibrium if for each i for which $X_i^c \neq \emptyset$

$$\begin{aligned} x_{i,j}^c - x_i + 2\delta p^+(\uparrow x_{i,j}^c)(x_i - x_m) \leq 0 & \qquad \forall j \text{ if } i < m \\ x_{i,j}^c - x_i + 2\delta p^-(\downarrow x_{i,j}^c)(x_i - x_m) \geq 0 & \qquad \forall j \text{ if } i > m \end{aligned}$$

Proof. See appendix 2.A1.

The intuition behind the proposition is as follows. Take player i with $i < m$. It is easy to see she will never offer a policy on the other side of the median. Furthermore, in the proof of the proposition we show that her expected utility function is non-decreasing on $(-\infty, x_i]$ and non-increasing on $[\hat{x}_i, x_m]$. But for the construction to be an equilibrium, we need a stronger result, namely that it is non-decreasing on $(-\infty, \hat{x}_i]$. Establishing that the expected overall utility is continuous, piecewise concave and piecewise differentiable allows us to focus only on the set of points in X_i^c. This is a set of points from B between the original and the induced bliss points of player i. A condition in the proposition then makes sure that the left derivative of the expected utility is non-negative. When the condition holds, we know that on $(-\infty, x_m]$ the expected utility of player i attains a maximum at $\hat{x}_i$ and decreases on $[\hat{x}_i, x_m]$. As a result, when $\hat{x}_i$ is not in median player's acceptance set for a given q, player i will offer as low a policy as possible, and if it is in the acceptance set, she will offer $\hat{x}_i$.

Notice also that the condition in proposition 2.2 is stronger than needed, as it would suffice for, say, player $i < m$, for the expected utility to attain a maximum at $\hat{x}_i$ irrespective of its shape on $[x_i, \hat{x}_i]$. This indeed motivates a condition in the next proposition, which is both necessary and sufficient, but we have decided to include the condition in proposition 2.2 as it is extremely simple to check, as shown by the next example.

Example 2.1 (continued)**.** *With $\hat{x} = \{1.6, 2, 3\}$ and a set of breaks $B =$*

$\{1.6, 2.4\}$,

$$C(q) = \begin{cases} \{1,2,3\} & \textit{for } q \in (-\infty, 1] \cup [3, \infty) \\ \{1,2\} & \textit{for } q \in [1, 1.6] \cup [2.4, 3] \\ \{2\} & \textit{for } q \in [1.6, 2.4] \end{cases}$$

and the probability correspondences are

$$\begin{aligned} p^-(q) &= \begin{cases} \frac{1}{3} & \textit{for } q \in [1.6, 2.4] \\ 0 & \textit{for } q \in (-\infty, 1.6] \cup [2.4, \infty) \end{cases} \\ p^+(q) &= \begin{cases} \frac{1}{3} & \textit{for } q \in [1, 3] \\ 0 & \textit{for } q \in (-\infty, 1] \cup [3, \infty) \end{cases} \end{aligned}.$$

The condition of proposition 2.2 holds as $X_i^c = \emptyset$ for $i = 1, 2, 3$. Indeed it is easy to show that the condition will hold for any model with $N = 3$, $x_m - x_1 = x_3 - x_m$ and $p_1 = p_3$ where players 1 and 3 denote the non-median ones.

We next state a condition that is both sufficient and necessary for the construction above to be an equilibrium. The proposition uses set Z_i which is a set of points in $X \setminus B$ for which the overall utility of player i has a zero derivative, indicating a local maximum.

Proposition 2.3 (Sufficient and necessary condition for equilibrium with $X = \mathbb{R}$)**.** *For $i \in N$ denote*

$$X_i^c = \begin{cases} ((B \cup Z_i) \cap (x_i, \hat{x}_i)) \cup \{x_i, \hat{x}_i\} & \textit{for } i < m \\ ((B \cup Z_i) \cap (\hat{x}_i, x_i)) \cup \{x_i, \hat{x}_i\} & \textit{for } i > m \end{cases}$$

with a typical element $x_{i,j}^c$. Arrange the elements of $X_{i,j}^c$ in a decreasing (increasing) order for $i < m$ ($i > m$) and denote the first element by $x_{i,0}^c$. Finally, denote by J_i' the number of elements in X_i^c.

The construction from (2.1) with the strategic bliss points given by algorithm 2.1 is then an equilibrium if and only if for each i

$$\begin{aligned} &\textstyle\sum_{j=1}^{J} \left[\frac{x^2}{2} c_1(x) + c_2(x) x\right]_{\downarrow x_{i,j}^c}^{\uparrow x_{i,j-1}^c} \geq 0 \qquad J = 1, \ldots, J_i' \textit{ if } i < m \\ &\textstyle\sum_{j=1}^{J} \left[\frac{x^2}{2} c_1(x) + c_2(x) x\right]_{\uparrow x_{i,j}^c}^{\downarrow x_{i,j-1}^c} \geq 0 \qquad J = 1, \ldots, J_i' \textit{ if } i > m \end{aligned}$$

where

$$c_1(x) = -\frac{2}{1-\delta(p^+(x)+p^-(x))}$$
$$c_2(x) = \begin{cases} c_1(x)[-x_i + 2\delta p^+(x)(x_i - x_m)] & \textit{for } i < m \\ c_1(x)[-x_i + 2\delta p^-(x)(x_i - x_m)] & \textit{for } i > m \end{cases}$$

Proof. See appendix 2.A1.

Proposition 2.3 checks that $U_i(\hat{x}_i)$ is higher than $U_i(x)$ for any x in the $[x_i, \hat{x}_i]$ interval. It turns out to be enough to check a finite set of points collected in X_i^c. The proposition does not require us to construct $U_i(q)$ explicitly, as it turns out to be easier to integrate its derivative and use the fact that $U_i(x) - U_i(y) = \left[\frac{\partial U_i(x)}{\partial x}\right]_y^x$, proceeding interval by interval due to the piecewise differentiability of U_i.

We close this section by an example in which the conditions explained above might fail depending on the value of δ. It is also easy to see that both of the conditions above hold in all the preceding examples 2.1 through 2.3.

Example 2.4 (Possible failure of conditions for equilibrium)**.** *Consider a model with* $N = 7$*,* $x_i = i$*,* $p_i = \frac{1}{N}$ *and* $\delta = 0.5$*. Then it is relatively straightforward to check that the algorithm 2.1 gives eight possible arrangements of strategic bliss points, i.e. eight conjectured equilibria, and that for all those, the condition of proposition 2.2 and condition of proposition 2.3, hold.*

For the same model with $\delta = 0.9$ *the set of conjectures reduces to two but both fail both conditions from above.*

Finally, for $\delta = 0.95$ *there are again two conjectured equilibria and for both of them the condition of proposition 2.2 fails while the condition of proposition 2.3 holds.*

2.5 Equilibria with $X \in \mathbf{R}^n$

In this section we extend results from the previous one to the models with a multi-dimensional policy space. The setup of the model is exactly the same, except for the policy space $X = \mathbb{R}^n$. The utility of a player i is taken to be quadratic over each dimension, i.e. $u_i(x) = \sum_{j=1}^n -(x^j - x_i^j)^2$ where x^j is taken to be a policy in dimension j. Original bliss points will be vectors x_i with the preferred policy along dimension j denoted by x_i^j. Notice that $u_i(x) = ||x - x_i||^2$ where $|| \cdot ||$ denotes the norm.

In order to proceed we make an assumption about the arrangement of x_is in the policy space. We assume that the original bliss points are arranged in a way that ensures the existence of a core, more specifically we assume that the Plott (1967) condition is satisfied. For N odd this condition is both necessary and sufficient for the existence of a core (Austen-Smith and Banks, 2000).

This allows us to denote the player with a bliss point at the core as the median with bliss point x_m. The Plott (1967) condition then states that for each player i different from the median, there is another player j with a bliss point on the line connecting x_i with x_m but on the other side of x_m relative to x_i. For simplicity, we assume that exactly three players lie on each such line, and without loss of generality, set the bliss point of the median to be an origin of X.

Next, we prove a result similar to the dynamic median voter theorem proven in proposition 2.1 for a multi-dimensional policy space.

Proposition 2.4 (Dynamic median voter theorem for $X = \mathbb{R}^n$)**.** *For any set of proposal and voting strategies and any status-quo policy, a proposal is accepted if and only if it is accepted by a player with a median bliss point x_m.*

Proof. See appendix 2.A1.

The proposition again allows us to focus on the median player who determines whether a given proposal will be accepted or not. With this result we can conjecture proposal strategies $r_i(q) : \mathbb{R}^n \to \mathbb{R}^n$ to be of the form

$$r_i(q) = \begin{cases} \frac{||q||}{||x_i||} x_i & \text{for } \frac{||q||}{||x_i||} \leq \hat{k}_i \\ \hat{k}_i x_i & \text{for } \frac{||q||}{||x_i||} \geq \hat{k}_i \end{cases}. \tag{2.2}$$

The proposal strategy of player i specifies, for a status-quo close to the origin, offering a policy on the line connecting origin and x_i with the same distance from the origin as the status-quo q. For a status-quo far away from origin, player i will be offering a fixed policy $\hat{k}_i x_i$, which we again term the strategic bliss point, for some $\hat{k}_i \in [0, 1]$.

The logic behind the proposal strategies is similar to the one-dimensional case. For a given status-quo q, the typical acceptance set will be a circle with the center at the origin and a radius $||q||$. There are again two forces at play.

The first force pushes players into offering policies as close as possible to their original bliss points. The second strategic force pushes them into offering policies closer to the origin in an attempt to constraint future policies of the other players. For values of q close to the origin, the first force dominates and players offer a policy on a line connecting origin and x_i with the same distance from the origin as q. For values of q further away from the origin, the second force dominates and players offer fixed policies given by $\hat{k}_i x_i$.

Determining $\hat{k}_i$s is again done via a similar algorithm as in the previous section. The algorithm uses $a(i,j)$ to denote an angle between x_i and x_j. Subsequently it is easy to see that $\cos(a(i,j)) = \frac{x_i' x_j}{||x_i|| \cdot ||x_j||}$.

Algorithm 2.2 (Strategic bliss points with $X = \mathbb{R}^n$)**.**

step 0 Set $\hat{k}_m = 0$ *and* $\mathbb{P}_1 = \{1, \ldots, N\} \setminus \{m\}$

step t For $i \in \mathbb{P}_t$ *compute*

$$\hat{k}_{i,t} = 1 - \delta \sum_{j \in \mathbb{P}_t} p_j [1 - \cos(a(i,j))]$$

and define $\mathbb{R}_t = \{i | \hat{k}_{i,t} < 0\}$.

If $\mathbb{R}_t = \emptyset$ *pick a player with the smallest* $\hat{k}_{i,t} ||x_i||$ *out of* $\mathbb{P}_t$. *If more than one player is chosen, pick one of them in an arbitrary way. Denote the chosen player by* j. *Then* $\hat{k}_j = \hat{k}_{j,t}$ *and* $\mathbb{P}_{t+1} = \mathbb{P}_t \setminus \{j\}$. *If* $\mathbb{P}_{t+1} \neq \emptyset$, *proceed to next step. If* $\mathbb{R}_t \neq \emptyset$, *proceed similarly except for picking player* j *out of* $\mathbb{R}_t$ *and setting* $\hat{k}_j = 0$.

To proceed, we need to define similar objects as in the previous section, collecting all the points at which the value function induced by the construction in (2.2) and algorithm 2.2 kinks, and splitting the players into those on the constant and variable part of an equilibrium. However, we need to be concerned only about the distances from origin, not about the specific location in X.

For this purpose define B to be the collection of distances of the induced bliss points from the origin and order elements in B in an increasing order. Naturally, the first element of B is equal to 0. Next, for a given distance from origin d, let us define $C(d)$ to be a set of players on a constant part of the equilibrium proposal strategies, i.e. those players with $d > \hat{k}_i ||x_i||$. For notational convenience, when we say $C(q)$ with $q \in \mathbb{R}^n$ we mean $C(||q||)$.

Again $C(d)$ is well defined for $d \notin B$. For $d \in B$ we regard $C(d)$ as a correspondence giving two sets of players, one with the player for whom $d = \hat{k}_i||x_i||$ and all the players with $d > \hat{k}_i||x_i||$, and one with only the latter group of players.

Finally it will be convenient to redefine both constructions in terms of relative-to-x_i distance for player i. Hence let us define $B_i = \frac{B}{||x_i||}$ and also $C_i(k) = C(k||x_i||)$ for all $i \neq m$ and $k \in [0, \infty)$. With this notation we can state a sufficient condition for the construction just explained to be an equilibrium.

Proposition 2.5 (Sufficient condition for equilibrium with $X = \mathbb{R}^n$)**.** *For $i \in N \setminus \{m\}$ denote*

$$K_i^c = B_i \cap (\hat{k}_i, 1)$$

with a typical element $k_{i,j}^c$. Then the construction from (2.2) with the strategic bliss points given by algorithm 2.2 is an equilibrium if for each i for which $K_i^c \neq \emptyset$

$$1 - k_{i,j}^c - \delta \sum_{j \in N \setminus C_i(\downarrow k_{i,j}^c)} p_j [1 - cos(a(i,j))] \leq 0 \; \forall j$$

Proof. See appendix 2.A1.

The intuition behind this result is simple. As we argue in the proof, it is enough to focus on player i offering policies on a ray starting at the origin and passing through x_i. For the construction to be an equilibrium, we want the expected utility function to first increase along this ray, until it reaches distance $\hat{k}_i||x_i||$, and then decrease. It turns out to be sufficient to focus on the interval $(\hat{k}_i||x_i||, ||x_i||)$ or in terms of the relative distance on $(\hat{k}_i, 1)$. Given the piecewise concavity of the expected utility function, the condition in the proposition ensures that at any break in B_i the expected utility is non-increasing.

Proposition 2.5 gives a sufficient condition which is stronger than needed but is easy to check. The next proposition states a condition that is both sufficient and necessary for equilibrium. We will again use set Z_i, which is a set of relative-to-x_i distances in $[0, \infty) \setminus B_i$ for which the expected utility function has a zero directional derivative along a ray starting at the origin and passing through x_i.

Proposition 2.6 (Sufficient and necessary condition for equilibrium with $X = \mathbb{R}^n$)**.** *For $i \in N \setminus \{m\}$ denote*

$$K_i^c = ((B_i \cup Z_i) \cap (\hat{k}_i, 1)) \cup \{\hat{k}_i, 1\}$$

with a typical element $k_{i,j}^c$. Arrange the elements of $K_{i,j}^c$ in an increasing order and denote the first element by $k_{i,0}^c$. Finally denote by J_i' the number of elements in K_i^c.

Then the construction from (2.2) with the strategic bliss points given by algorithm 2.2 is an equilibrium if and only if for each $i \in N \setminus \{m\}$

$$\sum_{j=1}^{J} \left[\frac{k^2}{2} c_1(k) + c_2(k) k \right]_{\uparrow k_{i,j}^c}^{\downarrow k_{i,j-1}^c} \geq 0 \;\; J = 1, \ldots, J_i'$$

where

$$\begin{aligned} c_1(k) &= -\frac{2||x_i||^2}{1-\delta \sum_{j \in N \setminus C_i(k)}} \\ c_2(k) &= c_1(k)[-1 + \delta \textstyle\sum_{j \in N \setminus C_i(k)} p_j [1 - cos(a(i,j))]] \end{aligned}$$

Proof. See appendix 2.A1.

We finish this section with two examples both of which assume $X = \mathbb{R}^2$.

Example 2.5 (Simplest example in $\mathbb{R}^2$)**.** *Consider a model with $N = 5$, $p_i = \frac{1}{N}$, $\delta = 0.9$ and the following bliss points*

player	*1*	*2*	*3*	*4*	*5*
x_i^1	*2*	*-2*	*0*	*0*	*0*
x_i^2	*0*	*0*	*2*	*-2*	*0*

Algorithm 2.2 offers four possible players to be eliminated in step 1, then 2 in steps 2 and 3. As a consequence there will be 16 possible equilibria. Eliminating players 1, 3, 2 and 4 respectively produces

player	*1*	*2*	*3*	*4*	*5*
$\hat{k}_i$	*0.28*	*0.82*	*0.46*	*1*	*0*

The set of distances at which players switch between constant and non-constant proposal strategies will be $B = \{0, 0.56, 0.92, 1.64, 2\}$ and can be *translated into relative-to-x_i distance $B_i = \{0, 0.28, 0.46, 0.82, 1\}$. With these we have*

$$C(d) = \begin{cases} \{1,2,3,4\} & \textit{for } d \in [0, 0.56] \\ \{2,3,4\} & \textit{for } d \in [0.56, 0.92] \\ \{2,4\} & \textit{for } d \in [0.92, 1.64] \\ \{4\} & \textit{for } d \in [1.64, 2] \\ \emptyset & \textit{for } d \in [2, \infty] \end{cases}$$

and K_i^c from proposition 2.5 will be $K_1^c = \{0.46, 0.82\}$, $K_3^c = \{0.82\}$ and an empty set for the remaining players. It is easy to check that both conditions from propositions 2.5 and 2.6 hold.

Example 2.6 (Duggan and Kalandrakis (2011) parametrization)**.** *Consider a model with $N = 9$, $p_i = \frac{1}{N}$, $\delta = 0.7$ and bliss points*

player	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
x_i^1	*-0.8*	*0.3*	*-0.2*	*0.9*	*0.1*	*-0.15*	*0.3*	*-0.9*	*0*
x_i^2	*0*	*0*	*0.2*	*-0.9*	*0.6*	*-0.9*	*0.2*	*-0.6*	*0*

Algorithm 2.2 produces a unique set of bliss points (numbers rounded)

player	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
$\hat{k}_i$	*0.79*	*0.51*	*0.38*	*1*	*0.50*	*0.94*	*0.48*	*0.91*	*0*

for which conditions from propositions 2.5 and 2.6 hold.

2.6 Discussion

In this section we discuss several topics related to the results presented so far. First, we look at the comparative static properties of the construction above. Although we do not conduct an explicit comparative static exercise, it is obvious that two variables have a strong influence on the shape of an equilibrium. Firstly, there is the discount factor δ. With the future becoming more important, all the equilibria above will be more concentrated around the bliss point of the median player. Secondly, there is a vector of recognition probabilities p. These vectors influence the equilibria in a complex way, but in general the higher the probability of recognition of a given player, the closer her opposition will be to the median.

Another observation regards the position of the most extreme player judged by the original bliss point. Notice that making player more extreme, i.e. further away from median, if this player is the last to be eliminated by the algorithms above, does not change the equilibrium behaviour of the other players. To a certain extent the same observation applies for other players. If the shape of the equilibrium remains the same, changing the bliss point of player i does not change the behaviour of other players.

Related to the shape of the equilibria explained above is the behaviour of policies over time. It is easy to see that starting from any status-quo, policies will always converge to the most preferred policy of the median player. Nevertheless, the pattern of policies during the convergence can be rather complex.

At the same time, the convergence to the median prediction is easy to avoid. Consider a similar model as above but with some probability the policy approved today will not become the next status-quo. Instead, the next status-quo would be drawn from some distribution with the cumulative distribution $F(q)$. Assuming that the distribution $F(q)$ is independent of the policy approved today, the equilibria constructed above will be the equilibria in this extended model (given some adjustment to δ). But policies in the model will not converge to the median over time.

Our construction has also uncovered the possible multiplicity of equilibria in certain environments. While not explicitly proven, it should be obvious that environments giving rise to the multiplicity are those with symmetric bliss points and equal recognition probabilities. We want to highlight this observation as the multiplicity can prove problematic for computer simulations in which a researcher chooses to simulate an often tempting symmetric environment.

