A Macro-Finance Approach to the Term Structure of Interest Rates

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Declaration

I certify that the thesis I have presented for examination for the Ph.D. degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I declare my thesis consists of 32,954 words.

Statement of conjoint work

I confirm that Chapter 3 was jointly co-authored with Dr. Martin M. Andreasen and Dr. Pawel Zabczyk and I contributed 60% of this work.

Abstract

This thesis contributes to the literature that analyses the term structure of interest rates from a macroeconomic perspective. Chapter 1 studies the transmission of monetary policy shocks to the US macroeconomy and term structure. Based on estimates of a Macro-Affine model, it shows that monetary policy shocks trigger relevant movements in bond premia, which in turn feed back into the macroeconomy. This channel of monetary transmission shows up importantly in the pre-Volcker period, but becomes irrelevant later. This chapter concludes with an analysis of the macroeconomic implications of shocks to expectations about *future* monetary policy actions.

Chapter 2 proposes a regime-switching approach to explain why the U.S. nominal yield curve on average has been steeper since the mid-1980s than during the Great Inflation of the 1970s. It shows that, once the possibility of regime switches in the short-rate process is incorporated into investors' beliefs, the average slope of the yield curve generally will contain a new component called 'level risk'. Level risk estimates were found to be large and negative during the Great Inflation, but became moderate and positive afterwards. These findings are replicated in a Markov-Switching DSGE model, where the monetary policy rule shifts between an active and a passive regime with respect to inflation fluctuations.

Chapter 3 develops a DSGE model in which banks use short-term deposits to provide firms with long-term credit. The demand for long-term credit arises because firms borrow in order to finance their capital stock which they only adjust at infrequent intervals. The model shows that maturity transformation in the banking sector in general attenuates the output response to a technological shock. Implications of long-term nominal contracts are also examined in a New Keynesian version of the model. In this case, maturity transformation reduces the real effects of a monetary policy shock.

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To

Clarissa Rahmeier

and

Frederico Rahmeier Ferman

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Preface

Long-term bonds carry a wealth of information that can provide important insights for economists. The prices of these assets reflect average future expected short-rates, information that is crucial for the investment decisions of firms, for the savings decisions of consumers, and for the formulation of monetary policy. They also reflect premia that summarize the investors' assessment of the risks they face during the period for which they hold the long-term bond. As a result of this, a literature has developed that analyzes the determinants of bond prices.

The models employed in this literature are designed to explain the cross-section of yields (bond yields of many different maturities) in parallel with their dynamic evolution. No-arbitrage arguments are used to link bond yields of different maturities and thus reduce the dimensionality of the problem. In the first models, developed in the late 1970's, the dynamic and cross-sectional properties of the term structure depended on a set of estimated unobserved factors, which were extracted from the term structure data itself. These factors were usually identified as level, slope and curvature factors – closely related to the first three principal components of the term structure data – and, in general, explain almost all variation in bond yields of different maturities. However, as these models depend solely on yields, they appear to lack macroeconomic structure. In fact, there is plenty of evidence that bond prices react significantly to macroeconomic news[1].

More recently, many economists have succeeded in incorporating macroeconomic fundamentals into empirically relevant models of bond price determination. The gains from this are twofold. On the one hand, in some situations macroeconomic fundamentals may clarify observed movements in bond yields. On the other, macroeconomic models that do not generate reasonable implications for asset prices should be reconsidered, after all:

[1] See, for example, Balduzzi, Elton, and Green (2001), Green (2004), and Gürkaynak, Levin, and Swanson (2006).

"The centerpieces of dynamic macroeconomics are the equation of savings to investment, the equation of marginal rates of substitution to marginal rates of transformation, and the allocation of consumption and investment across time and states of nature. Asset markets are the mechanism that does all this equating" [Cochrane (2005)].

This thesis consists of three chapters, each of which analyze some of the issues that arise when the links between long-term bond prices and macroeconomic fundamentals are explored. The chapters are ordered according to the degree of complexity of the economic system that determines bond prices.

In Chapter 1, no-arbitrage bond prices are linked to a simple Vector Autoregression (VAR) on real economic activity, inflation, and commodity prices. This reduced-form system of macro-finance relationships is then used to study the implications of unexpected monetary policy changes. The term-structure model I employed allows for flexible bond premia that react to the monetary shock and feed back into the macroeconomic VAR. I found that, following a monetary policy shock, these feedback effects from bond premia to the macroeconomy were empirically relevant during the pre-Volcker period. In the post-Volcker sample, these effects were virtually inexistent.

Adding complexity to the linkages between bond prices and the macroeconomy, Chapter 2 analyzes the term structure in a fully specified general equilibrium system. The model is a version of the standard New-Keynesian framework of Woodford (2003) that allows the monetary policy regime to switch over time. More specifically, an exogenous Markov chain dictates whether the central bank adopts an active or a passive stance with respect to inflation deviations from the target. In the passive regime, which is characterized by stronger macroeconomic volatility than when policy is active, investors demand larger premia in order to hold long-term bonds. Additionally, because agents acquire different levels of precautionary savings in each regime, the average level around which the short-term interest rate fluctuates differs across regimes. The implications that this short-rate wedge across regimes has for long-term bond prices are carefully explored in this chapter. The empirical motivation for this chapter is the fact that the U.S. nominal yield curve has on average been steeper since the mid-1980s than during the Great Inflation of the 1970s, a feature that is very well explained in the Markov-Switching model.

Finally, Chapter 3 is the result of work done jointly with Martin M. Andreasen and Pawel Zabczyk at the Bank of England. Our objective was to produce a framework in which the microstructure of the market for long-term assets was explicitly modelled. This was done by assuming that banks use short-term deposits to provide firms with long-term credit. The demand for long-term credit arises because firms borrow in order to finance their capital stock, which they adjust only at infrequent intervals. We show that, in this general equilibrium framework, the presence of maturity transformation in the banking sector has important business cycle implications. In particular, the presence of maturity transformation in the banking sector in general attenuates the output response to a technological shock. Implications of long-term nominal contracts are also examined in a New Keynesian version of the model, where we find that maturity transformation reduces the real effects of a monetary policy shock.

1 The Monetary Transmission Mechanism in a Term-Structure Model with Unspanned Macro Risks

1.1 Introduction

To a large extent, bond prices reflect the expectations that private agents form regarding the future path of the short-term interest rate. Not surprisingly, many recent papers in the macro-finance literature have tried to incorporate the information contained in these prices into the study of the monetary policy transmission mechanism[2]. There is also overwhelming evidence that, beyond short-rate expectations, bond prices reflect time-varying and potentially sizeable premia components[3]. Currently, the literature offers only scarce evidence regarding the role of these premia in the monetary transmission mechanism.

How exactly do bond premia, namely term premia[4], respond (if at all) to monetary policy shocks? Is there any feedback from these responses to the macroeconomy? Do term premia responses change across different subsamples of the US data? And finally, what happens when, after isolating term premia, a shock to future monetary policy expectations occurs? Based on an empirical analysis of US data, I provide in this chapter answers to all of these questions.

In order to succeed, I must overcome two crucial identification issues here. First, there is the issue of identifying the unobserved term premium component implicit in bond yields of different maturities. Then, I must devise a procedure for identifying

[2]E.g. Evans and Marshall (1998), Kuttner (2001), Evans and Marshall (2007), and Mumtaz and Surico (2009).

[3]E.g. Fama and Bliss (1987), Campbell and Shiller (1991), Dai and Singleton (2002), and Cochrane and Piazzesi (2005)

[4]Throughout this paper, I define term premia as the deviations of the actual bond yields from the term-structure Expectations Hypothesis. A formal definition can be found in Appendix 1.D.

unobserved monetary policy shocks. The framework I propose overcomes these two identification issues simultaneously.

To address the first issue, I model the joint reduced-form dynamics of the macroeconomy and the yield curve according to a Macro-Affine Term Structure Model (MTSM) similar to Ang and Piazzesi (2003). This family of models explores the discipline imposed by no-arbitrage in order to clarify the links between macroeconomic shocks and the entire yield curve. The identification of term premia in these models follows naturally from the no-arbitrage conditions that are in the core of these frameworks.

There are several variants of the Ang and Piazzesi (2003) framework available in the literature. In this chapter I chose to follow the one proposed by Joslin, Priebsch, and Singleton (2010). This framework is attractive for at least two reasons. First, it is more general than Ang and Piazzesi (2003) in that it allows for two-way feedback effects between bond prices and macroeconomic variables. As a result, movements in term premia have the potential to affect not only bond prices but also the macroeconomic variables included in the model. Second, the Joslin, Priebsch, and Singleton (2010) framework does not have the property, shared by most models in the Ang and Piazzesi (2003) tradition, that the macroeconomic variables in the model are spanned by the yield curve[5]. Joslin, Priebsch, and Singleton (2010) show that this spanning property is at odds with the US data.

Turning now to the identification of monetary policy shocks, the difficulty lies in finding a procedure for differentiating the truly exogenous movements in the monetary policy instrument from those movements that arise endogenously as the monetary authority responds to changes in the state of the economy. To overcome this issue, I design the MTSM in such a way that identification through the *recursiveness assumption* of Christiano, Eichenbaum, and Evans (1999) can be applied

[5] That is, most models in the Ang and Piazzesi (2003) tradition impose that a combination of yields explains all of the variation in the macro variables.

easily. More specifically, I include the monetary policy instrument, which I assume to be the short-term interest rate, in the state vector of the MTSM. Then I impose a particular ordering of the elements in this vector in order to give rise to a recursive causal relationship among the macro and term-structure variables.

The model is fitted to quarterly US data on inflation, economic activity, commodity prices, and long-term yields. To evaluate possible changes in the monetary transmission mechanism, I split the data sample into two periods: 1959:1-1979:3 (pre-Volcker) and 1979:4-2007:4 (post-Volcker). My main finding is that monetary policy shocks trigger relevant movements in long-term bond premia in both subsamples. These movements feed back into the macroeconomy, giving rise to what I refer to as the *term-premium channel of monetary transmission*.

More specifically, I find that an exogenous increase in the short rate temporarily raises term premia across different maturities by a statistically significant amount. This result holds for both samples. However, the responses of term premia are more pronounced and persistent in the pre-Volcker than in the post-Volcker subsample. I further show that the responses of the macro variables after a monetary shock can be decomposed into one portion due to term premia movements and another due to movements in the term-structure that are consistent with the Expectations Hypothesis. That is, I quantify the term-premium channel of monetary transmission. Interestingly, my estimates show that the term-premium channel was particularly important in the pre-Volcker period, while this channel turns out to be empirically irrelevant in the latter period.

I then analyze how shocks to *future monetary policy expectations* affect the macroeconomy. This is motivated by the crucial role that modern central banks across the globe give to efficiently managing private agents' expectations. My framework is convenient for this analysis because it allows for the isolation of policy expectations from term premia – i.e. it guarantees that the proposed shock affects the future policy expectations implied by the model, and not term premia.

In the post-Volcker subsample, I find that a shock to monetary policy expectations up to one year ahead will lead to more pronounced and more intuitive responses for the macroeconomic variables than would standard shocks to the contemporaneous value of the monetary policy instrument. In other words, while a contractionary shock to the current value of the short rate leads to counterintuitive rises in inflation and economic activity in the post-Volcker sample, a shock to policy expectations one-year ahead leads to declines in both macro variables. I further show that if one directly shocks the long-term yield, which includes both policy expectations *and* term premia, then identification of the expectations shock becomes biased. In particular, the responses of the macro variables after a monetary policy shock become less pronounced than when I control for term premia.

There have been several earlier contributions to the macro-finance literature that, in some sense, tried to address one or more questions raised in the beginning of this chapter. In particular, Evans and Marshall (1998) were among the first to study the monetary policy transmission in a system containing both macroeconomic and term structure variables[6]. Their model consisted of a standard Vector Autoregression on macroeconomic variables and nominal yields that did not allow for feedback effects from bond yields to the macroeconomic variables. More recently, in a context similar to mine, Diebold, Rudebusch, and Aruoba (2006) and Mumtaz and Surico (2009)[7] used the recursiveness assumption to identify monetary policy shocks in a MTSM based on Nelson and Siegel (1987). My approach differs from these three papers in that my model rules out arbitrage opportunities across bonds of different maturities. In addition, none of them quantify the term premium channel of monetary transmission or study shocks to policy expectations.

This chapter is also related to a literature that uses Fed funds futures' data to identify monetary policy shocks. Kuttner (2001), for example, finds that long-term yields respond significantly to movements in the Fed funds rate that are not

[6] See also Evans and Marshall (2007).
[7] See also Bianchi, Mumtaz, and Surico (2009).

anticipated by Fed funds futures. On the other hand, anticipated changes in the Fed funds rate have only minimal effects on long-term yields. More recently, Piazzesi and Swanson (2008) show that accounting for premia in the Fed funds futures' data is crucial in pursuing the identification scheme proposed by Kuttner (2001). Although this branch of the literature offers important insights into the identification of monetary policy shocks, it does not address the effects of these shocks on the rest of the economy. This chapter focuses precisely on these effects.

This chapter is organized as follows: Section 1.2 describes the data used to fit the model; Section 1.3 describes how I use the MTSM to identify both term premia and monetary policy shocks; Section 1.4 evaluates the model implied macro and term-structure responses to a standard monetary policy shock; Section 1.5 analyzes the effects of a shock to future monetary policy expectations; Section 1.6 concludes.

1.2 Data

My analysis focuses on quarterly data for the U.S. ranging from 1959:1 to 2007:4. The term-structure series that I include in my analysis are the nominal yields on 6-month and 1, 2, 3, 4, and 5-year zero-coupon bonds obtained from the CRSP database. For the short-term interest rate, I use the 3-month riskfree rate, also from CRSP. All of the yields are compounded continuously, and are observed on the last trading day of each quarter.

Three macro variables are included in the term structure model: the output gap (GAP); the rate of inflation (INF); and a measure of commodity prices ($COMM$). Although many term-structure models in the literature already incorporate measures of GAP and INF[8], I add $COMM$ as in an extensive branch of the macro literature

[8] See, for example, Ang and Piazzesi (2003), Bikbov and Chernov (2010), and Joslin, Priebsch, and Singleton (2010).

that aims at identifying monetary policy shocks using structural VARs[9].

Inspired by Bernanke and Boivin (2003), Ang and Piazzesi (2003) and Moench (2008), I extract measures of GAP and INF from rich datasets that include several different output gap and inflation indicators. The motivation for this approach is that, in practice, central banks use many different economic indicators in order to form their views about the underlying levels of economic slack and inflation in the economy (in other words, central banks act in a "data-rich environment"). Therefore, I use the methodology proposed by Stock and Watson (1988)[10], and measure GAP as the common factor extracted from a set of seven different economic slack indicators[11]. The same methodology is applied to compute INF based on five different quarterly inflation indicators[12,13]. Finally, $COMM$ is a detrended and smoothed measure of commodity prices based on the CRB spot commodity prices index[14].

Figure 1.1 depicts the time series described above. The units associated with GAP and INF cannot be interpreted, because these factors were normalized to have zero mean and unit conditional variance. Nevertheless, the dynamics of GAP and INF capture the timing of the NBER recessions, and the inflation peaks associated

[9] Structural VARs estimated on post-war U.S. data in general give rise to a "pricing puzzle". That is, they have the counterintuitive property that inflation increases in response to a contractionary monetary policy shock. According to Christiano, Eichenbaum, and Evans (1996) and Sims and Zha (2006), this puzzle can be solved by including $COMM$ as an additional endogenous variable in structural VARs. See also Christiano, Eichenbaum, and Evans (1999).

[10] See Appendix 1.A for details.

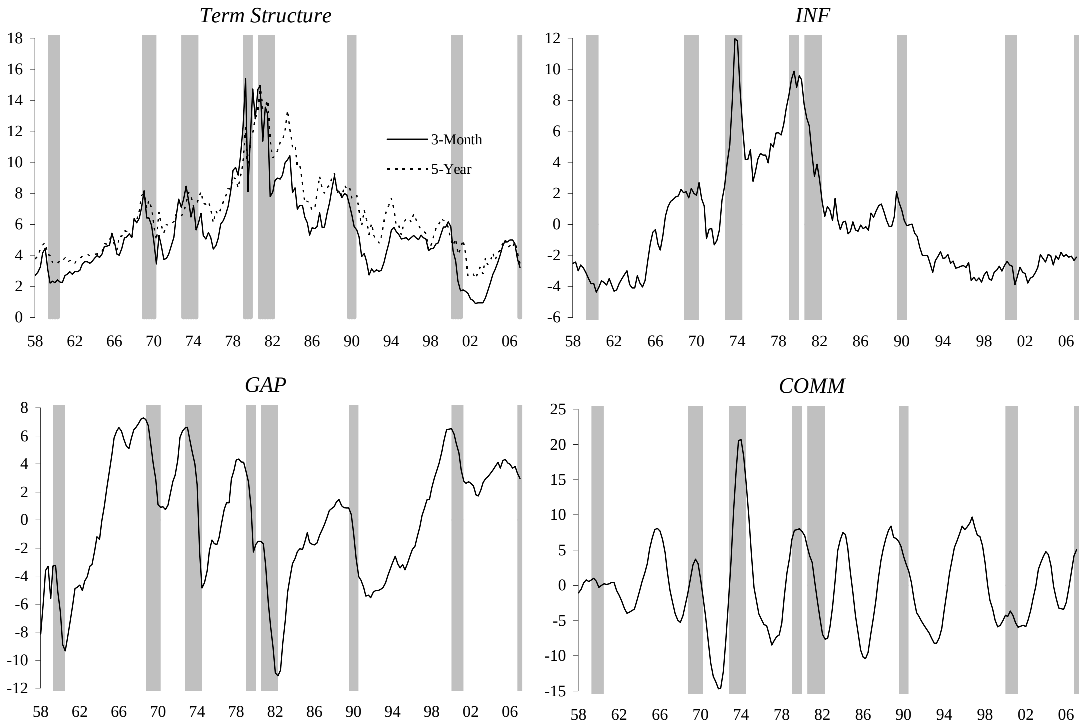
[11] The series used to compute GAP are: (i) industrial production index, (ii) total nonfarm payrolls, (iii) real personal consumption expenditures, (iv) real GDP index, (v) the new orders component of the ISM manufacturing index, (vi) total housing starts, and (vii) civilian unemployment rate. (i)-(iv) were detrended using linear and quadratic deterministic trends, whereas (v)-(vii) were used directly in levels. All series were obtained from the St. Louis Fed.

[12] The series used to compute INF are: (i) CPI fess food & energy, (ii) finished goods PPI less food & energy, (iii) personal consumption expenditures deflator less food and energy, (iv) GDP deflator, (v) average hourly earnings. All series were obtained from the St. Louis Fed, and were transformer into quarterly growth rates before applying the Kalman Filter.

[13] The quarterly series used to construct GAP and INF represent figures observed in the first month of each quarter. The only two exception are the GDP and GDP deflator series, which are not observed on a monthly frequency and were therefore proxied by their one-quarter lag. This way the plausibility of the recursive identification scheme described in Section 1.3.2 is guaranteed.

[14] More specifically, I detrend the CRB index (expressed in logs) by applying the standard Hodrick-Prescott filter. Then, in order to improve the fit of the model, I take a moving average of the detrended CRB index.

Figure 1.1: The Time Series Used to Fit the Term-Structure Model



Notes: The 3-month and 5-year yields from CRSP are expressed in percent per annum. GAP and INF are common factors respectively extracted from several economic slack and quarterly inflation series. By construction, these common factors have zero mean and unit variance. $COMM$ smoothed log-deviations of the CRB commodity index from an HP trend. The shaded areas represent NBER recessions.

with the 1973 and 1979 oil shocks, very well.

In the remainder of this chapter, the m-period bond yield is denoted by $y_{m,t}$. The short-rate (i.e. the 3-month rate) is denoted by $y_{1,t} \equiv r_t$. The yields used to evaluate the fit of the model are collected in the 7×1 vector $y_t \equiv [\ r_t \quad y_{2,t} \quad y_{4,t} \quad \cdots \quad y_{20,t}\ ]'$. Finally, the macro variables are arranged in $M_t \equiv [\ GAP_t \quad INF_t \quad COMM_t\ ]'$.

1.3 The Macro-Affine Term Structure Model

I model the joint reduced-form dynamics of the macroeconomy and the yield curve according to a Macro-Affine Term Structure Model (MTSM) similar to Ang and Piazzesi (2003). This family of models explores the discipline imposed by no-arbitrage in order to clarify the links between macro shocks and the entire yield curve. The particular framework I adopt in this chapter follows Joslin, Priebsch, and Singleton (2010).

Section 1.3.1 describes the core equations of the model and shows how it can be used to identify term premia over the term structure. Section 1.3.2 then shows how the recursiveness assumption is used to identify monetary policy shocks in the model. Finally, Section 1.3.3 describes the econometric methodology used to fit the model to the U.S. data.

1.3.1 Identifying Term Premia

I will now describe the details of the model that will allow me to identify term premia over the term structure. Suppose that the state of the economy is summarized by the three macro variables described in Section 1.2 plus N additional yield-based factors. More specifically, at any time t, the state-vector of the economy is given by $Z_t \equiv [\ M_t' \quad \mathcal{P}_t'\ ]' \in \mathbb{R}^{3+N}$, where M_t was already defined in the previous section and the N-column vector $\mathcal{P}_t$ contains the yield-based factors. Following Joslin, Singleton, and Zhu (2011), I assume that $\mathcal{P}_t$ consists of returns on *observed* bond portfolios. More precisely, for a full-rank matrix of portfolio weights P, I define $\mathcal{P}_t \equiv P y_t$. My particular choice for P will be described in Section 1.3.2.

The Macro-Affine Term Structure Model is summarized by three equations:

$$r_t = \rho_0 + \rho_1' \mathcal{P}_t \tag{1.1}$$

$$\Delta \mathcal{P}_t = \Theta_{0\mathcal{P}}^{\mathbb{Q}} + \Theta_{1\mathcal{P}}^{\mathbb{Q}} \mathcal{P}_{t-1} + \sqrt{\Sigma_{\mathcal{P}}} \epsilon_{\mathcal{P}t}^{\mathbb{Q}} \tag{1.2}$$

$$\Delta Z_t = \Theta_{0Z}^{\mathbb{P}} + \Theta_{1Z}^{\mathbb{P}} Z_{t-1} + \sqrt{\Sigma_Z} \epsilon_{Zt}^{\mathbb{P}} \tag{1.3}$$

where $\epsilon_{\mathcal{P}t}^{\mathbb{Q}} \sim N(0, I_N)$ and $\epsilon_{Zt}^{\mathbb{P}} \sim N(0, I_{3+N})$.[15] According to equation (1.1), the

[15] The dimensions of the unknown coefficients present in the model are: ρ_0 is a scalar, ρ_1 and

short-term interest rate, r_t, is assumed to be a linear function of $\mathcal{P}_t$. In addition, the dynamics of the yield portfolios, $\mathcal{P}_t$, under the risk-neutral probability measure ($\mathbb{Q}$), follow the Gaussian process described in equation (1.2). The model is completed by assuming that the evolution of the complete state vector Z_t under the historical probability measure ($\mathbb{P}$) is given by the Gaussian process in equation (1.3).

In the absence of arbitrage opportunities, bond prices are determined by equations (1.1) and (1.2). More specifically, letting $V_{m,t} \equiv \exp(-m \times y_{m.t})$ represent the time t price of a bond that repays the investor at time $t+m$, it can be shown that the no-arbitrage bond price must respect $V_{m,t} = E_t^{\mathbb{Q}}\left[e^{-r_t} V_{m-1,t+1}\right]$. Appendix 1.B shows that combining the bond-pricing condition to equations (1.1) and (1.2) yields the following solution for bond yields:

$$y_{m,t} = A_m + B_m \mathcal{P}_t \tag{1.4}$$

where A_m and B_m are determined by the first-order difference equations described in Appendix 1.B. Importantly, because of the assumed short-rate equation (1.1), the bond yields of different maturities are affine on $\mathcal{P}_t$ and not on M_t. This means that only the risks associated with $\mathcal{P}_t$ are priced explicitly by the model. Although macroeconomic risks are not priced explicitly, they may have important implications for bond prices, because under $\mathbb{P}$ the dynamics of M_t interact with those of the yield portfolios (see equation (1.3)).

Standard models in the tradition of Ang and Piazzesi (2003) substitute equation (1.1) for an equation in which r_t is a linear function of *both* $\mathcal{P}_t$ and M_t. The bond prices implied by these more traditional models are then affine not only in $\mathcal{P}_t$, but also in M_t. There are at least two reasons why having bond yields follow equation (1.4) is preferable to those implied by traditional models. First, having yields determined by equation (1.4) is consistent with the fact that a low-dimensional factor

$\Theta_{0\mathcal{P}}^{\mathbb{Q}}$ are $N \times 1$, $\Theta_{1\mathcal{P}}^{\mathbb{Q}}$ and $\Sigma_{\mathcal{P}}$ are $N \times N$, $\Theta_{0Z}^{\mathbb{P}}$ is $3+N \times 1$, and finally $\Theta_{1Z}^{\mathbb{P}}$ and Σ_Z are $3+N \times 3+N$. The matrix $\sqrt{\Sigma_{\mathcal{P}}}$ is equal to the $N \times N$ lower right corner of $\sqrt{\Sigma_Z}$.

structure is sufficient to explain most of the variation in yields[16]. This avoids the problems that are likely to arise with estimating over-parameterized models, which will probably be the case when all variables in Z_t are explicitly priced. Furthermore, Joslin, Priebsch, and Singleton (2010) show that in models where bond prices are affine on both $\mathcal{P}_t$ and M_t, the macro factors are spanned by the term structure (i.e. a combination of yields explains all of the variation in M_t). In a setup similar to the one developed here, they show that this spanning property is empirically rejected in the U.S. data (more details follow in Section 1.4.1).

As in Diebold, Rudebusch, and Aruoba (2006), another important property of the model (1.1) - (1.3) is that it allows for two-way feedback effects between the macro variables and the yield curve. Mechanically, the interaction between M_t and $\mathcal{P}_t$ occurs because the conditional covariance matrix of Z_t, Σ_Z, and the matrix of the slope coefficients $\Theta_{1Z}^{\mathbb{P}}$ are potentially full. The model in which these feedback effects are not present is nested as a constrained version of equations (1.1) - (1.3).

To understand how this model can be used to identify term premia contained in bond prices, let us consider a risk neutral world. In this world, the risk-adjusted ($\mathbb{Q}$) and the historical ($\mathbb{P}$) probability measures coincide[17]. Appendix 1.D shows that in this case bond yields will be given by $y_{m,t}^{EH} = A_m^{EH} + B_m^{EH} Z_t$, where A_m^{EH} and B_m^{EH} follow the recursions shown in the appendix[18]. Interestingly, it can be shown that, up to a convexity term, the m-period bond yield in this hypothetical world, $y_{m,t}^{EH}$, behaves according to the EH. In other words, the dynamics of $y_{m,t}^{EH}$ are determined by the expected dynamics of the short-rate. Letting $tp_{m,t}$ capture the deviations of the m-period yield from the EH, I ultimately obtain the following yield decomposition:

$$y_{m,t} \equiv y_{m,t}^{EH} + tp_{m,t} \tag{1.5}$$

[16]Traditionally, a 3-factor structure consisting of Level, Slope and Curvature factors is sufficient to explain most of the cross-sectional variation in the term structure. See Litterman and Scheinkman (1992).

[17]In other words, all prices of risk are zero.

[18]Importantly, the values of A_m^{EH} and B_m^{EH} depend only on the parameters contained in equations (1.1) to (1.3).

Following the macro and finance literatures, I will call $tp_{m,t}$ the "m-period term premium". A positive value for $tp_{m,t}$ indicates that investing in the long-term bond is riskier than investing in a sequence of short-term bonds for m periods.

1.3.2 Identifying Monetary Policy Shocks

Because the model described in the previous section consists of a reduced-form economic system, an identification scheme is needed in order to distinguish exogenous monetary impulses from those systematic responses of the Fed to changes in the state of the economy. In this chapter, the identification of monetary policy shocks follows the *recursiveness assumption* of Christiano, Eichenbaum, and Evans (1999). According to this identification scheme, a particular ordering of the variables in Z_t is imposed in order to give rise to a recursive causal relationship among these variables.

Assume that the Fed's monetary policy instrument is one of the endogenous variables included in Z_t. Then the recursive identification scheme of Christiano, Eichenbaum, and Evans (1999) can be applied to equation (1.3) of the term-structure model, just as in any standard VAR. In particular, assume that $\sqrt{\Sigma_Z}$ is the Cholesky factor associated with Σ_Z. Then, the lower-triangular shape of $\sqrt{\Sigma_Z}$ implies that the ordering of the variables in Z_t establishes a causal relation among the state variables. In particular, the variables that are ordered in Z_t above the monetary policy instrument do not move instantly when a monetary shock occurs. The values of these variables in a given period are assumed to be observed by the Fed before its monetary policy decision is taken. On the other hand, the variables ordered in Z_t below the policy instrument move instantly when a monetary shock occurs; thus, their values in a given period are assumed to be observed only after the Fed's policy decision. As a result, this identification scheme implies that the policy shocks are orthogonal to the variables assumed to be included in the Fed's information set[19,20].

To implement the recursive identification scheme in the model of Section 1.3.1, first one needs to choose the variable that will represent the monetary policy instrument of the Fed. A second decision must be made regarding the particular state variables that are assumed to be included in the information set available to the Fed before its policy decision (that is, one must choose whether each variable included in Z_t should appear above or below the policy instrument).

I assume that the short-rate, r_t, represents Fed's the monetary policy instrument. This choice is motivated by Bernanke and Mihov (1998) who find that, except for Volcker's reserve targeting experiment (1979-1982), in practice the Fed actually has targeted the interest rate since the 1950s.[21] Moreover, many recent empirical analyses of the term-structure – such as Ang, Dong, and Piazzesi (2007), Ang, Boivin, Dong, and Loo-Kung (2009), and Mumtaz and Surico (2009) – also view the short rate as the Fed's monetary policy instrument. I introduce the policy instrument in the state vector Z_t by assuming that one bond portfolio contained in $\mathcal{P}_t$ simply replicates r_t. More specifically, I set one line of the matrix of portfolio weights P to $\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$.

In contrast to my approach, Diebold, Rudebusch, and Aruoba (2006) introduce r_t in the state vector through M_t and not through $\mathcal{P}_t$. However, their approach does not rule out arbitrage opportunities in bond prices. By introducing the short-rate as a portfolio in $\mathcal{P}_t$, I guarantee the absence of arbitrage opportunities across short-term and longer-term bonds.

[19] It can be shown that the monetary policy shocks identified through this recursive scheme do not depend on (i) the particular ordering of the variables above the policy instrument in Z_t, and (ii) the particular ordering of the variables below the policy instrument in Z_t. See Christiano, Eichenbaum, and Evans (1999).

[20] The Cholesky factorization of Σ_Z actually implies a just-identification scheme. Therefore, it provides a simple recursive identification to $3 + N$ "structural" shocks in the model. In this paper I focus on monetary policy shocks, because in this case the recursive identification scheme is supported by many previous theoretical and empirical papers in the macro literature.

[21] According to Cook (1988), movements in the fed funds rate followed judgemental actions of the Fed even during Volcker's reserve targeting experiment.

With respect to the causal relations in the state vector, I choose the following ordering for the elements of Z_t:

$$Z_t = \begin{bmatrix} GAP_t & INF_t & COMM_t & r_t & \mathcal{P}_{2,t} & ... & \mathcal{P}_{N,t} \end{bmatrix}'$$

where $\mathcal{P}_{i,t}$ represents the i^{th} element of $\mathcal{P}_t$ (with $\mathcal{P}_{1,t} \equiv y_{1,t} \equiv r_t$). Note that the bond portfolio that replicates r_t is ordered below M_t and above all remaining $N-1$ bond portfolios. This implies that all elements of M_t are included in the Fed's time t information set. As a result, M_t responds with a lag to exogenous movements in the short-rate. On the other hand, the bond portfolios $\mathcal{P}_{2,t}, ..., \mathcal{P}_{N,t}$ are not in the Fed's time t information set, and therefore are allowed to adjust instantly to monetary shocks.

The motivation behind my ordering in Z_t is as follows: because bond portfolios reflect asset prices that are purely forward-looking (see the bond pricing equation in Section 1.3), it is reasonable to assume that an exogenous change in r_t triggers instant movements in $\mathcal{P}_{2,t}, ..., \mathcal{P}_{N,t}$. In other words, as soon as investors' expectations are revised to incorporate the new level of r_t, the observed bond prices will be affected. In contrast, in case of M_t, the same policy shock in general will "affect economic conditions only after a lag that is both long and variable"[22]. This lag could be rationalized in terms of the economic costs related, for example, to changes in production plans, revising goods' prices, and etc. As a result, the policy shock will take longer to show up in the aggregate macroeconomic data.

Finally, note that the normalizations imposed to obtain econometric identification (see Appendix 1.C) result in the coefficients of the short-rate equation (1.1) being $\rho_0 = 0$ and $\rho_1 = [\ 1 \quad 0 \quad \cdots \quad 0\ ]'$. Therefore, in my identification scheme the short-rate dynamics are actually determined by the state equation (1.3) rather than equation (1.1). Following Christiano, Eichenbaum, and Evans (1999), the short-rate

[22] Friedman (1961).

process implied by my framework therefore can be interpreted as an interest-rate feedback rule of the sort proposed by Taylor (1993) (expressed in reduced form). According to this view, endogenous short-rate movements would occur in response to changes in M_t, while all residual movements would be interpreted as monetary policy shocks.

1.3.3 Estimation Methodology

Because the first bond portfolio to enter $\mathcal{P}_t$ was already chosen in the previous section, it only remains to choose the other $N-1$ yield-based factors (bond portfolios) in order to complete the model specification. The finance literature finds that most of the variation in bond yields is explained well by three unobserved factors usually referred to as level, slope and curvature. As Joslin, Singleton, and Zhu (2011) show, these estimated unobserved factors in general are similar to the first three principal components (PCs) of the term-structure data.

Accordingly, in this chapter I allow for $N = 3$ yield-based pricing factors. As explained before, the first of these simply replicates the short-rate, r_t. The two remaining factors are given by the second and third term-structure PCs, PC_2 and PC_3, which were extracted from my term-structure dataset. More specifically, the matrix of portfolio weights is given by:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.51 & -0.45 & -0.22 & 0.09 & 0.28 & 0.40 & 0.49 \\ 0.60 & -0.01 & -0.61 & -0.34 & -0.11 & 0.17 & 0.33 \end{bmatrix} \tag{1.6}$$

Note that the loadings associated with the second bond portfolio roughly replicate the slope of the yield curve, while the third portfolio has the shape of a curvature factor with a trough on the 1-year maturity. I will therefore refer to PC_2 and PC_3 as the slope and curvature factors. In my dataset, the correlation between r_t and the first term-structure PC is above 0.97. Therefore, the fit of the model with my choice of P must be similar to that of a model where $\mathcal{P}_t$ contains the first three term-structure PCs.

The model is estimated by Maximum Likelihood (ML) after imposing the identifying normalizations proposed by Joslin, Singleton, and Zhu (2011). The bond portfolios $\mathcal{P}_t$ are assumed to be perfectly priced by the model. However, each observed yield $y_{m,t}^{obs}$ (except for the short-rate) is assumed to be priced with a measurement error $u_{m,t} \equiv \left(y_{m,t}^{obs} - y_{m,t}\right) \sim N\left(0, \omega^2\right)$. The ML estimates of the $\mathbb{P}$-dynamics of Z_t (except for Σ_Z) can be conveniently obtained by OLS. Conditional on these estimates, the likelihood function is optimized with respect to the parameters determining the $\mathbb{Q}$-dynamics of $\mathcal{P}_t$ (Σ_Z included). For more details see Appendix 1.C.

1.4 Model Analysis

This section analyzes the model estimation results. I show the results from using two different subsamples of my dataset, namely 1959:1-1979:3 and 1979:4-2007:4. This follows from Boivin and Giannoni (2006); in the context of a VAR similar to the state equation (1.3) of my model, they find evidence of a structural break in the U.S. data in 1979:4.

Section 1.4.1 compares the model fit to alternative model specifications. Section 1.4.2 discusses the implications for the monetary policy transmission mechanism across the two samples used to fit the model.

1.4.1 Bond Pricing Errors

In order for the analysis carried out in the remainder of this chapter to be meaningful, it is required that the long-term yields implied by the model track their observed counterparts reasonably well. Therefore, it is important to compare the fit of the

MTSM described in Section 1.3 to other standard benchmark models in the literature. To form a fair comparison with the model from Section 1.3, all alternative models considered here have exactly three pricing factors[23].

The first alternative model is an Affine Term Structure Model with three yield-based pricing factors and no macro factors. As is standard in the finance literature, this yields-only model assumes that the pricing factors are the first three PCs of the term structure data. Based on their loadings on the term-structure data, these three PCs follow the usual Level, Slope and Curvature interpretation. Note that, unlike the model from Section 1.3, I use the first PC (i.e. the Level factor) in this case instead of r_t for the first yield-based factor.

The second model that I consider is an MTSM, as described in Section 1.3, with the exception that the macro factors in this case are assumed to be spanned by the term structure. More specifically, the model with spanned macro factors substitutes the short-rate equation (1.1) for another specification where r_t is a linear function of both $\mathcal{P}_t$ *and* M_t (i.e. both yield-based and macro factors are explicitly treated as pricing factors). In this case, it can be shown that the model-implied yields are linear in $\mathcal{P}_t$ and M_t. Importantly, Joslin, Le, and Singleton (2011) show that this model has the property that M_t can be replicated by appropriately chosen bond portfolios[24] – i.e. the macro variables are spanned by the information contained in the term structure. Models that have this spanning property include Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), and Bikbov and Chernov (2010). Because I am only focusing on three-factor models, the particular spanned MTSM that I estimate includes GAP, INF and $COMM$ as pricing factors; no yield-based factor was included[25].

[23] In the model from Section 1.3 only the risks associated with the 3×1 vector $\mathcal{P}_t$ were explicitly priced.

[24] More precisely, the Spanned-Macro model implies that $M_t = \zeta_0 + \zeta_1 \widetilde{\mathcal{P}}_t$, where $\widetilde{\mathcal{P}}_t$ is a vector of bond portfolios with as many entries as the number of priced factors in the model. See Joslin, Le, and Singleton (2011).

[25] Keeping the three-factor specification, I also compared the unspanned MTSM from Section 1.3 to a spanned MTSM with the following pricing factors: GAP, INF and PC_1 (instead of

Table 1.1: Comparing Price Errors Across Different Term-Structure Models

	1959:1 - 1979:3			1979:4 - 2007:4		
	$\frac{1}{T}\sum_{t=1}^{T}\lvert u_{m,t}\rvert$	$\underset{(u_{m,t})}{\min}$	$\underset{(u_{m,t})}{\max}$	$\frac{1}{T}\sum_{t=1}^{T}\lvert u_{m,t}\rvert$	$\underset{(u_{m,t})}{\min}$	$\underset{(u_{m,t})}{\max}$
(I) Unspanned MTSM:						
$m = 4$	4.9	-15.3	19.5	4.9	-17.3	15.9
$m = 12$	5.4	-14.6	24.0	5.4	-14.0	19.8
$m = 20$	6.6	-21.7	18.9	6.6	-23.4	22.6
(II) Yields-only model:						
$m = 4$	4.8	-17.4	19.6	4.1	-25.1	12.4
$m = 12$	4.8	-16.5	18.4	4.1	-13.4	13.5
$m = 20$	4.5	-15.8	9.2	4.8	-28.9	17.5
(III) Spanned MTSM:						
$m = 4$	62.7	-245.5	248.2	135.1	-612.3	413.1
$m = 12$	50.5	-191.7	191.7	126.3	-477.2	375.6
$m = 20$	46.7	-187.0	178.2	118.5	-406.5	378.2

Notes: "Unspanned MTSM" is the model described in Section 1.3, with 3 yield-based pricing factors given by the portfolio weights in (1.6) and 3 unspanned macro factors; "Yields-only model" is an Affine Term-Structure Model with 3 yield-based pricing factors (the first 3 PCs) and no macro factors; "Spanned MTSM" is a 3-factor Macro-Affine Term Structure Model where the pricing factors are given by M_t. All figures are expressed in annualized basis points.

Panels (I) to (III) of Table 1.1 compare the fit of the three models discussed above for the 1959:1-1979:3 and 1979:4-2007:4 samples. The fit of each model is evaluated according to the mean absolute bond pricing error, $\frac{1}{T}\sum_{t=1}^{T}\lvert\widehat{u}_{m,t}\rvert$, as well as the minimum and maximum estimated pricing error within each sample. All figures are expressed in annualized basis points. Table 1.1 focuses on 1-,3-, and 5-year maturities.

The pricing errors associated with models "yields-only" and the unspanned MTSM are very similar in both samples. For these models, the pricing errors on average are small in absolute terms and they fluctuate inside a relatively narrow

GAP, INF and $COMM$). Still in this case, the unspanned MTSM fits the term-structure data significantly better than this specification of the spanned MTSM.

interval. There is little deterioration in fit as we go from the yields-only model to the unspanned MTSM. This is because the former assumes that the pricing factors are the first three term-structure PCs, whereas the latter substitutes the first PC for the short-rate. Therefore, the cost of having the short-rate in the state vector to allow for monetary policy identification is very small.

Comparing now the unspanned and spanned MTSMs, observe that the latter displays pricing errors that are an order of magnitude larger in both samples. This is because most of the variation in yields can be explained by the first three term-structure PCs, and GAP, INF and $COMM$ fail to replicate the variation on these PCs[26]. Only at the cost of increasing the dimension of the vector of pricing factors (in particular, if extra yield-based factors are added) will the spanned MTSM fit the data as well as the model unspanned MTSM.

