London School of Economics and Political Science

Essays in Wealth Inequality

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A thesis submitted to the Department of Economics of the London School of Economics for the degree of Doctor of Philosophy.

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I declare that my thesis consists of 40,000 words.

Statement of co-authored work

I confirm that Chapter 3 is joint work with Tomer Ifergane and I contributed 50% of this work.

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Abstract

Wealth inequality in the United States is large and has been increasing in the last five decades. In my thesis I try to help our understanding of both of these facts.

The first chapter shows that portfolios of US households changed recently, and argues that these changes contributed to the increase in wealth inequality. The portfolio changes are characterised by an increase in investment in equity in the top 1% of the wealth distribution, and an increase in debt to invest in real estate at the bottom 50%. With an accounting exercise I show that, as equity had a higher return than other assets in the same period, wealth inequality increased as a result of these portfolio movements.

In the second chapter I propose that portfolio choices are important for understanding the impact of income taxes on wealth inequality. I develop an analytical model and show that when taxes fall households want to invest more in high-risk, high-return assets, just like households at the top 1% did with equity. I then construct a quantitative model of US households' portfolio choice and wealth inequality and show that portfolio choices amplify the impact of the observed fall in top income taxes on wealth inequality in a quantitatively significant way.

Finally, one of the most striking features of the wealth distribution in the US is the racial wealth gap between Black and White households. In the third chapter we develop a model with endogenous entrepreneurship choice and wealth inequality, and use it to quantify the impact of racial gaps in wages and business start-up costs. We find that the wage gap is particularly important in explaining gaps at the bottom of the wealth distribution, while entrepreneurship is essential to explain gaps at the top. Furthermore, changes in the racial wealth gap are slow in our setting, due to the long time required for Black households to reach the top 10% of wealth. Finally, we discuss the impact of reparations and demonstrate that, while they are not successful at speeding the transition towards permanent equality, they help make inequality smaller during the transition.

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Chapter 1

Portfolio Changes and Wealth Inequality Dynamics in the US from 1989 to 2019

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Abstract

This paper highlights how portfolios of US households have changed from 1989 to 2019 and investigates the role played by these changes on the evolution of wealth shares. First, it shows that there were different portfolio movements across the wealth distribution: the Bottom 50% increased their leverage to invest in Real Estate, while the Top 1% shifted their portfolio towards Public and Private Equity. Second, it uses an accounting exercise to investigative the effect of portfolio changes on wealth inequality. The results show that portfolio change effects are large, and account for 0.9 p.p. of the total 1.5 p.p. decline in the wealth share of the Bottom 50%, and for 2.4 p.p. of the total 7.8 p.p. increase in the wealth share of the Top 1%.

Keywords: Wealth Distribution, Household Finance.
J.E.L. codes: E21, G50.

1.1 Introduction

The study of wealth inequality has received a renewed interest, partly due to the increase in the share of aggregate wealth held by individuals at the top of the distribution in the US. Direct explanations for it have included the increase in labor income inequality, changes in saving rates by wealth groups, and a rising capital income inequality, while more fundamental explanations consider technological change, tax reforms and a fall in the risk-free rate.[1] There has also been a great focus on the role of portfolios in explaining wealth inequality in a static sense. Instead, this paper investigates the role of changes in households' portfolio choices in explaining trends in inequality, and how they magnify the impact of tax changes.

Households' portfolio in the US is mostly composed of real estate, equity and safe assets, but it is not static. In this paper I document how the aggregate portfolio of US households has changed in a sizeable way from 1989 to 2019, and also in different ways along the wealth distribution. Given that capital income is important for the accumulation of wealth, particularly at the top, these movements could have had an impact on wealth inequality. To that end, I perform an accounting exercise that allows me to estimate a counterfactual in which portfolios compositions are kept fixed at their 1989 levels. The results confirm that the portfolio change effect is large relative to the actual wealth share changes, and they were mostly drive by the Top 1% of the wealth distribution moving towards high-risk, high-return assets more than those households in the other parts of the distribution.

The recent increase in wealth inequality in the US can be observed in Figure 1. While the exact magnitude of the increase in inequality is still a matter of debate,[2] it is clear

[1]See Eisfeldt, Falato, and Xiaolan (2022), Kaymak and Poschke (2016), Sargent, Wang, and Yang (2021) for papers that highlight the importance of labor income and human capital; De Nardi and Fella (2017), Mian, Straub, and Sufi (2021) and Saez and Zucman (2016) for examples of papers that focus on saving rates; Moll, Rachel, and Restrepo (2021) for the role of technological change; and Hubmer, Krusell, and Smith (2021) for the role of changes in income taxes.

[2]See Bricker et al. (2016) and Kopczuk (2015) for a discussion on the merits of the "capitalisation method" of Saez and Zucman (2016) compared to survey data from the Survey of Consumer Finances (SCF). Smith, Zidar, and Zwick (2020) highlight the importance of heterogeneous returns when attributing wealth to income and Saez and Zucman (2020) provide updated estimates.

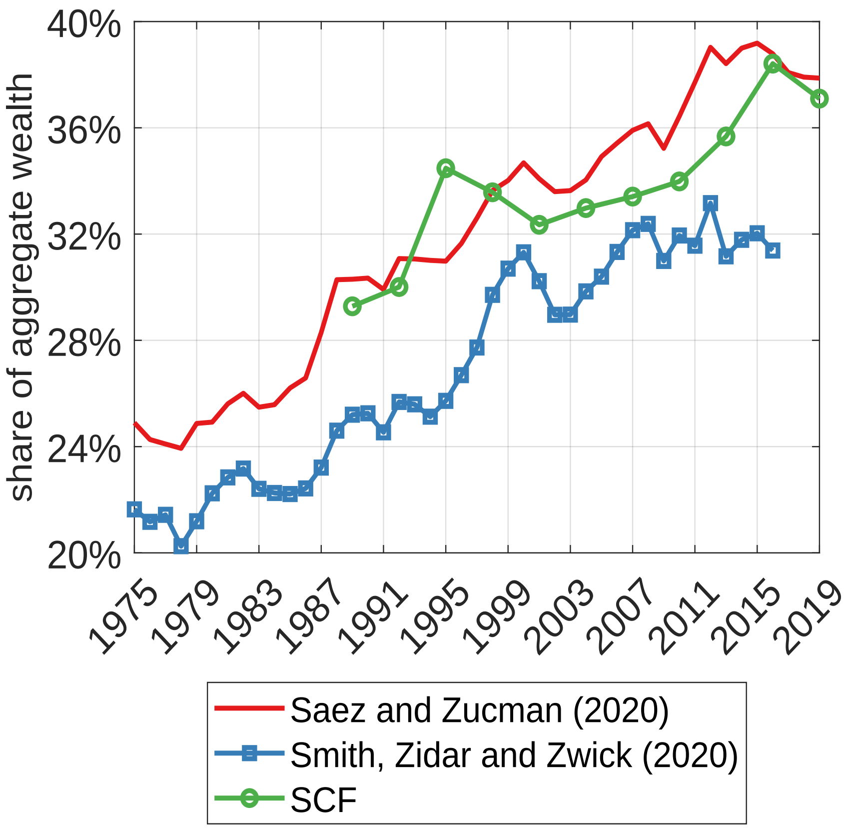

Figure 1.1: Wealth share of the top 1% in the US

Notes: This figure shows the evolution over time of the share of aggregate wealth held by the Top 1% of the wealth distribution. *Source*: Saez and Zucman (2020), Smith, Zidar, and Zwick (2020) and author's calculation using SCF data.

that the shares of aggregate wealth held by those at the top of the wealth distribution started to increase around the late 1970's and early 1980's, and have not stopped since. This paper makes two main contributions.

First, using the SCF from 1989 to 2019 I document how not only the aggregate portfolio has changed, but also that portfolios have changed in different ways along the wealth distribution. On one hand, in the Bottom 50% of the distribution there was an increase in leverage to invest in housing assets, which was already the major asset. On the other hand, at the very top of the distribution there was an even larger concentration on equity investments, which is historically the asset with most risk and return.

Second, I combine the SCF with external data sources to calculate the return on different asset classes at different points of the wealth distribution. This allows me to perform an accounting exercise to estimate the effect of portfolio changes, which captures how the wealth share of a particular group changed due to the fact that the composition of their portfolio has changed, keeping everything else fixed. I show how this effect was negative for most of the distribution, and positive only within the Top 1%, and that it is large when compared to the total change of wealth shares. For example, it explains 2.4 p.p. of the total 7.8 p.p. increase of the Top 1% wealth share, and -0.9 p.p. of the total -1.5 p.p. change of the Bottom 50% wealth share from 1989 to 2019.

Fundamentally, this study argues that changes in portfolio are important to understand the dynamics of wealth inequality, and thus the reasons behind these movements deserve more discussion. While some focus has been given to the importance of the movement in rates of return, I highlight how portfolios are not fixed, and that it is important to understand what caused them to change if one is interested in understanding or addressing the dynamics of inequality.

Moreover, I also make two contributions to the literature that estimates synthetic saving rates. First, I show how these estimates are affected by not taking into account movements in between wealth groups. With the panel version of the SCF between 2007 and 2009, the results reveal that estimated saving rates at the top fall, and at the bottom rise. Also, using an estimate of growth of wealth in the Top 1% due to between movements from Gomez (2021), the baseline estimate for annual saving rate at the top of distribution falls from 29.3 to 19.4% per year in the 1989 to 2019 period. Second, I also show how the particular choice for partitioning the wealth distribution is relevant for analyses that interact the saving rate with other variables that they might be correlated with along the wealth distribution, e.g., rates of return.

1.1.1 Literature Review

Many authors have posed explanations for the high levels of wealth inequality that also highlight the effects of capital income. Examples include Bach, Calvet, and Sodini (2020), Benhabib, Bisin, and Luo (2019), Cagetti and De Nardi (2006), Fagereng et al. (2020), Hubmer, Krusell, and Smith (2021), Kacperczyk, Nosal, and Stevens (2019), and Xavier (2021). However, they focus on explaining the high level of wealth inequality at a given point in time, and not on the dynamics of wealth inequality, which is the focus of this paper.

In addition to the papers just mentioned, there are other authors that highlight how certain shocks affect wealthier and poorer households in different ways because the composition of their portfolios is different, which then has consequences for wealth inequality dynamics. For example, the impact of a change in the risk-free rate on households depends on the duration of their portfolios (Greenwald et al., 2021), and the evolution of the volatility of private firms is specially important at the top of the wealth distribution (Atkeson and Irie, 2020). Also, the relative movement over time for the rates of the return of equity and housing is an important variable since the Bottom 90% of the distribution is heavily invested in housing, while the top is more concentrated in equity (see Cioffi, 2021; Diwan, Duzhak, and Mertens, 2021; Feiveson and Sabelhaus, 2019; Kuhn, Schularick, and Steins, 2020; Martínez-Toledano, 2022).

In particular, Kuhn, Schularick, and Steins (2020) and Martínez-Toledano (2022) perform counterfactual empirical exercises that are similar to the decomposition of Section 2. However, they are mostly interested in what wealth inequality would have been had rates of return been different but portfolio compositions kept fixed as seen in the data. This is fundamentally different from the exercise of this paper, which is to keep rates of return as observed in the data, while using counterfactual portfolio compositions, highlighting how households changed their portfolios in the way that they did, and asking what was their impact on inequality. Martínez-Toledano (2022) also analyses the impact of changes in portfolio compositions for the case of Spain, and argues that wealthier households are better able to shift their portfolios after housing busts. I find similar results for the US, but also focus on the impact of longer-term trends in asset composition all along the wealth distribution.

The rest of the paper is organised as follows. Section 2 describes the data used to construct portfolios and rates of return across the wealth distribution and across time. Section 3 presents the estimates of the effect of portfolio changes on wealth shares, and several possible extensions. Section 4 concludes.

1.2 Data

The Survey of Consumer Finances (SCF) by the Federal Reserve in the United States is the main data source for this paper. It runs every three years, with eleven total waves from 1989 to 2019, which covers most of the period of the recent increase in wealth inequality. The SCF is specially useful for this study because it has extensive information on the wealth components of its respondents, which allows researchers to construct detailed portfolios for each part of the wealth distribution in the US.

One of the main concerns with using survey data when looking at the increase in wealth inequality is that it is necessary to have good data quality for the wealthiest individuals, given that a great share of the increase in wealth inequality in the last decades went to those at the very top. The SCF is particularly designed to get a clear picture of the wealthiest households, and it oversamples the top of the distribution - more than 10% of its respondents are in the top 1% of the wealth in recent waves. One might worry then that the SCF incorrectly measures the top of the distribution due to higher non-response rates and that it does not correct for it with its weighting scheme, but Kennickell (2009) finds no evidence of non-response bias on observables, including wealth.

However, for privacy concerns, individuals in the Forbes 400 are explicitly excluded from it.[3] One possible avenue chosen by other studies is to complement the data of the SCF with data from the Forbes 400 lists (e.g., see Smith, Zidar, and Zwick, 2020) by adding their wealth to the SCF total. The focus of this study is the portfolio composition of households' wealth and there is no detailed information on the portfolio holding of the members of the Forbes 400, so I do not add their wealth to the SCF total.

1.2.1 Wealth and Portfolios

Wealth is defined as total marketable assets minus total liabilities of a household, which captures best how well they would be able to use their wealth to weather negative income shocks. Importantly, it does not include human capital in the form of discounted future wages, nor promised future transfers, as government services.[4] Keeping in line

[3] Although in many years there are households in the SCF that are wealthy enough to have been included in the Forbes 400.

[4] See Sabelhaus and Volz (2020) for what happens when one includes Social Security wealth.

with the SCF definitions and the concept of marketable wealth, defined contribution pension plans are included, as well as defined benefit plans that households are able to draw from or borrow against.

Using the disaggregated information for each household, I classify wealth components into six asset categories, and those are the most disaggregated ones that allow me to calculate returns over time, in a methodology that mostly follows Xavier (2021). Moreover, debt is classified as a separate liability and not as negative asset, even if attached to a collateral. Thus, "Real Estate" wealth includes only the value of real estate properties, while mortgages are included with other liabilities in "Debt". The choice for gross exposure instead of net exposure is to better captures the risk and return characteristics of the portfolio: a leveraged household with a 75% loan-to-value ratio in a $400,000 house has a very different portfolio than another household with no debt and a house worth $100,000, even though they have the same net worth.

Furthermore, in comparison to other studies that use income tax data, the SCF allows for the inclusion of durable goods (mostly vehicles), which do not generate income. Although they are not large at the aggregate level, they are an important component of wealth at the bottom of the distribution, especially in recent years, which makes the distribution slightly less unequal. Table 1.1 shows the six categories for asset classes and its main components.

With the classification from Table 1.1, I construct portfolios for the aggregate economy and also along the wealth distribution. Figure 1.2 shows the composition over time of the aggregate portfolio of US households. The main takeaway is that portfolios are not static over time. For example, Public Equity has show a marked increase in importance of 17 p.p. from 1989 to 2019, while Real Estate displays a more cyclical pattern: decrease in the 1990's and 2010's, but an increase leading up to the housing boom before the Great Financial Crisis.

However, what Figure 1.2 does not show is the heterogeneous portfolio movements along the wealth distribution. Figures 1.A.3 and 1.A.4 in the Appendix show the evolution of the portfolio shares for each assets class in each wealth group, and we can observe that portfolio share not only are different across the wealth distribution, but they move in

Asset Class	Main Components
Real Estate	primary residence, second houses, investment properties, commercial real estate
Fixed Income	checking and savings accounts, directly held bonds and certificates of deposit, fixed income in pensions and fixed income in mutual funds
Debt	mortgages, home equity credit, student debt, credit card, vehicle debt, consumer credit
Other	vehicles, art, other financial assets
Public Equity	equity in hedge funds, directly owned stocks, equity in pensions
Private Equity	privately owned businesses, managed or not

Table 1.1: Classification of assets and liabilities from the SCF

Notes: This table shows the methodology used to group components of wealth from the SCF into the six main asset classes used in this paper.

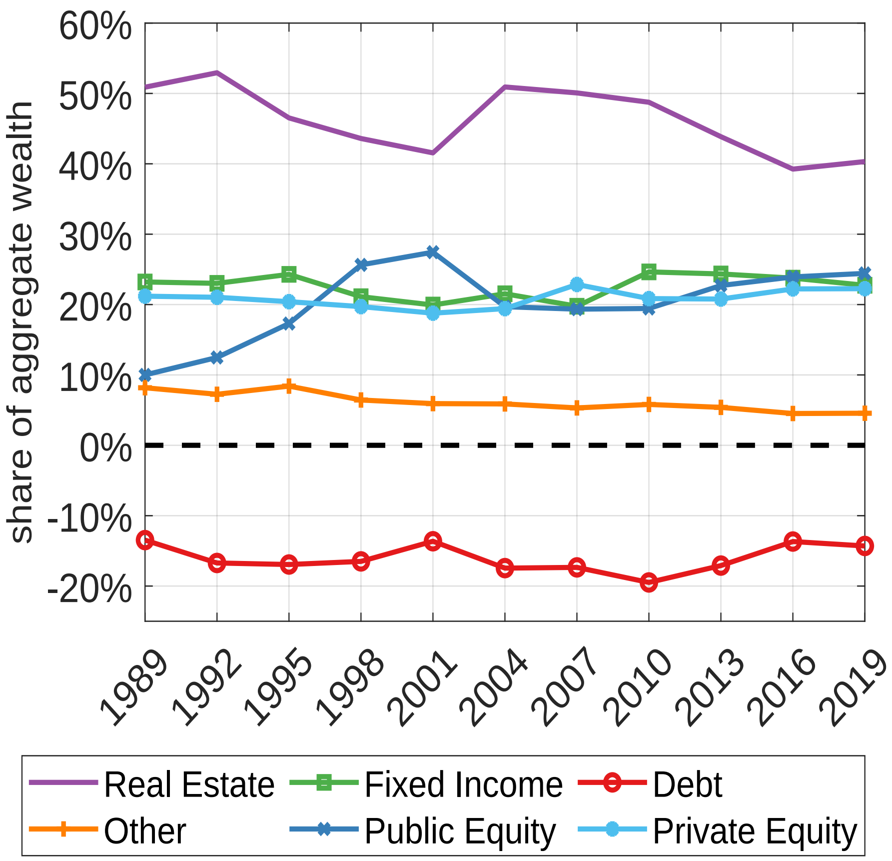

Figure 1.2: Composition of the aggregate portfolio of US households

different ways as well. Figure 1.3 summarises the total change in portfolio shares from 1989 to 2019, and we can see that those movements can be quite different at opposite ends of the distribution.[5] Panel B shows that the Top 1% had a sizeable increase in their allocation in Equity from 1989 to 2019. That was compensated by a reduction in the importance of all the other assets, particularly Real Estate. For fixed rates of return, this movement increased the expected return and volatility of their portfolio, contributing for an increase in wealth inequality, as we are going to see in Section 1.3.

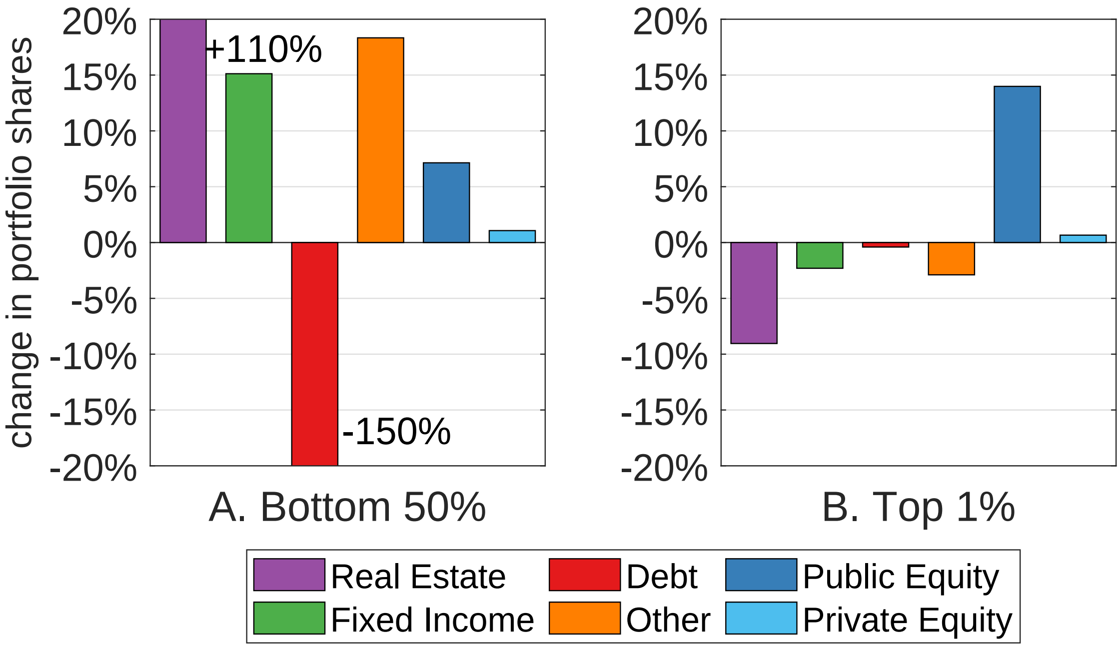

Figure 1.3: Change in portfolio share in each asset class, between 1989 and 2019

Notes: This figure shows the change in the wealth share of each asset, class from 1989 to 2019, in the portfolio of those in the Bottom 50% and those in the Top 1% of wealth.

Furthermore, Panel A of Figure 1.3 shows that the evolution of the portfolio for the bottom 50% was completely different. Even though there was a (smaller) increase in the allocation in Equity as well, the dominating feature was the increase in the importance of Real Estate, financed by an increase in leverage.

One might wonder whether these movements can be fully explained alone by capital gains, or if they were active decisions of households. First, it would be unreasonable to expect capital gains to be so different across the wealth distribution as to generate the

[5]Figure 1.A.5 does the same for those individuals between the 50th and 90th percentiles and those between the 90th and 99th percentiles. Figure 1.A.2 complements it, and compares the composition of the portfolio along the whole distribution in 1989 and 2019.

diverging movements of Figure 1.3. Second, as Figure 1.A.3 in the Appendix shows, the largest increase in the portfolio share of Real Estate for the Bottom 50% came in the years between 2007 to 2010 in the aftermath of the housing crisis, when house prices moved in the opposite direction and fell in the US. This rise in the portfolio share is explained by the increase in debt in the three year period, that reduced total wealth at the Bottom 50% more than the fall in real estate prices.[6] Third, most of the increase across the whole period is also explained by larger mortgages and debt at the Bottom 50%, which financed more expensive houses. For example, according to the SCF, for the Bottom 50% mortgages and home equity lines represented on average 56.1% of the value of their primary residences in 1989. This value increased to 66.7% in 2019, at the same time that Real Estate increased from 127.6% to 237.7% of wealth. Thus, arguably it is the increase in leverage at the bottom of the distribution that can explain in large part the dynamics of the portfolio share of Real Estate, and not capital gains.

1.2.2 Returns

Total return is composed of both dividends and capital gains. For dividends, the SCF has many questions that can be used to estimate them. Capital gains, however, are harder to measure because the SCF does not have a panel structure, nor asks questions about investment expenditures that are necessary to uncover true capital gains. Thus, I use many outside sources to complement the SCF and estimate total returns for different asset classes. Broadly speaking, because most of the dividend information comes from the SCF I am able to estimate them on a per-wealth group basis and to have heterogeneous rates of return for the same asset class, while capital gain comes from outside sources and are assumed to be the same for all households, for a given asset class.

I follow the methodology of Xavier (2021) for most asset classes, and the strategy for private equity in particular borrows from Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014). I make only small deviations to previous methodologies: (i) for the yield (rental rate) on housing, I use a series from Jordà et al. (2019) that has the

[6] Martínez-Toledano (2022) highlights how wealthier households are better able to move away from real estate assets after a housing bust, when compared to poorer households.

benefit on including owner-occupied residences; (ii) I assume negative depreciation on vehicles, which are an important component of wealth at the bottom of the distribution, using an estimate from Gitiaux et al. (2012); and (iii) I exclude businesses with negative valuation.[7]

Using data from the SCF and external sources, the construction of asset returns is as follows:

Real Estate: Capital gains come from the Case-Shiller Home Price Index. For dividends, the SCF provides rental income only for rental properties. However, it is important to impute rent to homeowners as well. I use the rental rate series from Jordà et al. (2019), who estimate it for the US for the period of this analysis and take into account owner-occupied dwellings.

Fixed Income: Capital gains are assumed to be equal to zero, while dividends come from the income generated by interest-earning assets as reported by the SCF.

Debt: Capital gains are assumed to be equal to zero, and the interest rate on debt is a weighted average of the annual interest rates reported by households in the SCF on their different types of debt.

Public Equity: Capital gains are derived from the Wilshire 5000 price return index. Dividend income comes from the SCF, which reports the dividends from stocks, mutual funds, etc., paid to shareholders.

Private Equity: Capital gains come from the change in value of private businesses in the SCF, which I then correct for the creation of new businesses (those founded less than three years before, as reported in the SCF as well), and for new IPOs (which comes from Jay Ritter's website). Dividends are the total profits generated by these businesses as reported by the SCF, which I then correct for corporate taxes, reinvestment and imputed wages to active managers in the same way as in Moskowitz and Vissing-Jørgensen (2002).

[7] Businesses with negative net worth are excluded because of the impact they have when estimating rates of return for equity at the bottom of the distribution, which would be negative with an unreasonably large magnitude. As private equity is a small component of the wealth of the bottom 50%, it does not affect the estimation of the overall rate of return on their wealth. However, when combined with changes in composition over time, it can impact the accounting exercise. Taking into account businesses with negative valuations would make the portfolio change effect found later in Section 1.3 even more negative at the bottom 50% as it increased in importance over time, while not affecting directly the other wealth groups.

Other: The largest component of "Other" assets for most of the wealth distribution is vehicles, and they are an important component of wealth at the bottom of the distribution. For its total return I use an estimate of -8% for the depreciation of used cars from Gitiaux et al. (2012). For the non-vehicle component of "Other" assets I assume a zero total return, for lack of better information. Different assumptions should not change the results significantly, as these assets are not a significant component of wealth in any part of the wealth distribution.

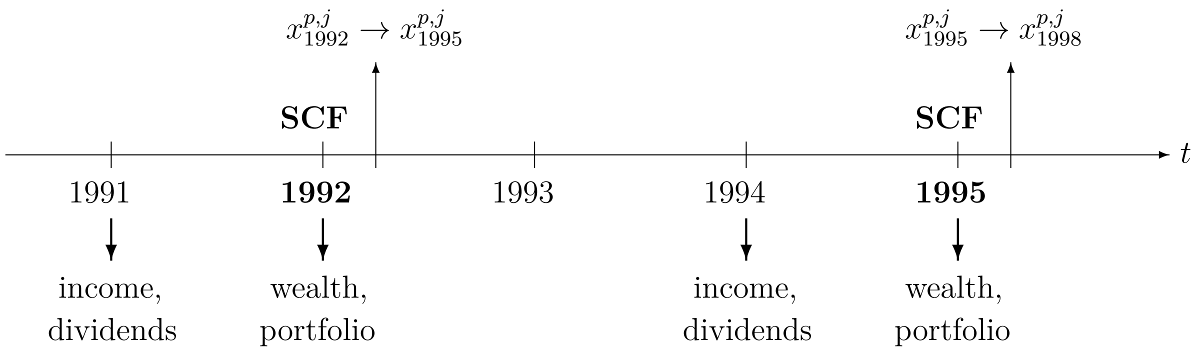

Figure 1.4: Timing of information in the SCF and in the Accounting Exercise

The timing of the information across SCF surveys is shown in Figure 1.4. For example, 1992 was a year with a wave of the SCF, in which households were asked about their current level of wealth in 1992. They were also asked about their income in 1991, including labour income, transfers and income (dividends) derived from assets such as bonds, public equity and private equity. Let $W_t^{p,j}$ be the total wealth of group p (e.g., the Top 1% in the wealth distribution) held in asset class j at the time of a SCF wave t; and let $D_t^{p,j}$ denote the dividends generated by the asset in the previous year, adjusted for inflation. In a methodology similar to Kartashova (2014) and Xavier (2021), I estimate the annual dividend yield $r_{t+1}^{p,j}$ between waves t and $t+1$ of the SCF in the following way:

$$\begin{aligned} 1+\tilde{r}_t^{p,j} &= D_t^{p,j}/W_t^{p,j}, \\ 1+\tilde{r}_{t+1}^{p,j} &= D_{t+1}^{p,j}/W_{t+1}^{p,j}, \\ r_{t+1}^{p,j} &= \sqrt{(1+\tilde{r}_t)(1+\tilde{r}_{t+1})}. \end{aligned} \tag{1.1}$$

Thus, the yield between waves $r_{t+1}^{p,j}$ is the geometric average of the yields $\tilde{r}_t^{p,j}$ and $\tilde{r}_{t+1}^{p,j}$

for each of the two waves. Because I am able to compute capital gains exactly for the period between two waves of the SCF there is no adjustment needed. Let q^j_{t+1} denote the annualised capital gains between waves t and $t+1$ (notice that they do not differ along the wealth distribution). The total annual return is then equal to $(1 + r^{p,j}_{t+1})(1 + q^j_{t+1}) - 1$.

Figure 1.5 shows the annual nominal realized returns in the years between two waves of the SCF using the methodology described above.[8] It shows that both types of equity are by far the asset classes with the more volatile returns, and that private equity was the best asset class in terms of average returns over the period, as documented by Kartashova (2014). It is also interesting to see the decline in the nominal returns for fixed income and debt, which were larger than the decline in inflation, and points to lower real rates of return now than in 1989.

1.2.3 Heterogeneous Returns

One important feature of the rates of return in the data highlighted by Bach, Calvet, and Sodini (2020) and Fagereng et al. (2020) for Scandinavian countries, and by Xavier (2021) for the US using the SCF as well, is that they are increasing in wealth, even within asset classes as defined in Table 1.1. Table 1.2 shows the estimate average annual returns for the period of 1989 to 2019 for different wealth groups and assets classes. Notice that, by assumption, capital gains are the same across wealth groups; that capital gains for Fixed Income, Debt and Other assets are all equal to zero; and that dividends for Real Estate are the same across the wealth distribution.

We can see from Table 1.2 that in the exercise of Section 1.3 the total rates of return will indeed be increasing in wealth due to dividends being increasing in wealth for most asset classes. That is the case for Fixed Income and Public Equity, also the cost of Debt is decreasing in wealth, and for Other assets the return is less negative at the top due to a decreasing share of Vehicles as a component of Other assets along the wealth distribution.

The results for Private Equity warrant a brief discussion. First, the two main factors

[8] Figure 1.A.6 compares my resulting series of returns to other sources.

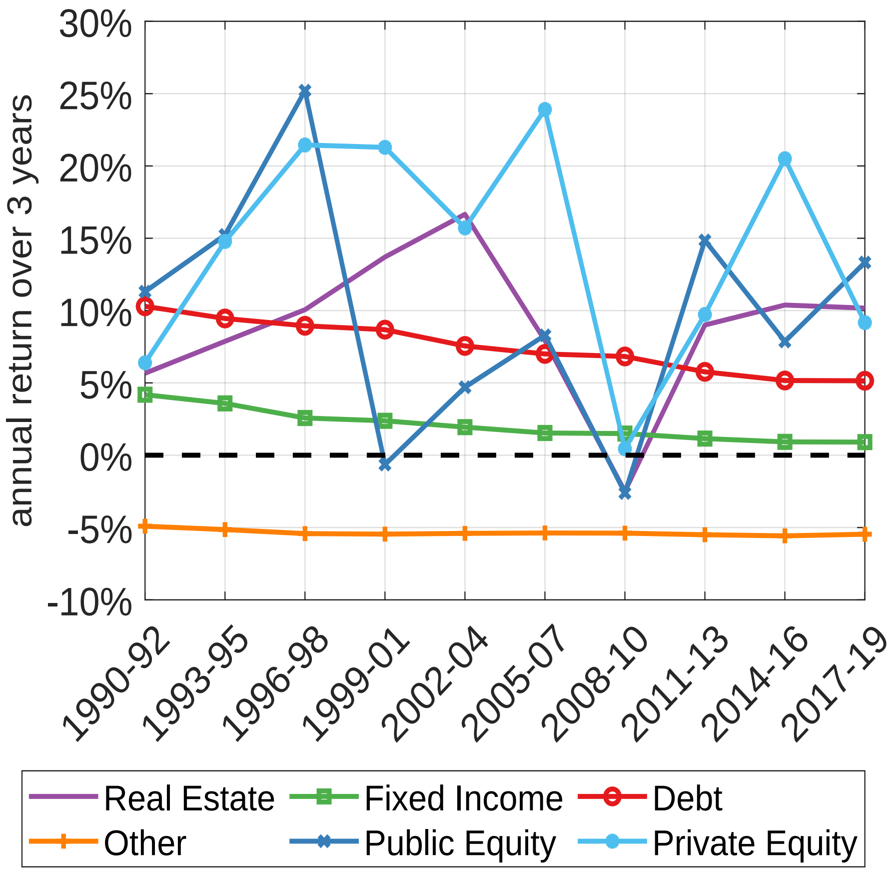

Figure 1.5: Annualised total nominal returns

Notes: This figure shows the annualised total (including capital gains and dividends) nominal return in between SCF waves for the aggregate portfolio of US households, using the methodology described in Section 1.2.2.

Dividends	Real Estate	Fixed Income	Debt	Other	Public Equity	Private Equity
Bottom 50%	5.3	1.2	8.7	-7.5	1.3	-57.0
50^{th}-90^{th}	5.3	1.4	7.7	-6.5	1.3	-2.0
90^{th}-99^{th}	5.3	2.3	7.0	-4.0	1.8	12.8
Top 1%	5.3	3.3	6.5	-1.9	2.1	8.7
Capital Gains	3.4	0.0	0.0	0.0	7.7	4.9

Table 1.2: Annualised dividend rates and capital gains by asset classes, in p.p.

Notes: Real Estate dividends are assumed to be the same across the wealth distribution, and capital gains are assumed to be equal to zero for Fixed Income, Debt and Other. See Section 1.2.2 for the detailed description on the estimation of these returns.

explaining its highly negative return at the bottom is that there are some large businesses posting big losses, and also that imputed wages for entrepreneurs are larger than the profits of the firm for many businesses, thus driving down economic profits.[9] However, because Private Equity is not an important component of the portfolio at the bottom of the distribution, these massive negative returns do not reduce their aggregate return significantly. Second, notice that dividends for Private Equity fall at the Top 1%. It could be that the mix of total return between capital gains and dividends of Private Equity (and other asset classes) differs along the wealth distribution, with a higher share of it being Capital Gains at the very top (for example, due to tax planning). If that is the case, then the portfolio effects in Section 1.3 are underestimated, as the Top 1% has moved their portfolios towards Private Equity, but I would be underestimating the benefit of that.

Finally, one possible way of measuring increasing returns to wealth over the whole portfolio (as opposed to within asset classes) is to calculate what would be the total return over the wealth distribution if all wealth groups held the same portfolio, equal to the aggregate. Thus, any differences in total returns are not due to portfolio composition, which is being kept fixed across the distribution, but due to return heterogeneity. Figure 1.6 shows the result of this exercise, and how households at the top of the wealth distribution can achieve a premium of more than +2% when compared to the bottom 50%, when both are holding the same portfolio.[10]

[9] It could be that when entrepreneurs start firms they earn profits that are smaller than what they would earn as wages on the labour market, but that investment pays off in the long term when the business grows and the owner is now observed at the top of the distribution. Unfortunately, due to the data not having a panel structure this a dimension that the exercise in this paper cannot capture.

[10] One important detail is that for Figure 1.6 I use the rates of return on Public Equity for Private Equity as well. This is because the return for Private Equity at the bottom of the distribution is negative and quite large in magnitude. Given that Private Equity is a small share of the portfolio of the bottom 50%, this does not affect significantly the calculations for the total return of the bottom 50%, nor the Accounting Exercise. However, when applied to the average portfolio across the wealth distribution, the Private Equity share increases and its impact is large. If I had used the returns for Private Equity, the gradient would be greater at the bottom of the distribution, and flatter at the top.

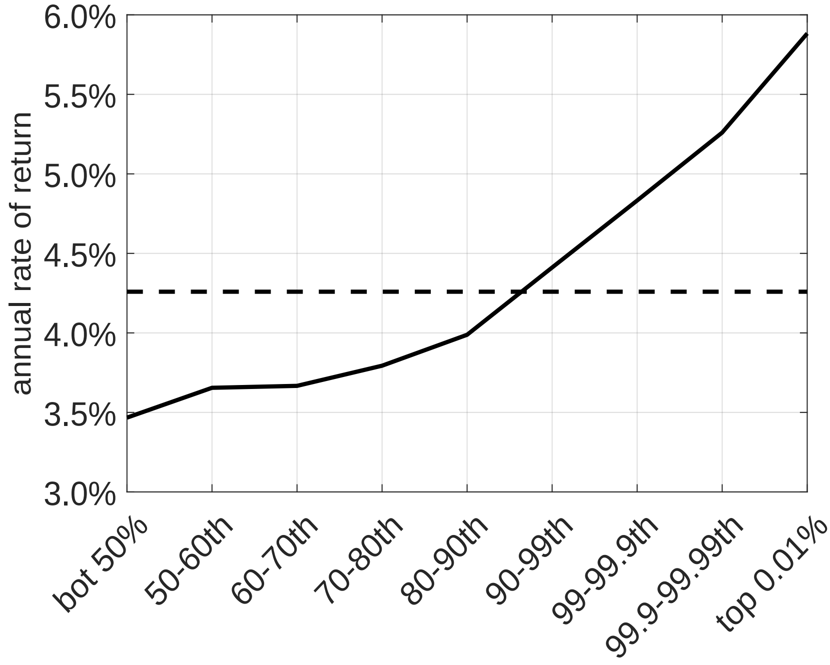

Figure 1.6: Increasing returns to wealth in the SCF

Notes: This figure plots the average annual rate of return from 1989 to 2019 that each wealth group would have achieved if they held the aggregate portfolio during each 3-year period between SCF waves. See Footnote 10 for details.

1.3 Accounting Exercise

The main goal of this section is to investigate the role that the portfolio movements highlighted in Section 1.2 played in explaining wealth inequality dynamics in the US from 1989 to 2019, given the estimated rates of return from the data. In particular, the thought experiment is: had a given wealth group not shifted wealth from one asset to another in the way that they did, would they hold a larger or smaller share of overall wealth?

To tackle that question I use an accounting framework to estimate saving rates to then be able to simulate a counterfactual path for wealth shares in which capital income for each wealth group is different due to counterfactual portfolio holdings. In order to do that I use the following budget constraint identity:

$$W_{t+1}^p = W_t^p + W_t^p \sum_{j=1}^{6} (r_{t+1}^{p,j} + q_{t+1}^j) x_t^{p,j} + Y_t^p - C_t^p, \tag{1.2}$$

where W_t^p denotes the total wealth of group p (e.g., top 1%); $x_t^{p,j}$ are group p's portfolio shares in each asset class j; $r^{p,j}_{t+1}$ and q^j_{t+1} are the dividend yield and capital gains for each asset class; Y^p_t is total non-capital income, which includes labour income and transfers; and C^p_t is total consumption. With slight abuse of notation, let time t now denote a calendar year, and not waves of the SCF.

To better compare my results to what would have been predicted by models of consumption choice, I calculate saving rates out of expected income:

$$C^p_t = (1 - s^p_t) W^p_t \left[\sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x^{p,j}_t + y^p_t \right], \tag{1.3}$$

where $\bar{r}^{p,j}$ and $\bar{q}^j$ are the expected rates of return for each asset class (assumed not to vary over time), and y^p_t is the labour income to wealth ratio. If labour income is not uncertain but dividends $r^{p,j}_{t+1}$ potentially are, then under a CRRA utility function the optimal choice of the household would be to consume a constant fraction of their expected income just like in Equation 1.3 above.[11] Combining Equations 1.2 and 1.3 we get:

$$W^p_{t+1} = \left\{ 1 + s^p_t \left[\sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x^{p,j}_t + y^p_t \right] + \sum_{j=1}^{6} (r^{p,j}_{t+1} - \bar{r}^{p,j} + q^j_{t+1} - \bar{q}^j) x^{p,j}_t \right\} W^p_t, \tag{1.4}$$

which is the main equation of this exercise, and which will be used to estimate s^p_t, as all the other variables are observed.