At the same time it is obvious that the environments giving rise to the multiple equilibria are 'zero measure'. In example 2.1 we have two possible equilibria with $x = \{1, 2, 3\}$ and $p = \{1/3, 1/3, 1/3\}$. However, perturbing the environment to, say, $x = \{1, 2, 3 + \epsilon\}$ or $p = \{1/3, 1/3 + \epsilon, 1/3 - \epsilon\}$ for some small ϵ would make the equilibrium unique.

We have also made a series of simplifying assumptions. But we think the approach to the construction of simple equilibria in dynamic bargaining models taken here extends to more general environments. More specifically, we have assumed an equal δ for all players, but the construction would also be applicable to a model with player-specific δ_i. Specifying a model with a different non-quadratic utility function would also admit a similar approach, as well as an assumption in the multi-dimensional model that players attach different weights to different dimensions of a policy space, i.e. assuming utility of the form $u_i(x) = \sum_{j=1}^{n} -k_j(x^j - x_i^j)^2$ with some set of positive constants k_j. On the other hand, we do not think that the construction explained in this paper for a multi-dimensional policy space would extend to environments in which the Plott (1967) condition for core existence fails.

Finally, we admit that, for a specific parametrization of the model above, deriving strategic bliss points via algorithms 2.1 or 2.2 can be time-consuming and error prone. The same qualification applies when checking conditions for the constructions to be an equilibrium or, when deriving explicit formulas for the value functions $V_i(\cdot)$. For this reason, we have written Matlab routines for both one- and multi-dimensional models that derive the strategic bliss points, check for conditions ensuring equilibrium and derive the resulting value and expected utility functions all in a matter of seconds. Both routines are available upon request.

2.7 Conclusion

We provide an approach to constructing conjectured equilibria in dynamic bargaining models that produces simple and intuitive equilibrium strategies. Our results apply to models with both single and multi-dimensional policy spaces. We have also shown under which conditions the conjecture is indeed an equilibrium, some of which are straightforward to check for a given parametrization of the model.

The shape of the equilibria are in general driven by the interplay of two forces. One force pushes players into proposing policies that maximize their current utility. Another opposing and strategic force pushes players into proposing policies that constraint all the players in the future.

Our analysis shows that in dynamic bargaining models where a median is present, policies will always converge to the policy most preferred by the median. This, however, does not preclude the possibly complex behaviour of policies along the convergence path. We have also uncovered the possibility of multiple equilibria in certain symmetric environments. However none of the resulting equilibria found involves symmetric behaviour of otherwise symmetric players.

Despite the fact that our approach is not generally applicable and does not always produce equilibrium strategies, we nevertheless think it provides interesting insights into an environment with rather scarce analytical results.

2.A1 Proofs

2.A1.1 Proof of proposition 2.1

The proof of proposition 2.1 builds on similar proof found in Riboni and Ruge-Murcia (2008), namely on their appendix A proof.

Proof. Note that any set of strategies for an accepted policy r_0 generates a stochastic sequence of policies $\{r_0, r_1, \ldots\}$ with implied utility for player i given as

$$U_i(r_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (r_t - x_i)^2\right].$$

Similarly accepting r'_0 generates $\{r'_0, r'_1, \ldots\}$ and gives player i

$$U_i(r'_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (r'_t - x_i)^2\right].$$

Differentiating the difference in utility that the two policies bring, with respect to bliss point of player i, gives

$$\frac{\partial [U_i(r_0) - U_i(r'_0)]}{\partial x_i} = \mathbb{E}\left[2\sum_{t=0}^{\infty} -\delta^t (r'_t - r_t)\right]$$

which is independent of x_i and hence $U_i(r_0) - U_i(r'_0)$ is linear in x_i. If, on the one hand, the median player prefers r_0 to r'_0, either all the players with a $x_i \geq x_m$ or all the players with $x_i \leq x_m$ also prefer r_0 to r'_0. As a result r_0 is accepted. If, on the other hand, the median player rejects r_0 to r'_0, either all the players with $x_i \geq x_m$ or all the players with $x_i \leq x_m$ also reject r_0 to r'_0. As a results r_0 is rejected. □

2.A1.2 Proof of proposition 2.2

We proceed is several steps, first establishing some properties of the value function induced by the construction in (2.1), next proving properties of the expected value function and then showing the rationale behind the condition in the proposition.

Proof. First observe that the continuation value function induced by construction (2.1) is continuous, piecewise concave, piecewise quadratic and symmetric around x_m. Some algebra shows that it can be written as

$$V_i(q) = \frac{h_i(q) + \sum_{j \in N \setminus C(q)} p_j u_i(r_j(q))}{1 - \delta \sum_{j \in N \setminus C(q)} p_j}$$

where $h_i(q)$ is a correspondence ensuring that $V_i(q)$ is continuous and that $V_i(x_m) = \frac{u_i(x_m)}{1-\delta}$. Its shape is similar to the shape of $p^+(q)$ in that it attains a unique value at $q \in X \setminus B$ and two values at $q \in B$. Moreover, for every $b \in B$ we require one value of $h_i(b)$ to correspond to one of the sets in $N \setminus C(b)$ and the second value of $h_i(b)$ to correspond to the other set in $N \setminus C(b)$. While this is a somewhat unusual construction, it allows us to economize on the notation later on.

With this notation it is easy to see that the derivative of the expected utility function with respect to status-quo is

$$\frac{\partial U_i(q)}{\partial q} = \begin{cases} -\frac{2}{1-\delta(p^+(q)+p^-(q))}\left[q - x_i + 2\delta p^+(q)(x_i - x_m)\right] & \text{for } q \leq x_m \\ -\frac{2}{1-\delta(p^+(q)+p^-(q))}\left[q - x_i + 2\delta p^-(q)(x_i - x_m)\right] & \text{for } q \geq x_m \end{cases}$$

where again the possibility of two values at $q = x_m$ and at all the $q \in B$ reflects the fact that the overall utility is not differentiable at those points. But it is easy to see that it possesses left and right derivatives, so we regard one value of $\frac{\partial U_i(q)}{\partial q}$ at, say, $q = x_m$ as the left derivative and the other value as the right derivative.

Now notice that the expected utility of the median player is strictly increasing on $(-\infty, x_m)$, strictly decreasing on (x_m, ∞), has unique global maximum at x_m and is symmetric around x_m. As a result, the acceptance set for the general value of q will be $[q, 2x_m - q]$ or $[2x_m - q, q]$ depending on which of the two values is larger.

Faced with the symmetry of the acceptance sets, player i never offers a policy that is on the other side of x_m compared to her x_i, due to the symmetry of the $V_i(q)$ function. As a result, we can focus only on an interval $(-\infty, x_m]$ for players $i < m$ and on an interval $[x_m, \infty)$ for players $i > m$. We will only do the former as the latter relies on a similar argument.

Take player with $i < m$. For the construction in (2.1) to be an equilibrium, we need the expected utility to be non-decreasing on $(-\infty, \hat{x}_i]$ and non-increasing on $[\hat{x}_i, x_m]$. The latter part is easy and follows directly from the construction of algorithm 2.1. Non-decreasing on $(-\infty, x_i]$ is also immediate inspecting the expression for derivative above.

This leaves $[x_i, \hat{x}_i]$ to be inspected. However, there is no need to inspect the whole interval but, given the piecewise concavity, it is enough to check the upper boundary of each interval on which the expected utility function is differentiable. We can also omit $\hat{x}_i$ as the derivative of $U_i(q)$ is zero at that point.

Proposition 2.2 is then the mathematical restatement of what we have just explained. Set X_i^c is a set of all points in $(x_i, \hat{x}_i)$ at which $U_i(q)$ kinks and the condition of the proposition ensures that the left derivative is non-negative at all those points. □

2.A1.3 Proof of proposition 2.3

Proof of proposition 2.3 uses the fact that for a differentiable continuous function $f(x)$ we have $f(x) - f(z) = [f'(a)]_z^x$. Extending this result to a continuous but only piecewise differentiable function is straightforward. If, for example, $x < y < z$ and $f(x)$ is not differentiable at y but has a left and a right derivative, we have $f(x) - f(z) = [f'(a)]_y^x + [f'(a)]_z^y$.

Proof. We show the result for $i < m$ as the argument is similar for $i > m$. We also omit repeating arguments from proof of proposition 2.2 and hence focus solely on the $[x_i, \hat{x}_i]$ interval.

First notice that the construction in (2.1) along with the bliss points derived by algorithm 2.1 is an equilibrium if and only if $U_i(\hat{x}_i) \geq U_i(x) \; \forall \; x \in [x_i, \hat{x}_i]$. One possible approach would be to construct the function $U_i(q)$ explicitly. Nevertheless, we propose a simpler approach as we already know the derivative of $U_i(q)$ from the proof of the previous proposition and are only interested in relative, not absolute, values.

Next, it is easy to show that the condition $U_i(\hat{x}_i) \geq U_i(x)$ fails if and only if it fails at some point of the X_i^c set, which includes all the kinks and all local maxima of the expected utility function on the $[x_i, \hat{x}_i]$ interval (the if part is obvious, the only if part follows from piecewise concavity). Hence we need to check the condition only at the points in X_i^c.

Using the derivative of $U_i(q)$ from the proof of the previous propositions, it can be written as $c_1(q)q + c_2(q)$, where $c_1(q)$ and $c_2(q)$ are given in the statement of this proposition. Integrating the derivative yields the expression $\frac{x^2}{2}c_1(x) + c_2(x)x$ and the sum then proceeds from $x^c_{i,0} = \hat{x}_i$ checking all points from X^c_i for the $U_i(\hat{x}_i) \geq U_i(x)$ condition. □

2.A1.4 Proof of proposition 2.4

Proof. The approach to the proof of proposition 2.4 is analogous to the proof of proposition 2.1. Accepting policy r_0 generates stochastic sequence of policies $\{r_0, r_1, \ldots\}$ with the implied utility

$$U_i(r_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} -\delta^t (r_t - x_i)'(r_t - x_i)\right].$$

Differentiating the difference in utility that r_0 and r'_0 provide, gives

$$\frac{\partial[U_i(r_0) - U_i(r'_0)]}{\partial x_i} = \mathbb{E}\left[2\sum_{t=0}^{\infty} -\delta^t (r'_t - r_t)\right]$$

which is again independent of x_i and hence $U_i(r_0) - U_i(r'_0)$ is linear in x_i. As a consequence, the derivative defines a hyperplane in $\mathbb{R}^n$ which gives all the bliss points such that any player j with a bliss point on this hyperplane will choose between r_0 and r'_0 in exactly the same way as player i. The result should now be obvious, realizing that any hyperplane going through x_m will split the remaining players into two equal size groups, at least one of which votes in the same way as the median. □

2.A1.5 Proof of proposition 2.5

The proof of this proposition is very similar to the proof of proposition 2.2 so we keep it brief.

Proof. First notice that the acceptance sets are circles with a center at the origin and that the continuation value function of all the players has level sets of a similar shape. It follows that player i will never offer any other policy than the policy on the line starting at the origin and going through her bliss point x_i, we call this line the i-ray.

As a consequence, we can work with the continuation value function V_i and the expected utility function U_i, mapping the i-ray into $\mathbb{R}$, instead of both functions having to map the whole policy space X into $\mathbb{R}^n$. It is then convenient to define the argument of both functions to be the distance of a policy on the i-ray from the origin, relative to $||x_i||$. With this notation, some algebra gives

$$V_i(k) = \frac{h_i(k) + \sum_{z \in N \setminus C_i(k)} p_z \sum_{j=1}^{n} - \left(\frac{x_z^j k ||x_i||}{||x_z||} - x_i^j \right)^2}{1 - \delta \sum_{z \in N \setminus C_i(k)} p_z}$$

for $k \in [0, \infty)$, with $h_i(k)$ defined similarly as in proposition 2.2, ensuring $V_i(k)$ is continuous and $V_i(0) = \frac{u_i(0)}{1-\delta}$. It is also immediate that $V_i(k)$ is differentiable except at points in B_i.

With this it is easy to see that the derivative of the overall utility function with respect to k is

$$\frac{\partial U_i(k)}{\partial k} = \frac{2||x_i||^2}{1 - \delta \sum_{j \in N \setminus C_i(k)} p_j} \left[1 - k - \delta \sum_{j \in N \setminus C_i(k)} p_j [1 - cos(a(i,j))] \right]$$

and that $U_i(k)$ is piecewise concave.

For the construction in (2.2), along with the bliss points derived via algorithm 2.2, to be an equilibrium, we need $U_i(k)$ to be non-decreasing on $[0, \hat{k}_i]$ and non-increasing on $[\hat{k}_i, \infty)$. Non-decreasing on $[0, \hat{k}_i]$ comes again from the construction of algorithm 2.2. Non-increasing on $[1, \infty)$ is also easy by inspecting the derivative of $U_i(k)$ above. This leaves $[\hat{k}_i, 1]$ to be inspected and, given piecewise concavity, it is enough to check the right derivative of $U_i(k)$ at each break B_i that falls into $(\hat{k}_i, 1)$. □

2.A1.6 Proof of proposition 2.6

The proof of proposition is very similar to the proof of proposition 2.3 so we only include a brief outline.

Proof. As in proposition 2.3 we want to make sure that $U_i(\hat{k}_i) \geq U_i(k)$ for all $k \in [\hat{k}_i, 1]$. Again, we can focus only on the points at which either $U_i(k)$ kinks or has a local maximum. This is what the set K_i^c collects. Integrating the derivative of $U_i(k)$ from the previous proposition gives $\frac{k^2}{2} c_1(k) + c_2(k) k$

and the condition of this proposition checks that $U_i(\hat{k}_i)$ is higher than $U_i(k)$ for any $k \in K_i^c$. $\square$

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Chapter 3

Central Banks' Voting Records and Future Policy[1]

[1] This chapter is a joint work with Roman Horváth and Kateřina Šmídková. Currently published as Czech National Bank working paper 11/2010.

3.1 Introduction

Monetary policy transparency has increased dramatically over the last two decades (Geraats, 2009; Posen, 2003). Nowadays, central banks typically communicate effectively with the public and explain their policies in great detail. Every monetary policy decision is accompanied by minutes or press releases that outline the arguments that central bankers expressed during the monetary policy meeting. The most transparent central banks where bank boards[2] decide by majority vote also release attributed voting records, typically together with the minutes.[3] In this paper we aim to examine whether voting records are informative about future policy. From the voting records, we are able to calculate an indicator called *skew*, defined as the difference between the average policy rate voted for by the individual board members and the policy rate that is the outcome of the majority vote. Our theoretical model examines under which conditions it is more likely that there will be a rate hike (reduction) in the future when there is a minority vote for higher (lower) rates than the decided-on rate. In addition, an extended empirical model tests whether the *skew* conveys new information in addition to all the other information already incorporated into financial market expectations prior to the monetary policy meeting.

While some previous research has examined the information content of voting records in the case of the UK (Gerlach-Kristen, 2004), many other central banks' voting records have not been examined empirically yet. Similarly, there is also a lack of theoretical studies examining whether voting results are useful for understanding future monetary policy.

On the theoretical side, we fully specify a model of the central bank committee decision-making process, simulate the decisions taken by the model committee and assess the informative power of the voting pattern for future monetary policy. The basic version of our model is similar to the model of Riboni and Ruge-Murcia (2008a) in acknowledging the endogenous nature of the status-quo decision in the central bank decision-making process. Besides the endogenous status quo, our model also incorporates uncertainty and time dependence in optimal monetary policy as well as the private information of

[2] The decision-making bodies in central banks are typically called either monetary policy committees or bank boards. We use the two terms interchangeably in our paper.

[3] Fry, Julius, Mahadeva, Roger, and Sterne (2000) report that approximately 90% of central banks around the world make decisions in committees.

individual committee members, in a way similar to Gerlach-Kristen (2008). Our approach, specifying the model and then proceeding using computer simulations, is also similar to the article just mentioned. We use several alternative models of monetary policy committee decision-making that differ (among other things) in the degree of informational influence among its members and that are related to the models already found in the relevant literature (Gerlach-Kristen, 2008; Riboni and Ruge-Murcia, 2010; Weber, 2010).

What distinguishes our model from the already existing ones is combination of endogeneity of the status-quo policy with time varying heterogeneity of preferences of the monetary policy committee members. The first feature allows us to talk about the *skew* variable in the first place as the committee decisions are made by vote between two alternatives. Second feature then ensures that typical outcome of such vote will not be unanimous or in other words that the *skew* variable will attain non-zero values.

Our theoretical model shows that the voting record contains important information about future monetary policy provided that the signals about the optimal policy rate are noisy and a sufficient degree of information independence exists among the committee members. Even if both of those conditions hold, the informative power of the voting record can be overridden by high volatility of the economic environment or by enough noise in the committee members' information, with a larger committee size counteracting both of those effects.

In the empirical part, this paper examines the informative power of voting results in five inflation-targeting countries - the Czech Republic, Hungary, Poland, Sweden and the UK - and in the U.S., where monetary policy is decided by a majority vote of at least formally independent committee or board members. In consequence, our research gives a greater international perspective than previously published case studies and is able to draw conclusions that are not country-specific.

Our empirical results confirm the theoretical conclusions. The voting record is informative of future monetary policy changes in all the sample countries. It adds news to the information set used in financial market expectations prior to the voting record announcement. This result is robust to the measure of disagreement in the committee as well as to different sample periods. The result is also robust to the timing and style of the voting record announcement. Our dataset provides two 'natural experiment' setups, where we can quantify the effect of publicly unavailable voting results (for the cases of Poland and the U.S.) and the effect of publicly unavailable names of voting members (for the Czech case). The voting record is informative about future policy in these two setups as well. This implies that that releasing the names themselves is less important for transparency than releasing the voting outcome itself.

The paper is organized as follows. Section 3.2 contains the related literature. Section 3.3 introduces a theoretical model of central bank board decision-making. Section 3.4 presents the institutional background of monetary policy decision-making in our sample countries. The empirical methodology is discussed in section 3.5. Section 3.6 gives the results. Section 3.7 offers concluding remarks. Appendices containing details of the theoretical model (Appendix 3.A1), details of the institutional background of monetary policy decision-making (Appendix 3.A2) and a data description (Appendix 3.A3) follow.

3.2 Related Literature

On the most general level the question of whether the voting records of central bank boards and monetary policy committees (MPCs) reveal information about future changes in monetary policy is related to the literature on central bank communication and central bank transparency, surveyed by Blinder, Ehrmann, Fratzscher, De Haan, and Jansen (2008) and Geraats (2002, 2009) respectively. The general conclusion of both strands of literature is that the way central banks communicate to the public and their degree of transparency matters for monetary policy. Most of the theoretical and empirical studies also indicate the benefits of more open and more transparent central bank behaviour. However, not all the studies reach unequivocal conclusions. For example, the model in Morris and Shin (2002) leaves open the possibility that more information provided by a central bank is welfare reducing, while Meade and Stasavage (2008) show that the Federal Reserve's decision to release full transcripts of Federal Open Market Committee (FOMC) meetings decreased the incentives of its participants to voice dissenting opinions. Swank, Swank, and Visser (2008) analyse the reputational issues in expert committees and disincentive to dissent. Winkler

(2000) draws similar conclusions and puts forward a conceptual framework to distinguish different aspects of transparency.

From the theoretical side, the question of whether the voting records of bank board members are informative about future monetary policy is virtually untouched. One of the reasons is the difficulty of modelling committee decision-making with members who hold possibly different beliefs and objectives in the uncertain monetary environment. Another difficulty is the dynamic nature of central bank decision-making, as a policy rate adopted today becomes the status-quo policy for the next meeting.