Therefore, in terms of fit to the term-structure data, the unspanned MTSM proposed in Section 1.3 is comparable to the standard yields-only model. The advantage of the unspanned MTSM *vis-à-vis* the yields-only model is that the former allows for interactions between the term structure and the macroeconomy, crucial for the purposes of this chapter. Additionally, in comparison to the three-factor spanned MTSM, the unspanned model delivers a much better fit to the term-structure data. I therefore conclude that the unspanned MTSM is an adequate tool for the study of monetary policy shocks carried out in the next sections.

1.4.2 The Monetary Transmission Mechanism in the Unspanned MTSM

In this subsection, I use the unspanned MTSM from Section 1.3 to study how shocks to the assumed monetary policy instrument, r_t, transmit to the macroeconomy and to the term structure. I begin by computing the model-implied impulse-response functions (IRFs) to a monetary policy shock. In so doing, I pay particular attention

[26] This point was first made by Joslin, Le, and Singleton (2011).

to the differences that emerge between the two samples used to estimate the model, namely the 1959:1-1979:3 sample and the more recent 1979:4-2007:4 sample. In order to make the IRFs comparable across samples, I normalize the monetary policy shock to be an exogenous increase of 100 basis points in the (annualized) short-rate.

Figure 1.2 shows the movements of the state vector $Z_t \equiv [\ M_t' \quad \mathcal{P}_t'\ ]'$ following the monetary policy shock defined above. Model-implied IRFs based on the 1959:1-1979:3 sample are shown in the top panel, whereas the bottom panel shows the same for the 1979:4-2007:4 sample. In each chart the solid line corresponds to the mean response, whereas the shaded areas represent small-sample 95% confidence intervals[27].

I begin by analyzing the IRFs for the macro variables GAP, INF, and $COMM$. Note that the IRFs for these variables differ substantially across the two samples. In the model estimated using the first sample, the policy shock leads to statistically significant and persistent movements in GAP, INF, and $COMM$. For the second sample, the responses are much smaller – in fact, all three IRFs in the second sample are not significantly different from zero.

This result is in line with Boivin and Giannoni (2002), who find that the impacts of monetary policy shocks on output and inflation are much smaller since the beginning of the 1980s than in the period before. Based on estimated general equilibrium models, they associate these reduced monetary policy impacts to an increase in the Fed's responsiveness to inflation expectations that began in the 1980s.

Note that for the model estimated over the 1959:1-1979:3 sample, the response of INF stays positive for some quarters before turning negative. In the literature

[27]The confidence intervals were computed using the following bootstrap method: (i) resample, with replacement, T residuals from the estimated model; (ii) given the inital state vector Z_0, use the resampled residuals and the estimated model parameters to construct new time series for the state-vector and the yield curve; (iii) re-estimate the model using the resampled data; (iv) repeat steps (i) - (iii) 500 times. The dashed lines report the 2.5 and 97.5 percentiles of the distribution of estimated impulse responses.

this is known as the "price puzzle"[28], and is usually treated by including a measure of commodity prices in the model. In fact, excluding $COMM$ from the unspanned MTSM amplifies this effect significantly.

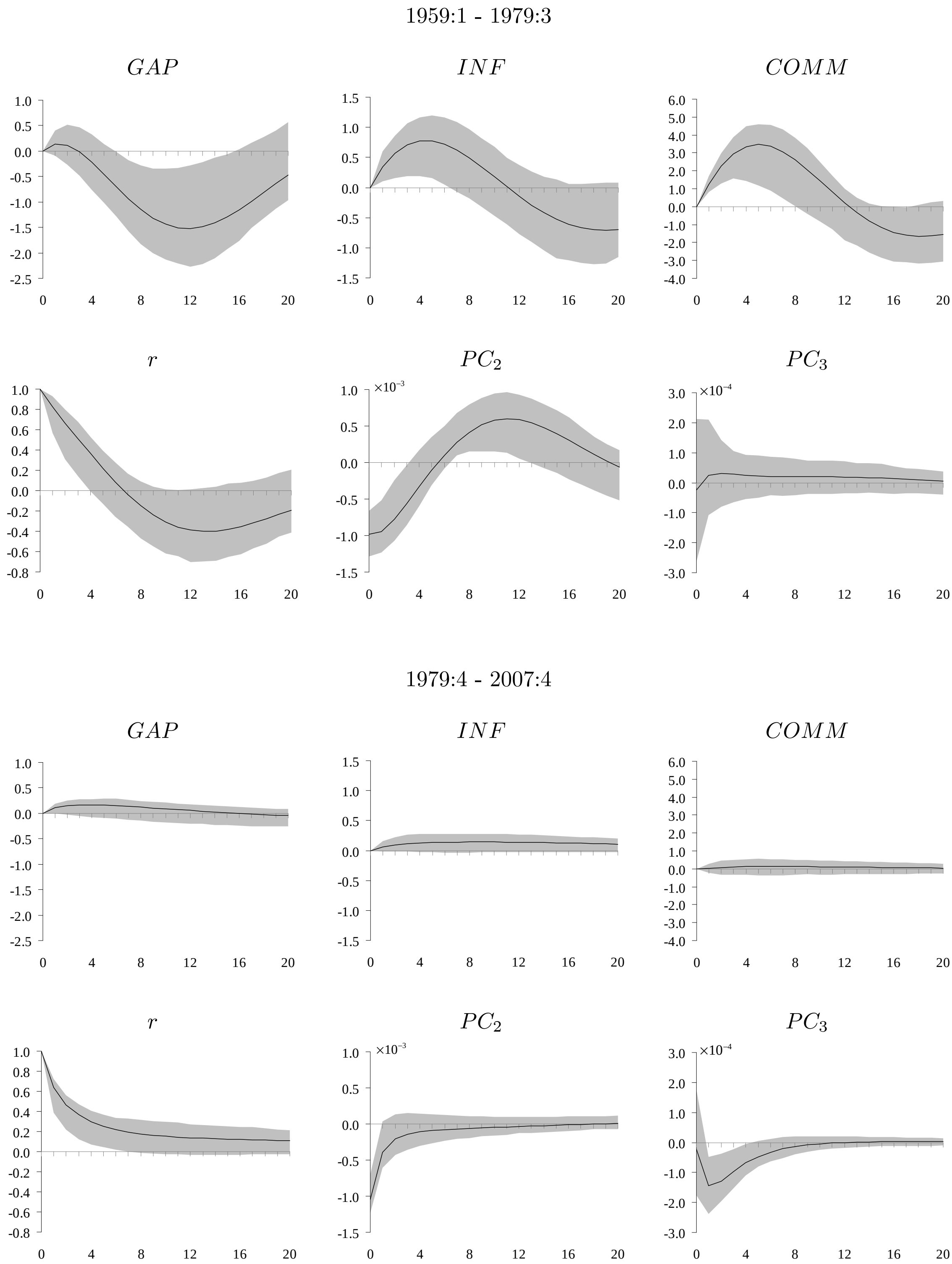
Returning to Figure 1.2, the bottom row of charts in each panel shows the IRFs for the pricing factors r, PC_2, and PC_3. In both samples the monetary policy shock triggers a persistent reaction of the short rate. In the 1959:1-1979:3 sample, the short-rate reaction stays positive for two years, turns negative and then approaches zero from below. In contrast, for the 1979:4-2007:4 sample the reaction stays positive for the entire time until it disappears. The slope factor, PC_2, falls after the shock in both samples, implying that long-term yields react to the shock by less than the short-rate. Finally, the curvature factor, PC_3, does not react significantly to the shock in the 1959:1-1979:3 sample, whereas this factor decreases two periods after the shock hits in the 1979:4-2007:4 sample.

According to equation (1.4), as the three pricing factors respond to the shock, the model-implied yields will also move in general. Moreover, because of the recursive ordering that I proposed in Section 1.3.2, all of the pricing factors, and consequently all individual bond yields, are allowed to move instantly with the shock. I now study how monetary policy shocks are transmitted to the term structure; in particular, I consider the model-implied decomposition of bond yields into premium and Expectations Hypothesis (EH) components.

Figures 1.3 and 1.4 show how the term-structure and its decomposition react to the same monetary policy shock that was considered earlier. Each figure refers to a different sample over which the model was estimated. To make the comparison across samples easier, I also plot Figure 1.5, which juxtaposes the IRFs for the two samples. In all charts, the horizontal axes measure maturity going from one quarter to five years. The black lines represent mean responses across maturities and the shaded

[28] See Christiano, Eichenbaum, and Evans (1996) and Sims and Zha (2006).

Figure 1.2: A Contractionary Monetary Policy Shock: The State Vector



Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures time in quarters. $COMM$ is expressed in % deviation from an HP trend, whereas r is expressed in percent per annum. The units of GAP, INF, PC_1 and PC_2 are not interpretable. The dashed lines are 95% bootstrapped confidence intervals.

areas are the 95% bootstrapped confidence intervals (for presentational reasons, I ignored the confidence intervals in Figure 1.5). Each row of charts in Figures 1.3 to 1.5 corresponds to a given number of periods after the shock hits. E.g. the first row displays IRFs for the quarter when the shock hits; the second row shows IRFs for two quarters later; and etc. Finally, the columns correspond to the yield decomposition of equation (1.5), i.e. $y_{m,t}$, $y^{EH}_{m,t}$ and $tp_{m,t}$.

I begin by analyzing the IRFs for the model-implied yields, $y_{m,t}$, as shown in the first columns of Figures 1.3 and 1.4. [29] To help organize the discussion, I begin with my first result:

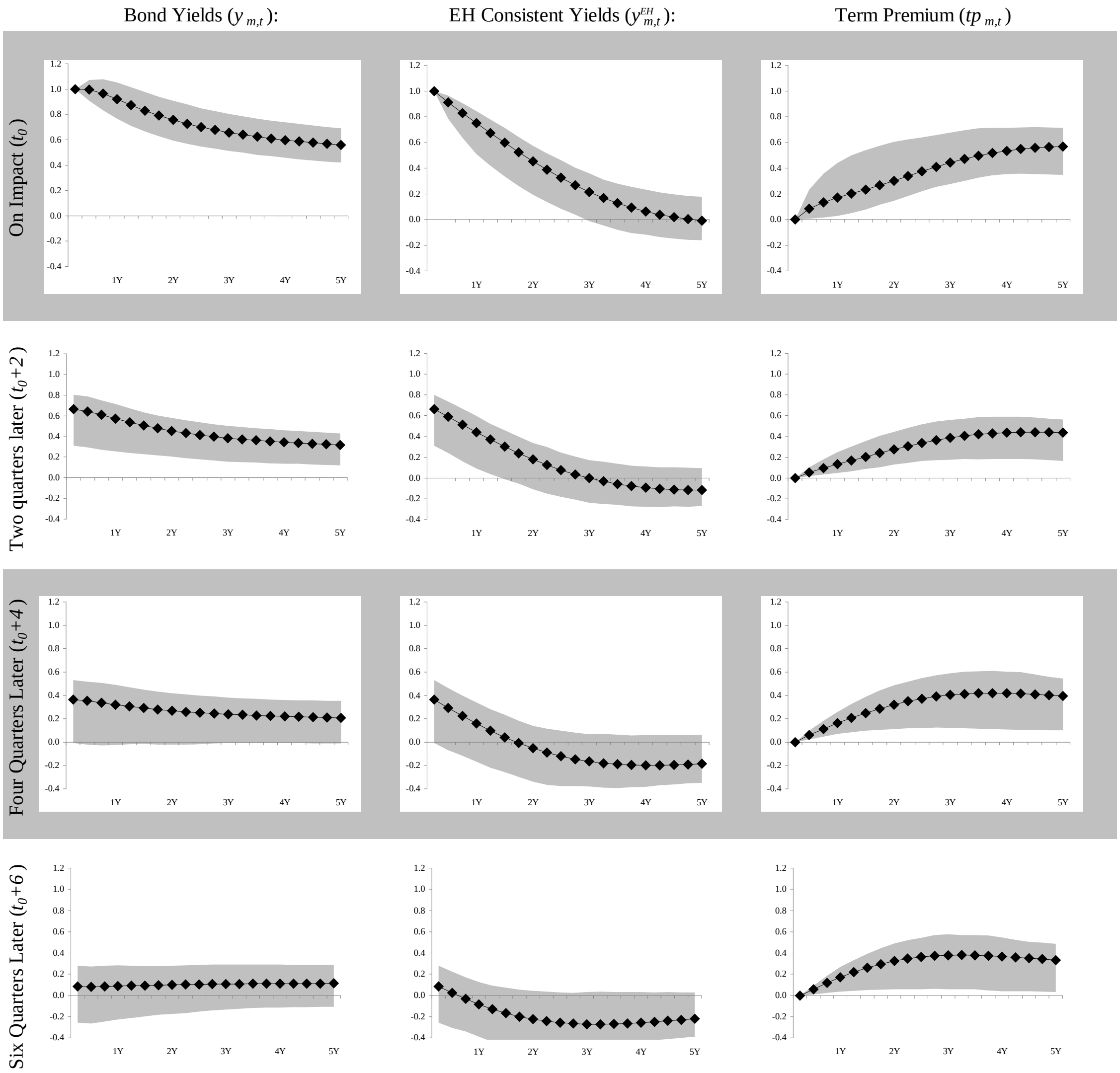
Result I: *In both samples, an exogenous increase in the short rate moves long-term yields by a significantly positive amount, at the same time reducing the yield curve slope. The IRFs for the two samples are remarkably similar across maturities and over time.*

As soon as the monetary policy shock hits the economy, all long-term yields reported increase by a significantly positive amount in both samples. The impact of the shock on yields declines as maturity increases. In fact, whereas the short rate increases by 100 basis points, the 5-year yield increases by only about 50-to-60 basis points in both samples. This means that the monetary shock considered here leads to a drop of about 40-to-50 basis points in the 5-year slope of the yield curve. Going back to Figure 1.2, this is in line with the fact that the slope factor PC_2 drops in response to the shock.

Over the quarters following the shock, the IRFs of bond yields die out in both samples. Note, in particular, that these responses over time and at different maturities are remarkably similar across the two samples (see the first column of Figure

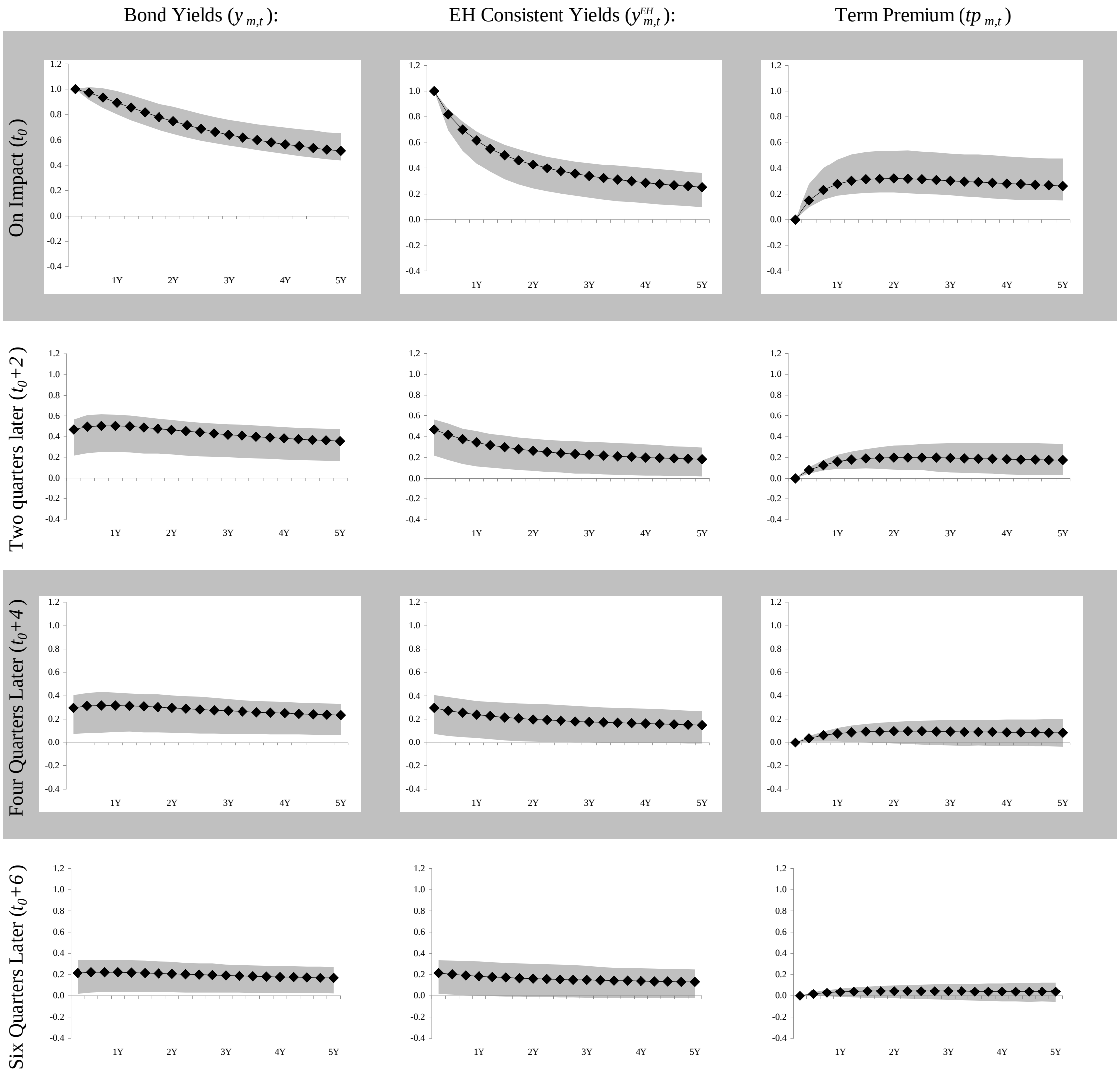
[29] Impulse response functions and variance decompositions of $y_{m,t}$, $y^{EH}_{m,t}$ and $tp_{m,t}$ are trivial to compute in the unspanned MTSM because these variables are all linear functions of the state vector Z_t.

Figure 1.3: 1959:1-1979:3; Yield Curve Decomposition after a Monetary Policy Shock



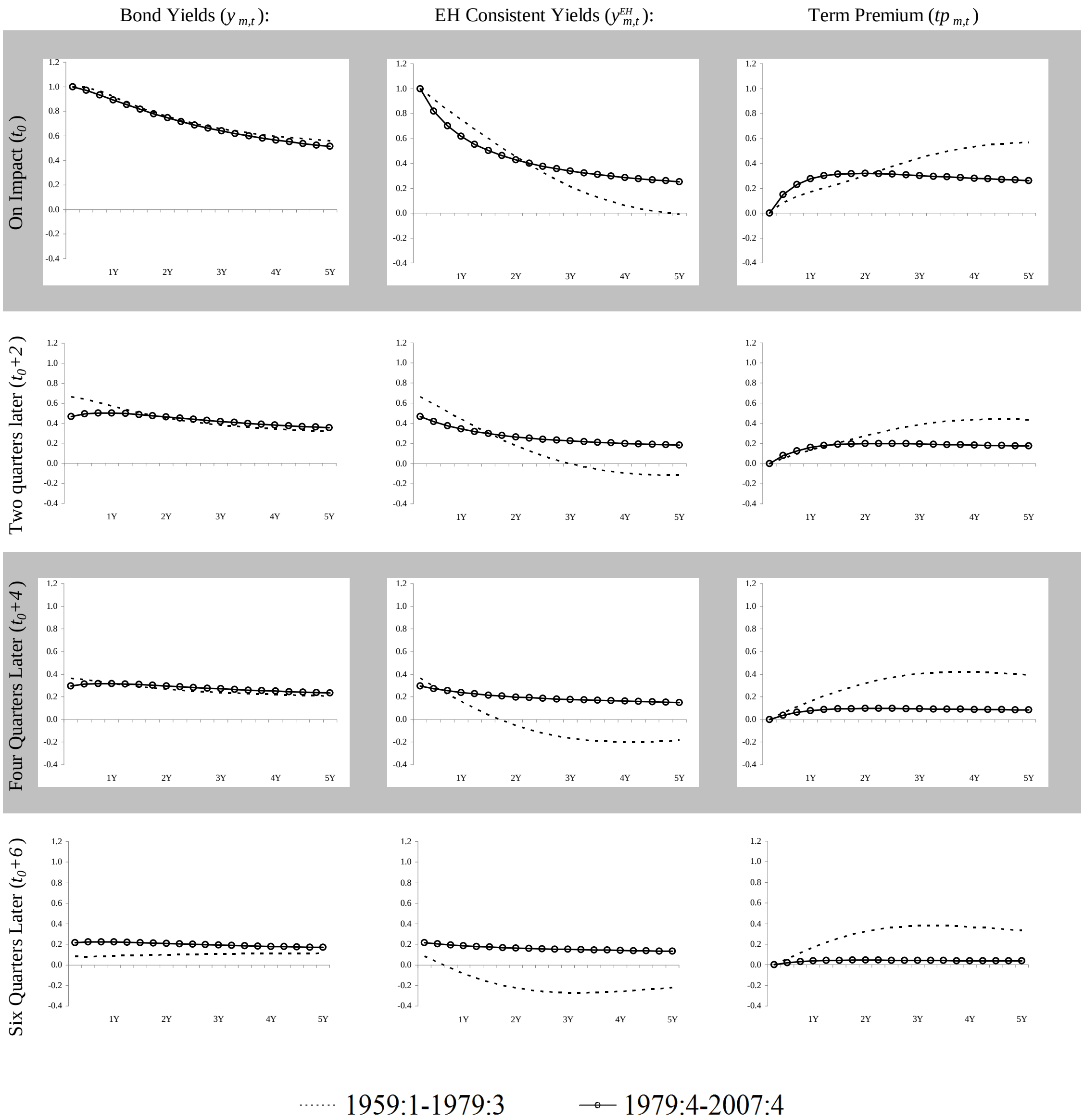
Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum. The gray areas are 95% bootstrapped confidence intervals.

Figure 1.4: 1979:4-2007:4; Yield Curve Decomposition after a Monetary Policy Shock



Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum. The gray areas are 95% bootstrapped confidence intervals.

Figure 1.5: Yield Curve Decomposition after a Monetary Policy Shock; Comparing Samples



Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum.

1.5). At the same time, we know that the short-rate IRFs differ significantly across samples (Figure 1.2). Consequently, it is very likely that the EH and term premia components of the term structure respond differently to the shock depending on the sample analyzed. This leads me to formalize my second result:

Result II: *In both samples an exogenous increase in the short rate leads to statistically significant increases in term premia across maturities. These responses are more pronounced and persistent in the 1959:1-1979:3 than in the 1979:4-2007:4 sample.*

In fact, the last column of Figures 1.3 and 1.4 show that term premia of all maturities increase by a significant amount following the shock. However, this increase is larger in general for the model estimated over the 1959:1-1979:3 samples than when I use data from the later sample (see the third column of Figure 1.5). For example, while in the 1979:4-2007:4 sample the 5-year term premium instantly increases by 26 basis points on average, in the earlier sample it increases by more than double that amount. Moreover, in the first sample the term premia responses of all maturities persistently stay above zero for more than six quarters after the shock. For the second sample, on the other hand, the term premia responses statistically become zero after about four quarters following the shock.

The fact that term premia move significantly following the shocks may have crucial implications for the monetary policy transmission mechanism. Remember from Section 1.3 that the term-structure model I analyze allows for two-way feedback effects between the macro variables and the yield curve. As a result, there is a *term-premium channel* through which monetary policy affects output and inflation. More specifically, movements in term premia following a monetary shock in general will have an impact on the pricing factors contained in $\mathcal{P}_t$, which then will affect the dynamics of the macro variables contained in M_t. Interestingly, the fact that term premia respond more to the shock in the 1959:1-1979:3 than in the 1979:4-2007:4 sample suggests that the term-premium channel may be particularly important in the earlier sample.

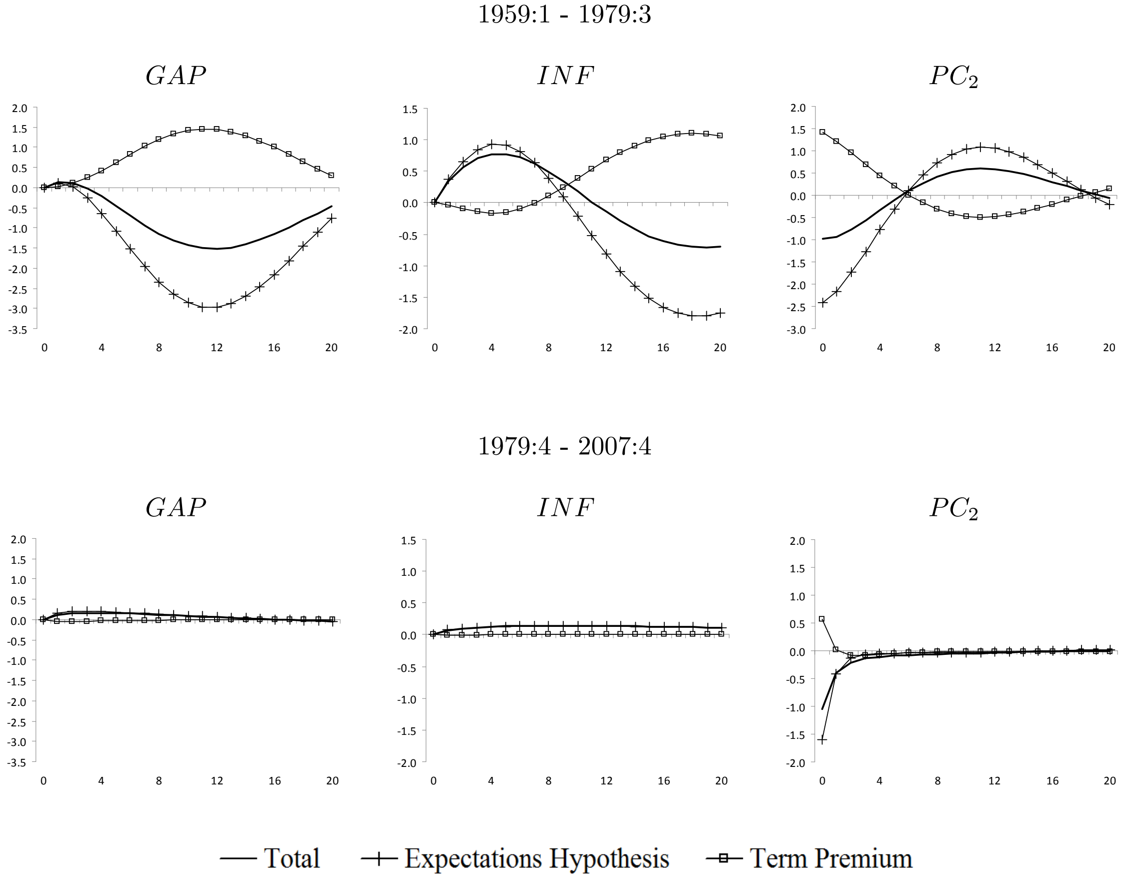
I now try to quantify the term-premium channel of monetary transmission. Let $\Psi_{Z,h}$ be the impulse response of the state vector h periods after a monetary policy shock occurs. Appendix 1.E shows that $\Psi_{Z,h}$ can be decomposed into two components: $\Psi^{EH}_{Z,h}$ and $\Psi^{TP}_{Z,h}$. The first component, $\Psi^{EH}_{Z,h}$, measures what the response of Z_t would be to a monetary policy shock if the term premium were held constant. In this case, the response of Z_t will be affected only by the EH component of the term structure. I refer to this as the EH channel of monetary transmission. The second component, $\Psi^{TP}_{Z,h}$, holds the EH component of the term structure fixed and accounts only for the change in Z_t that is caused by movements in term premia; this is the term-premium channel mentioned above.

Figure 1.6 shows the decomposition of the impulse responses of GAP, INF and PC_2 into their EH and term-premium components. The top panel shows IRFs for the 1959:1-1979:3 sample; the bottom panel refers to the 1979:4-2007:4 sample. Based the decomposition of the IRFs shown in this figure, I now formalize my third result:

Result III: *In the 1959:1-1979:3 sample, the term-premium channel is responsible for a large fraction of the movements in the macro variables following a monetary policy shock. In this case, the term premium channel acts to diminish the effectiveness of monetary policy shocks. The opposite is true for the 1979:4-2007:4 sample, for which the term-premium channel is close to being non-existent.*

The top panel of Figure 1.6 shows that a large portion of the responses of GAP and INF in the 1959:1-1979:3 sample are due to the term premium channel. The

Figure 1.6: Decomposing the Impulse Responses of the Macro Variables



Notes: Impulse responses to an exogenous increase of 100 basis points in the short-rate. The horizontal axes measures time in quarters. The units of GAP and INF are not interpretable.

exact mechanism behind this result is as follows: movements in term premia caused by the monetary impulse are reflected directly into the pricing (yield-based) factors included in Z_t. These in turn feed back into the macro variables through equation (1.3) which, in the 1959:1-1979:3 sample, causes large movements in GAP and INF.

Going one step further, note that in the first sample the term-premium and EH channels of GAP and INF move in opposite directions. For example, the drop in GAP in response to the shock is the result of a sharp drop in activity due to the EH channel, which is significantly moderated by the term-premium channel. For both GAP and INF, the term premium channel in the first sample acts to diminish the effectiveness of monetary policy shocks.

To understand this last point, let me focus for a moment on the IRF of PC_2 shown in the top right corner of Figure 1.6. This chart shows that the movements in term premia following a monetary impulse pressure the slope factor to move up (not down, as the EH channel would predict). In fact, going back to Figure 1.3, note that in this sample the IRFs of term premia are increasing in bond maturity – thus they exert upward pressure on the yield curve slope. In turn, this positive pressure on the slope factor feeds back into the macroeconomy through equation (1.3), causing the term premium channel to pressure GAP and INF in the direction opposite of the EH channel.

In terms of the 1979:4-2007:4 sample, note that the term-premium channel barely moves in the responses of both GAP and INF. In fact, for these variables the lines corresponding to the total IRF and the EH channel are almost exactly on top of each other, as shown in the bottom panel of Figure 1.6. This striking difference across samples occurs mostly because the term-premium channel pressures the slope factor PC_2 by much less in the second than in the first sample (see the bottom right graph in Figure 1.6).

To conclude this section, I study the variance decomposition of the yield curve

Table 1.2: Forecast Error Variance Due to Monetary Policy Shocks

	1959:1 - 1979:3			1979:3 - 2007:4		
	$h=4$	$h=12$	$h=20$	$h=4$	$h=12$	$h=20$
y_{20}	46.0 $(21.2, 59.2)$	24.2 $(9.8, 48.0)$	21.0 $(9.2, 45.2)$	49.4 $(25.8, 63.6)$	36.8 $(15.1, 50.4)$	29.8 $(10.9, 43.6)$
y_{20}^{EH}	5.2 $(0.7, 31.6)$	15.8 $(1.0, 48.5)$	16.8 $(1.0, 47.1)$	28.8 $(1.9, 50.1)$	22.3 $(1.0, 43.9)$	21.2 $(0.8, 40.8)$
tp_{20}	65.2 $(32.3, 76.8)$	60.9 $(22.7, 68.1)$	54.9 $(20.4, 63.6)$	21.3 $(5.2, 39.8)$	17.5 $(4.7, 34.7)$	14.0 $(5.0, 30.1)$

Notes: Numbers in parentheses are 95% bootstrapped confidence intervals.

components. Table 1.2 shows the portion of the variance of h-period ahead forecasts of $y_{m,t}$, $y_{m,t}^{EH}$ and $tp_{m,t}$ that is due to identified monetary policy shocks. For purposes of presentation, I only report here the cases where m is equal to 20 quarter and h is set to 4, 12, and 20 quarters. The left panel corresponds to the 1959:1-1979:3 sample; the right panel refers to the 1979:4-2007:4 sample. The numbers in parentheses are bootstrapped 95% confidence intervals.

Even though the bootstrapped confidence intervals are quite wide, at all horizons considered in Table 1.2 monetary policy shocks explain a substantial fraction of the forecast error variance of the 5-year yield and its premium and EH components. In the 1959:1-1979:3 sample, in particular, monetary policy shocks explain about 60% of the 5-year term-premium forecast error variance. As we look at the more recent sample, though, this fraction drops quite substantially, to around 15 to 20%. This leads me to my fourth result:

Result IV: *A substantial portion of the term-premia forecast error variance is due to monetary policy shocks. In the 1959:1-1979:3 sample, these shocks explain around 60% of the forecast error variance of the 5-year term-premium. In the second sample, this fraction drops to around 15 to 20%.*

This result is surprising in light of some recent theoretical results obtained from general equilibrium models. Rudebusch and Swanson (2008) study the nominal term premium in the context of a dynamic stochastic general equilibrium model (DSGE) with long-run risks as in Bansal and Yaron (2004). They find that the variability of term premia is dictated by technological shocks. Monetary policy shocks are responsible for only a small portion of the variability in term premia. In contrast, the results in Table 1.2 show that monetary policy shocks, at least in the context of the model developed here, are crucial to understanding the variability in term-premia.

1.5 Shocks to Future Monetary Policy Expectations

Up until now I have considered only shocks that directly affect the short-term interest rate r_t. However, one of the major challenges of central banking is to coordinate the public's *future* expectations, allowing the monetary authority to reach its objectives with minimum costs[30]. In particular, modern macroeconomic theory shows that expectations about the future directions of key macroeconomic variables such as GDP, inflation and the short-rate are crucial to determining the success of current monetary policy actions. In fact, the introduction to Michael Woodford's classic book states that:

> "For successful monetary policy is not so much a matter of effective control of overnight interest rates as it is shaping market expectations of the way in which interest rates, inflation, and income are likely to evolve over the coming year and later". [Woodford (2003)]

[30] The importance of expectations' coordination became clear with the advent of micro-founded models for the analysis of monetary policy. These models highlight that future expectations play a crucial role in the decision making of private agents, which will in turn affect the macroeconomic equilibrium. See Woodford (2003).

It follows that central banks' communication with the private sector is now considered to be one of the most important instruments for the conduct of monetary policy. In the words of Woodford (2003):

> "... insofar as it is possible for the central bank to affect expectation, this should be an important tool of policy stabilization... Not only do expectations about policy matter, but, at least under current conditions, very little *else* matters" (author's italic).

In this section I study the macroeconomic implications of shocks to expectations about the future path of the monetary policy instrument r_t. The MTSM from Section 1.3 is a particularly convenient tool for this study because it allows for the decomposition of long-term yields into policy expectations (i.e. the EH component of $y_{m,t}$) and term premia.

Section 1.5.1 shows how the unspanned MTSM can be used to identify the future policy expectations shock discussed above. Section 1.5.2 studies the transmission of shocks to future policy expectations to the macroeconomy. Since the consensus that expectations are crucial for monetary policy started to form in the mid 1980's, I will only show results for the more recent sample, 1979:4-2007:4[31], in this section.

1.5.1 Identifying Shocks to Monetary Policy Expectations

I define a shock to policy expectations m periods ahead as an exogenous impulse in $y_{m,t}^{EH}$ that is neither accompanied by an instantaneous movement in the short-rate r_t nor by an instantaneous change in the macro variables in M_t. Because the EH states that (up to a Jensen's inequality term) the m-period yield equals the average short-rate from t until maturity, the expectations shock that I consider

[31] Results for the pre-Volcker sample are available upon request.

directly affects the *future* path of the traditional monetary policy instrument, r_t, from the current period until maturity. The requirement that r_t does not move on impact guarantees that the shock will not be confused with the standard monetary policy shock considered in Sections 1.3 and 1.4. Also, the assumption that M_t does not move on impact maintains a coherence with the monetary policy transmission mechanism discussed in the previous sections.

Two points about my shock definition are worth mentioning. Firstly, although the short rate is assumed to remain constant on impact, it is allowed to move endogenously in the periods after the shock hits. Second, I allow the term premia to move instantly when the shock hits.

To make the discussion more concrete, I now show how to identify shocks to monetary policy expectations in the MTSM from Section 1.3. [32] The main idea is to use a recursive identification scheme as described in Section 1.3.2, except that instead of using the state equation (1.3) to identify the shocks, I now use a rotated version of this equation. In the rotated model, the quantity I want to shock, $y^{EH}_{m,t}$, appears explicitly as an endogenous variable in the rotated state vector.

More precisely, let the rotated state vector be given by $\widetilde{Z}_t \equiv \begin{bmatrix} M_t' & r_t & y^{EH}_{m,t} & tp_{m,t} \end{bmatrix}'$. Appendix 1.F shows that there exists a 6×1 vector W_0 and an invertible 6×6 matrix W_1 such that

$$\underbrace{\begin{pmatrix} M_t \\ r_t \\ y^{EH}_{m,t} \\ tp_{m,t} \end{pmatrix}}_{\equiv \widetilde{Z}_t} \equiv W_0 + W_1 \underbrace{\begin{pmatrix} M_t \\ r_t \\ PC_{1,t} \\ PC_{2,t} \end{pmatrix}}_{\equiv Z_t} \tag{1.7}$$

The choices of W_0 and W_1 rely on the fact that in the unspanned MTSM $y^{EH}_{m,t}$ and $tp_{m,t}$ are linear functions of Z_t, namely $y^{EH}_{m,t} = A^{EH}_m + B^{EH}_m Z_t$ and $tp_{m,t} = \left(A_m - A^{EH}_m\right) + \left([0\ B_m] - B^{EH}_m\right) Z_t$. Note that the decomposition of $y_{m,t}$ into policy

[32] See Appendix 1.F for more details of the identification methodology.

expectations, $y^{EH}_{m,t}$, and term premium, $tp_{m,t}$, appears explicitly in the last two entries of the rotated state vector. Appendix 1.F shows that $\widetilde{Z}_t$ evolves according to

$$\widetilde{Z}_t = \widetilde{\Theta}^{\mathbb{P}}_{0Z} + \widetilde{\Theta}^{\mathbb{P}}_{1Z}\widetilde{Z}_{t-1} + \sqrt{W_1\Sigma_Z W_1'}\epsilon^{\mathbb{P}}_{zt}$$

where $\widetilde{\Theta}^{\mathbb{P}}_{0Z}$ and $\widetilde{\Theta}^{\mathbb{P}}_{1Z}$ are known functions of W_0, W_1, $\Theta^{\mathbb{P}}_{0Z}$ and $\Theta^{\mathbb{P}}_{1Z}$.

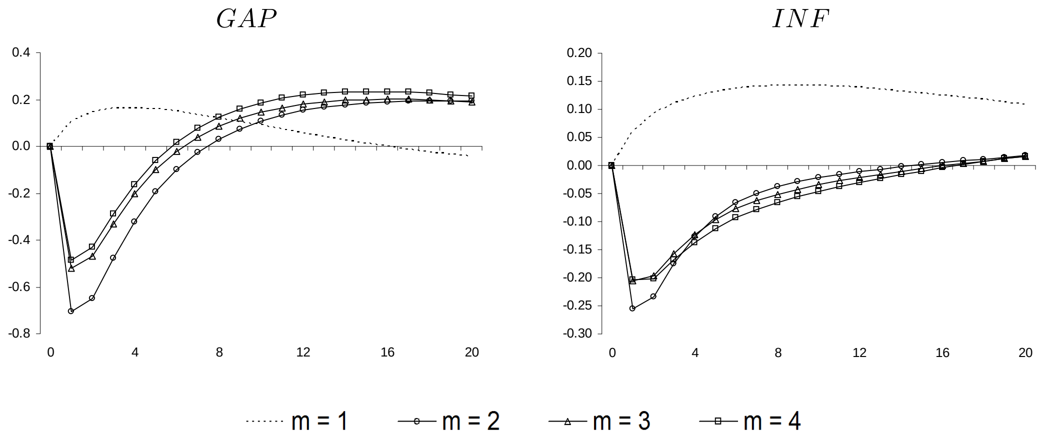
As before, I identify shocks using the recursiveness assumption and thus assume that $\sqrt{W_1\Sigma_Z W_1'}$ is the Cholesky factor associated with matrix $W_1\Sigma_Z W_1'$. The ordering of the variables in $\widetilde{Z}_t$ is again crucial. In the particular way I defined $\widetilde{Z}_t$ in equation (1.7), an exogenous shock to $y^{EH}_{m,t}$ agrees with my definition of a policy expectations shock. That is, on impact all three variables in M_t and the short rate shows no response to the shock; on the other hand, the term premium is allowed to move instantly. Only one period after the shock hits, all variables in $\widetilde{Z}_t$ will be allowed to respond to the shock.

1.5.2 The Transmission of Policy Expectations Shocks

Figure 1.7 plots the IRFs of GAP and INF to a +100-basis-point shock to future monetary policy expectations. In each chart, the lines correspond to alternative choices for m. In particular, the dotted lines refer to IRFs when m equals 1, in which case the shock collapses to the standard monetary policy shock studied in Section 1.4 (note that the scales of the vertical axes in this figure are different from the ones in Figure 1.2). The marked lines, on the other hand, refer to IRFs to policy expectations shocks for the cases where m is equal to 2, 3 and 4.