Because the SCF does not have a panel structure, when taking Equation 1.4 to the data I need to do it at the wealth group level. Thus, several assumptions are needed for it to hold: (i) no populational growth; (ii) no movement of agents between wealth groups; (iii) constant saving rates, rates of return and labour to income ratios for all individuals within a wealth group. Assumptions (ii) and (iii) will be discussed in Sections 1.3.2.1 and 1.3.2.2, respectively. To deal with (i), I use average wealth within wealth groups to estimate the saving rates, and not total wealth.

[11]This fraction s^p_t could be different along the wealth distribution due to differential rates of return, for example. See Merton (1969) for a derivation of optimal consumption and savings rate when capital income is uncertain but labour income is not. As highlighted by Fagereng et al. (2021) the role of capital gains q^j_{t+1} is more complicated and whether they imply an increasing or flat profile for gross saving rates depends on dividends or discount rates being the main drivers of price increase.

Furthermore, remember that there are SCF surveys every three years. I can then back out saving rates for each wealth group from the data using Equation 1.4 for a three year period:

$$s_t^p = \left\{ \left(\frac{W_{t+3}^p}{W_t^p} \right)^{\frac{1}{3}} - \left[1 + \sum_{j=1}^{6} (r_{t+1}^{p,j} - \bar{r}^{p,j} + q_{t+1}^j - \bar{q}^j) x_t^{p,j} \right] \right\} \frac{1}{y_t^p + \sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x_t^{p,j}}, \tag{1.5}$$

in which I assume that annual saving rates s_t^p, rates of return $r_{t+1}^{p,j}$ and q_{t+1}^j, labour to income ratios y_t^p and portfolio composition $x_t^{p,j}$ are kept constant in the years between SCF surveys. The rates of return are estimated from the data as explained in Section 1.2.2, and labour income to wealth ratios y_t^p are assumed to be the average of those ratios in subsequent waves of the SCF in a similar fashion to dividends. Portfolio composition is directly observable and I assume that the portfolio that was prevalent between t and $t+3$ is the one observed at $t+3$.[12] Finally, I estimate $\bar{r}^{p,j}$ and $\bar{q}^j$ as the average return over the whole 1989 to 2019 period.

The results of the estimation are displayed in Panel A of Table 1.3. The average saving rates from 1989 to 2019 estimated are similar in magnitudes to those from Martínez-Toledano (2022) for Spain and Saez and Zucman (2016) for the US, which use similar techniques. In particular, the saving rates in Equations 1.3 are gross saving rates (inclusive of capital gain income) and they are increasing in wealth. As highlighted by Fagereng et al. (2021), gross saving rates that are increasing in wealth can be explained by capital gains that are driven mainly by a fall in the discount rate applied to its future stream of dividends. This would be in line with the evidence from Figure 1.5 that the rate of safe assets has declined over time (even after taking into account the decline in inflation). Finally, notice that the fact that they are increasing in wealth does not necessarily imply an ever-diverging wealth distribution, as income to wealth ratios at the bottom of the distribution are larger than at the top, due in large part to labour income being more relevant.

[12] It is not clear which one would be best, but I use the one observed at $t+3$ so that I can use the period between the first and second waves from 1989 to 1992 too, as in the counterfactual scenario portfolios will be kept at 1989 levels. I discuss this assumption in Section 1.3.2.4.

Saving Rates in p.p.	Bottom 50%	50-90th	90-99th	Top 1%
A. Baseline	0.9	4.6	18.5	29.3
B. Exit and Entry				
0.5%				24.4
1.0%				19.4
2.0%				9.4
C. Alternative Timing	0.2	4.9	19.6	31.6
D. Finer Partition		5.3	18.6	29.5

Table 1.3: Annualised estimated saving rates by wealth groups (average over 1989-2019)

Notes: (A) refers to the baseline exercise that uses Equation 1.5 to estimate saving rates; (B) allows for different estimates for the annual growth of wealth that is due to net entry of households into the Top 1% in the estimation of saving rates (Section 1.3.2.1); (C) assumes that in between two waves of the SCF households keep their portfolios fixed at the average of what is observed in each of the two waves, instead of the one observed at the later wave (Section 1.3.2.4); and (D) divides the groups above the median into smaller groups, to allow for different saving rates within the original groups, and then aggregates them (Section 1.3.2.2).

1.3.1 The Effect of Portfolio Changes

Having estimated the saving rates we are now able to perform counterfactuals with respect to the portfolio choices of households. The idea is to highlight the importance of portfolio movements in the US in the last 30 years for wealth inequality dynamics by estimating what would have happened if portfolios had not changed at all. In order to do that I construct a counterfactual series for wealth $\widehat{W}_t^{p,j}$ that is given by:

$$\widehat{W}_{t+1}^p = \left\{ 1 + s_t^p \left[\sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x_{1989}^{p,j} + y_t^p \right] + \sum_{j=1}^{6} (r_{t+1}^{p,j} - \bar{r}^{p,j} + q_{t+1}^j - \bar{q}^j) x_{1989}^{p,j} \right\} \widehat{W}_t^p, \tag{1.6}$$

with $\widehat{W}_{1989}^p = W_{1989}^p, \forall p$. Notice that the only difference between Equations 1.4 and 1.6 is that in the latter one portfolios are assumed to be fixed at their 1989 levels for all wealth groups and all asset classes (rates of return, saving rates and labour to income rates are all the same as before). Thus, any differences between the two series $W_t^p, \widehat{W}_t^p$, is due to capital income being different if portfolios had not changed in the way that they did.

Once the counterfactual wealth series $\widehat{W}_t^p$ is constructed, I compare the wealth shares $\omega_t^p = W_t^p/W_t$ in the data and in the counterfactual scenario $\widehat{\omega}_t^p = \widehat{W}_t^p/\widehat{W}_t$, where W_t and $\widehat{W}_t$ are the aggregate wealth of US households. The total effect of the changes in portfolio from 1989 to 2019 is measured as $\omega_{2019}^p - \widehat{\omega}_{2019}^p$ or, in words, how bigger the wealth share of group p is because their portfolios have moved since 1989.

Panel A in Table 1.4 summarises the final results of this exercise, and shows that the effects of portfolio changes are particularly important at the top and the bottom of the wealth distribution. In the Bottom 50%, the changes in portfolio choices can explain 0.9 p.p of the total 1.5 p.p in drop of wealth shares from 1989 to 2019, which is equal to 60% of the total movement. For the Top 1%, the better choices for portfolio can explain 2.4 p.p. of the 7.8 p.p increase in wealth shares, or 30.7% of the total change. Therefore, the contribution of changes in portfolio choices for the increase in wealth inequality is large and the exercise suggests that wealth inequality today would be smaller had it not been for the changes in portfolios.

The next question is why have the portfolio effects been positive at the top, but negative at the bottom? The way that the accounting exercise works is that, assuming rates of return are positive, increasing the allocation to any asset contributes positively to accumulating wealth, while reducing contributes negatively. Obviously, any increase in the portfolio share of an asset needs to be compensated by a decrease in another one, and the net effect over the changes for all the assets is the portfolio change effect. With this in mind, the answer for the top of the distribution is simple: these groups shifted their portfolios over time into equity, which yielded the highest return among all the assets during the period of the analysis. Thus, the net effect was positive at the top.

However, this does not explain why the effect was negative at the bottom, since there was an increase in leverage, which also increases expected returns. To shed light in this issue it helps to decompose the effect of portfolio changes over the whole time period and also across the different asset classes. To decompose the effect of portfolio changes across asset classes I perform a counterfactual in which I keep each portfolios fixed at

Effect of Portfolio Change in p.p.	Bottom 50%	50-90th	90-99th	Top 1%
A. Baseline	-0.9	-0.6	-0.9	2.4
B. Exit and Entry				
0.5%				2.2
1.0%				2.1
2.0%				1.8
C. Alternative Timing	0.1	-0.4	-1.2	1.6
D. Finer Partition	-0.9	-0.4	-1.6	2.9
E. Capital Gains Only	-1.5	-1.2	1.0	1.8
Changes in Wealth Shares from 1989-2019	-1.5	-8.1	1.8	7.8

Table 1.4: Estimated Effect of Portfolio Changes on Wealth Shares, from 1989 to 2019

Notes: (A) refers to the baseline exercise that uses Equation 1.5 to estimate saving rates; (B) allows for different estimates for the annual growth of wealth that is due to net entry of households into the Top 1% in the estimation of saving rates (Section 1.3.2.1); (C) assumes that in between two waves of the SCF households keep their portfolios fixed at the average of what is observed in each of the two waves, instead of the one observed at the later wave (Section 1.3.2.4); (D) divides the groups above the median into smaller groups, to allow for different saving rates within the original groups, and then aggregates them (Section 1.3.2.2); and (E) used counterfactual capital gains only, and the same dividend returns as observed in the data (Section 1.3.2.3).

1989 levels only for one asset class $\widehat{j}$ at a time and then calculate how much higher/lower the wealth share of a group becomes.[13]

For the effect of portfolios over time, Figure 1.7 shows the counterfactual evolution of wealth shares for the bottom 50% and the top 1%, in which the portfolio change effect up to that period was subtracted. If the effect is important, we would expect the two

[13] Let $\widehat{W}_t^{p,\widehat{j}} = \left\{1 + s_t^p \left[\sum_{j=1}^{6}(\bar{r}^{p,j} + \bar{q}^j)\widehat{x}_t^{p,j,\widehat{j}} + y_t^p\right] + \sum_{j=1}^{6}(r_{t+1}^{p,j} - \bar{r}^{p,j} + q_{t+1}^j - \bar{q}^j)\widehat{x}_t^{p,j,\widehat{j}}\right\}\widehat{W}_t^p$ be the counterfactual series for wealth to uncover the effect of asset class $\widehat{j}$, where $\widehat{x}_t^{p,j,\widehat{j}} = x_{1989}^{p,\widehat{j}}$ if $j = \widehat{j}$, and $= x_t^{p,j}$ otherwise. The estimated effect is then measured as $\omega_{2019}^p - \widehat{W}_{2019}^{p,\widehat{j}}/W_{2019}$. Notice that the effects of the different asset classes when estimated separately do not need to sum up to the aggregate effect in Table 1.4 because negative/positive effects of different asset classes can compound on each other. I then rescale the final individual asset class effects so that they add up to the total effect.

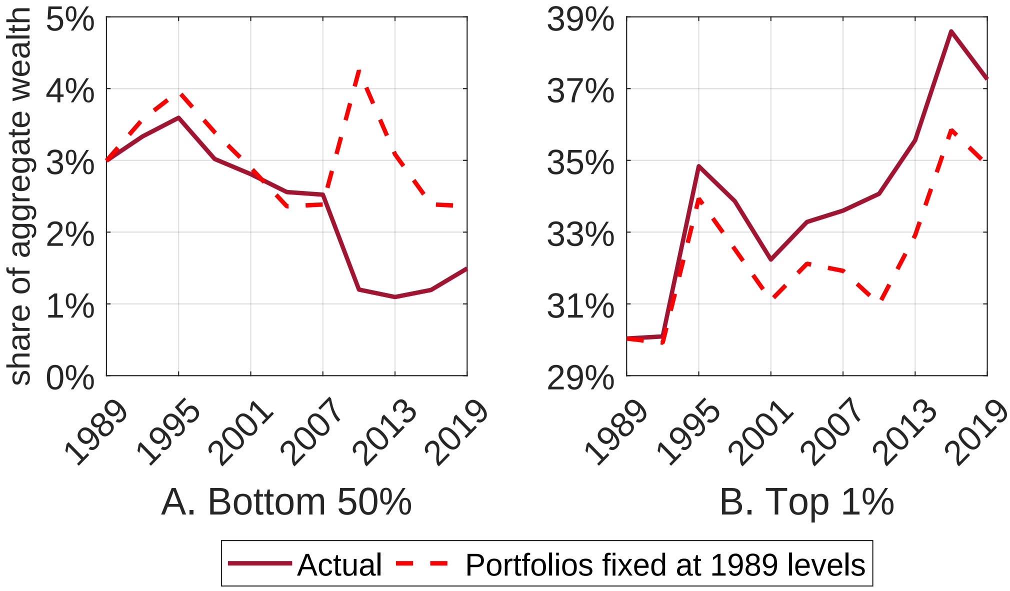

Figure 1.7: Evolution of Counterfactual Wealth Shares, without Portfolio Effect

series to increasingly diverge over time, simply due to the fact that in the beginning of the sample they are assumed to be the same, and at the end there are 30 years of accumulated portfolio movements.

Figure 1.7 highlights how the behaviour of the portfolio change effects is different depending on where in the distribution we look at. Panel B shows an effect that is not concentrated in any particular year:[14] the top percentile has been steadily improving its portfolio by shifting its allocation into equity, which had a better return over the period.

This interpretation is supported by Figure 1.8, which further decomposes the portfolio change effect into each of the 6 asset classes. The portfolio movement at the top was a net positive one: movement away from Real Estate and into Public and Private Equity. There was also a gain from reducing the importance of Other assets, which carry negative returns.

The picture for the bottom 50%, however, is more complicated. What is most striking from Figure 1.8 is that the component for Real Estate is basically zero, even though it had positive returns on average and that it had a massive increase in importance on

[14]That is true for the other wealth groups above the median as well, which are displayed in Figure 1.A.7.

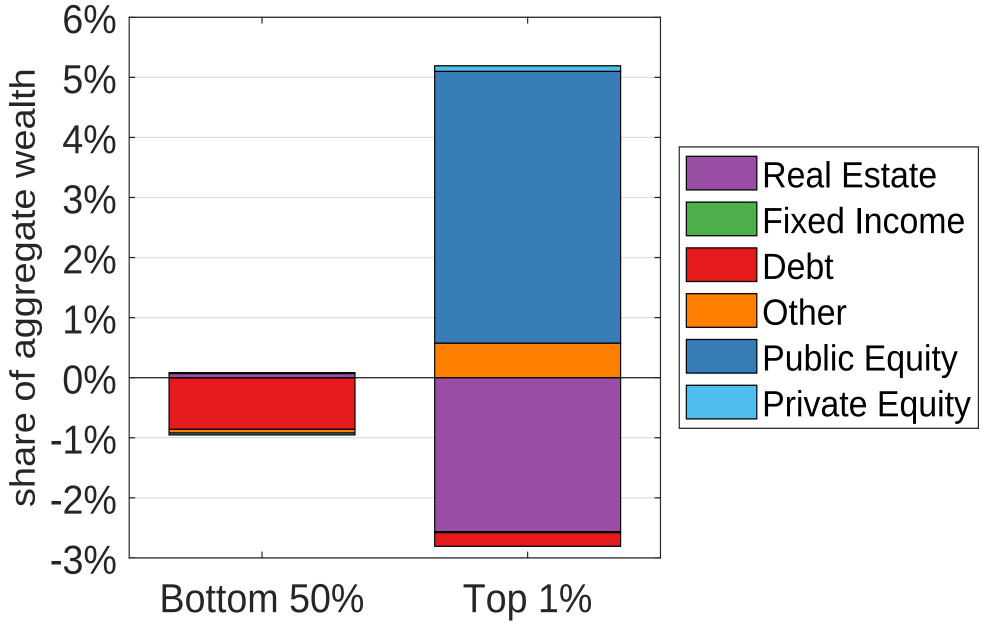

Figure 1.8: Breakdown of portfolio change effect

the portfolio of the bottom 50%. The answer is that the main story for the Bottom 50% is bad timing: most of the increase was from 2007 to 2010, when Real Estate had a negative return. Also, after 2010 the bottom of the distribution has been moving away from it, even though it has displayed positive returns. One can see in Panel A of Figure 1.7 that the portfolio effect had been mostly neutral until 2007, and it peaked in 2010. As discussed in Section 2.1, these movements can be explained by the build-up of leverage before and during the GFC, and by the deleveraging that has been happening since. When one takes into account that the move into Real Estate has been financed by the increase in debt, the decision looks even worse, and it becomes net negative, as Figure 1.8 shows.

1.3.2 Additional Discussions

I now discuss relaxing some of the assumptions made for the accounting exercise and alternative settings.

1.3.2.1 Entry and Exit

Equation 1.2 holds under the assumption of no movements between groups. However, there is wealth mobility in the US and it might be important to take those movements into account. In particular, at the very top of the distribution this can be a bigger issue as portfolios are riskier, which would lead to more idiosyncratic movements across agents. Indeed, Kennickell and Starr-Mccluer (1997), Gomez (2021), and Zheng (2021) argue that transition between groups at the top of the wealth distribution is an important phenomenon.

To see what would happen if that was not the case, let N^p_{t+1} be the total wealth at time $t+1$ of individuals that were not in group p at time t, but entered at time $t+1$; and O^p_{t+1} the wealth of individuals at time $t+1$ that were part of group i in time t, but exited at time $t+1$. Then Equation 1.2 becomes

$$W^p_{t+1} = W^p_t + W^p_t \sum_{j=1}^{6} (r^{p,j}_{t+1} + q^j_{t+1}) x^{p,j}_t + Y^p_t - C^p_t + N^p_{t+1} - O^p_{t+1}, \tag{1.7}$$

which leads to a new version of Equation 1.4:

$$W^p_{t+1} = \left\{ 1 + s^p_t \left[\sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x^{p,j}_t + y^p_t \right] + \sum_{j=1}^{6} (r^{p,j}_{t+1} - \bar{r}^{p,j} + q^j_{t+1} - \bar{q}^j) x^{p,j}_t \right\} W^p_t + N^p_{t+1} - O^p_{t+1}. \tag{1.8}$$

The equation above shows that there is an extra term $N^p_{t+1} - O^p_{t+1}$ which is not taken into account in the original estimation. Because saving rates s^p_t are the residual estimated from the equation, whenever $N^p_{t+1} \neq O^p_{t+1}$ their value would bias the estimation of s^p_t. Notice that for top groups (top 10%, top 1%, top 0.1%, ...) it is always the case that $N^p_{t+1} \geqslant O^p_{t+1}$, which means that the original exercise is overestimating saving rates at the top (and underestimating at the Bottom 50%). One then might worry that the effect of portfolios on the counterfactual are overestimated as well, as the impact of counterfactual income on wealth accumulation depends exactly on the saving rate.

Unfortunately, the SCF is not a panel data set, so it is not possible to properly take into account the movement in between groups. However, there were two limited panel versions of the SCF between 1983 and 1989, and between 2007 and 2009. Because the information is not as detailed as in the regular waves of the SCF (specially on income in 1986 and 2009), they were not included in the baseline exercise. But it is possible to use the later panel data set to calculate saving rates with and without the entry and exit term.[15]

Saving Rates in p.p.	Bottom 50%	50-90th	90-99th	Top 1%
A. Baseline	7.7	10.7	22.0	23.0
B. Exit and Entry	31.0	20.0	26.9	-27.9
Wealth Mobility				
C. Share that Remains	88.1%	80.2%	72.6%	65.1%

Table 1.5: Saving rates and wealth mobility by wealth groups (average over 2007-2009)

Notes: This table shows the annualised estimated saving rates and wealth mobility using the special panel version of the SCF from 2007 to 2009. (A) refers to the baseline exercise that uses Equation 1.5 to estimate saving rates; (B) takes into account entry and exit as explained in Equation 1.7. It is the estimated saving rate for those in each wealth group at the time of the 2007 interviews, irrespective of where in the wealth distribution they were in 2009; (C) displays the share of households that remained in the same wealth group from 2007 to 2009.

Table 1.5 shows the results of this estimation. In Panel A, we have the estimated saving rates without taking entry and exit into account, as in the previous baseline exercise. In Panel B, I correct for the entry and exit term, and calculate the saving rates for the households in a given wealth group in 2007 irrespective of where in the wealth distribution they end up in 2009. The estimates in Panel B are quite different from the baseline ones: as expected, the saving rate at the Top 1% falls, and increases for the Bottom 50%. Because the net movement between groups contributed negatively for the other wealth groups, the new estimates also increase from the 50^{th} to the 99^{th} percentiles.

As one can see, the estimates are sensitive to the assumption over entry and exit, specially at the ends of the distribution. One then might be worried that this is a special period with high wealth mobility because the panel includes the Great Financial Crisis.

[15]Given that there is not as much information in the 2009 re-interview, I assume that dividend yields and labour to income ratios of the households along the wealth distribution in the panel data set are the same as those for the cross-section version of the SCF in 2007. I adjust capital gains to reflect the 2007 to 2009 period only, excluding 2010.

However, as Panel C shows, the share of households that stay in the same wealth group in the two-year period does not seem to be lower than the estimates for the longer 1998 to 2010 period from Kuhn, Schularick, and Steins (2020), who use the Panel Survey of Income Dynamics. Still, as Equation 1.7 makes clear, what matters is how much wealthier are the people that enter a given group compared to those that leave it, not how many people are changing groups. Also, it would be ideal to be able to correct the estimated saving rates for the whole period of 1989 to 2019.

With that in mind, assume that we know that $N^p_{t+1} - O^p_{t+1} = \gamma^p_t W^p_t$, or that new entrants being wealthier than those that are exiting the group is responsible for γ^p_t p.p. of the annual increase in wealth for group p. This is exactly the "displacement" term of Gomez (2021), that estimate it for top wealth groups in the US in recent decades. I can then re-write Equation 1.8 as

$$W^p_{t+1} = \left\{ 1 + \gamma^p_t + s^p_t \left[\sum_{j=1}^{6} (\bar{r}^{p,j} + \bar{q}^j) x^{p,j}_t + y^p_t \right] + \sum_{j=1}^{6} (r^{p,j}_{t+1} - \bar{r}^{p,j} + q^j_{t+1} - \bar{q}^j) x^{p,j}_t \right\} W^p_t. \tag{1.9}$$

Gomez (2021) estimates that approximately 1 p.p. per year of the increase in the wealth of the Top 1% in recent decades is due to new entrants, thus I set $\gamma^{\text{Top 1\%}}_t = 1\%$ in Equation 1.9, but I also report results for values of $\gamma^{\text{Top 1\%}} = 0.5\%, 2\%$. I do that only for the top wealth group both because this is the part of the distribution where we should be more concerned about movements between groups contributing to wealth growth, and also because of lack of available estimates for the bottom groups for the longer period.

Panel B of Table 1.3 shows the new estimates for the saving rates for the Top 1% (no saving rates are show for the other groups because they did not change by construction). As expected, the saving rate of the Top 1% falls again. The fall is smaller than the fall using the panel from 2007-2009,[16] but it is still large: it decreases by a third from 29.3 to 19.4 p.p. under the preferred estimate of $\gamma^{\text{Top 1\%}}_t = 1\%$. When using this new

[16]This is because, although there did not seem to be particularly high wealth mobility in the 2007 to 2009 period as measured by percentage of households that move in between groups, the wealth of households that entered the Top 1% in 2009 compared to those that left was larger than the relative wealth of entrants compared to leavers reported by Gomez (2021) for the longer period.

estimated saving rate in the counterfactual scenario, it could then be that the portfolio change effect would fall a lot as well.

However, Panel B of Table 1.4 shows that is not the case, and the effect of portfolio changes falls from 2.4 to 2.1 p.p., and it still accounts for 27.0% of the increase in the Top 1% wealth share in recent decades. The reason why the portfolio effect did not change by as much as the saving rate did is that if on one hand now there is less accumulation due to lower savings, on the other hand now there is increased accumulation due to the entry of wealthier households into the Top 1%, and these effects almost cancel each other.[17]

One important final consideration then is whether the portfolios of those that are entering the Top 1% are significantly different than those that are leaving it. Figure 1.A.2 in the Appendix shows that, even though the portfolios at the bottom and at the top of the distribution are quite different, they do not vary significantly between subsequent groups of wealth, both in 1989 and in 2019 (and in other years the pattern is the same). As movements into the Top 1% are likely to come from households that were in percentiles close to it in the previous wave, then it is unlikely that the portfolios would be significantly different.

1.3.2.2 Finer Partition

Another important assumption behind equation 1.2 is that individuals within a group p have the same saving rate, rates of return and labour income to wealth ratio. Unfortunately, due to the nature of the SCF I am not able to calculate individual saving rates.

The strategy taken in this section will be to use a finer partition than the one in the baseline. With smaller groups, the assumption of constant saving and return rates within wealth groups should be less problematic and how the results change should give us an indication on what would happen if we could calculate individual saving rates. The baseline partition of the population in terms of percentiles was: {“0-50”,“50-

[17]How the changes in portfolio impact the savings rates or the movement in between wealth groups is beyond the scope of this paper, but see Chapter 2 for why we would expect that both would increase when portfolios shift towards assets with higher risk and return, just like the movement observed for the Top 1%.

90", "90-99", "99-100"}. The finer partition now is: {"0-50","50-60", "60-70", "70-80", "80-90","90-95", "95-97.5", "97.5-99", "99-99.3", "99.3-99.6", "99.6-99.9", "99.9-100"}. Thus every original group is divided into at least three subgroups, with the exception of the "0-50", which stays the same due to the difficulty in estimating saving rates out of negative wealth in some years at the very bottom of the distribution.

Figure 1.9 shows the results of the estimation. We can see that there is heterogeneity on saving rates within the original wealth groups, as saving rates are (mostly) increasing within these groups as well. However, this heterogeneity per se does not imply that the original exercise is flawed.

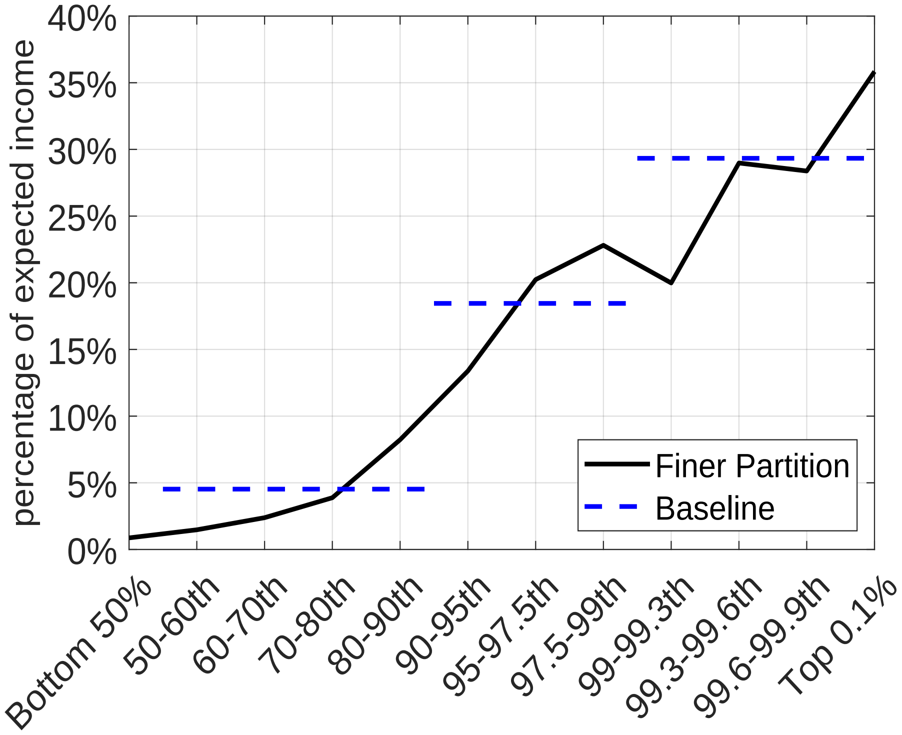

Figure 1.9: Saving rates across the wealth distribution

Notes: This figure shows the average annual saving rate for each group in the wealth distribution for the finer partition (solid line), and compares it to the numbers from their original broader groups in the baseline estimation (dashed line).

Panel D of Table 1.3 show what happens when the saving rates estimated for each of the smaller groups in the new partition are aggregated back to the original groups weighted by their wealth for each year, and then averaged across years. The results show that saving rates are not affected by the specific choice of wealth groups once they are aggregated back. Thus it would seem that the aggregation assumed by Equation 1.4 is not an issue.

But the results for the portfolio change effects shown in Table 1.4 show that the aggregation can actually be relevant. We see that now the effect of using smaller groups of wealth actually increases the final impact on Top 1% wealth share. This is due to the correlation between saving rates and returns: those at the top of the distribution save more and earn higher returns, thus in the counterfactual with a finer partition the people at the very top earn higher capital income and save a bigger share of it, increasing the effect of portfolio changes.

Given the results of this exercise, we would expect that going from the finer partition to individual data would increase even more the effect of portfolio changes at the top. Also, the aggregation assumed did not seem problematic for estimating group-level saving rates, but it can be important when interacting saving rates with variables that are correlated with it.

1.3.2.3 The Effect of Capital Gains

Several papers have highlighted the importance of changes in asset prices for recent increase in wealth inequality (see Cioffi, 2021; Diwan, Duzhak, and Mertens, 2021; Feiveson and Sabelhaus, 2019; Greenwald et al., 2021; Kuhn, Schularick, and Steins, 2020; Martínez-Toledano, 2022). One interesting question about the current baseline exercise is then whether most of the gains from portfolio changes is due to returns in term of dividends or capital gains as highlighted by the previous papers.

To shed light to this question I calculate a series for wealth $\widehat{W}_t^p$ in which the counterfactual only contains counterfactual capital gain income, but the same series for dividend returns as the original one: $\widehat{W}_{t+1}^p = \Big\{1 + s_t^p \Big[\sum_{j=1}^6 (\bar{r}^{p,j} x_t^{p,j} + \bar{q}^j x_{1989}^{p,j}) + y_t^p\Big] + \sum_{j=1}^6 \Big[(r_{t+1}^{p,j} - \bar{r}^{p,j}) x_t^{p,j} + (q_{t+1}^j - \bar{q}^j) x_{1989}^{p,j}\Big]\Big\} \widehat{W}_t^p$. In this way, if wealth shares are different due to differences in portfolio choices, it is only because capital gain income has changed.

Panel E of Table 1.4 shows that taking into consideration only capital gains, the effects of portfolio change are even larger at the Bottom 50% (-1.5 p.p. vs -0.9 p.p.), while at the top they are smaller (1.8 p.p. vs 2.4 p.p.). The fact that the effect at the bottom became worse might seem puzzling, but that is because a large share of the return on the asset that grew the most in importance at the bottom, housing, is rent (and imputed rent), and not capital gains. For the top of the distribution, we see that capital gains explain 1.8/2.4 = 75% of the effect of portfolio changes. Therefore, we see that capital gains are indeed important for understanding recent changes in wealth inequality (although see Fagereng et al. (2022) for a thorough discussion on when capital gains have welfare impacts).

1.3.2.4 Alternative Timing

On the top of Figure 1.4 we have the baseline timing assumption with respect to portfolios. As mentioned before, the assumption is that, for example, portfolios observed in 1992 are the ones that are prevalent during the period of 1989 to 1992. The alternative assumption that portfolios observed in 1989 are the ones prevalent during the following three years is undesirable because then it would not be possible to calculate a counterfactual for the first three years, since I do not observe portfolios before 1989, and that would mean losing 10% of the already not so large sample.

One possible solution is to assume that households keep their portfolios at the average between the 1989 and 1992 levels in between the two waves of the SCF (and analogously for other waves), in which case it would be possible to calculate counterfactuals even for the three years in between the first two waves. Panel C of Table 1.3 shows that estimated saving rates are not really affected by this timing assumption. However, looking at Panel C of Table 1.4 we see that the final effect of portfolio changes are now basically neutral at the Bottom 50%, and smaller at the Top 1% (although still large for understanding wealth inequality dynamics at the top).

Given that the saving rates estimated are similar under the two different timing assumptions, if the results for the effect of portfolio changes was similar as well, we could conclude that almost all of the effect of portfolio changes is coming from long-run movements in portfolios and return differentials. The fact that the magnitude of the effect is sensitive to this assumption points in the direction that part of the results could be due to good timing of portfolio movements in short time horizons by the Top 1%, which is in line with findings of Martínez-Toledano (2022).

1.4 Conclusion

The possible causes for the increase in wealth inequality in the US have been the center of a great debate in the last couple of decades. However, one dimension has been understudied so far, namely the impact of the changes in households' portfolio choices.

First, I used the Survey of Consumer Finances to highlight how portfolios have changed significantly in the period from 1989 to 2019. Moreover, not only the aggregate portfolio of US households has changed, but also the portfolio of households at different parts of the wealth distribution has changed in different ways: the Bottom 50% has increased their leverage, by taking lager mortgages and investing more into Real Estate, while the Top 1% reduced their investment into Real Estate, and increased their investment into Public and Private Equity.

Second, to uncover the impact of these differential portfolio movements on wealth shares I used an accounting framework and a budget constraint identity to estimate saving rates across the wealth distribution. This allowed me to calculate what would happen to wealth shares in a counterfactual scenario in which portfolio shares where kept constant at their 1989 levels.

The baseline results show that portfolio movements were particularly important at the Bottom 50% of the distribution, where they explained 60% (0.9 out of 1.5 p.p.) of the decline in its wealth share from 1989 to 2019, and at the Top 1%, where they explained 30.7% (2.4 out of 7.8 p.p.) of the gain in wealth share by the wealthiest households.

The results highlight the need to better understand not only how households make portfolio choices, but why they change over time in the way see in the data. If changes in portfolio have impacts on wealth inequality, they can also be important for understanding changes in welfare and the dynamics of other important macroeconomic aggregates, as consumption and savings.

Appendices

1.A Additional Figures

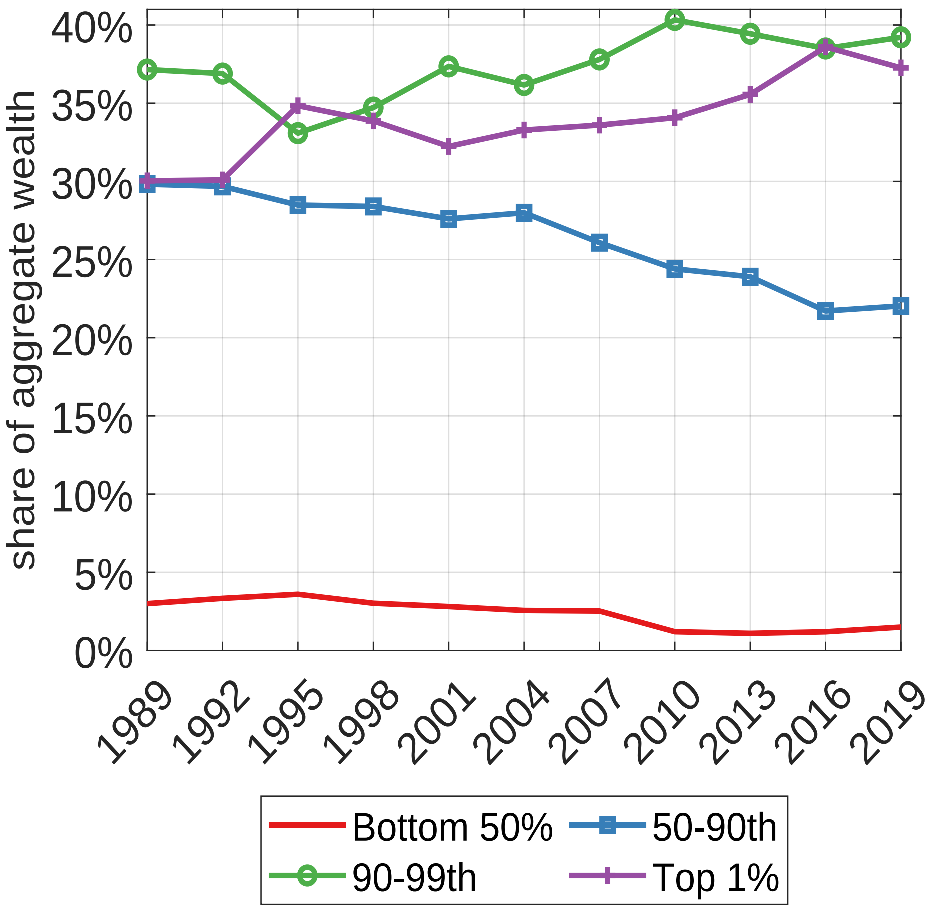

Figure 1.A.1: Evolution of Wealth Shares in the United States (SCF)

Notes: This figure show the evolution of share of aggregate wealth held by each of the following wealth groups: the Bottom 50% of wealth, those between the 50^{th} to 90^{th} percentile, those between the 90^{th}-99^{th} percentile, the Top 1%.

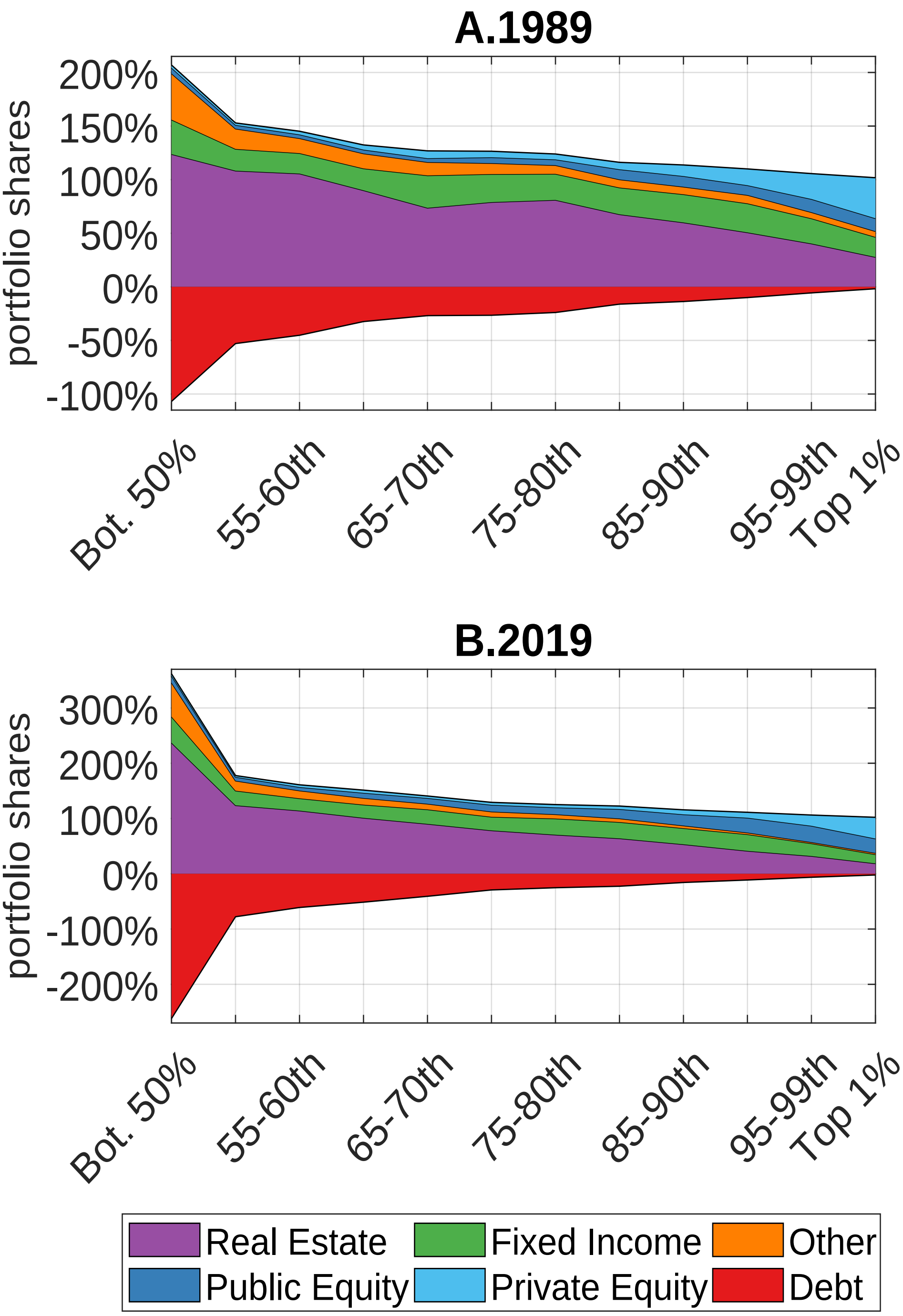

Figure 1.A.2: Portfolio Composition Along the Wealth Distribution

Notes: The wealth groups in the figure above are the Bottom 50%; then the ventiles (groups of 5%) from 50^{th}-55^{th} percentile up to 90^{th}-95^{th} percentile; those between the 95^{th} and 99^{th} percentiles; and the Top 1%.