Furthermore, it is not entirely clear what is the appropriate assumption to be made about the way bank boards reach decisions. While in reality the chairman usually holds most of the proposal power, empirical evidence in Riboni and Ruge-Murcia (2010) suggests that the real-world features are better captured by what they call a consensus model in which the adopted policy is equal to the most preferred policy of the next-to-median member.

Riboni and Ruge-Murcia (2008a) try to model central bank decision-making taking into account its dynamic nature. They show that even in periods in which policy-makers' preferences do not differ, policy-makers may fail to reach a consensus and change the policy from the status quo, due to the possibility of future disagreement. However, it is not clear whether their model can support the information content of voting behaviour, despite the fact that it produces persistence and strong autocorrelation of policy rates.

Disregarding the dynamic nature of central bank policy-making, Gerlach-Kristen (2008) investigates the role of the MPC chairman in committee decision-making in a model that generates real-world-like dissenting frequencies. The possibility of dissent arising is due to the fact that individual policy-makers receive private information about the unobserved optimal interest rate. Differences in private information sets among the MPC members then give rise to different votes by the time the policy decision is made. In a similar vein, Farvaque, Matsueda, and Mon (2009) examines how different decision rules in monetary policy committees affect the volatility of interest rates.

The model in Weber (2010) then supports the basic intuition that the publication of voting records reveals the bank board's opinion heterogeneity and thus provides more information to the financial markets than the publication of the final decision only. Better informed financial markets are then able to better predict the central bank's future behaviour, providing a rationale for the publication of voting records.

Similarly, the empirical literature investigating the informative power of voting records is rather scant. This is mainly due to the fact that the practice of publishing the voting records of board members has been adopted relatively recently and several central banks make their voting records public only in the transcripts of their monetary policy meetings, published with a several-year lag.

For the MPC of the Bank of England, Gerlach-Kristen (2004) shows that for the period 1997-2002 the difference between the average voted-for and actually implemented policy rate is informative about changes in the policy rate in the future, a conclusion robust to the inclusion of different measures of market expectations. In a similar spirit and using the same measure of dissent in the MPC, Fujiki (2005) reaches a similar conclusion for the Bank of Japan, and Andersson, Dillen, and Sellin (2006) do likewise for the Riksbank.

The empirical literature trying to estimate the reaction functions of individual bank board members using information about their voting behaviour is closely related. In this case, information about the individual members' votes is used to predict their preferred policy rate given the state of the economy and hence to better forecast future monetary policy decisions. For the Federal Reserve, Chappell et al. (2005) estimate the individual reaction functions of FOMC members. For the Bank of England MPC, Bhattacharjee and Holly (2005, 2006), Brooks, Harris, and Spencer (2008), Besley, Meads, and Surico (2008) and Riboni and Ruge-Murcia (2008b) conduct a similar exercise.

The general conclusion emerging from these studies is that there is often significant evidence of heterogeneity among bank board members. In combination with the assumption that monetary policy is better conducted in an environment with no information asymmetry between the central bank and the markets, the publication of voting records revealing the heterogeneity of the bank board members is desirable.

3.3 A Model of Central Bank Board Decision-Making

In this section we introduce a theoretical model of the central bank board decision-making process and investigate under which conditions the voting pattern can be informative about future policy. The general objective is to fully specify the model, simulate the path of the decisions, recording the preferences of the individual committee members, and use those in a regression similar to our benchmark study Gerlach-Kristen (2004), which is also the starting point of our empirical analysis. In this regression, we test whether the *skew* indicator is informative about future interest rate changes.

Model setup

The model is set in an infinite horizon with discrete periods denoted by $t = 0, 1, \ldots$, in each of which the monetary policy committee or board takes a decision about the policy instrument with a policy adopted at t denoted by p_t. Although we call p_t the interest rate, it can stand for any standard monetary policy instrument.

There are N (N being an even number) 'normal' board members P (each referred to as 'he') and one proposer or chairman C (referred to as 'she'). Therefore, the committee size is odd. In each period t, decision-making is done by a standard majority rule with two alternatives pitched against each other. The first alternative is the current status-quo policy x_t, which is equal to the policy adopted at $t-1$, i.e. $x_t = p_{t-1}$. The second alternative is the policy proposed by the chairman, which we denote by y_t. The alternative that gains a majority of the votes then becomes the new policy p_t. For mathematical convenience we assume that a C who cannot propose anything better than x_t indeed proposes x_t (instead of proposing a policy that would be rejected for certain).

The committee tries to set policy p_t so as to match the uncertain 'state of the world' denoted by i^*_t, where for inflation-targeting central banks i^*_t can be interpreted as the interest rate that is compatible with achieving the inflation target over time. We assume that the per-period utility function of all committee members is quadratic around i^*_t and is given by $-(p_t - i^*_t)^2$. Note that even though the board members share an equal goal embedded in a common utility function, their behaviour can (and will) depend on their private information, which is not necessarily homogeneous.

We assume that the unobserved state of the world follows an $AR(1)$ process given by $i_t^* = \rho i_{t-1}^* + u_t$, where $\rho \in (0, 1)$, with u_t being an *i.i.d.* shock with distribution $N(0, \sigma_u^2)$. That is, the optimal monetary policy changes over time, with the current optimal interest rate being influenced by the previous-period optimal interest rate and eventually converging to some long-run value compatible with a stable state of the economy. With our interpretation of i_t^* as the optimal interest rate it might seem unrealistic to assume that it can attain negative values, but the whole model and all the results are invariant to adding a constant to the optimal interest rate. In Appendix 3.A1, we provide a robustness check to show that the $AR(1)$ assumption can be changed into $AR(2)$ without altering the conclusions.

To generate non-homogeneous votes among the committee members we assume that each member j has an imperfect signal i_t^j about i_t^* given by $i_t^j = i_t^* + v_t^j$, where the noise v_t^j is *i.i.d.* with distribution $N(0, \sigma_j^2)$. The assumption of non-homogeneous views of the individual committee members about the state of the economy is perfectly in line with the observed practice. Individual committee members often rely both on a staff forecast and on their privately formed views about which risks should be attached to the staff forecast and additional privately collected information about the state of the economy (Budd, 1998). It is assumed that for all Ps $\sigma_j = \sigma_P$ and that C has $\sigma_j = \sigma_C$. We assume that the chairman has the same or a higher capacity to collect private information compared to the other committee members and hence the same or a higher capacity to reduce noise. It follows that $\sigma_C \leq \sigma_P$.[4]

The proposal power of chairman along with heterogeneous preferences among the committee members generated by different signals implies that interpretation of our model fits best final stage of a typical monetary policy meeting. Common practice in many central banks is to start with a free format discussion of economic developments after which, typically the chairman, proposes policy which is then approved or rejected in a formal

[4] We could have generated heterogeneous preferences among the committee member by assuming fixed innate differences in their preferences. But with fixed pattern of heterogeneity, there is no reason why voting record should predict future decisions. On the other hand our assumption of private signals generating heterogeneous preferences can be alternatively viewed as an assumption of different innate preferences among the committee members, but one following stochastic pattern.

vote.

Next we make a strong assumption in order to make the model tractable. We assume that the whole committee learns the previous state of the world at the beginning of each period before making its next decision, i.e. i^*_{t-1} is known by the time the t-period decision is being made. The alternative to this assumption would be not to reveal i^*_{t-1} and have the board members use Kalman filtering to update their beliefs about the optimal interest rate. While this extension is possible, we think it would add no substantive insight while greatly complicating the analysis.

The timing of events in period t is as follows: i) the last-period state of the world i^*_{t-1} is revealed, ii) nature determines all the random variables in the model, hence setting i^*_t and all the signals of the board members, iii) the signals about the current state of the world i^j_ts are revealed to all the members and remain their private information, iv) C makes proposal y_t, v) voting takes place between y_t and the status quo (i.e. the last-period policy) $x_t = p_{t-1}$ and the winning alternative becomes the new policy p_t, and finally, vi) the players collect their utilities and the decision-making process moves to $t+1$.

We will focus on a Stationary Markov Perfect equilibrium in which strategies are measurable only with respect to payoff-relevant variables (histories) and do not depend on time (Maskin and Tirole, 2001). This allows us to drop the time subscripts and the notation becomes x for the status quo, y for the proposal, i^* for the previous-period optimal interest rate, and i^j for signals about the current optimal interest rate. The current optimal interest rate will be denoted by $\bar{i}^*$, with the bar denoting variables that will become known in the next period (the same applies to the other variables, i.e. $\bar{i}^j$ is the signal about the next-period optimal interest rate player j receives at the beginning of the next period). With this notation the $AR(1)$ process for the optimal interest rate becomes $\bar{i}^* = \rho i^* + \bar{u}$ and the signals are determined according to $i^j = \bar{i}^* + \bar{v}^j$. The information set of each player j is thus $I_j = \{i^*, i^j\}$.

C's strategy in this game is to offer the proposal, depending on information set variables and denoted by $y(x, I_C)$, that maximizes her expected utility. It will be a solution to

$$U_C(x, I_C) = \max_{y \in Y} \mathbb{E}_M \left[-(p(x,y) - \bar{i}^*)^2 + \delta U_C(p(x,y), \bar{i}^*, \bar{i}^C) | I_C \right] \tag{3.1}$$

where δ is a discount factor common to all board members and $p(x, y)$ denotes the policy adopted, depending on the status quo x and proposal y. Set Y is assumed to be a set of discrete values in which the interest rate can be set, i.e. Y is a set of integer multiples of some value $\bar{s}$. The notation for the expectation operator $\mathbb{E}_M[\cdot]$ captures the idea that C will calculate her expectations differently based on a model of the committee members' behaviour, which we specify below. Finally, $U_C(x, I_C) = U_C(x, i^*, i^C)$ is C's continuation value utility from a game starting with the status quo x, the last-period optimal interest rate i^* and a signal about the current optimal interest rate i^C.

The strategy of each P member j is a simple binary decision to vote for or reject C's proposal given the status quo x and all the remaining variables in information set I_j. We restrict our attention to stage-undominated strategies (Baron and Kalai, 1993) in which player j simply votes for an alternative providing higher expected utility. This avoids equilibria in which players vote for an alternative they do not prefer simply because their vote cannot change the final decision. Along with the assumption above, this implies that j, given the status quo x, C's proposal y and j's signal i^j, votes for y if and only if

$$\begin{aligned} &\mathbb{E}_M\left[-(y - \bar{i}^*)^2 + \delta U_j(y, \bar{i}^*, \bar{i}^j) | I_j\right] \geq \\ &\qquad\qquad \mathbb{E}_M\left[-(x - \bar{i}^*)^2 + \delta U_j(x, \bar{i}^*, \bar{i}^j) | I_j\right] \end{aligned} \tag{3.2}$$

where again $U_j(y, i^*, i^j)$ is the continuation value utility of player j from a game starting with the status quo y, with the previous-period optimal interest rate i^* and signal i^j. Notice that the voting rule specifies that an indifferent j votes for C's proposal. Hence, when C's offer y equals the current status quo x, pro-forma voting takes place within the committee and C's proposal is unanimously approved.

Committee members' behaviour

One way to proceed would be to assume full rationality on the part of all the committee members in the standard sense, solve for the model equilibrium (which would involve complicated expectation updating and signal extraction problems) and then simulate the path of the decisions for a random draw of model stochastic variables. However, the presence of information asymmetry among the board members, along with the infinite horizon framework, makes derivation of a full solution unfeasible.

Besides technical complexity, such a model does not capture the different modes or codes of conduct found among real-world central bank committees (see Blinder, 2004; Chappell et al., 2005, for a discussion) and the possible degrees of informational influence among their members. A purely rational model also implicitly assumes a lack of other motives on the part of central bank committee members, such as acknowledgement of the chairman's authority and better expertise or career concerns manifested by a willingness to adopt the chairman's opinion. In effect we view the fully rational model as an unrealistic description of reality.

For this reason we specify four different models of committee behaviour, for which we solve for equilibrium and then proceed with the simulations. The first three models, which we label as *democratic, consensual and opportunistic* based on C's behaviour, assume that the committee members do not take into account the impact of their actions on their future decisions. Formally, this is achieved by assuming $\delta = 0$.[5] By making this assumption we break the first intertemporal link in the committee decision-making mentioned above. Current policy still determines the future status quo, but the committee members do not take this fact into account. This assumption, for environments with $\sigma_C = \sigma_P$, implies that the policy proposal could in fact come from a different board member at each meeting, so that the role of the chairman is not institutional. When $\sigma_C < \sigma_P$, that is, when chairman C is better informed, her proposal power reflects her position as the best-informed board member. The fourth and last model, which we label *intertemporal democratic*, maintains the first intertemporal link but breaks the second one, i.e. it assumes that the optimal monetary policy is independent across periods. Formally, this is achieved by assuming $\rho = 0$ in the $AR(1)$ process determining the optimal monetary policy rate i^*. Below we describe the models, relegating the formal details to Appendix 3.A1.

Notice that the first three models, described below, embed different de-

[5] Assumption that policy-makers ignore effect of their current actions on their future decisions is common (Gerlach-Kristen, 2008; Riboni and Ruge-Murcia, 2008b, 2010; Weber, 2010) although it manifests through the $\delta = 0$ assumption only in the first paper. In Riboni and Ruge-Murcia (2008b) and in Weber (2010) there is no decision making between status-quo and proposed alternative. And in Riboni and Ruge-Murcia (2010) even though policy-makers are forward looking when determining their most preferred interest rate, their do not take into account effect of their vote on future status-quo.

grees of informational influence among chairman C and the remaining P members. In the democratic model there is little or no influence, as C is not influenced by the information that the P members have, and they are not influenced by C's proposal. In the consensual model, C is informationally independent, while the P members are influenced by her proposal. Finally, in the opportunistic model it is C who is influenced by the other P members by basing her proposal on their preferences and disregarding her own preference to a certain extent.

Democratic model

In this model of committee behaviour, chairman C plays the role of a democratic leader whose only special power is a proposal-making one. The proposal is based solely on C's own information set. The other committee members are free to express their own will by voting on her proposal, and C's behaviour has no effect on their own. In the language of our model, each P member j is assumed to vote based on the voting rule (3.2) using information set $I_j = \{i^*, i^j\}$ and extracting no information content from C's proposal. Given this behaviour, C solves her optimization problem (3.1) using information $I_C = \{i^*, i^C\}$ and forming her expectation in a standard rational manner, i.e. $\mathbb{E}_M[\cdot] = \mathbb{E}[\cdot]$, where $\mathbb{E}[\cdot]$ is a standard expectation operator. Notice that this does not mean C offers her expected optimal policy rate $\mathbb{E}[\,\bar{i}^*|i^*, i^C]$ given her information set; she offers her proposal y taking into account the fact that its eventual acceptance (as opposed to the acceptance of the status quo x) reveals information about the unobserved $\bar{i}^*$.

Consensual model

In this model, chairman C is assumed to have a dominant position beyond her proposal-making power. Her dominant position makes the other P members too keen to adopt her point of view, since they assume that the information available to the chairman is superior. In the language of our model, C's proposal is a solution to (3.1) given information $I_C = \{i^*, i^C\}$, but with the expectation operator $\mathbb{E}_M[\cdot]$ not taking into account the fact that possible rejection or acceptance of y contains information about unknown $\bar{i}^*$. In other words, C's proposal is the policy in Y closest to C's expectation of $\bar{i}^*$, i.e. closest to $\mathbb{E}[\,\bar{i}^*|i^*, i^C]$. While not fully rational, this specification of the way in which C forms her expectations captures the notion that because she knows that the other committee members' voting behaviour is strongly influenced by her own proposal she disregards the possible information content of that behaviour and proposes her optimal policy.

To capture the notion that the P members adopt C's point of view, we assume that each P member j votes based on voting rule (3.2), but when calculating the expected value of $\bar{i}^*$, j extracts information from C's proposal. It is easy to see that the expectation can be written as $\mathbb{E}[\,\bar{i}^*|i^*, i^j, i^C \in \langle i_l^C, i_u^C\rangle]$, where i_l^C and i_u^C are, respectively, the lower and upper bounds on C's signal, as revealed by her proposal. We have decided to label this model consensual, since the extraction of information from C's proposal considerably reduces the level of heterogeneity of opinions within the committee.

Opportunistic model

In this model, we assume that C is opportunistic in consulting the other P members before the actual committee meeting. Once at the meeting, C then knows the most preferred policies of the other members and offers the policy she knows will be adopted by a supermajority of $\frac{N}{2}+2$ of them. In the appendix, we provide a robustness check for a mere majority case to illustrate that this assumption is not binding for our results. In terms of our model, we assume that C knows the most preferred policy of each member j, which is the policy in Y closest to $\mathbb{E}[\,\bar{i}^*|i^*, i^j]$. Ordering those policies such that $y_1 \leq \ldots \leq y_m \leq \ldots \leq y_{N+1}$, where y_m is the policy preferred by the median committee member, offering the policy adopted by a supermajority of $\frac{N}{2}+2$ amounts to, for the $x \leq y_m$ case, offering y_{m-1} if $x \leq y_{m-1}$ and offering x if $x \geq y_{m-1}$. The $x \geq y_m$ case is analogous. An implicit assumption about the behaviour of each P member j is that his voting is given by voting rule (3.2) with the expectation computed using information set $I_j = \{i^*, i^j\}$ and ignoring the information content of C's proposal.

This model is inspired by Riboni and Ruge-Murcia (2010), who in their empirical investigation of several descriptive monetary policy committee decision-making models show that their 'consensual' model fits the real world data best. In their model, the adopted policy is equal to the most preferred policy of the next-to-median member (the side depending on the position of the status quo) when this policy is sufficiently far away from the status quo. When this policy is close to the status quo, the adopted policy is indeed the status quo. This is what our opportunistic model does, except that we label it differently, as in our model it captures the idea that the chairman's objective is to offer a policy which would never be rejected and she achieves this by using her authority to consult individual committee members or, in an alternative interpretation, to speak last during the committee discussion, after the remaining members have expressed their preferred policies.

The results from the experiments with the opportunistic model in which C offers the policy accepted by a mere majority $\frac{N}{2}+1$ of the members are reported only for completeness in Appendix 3.A1 (Tables 3.15 and 3.16). They are largely similar to the results of the opportunistic model presented above. Notice that a mere majority model is often used in the literature, as the accepted policy is equal to the policy most preferred by the median committee member.

The three models just explained are also related to some of the models found in the existing literature. As already noted, our opportunistic model is similar to the 'consensual' model of Riboni and Ruge-Murcia (2010). Our simple majority version of the opportunistic model mentioned above is similar to the 'frictionless' model in Riboni and Ruge-Murcia (2010), to the 'individualistic' model in Gerlach-Kristen (2008) and to the model in Weber (2010). In all those models, the adopted policy is equal to the policy preferred by the median committee member. Furthermore, our democratic model is similar to the 'agenda-setting' model of Riboni and Ruge-Murcia (2010) in that the chairman proposes the policy that maximizes her expected utility among the policies she knows would be accepted. The key difference in our democratic model is that the acceptance is only probabilistic, as C does not know the signals of the other committee members. Finally, our consensual model is similar to the 'autocratically collegial' model in Gerlach-Kristen (2008) in that chairman proposes her most preferred policy and her authority makes the other committee members vote for her proposal. In the autocratically collegial model this is modelled as the other committee members having a 'tolerance interval' around their preferred policy, but in our model it is modelled as the other members considering the chairman's point of view by extracting information from her signal.

Intertemporal democratic model

The fourth model is similar to the democratic model specified above in that each P committee member votes based on his private information only and does not extract any information from C's proposal, with C solving her optimization problem in a fully rational manner. As opposed to the democratic model, this model maintains the intertemporal link in the committee decision by assuming that all the committee members take into account the effect of their current behaviour on their future decisions. This effect works through current policy determining the future status quo. Formally, this is achieved by setting $\delta > 0$.

