The London School of Economics and Political Science

Essays on Macroeconomics

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A thesis submitted to the Department of Economics
for the degree of Doctor of Philosophy

November 2023

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 45936 words.

Acknowledgment

I wish to extend my deepest gratitude to my supervisor, Wouter den Haan, for his unparalleled guidance throughout this journey. Our discussions on economics have not only honed my academic understanding but have also shaped the way I approach research. His patience, encouragement, and unwavering courage, especially as a human rights advocate, have been instrumental to my growth. Additionally, my heartfelt thanks go to Ricardo Reis and Cameron Peng, whose insightful advice and support enriched my academic journey.

I'm also profoundly grateful to the LSE macro faculty. Special mentions to Maarten De Ridder, Matthias Doepke, Jonathon Hazell, Benjamin Moll, Dmitry Mukhin, Rachel Ngai, and Shengxing Zhang for their stimulating discussions and invaluable inspirations.

To my friends and colleagues, who made the Ph.D. experience memorable, I owe a debt of gratitude. I cherish the camaraderie with Marco Bellifemine, Jingxuan Chen, Adrien Couturier, Marina Feliciano, Yuan Hu, Xitong Hui, Stephan Hobler, Seyed Hosseini, Tomer Ifergane, Tiago Paúl, Alberto Pellicioli, Hugo Reichardt, Yannick Schindler, Aditya Soenarjo, Bo Tang, Wei Wang, Linchuan Xu, Jialin Yi, and Xiang Yin. Your presence has enriched both my professional and personal life.

Last, but most certainly not least, my deepest appreciation goes to my parents. Their love and support have been the bedrock upon which this journey was built.

Abstract

This thesis comprises three chapters on macroeconomics.

Chapter 1 studies over/under-reaction and judgment noise in expectations formation. In forecast surveys of aggregate macroeconomic and financial variables, the correlation between forecast errors and forecast revisions is positive at the consensus level, but negative at the individual level. I argue that noise in predictive judgment can account for the difference. Using forecast survey data, I provide evidence that judgment noise is large enough to reconcile the difference between the two coefficients. The structural parameter measuring over-/underreaction mainly points to underreaction, regardless of whether the model matches correlation coefficients at the individual or aggregate level.

Chapter 2 looks at adaptive expectations and over-/under-reaction to new information. It is shown that the occurrence of over- or under-reaction using adaptive expectations is contingent on both the weighting parameter used in forecasts and the persistence of the associated actual variable. Furthermore, compared to the generalized diagnostic expectations model, the adaptive expectations framework can better match the under-reaction to new information for several variables, as measured by the correlations between forecast errors and forecast revisions. This advantage stems from the capacity of adaptive expectations to allow varying degrees of stickiness in expectations.

Chapter 3 explores the effects of an earnings-based borrowing constraint on long-term productivity growth and employment within an economy characterized by endogenous growth, nominal rigidities, and the presence of a zero lower bound on the nominal interest rate. My findings indicate a bifurcated impact from the tightening of the borrowing constraint, contingent upon the position of the nominal interest rate. Particularly, when the nominal interest rate is at the zero lower bound, the tightening of borrowing constraints displays a neutral impact on growth but surprisingly leads to a rise in employment.

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Chapter 1

Over/Under-reaction and Judgment Noise in Expectations Formation

1.1 Introduction

> *To understand error in judgment, we must understand both bias and noise. Sometimes, as we will see, noise is the more important problem. But in public conversations about human error and in organizations all over the world, noise is rarely recognized. Bias is the star of the show. Noise is a bit player, usually offstage.*
>
> *—Noise: A Flaw in Human Judgment*
> *By Daniel Kahneman, Olivier Sibony and Cass R. Sunstein.*

Are people overreacting or underreacting to new information when forming expectations about aggregate macroeconomic and financial variables? Researchers have found mixed evidence and have come up with different models featuring either overreaction or underreaction. Early models introduce costs in information acquisition or processing (Sims (2003), Woodford (2003)), leading to underreaction to new information and sluggish price movements (Mankiw and Reis (2002)). A series of recent papers (e.g., Bordalo, Gennaioli, Ma, and Shleifer (2020)) develop the diagnostic expectation theory, a pyschologically founded non-Bayesian model of belief formation featuring overreaction. They find that overreaction helps explain long-standing empirical puzzles in macroeconomics and finance, such as the large volatility of stock prices and bond yields, and return predictability.

This chapter revisits two pieces of crucial but puzzling empirical facts. Starting from Coibion and Gorodnichenko (2015), henceforth CG, the "forecast error on forecast revision" regression has been the off-the-shelf methodology to identify over-/underreaction to new information in expectation formation using survey data of aggregate macroeconomic and financial variables. The forecast error is defined as the future realization minus the current forecast. The forecast revision is defined as the current forecast minus the forecast made in the last period about the same realization. Under Full Information Rational Expectation (FIRE) hypothesis, current information has no predictive power of future forecast errors. The correlation between forecast errors and forecast revisions should be zero. Positive correlations mean that upward forecast revisions predict higher realizations relative to current forecasts. When there is upward revision, the revision is on average not large enough. Thus, there is underreaction. Similarly, negative correlations mean that upward forecast revisions predict lower realizations relative to current forecasts. Thus, there is overreaction. Researchers run the regression at the consensus level and individual forecaster level using several forecast surveys of aggregate macroeconomic and financial variables.[1] The consensus forecast is the average across all individual forecasts. Mostly, correlations at the consensus level are positive, which is interpreted as underreaction at the aggregate level. Correlations at the individual level are usually negative, which is regarded as evidence of overreaction at the individual level.[2] Angeletos, Huo, and Sastry (2021), henceforth AHS, and Bordalo, Gennaioli, Ma, and Shleifer (2020), henceforth BGMS, show that incorporating individual level overreaction modeling elements (overextrapolation for the former, diagnostic expectation for the latter) into a noisy information environment can simultaneously match those two pieces of evidence.

I take a step back and challenge the previous interpretation of the signs of those correlations. First, I ask the question: do negative regression coefficients at the individual level necessarily imply overreactions in individual level expectation formation? By formally examining how judgment noise in forecast impacts the correlation coefficients in a stylized model of expectation formation, my answer is no. The current forecast appears both in forecast errors and in forecast revisions, but with opposite signs. Due to the existence of idiosyncratic noise in individual forecasts, it is mechanical that the correlation coefficients at the consensus level are greater than those at the individual level.

"Judgment noise" is a terminology inherited from the book *Noise: A Flaw in Human Judgment* (2021) by Kahneman et al. Judgment noise is the disagreement among people making judgments using the same information. Making a forecast is predictive judgment. This noise is further decomposed into individual fixed heterogeneity and idiosyncratic random noise.[3] Individual fixed heterogeneity is forecaster specific: some forecasters might always be more optimistic, while others are always more pessimistic. Idiosyncratic random noise varies over time: it can be due to forecasters' unique interpretation of the current economy at that particular time,

[1]See Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma, and Shleifer (2020), Reis (2020), Angeletos, Huo, and Sastry (2021), Fuhrer (2018), etc.