Consider first the IRFs to a standard monetary policy shock, i.e. $m = 1$. In this model, this shock generates counter-intuitive responses for the macro variables shown in Figure 1.4. That is, both GAP and INF *increase* persistently following the standard monetary policy shock (even though we showed in Section 1.4 that

Figure 1.7: Impulse Responses to an Expected Monetary Policy Shock, 1979:4-2007:4



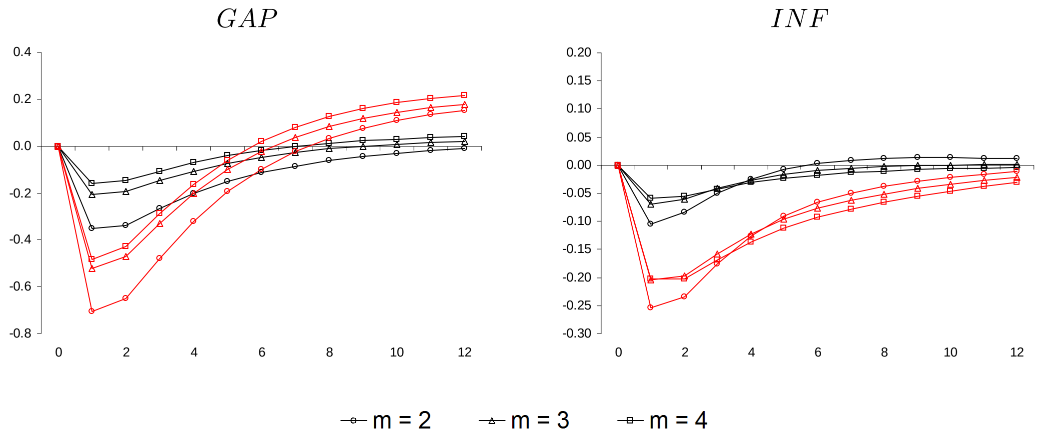
Notes: Impulse responses to an increase of 100 basis points in $y^{EH}_{m,t}$. The horizontal axes measure the number of quarters after the shock. The units of GAP and INF are not interpretable.

these movements were not statistically significant), contrary to what is suggested by monetary theory. On the other hand, a shock of the same magnitude to policy expectations 2, 3 or 4 periods ahead causes both GAP and INF to *drop* persistently. Intuitively, once private agents learn that monetary policy on average will be tighter from now until m, both activity and inflation drop for several quarters. Interestingly, these drops occur quite quickly, with the trough of the responses occurring just one quarter after the shock hits. Also, increasing the shock horizon, m, from 2 to 4 reduces the initial impact of the shock on both GAP and INF. I summarize these results as follows:

Result V: *In the 1979:4-2007:4 sample, an exogenous shock to monetary policy expectations leads to drops in activity and inflation. The troughs of these responses occur only one quarter after the shock hits. Increasing the horizon of the expected future policy shock from 2 to 4 quarters attenuates the initial drops in GAP and INF.*

As mentioned before, an important advantage of using the MTSM to study

Figure 1.8: IRF to Shocks to $y_{m,t}^{EH}$ and $y_{m,t}$, 1979:4-2007:4



Notes: Impulse responses to an increase of 100 basis points in $y_{m,t}$ (black lines) and $y_{m,t}^{EH}$ (red lines) for different values of m. The horizontal axes measure the number of quarters after the shock. The units of GAP and INF are not interpretable.

policy expectations shocks lies in the fact that this model allows for the separation of premium and EH component of yields[33]. Therefore, I now ask the following question: what would be the difference in the IRFs shown in Figure 1.7 had I ignored the term premium component of long-term yields and directly shocked $y_{m,t}$ instead of $y_{m,t}^{EH}$?

To answer this question, I compare the IRFs to the policy expectations shocks shown in Figure 1.7 to the new IRFs following a shock to $y_{m,t}$. Figure 1.8 plots the results. The IRFs to $y_{m,t}^{EH}$ shocks correspond to the red lines; the black lines are responses to shocks to $y_{m,t}$. For each shock, I continue to show responses for the case where m is set to 2, 3 and 4.

Note that for all policy horizons (m) shown in Figure 1.8, the IRFs following a shock to $y_{m,t}$ are less pronounced than those following a shock to $y_{m,t}^{EH}$. In general, the responses to a $y_{m,t}^{EH}$ shock are about two times stronger on impact than the corresponding IRFs to a $y_{m,t}$ shock. Therefore, if I had ignored the term-premium

[33]The importance of separating policy expectations and premia in the context of monetary policy shocks' identification was also highlighted by Piazzesi and Swanson (2008).

component of long-term yields when identifying policy expectations shocks, I would have gotten substantially weaker IRFs than in the case where this component is accounted for. This result is summarized below:

Result VI: *If the objective is to identify shocks to future policy expectations, then shocking* $y_{m,t}$ *instead of the true model-implied policy expectations* $y_{m,t}^{EH}$ *leads to biased IRFs of* GAP *and* INF*. These IRFs following a shock to* $y_{m,t}$ *are substantially weaker than the corresponding ones for the case where* $y_{m,t}^{EH}$ *is shocked. This is because the shock to* $y_{m,t}^{EH}$ *controls for the term premium component of the long-term yield.*

1.6 Conclusions

This chapter has shown that, in the context of a Macro-Affine Term Structure Model, movements in bond premia may play an important role in the monetary transmission mechanism. I found that, during the pre-Volcker sample, a large portion of the movements in inflation and economic activity following a monetary policy shock are due to movements in term premia that feed back into the economy. In the post-Volcker period, in contrast, this channel of monetary transmission is empirically irrelevant. I also have shown that in the post-Volcker period, shocks to policy expectations produce more pronounced and more plausible responses for the macroeconomic variables than do standard shocks to the contemporaneous value of the monetary policy instrument.

My findings show that accounting for premia is important not only to understanding movements in bond prices, as highlighted by Dai and Singleton (2002), but also to understanding the dynamics of key macroeconomic variables, such as inflation and output. This reaffirms, in the context of a monetary policy transmission analysis, the crucial role that financial assets play in the behavior of the macroeconomy.

Appendix 1.A Extraction of *GAP* and *INF* from a Rich Dataset

This appendix describes the details behind the estimates of GAP and INF shown in Section 1.2. Suppose that one wants to extract a common factor from a set of K different observed economic indicators $\{x_{1,t}, ..., x_{K,t}\}$. Following Stock and Watson (1988), one way to estimate this common factor is to consider the following state-space model:

Signal Equation:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{K,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_K \end{bmatrix} + \begin{bmatrix} \gamma_1 & 1 & 0 & \cdots & 0 \\ \gamma_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_K & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \mathcal{C}_t \\ \lambda_{1,t} \\ \vdots \\ \lambda_{K,t} \end{bmatrix}$$

Transition Equation:

$$\begin{bmatrix} \mathcal{C}_t \\ \lambda_{1,t} \\ \vdots \\ \lambda_{K,t} \end{bmatrix} = \begin{bmatrix} \phi_{\mathcal{C}} & 0 & \cdots & 0 \\ 0 & \phi_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_K \end{bmatrix} \begin{bmatrix} \mathcal{C}_{t-1} \\ \lambda_{1,t-1} \\ \vdots \\ \lambda_{k,t-1} \end{bmatrix} + \begin{bmatrix} \nu_{\mathcal{C},t} \\ \nu_{1,t} \\ \vdots \\ \nu_{K,t} \end{bmatrix}$$

where $\mathcal{C}_t$ is a common factor, $\lambda_{i,t}$ for $i = 1, ..., K$ are idiosyncratic factors (one for each economic indicator). Also, the *iid* innovations follow $\nu_{\mathcal{C},t} \sim N(0, 1)$ and $\nu_{i,t} \sim N\left(0, \sigma_i^2\right)$ for $i = 1, ..., K$, and are assumed to be uncorrelated with each other at all lags. Therefore, each observed economic indicator loads both on the common and on an idiosyncratic factors.

For simplicity, I assume that both the common and the idiosyncratic factors follow AR(1) processes, even though more complex lag structures could also be considered. The common-factor model is estimated via Maximum Likelihood using standard Kalman filtering techniques. Details about the economic indicators used to extract GAP and INF are shown in Table 1.3. Note that the Kalman Filter is particularly convenient in handling missing observations in the economic indicators' time series, a feature that occurs often in my dataset.

Appendix 1.B No-Arbitrage Bond Pricing

This appendix shows how to derive the first-order difference equations that determine A_m and B_m in equation (1.4). No-arbitrage pricing implies that the price of an m-period bond, $V_{m,t}$, is determined by $V_{m,t} = E_t^{\mathbb{Q}}\left[e^{-i_t} V_{m-1,t+1}\right]$. Start by guessing that the model-implied $V_{m,t}$ follows an affine function of the pricing factors, that is $V_{\tau,t} = \exp(\tilde{A}_\tau + \tilde{B}_\tau \mathcal{P}_t)$. Using equations (1.1) and (1.2) this guess is verified as follows:

$$\begin{aligned} V_{m,t} &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t + \tilde{A}_{m-1} + \tilde{B}_{m-1}\mathcal{P}_{t+1}\right)\right] \\ &= E_t^{\mathbb{Q}}\left[\exp\left(-\rho_0 - \rho_1'\mathcal{P}_t + \tilde{A}_{m-1} + \tilde{B}_{m-1}\Theta_{0\mathcal{P}}^{\mathbb{Q}} + \tilde{B}_{m-1}\left(\Theta_{1\mathcal{P}}^{\mathbb{Q}} + I_N\right)\mathcal{P}_t + \tilde{B}_{m-1}\Sigma_{\mathcal{P}}\epsilon_{\mathcal{P}t+1}^{\mathbb{Q}}\right)\right] \\ &= \exp\left(\tilde{A}_{m-1} + \tilde{B}_{m-1}\Theta_{0\mathcal{P}}^{\mathbb{Q}} + \frac{1}{2}\tilde{B}_{m-1}\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}'\tilde{B}_{m-1}' - \rho_0 + \left[\tilde{B}_{m-1}\left(\Theta_{1\mathcal{P}}^{\mathbb{Q}} + I_N\right) - \rho_1'\right] X_t\right) \end{aligned}$$

Table 1.3: Economic Indicators Used to Construct the Commmon Macro Factors

long name:	short name:	series ID*:	sample:	transf.:
(1) SERIES USED IN GAP:				
(i) Industrial Production Index	INPROD	INDPRO	58:4 - 07:4	DT
(ii) Total Nonfarm Payrolls	PAYROLL	PAYEMS	58:4 - 07:4	DT
(iii) Real Personal Consumption Expenditures	PCE	PCEC96	59:1 - 07:4	DT
(iv) Real GDP Index	GDP	GDPC96	58:4 - 07:4	DT
(v) ISM manufacturing index (New Orders)	ISM	NAPMNOI	58:4 - 07:4	LEV
(vi) Total Housing Starts	HOUST	HOUST	59:1 - 07:4	LEV
(vii) Civilian Unemp. Rate	URATE	UNRATE	58:4 - 07:4	LEV
(2) SERIES USED IN INF:				
(i) CPI less food & energy	CPI	CPILFESL	58:4 - 07:4	QG
(ii) Finished Goods PPI less food & energy	PPI	PPILFE	74:2 - 07:4	QG
(iii) PCE Deflator less food and energy	PCEDEF	PCEPILFE	59:2 - 07:4	QG
(iv) GDP deflator	GDPDEF	GDPDEF	58:4 - 07:4	QG
(v) Average Hourly Earnings	EARN	AHEMAN	58:4 - 07:4	QG

Notes: The transformations applied to the series are the following: DT = quadratic deterministic trend removed, QG = series transformed into quarterly growth rates , and N = no transformation applied. All series were obtained from the St. Louis Fed database. * corresponds to ID codes for the St. Louis Fed database.

where the third equality uses the properties of the log-normal distribution. It follows that:

$$\begin{aligned}\tilde{A}_m - \tilde{A}_{m-1} &= \tilde{B}_{m-1}\Theta^{\mathbb{Q}}_{0\mathcal{P}} + \frac{1}{2}\tilde{B}_{m-1}\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}}\tilde{B}'_{m-1} - \rho_0 \\ \tilde{B}_m - \tilde{B}_{m-1} &= \tilde{B}_{m-1}\Theta^{\mathbb{Q}}_{1\mathcal{P}} - \rho'_1\end{aligned}$$

where $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0$. The m-period continuously compounded yield, $y_{m,t} \equiv -\frac{P_{m,t}}{m}$, is then given by:

$$y_{m,t} = A_m + B_m X_t$$

where $A_m \equiv -\frac{\tilde{A}_m}{m}$ and $B_m \equiv -\frac{\tilde{B}_m}{m}$.

Appendix 1.C Details of the Econometric Methodology

Joslin, Singleton, and Zhu (2011) propose a set of normalizations that not only allow for identification of the model described in Section 1.3.1, but also simplifies the task of finding a global maximum of the likelihood function. More specifically, the model with observable pricing factors has the property that the parameters governing bond pricing, $\left(\Theta^{\mathbb{Q}}_{0\mathcal{P}}, \Theta^{\mathbb{Q}}_{1\mathcal{P}}, \rho_0, \rho_1\right)$, are uniquely mapped into a smaller set of parameters $\left(r^{\mathbb{Q}}_{\infty}, \lambda^{\mathbb{Q}}, \Sigma_{\mathcal{P}\mathcal{P}}\right)$, where $r^{\mathbb{Q}}_{\infty}$ represents the long-run mean of the short rate under $\mathbb{Q}$, and $\lambda^{\mathbb{Q}}$ is the N-vector of ordered eigenvalues of $\Theta^{\mathbb{Q}}_{1\mathcal{P}}$.

The bond portfolios contained in $\mathcal{P}_t$ are assumed to be priced perfectly by the model. The observed yields, except the short-rate (which is included in $\mathcal{P}_t$), are allowed to differ from their model-implied counterpart through a $(J-1)$-vector of measurement errors $u_t \sim N\left(0, \omega^2 I_{J-1}\right)$. Note that it is assumed for simplicity that the variance of the measurement errors is the same across all long-term yields used to fit the model.

The likelihood function of the model is then given by

$$\mathcal{L}\left(y^{obs}_t, Z_t|Z_{t-1}; \Phi\right) = \mathcal{L}\left(y^{obs}_t|Z_t, Z_{t-1}; r^{\mathbb{Q}}_{\infty}, \lambda^{\mathbb{Q}}, \Sigma_Z, \omega\right) \times \mathcal{L}\left(Z_t|Z_{t-1}; \Theta^{\mathbb{P}}_{0Z}, \Theta^{\mathbb{P}}_{1Z}, \Sigma_Z\right)$$

where y^{obs}_t contains the yields observed with measured errors. A convenient feature of the normalization proposed by Joslin, Singleton, and Zhu (2011) is that the ML estimate of $\Theta^{\mathbb{P}}_{0Z}$ and $\Theta^{\mathbb{P}}_{1Z}$, that is $\widehat{\Theta}^{\mathbb{P}}_{0Z}$ and $\widehat{\Theta}^{\mathbb{P}}_{1Z}$, are obtained by OLS. Conditional on $\widehat{\Theta}^{\mathbb{P}}_{1}$, an optimization algorithm searches for the values of $r^{\mathbb{Q}}_{\infty}$, $\lambda^{\mathbb{Q}}$, Σ_Z, and ω in order to find the global maximum of the likelihood function.

Note that the search algorithm in the second stage of the estimation procedure usually converges very quickly because good starting values are easy to obtain. First, good starting values for the parameters in Σ_Z can be obtained by running an OLS regression of Z_t on Z_{t-1}. Also, good starting values for $r^{\mathbb{Q}}_{\infty}$ and $\lambda^{\mathbb{Q}}$ are not difficult to obtain because these parameters are rotation-invariant and therefore carry economic interpretation.

Appendix 1.D Risk Premium Accounting in the MTSM

The most general form of the MTSM described in Section 1.3 is given by[34]

$$\begin{aligned} Z_t &= \Theta_{0Z}^{\mathbb{Q}} + \Theta_{1Z}^{\mathbb{Q}} Z_{t-1} + \sqrt{\Sigma_Z}\epsilon_{Zt}^{\mathbb{Q}} \\ Z_t &= \Theta_{0Z}^{\mathbb{P}} + \Theta_{1Z}^{\mathbb{P}} Z_{t-1} + \sqrt{\Sigma_Z}\epsilon_{Zt}^{\mathbb{P}} \\ r_t &= \rho_0 + \rho_{1Z}' Z_t \end{aligned}$$

where $Z_t \equiv [\ M_t' \quad \mathcal{P}_t'\ ]'$. Note that, differently from the model described in Section 1.3, here I let all factors, macro and yield-based, be treated as pricing factors (i.e. I do not impose that the macro factors are unspanned by the yield curve). The case of unspanned macro factors is easily recovered by setting $\rho_{1Z} = \begin{bmatrix} 0 & \rho_1 \end{bmatrix}'$.

In a risk neutral world, the risk-adjusted ($\mathbb{Q}$) and the actual ($\mathbb{P}$) probability measures coincide, which implies that bond prices would be given by

$$V_{m,t}^{EH} = E_t^{\mathbb{P}}\left[e^{-r_t} V_{m-1,t+1}^{EH}\right]$$

Note that this would imply that, up to a convexity term, the EH holds. I follow the derivations in Appendix 1.B and, also in the risk neutral world, I guess and verify that the log of bond price is an affine function of Z_t, i.e. $V_{m,t}^{EH} = \exp(\tilde{A}_m^{EH} + \tilde{B}_m^{EH} Z_t)$. I therefore obtain

$$\begin{aligned} y_{m,t}^{EH} &= A_m^{EH} + B_m^{EH} Z_t \\ \tilde{A}_m^{EH} &= \tilde{A}_{m-1}^{EH} + \tilde{B}_{m-1}^{EH}\Theta_{0Z}^{\mathbb{P}} + \frac{1}{2}\tilde{B}_{m-1}^{EH}\sqrt{\Sigma_Z}\left(\sqrt{\Sigma_Z}\right)'\left(\tilde{B}_{m-1}^{EH}\right)' - \rho_0 \\ \tilde{B}_m^{EH} &= \tilde{B}_{m-1}^{EH}\Theta_{1Z}^{\mathbb{P}} - \rho_{1Z} \end{aligned}$$

where $A_m^{EH} \equiv -\frac{\tilde{A}_m^{EH}}{m}$ and $B_m^{EH} \equiv -\frac{\tilde{B}_m^{EH}}{m}$. Finally, defining the term premium at maturity m, $tp_{m,t}$, as the deviation of $y_{m,t}$ from m-period yield consistent with the EH, it is easy to see that

$$\begin{aligned} tp_{m,t} &\equiv y_{m,t} - y_{m,t}^{EH} \\ &= \left(A_m - A_m^{EH}\right) + \left(\begin{bmatrix} 0 & B_m \end{bmatrix} - B_m^{EH}\right) Z_t \end{aligned}$$

Appendix 1.E Decomposing the Impulse Response Functions

Let the column of $\sqrt{\Sigma_Z}$ that corresponds to the short-rate be given by $\begin{pmatrix} \underset{1\times 3}{0'} & \underset{1\times 3}{\omega'} \end{pmatrix}'$. This follows from the assumption that the short-rate is ordered as the first bond portfolio and is below the macro variables in M_t. The impulse response of Z_t after h periods since a one standard deviation monetary policy shock hits, $\Psi_{Z,h}$, is given by:

$$\Psi_{Z,h} = \left(\Theta_{1Z}^{\mathbb{P}}\right)^h \begin{pmatrix} 0 \\ \omega \end{pmatrix}$$

[34] In this paper I am restricting attention to models where the factors, Z_t, are markovian. More general models can be obtained by relaxing this assumption.

From Section 1.3, it is easy to write the model-implied yields as an affine function of the state-vector Z_t:

$$\begin{aligned} y_t &= \underset{7\times 1}{A} + \begin{pmatrix} \underset{7\times 3}{0} & \underset{7\times 3}{\widetilde{B}} \end{pmatrix} Z_t \\ &= \underset{7\times 1}{A} + \underset{7\times 6}{B} Z_t \end{aligned}$$

where $A = \begin{pmatrix} A_1 & A_2 & A_4 & \cdots & A_{20} \end{pmatrix}'$, $\widetilde{B} = \begin{pmatrix} B_1' & B_2' & B_4' & \cdots & B_{20}' \end{pmatrix}'$, and $B = \begin{pmatrix} 0 & \widetilde{B} \end{pmatrix}$. Setting the prices of risk to zero, the version of y_t that is consistent with the EH is an affine function of Z_t, which implies that the residual term premium is also affine in Z_t:

$$\begin{aligned} y_t^{EH} &= \underset{7\times 1}{A}^{EH} + \underset{7\times 6}{B}^{EH} Z_t \\ tp_t &= \underset{7\times 1}{A}^{tp} + \underset{7\times 6}{B}^{tp} Z_t \end{aligned} \tag{1.8}$$

where $y_t^{EH} \equiv \begin{pmatrix} y_{1,t}^{EH} & y_{2,t}^{EH} & y_{4,t}^{EH} & \cdots & y_{20,t}^{EH} \end{pmatrix}'$ and $tp_t \equiv \begin{pmatrix} tp_{1,t} & tp_{2,t} & tp_{4,t} & \cdots & tp_{20,t} \end{pmatrix}'$. Note that $A^{tp} = A - A^{EH}$ and $B^{tp} = B - B^{EH}$.

Since $\mathcal{P}_t = Py_t$, the impulse response of Z_t when $h = 0$ is given by:

$$\begin{aligned} \Psi_{Z,0} &= \begin{pmatrix} 0 \\ \omega \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ P\Psi_{y,0} \end{pmatrix} \end{aligned}$$

where $\Psi_{y,0}$ is the impulse response of y_t on impact. Moreover, $\Psi_{y,0}$ can be decomposed into an EH and a term premium component using equations (1.8):

$$\begin{aligned} \Psi_{y,0} &= B\begin{pmatrix} 0 \\ \omega \end{pmatrix} \\ &= \Psi_{y^{EH},0} + \Psi_{tp,0} \\ &= B^{EH}\begin{pmatrix} 0 \\ \omega \end{pmatrix} + B^{tp}\begin{pmatrix} 0 \\ \omega \end{pmatrix} \end{aligned}$$

Therefore, $\Psi_{Z,0}$ can be decomposed into

$$\Psi_{Z,0} = \begin{pmatrix} 0 \\ P\Psi_{y^{EH},0} \end{pmatrix} + \begin{pmatrix} 0 \\ P\Psi_{tp,0} \end{pmatrix}$$

and it follows that the impulse response of Z_t to a monetary policy shock h periods after the shock occurs is given by

$$\Psi_{Z,h} = \underbrace{\left(\Theta_{1Z}^{\mathbb{P}}\right)^h \begin{pmatrix} 0 \\ P\Psi_{y^{EH},0} \end{pmatrix}}_{\Psi_{Z,h}^{EH}} + \underbrace{\left(\Theta_{1Z}^{\mathbb{P}}\right)^h \begin{pmatrix} 0 \\ P\Psi_{tp,0} \end{pmatrix}}_{\Psi_{Z,h}^{tp}}$$

where $\Psi_{Z,h}^{EH}$ and $\Psi_{Z,h}^{tp}$ are respectively the EH and the term premium components of the impulse response to a monetary policy shock.

Appendix 1.F Identifying Shocks to $y_{m,t}^{EH}$

To be able to shock monetary policy expectations and term premia, I will rotate $Z_t \equiv [\ M_t' \quad \mathcal{P}_t'\ ]'$ so that the quantities shocked appears explicitly in the rotated state vector $\widetilde{Z}_t$. Let W_0 be a $3+N \times 1$ vector and W_1 be an invertible $3+N \times 3+N$ matrix such that

$$\widetilde{Z}_t \equiv W_0 + W_1 Z_t$$

i.e. W_0 and W_1 rotate Z_t linearly. If, for example one wants to rotate $Z_t \equiv [\ M_t' \quad \mathcal{P}_t'\ ]'$ in order to obtain $\widetilde{Z}_t = [\ GAP_t \quad INF_t \quad COMM_t \quad r_t \quad y_{m,t}^{EH} \quad tp_{m,t}\ ]'$, the choices of W_0 and W_1 would be

$$W_0 \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ A_m^{EH} \\ A_m - A_m^{EH} \end{pmatrix} \qquad W_1 \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ B_m^{(1)} & B_m^{(2)} & B_m^{(3)} & B_m^{(4)} & B_m^{(5)} & B_m^{(6)} \\ B_m^{tp(1)} & B_m^{tp(2)} & B_m^{tp(3)} & B_m^{tp(4)} & B_m^{tp(5)} & B_m^{tp(6)} \end{pmatrix}$$

where $B_m^{(i)}$ is the i^{th} entry in the vector B_m and $B_m^{tp} \equiv [0 \quad B_m] - B_m^{EH}$ with $B_m^{tp(i)}$ being the i^{th} entry in B_m^{tp}.

Using the $\widetilde{Z}_t$ definition, equation (1.3) in the text can be written in terms of the rotated state vector $\widetilde{Z}_t$:

$$\begin{aligned} W_1^{-1}\left(\widetilde{Z}_t - W_0\right) &= \Theta_{0Z}^{\mathbb{P}} + \left[\Theta_{1Z}^{\mathbb{P}} + I\right] W_1^{-1}\left(\widetilde{Z}_{t-1} - W_0\right) + \sqrt{\Sigma_Z}\epsilon_{zt}^{\mathbb{P}} \\ &\Leftrightarrow \\ \widetilde{Z}_t &= \widetilde{\Theta}_{0Z}^{\mathbb{P}} + \widetilde{\Theta}_{1Z}^{\mathbb{P}} \widetilde{Z}_{t-1} + \left(W_1\sqrt{\Sigma_Z}\right)\epsilon_{zt}^{\mathbb{P}} \end{aligned}$$

where $\widetilde{\Theta}_{0Z}^{\mathbb{P}} = W_1\left(\Theta_{0Z}^{\mathbb{P}} - \Theta_{1Z}^{\mathbb{P}} W_1^{-1} W_0\right)$ and $\widetilde{\Theta}_{1Z}^{\mathbb{P}} = W_1\left(\Theta_{1Z}^{\mathbb{P}} + I\right) W_1^{-1}$. A recursive identification of the shocks that hit the rotated state vector can therefore be obtained by taking the Cholesky decomposition of the variance of the rotated residuals $W_1 \Sigma_Z W_1'$. Moreover, the dynamics following the shock can be obtained through the rotated matrices of VAR coefficients, namely $\widetilde{\Theta}_{0Z}^{\mathbb{P}}$ and $\widetilde{\Theta}_{1Z}^{\mathbb{P}}$.

2 Switching Monetary Policy Regimes and the Nominal Term Structure

2.1 Introduction

The U.S. nominal yield curve on average has been steeper since the mid-1980s than during the Great Inflation of the 1970s. This is puzzling because, in general, the *average* slope of the yield curve is thought to reflect risk premia demanded by bond investors. Therefore, in periods of high macroeconomic uncertainty such as the Great Inflation, in principle one should expect bonds to pay higher premia – that is, term premia – than they would from the mid-1980s until 2007 when macroeconomic uncertainty had reached historically low levels (the 'Great Moderation'). I refer to this apparently inconsistent relation between the yield curve's slope and macroeconomic uncertainty as the '*Slope-Volatility Puzzle*'. In this chapter, I propose a theory that is based on switching macroeconomic regimes to explain this puzzle.

In the proposed framework, investors incorporate the possibility that the economy switches across different regimes into their beliefs. In particular, if the nominal short-rate fluctuates around different means across regimes, then the average slope of the yield curve in general will reflect not only the standard term premium, but also a new term attributable to the Expectations Hypothesis in the presence of regime shifts. I call this '*level risk*', and it reflects the risk that, conditional on the economy being in a low short-rate regime, long-term bonds will lose value in the case of a shift to higher short-rate regimes. The level risk will be positive in this case. Similarly, a negative level risk occurs when the short-rate is currently high and there is a possibility of switching to low short-rate regimes, which represents gains for bond holders.

I estimate the level risk during the post-World War II period using a simple

Markov-Switching Vector Autoregression of the U.S. economy. This model identifies a high macroeconomic volatility regime that corresponds broadly to the Great Inflation of the 1970s. This regime is also characterized by a high average short-rate (and high inflation) and is not very persistent, with an average duration of only 7.7 years. On the other hand, the Great Moderation period appears in the model as the realization of a highly persistent regime (average duration of almost 35 years) associated with low macroeconomic volatility and a low average short-rate. If investors form beliefs according to this model, then the level risk was large and negative during the Great Inflation. Intuitively, because agents perceived this regime as relatively short-lived, there was as high probability that the short-rate would switch to lower levels in the future, causing nominal bonds to gain value. Since the mid-1980s, in contrast, the level risk has been more moderate in magnitude and positive. In this case, investors ascribed a relatively low probability to a switch back to the high short-rate regime of the 1970s, which would have caused nominal bonds to lose value. Extracting the estimated level risks from observed yield curve slope measures, I find that term premia on average were substantially *higher* during the Great Inflation of the 1970s than they have been since the mid-1980s. Therefore, my first main conclusion in this chapter is that the Slope-Volatility Puzzle can be explained by differences in level risks across regimes.

One important implication of the level risk is that term structure models that do not allow for Markov-Switching regimes tend to generate biased estimates of term premia. In other words, if agents consider the possibility of regime shifts when forming expectations, then models that do not take this into account will have the term premium explain too much of the yield curve's slope. Because the level risk operates exclusively through investor's expectations, *this bias will appear even if the term structure model is fitted to particular subsamples of the data that correspond to a single economic regime.*

Next I ask: what fundamental macroeconomic changes could reproduce the

Slope-Volatility Puzzle? My answer is based on a simple Markov-Switching dynamic general equilibrium model, calibrated to replicate the U.S. economy. Following Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004) and many others, I assume that monetary policy switches between a regime where the central bank accommodates inflation pressures (passive policy) and a regime where the central bank fights these pressures in a proactive manner (active policy). What sets this model apart from standard ones is that here, as in Davig and Leeper (2007), agents incorporate into their expectations the possibility of regime switches. In this model, agents acquire different levels of precautionary savings depending on the current policy regime. As a result, the average short-term interest rate differs across regimes, giving rise to potentially sizeable level risks over the yield curve.

Under my calibration, households hold more precautionary savings when the passive regime occurs. The intuition behind this is that consumption growth is more volatile in the passive than in the active regime; therefore, risk averse agents will want to hold more "insurance" when the passive regime realizes. Thus, the nominal short-rate in the passive policy regime is higher than in the active regime, giving rise to level risks. In the passive regime, level risks are large and negative as I estimated for the Great Inflation regime. In the active regime, level risks are moderate and positive, replicating my estimate for the post-1985 period. Therefore, my second main conclusion in this chapter is that a general equilibrium model with a Markov-Switching monetary policy rule is capable of explaining the Slope-Volatility Puzzle.

Many economists, including Gürkaynak, Levin, and Swanson (2006, 2010), Wright (2008), and Capistrán and Ramos-Francia (2010), have highlighted gains from the adoption of an explicit inflation targeting framework. Under this policy arrangement, improved communication between the central bank and the public would reduce uncertainty about the particular way that the monetary authority will deal with inflationary pressures. This can be rationalized in the context of the model

I present here. All else constant, agents would perceive a change from the active towards the passive regime as less likely under explicit inflation targeting than if the central bank did not adopt this framework. According to the model, increasing the persistence of the active regime on average flattens the yield curve in that regime, because both level risks and term premia fall. Using the Wright (2008) international dataset, I show that this prediction of the model is corroborated by the data. That is, measures of average yield curve slope in developed economies that adopted explicit inflation targeting are systematically lower than in economies that did not adopt such a framework. I interpret this as additional evidence supporting the model with a Markov-Switching monetary policy rule.

This chapter is organized as follows. Section 2.2 includes a brief review of the literature. Section 2.3 documents the Slope-Volatility puzzle. Section 2.4 describes how allowing for a regime switching approach gives rise to the level risk. Section 2.5 provides level risk estimates for the United States which are shown to explain the Slope-Volatility Puzzle. In Section 2.6, the general equilibrium term-structure model with a Markov-Switching monetary policy rule is presented and shown to replicate the main features of this puzzle. I also show that the model replicates the yield curve evidence from inflation targeting countries. Section 2.7 concludes.

2.2 Related Literature

In this chapter, I focus on how level risks on average affect the nominal U.S. yield curve. Level risks naturally emerge from the Expectations Hypothesis component of the term structure, once the short-rate is modelled as a Markov-Switching (MS) process. This insight draws on two important earlier contributions.

Hamilton (1988) pioneered in studying term structure behavior when the short rate follows a simple autoregressive MS process. He found that when regime shifts were incorporated into agents' beliefs, violations of the Expectations Hypothesis of the term structure were less severe than in single regime models. Bekaert, Hodrick, and Marshall (2001) proposed the peso problem theory which I also draw on. They noted that violations of the Expectations Hypothesis in the United States, to a large extent, are due to a 'peso problem' which is associated with the short-rate process: observed long-term yields in the United States largely can be reconciled with short-rate behavior if investors' beliefs allow for short-rate levels not observed *ex-post* in the data. This theory can be formalized by first assuming a reduced-form MS process for the short rate and then letting long-term bonds be priced by rational agents who form beliefs taking this MS process into account.

Both Hamilton (1988) and Bekaert, Hodrick, and Marshall (2001) focus on reduced-form short-rate processes, but I go one step further here and, in a no-arbitrage general equilibrium framework, relate the behavior of short- and long-term nominal yields in the United States to macro factors. In a structural micro-founded framework, I claim that changes in monetary policy regimes can explain observed changes in key U.S. macro and term structure moments over time.

There is substantial reduced-form empirical evidence in the literature showing that changes in monetary policy did affect the behavior of the U.S. term structure (and the macroeconomy) over time[35]. For example, Bikbov and Chernov (2008) have shown that the information contained in the nominal term structure can be crucial in identifying regimes in which the Fed adopted an active or passive stance for inflation. Similarly, in the context of identified time-varying VARs with no-arbitrage bond prices, both Ang, Boivin, Dong, and Loo-Kung (2009) and Mumtaz and Surico (2009) find evidence of large movements in the Fed's response to inflation

[35]The literature on term-structure models with only yield-related factors also provides ample support for the view that the stochastic behavior of the U.S. yield curve has varied over the past decades. See, for example, the latent-factors, regime-shifting, no-arbitrage frameworks of Bansal and Zhou (2002) and Dai, Singleton, and Yang (2007). Without relying on no-arbitrage restrictions, Ang and Bekaert (2002) show that regime shifts in the nominal term structure are present in the US, UK and Germany datasets. Evidence of regime shifts in real term structures in the US and the UK respectively also has been provided by Ang, Bekaert, and Wei (2008) and Evans (2003).

over the last six decades. Bianchi, Mumtaz, and Surico (2009) find evidence for the United Kingdom that monetary policy shocks contributed significantly more to the variability of key macro and term structure time series before than after the adoption of inflation targeting[36]. My contribution to this strand of the literature is a structural, micro-founded no-arbitrage approach to modelling the term structure in the presence of monetary policy shifts.

Interestingly, my proposed level-risk theory is related to the peso problem (or 'rare disaster') as suggested by both Rietz (1988) and Barro (2006) in the context of the equity premium puzzle. In their formulation, equity becomes very risky because there is a small probability in every period of a sudden and very sharp drop in the economy's productive capacity. In my case, a change from an active to a passive monetary policy regime, where holding nominal bonds involves significantly more risk, could be seen as a rare disaster by bond traders. As in Bansal and Yaron (2004), I also show that when shifts to a passive monetary policy regime are possible, the amount of long-run risk in the economy during the active regime is substantially higher than in a single active regime model.

The available literature on MS Dynamic Stochastic General Equilibrium (MS-DSGE) models, including Davig and Leeper (2007), Farmer, Waggoner, and Zha (2007), Davig and Doh (2008), Liu, Waggoner, and Zha (2009) and Liu and Mumtaz (2010), relies on linear approximations to the true model solution, which by construction rule out precautionary savings and premia in financial assets. Amisano and Tristani (2010a,b) considered non-linear solutions to MS-DSGE models with MS shocks' volatilities, but their method does not allow for changes in the Fed's response to inflationary pressures. I also contribute to this branch of the literature by

[36]The existence of shifts in the U.S. monetary policy regime are supported by a well-known macro literature that tries to identify the main reason behind the Great Moderation of the post mid-1980s. See, for example, Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006). Romer and Romer (2004) also produce compelling narrative evidence of changes in the Fed's monetary policy conduct during the 20^{th} century. For other explanations for the Great Moderation, see for example, McConnell and Perez-Quiros (2000), Primiceri (2005), Sims and Zha (2006), and Justiniano and Primiceri (2008).

offering a non-linear perturbation solution method to the standard New-Keynesian model with a Markov-Switching monetary policy rule[37].

2.3 The Slope-Volatility Puzzle

I now document the '*Slope-Volatility Puzzle*'. In the following subsection I derive results from the no-arbitrage theory linking the slope of the nominal yield curve to the underlying level of uncertainty in the economy. I then go on to show that these theoretical predictions are at odds with the U.S. data.

2.3.1 The Slope of the Yield Curve and Macroeconomic Uncertainty

Consider a long-term nominal bond that costs $B_{\tau,t}$ at time t and promises to repay the investor one dollar in $t+\tau$ (throughout this chapter, I assume that bonds are zero-coupon and default-free). The continuously compounded τ-period yield to maturity is defined as $i_{\tau,t} \equiv -\frac{1}{\tau}\log B_{\tau,t}$. The economy's short-term nominal interest rate is given by $i_t \equiv i_{1,t}$. Following Dai and Singleton (2002) and letting E_t be the expectations operator conditional on date t information, I define the *τ-period nominal term premium* as $NTP_{\tau,t} \equiv i_{\tau,t} - \frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t\left[i_{t+j}\right]$. This measure captures the deviations of $i_{\tau,t}$ from the pure expectations hypothesis and is positive when it is riskier to invest in the long-term bond than to invest in a sequence of short-term bonds for τ periods.

Rearranging the term premium definition, it follows that the *τ-period yield curve*

[37] Chib, Kang, and Ramamurthy (2011) estimate a MS-DSGE model for the US nominal term structure. Their solution method relies on the linear/log-normal approach of Bekaert, Cho, and Moreno (2010), which assumes that the short-rate is not affected by precautionary savings effects (i.e. the average short rate is the same across regimes). As a result, their solution method by construction rules out the existence of level risks along the yield curve.

slope, $i_{\tau,t} - i_t$, can be written as[38]

$$i_{\tau,t} - i_t = \left(\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t \left[i_{t+j} \right] - i_t \right) + NTP_{\tau,t} \ . \tag{2.1}$$

I call the term in parentheses in equation (2.1) the expectations hypothesis (EH) component of the slope. If the nominal term premium remains constant, then whenever investors revise their short-rate forecasts up (down), the EH component of the slope increases (decreases). Similarly, if the EH component remains unchanged, then an increase (decrease) in the nominal term premium increases (decreases) the slope.

The term premium definition can be used to gain some insights about the determinants of the yield curve slope in the long run. Taking unconditional expectations on both sides of equation (2.1), I obtain

$$E \left[i_{\tau,t} - i_t \right] = \frac{1}{\tau} \sum_{j=0}^{\tau-1} \left(E \left[i_{t+j} \right] - E \left[i_t \right] \right) + E \left[NTP_{\tau,t} \right]$$

because of the law of iterated expectations. From the assumption that the short rate follows a covariance-stationary process it follows that $\sum_{j=0}^{\tau-1} \left(E \left[i_{t+j} \right] - E \left[i_t \right] \right) = 0 \ \forall j$. Intuitively, when calculating the mean, periods where the short rate is expected to increase will cancel out those where the short rate is expected to decrease, and the EH component of the slope is equal to zero on average. As a result $E \left[i_{\tau,t} - i_t \right] = E \left[NTP_{\tau,t} \right]$, which means that if the nominal yield curve is *unconditionally* positively sloped, it is because the term premium is positive on average. Therefore, to understand why yield curves in general are positively slopped, it is important to understand the determinants of the term premium.

[38] This slope definition sometimes has been refered to in the literature as 'term spread', but is not to be confused with the 'term premium'.

It can be shown that the no-arbitrage price of a τ-period bond is given by

$$B_{\tau,t} = E_t\left[M_{t,t+1}\frac{1}{\Pi_{t,t+1}}B_{\tau-1,t+1}\right] \tag{2.2}$$

where $M_{t,t+1}$ and $\Pi_{t,t+1}$ are the *real* stochastic discount factor (SDF) and the inflation rate between periods t and $t+1$. All else constant, an increase in the expected rate of inflation will reduce the price of the bond today because its expected resale value, measured in real terms, falls.

Taking a second-order expansion of the Euler condition above around the deterministic steady state, the nominal term premium is given by[39]

$$\begin{aligned} NTP_{\tau,t} \;&\cong\; RTP_{\tau,t} + Convexity^{\pi}_{\tau,t} \\ &+\left(\frac{1}{\tau}Cov_t\left[\widehat{m}_{t,t+\tau},\widehat{\pi}_{t,t+\tau}\right] - \frac{1}{\tau}\sum_{j=0}^{\tau-1}E_t\left\{Cov_{t+j}\left[\widehat{m}_{t+j,t+j+1},\widehat{\pi}_{t+j,t+j+1}\right]\right\}\right) \end{aligned} \tag{2.3}$$

where for any variable X_t with steady state $\overline{X}$ define $\widehat{x}_t \equiv \log\left(X_t/\overline{X}\right)$, while Cov_t represents the conditional covariance operator. Therefore, the nominal term premium can be separated into three parts. The first corresponds to the *τ-period real term premium*. This is defined as $RTP_{\tau,t} \equiv r_{\tau,t} - \frac{1}{\tau}\sum_{j=0}^{\tau-1}E_t\left[r_{t+j}\right]$, where $r_{\tau,t}$ is the yield to maturity on a τ-period inflation indexed bond[40] and r_t is the real short-rate. The real term premium captures deviations of the long-term *real* yield from the EH which, as shown in Appendix 2.A, depends only on the autocorrelation structure of the SDF[41]. The second component of $NTP_{\tau,t}$, $Convexity^{\pi}_{\tau,t}$, represents an inflation convexity term that in practice is not very relevant.