A key problematic aspect in simulating the equilibrium of this model is the fact that C's proposal strategy maps $\mathbb{R}^2 \times Y$ into Y, and we would have to estimate the value functions $U_C(\cdot)$ at each point of this space. With standard value function iteration on the discrete version of $\mathbb{R}^2 \times Y$ the computational costs are prohibitive. To overcome this complication we set $\rho = 0$, breaking the intertemporal link in the optimal monetary policy. As a result, C's proposal strategies will be a function of the current status quo x along with her signal i^C mapping $\mathbb{R} \times Y$ into Y, which is considerably easier to simulate. We still have to derive the equilibrium value function $U_j(\cdot)$ for all the board members, but we only need to know $U_j(\cdot)$ at a discrete and rather coarse set of points sufficient for numerical integration over $\mathbb{R}$, as the Y set is already discrete.

Model simulations

For each version of the model of committee behaviour we generate 101 different random 100-period-long paths. These are chosen so as to gain insights into the results and avoid inference based either on a low number or on short paths while still keeping the simulations manageable. With the simulation of one path in the (intertemporal) democratic model taking approximately one hour for $N = 4$ on a standard desktop computer (twice as much for $N = 6$) we see the choice of the number and length of paths as an appropriate trade-off between validity and manageability (simulations of the other models take considerably less time, while simulations of the intertemporal democratic model require an additional several days for estimation of the continuation value function).[6]

Along each path for every period we record the status quo x_t, the proposal y_t and the final policy p_t and calculate the $skew_t$ variable as defined in the introductory part. It is given by

$$skew_t = \frac{\#(\text{voting for } y_t) \cdot y_t + \#(\text{voting for } x_t) \cdot x_t}{N+1} - p_t \tag{3.3}$$

and allows us to run an ordered probit regression analogous to the one from the benchmark study Gerlach-Kristen (2004), which we use later in our empirical part, in which the estimate of a_1 shows the informative power of the $skew$ variable for future policy changes

$$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}. \tag{3.4}$$

In order to make the results more comparable among the different models, we keep the values of the random variables fixed across the simulations of those models. That is, when simulating, say, the first path in the democratic model, the random values in the model are the same as when simulating the first path in the consensual, opportunistic or democratic intertemporal model.

Following the discussion above, the simulation values of the parameters in the models are $\rho = 0.95$ and $\delta = 0$ for the democratic, consensual and opportunistic models (however, see the simulation robustness checks in Appendix 3.A1 for the results with different values of ρ) and $\rho = 0$ with $\delta = 0.95$ for the intertemporal democratic model. In all the models, we assume that the interest rate is set in steps of a quarter of a percentage point, that is, in all the models $\bar{s} = 0.25$.

Next, we need to specify values for the distributions of random shocks. The choice of σ_u is driven by our attempt to match the standard deviation of the changes in the monetary policy rate in our empirical data. As p_t in our model eventually follows a similar process as i_t^*, Δp_t will follow a similar process as Δi_t^*. With the standard deviation of Δi_t^* equal to $\sqrt{2/(1+\rho)}\sigma_u$ and the empirically observed standard deviation of changes in the monetary

[6] We also tested stability of our results across sub-samples of the 101 paths with satisfactory results. For example, if we split the 101 paths into two halves, out of 292 p-values reported for the simulation exercise, only 13 cross the 10 percent significance level in either of the two halves, relative to the results reported. Full results are available upon request.

policy rate between 0.25 and 0.5, we set σ_u to those two values.

For the standard deviation of the board members' signals σ_P and σ_C, we assume those to be either 0.25 or 0.5, implying that approximately 70% of the board members' signals are within 25 or 50 basis points of the optimal interest rate.

From the values above we construct several scenarios. Our baseline scenario assumes $\sigma_u = 0.25$, $\sigma_C = 0.25$ and $\sigma_P = 0.25$. Interested in the comparative static properties, we further take $\sigma_u = 0.5$, $\sigma_C = 0.25$ and $\sigma_P = 0.25$ in a 'high volatility' scenario, $\sigma_u = 0.25$, $\sigma_C = 0.5$ and $\sigma_P = 0.5$ in a 'bad information' scenario, and finally $\sigma_u = 0.25$, $\sigma_C = 0.25$ and $\sigma_P = 0.5$ in a 'P bad information' scenario. Note that we could call this 'P bad information' scenario also 'C superior information' scenario, since we consider the relative noise in the C and P information sets. For the four scenarios just explained, we simulate the models for both $N = 4$ and $N = 6$ in order to see the effect of increasing committee size on the results. We have chosen committee sizes of 5 and 7, as those are the most common central bank monetary policy committee sizes (Mahadeva and Sterne, 2000).

Simulation results

Tables 3.1-3.3 show the results of our simulation exercise. Besides estimates of coefficients a_1 and a_2 from (3.4) averaged over the 101 paths, we include average standard errors and average p-values. The row labelled *MSE* is the average mean squared error between the enacted and optimal monetary policy, *Votes proposal* is the average number of votes for C's proposal, and *No change* is the average fraction of meetings resulting in no change in policy. Tables 3.1 and 3.2 show the results for the democratic, consensual, opportunistic and mechanical (see below) models for $N = 4$ and $N = 6$ respectively. Table 3.3 shows the results for the intertemporal democratic model.

Before proceeding to the discussion of our results, we were interested to see whether we could generate the informative power of *skew* with a purely mechanical model. In this model, policy p in each period is equal to the policy in Y closest to the optimal policy $\bar{i}^*$ and we calculate *skew* assuming that there are $d \in \{1, \ldots, N/2\}$ dissenting members voting for the status quo x. We take d to be a random variable drawn anew for each committee meeting, with each value from $\{1, \ldots, N/2\}$ being equally likely.

What is apparent from this mechanical model is that it cannot generate data in which *skew* holds information about future monetary policy changes. This is because there is no uncertainty about the optimal policy in this model, and there is nearly no difficulty in deciding where to set interest rates (the only difficulty being the fixed size of the minimum policy rate change). What the row MSE also shows is the benchmark or minimum error in monetary policy stemming from the fact that the monetary policy rate is set in discrete steps.

Looking at the democratic model results for the baseline scenario and $N = 4$ in Table 3.1, the average estimate of a_1 shows the informative power of the *skew* variable for future policy changes. The intuition for this result is the following. Assume that the optimal policy rate i^* has been constant for several periods at some value i_1^* and that the committee has been setting its policy p_1 at the same level. Assume now that the optimal policy rate increases to some value i_2^*. The committee members receive imperfect information about this shock and several courses of action follow. If C's signal does not prompt her to offer a policy different from the current status quo p_1, the new policy p_2 will be equal to the current status quo and hence the *skew* variable will be equal to zero.

If, on the other hand, C offers proposal y_2 close to the new optimal policy rate i_2^*, her proposal will be higher than the current status quo p_1. Depending on the votes of the other committee members, two possibilities arise. The first one is that C's proposal is approved. The new policy p_2 will then be approximately equal to the optimal rate i_2^* and the *skew* variable will be negative. But due to the fact that the optimal policy rate is an $AR(1)$ process with relatively large ρ, it is approximately equally likely that the optimal rate will increase or decrease in the future. With monetary policy eventually following the optimal rate, it is then equally likely that the policy will increase or decrease in the future. The second possibility is that C's proposal is rejected. The new policy p_2 will then be equal to the status quo p_1 and the *skew* variable will be positive. It is also more likely than not that the interest rate will increase in the future if it follows the optimal rate. The combination of an equal probability of increase and decrease in policy when $skew < 0$ and a higher probability of increase when $skew > 0$ is what gives the positive estimate of a_1 (see also Figure 3.1 below and the surrounding text).

The intuition just explained also reveals two conditions under which *skew* holds information about future policy changes. The first condition is that monetary policy cannot follow the optimal rate precisely. This is apparent from the estimates for the mechanical model. The second condition is that there has to be a certain minimum degree of dissent in the committee. If all the committee members vote in the same way, the *skew* variable will always be zero and hence cannot be informative about future policy changes. This is revealed by the estimates for the consensual and opportunistic models. In both of those models, information is shared among the committee members and hence their decision-making shows a low degree of dissent. This is also apparent from the high average votes for the proposal, which for both models is around 4.5 in a five-member committee.

Nevertheless, the two conditions just explained are not enough for *skew* to be informative about future policy changes. Inspecting the first column of Table 3.1 for the democratic model across the different scenarios, the informative power of *skew* can disappear either in a volatile economic environment (the high volatility scenario) or in an environment in which central bankers possess imprecise information (the bad information scenario). Comparing the results for the bad information and P bad information scenarios then suggests that it is the precision in C's signal that is important for the informative power of *skew*.

As already noted, the results for the other two models in Table 3.1 - the consensual and opportunistic ones - do not show any informational content in the *skew* variable, despite the fact that some of the estimates for the consensual model come close to statistical significance on average. This holds despite the fact that the policy in these models is on average further away from its optimum than in the democratic model, or, in other words, the first condition for *skew* to be informative explained above holds. What both of these models lack is the second condition - independence in the behaviour of the committee members.

We have already mentioned that high volatility of the economic environment or a lot of noise in the information of committee members can render *skew* uninformative about future monetary policy changes even in the democratic model. However, turning our attention to Table 3.2, it is apparent that both effects can be overcome by increasing the committee size. The estimates of a_1 for the democratic model now become significant on average even in the high volatility and bad information scenarios. At the same time, an increase in the committee size does not change the insignificance of the estimates of a_1 in the consensual and opportunistic models despite the fact that the average p-values increase for both models and all scenarios.

Finally, with one exception the average estimates of a_2 are not significant in Tables 3.1 and 3.2, suggesting that past changes in the interest rate do not predict future change in the interest rate in our model, despite the fact that some of the estimates for the opportunistic models, and for two scenarios also for the democratic model, come close to statistical significance. As further discussed in the empirical part, a significant estimate of a_2 suggests an interest rate smoothing motive on the part of the monetary policy committee. It is then not surprising that the estimates are not significant, as the interest rate smoothing motive is not built into any of the theoretical models. An alternative explanation of the lagged policy change insignificance is that it is driven by the $AR(1)$ assumption for the optimal policy rate. This is what the results of our simulation robustness checks suggest, as the lagged policy change becomes significant when the $AR(1)$ assumption is changed to $AR(2)$. Whether, both in theory and in reality, the significance of the lagged policy change is driven by the smoothing motive or by the structure of the underlying economic environment is beyond the scope of this study.

Table 3.1: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data with $N = 4$ and $\rho = 0.95$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	4.10 *	5.93	5.25	0.60
	[1.63] (0.089)	[3.29] (0.175)	[6.40] (0.418)	[3.99] (0.435)
Lagged policy	0.75	0.07	1.63	-0.14
change (a_2)	[0.53] (0.259)	[0.45] (0.520)	[0.74] (0.108)	[1.26] (0.451)
MSE	0.027	0.033	0.033	0.005
Votes proposal	2.92	4.64	4.83	—
No change	0.43	0.41	0.59	0.36
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.19	2.59	0.69	0.21
	[1.04] (0.124)	[2.43] (0.364)	[3.65] (0.585)	[2.05] (0.444)
Lagged policy	0.24	0.01	0.60	0.00
change (a_2)	[0.24] (0.362)	[0.21] (0.515)	[0.29] (0.112)	[0.64] (0.462)
MSE	0.043	0.049	0.044	0.005
Votes proposal	3.46	4.62	4.78	—
No change	0.28	0.24	0.37	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.43	6.84	6.05	—
	[1.50] (0.106)	[3.59] (0.148)	[6.47] (0.385)	—
Lagged policy	0.29	0.08	1.04	—
change (a_2)	[0.48] (0.435)	[0.46] (0.507)	[0.64] (0.203)	—
MSE	0.048	0.052	0.053	—
Votes proposal	3.00	4.69	4.85	—
No change	0.43	0.41	0.56	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	4.97 *	9.98	6.53	—
	[1.79] (0.055)	[6.24] (0.228)	[6.43] (0.356)	—
Lagged policy	0.82	-0.19	1.25	—
change (a_2)	[0.54] (0.228)	[0.42] (0.490)	[0.67] (0.128)	—
MSE	0.041	0.036	0.049	—
Votes proposal	2.74	4.88	4.84	—
No change	0.50	0.38	0.57	—

Note: Average ordered probit estimates over 101 random 100-period-long paths. [Average standard errors] and (average p-value). * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level based on average p-value. MSE is average mean squared difference between adopted and optimal policy. Votes proposal is average number of votes for chairman's proposal. No change is proportion of committee meetings with no policy change.

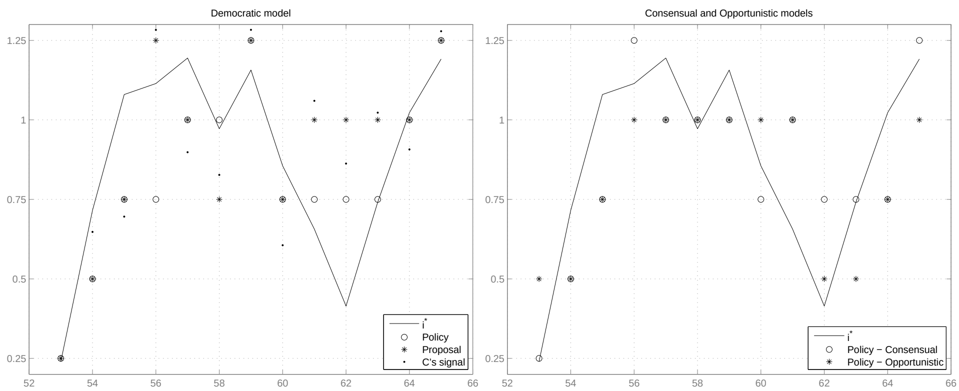
Table 3.2: Does the Voting Record Predict Policy Rate Changes? Estimates Using Simulated Data with $N = 6$ and $\rho = 0.95$ $\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	5.15 **	6.57	7.41	0.01
	[1.66] (0.025)	[3.30] (0.156)	[5.30] (0.256)	[3.45] (0.510)
Lagged policy	1.08	0.14	2.05 *	-0.30
change (a_2)	[0.56] (0.125)	[0.46] (0.490)	[0.81] (0.065)	[1.06] (0.541)
MSE	0.026	0.032	0.030	0.005
Votes proposal	3.90	6.46	6.62	—
No change	0.45	0.42	0.57	0.36
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.37 *	3.01	1.69	-0.03
	[1.02] (0.085)	[2.43] (0.328)	[2.99] (0.449)	[1.76] (0.521)
Lagged policy	0.28	0.02	0.63	-0.07
change (a_2)	[0.24] (0.311)	[0.21] (0.499)	[0.30] (0.115)	[0.54] (0.551)
MSE	0.040	0.049	0.036	0.005
Votes proposal	4.66	6.42	6.52	—
No change	0.31	0.24	0.33	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.57 *	7.60	7.44	—
	[1.47] (0.079)	[3.67] (0.140)	[5.09] (0.261)	—
Lagged policy	0.40	0.15	1.16	—
change (a_2)	[0.49] (0.421)	[0.47] (0.460)	[0.65] (0.184)	—
MSE	0.047	0.052	0.051	—
Votes proposal	4.04	6.54	6.67	—
No change	0.44	0.41	0.52	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	5.31 **	12.42	7.86	—
	[1.71] (0.032)	[6.61] (0.174)	[5.08] (0.231)	—
Lagged policy	1.00	-0.13	1.36	—
change (a_2)	[0.55] (0.147)	[0.42] (0.517)	[0.68] (0.128)	—
MSE	0.041	0.036	0.048	—
Votes proposal	3.66	6.82	6.66	—
No change	0.51	0.37	0.54	—

Note: See Table 3.1.

To provide further intuition behind our results, Figure 3.1 shows a fraction of a typical simulated policy path for the three models from Tables 3.1 and 3.2. The solid line in both figures is the optimal monetary policy rate unknown to the central bank committee. The left figure then shows enacted policy in the democratic model along with C's proposals, and the right figure shows enacted policy in the consensual and opportunistic models. We do not show the proposals for the two latter models, as they are always accepted in the opportunistic model and very often accepted in the consensual model (always accepted for the particular policies shown). We choose this particular path as it produces the estimates closest to the average estimates shown for the democratic model in Table 3.1 for the baseline scenario.

Figure 3.1: Simulated Policy Paths



Focusing first on the left figure shows why the *skew* variable is informative about future changes of monetary policy in the democratic model. For periods 53-55 the enacted policy closely follows the optimal one, but then in period 56 the committee fails to increase the policy further to C's proposal of 1.25 because the proposed step seems too large to the other committee members. This generates a positive value for *skew* in this period and suggests an increase of policy to 1.00 in period 57. The right figure then shows why the *skew* variable is not informative about future policy changes in the consensual and opportunistic models. In the consensual model chairman C gets her proposal of 1.25 in period 56 approved, as her proposal reveals her high signal to the other committee members, who are influenced by it, so that the policy does not need to 'catch up' in the period 57. In the opportunistic model a similar thing happens, but with a policy of 1.00 adopted instead.

Intuitively, it might seem that the democratic model generates an informative value of *skew* due to the failure of the committee to adopt higher policy in the period 56. While this is certainly true, notice that the other two models err in different situations. The consensual model errs in that C's proposals are too often accepted and hence the enacted policy reflects too much noise in C's signals. This is evident from C's failure to propose higher policy in period 59 or the eventually accepted proposal for higher policy in period 61. The democratic model, on the other hand, guards against strong influence of the chairman, as evident by the rejection of the proposals in periods 58 and 61-63, all of which would have taken the policy further away from the optimal one.

The opportunistic model errs in that it takes too long to form the supermajority of the committee needed to change the policy. This is evident from the no policy change in period 59 and then the maintenance of policy at the 1.00 level until period 61 before changing it to 0.50 in period 62, with a smoother transition being more appropriate.

What the figure also shows is that both the democratic and the consensual models generate policy paths that are somewhat more volatile than the policy path generated by the opportunistic model. From Tables 3.1 and 3.2, the democratic and consensual models on average, excluding the high volatility scenario, generate somewhere between 40 and 50 per cent of no policy change meetings, while the opportunistic model generates somewhere between 50 and 60 per cent of no policy change meetings. However, with the fraction of no policy change meetings in our data being 52% for the USA, 61% for Poland, 62% for Sweden, 65% for Hungary, 66% for the Czech Republic and 69% for the United Kingdom, this does not seem to be significant weakness of either of the two models.

What the figure does not show, however, is the source of the no policy change meetings. As already noted in both consensual and opportunistic models, C's proposal is often accepted, implying that the source of no changes in policy is C's proposal being equal to the status quo. This, along with the voting behaviour, implies a high percentage (equal to the fraction of no change meetings for the opportunistic model and very close to the fraction of no change meetings for the consensual model in Tables 3.1 and 3.2) of meetings with no change in the policy rate with the decision being reached unanimously. On the other hand, in the democratic model C's proposal is almost never equal to the status quo policy and hence almost all the no change meetings are a result of C's proposal being rejected. As at least she votes for her proposal, none of the no change meetings reach this decision unanimously, which more closely resembles the empirically observed stylized facts.

Source of meetings with no policy change in the three models also reveals another intuition for the explanatory power of the *skew* variable. Any meeting with unanimous decision generates zero *skew* and as a result consensual and democratic models are associated with *skew* variable with less variance relative to the democratic model.