[2]For example, see Reis (2020).

[3]In *Noise: A Flaw in Human Judgment*, individual fixed heterogeneity is called level noise and idiosyncratic random noise is called pattern noise.

their different forecasting models , or simply their mood, etc.[4] The noise part can be pretty large. There can be many reasons behind such idiosyncratic random noise. No matter what the reason is, the gap between the correlation coefficients at the consensus level and the individual level is mechanical.

Before diving into a parsimonious model of expectation formation, I show that the gap between correlations at the consensus and individual level can be due to idiosyncratic random noise, using a simple econometric example. The intuition is the following: since the current forecast appears in both the forecast revisions and the forecast errors with opposite signs, the stochastic idiosyncratic random noise drives correlation coefficients at the individual level downward. However, the idiosyncratic random noise component is averaged out when calculating the consensus forecast, assuming the number of forecasters is large enough. Thus, the correlation coefficient at the consensus level is always larger than at the individual level.

To fully understand what those two correlation coefficients reveal about expectation formation, a parsimonious model of expectation formation with the following features is presented: First, the key target of this chapter, a structural parameter, θ, measures degree of systematic overreaction or underreaction. The modeling element is very similar to diagnostic expectation theory (Bordalo, Gennaioli, and Shleifer (2018), Bordalo, Gennaioli, Porta, and Shleifer (2019), etc.). However, I allow overreaction ($\theta > 0$), underreaction ($-1 < \theta < 0$) and neither over- nor underreaction ($\theta = 0$) during estimation. Second, there is judgment noise across different forecasters. The judgment noise consists of individual fixed heterogeneity and idiosyncratic random noise. There is no information friction, so two types of judgment noise are the reason behind forecast dispersion in the model.

Under this framework, analytic expressions for "forecast error on forecast revision" regression coefficients at both consensus and individual levels are derived. Individual fixed effect controls the individual fixed heterogeneity in the panel regression. However, idiosyncratic random noise pushes individual correlation coefficients downward while having no impact on consensus correlation coefficients, as argued above. The literature has interpreted positive consensus correlation coefficients and negative individual correlation coefficients as underreaction to new information at the aggregate level and overreaction at the individual level. According to my theory, however, the discrepancies are simply due to the idiosyncratic random noise. If the number of forecasters is large, consensus correlation coefficients are not affected by idiosyncratic random noise. They can be directly used to infer the degree of over-/underreactions: when agents exhibit overreactions (underreactions), consensus correlation coefficients are negative (positive).

[4]The idiosyncratic random noise is conceptually different from measurement error. Measurement error is the deviation from the true value that econometricians try to observe. Nevertheless, noise plays a vital role in forecasters' beliefs when making a forecast. In the model, idiosyncratic random noise can be seen as including classic measurement error.

In professional forecaster surveys, the number of forecasters is limited rather than very large. As a result, in practice, the idiosyncratic random noise may not be averaged out when generating the consensus forecast. I show quantitatively that the negative impact of idiosyncratic random noise on correlation coefficients at the consensus level is non-trivial in some cases, and my estimation considers this.

In summary, seeing forecast dispersion as judgment noise changes our interpretation of the two correlation coefficients: First, negative correlation coefficients at the individual level do not reveal overreactions. Second, positive correlation coefficients at the consensus level underestimate the degree of underreaction.

Using seven macroeconomic and financial variables of the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia, I identify individual fixed heterogeneity and idiosyncratic random noise by exploiting their nature: out of disagreement among forecasters, individual fixed heterogeneity stays constant over time, and the rest of this disagreement is due to random noise. In the data, individual fixed effect explains 16% to 24% of variation in disagreement across forecasters. In other words, idiosyncratic random noise explains 76% to 84% of forecast disagreement. Then I show quantitatively that the estimated magnitude of idiosyncratic random noise is large enough to reconcile the gap between correlation coefficients at the consensus and individual level.

Then I turn to the estimation of the critical parameter θ measuring the degree of over-/underreaction. Much less impacted by idiosyncratic random noise, correlation coefficients at the consensus level being positive reveals that most variables yield negative θ. By matching the empirically estimated and model implied correlation coefficients, I obtain the degree of over-/underreaction: out of seven macroeconomic and financial variables, five show negative θs. θs for the other two variables are very close to zero. More importantly, estimation results for θs are similar whether we match correlation coefficients at the individual level or the consensus level. The result starkly contrasts the estimation result of the diagnosticity parameter in BGMS, which obtain mostly positive values. The reason behind this difference is that their paper identifies this parameter by matching individual-level correlation coefficients without considering the role of judgment noise.[5] Naturally, negative correlation coefficients would have yielded positive θs. It is worth emphasizing that neat estimation of θs is crucial for the literature. Papers either borrow existing estimation of θs or use them as priors, to study the implication of over-/underreactions in macroeconomic and financial models.

I perform various robustness checks and show: different correlation coefficients across variables are not due to different time coverage for different variables; allowing for serial correlated idiosyncratic random noise doesn't affect the results; there are similar patterns in ECB's Survey of Professional Forecasters, etc.

[5]They also use another method: estimating θs by matching variance of forecast errors and forecast revisions. However, in this method, they restrict that $\theta \geq 0$.

The chapter proceeds as follows. First, I illustrate how the idiosyncratic random noise generates the gap between consensus and individual correlation coefficients using a general econometric example. Second, a parsimonious model of expectation formation is described and estimated from the data. The model reconciles the empirical patterns while yielding negative θs. In the end, further discussion is presented.

Literature Review

This chapter is related to recent literature about people's overreaction and underreaction to new information. Specifically, it contributes to the literature that uses "forecast error on forecast revision" regressions to study over-/underreactions to new information in survey data. CG utilizes the method for a wide range of macroeconomic and financial variables in different forecast surveys but only at the consensus level. BGMS runs the regression at both consensus and individual-level data. Bordalo, Gennaioli, La Porta, and Shleifer (2019) use the method in long-term stock earnings growth forecast data. Wang (2021) and D'Arienzo (2020) apply the methodology to bond market data. Bouchaud, Kruger, Landier, and Thesmar (2019) apply the methodology to earnings per share forecasts. Fuhrer (2018) uses this methodology in several surveys covering professional forecasters and households. Angeletos, Huo, and Sastry (2021) and Reis (2020) point out the general pattern of regression coefficients being positive at the consensus level and negative at the individual level. Since an increasing number of papers rely on this methodology, we must understand what those two correlation coefficients tell us about people's expectation formation process. First, this chapter challenges the traditional interpretation of forecast errors/revisions correlation coefficients. Second, I obtain different values for the critical parameter, θ.

Moreover, this chapter is closely related to the literature discussing the large magnitude of disagreement among forecasters in all kinds of forecast surveys (Mankiw, Reis, and Wolfers (2003), Giglio, Maggiori, Stroebel, and Utkus (2021), etc.). Many papers rely on information friction to generate disagreement. In this chapter, I turn to judgment noise instead of the traditional wisdom of information friction. Numerous experiments have identified large magnitude of judgment noise (Kahneman, Olivier, and Cass R. (2021)).