The third component appearing in parenthesis in equation (2.3) is what I focus

[39] The following expression holds exactly if SDF and inflation are jointly log-normally distributed. Detailed derivations can be found in Appendix 2.A.

[40] Inflation indexation is assumed to be perfect.

[41] There is evidence that the real term premium over the US term structure is close to zero or slightly negative. Compared to premia related to inflation, however, real premia are thought to be quantitatively less relevant. See Buraschi and Jiltsov (2005), Piazzesi and Schneider (2006) and Ang, Bekaert, and Wei (2008).

on. This term corresponds to compensation for inflation risk. Its first part is positive when inflation from t until maturity co-varies positively with the investor's SDF. In this case, nominal bonds lose value exactly when wealth is most important to the investor. The second component of the inflation premium measures the (average expected) one-period-ahead inflation co-variability risk. Because inflation uncertainty is relatively low in such a short horizon, the first part of the inflation premium tends to dominate.

I use these results to relate the term premium to the underlying level of uncertainty in the economy. The dominant part of the inflation premium in equation (2.3) can be rewritten as

$$\frac{1}{\tau}Cov_t\left[\widehat{m}_{t,t+\tau},\widehat{\pi}_{t,t+\tau}\right]=\frac{1}{\tau}Corr_t\left[\widehat{m}_{t,t+\tau},\widehat{\pi}_{t,t+\tau}\right]\left(Var_t\left[\widehat{m}_{t,t+\tau}\right]Var_t\left[\widehat{\pi}_{t,t+\tau}\right]\right)^{1/2} \quad (2.4)$$

where Var_t and $Corr_t$ are the conditional variance and correlation coefficient. The square root term on the right hand side of this equation represents the product of the *ex-ante* volatilities of inflation and the SDF. If the conditional correlation between the SDF and inflation is positive (i.e. if the inflation premium is positive) and relatively constant over time, then periods associated with higher levels of inflation uncertainty will also be associated with higher inflation compensations and higher term premia. Intuitively, when inflation uncertainty is high, the future payoffs from investing in long-term nominal bonds are very uncertain if evaluated in real terms. As a result, investors will demand higher premia to hold long-term nominal bonds than when uncertainty is low.

Equation (2.4) also reveals that, in addition to inflation uncertainty, real uncertainty matters for term premia. To see this, note that macro models usually have the SDF be a function of real variables, such as consumption, hours worked, etc. Therefore, an increase in the level of uncertainty surrounding these real variables will increase the *ex-ante* volatility of the SDF, which in turn will have an impact on the nominal term premium through its inflation compensation component[42]. Intuitively, the investor cares about how the *real* payoff from investing in the long-term bond co-varies with his future path of consumption. If consumption becomes more difficult to predict, then investors will perceive long-term bonds as riskier and demand higher premia.

2.3.2 Evidence for the US

From the discussion above follows that periods characterized by high levels of macroeconomic uncertainty should be associated with higher term premia than periods of low uncertainty. Accordingly, differences in average term premia across periods are entirely reflected in the yield curve slope (provided that there are enough observation from each period)[43]. Do these theoretical predictions hold for the post-World War II U.S. data?

To address that question, I construct slope measures using the 5 and 10-year zero-coupon nominal yields from the CRSP Fama-Bliss and the Gurkaynak, Sack, and Wright (2007) databases[44]. To measure the short-rate I use the 3-month T-Bill returns taken from the CRSP Fama riskfree rate file. All of the series are arranged at a quarterly frequency, and the sample goes from 1952:2 to 2008:4. Interest rates

[42] The real term premium component of NTP_t^{τ} may also respond to the increase in real uncertainty. See Appendix 2.A.

[43] In Section 2.3.1 I characterized the relation between the slope of the yield curve and macroeconomic volatility in terms of *unconditional* moments, resulting in $E\left[i_{\tau,t} - i_t\right] = E\left[NTP_{\tau,t}\right]$. Although applying unconditional moments simplifies the exposition, it fails to agree precisely with my empirical analysis, which is interested in the co-movements between the slope and macro volatility across different *subsamples* of the U.S. data. A more appropriate characterization would then be the following. Assume that the economy switches across different regimes over time, and yet private agents believe the current regime to last indefinitely. This is the implicit assumption in a great number of papers in the macro literature, such as Lubik and Schorfheide (2004) and Smets and Wouters (2007). It then follows that $E\left[i_{\tau,t} - i_t/s_t\right] = E\left[NTP_{\tau,t}/s_t\right]$, that is the slope and term premium are on average equal conditional on each regime s_t. Section 2.4 shows that this equality is not true once agents incorporate the possibility of regime shifts into their beliefs, which will help to explain the Slope-Volatility Puzzle.

[44] Although the Gurkaynak, Sack, and Wright (2007) database also contains the 5-year zero-coupon yield, it only starts in 1961. The CRSP Fama-Bliss file starts in 1952, but does not contain the 10-year maturity.

are continuously compounded, expressed in annualized terms, and observed on the last working day of each quarter.

The measures of macroeconomic uncertainty that I focus on are based on real consumption growth and inflation. The former is measured by the quarterly growth in the Real Personal Consumption Expenditures index (PCE); the latter is measured by the quarterly growth in the core PCE deflator[45]. All growth rates are continuously compounded and multiplied by 400 to be expressed in percent per annum. Consumption growth is intended as a *proxy* for the unobservable SDF.

Because bond prices are fully forward looking, inflation and consumption uncertainty must be quantified according to an *ex-ante* concept (only using information up to t to measure uncertainty from period $t+1$ onwards). Therefore I estimate univariate GARCH processes for the inflation and consumption growth series over the 1952:2-2008:4 sample, and then measure uncertainty by the GARCH-based one-quarter-ahead forecast for the conditional variance[46].

The 5- and 10-year slope measures, together with the conditional variance forecasts for inflation and consumption growth, are shown in Figure 2.1. The vertical dashed lines in this figure identify subperiods of the U.S. sample where, according to Romer and Romer (2004), the Fed followed different monetary policy regimes. Each subsample is identified with the names of the Federal Reserve chairmen in office at the time[47]. I abandon these pre-specified subsamples in Section 2.5 and use more

[45] Consumption growth is measured by the quarterly change in the real PCE index (series code: PCECC96). For inflation I use the 'PCE deflator excluding food and energy' obtained from the St. Louis Fed webpage (series code: JCXSE). For the latter, the observations from 1952 until 1959 were estimated using the 'CPI excluding food and energy' and the 'PCE deflator all items' (series codes: CPILFESL and PCECTPI) and applying the principal components method suggested by Walczak and Massart (2001).

[46] The conditional mean of inflation and consumption growth are modelled respectively as an AR(2) and an ARMA(1,4) process. For both series the best fitting model for the conditional variance was a GARCH (1,1).

[47] Romer and Romer (2004) view monetary policy in the years where Paul Volcker and Alan Greenspan were Federal Reserve chairmen as based on the same principles. Here, I separate the Volcker era from the subsequent one, because the general macroeconomic environment in the two periods was substantially different. In particular, while Volcker inherited an ambience of high/volatile inflation where credibility in monetary policy was very weak (see Goodfriend and King (2005)), the same is not true for Greenspan.

rigorous statistical methods to identify the U.S. regimes.

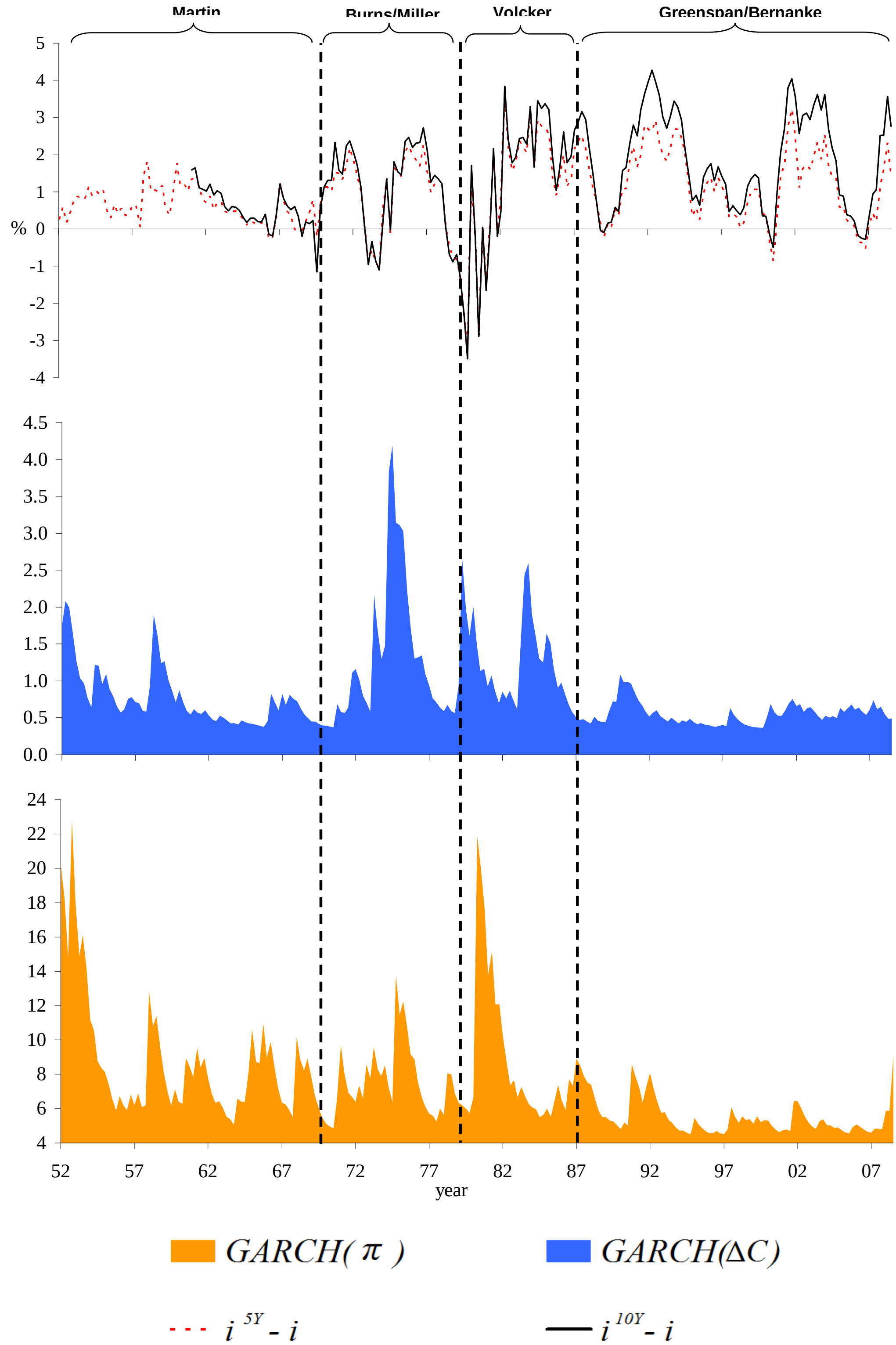
The two bottom charts in Figure 2.1 show a well-known stylized fact in the macro literature: from the mid-1980s until 2007, a period that corresponds roughly to my Greenspan / Bernanke subsample, the levels of macroeconomic uncertainty (in this case inflation and consumption growth uncertainty) were historically low. This is referred to in the literature as the Great Moderation[48], and usually is portrayed in terms of *ex-post* measures of uncertainty, such as the realized standard deviation of key macro time series. Figure 2.1 makes a similar point but uses GARCH-based volatilities which, as discussed before, better capture the level of *ex-ante* uncertainty faced by forward looking bond traders.

The subsamples depicted in Figure 2.1 can be interpreted as different regimes characterized by different levels of macroeconomic uncertainty. The Greenspan / Bernanke subsample might be viewed as a low uncertainty state, whereas the Burns / Miller and the Volcker subsamples can be seen as high uncertainty regimes. This classification is in line with the more rigorous estimates shown in Section 2.5. According to the theory developed in subsection 2.3.1, term premia and consequently yield curve slope measures in principle should reflect the underlying level of macro uncertainty. Following this logic, the yield curve slope on average should be flatter in the Greenspan / Bernanke subsample than in the Burns / Miller and Volcker subsamples.

As the top chart in Figure 2.1 shows, this does not hold in the data. During the Greenspan / Bernanke years, both the 5 and 10-year slope measures on average seem higher than in all three previous subsamples. If the theory above is correct, then these slope measures tell us that investing in nominal bonds during the Greenspan / Bernanke period is riskier than in all previous subsamples. However, this is difficult to reconcile with the low levels of macroeconomic uncertainty that characterize this

[48] See Kim and Nelson (1999) and McConnell and Perez-Quiros (2000). In addition, Stock and Watson (2002) conduct a detailed review of the literature and present some new evidence.

Figure 2.1: The Yield Curve Slope and Macroeconomic Uncertainty in the U.S.



Notes: $GARCH(\pi)$ and $GARCH(\Delta c)$ are the one-quarter-ahead forecasts of the conditional variance of inflation (π) and consumption growth (Δc), estimated through univariate GARCH(1,1) processes. The conditional means of inflation and consumption growth are modelled respectively as an AR(2) and an ARMA(1,4). The two yield curve slope measures are expressed in percent per annum.

subsample.

This point is reinforced in Table 2.1, which displays in more detail certain properties of the slope, inflation, and consumption growth time series across the previously defined subsamples. The first two lines of this table show the average 5 and 10-year slope measures in each subsample, together with their associated standard errors. Two alternative measures of *ex-post* uncertainty are shown in lines three through six. These can be compared to the *ex-ante* GARCH-based measures for the sake of robustness. In particular, the third line displays the realized standard deviation of inflation, whereas lines four through six display the root mean squared forecast error (RMSFE) of inflation at three different horizons, based on a simple random-walk model[49]. The same measures of uncertainty for the case of consumption growth are shown in lines seven to ten. Finally, the last line shows the contemporaneous correlation coefficient between consumption growth and inflation.

Note that the 10-year slope on average is 179 basis points during the Greenspan / Bernanke period, the highest across all subsamples. At the same time, all ex-post measures of inflation and consumption growth uncertainty are unambiguously lower in this period than in all other subsamples.

I can now formulate the main stylized fact that motivates the remainder of this chapter:

- *The Slope-Volatility Puzzle:*

  Although the inflation and consumption-based uncertainty measures suggest that the Greenspan / Bernanke subsample is the least risky for investors holding long-term nominal bonds, the nominal yield curve on average is steeper during that period than in the Martin, Burns / Miller and Volcker subsamples.

[49]The RMSFE we report are based on Atkeson and Ohanian (2001) and Stock and Watson (2007). To be more precise, let π_t be the annualized quarterly inflation rate at time t. To compute the forecast errors for inflation accumulated h periods ahead, I use the following model: $E_t\left[h^{-1}\sum_{j=1}^{h}\pi_{t+j}\right] = \frac{1}{4}\left(\pi_t + ... + \pi_{t-3}\right)$. An equivalent methodology is used for consumption growth.

Table 2.1: The Slope-Volatility Puzzle

		Martin	Burns / Miller	Volcker	Bernanke / Greenspan
		(52:2-69:4)	(70:1-79:2)	(79:3-87:2)	(87:3-08:4)
$E(i_{5Y} - i)$		0.61	0.93	1.10	1.19
		(0.06)	(0.21)	(0.31)	(0.13)
$E(i_{10Y} - i)$		$0.51^{\dagger}$	1.09	1.32	1.79
		(0.11)	(0.25)	(0.35)	(0.18)
$SD(\pi)$		1.23	2.00	2.16	1.01
$RMSFE\,(\pi)$	$h = 4$	0.75	1.88	1.11	0.48
	$h = 8$	0.83	2.10	1.40	0.48
	$h = 12$	0.93	2.04	1.67	0.55
$SD(\Delta c)$		3.26	3.30	3.50	2.10
$RMSFE\,(\Delta c)$	$h = 4$	3.10	3.16	2.26	1.49
	$h = 8$	2.54	3.39	2.15	1.55
	$h = 12$	2.31	3.10	2.37	1.62
$Corr\,(\Delta c, \pi)$		-0.25	-0.51	-0.46	-0.17

Notes: The measures of average slope are expressed in percent per annum and the values in parenthesis are Newey-West HAC consistent standard errors, calculated using monthly slope series. SD(Δc) and inflation SD(π) are the standard deviation of consumption growth (Δc) and inflation (π), also expressed in percent per annum. $RMSFE(\pi)$ and $RMSFE(\Delta c)$ are the random-walk-based, root mean squared forecast errors h periods ahead for consumption growth and inflation. Corr(π,Δc) is the contemporaneous correlation coefficient between inflation and consumption. † sample starts in 1961:2.

Returning to equation (2.4), one possible explanation for the puzzle is that $Corr_t\left[\widehat{m}_{t,t+\tau}, \widehat{\pi}_{t,t+\tau}\right]$ on average is sufficiently higher during the Greenspan / Bernanke subsample than in the previous ones. An increase in this correlation coefficient might offset the drop in ensuing uncertainty as the economy moves from the Volcker to the Greenspan / Bernanke periods. To rule out this possibility, the last line of Table 2.1 reports the realized contemporaneous correlation coefficient between inflation and consumption growth across all subsamples. This measure is a *proxy* for the unobserved conditional correlation in equation (2.4). Because the SDF is negatively related to consumption growth in most economic models, the correlation coefficient being negative in all subsamples is consistent with positive term premia. Note that the absolute value of the correlation coefficient changes across subsamples in the same direction as changes in uncertainty. This suggests that movements in the correlation coefficient across subsamples reinforce the puzzle, rather than helping to explain it.

2.4 The Level Risk

This section puts forth a theoretical approach that potentially can explain the Slope-Volatility Puzzle. It hinges on the assumption that, over time, the economy switches across different macroeconomic regimes characterized by different short-rate levels. By letting investors explicitly incorporate into their beliefs the possibility of regime shifts, I show that this assumption has important implications for the EH component of the yield curve slope.

Section 2.4.1 begins by assuming, for simplicity, that regimes evolve according to an exogenous Markov chain. Section 2.4.2 then incorporates the MS process into investors' beliefs. Conditional on a given regime, I show that in general the mean slope and term premium are not equal. The wedge between the two is what I call '*level risk*': that is, the risk of a level shift in the short-rate process in case the economy switches to a new regime. I conclude by showing that the level risk potentially can explain the Slope-Volatility Puzzle.

2.4.1 A Simple Markov-Switching Environment

Suppose that the short-rate, i_t, follows a regime-switching process. In particular, I assume that:

- **Assumption (1) – Markov-Switching Environment:**

  The short-rate i_t follows a Markov-Switching process with two possible states: $s_t \in \{1, 2\}$. Regimes evolve according to an exogenous Markov Chain with constant regime-switching probabilities arranged in the 2×2 matrix P. The element in the i^{th} row and j^{th} column of P represents $\Pr(s_{t+1} = j/s_t = i) \equiv p_{ij}$ for $i, j \in \{1, 2\}$. Accordingly, each line of P must sum to one. The regime-switching probabilities are known to all agents, who are also assumed to observe the realization of s_t in the beginning of period t. [50]

Assumption (1) implies that the short rate follows a potentially different dynamic, conditional on each regime. Particularly important for the results below, the short rate may fluctuate around different means across regimes. The assumptions that the Markov Chain is exogenous and features a constant transition matrix are made in order to simplify the derivations below. Additionally, I focus on the 2-regime case for expositional purposes; generalizations for an arbitrary number of regimes are simple to obtain.

I further assume that:

[50] This is different from empirical applications of the Hamilton (1989) filter, where agents use the available information to filter out the probability of being in each regime. See Hamilton (1993).

- **Assumption (2) – Past State Dependence:**

  The MS process for the short-rate is covariance-stationary in each regime and has the following property:

  $$E\left[i_t/\mathcal{S}_t\right] = E\left[i_t/s_t\right] \quad \text{for } t = 1, 2, 3, ...$$

  where $S_t \equiv \{s_0, s_1, ..., s_t\}$ *corresponds to the history of regimes realized up to period* $t-1$.

Assumption (2) means that the average level of the short rate depends only on the current regime, not on regimes realized in previous periods. The intuition is: if, for example, regime shifts imply changes of chairman of the Fed, then the average short-rate level chosen by a new chairman does not depend on the economic dynamics during previous regimes. Although this assumption is debatable, I adopt it on the grounds that it greatly simplifies the derivations that follow. Additionally, this assumption arises naturally as a property of the reduced form of the model which I analyze in Section 2.6. [51]

2.4.2 Accounting for Markov-Switching Probabilities in Agents' Beliefs

Given the proposed MS environment, I now compute the average yield curve slope, conditional on each regime. Let Ω_t represent the complete information set available to investors in period t, which summarizes all aspects of history that are relevant to the economy's future evolution, including s_t. To build the intuition, I start by considering the 2-period slope and later generalize for the slope at the τ-period horizon. In case τ equals 2, taking expectations on both sides of equation (2.1),

[51] See equation (2.17).

conditional on $s_t = s$, yields:

$$\begin{aligned} E\left[i_{2,t} - i_t/s_t = s\right] &= \frac{E\left[i_t/s_t = s\right] + E\left[E\left[i_{t+1}/\Omega_t\right]/s_t = s\right]}{2} - E\left[i_t/s_t = s\right] \\ &\quad + E\left[NTP_{2,t}/s_t = s\right] \end{aligned}$$

for $s = 1, 2$. Since s_t is contained in Ω_t, the law of iterated expectations implies that:

$$\begin{aligned} E\left[E\left[i_{t+1}/\Omega_t\right]/s_t = s\right] &= E\left[i_{t+1}/s_t = s\right] \\ &= p_{s1}E\left[i_{t+1}/s_t = s, s_{t+1} = 1\right] + p_{s2}E\left[i_{t+1}/s_t = s, s_{t+1} = 2\right] \end{aligned}$$

where the second equality uses the probabilities contained in $\mathbf{P}$ to parameterize expectations so that regime switches are taken into account explicitly. Combining the last two equations and using the Assumption (2) above[52] yields:

$$\begin{aligned} E\left[i_{2,t} - i_t/s_t = s\right] &= E\left[NTP_{2,t}/s_t = s\right] \\ &\quad + \left(\frac{p_{s1}E\left[i_{t+1}/s_{t+1} = 1\right] + p_{s2}E\left[i_{t+1}/s_{t+1} = 2\right] - E\left[i_t/s_t = s\right]}{2}\right) \end{aligned} \tag{2.5}$$

for $s \in \{1, 2\}$. Therefore, unlike the case analyzed in Section 2.3.1, once investors' beliefs incorporate the regime-switching probabilities, the mean slope conditional on regime s in general is not equal to the mean term premium in that particular regime. Instead, it is equal to the mean term premium *plus* the expression in parentheses in equation (2.5). This new term arises from the EH component of the slope definition in equation (2.1). Unlike the case analyzed in Section 2.3.1, conditional on a given regime, the EH component now does not cancel out in expectations. Intuitively, the new term takes into account the risk that the average level of the short-rate process switches due to a regime change. I refer to the expression in parentheses in equation (2.5) as the *'level risk'*.

[52] See Appendix 2.C for the case where Assumption 2 is dropped.

For a short-rate process that is covariance-stationary in each regime, it is easy to show that equation (2.5) can be written more compactly as:

$$E\left[i_{2,t} - i_t/s_t = 1\right] = \frac{\left(1 - p_{11}\right)\mathcal{D}_s E\left[i_t/s_t\right]}{2} + E\left[NTP_{2,t}/s_t = 1\right] \tag{2.6}$$

$$E\left[i_{2,t} - i_t/s_t = 2\right] = \frac{-\left(1 - p_{22}\right)\mathcal{D}_s E\left[i_t/s_t\right]}{2} + E\left[NTP_{2,t}/s_t = 2\right] \tag{2.7}$$

where $\mathcal{D}_s E\left[i_t/s_t\right]$ is defined as the average short-rate differential across regimes, i.e. $\mathcal{D}_s E\left[i_t/s_t\right] \equiv E\left[i_t/s_t = 2\right] - E\left[i_t/s_t = 1\right]$. Without loss of generality, let $E\left[i_t/s_t = 2\right] \geq E\left[i_t/s_t = 1\right]$, which implies that $\mathcal{D}_s E\left[i_t/s_t\right]$ is non-negative. It follows that, because $p_{11}, p_{22} \in [0,1]$, the level risk is non-negative in the low-average short-rate regime 1 and non-positive in the high-average short-rate regime 2. Intuitively, conditional on the economy being in regime 1, the possibility of future regime changes introduces a risk of increase in the average short-rate level, which would in turn reduce bond prices. Conditional on regime 2, the opposite would be true: regime switches would represent a risk of reduction in the average level of the short rate, which would increase bond prices. If p_{11} increases, everything else being constant, then the level risk associated with regime 1 falls, because switching from regime 1 becomes less likely. Similarly, the level risk associated with regime 2 falls in absolute value as p_{22} increases.

Note that only in two particular cases does the level risk equal zero. First, if $E\left[i_t/s_t = 2\right] = E\left[i_t/s_t = 1\right]$, then switching regimes implies no change in the average level of the short rate. As a result level risks are zero in each regime. Also, if investors believe regime $s \in \{1, 2\}$ to be an absorbing state, that is if $p_{ss} = 1$, then the level risk conditional on this regime is zero: once regime s is reached, investors perceive it as lasting indefinitely; consequently, further regime shifts are not considered when investors form beliefs about the future.

Generalizing equations (2.6) and (2.7) for $\tau > 2$ is simple. Following Hamilton

(1994), the probability that an observation of regime i will be followed k periods ahead by an observation of regime j, that is $\Pr(s_{t+k} = j/s_t = i)$, is the element in the i^{th} row and j^{th} column of $\mathbf{P}^k$. I denote this probability by $\left[\mathbf{P}^k\right]_{ij}$. The generalizations of equations (2.6) and (2.7) for the case $\tau \geq 2$ are then:

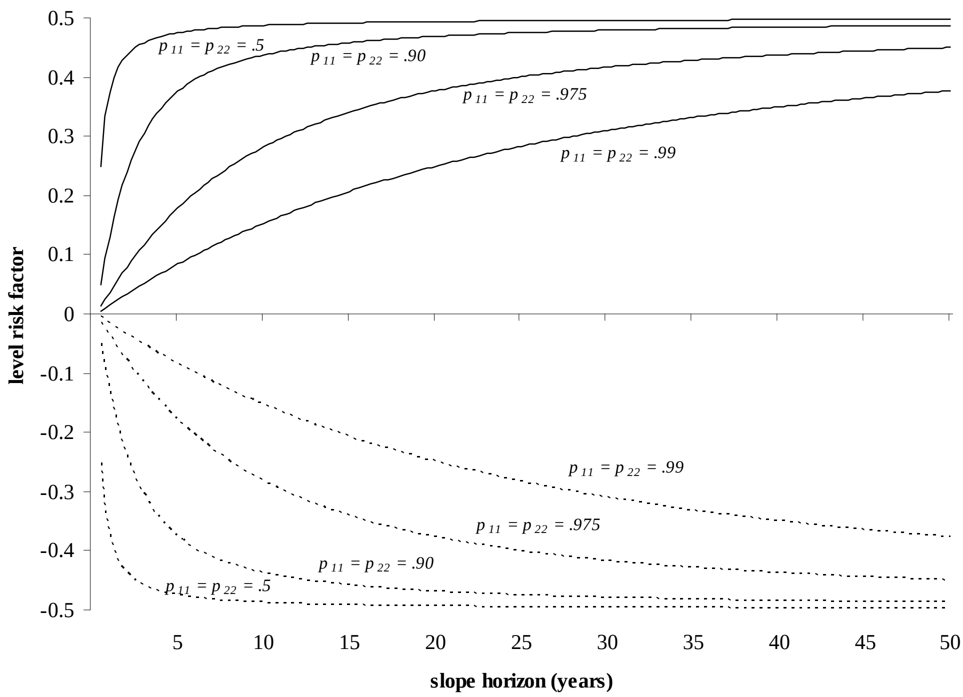
$$E\left[i_{\tau,t} - i_t/s_t = 1\right] = \left(\frac{(\tau-1)}{\tau} - \frac{1}{\tau}\sum_{k=1}^{\tau-1}\left[\mathbf{P}^k\right]_{11}\right)\mathcal{D}_s E\left[i_t/s_t\right] + E\left[NTP_t^\tau/s_t = 1\right] \tag{2.8}$$

$$E\left[i_{\tau,t} - i_t/s_t = 2\right] = -\left(\frac{(\tau-1)}{\tau} - \frac{1}{\tau}\sum_{k=1}^{\tau-1}\left[\mathbf{P}^k\right]_{22}\right)\mathcal{D}_s E\left[i_t/s_t\right] + E\left[NTP_t^\tau/s_t = 2\right] \ . \tag{2.9}$$

It can be shown that as $\tau \to \infty$, the factors multiplying the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ in equations (2.8) and (2.9) converge to $1 - p^{erg}$ and $-p^{erg}$ respectively, where p^{erg} is the ergodic probability associated with regime 1 (and the one associated with regime 2 is $1 - p^{erg}$). In case $p_{11} = p_{22} < 1$, the ergodic probability of the Markov-Chain is $p^{erg} = 0.5$, which implies that in the limit, where $\tau \to \infty$, half of the interest rate gap $\mathcal{D}_s E\left[i_t/s_t\right]$ is reflected in the average slope in each regime. For example, for an interest rate differential of 4% across the two regimes, in the limit as $\tau \to \infty$ the level risk in regimes 1 and 2 will be sizeable – namely 2% and -2% respectively.

Is the level risk quantitatively relevant for values of τ far form the limit? Figure 2.2 plots on the vertical axis the factors multiplying the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ in equations (2.8) and (2.9) against the slope horizon τ on the horizontal axis. Each full line corresponds to factors associated with equation (2.8) for a particular choice of the regime-switching probabilities. The dashed lines show the same, but in case of equation (2.9). Figure 2.2 displays only the case where $p_{11} = p_{22}$, which implies that the level risk associated with regime 2 is the mirror image of that associated with regime 1. For example, the line labeled $p_{11} = p_{22} = 0.90$ says that, at the 5-year horizon (i.e. $\tau = 20$), slightly less than 40% of the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ is going to be reflected in the average slope conditional on

Figure 2.2: The Level Risk for Different Slope Horizons



Notes: Each full line corresponds to the factors multiplying the mean interest rate gap from equation (2.8) as a function of $\mathcal{T}$ for a different choice of regime-switching probabilities. The dashed lines represent the same for equation (2.9).

regime 1. The dashed line shows that the same quantity, but with a negative sign, appears in the slope conditional on regime 2.

Note that the level risk increases in absolute value as the slope horizon increases. Intuitively, the probability of a regime switch happening during the life of the long-term bond considered in the slope measure increases in τ. Additionally, as the probability of remaining in the same regime gets closer and closer to one, the level risk takes longer to reach its limiting value as $\tau \to \infty$. As a result, it becomes quantitatively less relevant for values of τ far from the limit. However, even for the case where $p_{11} = p_{22} = 0.99$, the magnitude of the level risk at the 10-year horizon corresponds to a sizeable 15% of the mean interest rate differential across regimes.

One important implication of the level risk is that, when the true data-generating process displays regime switches, term structure models that do not allow for MS regimes probably will generate biased term premia estimates. In other words, if in reality agents consider the possibility of regime shifts when forming expectations, then models that do not take this into account will tend to force the term premium to explain too much of the cross section of yields. I emphasize that, because the level risk operates exclusively through investor's expectations, *this bias appears even if the term structure model is fitted to particular subsamples of the data that correspond to a single economic regime.* Consider, for example, the case where a researcher tries to fit a model that does not allow for regime switches to a particular subsample of the data encompassing a regime characterized by a relatively low short rate. If this regime is not an absorbing state, then the existence of a positive level risk implies that term premia estimates based on this model could be biased upwards.

To conclude, let me return to Figure 2.1. Can the level risk help to explain the Slope-Volatility Puzzle? In principle the answer is yes. When agents incorporate the MS short-rate process into their beliefs, the average slope in each subsample depicted in this figure may represent a combination of term premium and level risk. If, for example, there was a negative level risk during the Burns / Miller and Volcker subsamples and a positive level risk in the Greenspan / Bernanke years, then the puzzle potentially could be resolved. For this to be true, the mean short-rate in the regime represented by the Burns / Miller and Volcker subsamples must be higher than in the Greenspan / Bernanke regime. Additionally, both regimes must not be perceived by investors as absorbing states.

2.5 Level Risks and the Slope-Volatility Puzzle

This section provides level-risk estimates for different regimes of the U.S. economy. Subsection 2.5.1 models the dynamics of the U.S. economy according to a two-states Markov-Switching Vector Autoregression (MS-VAR). Subsection 2.5.2 demonstrates that, if investors form expectations based on the estimated MS-VAR, the level risks were moderate and positive in the Greenspan / Bernanke years and large and negative in the Burns / Miller and Volcker subsamples. After controlling for the level risk, the term premium in the Greenspan / Bernanke years were substantially smaller than in the Burns / Miller and Volcker periods. Accounting for level risks thus solves the Slope-Volatility Puzzle.

2.5.1 A Simple MS-VAR of the U.S. Economy

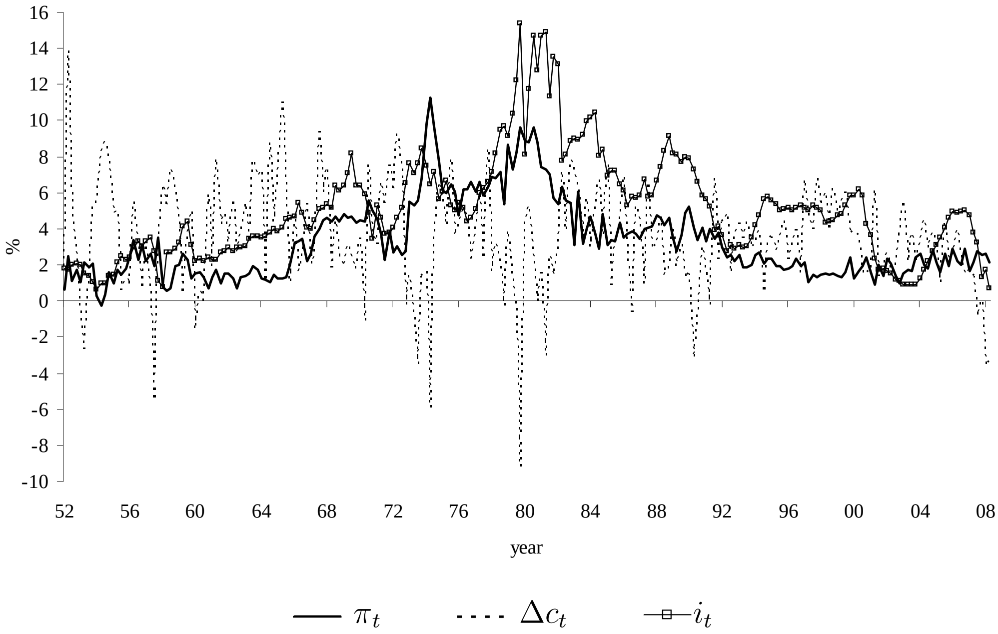
I model the dynamics of the U.S. economy during the post-World War II period according to a quarterly MS-VAR which includes three variables: the inflation rate π_t; the growth rate of consumption Δc_t; and the short-term nominal interest rate i_t. The MS-VAR can be written as

$$\begin{aligned} \mathbf{Y}_t &= \Phi_{0(s_t)} + \Phi_{1(s_t)}\mathbf{Y}_{t-1} + ... + \Phi_{q(s_t)}\mathbf{Y}_{t-q} + \mathbf{u}_t \\ \mathbf{u}_t &\sim N\left(\mathbf{0}, \mathbf{\Sigma}_{(s_t)}\right) \end{aligned} \quad (2.10)$$

where $\mathbf{Y}_t \equiv (\pi_t, \Delta c_t, i_t)'$ and $\mathbf{u}_t$ is a 3×1 vector of *iid* reduced-form innovations. The unknown regime-switching parameters are organized in the 3×1 vector of intercepts $\Phi_{0(s_t)}$, in the 3×3 matrices of autoregressive coefficients $\Phi_{k(s_t)}$ for $k = 1, ..., q$ and finally in the 3×3 covariance matrix of error terms $\mathbf{\Sigma}_{(s_t)}$. In this empirical application I consider the more general case where there are potentially more than two regimes indexed by $s_t \in \{1, 2, ..., K\}$. As before, regimes switch over time according to an exogenous and ergodic Markov-Chain with a $K \times K$ transition matrix given by $\mathbf{P}$.

This MS-VAR can be viewed as a reduced form of the general equilibrium model that I develop in Section 2.6, where regime switches trigger changes in the coefficients of the monetary policy rule followed by the monetary authority. As noted by Benati and Surico (2009), changes in the coefficients of the monetary policy rule can affect

Figure 2.3: Time Series Included in the MS-VAR



Notes: Inflation (π_t), consumption growth (Δc_t) and the short-rate (i_t) are measured in percent per annum.

both the autoregressive coefficients and the covariance matrix of the innovations in the model's reduced-form[53].

The time series included in the MS-VAR were described in Section 2.3.2 and are shown in Figure 2.3. I use data from 1952:2 until 2008:4 in order to be consistent with the available yield curve slope data shown previously. I also include in the dataset q observations before 1952:2 as an initial condition for the MS-VAR.

Assuming that agents form expectations based on the knowledge of the current regime realization and all of the parameters in equation (2.10), the level risk can be computed easily using the method discussed in Section 2.4. In the two-regimes case, the estimated transition matrix and the expected value of the variables included in the MS-VAR, $E[\mathbf{Y}_t/s_t]$, can be used to evaluate the level risk terms in equations

[53]In reality, the reduced form associated with the DSGE model from Section 2.6 has both linear and quadratic terms (the model is solved to a 2^{nd} order approximation), while for simplicity the MS-VAR has only linear terms. This does not represent a serious problem, because in the context of the DSGE model the variables included in the MS-VAR are well approximated by a linear solution.

(2.8) and (2.9).

2.5.2 Estimation Results

I estimated equation (2.10) via Maximum Likelihood by applying the Hamilton (1989)[54] filter. To choose the number of regimes K and the MS-VAR lag length q, I used standard information criteria. Table 2.2 shows some important model selection information for different choices of K and q. The first column reports the number of estimated parameters in each specification. The second through fourth columns respectively report the maximized value of the log-likelihood, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). According to the AIC and BIC criteria, the best fitting models are highlighted in the table.

Both the AIC and BIC select models with $K > 1$. Note that the number of estimated parameters in case $K > 1$ increases dramatically with the MS-VAR's lag-length. As a result, both information criteria point to models with low lag-lengths. It is important to note that as the number of estimated parameters increase, it is more likely that the optimization algorithm used to estimate the model will get stuck in local maxima. In Table 2.2, for the model with $K = 3$, when I increase the lag length from $q = 2$ to $q = 3$, the log-likelihood decreases. This is a sign that the algorithm is stuck in a local maximum. To avoid this problem, I choose the best fitting model according to the BIC criterion ($K = 2$ and $q = 1$). Apart from having desirable large sample properties, this criterion penalizes models with an excessive number of parameters more heavily than does the AIC.

[54] It was assumed in Section 2.4.2 that the agents in the economy observe the current and all past regime realizations. The econometrician, on the other hand, needs to filter out probabilities for the regime realizations conditional on the available information. That is, since at any point in the time series the researcher does not know *ex-ante* the state of the Markov-Chain, the best she can do is to use an optimal filter to ascribe probabilities for each state. This filter is described in Hamilton (1989). Here, the estimation algorithm was implemented via Krolzig's MSVAR package for OX that uses the EM methods discussed in Krolzig (1997).

Table 2.2: MS-AR Model Selection Criteria

		number of parameters	Log-likelihood	AIC	BIC
$K = 1$	$q = 1$	15	-1136.8	10.18	10.45
	$q = 2$	21	-1121.8	10.12	10.53
	$q = 3$	27	-1112.6	10.12	10.66
	$q = 4$	33	-1092.9	10.03	10.70
$K = 2$	$q = 1$	38	-1030.1	9.41	$\boxed{9.98}$
	$q = 2$	56	-1007.6	9.37	10.22
	$q = 3$	74	-992.9	9.40	10.52
	$q = 4$	92	-965.7	9.32	10.71
$K = 3$	$q = 1$	60	-989.8	9.25	10.15
	$q = 2$	87	-939.0	$\boxed{9.04}$	10.35
	$q = 3$	114	-939.4	9.28	11.00
	$q = 4$	141	-917.6	9.33	11.45

Notes: In the lines, K is the number of regimes and q is the MS-VAR's lag-length. In the columns, AIC is the Akaike Information Criterion and BIC is the (Schwartz) Bayesian Information Criterion.

Table 2.3: MS-VAR Conditional Moments

	Regime 1	Regime 2
Average regime duration in years	34.6	7.7
Ergodic Probabilities	0.818	0.182
$E\left[\pi_t/s_t\right]$	2.38	4.93
$E\left[\Delta c_t/s_t\right]$	3.30	4.33
$E\left[i_t/s_t\right]$	3.96	6.80
$SD\left[\pi_t/s_t\right]$	1.39	2.90
$SD\left[\Delta c_t/s_t\right]$	2.65	3.73
$SD\left[i_t/s_t\right]$	2.23	3.29

Notes: The conditional moments were computed using numerical simulation of the MS-VAR with K=2 and q=1. The average regime duration and ergodic probabilities were computed according to Hamilton (1994).