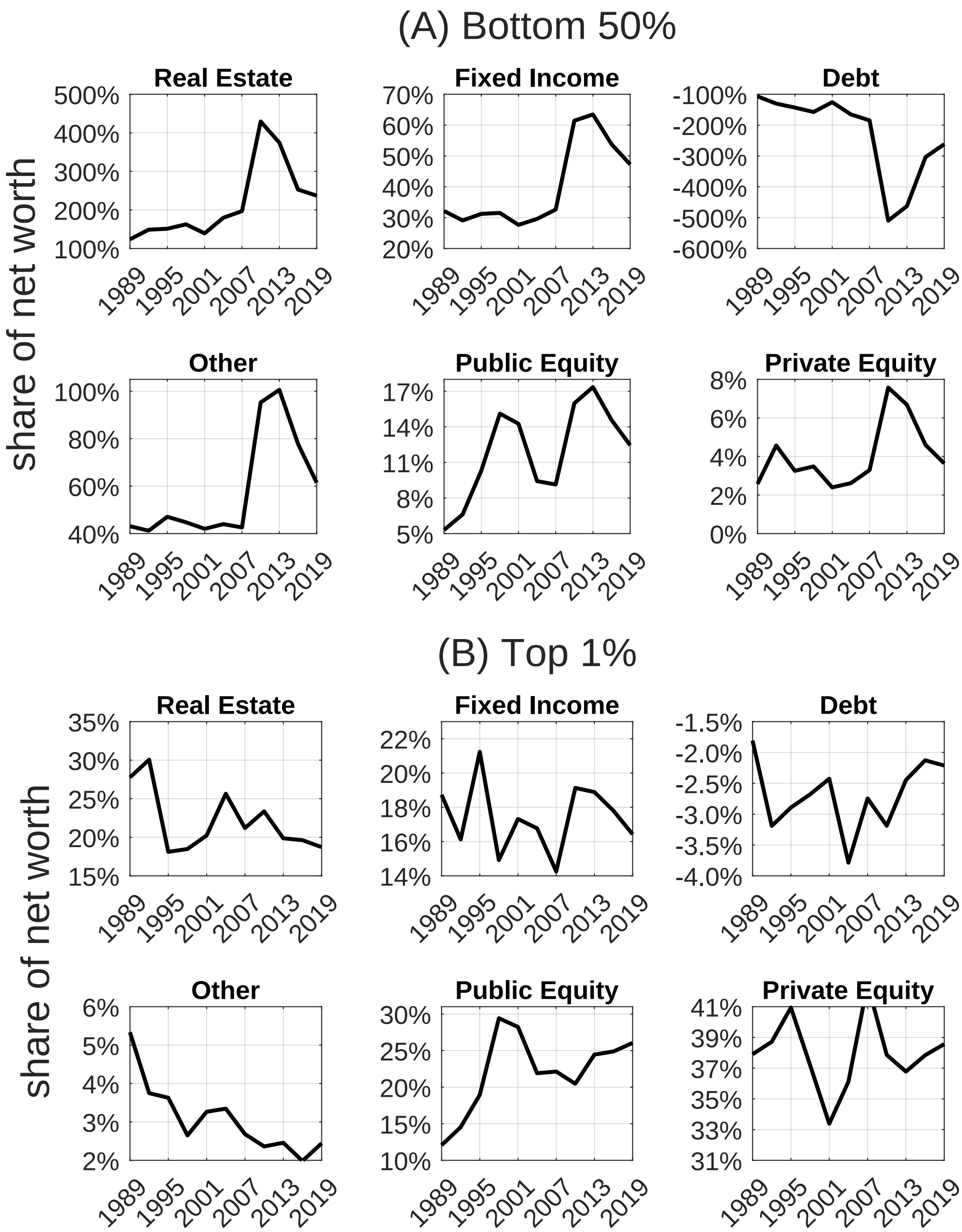

Figure 1.A.3: Portfolio shares over time for each asset class

Notes: This figure shows the portfolio shares of each of the six asset classes for those in the Bottom 50% of wealth in Panel (A), and those in the Top 1% in Panel (B).

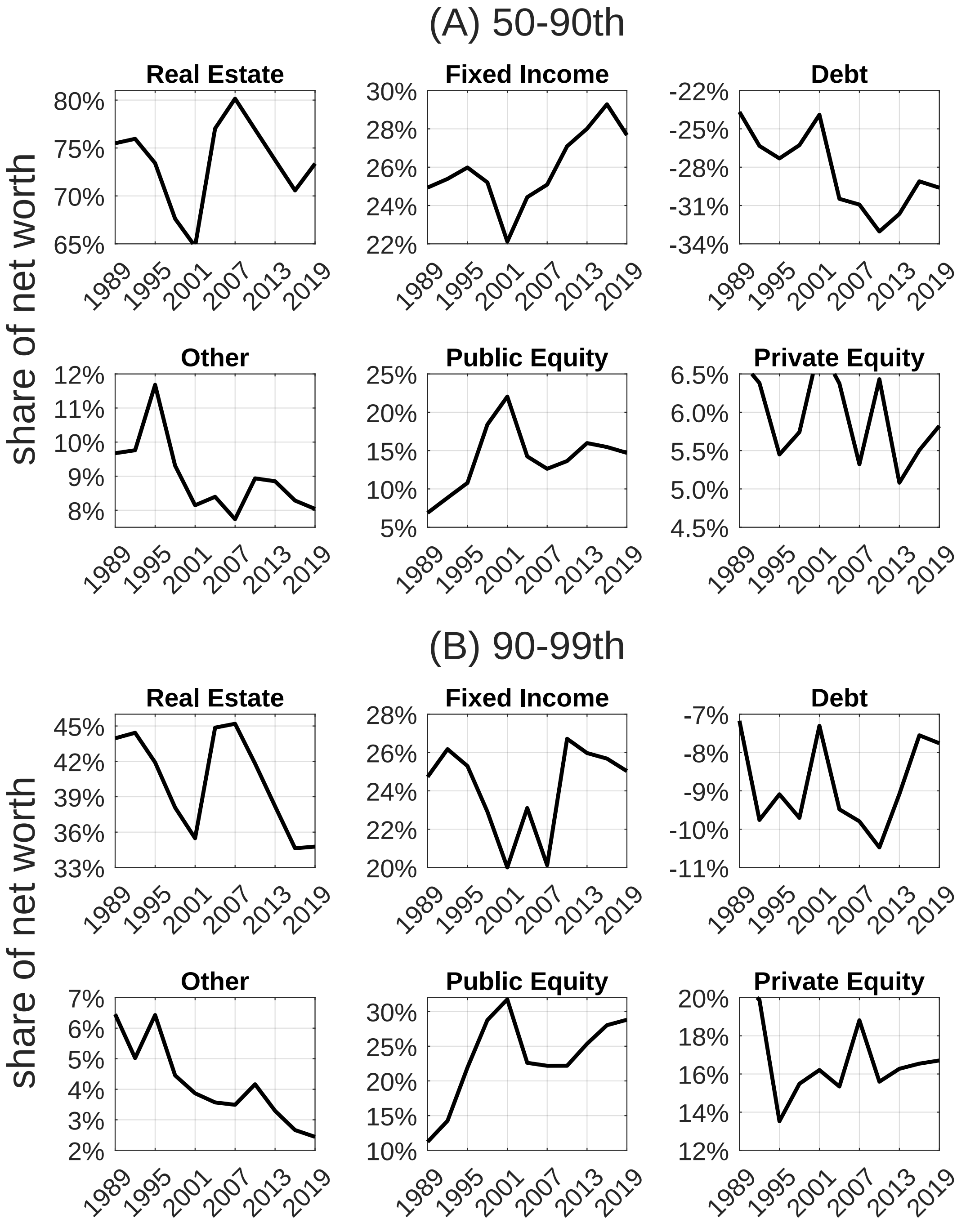

Figure 1.A.4: Portfolio shares over time for each asset class

Notes: This figure shows the portfolio shares of each of the six asset classes for those in between 50$^{\text{th}}$ and 90$^{\text{th}}$ percentiles of wealth in Panel (A), and those in between the 90$^{\text{th}}$ and 99$^{\text{th}}$ percentiles of wealth in Panel (B).

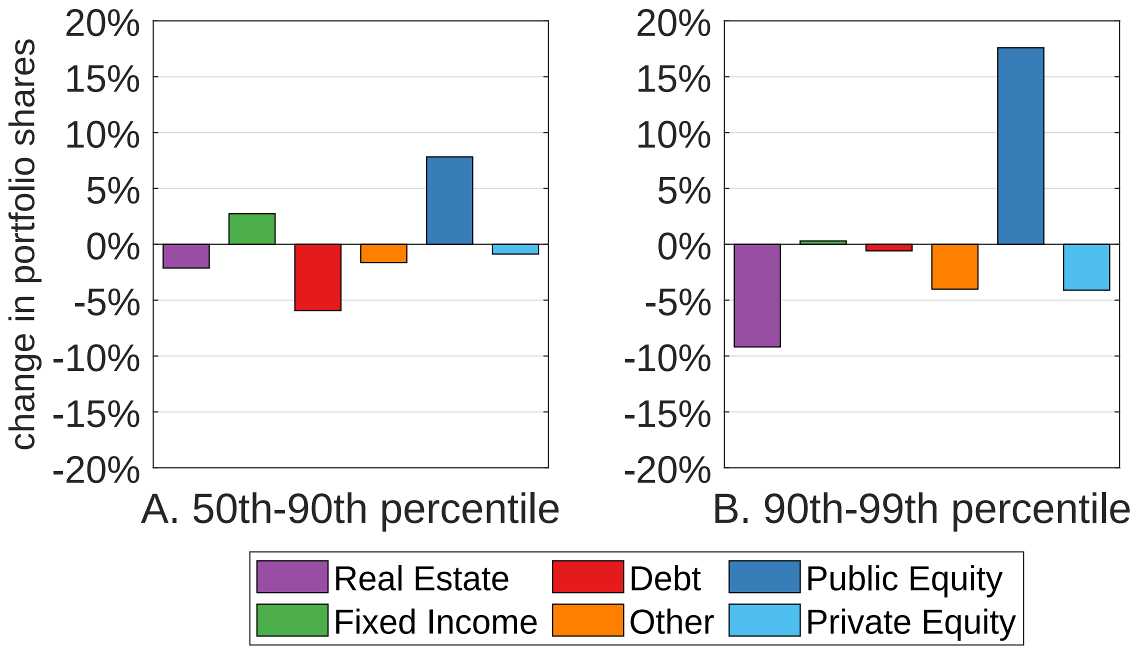

Figure 1.A.5: Change in portfolio share in each asset class, between 1989 and 2019

Notes: This figure shows the change in the wealth share of each asset, class from 1989 to 2019, in the portfolio of those in between the 50^{th} and 90^{th} percentiles, and those in between the 90^{th} and 99^{th} percentiles of wealth.

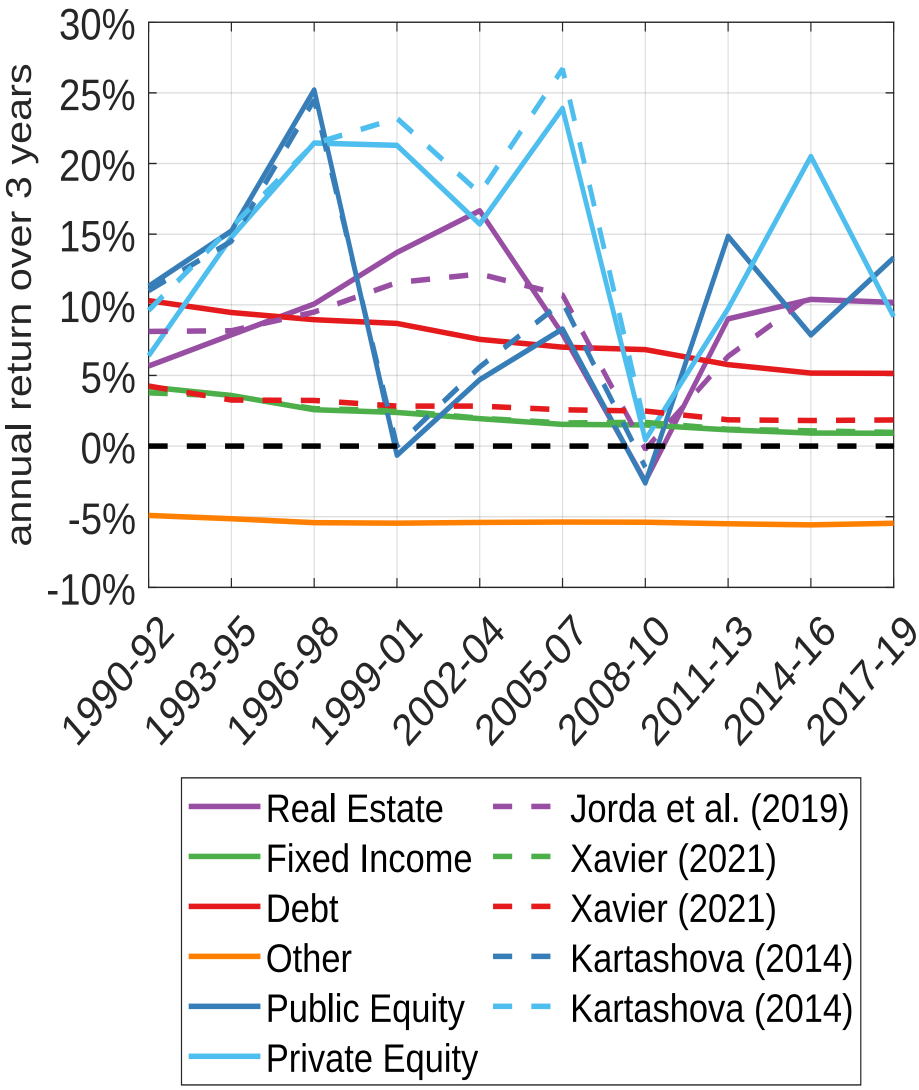

Figure 1.A.6: Comparison of annualised total nominal returns to other sources

Notes: The solid lines show the annualised total (including capital gains and dividends) nominal return in between SCF waves for the aggregate portfolio of US households, using the methodology described in Section 1.2.2; and the dashed lines compare it to other sources.

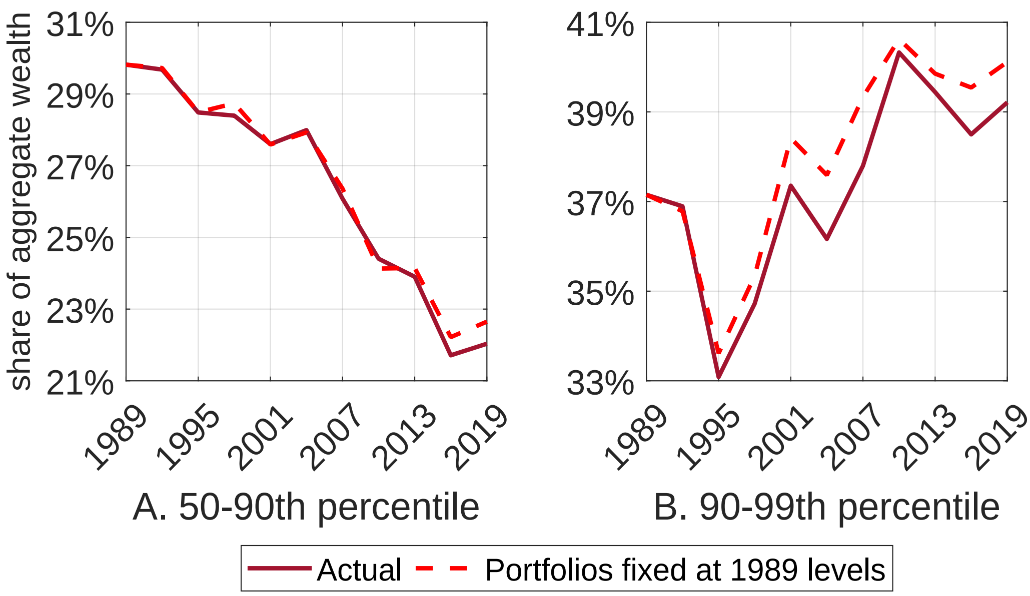

Figure 1.A.7: Evolution of counterfactual wealth shares, without portfolio effect

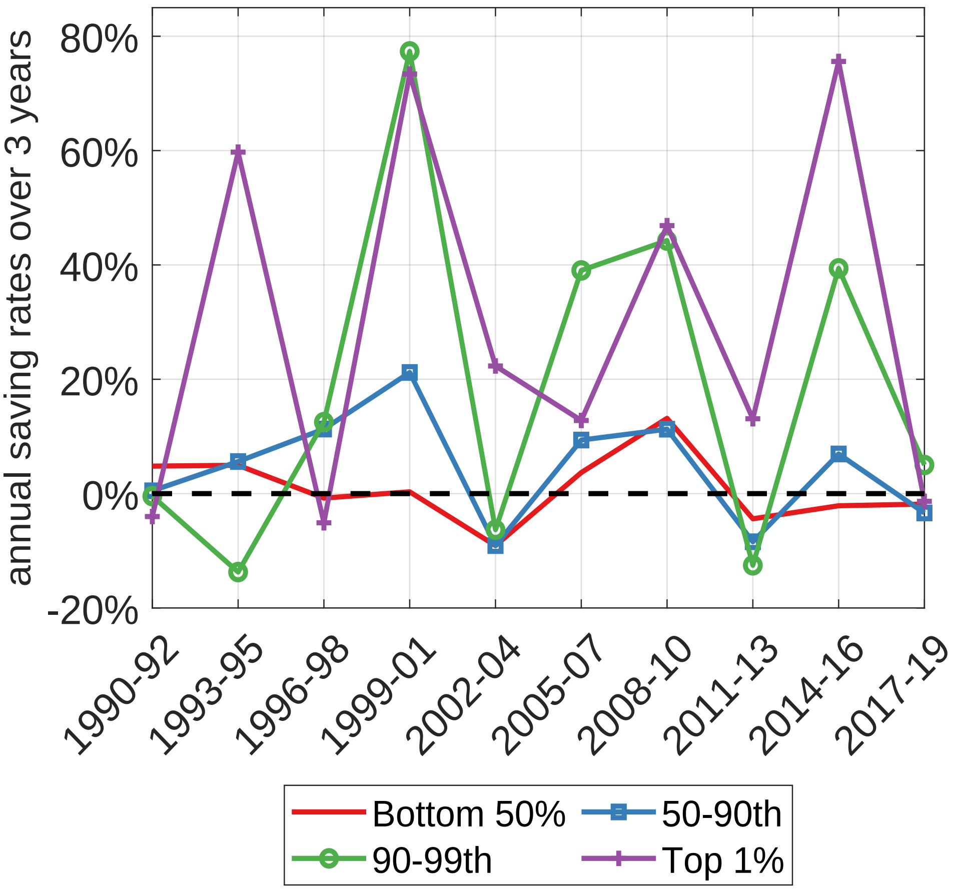

Figure 1.A.8: Baseline estimates for annualised saving rates in between waves of the SCF

Notes: This figure plots the saving rates in between waves of the SCF as estimated in Section 1.3. Table 1.3 shows the average over the whole period (1989-2019) of the saving rates displayed here.

Chapter 2

Income Taxes, Portfolio Choice and Wealth Inequality

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Abstract

This paper revisits the impact of income taxes on wealth inequality, now allowing for households to respond by changing their portfolio composition. First, an analytical model shows that portfolio adjustments magnify the impact of a fall in tax rates on wealth inequality beyond what their direct impact in savings would predict. This is because the decrease in tax rates also induces a shift towards high-return and high-risk assets, which further increases the saving rate and the volatility of households' portfolio, leading to higher wealth inequality. Second, I develop a quantitative model of households' saving and portfolio choice and analyse the decrease in US tax rate progressivity since 1975. Relative to when portfolios are kept fixed, allowing households to react by changing their portfolio composition amplifies the impact of taxes on the Top 1% wealth share by approximately 20% from 1975 to 2019, and by 25% when in the new steady state.

Keywords: Wealth Distribution, Household Finance, Taxation.
J.E.L. codes: E21, G50, H31.

2.1 Introduction

The increase in wealth inequality in the United States documented by Saez and Zucman (2020) and Smith, Zidar, and Zwick (2020), in particular the increase in the wealth shares held by the wealthiest households, has generated a big debate on what were its main causes.

One of the possible explanations proposed is that the increase in inequality was caused by the fall in the effective tax rates paid by those at the top of the wealth distribution. As shown in Figure 2.1, the taxes paid by those in top 1% of the taxable income distribution have fallen from the 1970's to the 1990's, and picked up somewhat afterwards. For the top 0.01%, they remain markedly lower than what they were 5 decades ago.

To investigate how important are changes in income tax rates for wealth inequality, I develop an analytical and a quantitative model in which households decide how much wealth to save, but also the composition of their portfolios. I find that, while taxes have an effect on the wealth distribution by changing savings rates through their impact on the rate of return on wealth, portfolio composition effects that were previously unaccounted for are large. That is because the fall in taxes also increases investment in risky assets, causing the expected after-tax rate of return and the volatility of future wealth paths to increase, both of which contribute to even higher savings rate and wealth inequality. The increase in the volatility of portfolios directly increases wealth inequality as well.

A key result is that the portfolio effects are large even though the model features households that face a simple portfolio decision between assets that capture the risk/return trade-off observed in the data, summarised by three assets: real estate, equity and net safe. Thus a contribution of the paper is to show how this decision, which is embedded in many household-finance models, is an important channel for explaining wealth inequality dynamics.

The starting point of the paper is to use the Survey of Consumer Finances (SCF) to classify assets into three broad classes that capture their risk-return trade-offs: Real Estate, Net Safe and Equity. Figure 2.2 shows how from 1989 to 2019 the composition

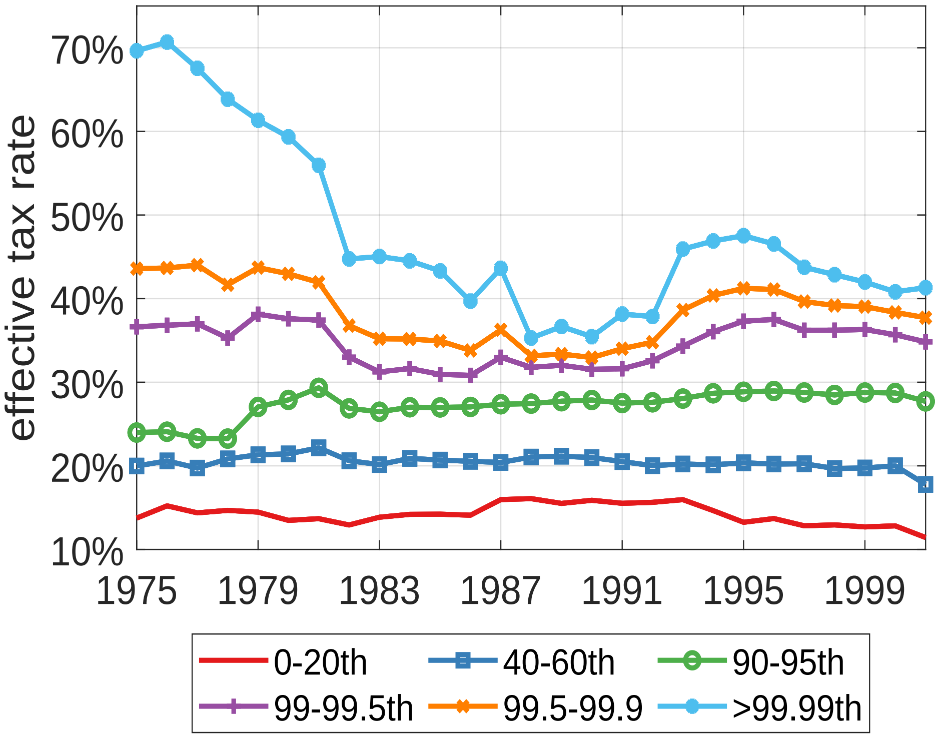

Figure 2.1: Effective tax rates for selected taxable income groups, from Piketty and Saez (2007)

Notes: Effective tax rates, as calculated by Piketty and Saez (2007), include all federal tax rates: income, corporate, payroll, capital gains and estate taxes. Individuals are ranked by taxable income excluding capital gains, although they are added back when calculating effective tax rates.

of the aggregate portfolio has changed, with a large increase in the importance of Equity in the period. As we will see later, that increase was even stronger for the wealthiest households as well, at the same time that the effective tax rate that they were paying was changing too.

Second, I use a random growth model of wealth augmented with portfolio choice to analyse how the impact of a change in taxes can be amplified by portfolio decisions. The model admits an analytical solution in an asymptotic analysis when wealth approaches infinity, both to the policy functions of the households' problem and to the thickness of the right tail of the distribution of wealth. This allows me to decompose the total impact of taxes on the top wealth inequality with and without portfolio choices into three separate effects (1-3). Without portfolio choices, a fall in taxes (1) increases the savings rate both because the return on savings has increased, and also due to a behavioural response that reduces the consumption rate. Wealth inequality then increases because

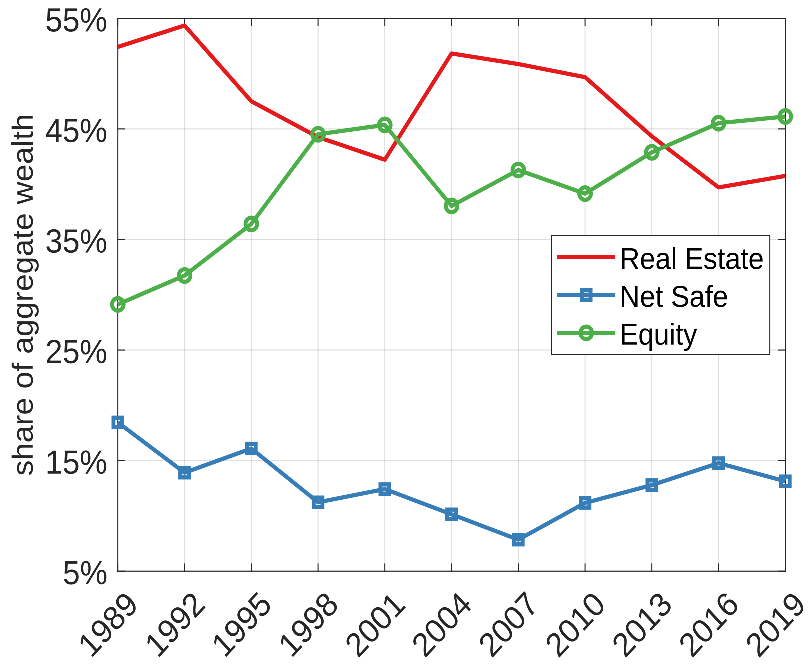

Figure 2.2: Evolution of portfolio shares for the aggregate portfolio of US households

the rate of wealth accumulation increases.

With portfolio choice the response is richer. When taxes fall, the spread between risky and riskless assets increase, which pushes households to invest more in risky assets. The increase in the allocation to risky assets has two direct consequences: the rate of return on wealth increases further and the riskiness of the portfolio also rises. Both these factors contribute to (2), a further reduction in the consumption rate, pushing inequality even higher. Finally, there is also an effect (3) not mediated by the savings rate, as the rise in the riskiness of portfolios directly increases wealth inequality because portfolios get more volatile.

Third, leveraging on the intuition of the analysis from the analytical model, I develop a quantitative, partial equilibrium, model of households' portfolio choice over three asset classes mentioned before: Real Estate, Net Safe and Equity. I allow assets to differ in terms of their risk and return profile, and also how liquid they are as Real Estate adjustment incur fixed and proportional transaction costs. The model generates a distribution of wealth and portfolio choices across the distribution comparable to those in the data, which was not possible with the analytical model.

Importantly, I also input a realistic version of the progressive tax schedule faced by households in the US since the 1970's. To do so, I use the estimates from Piketty and Saez (2007) in Figure 2.1, which shows that the effective tax rate paid by the very top (within the top 1%) of the income distribution has fallen relative to what is was in 1975.

I then perform my main exercise: comparing the impact of the changes in the tax rates in the US from 1975 to 2001 on wealth inequality, with and without the response of portfolio choices. In line with other studies, both cases show that the change in the tax system had a sizeable impact on wealth inequality, but allowing households' to respond by adjusting their portfolio makes the final impact significantly larger. It increases the impact of a change in taxes on the top 1% wealth shares by 20% in the transition and by 25% in the new steady-state. The results shown how important it is to take portfolio choices into account when evaluating quantitatively the possible drivers of wealth inequality.

2.1.1 Literature Review

The paper closest to this one, and probably the most exhaustive analysis of the determinants of the increase in wealth inequality in the US using a quantitative macro model, is the one by Hubmer, Krusell, and Smith (2021). They also share the preoccupation of looking at the whole distribution of wealth, and analyse the impact of many different potential explanations to the increase in wealth inequality, including the same changes in taxes rates analysed in this study. My main contribution relative to their paper is to explicitly model households' portfolio choice and show how this dimension is an important factor in determining the impact of tax rates on wealth inequality.

Hubmer, Krusell, and Smith (2021) incorporate households' portfolios in their framework by imputing the risk and return profile that individuals face at different levels of wealth and at different points in time. However, by not explicitly modeling portfolio choices (only consumption/saving), when they perform counterfactuals households are not able to respond by changing the composition of their portfolios. As I show in my main results, this can be an important source of amplification of the impact of taxes. Thus one can see the quantitative model in Section 5 of this paper as a step forward in this dimension (although I make simplifications in other dimensions when compared to their study, e.g., partial vs general equilibrium).

There is also a group of papers including Aoki and Nirei (2017), Moll, Rachel, and Restrepo (2021), Nirei and Aoki (2016) and Gomez and Gouin-Bonenfant (2020) that analyse how the portfolio decision of entrepreneurs change when a shock hits the economy, and how this impacts wealth inequality, specially at the top of the distribution. Gomez and Gouin-Bonenfant (2020) look at the impact of falling risk-free rates; Aoki and Nirei (2017) and Nirei and Aoki (2016) look at the impact of a fall in top tax rates, although their focus is on income inequality; and Moll, Rachel, and Restrepo (2021) analyse the impact of a technological change.

Relative to the papers mentioned just above, my contribution is to show how the simple portfolio decision between three standard assets faced by households all over the wealth distribution is an important channel that can magnify the impact of changes in the environment - in particular, taxes - without the need to consider entrepreneurship choices. Also, I look at the impact of the changes in tax rates on the US over the whole distribution, not just the top wealth brackets.

The rest of the paper is organised as follows. Section 2 describes the data from the SCF used to construct portfolios across the wealth distribution and over time. Section 3 presents the analytical model and uncovers the impact of taxes on wealth inequality through portfolio choices. Section 4 develops the full quantitative model. Section 5 shows the results for the impact of changes in taxes on wealth inequality. Section 6 concludes.

2.2 Data

2.2.1 Wealth and Portfolios

The main data source for this paper is the SCF, and I use it to calculate wealth and portfolio compositions across the wealth distribution. The details of the construction are the same as in Chapter 1, so I do not repeat them here.

The one difference in this study is that, for calibration purposes in the quantitative

Chapter 2	Chapter 1	Main Components
Real Estate	Real Estate	primary residence, second houses, investment properties, commercial real estate
Net Safe	Fixed Income	checking and savings accounts, directly held bonds and certificates of deposit, fixed income in pensions and fixed income in mutual funds
	Debt	mortgages, home equity credit, student debt, credit card, vehicle debt, consumer credit
	Other	vehicles, art, other financial assets
Equity	Public Equity	equity in hedge funds, directly owned stocks, equity in pensions
	Private Equity	privately owned businesses, managed or not

Table 2.1: Classification of Assets and Liabilities

Notes: This table shows the methodology used to group components of wealth from the SCF into the three main asset classes used in this paper, and its comparison to the six asset classes from Chapter 1.

model, some asset classes are bundled together to not burden excessively the numerical solution (from originally six asset classes to three now). The bundling is chosen to try and preserve as much as possible the risk/return characteristics of the assets classes. Table 2.1 shows the categories used in the portfolio choice models in this paper, compared to those in the accounting exercise of Chapter 1, and also its main components.

Figure 2.2 showed how the aggregate portfolio of US households is changing according to the new classification of assets. What is clear from that figure is that the importance of Equities has increased since 1989, while Real Estate shows a more cyclical pattern, and Net Safe assets have fallen in importance.

What it does not show, however, is how portfolios changed differently across the wealth distribution. Figure 2.1 remedies that and shows how the importance of Equity, the asset with the highest risk and return, has evolved for different wealth groups.[1] Interestingly, even though its share has increased for all wealth groups, it increased the most for the wealthiest agents: from 1989 to 2019, the share of Equity increased by 17.7 p.p. and 14 p.p for those in the Top 1% and those between the 90^{th} and 99^{th} percentiles, respectively; but only 7.8 and 9.1 p.p. for those in the 50^{th} to 90^{th} and Bottom 50%

[1]Figure 2.A.1 shows how the portfolio shares of all asset classes evolved for the Bottom 90% and the Top 1%.

groups, respectively. Given that taxes changed differently across the income distribution, the models in the following sections will investigate whether this two trends might be linked.

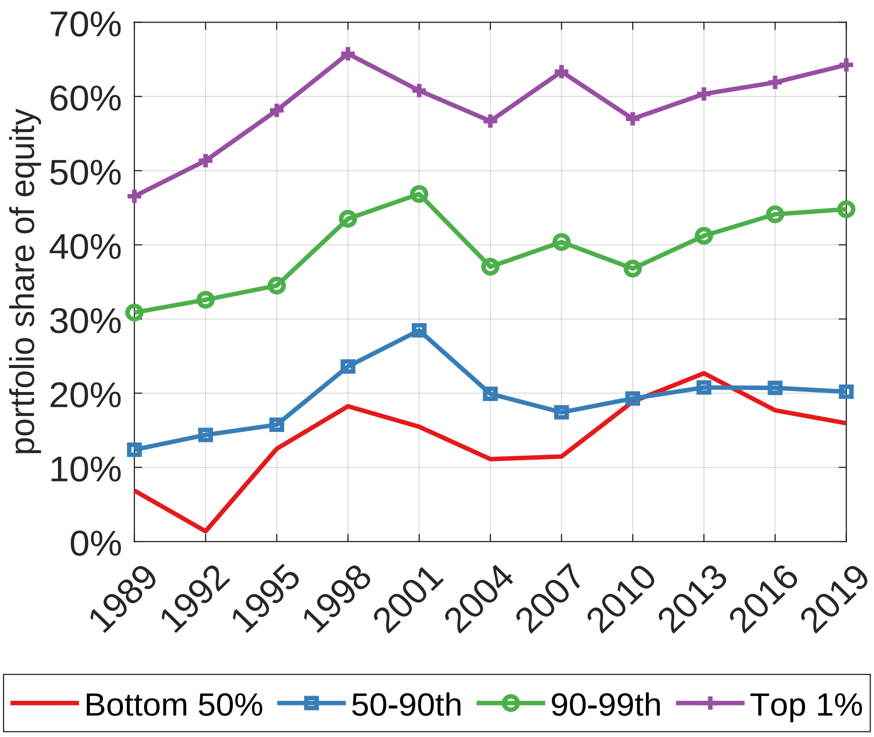

Figure 2.1: Portfolio share of equity for different wealth groups

2.2.2 Tax Rates

The data source for tax rates is the work by Piketty and Saez (2007), who calculate the effective tax rates as a share of taxable, for many groups along the distribution of taxable income as shown in Figure 2.1. Their estimates include all federal taxes (income, corporate, payroll, capital gains and estate taxes) and take into account some tax benefits for Real Estate, such as deduction from interest rate paid on mortgages. The main benefit of using their tax rates is that it is the effective tax paid, so changes to the tax system that are not reflected on marginal tax rates (e.g., changes in regulation about deductions, better tax planning from households, or changes in tax evasion) are still captured by their measure.

However, when thinking about the impact of taxes on portfolio allocation, the natural question to ask is: which taxes are important for portfolio choices? After all, the models in Sections 2.3 and 2.4 feature a simplifying assumption with a single tax applied to all the income, while in reality there are different types of taxes for different kinds of income.

To find out which taxes matter, we must then look at where are the returns to portfolio choices coming from, and the answer naturally depends on which asset we are looking at. Using the rates of return from Chapter 1 for the period of 1989 to 2019, which use a mix of SCF and external sources, I estimate that between 65 and 70% of the return in equity comes from capital gains, while the rest is dividends. On the other hand, using data from Jordà et al. (2019), for real estate the importance of capital gains is much lower, smaller than 20% of the total return. Of course, owner-occupied real estate does not pay taxes on imputed rent, so most of the return is untaxed (although the importance of mortgage deductions is increasing in the regular income tax), which will be true in the model of Section 2.4 as well. Finally, capital gains taxes should not be important for the return of safe assets.

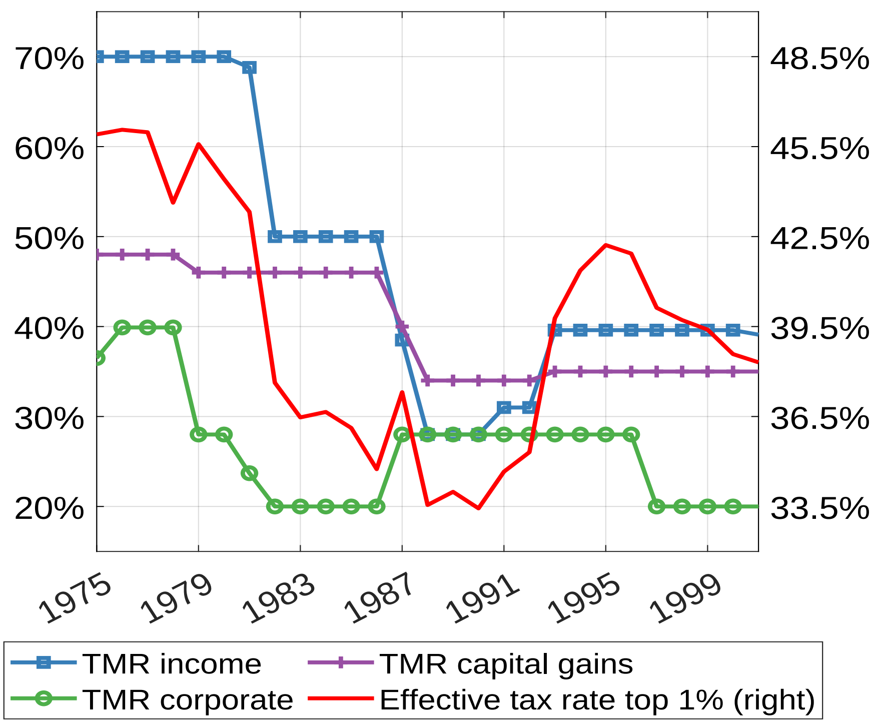

Figure 2.2: Effective Tax Rate and Top Marginal Rates (TMR) for Different Taxes

Notes: This figure plot the Top Marginal Rates (TMR) for different types of taxes (income tax, corporate income tax and capital gains tax) and the effective tax rate for the top 1%. Effective tax rate is from Piketty and Saez (2007) and TMRs are from Saez, Slemrod, and Giertz (2012)

Given that capital gains account for a substantial part of the return of equity, one might worry that the decline in tax rates from Figure 2.1 might not have a big impact on portfolio choice, if it is the result solely of a decline in non-capital gain income tax. However, (i) short-term capital gains are taxed at the same rate as other income; and (ii) as Figure 2.2 shows, the movements of the top marginal tax rates (TMR) on long-term capital gains, corporate taxes and income taxes have all been similar to those of the effective tax rates at top percentiles of income: they were high in the mid-70s, then they fell a lot. There is a rebound either in the late 80s or early 90s (except for corporate taxes), followed by another decrease in the late 90s. Therefore, I believe that the series of effective tax rates used in this paper is a good summary of how the tax rate that households were facing when making portfolios choices changed throughout the period of the analysis.[2]

2.3 Analytical Model

The goal for this section is to better understand how portfolio choices respond to changes in the environment and impact wealth inequality dynamics. Therefore, it is instructive to start with a basic model for portfolio choice to help us build intuition. I start with a standard random-growth model for wealth, augmented with a portfolio choice an is Merton (1969), a simple flat tax rate that applies to all income, and possibly stochastic labor income. There will be an analytical solution for the right tail of the distribution, and that will be the focus of the analysis in this section.