Third, this chapter is generally related to the literature about how people's expectation formation deviates from the Full Information Rational Expectation hypothesis (FIRE). FIRE consists of two parts: full information and rational expectations. Deviations from the first half, the full information hypothesis, have been explored by many, including Mankiw, Reis, and Wolfers (2003), Reis (2006a), Reis (2006b), and Sims (2003). The literature focusing on deviations from rational expectations is also huge, with various modeling assumptions that capture people's cognitive bias from rationality in expectation formation, notably Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), etc. More recent papers include Adam, Marcet, and Nicolini (2016), Adam, Marcet, and Beutel (2017), Gabaix (2020), etc. This chapter focuses on over-/underreaction to new information and noise in forecasts.

1.2 An Econometric Illustration

Before diving into a model of expectation formation, I illustrate why the correlation coefficients between individual forecast errors and forecast revisions should not be used as evidence of overreaction to new information at the individual level, using a simple econometric example. Denote the macroeconomic state variable as w_t, e.g., inflation. The following two equations are the "forecast errors on forecast revisions", henceforth Error-on-revision, regressions:

$$w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+h} - \bar{\mathcal{F}}_{t-1} w_{t+h}) + u_{t,t+h}, \tag{1.1}$$

$$w_{t+h} - \mathcal{F}_{i,t} w_{t+h} = \beta_0^I + \beta^I(\mathcal{F}_{i,t} w_{t+h} - \mathcal{F}_{i,t-1} w_{t+h}) + u_{i,t,t+h}. \tag{1.2}$$

Equation 1.1 is the consensus level regression, and 1.2 is the individual level regression. The difference is that in equation 1.1, $\bar{\mathcal{F}}_t w_{t+h}$ is the average forecast across all forecasters, while in equation 1.2, $\mathcal{F}_{i,t} w_{t+h}$ is the individual forecast. On the left-hand side of equations 1.1 and 1.2 are forecast errors at the consensus and individual level. On the right-hand side of equations 1.1 and 1.2 are the forecast revisions at the consensus and individual level, respectively. Researchers interpret a positive β^C as underreaction: when there is a positive forecast revision, the subsequent forecast error tends to be positive, which means that the upward forecast revision is not large enough compared to the true realization of w_{t+h}. A negative β^C is seen as overreaction: when there is a positive forecast revision, forecast error tends to be negative, which means that the upward forecast revision is too large compared to the true realization of w_{t+h}. Similar intuition applies to why $\beta^I < 0$ and $\beta^I > 0$ are interpreted as overreaction and underreaction on the individual level separately. Most of the literature finds: $\hat{\beta}^C > 0$ and $\hat{\beta}^I < 0$. To illustrate the role of idiosyncratic random noise on the estimation of β^I, let us assume for now that the idiosyncratic random noise is an idiosyncratic shock i.i.d. across forecasters and time, denoted by $\eta_{i,t}$. Consider $h = 1$ specifically. Assume individual forecast is given by:

$$\mathcal{F}_{i,t} w_{t+1} = \bar{E}_t w_{t+1} + \eta_{i,t}, \tag{1.3}$$

$$\mathcal{F}_{i,t-1}w_{t+1} = \bar{E}_{t-1}w_{t+1} + \eta_{i,t-1}, \tag{1.4}$$

where $\eta_{i,t} \sim N(0, \sigma_\eta^2)$ is i.i.d. across forecasters and time.[6] We can think of $\bar{E}_t w_{t+1}$ and $\bar{E}_{t-1}w_{t+1}$ as the common component of all forecasters' forecasts. There have not been any assumptions imposed on the structure of this common component. Assume the number of forecasters N is huge. By the law of large numbers, the consensus forecast is given by

$$\bar{\mathcal{F}}_t w_{t+1} = \frac{\sum_{i=1}^N \mathcal{F}_{i,t}w_{t+1}}{N} \xrightarrow{N\to\infty} \bar{E}_t w_{t+1},$$

$$\bar{\mathcal{F}}_{t-1} w_{t+1} = \frac{\sum_{i=1}^N \mathcal{F}_{i,t-1}w_{t+1}}{N} \xrightarrow{N\to\infty} \bar{E}_{t-1} w_{t+1},$$

By the OLS coefficients formula, the estimation of β^C is equal to

$$\beta^C_{OLS} = \frac{cov(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}, w_{t+1} - \bar{E}_t w_{t+1})}{var(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1})}. \tag{1.5}$$

At the same time, we can express the estimation of β^I as follows:

$$\begin{aligned} \beta^I_{OLS} = & \frac{cov(\bar{E}_t w_{t+1} + \eta_{i,t} - \bar{E}_{i,t-1}w_{t+1} - \eta_{i,t-1}, w_{t+1} - \bar{E}_t w_{t+1} - \eta_{i,t})}{var(\bar{E}_t w_{t+1} + \eta_{i,t} - \bar{E}_{t-1}w_{t+1} - \eta_{i,t-1})} \\ & = \frac{cov(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}, w_{t+1} - \bar{E}_t w_{t+1}) - \sigma_\eta^2}{var(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}) + 2\sigma_\eta^2}. \end{aligned} \tag{1.6}$$

When the covariance in equation 1.5 is positive, and the variance of idiosyncratic random noise σ_η^2 is large enough, we have $\beta^C_{OLS} > 0$ and $\beta^I_{OLS} < 0$. σ_η^2 has two impacts on β^I_{OLS}: first, it appears in the numerator and pushes β^I_{OLS} downward; second, it appears in the denominator and attenuates β^I_{OLS} towards zero. It can be verified that, when $cov(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}, w_{t+1} - \bar{E}_t w_{t+1}) > 0$, or equivalently, $\beta^C_{OLS} > 0$, β^I_{OLS} is monotonically decreasing in σ_η^2.[7] In other words, σ_η^2 drives β^I_{OLS} downwards. A negative β^I_{OLS} can be purely a result of a large variance of idiosyncratic random noise instead of overreaction to new information on the individual level. Those results are summarized in lemma 1

Lemma 1 *When $\sigma_\eta^2 > cov(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}, w_{t+1} - \bar{E}_t w_{t+1}) > 0$, we have*

1.

$$\beta^C_{OLS} > 0 > \beta^I_{OLS}.$$

[6] In the extension of the full model, I show that allowing for serially correlated $\eta_{i,t}$ does not affect the results of this chapter quantitatively.

[7] This is obvious by looking at the first order derivative of β^I_{OLS} with respect to σ_η^2:

$$\frac{\partial \beta^I_{OLS}}{\partial \sigma_\eta^2} = \frac{-var(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}) - 2cov(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}, w_{t+1} - \bar{E}_t w_{t+1})}{[var(\bar{E}_t w_{t+1} - \bar{E}_{t-1}w_{t+1}) + 2\sigma_\eta^2]^2}$$

2.

$$\frac{\partial \beta^I_{OLS}}{\partial \sigma^2_\eta} < 0$$

Note that the formulation in equation 1.3 and 1.4 is general. It can capture forecast dispersion due to noise in judgment, which is the focus of this chapter. It can also capture forecast dispersion resulted from dispersed information. Dispersed information is the reason behind forecast dispersion in many other papers, including BGMS and AHS. No matter what the reason is behind $\eta_{i,t}$, the fact $\beta^C_{OLS} > \beta^I_{OLS}$ is mechanical. When σ^2_η is large enough, β^C_{OLS} and β^I_{OLS} can have opposite signs.