To check how robust our simulation results are, we repeated the simulations for the democratic, consensual, opportunistic and mechanical models either for different values of ρ compared to the benchmark results or changing the $AR(1)$ process to an $AR(2)$ process. The results are given in Tables 3.9-3.14 in Appendix 3.A1. To summarize, the results change very little when we change $\rho = 0.95$ from the benchmark results to either $\rho = 0.90$ (Tables 3.9 and 3.10) or $\rho = 0.99$ (Tables 3.11 and 3.12). When we change the benchmark $AR(1)$ process to an $AR(2)$ process (Tables 3.13 and 3.14), the most notable change is that the average estimate of the lagged policy change becomes significant in most cases. Nevertheless, *skew* still is informative about future policy changes only in the democratic model.

Table 3.3: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Intertemporal Democratic	
	$N = 4$	$N = 6$
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)		
Skew (a_1)	1.98	2.36
	[1.64] (0.338)	[1.66] (0.271)
Lagged policy	-2.31 ***	-2.23 ***
change (a_2)	[0.56] (0.002)	[0.58] (0.008)
MSE	0.028	0.027
Votes proposal	2.89	3.89

Note: See Table 3.1.

Before we conclude the theoretical section, we turn our attention to the results for the intertemporal democratic model. Table 3.3 shows the simulation results for this model and the baseline scenario for both $N = 4$

and $N = 6$. We decided not to include more results, as those come with considerable time costs and even the estimates for the baseline scenario show the main weakness of this model, which is a negative estimate of a_2. Intuitively, this result is driven by the fact that $\rho = 0$. When the optimal rate increases to some value and monetary policy follows it, giving a positive policy change, it is highly likely that in the next period monetary policy will have to be reversed, as the optimal rate is normally distributed around zero for $\rho = 0$. Additionally, the average estimate of a_1 in Table 3.3 is not significant, showing that breaking the intertemporal link in the optimal interest rate renders the *skew* variable uninformative about future monetary policy.

Overall, the model delivers several interesting policy implications. First, publishing the voting pattern of the monetary policy committee members is important if monetary policy is not always at its optimal level. This allows other economic agents to gain information about the future course of monetary policy in the form of the *skew* variable.

Second, the informative power of the *skew* variable is not guaranteed automatically. What is needed is informational independence of the committee members. If all the committee members behave based on the same information or one of the committee members has enough authority for the other committee members to adopt his or her point of view, a high degree of consensus ensues and the *skew* variable is rarely different from zero.

Third, even with independently behaving central bankers, the *skew* variable might not be informative. In a volatile economic environment, or when the monetary policy committee members possess imprecise information, it is important for the committee to have a sufficient number of members, as every additional committee member brings new information.

3.4 Institutional Background

This section gives information on the background of central bank committees' decision-making about monetary policy. The bank boards typically meet on a monthly frequency and decide on the level of the repo rate. The frequency of monetary policy meetings varies. For example, the Bank of England and the Hungarian and Polish central banks meet monthly. The Czech National Bank used to meet monthly up to 2007 but has met eight times a year since 2008, the same as the U.S. Fed and Riksbank for the large part of our sample period. Occasionally, the central banks hold extraordinary policy meetings.

The boards take decisions based on a majority vote. In the event of a tie, the chairperson (the governor, if present at the meeting) has the casting vote. The policy decision is announced on the same day. Minutes explaining the monetary policy decision, i.e. the voting of central bankers, are published approximately one or two weeks later. Except for Poland, the voting record is an integral part of the minutes and summarizes the qualitative information contained in the minutes. In the case of Poland, the voting record appears no sooner than 6 weeks (and no later than 12 weeks) after the policy meeting.[7] In the U.S. case, we use the data for 1970-1996 (Burns and Greenspan chairmanships) collected and coded by Chappell et al. (2005). The voting records for the U.S. are primarily based on transcripts that are published several years later. Appendix 3.A2 contains further details on the U.S. data. Both U.S. and Polish case studies document that the informative power of the voting records does not depend on the ex ante known publishing time lag. An in-depth study on voting records in Poland is provided by Sirchenko (2010).

The voting results are typically attributed, but not always. For example, the voting ratio was released without an explicit statement on how the individual board members voted for the monetary policy decisions in the Czech Republic in 2000-2007. From mid-2000 to January 2006 the (unattributed) voting record was published in the minutes only, while since February 2006 the voting record has been released at the press conference held about 3 hours after the announcement of the interest rate decision. In addition, the Czech National Bank has recently published the transcripts of its monetary policy meetings in 1998-2001, which include the voting record as well. Hence, the Czech case offers us a second natural experiment set-up in which we can test whether the voting ratio has a similar informative power to the full voting record. The results show that this is the case. The lesson learnt from the Czech case is therefore to publish at least the voting ratio if there are serious concerns about naming names.

[7] More specifically, if the repo rate was changed, the voting record is first published in the Court and Economic Gazette of the Ministry of Justice and only after that in the inflation report. Voting records have to be published in the Court and Economic Gazette no sooner than 6 weeks and no later than 12 weeks after the voting took place.

Disagreement among central bankers is common. The voting was not unanimous in 46% of cases for the Czech central bank, 70% for the Hungarian central bank, 46% for the Polish central bank, 19% for the Swedish central bank and 59% for the Bank of England during our sample period. The frequency of unanimous voting depends to a certain extent on the size of the bank board, with Hungary having more than 10 members in the board during our sample. The typical magnitude of monetary policy rate change is 25 basis points. Other magnitudes are less common, although central banks decreased policy rates quite aggressively during the recent financial crisis, often by 50 or even 100 basis points at the meeting. Substantial policy rate changes of similar magnitude were also observed in the Czech Republic, Hungary and Poland during the period of transition to a market economy, which was characterized by more volatile macroeconomic development. The data are further described in Appendices 3.A2 and 3.A3.

Figure 3.2: Actual Voting Record Skew and Future Policy Rate Change

Note: *skew*, plotted on the x-axis, is calculated as the difference between the average repo rate voted for by the individual board members and the actual repo rate at the next meeting. The future monetary policy rate change is plotted on the y-axis. Jitter is used for overlapping observations for expositional purposes.

Figure 3.2 presents the link between the actual voting record *skew* and the future policy rate change. In all countries, the link seems to be positive, although there are cases where *skew* can give a noisy signal about future policy, for example when the rates are not changed and one board member dissents. When we look at the various signal-to-noise ratios, we see that there is a certain level of noise in an individual member's voting record, but when more than one member dissents at the same policy meeting, the level of noise declines and is typically well above 50%.[8] We perform a regression analysis in the following section to shed light on the extent to which the voting record gives systematic information for future policy. For the regression analysis, the future policy rate change is stacked in fewer categories, as large-magnitude policy changes happen rarely (more on this below).

3.5 Empirical Methodology

Our theoretical model shows when the voting record is likely to be informative for future policy changes. As regards the empirical methodology we follow the approach developed by Gerlach-Kristen (2004) to assess the predictions of our model. Gerlach-Kristen (2004) analyses the voting record of the MPC of the Bank of England over the period 1997-2002, while we provide a more comprehensive international comparison. More specifically, we focus on the following five countries that conduct their policies within an inflation-targeting regime: the Czech Republic, the United Kingdom, Hungary, Poland and Sweden. For comparison, we estimate similar models for the U.S.

Following our benchmark study Gerlach-Kristen (2004), we define a measure of disagreement in the bank board, the variable *skew*, as

$$skew_t = average(i_{j,t}) - i_t \tag{3.5}$$

[8] More specifically, we calculate the signal-to-noise ratio as follows. When at least 25% of board members dissent - for example at least two members out of seven vote for higher rates - at a particular meeting and the rates are not changed, we classify the *skew* variable as giving the correct signal when the rates are increased at the next policy meeting. Calculating the signal-to-noise ratio in this way, the ratio is 71% for the Czech Republic, 67% for Hungary, 64% for Poland, 80% for Sweden and 54% for both the UK and the USA. The ratio is above 50%, indicating that the voting record gives more often a correct, rather than noisy, signal.

where $i_{j,t}$ is the interest rate voted for by bank board member j at a monetary policy meeting at time t, and i_t denotes the monetary policy rate. This is an identical definition to equation (3.3) used in our theoretical models. However, for the sake of comparability with the benchmark study, we use here the benchmark notation for the policy interest rate i_t, while in the theoretical models we kept the notation typical for that stream of literature, where the policy tool is denoted p_t. We follow the benchmark study and assess whether the voting record reveals information on future monetary policy by estimating the following baseline regression model for each country separately.

$$\Delta i_{t+1} = a_0 + a_1 skew_{\tau(t)} + a_2 \Delta i_t + u_{t+1} \tag{3.6}$$

This equation is identical to equation (3.4) used in the theoretical part. Again, for the sake of comparability, we altered the notation for the policy interest rate. It is assumed in (3.6) that the interest rate decision is taken at time t. The votes are released at time $\tau(t)$, i.e. in the period between the interest rate decisions at t and $t+1$ (often together with the minutes, typically about two weeks after the interest rate decision at t; it is worth emphasizing that we focus on the voting record, as this is the only quantitative information in the minutes; alternatively, one would have to classify the qualitative information contained in the minutes). Analogously to the theoretical models, we estimate (3.6) by an ordered probit technique to reflect the discrete nature of monetary policy rate changes. It is important to emphasize that the discrete dependent variable has been stacked in fewer categories, as some policy change magnitudes, such as 75 basis points, happened rarely. Therefore, the dependent variable was coded in four to five categories depending on the country and defined as follows: large decrease, decrease, no change, hike and large hike (-50, -25, 0, +25 and +50 basis point changes respectively).[9]

According to our theoretical model, the coefficients a_1 and a_2 are expected to take positive values. As regards the sign of a_1, if some bank board

[9] The number of categories is set according to the log-likelihood of competing models. An alternative way would be to test whether the thresholds estimated within the ordered probit model differ significantly from each other. Note that the coding of the dependent variable substantially lowers the potential impact of vertical outliers. As concerns the potential impact of horizontal outliers, we estimate the regressions based on various subsamples, with the results being affected minimally.

members favour higher rates, *skew* is positive and a future interest rate hike is more likely, conditional upon the voting record being informative for future policy. As regards the coefficient a_2, it reflects interest rate smoothing and the attempt of central bankers to avoid sudden policy reversals. If a_1 is significant, we can infer that the voting record improves the explanatory power of a 'naive' model which assumes only smoothing and reactions to shocks. We can also infer that the conditions identified by our theoretical model have been fulfilled and that the voting mechanism has been democratic.

Our second baseline model extends this naive model by considering the information set available to the financial markets. We approximate their information set from the yield curve. While the naive model is directly comparable to the outcomes from our theoretical models, the second baseline model should be viewed as its extension. In this extension, we can test whether the information set available to the financial markets contains all the information sets available to the individual committee members. If the financial markets have an identical information set and evaluate the information at least as effectively as the central bank, the information content of the *skew* indicator should be built into the slope of the term structure of interest rates. In that case, parameter b_1 would be insignificant in our second baseline model (as would b_2 if interest rate smoothing is fully priced into the term structure). In the opposite case, the voting record reveals additional information to the financial markets. Our theoretical models also suggest other situations when *skew* could be insignificant. Specifically, in periods of high volatility or under certain voting mechanisms the *skew* may be insignificant despite the fact that individual board members have valuable information sets. To assess these considerations formally, we estimate a regression of the following form:

$$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3 (i_{\chi(t),L} - i_{\chi(t),S}) + u_{t+1}. \qquad (3.7)$$

As compared to (3.6), equation (3.7) now includes an additional term to control for financial market expectations. $i_{\chi(t),L} - i_{\chi(t),S}$ represents the slope of the term structure, where L and S denote the respective money market maturities[10] and it is assumed that $L > S$ (following Gerlach-Kristen, 2004,

[10] An alternative would be to include interest rate futures or forwards, but these were

we will consider various maturities). $\chi(t)$ denotes the time period between the interest rate decisions, and the data on $i_{\chi(t),L}$ and $i_{\chi(t),S}$ will be from the day before the release of the voting record (thus, $\chi(t) < \tau(t)$).

Regarding our two natural experiment set-ups, we can test whether *skew* is informative in the period when voting records are disclosed with a considerable time lag, as in the aforementioned cases of Poland and the U.S. We can also test whether the voting ratio is informative when only unattributed voting records are available, as in the aforementioned case of the Czech Republic.

We add two robustness checks to our baseline models. First, we extend the empirical specification by Gerlach-Kristen (2004) to include a measure of dispersion in the voting records, which can serve as an indicator of the degree of uncertainty the board members face. We measure the dispersion of the voting results by the standard deviation of the individual votes.[11]

$$\begin{aligned}\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3(i_{\chi(t),L} - i_{\chi(t),S}) + \\ b_4 dispersion_t + u_{t+1}\end{aligned} \tag{3.8}$$

The sign of b_4 is not clear-cut, although more uncertainty may trigger looser monetary policy (Soderstrom, 2002; Bekaert, Hoerova, and Lo Duca, 2010). Second, we also estimate equation (3.7) based on the data before the 2008-2009 financial crisis in order to test the sensitivity of the results.

Finally, we estimate the empirical model for the U.S. Fed - equation (3.9), where we additionally include the *skew* for alternate members - i.e. those who do not have voting power but are present at the meeting - as well as the committee bias. The committee bias is the official statement of the Fed on how the Fed is leaning in terms of its next interest rate move. The variable is coded so that a higher value indicates an upward move of interest rates. Financial market expectations data are not included in the empirical model for the U.S. due to significant lags in publishing the minutes, which were available only after the subsequent meeting in our 1970-1996 sample. More information on the U.S. data is available in Appendix 3.A2.

$$\begin{aligned}\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3 dispersion_t + \\ b_4 committee\ bias_t + b_5 skew\ alternates_t + u_{t+1}\end{aligned} \tag{3.9}$$

not available for all the sample countries.

[11] The share of the largest minority could serve as an alternative measure.

3.6 Empirical Results

This section gives the empirical results on whether the voting record is informative about future monetary policy. We first present our baseline estimates (equations (3.6) and (3.7)) for all countries. Alternative specifications follow.

Table 3.4: Does the Voting Record Predict Repo Rate Changes?
Baseline Estimates
$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3(i_{\chi(t),L} - i_{\chi(t),S}) + u_{t+1}$

Country		Czech Rep.		Hungary		Poland		Sweden		UK	
Sample		2000:7-2008:12		2005:10-2009:2		1998:2-2009:12		1999:1-2009:2		1997:6-2009:2	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Skew	(b_1)	1.74***	1.14***	0.62*	0.62*	0.33***	0.63***	1.58***	1.27***	1.22***	1.58***
		(0.33)	(0.40)	(0.33)	(0.33)	(0.08)	(0.14)	(0.36)	(0.39)	(0.39)	(0.29)
Lagged repo changes	(b_2)	1.34***	0.46	1.44***	0.97***	0.73***	0.66***	1.03***	0.87***	0.87***	0.99***
		(0.27)	(0.42)	(0.32)	(0.38)	(0.12)	(0.17)	(0.19)	(0.21)	(0.23)	(0.19)
Term structure	(b_3)	.	2.53**	.	3.92***	.	1.97***	.	1.26*	.	1.52***
		.	(1.15)	.	(1.36)	.	(0.36)	.	(0.74)	.	(0.49)
Adj. pseudo R^2		0.24	0.20	0.34	0.49	0.13	0.33	0.24	0.25	0.20	0.32
Observations		100	75	40	40	142	108	90	90	142	142

Note: * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level. Standard errors in parentheses. Ordered probit estimation. Term structure stands for difference between 1Y and 3M interbank rate in given country. Data for Czech Republic in column 2 until 2006:7 only. Data on 12M interbank rate in Poland is available only from 2001 onwards, therefore number of observations in column (6) is smaller than that in (5).

The results reported in Table 3.4 suggest that the voting record is indeed informative about future policy rate changes. The lagged repo rate change is typically significant, suggesting that the central banks smooth interest rates to a certain extent and try to avoid sudden reversals in their policies. The variable *skew* is statistically significant at conventional levels in all countries in the first baseline 'naive' model as well as in the second baseline model with financial market expectations. The pseudo R^2 - the measure of regression fit - varies from 0.13 to 0.49. Our results for the UK confirm the previous empirical findings by Gerlach-Kristen (2004). The significance of *skew* indicates that the conditions identified by our theoretical model have been fulfilled. First, the chairmen in these central banks probably act as democratic leaders whose only special power is the proposal-making one and other committee members are free to express their own will by voting on the proposals of the chairmen, and the chairmen consider the voting of the other committee members informative. In other words, although we do not want to overemphasize our results it suggests that the democratic version of our theoretical model describes the real world data most closely. Second, it is likely that in our sample period there was enough noise in the signals, and at the same time the committee members' information sets were not distorted by excessively high economic volatility, given the size of the committee.

In the case of Poland, where the voting record is published with a significant lag separately from the minutes and is not available before the next policy meeting, *skew* carries additional information available only to board members, not to the financial markets. The adjusted pseudo R^2 increases from 0.23 in the specification with lagged policy rate changes and term structure to 0.33 in the specification with lagged policy rate changes, term structure and *skew*. We therefore conclude that despite the time lag the *skew* indicator contains additional information that can be used by board members. Releasing voting records faster would be beneficial for transparency of monetary policy.

The results for the Czech Republic use the data until 2006:7 in the specification with financial market expectations (column 2 in Table 3.4). The reason is that from this period onwards the voting record was released only about 3 hours after the monetary policy decision was announced. The monetary policy decision was typically announced at around 1 p.m. and the voting ratio was released at around 3.30 p.m. at a press conference. In principle, we could collect the interbank rates at say 2 p.m. and therefore use more recent data as well, but it has to be emphasized that the interbank market was not very liquid during the financial crisis. Therefore, we preferred to restrict the sample to 2006:7. The results for the Czech Republic also suggest that publishing the voting ratio (without an attributed voting record) may be sufficient to foster a better understanding of the future course of monetary policy.

Table 3.5: Does the Voting Record Predict Repo Rate Changes?
Alternative Specifications - Different Maturities in Term Structure and Uncertainty
$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3(i_{\chi(t),L} - i_{\chi(t),S}) + b_4 dispersion_t + u_{t+1}$

Country		Czech Rep.		Hungary		Poland		Sweden		UK	
Sample		2000:7-2008:12		2005:10-2009:2		1998:2-2009:12		1999:1-2009:2		1997:6-2009:2	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Skew	(b_1)	0.89**	1.14***	0.50*	0.48	0.35***	0.60***	1.48***	1.29***	1.70***	1.54***
		(0.41)	(0.40)	(0.28)	(0.36)	(0.09)	(0.14)	(0.37)	(0.41)	(0.29)	(0.31)
Lagged repo changes	(b_2)	0.08	0.45	1.22***	0.88**	0.63***	0.69***	0.92***	0.87***	1.15***	0.99***
		(0.43)	(0.42)	(0.37)	(0.40)	(0.13)	(0.18)	(0.19)	(0.21)	(0.18)	(0.19)
Term structure	(b_3)	10.24***	2.48**	2.10	4.67***	1.61***	1.75***	3.23**	1.24*	0.41***	1.58***
		(2.87)	(1.15)	(1.96)	(1.73)	(0.30)	(0.41)	(1.45)	(0.74)	(0.67)	(0.50)
Dispersion	(b_4)	.	-0.93	.	-7.88*	.	-1.03	.	0.93	.	-3.99*
		.	(2.54)	.	(4.51)	.	(0.88)	.	(2.85)	.	(2.28)
Adj. pseudo R^2		0.27	0.20	0.35	0.54	0.24	0.41	0.27	0.25	0.29	0.33
Observations		75	75	40	40	142	60	90	90	142	142

Note: * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level. Standard errors in parentheses. Ordered probit estimation. Term structure stands for difference between 3M and 1M (1Y and 3M) interbank rate in odd (even) columns in given country. Data for Czech Republic in columns 1 and 2 until 2006:7 only. Uncertainty stands for standard deviation of individual votes in bank board. Data on 12M interbank rate in Poland is available only from 2001 onwards, therefore number of observations in column (6) is smaller than that in (5).