In this environment there are only 2 assets: a risk free asset b which pays a fixed rate r_b and a risky asset with a rate of return of $r_e dt + \sigma_e dB_t^e$, where r_e is the average return, dB_t^e is a idiosyncratic Brownian motion shock, and σ_e is the volatility of the return.

[2] To the extent that households not in the top of the income distribution were also making long-term capital gains and their effective tax rates were not falling because the income tax rate did not fall, I could be underestimating the portfolio effect outside of the top percentiles. If that is the case, one might worry that the reason why there is an increase in wealth inequality in the baseline model is because the top is investing more in risky assets and the bottom is not, while the tax on long-term capital gains rate fell for everyone. However, I do not believe this to be driving the results because, as discussed above, capital gains are not the main source of return for real estate, which is the major asset by far outside of the top 10%. Furthermore, as discussed in Section 2.3, even if households all across the distribution increase their investment in risky assets by the same amount the wealth inequality increases because both rates of return and riskiness of portfolio increase.

Here we will assume that both assets are fully liquid, which will be crucial for the existence of analytical results. There is a flat tax τ_y on labor income and another flat tax τ on the non-stochastic component of the return on assets.[3]

Finally, households have a discount rate of ρ and die at rate δ, being replaced by newborns with zero wealth; labor income y can be risky (for example, has some discrete probability distribution as in an Aiyagari model, or follows an Ornstein–Uhlenbeck process, the equivalent of an AR(1) in continuous time); households can borrow up to some limit θ_u; and cannot short nor leverage their risky asset positions.

With a CRRA utility function $u(c) = c^{1-\gamma}/(1-\gamma)$ on consumption c, the household's problem can be described by a Hamiltonian-Jacobi-Bellman (HJB) equation of the form:

$$\begin{aligned}
&(\rho+\delta)V(b,y) = \\
&\quad \max_{c,x}\left\{\frac{c^{1-\gamma}}{1-\gamma} + V_b[(1-\tau)(r_b b + (r_e - r_b)x) + (1-\tau_y)y - c] + \frac{V_{bb}}{2}(x\sigma_e)^2 + \frac{1}{dt}E[V_y]\right\}, \\
&\quad \text{subject to} \qquad\qquad (2.1) \\
&\quad b \geqslant -\theta_u, \text{ and } 0 \leqslant x \leqslant \max\{b, 0\}.
\end{aligned}$$

where the term $(1/dt)E_t[V_y]$ depends on the specific process chosen for labor income.

With no labor income risk in the problem above we would have analytical solutions for the policy functions for savings and equity investment, and both of them would be linear in total wealth b. However, the policy functions that are solutions to the problem in (2.1) are also linear in the asymptotic case $b \to \infty$, as the risk for labor income becomes negligible.[4]

[3] In this case, the optimal investment choice will be $x = (1-\tau)(r_e - r_b)/\gamma\sigma_e^2$. However, if there was a tax on the stochastic component of the risky asset as well the effective volatility of its returns would be equal to $\sigma_e(1-\tau)$, meaning that the investment choice would instead be equal to $\tilde{x} = (r_e - r_b)/\gamma((1-\tau)\sigma_e)^2$. Thus, the choice of whether to apply taxes to the stochastic component is crucial since it determines whether higher taxes decrease (x) or increase ($\tilde{x}$) the allocation to risky assets, although in both cases a change in taxes means that portfolio choices will have a sizable impact on wealth inequality (but with different directions). I believe not applying taxes on the stochastic component is the best choice for modelling the investment decision because the tax system gives incentives to households to not report gains in every period, thereby smoothing the return over time and reducing the possibility of taxes representing a reduction in the volatility. Households have incentives to not report capital gains every period as: (i) capital gains taxes are lower for longer holding periods; (ii) there is a progressive schedule of capital and income taxation.

[4] See Benhabib, Bisin, and Zhu (2015) for a proof of the asymptotic linearity of the saving

Furthermore, one can show that in this case the right tail of the wealth distribution will converge to a Pareto distribution, with a know parameter ζ governing its shape. This result is convenient because since the work of Pareto (1896) we know that power-law probability distributions are well-suited for modelling the top of the distribution of wealth and income in a society.

Therefore, the model analysed in this section is useful since we will be able to have analytical solutions for the portfolio and savings decision, and also for their impact on wealth inequality through their impact on tail inequality ζ.

2.3.1 The Effect of Portfolio Choices

In the asymptotic case $b \to \infty$ the savings rate $s = (1-\tau)(r_b b + (r_e - r_b)x) + (1-\tau_y)y - c$ and portfolio choice x converge to $\overline{s}b$ and $\overline{x}b$ respectively, where:

$$\overline{s} = \frac{(1-\tau)(r_b + (r_e - r_b)\overline{x}) - (\rho+\delta)}{\gamma} + \frac{\gamma - 1}{2}(\overline{x}\sigma_e)^2, \text{ and} \tag{2.2}$$

$$\overline{x} = \frac{(1-\tau)(r_e - r_b)}{\gamma\sigma_e^2}, \tag{2.3}$$

while the right tail of the distribution converges to a Pareto distribution with shape parameter equal to

$$\zeta = \frac{1}{2} - \frac{\overline{s}}{(\overline{x}\sigma_e)^2} + \sqrt{\left(\frac{1}{2} - \frac{\overline{s}}{(\overline{x}\sigma_e)^2}\right)^2 + \frac{2\delta}{(\overline{x}\sigma_e)^2}}. \tag{2.4}$$

From Equation (2.3) the impact of taxes on portfolio choice is clear: higher taxes reduce the premium of investing in the riskier asset, thereby reducing the allocation to riskier assets. The direct impact of taxes on the savings rate is also clear: keeping $\overline{x}$ constant, we have $\partial\overline{s}/\partial\tau = -(r_b + (r_e - r_b)\overline{x})/\gamma$, and the savings rate falls when taxes increase because the return to savings has fallen. Notice that this includes both the mechanical effect of the increase in taxation and a behavioral response of the consumption rate for

rate, and Benhabib and Bisin (2018) for a thorough discussion on its implications for the shape of the wealth distribution. See Achdou et al. (2021) for a proof of all the asymptotic results above in continuous time. The linearity of the optimal investment decision with no labor income and CRRA utility function has been know since Merton (1969); see also Krusell and Smith (1998) for an example that uses the asymptotic linearity of the savings policy function for its solution algorithm with aggregate shocks. In Appendix C I provide a full derivation of the savings function, the investment decision and the Pareto parameter in the case of no labor income.

a given $\overline{x}$.

However, the full response of taxes on savings rates when allowing for portfolios to change is equal to:

$$\frac{d\overline{s}}{d\tau} = \underbrace{\frac{\partial \overline{s}}{\partial \tau}}_{\substack{\text{partial effect}\\ \text{of taxes}}} + \underbrace{\frac{(1-\tau)(r_e - r_b)}{\gamma}\frac{d\overline{x}}{d\tau}}_{\substack{\text{effect of portfolio}\\ \text{change on return}}} + \underbrace{(\gamma - 1)\sigma_e^2 \frac{d\overline{x}}{d\tau}}_{\substack{\text{effect of portfolio}\\ \text{change on risk}}}. \tag{2.5}$$

Allowing for $\overline{x}$ to change thus adds two extra terms. The two additional effects arise because the increase in taxes further lowers the expected return of the portfolio due to a reduction in the allocation to risky assets; and also now the return on wealth is less risky, thus consumption increases given that future paths for wealth are less volatile. All three terms in Equation (2.5) above are negative, thus there is no difference in sign between $\partial \overline{s}/\partial \tau$ and $d\overline{s}/d\tau$, but the magnitudes can be quite different, as we will see in a numerical example later.

In a similar way, the final impact of taxes on the thickness of the right tail of the distribution is going to be larger in magnitude once we allow for portfolio reallocation. For a fixed $\overline{x}$, the impact is equal to $d\zeta/d\tau = (\partial \zeta/\partial \overline{s})(\partial \overline{s}/\partial \tau)$, while with changing portfolios the total effect becomes:

$$\frac{d\zeta}{d\tau} = \underbrace{\frac{\partial \zeta}{\partial \overline{s}}\frac{\partial \overline{s}}{\partial \tau}}_{\substack{\text{(A) keeping}\\ \overline{x}\text{ fixed}}} + \frac{d\overline{x}}{d\tau}\left[\underbrace{\frac{\partial \zeta}{\partial \overline{s}}\frac{\partial \overline{s}}{\partial \overline{x}}}_{\substack{\text{(B) effect of}\\ \overline{x}\text{ through }\overline{s}}} + \underbrace{\frac{\partial \zeta}{\partial \overline{x}}}_{\substack{\text{(C) direct effect}\\ \text{of }\overline{x}}}\right]. \tag{2.6}$$

Equation (2.6) summarises the importance of taking into account portfolio choice responses when looking at the impact of taxes on wealth inequality. With $\overline{x}$ fixed, we only have the direct effect of taxes on savings impacting wealth inequality (effect (A)). On the other hand, as $d\overline{x}/d\tau \neq 0$, changes in portfolio further reduce savings rates as explained in Equation 2.5 (effect (B)), but there is also a new term (effect (C)) because lower shares of the risky asset reduces the riskiness of the portfolio, which directly reduces the tail of the distribution of wealth through a channel that is independent of the savings rate.[5]

[5] I show in Appendix C that the terms $\frac{\partial \zeta}{\partial \overline{s}}$ and $\frac{\partial \zeta}{\partial \overline{x}}$ are both negative.

Moreover, it is important to emphasize that the assumption in this section is that there is a single tax rate for all individuals τ, thus the changes in portfolio composition at the right tail of the distribution are the same for all households. In other words, the increase in inequality through portfolio changes is not due to taxes changing in different ways at different levels of wealth, and impacting portfolios differently. Therefore, wealth inequality is impacted even when all households change their portfolio in the same way, through its impact on savings rate and the overall riskiness of the portfolio, as made clear in effects (B) and (C) above.

Finally, whether effects (B) and (C) are economically meaningful or not to understand inequality dynamics is ultimately a quantitative question, and to answer that I first need a measure of ζ from the data.

2.3.2 Pareto Inequality

If the whole distribution of wealth was exactly a Pareto distribution, its cdf would be $H(a) = 1 - (a/\underline{a})^{\zeta}$ in $a \geqslant \underline{a} > 0$, and the shape parameter $\zeta > 0$ would summarize all the information on top wealth shares, with a lower (higher) ζ associated with a thicker (thinner) right tail of the distribution, and higher (lower) wealth inequality. For example, one can show that the top percentile p of the population would hold a share $p^{(\zeta-1)/\zeta}$ of total wealth (e.g., if $p = 1\%$ and $\zeta = 1.5$, we have that the the top 1% wealth share is equal to 21.5%).

Because only the right tail of the empirical wealth distribution resembles a Pareto, it would not be appropriate to use the series of the top 1% wealth share over time on its own to back out ζ. However, it is possible to back out ζ in a different way,[6] using the relative values of the top wealth shares of different groups. For example, assuming that wealth follows a Pareto distribution for the top 1%, one can show that:

$$\zeta = \frac{1}{1 + \log_{10}(\omega_{0.1\%}/\omega_{1\%})}, \tag{2.7}$$

where $\omega_{0.1\%}/\omega_{1\%}$ is the ratio of wealth held by those in the top 0.1% relative to those in

[6] See Vermeulen (2018) for a recent example of a paper which main focus is the estimation of ζ for many advanced economies.

the top 1%. I use Equation (2.7) to construct a series of ζ over time for the US based on empirical top wealth shares from Smith, Zidar, and Zwick (2020),[7] which is displayed in Panel B of Figure 2.1. Next, I compare this empirical series with the predictions from the model.

2.3.3 Results for Analytical Model

Equation (2.6) provided intuition on why portfolio changes can be important when thinking about changes in wealth inequality, and this section will provide a first quantitative answer on whether this effect is economically meaningful.

To do so, I compare the paths for ζ as a function of the tax rate τ according to Equation (2.4), under two different assumptions: allowing the risky portfolio share $\overline{x}$ to react to the change in taxes as in Equation (2.3), or not. Panel A of Figure 2.1 plots the two series and the results of this exercise.[8] It shows that allowing the tax rate on the top 1% to fall from 46% to 38% (notice that the x-axis is reversed) is able to generate a large fall in ζ from 2.03 to 1.41 when portfolios are adjusting, but only to 1.67 when portfolios are fixed.

To better understand the significance of this result, Panel B plots the measure of ζ from the data according to Equation (2.7), and the effective tax rate on the 1% according to Piketty and Saez (2007) from 1975 to 2001.[9] During this period, there was an overall increase in wealth inequality, as shown by a fall in ζ, at the same time taxes on the top of the income distribution were falling. One can see that the fall in taxes from 46% to 38% in Panel A was not chosen at random, it is the total decrease in the data from 1975 to 2001. Furthermore, we see that ζ fell from 2.03 to 1.47 in the same period according to the data, and it was equal to 1.66 in 1988. Therefore, the magnitude of allowing portfolios to change when taxes in the top 1% fall from 46 to 38% is equivalent to the

[7]Given that what matters for ζ in Equation (2.7) is the wealth share of the top 0.1% relative to the wealth share of the top 1%, not their levels, using data from Saez and Zucman (2016) would not change significantly the magnitude of the fall in ζ.

[8]For this exercise the parameters used were: $\gamma = 1.5, r_b = 3\%, r_e = 7.6\%, \rho = 1.6\%, \delta = 3\%, \sigma_e = 0.19$. All parameters, except for r_e and ρ, are exactly the same as in the calibration of the full model in Section 2.4. The parameters r_e and ρ were changed to be able to target $\overline{x} = 46\%$ (the share of equity in the portfolio of the top 1% in 1989) and $\zeta = 2.03$ when $\tau = 46\%$.

[9]This is the period of the analysis because top tax rates start to fall just after 1975, and the data stops at 2001.

evolution of top wealth inequality from 1988 to 2001, which is a large effect.

Furthermore, in Panel A we can also see the evolution of $\overline{x}$ as implied by the model, when it is allowed to react to taxes. It makes clear that the increase in wealth inequality generated by changes in portfolio allocation does not rely on an unreasonably large increase in investment in risky assets. When taxes fall, the the share of risky assets increases by 6.5p.p, from 45.9% to 52.4%. As a comparison, Panel B of Figure (deleted) shows that the share of total equity in the portfolio of the top 1% increased by more than 15p.p. from 1989 to 2019. Thus, the movements in portfolio shares implied by the model as a result of a change in taxes seem reasonable.

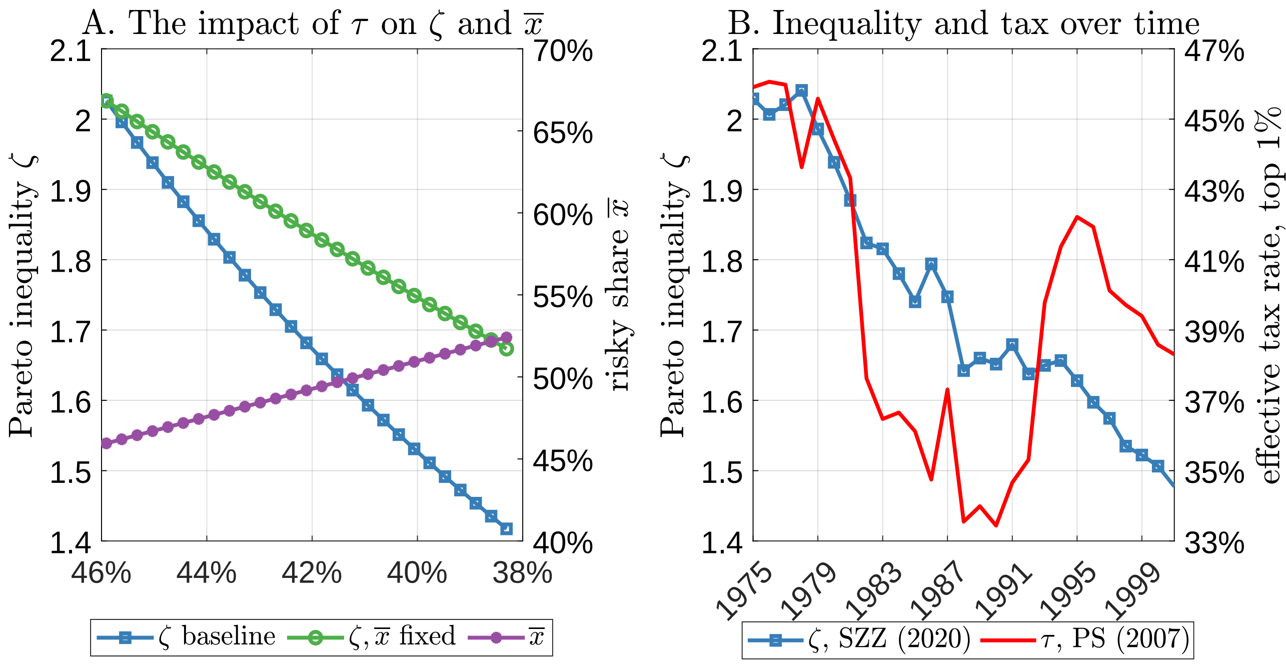

Figure 2.1: Pareto Inequality ζ in the analytical model (A), and in the data (B)

Notes: Panel A compares ζ from Equation (2.4) as a function of tax rate τ when the share of risky assets $\overline{x}$ is allowed to change (baseline) to when it is fixed; it also shows the evolution of $\overline{x}$ when it is allowed to change. Notice that the x-axis is reversed. Panel B shows the evolution of Pareto inequality ζ constructed from the relative wealth shares of the top 0.1% and top 1% series from Smith, Zidar, and Zwick (2020) as explained in Section 2.3.2, and the effective tax rate for the top 1% from Piketty and Saez (2007).

Finally, it is also possible to decompose the effect of taxes on ζ in the effects show in Equation (2.6). Figure 2.A.2 shows the results of this exercise, with the total effect as in Figure 2.1, and then the paths for ζ when only one of the effects (A), (B) or (C) are allowed to operate. It shows that the direct of taxes (A) is the largest effect, followed by the effect through portfolio changes on the rate of return of wealth (B) and the effect through portfolio changes on the volatility of the portfolio (C). Is is important to highlight that effects (B) and (C), when taken into account together, are as important as the direct effect of taxes. Also, even the smallest effect (C) is significant, representing more than 20% of the total effect effect of taxes.

While the model in this section is stylised in several dimensions, it provided us with the intuition on how portfolio changes can impact wealth inequality dynamics, and also a first quantitative answer that impact can be significant. The next sections aims at developing a fully quantitative model, and shows that the answer stays the same.

2.4 Full Model

The model in this section extends the model from Section 3 mainly in two directions: realistic portfolio choices and tax structure. With respect to portfolio choices, it is closest to a series of papers that share the preoccupation that to model the demand for any of the three main categories of assets (equity, fixed income and housing) it is important to consider all three assets at the same time. Recent examples include Catherine (2020) and Vestman (2018), and earlier ones include Cocco (2005), Flavin and Yamashita (2002) and Yao and Zhang (2005).

With respect to taxation, I follow the strategy of Hubmer, Krusell, and Smith (2021) of using data from Piketty and Saez (2007) on effective tax rates paid by different groups of the income distribution. Given the differential tax treatment of housing, I also allow individuals to claim mortgage deduction on their tax bill, and to not pay taxes on rent imputed to owner-occupied housing, with further details below.

Furthermore, because of the level of tax progressivity in the US, it would not be possible to generate enough wealth inequality in the model without some level of increasing returns to wealth. Following the evidence from Bach, Calvet, and Sodini (2020), Fagereng et al. (2020) and Xavier (2021) and the results from Section 2.2, the returns on all assets are increasing in total wealth, in a way that does not distort portfolio choices.

The economy in the model is populated by a continuum of ex-ante identical households that face idiosyncratic shocks to their labor income and risky investments. Households make portfolio and saving decisions to optimize their discounted, expected utility. Moreover, households die at rate δ and are replaced by newborns with zero wealth and labor income drawn from the stationary distribution of labor income. Finally, the model is partial equilibrium, thus rates of return for the different assets are calibrated parameters. This significantly simplifies the numerical solution, which is already complicated given the presence of multiple assets and the necessity to model all individuals, including very wealthy ones.

2.4.1 The Households' Problem

Households have a discount rate ρ and can invest into liquid b and illiquid h assets. Liquid assets can be further divided into risky equity x or risk-free asset $b - x$. The illiquid asset is interpreted to be housing and to adjust its level households must pay a fixed cost κ_0 and a proportional cost $\kappa_1 h$ when buying or selling a property. Housing assets have the advantage of being accept as collateral for debt (mortgages) up to a value of θh, where θ is the loan-to-value ratio. Households also have access to a small amount of uncollateralised debt θ_u in the case they do not own a house.

Households also pay taxes in all their income, with an increasing schedule, and there are transfers for the poorest households. I denote total taxes net of transfers $T(b, h, y)$ and the increasing effective tax rate that households have to pay on their income as $\tau(b, h, y)$. The notation with taxation would be too convoluted, thus I present the model below noting that there are net taxes T to be paid (which are not lump-sum), and then discuss in detail how taxes work in Section 2.4.4.1.

The household's problem is the following:

$$
\begin{aligned}
V(b_t, h_t, y_t) = \max_{\{c_t, x_t\}_{t\geqslant 0}, S} E_0 \left[\int_0^S e^{-(\rho+\delta)t} u(c_t) dt + e^{-(\rho+\delta)T} V^*(b_T, h_T, y_T) \right] \\
\text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.8)
\end{aligned}
$$

$$
\begin{aligned}
db_t &= \tilde{r} b_t dt + (r_e - \tilde{r}) x_t dt + r_r h_t + x_t \sigma_e dB_t^e + (y_t - c_t - T_t) dt, \\
dh_t &= r_h h_t dt + h_t \sigma_h dB_t^h, \\
b_t &\geqslant \min\{-\theta h_t, -\theta_u\} \text{ , and } 0 \leqslant x_t \leqslant \max\{b_t, 0\},
\end{aligned}
$$

and also subject to the stochastic process for labor income y_t, to be specified later. The value of reshuffling the composition of the portfolio between liquid and illiquid assets is given by:

$$V^*(b,h,y) = \max_{\tilde{b},\tilde{h}} V(\tilde{b},\tilde{h},y) \text{ subject to } b + h(1-\kappa_1) - \kappa_0 = \tilde{b} + \tilde{h}(1+\kappa_1)\ ,\ \tilde{h} \geqslant 0, \tilde{b} \geqslant -\theta\tilde{h},$$

where households can use their total wealth after paying the fixed cost $b+h(1-\kappa_1)-\kappa_0$ to freely readjust to new values of liquid and illiquid assets, subject to the collateral constraint.

In Equation (2.8), the choices available for the household are consumption c_t, the amount of liquid assets allocated to equity x_t and when to readjust S. Choices are restricted by the borrowing constraint $b_t \geqslant \min\{-\theta h_t, -\theta_u\}$, and a constraint $0 \leqslant x_t \leqslant \max\{b_t, 0\}$ meaning that households cannot short sell nor leverage on equity. Equity has an expected return of r_e and is subjected to Brownian motion shocks dB_t^e with volatility σ_e. Housing experiences capital gains with average return r_h, also with a stochastic component dB_t^h with volatility σ_h. Because I do not model housing utility services (so that I do not have to separate housing value h into price and quantity, including an extra state variable), it is important to input to homeowners the value of rent they would pay in their houses, and this is assumed to be fixed at r_r. Finally, the return on the risk-free asset is equal to $\tilde{r}$, which depends on the value of the liquid asset:

$$\tilde{r} = \begin{cases} r_b, & \text{if } b \geqslant 0 \\ r_m, & \text{if } 0 > b \geqslant -\theta h \\ r_u, & \text{if } b < -\theta h \end{cases} \cdot$$

The parameters r_b, r_m and r_u represent, respectively, the return on risk-free bonds, the cost of mortgages and the cost of uncollateralized debt. In my parametrisation, $r_b < r_m < r_u$. For solving numerically the problem in Equation (2.8) it is more convenient to work with its Hamilton-Jacobi-Bellman equation form, which can be written as:

$$
\begin{aligned}
(\rho+\delta)V(b,h,y) =& u(c)+V_b(\tilde{r}b+(r_e-\tilde{r})x+r_rh+y-c-T)+\frac{V_{bb}}{2}(x\sigma_e)^2 \\
&+V_hr_hh+\frac{V_{hh}}{2}(h\sigma_h)^2+\frac{1}{dt}E_t[V_y],
\end{aligned} \tag{2.9}
$$

with the following state constraints

$$
\begin{aligned}
V_b(-\theta h,h,y) &\geqslant u'(y-(r_m\theta-r_r)h-T), \forall b,h,y, \\
V_b(-\theta_u,0,y) &\geqslant u'(y-r_u\theta_u-T), \\
V(b,h,y) &\geqslant V^*(b+h,y), \forall b,h,y;
\end{aligned} \tag{2.10}
$$

and the following first-order conditions (FOCs):

$$
\begin{aligned}
u'(c) &= V_b(b,h,y), \\
x &= \max\left\{\min\left\{-\frac{V_b}{V_{bb}}\frac{(r_e-\tilde{r})(1-\tau)}{\sigma_e^2},b\right\},0\right\}.
\end{aligned} \tag{2.11}
$$

Notice above that the constraint on x in Equation (2.8) is already embedded in its FOC. Furthermore, the borrowing constraint is taken into account by the two first state constraints in Equation (2.10), where I use the fact that if $b<0$ (which is true when the borrowing constraint is satisfied with equality), then $x=0$. Also, remember that the effective tax rate $\tau(b,h,y)$ that households face is important to determine its allocation to equity assets in Equation 2.11, and it is a function of total taxable income.

The overall strategy for solving numerically the problem summarised by Equations (2.9), (2.10) and (2.11) is the same as the upwinding scheme in Achdou et al. (2021), but there is one further complication, which is dealing with the possibility of readjustment of the illiquid asset. The existence of a fixed cost of transaction κ_0 makes the adjustment cost function non-convex and introduces a stopping-time component to the problem (e.g., see Kaplan, Maxted, and Moll, 2016). To deal with the extra constraint $V(b,h,y)\geqslant$

$V^*(b,h,y)$ I can rewrite the problem as

$$0 = \min\Big\{(\rho+\delta)V(b,h,y) - \Big[u(c) + V_b(\tilde{r}b + (r_e - \tilde{r})x + r_r h + y - c - T) + V_{bb}(x\sigma_e)^2/2 \tag{2.12}$$

$$+ V_h r_h h + V_{hh}(\sigma_h h)^2/2 + \frac{1}{dt}E_t[V_y]\Big], V(b,h,y) - V^*(b+h,y)\Big\},$$

still subject to the other constraints in Equation (2.10) and the FOCs in (2.11). I recast Equation (2.12) above as a Linear Complementarity Problem (LCP) to solve it numerically, with further details in Appendix 2.C.

2.4.2 Labour Income

Labor income is assumed to be a mean-reverting process with a transitory and idiosyncratic ε_{it} shock. The law of motion for the log of labor income $w_t = \log(y_t)$ for a given individual individual is

$$dw_t = -\mu_w w_t dt + \sigma_w dB_t^w, \tag{2.13}$$

where $(1-\mu_w)$ is the degree of mean reversion, dB_t^w are Brownian motion innovations, and σ_w is the volatility of the idiosyncratic shock. One can then rewrite the problem as function of w instead of y, and we have:

$$\frac{1}{dt}E[V_w] = -\mu_w w V_w + \frac{(\sigma_w)^2}{2}V_{ww}. \tag{2.14}$$

I choose $\mu_w = 0.1$ and $\sigma_w = 0.2$, which are typical parameter values used in the literature. Finally, I normalise labor income such that average pre-tax labor earnings are equal to 1.

2.4.3 Stationary Distribution

Let $f_t(b,h,y)$ denote the distribution at time t over all the state variables. If households were not allowed to readjust ($\kappa_0 \to \infty$), then its KFE for $(b,h) \neq (0,0)$ would be given by:[10]

$$\frac{\partial}{\partial t}f_t(b,h,y) = \\
-\frac{\partial}{\partial b}(s(b,h,y)f_t(b,h,y)) + \frac{\partial^2}{2\partial b^2}(\sigma_e^2 x^2 f_t(b,h,y)) - \frac{\partial}{\partial h}(r^h h f_t(b,h,y)) \\
+\frac{\partial^2}{2\partial h^2}(\sigma_h^2 h^2 f_t(b,h,y)) + \frac{\partial}{\partial y}(\mu_w f_t(b,h,y)) + \frac{\partial^2}{2\partial y^2}(\sigma_w^2 f_t(b,h,y)) \\
-\delta f_t(b,h,y), \tag{2.15}$$

where $s(b,h,y) = \tilde{r}b + (r_e - \tilde{r})b + r_r h + y - c(b,h,y) - T$ is the average savings rate for the liquid asset when households are not paying the cost to adjust.

However, the introduction of the stopping-time component to the problem makes finding the stationary distribution slightly harder than usual. That is because when households reach a state in which they would want to readjust the composition of their wealth between liquid and illiquid assets their movement is not captured by the continuous evolution of $f_t(b,h,y)$ in the KFE above.

Instead, when readjusting they "jump" from one point (b,h,y) in the state-space to another $(\tilde{b},\tilde{h},y)$. Therefore, my numerical solution for the evolution of f_t over time will have two steps: first, find out how the distribution would change if households were not allowed to readjust; second, if they enter a region in which they would want to readjust, allow them to move according to their optimal choice of $(\tilde{b},\tilde{h})$. Notice that, because the movement described by the KFE in Equation (2.15) is continuous, with this procedure households will always adjust from the boundary of the adjustment region, not from the interior of it.

2.4.4 Calibration

Table 2.1 shows the values used for all the parameters in the model. To start with, for preference parameters I use a standard value of $\gamma = 1.5$ for relative risk aversion, while the values for $\delta = 1.2\%$ and $\rho = 3.3\%$ were chosen to target the wealth share of the top 1% and the aggregate level of capital relative to pre-tax labor income. Table 2.1 shows that the model achieves a top 1% wealth share of 20.2%, while the aggregate level of

[10] For $(b,h) = (0,0)$ there is an extra term $+\delta$ representing the households that are born with zero wealth.

capital to pre-tax labor income is equal to 12.4. This is is line with the most recent years according to the SCF, but higher than the levels seen in 1989.

Preference Parameters		
Coeff. of relative risk aversion	γ	1.5
Discount rate	ρ	3.3%
Death rate	δ	1.2%
Housing Parameters		
Capital gains	r_h	1.0%
Rental rate	r_r	5.3%
Volatility of cap. gains	σ_h	11.0%
Max LTV ratio	θ	0.6
Cost of mortgages	r_m	4.5%
Fixed cost of transaction	κ_0	0.03
Proportional cost of transac.	κ_1	5.0%
Other Assets		
Return on equity	r_e	9.0%
Volatility of equity	σ_e	19.0%
Risk-free rate	r_b	3.0%
Cost of uncollateralised debt	r_u	5.5%
Max uncollateralised debt	θ_u	0.4

Table 2.1: Calibration of the parameters of the model

Moving on to assets, for real estate I use $r_h = 1\%$ and $r_r = 5.3\%$, which comes from Jordà et al. (2019), and $\sigma_h = 11\%$. This is higher than the usual value of 9% from the same source and usually used in the literature, but it reflects that not only r_h is risky, but r_r should be as well. Furthermore, in the model mortgages are actually more advantageous than what they are in practice, because households can decide to reduce or increase their mortgages without any cost. Thus I choose a maximum loan-to-value ratio of $\theta = 0.6$, otherwise there would be too much leverage at the bottom of the distribution. The cost of mortgages of $r_m = 4.5\%$ I estimated using the mortgages on primary or secondary homes in the SCF in 1989. Finally, households need to pay a fixed adjustment cost of $\kappa_0 = 0.03$, which would amount to approximately \$ 1,800 in 2019, and a proportional one of $\kappa_1 = 5\%$ when buying or selling a house. Typically, others studies use a total transaction cost for selling the house between 6 to 10%, thus my total proportional cost of 10% is on the upper end of that range.

For the other assets, I use a risk-free rate equal to 3%, and for equity $r_e = 9\%$ and

$\sigma_e = 19\%$. The equity volatility is also in line with Jordà et al. (2019), and estimates for the volatility of the S&P index. The equity risky premium implied by $r_e = 9\%$ is larger than what is usually used in the literature, but already smaller than my estimate of 9.6% for the return on overall equity in the SCF from 1989 to 2019. The cost of uncollateralised debt is equal to 5.5% and households can borrow a maximum of 0.4 (or $24,000 in 2019) in case they do not have a house to use as a collateral. These values were chosen to target, respectively, a bottom 50% wealth share of 2.7% and a mass of 4% of households with negative wealth. Finally, the tax system and the increasing returns to wealth require a more detailed discussion, which I turn to next.

2.4.4.1 Taxes and Transfers

The tax system is composed of a single increasing tax schedule on taxable income and transfers to some qualifying individuals. The transfers are assumed to be such that the government guarantees a minimum level of labor income for the poorest households. Thus if $\tilde{y}_t$ is true labor market income, final income is equal to $y = \max\{0.3, \tilde{y}\}$ if total wealth $a = b + h \leqslant 1$, and $y = \tilde{y}$ otherwise. This is meant to capture a mix of social security, unemployment insurance and benefit programs, which are usually means-tested. The model is normalised such that annual average pre-tax labor earnings is equal to 1, and the value of a minimum income of 0.3 of average labor earnings is largely in line with the literature.

Taxable income in the model is assumed to be equal to: $r \max\{b, 0\} + (r_e - r_b)x + r_r h + y + r_h h$. Several important comments to the definition above: (i) notice that interest payments on general debt do not lower taxable income in the equation above, but interest payments on mortgages might, with further details below; (ii) capital gains on housing r_h are taxed because there are no explicit capital gains taxes in the model (which would require the inclusion of another state variable - value of the house when bought), and the tax schedule includes capital gain taxes. I choose to subtract capital gain taxes from liquid wealth instead of reducing the after tax capital gain so that the value of the house keeps changing according to the true rate $r_h + \sigma_h dB_t^h$, and the imputed rent also grows/falls accordingly; (iii) in the same way as in Section 2.3, the stochastic component to returns is not taxed, but that is equal to zero on expectation and does not affect the final tax revenue.[11]

For calibrating the tax schedule applied to taxable income, my strategy is similar to Hubmer, Krusell, and Smith (2021). The source of data for tax rates is Piketty and Saez (2007), who calculate the total effective annual federal tax rates (inclusive of capital gains) in the US for 11 different groups of taxable income, from 1960 to 2001, shown previously in Figure 2.1. In a similar way, I create a step-wise increasing tax schedule on taxable income with 11 brackets. Panel A of Figure 2.1 compares the resulting tax system in the model to Piketty and Saez (2007) estimates for 1975, the initial year of the quantitative exercise. It shows that the model does quite well in generating enough income inequality so that the tax rates can be calibrated as seen in the data, except maybe for the very top 0.01% of the distribution.

To be able to realistically match the incentives that households face when choosing in which assets to invest, I modify the simple tax system in two different ways. First, households receive a rent r_r on their properties owned, but they should not pay taxes if the return stands for owner-occupied rent. I then calculate in the SCF the increasing relationship between the total value of real estate a household owns and the percentage of it that is used for rental property as opposed to primary or secondary houses. In 1989, for example, 10% of real estate is used for renting at the 85^{th} percentile of the distribution of total real estate value owned by households, while that increases to 50% at the 99.9^{th} percentile. Therefore, households in the model in the 85^{th} percentile of the real estate distribution only pay taxes on 10% of the imputed rent, while those at the 99.9^{th} percentile pay on 50%. Panel B of Figure 2.1 compares the tax-deductability of rent on owned properties in the model and in the data.

Second, I deduct mortgages payments from the tax bill. Households in the US were able to claim deduction in taxes for mortgage payments on up to two mortgages worth a total of \$1mi before 2017 and \$750,000 after that. I calculate in the SCF in 1989

[11] Piketty and Saez (2007) do not include capital gains when classifying individuals to tax brackets, but include them when calculating the final effective tax rate. I do the same when calculating taxable income for classification purposes. The division of the total return on housing between capital gains and dividends is explicit, but for equity I assume that 68.5% of the total return is due to capital gains, which is my estimate for the period of 1989 to 2019 from Section 2.2. Fixed income is assumed to have no capital gains. Finally, I only include non-owner occupied rent when classifying households, in the same way as explained in the main text.

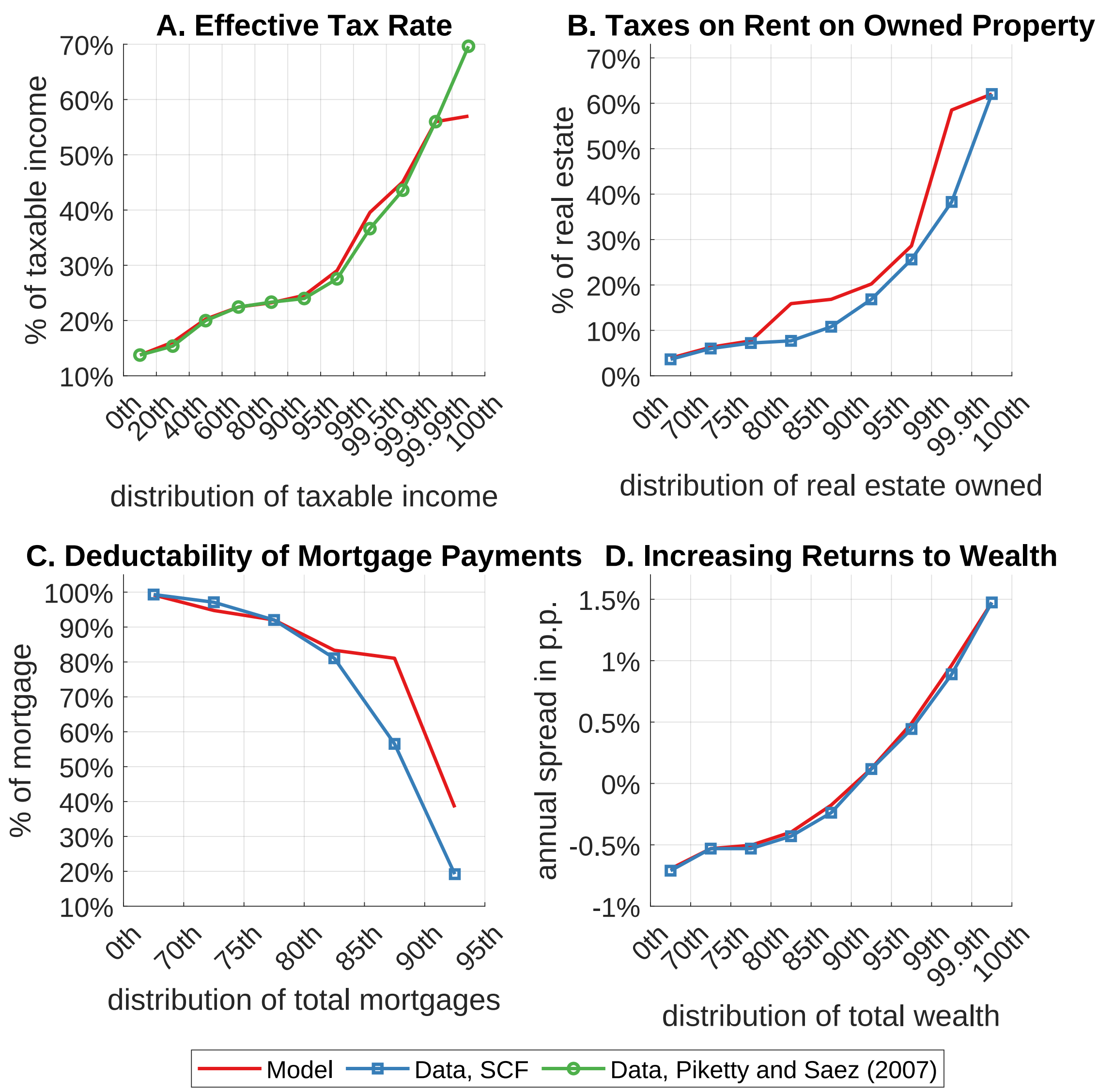

Figure 2.1: Comparison of the tax system and increasing returns to wealth: model vs data

Notes: Panel A compares the effective tax rate (not including transfers) in the model and in the data as a share of total taxable income, for different groups of taxable income. Panel B shows the share of real estate owned that is used as rental property along the distribution of total value of real estate owned. While in the model there are not different types of real estate, I input this increasing schedule so that households do not pay taxes on imputed rent for non-rental properties. Panel C shows the share of mortgages owned for which interest payments can be claimed for tax-deductability purposes. Panel D shows the spread over the average interest rate earned as a function of total wealth, controlling for portfolio composition effects.

what percentage of the total mortgage that households could claim for tax-deductability purposes as a function of the overall value of their mortgages held and use it to input to the model. Panel C of Figure 2.1 compares the deductability of mortgages payments as a function of the total size of mortgages in the model and in the data. The model does not do well in matching the highest levels of mortgages observed in the data because households in the model would not hold both mortgages and invest in fixed income assets at the same time, but notice that this is only the case for the top 1% of mortgages.