1.3 A Parsimonious Model of Expectation Formation

In this section, I first present a parsimonious model of expectation formation. The model allows both overreaction to new information and underreaction. It captures the two types of judgment noise in the spirit of Kahneman et al. (2021). Second I discuss how the degree of over-/underreaction and two types of judgment noise in the model can be identified. Third, the regression coefficients of the error-on-revision regression are derived analytically. We can see in the analytic expressions how the regression coefficients are determined and how judgment noise might confound our inference on the degree of over-/underreaction in expectation formation.

1.3.1 Model Setup

For simplicity, assume the state variable, e.g., inflation, follows an AR(1) process. In the appendix I derive the model under an AR(2) process. The model implication is the same under an AR(2) process.

$$w_t = \rho w_{t-1} + e_t, e_t \sim N(0, \sigma^2_e). \tag{1.7}$$

This model of expectation formation consists of three components: one capturing over/under-reaction, one denoting idiosyncratic noise, and one capturing individual fixed heterogeneity. Let me first introduce the diagnostic expectation theory, which the component capturing over/under-reaction in the model will be based on. The diagnostic expectation theory is a theory of overreaction. However, my model allows for both overreaction and underreaction. The idea of diagnostic expectation is that, while forming expectations, people attach higher conditional density to those states whose probability increases the most upon observing the current state w_t. In other words, people oversample those more representative states in their minds. Representativeness is measured by the increase in conditional density before and after seeing the current state. When w_t follows AR(1) process, the distorted conditional density of a specific state $\hat{w}_{t+1}$, upon seeing the most recent realization $\hat{w}_t$ is given by

$$h_t^{\theta}(\hat{w}_{t+1}) = h(\hat{w}_{t+1}|w_t = \hat{w}_t)\left[\frac{h(\hat{w}_{t+1}|w_t = \hat{w}_t)}{h(\hat{w}_{t+1}|w_t = \rho\hat{w}_{t-1})}\right]^{\theta}\frac{1}{Z} \tag{1.8}$$

$h_t^{\theta}(*)$ is the perceived density function under diagnostic expectation, $h_t(*)$ is the objective density function. θ is the key structural parameter that measures how much the expectations formation deviates from rational expectations systematically. θ is restricted to be positive, consistent with the idea that representative states are overweighted. Z is the normalization to ensure that the density function integrates to 1 in the whole domain of w_t. Bordalo, Gennaioli, and Shleifer (2018) show that when the state variable w_t follows an AR(1) process with normally distributed innovations, the diagnostic expectation is given by

$$E^{Diagnostic}w_{t+1} = E_t w_{t+1} + \theta(E_t w_{t+1} - E_{t-1}w_{t+1}) = \rho w_t + \rho\theta e_t. \tag{1.9}$$

The expression for $E^{Diagnostic}w_{t+1}$ above results from this representativeness heuristic and reflects the "kernel of truth" logic: on top of rational expectation, people overreact to new information they observe in period t by the term $E_t w_{t+1} - E_{t-1}w_{t+1}$. θ gauges this degree of overreaction. I use the same reduced form expression in the model. Instead of restricting $\theta > 0$, I generalize it and allow for overreaction, underreaction and neither over- nor underreaction, by allowing θ to be from the domain $[-1, \infty)$.[8] Such underreaction can be justified by the phenomenon documented by psychologists: conservatism. Conservatism means that individuals are slow to change their beliefs in the face of new evidence.[9] Edwards (1968) documents such a phenomenon in his experiments, where he compares a subject's reaction to new evidence against that of an idealized rational Bayesian agent. He finds that individuals update their posteriors too little compared with the rational Bayesian benchmark: "It turns out that opinion change is very orderly, and usually proportional to numbers calculated from the Bayes Theorem — but it is insufficient in amount. A conventional first approximation to the data would say that it takes anywhere from two to five observations to do one observation's worth of work in inducing a subject to change his opinions."

Having introduced the reduced form expression to capture over-/underreaction to new information, I can present the parsimonious model of expectation formation. Forecasters form expectations of the next period's outcome following the parsimo-

[8] Bordalo, Gennaioli, Ma, and Shleifer (2020) point out "Diagnostic expectations are a theory of overreaction and thus require $\theta > 0$... can also be used as a parsimonious general formalization of distorted beliefs, including underreaction to news for $\theta \in [-1, 0)$." (p. 2764)

[9] See the discussion in Barberis, Shleifer, and Vishny (1998)

nious model as:[10]

$$E^{\theta}_{i,t}w_{t+1} = \rho w_t + \rho\theta e_t + \eta_{i,t} + \phi_i \tag{1.10}$$

where w_t is the underlying state variable (e.g., inflation) which is assumed to follow an AR(1) process as in equation 1.7. In equation 1.10, $\rho w_t + \rho\theta e_t$ is inherited from equation 1.9. ρw_t is the rational expectation of w_{t+1} given the AR(1) process. $\rho\theta e_t$ captures the systematic over/under-reaction to the current innovation e_t while forming expectations, and θ measures the degree of over- and under-reaction. $\theta > 0$, $\theta = 0$, and $-1 \leq \theta < 0$ mean overreaction, neither overreaction nor underreaction, and underreaction, respectively.[11] The last two terms $\eta_{i,t} + \phi_i$ captures the judgment noise in forecast. $\eta_{i,t}$ is the idiosyncratic random noise. I assume it to be i.i.d. distributed across forecasters and time for now, $E(\eta_{i,t}) = 0$, $Var(\eta_{i,t}) = \sigma^2_\eta$. In the extension of the model, I allow serial correlation for the idiosyncratic random noise $\eta_{i,t}$, and it does not affect the chapter's results. ϕ_i is the individual fixed heterogeneity, fixed over time for each individual, $E_\phi(\phi_i) = 0$.

Kahneman et al. (2021) classify judgment noise, the disagreement among people when forming judgment, into pattern noise and level noise. Level noise, in the context of forecasting, is the variability in how an individual forecaster deviates from the consensus forecast on average: some forecasters are consistently more optimistic about the economy in the future, while others are more pessimistic. People exhibit such an individual fixed deviation from the consensus while making all kinds of judgment: some judges are more lenient than others facing the same legal cases; some orthopedists are more aggressive than others when providing recommendations for back surgeries, etc. To make the terminology closer to the economic literature, I call "level noise" individual fixed heterogeneity. The pattern noise is defined as the residual disagreement except for the level noise. It can be due to entirely random factors like mood, or it can be due to "the persistent personal reactions of particular individuals to a multitude of features, which determine their reactions to specific cases." If we think about how forecasters develop forecasts about inflation: after observing a broad range of economic indicators/ policy reports released by the Fed over time, they might use different models either in their minds or on computers that attach different coefficients/weights to different ingredients of these information. So forecasters would disagree with each other. Individual forecast deviation from consensus varies over time. I call this "pattern noise" idiosyncratic random noise.[12] In the literature, people argue that both components are critical in cap-

[10] I also consider an aggregate shock to over-/underreaction parameter θ in equation (2): $\theta + \varepsilon_t$. $\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$. ε_t captures the changes in aggregate over-/under-reaction over time. However, the structural estimation nearly always yields a zero σ^2_ε, except for the ten-year Tbond yield. So I eliminate the discussion of ε_t in the main body of the chapter.