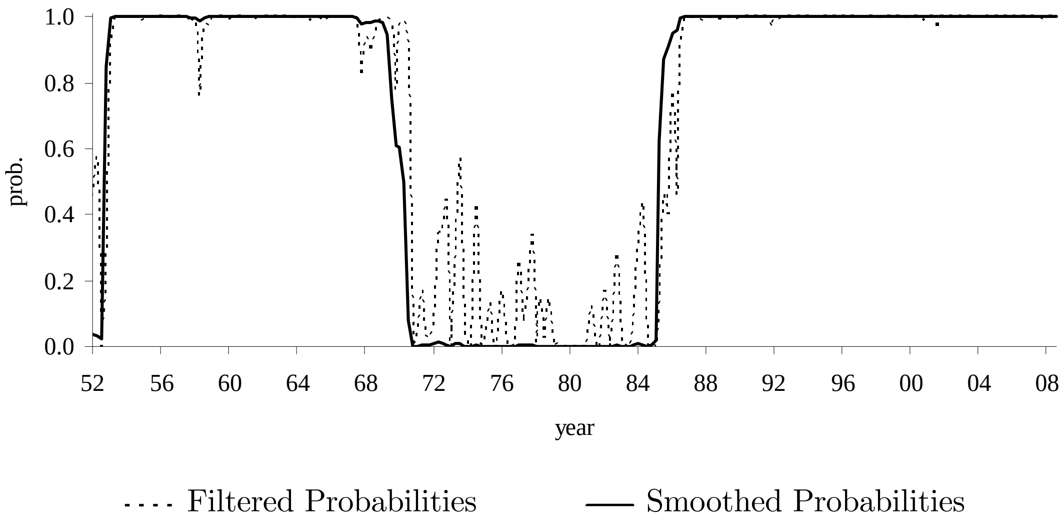
All of the parameter estimates for the best fitting MS-VAR with $K = 2$ and $q = 1$ are reported in Appendix 2.B. Some selected conditional moments for the MS-VAR also are shown in Table 2.3, while the filtered and smoothed regime probabilities are shown in Figure 2.4.

For convenience I reproduce here the estimated MS-VAR transition matrix

$$\begin{pmatrix} \widehat{p}_{11} & \widehat{p}_{12} \\ \widehat{p}_{21} & \widehat{p}_{22} \end{pmatrix} = \begin{pmatrix} \underset{(0.007)}{0.993} & 0.007 \\ 0.033 & \underset{(0.023)}{0.967} \end{pmatrix}$$

where the numbers in parentheses are standard errors. Note that, since p_{11} is close to unity, regime 1 is very persistent and close to being an absorbing state. According to Table 2.3, this regime has an average duration of about 35 years. On the other hand, the probability of remaining in regime 2, p_{22}, is considerably smaller, implying an average duration of only 7.7 years for regime 2. Note, however, that the estimate

Figure 2.4: MS-VAR Filtered and Smoothed Probatilities of Regime 1



Notes: Probabilities computed using the filters described in Hamilton (1994) for the MS-VAR with $K=2$ and $q=1$.

for p_{22} is not very precise.

Based on the smoothed regime probabilities, regime 1 almost exactly encompasses the Martin and the Greenspan / Bernanke subsamples from Section 2.3. In particular, the MS-VAR interprets the Great Moderation years as realizations of this regime. On the other hand, regime 2 roughly covers the Burns / Miller and Volcker subsamples from Section 2.3 (this regime also appears with a high probability very briefly from 1952:2 to 1953:1) - it therefore encompasses the Great Inflation years. The conditional moments in Table 2.3 reveal that when regime 1 occurs, the inflation rate and the short-term interest rate fluctuate around significantly lower levels than when regime 2 is realized. Moreover, confirming the results obtained in Section 2.3.2, the economy conditional on regime 1 is substantially more stable than when regime 2 realizes[55].

I now use the mean short-rate gap across regimes and the regime-switching probabilities estimated above to parameterize investors' beliefs. Using equations (2.8)

[55]Interestingly, the regime classification that emerges from Figure 2.4 is very similar to the one estimated by Ang and Bekaert (2002).

and (2.9), it is straightforward to compute the level risk conditional on each regime.

Using the information reported in Table 2.3, we note that the mean interest rate differential across regimes 1 and 2, $\mathcal{D}_s E\left[i_t/s_t\right]$, is equal to 2.84%. Using the estimated ergodic probabilities reported in the same table, it is easy to show that in the limit as $\tau \to \infty$ the level risk conditional on regime 1 is given by $(1-0.818)\times 2.84\% = 0.52\%$. A similar calculation for regime 2 yields a limiting level risk of $-0.818 \times 2.84\% = -2.32\%$. Therefore, in the limit the estimated level risk becomes sizeable, especially for the less persistent regime 2. Observe the sign of the level risk in each regime. In regime 2 investors anticipate that a future switch to regime 1 is possible. This switch would imply that the short rate fluctuates around a significantly lower level than the current one. As a result, there is a *negative* level risk. In regime 1, on the other hand, investors believe that there is a possibility of switching to the highly volatile regime 2, which implies a shift towards higher levels of the short rate. In this case a *positive* level risk arises. Because regime 1 is more persistent than regime 2, the level risk is larger in absolute value in the latter than in the former regime.

For values of τ far away from the limit, the estimated level risks at different maturities are reported in the second (regime 1) and third (regime 2) columns of Table 2.4. In both regimes, the level risk is relevant in terms of magnitude: in regime 2, the level risk grows quickly to -115 basis points at the 10-year horizon, while in regime 1 it is still capable of increasing the average 10-year slope by 26 basis points.

By rearranging equations (2.8) and (2.9), we can see that subtracting the estimated level risk from the average slope in each regime makes it possible to recover the term premium. However, because my MS-VAR does not include the yield curve slope, I cannot directly obtain the mean value of the slope conditional on each regime. I therefore use the sample average of observed slope measures as a *proxy*.

Sample averages of slope measures at different maturities[56] are reported in panel

[56] In addition to the 5 and 10-year slope measures discussed before, I consider in this section the 1, 2, 3, and 4-year slope measures calculated again using the CRSP database.

Table 2.4: Yield Curve Slope Decomposition

			Regime 1	Regime 2	*Great Moderation* (85:3-08:4)	*Great Inflation* (70:3-85:2)
(1)	Average Slope	$\tau = 1$ year	0.38	0.46	0.40	0.48
		$\tau = 3$ years	0.75	0.78	0.92	0.80
		$\tau = 5$ years	0.99	0.91	1.25	0.94
		$\tau = 10$ years	1.47	1.08	1.84	1.08
(2)	Level Risk	$\tau = 1$ year	0.03	-0.14	0.03	-0.14
		$\tau = 3$ years	0.10	-0.45	0.10	-0.45
		$\tau = 5$ years	0.16	-0.70	0.16	-0.70
		$\tau = 10$ years	0.26	-1.15	0.26	-1.15
(3)	Residual [(1)-(2)]	$\tau = 1$ year	0.35	0.60	0.37	0.61
		$\tau = 3$ years	0.65	1.22	0.82	1.25
		$\tau = 5$ years	0.84	1.61	1.09	1.64
		$\tau = 10$ years	1.22	2.23	1.58	2.23

Notes: Average slope measures conditional on regimes 1 and 2 were computed for the sample 1985:3-2008:4 and 1970:3-1985:2 respectively. Level risk measures are based on the estimated MS-VAR.

(1) of Table 2.4. In the second column I report average yield curve slopes for the quarters when regime 1 was associated with a smoothed probability greater that 50%. The third column reports the same measures for quarters where regime 2 was the predominant regime according to the smoothed probabilities. The fourth and fifth columns of the table report average slope measures for two particular subsamples of the data of interest: first, the 1985:3-2008:4 subsample, which is the portion of regime 1 that corresponds to the 'Great Moderation'; second, the 1970:3-1985:2 subsample, which corresponds to the 'Great Inflation' years of regime 2. Panel (3) of Table 2.4 shows the residuals obtained after subtracting the level risk from these average slope measures.

The slope averages across regimes restate the puzzle from Section 2.3: conditional on the high volatility regime 2, the slope on average is *lower* than in regime 1. This is true for the 5- and 10-year maturities reported in the second and third columns of Table 2.4, but if we restrict our attention to the Great Inflation and

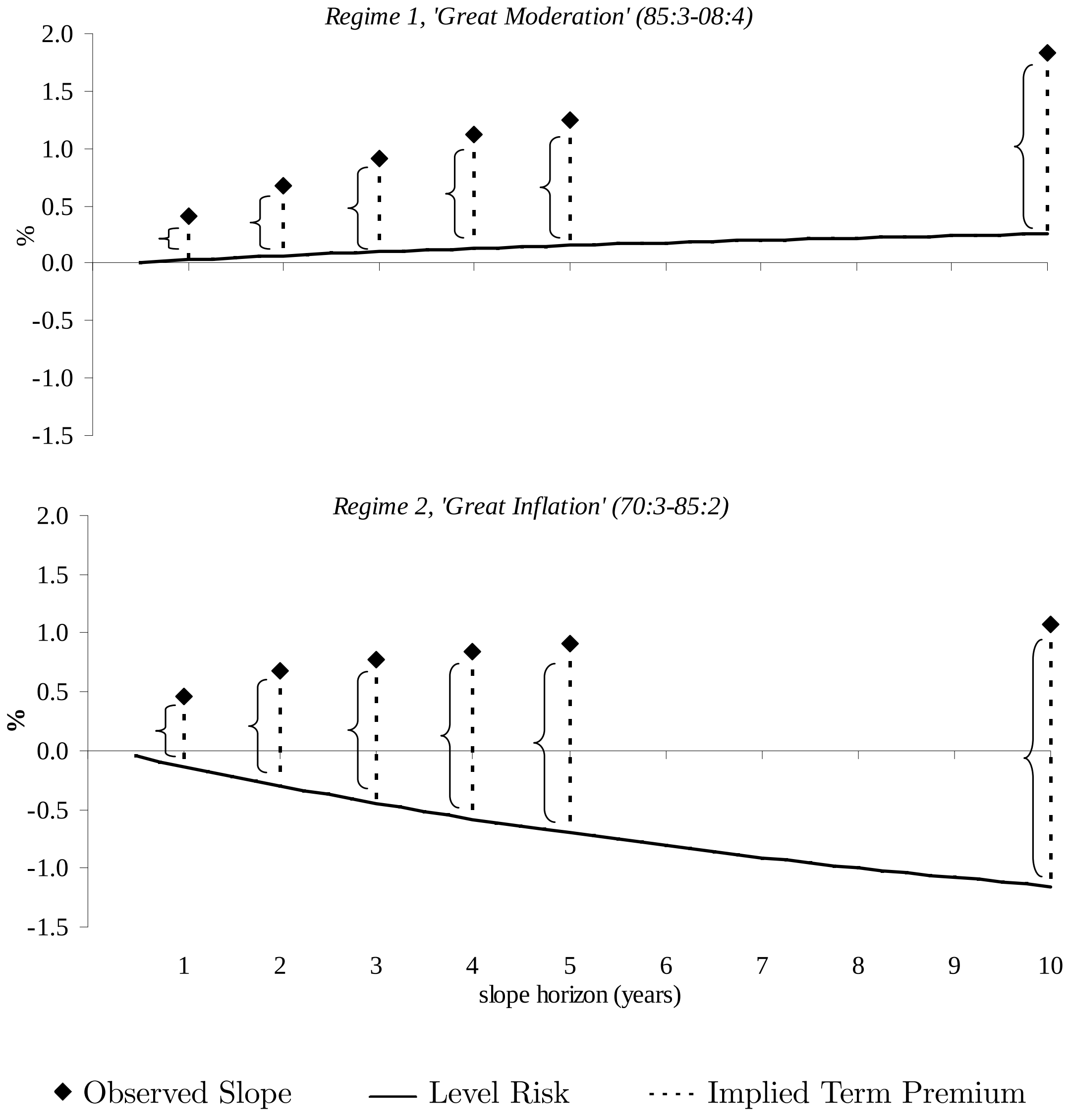
Great Moderation subsamples (fourth and fifth columns of Table 2.4), this is true for almost all reported maturities. Importantly, once I control for the level risk, the puzzle disappears completely. That is, once level risks are subtracted from the slope measures, regime 2 displays substantially higher premia than regime 1 at all maturities analyzed[57]. The same is true when I restrict my attention to the Great Inflation and Great Moderation subsamples. In other words, based on slope measures nominal bonds seem less risky in the Great Inflation than in the Great Moderation not because of a seemingly abnormal pattern of term premia across regimes, but rather simply because of an outcome of beliefs that incorporate the possibility of regime switches.

An alternative way to illustrate this last result is shown in Figure 2.5. In the top chart, the full line plots the estimated level risks in regime 1 against increasing maturities in the horizontal axis. The diamond markers correspond to the average slope measures from Table 2.4 for the 'Great Moderation' subsample. The bottom chart shows the same analysis but for the level risks conditional on regime 2 and the average slope measures that correspond to the 'Great Inflation' subsample.

Although the yield curve in both regimes on average is positively sloped at all available horizons, the decomposition of the slope is quite different. In regime 1 the slope measures are a combination of positive term premia and positive level risks, but in regime 2 the yield curve is positively sloped *in spite of substantially negative level risks*. Conditional on regime 2, it takes very high term premia to offset the negative level risks and to generate the observed slope. The residual term premia, indicated in Figure 2.5 by the curly brackets, are in fact significantly larger in regime 2 than in regime 1.

[57] This is similar to Bekaert, Hodrick, and Marshall (2001). They find that some term structure 'anomalies' can be explained by a combination of term premia and a 'peso problem' in the short-rate process.

Figure 2.5: Decomposing the Observed Mean Slope



2.6 Level Risks in a Structural Model with MS Monetary Policy Regimes

In the MS-VAR estimated in the previous section, what makes the short-rate fluctuate around different levels across regimes, giving rise to level risks? Similarly, what makes the economy less volatile in regime 1 than in regime 2? The reduced-form MS-VAR is silent about the possible structural changes experienced by the macroeconomy once it switches to a new regime.

In this section, I investigate whether differences in how the Fed conducts monetary policy jointly can explain the different macro dynamics and the yield curve shape across regimes. I find that the main features of the data explored in the previous sections can be replicated in a simple MS-DSGE that allows the economy to switch over time between active and passive monetary policy regimes. Private agents incorporate the MS possibility into their beliefs, which in turn has important implications for both the macroeconomic equilibrium and the yield curve.

In Section 2.6.1 I start by describing the MS-DSGE model and proposing an approximate non-linear solution method. The proposed solution method allows agents to accumulate precautionary savings, thus giving rise to non-trivial bond premia. In particular, the model endogenously generates a short-rate differential across regimes as private agents acquire different levels of precautionary savings depending on the monetary policy stance. As a result, level risks appear along the yield curve. In Section 2.6.2 I show that under a plausible choice of parameters the model is able to replicate the Slope-Volatility Puzzle. In other words, when there is a passive policy regime, the amount of macro uncertainty is substantially higher than in the active regime; as a result, term premia are higher. However, because of level risks generated endogenously through precautionary savings, during the passive regime the yield curve is less steep than in the active one, thus reproducing the Slope-Volatility Puzzle.

2.6.1 Model Description and Solution

Monetary Policy Regimes Following Davig and Leeper (2007) and many others[58], I assume that the Fed sets the short-term nominal interest rate i_t according to the following MS feedback rule

$$i_t = \bar{i} + \phi_{\pi(s_t)}\widehat{\pi}_t + \phi_{y(s_t)}\widehat{y}_t$$

where $\widehat{\pi}_t$ and $\widehat{y}_t$ are the log deviations of aggregate inflation and output from the deterministic steady state[59]. The crucial difference between this formulation and more standard Taylor rules is that here the policy reaction coefficients $\phi_{\pi(s_t)}$ and $\phi_{y(s_t)}$ at time t depend on the regime realization $s_t \in \{1, 2\}$. [60] As a result, a regime switch can trigger changes in how the Fed sets the short rate in order to fight deviations of inflation and output from their steady-state levels.

Without loss of generality, I set $\phi_{\pi(1)} \geq \phi_{\pi(2)}$, meaning that monetary policy in regime 1 is at least as 'active' with respect to inflation deviations as in regime 2. In particular, when $\phi_{\pi(1)} > \phi_{\pi(2)}$, regime 2 is considered 'less active' than regime 1. When $\phi_{\pi(2)} < 1$, policy in regime 2 is said to be 'passive'.

In a model with fixed parameters, a passive monetary policy rule implies indeterminacy of the equilibrium solution[61]. In that case, the policy rate increases less than one-to-one with an increase in inflation; consequently, the ex-post short-term real interest rate falls. However, in the case of a regime-switching Taylor rule, this is not necessarily true. As Davig and Leeper (2007) show, although monetary policy

[58] See, for example, Farmer, Waggoner, and Zha (2007), Davig and Doh (2008), Liu, Waggoner, and Zha (2009) and Liu and Mumtaz (2010).

[59] Adding an autocorrelated monetary policy shock to the monetary policy rule does not significantly change any of my results.

[60] A potentially interesting extension would be to analyze models with a regime-dependent inflation target. However, in a recent study, Liu, Waggoner, and Zha (2010) found no empirical support for this specification in the U.S. data.

[61] When $\phi_y \neq 0$, the threshold above which ϕ_π respects the Taylor principle is slightly below but still very close to unity for reasonable Taylor-rule parameterizations. See Bullard and Mitra (2002).

is passive in one regime, if the probability of switching to an active enough regime is sufficiently high, then the model has a unique stable solution.

For consistency with my previous results, I assume an MS environment in line with Assumption (1) from Section 2.5. That is, I again let the economy switch between two different regimes over time. These regimes evolve according to an exogenous Markov Chain indexed by $s_t \in \{1, 2\}$, with transition matrix $\mathbf{P}$. Finally, private agents again are assumed to observe the current regime realization s_t before making decisions; accordingly, the complete information set available to private agents at date t will be denoted by $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$.

Private Agents The macro model contains four agents: households, final and intermediate good producers and a monetary authority. The latter was described in the previous section. I now analyze the behavior of each remaining agent in turn. Detailed model derivations can be found in Appendix 2.D.

Following Rudebusch and Swanson (2008), the household sector has a representative infinitely-lived agent endowed with Epstein and Zin (1989, 1991) and Weil (1990) preferences. Letting C_t and N_t represent the household's consumption and labor supply, those preferences are described by:

$$V_t = \begin{cases} u\left(C_t, N_t\right) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if} \quad u\left(C_t, N_t\right) \geq 0 \text{ everywhere} \\ \\ u\left(C_t, N_t\right) - \beta \left[E_t \left(-V_{t+1}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if} \quad u\left(C_t, N_t\right) \leq 0 \text{ everywhere} \end{cases} \tag{2.11}$$

where V_{t+1} denotes the utility continuation value to the household. The period utility is given by $u\left(C_t, N_t\right) = e^{b_t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta}\right)$, where b_t represents a time-preference shock. As Epstein and Zin (1989) show, these preferences disentangle the coefficient of risk aversion from the elasticity of intertemporal substitution (EIS), which are constrained in standard expected utility preferences to be the reciprocal of one another. In the particular parametrization above, the degree of risk aversion is associated with (but not equal to) $\alpha \in \mathbb{R}$, whereas the EIS is given by $1/\gamma$. When $\alpha = 0$, the standard expected utility case is recovered.

The representative household maximizes (2.11) subject to the budget constraint

$$P_t C_t + E_t \widetilde{M}_{t+1} X_{t+1} \leq X_t + P_t W_t N_t + T_t$$

where P_t is the aggregate price level and $E_t \widetilde{M}_{t+1} X_{t+1}$ is the value of a complete portfolio of state-contingent assets with $\widetilde{M}_{t+1}$ representing the *nominal* stochastic discount factor and X_{t+1} the portfolio holdings from period t to $t+1$. Additionally, W_t represents the real wage rate and T_t summarizes all lump-sum transfers to the household. It follows from the household's optimization problem that the one-period *real* SDF and the labor supply are respectively given by

$$\begin{aligned} M_{t,t+1} &= \beta \left[\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} e^{b_{t+1}-b_t} \\ W_t &= \chi \frac{N_t^\eta}{C_t^{-\gamma}} \end{aligned} \tag{2.12}$$

where, under complete markets, the SDF can be used to price nominal bonds of different maturities recursively. More specifically, letting $\Pi_{t+1} \equiv P_{t+1}/P_t$, the price of a τ-period nominal bond is given by $B_{\tau,t} = E_t \left[M_{t,t+1} B_{\tau-1,t+1} \Pi_{t+1}^{-1} \right]$. In the specific case of a one-period bond, the pricing condition becomes $B_{1,t} = E_t \left[M_{t,t+1} \Pi_{t+1}^{-1} \right]$. Note that the term in parenthesis in the SDF expression, which also appears in expected utility models, captures the current consumption risk. The term in square brackets containing the continuation utility value introduces aversion to long-run consumption and labor risks.

Final good firms operate under perfect competition, and the representative producer is endowed with the following technology

$$Y_t = \left(\int_0^1 Y_{t(f)}^{\frac{1}{1+\lambda_t}} di \right)^{1+\lambda_t}$$

where Y_t is the quantity of final goods produced through a combination of $Y_{t(f)}$ of each intermediate good $f \in [0,1]$. Following Steinsson (2003) and Smets and Wouters (2003), I allow the degree of substitutability across differentiated intermediate goods λ_t to vary over time. A decrease in λ_t reduces the monopoly power of intermediate producers, which in turn reduces their price markup. Profit maximization in the final-good sector yields a demand curve for each intermediate good

$$Y_{t(f)} = \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t}} Y_t \tag{2.13}$$

where $P_{t(f)}$ is the price of intermediate good f. The aggregate price level is then given by $P_t = \left(\int_0^1 P_{t(f)}^{-\frac{1}{\lambda_t}} di \right)^{-\lambda_t}$.

In the intermediate-good sector, all firms have identical Cobb-Douglas production functions given by

$$Y_{t(f)} = A_t \overline{K}^{\theta} N_{t(f)}^{1-\theta} \tag{2.14}$$

where the level of capital $\overline{K}$ is assumed for simplicity to be fixed, $N_{t(f)}$ is the amount of labor employed by firm f, and $Y_{t(f)}$ is its level of output. The aggregate level of technology is denoted by A_t. As in Rotemberg (1982), firms can reset the prices of each differentiated good in every period, but incur intangible quadratic adjustment costs in doing so

$$\frac{\xi}{2} \left(\frac{P_{t(f)}}{P_{t-1(f)}} \frac{1}{\overline{\Pi}} - 1 \right)^2 P_t Y_t$$

where $\overline{\Pi}$ is the steady state rate of inflation. These costs do not affect the firm's cash-flows, but must be considered in the optimization problem. Therefore, each intermediate firm f chooses $P_{t(f)}$ so as to maximize the expected discounted sum of future profits corrected by the adjustment costs

$$E_t\left\{\sum_{j=0}^{\infty} M_{t,t+j}\frac{P_t}{P_{t+j}}\left[D_{t+j(f)}-\frac{\xi}{2}\left(\frac{P_{t+j(f)}}{P_{t+j-1(f)}}\frac{1}{\overline{\Pi}}-1\right)^2 P_{t+j}Y_{t+j}\right]\right\}$$

where $D_{t(f)}$ represents the period t profit which is given by $P_{t+j(f)}Y_{t+j(f)}-P_{t+j}W_{t+j}N_{t+j(f)}$. Since firms are owned by households, $M_{t,t+j}\equiv\prod_{k=1}^{j}M_{t+k}$ is used to discount future profits. The optimization problem is constrained by sequences of equations (2.13) and (2.14) starting from period t onwards.

The market clearing condition in the final good market is given by

$$Y_t = C_t + G_t + \delta\overline{K}$$

where G_t represents a shock to the market clearing condition and, following Rudebusch and Swanson (2008), a constant amount $\delta\overline{K}$ of the final good is used to repair the depreciated capital. If G_t is interpreted as government purchases, then it is assumed that the government runs a balanced budget financed through lump-sum taxes obtained from the household sector.

To complete the model description, assume that the four exogenous shocks follow AR(1) processes:

$$\begin{aligned}
b_t &= \rho_b b_{t-1}+\sigma_b\varepsilon_t^b\\
\log A_t &= \rho_A\log A_{t-1}+\sigma_A\varepsilon_t^A\\
\log\left(1+\lambda_t\right) &= \left(1-\rho_\lambda\right)\log\left(1+\overline{\lambda}\right)+\rho_\lambda\log\left(1+\lambda_{t-1}\right)+\sigma_\lambda\varepsilon_t^\lambda\\
\log G_t &= \left(1-\rho_G\right)\log\overline{G}+\rho_G\log G_{t-1}+\sigma_G\varepsilon_t^G
\end{aligned}$$

with independently and identically distributed innovations $\varepsilon_t^k\sim N\left(0,1\right)$ for $k\in\{b,A,\lambda,G\}$.

Model Solution The macro model has five (exogenous) predetermined variables: b_t, A_t, λ_t, G_t and s_t. The first four are standard continuously differentiable variables, whereas the last is a discrete variable that follows a Markov chain with transition matrix $\mathbf{P}$. In order to write the Markov chain compactly, I define a 2×1 vector $\boldsymbol{\xi}_t \equiv (1[s_t = 1], 1[s_t = 2])'$ where $1[s_t = j]$ is an indicator function which is equal to one if $s_t = j$. Hamilton (1994) shows that the Markov chain can be represented in terms of $\boldsymbol{\xi}_t$ by the autoregressive process $\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t$, where $\boldsymbol{\nu}_t$ is a heteroskedastic zero mean vector of innovations that can only assume discrete values. Collecting the differentiable predetermined variables in a $n_x \times 1$ vector $\mathbf{x}_t \equiv (b_t, \log A_t, \log(1+\lambda_t), \log G_t)'$ and letting $\mathbf{y}_t$ represent the $n_y \times 1$ vector of (natural logarithms of the) non-predetermined variables, the macroeconomic system together with the bond pricing equations can be written as

$$
\begin{aligned}
&E_t\left[f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \boldsymbol{\xi}_t\right)\right] = 0 \\
&\mathbf{x}_{t+1} = \left(I_4 - \boldsymbol{\Lambda}\right)\overline{\mathbf{x}} + \boldsymbol{\Lambda}\mathbf{x}_t + \Sigma\sigma\boldsymbol{\varepsilon}_{t+1} \\
&\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t
\end{aligned}
\tag{2.15}
$$

where $\boldsymbol{\varepsilon}_t \equiv \left(\varepsilon_t^b, \varepsilon_t^A, \varepsilon_t^\lambda, \varepsilon_t^G\right)'$ is the vector of *i.i.d.* standard normal shocks. The coefficient matrices are given by $\boldsymbol{\Lambda} = diag(\rho_b, \rho_A, \rho_\lambda, \rho_G)$, $\Sigma = diag(\sigma_b, \sigma_A, \sigma_\lambda, \sigma_G)$ and $\overline{\mathbf{x}} = \left(0, 0, \log\left(1+\overline{\lambda}\right), \log\overline{G}\right)'$, while σ is a perturbation parameter that scales uncertainty in the model.

The model above forms a system of Markov-Switching non-linear rational expectations equations for which an analytical solution is not known. Solution methods based on standard linear approximations have been proposed in the literature – e.g. Davig and Leeper (2007) and Farmer, Waggoner, and Zha (2010b)–, but they would give rise to zero risk premia implied in the prices of financial assets and therefore are not useful in my context. I will therefore look for a second-order approximation to the true model solution using the perturbation techniques suggested by Schmitt-

Grohe and Uribe (2004)[62].

In order to be able to apply perturbation techniques to the system (2.15), one needs to be able to differentiate it with respect to all state variables. But because $\boldsymbol{\xi}_t$ is discrete, it is impossible to apply these techniques directly to the system above. Therefore I define an *extended system*[63] of equations in which the dependence of the control variables on regimes is made explicit. It is then straightforward to implement perturbation methods to the extended system, because it is fully differentiable in the state variables.

In order to write the extended system, I introduce a state-contingent notation. That is, I denote the value of the vector of endogenous variables $\mathbf{y}_t$ contingent on $s_t = s \in \{1, 2\}$ by $\mathbf{y}_{t(s)}$. Using this notation, the expected value of $\mathbf{y}_{t+1}$ conditional on Ω_t can be parameterized as follows: $E\left[\mathbf{y}_{t+1}/\Omega_t^{-s}, s_t = s\right] = p_{s1}E\left[\mathbf{y}_{t+1(1)}/\Omega_t^{-s}\right] + p_{s2}E\left[\mathbf{y}_{t+1(2)}/\Omega_t^{-s}\right]$. The extended non-linear system therefore can be written as:[64]

$$
\begin{aligned}
F\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{y}_{t(2)}, \mathbf{x}_t\right) \equiv E\left[\begin{array}{c} f_1\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{x}_t\right) \\ f_2\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(2)}, \mathbf{x}_t\right) \end{array} /\Omega_t^{-s}\right] = 0 \\
\mathbf{x}_{t+1} = \left(I_4 - \boldsymbol{\Lambda}\right)\overline{\mathbf{x}} + \boldsymbol{\Lambda}\mathbf{x}_t + \Sigma\sigma\boldsymbol{\varepsilon}_{t+1} \\
\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t
\end{aligned}
\tag{2.16}
$$

where each equilibrium condition in the original system (2.15) is represented by two entries in $F(\cdot)$, each contingent on one possible realization of $\boldsymbol{\xi}_t$. Note that the expectations operator does not condition on s_t because the MS probabilities are already dealt with by the state-contingent notation. Observe too that $f_1(\bullet)$, for

[62] In a model where only the shock volatilities follow exogenous MS processes, Amisano and Tristani (2010a,b) show that the perturbation solution is particularly easy to obtain. However, their solution method does not apply to the general case where other model parameters, such as the ones in the policy rule, are allowed to follow MS processes.

[63] Note that I rewrite the *non-linear* Markov Switching model represented by system (2.15) into the extended form. Davig and Leeper (2009) apply a similar transformation to a Markov-Switching model *after* log-linearizing the equilibrium conditions and refer to this as the "linear representation". Since here the model remains non-linear after being transformed into the extended form, I refer to it as the "extended system" to avoid confusion.

[64] See Appendix 2.E for more details on the extended non-linear system.

example, depends on the $t+1$ vector of control variables contingent on regime 1 and 2. Writing the system this way makes explicit the fact that, in general, the solution in each regime will depend crucially on the behavior of the economy in the alternative regime. Expectations that policy may switch in the future will affect households' and firms' decisions today and will lead to a different equilibrium relative to a model without switching regimes. Only when both regimes are absorbing states, that is $p_{11} = p_{22} = 1$, will the solution in each regime be independent of the behavior of the economy in the alternative regime. Unlike in (2.15), the extended system only has differentiable predetermined variables. Therefore a perturbation solution can be obtained easily[65].

The model solution I seek takes the form

$$\begin{aligned}
\begin{pmatrix} \mathbf{y}_{t(1)} \\ \mathbf{y}_{t(2)} \end{pmatrix} &= \begin{pmatrix} g^1(\mathbf{x}_t, \sigma) \\ g^2(\mathbf{x}_t, \sigma) \end{pmatrix} \\
\mathbf{x}_{t+1} &= (I_4 - \mathbf{\Lambda})\,\overline{\mathbf{x}} + \mathbf{\Lambda}\mathbf{x}_t + \Sigma\sigma\boldsymbol{\varepsilon}_{t+1} \\
\boldsymbol{\xi}_{t+1} &= \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t \ .
\end{aligned}$$

My aim is to approximate $g^1(\bullet)$ and $g^2(\bullet)$ around the deterministic steady state defined by $\mathbf{x}_{t+1} = \mathbf{x}_t = \overline{\mathbf{x}}$ and $\sigma = 0$, which implies that $\overline{g}^j = g^j(\overline{\mathbf{x}}, 0)$ for $j \in \{1, 2\}$. This is a convenient approximation point because an analytical solution to the non-linear system can be found easily. Note that the regime-switching parameters $\phi_{\pi(s_t)}$ and $\phi_{y(s_t)}$ do not affect the economy in the deterministic steady state, which implies that $\overline{g}^1 = \overline{g}^2 = \overline{g}$.

Using insights from Schmitt-Grohe and Uribe (2004), a second-order approximate

[65] To write the extended system, it is crucial that there be no regime-dependent state variables in the model. Imagine, for example, that time-varying capital is included in the model. Then, in order to rewrite the system in extended form, we would need to keep track of all the history of realized regimes. It then would be impossible to solve the model using the method proposed here.

solution to the vector of control variables conditional on $s_t = s$ is given by

$$\mathbf{y}_{t(s)} \cong \overline{g} + \overline{g}_x^s (\mathbf{x}_t - \overline{\mathbf{x}}) + \frac{1}{2} \begin{bmatrix} (\mathbf{x}_t - \overline{\mathbf{x}})' \overline{g}_{xx[1]}^s (\mathbf{x}_t - \overline{\mathbf{x}}) \\ \cdots \\ (\mathbf{x}_t - \overline{\mathbf{x}})' \overline{g}_{xx[n_y]}^s (\mathbf{x}_t - \overline{\mathbf{x}}) \end{bmatrix} + \frac{1}{2} \overline{g}_{\sigma\sigma}^s \sigma^2 \quad \text{for } s = 1, 2 \tag{2.17}$$

where $\overline{g}_x^s$ is a $n_y \times n_x$ matrix of first derivatives of $g^s(\bullet)$ with respect to the state variables and $\overline{g}_{xx[k]}^s$ for $k = 1, ..., n_y$ are symmetric $n_x \times n_x$ matrices of second derivatives of $g^s(\bullet)$ again with respect to the state variables. The $n_y \times 1$ vector $\overline{g}_{\sigma\sigma}^s$ denotes the second derivatives of $g^s(\bullet)$ with respect to the scalar σ. All matrices of first and second derivatives are evaluated at the deterministic steady state. Finally, note that the $\overline{g}_\sigma^s$ and $\overline{g}_{x\sigma}^s$ terms omitted from equation (2.17) are proven to be equal to zero by Schmitt-Grohe and Uribe (2004).

From equation (2.17), some important properties of the model solution emerge. First, uncertainty as measured by σ only shifts the constant term of the policy function by $\overline{g}_{\sigma\sigma}^s$. This shift causes the control variables to fluctuate around a *stochastic steady state*, which corrects for the precautionary savings motive and gives rise to risk premia in financial assets. Because the cross derivatives between the state variables and σ are all zero up to a second-order, the risk premia *within any given regime* are constant. Note, however, that the second derivative term with respect to σ will in general depend on the current regime realization which in turn will give rise to discrete changes in risk premia as regimes alternate.

Standard perturbation methods solve for the unknown coefficients of the above Taylor expansion by taking derivatives of (2.16) with respect to $\mathbf{x}_t$ and σ, which are equal to zero and can be evaluated easily at the deterministic steady state. All unknown coefficients of equation (2.17) are then determined by solving relatively simple systems of equations[66].

[66] Conditions for uniqueness of a bounded solution in Markov-Switching DSGE models have been established by Farmer, Waggoner, and Zha (2010a), but apply to the case of linearized models. More general conditions that apply to our original non-linear system (2.15) are not yet available in

2.6.2 Model Analysis

Here I start by calibrating the parameters of the MS-DSGE model. I then analyze how well the model that is based on switching monetary policy regimes is able to replicate the key empirical macro and yield curve moments analyzed in Section 2.5.

For convenience, the empirical moments I focus on in this section are reproduced in Table 2.5. In particular, I am interested in simultaneously replicating two sets of empirical moments. First, the MS-DSGE model should be able to reproduce the variances of inflation, consumption growth, and the short rate, conditional on each regime of the MS-VAR. These are displayed in the top panel of Table 2.5. Second, I require that the MS-DSGE replicates the yield curve slope decomposition based on the MS-VAR, which I reproduce in the bottom panel of Table 2.5 (for simplicity, I focus here only on the 10-year maturity). For the latter set of moments, I focus in this section on the empirical moments shown in the fourth and fifth columns of Table 2.4, which correspond to the Great Inflation and Great Moderation subsamples.

The objective of this calibration exercise is to verify whether the model implied moments, conditional on the monetary policy regimes 1 (more active) and 2 respectively, replicate the empirical moments for the Great Moderation and Great Inflation subsamples shown in Table 2.5. Put more formally:

- **The Proposed Calibration Exercise:**

  Consider the MS-DSGE model described above where $\phi_{\pi(1)} > \phi_{\pi(2)}$. Do the model implied moments conditional on regime 1 (regime 2) replicate the empirical Great Moderation (Great Inflation) moments shown in Table 2.5?

The spirit of this exercise is to study how well the model, relying *only* on shifts in monetary policy, replicates the macro and yield curve moments in Table 2.5. To

the literature. In what follows, I consider only bounded equilibria that are unique for a linearized version of the model using the Farmer, Waggoner, and Zha (2010a) method.

Table 2.5: Summary of the Empirical Moments

	Great Moderation	*Great Inflation*
(1) Macro Moments:		
$SD\left[\pi_t/s_t\right]$	1.39	2.90
$SD\left[\Delta c_t/s_t\right]$	2.65	3.73
$SD\left[i_t/s_t\right]$	2.23	3.29
(2) Yield Curve Moments:		
$E\left[i_{10Y,t} - i_t/s_t\right]$	1.84	1.08
of which:		
$E\left[NTP_{10Y,t}/s_t\right]$	1.58	2.23
$LevelRisk_{10Y}$	0.26	-1.15

Notes: The macro moments in panel (1) are reproduced from table 2.3, whereas the decomposition of the 10-year average slope in panel (2) is reproduced from the fourth and fifth columns of table 2.4.

keep the results easy to interpret I refrain from analyzing models with MS volatilities, even though this extra feature potentially could improve the model's fit.

Choice of Parameters I calibrate the model such that, conditional on regime 1, it fits the Great Moderation moments in Table 2.5. The parameter choices shown in Table 2.6 follow estimated DSGE models such as Lubik and Schorfheide (2004), Smets and Wouters (2007) and Justiniano and Primiceri (2008).

The parameters in the monetary policy rule conditional on the more active regime are set according to the post-1982 estimates in Lubik and Schorfheide (2004), i.e. $\phi_{\pi(1)} = 2.19$ and $\phi_{y(1)} = 0.075$. In regime 2, the Fed's response to inflation is set to the lowest value that guarantees the existence of a unique stable model equilibrium, i.e. $\phi_{\pi(2)} = 0.948$. For simplicity, I also set $\phi_{y(2)} = 0.075$. [67] As in Section 2.5, the transition probabilities were set to $p_{11} = 0.993$ and $p_{22} = 0.967$.

For the household's preferences, I choose $\beta = 0.99$ that implies an annualized real discount rate of 4% in the deterministic steady state. The utility consumption curvature γ is set to 2: this implies an EIS of one half, in line with micro data estimates such as Vissing-Jorgensen (2002). Following Smets and Wouters (2007), I set the inverse Frisch elasticity of the labor supply η to 0.4. I set α according to the 'best fit' specification in Rudebusch and Swanson (2008). A traditional measure of risk aversion suggested by Epstein and Zin (1989), $\gamma + \alpha(1 - \gamma)$, would then imply a coefficient of relative risk aversion of 110. Although high, this risk aversion measure applies only to endowment economies and, in models with a flexible labor supply, suffers from a substantial upward bias (see Swanson (2009)). Moreover, estimated DSGE models with recursive preferences fitted to U.S. bond prices usually feature

[67] In a reduced form macro-term structure model, Ang, Boivin, Dong, and Loo-Kung (2011) find that the Fed's policy response to the output gap was roughly stable over the sample period I analyze.

Table 2.6: The Benchmark Calibration

Monetary Policy Rule:				Exogenous Processes:			
$\phi_{\pi(1)}$	2.19	$\phi_{\pi(2)}$	0.948	ρ_b	0.83	σ_b	0.020
$\phi_{y(1)}$	0.075	$\phi_{y(2)}$	0.075	ρ_A	0.98	σ_A	0.005
p_{11}	0.993	p_{22}	0.967	ρ_λ	0.18	σ_λ	0.051
				ρ_G	0.94	σ_G	0.008
Structural Parameters:				**The Steady State:**			
β	0.99	θ	0.33	$\overline{\Pi}$	1.004		
γ	2	$\overline{\lambda}$	0.2	$\overline{K}/\left(4\overline{Y}\right)$	2.5		
η	0.40	ξ	233	$\overline{G}/\overline{Y}$	0.2		
α	-108	δ	0.02				

a high level of risk aversion (e.g. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2011)).

The capital share in the production function θ is set to 0.33. Also, I choose $\overline{\lambda} = 0.2$, implying a steady-state price markup of 20%. The price adjustment costs parameter ξ is set to 233 which, for a linearized version of my model, corresponds to a Calvo coefficient of 0.75. [68]

The four exogenous shock processes were calibrated as follows: the parameters associated with the shocks to the household's preferences and to the price markup follow the estimates in Justiniano and Primiceri (2008) and Smets and Wouters (2007) respectively for the post-Great Inflation subsamples[69]. For the technology and government expenditure shocks, I fit an AR(1) model to a constructed Solow-residuals series and to the Real Government Consumption series over the 1985:3-2008:4 sample.