2.4.4.2 Increasing Returns to Wealth

The model endogenously generates increasing returns to wealth because wealthier households invest more in higher-yielding assets. However, this dimension is not enough to generate as much wealth inequality as observed in the data when taking into account the progressivity of the tax system in the US, specially in the 1970's. Therefore, I include in the model increasing returns to wealth also within asset categories, in line with ample empirical evidence (Bach, Calvet, and Sodini (2020) and Fagereng et al. (2020) for Scandinavian countries, and Xavier (2021) for the US).

The increasing returns are chosen to match the results for the SCF detailed in Chapter 1. Depending on the level of total wealth a households earn a premium over the average return of the asset. The premium is slightly negative for the poorest individuals, but increasing in wealth and positive at the top of the distribution. All assets feature increasing returns to wealth in the same way, so as not to dictate the portfolio decisions of households. For example, the equity risk premium $(r_e(a) - r_b(a))$ is fixed for all levels of wealth, even though $r_e(a)$ and $r_b(a)$ are both increasing in total wealth a. For real estate, I split the increasing schedule into r_r and r_h in a way that is proportional to their values, so that total return on housing $r_r + r_h$ increases in wealth in the same way as r_e and r_b do. Panel D of Figure 2.1 compares the calibration in the model to the increasing returns to wealth from the data.

2.5 Results

Before looking at the main results after a change in taxes it is useful to understand the mechanics of the model in the steady state.

2.5.1 Steady State

The main dimensions that the portfolio model should match are the high level of wealth inequality seen in the data, the portfolio composition of households across the wealth distribution and the progressive tax schedule, with the deductions for mortgage payments and imputed rent for home owners. Figure 2.1 showed that the model does well in matching the targets related to the tax system.

	Bottom 90%	90^{th}-99^{th}	Top 1%
Data 1975 (SZ)	30.8%	44.3%	24.9%
Data 1975 (SZZ)	33.7%	44.7%	21.6%
Model	36.1%	43.7%	20.2%

Table 2.1: Wealth shares in the steady-state of the model and in the data

Table 2.1 shows the high level of wealth inequality in the steady-state of the model, which is in line with levels observed in 1975. Unsurprisingly, there is a bit less inequality at the very top, given that there is no other source of inequality other than capital income. Although the labor income process used generates a reasonable amount of wage inequality, it is not as big as that seen in the data. That is not the focus of this paper, but if the model was augmented in that direction it could match the data even better. Finally, not shown in the table, the presence of uncollateralised debt allows it to match well the low wealth share of the bottom 50% seen in the SCF since 1989, which has been below 3% and is equal to 2.8% in the model.

Moving on to the portfolio composition, Figure 2.1 compares the steady-state to the data in 1989, since that is the first year available from the SCF. We see that the model matches quite well the main qualitative features of the data: on the one hand real estate dominates portfolios at the bottom of the distribution, and its importance decreases monotonically as we move to wealthier groups. On the other hand, both net safe assets and equity are increasing over the wealth distribution. Finally, households are leveraged at the bottom of the distribution, but have positive risk-free assets at the top of the distribution.

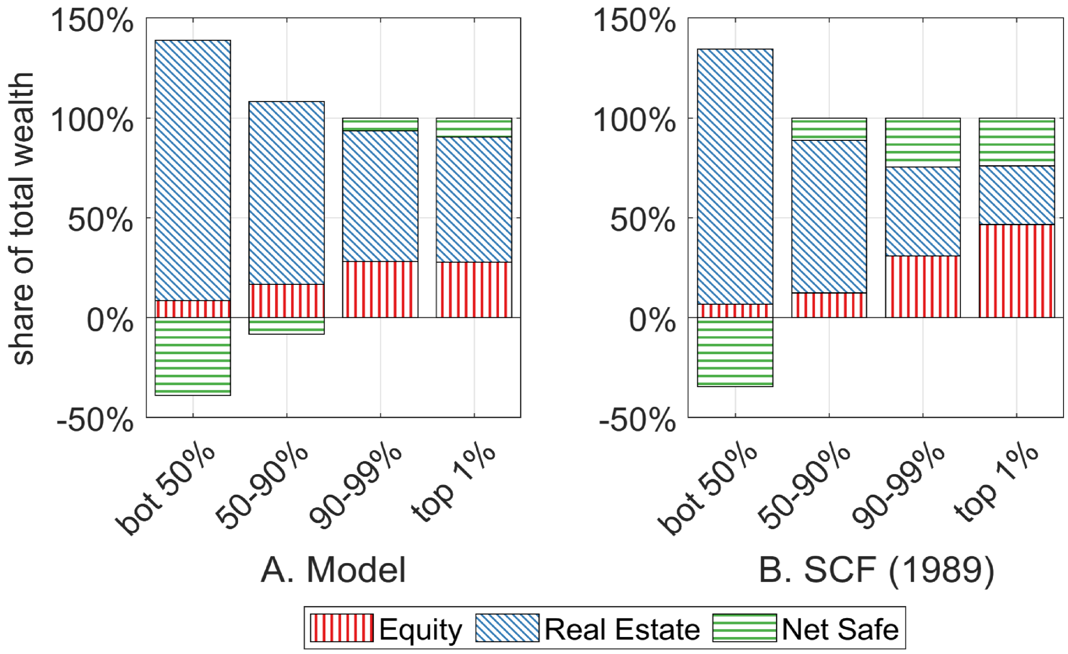

Figure 2.1: Portfolio composition in the model and in the data

The main theme with respect to portfolio choices is the dominance of real estate at the bottom of the distribution. This is not something new to this model, and has been present in other models since Cocco (2005), Flavin and Yamashita (2002) and Yao and Zhang (2005), which highlight how real estate effectively crowds out the other asset categories for low levels of wealth. In fact, it is the presence of real estate that explains how the allocation towards equity and safe assets are increasing in wealth. Without the illiquid asset, the share of wealth allocated to equity would actually be falling across the wealth distribution, which is a common feature of any model with a simple portfolio choice as in Merton (1969) and stochastic labor income.[12]

There are several reasons for why real estate is so appealing at the bottom of the distribution. First, it can be bought with a down payment of $(1-\theta)$ for every dollar, which allows households to leverage and explores the positive expected spread between

[12]Intuitively, this happens because for low levels of wealth the riskiness of the portfolio is irrelevant when compared to the riskiness of labor income. In contrast, wealthier households do not want to invest large share of their portfolios into risky assets, given that is their main source of income.

the total return on housing and mortgage rates. Second, if the household is not constrained, it serves as collateral for borrowing. This allows households to weather negative labor income shocks and keep higher levels of consumption by borrowing more and at a cheaper rate against their houses, which is not permitted with equity. Third, there is the tax system that is imputed to the model, which explicitly makes housing a better investment at the bottom of the distribution by allowing households to deduct mortgage payments from tax bills for low mortgages (according to the actual tax system), and also by taxing more the imputed rent at the top of the distribution (according to the data). The latter can be seen as a way to impute to the model the endogenous decision of households to first buy houses to live on, and then invest in rental real estate, without the need to model housing services and rental market decisions. To show how important the imputed tax system is for portfolio decisions, Figure (deleted) in the Appendix shows the portfolio decisions of households if there was no mortgage deduction and tax was charged on imputed rent for homeowners. In that case, equity would dominate everywhere and would actually be decreasing as a share of portfolio over the wealth distribution.

2.5.2 Transition

With the results for the steady-state, we are now able to analyse the main quantitative results of this paper - the effects of tax rates changes on wealth inequality when allowing for portfolio changes. In particular, what would be the dynamics of wealth inequality when tax rates changed as they did in the US from 1975 to 2001, according to Piketty and Saez (2007), and assuming it to be constant since then.

This transition dynamics exercise assumes that households are myopic, thus every year from 1975 to 2001 the economy is shocked with a change in taxes, but households always assume that taxes will be constant from then onward. To uncover the impact of portfolio changes I will compare the baseline results that allow for households to react to the tax shocks by changing their portfolios with another in which portfolio choices are fixed at their steady-state levels. It is important to notice that what is being kept fixed is not the portfolio of a specific individual, but the policy functions for portfolio allocations, i.e., the demand for each different asset given the state variables. Thus for the liquid asset choice I fix $x(b,h,y)$, and for the illiquid asset I fix $\tilde{h}(b,h,y)$ and $\tilde{b}(b,h,y)$, the demands when choosing to pay the transaction costs.

Figure 2.2 shows the evolution of top wealth shares over time in the data in the baseline scenario of the model, and in the counterfactual in which portfolio policy functions are fixed to their levels in 1975. It shows that the actual tax changes observed in the US can explain a great deal of the changes in wealth inequality when embedded into a quantitative macro model of portfolio and savings choices. Looking at Panel B, it is clear that allowing portfolios to change magnifies the impact of taxes: after 45 years of transition the wealth share of the top 1% increases by 4.9% in the baseline case while only 4.1% in the counterfactual in which portfolios are not readjusting, or an increase of 20% of the impact of taxes. This effect is increasing over time, as it will be clear when analysing the final steady-state.

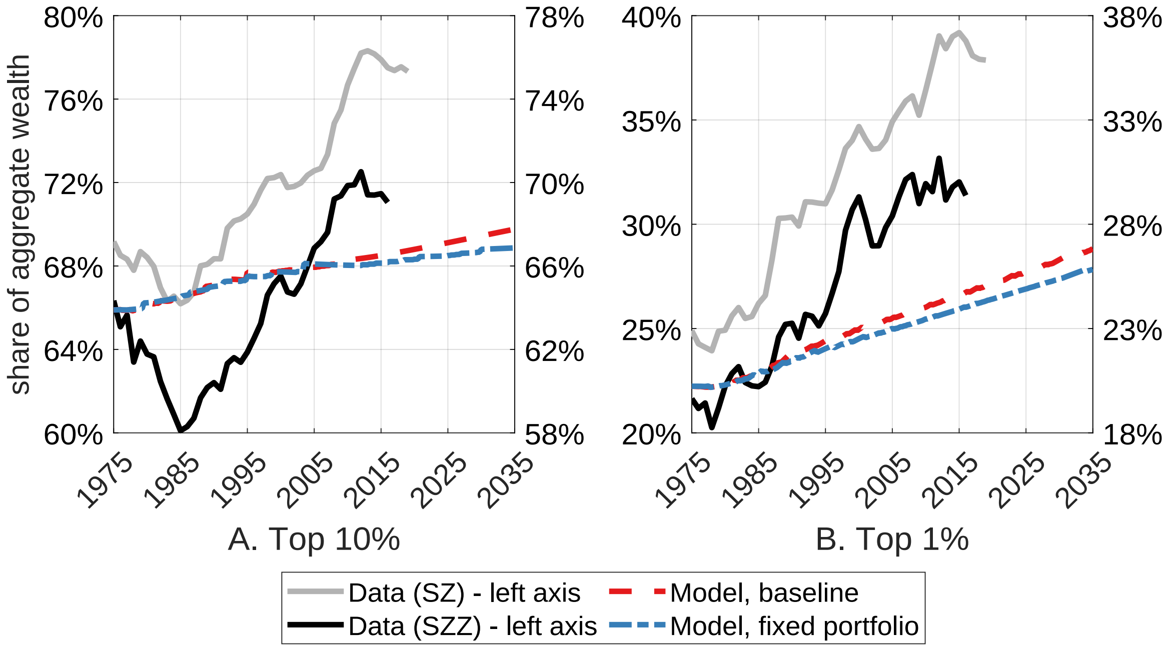

Figure 2.2: Top wealth shares over time

Notes: This figure shows the evolution of top wealth shares both in the model and in the data. The data is plotted on the left axis of each graph, while both model series are plotted on the right axis.

Looking at Panel A of Figure it seems that fixing the portfolio choices at 1975 levels or not does not have an impact for most of the transition period. However, as shown previously in Figure 2.1, taxes are falling over time only for households within the top

1% of the income distribution. Therefore, taxes within the top 10% of wealth in Panel A are actually increasing for the top 10-1% and mostly falling over time for the top 1%, which means that the portfolio effect contributes positively for the top 1%, but contributes negatively for the rest, and for most of the transition these two effects cancel each other. Thus, to evaluate the impact of a fall in tax rate we need to focus in the top 1% in Panel B.

With respect to the bottom of the distribution, as the top wealth shares increase, the bottom 90% gets squeezed and loses wealth share, in the data and in the model. However, because the poorest individuals were not affected by an increase in taxes, the wealth share of the bottom 50% does not change much over time in the model, while those in between the 50^{th} and 90^{th} percentiles are the ones that suffer the most.

To better understand the effects of keeping the portfolios fixed or not over the transition, it is informative to look at how portfolios evolved over time under those two different scenarios. Notice that even though in one of the exercises I keep portfolio policy functions fixed, the portfolio composition of different wealth groups are still changing over time as portfolios differ along the wealth distribution, and individuals get wealthier or poorer due to the changes in taxes.

Figure 2.3 shows the share of fixed income assets for the the top 10% and top 1% under different assumptions. The prediction from the model in Section 3 is that the allocation to riskier assets should increase when taxes fall, but now that there are two different risky assets, it is not clear to which one households should move to. Thus it is better to look at the portfolios shares of riskless assets, keeping in mind that the rest is composed of risky assets.

First, notice that the share of net safe assets is always higher for the top 1% than for the top 10%, in line with the evidence from the data and the steady state of the model. Second, the portfolio shares under the baseline exercise that allows households to adjust their portfolios largely follow the movement in taxes, which is the prediction from Section 3. These shares fell from 1975 until 1990, rebounded in the beginning of the 1990s and then started falling again after 1995, just like what happened with tax rates at the top as seen in Figure 2.1.

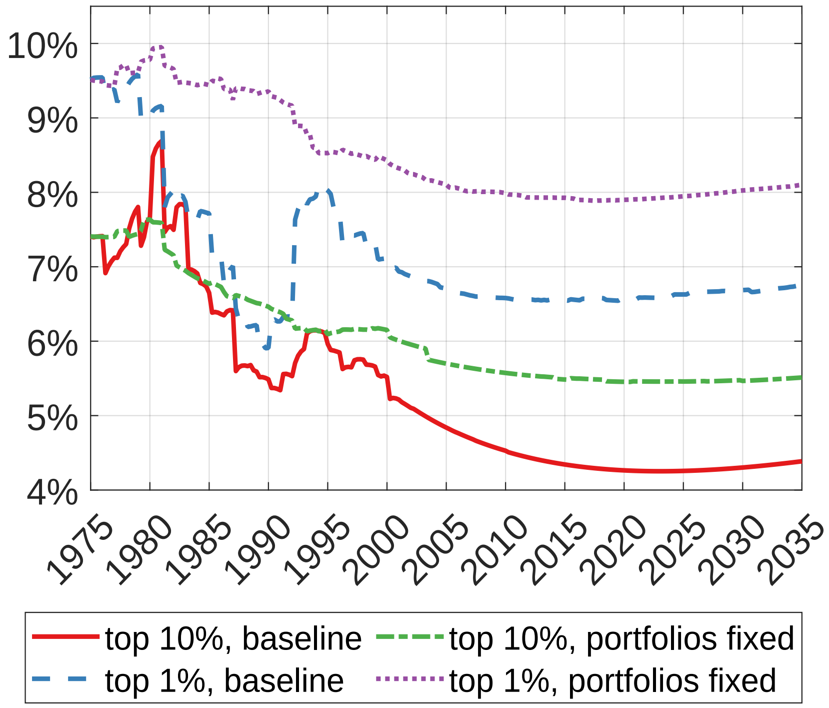

Figure 2.3: Portfolio share of net safe assets over the transition

Third, the movements with the assumption of portfolios fixed do not follow the same pattern for the top 1%, but are closer for the top 10%. Remember that for the top 10% taxes increased for the top 10-1%, but decreased for the top 1%, thus there are effects on different directions that mostly cancel each other. However, for the top 1% the share invested in safe assets is clearly larger when households are not allowed to readjust their portfolios, and that is what is driving the different in results when compared to the baseline scenario.

2.5.3 Final Steady State

As the figures above show, if we allowed the economy to continue to run for longer with the tax rates that were set in 2001 the top wealth shares would keep increasing, and the effect of portfolio changes would become even clearer. In fact, it is well known that transition dynamics for a large class of model with random growth, which include the ones in this paper, are quite slow when compared to the data (see Gabaix et al. (2016)).[13] Thus, it is instructive to look at what would be the final steady state of the

[13] I experimented with allowing for type dependence, on top of increasing returns to wealth, as suggested by Gabaix et al. (2016). However, for low levels of extra return by type, around

model under the new tax regime.

	Bottom 90% (p.p.)	90^{th}-99^{th} (p.p.)	Top 1% (p.p.)
Data, 1975-2019 (SZ)	-8.1	-4.8	12.9
Data, 1975-2016 (SZZ)	-4.7	-5.0	9.7
Model, transition 1975-2019			
Baseline	-2.9	-2.1	5.0
Portfolios fixed	-2.4	-1.8	4.2
Model, final steady state			
Baseline	-7.7	-7.0	14.7
Portfolios fixed	-6.3	-5.4	11.7

Table 2.2: Change in wealth shares: model vs data

Table 2.2 summarises all the information with respect to changes in wealth shares in the model and in the data. When looking at the values under the final steady state of the model, we see that the changes in tax rates produce an increase in the wealth shares of the top 1% that is even slightly larger than the one observed in the data, once we allow households to adjust by changing their portfolios.

Comparing the baseline scenario with the ones with portfolios fixed, the wealth share of the top 1% increases by 14.7 p.p. between steady-states in the former, and by 11.7 p.p. in the latter. This implies that accounting for portfolio responses increases the impact of taxes by 25%, which suggests that this impact was increasing over time during the experiment. This is not surprising, given that the final top tax rates in 2001 are below the average of the period from 1975-2001.

2.6 Conclusion

The possible causes for the increase in wealth inequality in the US have been the center of a great debate in the last couple of decades. However, one dimension has been understudied so far, namely the impact of the changes in households' portfolio choices. This study attempted to highlight that it is important to think about how households'

3%, the speed of transition did not change.

portfolio choices will react to shocks (in particular, changes in the tax rate) if one is interested in understanding wealth inequality dynamics.

First, it revisited the impact of taxes on wealth inequality using an analytical model. Thanks to the existence of analytical solutions to the portfolio and savings decisions, it showed how allowing households to rebalance their portfolio in response to changes in taxes can magnify their final impact, even after modest portfolio movements.

Second, it developed a full quantitative model of households' portfolio choice that can generate the distribution of portfolio and wealth inequality seen in the data, and that also takes into account the progressive tax system present in the US. Even though the setting was different, the results also showed how it is important to account for portfolio changes when calculating the impact of taxes on wealth inequality.

The results of the paper highlight the importance of understanding better the portfolio decision of households, in particular with respect to tax rates. Finally, this paper also highlights the need for more empirical studies that uncover how households readjust their portfolios in response to changes in the environment that would help discipline the portfolio dynamics embedded in quantitative macro models.

Appendices

2.A Additional Figures

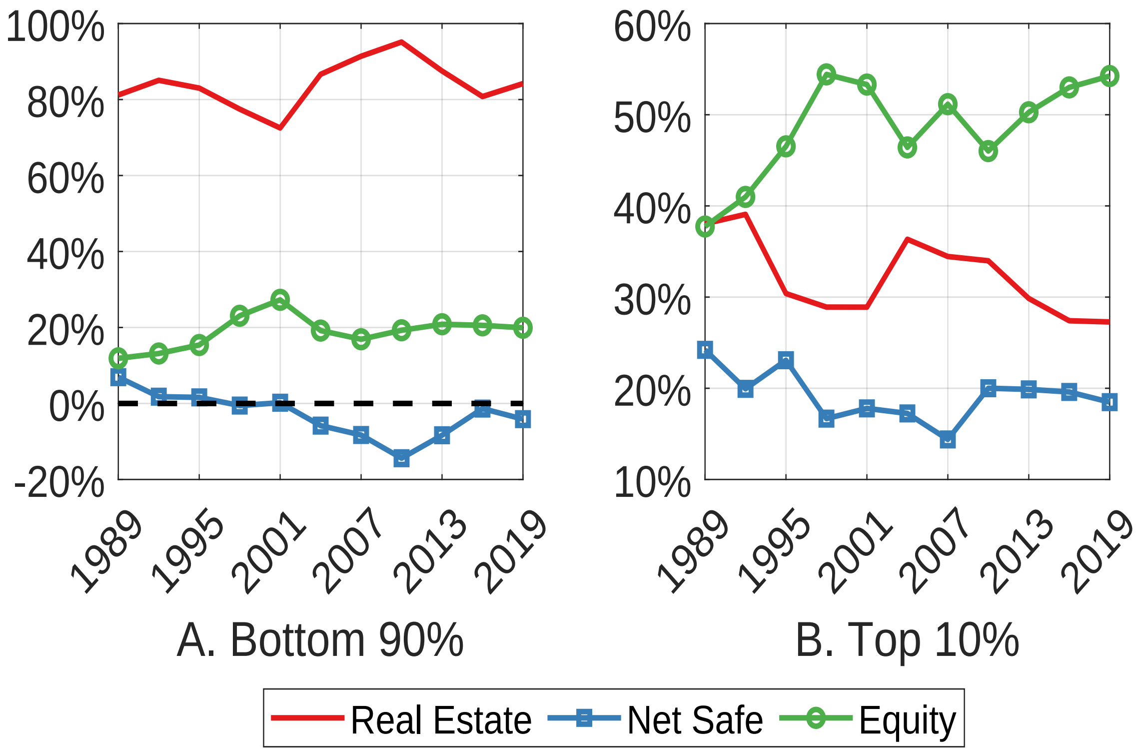

Figure 2.A.1: Portfolio Composition Over Time for the Bottom 90% and the Top 10%

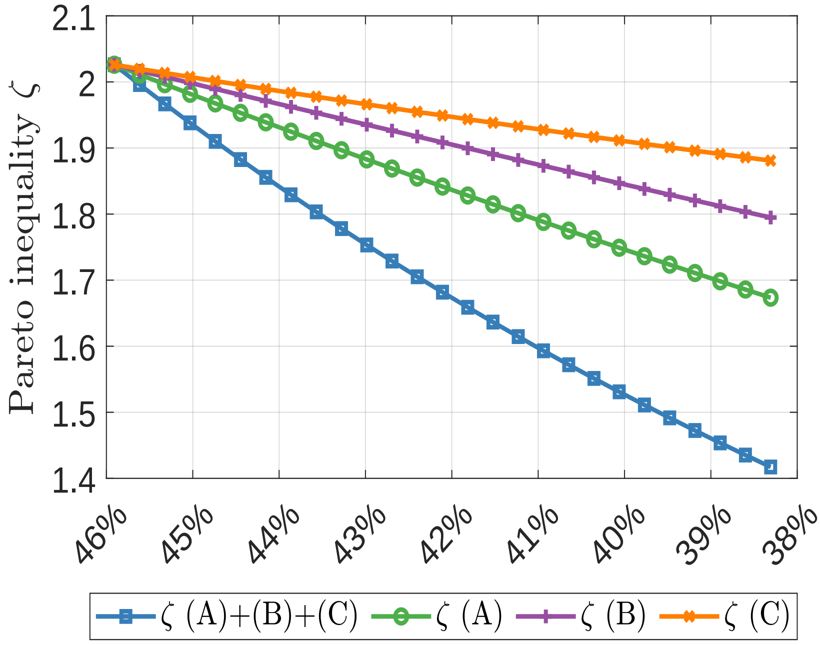

Figure 2.A.2: Decomposition of the impact of taxes on ζ

Notes: This figure shows the decomposition of the total effect of taxes on ζ according to the terms (A), (B) and (C) from Equation (2.6)

2.B Proofs for the Analytical Model

Consider again the analytical model described in Section 3:

$$(\rho+\delta)V(b,y)=\max_{c,x}\left\{\frac{c^{1-\gamma}}{1-\gamma}+V_b[(1-\tau)(r_b b+(r_e-r_b)x+y)-c]+\frac{V_{bb}}{2}(x\sigma_e)^2+\frac{1}{dt}E[V_y]\right\},$$

subject to

$b\geqslant-\theta_u$, and $0\leqslant x\leqslant\max\{b,0\}$.

where the term $(1/dt)E_t[V_y]$ depends on the specific process chosen for labor income. As mentioned in the main text, as $b\to\infty$ the policy functions for savings rate $s(b,y)=(1-\tau)(r_b b+(r_e-r_b)x+y)-c(b)$ and optimal share invested in risky assets $x(b,y)$ are asymptotically linear in wealth: $s(b,y)\to\overline{s}b$ and $x(b,y)\to\overline{x}b$. Furthermore, the right tail of the distribution converges to a Pareto distribution. Achdou et al. (2021) provide proofs of these statements,[14] here I provide a proof for the specific case in which $y=0$, which conveys all the intuition.

2.B.1 Finding $\overline{s}$ and $\overline{x}$

Now consider the simpler version of the problem above, without labor income:

$$(\rho+\delta)V(b)=\max_{c,x}\left\{\frac{c^{1-\gamma}}{1-\gamma}+V_b[(1-\tau)(r_b b+(r_e-r_b)x)-c]+\frac{V_{bb}}{2}(x\sigma_e)^2\right\},$$

subject to

$b\geqslant-\theta_u$, and $0\leqslant x\leqslant\max\{b,0\}$.

The FOCs are:

$$c^{-\gamma}=V_b,\text{ and } x=-\frac{V_b}{V_{bb}}\frac{(1-\tau)(r_e-r_b)}{\sigma_e^2}.$$

Now using a guess-and-verify approach, guess that $V(b)=\kappa^{-\gamma}b^{1-\gamma}/(1-\gamma)$ for some parameter κ. We then have $V_b=\kappa^{-\gamma}b^{-\gamma}$ and $V_{bb}=-\gamma\kappa^{-\gamma}b^{-\gamma-1}$. Substituting our guess in the FOCs we get:

$$c=\kappa b,\text{ and } x=(1-\tau)(r_e-r_b)b/(\gamma\sigma_e^2),$$

[14] The only difference being that in their case there are no deaths.

thus we already found that $\overline{x} = (1-\tau)(r_e - r_b)/(\gamma\sigma_e^2)$. To find out κ, substitute everything back into the HJB equation, which becomes:

$$(\rho+\delta)\kappa^{-\gamma}\frac{b^{1-\gamma}}{1-\gamma} = \kappa^{1-\gamma}\frac{b^{1-\gamma}}{1-\gamma} + \kappa^{-\gamma}b^{1-\gamma}[(1-\tau)(r_b + (r_e - r_b)\overline{x}) - \kappa] - \gamma\kappa^{-\gamma}b^{1-\gamma}\frac{(\overline{x}\sigma_e)^2}{2}.$$

For the equation above to hold for any b it must be that:

$$\frac{\rho+\delta}{1-\gamma} = \frac{\kappa}{1-\gamma} + ((1-\tau)(r_b + (r_e - r_b)\overline{x}) - \kappa) - \gamma\frac{(\overline{x}\sigma_e)^2}{2}.$$

Solving the above for κ gives us:

$$\kappa = \frac{(\rho+\delta) - (1-\gamma)(1-\tau)(r_b + (r_e - r_b)\overline{x})}{\gamma} - (1-\gamma)\frac{(\overline{x}\sigma_e)^2}{2},$$

which verifies our guess. We can then rewrite $s(b) = (1-\tau)(r_b b + (r_e - r_b)x) - c = [(1-\tau)(r_b + (r_e - r_b)\overline{x}) - \kappa]b$, thus:

$$\overline{s} = \frac{(1-\tau)(r_b + (r_e - r_b)\overline{x}) - (\rho+\delta)}{\gamma} + \frac{\gamma-1}{2}(\overline{x}\sigma_e)^2.$$

2.B.2 Finding ζ

The KFE for the stationary distribution $f(b)$ away from $b = 0$ where households are reborn is given by:

$$0 = -\frac{\partial}{\partial b}(\overline{s}bf(b)) + \frac{\partial^2}{2\partial b^2}((\overline{x}b\sigma_e)^2 f(b)) - \delta f(b).$$

Guess that $f(b) \propto b^{-\zeta-1}$, which is a Pareto distribution. In this case the KFE implies that:

$$0 = \zeta\overline{s} - \frac{1}{2}(1-\zeta)\zeta(\overline{x}\sigma_e)^2 - \delta. \tag{2.16}$$

Solving the quadratic equation above for the only positive root (necessary for the existence of the Pareto distribution) leads to:

$$\zeta = \frac{1}{2} - \frac{\overline{s}}{(\overline{x}\sigma_e)^2} + \sqrt{\left(\frac{1}{2} - \frac{\overline{s}}{(\overline{x}\sigma_e)^2}\right)^2 + \frac{2\delta}{(\overline{x}\sigma_e)^2}}.$$

2.B.3 The effect of taxes

From Equations (2.2) and (2.3) it is easy to see that:

$$\frac{\partial \overline{s}}{\partial \tau} = -\frac{(r_b + (r_e - r_b)\overline{x})}{\gamma} < 0,$$

and

$$\frac{d\overline{x}}{d\tau} = -\frac{r_e - r_b}{\gamma \sigma_e^2} < 0,$$

which confirms that all the terms in Equation (2.5) are negative.

Furthermore, it was mentioned in the main text that both $\partial\zeta/\partial\overline{s}$ and $\partial\zeta/\partial\overline{x}$ are negative, meaning that all the terms in Equation (2.6) are positive. To see that, consider Equation (2.16), which implicitly defines ζ as a function of $\overline{s}$ and $\overline{x}$. Differentiating it with respect to $\overline{s}$ we arrive at:

$$0 = \frac{\partial \zeta}{\partial \overline{s}} \left(\overline{s} - \frac{(\overline{x}\sigma_e)^2}{2} + \zeta(\overline{x}\sigma_e)^2 \right) + \zeta + 2\zeta \frac{(\overline{x}\sigma_e)^2}{2} \frac{\partial \zeta}{\partial \overline{s}}.$$

Let $\Gamma = \overline{s} - (\overline{x}\sigma_e)^2/2 + \zeta(\overline{x}\sigma_e)^2$. Then we can rewrite the equation above as:

$$\frac{\partial \zeta}{\partial \overline{s}} = -\frac{\zeta}{\Gamma}.$$

Notice that $\Gamma > 0 \iff \zeta > \frac{1}{2} - \frac{\overline{s}}{(\overline{x}\sigma_e)^2}$, which is true from the definition of ζ in Equation (2.4). Thus, $\partial\zeta/\partial\overline{s} < 0$. We also had that $\partial\overline{s}/\partial\tau < 0$, which proves that the effect (A) in Equation (2.6) is positive.

We now differentiate Equation (2.16) with respect to $\overline{x}$ which, after rearranging, can be written as:

$$\frac{\partial \zeta}{\partial \overline{x}} \Gamma = -\zeta \overline{x} \sigma_e^2 (\zeta - 1).$$

We will only consider parameter combinations such that $\zeta > 1$, because otherwise the first moment of the distribution would not exist. Thus we conclude that $\partial\zeta/\partial\overline{x} < 0$. We had that $d\overline{x}/d\tau < 0$, thus the effect (C) in Equation (2.6) is also positive.

Finally, for effect (B) notice that

$$\begin{aligned}\frac{\partial \overline{s}}{\partial \overline{x}} = (1-\tau)\frac{r_e - r_b}{\gamma} + (\gamma-1)\overline{x}\sigma_e^2 > 0 &\iff (1-\tau)\frac{r_e - r_b}{\gamma} > -(\gamma-1)\overline{x}\sigma_e^2 \\ &\iff \overline{x} > -(\gamma-1)\overline{x} \\ &\iff 1 > -\gamma + 1,\end{aligned}$$

which is true as $\gamma > 0$. We had that $\partial\zeta/\partial\overline{x} < 0$ and $d\overline{x}/d\tau < 0$, thus the effect (B) is also positive, which concludes the proof that all the terms in Equation (2.6) are positive.

2.C Numerical Solution

This Appendix explains how to solve the model from Section 4 numerically. I use the upwind scheme as described in Achdou et al. (2021), while I follow Kaplan, Maxted, and Moll (2016) in how to deal with the illiquid asset.

2.C.1 Overall Strategy

The basic idea of the convergence algorithm is summarised bellow. First, I solve the problem without allowing households to readjust their illiquid assets holdings, and then I use the resulting value function as a guess for solving the full problem, where each iteration is solved as a LCP problem. Using the good guess that is the first step is crucial for guaranteeing convergence in the second step, and solving it as an LCP problem is the best choice in terms of speed.

The state-space (b, h, w), where w = log(y), is discretised in $\{b_i\}_{i=1}^{I}, \{h_j\}_{j=1}^{J}, \{w_k\}_{k=1}^{K}$, in non-linear grids in all dimensions. The discretised version of Equation 2.12 becomes:

$$\min\{(\rho+\delta)v - u(v) - \mathbf{A}(v)v, v - v^*(v)\} = 0, \tag{2.17}$$

where v is a stacked vector of the value function $V_{i,j,k}$ over the grids space of size $L \times 1$, and $L = I \times J \times K$. I proceed as follows:

1. Find v^0 as the solution to the problem without adjustment:

$$(\rho+\delta)v^0 - u(v^0) - \mathbf{A}(v^0)v^0 = 0.$$

   This is done with standard value function iteration.

2. given v^n, find v^{n+1} by solving

$$\min\left\{\frac{v^{n+1}-v^n}{\Delta t} + (\rho+\delta)v^{n+1} - u(v^n) - \mathbf{A}(v^n)v^{n+1}, v^{n+1} - v^*(v^n)\right\} = 0$$

   and stop when v^n and v^{n+1} are close. Each step above is solved numerically as a Linear Complementary Problem (LCP),[15] and $v^*_{i,j,k}(v^n)$ is found by maximizing over the all the points in v^n that an agent at (i,j,k) could afford.

2.C.2 Specifics of the Discretisation

Let $\Delta b_i^+ = b_{i+1} - b_i, \Delta b_i^- = b_i - b_{i-1}$, and analogously for h and y. For $u(c) = (c^{1-\gamma}-1)/(1-\gamma)$ and iteration n define:

$$\begin{aligned}
V^{n,F}_{b,i,j,k} &= (V^n_{i+1,j,k} - V^n_{i,j,k})/\Delta b_i^+,\\
V^n_{bb,i,j,k} &= \frac{\Delta b_i^- V^n_{i+1,j,k} - (\Delta b_i^+ + \Delta b_i^-)V^n_{i,j,k} + \Delta b_i^+ V^n_{i-1,j,k}}{\frac{1}{2}\Delta b_i^+ \Delta b_i^- (\Delta b_i^+ + \Delta b_i^-)},\\
c^{F,n}_{i,j,k} &= \left(V^{F,n}_{b,i,j,k}\right)^{-1/\gamma},\\
x^{F,n}_{i,j,k} &= \max\{\min\{-V^{F,n}_{b,i,j,k}(r_e-\tilde{r})/(\sigma_e^2 V^n_{bb,i,j,k}), b_i\}, 0\},\\
s^{F,n}_{i,j,k} &= \tilde{r}b_i + (r_e-\tilde{r})x^{F,n}_{i,j,k} + r_r h_j + e^{w_k} - c^{F,n}_{i,j,k} - T_{i,j,k},\\
\bar{x}^n_{i,j,k} &= \alpha^n_{i,j,k}x^{F,n}_{i,j,k} + (1-\alpha^n_{i,j,k})x^{B,n}_{i,j,k},\\
x^n_{i,j,k} &= x^{F,n}_{i,j,k}\mathbb{1}\{s^{F,n}_{i,j,k} > 0\} + x^{B,n}_{i,j,k}\mathbb{1}\{s^{B,n}_{i,j,k} < 0\} + \bar{x}^n_{i,j,k}\mathbb{1}\{s^{F,n}_{i,j,k} \leqslant 0 \leqslant s^{B,n}_{i,j,k}\},\\
\bar{c}^n_{i,j,k} &= \alpha^n_{i,j,k}c^{F,n}_{i,j,k} + (1-\alpha^n_{i,j,k})c^{B,n}_{i,j,k},\\
c^n_{i,j,k} &= c^{F,n}_{i,j,k}\mathbb{1}\{s^{F,n}_{i,j,k} > 0\} + c^{B,n}_{i,j,k}\mathbb{1}\{s^{B,n}_{i,j,k} < 0\} + \bar{c}^n_{i,j,k}\mathbb{1}\{s^{F,n}_{i,j,k} \leqslant 0 \leqslant s^{B,n}_{i,j,k}\},
\end{aligned}$$

[15] A LCP aims to find vector z that satisfies: $Mz+q, z \geqslant 0, z'(Mz+q) = 0$. In the problem above, $z = v^{n+1} - v^*, M = I(1/\Delta t + \rho + \delta) - A, q = -u - \frac{v^n}{\Delta t} + v^*(I(1/\Delta t + \rho + \delta) - A)$. With v^n and v^* one finds v^{n+1}.

where F stands for the forward approximation of 1[st] order derivatives, and equivalently for backward B, and equivalently for the derivatives with respect to h and w as well.[16] Notice that the case $\mathbb{1}\{s^{F,n}_{i,j,k} \leqslant 0 \leqslant s^{B,n}_{i,j,k}\}$ is slightly more complicated than a simpler model without portfolio choice because there are many $(\bar{x}, \bar{c})$ such that $\bar{c} = \tilde{r}b + (r_e - \tilde{r})\bar{x} + e^w + r_r h \to s = 0$. However, I would like $\bar{x} \in [x^F, x^B]$ and $\bar{c} \in [c^B, c^F]$, thus I look for the solutions such as the ones above.[17] Once we impose that $\bar{c} = \tilde{r}b + (r_e - \tilde{r})\bar{x} + e^w + r_r h$, we get $\alpha^n_{i,j,k} = s^{B,n}_{i,j,k}/(s^{B,n}_{i,j,k} - s^{F,n}_{i,j,k})$. Because we have $s^B \geqslant 0 \geqslant s^F$, then $\alpha \in [0, 1]$.