[11] I make the restriction that $-1 \leq \theta$. When $\theta < -1$, conceptually, forecasters are not just underreacting to new information. Instead, people are adjusting their forecast in the opposite direction of new information, which does not have any behavioral science support.

[12] Note that, although randomness is a substantial part, "pattern noise" is not mere chance: if

turing the variation in forecast survey data. One recent example is Giglio et al. (2021). Kahneman and his coauthors point out that the noise part is crucial in understanding forecast error, and its size is more significant than people would have expected. $\eta_{i,t}$ can also include other factors that add random errors to individually reported forecasts except for forecasters' true belief, for example, measurement error.

1.3.2 Identification of Over-/Underreaction and Judgment Noise

One key goal of this chapter is to identify the degree of over-/underreaction θ from the data. In the following subsection, I show analytically and numerically how the existence of judgment noise confounds our identification of θ. First, I discuss how $\eta_{i,t}$, ϕ_i and θ can be separately identified from the data. I run a regression from individual forecasts on time and individual fixed effect.

$$\mathcal{F}_{i,t}w_{t+3} = \alpha_t + \phi_i + \eta_{i,t} \tag{1.11}$$

The time fixed effect α_t captures the common component across individual forecasters. In the context of the model, it captures the term $\rho w_t + \rho\theta e_t$. The individual fixed effect accounts for the individual fixed heterogeneity. The residual of the regression $\eta_{i,t}$ represents the idiosyncratic noise. Identifying the key parameter θ relies on establishing the link between the structural coefficient θ and the regression coefficients, which is discussed in the following subsection.

1.3.3 Error-on-revision Regression Coefficients

In this section, I derive the model implied regression coefficients β^C_{OLS} and β^I_{OLS}, under the expectation formation framework laid out in section 1.3.1.

I assume in equation 1.10 that people know the true persistence ρ. As a result, it is natural to write down the subjective expectation of the h-period ahead forecast as

$$E^{\theta}_{i,t}w_{t+h} = E_t w_{t+h} + \rho^h\theta e_t + \rho^{h-1}(\eta_{i,t} + \phi_i) \tag{1.12}$$

Implicitly in equation 1.12, I make a simplifying assumption that at time t, the judgment noise across adjacent forecast horizons diminishes at rate ρ. Relaxing this assumption does not affect the main idea of the chapter. The following derivation takes $h = 1$ for simplicity. In the empirical results in section 1.5.1, I focus on $h = 3$ to be consistent with BGMS so that the results are comparable.

the same macroeconomic condition arises again we would expect the individual forecaster to make the same forecast (controlling for real random factor like mood). However, the economy is such a complex system. The same economic condition never appears twice. For now I see "pattern noise" as "random".

The forecast error of forecaster i made at period t is

$$w_{t+1} - E^{\theta}_{i,t} w_{t+1} = e_{t+1} - \rho\theta e_t - \eta_{i,t} - \phi_i \tag{1.13}$$

The forecast revision of i at period t is

$$E^{\theta}_{i,t} w_{t+1} - E^{\theta}_{i,t-1} w_{t+1} = \rho(\theta+1)e_t + \eta_{i,t} + \phi_i - \rho^2\theta e_{t-1} - \rho\eta_{i,t-1} - \rho\phi_i \tag{1.14}$$

Looking at expressions 1.13 and 1.14, $\eta_{i,t}$ shows up in both the forecast error and the forecast revision with opposite signs. So the idiosyncratic random noise pushes the correlation between forecast error and forecast revision on the individual level downward. ϕ_i stays constant over time for each forecaster, so it doesn't enter into the individual level covariance of forecast errors and revisions. In a panel regression, ϕ_i can be controlled by an individual fixed effect.

$$cov(w_{t+1} - E^{\theta}_{i,t} w_{t+1}, E^{\theta}_{i,t} w_{t+1} - E^{\theta}_{i,t-1} w_{t+1}) = -\rho^2\theta(1+\theta)\sigma^2_e - \sigma^2_\eta \tag{1.15}$$

When the number of forecasters N is large enough, the average forecast error, or forecast error at the consensus level, at period t is

$$w_{t+1} - E^{\theta}_{t} w_{t+1} = e_{t+1} - \rho\theta e_t - \frac{\sum_i^N \eta_{i,t}}{N} - \frac{\sum_i^N \phi_i}{N} \xrightarrow{N\to\infty} e_{t+1} - \rho\theta e_t \tag{1.16}$$

The average forecast revision is

$$\begin{aligned} E^{\theta}_{t} w_{t+1} - E^{\theta}_{t-1} w_{t+1} &= \rho(\theta+1)e_t + \frac{\sum_i^N \eta_{i,t}}{N} + \frac{\sum_i^N \phi_i}{N} - \rho^2\theta e_{t-1} - \rho\frac{\sum_i^N \eta_{i,t-1}}{N} - \rho\frac{\sum_i^N \phi_i}{N} \\ &\xrightarrow{N\to\infty} \rho(\theta+1)e_t - \rho^2\theta e_{t-1} \end{aligned}$$

$\lim_{N\to+\infty} \frac{\sum_i^N \eta_{i,t}}{N} = 0$ and $\lim_{N\to+\infty} \frac{\sum_i^N \phi_i}{N} = 0$ by law of large number. When the number of forecasters is huge, idiosyncratic random noise is averaged out when we calculate the consensus forecast. In this case, idiosyncratic random noise drives downward β^I_{OLS} but has no impact on β^C_{OLS}.

$$cov(w_{t+1} - E^{\theta}_{t} w_{t+1}, E^{\theta}_{t} w_{t+1} - E^{\theta}_{t-1} w_{t+1}) = -\rho^2\theta(1+\theta)\sigma^2_e \tag{1.17}$$

Now we can derive the "forecast error on forecast revision" regression coefficients:

Proposition 1 *When the number of forecasters in the sample is large, the OLS coefficients of consensus level regression and individual level regression with individual fixed effect are given by*

$$\beta^C_{OLS} = \frac{cov(w_{t+h} - E^{\theta}_{t} w_{t+h}, E^{\theta}_{t} w_{t+h} - E^{\theta}_{t-1} w_{t+h})}{var(E^{\theta}_{t} w_{t+h} - E^{\theta}_{t-1} w_{t+h})} = \frac{-\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2} \tag{1.18}$$

$$\beta^I_{OLS} = \frac{cov(w_{t+h} - E^\theta_{i,t}w_{t+h}, E^\theta_{i,t}w_{t+h} - E^\theta_{i,t-1}w_{t+h})}{var(E^\theta_{i,t}w_{t+h} - E^\theta_{i,t-1}w_{t+h})} = \frac{-(1+\theta)\theta - \frac{\sigma^2_\eta}{\rho^2\sigma^2_e}}{(1+\theta)^2 + \rho^2\theta^2 + (1+\rho^2)\frac{\sigma^2_\eta}{\rho^2\sigma^2_e}} \tag{1.19}$$

When $-(1+\theta)\theta < \frac{\sigma^2_\eta}{\rho^2\sigma^2_e}$ *and* $-1 < \theta < 0$*, we have* $\beta^I_{OLS} < 0$ *and* $\beta^C_{OLS} > 0$*.*

The difference between β^I_{OLS} and β^C_{OLS} is due to σ^2_η. In equation 1.19, σ^2_η has two impacts: first, it appears in the numerator and pushes β^I_{OLS} downward; second, it appears in the denominator and attenuates β^I_{OLS} towards zero.