We also carried out a number of robustness checks. In the baseline specifications, the term structure was defined as the difference between the 12-month and 3-month interbank rate. Alternatively, the term structure is based on different maturities, defined in the regressions presented in Table 3.5 as the difference between the 3-month and 1-month interbank rate. The results remain largely unchanged. *skew* remains statistically significant and its estimated size is largely similar. Similarly, introducing dispersion - a measure of disagreement in the board - as an additional explanatory variable does not change the interpretation of the baseline estimates. The dispersion is statistically significant at 10% level in Hungary and the UK. This suggests that a more dispersed opinion about policy rates is associated with a loosening of policy in these two countries. The dispersion in the other countries is insignificant. Table 3.6 reports the results based on the sample excluding the financial crisis period (up to 2007:7). Again, the results remain largely stable. Finally, we included the level of interest rates as additional regressor to tackle the issue that the increase in the policy rate by 0.25 if the rate is at, for example, 1% or when it is at 5% can give different message to the public. Even after the inclusion of the level of interest rate, *skew* remains statistically significant (these results are available upon request).

Table 3.6: Does the Voting Record Predict Repo Rate Changes?
Alternative Specifications - Data until Financial Crisis Only
$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3(i_{\chi(t),L} - i_{\chi(t),S}) + u_{t+1}$

Country		Czech Rep.		Hungary		Poland		Sweden		UK	
Sample		2000:7-2008:12		2005:10-2009:2		1998:2-2009:12		1999:1-2009:2		1997:6-2009:2	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Skew	(b_1)	1.66***	1.14***	0.47	1.94**	0.28***	0.62***	1.39***	0.84*	1.57***	1.28***
		(0.35)	(0.40)	(0.47)	(0.92)	(0.08)	(0.15)	(0.28)	(0.44)	(0.29)	(0.32)
Lagged repo changes	(b_2)	1.24***	0.46	1.50***	1.22	0.64***	0.49**	1.01***	0.67***	0.99***	0.46*
		(0.31)	(0.42)	(0.47)	(0.80)	(0.13)	(0.20)	(0.23)	(0.27)	(0.21)	(0.25)
Term structure	(b_3)	.	2.53**	.	8.08**	.	2.44***	.	2.24**	.	2.99***
		.	(1.15)	.	(3.19)	.	(0.47)	.	(0.88)	.	(0.68)
Adj. pseudo R^2		0.19	0.20	0.35	0.71	0.11	0.37	0.24	0.25	0.23	0.33
Observations		87	75	22	22	114	80	79	79	123	123

Note: * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level. Standard errors in parentheses. Ordered probit estimation. Term structure stands for difference between 1Y and 3M interbank rate in given country. Data until 2007:7 exclude global financial crisis period. Data for Czech Republic in column 2 until 2006:7 only. Data on 12M interbank rate in Poland is available only from 2001 onwards, therefore number of observations in column (6) is smaller than that in (5).

The results for the U.S. Fed support our findings for the inflation-targeting countries. *skew* is statistically significant in all cases at the 1% level even with the measure of committee bias, which in principle carries the same piece of information. The results suggest that the FOMC still has informationally independent members, despite the common perception of chairman dominance (see Chappell et al., 2005),[12] which would have been closer to our consensual model, in which, however, the significance in the *skew* indicator is not indicated. Indeed, this is supported by the non-significance of *skew alternates*, who arguably do not put great weight on their private information in their voting decision. The finding that *skew alternates* is not significant in any specifications is broadly consistent with Tillmann (2011), who shows that alternate members systematically exaggerate their views to influence policy deliberation.[13]

[12] *skew* remains significant even if we exclude the first years of Greenspan chairmanship, i.e. the period for which it could eventually be argued that Greenspan did not build his reputation yet.

[13] As regards the insignificance of *skew alternates*, it is noteworthy that the voting of alternate members is much more in line with the chairman under the Greenspan chairmanship than the voting of the FOMC members with voting power. The sample average difference between Greenspan's preferred policy rate and the alternates' preferred rate is only 0.01, while this difference is 0.17 for the FOMC members with voting power. This may explain the insignificance of *skew alternates* and, in line with our democratic model, it suggests that independence among voters is needed in order to generate signalling power for skew. On the other hand, these results (the magnitude of the difference between the voting records of the chairman and the remaining FOMC members with and without voting power) do not hold for the Burns chairmanship period. One hypothesis that might be put forward is that there was more data imputation (for the preferred policy rate) for the Burns era than for the Greenspan era; see Appendix 3.A2 on how the raw data were coded by Chappell et al. (2005).

Table 3.7: Does the Voting Record Predict Repo Rate Changes in the USA?
Chappell et al. (2005) Data for Burns and Greenspan Era
$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3 dispersion_t + b_4 committee\ bias_t + u_{t+1}$

Country		Full sample		Burns era		Greenspan era		Greenspan era	
Sample		1970:2-1996:12		1970:2-1978:2		1987:8-1996:12		1987:8-1996:12	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Skew	(b_1)	4.59***	4.61***	2.85**	2.87***	12.25***	12.54***	9.19***	8.79***
		(1.02)	(1.02)	(1.12)	(1.12)	(2.64)	(2.65)	(2.83)	(2.90)
Lagged repo changes	(b_2)	0.42***	0.42***	0.45***	0.45***	0.41***	0.38***	0.11	0.06
		(0.07)	(0.07)	(0.08)	(0.09)	(0.13)	(0.12)	(0.15)	(0.15)
Dispersion	(b_3)	.	0.53	.	0.20	.	1.14	.	2.70
		.	(0.85)	.	(1.08)	.	(2.01)	.	(2.14)
Committee bias	(b_4)	.	.	.	.	.	.	1.08***	1.19***
		.	.	.	.	.	.	(0.28)	(0.28)
Adj. pseudo R^2		0.12	0.12	0.13	0.13	0.18	0.18	0.27	0.28
Observations		172	172	98	98	74	74	74	74

Note: * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level. Standard errors in parentheses. Ordered probit estimation. Committee bias indicates how Fed is leaning in terms of its next interest rate move; variable is coded so that higher value indicates upward move of interest rates.

Table 3.8: Does the Voting Record Predict Repo Rate Changes in the USA?
Chappell et al. (2005) Data for Burns and Greenspan Era
Skew for Alternate Members Added

$$\Delta i_{t+1} = b_0 + b_1 skew_{\tau(t)} + b_2 \Delta i_t + b_3 dispersion_t + b_4 committee\ bias_t + b_5 skew\ alternates_t + u_{t+1}$$

Country		Burns era		Greenspan era		Greenspan era	
Sample		1970:2-1978:2		1987:8-1996:12		1987:8-1996:12	
		(3)	(4)	(5)	(6)	(7)	(8)
Skew	(b_1)	2.89***	2.90***	11.39***	10.88***	11.21***	10.39***
		(1.12)	(1.12)	(3.36)	(3.44)	(3.50)	(3.59)
Lagged repo changes	(b_2)	0.45***	0.45***	0.38***	0.37***	0.07	0.05
		(0.09)	(0.09)	(0.12)	(0.12)	(0.15)	(0.15)
Dispersion	(b_3)	.	0.17	.	1.39	.	2.48
		.	(1.08)	.	(2.04)	.	(2.16)
Committee bias	(b_4)	.	.	.	.	1.22***	1.26***
		.	.	.	.	(0.29)	(0.30)
Skew alternates	(b_5)	0.06	0.05	1.16	1.37	-1.82	-1.53
		(0.15)	(0.15)	(1.79)	(1.82)	(1.99)	(2.02)
Adj. pseudo R^2		0.13	0.13	0.19	0.19	0.28	0.29
Observations		98	98	74	74	74	74

Note: * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level. Standard errors in parentheses. Ordered probit estimation. Committee bias indicates how Fed is leaning in terms of its next interest rate move; variable is coded so that higher value indicates upward move of interest rates. Skew alternates - defined as difference between average policy rate voted for by individual alternate committee members and policy rate that is outcome of majority vote. Note that even though alternate members are not voting members, they actively participate in discussions during monetary policy meeting.

All in all, the results suggest that the voting record bears relevant information about future monetary policy for all the countries in our sample and, in consequence, serves as a useful tool for improving the transparency of monetary policy.

3.7 Concluding Remarks

In this paper, we examine whether the voting records of central bank boards or monetary policy committees are informative about future monetary policy. We approach this issue from two angles. First, we develop a theoretical model of central bank decision-making where board members have non-homogeneous information sets and try to set policies so as to match the uncertain 'state of the world'. The model contains an intertemporal link between decisions taken at different board meetings to reflect the nature of monetary policy-making in which the interest rate adopted at one board meeting becomes the status quo for the next board meeting. The model also assumes an intertemporal link in optimal policies that change only slowly over time. We investigate whether the voting pattern is informative about changes in the interest rate based on data simulated from this model. Three different versions of model are estimated with the simulated data: 1) democratic, 2) consensual and 3) opportunistic. In essence, these versions differ in the extent to which the chairman influences the voting of the other board members. In version 1, the chairman allows the other board members to express their opinions democratically, and there is sufficient independence in the voting across the board members. In version 2, the chairman has a dominant enough position to bring about a consensus. And in version 3, the chairman votes opportunistically according to the majority of the other board members. The results show that only the democratic version of our model is able to generate significant correlations between the voting pattern and future policy changes. The results also show that the voting pattern resulting from democratic voting is informative only if there is sufficient noise in the signals.

Second, the model predictions are tested on real data. For this reason, data on six countries (the Czech Republic, Hungary, Poland, Sweden, the United Kingdom and the United States) that release voting records are collected. It is found that in all these countries the voting records are indeed informative about future monetary policy and thus in principle improve monetary policy transparency. More specifically, it is found that if a minority votes for higher rates than the majority, it is more likely that there will be a rate hike at the following meeting. This result is robust to controlling for financial market expectations as well as different sample periods. The results for Poland and for the U.S. under the Burns and Greenspan chairmanships suggest that committee members tend to put the same effort into forming their views no matter whether their voting is published soon after the meeting or after a longer period of time. Hence, releasing voting records faster would be beneficial for both the public and the central bank, which could gain credibility.

Similarly to Gerlach-Kristen (2004) the results in this paper hold regardless of whether the voting record is attributed or not. In consequence, where there are concerns that attributed voting records might expose individual board members to some external pressure (such as in the case of a monetary union with board members not voting for national interests), the voting results can be published as non-attributed and still contribute to a better understanding of monetary policy. All in all, monetary policy transparency can be improved by releasing the voting record in a timely fashion.

3.A1 Derivation of Central Bank Board Decision-Making Model and Simulation Robustness Checks

In this appendix we explain the models from the third part of the paper in more detail so that it becomes apparent how to generate C's proposals and Ps' voting behaviour. We further explain several aspects of our simulation exercise and the methods we used.

First note that for all the models, equilibrium exists. This can be established for the three models with $\delta = 0$ using the simple backward induction argument. As there is no intertemporal link in the decisions, we can focus on a single period. Within this period, the P members move last and their behaviour is given by the specified voting condition. Knowing this, C derives her proposal y as a solution to her optimization problem. Finally, for the intertemporal democratic model, note that the policy space is finite and the existence of a Stationary Markov Perfect equilibrium follows from the arguments in Maskin and Tirole (2001).

Throughout the explanation we will often work with a vector of random variables in our model. All those variables form a random vector $r = \{\bar{i}^*, i^{P1}, \ldots, i^{PN}, i^C\}'$ that has a multivariate normal distribution with - conditional on the information embedded in i^* - a mean equal to ρi^* and a variance-covariance matrix equal to a matrix with the vector $\{\sigma_u^2, \sigma_u^2 + \sigma_P^2, \ldots, \sigma_u^2 + \sigma_P^2, \sigma_u^2 + \sigma_C^2\}$ on the main diagonal and all the off-diagonal elements equal to σ_u^2. Often we will need to compute the conditional expectation of r given the specific value of one or more of its elements. For this we use the well known result for the multivariate normal distribution that states that for a vector of (possibly more than two) random variables $\{x_1, x_2\}'$ distributed according to $N(\mu, \Sigma)$ with $\mu = \{\mu_1, \mu_2\}'$ and $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$, where the partitioning of μ and Σ conforms to the partition of $\{x_1, x_2\}'$, the conditional distribution of x_1 given a specific value of x_2 is $N(\mu_1', \sigma_{11}')$, where $\mu_1' = \mu_1 + \sigma_{12}\sigma_{22}^{-1}(x_2 - \mu_2)$ and $\sigma_{11}' = \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{21}$.

To simulate each of the models, we start in the first period, with the previous optimal interest rate and monetary policy rate being zero. In the simulations of the democratic, consensual and opportunistic models we restrict the policy space to be in the interval $\langle -10, 10\rangle$ so that with our choice of $\bar{s}$ the policy space is equal to $Y = \{-10, -9.75, \ldots, 9.75, 10\}'$. For the intertemporal democratic mode we restrict the policy space to be in $\langle -3.5, 3.5\rangle$ for the baseline scenario. We do not need to look at a larger policy space, as the optimal interest rate and players' signals stay well away from its border. As explained in the text, it is also inconsequential that we allow the optimal interest rate and the monetary policy rate to attain negative values, as all the results and estimates are invariant to adding a constant to the optimal interest rate.

The values of the random variables used in the simulations are kept constant across the different models. That is, when we simulate, say, the first path for the baseline scenario of the democratic model, the random variables used are the same as when simulating the first path of any other scenario for the same model or of any other model for the same scenario. This holds even across the $N = 4$ and $N = 6$ simulations, where we naturally have to add two more random variables for the two extra players, but the remaining random variables are kept the same.

In the democratic model with $\rho = 0.95$, $\delta = 0$ and $\bar{s} = 0.25$, at the beginning of each period with status quo x, last-period optimal interest rate i^* and fresh draw of $r = \{\bar{i}^*, i^{P1}, \ldots, i^{PN}, i^C\}'$, we first need to derive C's proposal y. This will be given as a solution to the optimization problem

$$\max_{y \in Y} \mathbb{E}\left[-(p(x,y) - \bar{i}^*)^2 | i^*, i^C\right] \tag{3.10}$$

where $p(x, y)$ is the policy adopted given proposal y and status quo x. The optimization problem can be rewritten as

$$\max_{y \in Y} p_a \mathbb{E}[-(y - \bar{i}^*)^2 | i^*, i^C, a] + (1 - p_a)\mathbb{E}[-(x - \bar{i}^*)^2 | i^*, i^C, \hat{a}] \tag{3.11}$$

where a is the event of y being accepted, $\hat{a}$ is the event of y being rejected and p_a is the probability of event a.

Next, we will need to calculate the probability of offer y being accepted against status quo x, p_a. In order to do so, chairman C knows, and we show below, that the remaining players will vote for y if and only if their signal is above (or below, but this case is symmetric) a certain cut-off that we denote here by k. The other relevant information that C has is her own signal i^C and the previous optimal interest rate i^*, hence we need to calculate the probability of at least $\frac{N}{2}$ P members voting for y given i^C and i^*. The probability of, say, the first N' members voting for y is equal to $\mathbb{P}(\#|i^P \geq k| = N', \#|i^P < k| = N - N' | i^*, i^C)$ and is straightforward to calculate, as we know the distribution of the random vector $\{i^{P1}, \ldots, i^{PN}\}'$ and can always transform it into a problem of calculating $\mathbb{P}(\#|i^P \leq k| = N | i^*, i^C)$ by multiplying the whole problem (that is the mean and variance-covariance matrix) by $\{-1, \ldots, -1, 1, \ldots, 1\}'$, where there are N' negative ones and $N - N'$ positive ones. The probability can then be calculated using the standard cumulative distribution function of the multivariate normal distribution. Denoting the probability of the first N' members accepting by $\mathbb{P}_{N'}$, the probability of accepting becomes $\sum_{i=N/2}^{N} \mathbb{P}_i \begin{pmatrix} N \\ i \end{pmatrix}$.

The key computational problem in simulating the democratic model is computing the expected value of $\bar{i}^*$ given C's signal i^C, i^* and the event of y being accepted, as the event of accepting y means that the signals i^P of $\frac{N}{2}$ or more P members must have been above (or below) a certain threshold

k. There are two results we use to make the computation simpler that are straightforward to prove. First, for random variable X and two mutually exclusive and exhaustive events A and B we have

$$\mathbb{E}[X] = \mathbb{E}[X|A]\mathbb{P}(A) + \mathbb{E}[X|B]\mathbb{P}(B)$$

and the similar result for variance states that

$$\begin{aligned} var(X) = \; & var(X|A)\mathbb{P}(A) + var(X|B)\mathbb{P}(B) \\ & + (\mathbb{E}[X|A] - \mathbb{E}[X])^2\mathbb{P}(A) + (\mathbb{E}[X|B] - \mathbb{E}[X])^2\mathbb{P}(B) \end{aligned}$$

which greatly simplifies the calculation of some of the expressions below.

Nevertheless, the key problem remains, as we need to calculate an expectation of the form $\mathbb{E}[\ \bar{i}^*|i^*, i^C, \#|i^P \geq k| = N', \#|i^P < k| = N - N']$. The first step is simple and amounts to calculating the distribution of $\{\bar{i}^*, i^{P1}, \ldots, i^{PN}\}'$ given i^* and i^C, which is $N(\mu, \Sigma)$, with each element of μ being equal to $\frac{\rho i^* \sigma_C^2 + i^C \sigma_u^2}{\sigma_u^2 + \sigma_C^2}$ and Σ being a matrix with the vector $\left\{\sigma', \sigma' + \sigma_P^2, \ldots, \sigma' + \sigma_P^2\right\}'$ on the main diagonal and $\sigma' = \frac{\sigma_u^2 \sigma_C^2}{\sigma_u^2 + \sigma_C^2}$ off the main diagonal. We then convert the problem into one of finding $\mathbb{E}[\ \bar{i}^*|\#|i^P \geq k| = N]$ using the technique just explained for the calculation of p_a. This leaves us with a multivariate truncated normal random vector with known mean and variance. To calculate the expectation we used the results in Tallis (1961) and Lee (1979) and wrote our own MATLAB function which calculates the expectation. We checked its correctness using the 'tmvtnorm' R-software package (see Wilhelm, 2010).

With those results, we can expand the maximand in (3.11) and use the rules for conditional expectations and variance given above, then we determine the value of the objective function for each $y \in Y$ using the function for the expectation of the truncated multivariate normal, finally determining the solution to C's optimization problem and hence her proposal.

With C's proposal y determined, we can determine the voting behaviour of the remaining P committee members. For each member j we use the voting rule (3.2) from the text adapted to the democratic model

$$\mathbb{E}\left[-(y - \bar{i}^*)^2|i^*, i^j\right] \geq \mathbb{E}\left[-(x - \bar{i}^*)^2|i^*, i^j\right] \tag{3.12}$$

which rewrites as

$$x^2 - y^2 \geq 2(y - x)\mathbb{E}[\ \bar{i}^*|i^*, i^j] \tag{3.13}$$

with $\mathbb{E}[\ \bar{i}^*|i^*, i^j] = \frac{\rho i^* \sigma_j^2 + i^j \sigma_u^2}{\sigma_u^2 + \sigma_j^2}$. This result also proves that each P member votes for y if and only if his signal is above (or below, depending on the position of the status quo) a certain cut-off. With the voting pattern determined, we can calculate the *skew* variable and proceed to the next period.