The upwind scheme on Equation 2.17 becomes:

$$\min\Bigg\{(\rho+\delta)V^{n+1}_{i,j,k} - u(c^n_{i,j,k}) - V^{F,n+1}_{b,i,j,k}(s^{F,n}_{i,j,k})^+ - V^{B,n+1}_{b,i,j,k}(s^{B,n}_{i,j,k})^- - V^{n+1}_{bb,i,j,k}(\sigma_e x^n_{i,j,k})^2/2$$
$$- V^{F,n+1}_{h,i,j,k} r_h h_{i,j,k} - V^{n+1}_{hh,i,j,k}(\sigma_h h_{i,j,k})^2/2 - E[V_w]/dt + \frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta t}, V^{n+1}_{i,j,k} - V^{*,n}_{i,j,k}\Bigg\} = 0,$$

where $V^{*,n}_{i,j,k}$ is found by looking in the grid for the best option after adjusting, and $E[V_w]/dt$ is defined further below. We want to write the equation above in block form, thus define:

$$wb^-_i = \frac{\Delta b^-_i}{\Delta b^+_i \Delta b^-_i (\Delta b^+_i + \Delta b^-_i)}, wb^+_i = \frac{\Delta b^+_i}{\Delta b^+_i \Delta b^-_i (\Delta b^+_i + \Delta b^+_i)}$$
$$wh^-_j = \frac{\Delta h^-_j}{\Delta h^+_j \Delta h^-_j (\Delta^+_h + \Delta b^-_i)}, wh^+_j = \frac{\Delta h^+_j}{\Delta h^+_j \Delta h^-_j (\Delta^+_h + \Delta h^+_j)}.$$

Therefore, the terms involving derivatives not related to the labor income process can

[16] Notice that $V^F_b < V^B_b$ if $V_{bb} < 0$, which should be the case. Therefore $c^F > c^B$ and $x^F < x^B$ according to their FOC's, and we have $s^F < s^B$.

[17] The α's for $\bar{x}, \bar{c}$ need not be the same, but if they are then we get the equation above for it.

be rewritten in block form as:

$$V^{F,n+1}_{b,i,j,k}(s^{F,n}_{i,j,k})^+ + V^{B,n+1}_{b,i,j,k}(s^{B,n}_{i,j,k})^- + V^{n+1}_{bb,i,j,k}(\sigma_e x^n_{i,j,k})^2/2 + V^{F,n+1}_{h,i,j,k} r_h h_{i,j,k} + V^{n+1}_{hh,i,j,k}(\sigma_h h_{i,j,k})^2/2$$
$$= \begin{bmatrix} -\frac{(s^{B,n}_{i,j,k})^-}{\Delta b^-_i} + wb^+_i(\sigma_e x^n_{i,j,k})^2 & \frac{(s^{B,n}_{i,j,k})^-}{\Delta b^-_i} - (wb^+_i + wb^-_i)(\sigma_e x^n_{i,j,k})^2 - \frac{(s^{F,n}_{i,j,k})^+}{\Delta b^+_i} & \frac{(s^{F,n}_{i,j,k})^+}{\Delta b^+_i} + wb^-_i(\sigma_e x^n_{i,j,k})^2 \end{bmatrix}$$
$$\begin{bmatrix} V^{n+1}_{i-1,j,k} \\ V^{n+1}_{i,j,k} \\ V^{n+1}_{i+1,j,k} \end{bmatrix} + \begin{bmatrix} wh^+_j(\sigma_h h_{i,j,k})^2 & -\frac{r_h h_j}{\Delta h^+_j} - (wh^+_j + wh^-_j)(\sigma_h h_{i,j,k})^2 & \frac{r_h h_j}{\Delta h^+_j} + wh^-_j(\sigma_h h_{i,j,k})^2 \end{bmatrix} \begin{bmatrix} V^{n+1}_{i,j-1,k} \\ V^{n+1}_{i,j,k} \\ V^{n+1}_{i,j+1,k} \end{bmatrix}$$
$$= \mathbf{A}^n_c \mathbf{v}^{n+1}. \tag{2.18}$$

Then we can see that the proper discretisation of the upwind scheme of equation 2.17 is of the form:

$$\min\left\{ \frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} + (\rho + \delta)\mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}^n \mathbf{v}^{n+1}, \mathbf{v}^{n+1} - \mathbf{v}^{*,n} \right\} = 0, \tag{2.19}$$

and $\mathbf{A}^n = \mathbf{A}^n_c + \mathbf{A}_y$, where $\mathbf{A}_y$ is defined below.

2.C.2.1 Labor Income Dimension

The discretised version of $E[V_w]/dt$ as it appears in Equation (2.14) is equal to

$$\frac{1}{dt}E[V_y] = -\mu_w w_k V_{w,i,j,k} + \frac{\sigma^2_w}{2} V_{ww,i,j,k}.$$

The equation above can be rewritten in block form as:

$$\frac{1}{dt}E[V_{w,i,j,k}] = \begin{bmatrix} ww^+_k(\sigma^2_w) + \mu_w w_k & -(ww^+_k + ww^-_k)(\sigma^2_w) - \mu_w w_k & ww^-_k(\sigma^2_w) \end{bmatrix} \begin{bmatrix} V^{n+1}_{i,j,k-1} \\ V^{n+1}_{i,j,k} \\ V^{n+1}_{i,j,k+1} \end{bmatrix}$$
$$= \mathbf{A}_y \mathbf{v}^{n+1},$$

where $\mathbf{v}$ is the usual $L \times 1$ vector with all the dimensions stacked, and ww^-_k, ww^+_k are defined in an analogous way as wb^-_i, wb^+_i.

2.C.2.2 Approximations at the Top

There are two complications of having a maximum value b_I for b in the grid. First, I need to impose $s^F_{I,j,k} \leqslant 0$ so as not to break the upper bound. Second, with x as a choice I need to find $V^{n+1}_{bb,i,j,k}$, which uses both $V^{n+1}_{i+1,j,k}$ and $V^{n+1}_{i-1,j,k}$. Thus, it's not clear what to use at $i = I$ since there is no $I + 1$ point in the grid.[18]

I follow Achdou et al. (2020), and assume that the value function converges to being of the form $V(b) \propto b^{(1-\gamma)}$.[19] If that's the case, one can write $V_{bb}(b_I) = -\gamma V_b(b_I)/b_I$, which implies $x(b_I) = b_I \frac{r^e - \tilde{r}}{\gamma \sigma_e^2}$, which I use instead of using an approximation for the second derivative. I also impose $s(b_I) \leqslant 0$ for the forward approximation of the upwind scheme, which implies a bound of $V^F_b(b_I, h_j, y_k) = (rb_I + (r_e - r)x(b_I) - T(b_I, h_j, w_k) + r_r h_j + exp^{w_k})^{-\gamma}$ through the FOC (for the backward approximation, $V^B_{b,I,j,k}$ is calculated as usual). When constructing the $\mathbf{A}_c$ matrix, notice that we need $(\sigma_e x)^2 V_{bb}/2$, which is equal to $-\frac{\gamma(\sigma_e x(b_I))^2 V_b(b_I)}{2b_I}$ at $b = b_I$, according to the conjecture. Thus the first matrix of Equation 2.18 at $i = I$ is actually calculated as:

$$\begin{bmatrix} -\frac{(s^{B,n}_{I,j,k})^-}{b_I - b_{I-1}} + \frac{\gamma(\sigma_e x(b_I))^2}{2b_I(b_I - b_{I-1})} & \frac{(s^{B,n}_{i,j,k})^-}{b_I - b_{I-1}} - \frac{\gamma(\sigma_e x(b_I))^2}{2b_I(b_I - b_{I-1})} & 0 \end{bmatrix} \begin{bmatrix} V^{n+1}_{I-1,j,k} \\ V^{n+1}_{I,j,k} \\ 0 \end{bmatrix}.$$

2.C.3 Stationary Distribution

Without adjustment, the discretised version of the KFE in Equation (2.15), this time accounting for births,[20] would be equal to:

$$\frac{f_{t+\Delta t} - f_t}{\Delta t} = \mathbf{A}^T f_{t+\Delta} - \delta f_{t+\Delta} + \text{births},$$

where $\mathbf{A}$ comes from the HJB problem.

[18] At the bottom of the grid households do not have equity as the liquid wealth is all mortgage, so I do not need to find $V_{0,j,k}$ to use it at $i = 1$.

[19] Notice that while this is known to be the case without labor income (see Appendix 2.B), or as $b \to \infty$, and no illiquid assets, it is not clear that it should be the case with the adjustment decision as well. However, it works well in practice.

[20] The vector for births is a vector of mass equal to δ that is the stationary distribution over the y dimension for $(b, h) = 0$ and equal to zero otherwise. That is, households are born with zero wealth but according to the stationary labor income distribution.

However, as mentioned in Section 4, because of the adjustment in the illiquid assets households do not always move according to the equation above and it is necessary to construct an $L \times L$ matrix $\mathbf{M}$ that deals with those cases (see Kaplan, Maxted, and Moll (2016) for details).

The idea of the matrix $\mathbf{M}$ is that it will take agent that adjust from their point in the grid space (b, h) to their chosen destination $(\tilde{b}, \tilde{h})$. Let $m = 1, \ldots, L$. Then there are two cases. First, for all the points m in the space (b, h, y) that do not want to adjust we have $M_{m,m} = 1$. That is, the agent does not jump. Second, for those that want to adjust, denote $d(m)$ as their chosen destination. Then we have $M_{m,d(m)} = 1$ for these cases. Otherwise, $\mathbf{M}$ is full of zeros. In other words, matrix $\mathbf{M}$ would be the identity matrix without adjustments, but in the adjustment cases it moves mass from the adjustment region to the not adjustment region.

Finally, an iterative procedure leads to the stationary distribution. I proceed as follows:

1. Given f^n, find $f^{n+\frac{1}{2}}$ from
$$f^{n+\frac{1}{2}} = \mathbf{M}^T f^n$$

2. Given $f^{n+\frac{1}{2}}$ find f^{n+1}
$$\frac{f^{n+1} - f^{n+\frac{1}{2}}}{\Delta t} = (\mathbf{AM})^T f^{n+1} - \delta f^{n+1} + \text{births},$$

until f^{n+1} and f^n are close.

Chapter 3

Racial Representation Along the Wealth Distribution: the Roles of the Entrepreneurship and Wage Gaps

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Abstract

The racial wealth gap in the US is one of the most striking and persistent disparities between Black and White households. We develop an incomplete markets model with endogenous entrepreneurship choice and realistic wealth dispersion, and use it to quantify the impact of racial gaps in wages, business start-up costs, and labour market attachment on the racial wealth gap. We find that the wage gap is particularly important in explaining gaps at the bottom of the wealth distribution, while entrepreneurship is essential to explain gaps at the top. Furthermore, changes in the racial wealth gap are slow in our setting, due to the long time required for Black households to reach the top 10% of wealth. Finally, we discuss the impact of reparations and demonstrate that, while they are not successful at speeding the transition towards permanent equality, they help make inequality smaller during the transition.

Keywords: Racial wealth gap, Entrepreneurship, Racial disparities, Incomplete markets, Wealth accumulation.
J.E.L. codes: E21, J15, D52.

3.1 Introduction

The gap in average wealth between Black and White households in the United States, henceforth the racial wealth gap, is one of the most striking features of the wealth distribution and also one of the biggest racial gaps in economic outcomes. Possible explanations for it include the persistent effects of past slavery and expropriations and also current discrimination in wages, access to credit, and barriers to entrepreneurship.

Unsurprisingly, the large racial gaps in wealth and other social-economic factors lead to a substantial gap in welfare. Thus, understanding which factors are behind the current and future racial wealth gap is crucial to address present disparities and to understand the impact of different types of policies aimed at addressing it, including reparations.

In this paper, we start by documenting the past and current racial gaps in wages, entrepreneurship and wealth. In line with the literature, we show that they have been large and persistent since the 1980s, with the wealth gap larger than the wage gap.

Next, we develop a general equilibrium model with incomplete markets à la Bewley-Imrohoroglu-Hugget-Aiyagari that considers the gaps in wages and unemployment rates as exogenous. We augment it with an entrepreneurship choice to achieve a better fit for wealth inequality, as in Cagetti and De Nardi (2006) and Quadrini (2000). Our modelling of entrepreneurship choice with fixed costs is closer to Buera, Kaboski, and Shin (2011), and we allow for differential fixed costs for Black and White households to match the entrepreneurship rates observed in the data.

The exogenous gaps in wages, unemployment and cost of entrepreneurship generate endogenous differences in wealth across races. This modelling approach allows us to analyse their roles in explaining the current wealth gap and its possible future evolution, including what happens when we perform a one-off transfer of wealth from White to Black households (i.e., reparations). We have four main findings.

First, we present a stylised fact that, even though there is a substantial gap in entrepreneurship rates between Black and White households, that gap is significantly smaller than the gap in entrepreneurship between wealthy and poor households, even within races. We perform a decomposition exercise on the entrepreneurship gap across races into differences in entrepreneurship conditional on wealth and differences in wealth and show that the latter is more important to explain the racial entrepreneurship gap for different measures of entrepreneurship. We interpret this result as indicating that lack of wealth accumulation is a major barrier to Black entrepreneurship and that Black households would start more businesses if faced with higher wealth. This will be incorporated as an important mechanism in our model.

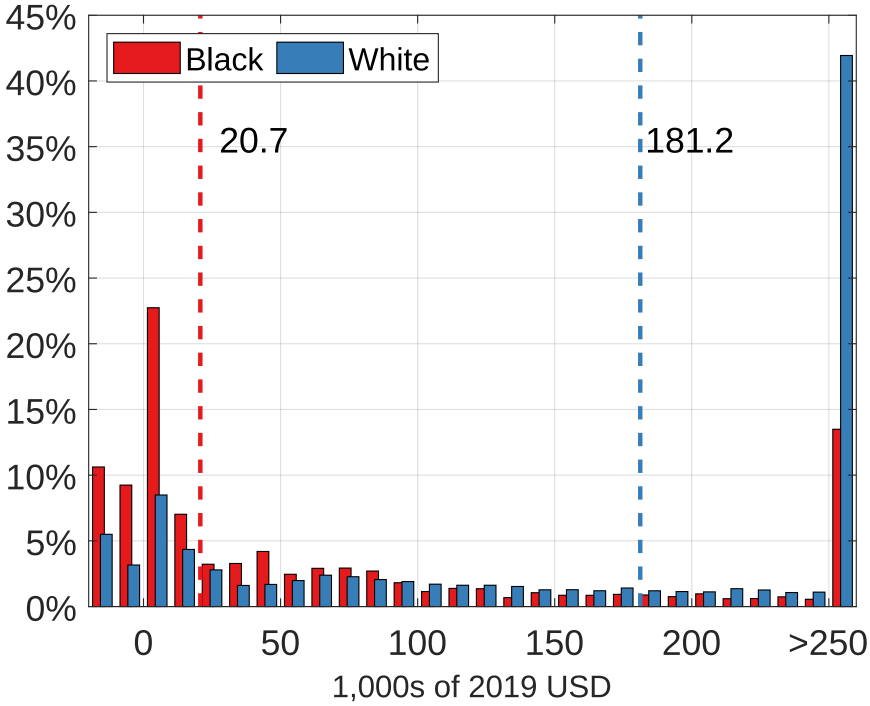

Figure 3.1: Histogram of the distribution of wealth for White and Black Households in 2019

Notes: The red and blue dashed lines indicate the median wealth for Black and White households, respectively. *Source*: SCF.

Second, we take the gaps in wage, business start-up cost, and unemployment as exogenous and ask if they can explain the racial wealth gap. We find that, quantitatively, the wage gap is the most important, followed by the entrepreneurship gap, and finally the unemployment gap, which is quantitatively unimportant. Specifically, the wage gap can explain on its own all the over-representation of Black households at the bottom 90% of the distribution. However, the entrepreneurship gap is key to achieving parity at the top of the distribution and closing the racial wealth gap without addressing it is impossible.

Third, we analyse the future path for the wealth gap when racial discrimination in wages and entrepreneurship disappear and find that convergence in average wealth across races is slow and that it takes time for Black households to catch up with the very top of the wealth distribution. If exogenous gaps close instantly, we find that it would take 200 years for the racial wealth gap to close; if they decay linearly over the next 100 years, the wealth gap takes 250 years to close. We argue that the convergence is slow because it takes time for Black households to start more firms, for those firms to grow, and for increased labour income and profits to be accumulated into wealth. Crucially, we show that it is important to look at measures of the wealth gap beyond average wealth, as Black households are no longer over-represented in the poorest 50% of households 100 years before the closure of the average wealth gap.

Fourth, we study the effect of reparations as a transfer of wealth from White to Black households that closes the gap immediately. We find that reparations significantly increase the representation of Black households among the middle class (those between the 50^{th} and 90^{th} percentiles of wealth). Still, they are not enough to make them equally represented in the top 10%. Also, as the convergence in income is not immediate even with reparations, the racial wealth gap reopens significantly in the first 25 to 50 years. Moreover, reparations do not help to permanently close the average wealth gap faster. However, they are important during the transition in reducing racial wealth disparities for 100 to 150 years, by which point these differences have been greatly reduced already.

Our contributions are in line with other findings from the literature that current conditions in racial wealth inequality are persistent and that any path towards permanent equality in average wealth for Black and White households will take time (Aliprantis, Carroll, and Young, 2019; Boerma and Karabarbounis, 2022), even with reparations.

However, we highlight that there are distinct determinants of the racial wealth gap at different parts of the wealth distribution, and thus the speed of convergence will depend on where one is looking. On the one hand, as the bottom 90% of the distribution is primarily dependent on labour income and does not have a high wealth-to-income ratio, the convergence at the bottom happens faster once the wage gap closes.

On the other hand, wealth at the top of the distribution comes mostly from accumulated capital income. Thus, convergence will be slower, as Black households need time to start businesses, grow their firms, and accumulate profits (it also takes longer to accumulate enough labour income). Moreover, because there is a high level of wealth inequality in the US and the top 10% own more than 70% of aggregate wealth, the slow speed of convergence at the top of the wealth distribution dominates, which explains the slow convergence of average wealth.

3.1.1 Related Literature

There is a long history of scholarship on the various racial gaps. The one that has arguably received the most attention is the gap in labour income, probably due to better access to high-quality data. Early examples include Freeman (1973), Card and Krueger (1992), and Donohue and Heckman (1991), who report on the relative gains for Black households in the decades following the passage of the Civil Rights Act. Their findings were revisited by Neal and Johnson (1996) and Carneiro, Heckman, and Masterov (2005), who tried to control for ability using the Armed Forces Qualification Test when explaining the wage gap, with the latter finding that it does not reduce the unexplained racial wage gap. Lang and Lehmann (2012) summarise the findings of the literature labour income gap literature as there being a wage and unemployment gap between Black and Whites and, while the wage gap has fallen somewhat from 1970-2010, the unemployment gap has remained constant.[1] More recently, Chandra (2003) and Bayer and Charles (2018) have highlighted the importance of other labour market outcomes such as non-participation in the labour force (partly explained by higher incarceration rates for Black men) and how considering those reduces the observed fall in the wage gap in the second half of the 20$^{\text{th}}$ century. Bayer and Charles (2018) find a racial wage gap that has been stable at around 40% since the 1970s for those working full-time.[2]

Even though the wage gap documented by the studies above is large and long-lasting, it pales in comparison to the size of the racial wealth gap. As shown in Figure 3.1, in

[1] See Darity and Mason (1998) and Altonji and Blank (1999) for other comprehensive literature reviews.

[2] See Blanchet, Saez, and Zucman (2022) for evidence of the gap in capital income, which is similar to that on wealth; Derenoncourt and Montialoux (2021) for the impact of minimum wage policies on the decline of the income gap in the 1960s and 1970s; and Althoof and Reichardt (2022) for the long-run effects of being tied to the Deep South.

2019, the median Black household owned only 11.4% of the median White household, a gap of 88.6%.[3] Good quality data on the wealth distribution for the US is mostly recent - the Survey of Consumer Finances (SCF) tracks wealth, but it started in the 1980s, while the Panel Survey of Income Dynamics (PSID) only tracked housing wealth before 1984.[4] Earlier examples of studies tracking the wealth gap are Higgs (1982) and Margo (1984), who are able to construct data for a few states in the US. More recently, Kuhn, Schularick, and Steins (2020) extend the SCF further back in time and documented that the wealth gap has been more or less stable in the last 70 years. Finally, Derenoncourt et al. (2022) are able to construct a series since the 1860s and report that, from an extremely high level in 1860, there was significant progress in closing the gap in the 50 years after the Emancipation and some progress from 1920 to 1950. However, progress has stalled since then.

Explanations for the wealth gap are varied, and our study focuses on the effects of wage, unemployment and entrepreneurship gaps. Entrepreneurship is an important contributor to wealth accumulation (Cagetti and De Nardi, 2006; Castaneda, Diaz-Gimenez, and Rios-Rull, 2003; Quadrini, 2000), but Black households have lower rates of entrepreneurship when compared to the overall population (Bogan and Darity Jr, 2008; Fairlie and Meyer, 2000). Thus, many authors have argued that increasing entrepreneurship rates among Black households is the most promising way to reduce the wealth gap (Bradford, 2014; Boston, 1998; Butler, 2012; Wallace, 1997).

The question then becomes, why are so few Black entrepreneurs? Bento, Hwang, et al. (2022) estimate a structural model of entrepreneurship and find that Black entrepreneurs face higher relative costs for starting and running a firm. Moreover, there is evidence that Black people face worse access to credit for their businesses. Studies have found that they face: lower approval rates for credit (Blanchflower, Levine, and Zimmerman, 2003; Blanchard, Zhao, and Yinger, 2008; Cavalluzzo and Wolken, 2005; García and Darity Jr, 2021); higher interest rates (Dougal et al., 2019; Hu et al., 2011); get access to smaller loans (Atkins, Cook, and Seamans, 2022; Bates and Robb, 2016); and have a harder time raising start-up capital (Fairlie, Robb, and Robinson, 2022).

[3]Figure 3.A.1 shows the wealth distribution in 1989.

[4]It is also not possible to try and infer wealth from income as in Saez and Zucman (2016) because tax records do not contain information on race.

Our paper is related to a growing literature that employs quantitative macro models to understand the drivers of the racial wealth gap. The focus of this literature so far has been to take the racial wage gap as given and ask how much of the wealth gap it can explain. White (2007) focuses on the role of educational choices and human capital accumulation in explaining the persistence over time of the wealth gap. Ashman and Neumuller (2020) highlight how the wage gap can generate large wealth gaps through their impact on savings, bequest and intergenerational transfers; and Aliprantis, Carroll, and Young (2019), using a general equilibrium model, find that the gap in earnings is the main cause of the persistent wealth gap and that reparations have only a temporary effect if the wage gap is not addressed. Relative to these papers, our contribution is to analyse the wage gap in conjunction with entrepreneurship choices. We show that not only do both the wage and entrepreneurship gap have important quantitative contributions to the wealth gap, but also that there are meaningful interactions between the two. Namely, the wage gap makes it harder for poorer Black households to accumulate wealth and start a business but also makes the outside option of paid work worse.

The papers closest to ours is probably Boerma and Karabarbounis (2022), which features a model with an endowment economy in which the wage gap is also exogenous, but households whose beliefs about the profitability of starting a firm depend on their past experiences.[5] Thus, as Black households were deprived of owning property in the past, they start fewer businesses now, and the wealth gap is persistent. This is different from our setting in which households when faced with an increase in wealth, would eventually increase their investment into firms, which we believe is in line with the evidence we present that entrepreneurship rates conditional on wealth are similar across races. This difference is crucial because in our setting, once the exogenous gaps close, the wealth gap will eventually close, even if slowly.

The rest of the paper is organised as follows. Section 2 describes the data used to measure the racial gaps in wage, entrepreneurship and wealth and displays the empirical findings. Section 3 develops the model used to analyse the racial wealth gap. Section 4 presents the results of the impact of wage and entrepreneurship gaps on the current racial

[5] See also Lipton et al. (2022) for an overlapping generations model in which different initial firm ownership is persistent over time through inheritance, which perpetuates the racial wealth gap.

wealth gap and in possible future scenarios. It also analyses the effect of reparations. Section 5 concludes.

3.2 Data

The primary data sources for this study are the Panel Survey for Income Dynamics (PSID) and the Survey of Consumer Finances (SCF). While there is ample and readily available data documenting income across races, there are fewer options for wealth. In the 1980s, both the SCF was created with the specific goal of being a survey of wealth, and the PSID started documenting wealth every five years (and then every two years when it became biennial in 1999). The SCF and the PSID can be viewed as complements: an objective of the SCF is to get a good picture of the top of the wealth distribution, and it oversamples households believed to be in that region, while the PSID is particularly good for the bottom of the income distribution. We will work with both surveys throughout the paper and show that the results for the wealth and entrepreneurship gaps are the same.

We define a household as the unit of observation in both surveys and restrict our sample to households where the main respondent classifies themselves as Black or White, excluding all households that also identified as Latinx or of Hispanic origin. Since we are interested in fitting our model to the current state of the wealth gap in the US, we focus on the most recent period between 2001 and 2019 to draw implications from the data.

The PSID is a better choice for income data due to its panel structure. This allows us to compare changes in income over time for the same household, which will be important when calibrating the income process imputed to the model. Given our focus on income changes of households and to better compare our results to those of the previous literature, the PSID sample is further restricted to male-led households between 25 and 65 years old. To be consistent, this is the same sample used for non-income variables with the PSID. However, we make no restrictions on gender or age when using the SCF. Thus, given their different focus, and our different sample choices, it is reassuring that the results for wealth and entrepreneurship gaps are similar across surveys.

3.2.1 Wage Gap

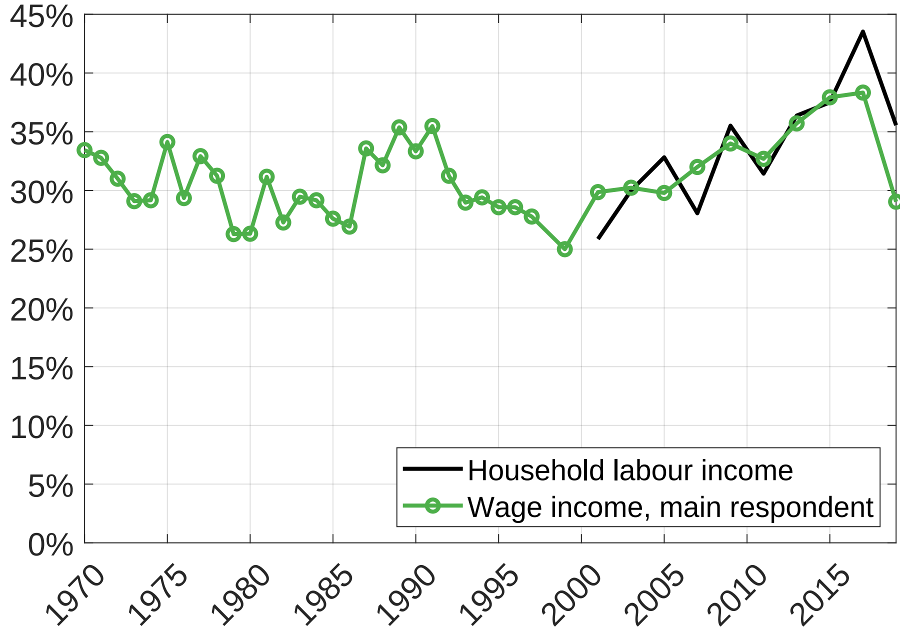

Figure 3.1: Gap in labour income between Black and White households

Notes: This figure illustrates the racial gap in households' labour income and in wage income of the main respondents. Household labour income includes the wages of the main respondent but also other sources of labour income, such as overtime pay, tips, bonuses, etc., for both the main respondent and their spouse. *Source*: PSID.

We use data from the PSID to calculate the gap in labour income between Black and White households. As the unit of observation is a household, our preferred measure of income includes the total labour income of the survey's main respondent and their spouse, if there is one. We restrict the sample to individuals between 25 and 65 years old to capture the working-age population. Additionally, to not mix gaps in labour income conditional on employment with differences in employment probabilities, we exclude those with earnings below 1/12 of the median wage of those with positive earnings.[6]

In the model in Section 3.3 our main focus is the entrepreneurship decision. We do not take educational choices into account explicitly nor consider important variables for explaining the observed wage gap, such as school quality. Thus, the appropriate measure of the wage gap is the raw, simple comparison of the wage in Black and White

[6]The reasoning being that the minimum wage historically is approximately 1/3 of the median wage, and we want households that worked for at least a full quarter in a given year.

households, with no controls. Therefore, when in our model we perform an exercise where the wage gap is closing over time, we interpret it as not just the wage gap conditional on observables closing but also, for example, the convergence of educational attainment across races.

Figure 3.1 shows the result of this exercise. Our preferred measure of the wage gap that takes into account all labour income of a given household is not available since the beginning of the sample. Thus we plot it in the period from 2001-2019. We see that the gap has actually widened recently. However, when we look at the gap in the wage income of the main respondent, which has been available since 1970, we might interpret the figure slightly differently. It seems that there is no clear trend in the longer horizon, in line with evidence from Bayer and Charles (2018).

3.2.2 Wealth Gap

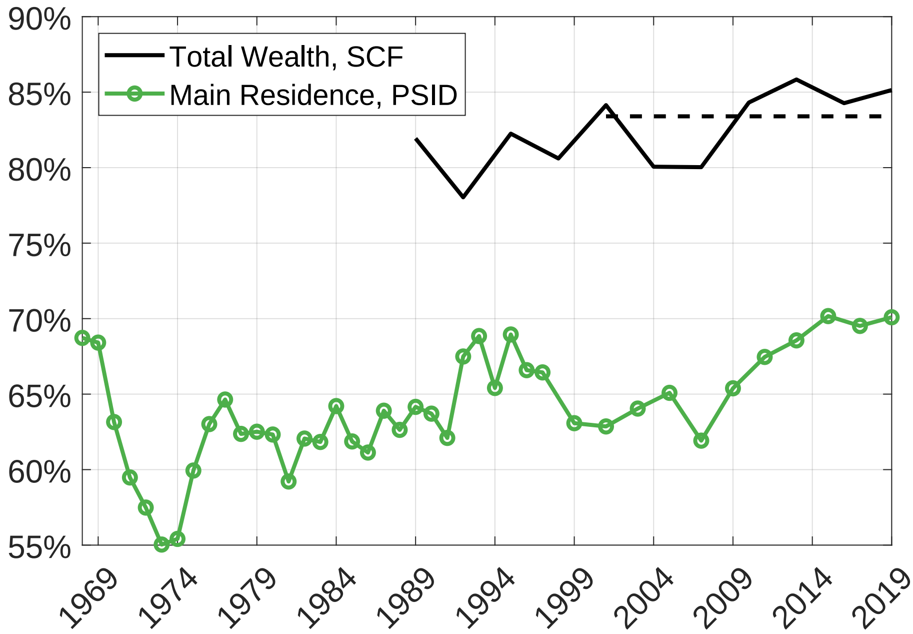

Figure 3.2: Racial Wealth Gap

Notes: This figure shows the racial wealth gap between Black and White households, defined as the gap in average wealth between races. The results are shown for two different measures of wealth: (i) total wealth; (ii) value of the main residence. The dashed line is the average from 2001-2019 of the gap in total wealth. *Source*: SCF and PSID.

One of the consequences of the wage gap, and other variables, is the wealth gap shown in Figure 3.2. As mentioned before, readily available data on wealth started in the

1980s, and the figure shows that the wealth gap has hovered between 80 and 85% since then. It also plots since 1968 the gap in the value of the main residence as a proxy for the wealth gap, which has also been large and without a clear trend in this longer horizon.

One might be worried that the wealth gap as measured by the ratio of average wealth across races could be high due to the presence of Black households with negative wealth that would push the denominator towards zero, but that otherwise, the distributions are similar. Figure 3.3 shows that this is not the case: Black households are underrepresented at the top of the distribution and over-represented at the bottom. If there was no difference between the distribution of wealth across races, all the bars in Figure 3.3 should be at the level of the dashed line, 16.6%, which is the share of Black households in the whole population.

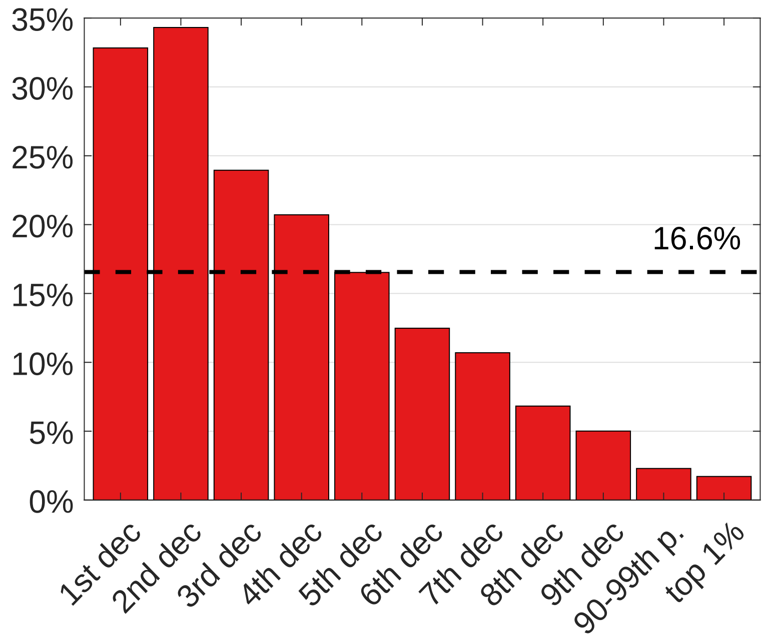

Figure 3.3: Share of Black households by group of wealth

Notes: This figure shows the share of Black households in different parts of the wealth distribution: 1^{st} decile of wealth, ..., 9^{th} decile of wealth, those between the 90^{th} and 99^{th} percentile, and those in the top 1% of wealth. The dashed line indicates the share of Black households over the whole distribution, which is equal to 16.6%. *Source*: SCF 2001-2019.

A feature of the wealth gap that can be striking at first is that it is higher than the wage gap. Other studies using a quantitative macro model have found that the wage gap can account for a large portion of the wealth gap (Aliprantis, Carroll, and Young, 2019; Ashman and Neumuller, 2020). We will document next the gap in entrepreneurship rates and argue that the entrepreneurship gap is also important to understand the differences in wealth.

3.2.3 Entrepreneurship Gap

Our preferred definition of an entrepreneur is a household that owns and actively manages a private business, as documented by the SCF. We do not consider households that own a business but do not manage it to not include households that made a portfolio choice of investing in a private business but are otherwise not entrepreneurs, for example, because their main source of income can be paid labour for another company. However, we show our results for other definitions of entrepreneurship as well.

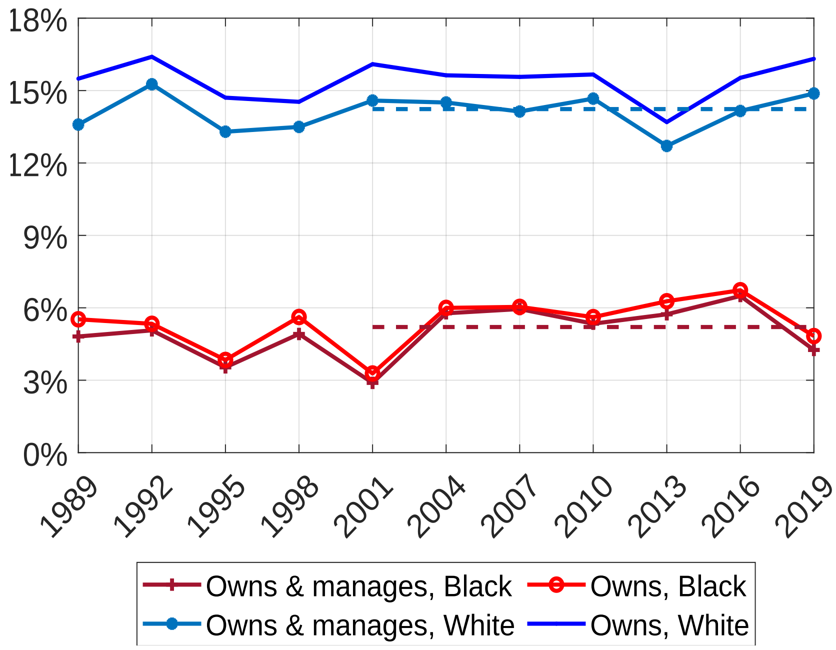

Figure 3.4: Entrepreneurship rates for Black and White households

Notes: This figure shows over time the share of Black and White households that are entrepreneurs according to two definitions: (i) owns a private business; (ii) owns and actively manages a private business. The dashed lines show the averages from 2001-2019 for "Owns & manages", which are equal to 5.2 and 14.2% for Black and White households, respectively. *Source*: SCF.

Figure 3.4 plots entrepreneurship rates in the last 30 years using the SCF. It shows that for different definitions, the racial gap has been stable and sizeable, around 9 p.p. (5.2 vs 14.2%), over the last decades. Figure 3.A.2 in the Appendix shows the same figure using PSID data. Looking at the "own a business" definition, paints a different picture than the SCF: the gap has been closing since the late 1980s because the ownership rate for White households has fallen, while the one for Black households has slightly increased. However, when we restrict to those that own an incorporated business, we see that the gap has been stable in the last 30 years, in line with our preferred measure using the SCF. We argue that this is the notion that matters most when thinking about wealth inequality because incorporated businesses are more likely to be present at the top of the wealth distribution and are the ones most associated with entrepreneurship activities. There is little switching from unincorporated businesses to incorporated ones (see Levine and Rubinstein, 2017). Thus, the PSID data concludes that the gap in entrepreneurship has been large and stable over the last three decades.

We now examine the relationship between entrepreneurship and wealth across races. As Figure 3.5 shows, entrepreneurship is highly correlated with wealth:[7] more than 60% of households in the top 1% of the overall wealth distribution are classified as entrepreneurs, while that number is smaller than 10% in the bottom half of the distribution, and that is true regardless of race. Importantly, it shows that the variance of entrepreneurship rates across the wealth distribution for both races seems to dominate the variance of entrepreneurship rates across races, but at the same part of the wealth distribution. This suggests that differences in the overall entrepreneurship rates between races could be due mostly to differences in wealth and not due to different likelihoods across races to start a business conditional on their level of wealth.

To shed some light on that question, we employ a decomposition exercise. Let x^i denote the overall entrepreneurship rate of race $i \in \{B, W\}$, x^i_j the entrepreneurship rate in group of wealth j (e.g., the top 1%), and p^i_j the percentage of race i total mass that is in group of wealth j. It is possible then to write the overall entrepreneurship rate as the weighted average of the entrepreneurship rates in each part of the wealth distribution:

$$x^i = \sum_j x^i_j p^i_j. \tag{3.1}$$

[7]Figures 3.A.3, 3.A.4 and 3.A.5 in the Appendix, using the other definitions of entrepreneurship, and data from the PSID as well, also show that entrepreneurship and wealth are highly correlated across race, and in a similar way to Figure 3.5.

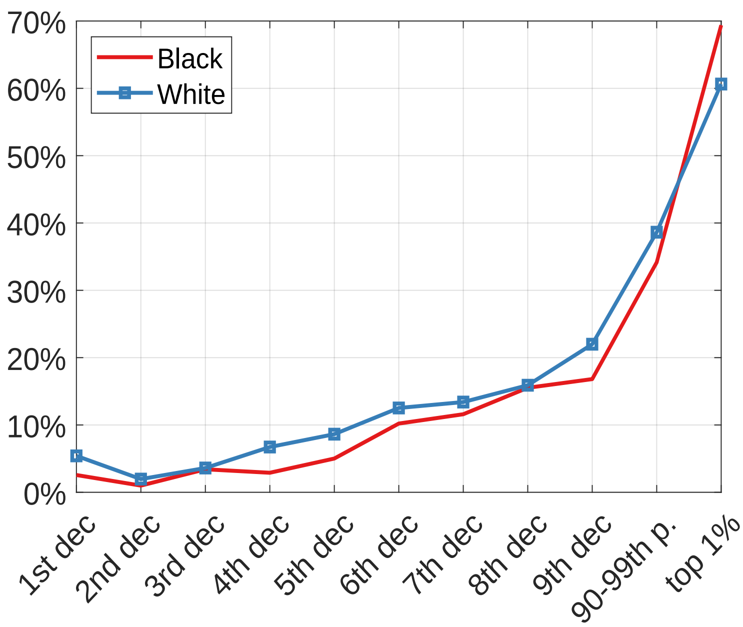

Figure 3.5: Entrepreneurship rates by wealth groups

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the overall wealth distribution: 1^{st} decile of wealth, ..., 9^{th} decile of wealth, those between the 90^{th} and 99^{th} percentile, and those in the top 1% of wealth. A household is classified as an entrepreneur if it owns and actively manages a private business. *Source*: SCF, 2001-2019.