Proposition 1 conveys two pieces of information. First, β^I_{OLS} is not a reliable indicator of the degree of over-/underreaction on the individual level. Equation 1.19 tells us that idiosyncratic random noise plays a role in pushing β^I_{OLS} downward. The reason is that $E^\theta_{i,t}w_{t+h}$ appears in both forecast error and forecast revision with opposite signs. The stochastic idiosyncratic random noise pushes this correlation downward. Even if agents underreact to new information while forming expectations, $-1 < \theta < 0$, as long as the variance of idiosyncratic random noise is large enough, β^I_{OLS} would be negative. If we do not consider the role of judgment noise in β^I_{OLS}, or equivalently $\sigma^2_\eta = 0$, we get the expression in BGMS:

$$\beta^I_{BGMS} = \frac{-(1+\theta)\theta}{(1+\theta)^2 + \rho^2\theta^2} \tag{1.20}$$

$\beta^I_{BGMS} < 0$ when $\theta > 0$, namely overreaction; $\beta^I_{BGMS} = 0$ when $\theta = 0$, namely neither overreaction nor underreaction; $\beta^I_{BGMS} > 0$ when $\theta < 0$, namely underreaction. Without fully acknowledging the role of idiosyncratic random noise and its magnitude, we cannot make inferences on θ by looking at β^I_{BGMS}. Since most empirically estimated β^I_{OLS}s are negative, the implied θs are mostly positive. People would conclude that there is overreaction in expectation on the individual level. Second, opposite signs of $\beta^C_{OLS} > 0$ and $\beta^I_{OLS} < 0$ do not imply different degrees of over-/underreaction on the consensus level and individual level. In proposition 1, we can infer the signs of θ directly from β^C_{OLS} since β^C_{OLS} is not affected by σ^2_η. In summary, in forecast surveys with judgment noise, β^C_{OLS} should be the statistic to look at when we try to understand over-/underreactions in expectations.

In the derivation of β^C_{OLS} in proposition 1, I assume a large enough number of forecasters N in the survey so that the sampling error can be ignored. That is why idiosyncratic random noise doesn't impact β^C_{OLS}. However, in commonly used forecast survey data like Survey of Professional Forecasters and Bluechip Financial Forecast, the number of forecasters is limited, on average around 30. The negative impact of idiosyncratic random noise on β^C_{OLS} is non-trivial. The case with limited N is formally formulated in the following corollary:

Corollary 1 *When the number of forecasters in the sample is $N < +\infty$, the estimated coefficients of consensus level regression and individual level regression with individual fixed effect are given by*

$$\beta^C_{OLS} = \frac{cov(w_{t+h} - E^\theta_t w_{t+h}, E^\theta_t w_{t+h} - E^\theta_{t-1} w_{t+h})}{var(E^\theta_t w_{t+h} - E^\theta_{t-1} w_{t+h})} = \frac{-\theta(1+\theta) - \frac{\sigma^2_\eta}{N\rho^2\sigma^2_e}}{(1+\theta)^2 + \rho^2\theta^2 + (1+\rho^2)\frac{\sigma^2_\eta}{N\rho^2\sigma^2_e}} \tag{1.21}$$

$$\beta^I_{OLS} = \frac{cov(w_{t+h} - E^\theta_{i,t} w_{t+h}, E^\theta_{i,t} w_{t+h} - E^\theta_{i,t-1} w_{t+h})}{var(E^\theta_{i,t} w_{t+h} - E^\theta_{i,t-1} w_{t+h})} = \frac{-(1+\theta)\theta - \frac{\sigma^2_\eta}{\rho^2\sigma^2_e}}{(1+\theta)^2 + \rho^2\theta^2 + (1+\rho^2)\frac{\sigma^2_\eta}{\rho^2\sigma^2_e}} \tag{1.22}$$

When $\frac{\sigma^2_\eta}{N\rho^2\sigma^2_e} < -(1+\theta)\theta < \frac{\sigma^2_\eta}{\rho^2\sigma^2_e}$, we have $\beta^I_{OLS} < 0$ and $\beta^C_{OLS} > 0$.

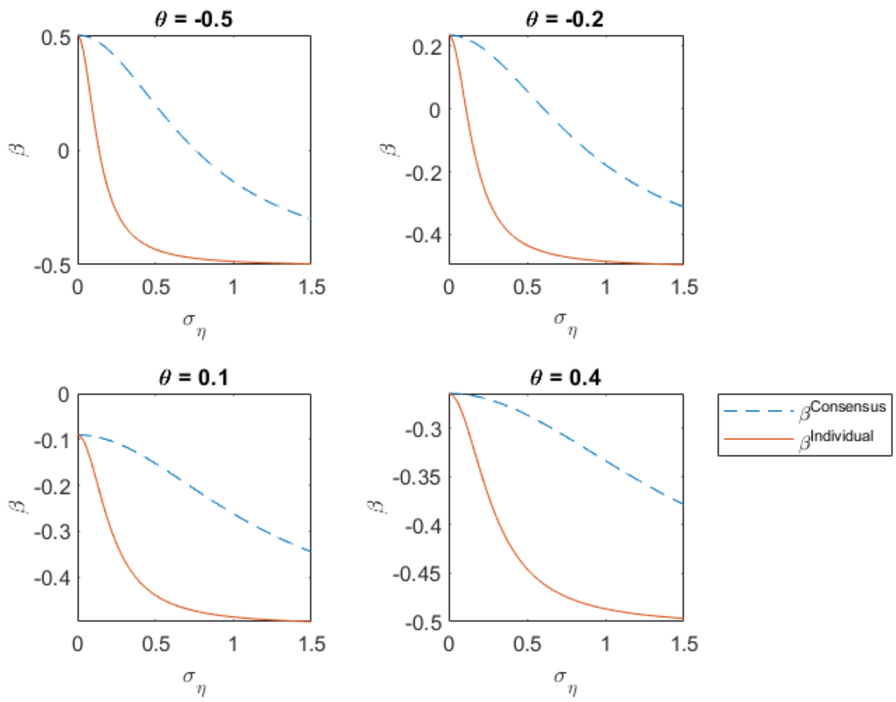
When the number of forecasters is limited, and the impact of the $\eta_{i,t}$ is not completely averaged out, then the existence of $\eta_{i,t}$ would push both β^C_{OLS} and β^I_{OLS} downward, but push β^I_{OLS} downward by more. When there is systematic underreaction, i.e., $\theta < 0$, then as long as σ^2_η and N are large enough, β^I_{OLS} will be negative and β^C_{OLS} will be positive. The intuition for the term $\frac{\sigma^2_\eta}{N\rho^2\sigma^2_e}$ in β^C_{OLS} is clear by looking at the covariance between forecast error and forecast revision at the consensus level:

$$cov(w_{t+1} - E^\theta_t w_{t+1}, E^\theta_t w_{t+1} - E^\theta_{t-1} w_{t+1}) = -\rho^2\theta(1+\theta)\sigma^2_e - \frac{\sigma^2_\eta}{N} \tag{1.23}$$