In the consensual model with $\rho = 0.95$, $\delta = 0$ and $\bar{s} = 0.25$, at the beginning of each period with status quo x, last-period optimal interest rate i^* and fresh draw of $r = \{\bar{i}^*, i^{P1}, \ldots, i^{PN}, i^C\}'$, proposal y will be the policy most preferred by C. This is equal to the policy in Y that is closest to C's expectation of $\bar{i}^*$ given her signal i^C and the previous optimal policy rate i^*. This expectation is equal to $\mathbb{E}[\ \bar{i}^*|i^*, i^C] = \frac{\rho i^* \sigma_C^2 + i^C \sigma_u^2}{\sigma_u^2 + \sigma_C^2}$.

Next, we need to determine the behaviour of the P committee members. In the consensual model, each P member j will vote based on the voting rule (3.2) using his information about the previous optimal interest rate i^*, his private signal i^j and the information embedded in C's proposal y, hence the voting rule rewrites as

$$x^2 - y^2 \geq 2(y - x)\mathbb{E}[\ \bar{i}^*|i^*, i^j, y]. \tag{3.14}$$

It is easy to confirm that the information embedded in C's proposal is equal to an event of $i^C \in \langle i_l^C, i_u^C \rangle$, where the lower bound of the interval is $i_l^C = \frac{1}{\sigma_u^2}\left[(y - \frac{\bar{s}}{2})(\sigma_u^2 + \sigma_C^2) - \sigma_C^2 \rho i^*\right]$ and the upper bound of the interval is $i_u^C = \frac{1}{\sigma_u^2}\left[(y + \frac{\bar{s}}{2})(\sigma_u^2 + \sigma_C^2) - \sigma_C^2 \rho i^*\right]$. Calculation of $\mathbb{E}[\bar{i}^*|i^*, i^j, i^C \in \langle i_l^C, i_u^C \rangle]$ is then easy using the law of iterated expectations, which allows us to rewrite the expression to $\mathbb{E}[\mathbb{E}[\bar{i}^*|i^*, i^j, i^C]|i^*, i^j, i^C \in \langle i_l^C, i_u^C \rangle]$. The inner expectations are equal to $\frac{\rho i^* \sigma_j^2 \sigma_C^2 + i^j \sigma_u^2 \sigma_C^2 + i^C \sigma_u^2 \sigma_j^2}{\sigma_u^2 \sigma_j^2 + \sigma_u^2 \sigma_C^2 + \sigma_C^2 \sigma_j^2}$. Moreover, we know that the distribution of i^C given i^* and i^j is normal, with mean $\frac{\rho i^* \sigma_j^2 + i^j \sigma_u^2}{\sigma_u^2 + \sigma_j^2}$ and variance $\frac{\sigma_u^2 \sigma_j^2}{\sigma_u^2 + \sigma_j^2} + \sigma_C^2$. The last result we use to calculate the expectations is that for random variable x_1 distributed according to $N(\mu, \sigma^2)$. The conditional expectation of x_1 given that $x_1 \in \langle a_l, a_u \rangle$ is given by $\mathbb{E}[x_1 | x_1 \in \langle a_l, a_u \rangle] = \mu + \sigma \frac{\phi(\frac{a_l - \mu}{\sigma}) - \phi(\frac{a_u - \mu}{\sigma})}{\Phi(\frac{a_u - \mu}{\sigma}) - \Phi(\frac{a_l - \mu}{\sigma})}$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the probability density and cumulative distribution functions of the univariate standard normal distribution. With the voting pattern determined, we can calculate the *skew* variable and proceed to the next period.

In the opportunistic model with $\rho = 0.95$, $\delta = 0$ and $\bar{s} = 0.25$, the chairman C knows the most preferred policies of all the committee members. For each player j, this policy will be the policy in Y closest to j's expectation of $\bar{i}^*$ given i^* and i^j, i.e. closest to $\mathbb{E}[\bar{i}^*|i^*, i^j] = \frac{\rho i^* \sigma_j^2 + i^j \sigma_u^2}{\sigma_u^2 + \sigma_j^2}$. At the beginning of each period with status quo x and last-period optimal interest rate i^*, given a fresh drawn of $r = \{\bar{i}^*, i^{P1}, \ldots, i^{PN}, i^C\}'$, we will then have the vector of most preferred policies $\{p_1^*, \ldots, p_{N+1}^*\}$, which we order so that $p_j^* \leq p_{j+1}^*$ for $j \in \{1, \ldots, N\}$, where we denote the policy most preferred by the median member by $p_m^* = p_{N/2+1}^*$.

In the opportunistic model, C's proposal will be the policy which receives a super-majority of at least $\frac{N}{2} + 2$ members. It is easy to see that this will be the policy in the interval (x, p_m^*) (where the order is reversed if the status quo x is larger than p_m^*) that is closest to p_m^* if such policy exists. Otherwise, the proposal will be equal to the status quo x.

Next, we need to calculate the *skew* variable. For this, we again use the voting rule (3.2) along with the assumption that player j does not extract any information content from proposal y and votes for y as opposed to voting for the status quo x if and only if

$$x^2 - y^2 \geq 2(x - y)\mathbb{E}[\ \bar{i}^*|i^*, i^j] \tag{3.15}$$

with $\mathbb{E}[\ \bar{i}^*|i^*, i^j] = \frac{\rho i^* \sigma_j^2 + i^j \sigma_u^2}{\sigma_u^2 + \sigma_j^2}$. By construction, C's proposal is always accepted, with the number of votes for y being at least $\frac{N}{2} + 2$. With the voting pattern and hence *skew* determined, we move to the next period.

Finally, in the intertemporal democratic model with $\bar{s} = 0.25$, $\rho = 0$ and $\delta = 0.95$, the previous-period optimal interest rate i^* plays no role and hence the only relevant information is the current status quo x. We again start each period of the simulation by drawing fresh values for $r = \{\bar{i}^*, i^{P1}, \ldots, i^{PN}, i^C\}'$. Next, we need to determine C's proposal. This will again be the solution to the optimization problem

$$\max_{y \in Y} \mathbb{E}\left[-(p(x,y) - \bar{i}^*)^2 + \delta U_C(p(x,y), \bar{i}^C)|i^C\right] \tag{3.16}$$

where $U_C(\cdot)$ is the continuation value function of a game starting with the status quo $p(x, y)$ and C's signal $\bar{i}^C$. The expression can again be rewritten as

$$\max_{y \in Y} \left[\begin{array}{l} p_a \mathbb{E}[-(y-\bar{i}^*)^2 | i^C, a] + (1-p_a)\mathbb{E}[-(x-\bar{i}^*)^2 | i^C, \hat{a}] \\ +\delta p_a V_C(y) + \delta(1-p_a) V_C(x) \end{array} \right] \tag{3.17}$$

where $V_C(x) = \int U_C(x, z) f(z) dz$, with $f(z)$ being a probability distribution function of univariate normal distribution with mean zero and variance equal to $\sigma_u^2 + \sigma_C^2$. With $V_C(\cdot)$ known (we explain its estimation below) we proceed similarly as in the democratic model, calculating the probability of y being accepted and the expected values in the maximand. The distribution of the random variables $\{\bar{i}^*, i^{P1}, \ldots, i^{PN}\}$ given i^C is again $N(\mu, \Sigma)$, with each element of μ being equal to $\frac{i^C \sigma_u^2}{\sigma_u^2 + \sigma_C^2}$ and Σ being a matrix with vector $\left\{\sigma', \sigma' + \sigma_P^2, \ldots, \sigma' + \sigma_P^2\right\}'$ on the main diagonal and $\sigma' = \frac{\sigma_u^2 \sigma_C^2}{\sigma_u^2 + \sigma_C^2}$ off the main diagonal.

The voting behaviour of P member j given status quo x, proposal y and signal i^j is again given by the voting rule (3.2), which for the intertemporal democratic model becomes

$$\mathbb{E}\left[-(y-\bar{i}^*)^2 + \delta U_P(y, \bar{i}^j) | i^j\right] \geq \mathbb{E}\left[-(x-\bar{i}^*)^2 + \delta U_P(x, \bar{i}^j) | i^j\right] \tag{3.18}$$

where $U_P(\cdot)$ is the continuation value function of the P member from a game starting with the given status quo and signal. Note that this function is equal for all P players. This condition can be rewritten as

$$x^2 - y^2 + \delta(V_P(y) - V_P(x)) \geq 2(y-x)\mathbb{E}[\ \bar{i}^* | i^j] \tag{3.19}$$

with $\mathbb{E}[\ \bar{i}^* | i^j] = \frac{i^j \sigma_u^2}{\sigma_u^2 + \sigma_j^2}$ and $V_P(x) = \int U_P(x, z) f(z) dz$, with $f(z)$ being a probability distribution function of univariate normal distribution with mean zero and variance equal to $\sigma_u^2 + \sigma_P^2$. With the voting pattern determined, we can calculate the *skew* variable and proceed to the next period.

It remains to explain how we determine the continuation value functions. Prior to running the simulations, we estimate the $V_C(\cdot)$ and $V_P(\cdot)$ functions by standard value function iteration. We start with $V_{C,0}(\cdot) = 0$ and $V_{P,0}(\cdot) = 0$ and determine both functions $V_{C,s}(\cdot)$ and $V_{P,s}(\cdot)$ in a general step s as follows.

For $V_{C,s}(\cdot)$ we use numerical integration via the standard Gaussian quadrature method using the 'compecon' toolbox described in Miranda and Fackler (2002). To determine $V_{C,s}(x)$ for a specific value of x, we determine the set of nodes for C's signals $\{i^1, \ldots, i^H\}$ (using $H = 9$ in practice) and for each signal $i^h \in \{i^1, \ldots, i^H\}$ calculate

$$U_{C,s}(x, i^h) = \max_{y \in Y} \left[\begin{array}{l} p_a \mathbb{E}[-(y - \bar{i}^*)^2 | i^h, a] + (1 - p_a) \mathbb{E}[-(x - \bar{i}^*)^2 | i^h, \hat{a}] \\ + \delta p_a V_{C,s-1}(y) + \delta(1 - p_a) V_{C,s-1}(x) \end{array} \right] \tag{3.20}$$

using the approach described above. The cut-off values for the calculation of the probability of acceptance are derived from the voting rule, which uses the $V_{P,s-1}(\cdot)$ function.

For $V_{P,s}(\cdot)$ we use the same numerical integration approach, generating the set of P's signals and numerically integrating $V_{P,s}(x) = \int U_{P,s}(x,z) f(z) dz$. The only complication is that we need to determine the $U_{P,s}(\cdot)$ function. For the specific status quo x and signal i^j of P player j, $U_{P,s}(x, i^j)$ gives the continuation value from the game starting with x and i^j and hence can be written as

$$U_{P,s}(x, i^j) = \mathbb{E}\left[-(p - \bar{i}^*)^2 + \delta U_{P,s-1}(p, \bar{i}^j) | i^j \right] \tag{3.21}$$

but the expectation operator hides considerable complexity. First, j does not know C's signal and hence her proposal. Second, j does not know whether the proposal will be accepted or not, and third, j does not know the next-period signal. Reconciling the third source of uncertainty is straightforward and the whole expression can be rewritten as

$$U_{P,s}(x, i^j) = \mathbb{E}\left[-(p - \bar{i}^*)^2 + \delta V_{P,s-1}(p) | i^j \right] \tag{3.22}$$

with only the first two sources of uncertainty remaining.

To resolve those we need to take expectations over C's signal, which will determine her proposal as well. Expanding the expectations operator thus gives

$$U_{P,s}(x, i^j) = \int \left[\begin{array}{l} p_a(z) [\mathbb{E}[-(y(z) - \bar{i}^*)^2 | i^j, z, a] + \delta V_{P,s-1}(y(z))] \\ +(1 - p_a(z)) [\mathbb{E}[-(x - \bar{i}^*)^2 | i^j, z, \hat{a}] + \delta V_{P,s-1}(x)] \end{array} \right] f(z) dz \tag{3.23}$$

where the variable of integration is C's signal, $y(z)$ is C's proposal given her signal, $p_a(z)$ is the probability of this proposal being accepted, and $f(z)$ is the probability distribution function of C's signal.

Hence, in order to get $V_{P,s}(\cdot)$ we need to integrate twice, once over the distribution of j's signal (which will be normal, with mean zero and variance $\sigma_u^2 + \sigma_P^2$) and once over the distribution of C's signal (which for a given value of i^j will be normal, with mean equal to $\frac{i^j \sigma_u^2}{\sigma_u^2+\sigma_P^2}$ and variance equal to $\frac{\sigma_u^2\sigma_P^2}{\sigma_u^2+\sigma_P^2} + \sigma_C^2$). Integrating numerically then amounts to generating a grid of discrete nodes in $\mathbb{R}^2$ with one dimension for j's signals and nodes $\{i^1, \ldots, i^M\}$ and the second dimension for C's signal and nodes $\{i^1, \ldots, i^H\}$ (again we use $H = M = 9$ in practice).

For each node in $\mathbb{R}^2$ consisting of $\{i^m, i^h\}$ we calculate

$$\begin{aligned} & p_a(i^h)[\mathbb{E}[-(y(i^h) - \bar{i}^*)^2 | i^m, i^h, a] + \delta V_{P,s-1}(y(i^h))] \\ & +(1 - p_a(i^h))[\mathbb{E}[-(x - \bar{i}^*)^2 | i^m, i^h, \hat{a}] + \delta V_{P,s-1}(x)] \end{aligned} \tag{3.24}$$

which then allows us to calculate $V_{P,s}(\cdot)$. To calculate the expression, we first calculate C's proposal $y(i^h)$. Given the proposal, we can calculate the probability of the proposal being accepted (with j taking into account his own voting behaviour) and, finally, the remaining expectations given acceptance or rejection. In the whole expression, j will condition on the information embedded in $\{i^m, i^h\}$ and hence the appropriate conditional distribution of the remaining random variables in the model $\{\bar{i}^*, i^{P1}, \ldots, i^{PN-1}\}$ is multivariate normal $N(\mu, \Sigma)$, with each element of μ being equal to $\frac{\sigma_u^2(\sigma_C^2 i^m + \sigma_P^2 i^h)}{\sigma_u^2(\sigma_C^2+\sigma_P^2)+\sigma_C^2\sigma_P^2}$ and Σ being a matrix with vector $\left\{\sigma', \sigma' + \sigma_P^2, \ldots, \sigma' + \sigma_P^2\right\}'$ on the main diagonal and $\sigma' = \frac{\sigma_u^2\sigma_C^2\sigma_P^2}{\sigma_u^2(\sigma_C^2+\sigma_P^2)+\sigma_C^2\sigma_P^2}$ off the main diagonal.

We iterate on s until $\max\{||V_{P,s} - V_{P,s-1}||, ||V_{C,s} - V_{C,s-1}||\} \geq 0.001$, where $||\cdot||$ is the usual sup norm. We experienced no problems with convergence, and the typical s needed was around 70 iterations.

Next, we re-run the simulations of the democratic, consensual, opportunistic and mechanical models for different parameter values compared to those in the main part of the paper. First, we changed the benchmark $\rho = 0.95$ to $\rho = 0.90$, second we changed the benchmark $\rho = 0.95$ to $\rho = 0.99$, and third we changed the underlying $AR(1)$ process for optimal policy to $AR(2)$. In this specification, it evolves according to $i_t^* = \rho_1 i_{t-1}^* + \rho_2 i_{t-2}^* + u_t$ and we picked $\rho_1 = 1.95$ and $\rho_2 = -0.98$ following Gerlach-Kristen (2008).

Note that the model changes only in that ρi^*_{t-1} in the expressions for expectations changes to $\rho_1 i^*_{t-1} + \rho_2 i^*_{t-2}$. With the $AR(2)$ process governing the optimal policy we also had to change the standard deviation of the underlying shocks, but followed the same rationale as in the benchmark model. That is, Δi^*_t has a standard deviation of $\sqrt{\frac{2}{(1+\rho_2)(1+\rho_1-\rho_2)}}\sigma_u$, hence in order to match the observed standard deviation of the policy changes between 0.25 and 0.50 we set $\sigma_u = 0.05$ in the baseline scenario and correspondingly decreased the noise in the committee signals to $\sigma_C = 0.05$ and $\sigma_P = 0.05$, doubling those values when appropriate for the other scenarios. The results of the simulations are given in Tables 3.9-3.14, with the following Tables 3.15 and 3.16 showing the results of the simulations for the opportunistic model with a simple majority as opposed to the super-majority used in the benchmark simulations.

Table 3.9: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data with $N = 4$ and $\rho = 0.90$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	4.16 *	5.90	6.22	0.05
	[1.64] (0.074)	[3.24] (0.159)	[6.56] (0.391)	[3.94] (0.438)
Lagged policy	0.65	-0.02	1.62	-0.38
change (a_2)	[0.54] (0.305)	[0.46] (0.548)	[0.77] (0.107)	[1.24] (0.459)
MSE	0.027	0.033	0.034	0.005
Votes proposal	2.91	4.63	4.83	—
No change	0.43	0.41	0.59	0.36
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.20	3.05	0.91	0.21
	[1.06] (0.118)	[2.50] (0.303)	[3.65] (0.506)	[2.03] (0.424)
Lagged policy	0.17	-0.04	0.54	-0.04
change (a_2)	[0.24] (0.454)	[0.21] (0.534)	[0.29] (0.155)	[0.64] (0.455)
MSE	0.041	0.050	0.045	0.005
Votes proposal	3.48	4.62	4.78	—
No change	0.28	0.24	0.37	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.43 *	6.79	6.50	—
	[1.52] (0.100)	[3.58] (0.164)	[6.53] (0.349)	—
Lagged policy	0.19	-0.03	1.00	—
change (a_2)	[0.49] (0.514)	[0.47] (0.505)	[0.67] (0.248)	—
MSE	0.048	0.052	0.052	—
Votes proposal	2.97	4.69	4.85	—
No change	0.43	0.42	0.57	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	4.93 **	8.86	7.36	—
	[1.78] (0.036)	[6.25] (0.250)	[6.57] (0.335)	—
Lagged policy	0.76	-0.28	1.22	—
change (a_2)	[0.56] (0.284)	[0.42] (0.473)	[0.70] (0.181)	—
MSE	0.041	0.036	0.050	—
Votes proposal	2.70	4.88	4.84	—
No change	0.51	0.37	0.58	—

Note: Average ordered probit estimates over 101 random 100-period-long paths. [Average standard errors] and (average p-value). * statistically significant at 10% level, ** statistically significant at 5% level, *** statistically significant at 1% level based on average p-value. MSE is average mean squared difference between adopted and optimal policy. Votes proposal is average number of votes for chairman's proposal. No change is proportion of committee meetings with no policy change.

Table 3.10: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data with $N = 6$ and $\rho = 0.90$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	5.07 **	6.57	7.65	-0.02
	[1.65] (0.025)	[3.25] (0.130)	[5.40] (0.257)	[3.39] (0.513)
Lagged policy	1.00	0.06	1.98	-0.39
change (a_2)	[0.56] (0.172)	[0.46] (0.516)	[0.85] (0.102)	[1.04] (0.530)
MSE	0.026	0.033	0.030	0.005
Votes proposal	3.85	6.45	6.62	—
No change	0.47	0.41	0.57	0.36
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.38 *	3.25	1.86	-0.05
	[1.03] (0.096)	[2.45] (0.302)	[3.04] (0.421)	[1.75] (0.514)
Lagged policy	0.21	-0.03	0.54	-0.12
change (a_2)	[0.24] (0.451)	[0.21] (0.529)	[0.30] (0.164)	[0.54] (0.555)
MSE	0.039	0.049	0.036	0.005
Votes proposal	4.69	6.42	6.53	—
No change	0.30	0.24	0.34	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.62 *	7.53	6.70	—
	[1.49] (0.079)	[3.63] (0.133)	[5.23] (0.305)	—
Lagged policy	0.33	0.05	1.11	—
change (a_2)	[0.51] (0.450)	[0.48] (0.500)	[0.69] (0.218)	—
MSE	0.047	0.052	0.050	—
Votes proposal	3.94	6.53	6.67	—
No change	0.46	0.42	0.54	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	5.56 **	11.17	7.73	—
	[1.72] (0.017)	[6.57] (0.206)	[5.26] (0.274)	—
Lagged policy	0.94	-0.22	1.28	—
change (a_2)	[0.57] (0.194)	[0.43] (0.518)	[0.72] (0.182)	—
MSE	0.041	0.036	0.048	—
Votes proposal	3.60	6.82	6.66	—
No change	0.52	0.37	0.55	—

Note: See Table 3.9.