We can then decompose the gap in entrepreneurship between Black and White households in the following way:

$$
\begin{aligned}
x^W - x^B &= \sum_j x_j^W p_j^W - \sum_j x_j^B p_j^B + \sum_j x_j^B p_j^W - \sum_j x_j^B p_j^W \\
&= \underbrace{\sum_j \left(x_j^W - x_j^B\right) p_j^W}_{\substack{\text{(1) differences in} \\ \text{entrep. given wealth}}} + \underbrace{\sum_j x_j^W (p_j^W - p_j^B)}_{\substack{\text{(2) differences} \\ \text{in wealth}}},
\end{aligned}
\tag{3.2}
$$

where in the first line we just used the definition for x^W, x^B, from Equation 3.1 and added and subtracted the term $\sum_j x_j^B p_j^W$. Equation 3.2 shows that we can decompose $x^W - x^B$ into two terms: (1) differences in entrepreneurship given wealth x_j^i, which calculates what the entrepreneurship gap would be if Black households had the same wealth distribution p_j^i as White households; and (2) differences in wealth distribution p_j^i, which calculates what the entrepreneurship gap would be if Black households had the same entrepreneurship rate conditional on wealth x_j^i, but different wealth distributions

p_j^i observed in the data.

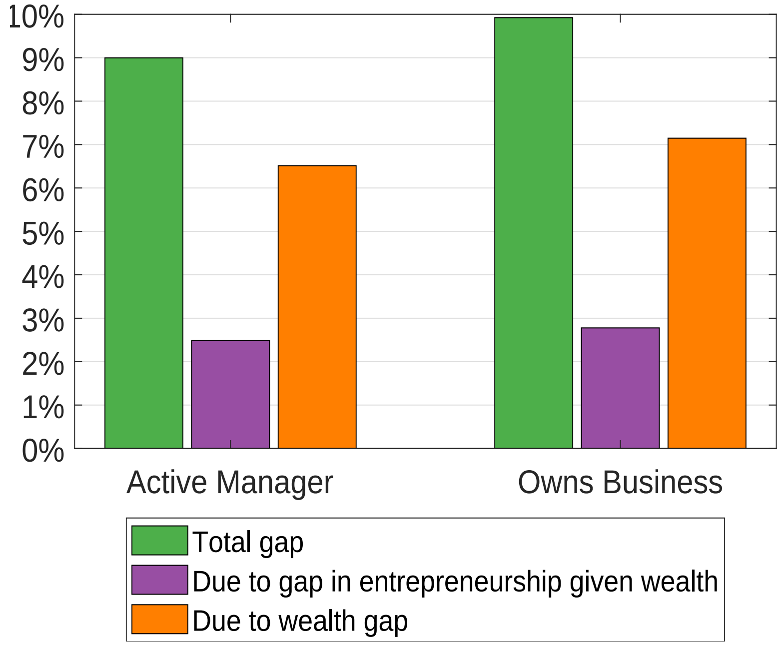

Figure 3.6: Decomposition of the gap in entrepreneurship rate between Black and White households

Notes: This figure shows the decomposition of the gap in entrepreneurship rates between Black and White households from Equation 3.2 into: (1) differences in entrepreneurship conditional on wealth; (2) differences in wealth. It shows the decomposition for entrepreneurship defined as "owns and actively manages a business" and also as "owns a business". *Source*: SCF, 2001-2019.

Figure 3.6 and Figure 3.A.6 in the Appendix show the results of this decomposition for the SCF and PSID, and using different definitions of entrepreneurship. The results are the same: the Black-White entrepreneurship gap is more related to differences in wealth than to differences in entrepreneurship conditional on wealth. In our preferred definition of being an active manager, differences in wealth explain 72% of the overall gap (or 6.5 p.p. of the total 9 p.p.).

We interpret these results as suggestive that factors such as the wage gap prevent Black households from accumulating wealth, which leads to lower business creation, which further contributes to harder wealth accumulation. Also, it could imply that Black households would create more businesses and reduce the wealth gap if faced with increased wealth from reparations. We discuss these possibilities with the help of our model in Section 3.3.

3.3 Model

Our model utilizes the workhorse incomplete market model à la Bewley (1986), Imrohoroglu (1989), Huggett (1993), and Aiyagari (1994) set in general equilibrium. Unlike in the standard Aiyagari (1994) set-up, we allow for a non-degenerate distribution of firms, each exhibiting decreasing returns to scale, which allows for positive profits in equilibrium. This modelling approach is motivated by the works of Quadrini (2000) and Cagetti and De Nardi (2006), which illustrate the significance of entrepreneurship for modelling wealth concentration and wealth mobility. Generally speaking, entrepreneurs will enjoy higher upward wealth mobility than workers.

Modelling the racial wealth gap requires us to model two groups, Black and White households, as distinct agent types that face different market conditions. Because this paper focuses on exploring the racial wealth gap, we will take a reduced-form modelling approach to other racial gaps. Thus, we take an agnostic approach to the sources of other racial gaps and treat them as fundamentals. These include the wage, business start-up cost, and unemployment or labour-force attachment gap. We begin with the model description and follow with an in-depth explanation of our modelling of racial disparities as this is the non-standard element in our modelling approach.

Time t is continuous, and there is a unit mass of households, which are ex-ante identical except for their race $i \in \{B, W\}$, where B denotes a Black household and W a White one. The mass of households of each race is denoted by m^i, and it is by assumption constant over time. Households can be either entrepreneurs or workers. Workers face uninsurable idiosyncratic shocks to their labour productivity z_L and to their employment status l, where $l = 1$ denotes employed and $l = 0$ unemployed.[8] Total labour income of workers in the model is thus wlz_L, where w is the wage per unit of labour productivity that is determined in general equilibrium.

Workers can also choose whether to become an entrepreneur at a cost κ^i and start a firm with the lowest productivity level $\underline{z}_F$. Entrepreneurs face idiosyncratic shocks to the productivity of their firms z_F and thus to their flow income, i.e., profits. Additionally,

[8] We do not model non-participation explicitly, so we interpret the $l = 0$ more broadly as non-employment, which includes unemployment but also non-participation.

their firm may exit at an exogenous rate λ_D, in which case the entrepreneur gets re-injected into the worker pool. All households in the model can save and accumulate wealth a subject to a borrowing constraint $a \geqslant \underline{a}$.

Households are infinitely lived, and we interpret them as dynasties. This modelling choice is equivalent to households having perfect "warm glow" motives towards their offspring and leaving bequests, which is important in our setting to generate intergenerational transmission of wealth and the persistence of racial wealth inequality observed in the data. Furthermore, we assume stochastic discount factors ρ as in Krusell and Smith (1998) to generate more wealth dispersion (see also Toda, 2019).

3.3.1 Discrimination

We model three fundamental racial gaps that will act as determinants of the racial wealth gap. First, we model a racial wage gap as a proportional gap in labour income. A White worker in the model will earn a labour income of wz_L, whereas the Black worker will earn $wz_L(1-\omega^B)$, with $0 < \omega^B \leqslant 1$. Second, Black workers and White workers face different transition rates from employment to unemployment, which, when estimated from the data, will lead to higher unemployment rates amongst Black workers. We denote the exit hazard from state l into state l' for a person of race i as $\lambda^i_{ll'}$. There is extensive literature documenting the different outcomes of Black households and White households in the labour market in the US (e.g., the literature reviewed in Lang and Lehmann, 2012). The most recent evidence (Bayer and Charles, 2018) points to a persistent wage gap conditional on employment and different labour market participation rates, which supports our modelling approach. Third, we assume a higher cost $\kappa^B > \kappa^W$ for Black households to start a firm. This assumption is supported by the fact that there is a significant gap in entrepreneurship between races and that there is ample evidence in the literature that Black entrepreneurs face higher barriers to setting up and running a firm (e.g., Bento, Hwang, et al., 2022; Fairlie, Robb, and Robinson, 2022; García and Darity Jr, 2021).

We model the wage gap ω^B for Black workers without modelling a segmented labour market for Black workers and White workers. Each unit of labour productivity is paid by the firm the wage rate w, which is equal to the value of the marginal product for that unit. Thus, in the model, firms are colourblind and cannot distinguish between workers of different races. We reconcile both of these assumptions by diverting a fraction ω^B of the labour income of Black households in the model to benefit White households that own capital in a way that is proportional to their wealth. We make this assumption because if firms knew they could pay less to Black workers, they would prefer hiring them, which is not in line with the data. Thus, we would need to include a mechanism for why racial discrimination is sustained in equilibrium, which is beyond the scope of this paper.[9] Alternatively, we could have introduced an extra agent, for example, a union that receives the wages that firms pay and then takes a share ω^B from the Black workers' wages before passing it to them because of differences in bargaining power. We believe that such a set-up would not add any clarity.

To conclude our modelling of discrimination, we formally define the gains from discrimination $\hat{r}^W$ originating from the racial wage gap as follows: let m_w^B be the mass of Black workers; $n(z_L, l|i)$ be the probability density function (PDF) over workers' labour productivity and employment status conditional on race; and $f(a|i)$ the PDF of asset holdings conditional on race. Then the gains from discrimination $\hat{r}^W$ must satisfy:

$$\hat{r}^W m^W \int_0^\infty af(a|W)da = w\omega^B m_w^B \int_{\underline{z}_L}^{\overline{z}_L} zn(z, 1|B)dz. \tag{3.3}$$

As a result of this set-up, Black households receive a lower gross return on their wealth: $r \leqslant r + \hat{r}^W$.

3.3.2 Households

Workers and entrepreneurs choose how much to consume c, and workers face the additional choice of whether to leave the labour market and start a firm. Labour productivity z_L, employment status l, discount rates ρ and firm productivity z_F are exogenous processes to be detailed below. Workers receive the opportunity to start a firm at a rate η, in which case they need to pay a fixed cost κ^i to set up the firm, as in

[9]See Becker (1971), Arrow (1972), and Lang, Manove, and Dickens (2005) for taste-based theories of discrimination in labour markets and Aigner and Cain (1977), Altonji and Pierret (2001), and Coate and Loury (1993) for theories based on statistical discrimination.

Buera, Kaboski, and Shin (2011). Omitting time notation where it is not necessary, let $V(a, z_L, l, \rho, i)$ denote the value of being a worker. Workers face the following problem:

$$\begin{aligned}
&\rho V(a, z_L, l, \rho, i) \qquad (3.4)\\
&= \max_c \Bigg\{ u(c) + V_a s_V(a, z_L, l, \rho, i) + \eta \max \Big\{ F(a - \kappa^i, \underline{z}_F, \rho, i) - V(a, z_L, l, \rho, i), 0 \Big\} \\
&\quad + \lambda^i_{ll'}(V(a, z_L, l', \rho, i) - V(a, z_L, l, \rho, i)) + \sum_{\rho'} (V(a, z_L, l, \rho', i) - V(a, z_L, l, \rho, i)) M^{\rho}_{\rho\rho'} \\
&\quad - V_{z_L}(z_L \log(z_L)\mu_L) + \lambda_L \int_{-\infty}^{\infty} (V(a, e^q, l, \rho, i) - V(a, z_L, l, \rho, i))\, \phi(q|\sigma^2_L) dq \Bigg\},
\end{aligned}$$

subject to the borrowing constraint $a \geqslant \underline{a}$, and where $u(c)$ is the flow utility from consumption that is assumed to display CRRA with relative risk aversion parameter γ. We denote $\phi(\cdot|\sigma^2_L)$ as the PDF of a normal distribution with a zero mean and variance of σ^2_L. We also use a shorthand notation for the partial derivative $V_a = \partial V(a, z_L, l, \rho, i)/\partial a$, and analogously for the other state variables. $F(a - \kappa^i, \underline{z}_F, \rho, i)$ denotes the value of being an entrepreneur after paying the fixed cost κ^i out of pocket. The law of motion for assets $\dot{a} = s_V(\cdot)$ is

$$s_V(a, z_L, l, \rho, i) = w z_L l(1 - \omega^i)(1 - \tau) + (r - \delta)a + \hat{r}^i a - c + T, \qquad (3.5)$$

where the wage per unity of productivity is w and the net return for asset holdings is given by $r - \delta$, with r being the rental rate of assets and δ the depreciation rate. The wage gap is captured by ω^B and the gains from discrimination by $\hat{r}^W$. For ease of notation, we also define $\omega^W = 0$, and $\hat{r}^B = 0$. All households face a proportional tax rate τ on labour income and receive a lump-sum transfer benefit of T.

The process for the log of the labour productivity $y_{L,j,t} = \log(z_{L,j,t})$ for a given household j follows a jump-drift process similar to that in Kaplan, Moll, and Violante (2018). Its law of motion is given by:

$$dy_{L,j,t} = -\mu_L y_{L,j,t} dt + dJ_{L,j,t}, \qquad (3.6)$$

within bounds $[\underline{y}_L, \overline{y}_L]$, where $dJ_{L,j,t}$ is an idiosyncratic jump process with an arrival rate of λ_L, in which case $y_{L,j,t}$ is redrawn from a normal distribution with mean equal to zero and variance equal to σ_L^2. Moreover, it is a mean-reverting process, similar to an AR(1) in discrete time with persistence $(1-\mu_L)$. However, instead of shocks arriving at every period, $y_{L,j,t}$ jumps with probability $\Delta t \lambda_L$ in an interval of time Δt.

Finally, l_t and ρ_t are idiosyncratic jump processes with a constant Poisson arrival rate. The rate at which households of race i switch from employment status l to l' is denoted by $\lambda^i_{ll'}$ thus we have λ^i_{10} and λ^i_{01} for $i \in \{B, W\}$. We denote as M^ρ the matrix whose entries $M^\rho_{\rho\rho'}$ denote the transition rate from discount rate ρ to ρ'.

The other group of households in the model are entrepreneurs. Their optimisation problem is as follows:

$$
\begin{aligned}
&(\rho+\lambda_D)F(a,z_F,\rho,i) = \\
&\qquad \max_c \Bigg\{ u(c) + F_a s_F(a,z_F,\rho,i) + \lambda_D E_{z,l}[V(a,z,l,\rho,i)] + F_{z_F}(\mu_F z_F) \\
&\qquad\qquad + \frac{(z_F\sigma_F)^2}{2} F_{z_F z_F} + \sum_{\rho'} (F(a,z_F,\rho',i) - F(a,z_F,\rho,i)) M^\rho_{\rho\rho'} \Bigg\},
\end{aligned} \tag{3.7}
$$

with the associated law of motion of assets $\dot{a} = s_F(\cdot)$ being

$$
s_F(a,z_F,\rho,i) = \pi(z_F) + (r-\delta)a + \hat{r}^i a - c + T, \tag{3.8}
$$

where entrepreneurs are subject to the same borrowing constraint $a \geqslant \underline{a}$, and the same process for ρ_t.

The firms owned by entrepreneurs die with rate λ_D, in which case the household becomes a worker again, and $E_{z,l}[V(a,z,l,\rho,i)]$ in the expected value of this transition. Let $n(z_L)$ denote the PDF of the stationary distribution of the process described in Equation (3.6) and notice that if there were no entrepreneurship choice, the steady-state share of employed workers would be equal to $\frac{\lambda^i_{01}}{\lambda^i_{10}+\lambda^i_{01}}$. We assume that entrepreneurs get reintroduced into the labour productivity and employment status according to the distribution given by $n(z_L)$ and the transition rates $\lambda^i_{ll'}$.[10] As such, we have that

[10]The stationary distribution of workers over (z_L, l) will not be given exclusively by $n(z_L)$ and the $\lambda^i_{ll'}$'s because entrepreneurship decisions will depend on both (z_L, l) and on race as well. We use this assumption because it makes the numerical solution easier and because the transition rates across labour statuses within workers dominate those between workers and entrepreneurs in our model. Thus, these two distributions will be approximately the same.

$E_{z,l}[V(a,z,l,\rho,i)] = \int_{\underline{z}_L}^{\overline{z}_L} \left(\frac{\lambda_{01}^i}{\lambda_{10}^i+\lambda_{01}^i} W(a,q,1,\rho,i) + \frac{\lambda_{10}^i}{\lambda_{10}^i+\lambda_{01}^i} W(a,q,0,\rho,i) \right) n(q)dq.$

Conditional on staying in business, the firm's productivity $z_{j,t}^F$ of a given firm j follows a random growth process with average growth rate μ_F and variance σ_F^2 given by:

$$dz_{j,t}^F = \mu_F z_{F,j,t} dt + \sigma_F dB_{t,j}, \tag{3.9}$$

in $z_F \in [\underline{z}_F, \infty)$, where $dB_{t,j}$ denotes a Brownian motion process.

3.3.3 Firms

Firms are each owned by a single entrepreneur and differ by their productivity level z_F. These firms produce a single homogeneous final consumption good output y by renting physical capital, and labour from households using a production function $y = z_F k^\alpha h^\beta$, where h denotes effective units of labour and k capital. Firms display decreasing returns to scale, i.e., $\alpha + \beta < 1$, which generates positive profits in equilibrium. This assumption is necessary for households to be willing to pay the fixed cost and become entrepreneurs.

As mentioned previously, we assume, for the sake of tractability, that firms are colourblind and pay the same wage w for each unit of effective labour. Thus, they are indifferent between hiring Black workers or White workers, and the profit maximisation problem of a firm with productivity z_F is:

$$\pi(z_F) = \max_{k,h} z_F k^\alpha h^\beta - wh - rk, \tag{3.10}$$

with first-order conditions

$$\alpha \frac{y}{k} = r, \text{ and } \beta \frac{y}{h} = w. \tag{3.11}$$

These first-order conditions imply that profits will be a share $(1 - \alpha - \beta)$ of the total output of each firm.

Given the process for the firm's productivity in Equation (3.9), we can determine the stationary distribution for firm productivity z_F and size analytically. Let $g(z_F)$ denote the PDF of z_F. Then the steady-state Kolmogorov Forward Equation (KFE) for z_F

implies that $g(z_F)$ must satisfy:

$$0 = -\frac{\partial}{\partial z_F}\left[g(z_F)\mu_F z_F\right] + \frac{1}{2}\frac{\partial^2}{\partial z_F z_F}\left[(\sigma_F z_F)^2 g(z_F)\right] - \lambda_D g(z_F), \tag{3.12}$$

for $z_F > \underline{z}^F$. Through guess-and-verify, one can show that guessing that z_F follows a Pareto distribution with shape parameter ζ, i.e. $g(z_F) \propto z_F^{-(\zeta+1)}$, solves the equation above with:

$$\zeta = \frac{1}{2} - \frac{\mu_F}{\sigma_F^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu_F}{\sigma_F^2}\right)^2 + \frac{2\lambda_D}{\sigma_F^2}}. \tag{3.13}$$

Moreover, output will not be linear in productivity. Solving for the labour and capital demands h, k, one finds $y = z_F^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}}$. Thus, the size distribution of firms, as given by either profits, output or employment, will actually be a Pareto distribution with a shape parameter $\tilde{\zeta} = \zeta(1-\alpha-\beta)$ as all three will be proportional to $z_F^{\frac{1}{1-\alpha-\beta}}$. Therefore, this distribution will also be a Pareto distribution that will exhibit more inequality than the productivity distribution, with the ratio of outputs between a more productive firm and another less productive firm being larger than their ratios in productivity. This relationship will be exploited later to calibrate the model.[11]

3.3.4 Equilibrium

There are three markets in the economy: capital, labour and goods. Equilibrium in the capital market requires that the total asset holdings in the economy equal the capital demanded by firms.[12] Remember that $g(z_F)$ is the stationary distribution of firms' productivity, and $f(a|i)$ is the distribution of wealth conditional on race. Let m_e^i be the mass of entrepreneurs in each race, and denote the capital demand of a firm with productivity z_F as $k(z_F) = z_F^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{r}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}}$, which can be derived from Equation (3.11). Equilibrium in capital markets requires:

$$m^B \int_{\underline{a}}^{\infty} af(a|B)da + m^W \int_{\underline{a}}^{\infty} af(a|W)da = (m_e^B + m_e^W) \int_{\underline{z}_F}^{\infty} k(z)g(z)dz. \tag{3.14}$$

[11] Many papers have dealt with this feature of the firm size distribution in decreasing returns to scale economies (e.g., Hopenhayn, 2014; Carvalho and Grassi, 2019). We do not cover the issue in depth as this is not the main focus of the paper and the setup is conventional.

[12] Total asset holdings in the economy will include negative and positive positions. Thus, negative positions are treated as loans from households with positive positions at the same rate of return that these households could get by renting out capital to the firms.

In the labour market, households do not choose how many hours to work, but they differ in their productivity z_L and can also choose to leave the labour market and become entrepreneurs. Recall that $n(z_L, l|i)$ is the joint PDF of (z_L, l) conditional on race i, and notice that the demand for labour of a firm with productivity z_F is equal to $h(z_F) = z_F^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}$. For labour markets to clear we need:

$$(1-m^B)(1-m_e^B)\int_{\underline{z}_L}^{\overline{z}_L} zn(z,1|B)dz + m^B(1-m_e^W)\int_{\underline{z}_L}^{\overline{z}_L} zn(z,1|W)dz = m_e\int_{\underline{z}_F}^{\infty} h(z)g(z)dz, \tag{3.15}$$

where m_e denotes the overall mass of entrepreneurs.

For the goods market to clear, the total output produced[13] must equal the sum of aggregate consumption, net investment in assets and total costs paid for starting firms.

Finally, we assume that the government must balance its flow budget at every period, thus

$$T = w\tau\left(m_w^B(1-\omega^B)\int_{\underline{z}_L}^{\overline{z}_L} zn(z,1|B)dz + m_w^W\int_{\underline{z}_L}^{\overline{z}_L} zn(z,1|W))dz\right), \tag{3.16}$$

where m_w^W is the mass of White workers. For brevity, we relegate the formal definition of recursive stationary equilibrium in the model to Appendix 3.C.

3.3.5 Solution Algorithm

The entire problem can be boiled down to a system of four equations, namely Equations (3.3), (3.14), (3.15), and (3.16), in the four unknowns r, w, T, and $\hat{r}^W$. However, given that the mass of entrepreneurs m_e is an equilibrium object that can be zero for some prices, it is more convenient to solve this system given a guess for this mass and to add the additional consistency equation for the mass of entrepreneurs. The detailed solution algorithm is given in Appendix 3.D.

[13]Total output is given by $m_e\int_{\underline{z}_F}^{\infty} y(z)g(z)dz$.

3.3.6 Calibration

This section details the calibration procedure of the model and reports the model fit and performance. The calibrated model is consistent with the patterns in the data. We obtain a good fit for the overall population's wealth distribution and entrepreneurship rate. The model also adequately captures racial disparities in representation along the wealth distribution and the entrepreneurship rate for Black and White households. However, it slightly understates the racial wealth gap and the entrepreneurship gap. In what follows, we discuss our calibration strategy for each model component and the resulting fit. We choose the model's frequency to be annual, and all hazard rates are given in annual terms. All internally calibrated parameters and targets are summarized in Table 3.3 and the externally calibrated parameters are summarized in Table 3.4.

Household All households have log flow utility from consumption ($\gamma = 1$). We specify three discrete discount rate types at the levels $[\bar{\rho} - \Delta_\rho, \bar{\rho}, \bar{\rho} + \Delta_\rho]$ and calibrate $\bar{\rho} = 0.072$ to target a net return on assets of four percent annually. The increment size Δ_{rho} is set to target the wealth distribution as measured by the shares of aggregate wealth held by different quantiles. The wealth shares and the model's overall fit with respect to the wealth distribution is reported in Table 3.1.[14] We follow Krusell and Smith (1998) in assuming that 80% of the population has a discount rate of $\bar{\rho}$ and the rest are equally distributed at the extremes of the discount rate distribution, and in calibrating the persistence of the stochastic discount factors. Discount factors are assumed to be uncorrelated with race. The borrowing limit is calibrated to $\underline{a} = -0.31$, which is half of the median annual wage in the model, to target the share of households with negative net wealth of 11%.

Wage process estimation We estimate the parameters $\mu_L, \lambda_L, \sigma_L^2, \lambda_{01}^B, \lambda_{10}^B, \lambda_{01}^W, \lambda_{10}^W$ and ω^B which govern the labour productivity and employment status processes using data from the PSID from 2001-2019, which is annual data observed every other year. We restrict our sample to households where the main respondent is between 25 and 65

[14]Technically speaking, we use the Kolmogorov Smirnov distance between the wealth shares in the data and the model counterpart as a distance metric. The resulting Kolmogorov Smirnov distance is 0.049.

Table 3.1: Wealth Shares: Data vs Model

	Share of wealth held by the			
	Bottom 50%	50th-90th	90th-99th	Top 1%
Data	1.8%	25.0%	38.4%	34.8%
Model	3.3%	20.0%	40.9%	35.9%

Notes: This table reports the shares of aggregate wealth held by each wealth group in the U.S. wealth distribution based on SCF averages for 2001-2019 versus their model counterparts.

years old, is a male and is present in at least two subsequent waves of the PSID to track wage changes. We define wages as the total labour income of the main respondent and their spouse. We give an overview of the steps involved in the estimation procedure below and explain it in detail, including the moments targeted, in Appendix 3.B.

First, we estimate the racial wage gap ω^B as the simple ratio of the median wage of a Black household versus a White household. We include in this first step only households that earn wages above a threshold so that we do not confound different wages conditional on employment across races with different labour market participation rates and arrive at $\omega^B = 36.6\%$. Because we do not model dimensions such as educational choices or school quality, our measure of the wage gap must reflect that as well, and not only pure discrimination conditional on a worker's characteristics.

Second, we estimate the other parameters using a Simulated Method of Moments (SMM) procedure in which we simulate the processes for labour income $y_{L,j,t}$ and employment status $l_{j,t}$ for many households j over a long horizon, without discretizing the support.[15] We then estimate the parameters by jointly targeting moments from the PSID data.

Table 3.2 shows the results of this estimation. Given that there is only one labour income shock, it is not surprising that the mean reversion parameter μ_L is neither close to zero (which would be the case for a permanent shock) nor close to one (transitory). Overall, the parameters estimated seem to be in between those found for persistent and transitory shocks (see Guvenen et al., 2021; Kaplan, Moll, and Violante, 2018). Also, consistently with the data, we find the implied non-participation rate is higher for Black households than it is for White households: $\frac{\lambda_{10}^B}{\lambda_{10}^B+\lambda_{01}^B} = 9.6\% > 7.3\% = \frac{\lambda_{10}^W}{\lambda_{10}^W+\lambda_{01}^W}$. The

[15] Steps 2 and 3 follow Kaplan, Moll, and Violante (2018).

estimation attributes this to a higher separation rate for Black households but estimates similar job finding rates across races.

Finally, using the parameters already estimated, we choose eleven grid points for the labour income grid process and optimize the choice of the grid points (curvature and width) by targeting the same moments used to estimate the parameters.

Table 3.2: Estimated parameters for the wage and employment status processes

Labour income		
Racial wage gap	ω^B	36.6%
Mean reversion	μ_L	39.7%
Volatility of jumps	σ_L^2	1.1%
Rate of arrival of jumps	λ_L	18.1%
Employment status transition rates		
Employment $\rightarrow$ Unemployment, B	λ_{10}^B	9.1%
Employment $\rightarrow$ Unemployment, W	λ_{10}^W	6.7%
Unemployment $\rightarrow$ Employment, B	λ_{01}^B	86.2%
Unemployment $\rightarrow$ Employment, W	λ_{01}^W	85.9%

Notes: This table reports the estimated parameters of the labour income productivity $z_{L,t}$ and labour status l_t processes. All transition rates are at an annual frequency.

The entrepreneurship choice To complete the household's side of the model, we need to specify the costs of becoming an entrepreneur and the potential benefits of entry. We calibrate the opportunity arrival parameter η to 0.017 to target the population rate of entrepreneurs. The average entrepreneurship rate in the SCF between 2001-2019 is about 12.7%. We calibrate the death rate of firms to be $\lambda_D = 10\%$ annually, which is consistent with the literature.[16] The cost of entry κ^i excludes asset-poor households from participating in business formation, and the differential costs κ^B, κ^W allow us to target entrepreneurship rates by race. We set $\kappa^B = 0.82$ and $\kappa^W = 0.15$, which are equivalent to 1.32 and 0.24 of the annual median wage in the model.

[16]Under such parameter values, the maximum rate of entrepreneurs out of the general population is 14.5%. Note, however, that this is the absolute upper bar in our set-up, and the realised number depends on the wealth distribution.

The stochastic productivity process New businesses start with a normalised productivity of $z_F = 1$, and the productivity process follows a geometric Brownian motion as specified in Equation (3.12). We discretise this process by allowing ten discrete productivity levels along an exponentially spaced grid and set the upper bound as the level with at least 1% of firms at the steady-state productivity distribution. The volatility σ_F is set to target a profit volatility of 15%.[17] We also calibrate the decreasing returns to scale in the model to be $\alpha + \beta = 0.8$, where $\alpha = 0.288$ and $\beta = 0.512$, such that the profit share is 20% which is consistent with the literature. Last, we calibrate the shape parameter of profits to have the same shape parameter 1.5 as that of top income in the data, which, together with the degree of decreasing returns, yields a shape parameter of $\zeta = 7.5$ for the distribution of productivity. Using Equation (3.13), we can back out the value of μ_F and obtain our grid for productivity z_f.

Parameter	Value	Target	Model
$\bar{\rho}$	7.2%	net return of 4%	4.1%
Δ_ρ	1%	wealth distribution	see Table 3.1
η	1.7%	entrep. rate of 12.7%	12.0%
κ^B, κ^W	$[0.82, 0.15]$	entrep. rate by race of $[5.2\%, 14.2\%]$	$[5.7\%, 13.3\%]$
$\underline{a}$	-0.31	% of households with negative net wealth of 11%	12%

Table 3.3: Internally Calibrated Parameters

Notes: This table reports the internally calibrated parameter values in the baseline model and the targets to which they are set. All hazard rates are at an annual frequency.

Parameter	Value
γ	1.000
α	0.288
β	0.512
δ	0.048
ω^B	0.366
τ	0.250
ζ	7.500
λ_D	0.100

Table 3.4: Externally Calibrated Parameters

To conclude, the model performs well with respect to most targets. The model adequately captures wealth dispersion with a slight overshoot in the share of households with

[17]This number is consistent with recent estimates in the literature, e.g. see Gabaix (2011).

negative net wealth, which balances out the fact that our bottom 50% of the wealth distribution are richer than in the data. This error is, however, still small compared with the total wealth in the economy. The model is broadly consistent with the entrepreneurship patterns we observe in the data but not as starkly. Specifically, the entrepreneurship rate gap between Black and White households is nine percentage points. In contrast, the model delivers a gap of 7.6 percentage points which understates entrepreneurship among White households and overstates entrepreneurship for Black households. The model also slightly understates entrepreneurship overall, with the population rate lower by 0.7 percentage points than in the data. However, considering these differences, our model is consistent with the overall distribution of wealth in the US, has an empirically estimated income process that generates a realistic dispersion of income, and generates racial disparities that are quite stark.

3.4 Results

3.4.1 Steady State Decomposition

In this section, we use the calibrated model to analyse the contribution of each model gap to the racial wealth gap and the representation of Black households along the wealth distribution. Recall that the model allows for three gaps along which Black households are different from White ones: the wage gap, business start-up cost gap, and unemployment gap. These gaps affect the incentives to start a business and accumulate wealth. We conduct a comparative statics exercise where we shut down each gap separately and demonstrate its effect on the racial wealth gap and the representation of Black households along the wealth distribution. The results of this exercise are reported in Table 3.1, 3.2, and 3.3.

In the baseline model, the average Black household holds 64.7% less wealth than the average White household. Removing the wage gap between the groups would reduce most (72%) of the racial wealth gap. Table 3.1 demonstrates that the wage gap primarily affects the bottom of the wealth distribution, and closing it is essential for reducing the over-representation of Black households at the bottom of the wealth distribution.

Compared to the wage gap, the entry cost gap, which is a novel element of our model compared to the literature, operates on other margins. Eliminating the entry cost gap effectively eliminates the entrepreneurship gap between races, and its contribution to the representation of Black households in the top 1% of the wealth distribution is nearly as decisive as the wage gap. However, the entry cost gap is less potent than the wage gap in determining the overall racial wealth gap and accounts for 42% of it. Note that the effects are not additive since these results are obtained in general equilibrium, so prices are allowed to respond, and model forces may interact in a non-linear fashion.

Overall, the unemployment gap is a weak force that creates the opposite influence to that of the former two. Specifically, equaling the labour market risks of Black and White households would mean reducing the risks that Black households face. Thus, their precautionary savings motive is reduced, and they accumulate less wealth. This effect is quantitatively negligible when compared to the other margins.

Surprisingly, Table 3.1 illustrates that eliminating the wage gap in our model makes Black households slightly under-represented at the bottom 50% and over-represented between the 50th to 90th percentiles of the wealth distribution. This is because, in the absence of the wage gap, there are only two gaps between Blacks and Whites in the model: the unemployment gap and the entry cost gap. The unemployment gap gives Black households a stronger precautionary motive. Additionally, the entry cost gap provides a stronger incentive for middle-class Black households to save than middle-class White households. Thus, the overall economy is one where Black workers have equal means to accumulate wealth and stronger incentives to do so. However, this is not enough to close the racial wealth gap since the entry-cost gap still prevents the entry of Black entrepreneurs. To support this, observe that the gap in entrepreneurship is lower in the absence of a wage gap, but it is still substantial.

Eliminating the entry cost gap generates another surprising result - it increases the over-representation of Black households in the bottom 50% of the wealth distribution. Without the entry cost gap, Black households have a lower incentive to accumulate wealth as they can start a business at the same cost. Thus, the entry cost gap effectively incentivises wealth accumulation and its removal increase wealth dispersion within the

Black population. As a result of removing the entry cost gap, Black households are more over-represented at the bottom and less under-represented at the very top than before.

Table 3.1: The Racial Wealth Gap and Racial Representation Along the Wealth Distribution

	Racial Wealth Gap	Racial Entrep. Gap	Share of Black households in the			
			Bottom 50%	50th-90th	90th-99th	Top 1%
Data	83.4%	9.0%	25.7%	8.8%	2.3%	1.7%
Baseline	64.7%	7.6%	19.9%	15.1%	6.2%	4.9%
Counterfactual scenario - baseline without						
Wage gap	18.2%	4.7%	13.1%	21.4%	15.2%	11.4%
Entry cost gap	37.6%	0.2%	21.5%	11.8%	11.0%	10.4%
Unemp. gap	66.4%	7.7%	20.3%	14.7%	6.0%	4.8%
All gaps	0.0%	0.0%	16.6%	16.6%	16.6%	16.6%

Notes: This table reports each counterfactual scenario, the racial wealth gap and the entrepreneurship gap. The racial wealth gap is expressed in percentage terms of the average wealth of Whites. The entrepreneurship gap is expressed as the difference in entrepreneurship rate between the groups (Whites - Blacks). Additionally, for each scenario, we report the share of Blacks in different subgroups that compose the wealth distribution. The baseline model and the SCF data are also reported for comparison.

One of the advantages of our setting is that, due to its general equilibrium nature, we can analyse the counterfactual effects on prices and aggregate quantities. Table 3.2 shows that in a world without racial gaps, net return would increase by 20% while wages fall by approximately 1.6% and output falls by 2.2%. These movements are mainly due to three factors.

First, the decrease in start-up cost κ^B for Black households significantly increases their entrepreneurship rate, as is shown in Table 3.1. Even though White entrepreneurship falls slightly, the increase in Black entrepreneurship dominates and entrepreneurs as a share of the overall population increase from 12 to 13%. This increase in overall entrepreneurship and, consequently, in the number of firms increases the demand for capital, pushing net returns higher. It is interesting to note that, observing the decrease in entrepreneurship of White households, we can conclude that the pure change in prices makes workers better off relative to entrepreneurs. However, the opposite is true for

Table 3.2: Counterfactual General Equilibrium Implications

	$r-\delta$	m_e	K	Z_L	w	Y
Baseline	4.1%	12%	100%	100%	100%	100%
Counterfactual scenario - baseline without						
	p.p. deviations		% deviations			
Wage gap	0.8	0.3	-11.5%	-0.3%	-2.9%	-3.2%
Entry cost gap	0.1	1.0	0.2%	-1.1%	2.2%	1.1%
Unemp. gap	0.0	0.0	0.5%	0.4%	-0.1%	0.3%
All gaps	0.9	0.9	-11.1%	-0.6%	-1.6%	-2.2%

Notes: This table compares each counterfactual scenario to the baseline in terms of aggregate outcomes: net return $r-\delta$, the per productivity unit wage w, aggregate capital stock K, aggregate labour productivity Z_L, the mass of firms m_e, and aggregate output Y.

Table 3.3: Counterfactual General Equilibrium Implications

	Share of wealth held by the Bottom 50th	50th-90th	90th-99th	Top 1%
Baseline	3.3%	20.0%	40.9%	35.9%
Counterfactual scenario - baseline without				
Wage gap	3.9%	20.9%	38.9%	36.2%
Entry cost gap	3.9%	20.2%	41.2%	34.7%
Unemp. gap	3.2%	19.9%	41.0%	35.9%
All gaps	4.0%	20.3%	39.8%	35.9%

Notes: This table reports the wealth distribution for the same percentiles as in Table 3.1 in each of the counterfactual scenarios.

Black households because the exogenous racial gaps were so disadvantageous to Black entrepreneurs that their elimination dominates the effect of price changes.

Second, once the exogenous racial gaps close, there are no more gains from discrimination and $\hat{r}^W = 0$. This change reduces the incentive for White households to save, reducing the capital supply and the aggregate wealth to output ratio, which further contributes to the increase in interest rates. Quantitatively speaking, Table 3.2 shows that removing the wage gap alone accounts for most of the effect on capital stock and net returns.

Third, there are opposing movements in the labour supply. On the one hand, as the transition rates of Black households $\lambda^B_{ll'}$ converge to their White counterparts, their employment rate increases, and more households are working in the end, which increases the labour supply.[18] On the other hand, the size of the overall workforce, i.e., the model population excluding the entrepreneurs, decreased, which is the dominant effect.

Thus, in the capital market, there is an increase in demand and a decrease in supply, which translates to higher interest rates and lower aggregate wealth. At the same time, there is a decrease in the effective labour supply. The final result when there are no racial gaps is a fall in output due to a fall in both aggregate capital and labour.[19]

To conclude this section of the analysis, observe that these counterfactual shifts within the wealth distribution are obtained while very few changes in the overall wealth distribution. This fact is documented in Table 3.3, and it hinges on the fact that the first-order determinant of overall wealth dispersion in the model is the stochastic process governing the discount rates[20] which remains unchanged in all scenarios.

One aspect of this analysis is that it is a static exercise by nature and thus unable to account for the slow accumulation dynamics that wealth inequality entails. These dynamics will be explored in depth in the next section.

3.4.2 Transition

We now analyse the transition over time of the model from the initial steady state calibrated to the US in 2001-2019 to one in which there is no racial wealth gap due to all the exogenous gaps ω^B, κ^B and $\lambda^B_{ll'}$ closing. All the results are shown in Figure 3.1, and in this section, we analyse the results without any reparations under two scenarios: (i) the exogenous gaps close immediately at $t = 0$; and (ii) the exogenous gaps decrease linearly over the next 100 years.