Since $\frac{\sum_i^N \eta_{i,t}}{N}$ appears in both forecast errors and forecast revisions at the consensus level, its variance will show up in the correlation coefficient. Note that in the following parts of the chapter, I will focus on the expression for β^C_{OLS} in corollary 1. In figure 1.11 of the appendix, I plot how β^C_{OLS} in equation 1.21 changes as we increase N. From the graph, when the number of forecasters is fewer than around ten, an additional forecaster makes a big difference in attenuating the effect of idiosyncratic noise. The impact of one more forecaster is marginal when the number of forecasters exceeds 15.

To have an idea of the magnitude of the impact $\eta_{i,t}$ has on β^C_{OLS} and β^I_{OLS}, I show numerically how much impact idiosyncratic random noise can have on β^C_{OLS} and β^I_{OLS}, for σ^2_η and θ in an empirically reasonable range. As an example, I use the parameters estimated below for the AAA corporate bond yield. The following figure shows that, as the variance of idiosyncratic random noise increases, how β^C_{OLS} and β^I_{OLS} change given different values of θ.

Figure 1.1: The effect of idiosyncratic random noise on β^C_{OLS} and β^I_{OLS}



Notes: This figure plots a numerical example showing how β^I_{OLS} and β^C_{OLS} change while the variance of the idiosyncratic noise σ_η increases. I use the parameters from AAA corporate bond in this exercise. $\rho = 0.99$. $\sigma_e = 0.38$. $N = 27$.

There are three main takeaways we can learn from the numeral exercise. First, the downward sloping lines tell us that both β^C_{OLS} and β^I_{OLS} are driven downward as σ_η increases. β^I_{OLS} drops quickly as σ_η increases. The impact of σ_η on β^C_{OLS} is not trivial. When σ_η increases from 0 to the empirically estimated value 0.4 for AAA corporate bond, β^C_{OLS} decreases from 0.5 to 0.3 when $\theta = -0.5$, and it decreases from 0.2 to 0.1 when $\theta = -0.2$. Second, the gap between β^C_{OLS} and β^I_{OLS} can be sizable. When $\sigma_\eta = 0$, $\beta^C_{OLS} = \beta^I_{OLS}$. As the magnitude of idiosyncratic random noise increases, the gap increases. Let $\sigma_\eta = 0.4$, $\beta^C_{OLS} = 0.3$ and $\beta^I_{OLS} = -0.4$ when $\theta = -0.5$; $\beta^C_{OLS} = 0.1$ and $\beta^I_{OLS} = -0.4$ when $\theta = -0.2$. In both cases, we have the common empirical finding $\beta^C_{OLS} > 0$ and $\beta^I_{OLS} < 0$. In this particular numerical example, idiosyncratic random noise can reconcile the difference between the two β_{OLS}s. As σ_η increases to very large, the gap between two β_{OLS}s tends to close. The reason is that when σ_η is very large, both β^C_{OLS}(when N is finite) and β^I_{OLS} converge to $\frac{-1}{1+\rho^2}$, as we can see from the expressions in corollary 1.

Given the analytical framework, we have learned that idiosyncratic random noise confounds our inference on the degree of over-/underreaction in expectation formation. Below I will identify the magnitude of judgment noise in the data and quantitatively investigate how much it affects β^C_{OLS} and β^I_{OLS}. Can the size of idiosyncratic random noise reconcile the gap between the estimated $\hat{\beta}^C$ and $\hat{\beta}^I$ in the data? How much does idiosyncratic random noise affect our inference on the critical parameter

θ?

1.4 Data

The primary forecast data I use is the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia. SPF is a quarterly survey of professional forecasters on various macroeconomic and financial variables dating back to 1968. It is conducted around the end of the second month of each quarter. Given the timing of filling the survey, when forecasters are making a forecast at quarter t, they know the actual realization of various macroeconomic variables up until quarter *t-1*. Not every variable has a historical record dating back to 1968. In this chapter, not all variables in SPF are included in the empirical exercise. Instead, for the time being, I focus on seven popular variables: real GDP growth rate (RGDP), nominal GDP growth rate (NGDP), GDP price index inflation (GDP Price Index), CPI inflation (CPI), 3-month Tbill yield (Tbill), AAA corporate bond yield (AAA), Tbond corporate bond yield (Tbond). Forecasts are made for the current quarter and the next four quarters. SPF identifies individual forecasters anonymously by assigning them a unique ID.[13] The average number of forecasters in each survey wave ranges from 27 to 36, depending on which variable we look at. The total number of forecasters in the survey is 446 - 448. The mean number of quarters that each panelist participated in the survey is about 23. So SPF is an unbalanced panel dataset.

Because the release of macroeconomic statistics is subject to subsequent revision, I use vintage data for actual realization of key economic variables. This is to match forecasters' observation of economic statistics at the time of making forecasts. The initial release of macroeconomic statistics comes from the real-time dataset of the Federal Reserve Bank of Philadelphia. For financial variables like bond yields, they are never revised. The historical data on bond yields can be obtained from the Federal Reserve Bank of St. Louis.

Forecasts about macroeconomic variables in SPF focus mainly on the level rather than the growth rate, and I transform all levels (except for bond yields) into growth rates. For the majority of empirical exercises in this chapter, I focus on the forecast horizon, which is three quarters ahead. Then the growth rate is the yearly growth rate from quarter $t-1$ to quarter $t+3$. For example, when calculating the actual growth rate for real GDP from quarter $t-1$ to quarter $t+3$, real GDP data in the respective two quarters is obtained from the vintage data released at quarter

[13] However, the forecaster identification is not entirely accurate. In the documentation of SPF, it mentions: "In these surveys, we have noticed some occurrences in which an individual participates, suddenly drops out of the panel for a large number of periods, and suddenly re-enters, suggesting that the same identifier might have been assigned to different forecasters," and "it can be difficult to assign an identification number to an individual who changes his place of employment but remains in the survey."

$t+4$. For calculating the real GDP growth rate forecasts from $t-1$ to $t+3$, I use the forecast made at t and the initial release of real GDP at $t-1$ published at t. I follow the methods in BGMS (2020) in order to replicate their results. I drop all forecasters that appear fewer than ten times in the survey. The summary statistics for individual forecast errors and forecast revisions about $t+3$ are presented in table 1.1.