[68] Setting $\xi = \frac{\varphi\left(1-\theta+\theta\frac{1+\overline{\lambda}}{\overline{\lambda}}\right)\frac{1}{\overline{\lambda}}}{(1-\varphi)(1-\varphi\beta)(1-\theta)}$ in the model considered here yields the same linearized Phillips curve slope as in a Calvo (1983) version of this model where a fraction φ of the intermediate good producers are not able to reset prices in each period. See Keen and Wang (2007).

[69] Although the markup shock in Smets and Wouters (2007) follows an ARMA(1,1) process, here I consider a more standard AR(1) process. The parameters ρ_λ and σ_λ are calibrated so that the dynamics of the AR(1) are as close as possible to the ARMA(1,1) in Smets and Wouters (2007). None of the results change significantly if I had used instead the ARMA(1,1) process.

Finally, the non-stochastic steady state of the model is set as: $\overline{K}/\left(4\overline{Y}\right) = 2.5$, $\overline{G}/\overline{Y} = 0.20$ and $\delta = 0.02$. The value of χ makes labor in the deterministic steady state equal to one. In the stochastic steady state, $\overline{\Pi}$ makes the model fit the average short-rate in the Great Moderation regime.

Results

Result I: *The nominal term premium is higher in the passive than in the active monetary policy regime.*

Table 2.7 shows the empirical moments discussed above alongside comparable moments implied by the calibrated MS-DSGE model. Panel (1) focuses on the macro moments, while panels (2) and (3) focus on yield curve moments. In particular, panel (2) explores the decomposition of the average slope into term premium and level risk, whereas panel (3) decomposes the nominal term premium into a real term premium component and a compensation for inflation risk (see equation (2.3)). [70]

As expected, conditional on the active policy regime, the model replicates the relatively low macro volatilities observed during the Great Moderation period fairly well (the calibration above was tailored to fit these moments). When the model economy switches to the passive policy regime, however, both real and nominal volatilities become substantially higher. Although the model conditional on regime 2 overshoots the level of nominal volatility observed in the Great Inflation period, it qualitatively replicates the empirical macro moments in each subsample of the data adequately.

Note that the model-implied 10-year nominal term premium in regime 2 is 35 basis points higher than in regime 1. Even though the nominal term premium

[70] The model-implied nominal term premium decomposition was obtained as follows: let $\widetilde{B}_{\tau,t} \equiv exp(-\tau r_{\tau,t})$ represent the period t price of a bond that pays one unit of the final good at $t+\tau$. I include in the MS-DSGE model the following recursive pricing conditions: $\widetilde{B}_{\tau,t} = E_t\left[M_{t,t+1}\widetilde{B}_{\tau-1,t+1}\right]$ for $\tau \geq 1$ with initial condition $\widetilde{B}_{0,t} = 1\ \forall t$. The real term premium is then computed using the definition in Section 2.3.1.

conditional on regime 2 is not as high as my estimate for the Great Inflation regime, the model generates a higher nominal term premium in the regime associated with higher levels of macro uncertainty. Thus it replicates an important result obtained in Section 2.5.

To better understand this result, I explore the decomposition shown in panel (3) of Table 2.7. Consider, for example, an economy that switches from a passive to an active regime. Two forces pressure the 10-year nominal term premium in opposing directions. On the one hand, as the level of nominal uncertainty falls, the inflation risk portion of the nominal term premium – the inflation risk shown in table 2.7 includes an inflation convexity term, as shown in equation (2.3) – drops by 62 basis points. On the other hand, the 10-year real term premium increases by 27 basis points. Intuitively, as the Fed becomes more aggressive in fighting inflationary pressures, the volatility of the short-term *real* interest rate increases, making long-term real bonds riskier. Given my choice of parameters, the first effect dominates and, as a result, the nominal term premium *decreases* as the economy goes from a passive to an active regime.

Result II: *The model endogenously generates realistic level risks along the yield curve.*

Perhaps the most striking feature of Table 2.7 is that the MS-DSGE model is able to *endogenously* generate level risks very much in line with the ones estimated in Section 2.5. As discussed above, for this to be true the nominal short-rate process that results from the model must fluctuate around a different mean conditional on each regime. That is, $\mathcal{D}_s E\left[i_t/s_t\right]$ must be different from zero. Because the short-rate in both regimes is equal to $\log\left(\overline{\Pi}/\beta\right)$ in the deterministic steady state (in fact, the deterministic steady state is the same across regimes for all variables in the model), the existence of level risks implies that the model generates a short-rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ endogenously in the *stochastic* steady state.

Table 2.7: Actual vs. Model-Based Moments

	US Data		MS-DSGE Model	
	Great Moderation	*Great Inflation*	Regime 1 $\phi_{\pi(1)} = 2.19$	Regime 2 $\phi_{\pi(2)} = 0.95$
(1) Macro volatility:				
$SD\,[\pi_t/s_t]$	1.39	2.90	1.27	4.92
$SD\,[\Delta c_t/s_t]$	2.65	3.73	2.57	3.41
$SD\,[i_t/s_t]$	2.23	3.29	2.72	4.62
(2) Slope decomposition:				
$E\,[i_{10Y,t} - i_t/s_t]$	1.84	1.08	1.61	0.95
$=$				
$E\,[NTP_{10Y,t}/s_t]$	1.58	2.23	1.43	1.78
$+$				
$Level\ Risk_{10Y}$	0.26	-1.15	0.18	-0.83
(3) Term Premium decomposition:				
$E\,[NTP_{10Y,t}/s_t]$	1.58	2.23	1.43	1.78
$=$				
$E\,[RTP_{10Y,t}/s_t]$	–	–	0.66	0.39
$+$				
$Inflation\ Risk_{10Y}$	–	–	0.77	1.39

Notes: $E[\text{x}]$ and $SD(\text{x})$ respectively represent the mean and standard deviation of x. The empirical moments from Table 2.5 are reproduced here. The two last columns display model implied theoretical moments, except for the standard deviation of consumption growth, which was simulated. All variables are expressed in percent per annum. The inflation risk shown in the last row includes an inflation convexity term (see equation (2.3)).

Returning to equation (2.17), note that uncertainty causes the constant term in the approximate policy function to shift from the deterministic steady state $\overline{g}$ to the stochastic steady state $\overline{g} + \frac{1}{2}\overline{g}^s_{\sigma\sigma}$ for $s = 1, 2$. The size of this shift for any given variable may depend upon the monetary policy regime. That is, each regime may be characterized by different levels of precautionary savings. Accordingly, the conditional mean of the short rate in general will be different across regimes[71].

To better understand this mechanism, Table 2.8 displays first moments of key macro variables conditional on each regime. The model generates a nominal short-rate differential of 2.04%, close to the 2.84% estimate based on the MS-VAR from Section 2.5. This differential is a result of three mechanisms in the model:

1. The real short-rate mechanism:

   In Table 2.8, the real short-rate r_t on average is 0.79% higher in the passive than in the active regime, producing an upward pressure on i_t in the former regime. To understand this mechanism, remember that consumption uncertainty is higher in the passive than the active regime. The risk-averse household thus responds by forming more precautionary savings in the passive than in the active regime (in Table 2.8, the average level of consumption is lower in the passive regime), implying that the expected growth rate of consumption is positive in the passive, and negative in the active, regime. The short-term real rate therefore is higher in the passive regime in order to counter the household's desire to smooth consumption over time.

2. The inflation level mechanism:

   The short-term nominal rate is also higher in the passive than in the active regime because inflation on average is 2.04% higher in the former regime. Due

[71] Even though level risks are associated with the EH component of the slope in the presence of a MS short-rate process, they only appear in the MS-DSGE model because of the second-order term $\overline{g}^s_{\sigma\sigma}$ in equation (2.17). It follows that level risks are zero if one considers a standard first order approximation to the model solution, or if private agents are risk-neutral. Therefore, in the MS-DSGE model, level risks behave very much like standard premia.

Table 2.8: Understanding the Nominal Short-Rate Differential Across Regimes

	(a) Regime 1 $\phi_{\pi(1)} = 2.19$	(b) Regime 2 $\phi_{\pi(2)} = 0.95$	(b) - (a)
$E[i_t/s_t]$	3.90%	5.94%	2.04%
$E[r_t/s_t]$	2.93%	3.72%	0.79%
$E[\pi_t/s_t]$	0.87%	2.01%	1.14%
$E[C_t/s_t]$	1.90	1.89	-0.01
$E\left[\frac{C_{t+1}}{C_t}/s_t\right]$	-0.01%	0.06%	0.07%

Notes: Model-implied theoretical moments are expressed in percent per annum, except for consumption which is expressed in level.

to the real short-rate mechanism explained above, intermediate good producers face a higher discount on their future profits in the passive than in the active regime. Because current profit has a higher weight in their objective function, firms choose higher optimal prices in the passive than in the active regime. In other words, intermediate firms are more concerned with the future implications of their pricing decision today in the active regime.

3. The inflation risk mechanism:

   As discussed above, the difference between i_t and r_t represents compensation for inflation risk (again including an inflation convexity term). Because inflation uncertainty is higher in the passive than in the active regime, short-term nominal bonds pay higher premia in the former than in the latter regime. Using the moments reported in Table 2.8, note that the compensation for inflation risk falls from 0.21% to 0.10% as policy switches from passive to active.

What happens to the 10-year average yield curve slope when the economy switches from a passive to an active regime? First, as a result of the three channels explained above, the level risk switches from being highly negative to moderately positive, imposing an upward pressure on the 10-year slope. At the same time, because the short-term real rate is more volatile in the active than in the passive regime, the real term premium increases, also putting upward pressure on the 10-year slope. Finally, going in the opposite direction, the drop in inflation uncertainty as the economy switches to the active regime results in a sharp drop in the compensation for inflation risk that is paid by 10-year nominal bonds. Under the proposed calibration, the first two effects dominate and the 10-year slope actually increases as the economy switches from a passive to an active policy regime. Therefore I conclude that the MS-DSGE model in which monetary policy switches between active and passive regimes is able to replicate the Slope-Volatility Puzzle.

Result III: *The MS economy in the active regime is riskier than a corresponding economy with a fixed active policy.*

Table 2.9 compares the active regime of the MS-DSGE model under two different assumption for p_{11}: in the second column, p_{11} is set as in the benchmark calibration to 0.993, while in the third column the active regime is assumed to be an absorbing state, that is $p_{11} = 1$. All other model parameters are kept at the values showed in Table 3.1. In the case of $p_{11} = 1$, once the economy reaches regime 1 the MS-DSGE model behaves exactly as a simpler model with a fixed active monetary policy rule.

Note that the 10-year nominal term premium is higher in the active regime with $p_{11} = 0.993$ than with $p_{11} = 1$. This can be explained by a combination of two mechanisms. The first, in line with the Barro-Rietz rare disasters theory, is as follows. When $p_{11} = 0.993$ there is a risk that during the life of the bond a (relatively) rare bad event will occur and the economy switch to the passive regime in which nominal bonds are very risky. The risk of a sudden change in policy is

Table 2.9: Absorbing vs. Non-Absorbing Active Monetary Policy Regimes

	MS-DSGE conditional on $s_t = 1$	
	$p_{11} = 0.993$	$p_{11} = 1.000$
(1) Macro volatility:		
$SD\left[\pi_t/s_t\right]$	1.27	1.20
$SD\left[\Delta c_t/s_t\right]$	2.57	2.46
$SD\left[i_t/s_t\right]$	2.72	2.60
(2) Slope decomposition:		
$E\left[i_{10Y,t} - i_t/s_t\right]$	1.61	1.19
$=$		
$E\left[NTP_{10Y,t}/s_t\right]$	1.43	1.19
$+$		
$Level\ Risk_{10Y}$	0.18	0.00
(3) Term Premium decomposition:		
$E\left[NTP_{10Y,t}/s_t\right]$	1.43	1.19
$=$		
$E\left[RTP_{10Y,t}/s_t\right]$	0.66	0.67
$+$		
$Inflation\ Risk_{10Y}$	0.77	0.52

Notes: E[x] and SD(x) respectively represent the mean and standard deviation of x. The empirical moments are reproduced here from table 2.5. The two last columns display model implied theoretical moments, except for the standard deviation of consumption growth which was simulated. All variables are expressed in percent per annum. The inflation risk shown in the last row includes an inflation convexity term (see equation (2.3)).

priced into nominal bonds. As a result, term premia in the active regime are higher when $p_{11} = 0.993$ than when $p_{11} = 1$.

A second mechanism, which resembles the long-run risk theory of Bansal and Yaron (2004), operates in parallel to the one just described. In order to explain this new mechanism, Figure 2.6 plots impulse response functions to a negative technological shock[72]. The solid lines represent the active regime when $p_{11} = 1$, whereas the dashed lines correspond to the active regime when $p_{11} = 0.993$. In both cases, the technological shock reduces the price of the 10-year nominal bond (inflation expectations increase) exactly when the level of consumption falls, making this bond a risky asset. Comparing models under different p_{11} values, observe that both consumption and the bond price suffer more pronounced drops when this parameter is set to 0.993 than to 1, implying that investing is this bond is riskier in the former case[73]. Intuitively, when $p_{11} = 0.993$ the model equilibrium in the active regime is affected through private agents' expectations by the possibility of switching to the passive regime, generating more pronounced inflation responses than when $p_{11} = 1$. [74] Therefore, following the shock, an equally active central bank has to fight higher inflationary pressures when $p_{11} = 0.993$ than when $p_{11} = 1$, resulting in sharper drops in consumption in the former case. As in the case analyzed by Rudebusch and Swanson (2008), the possibility of regime switches can be seen as increasing the amount of long-run risk in the economy relative to a situation in which the active policy regime is perceived as an absorbing state[75].

Result IV: *The average yield curve slope is higher in IT than in non-IT countries.*

On Table 2.9, note that the 10-year yield curve slope is 42 basis points higher in

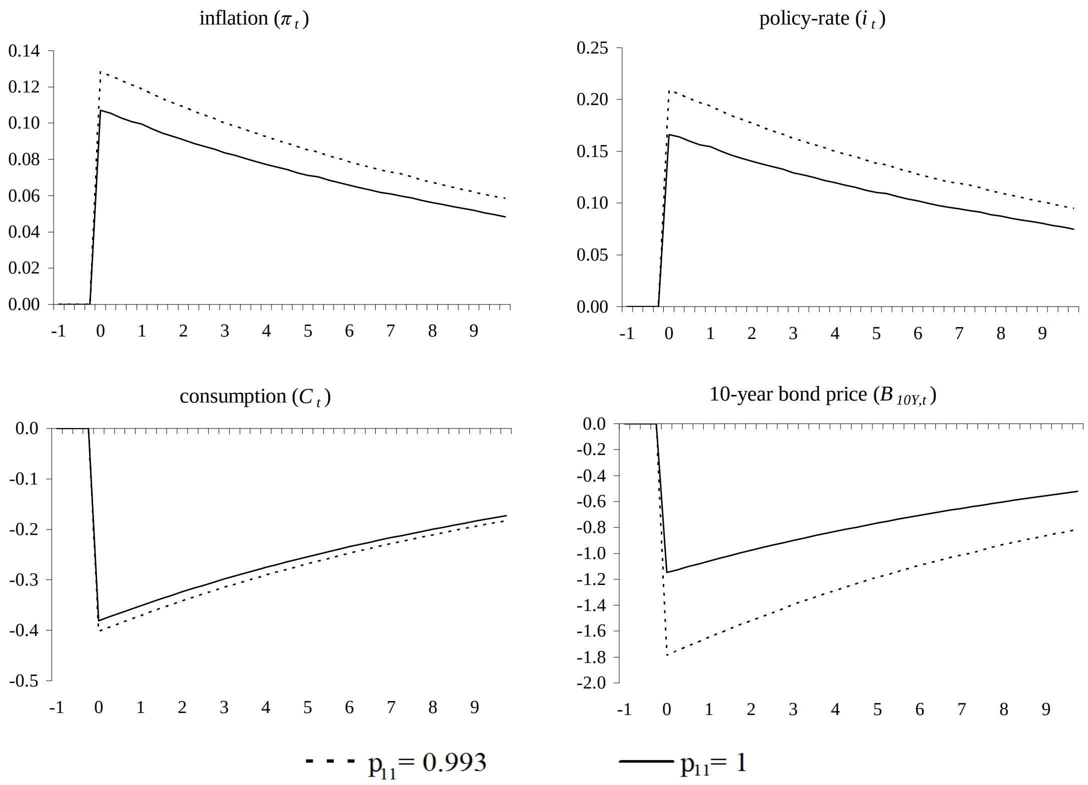
[72] As Rudebusch and Swanson (2008) show, technological shocks are the most important determinants of the term premium in general equilibrium models like the one considered here.

[73] This also can be observed in Table 2.9, which shows that the MS-DSGE model generates more macroeconomic volatility in the active regime with $p_{11} = 0.993$ than with $p_{11} = 1$.

[74] See Davig and Leeper (2007).

[75] It is interesting to note that the extra premium charged by investors when p_{11} is lower comes in the form of an extra compensation for bearing inflation risk, whereas the real term premium remains almost the same; see panel (3) of Table 2.9.

Figure 2.6: Impulse Responses to a Negative Technological Shock in the Active Regime



Notes: Impulse response functions to a negative one standard deviation shock to technology. Full and dashed lines correspond respectively to the MS-DSGE model under $p_{11}= 1$ and $p_{11}= 0.993$. The vertical axes represent percentage deviations from the stochastic steady state, where the deviations of the inflation rate and the policy rate are expressed in percent per annum. The numbers in the horizontal axes represent years following the shock.

an economy with $p_{11} = 0.993$ than in a similar economy with $p_{11} = 1$. Why? First, in light of Result III described above, the nominal term premium is higher when p_{11} is set to 0.993 than when it is set to 1. This channel alone is responsible for 24 out of the 42 basis-points difference in the slope across models. The remaining 18 basis points are explained by the level risk, which is positive when $p_{11} = 0.993$, but is equal to zero in an absorbing state (see Section 2.4.2).

To verify this model's prediction in the data, I compare the observed average yield curve slope across IT and non-IT countries. I focus only on developed economies after the mid-1980s, a period characterized by particularly benign inflation developments in both groups of countries. The general idea here is that, relative to the case where IT is not adopted, private agents in IT countries may believe that future switches to passive monetary policy regimes are less likely to occur[76]. In terms of the MS-DSGE model, the probability that the economy remains in the active regime, p_{11}, therefore would be higher when IT is adopted than otherwise. So, assuming all else constant, the yield curve in IT countries should be flatter on average than in non-IT countries.

There are many ways to justify why p_{11} would tend to be higher in active regimes with than without IT[77]. First, a higher p_{11} could represent improvements in communication between the Fed and the general public once IT is in place. According to this view, IT would make clearer to the public that shifts in inflation away from the objective are going to be dealt with actively. Second, in IT countries emphasis is shifted away from the particular individuals conducting monetary policy in a given point in time to the monetary policy framework itself. A higher p_{11} therefore could signal that, with IT, changes in the individuals in charge of the central bank are less likely to significantly modify the way that policy is conducted. Finally, increases in p_{11} could represent the gain in central bank accountability prompted by IT. Under

[76]This exercise implicitly assumes that, within an IT regime, the monetary authority responds to inflationary pressures actively.

[77]See, for example, Bernanke, Laubach, Mishkin, and Posen (1999).

IT, a central bank that decides to adopt a passive stance with respect to inflation would have to explain to the public the short and long-term implications of its decision; as a result, the probability of switching from the active regime decreases.

Using the Wright (2008) database of international monthly zero-coupon yields of up to ten years maturity, I compute slope measures for ten different countries: three non-IT countries (Germany[78], Japan, and US) and seven countries that adopted IT (New Zealand, Canada, UK, Sweden, Australia, Switzerland, and Norway). For the non-IT countries, I compute the average 10-year slope for the sample which corresponds to the Great Moderation subsample from Section 2.5, specifically Sep/1985 - Dec/2008. For each IT country, I instead consider the average 10-year slope from the month of IT adoption until Dec/2008.

Table 2.10 shows the results. The top panel reports the 10-year average slope for the non-IT countries; the bottom panel shows the measure for the IT countries. Also, to facilitate comparisons between IT and non-IT countries, I show in parentheses the average slope in each of the non-IT countries taken over the same sample as for the IT country. For example, the values in parentheses for New Zealand correspond to the average slope in the US, Germany and Japan taken over the Feb/90 - Dec/08 sample.

If the model's predictions are correct, then the non-IT countries should be associated with steeper yield curves than the IT countries. Indeed, Table 2.10 reveals that the average 10-year slope across the non-IT countries was 1.46%, more than 60% higher than in the IT countries. Using the numbers in parentheses shown in Table 2.10, I can compare pairs of IT and non-IT countries while fixing the same sample. Each time the slope in the non-IT country is higher than that of the IT country, I indicate it with a $*$ in the table. I find that in the great majority of

[78] Bernanke, Laubach, Mishkin, and Posen (1999) consider Germany as an early case of inflation targeting. I decided to include Germany in the non-IT group because there an explicit inflation objective only has been set for the long run whereas short- to medium-term inflation targets have not been announced (this is also true after the ECB started to operate in 1999).

Table 2.10: Yield Curve Slope in non-IT vs. IT Countries

Non-IT Countries:

	Sample	10-Year Slope
US	Sep/85 - Dec/08	1.90
Germany	Sep/85 - Dec/08	1.25
Japan	Sep/85 - Dec/08	1.23
Avg. non-IT		**1.46**

IT Countries:

	Sample	10-Year Slope	
		IT Country	(US, Germany, Japan)
New-Zealand	Feb/90 - Dec/08	-0.01	(1.91^*, 1.19^*, 1.37^*)
Canada	Mar/91 - Dec/08	1.60	(1.95^*, 1.24 , 1.49)
United Kingdom	Oct/92 - Dec/08	0.26	(1.81^*, 1.48^*, 1.61^*)
Sweden	Jan/93 - Dec/08	1.35	(1.78^*, 1.51^*, 1.61^*)
Australia	Sep/94 - Dec/08	0.64	(1.62^*, 1.65^*, 1.58^*)
Switzerland	Jan/00 - Dec/08	1.28	(1.83^*, 1.23 , 1.26)
Norway	Apr/01 - Dec/08	0.77	(2.07^*,1.30^* , 1.22^*)
Average IT		**0.89**	

Notes: The average measures of yield curve slope were computed using the Wright (2008) database of international zero-coupon yields. As a proxy for the short-rate, the 3-month zero-coupon yield was used in the slope computation.

the pair-wise comparisons, the IT countries had flatter yield curves than the non-IT countries.

When making cross-country comparisons, it should be noted that country-specific factors potentially could influence the results. For example, there could be specific features of some IT countries that make the yield curve flatter in those countries, even though perceptions about monetary policy are not significantly different there from in the non-IT countries. However, it is reassuring that in *almost all* pair-wise comparisons the model prediction was verified. That is, the results shown in Table 2.10 appear to be robust enough across different pairs of IT and non-IT countries, thus providing evidence in favor of the explanation that I propose which is based on monetary policy regimes.

2.7 Conclusions

In this chapter I studied how shifts in the monetary policy regime might have affected the average slope of the U.S. nominal term structure in the past decades. My first contribution was to show that, in the presence of a Markov-Switching short-rate process, measures of the average yield curve slope reflect not only term premia but also level risks. I provide level risk estimates based on a simple reduced form Markov-Switching Vector Autoregression: they are large and negative during the Great Inflation and moderate and positive after 1985. Controlling for level risks, the average slope measures imply that term premia in the Great Inflation were substantially higher than after 1985.

My second contribution was to show that a calibrated dynamic general equilibrium model, where the Taylor rule shifts between an active and a passive stance for inflation, replicates my U.S. level risks and term premia estimates. Because the model was solved using a second-order rather than a standard first-order approximation method, I can analyze the different levels of precautionary savings that characterize each policy regime. The model-implied differences in term premia and level risks across regimes are a result of the optimal behavior of private agents and therefore are entirely endogenous.

Appendix 2.A No-Arbitrage Bond Prices

In this appendix I will show how to derive the approximate no-arbitrage bond pricing formulas discussed in section 2.3.1. I will start by pricing inflation protected bonds and then will move on to nominal bonds.

(i) Pricing Inflation Protected Bonds:

Let $\widetilde{B}_{\tau,t}$ represent the period t price of a bond that pays one unit of the consumption good at $t+\tau$. Because their payoffs are already expressed in terms of consumption, these bonds are not subject to inflation risk. Under no-arbitrage $\widetilde{B}_{\tau,t}$ is determined by $\widetilde{B}_{\tau,t} = E_t\left[M_{t+1}\widetilde{B}_{\tau-1,t+1}\right]$. Denote the τ-period real yield to maturity by $r_{\tau,t} \equiv -\frac{1}{\tau}\log \widetilde{B}_{\tau,t}$. Then, taking a second order approximation to the Euler equation above we get:

$$\widehat{r}_{\tau,t} = -\frac{1}{\tau}\sum_{j=1}^{\tau} E_t\left[\widehat{m}_{t+j}\right] - \frac{1}{2\tau}Var_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + O\left(\epsilon^3\right) \tag{2.18}$$

where $O\left(\epsilon^3\right)$ contains the terms of order higher than two that are ignored.

Using the equation above, it can be shown that the second order approximate real term premium is given by:

$$RTP_{\tau,t} \equiv \widehat{r}_{\tau,t} - \tfrac{1}{\tau}\sum_{j=1}^{\tau} E_t\left[r_{t+j-1}\right] \cong$$
$$-\tfrac{1}{2\tau}Var_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + \tfrac{1}{2\tau}\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\}$$

where I made use of the law of iterated expectations to eliminate the first moments of the SDF. It is important to note that $E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\} \neq Var_t\left[\widehat{m}_{t+j}\right]$ for $j \geq 1$. I can also rewrite the real term premium as:

$$\begin{aligned}
RTP_{\tau,t} &\cong -\tfrac{1}{2\tau}Var_t\left[\textstyle\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\} \\
&= -\tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} Var_t\left[\widehat{m}_{t+j}\right] - \tfrac{2}{2\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j}, \widehat{m}_{t+k}\right] \\
&\quad + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\}
\end{aligned}$$

But note that:

(1) $Var_{t+j-1}\left[\widehat{m}_{t+j}\right] = E_{t+j-1}\left[\widehat{m}^2_{t+j}\right] - \left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2$

$$\Downarrow$$

$$E_t\left[Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] = E_t\left[\widehat{m}^2_{t+j}\right] - E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right]$$

(2) $Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] = E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right] - \left(E_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]\right)^2$

$$= E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right] - \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2$$

$$\Downarrow$$

$$Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] + \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2 = E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right]$$

Combining the two results above I get

$$\begin{aligned} E_t\left[Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] &= E_t\left[\widehat{m}^2_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] - \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2 \\ &= Var_t\left[\widehat{m}_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] \end{aligned}$$

Then the real term premium can be written as:

$$\begin{aligned} RTP_{\tau,t} &\cong -\tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} Var_t\left[\widehat{m}_{t+j}\right] - \tfrac{1}{\tau}\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j}, \widehat{m}_{t+k}\right] \\ &\quad +\tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau}\left\{Var_t\left[\widehat{m}_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]\right\} \\ &= -\tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j}, \widehat{m}_{t+k}\right] - \underbrace{\frac{1}{2\tau}\sum_{j=1}^{\tau} Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]}_{\text{SDF convexity term}} \end{aligned}$$

Which implies that:

$$RTP_{\tau,t} \cong -\tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j}, \widehat{m}_{t+k}\right] - \tfrac{1}{2\tau}\sum_{j=1}^{\tau} Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]$$

Therefore the real term premium depends on the autocorrelation structure of the SDF and a convexity term.

(ii) Pricing Nominal Bonds:

Taking a second order approximation to equation (2.2) I obtain:

$$\begin{aligned} \left(i_{\tau,t} - \overline{i_\tau}\right) &= -\tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau} E_t\left[\widehat{m}_{t+j}\right] + \tfrac{1}{\tau}\sum_{i=1}^{\tau} E_t\left[\widehat{\pi}_{t+j}\right] \\ &\quad -\tfrac{1}{2\tau} Var_t\left[\textstyle\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] - \tfrac{1}{2\tau} Var_t\left[\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] \\ &\quad +\tfrac{1}{\tau} Cov_t\left[\textstyle\sum_{j=1}^{\tau}\widehat{m}_{t+j}, \sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] + O\left(\epsilon^3\right) \end{aligned}$$

Combining this equation with equation (2.18) yields:

$$\left(i_{\tau,t} - \overline{i_\tau}\right) = \left(r_{\tau,t} - \overline{r_\tau}\right) + \tfrac{1}{\tau}\textstyle\sum_{i=1}^{\tau} E_t\left[\widehat{\pi}_{t+j}\right]$$

$$-\tfrac{1}{2\tau}Var_t\left[\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right]+\tfrac{1}{\tau}Cov_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j},\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right]+O\left(\epsilon^3\right)$$

which is a Fisher-type equation extended to take into account the risk premia implied in long-term bond prices. The nominal term premium according to the definition in the text can therefore be written as:

$$\begin{aligned}NTP_{\tau,t} &\cong RTP_{\tau,t}+Convexity^{\pi}_{\tau,t}\\ &+\tfrac{1}{\tau}Cov_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j},\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right]-\tfrac{1}{\tau}\sum_{j=1}^{\tau}E_t\left\{Cov_{t+j-1}\left[\widehat{m}_{t+j},\widehat{\pi}_{t+j}\right]\right\}\end{aligned}$$

where the first moments of inflation drop out due to the law of iterated expectations. The inflation convexity term is given by

$$Convexity^{\pi}_{\tau,t}\equiv-\tfrac{1}{\tau}\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau}Cov_t\left[\widehat{\pi}_{t+j},\widehat{\pi}_{t+k}\right]-\tfrac{1}{2\tau}\sum_{j=1}^{\tau}Var_t\left[E_{t+j-1}\left[\widehat{\pi}_{t+j}\right]\right]$$

Appendix 2.B MS-VAR Parameter Estimates

The parameter estimates for the best fitting MS-VAR model from Section 2.5 together with their respective standard errors (in parenthesis) are reported below:

$$\widehat{\mathbf{P}}=\begin{pmatrix}\underset{(0.007)}{0.993} & 0.007\\ 0.033 & \underset{(0.023)}{0.967}\end{pmatrix}\qquad \widehat{\Phi}_{0(1)}=\begin{pmatrix}\underset{(0.14)}{0.01}\\ \underset{(0.59)}{3.09}\\ \underset{(0.13)}{-0.17}\end{pmatrix}\qquad \widehat{\Phi}_{0(2)}=\begin{pmatrix}\underset{(0.65)}{0.80}\\ \underset{(1.61)}{9.15}\\ \underset{(0.81)}{0.44}\end{pmatrix}$$

$$\widehat{\Phi}_{1(1)}=\begin{pmatrix}\underset{(0.06)}{0.76} & \underset{(0.02)}{0.04} & \underset{(0.03)}{0.10}\\ \underset{(0.24)}{-0.53} & \underset{(0.08)}{0.27} & \underset{(0.14)}{0.14}\\ \underset{(0.05)}{0.08} & \underset{(0.02)}{0.07} & \underset{(0.03)}{0.93}\end{pmatrix}\quad \widehat{\Phi}_{1(2)}=\begin{pmatrix}\underset{(0.09)}{0.77} & \underset{(0.05)}{-0.02} & \underset{(0.06)}{0.07}\\ \underset{(0.23)}{-0.47} & \underset{(0.13)}{0.01} & \underset{(0.14)}{-0.39}\\ \underset{(0.12)}{0.15} & \underset{(0.07)}{0.03} & \underset{(0.07)}{0.82}\end{pmatrix}$$

$$\widehat{\mathbf{\Sigma}}_{(1)}=\begin{pmatrix}0.36 & -0.18 & 0.04\\ -0.18 & 5.75 & 0.39\\ 0.04 & 0.39 & 0.28\end{pmatrix}\quad \widehat{\mathbf{\Sigma}}_{(2)}=\begin{pmatrix}1.51 & -1.22 & 0.21\\ -1.22 & 9.32 & 0.99\\ 0.21 & 0.99 & 2.55\end{pmatrix}$$

Appendix 2.C Level Risks Without Assumption 2

I show in this appendix that without imposing assumption 2 from Section 2.4.1 level risks become substantially less tractable. For the case of $\tau=2$, I showed in Section 2.4.2 it is easy to see that the level risk is given by:

$$LR^{(s)}_{\tau=2} = \frac{p_{s1}E\left[i_{t+1}/\,^{s_t=s}_{s_{t+1}=1}\right]+p_{s2}E\left[i_{t+1}/\,^{s_t=s}_{s_{t+1}=2}\right]-E[i_t/s_t=s]}{2}$$

where $LR^{(s)}_{\tau} \equiv E\left[i_{\tau,t} - i_t/s_t = s\right] - E\left[NTP_{\tau,t}/s_t = s\right]$ is the level risk at the 2-period horizon conditional on regime s. For the case $\tau = 3$ the expressions get significantly more complicated. Start from the 3-period slope definition:

$$i_{3,t} - i_t = \frac{i_t+E[i_{t+1}/\Omega_t]+E[i_{t+2}/\Omega_t]}{3} - i_t + NTP_{3,t}$$

Taking conditional expectations on both side I get:

$$\begin{aligned} E\left[i_{3,t} - i_t/s_t\right] &= \frac{-2E[i_t/s_t]+E[E[i_{t+1}/\Omega_t]/s_t]+E[E[i_{t+2}/\Omega_t]/s_t]}{3} + E\left[NTP_{3,t}/s_t\right] \\ &= \frac{-2E[i_t/s_t]+E[i_{t+1}/s_t]+E[i_{t+2}/s_t]}{3} + E\left[NTP_{3,t}/s_t\right] \end{aligned}$$

where the second equality follow from the law of iterated expectations. But the conditionally expected short-rates can be written as:

$$E\left[i_{t+1}/s_t = s\right] = p_{s1}E\left[i_{t+1}/\,^{(s_t,s_{t+1})}_{=(s,1)}\right] + p_{s2}E\left[i_{t+1}/\,^{(s_t,s_{t+1})}_{=(s,2)}\right]$$

and

$$\begin{aligned} E\left[i_{t+2}/s_t = s\right] = {} & p_{s1}p_{11}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,1,1)}\right] + p_{s2}p_{21}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,2,1)}\right] \\ & +p_{s1}p_{12}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,1,2)}\right] + p_{s2}p_{22}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,2,2)}\right] \end{aligned}$$

As a result the 3-period level risk becomes:

$$LR^{(s)}_{\tau=3} = \frac{1}{3}\left[\begin{array}{c} -2E\left[i_t/s_t\right] + p_{s1}E\left[i_{t+1}/\,^{s_t=s}_{s_{t+1}=1}\right] + p_{s2}E\left[i_{t+1}/\,^{s_t=s}_{s_{t+1}=2}\right] \\ +p_{s1}p_{11}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,1,1)}\right] + p_{s2}p_{21}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,2,1)}\right] \\ +p_{s1}p_{12}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,1,2)}\right] + p_{s2}p_{22}E\left[i_{t+2}/\,^{(s_t,s_{t+1},s_{t+2})}_{=(s,2,2)}\right] \end{array}\right]$$

I will stop at $\tau = 3$, but for the usual long-term maturities of interest such as $\tau = 3$ or $\tau = 40$ it is easy to see that the level risk expressions become prohibitively large. In other words, without assumption 2 the number of different conditional expectations terms one needs to keep track of in order to compute an expression for the level risk increases very fast with maturity.

Appendix 2.D The New-Keynesian Model – Detailed Derivations

This appendix reports some more detailed derivations for the MS-DSGE model described in Section 2.6.1.

2.D.1 Households

Assuming $u\left(C_{t(h)}, N_{t(h)}\right) \geq 0$ everywhere, the representative household solves the following problem:

$$V\left(X_t\right) = \max_{C_t, N_t, X_{t+1}} \Big\{ u\left(C_t, N_t\right) + \beta \left[E_t V\left(X_{t+1}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$$
$$-\Lambda_t \left[P_t C_t + E_t \widetilde{M}_{t,t+1} X_{t+1} - X_t - P_t W_t N_t - D_t \right] \Big\}$$

Letting $V\left(X_t\right) \equiv V_t$, I take first order conditions (FOCs) to get:

$$\Lambda_t P_t = \frac{\partial u(C_t, N_t)}{\partial C_t} \qquad -\Lambda_t P_t W_t = \frac{\partial u(C_t, N_t)}{\partial N_t}$$
$$\Lambda_t \widetilde{M}_{t,t+1} = \beta \left(E_t V_{t+1}^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} V_{t+1}^{-\alpha} \frac{\partial V_{t+1}}{\partial X_{t+1}}$$

where the envelope condition is given by $\frac{\partial V(X_t)}{\partial X_t} = \Lambda_t$. Note that the optimized value function is given by $V_t = u\left(C_t, N_t\right) + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$. Therefore, substituting the Envelope Condition into the FOCs I get:

$$\Lambda_t P_t = \frac{\partial u(C_t, N_t)}{\partial C_t} \qquad -\Lambda_t P_t W_t = \frac{\partial u(C_t, N_t)}{\partial N_t}$$
$$\Lambda_t \widetilde{M}_{t,t+1} = \beta \left(E_t V_{t+1}^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} V_{t+1}^{-\alpha} \Lambda_{t+1}$$

Combining the first and second FOCs above yields the labor supply equation $\chi \frac{N_t^{\eta}}{C_t^{-\gamma}} = W_t$, whereas combining the first and third FOCs yields the nominal stochastic discount factor:

$$\widetilde{M}_{t,t+1} = \beta \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha} \right)^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \frac{\frac{\partial u\left(C_{t+1}, N_{t+1}\right)}{\partial C_{t+1}}}{\frac{\partial u(C_t, N_t)}{\partial C_t}} \frac{1}{\Pi_{t+1}}$$

Imposing $u\left(C_t, N_t\right) = e^{b_t} \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right)$ one obtains equations (2.12) in the text, where $\widetilde{M}_{t,t+1} \equiv M_{t,t+1} / \Pi_{t+1}$.

2.D.2 Firms

(a) Final Good Producers:

The representative final good producer chooses $Y_{t(f)}$ for $f \in [0,1]$ to solve:

$$\max_{Y_{t(f)}} P_t Y_t - \int_0^1 P_{t(f)} Y_{t(f)} df \qquad s.t. \qquad Y_t = \left(\int_0^1 Y_{t(f)}^{\frac{1}{1+\lambda_t}} di \right)^{1+\lambda_t}$$

The first order condition can be seen as the demand curve for the differentiated good f, i.e. $Y_{t(f)} = \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t}} Y_t$. Zero-profit in the final good sector implies that $P_t = \left(\int_0^1 P_{t(i)}^{-\frac{1}{\lambda_t}} di \right)^{-\lambda_t}$.

(b) Intermediate Good Producer:

Since capital is fixed, the real marginal cost of each firm f is given by W_t divided by the marginal product of labor, i.e. $MC_{t(f)} = \frac{W_t}{(1-\theta) A_t \overline{K}^{\theta} N_{t(f)}^{-\theta}}$. Using the demand for $Y_{t(f)}$, the real marginal cost of firm f can be written as:

$$MC_{t(f)} = \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{\theta}{1-\theta}\frac{1+\lambda_t}{\lambda_t}} \underbrace{\left[\frac{1}{(1-\theta)\, \overline{K}^{\frac{\theta}{1-\theta}}} \frac{W_t}{A_t} \left(\frac{Y_t}{A_t} \right)^{\frac{\theta}{1-\theta}} \right]}_{\equiv MC_t}$$

where MC_t represents the average real marginal in the intermediate good sector. In period t each firm f faces the following price-setting problem:

$$\max_{P_{t(f)}} E_t \left\{ \sum_{j=0}^{\infty} M_{t,t+j} \frac{P_t}{P_{t+j}} \left[P_{t+j(f)} Y_{t+j(f)} - P_{t+j} W_{t+j} N_{t+j(f)} - \frac{\xi}{2} \left(\frac{P_{t+j(f)}}{P_{t+j-1(f)}} \frac{1}{\Pi} - 1 \right)^2 P_{t+j} Y_{t+j} \right] \right\}$$

subject to $Y_{t+j(f)} = A_{t+j} \overline{K}^{\theta} N_{t+j(f)}^{1-\theta}$ and $Y_{t+j(f)} = \left(\frac{P_{t+j(f)}}{P_{t+j}} \right)^{-\frac{1+\lambda_{t+j}}{\lambda_{t+j}}} Y_{t+j}$, where $M_{t,t+j}$ is the real SDF between periods t and $t+j$. The FOC with respect to $P_{t(f)}$ is:

$$\left[\left(1 - \frac{1+\lambda_t}{\lambda_t}\right) \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t}} \frac{Y_t}{P_t} + \frac{1+\lambda_t}{\lambda_t} \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t}\frac{1}{1-\theta}-1} MC_t \frac{Y_t}{P_t} - \xi \left(\frac{P_{t(f)}}{P_{t-1(f)}} \frac{1}{\Pi} - 1 \right) \frac{1}{P_{t-1(f)}} \frac{1}{\Pi} Y_t \right]$$
$$+ E_t \left\{ M_{t+1} \left[\xi \left(\frac{P_{t+1(f)}}{P_{t(f)}} \frac{1}{\Pi} - 1 \right) \frac{P_{t+1(f)}}{P_{t(f)}^2} \frac{1}{\Pi} Y_{t+1} \right] \right\} = 0$$

where to simplify notation I let $M_{t+1} \equiv M_{t,t+1}$.