[18]We do not model the labour market explicitly; thus, we assume that each $\lambda^B_{ll'}$ converges to $\lambda^W_{ll'}$. In a search and matching model, the results could be different, with the transition rates for the Black and White households meeting somewhere in the middle, meaning that the overall employment rate would not increase by much or even stay constant.

[19]In our model, the initial 36.6% gap in wages is 100% due to discrimination in the labour market. However, in practice, part of this gap can be explained by differences in education, skills and school quality, with the unexplained wage gap being closer to 10% (Lang and Lehmann, 2012). Thus, our model overestimates the initial gains from discrimination for White households and assumes no gains from productivity from eliminating racial discrimination. We then believe that, if anything, our model overestimates the fall in output.

[20]See Toda (2019) for a formal treatment of the determinants of wealth inequality in the presence of stochastic discount rates.

Looking first at the case in which gaps close immediately, graph (1) in Figure 3.1 shows that it takes 200 years for the racial wealth gap to close, which illustrates the importance of initial conditions for the future path of racial wealth inequality. From $t = 0$ onwards, there are no exogenous gaps imputed to the model still, it takes two centuries and many generations for Black households to catch up to White ones, although how long it takes depends on the part of the wealth distribution on which one focuses. The convergence is slow due to the time needed for Black households to start firms and for those firms to grow, but also, once they have caught up in the labour income and profit distributions, it still takes time to accumulate those gains over time.

The second row of graphs provides more details by displaying the share of Black households at different parts of the wealth distribution. With no racial inequality, they should all be equal to the overall share of Black households in the US, which is equal to 16.6%. Graph (5) shows that between the 50^{th} and 90^{th} percentile of wealth that convergence occurs quite quickly, in 50 years. However, for the bottom 50%, it takes approximately 100 years while, for the top 10% of wealth, Black households are underrepresented up to 200 years after all the exogenous racial gaps close.[21] Because in the model (and in the data), the top 10% of the wealth distribution hold more than 70% of aggregate wealth, the fact that Black households are slow to enter the wealthiest group explains the slow convergence in average wealth. But it is important to highlight that the over-representation of Black households amongst the poorest of the population ends quite faster after 100 years.

Given that entrepreneurs are the wealthiest group of households, one might think that it is because it takes time for Black households to catch up to White ones in their business ownership that explains the slow convergence at the top. As graph (7) in Figure 3.1 shows, that is not the whole story: the entrepreneurship rate of Black households converges to 12.9%, which is the same for White households, after just 50 years. However, the firms that Black and White households own at that point in time are quite different. We plot on graph (8) the percentage deviation of average productivity of Black firms

[21] Of course, if there is not exactly 16.6% of Black households in one wealth group, then it must be that in at least another group, that is also true. However, as the mass of households in the top 10% is smaller than in the other groups analysed, big deviations from 16.6% in this group can be compensated by relatively small ones in the others.

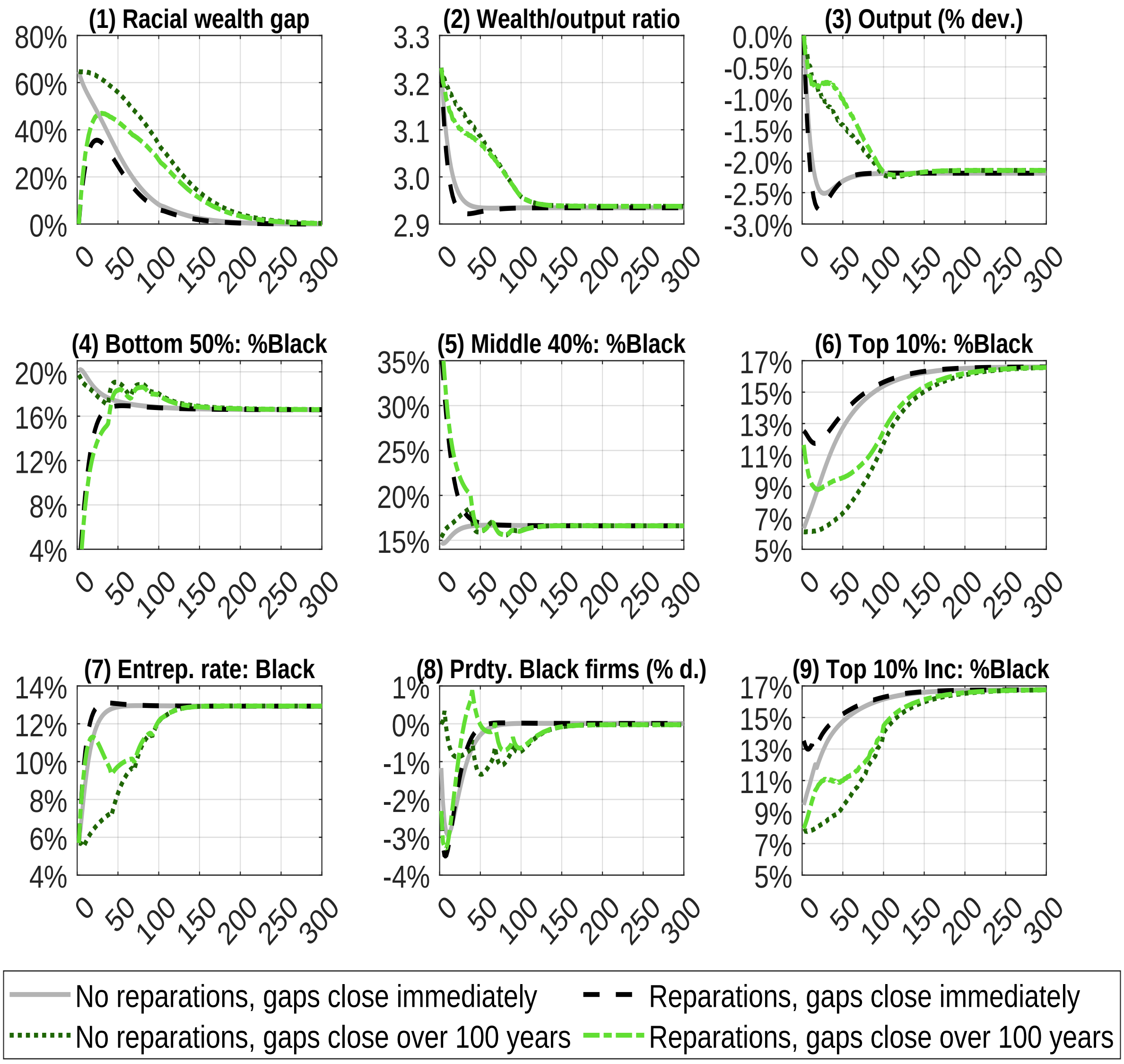

Figure 3.1: Transition towards no wealth gap under different scenarios

Notes: This figure shows the transition from the model's steady state with high racial wealth inequality to a future with no racial wealth gap. The transitions occur under two different scenarios for the exogenous racial gaps imputed to the model (ω^B, κ^B and $\lambda^B_{ll'}$): (i) they close immediately; or (ii) they close linearly over the next 100 years; and under two assumptions on reparations: (i) no reparations; (ii) reparations fully close the racial gap in average wealth immediately. The graphs show: (1) the gap in average wealth between Black and White households; (2) the wealth to output ratio; (3) the percentage deviation of output relative to the steady state; (4) the percentage of households in the bottom 50% of the wealth distribution that are Black; (5) the percentage of households between the 50$^{\text{th}}$ and 90$^{\text{th}}$ percentile of wealth that are Black; (6) the percentage of households in the top 10% of wealth that are Black; (7) the percentage of Black households that are entrepreneurs; (8) the percentage deviation of the average productivity of firms owned by Black entrepreneurs with respect to the initial steady state; (9) the percentage of households in the top 10% of the income distribution that are Black.

with respect to that in the steady state (notice that the stationary distribution of productivity is the same for Black and White entrepreneurs, both before and after the exogenous gaps close - they can be different during the transition only), and one can see that Black firms are less productive in the transition for 50 to 100 years, as there were many of them are recently founded and need time to grow to catch up with the other Black and White firms that existed already.

Furthermore, for firms of equal size, Black entrepreneurs have had less time to accumulate profits. The same reasoning applies to Black workers with high labour income: Black households catch up to White households faster in the income distribution, but it takes them longer to accumulate wealth, reach the top of the wealth distribution, and close the racial wealth gap. Graph (9) illustrates that after 150 years, Black households have caught up with White households at the top of the income distribution. However, it still takes another 50 years to materialise as equality in the wealth distribution.

The intuition is similar for the case in which the exogenous racial gaps close over the next 100 years, but now it takes between 250 and 300 years to close the racial wealth gap. In particular, due to the start-up cost gap ($\kappa^B > \kappa^W$) closing completely only in 100 years, now the entrepreneurship gap closes in 130 years, compared to 50 in the scenario in which all the exogenous gaps close immediately. Therefore, it is not surprising that it takes 250 years for equal representation among the top 10% of wealth and suggests that incentivising Black entrepreneurship during the transition is particularly important for the speed of closing the racial wealth gap.[22]

3.4.3 The Effect of Reparations

We now turn to the case where there are reparations, with the results also reported in Figure 3.1. Reparations take the form of a tax proportional to wealth imposed on White households, which is then redistributed lump-sum to all the Black households, independent of their wealth or other characteristics. The tax is chosen such that after the redistribution, the average wealth of Black and White households are the same. As a result, the average White household gets 10.7% of their wealth taxed, and the average wealth of Black households increases on impact by 152%. Using the median wage as a

[22] Boerma and Karabarbounis (2022) make a similar point.

numeraire, this transfer of wealth would amount to a gain of approximately $210,000 in 2019 terms for each Black household.[23] There are also two scenarios with reparations, with exogenous gaps closing immediately or linearly over 100 years.

Graph (1) in Figure 3.1 shows that in both cases with reparations, the average wealth gap falls to 0% on impact by construction, but many of the gains are reversed in the first 25 years as Black households consume a good portion of their wealth. In the end, the racial wealth gap only closes permanently at approximately the same time as it would have without reparations: 200 years if the exogenous racial gaps close immediately and 250 years if they close over the next 100 years. However, it is important to notice that the racial gap in average wealth never returns to the ones observed in the initial steady-state,[24] and that in the case of the exogenous gaps closing in 100 years, the economy with reparations presents lower racial inequality than the case with no reparations for at least 100 to 150 years, when the overall racial inequality is already much lower than it was before reparations.

While the increase in the wealth gap some years after reparations might seem puzzling, notice that the wealth of Black households has increased, but their income is still momentarily smaller on average than their White counterparts (shown in graph (9)) as their employment rate is increasing and as they own business that still provide lower profits than those owned by White households (shown in the lower productivity of Black-owned firms of graph (8)). Moreover, Black households have a sudden increase in wealth and know that in the distant future, when racial inequality has disappeared, they will have higher income and wealth again. Thus, during the initial part of the transition in which income has not increased yet, they consume beyond what would be allowed to maintain their wealth level but recover as income catches up.

Looking beyond what happens at the mean, in the second row of graphs in Figure 3.1 we can see the effect of reparations along the wealth distribution. Specifically, after the transfer of wealth, the rate of Black households among the 50% poorest

[23] Given the approximately 20.1 million Black households in 2019 according to our classification, this would amount to a total transfer of wealth of $4.2 trillion. Due to our model generating racial wealth gaps that are smaller than in the data, the true number would be even larger, which would make it more in line with other estimates: Boerma and Karabarbounis (2022) report a number of $10 trillion, Darity Jr and Mullen (2020) of $8 trillion.

[24] That would be the case if there was no closure of the exogenous racial gaps.

households plummets, and they are all transferred to the middle 40%, even if temporarily. Nonetheless, given the level of wealth inequality present in the US, reparations are not enough to close the gap at the top 10%: Black households are still underrepresented right after it. Therefore, while reparations would eliminate Black poverty temporarily on impact, it would nonetheless take 250 years for equal representation among the wealthiest members of society.

3.5 Conclusion

We study the racial wealth gap and its determinants in different parts of the wealth distribution. To do so, we develop an incomplete markets model with endogenous entrepreneurship choice subject to a fixed cost that generates a high level of wealth inequality, as observed in the data. Using exogenous gaps in wage, business start-up cost, and unemployment rate, we decompose the effect of each component on racial wealth inequality.

We find that the wage gap is the most quantitatively important component and is especially important for explaining why there are so many Black households at the bottom of the wealth distribution. However, the business start-up cost gap is also meaningful and is crucial for understanding why Black households are severely underrepresented at the very top of the wealth distribution.

Analysing the possible future path for wealth inequality under different scenarios, we find that current conditions are persistent, and any permanent equality in average wealth between races will take at least 200 years. That is because the top 10% of wealth hold most of the wealth in the economy, and it takes time for Black households to accumulate such wealth. At the bottom 90% of the distribution, convergence is faster, and it takes between 100 to 150 years.

We also analyse the case of reparations and, in line with the findings above, reparations are not helpful in permanently closing the wealth gap faster in our setting. This is the case because reparations in the form of equal lump-sum transfers to Black households that close the wealth gap immediately are not enough to push many households into the top 10%. However, they help in reducing racial wealth disparities for at least 100 years, by which point they are already much smaller.

Our analysis provides important insights for the debate on the racial wealth gap by re-framing it in terms of representation along the wealth distribution. If one is mainly interested in making sure that Black households are not over-represented at the bottom of the distribution, a programme targeted at these households could be successful in helping the gap close faster, as long as other gaps are closing as well. In contrast, if one is mainly concerned with closing the wealth gap at the top, policies targeted at subsidising Black entrepreneurship could be especially promising.

Appendices

3.A Additional Figures

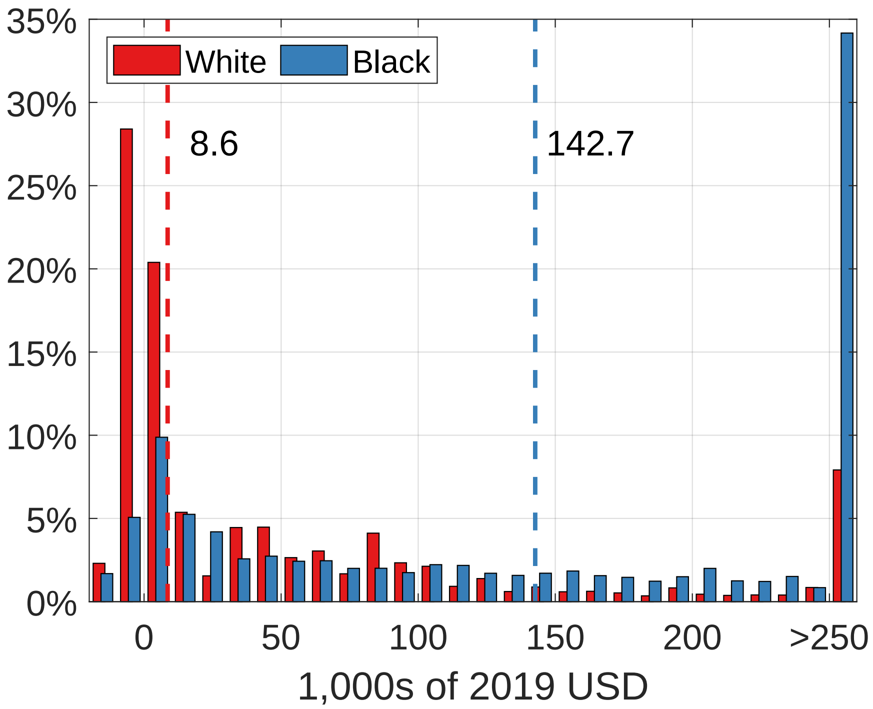

Figure 3.A.1: Histogram of the distribution of wealth for White and Black Households in 1989

Notes: The red and blue dashed lines indicate the median wealth for Black and White households, respectively. *Source*: SCF.

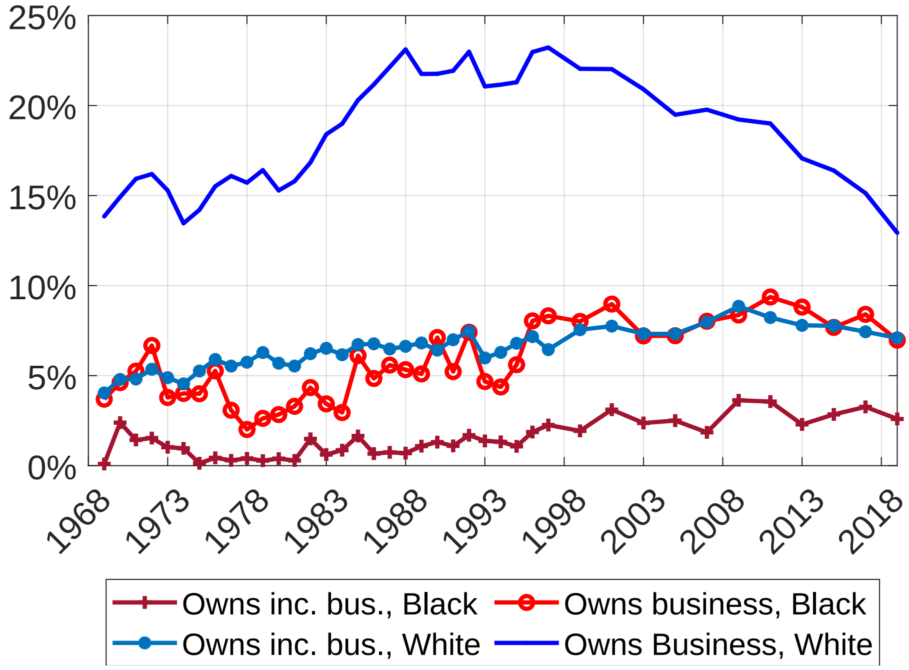

Figure 3.A.2: Entrepreneurship rate for Black and White households

Notes: This figure shows over time the share of Black and White households that are entrepreneurs according to two definitions: (i) owns an incorporated business; (ii) owns a business. *Source*: PSID.

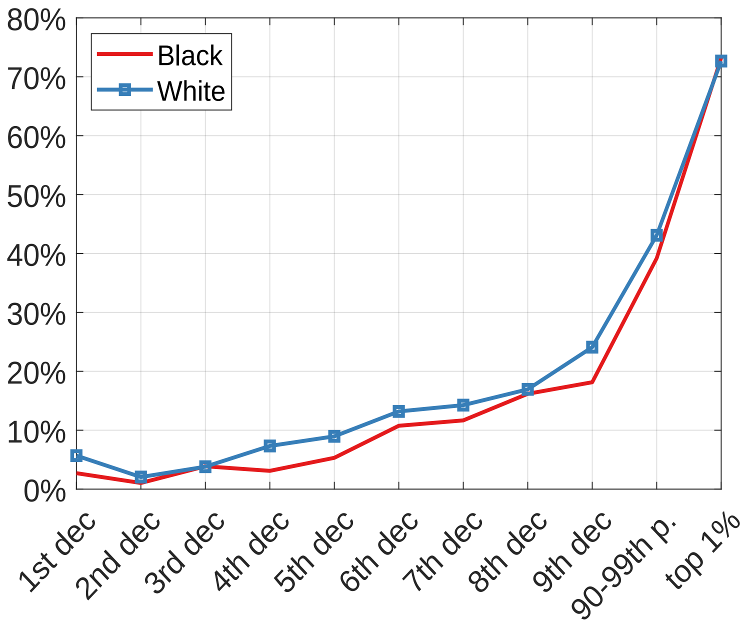

Figure 3.A.3: Entrepreneurship rates by wealth groups: owns a business, SCF

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the wealth distribution: 1^{st} decile of wealth, . . . , 9^{th} decile of wealth, those between the 90^{th} and 99^{th} percentile, and those in the top 1% of wealth. A household is classified as an entrepreneur if it owns a private business. *Source*: SCF, 2001-2019.

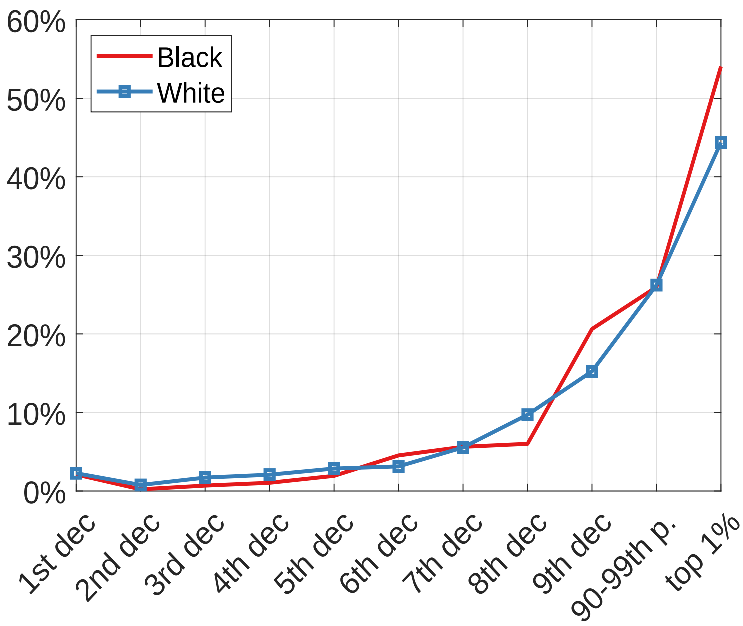

Figure 3.A.4: Entrepreneurship rates by wealth groups: owns an incorporated business, PSID

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the wealth distribution: 1^{st} decile of wealth, ..., 9^{th} decile of wealth, those between the 90^{th} and 99^{th} percentile, and those in the top 1% of wealth. A household is classified as an entrepreneur if it owns an incorporated business. *Source*: PSID, 2001-2019.

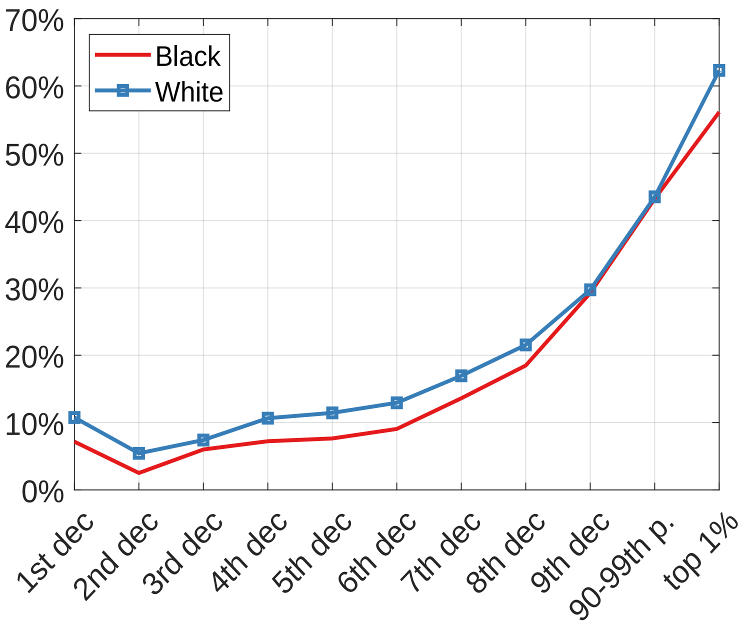

Figure 3.A.5: Entrepreneurship rates by wealth groups: owns a business, PSID

Notes: This figure shows the share of households of a given race that are classified as entrepreneurs in different parts of the wealth distribution: 1st decile of wealth, ..., 9th decile of wealth, those between the 90th and 99th percentile, and those in the top 1% of wealth. A household is classified as an entrepreneur if it owns a business. *Source*: PSID, 2001-2019.

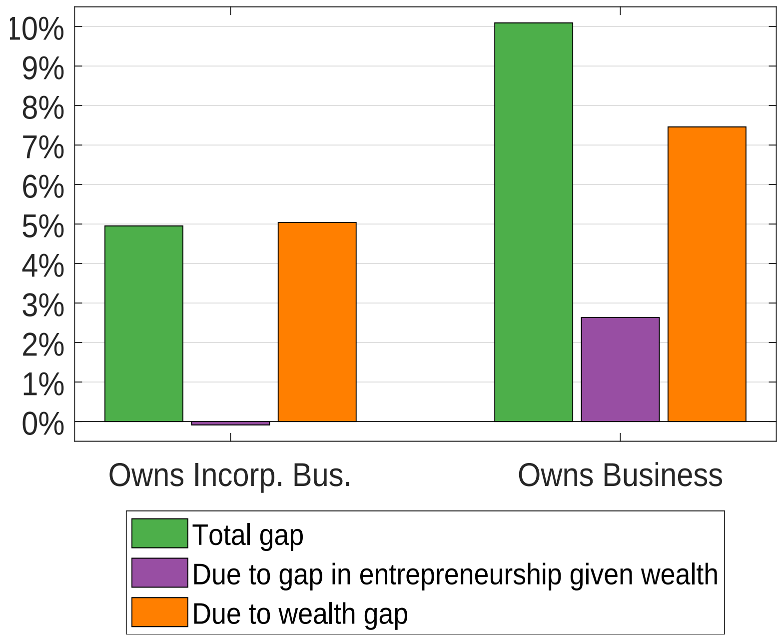

Figure 3.A.6: Decomposition of the gap in entrepreneurship rate between Black and White households: PSID

Notes: This figure decomposes the gap in entrepreneurship rates between Black and White households into: (i) differences in entrepreneurship conditional on wealth; (ii) differences in wealth. It shows the decomposition for entrepreneurship defined as "owns an incorporated business" and also as "owns a business". *Source*: PSID, 2001-2019.

3.B Wage Estimation

Here we explain in greater detail the estimation of the parameters involved in the labour productivity $y_{L,j,t}$ and employment status $l_{j,t}$ processes: $\{\mu_L, \lambda_L, \sigma_L^2, \lambda_{01}^B, \lambda_{10}^B, \lambda_{01}^W, \lambda_{10}^W, \omega^B\}$.

Our data source for the moments to be matched by the model is the PSID from 2001 to 2019. Because our unit of observation is a household, we define as "wage" the total labour income for both the main respondent to the survey and their spouse. We restrict the sample to those in working age between 25 and 65 years old and to male-led households only. Most of the moments we calculate will be based on changes in wages over time. Thus the panel structure of the PSID is very helpful. To calculate these wage changes, we construct a single database with all the households that appeared in at least two consecutive waves.

The first step in our procedure is to estimate the racial wage gap ω^B. Importantly, ω^B is the wage gap conditional on being employed. Thus we need some criteria to classify households as being employed in a given year. Accordingly, we only consider households that earned at least 1/12 of that year's annual median wage for households with strictly positive wage earnings. The reasoning is that the minimum wage is approximately 1/3 of the median wage, and we want to consider households employed during at least one full quarter in a year. We then compare the median wage of Black and White households above this cutoff and arrive at $\omega^B = 36.6\%$.

Second, we estimate all the other parameters jointly using a Simulated Method of Moments (SMM). The idea is to simulate the processes for $y_{L,j,t}$ and $l_{j,t}$ for a given combination of parameters, and calculate in the model the same moments that we estimated from the data. Then we optimise over the choice of parameters to minimise the sum of squared deviations between the moments simulated from the model and those from the data.

The moments that we use are in column (1) of Table 3.B.1. We target the share of Black and White households that earn exactly zero earnings over a given year and those that earn low wages, defined as smaller than 20% of the median wage but not equal to zero, to capture both those that are out of the labour force and stay there and those that might enter for short spells. We also target the standard deviation of 2, 4 and 6 years per cent changes of wages, and also the kurtosis, as in Guvenen et al. (2021); and also the fraction of households whose 2, 4 and 6 years per cent changes in wages were smaller than 5%, 10% or 20%, as in Kaplan, Moll, and Violante (2018). In total, we have 13 moments for the seven parameters that are left to be estimated, and we weigh all the moments equally.

	Moments		
	(1) Data	(2) Model	(3) Discretised Model
fraction wage = 0, Black	10.0%	9.6%	9.6%
fraction wage = 0, White	5.4%	6.7%	7.2%
wage low, Black	8.5%	3.6%	1.9%
wage low, White	3.9%	1.7%	1.9%
std Δ2y	0.49	0.68	0.52
std Δ4y	0.58	0.78	0.62
std Δ6y	0.63	0.78	0.65
kurtosis Δ2y	8.9	7.2	8.6
kurtosis Δ4y	7.3	6.4	6.0
kurtosis Δ6y	6.8	5.7	5.4
fraction $\Delta 2y < 5\%$	17.5%	21.6%	50.0%
fraction $\Delta 2y < 10\%$	35.5%	31.0%	57.2%
fraction $\Delta 2y < 20\%$	53.4%	42.1%	62.4%

Table 3.B.1: Labour income moments from data and model

Notes: This table shows the moments estimated from the data, simulated by the model without a grid constraint, and simulated by the model in the grid that we impute to the model, in columns (1), (2) and (3), respectively. The moments targeted are: the share of households with wage equal to zero over a year for each race; the share of households with wage below 20% of the median wage but above 0 for each race; the standard deviation and kurtosis of 2, 4 and 6 year wage changes; and the fraction of households that experience wage changes below 5, 10 and 20% over a 2-year period. *Source*: PSID, 2001-2019.

We start by simulating the model without the constraint of a specific choice of grid for the labour income process so that it does not affect the estimation of the parameters. We simulate 5000 households over a period of 1000 years to arrive at the stationary distribution and then calculate the necessary moments over the next six years. The simulated process for labour income is annual, but we calculate the implied 2, 4 and 6 year wage changes to match the data. The estimated parameters were reported in

Table 3.2, and the moments implied by the model are shown in column (2) of Table 3.B.1. It shows that the model does an overall good job in matching most moments, including different outcomes across races due to differences in the transition rates $\lambda^i_{ll'}$ and the characteristic high kurtosis highlighted by Guvenen et al. (2021), due to shocks not arriving at every period (Kaplan, Moll, and Violante, 2018). Where the model does not do so great is in generating households that have low wages but are not equal to zero, as the estimated job separation rates are low. One possible extension would be to introduce short-duration jobs in which individuals have a higher chance of getting fired in a few months, thus generating low wages over a whole year.

Finally, with the estimated parameters in hand, we estimate the best grid that, given the parameters, can generate the same moments. We choose 11 grid points for this state variable so as not to burden too much the numerical solution of the full model. In this step, we construct a grid for percentage deviations from the average wage, where there is a grid point exactly at zero and five other grid points above and below in a symmetric fashion. We then optimise over the width of the grid points furthest away from the average and the curvature of these points (that is, they are not uniformly distributed between zero and the points furthest away from it). The results for the moments constrained to this grid are shown in column (3) of Table 3.B.1. One can see that most of the moments are similar to those in columns (1) and (2), suggesting that the process imputed to the model does a good job at matching the data. The one dimension where it is not great is in matching the fraction of households in which their 2 and 4 year changes are below a certain threshold. This can be explained by the fact that, because there are not many grid points given that we have to constrain ourselves to only 11 in total, many households do not experience a shock of labour income in a given year. Otherwise, they would move far away from the average. If one were to include transitory shocks that arrive more frequently, it would be possible to match this moment better and generate more households that have some changes in wages every year.

3.C Recursive Stationary Equilibrium

Recursive stationary equilibrium in the model economy consists of value functions $V(a, z_L, l, \rho, i)$ and $F(a, z_F, \rho, i)$; saving rules $s_V(a, z_L, l, \rho, i), s_F(a, z_F, \rho, i)$ and the corresponding consumption policy function $c_V(a, z_L, l, \rho, i), c_F(a, z_F, \rho, i)$; entry choice policies $I_V(a, z_L, l, \rho, i)$[25]; stationary probability density functions $m(a, z_L, z_F, l, \rho, \mathbf{E}, i)$, where $\mathbf{E}$ is an indicator that takes the value of unity if the agent is an entrepreneur and zero otherwise; firm policy functions for capital $k(z_F)$ and labour $h(z_F)$; firm profit functions $\pi(z_F)$ rental rate of capital r; wage rate w; benefits T; and gains from discrimination $\hat{r}^W$, which jointly satisfy the following:

1. Consumer optimization - Given the rental rate r, the wage rate w, the transfer benefits T, the gains from discrimination $\hat{r}^W$, and the profit functions $\pi(z_F)$, the policy functions $c_V(a, z_L, l, \rho, i), c_F(a, z_F, \rho, i)$ and $I_V(a, z_L, l, \rho, i)$ solve the optimization problems given by problems (3.4) and (3.7) that are associated with the value functions $V(a, z_L, l, \rho, i)$ and $F(a, z_F, \rho, i)$.

2. Firm optimization - Given the rental rate r and the wage w, the policy functions for capital $k(z_F)$ and labour $h(z_F)$ are consistent with the firms maximizing instantaneous profits. These maximized profits are given by $\pi(z_F)$.

3. Capital market - the rental rate r satisfies the capital market clearing as given in Equation (3.14).

4. Labour market - the wage w satisfies the labour market clearing as given in Equation (3.15).

5. Discrimination - the gains from discrimination $\hat{r}^W$ satisfy Equation (3.3).

6. Government budget as given by Equation (3.16) is balanced.

7. Consistency - The distribution $m(a, z_L, z_F, l, \rho, \mathbf{E}, i)$ is the stationary distributions implied by the saving rules $s_V(a, z_L, l, \rho, i), s_F(a, z_F, \rho, i)$ and decision rule

[25] I_V is an indicator function that take the value of unity if the worker chooses to become an entrepreneur and zero otherwise for each state in the worker's state-space. In the main text this decision rule is replaced by the max operator for readability.

$I_V(a, z_L, l, \rho, i)$ and is consistent with the following KFE

$$\begin{aligned} &\frac{\partial m(a, z_L, z_F, l, \rho, \mathbf{E}, i)}{\partial t} = \\ &- \frac{\partial}{\partial a}[(1-\mathbf{E})\, m(\cdot, \mathbf{E}=0)\, s_V(\cdot) \, + \mathbf{E} m(\cdot, \mathbf{E}=1)\; s_F(\cdot)] \\ &+ (1-\mathbf{E})\Big[m(\cdot, \mathbf{E}=0)\, \Big(-\eta\, I_V(\cdot) + A_\rho + A_{z_L} + \lambda_{ll'}{}^i\Big) + \lambda_D\, m(\cdot, \mathbf{E}=1)\Big] \\ &+ \mathbf{E}[\, m(\cdot, \mathbf{E}=1)\; (-\lambda_D + A_\rho + A_{z_F}) + \; m(\cdot, \mathbf{E}=0)\; \eta\, \mathrm{I}_V(\cdot)\,]\,, \end{aligned} \tag{3.17}$$

where A_j denotes the infinitesimal generator of the process that governs the exogenous state variable j.

3.D Solution Algorithm

This appendix details the algorithm used to solve our model. The algorithm builds on the continuous-time methods of Achdou et al. (2021) and follows along the lines of the definition of the recursive stationary equilibrium in the model economy as given in Appendix 3.C.

3.D.1 Solution Algorithm for the Steady State

The solution algorithm solves a system of four equations, i.e, (3.3), (3.14), (3.15), and (3.16), in the four unknowns, r, w, T and $\hat{r}^W$. Since the model involves an entry decision, it is possible for the algorithm to run into regions where the capital demand and labour demand functions are flat since there is a zero mass of entrepreneurs that enter the market. As such, it is useful to guess also m_e, which is the total mass of entrepreneurs in equilibrium. This guess adds another auxiliary equation to the system, which is that m_e equals the mass of entrepreneurs implied by Equation (3.17). The algorithm follows from the definition of recursive stationary equilibrium.

1. **Initialization** Provide a grid for assets, parameter values for the model, and initial guesses for the values of $r, w, T, \hat{r}^W$, and m_e .

2. **Solve firm block** Using the values of r and w solve for the firms' demand for capital and labour $k(z_F)$ and labour $h(z_F)$ and for firm profits $\pi(z_F)$.

3. **Solve household block** Solve the household optimization problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations given in Achdou et al. (2021). Given the high dimensionality of the problem, we modify the algorithm as follows:

   (a) Provide the initial guess that the value function stays put (flow utility is constant) and solve the consumption savings problem as if all the exogenous state variables ρ, z_L, z_F, l are constant, and households can't become entrepreneurs.

   (b) Use the solution to the limited problem in step 3a as an initial guess to the consumption savings problem that allows for changes in ρ, z_L, z_F, l, but still prohibits the entrepreneurship choice.

   (c) Finally, use the solution to the limited problem in step 3b as the initial guess to the full HJBs given by Equations (3.4) and (3.7).

   This will allow us to obtain the distributions $m\left(a, z_L, z_F, l, \rho, \mathbf{E}, i\right)$, the policy functions $c_W\left(a, z_L, l, \rho, i\right), c_V\left(a, z_F, \rho, i\right)$ and $I_W\left(a, z_L, l, \rho, i\right)$, the equilibrium masses, the savings rules, and the mass of entrepreneurs m_e, the supply of effective labour by households, and the total aggregate asset supply.

4. **Compute capital and labour demand** Combine the masses from 3 with the capital and labour solutions from 2 to obtain the aggregate capital and labour demand by the firms.

5. **Compute gains from discrimination** Using the masses from 3, compute the total gains from discrimination by summing the wage income of Black households and compute the total asset holdings of White ones. These can be combined to compute the implied gains from discrimination by Equation (3.3).

6. **Compute government income** Using the tax rate and the total income in the economy, use Equation (3.16) to compute the government income.

7. **Clear markets** Using the results of steps 3, 4, 5, and 6 evaluate Equations (3.3), (3.14), (3.15), and (3.16), and check that the guess of m_e is sufficiently close to the one obtained in step 3. If the system is sufficiently close to zero, stop.

Else, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

Solver We use a quasi-Newton solver based on the Broyden method and evaluate the Jacobian of the system using finite differences. It is useful to relax the updated solution in the Newton direction, such that at the new guess, the value of $r - \delta$ lays between zero and the largest discount rate, and that $w, T, \hat{r}^W$ and m_e are strictly positive. We use backtracking to choose the largest relaxation parameter from a pre-specified set of values (all less than one), such that the new guess is well within these bounds. If the bounds are already violated, which can occur, we use a pre-set relaxation parameter, which, in many cases, leads the algorithm to return to its normal bounds. If the solver was unsuccessful, a new guess is randomized, and the procedure begins anew.

Stopping criterion and normalizations A convergence criterion of 10^{-3} yields fast results and performs well. All equations described in stage 7 are solved after normalization to obtain a meaningful stopping criterion. The labour and capital market clearing conditions are normalized such that they are expressed in percentage deviations of the aggregate supply. The government budget is normalized such that it is expressed as percentage deviations from the government's total tax revenue. The gains from discrimination and the mass of firms are already in percentage terms and do not require any normalization.

Grid for assets We use $n = 100$ grid points for assets. The grid is not uniform such that most grid points are concentrated near the borrowing constraint $\underline{a}$. The maximum value for assets is set at $a = 2,000$, which corresponds to asset holdings equivalent to around 3,300 unconsumed annual median wages. The asset vector $\bar{a}$ is set such that it has monotonically increasing increments as follows

$$\bar{a} = (a_{\max} - \underline{a}) \frac{(0, 1, \ldots, n-1)^5}{(n-1)^5} + \underline{a}. \tag{3.18}$$

This generates monotonically increasing increments with a grid point exactly on the borrowing constraint, which will have a positive mass of households on it. This point is treated throughout as a Dirac mass.

Modifications required outside of steady state To solve for the transition dynamics as in Section 3.4.2 and Section 3.4.3 one needs to solve for Equations (3.3), (3.14), (3.15) in every point in time such that for t periods one requires to solve $4t$ equations given guesses for the paths of r, w, T, and $\hat{r}^W$. Since the initial point and terminal points are known, there is no need to guess the mass m_e as it is positive in both. As shown in Achdou et al. (2021), the procedure would involve solving the HJB in every period backwards from the terminal condition and using the transition matrices from every period to move the distribution m forwards from the initial condition and clear the four markets in every period. Since we solve for long horizons, we use a non-uniform grid on time as follows. We solve in 50 increments of 2 years each for the first hundred years, and then we use 30 additional non-uniformly spaced increments similar to the grid for assets but with a lower power of 1.5 instead of 5.

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