Table 3.11: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data with $N = 4$ and $\rho = 0.99$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	4.30 *	6.33	7.06	0.52
	[1.65] (0.065)	[3.25] (0.152)	[6.31] (0.351)	[4.05] (0.435)
Lagged policy	0.78	0.15	1.86 *	-0.11
change (a_2)	[0.53] (0.237)	[0.45] (0.511)	[0.73] (0.068)	[1.28] (0.485)
MSE	0.027	0.033	0.033	0.005
Votes proposal	2.94	4.64	4.83	—
No change	0.42	0.41	0.58	0.37
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.16	2.68	0.77	0.05
	[1.05] (0.140)	[2.48] (0.373)	[3.60] (0.523)	[2.07] (0.438)
Lagged policy	0.29	0.04	0.66 *	-0.01
change (a_2)	[0.24] (0.344)	[0.21] (0.494)	[0.29] (0.090)	[0.65] (0.472)
MSE	0.041	0.050	0.045	0.005
Votes proposal	3.45	4.61	4.78	—
No change	0.28	0.24	0.37	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.58	7.01	5.67	—
	[1.53] (0.103)	[3.67] (0.160)	[6.19] (0.410)	—
Lagged policy	0.38	0.11	1.06	—
change (a_2)	[0.47] (0.390)	[0.45] (0.457)	[0.61] (0.207)	—
MSE	0.049	0.052	0.052	—
Votes proposal	3.05	4.70	4.84	—
No change	0.41	0.39	0.53	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	4.83 *	8.77	6.46	—
	[1.76] (0.056)	[6.08] (0.279)	[6.16] (0.334)	—
Lagged policy	0.87	-0.14	1.33	—
change (a_2)	[0.53] (0.228)	[0.41] (0.492)	[0.64] (0.134)	—
MSE	0.041	0.036	0.049	—
Votes proposal	2.78	4.88	4.83	—
No change	0.49	0.37	0.55	—

Note: See Table 3.9.

Table 3.12: Does the Voting Record Predict Policy Rate Changes?
Estimates Using Simulated Data with $N = 6$ and $\rho = 0.99$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	4.94 **	6.70	8.16	0.12
	[1.66] (0.029)	[3.24] (0.128)	[5.21] (0.255)	[3.49] (0.502)
Lagged policy	1.12	0.22	2.20 **	-0.23
change (a_2)	[0.55] (0.128)	[0.45] (0.470)	[0.80] (0.047)	[1.08] (0.504)
MSE	0.026	0.033	0.029	0.005
Votes proposal	3.92	6.46	6.61	—
No change	0.45	0.41	0.56	0.37
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	2.35	2.91	1.39	0.06
	[1.02] (0.108)	[2.48] (0.318)	[2.92] (0.466)	[1.78] (0.507)
Lagged policy	0.33	0.05	0.65	-0.01
change (a_2)	[0.25] (0.284)	[0.21] (0.500)	[0.31] (0.111)	[0.54] (0.535)
MSE	0.040	0.049	0.036	0.005
Votes proposal	4.64	6.41	6.50	—
No change	0.31	0.24	0.33	0.19
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	3.59 *	8.24	6.22	—
	[1.48] (0.069)	[3.66] (0.114)	[5.05] (0.292)	—
Lagged policy	0.46	0.21	1.15	—
change (a_2)	[0.48] (0.388)	[0.45] (0.422)	[0.63] (0.166)	—
MSE	0.048	0.039	0.050	—
Votes proposal	4.07	6.55	6.67	—
No change	0.44	0.39	0.51	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	5.23 **	11.56	6.69	—
	[1.70] (0.024)	[6.45] (0.174)	[5.04] (0.271)	—
Lagged policy	1.04	-0.07	1.38	—
change (a_2)	[0.54] (0.148)	[0.42] (0.516)	[0.66] (0.108)	—
MSE	0.041	0.036	0.047	—
Votes proposal	3.71	6.82	6.65	—
No change	0.51	0.36	0.52	—

Note: See Table 3.9.

Table 3.13: Does the Voting Record Predict Policy Rate Changes? Estimates Using Simulated Data with $N = 4$ and $\rho_1 = 1.95$, $\rho_2 = -0.98$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	12.88 ***	14.95	19.38	-0.31
	[3.03] (0.002)	[8.37] (0.138)	[11.81] (0.177)	[5.89] (0.550)
Lagged policy	4.40 ***	3.62 **	4.02 **	3.57
change (a_2)	[0.72] (0.008)	[0.85] (0.034)	[0.90] (0.032)	[1.90] (0.186)
MSE	0.006	0.006	0.006	0.005
Votes proposal	3.23	4.92	4.95	—
No change	0.47	0.47	0.48	0.48
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	5.96	15.21	19.04	0.14
	[2.50] (0.107)	[8.28] (0.145)	[10.77] (0.173)	[3.23] (0.489)
Lagged policy	4.10 ***	3.77 **	4.18 **	4.09 **
change (a_2)	[0.44] (0.000)	[0.72] (0.030)	[0.65] (0.020)	[1.07] (0.018)
MSE	0.007	0.007	0.007	0.005
Votes proposal	4.18	4.92	4.95	—
No change	0.27	0.28	0.29	0.27
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	13.69 ***	16.25	17.35	—
	[3.05] (0.006)	[11.07] (0.155)	[12.60] (0.259)	—
Lagged policy	4.18 ***	3.40 **	3.52 *	—
change (a_2)	[0.70] (0.006)	[0.83] (0.039)	[1.04] (0.053)	—
MSE	0.007	0.007	0.007	—
Votes proposal	3.15	4.94	4.96	—
No change	0.46	0.46	0.48	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	16.12 ***	14.59	19.76	—
	[3.68] (0.007)	[138.32] (0.470)	[12.08] (0.174)	—
Lagged policy	4.25 ***	2.63	3.75 **	—
change (a_2)	[0.70] (0.003)	[2.67] (0.239)	[0.77] (0.022)	—
MSE	0.007	0.007	0.007	—
Votes proposal	3.82	4.98	4.95	—
No change	0.48	0.46	0.48	—

Note: See Table 3.9.

Table 3.14: Does the Voting Record Predict Policy Rate Changes? Estimates Using Simulated Data with $N = 6$ and $\rho_1 = 1.95$, $\rho_2 = -0.98$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

Model	Democratic	Consensual	Opportunistic	Mechanical
Baseline scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$,$\sigma_P = 0.25$)				
Skew (a_1)	17.56 ***	16.49	18.38	0.26
	[3.55] (0.000)	[8.67] (0.106)	[9.53] (0.106)	[5.09] (0.525)
Lagged policy	5.02 ***	3.75 **	4.19 ***	3.71
change (a_2)	[0.79] (0.003)	[0.67] (0.014)	[0.72] (0.009)	[1.61] (0.125)
MSE	0.006	0.006	0.006	0.005
Votes proposal	4.13	6.88	6.90	—
No change	0.47	0.47	0.48	0.48
High volatility scenario ($\sigma_u = 0.5$, $\sigma_C = 0.25$, $\sigma_P = 0.25$)				
Skew (a_1)	10.09 **	15.50	16.70	-0.21
	[3.10] (0.025)	[8.52] (0.142)	[8.74] (0.114)	[2.78] (0.543)
Lagged policy	4.30 ***	3.83 **	4.37 ***	3.98 **
change (a_2)	[0.45] (0.000)	[0.62] (0.020)	[0.46] (0.000)	[0.90] (0.017)
MSE	0.007	0.007	0.006	0.005
Votes proposal	5.33	6.88	6.89	—
No change	0.27	0.28	0.29	0.27
Bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.5$, $\sigma_P = 0.5$)				
Skew (a_1)	16.22 ***	17.79	17.53	—
	[3.56] (0.000)	[9.88] (0.129)	[10.26] (0.163)	—
Lagged policy	4.27 ***	3.54 **	3.70 **	—
change (a_2)	[0.70] (0.005)	[0.65] (0.018)	[0.76] (0.023)	—
MSE	0.007	0.007	0.007	—
Votes proposal	4.31	6.90	6.92	—
No change	0.46	0.46	0.47	—
P bad information scenario ($\sigma_u = 0.25$, $\sigma_C = 0.25$, $\sigma_P = 0.5$)				
Skew (a_1)	17.63 ***	21.99	18.10	—
	[3.95] (0.001)	[192.17] (0.369)	[9.84] (0.153)	—
Lagged policy	4.33 ***	3.05	3.81 **	—
change (a_2)	[0.71] (0.002)	[1.75] (0.139)	[0.78] (0.022)	—
MSE	0.007	0.007	0.007	—
Votes proposal	4.93	6.97	6.91	—
No change	0.48	0.46	0.47	—

Note: See Table 3.9.

Table 3.15: Does the Voting Record Predict Policy Rate Changes? Opportunistic Model with Simple Majority, Estimates Using Simulated Data with $N = 4$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

	$\rho = 0.95$	$\rho = 0.90$	$\rho = 0.99$	$\rho_1 = 1.95$, $\rho_2 = -0.98$
Baseline scenario				
Skew (a_1)	6.58	6.53	6.58	13.65 *
	[3.19] (0.142)	[3.22] (0.135)	[3.14] (0.123)	[6.03] (0.060)
Lagged policy	1.51	1.45	1.65 *	4.19 **
change (a_2)	[0.72] (0.120)	[0.75] (0.133)	[0.72] (0.083)	[0.72] (0.011)
MSE	0.024	0.025	0.025	0.006
Votes proposal	4.46	4.46	4.45	4.87
No change	0.45	0.46	0.44	0.47
High volatility scenario				
Skew (a_1)	1.82	1.77	1.76	12.12 *
	[1.78] (0.365)	[1.78] (0.363)	[1.79] (0.392)	[5.35] (0.075)
Lagged policy	0.37	0.28	0.40	4.33 ***
change (a_2)	[0.27] (0.267)	[0.27] (0.340)	[0.28] (0.236)	[0.46] (0.000)
MSE	0.026	0.026	0.026	0.006
Votes proposal	4.38	4.38	4.37	4.87
No change	0.22	0.23	0.22	0.27
Bad information scenario				
Skew (a_1)	5.11	5.71	5.23	13.22
	[3.11] (0.193)	[3.22] (0.187)	[3.09] (0.197)	[6.65] (0.123)
Lagged policy	0.70	0.72	0.76	3.69 **
change (a_2)	[0.59] (0.317)	[0.62] (0.317)	[0.57] (0.280)	[0.67] (0.015)
MSE	0.048	0.048	0.047	0.007
Votes proposal	4.56	4.56	4.56	4.90
No change	0.43	0.45	0.42	0.46
P bad information scenario				
Skew (a_1)	5.90	5.89	5.65	13.24 *
	[3.13] (0.162)	[3.24] (0.181)	[3.07] (0.148)	[6.36] (0.098)
Lagged policy	0.95	0.95	1.00	3.82 **
change (a_2)	[0.63] (0.237)	[0.66] (0.244)	[0.60] (0.186)	[0.68] (0.014)
MSE	0.044	0.043	0.043	0.007
Votes proposal	4.53	4.53	4.53	4.89
No change	0.44	0.46	0.43	0.46

Note: See Table 3.9. Values of σ_u, σ_C and σ_P depend on ρ, but correspond to those in previous tables.

Table 3.16: Does the Voting Record Predict Policy Rate Changes? Opportunistic Model with Simple Majority, Estimates Using Simulated Data with $N = 6$
$\Delta p_{t+1} = a_0 + a_1 skew_t + a_2 \Delta p_t + u_{t+1}$

	$\rho = 0.95$	$\rho = 0.90$	$\rho = 0.99$	$\rho_1 = 1.95$, $\rho_2 = -0.98$
Baseline scenario				
Skew (a_1)	7.66	7.35 *	7.73	13.51 *
	[3.36] (0.109)	[3.37] (0.090)	[3.34] (0.115)	[5.90] (0.065)
Lagged policy	1.99 *	1.88 *	2.13 **	4.24 **
change (a_2)	[0.79] (0.052)	[0.81] (0.063)	[0.79] (0.047)	[0.73] (0.014)
MSE	0.024	0.024	0.024	0.006
Votes proposal	6.21	6.21	6.21	6.81
No change	0.46	0.47	0.46	0.47
High volatility scenario				
Skew (a_1)	2.11	2.03	2.10	12.43 *
	[1.86] (0.330)	[1.88] (0.333)	[1.88] (0.332)	[5.21] (0.060)
Lagged policy	0.44	0.38	0.50	4.39 ***
change (a_2)	[0.29] (0.237)	[0.29] (0.288)	[0.29] (0.202)	[0.46] (0.000)
MSE	0.023	0.023	0.023	0.006
Votes proposal	6.06	6.08	6.06	6.80
No change	0.22	0.23	0.22	0.27
Bad information scenario				
Skew (a_1)	5.65	6.08	5.58	13.23
	[3.16] (0.170)	[3.28] (0.166)	[3.13] (0.183)	[6.27] (0.104)
Lagged policy	0.85	0.86	0.92	3.76 **
change (a_2)	[0.62] (0.260)	[0.65] (0.305)	[0.59] (0.243)	[0.68] (0.015)
MSE	0.047	0.047	0.047	0.007
Votes proposal	6.34	6.35	6.35	6.85
No change	0.43	0.45	0.42	0.46
P bad information scenario				
Skew (a_1)	6.14	6.51	6.01	13.53 *
	[3.18] (0.156)	[3.30] (0.156)	[3.14] (0.147)	[6.11] (0.083)
Lagged policy	1.04	1.10	1.14	3.86 **
change (a_2)	[0.64] (0.216)	[0.69] (0.218)	[0.63] (0.170)	[0.69] (0.013)
MSE	0.044	0.044	0.044	0.007
Votes proposal	6.31	6.32	6.31	6.83
No change	0.43	0.46	0.43	0.46

Note: See Table 3.9. Values of σ_u, σ_C and σ_P depend on ρ, but correspond to those in previous tables.

3.A2 Data

Voting records

Voting records were collected from the following central banks (start and end dates of the sample in brackets): the Czech Republic (1998:1-2008:12), the United Kingdom (1997:6-2009:2), Hungary (2005:10-2009:2), Poland (2000:2-2008:12), Sweden (1999:1-2009:2) and the U.S. (1970:2-1996:12). Typically, voting data are available at a monthly frequency. Except for the U.S., the data are publicly available on the central banks' websites. The U.S. data come from Chappell et al. (2005) and are only partially available on the Fed website.

As regards the Czech Republic, the 1998:1-2000:4 voting results were available only in transcripts that are published with a 6-year delay. Therefore, the baseline estimates for this country are based on the data from 2000:7 onwards. In addition, the baseline estimates for the Czech Republic are restricted until 2006:7 in the specification with financial market expectations. The reason is that from this period onwards the voting record was released only about 3 hours after the monetary policy decision was announced. The monetary policy decision was typically announced at around 1 p.m. and the voting ratio was released at around 3.30 p.m. at a press conference. In principle, the interbank rates could have been collected at, say, 2 p.m. and therefore more recent data could have been used as well, but it has to be emphasized that the interbank market was not very liquid during the financial crisis. In light of this fact, we restrict the data for the Czech Republic to the period until 2006:7.

All the U.S. data are from Chappell et al. (2005), who code the policy preferences of individual FOMC members based on the transcripts of the FOMC monetary policy meetings. The desired federal funds rate is available directly from the records in 80.1% of cases under the Burns chairmanship and in 92.4% of cases under the Greenspan chairmanship. By available directly, Chappell et al. (2005) mean that the individual member explicitly stated the desired range for the policy rate or explicitly expressed a preference for the staff policy scenario or another committee member with an explicit target range for the federal funds rate. Each individual's desired funds rate is calculated as the mid-point of the reported range. In the remaining 19.9% and 7.6% of cases respectively, where the preferred policy rates are not observed, the textual record of committee deliberations (lean for ease, lean for tightness or assent with staff proposal) is used to code the member's policy positions. The coding is complemented with the estimation of individual reaction functions, where the reaction functions are used to calculate expected values for the desired funds rates, conditional on the information provided by leaning positions. For the U.S., we are able to calculate the *skew* both for voting members and for alternate members, who are present at the policy meeting but do not have voting power. Neither of these two skew measures is available to the public in a timely fashion. Nevertheless, the committee bias was announced from 1983 to 1999 in official Fed statements on how the Fed was leaning in terms of its next interest rate move, and the variable is coded so that a higher value indicates an upward move of interest rates.

Interbank rates

Interbank rates are collected to capture financial market expectations. The source of the data is Datastream. Specifically, we use PRIBOR rates for the Czech Republic, BUBOR rates for Hungary, WIBOR rates for Poland, STIBOR rates for Sweden and LIBOR rates for the UK for the following maturities: 1 month, 3 months and 12 months. U.S. interbank rates are not used due to significant lags in publishing the minutes and the transcripts (both were published after the subsequent meeting in our sample).

3.A3 Central Banks' Voting Record Release Schedules

Czech National Bank

The Bank Board meets on Thursdays.[14] A press conference with a presentation containing the voting ratio (without the names) takes place the same

[14] There are some exceptions to the described organization of monetary policy decision-making processes for all the central banks, typically because of national holidays. For example, in the case of the Czech National Bank, the board usually meets on a Thursday. In exceptional cases, however, it may meet on a Wednesday instead of a Thursday because of holidays. Since 4/2005, the minutes have been published 8 days after the meeting. In the case of holidays, the minutes can be published more than 8 days after the meeting.

day in the early afternoon.

Until 8/2006, the voting ratio was not disclosed at the press conference.

The minutes are released the next Friday (+8 days). They contain the voting ratio, and since 1/2008 have also included the names explicitly.

Until 4/2005, the minutes were released on Tuesdays, two weeks after the meetings (+12 days).

Bank of England

The Monetary Policy Committee decides during a two-day meeting that takes place on Wednesdays and Thursdays. A press release of the decision follows at midday on Thursday.

The minutes are released two weeks later, on Wednesdays (+13 days). They contain the voting record with names.

Magyar Nemzeti Bank

The Monetary Council meets on Mondays. A press release of the decision follows on Monday at 3 p.m.

The minutes are released 2-4 weeks after the decision, usually on Wednesdays. They contain the detailed voting record with names.

National Bank of Poland

The Monetary Policy Council decides during a two-day meeting that takes place on Tuesdays and Wednesdays. A press release of the decision follows on Wednesday.

The minutes are released on Thursdays in the week before the next MPC meeting, which means 3-4 weeks after the decision.

The MPC meeting minutes do not contain the voting records. The voting records are published only later, in the quarterly inflation reports. If the repo rate was changed, the voting record is first published in the Court and Economic Gazette of the Ministry of Justice and only after that in the inflation report. Voting records have to be published in the Court and Economic Gazette no sooner than 6 weeks and no later than 12 weeks after the voting took place.

Sveriges Riksbank

The Executive Board meets on Mondays or Wednesdays. A press release of the decision follows the same day.

The minutes are released approximately two weeks later (+14, or occasionally +15, days). They contain a detailed voting record with names.

U.S. Fed

All the U.S. data are from Chappell et al. (2005); see Appendix 3.A2 for details.

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