2.D.3 Market Clearing Conditions

(a) Symmetric Equilibrium:

Since firms in the intermediate good sector are identical in every aspects, I can now impose the condition for a symmetric equilibrium $P_{t(i)} = P_t \forall i$ to get:

$$MC_t = \frac{1}{1+\lambda_t} + \frac{\lambda_t}{1+\lambda_t}\xi\left(\frac{\Pi_t}{\overline{\Pi}} - 1\right)\frac{\Pi_t}{\overline{\Pi}} - \frac{\lambda_t}{1+\lambda_t}E_t\left\{M_{t+1}\left[\xi\left(\frac{\Pi_{t+1}}{\overline{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\overline{\Pi}}\frac{Y_{t+1}}{Y_t}\right]\right\}$$

(b) Labor Market Clearing Condition:

The labor market clears when:

$$N_t \equiv \int_0^1 N_{t(f)}df = \left(\frac{Y_t}{A_t\overline{K}^{\theta}}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_{t(f)}}{P_t}\right)^{-\frac{1+\lambda_t}{\lambda_t(1-\theta)}} df$$

Solving the last expression for Y_t one gets $Y_t = \left[\int_0^1 \left(\frac{P_{t(f)}}{P_t}\right)^{-\frac{1+\lambda_t}{\lambda_t(1-\theta)}} df\right]^{-(1-\theta)} A_t\overline{K}^{\theta}N_t^{1-\theta}$, where the term in square brackets is the index of price dispersion across firms. Under Rotemberg (1982) adjustment costs, all firms charge the same price and therefore this index is equal to one so that aggregate output follows $Y_t = A_t\overline{K}^{\theta}N_t^{1-\theta}$.

(c) Final Goods Market:

The goods market clearing condition is simply $Y_t = C_t + G_t + \delta\overline{K}$.

Appendix 2.E The Extended Non-Linear System

This appendix describes in detail the equations that form the extended system as in equation (2.16) from Section 2.6.1. For $s \in \{1, 2\}$:

1: Household's Preferences

$$V_{t(s)} = \frac{C_{t(s)}^{1-\gamma}}{1-\gamma} - \chi\frac{N_{t(s)}^{1+\eta}}{1+\eta} + \beta\left\{p_{s1}E\left[V_{t+1(1)}^{1-\alpha}/\Omega^{-s}\right] + p_{s2}E\left[V_{t+1(2)}^{1-\alpha}/\Omega^{-s}\right]\right\}^{\frac{1}{1-\alpha}}$$

2: Labor Supply

$$W_{t(s)} = \chi\frac{N_{t(s)}^{\eta}}{C_{t(s)}^{-\gamma}}$$

3: Short-term Nominal Bond Euler Equation

$$\begin{aligned} e^{-i_{t(s)}} = {} & p_{s1}E\left[\beta\left[\frac{V_{t+1(1)}}{\left(p_{s1}E\left[V_{t+1(1)}^{1-\alpha}/\Omega^{-s}\right]+p_{s2}E\left[J_{t+1(2)}^{1-\alpha}/\Omega^{-s}\right]\right)^{\frac{1}{1-\alpha}}}\right]^{-\alpha}\left(\frac{C_{t+1(1)}}{C_{t(s)}}\right)^{-\gamma}\frac{1}{\Pi_{t+1(1)}}/\Omega^{-s}\right] \\ & +p_{s2}E\left[\beta\left[\frac{V_{t+1(2)}}{\left(p_{s1}E\left[V_{t+1(1)}^{1-\alpha}/\Omega^{-s}\right]+p_{s2}E\left[V_{t+1(2)}^{1-\alpha}/\Omega^{-s}\right]\right)^{\frac{1}{1-\alpha}}}\right]^{-\alpha}\left(\frac{C_{t+1(2)}}{C_{t(s)}}\right)^{-\gamma}\frac{1}{\Pi_{t+1(2)}}/\Omega^{-s}\right] \end{aligned}$$

4: Optimal Pricing Equation in the Intermediate Sector

$$MC_{t(s)} = \frac{1}{1+\lambda_t} + \xi \frac{\lambda_t}{1+\lambda_t} \left(\frac{\Pi_{t(s)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t(s)}}{\overline{\Pi}}$$

$$-p_{s1} \frac{\lambda_t}{1+\lambda_t} E \left[\beta \left[\frac{V_{t+1(1)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha}\right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha}\right]\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(1)}}{C_{t(s)}} \right)^{-\gamma} \left[\xi \left(\frac{\Pi_{t+1(1)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t+1(1)}}{\overline{\Pi}} \frac{Y_{t+1(1)}}{Y_{t(s)}} \right] / \Omega^{-s} \right]$$

$$-p_{s2} \frac{\lambda_t}{1+\lambda_t} E \left[\beta \left[\frac{V_{t+1(2)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha}\right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha}\right]\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(2)}}{C_{t(s)}} \right)^{-\gamma} \left[\xi \left(\frac{\Pi_{t+1(2)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t+1(2)}}{\overline{\Pi}} \frac{Y_{t+1(2)}}{Y_{t(s)}} \right] / \Omega^{-s} \right]$$

5: Real Marginal Cost

$$MC_{t(s)} = \frac{1}{(1-\theta)\overline{K}^{\frac{\theta}{1-\theta}}} \frac{W_{t(s)}}{A_t} \left(\frac{Y_{t(s)}}{A_t} \right)^{\frac{\theta}{1-\theta}}$$

6: Monetary Policy Rule

$$i_{t(s)} = \log \frac{\overline{\Pi}}{\beta} + \phi_{\pi(s)} \left(\pi_{t(s)} - \overline{\pi} \right) + \phi_{y(s)} \left(y_{t(s)} - \overline{y} \right)$$

7: Market Clearing Condition in the Final Good Sector

$$Y_{t(s)} = C_{t(s)} + G_t + \delta \overline{K}$$

8: Market Clearing Condition in the Labor Market

$$Y_{t(s)} = A_t \overline{K}^{\alpha} N_{t(s)}^{1-\alpha}$$

9: No-Arbitrage Bond Prices

$$B_{t(s)}^{\tau} = p_{s1} E \left[\beta \left[\frac{V_{t+1(1)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha}\right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha}\right]\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(1)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(1)}} B_{t+1(1)}^{\tau-1} / \Omega^{-s} \right]$$

$$+ p_{s2} E \left[\beta \left[\frac{V_{t+1(2)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha}\right] + p_{s2} E\left[V_{t+1(1)}^{1-\alpha}\right]\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(2)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(2)}} B_{t+1(2)}^{\tau-1} / \Omega^{-s} \right]$$

for $\tau = 1, ..., T$

3 The Business Cycle Implications of Banks' Maturity Transformation

This chapter consists of work done jointly with Martin M. Andreasen and Pawel Zabczyk at the Bank of England. I was involved in all developments that led to this chapter, which includes carrying out the analytical and numerical analysis, writing the first drafts, and extending the model to include nominal contracts. I was responsible for about 60% of the work carried out in this chapter.

3.1 Introduction

The seminal contributions by Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999) show how financial frictions augment the propagation of shocks in otherwise standard real business cycle (RBC) models.[79] This well-known financial accelerator effect is derived without explicitly modelling the behavior of the banking sector and a growing literature has therefore incorporated this sector into a general equilibrium framework.[80] With a few exceptions, in this recent literature banks are assumed to receive one-period deposits which are instantaneously passed on to firms as one-period credit. Hence, most of the papers in this literature do not address a key aspect of banks' behavior, namely the transformation of short-term deposits into long-term credit.

The aim of this chapter is to examine how banks' maturity transformation affects business cycle dynamics. Our main contribution is to show how maturity transformation in the banking sector can be introduced in otherwise standard dynamic

[79] See also Berger and Udell (1992); Peek and Rosengren (2000); Hoggarth, Reis, and Saporta (2002); Dell'Ariccia, Detragiache, and Rajan (2008); Chari, Christiano, and Kehoe (2008); Campello, Graham, and Harvey (2009) for a discussion of the real impact of financial shocks.

[80] See for instance Chen (2001), Aikman and Paustian (2006), Goodfriend and McCallum (2007), Teranishi (2008), Gertler and Karadi (2009), Gertler and Kiyotaki (2009), and Gerali, Neri, Sessa, and Signoretti (2009).

stochastic general equilibrium (DSGE) models, including the models by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). We then illustrate the quantitative implications of maturity transformation, first in a simple RBC model with long-term real contracts and subsequently in a New Keynesian model with long-term nominal contracts.

Some implications of maturity transformation have been studied outside a general equilibrium framework. For instance, Flannery and James (1984), Vourougou (1990), and Akella and Greenbaum (1992) document that asset prices of banks with a large maturity mismatch on their balance sheets react more to unanticipated interest rate changes than asset prices of banks with a small maturity mismatch. Additionally, the papers by Gambacorta and Mistrulli (2004) and den Heuvel (2006) argue that banks' maturity transformation also affects the transmission mechanism of a monetary policy shock. In our context, however, a general equilibrium framework is necessary because we are interested not only in explaining how long-term credit affects the economy but also in the important feedback effects from the rest of the economy to banks and their credit supply.

Maturity transformation based on long-term credit has to our knowledge not been studied in a general equilibrium setting, although long-term financial contracts have been examined by Gertler (1992) and Smith and Wang (2006).[81] This may partly be explained by the fact that introducing long-term credit and maturity transformation in a general equilibrium framework is quite challenging for at least three reasons. Firstly, one needs to explain why firms demand long-term credit. Secondly, banks' portfolios of outstanding loans are difficult to keep track of in the presence of long-term credit. Finally, and related to the second point, model aggregation is often very difficult or simply infeasible when banks provide long-term credit.

The framework we propose overcomes these three difficulties and remains con-

[81] The paper by Gertler and Karadi (2009) implicitly allows for maturity transformation by letting banks receive one-period deposits and invest in firms' equity, which have infinite duration.

veniently tractable. Our novel assumption is to consider the case where firms face a constant probability α_k of being unable to adjust their capital stock in every period. The capital level of firms which cannot adjust their capital stock is assumed to slowly depreciate over time. This setup generates a demand for long-term credit when we impose the standard assumption that firms borrow in order to finance their capital stock. That is, firms require a given amount of credit for potentially many periods, because they may be unable to adjust their capital levels for many periods in the future.

Interestingly, our setup with infrequent capital adjustments implies heterogeneity at the firm level. In particular, the firm-level dynamics of capital in our model is in line with the main stylized fact which the literature on non-convex investment adjustment costs aims to explain, i.e. that firms usually invest in a lumpy fashion (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006). However, we show for a wide class of DSGE models without a banking sector that the dynamics of prices and *aggregate* variables are unchanged relative to the case where firms adjust capital in every period. This result relies on firms having a Cobb-Doublas production function, as the scale of each firm then becomes irrelevant for all prices and aggregate quantities. We refer to this result as the '*irrelevance of infrequent capital adjustments*'. This is a very important result because it shows that the constraint we impose on firms' ability to adjust capital does not affect the aggregate properties of many existing DSGE models. Accordingly, the aggregate effects of maturity transformation we obtain in a model with a banking sector are *not* a trivial implication of the infrequent capital adjustment assumption.

Our next step is to introduce a banking sector into the model. We specify the behavior of banks along the lines suggested by Gertler and Karadi (2009) and Gertler and Kiyotaki (2009). That is, banks receive short-term deposits from the household sector and face an agency problem in the relationship with households. Differently from Gertler and Karadi (2009) and Gertler and Kiyotaki (2009), banks' assets consist in our case of long-term credit contracts supplied to firms. As we match the life of the credit contracts to the number of periods the firm does not adjust capital, the average life of banks' assets in the economy as a whole is $\mathcal{D} \equiv 1/(1-\alpha_k)$. When $\alpha_k > 0$, this implies that banks face a maturity transformation problem because they use short term deposits and accumulated wealth to provide long-term credit. The standard case of no maturity transformation in the banking sector is thus recovered when $\alpha_k = 0$.

We first illustrate the quantitative implications of maturity transformation in a simple RBC model with long-term real contracts following a positive technological shock. Our analysis shows the existence of a *credit maturity attenuator effect*, meaning that the response of output to this shock is weaker the higher the degree of maturity transformation. The intuition for this result is as follows. The positive technological shock increases the demand for capital and its price. In the model without maturity transformation, the entire portfolio of loans in banks' balance sheets is instantly reset to reflect the higher price of capital. This means that firms now need to borrow more to finance the same amount of productive capital. Banks provide the extra funds to firms and consequently benefit from higher revenues. With maturity transformation, on the other hand, only a fraction of all loans in banks' balance sheets is instantly reset, creating a smaller increase in banks' revenues. As a result, the increase in banks' net-worth and consequently in output are weaker the higher the degree of maturity transformation.

Our second illustration studies the quantitative implications of maturity transformation in a New Keynesian model with nominal financial contracts. In the case of long-term lending, the distinction between nominal and real contracts is especially interesting because long-term inflation expectations directly affect firms' decisions. Here, we focus on how maturity transformation affects the monetary transmission mechanism.

We find that increasing the degree of maturity transformation attenuates the fall in output following a contractionary monetary policy shock. This result can be explained by three main channels. Firstly, the fall in real activity lowers the price of capital. As before, changes in the price of capital have weaker effects on banks' revenues for higher degrees of maturity transformation, and this reduces the fall in output following the monetary contraction. Secondly, there is a debt-deflation mechanism that interacts with the channel just described. The monetary contraction generates a fall in inflation and raises the *ex-post* real interest rate on loans. The aggregate value of loans fall by less in the presence maturity transformation (due to the first channel) and the higher *ex-post* real rate therefore has a larger positive effect on banks' balance sheets and output than without long-term loans. Finally, the smaller reduction in output (and income) following the shock implies that households' deposits fall by less with maturity transformation. Banks are therefore able to provide more credit and this reduces the contraction in output.

The remainder of the chapter is structured as follows. Section 3.2 extends the simple RBC model with infrequent capital adjustments and analyzes the implications of this assumption. This model is extended in Section 3.3 with a banking sector performing maturity transformation based on real financial contracts. The following section explores how maturity transformation and long-term nominal contracts affect the monetary transmission mechanism within a New Keynesian model. Concluding comments are provided in Section 3.5.

3.2 A Standard RBC Model with Infrequent Capital Adjustments

The aim of this section is to describe how a standard real business cycle (RBC) model can be extended to incorporate the idea that firms do not optimally choose capital in every period. We show that this extension does not affect the dynamics of any prices and aggregate variables in the model. This result holds under weak assumptions and generalizes to a wide class of DSGE models. We proceed as follows. Sections 3.2.1 to 3.2.3 describe how we modify the standard RBC model. The implications of this assumption are then analyzed in Section 3.2.4.

3.2.1 Households

Consider a representative household which consumes c_t, provides labor h_t, and accumulates capital k_t^s. The contingency plans for c_t, h_t, and i_t are determined by maximizing

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{(c_{t+j} - b\, c_{t+j-1})^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) \tag{3.1}$$

subject to

$$c_t + i_t = h_t\, w_t + r_t^k\, k_t^s \tag{3.2}$$

$$k_{t+1}^s = (1-\delta)\, k_t^s + i_t \left[1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] \tag{3.3}$$

and the usual no-Ponzi game condition. The left-hand side of equation (3.2) lists expenditures on consumption and investment i_t, while the right-hand side lists the sources of income. We let w_t denote the real wage and r_t^k be the real rental rate of capital. As in Christiano, Eichenbaum, and Evans (2005), the household's preferences are assumed to display internal habits with intensity parameter b. The capital depreciation is determined by δ, while the capital accumulation equation includes quadratic adjustment costs as in Christiano, Eichenbaum, and Evans (2005).

3.2.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$ and owned by the household. Profit in each period is given by the difference between firms' output and costs, where the latter are composed of capital rental fees $r_t^k\, k_{i,t}$ and the wage bill $w_t\, h_{i,t}$. Both costs are paid at the end of the period. We assume that output is produced from capital and labor according to a standard Cobb-Douglas production function

$$y_{i,t} = a_t k_{i,t}^{\theta} h_{i,t}^{1-\theta}. \tag{3.4}$$

The aggregate level of productivity a_t is assumed to evolve according to

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a, \tag{3.5}$$

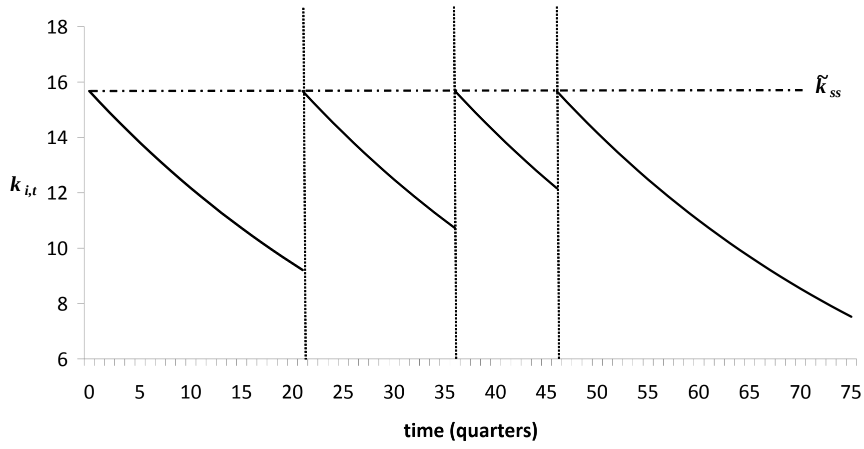
where $\varepsilon_t^a \sim \mathcal{NID}(0, \sigma_a^2)$ and $\rho_a \in (-1, 1)$.

The model has so far been completely standard. We now depart from the typical RBC setup by assuming that firms can only choose their optimal capital level with probability $1 - \alpha_k$ in every period. The probability $\alpha_k \in [0, 1[$ is assumed to be the same for all firms and across time. Capital for firms which cannot reoptimize is assumed to depreciate by the rate δ over time. All firms, however, are allowed to choose labor in every period as in the standard RBC model.

One way to rationalize the restriction we impose on firms' ability to adjust capital is as follows. The decision of a firm to purchase a new machine or to set up a new plant usually involves large fixed costs. These could be costs related to gathering information, decision making, and training the workforce. We do not attempt to model the exact nature of these costs and how firms choose which period to adjust capital, but our setup still captures the main macroeconomic implications of firms' infrequent changes in capital.

To see how this assumption affects the level of capital for the i'th firm, consider the example displayed in Figure 3.1 for an economy in steady state. The downward slopping lines denote the capital level for the i'th firm over time. The dashed horizontal line represents the optimal choice of capital for firms that are able to

Figure 3.1: Infrequent Capital Adjustments - Dynamics at the Firm Level



Notes: Bold lines represent the capital of the considered firm. Vertical lines mark the periods in which the firm is allowed to reoptimize capital. The dotted horizontal line represents the steady state level.

optimize ($\widetilde{k}_{ss}$), whereas vertical lines mark the periods in which the firm is allowed to reoptimize capital. In this example, the firm is not allowed to reoptimize capital from period zero until the first vertical line and simply sees its capital depreciate. Once the vertical line is reached the firm adjusts its capital stock and chooses $\widetilde{k}_{ss}$. In the following periods capital depreciates again until the firm is allowed to adjust capital once more. Note that the vertical lines are not equidistant, reflecting our assumption of random capital adjustment dates.

It is important to note that the dynamics of capital at the firm level implied by our assumption is in line with the key finding in the empirical literature on non-convex investment adjustment costs (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006). This literature uses micro data to document that firms usually invest in a lumpy fashion, i.e. there are many periods of investment inaction followed by spikes in the level of investment and capital.

Our assumption on firms' ability to adjust their capital level implies that there are two groups of firms in every period : i) a fraction $1 - \alpha_k$ which potentially change their capital level and ii) the remaining fraction α_k which produce using the depreciated capital chosen in the past. All reoptimizing firms choose the same level of capital due to absence of cross-sectional heterogeneity. We denote this capital level by $\widetilde{k}_t$. By the same token, all firms that produce in period t using capital chosen in period $t-m$ also set the same level of labor which we denote by $\widetilde{h}_{t|t-m}$ for $m = \{1, 2, ...\}$.[82] Hence, firms adjusting capital in period t solve the problem

$$\max_{\widetilde{k}} \ \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[a_{t+j} \left((1-\delta)^j \widetilde{k}_t \right)^{\theta} \widetilde{h}_{t+j|t}^{1-\theta} - r_{t+j}^k (1-\delta)^j \widetilde{k}_t - w_{t+j} \widetilde{h}_{t+j|t} \right]. \tag{3.6}$$

We see that firms account for the fact that they might not adjust capital for potentially many periods. Note that capital depreciates while the firm does not adjust its capital level, and the amount of capital available in period $t+j$ for a firm that last optimized in period t is $(1-\delta)^j \widetilde{k}_t$.

The first-order condition for the choice of capital $\tilde{k}_t$ is given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \theta (1-\delta)^{j\theta} \tilde{k}_t^{\theta-1} \widetilde{h}_{t+j|t}^{1-\theta} - r_{t+j}^k (1-\delta)^j \right) = 0. \tag{3.7}$$

If $\alpha_k > 0$, the optimal choice of capital now depends on the discounted value of all future expected marginal products of capital and rental rates. Note also that the discount factor between periods t and $t+j$ incorporates α_k^j which is the probability that the firm cannot adjust its level of capital after j periods. If $\alpha_k = 0$, equation (3.7) reduces to the standard case where the firm sets capital such that its marginal product equates the rental rate.

The first-order condition for labor is given by

$$h_{i,t} = \left(\frac{w_t}{a_t (1-\theta)} \right)^{-\frac{1}{\theta}} k_{i,t} \quad \text{for } i \in [0,1]. \tag{3.8}$$

Here, we do not need to distinguish between optimizing and non-optimizing firms because all firms are allowed to optimally set their labor demand each period. It is important to note that the capital-labor ratio only depends on aggregate variables

[82] A similar notation for capital implies $\widetilde{k}_{t|t-m} \equiv \widetilde{k}_{t-m} (1-\delta)^m$.

and is therefore identical for all firms.

3.2.3 Market Clearing and Aggregation

In equilibrium, the aggregate supply of capital must equal the capital demand of all firms, i.e.

$$k_t^s = \int_0^1 k_{i,t} di. \tag{3.9}$$

A fraction of $1 - \alpha_k$ firms choose $\tilde{k}_t$ in period t. The capital demand among non-reoptimizing firms is equal to the aggregate capital in period $t - 1$ rescaled by α_k and adjusted for depreciation. This is because all firms face the same probability of being allowed to adjust capital. Market clearing in the rental market for capital is therefore given by

$$k_t^s = (1 - \alpha_k)\, \tilde{k}_t + \alpha_k\, (1 - \delta)\, k_{t-1}^s. \tag{3.10}$$

Note that $k_t^s = \tilde{k}_t$ when $\alpha_k = 0$ and all firms are allowed to adjust their capital level in every period.

Market clearing in the labor market implies

$$h_t = \int_0^1 h_{i,t} di, \tag{3.11}$$

and (3.8) therefore gives

$$h_t = \left(\frac{w_t}{a_t\, (1 - \theta)} \right)^{-\frac{1}{\theta}} k_t^s. \tag{3.12}$$

Finally, the goods market clears when

$$y_t \equiv \int_0^1 y_{i,t} di = c_t + i_t. \tag{3.13}$$

3.2.4 Implications of Infrequent Capital Adjustments

The parameter α_k determines the fraction of firms reoptimizing capital in a given period, or equivalently the average numbers of periods that the i'th firm operates without adjusting its capital level. It is therefore natural to expect that different values of α_k result in different business cycle implications for prices and aggregate variables in the model. For instance, large values of α_k imply that adjusting firms are more forward-looking compared to the case where α_k is small, and this could potentially give rise to different dynamics for prices and aggregate variables. This simple intuition turns out *not* to be correct: different values of α_k actually give exactly the same aggregate model dynamics[83]. We summarize this result in Proposition 1.

Proposition 1 *The parameter α_k has no impact on the law of motions for c_t, i_t, h_t, w_t, r_t^k, k_t^s, and a_t.*

Proof. The model consists of eight variables $c_t, i_t, h_t, w_t, r_t^k, k_t^s,\ a_t, \tilde{k}_t$ and eight equations. The parameter α_k only enters in (3.7) and (3.10). The dynamics of k_t^s follows from $\tilde{k}_t$ and the system can therefore be reduced to seven equations in seven variables $c_t, i_t, h_t, w_t, r_t^k, \tilde{k}_t, a_t$. Note also that (3.12) implies $\tilde{k}_t^{\theta-1}\tilde{h}_{t+j|t}^{1-\theta} = \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}$ which allow us to simplify the algebra. To prove the proposition, we need to show that the first-order condition for capital when $\alpha_k = 0$ is equivalent to the first-order condition for capital when $\alpha_k > 0$, i.e.

$$\forall t : a_t\theta\left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1-\theta}{\theta}} = r_t^k \Leftrightarrow$$

$$\forall t : E_t \sum\nolimits_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t}(1-\delta)^j \left(a_{t+j}\theta\left(\frac{w_{t+j}}{a_{t+j}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}} - r_{t+j}^k\right) = 0.$$

[83] Note that the implications of infrequent capital adjustments differ substantially from the well-known real effects of staggered *nominal price* contracts when specified following Calvo (1983).

To show $\Rightarrow$ we observe that $a_t\theta\left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}=r_t^k$ implies that each of the elements in the infinite sum is equal to zero and so is the conditional expectations. To prove $\Longleftarrow$ we first lead the infinite sum by one period and multiply the expression by $\alpha_k\beta(1-\delta)\frac{\lambda_{t+1}}{\lambda_t}>0$. This gives

$$E_{t+1}\left[\sum\nolimits_{i=1}^{+\infty}\alpha_k^i\beta^i\frac{\lambda_{t+i}}{\lambda_t}(1-\delta)^i\left(a_{t+i}\theta\left(\frac{w_{t+i}}{a_{t+i}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}-r_{t+i}^k\right)\right]=0$$

and by the law of iterated expectations

$$E_t\left[\sum\nolimits_{i=1}^{+\infty}\alpha_k^i\beta^i\frac{\lambda_{t+i}}{\lambda_t}(1-\delta)^i\left(a_{t+i}\theta\left(\frac{w_{t+i}}{a_{t+i}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}-r_{t+i}^k\right)\right]=0. \tag{3.14}$$

Another way to express the infinite sum is by

$$E_t\left[a_t\theta\left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}-r_t^k\right]+$$

$$E_t\left[\sum\nolimits_{j=1}^{+\infty}\alpha_k^j\beta^j\frac{\lambda_{t+j}}{\lambda_t}(1-\delta)^j\left(a_{t+j}\theta\left(\frac{w_{t+j}}{a_{t+j}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}-r_{t+j}^k\right)\right]=0$$

Using (3.14), this expression reduces to

$$a_t\theta\left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1-\theta}{\theta}}=r_t^k$$

as required. ■

The intuition behind this irrelevance proposition is simple. When the capital supply is predetermined, it does not matter if a fraction of firms cannot change their capital level because the other firms have to demand the remaining amount of capital to ensure equilibrium in the capital market. The fact that the capital-labor ratio is the same across firms further implies that aggregate labor demand is similar to the case where all firms can adjust capital. The aggregate output produced by firms is also unaffected due to the presence of constant returns to scale in the production function. The result in theorem 1 is thus similar to the well-known result from microeconomics for a market in perfect competition and constant returns to scale, where only the aggregate production level can be determined but not the production level of the individual firms.

There are at least two interesting implications of the infrequent capital adjustments at the firm level. Firstly, the distortion on firms' ability to change their capital level does not break the relation from the standard RBC model, where the marginal product of capital equals its rental price. In other words, the induced distortion in the capital market does not lead to any inefficiencies because the remaining part of the economy is sufficiently flexible to compensate for the imposed friction.

Secondly, the infrequent capital adjustments give rise to firm heterogeneity. There will be firms which have not adjusted their capital levels for a long time and hence have small capital levels due to the effect of depreciation. These firms will therefore produce a small amount of output and will also have a low labor demand due to (3.8). Similarly, there will also be firms which have recently adjusted their capital levels and therefore produce relatively high quantities and have high labor demands. This firm heterogeneity relates to the literature on firm specific capital as in Sveen and Weinke (2005), Woodford (2005), among others.

When proving Proposition 1 we only used two assumptions from our RBC model, besides a predetermined capital supply. Hence, the irrelevance result for α_k holds for all DSGE models with these two properties. We state this observation in Corollary 1.

Corollary 1 *Proposition 1 holds for any DSGE model with the following two properties:*

1. *The capital labor ratio is identical for all firms*

2. *The parameter α_k only enters into the equilibrium conditions for capital*

Examples of DSGE models with these properties are models with sticky prices, sticky wages, monopolistic competition, habits, to name just a few. The three most obvious ways to break the irrelevance of the infrequent capital adjustments can be inferred from (3.8). That is, if firms i) do not have a Cobb-Douglass production function, ii) face firm-specific productivity shocks, or iii) face different wage levels due to imperfections in the labor market.

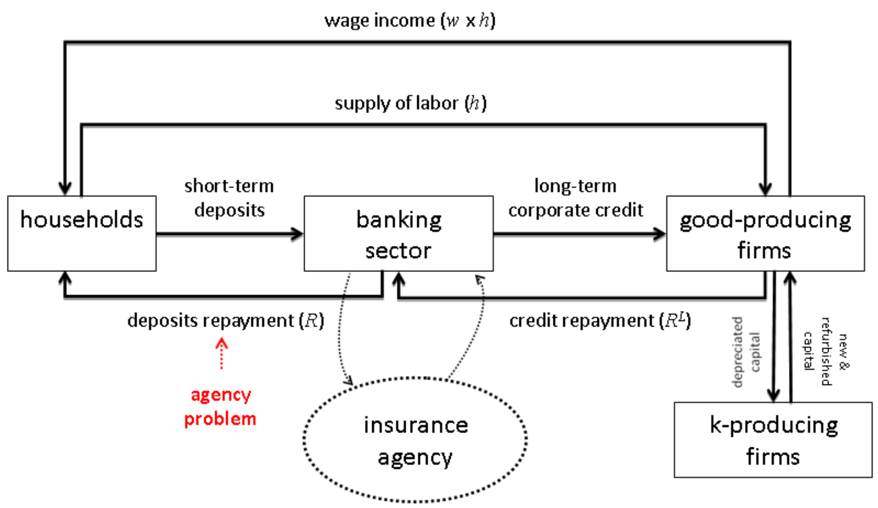
Another way to break the irrelevance of infrequent capital adjustments is to make α_k affect the remaining part of the economy. We will in the next section show how this can be accomplished by introducing a banking sector into the model.

3.3 An RBC Model With Banks and Maturity Transformation

This section incorporates a banking sector into the RBC model developed above. Here, we impose the standard assumption that firms need to borrow prior to financing their desired level of capital. This requirement combined with infrequent capital adjustments generate a demand for long-term credit at the firm level. Banks use one-period deposits from households and accumulated wealth (i.e. net worth) to meet this demand. As a result, banks face a maturity transformation problem because they use short-term deposits to provide long-term credit.

Having outlined the novel feature of our model, we now turn to the details. The economy is assumed to have four agents: i) households, ii) banks, iii) good-producing firms, and iv) capital-producing firms. The latter type of firms are standard in the literature and introduced to facilitate the aggregation (see for instance Bernanke, Gertler, and Gilchrist (1999)).

Figure 3.2: RBC Model With Banks and Maturity Transformation



The interactions between the four types of agents are displayed in Figure 3.2.[84] Households supply labor to the good-producing firms and make short-term deposits in banks. Banks then use these deposits together with their own wealth to provide long-term credit to good-producing firms. The good-producing firms hire labor and use credit to obtain capital from the capital-producers. The latter firms simply repair the depreciated capital and build new capital which they provide to good-producing firms.

We proceed as follows. Sections 3.3.1 and 3.3.2 revisit the problems for the households and good-producing firms when banks are present. Sections 3.3.3 and 3.3.4 are devoted to the behavior of banks and the capital-producing firm, respectively. Market clearing conditions and the model calibration are discussed in Section 3.3.5. We then study the quantitative implication of maturity transformation following a technology shock in Section 3.3.6.

[84] For simplicity, Figure 3.2 does not show profit flows going from firms and banks to households.

3.3.1 Households

Each household is inhabited by workers and bankers. Workers provide labor h_t to good-producing firms and in exchange receive labor income $w_t h_t$. Each banker manages a bank and accumulates wealth that is eventually transferred to his respective household. It is assumed that a banker becomes a worker with probability α_b in each period, and only in this event is the wealth of the banker transferred to the household. Each household postpones consumption from periods t to $t+1$ by holding short-term deposits in banks.[85] Deposits b_t made in period t are repaid in the beginning of period $t+1$ at the gross deposit rate R_t.

The households' preferences are as in Section 3.2.1. The lifetime utility function is maximized with respect to c_t, b_t, and h_t subject to

$$c_t + b_t = h_t\, w_t + R_{t-1} b_{t-1} + T_t. \tag{3.15}$$

Here, T_t denotes the net transfers of profits from firms and banks. Note that the households are not allowed to accumulate capital, as in the previous model, but are forced to postpone consumption through deposits in banks.

3.3.2 Good-Producing Firms

We impose the requirement on good-producing firms that they need credit to finance their capital stock. With infrequent capital adjustments these firms therefore demand long-term credit which we assume is provided by banks.

It is convenient in this setup to match the number of periods a firm cannot adjust capital to the duration of its financial contract with the bank. That is, the

[85] As in Gertler and Karadi (2009), it is assumed that a household is only allowed to deposit savings in banks owned by bankers from a different household. Additionally, it assumed that within a household there is perfect consumption insurance.

financial contract lasts for all periods where the firm cannot adjust its capital level, and a new contract is signed whenever the firm is allowed to adjust capital. Since the latter event happens with probability $1-\alpha_k$ in each period, the exact maturity of a contract is not known *ex-ante*. The average maturity of all existing contracts, however, is known and given by $\mathcal{D} = 1/(1-\alpha_k)$.

The specific obligations in the financial contract are as follows. A contract signed in period t specifies the amount of capital $\widetilde{k}_t$ that the good-producing firm wants to finance for as long as it cannot reoptimize capital. As in section 3.2.2, capital depreciates over time, meaning that after j periods the firm only needs funds for $(1-\delta)^j \widetilde{k}_t p_t^k$ units of capital. Here, p_t^k denotes the real price of capital. The bank provides credit to finance the rental of capital throughout the contract at a constant (net) interest rate $r_t^L + \delta$. The first component of the loan rate r_t^L reflects the fact that firms need external finance, whereas the second component δ refers to the depreciation cost associated with capital usage. It should be emphasized that we do not consider informational asymmetries between banks and the firm, implying that the firm cannot deviate from the signed contract or renegotiate it as considered in Hart and Moore (1998).

As in the standard RBC model, good-producing firms also hire labor which is combined with capital in a Cobb-Douglas production function. We continue to assume that the wage bill is paid after production takes place, implying that demand for credit is uniquely associated with firms' capital level.

The assumptions above are summarized in the expression for $profit_{t+j|t}$, i.e. the profit in $t+j$ for a firm that entered a financial contract in period t:

$$profit_{t+j|t} = \underbrace{a_{t+j}\left[(1-\delta)^j \widetilde{k}_t\right]^{\theta} h_{t+j|t}^{1-\theta}}_{\text{production revenue}} - \underbrace{w_{t+j}\widetilde{h}_{t+j|t}}_{\text{wage bill}} - \underbrace{\left(r_t^L+\delta\right) p_t^k \left[(1-\delta)^j \widetilde{k}_t\right]}_{\text{capital rental bill}}. \quad (3.16)$$

Note that all future cash flow between the firm and the bank are determined with certainty for the duration of the contract. That is, the firm needs to fund $\widetilde{k}_t$ units of capital based on a fixed price p_t^k, which is done at the fixed loan rate r_t^L.

The good-producing firm determines capital and labor by maximizing the net present value of future profits. Using the households' stochastic discount factor, the first-order condition for the optimal level of capital $\widetilde{k}_t$ is given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\theta a_{t+j} (1-\delta)^{j\theta} \left(\widetilde{k}_t \right)^{\theta-1} h_{t+j|t}^{1-\theta} - \left(r_t^L + \delta \right) p_t^k (1-\delta)^j \right] = 0. \quad (3.17)$$

The price for financing one unit of capital throughout the contract is thus constant and given by $\left(r_t^L + \delta \right) p_t^k$. The first-order condition for the optimal choice of labor is exactly as in the standard RBC model, i.e. as in (3.8).

3.3.3 The Banking Sector

We incorporate banks following the approach suggested by Gertler and Kiyotaki (2009) and Gertler and Karadi (2009). Their specification has two key elements. The first is an agency problem that characterizes the interaction between households and banks and limits banks' leverage. This in turn limits the amount of credit provided by banks to the good-producing firms. The agency problem only constrains banks' supply of credit as long as banks cannot accumulate sufficient wealth to be independent of deposits from households. The second key element is therefore to assume that bankers retire with probability α_b in each period, and when doing so, transfer wealth back to their respective households. The retired bankers are assumed to be replaced by new bankers with a sufficiently low initial wealth to make the aggregate wealth of the banking sector bounded.[86]

Although our model is very similar to the model by Gertler and Karadi (2009), the existence of long-term financial contracts complicates the aggregation. This is

[86] Note that their second assumption generates heterogeneity in the banking sector and there does not exist a representative bank.

because new bankers must inherit the outstanding long-term contracts from the retired bankers, but the new bankers may not be able to do so with a low initial wealth. We want to maintain the assumption of bankers having to retire with probability α_b, because this justifies the transfer of wealth from the banking sector to the households and in turn to consumption. Our solution is to introduce an insurance agency financed by a proportional tax on banks' profit. When a banker retires, the role of this agency is to create a new bank with an identical asset and liability structure and effectively guarantee the outstanding contracts of the old bank. This agency therefore ensures the existence of a representative bank and that the wealth of this bank is bounded with an appropriately calibrated tax rate.

We next describe the balance sheet of the representative bank in Section 3.3.3 and present the agency problem in Section 3.3.3.

Banks' Balance Sheets As mentioned earlier, the representative bank uses accumulated wealth n_t and short-term deposits from households b_t to provide credit to good-producing firms. This implies the following identity for the bank's balance sheet

$$len_t \equiv n_t + b_t, \tag{3.18}$$

where len_t represents the amount of lending.

The net wealth generated by the bank in period t is given by

$$n_{t+1} = (1 - \tau)\left[rev_t - R_t b_t\right], \tag{3.19}$$

where τ is the proportional tax rate and rev_t denotes revenue from lending to good-producing firms. The term $R_t b_t$ constitutes the value of deposits repaid to consumers. Combining the last two equations gives the following law of motion for the bank's net wealth

$$n_{t+1} = (1-\tau)\left[rev_t - R_t len_t + R_t n_t\right]. \tag{3.20}$$

The imposed structure for firms' inability to adjust capital implies simple expressions for len_t and rev_t. Starting with the total amount of lending in period t, we have

$$\begin{aligned} len_t &\equiv \textstyle\int_0^1 p_{i,t}^k k_{i,t} di \\ &= \underbrace{(1-\alpha_k)\, p_t^k \widetilde{k}_t}_{\text{adjust in period } t} + \underbrace{(1-\alpha_k)\,\alpha_k\,(1-\delta)\, p_{t-1}^k \widetilde{k}_{t-1}}_{\text{adjust in period } t-1} + ... \\ &= (1-\alpha_k) \sum_{j=0}^{\infty} \left((1-\delta)\,\alpha_k\right)^j p_{t-j}^k \widetilde{k}_{t-j} \end{aligned} \tag{3.21}$$

where simple recursions are easily derived. Similarly, for the total revenue we have

$$rev_t = (1-\alpha_k) \sum_{j=0}^{\infty} \left((1-\delta)\,\alpha_k\right)^j R_{t-j}^L p_{t-j}^k \widetilde{k}_{t-j}. \tag{3.22}$$

Here, $R_t^L \equiv 1 + r_t^L$ is the gross loan rate. The intuition for these equations is as follows. A fraction $(1-\alpha_k)$ of the bank's lending and revenue in period t relates to credit provided to adjusting firms in the same period. Likewise, a fraction $(1-\alpha_k)\,\alpha_k\,(1-\delta)$ of lending and revenue relates to credit provided to firms that last adjusted capital in period $t-1$, and so on. For all contracts, the loans made j periods in the past are repaid at the rate R_{t-j}^L. Thus, a large values of α_k makes the bank's balance sheet less exposed to changes in R_t^L compared to small values of α_k. The most important thing to notice, however, is that α_k affects the bank's lending and revenue and thereby its balance sheet, implying that the irrelevance theorem of infrequent capital adjustments in Section 3.2.4 does not hold for this model.

The Agency Problem As in Gertler and Karadi (2009), we assume that bankers can divert a fraction Λ of their deposits and wealth at the beginning of the period, and transfer this amount of money back to their corresponding households. The cost for bankers of diverting is that depositors can force them into bankruptcy and recover the remaining fraction $1 - \Lambda$ of assets. Bankers therefore choose to divert whenever the benefit from diverting, i.e. Λlen_t, is greater than the value associated with staying in business as a banker, i.e. V_t. This gives the following incentive constraint

$$\underbrace{V_t}_{\substack{\text{banker's loss} \\ \text{from diverting}}} \geq \underbrace{\Lambda len_t}_{\substack{\text{banker's gain} \\ \text{from diverting}}} . \tag{3.23}$$

for households to have deposits in banks. The continuation value V_t of a bank is given by

$$V_t = \mathbf{E}_t \sum_{j=0}^{+\infty} (1 - \alpha_b)\, \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} n_{t+j+1}. \tag{3.24}$$

This expression reflects the idea that bankers attempt to maximize their expected wealth at the point of retirement where they transfer n_t to their respective household. Note that the discount factor in (3.24) is adjusted by $(1 - \alpha_b)\, \alpha_b^j$ to reflect the fact that retirement itself is stochastic and therefore could happen with positive probability in any period.

We assume that lending to the good-producing firms is profitable for banks. This implies that banks lend up to the limit allowed by the incentive constraint, which therefore is assumed to hold with equality. Consequently, the amount of credit provided by the representative bank is limited by its accumulated wealth through the relation

$$len_t = (lev_t)\, n_t \tag{3.25}$$

where

$$lev_t \equiv \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}} \tag{3.26}$$

is the bank's leverage ratio. The two control variables $x_{1,t}$ and $x_{2,t}$ follow simple recursions derived in Appendix 3.B.1.

3.3.4 Capital-Producing Firms

A capital-producing firm is assumed to control the aggregate supply of capital. This firm takes depreciated capital from all good-producing firms and invests in new capital before sending the 'refurbished' capital back to these firms. The decisions by the capital-producing firm are closely related to the financial contract provided by the representative bank. This is because the capital-producing firm trades capital at individual prices with each of the good-producing firms. That is, throughout a given financial contract, capital is traded at the price when this contract was signed. For instance, if a contract was signed in period $t-4$, then the capital-producing firm trades capital with this particular firm at the price p_{t-4}^k throughout the contract. That is, when the good-producing firm enters a financial contract, it obtains the right to borrow at the constant rate r_t^L based on the current value of its capital stock p_t^k. By doing this we ensure that within each financial contract the cash flows between banks and good-producing firms are known with certainty[87].

More specifically, the net present value of profit for the capital-producing firm is given by

$$profit_t^k = \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[v_{t+j} - v_{t+j}(1-\delta) - i_{t+j} \right]. \tag{3.27}$$

Here, v_t is a value aggregate given by

$$v_t \equiv (1-\alpha_k) \sum_{j=0}^{+\infty} \alpha_k^j p_{t-j}^k (1-\delta)^j \widetilde{k}_{t-j}, \tag{3.28}$$

or equivalently

$$v_t = (1-\alpha_k)\, p_t^k \widetilde{k}_t + \alpha_k \,(1-\delta)\, v_{t-1}. \tag{3.29}$$

According to (3.27), the capital-producing firm obtains depreciated capital from good-producing firms $v_t(1-\delta)$ and allocates resources to investments i_t. The output

[87] Another way to justify this assumption is to consider the bank and the capital-producing firm as a joint entity.

from this production process is an upgraded capital stock, which is send to the good-producing firms resulting in revenue v_t.

When maximizing profits, the firm is constrained by the evolution of $\tilde{k}_t$, i.e.

$$k_t = (1 - \alpha_k)\, \tilde{k}_t + \alpha_k\, (1 - \delta)\, k_{t-1}, \tag{3.30}$$

and the law of motion for aggregate capital:

$$k_{t+1} = (1 - \delta) k_t + i_t \left[1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right]. \tag{3.31}$$

The optimization of (3.27) is described in Appendix 3.B.2. An important point to note is that the Lagrange multiplier for (3.31), i.e. q_t, is the standard Tobin's Q and indicates a marginal change in profit following a marginal change in the next period capital k_{t+1}. On the other hand, the price of capital p_t^k denotes the marginal change in profit for a marginal change in current capital k_t.

3.3.5 Market Clearing and Calibration

Market clearing conditions in the capital, labor, and good markets are similar to those derived in Section 3.2.3, and technology evolves according to the AR(1) process in (3.5).[88]

The model is calibrated to the post-war US economy in Table 3.1. We chose standard values for the discount factor $\beta = 0.9926$, the capital share $\theta = 0.36$, the coefficient of relative risk-aversion $\phi_0 = 1$, and the rate of depreciation $\delta = 0.025$. In line with the estimates in Christiano, Eichenbaum, and Evans (2005), we set the intensity of habits to $b = 0.65$ and investment adjustment costs to $\kappa = 2.5$. The inverse Frisch elasticity of the labor supply ϕ_1 is set to 1/3. This is slightly below

[88] The complete list of equations in the model is shown in Appendix 3.B.3.

Table 3.1: Baseline Calibration

β	0.9926	Λ	0.2
b	0.65	α_b	0.972
ϕ_0	1	τ	0.017
ϕ_1	1/3	κ	2.5
θ	0.36	δ	0.025
α_k	*free*	ρ_a	0.90
		σ_a	0.7%

the value estimated in Smets and Wouters (2007) but preferred to account for the fact that there are no wage rigidities in our model. The parameters affecting the evolution of technological shocks are set to $\rho_a = 0.90$ and $\sigma_a = 0.007$.

There are three parameters that directly affect the behavior of banks: i) the fraction of banks' assets that can be diverted Λ, ii) the probability that a banker retires α_b, and iii) the tax rate on banks' wealth τ. We calibrate these parameters to generate an external financing premium of 100 annualized basis points and a steady state leverage ratio of 4 in the banking sector as in Gertler and Karadi (2009).[89] The value of α_k determines the average duration of financial contracts and is left as a free parameter to explore the implications of maturity transformation. Finally, we compute the model solution by a standard log-linear approximation.[90]

3.3.6 Implications of Maturity Transformation: A Shock to Technology

Figure 3.3 shows impulse response functions to a positive technological shock. In each graph, the continuous line shows the model with banks and no maturity transformation, i.e. in case the average duration of contracts in the economy, $\mathcal{D}$, is set equal to 1. The dashed lines, on the other hand, correspond to two different calibrations of the model with maturity transformation – $\mathcal{D} = 4$ and $\mathcal{D} = 12$.

[89] Simple algebra shows that the steady state level of the external financing premium implied by our model does not depend on α_k.

[90] All versions of the model are implemented in Dynare. Codes are available on request.

We start by analyzing the model without maturity transformation. As in standard RBC models, the shock generates an increase in consumption, investment, and output. Households become temporarily richer and therefore raise their deposits b_t while r_t falls. With a higher level of deposits, banks increase their supply of credit, resulting in a fall in the loan rate r_t^L. Firms demand more capital and therefore its price p_t^k increases. This means that they now need to borrow more in order to finance each unit of capital, and firms therefore increase their demand for credit. These combined effects generate an increase in banks' net worth as shown in Figure 3.3. As banks' financial position is strengthened following the shock, restrictions to credit provision are relaxed and banks' leverage ratio increases. We therefore obtain a financial accelerator effect in the sense of Bernanke, Gertler, and Gilchrist (1999).

The business cycle implications of maturity transformation can be considered by comparing the full and dashed lines in Figure 3.3. We see that increasing the average duration of loans to $\mathcal{D} = 4$ and $\mathcal{D} = 12$ generates weaker responses in output following the shock. Accordingly, our model predicts a *credit maturity attenuator effect*. To understand why, consider banks' balance sheet equations (3.20) to (3.22). The presence of maturity transformation ($\alpha_k > 0$) implies that only a fraction of all loans is reset to reflect a higher price of capital p_t^k following the shock. The remaining fraction of contracts was signed in the past and does not respond to changes p_t^k. Consequently, good-producing firms increase their demand for credit by a smaller amount the higher the degree of maturity transformation. Banks' revenues and net-worth therefore increase by less, which in turn results in a weaker response of output to the shock.

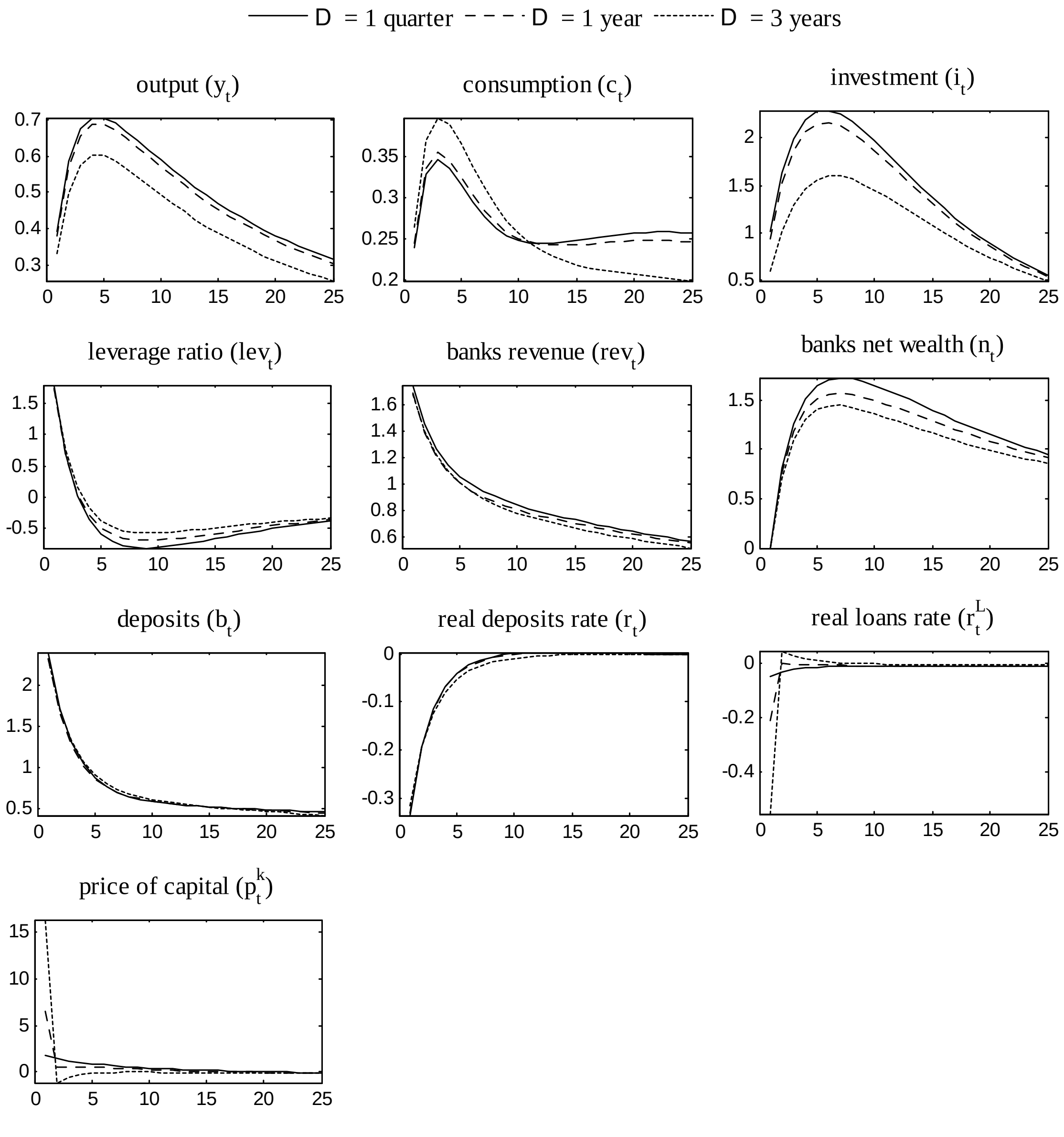
Interestingly, in our general equilibrium setup, the effects of different degrees of maturity transformation are felt not only in the relation between banks and good-producing firms, but also in the behavior of all agents in the economy. Capital producers, for example, know that higher degrees of maturity transformation are associated with weaker increases in the demand for capital after the shock. They therefore raise investment by less compared to the case without maturity transformation, resulting in more room for households' consumption to increase. Over time, however, the smaller increase in investment affects households' income and, consequently, consumption goes back to the steady state faster the higher the degree of maturity transformation.

3.4 A New Keynesian Model: Nominal Financial Contracts

The analysis has so far focused on long-term financial contracts set in real terms, i.e. with inflation protection. Such insurance against inflation is often not available in reality and most lending is therefore conducted based on nominal contracts. The distinction between nominal and real contracts is especially interesting in our setup, because long-term inflation expectations here have a larger impact on firms' decisions compared to one-period nominal contracts as considered in Christiano, Motto, and Rostagno (2003) and Christiano, Motto, and Rostagno (2007). The aim of this section is therefore to extend the model presented in Section 3.3 to nominal contracts and study how maturity transformation affects the monetary transmission mechanism in an otherwise standard New Keynesian model.

We proceed as follows. Sections 3.4.1 and 3.4.2 revisit the problems for the good-producing firms and banks, respectively, when we have long-term nominal contracts. To introduce price stickiness into the model, Section 3.4.3 follows Gertler and Karadi (2009) and adds retail firms to the economy. Monetary policy and market clearing conditions are outlined in Section 3.4.4. Section 3.4.5 then studies the quantitative implications of maturity transformation following a monetary policy shock.

Figure 3.3: Impulse Responses to a Positive Technological Shock



Notes: Impulse response to a one standard deviation positive shock to technology. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

3.4.1 Good-Producing Firms

The basic setup for the good-producing firms is similar to the one presented in Section 3.3.2, except firms now need to borrow based on the nominal price of their capital stock when signing the contract. To see the implications of this assumption, let P_t denote the nominal price level of aggregate output (defined below) and let P_t^k be the nominal price of capital. The expression for real profit in period $t+j$ for a firm that entered a contract is period t is then

$$profit_{t+j|t} = \underbrace{\frac{P_{t+j}^{int}}{P_{t+j}} a_{t+j} \left[(1-\delta)^j \widetilde{k}_t\right]^\theta h_{t+j|t}^{1-\theta}}_{\text{production revenue}} - \underbrace{w_{t+j}\widetilde{h}_{t+j|t}}_{\text{wage bill}} - \underbrace{\left(r_t^L+\delta\right)(1-\delta)^j \widetilde{k}_t \frac{P_t^k}{P_{t+j}}}_{\text{capital rental bill}}, \quad (3.32)$$

where P_t^{int} is the nominal price of the good produced by the firm. That is, the firm borrows $\widetilde{k}_t P_t^k$ units of *cash* throughout the contract, and the interest rate on this loan $r_t^{L,nom}$ is now expressed in nominal terms. Importantly, changes in the price level P_t affects the *real* value of the loan and thereby its implied real interest rate. This effect is easily seen by rewriting the firm's profit as

$$\begin{aligned} profit_{t+j|t} = {} & p_{t+j}^{int} a_{t+j} (1-\delta)^{\theta j} \left(\widetilde{k}_t\right)^\theta h_{t+j|t}^{1-\theta} - w_t h_{t+j|t} \\ & - \left(r_t^{L,nom}+\delta\right)\left(\textstyle\prod_{i=1}^j \pi_{t+i}\right)^{-1} p_t^k \widetilde{k}_t (1-\delta)^j , \end{aligned} \quad (3.33)$$

where we define the real prices $p_t^{int} \equiv P_t^{int}/P_t$ and $p_t^k \equiv P_t^k/P_t$. Moreover, $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate. Hence, higher inflation during the contract erodes the real value of the loan and hence lowers its real interest rate $\left(r_t^{L,nom}+\delta\right) \times \left(\prod_{i=1}^j \pi_{t+i}\right)^{-1}$, and vice versa for lower inflation. The firm and the bank are aware of this effect when signing the contract, and $r_t^{L,nom}$ therefore accounts for long-term inflation expectations.

As in Section 3.3.2, the good-producing firm determines capital and labor by maximizing the net present value of future profits. Applying the households' stochastic discount factor, the first-order condition for the optimal level of capital $\widetilde{k}_t$ is now

$$\mathbf{E}_t \sum_{j=0}^{\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(p_{t+j}^{int} \theta a_{t+j} (1-\delta)^{\theta j} \left(\widetilde{k}_t\right)^{\theta-1} h_{t+j|t}^{1-\theta} - \frac{\left(r_t^{L,nom} + \delta\right)}{\prod_{i=1}^{j} \pi_{t+i}} p_t^k (1-\delta)^j \right) = 0. \tag{3.34}$$

The first-order condition for labor remains unchanged as in equation (3.8).

3.4.2 The Banking Sector

The behavior of the representative bank is similar to the case with real contracts. However, the fact that contracts are set in nominal terms introduces a debt-deflation channel following Fisher (1933). We briefly describe how this effect operates via banks' balance sheet within our model.

Redoing the arguments in Section 3.3.3 for nominal variables imply that

$$N_{t+1} = (1-\tau)\left[REV_t - R_t^{nom} LEN_t + R_t^{nom} N_t\right], \tag{3.35}$$

where N_t is *nominal* net worth, REV_t is *nominal* revenue, and LEN_t is *nominal* lending. Re-expressing this equation in real terms implies

$$n_{t+1} = (1-\tau)\left[\frac{rev_t}{\pi_{t+1}} - R_t^{nom} \frac{len_t}{\pi_{t+1}} + R_t^{nom} \frac{n_t}{\pi_{t+1}}\right]. \tag{3.36}$$

where $rev_t \equiv REV_t/P_t$, $len_t \equiv LEN_t/P_t$ and $n_t \equiv N_t/P_t$. The important difference compared to the corresponding equation based on real contracts in (3.20) is the correction for inflation. Hence, a reduction in inflation increases the real value of banks' net worth from the previous period n_t/π_{t+1} and their revenue rev_t/π_{t+1}. The real value of deposits len_t/π_{t+1} also increase, but the combined effect is likely to be positive, in so far as banks are running a surplus in period t.

This effect from inflation introduces a debt-deflation mechanism whereby fundamental macroeconomic shocks affect real activity. The channel operates in the following way. Unpredictable macro shocks may move inflation temporarily away from what was expected when contracts were signed, resulting in changes in the *ex-post* real revenue of long-term loans. This in turn affects banks net worth and therefore also the supply of credit.

The remaining equations for the banking sector are as in Section 3.3.3, given appropriate corrections for inflation (see Appendix 3.C.1).

3.4.3 Retail Firms

The final output in the economy is assumed to be a CES composite produced from differentiated retail goods, i.e.

$$y_t = \left[\int_0^1 y_{f,t}^{\frac{\eta-1}{\eta}} df \right]^{\frac{\eta}{\eta-1}}, \tag{3.37}$$

where $\eta > 1$ and $y_{f,t}$ is the product from retail firm f. Cost minimization implies the standard demand function

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\eta} y_t, \tag{3.38}$$

where $P_{f,t}$ is the price of the retail good from firm f. The aggregate price level is thus given $P_t = \left[\int_0^1 P_{f,t}^{1-\eta} df \right]^{\frac{1}{1-\eta}}$.

The role of the individual retail firms is to re-package the good from the good-producing firms using a linear production technology. Nominal rigidity is introduced based on a Calvo-style formulation, where only a fraction $1 - \alpha_p$ of retail firms can reset their prices every period. This price is denoted by P_t^*. The remaining fraction α_p of retail firms simply let $P_{f,t} = P_{f,t-1}$. Accordingly, the problem for retail firms adjusting prices in period t is given by

$$\max_{P_t^*} \mathbf{E}_t \sum_{i=0}^{\infty} (\alpha_p \beta)^i \frac{\lambda_{t+i}}{\lambda_t} \left[\frac{P_t^*}{P_{t+i}} - p_{t+i}^{int} \right] y_{f,t+i} \tag{3.39}$$

subject to (3.38).

3.4.4 Monetary policy and Marked Clearing Conditions

Monetary policy is specified by a standard Taylor-rule

$$r_t^{nom} = \rho r_{t-1}^{nom} + (1-\rho) \left(r_{ss}^{nom} + \phi_\pi \log \left(\frac{\pi_t}{\pi_{ss}} \right) + \phi_y \log \left(\frac{y_t}{y_{ss}} \right) \right) + \varepsilon_t^r \tag{3.40}$$

where $R_t^{nom} \equiv 1 + r_t^{nom}$ and $\varepsilon_t^r \sim \mathcal{NID}\,(0, \sigma_r^2)$. That is, central bank aims to close the inflation and output gaps, while potentially smoothing changes in the policy rate.

The market clearing conditions are standard and stated in Appendix 3.C.1.

3.4.5 Implications of Maturity Transformation: A Monetary Policy Shock

This section examines effects of maturity transformation following a positive monetary policy shock, i.e. an exogenous increase in r_t^{nom}. The real part of the model is calibrated as in Table 3.1. The parameters associated to the nominal frictions are calibrated as follows. Inflation in the steady state is assumed to be zero, while we let $\alpha_p = 0.75$ so that retail firms on average change their prices once every year. The value of η is set to 6, consistent with a 20% price markup as implied by the benchmark estimate in Christiano, Eichenbaum, and Evans (2005). Finally, the coefficients in the Taylor-rule are taken from the post-1984 estimates in Justiniano and Primiceri (2008), i.e. $\rho = 0.84$, $\phi_\pi = 2.37$, and $\phi_y = 0.02$. Figure 3.4 displays the impulse response functions to a monetary policy shock of 25 basis points (equivalent to an annualized 100 basis points shock). As before, the continuous line represents the model without maturity transformation ($\mathcal{D} = 1$), whereas dashed lines refer to different calibrations of maturity transformation with $\mathcal{D} = 4$ and $\mathcal{D} = 12$.

Starting with the simpler model where $\mathcal{D} = 1$, the policy shock generates an increase in the implied real deposit rate (r_t^{nom} increases and π_t decreases) which results in the familiar contraction in consumption, investment, output, and inflation. The reduction in inflation increases the real value of banks' nominal assets and banks are therefore better off on impact. However, the fall in the demand for capital and the associated fall in p_t^k reduces banks' real revenues, lowering their net-worth from the second period onwards.[91] The positive co-movement between net-worth and output generates a financial accelerator effect as in Bernanke, Gertler, and Gilchrist (1999).

We next study how maturity transformation affects the monetary transmission mechanism. Our model predicts that the fall in output is weaker the higher the degree of maturity transformation. In other words, we also obtain a *credit maturity attenuator effect* in the case of a monetary policy shock. This is in contrast to the "bank capital channel" analyzed in the context of partial equilibrium models by den Heuvel (2006). According to this theory, the presence of maturity mismatches in banks' balance sheets implies that only a small fraction of loans can be quickly adjusted following a monetary policy shock, whereas deposits are almost entirely adjusted on impact. This means that an increase in the policy rate would have a negative impact on banks' profits and consequently on the supply of credit, potentially exacerbating the real effects of the shock. To explain the difference between this theory and our result, we focus on how maturity transformation affects banks' net-worth within our model. Here, we emphasize three general equilibrium effects, which are not present in the partial equilibrium analysis behind the bank capital

[91] Note in equation (3.36) that on impact movements in n_t following any shock are only a result of the change in inflation. Changes in rev_t, len_t and R_t^{nom} can only affect banks' net-worth from the second period and onwards.

channel.

First, in the model without maturity transformation the fall in the price of capital p_t^k implies a reduction in the value of *all* loans, and banks therefore see a fall in their revenues. However, with maturity transformation only a fraction $1 - \alpha_k$ of loans are reset every period to reflect the fall in p_t^k. Accordingly, banks revenues do not fall as much the higher the degree of maturity transformation.

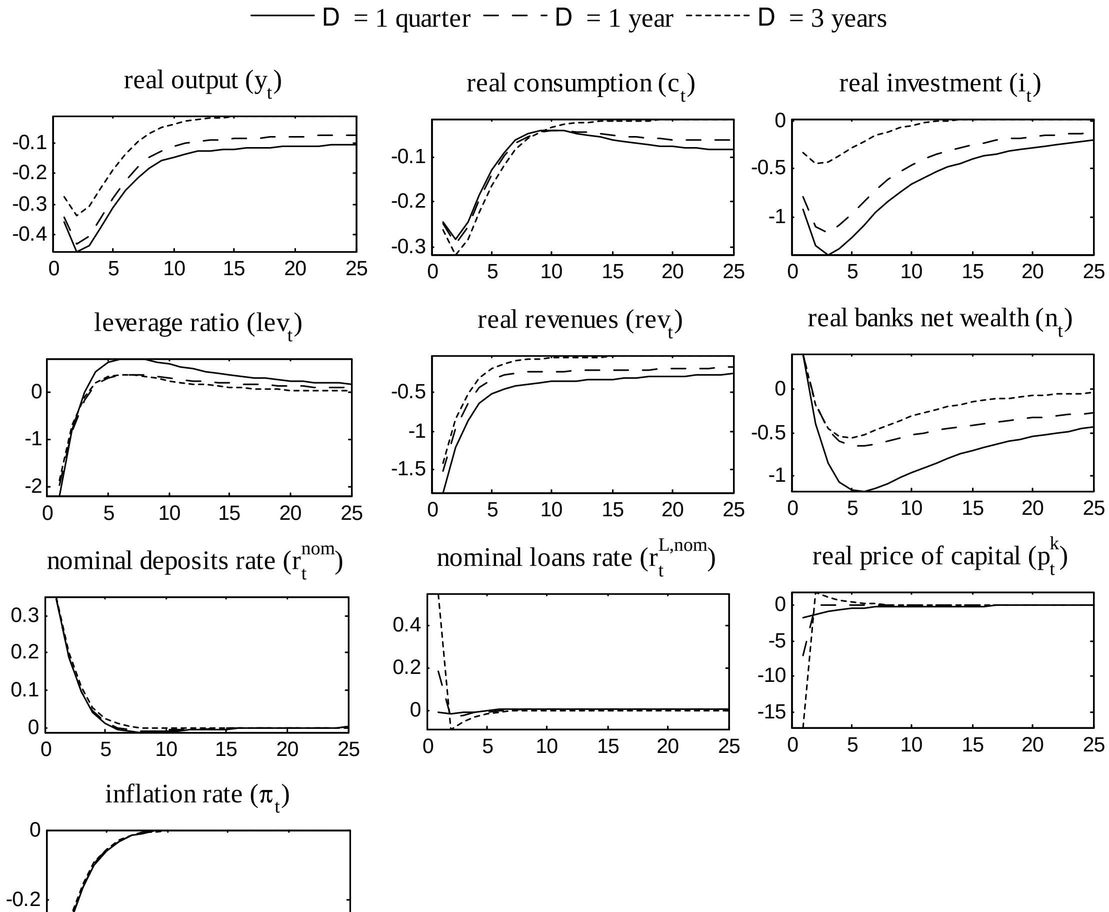
A second general equilibrium effect occurs as a result of the debt-deflation channel discussed in Section 3.4.2. The reduction in inflation following the shock raises the *ex-post* real interest rates paid by the good-producing firms. The aggregate value of loans fall by less in the presence maturity transformation (due to the first channel) and the higher *ex-post* real rate therefore has a larger positive effect on banks' balance sheets and output than without long-term loans.

The third general equilibrium effect is as follows. With maturity transformation, the smaller reduction in banks' net-worth n_t implies that output (and income) does not fall as much as in the case without long-term contracts. Hence, the decline in households' deposits is smaller, and banks are able to provide more credit to good-producing firms. As a result, this effect also reduces the contraction in output following the shock.

3.5 Conclusion

This chapter shows how to introduce a banking sector with maturity transformation into an otherwise standard DSGE model. Our novel assumption is to consider the case where firms face a constant probability of being unable to reset their capital level in every period. We first show that this restriction on firms' ability to adjust capital does not effects prices and aggregate quantities in a wide range of DSGE models. Importantly, the considered friction generates a demand for long-term credit when

Figure 3.4: Impulse Responses to a Positive Monetary Policy Shock



Notes: Impulse response to a 25 basis points positive monetary policy shock. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

we impose the standard requirement that firms borrow when financing their capital stock. As a result, banks face a maturity transformation problem because they use short term deposits and accumulated wealth to fund the provision of long-term credit. Within an RBC model featuring long-term contracts and banks, we then analyze the quantitative implications of maturity transformation following a positive technological shock. Our model suggests that the responses of the model economy to this shock are in general weaker the higher the degree of maturity transformation in the banking sector.

The final part of this chapter studies implications of maturity transformation when financial contracts are set in nominal terms. We therefore extend the considered RBC model with sticky prices, long-term nominal contracts, and a central bank. Effects of maturity transformation within the banking sector are then analyzed following a positive monetary policy shock. We once again conclude that responses in the economy in general are weaker the higher the degree of maturity transformation in the banking sector.

Our way of incorporating maturity transformation is only a first step in analyzing this topic in a dynamic stochastic general equilibrium setup. Interesting extensions could introduce extra financing options for firms, possibly by breaking the match between the duration of firms' exposure and their financial contract. This would also have the potential to create a time-varying maturity transformation problem within the banking sector. Studying higher-order effects and the impact of risk on banks' behavior would also make for an interesting extensions.

Appendix 3.A A Standard RBC Model with Infrequent Capital Adjustments

3.A.1 Households

The representative household's problem can be summarized by the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} = & \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{(c_{t+j} - b\, c_{t+j-1})^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) + \\
& \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \lambda_{t+j} [h_{t+j}\, w_{t+j} + R_{t+j}^k\, k_{t+j} - c_{t+j} - i_{t+j}] + \\
& \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j q_{t+j} \lambda_{t+j} \left[(1-\delta)\, k_{t+j}^s + i_{t+j} \left[1 - S\left(\frac{i_{j+j}}{i_{t-1+j}} \right) \right] - k_{t+1+j}^s \right],
\end{aligned}
$$

where λ_t is the Lagrange multiplier associated with the budget constraint. The first order conditions are:

i **Consumption,** c_t:

$$
\lambda_t = \mathbf{E}_t \left[\frac{1}{(c_t - bc_{t-1})^{\phi_0}} - \frac{\beta b}{(c_{t+1} - bc_t)^{\phi_0}} \right]
$$

ii **Labor,** h_t:

$$
\phi_2 h_t^{\phi_1} = \lambda_t w_t
$$

iii **Physical capital stock,** k_{t+1}^s:

$$
1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{R_{t+1}^k + q_{t+1}(1-\delta)}{q_t} \right) \right]
$$

iv **Investment,** i_t :

$$
q_t = \frac{1 - \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 S'\left(\frac{i_{t+1}}{i_t} \right) \right]}{\left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}} \right) \right]}
$$

3.A.2 Firms

The profit of firm i in period $t+j$ is

$$
a_t k_{i,t+j}^{\theta} h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j},
$$

and the firm seeks to maximize its expected discounted value of profits given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} k_{i,t+j}^{\theta} h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j} \right).$$

This problem is divided in two steps. We first derive the i^{th} firm's demand of labor, which takes the standard form since labor is optimally chosen in every period. In the second step, we derive the optimal value of capital $\widetilde{k}_{i,t}$ for firms that are able to adjust capital in period t. Note that a firm adjusting capital in period t faces a probability α_k^j of not being able to reoptimize after j periods in the future and hence have $(1-\delta)^j \widetilde{k}_{i,t}$ units of capital in period $t+j$.

i **Labor,** h_t :

In every period $t+j$, for $j = 0, 1, 2, ...,$ all firms are allowed to adjust their labor demand. Hence, we can ignore the dynamic dimension of the firm's problem which implies

$$h_{i,t+j} = \left(\frac{w_{t+j}}{a_{t+j} (1-\theta)} \right)^{-\frac{1}{\theta}} k_{i,t+j}.$$

The period $t+j$ demand for labor for a firm that last reoptimized in period t, $\tilde{h}_{i,t+j|t}$, is given by

$$\tilde{h}_{i,t+j|t} = \left(\frac{w_{t+j}}{a_{t+j} (1-\theta)} \right)^{-\frac{1}{\theta}} (1-\delta)^j \widetilde{k}_{i,t}$$

ii **Capital,** $\widetilde{k}_t$:

A firm adjusting capital in period t chooses $\widetilde{k}_{i,t}$ to maximize the present discounted value of profits. This firm therefore solves

$$\max_{\widetilde{k}_{i,t}} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \left((1-\delta)^j \widetilde{k}_{i,t} \right)^{\theta} \widetilde{h}_{i,t+j|t}^{1-\theta} - R_{t+j}^k (1-\delta)^j \widetilde{k}_{i,t} - w_{t+j} h_{i,t+j|t} \right)$$

Using the demand for labor derived above, the optimality condition associated with the capital choice for firm i can be written as

$$\mathbf{E}_t \sum_{l=0}^{+\infty} (\alpha_k \beta (1-\delta))^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \theta \left(\frac{w_{t+j}}{a_{t+j} (1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - R_{t+j}^k \right) = 0$$

Appendix 3.B An RBC Model With Banks and Maturity Transformation

3.B.1 Recursions for $x_{1,t}$ and $x_{2,t}$

The expected discounted value of bank equity V_t can be expressed as

$$\begin{aligned}
V_t &= \mathbf{E}_t \sum_{i=0}^{\infty} (1-\alpha_b)\, \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} (1-\tau) \left[rev_{t+i} - R_{t+i} len_{t+i} + R_{t+i} n_{t+i} \right] \\
&= (1-\tau) \left\{ len_t \left(\mathbf{E}_t \sum_{i=0}^{\infty} (1-\alpha_b)\, \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \left[\frac{rev_{t+i}}{len_t} - R_{t+i} \frac{len_{t+i}}{len_t} \right] \right) \right. \\
&\left. + n_t \left(\mathbf{E}_t \sum_{i=0}^{\infty} (1-\alpha_b)\, \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \frac{R_{t+i} n_{t+i}}{n_t} \right) \right\} \\
&= (1-\tau) \left[len_t x_{1,t} + n_t x_{2,t} \right]
\end{aligned}$$

where we have defined

$$x_{1,t} \equiv \mathbf{E}_t \sum_{i=0}^{\infty} (1-\alpha_b)\, \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \left[\frac{rev_{t+i}}{len_t} - R_{t+i} \frac{len_{t+i}}{len_t} \right]$$

$$x_{2,t} \equiv \mathbf{E}_t \sum_{i=0}^{\infty} (1-\alpha_b)\, \alpha_b^i \beta^{i+1} \frac{\lambda_{t+i+1}}{\lambda_t} \frac{R_{t+i} n_{t+i}}{n_t}$$

Straightforward algebra then implies the following recursions:

$$x_{1,t} = E_t (1-\alpha_b)\, \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{rev_t}{len_t} - R_t \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right]$$

$$x_{2,t} = (1-\alpha_b)\, E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] R_t + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]$$

3.B.2 First-order conditions for the capital-producing firm

To simplify the optimization, we isolate $\tilde{k}_t$ from (3.30) and substitute it into (3.29). Hence, we need to optimize (3.27) with respect to v_t, k_t, and i_t subject to (3.30) and (3.31). The Lagrange function then reads:

$$\begin{aligned}
\mathcal{L} &= \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\delta v_{t+j} - i_{t+j} \right] \\
&+ \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} u_{1,t+j} \left[\left(k_{t+j} - \alpha_k (1-\delta)\, k_{t-1+j} \right) p_{t+j}^k + \alpha_k (1-\delta)\, v_{t-1+j} - v_{t+j} \right] \\
&+ \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} q_{t+j} \left[(1-\delta) k_{t+j} + i_{t+j} \left[1 - \frac{\kappa}{2} \left(\frac{i_{t+j}}{i_{t+j-1}} - 1 \right)^2 \right] - k_{t+j+1} \right]
\end{aligned}$$

The first-order conditions are:

i. **The value-aggregate** v_t:

$$u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} \alpha_k (1-\delta) \right]$$

ii. **Capital** k_t :

$$q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1-\delta) p^k_{t+2} \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p^k_{t+1} \right] + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1-\delta) \right].$$

iii. **Investment** i_t:

$$1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i^2_{t+1}}{i^2_t} \right]$$

Notice that q_t is the standard Tobin's Q, i.e. indicating the marginal change in profit of a marginal change in k_{t+1}. On the other hand, p^k_t is the marginal change in profit of a marginal change in k_t.

3.B.3 Model summary

Household:

1) $\lambda_t = \mathbf{E}_t \left[(c_t - bc_{t-1})^{-\sigma_c} - \beta b \left(c_{t+1} - bc_t \right)^{-\sigma_c} \right]$
2) $1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} R_t \right]$
3) $\phi_2 h_t^{\phi_1} = \lambda_t w_t$

Good-Producing Firms:

4) $h_t = \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1}{\theta}} k_t$
5) $z_{1,t} = \left(r_t^L + \delta \right) p_t^k z_{2,t}$
6) $z_{1,t} = \theta a_t \left(\frac{w_t}{a_t(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} z_{1,t+1} \left((1-\delta) \alpha_k \beta \right) \right]$
7) $z_{2,t} = 1 + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} ((1-\delta)\beta\alpha_k) z_{2,t+1} \right]$
8) $k_t = (1-\alpha_k) \widetilde{k}_t + \alpha_k (1-\delta) k_{t-1}$

Banking sector:

9) $n_{t+1} = (1-\tau) \left[rev_t - R_t len_t + R_t n_t \right]$
10) $rev_t = (1-\alpha_k) R_t^L p_t^k \widetilde{k}_t + (1-\delta) \alpha_k rev_{t-1}$
11) $len_t = (1-\alpha_k) p_t^k \widetilde{k}_t + (1-\delta) \alpha_k len_{t-1}$
12) $lev_t \equiv \frac{len_t}{n_t} = \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}}$
13) $V_t = (1-\tau) \left[len_t x_{1,t} + n_t x_{2,t} \right]$
14) $x_{1,t} = \mathbf{E}_t (1-\alpha_b) \beta^1 \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{rev_t}{len_t} - R_t \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right]$
15) $x_{2,t} = (1-\alpha_b) \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] R_t + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]$

Capital-Producing Firm:

16) $k_{t+1} = (1-\delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right]$
17) $u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} \alpha_k (1-\delta) \right]$
18) $1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right]$
19) $q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1-\delta) p_{t+2}^k \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p_{t+1}^k \right] + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1-\delta) \right]$
20) $v_t = (1-\alpha_k) \widetilde{k}_t p_t^k + \alpha_k (1-\delta) v_{t-1}$

Market Clearing Conditions:

21) $y_t = a_t k_t^\theta h_t^{1-\theta}$
22) $y_t = c_t + i_t$

Exogenous Processes:

23) $\log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$

Appendix 3.C The New Keynesian Model With Banks and Maturity Transformation

3.C.1 Model summary

Household:

1) $\lambda_t = \mathbf{E}_t \left[(c_t - bc_{t-1})^{-\sigma_c} - \beta b \left(c_{t+1} - bc_t \right)^{-\sigma_c} \right]$

2) $1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^{nom}}{\pi_{t+1}} \right]$

3) $\phi_2 h_t^{\phi_1} = \lambda_t w_t$

Intermediate Goods Producing Firms:

4) $h_t = \left(\frac{w_t}{p_t^{int} a_t (1-\theta)} \right)^{-\frac{1}{\theta}} k_t$

5) $z_{1,t} = \left(r_t^{L,nom} + \delta \right) p_t^k z_{2,t}$

6) $z_{1,t} = p_t^{int} \theta a_t \left(\frac{w_t}{p_t^{int} a_t (1-\theta)} \right)^{-\frac{1-\theta}{\theta}} + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} z_{1,t+1} \left((1-\delta) \alpha_k \beta \right) \right]$

7) $z_{2,t} = 1 + \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} ((1-\delta) \beta \alpha_k) z_{2,t+1} \right]$

8) $k_t = (1-\alpha_k) \widetilde{k}_t + \alpha_k (1-\delta) k_{t-1}$

Financial Intermediaries:

9) $n_{t+1} = (1-\tau) \pi_{t+1}^{-1} \left[rev_t - R_t^{nom} len_t + R_t^{nom} n_t \right]$

10) $rev_t = (1-\alpha_k) R_t^{L,nom} p_t^k \widetilde{k}_t + (1-\delta) \alpha_k rev_{t-1} \pi_t^{-1}$

11) $len_t = (1-\alpha_k) p_t^k \widetilde{k}_t + (1-\delta) \alpha_k len_{t-1} \pi_t^{-1}$

12) $lev_t \equiv \frac{len_t}{n_t}$

13) $lev_t = \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}}$

14) $x_{1,t} = \mathbf{E}_t (1-\alpha_b) \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} \left[\frac{rev_t}{len_t} - R_t^{nom} \right] + \mathbf{E}_t \left[\alpha_b \beta x_{1,t+1} \frac{len_{t+1}}{len_t} \frac{\lambda_{t+1}}{\lambda_t} \right]$

15) $x_{2,t} = (1-\alpha_b) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} \right] R_t^{nom} + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]$

Capital Producing Firms:

16) $k_{t+1} = (1-\delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right]$

17) $u_{1,t} = \delta + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1} u_{1,t+1} \alpha_k (1-\delta) \right]$

18) $1 = q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) + \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right]$

19) $q_t + \mathbf{E}_t \left[\beta^2 \frac{\lambda_{t+2}}{\lambda_t} u_{1,t+2} \alpha_k (1-\delta) p_{t+2}^k \right] = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} u_{1,t+1} p_{t+1}^k \right] + E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1-\delta) \right]$

Retail Firms:

20) $\frac{P_t^*}{P_t} = \frac{num_t}{den_t}$

21) $num_t = \mu p_t^{int} y_t + \mathbf{E}_t \left[\alpha_p \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta} num_{t+1} \right]$

22) $den_t = y_t + \mathbf{E}_t \left[\alpha_p \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{\eta-1} den_{t+1} \right]$

23) $\pi_t = \left[(1-\alpha_p) \left(\frac{P_t^*}{P_t} \pi_t \right)^{1-\eta} + \alpha_p \right]^{\frac{1}{1-\eta}}$

Market Clearing Conditions:
24) $y_t^{int} = a_t k_t^{\theta} h_t^{1-\theta}$
25) $y_t = \Delta_t^{-1} y_t^{int}$
26) $\Delta_t = (1-\alpha_p)\left(\frac{P_t^*}{P_t}\right)^{-\eta} + \alpha_p (\pi_t)^{\eta} \Delta_{t-1}$
27) $y_t = c_t + i_t$
28) $r_t^{nom} = \rho r_{t-1}^{nom} + (1-\rho)\left[r_{ss}^{nom} + \phi_\pi \pi_t + \phi_y (\log y_t - \log y_{ss})\right] + \varepsilon_t^r$

Exogenous Processes:
29) $\log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$

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