The standard deviation in table 1.1 is defined as the standard deviation of individual forecast errors/revisions when pooled across quarters and forecasters. The average standard deviation is obtained by first calculating the standard deviation for each quarter and then averaging across quarters. From the summary statistics, we can see a large number of observations for individual forecast errors and revisions for each variable. This is due to the survey's long time series and panel structure. The reason why the number of observations for forecast errors is larger than the number of forecast revisions is the following: to calculate forecast revisions, I need both the forecast for $t+3$ and the lagged $t+4$ forecast. But in SPF, some forecasters provide forecasts for $t+3$ without providing those for $t+4$. This results in more empty cells for $t+4$ relative to $t+3$. Another pattern is that the mean forecast errors and revisions[14] are mostly indistinguishable from zero. This means there is no evidence for either of the following: first, forecasts are systematically biased; second, forecast revisions are asymmetric. The average standard deviation tells us that there is systematic disagreement among forecasters, which supports the existence of judgment noise in the spirit of Kahneman et al. (2021). The periods covered for each variable vary: for RGDP, NGDP, and GDP price index inflation, the survey started as early as 1968, while the survey started much later for Tbond, in 1991.

[14]The mean is calculated across forecasters and time.

Table 1.1: Summary Statistics

	RGDP	NGDP	GDP Price Index	CPI	Tbill	AAA	Tbond
Individual Forecast Error of $t+3$							
Number of Obs.	7505	7523	7455	5258	5054	4305	4039
Mean	-0.37	-0.24	0.12	-0.22	-0.56	-0.48	-0.53
Standard Dev.	2.31	2.48	1.59	2.41	1.18	0.97	0.81
Average Standard Dev.	1	1.19	0.77	0.73	0.48	0.53	0.4
Individual Forecast Revision of $t+3$							
Number of Obs.	5696	5710	5712	4224	4055	3409	3311
Mean	-0.14	-0.11	0.03	-0.07	-0.2	-0.13	-0.14
Standard Dev.	1.37	1.52	0.98	0.77	0.67	0.6	0.51
Average Standard Dev.	0.91	1.09	0.72	0.66	0.44	0.48	0.36
Number of Forecasters	36	36	36	33	32	27	35
Time Periods	1968-2022			1981-2022			1991-2022

Notes: This table reports the summary statistics of individual forecast errors and forecast revisions. Standard Dev. is the standard deviation after pooling all the observations. Average Standard Dev. is obtained by first calculating the standard deviation for each quarter and then averaging across quarters. Number of forecasters is the average number of forecasters across different waves of survey. All forecast errors and revisions are calculated at horizon $t+3$.

1.5 Estimation

This section first reports the "forecast error on forecast revision" regression results across various macroeconomic and financial variables. Second, I provide some evidence for the relative magnitude of idiosyncratic random noise and fixed heterogeneity. Third, after accounting for judgment noise, I formally identify two types of judgment noise and uncover the critical parameter of interest, namely the degree of over-/underreaction θ.

1.5.1 Error-on-revision Regression Results

In the benchmark "forecast error on forecast revision" regression, I use $t+3$ as the forecast horizon. The reason for choosing this specific forecast horizon is that BGMS uses this forecast horizon, so regression results will be comparable to theirs. Table 1.2 reports regression results for both consensus level and individual level regression. For consensus time series regressions, standard errors are corrected following Newey-West (1994) with automatic bandwidth selection. For individual panel regressions, standard errors are clustered by forecaster and time. Most estimations for β^I are significantly negative except for the three-month Tbill yield, which is significantly positive. The point estimations for β^C vary across different variables. For long-maturity bonds, including the ten-year Tbond and the AAA corporate bond yields, $\hat{\beta}^C$s are insignificant. For inflation and the Tbill yield, $\hat{\beta}^C$ is significantly positive. $\hat{\beta}^C$ is positive but insignificant for nominal and real GDP growth. Those regression results are mostly similar to those in BGMS, with some differences. For example, one difference is that in BGMS, $\hat{\beta}^C$ is significant for both real and nominal GDP. Such difference is due to the extra six years of data I have: their data coverage is until 2016, but the empirical exercises include data until 2022. When running regressions on data covering the same periods, I obtain nearly identical results as BGMS. To be consistent with the discussion of the literature, the general case I will be focusing on in this chapter is that $\hat{\beta}^C > 0$ and $\hat{\beta}^I < 0$.

Under the Full Information Rational Expectation(FIRE) hypothesis, both β^C and β^I should be zero. $\beta^C > 0$ and $\beta^I < 0$ are both strong evidence rejecting FIRE. BGMS argues that $\beta^C > 0$ could be a result of a combination of the information friction and a violation of rationality, whereas $\beta^I < 0$ indicates overreaction to information in expectation formation at the individual level. One of the goals of this chapter is to show that $\beta^I_{OLS} < 0$ should not be seen as evidence of overreaction on the individual forecaster level, since the idiosyncratic random noise component could drive it.

As we can see in table 1.1, surveys for different variables in SPF started in different years, which results in different time coverage for different variables. Does the variation in $\hat{\beta}^C$ and $\hat{\beta}^I$ across different variables result from this different time coverage? Indeed, in the 70s and 80s, several historical episodes of economic and financial turmoil might structurally affect how people react to information. Table 1.13 in the appendix reports the results of this robustness test. For those variables whose initial forecast release was in 1968, I rerun the regressions, first dropping 1968 to 1980 and then dropping 1968 to 1990. Similarly, for those variables whose initial forecast release was in 1981, the regressions are rerun while dropping 1981 to 1990. As we can see from the table, quantitatively $\hat{\beta}^C$ and $\hat{\beta}^I$ do differ but not much. Qualitatively, $\hat{\beta}^C$ and $\hat{\beta}^I$ for each variable are consistent when time coverage is different. For example, $\hat{\beta}^C$ is always significantly positive for the GDP deflator, while its $\hat{\beta}^I$ is always significantly negative. The only exception is $\hat{\beta}^C$ of CPI: dropping 1981-1990 switches its sign, although both estimates are insignificant.

1.5.2 Evidence on Two Types of Judgment Noise

Before formally identifying the importance of idiosyncratic random noise and fixed heterogeneity, I provide evidence on the relative magnitude of these two types of judgment noise by running the following two regressions. To know how much of the variation of disagreement in forecast surveys can be explained by individual fixed heterogeneity, and how much of the variation can be explained by idiosyncratic random noise, I run the regression

$$\mathcal{F}_{i,t}w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = f_i + u_{i,t}, \tag{1.24}$$

Table 1.2: Error-on-revision Regression Coefficients

	$\hat{\beta}^C$			$\hat{\beta}^I$		
	Point Estimate	SE	p-value	Point Estimate	SE	p-value
RGDP	0.11	0.31	0.73	-0.28	0.12	0.02
GDP Price Index	1.26	0.41	0.00	-0.15	0.07	0.04
NGDP	0.14	0.25	0.56	-0.32	0.12	0.01
CPI	1.04	0.76	0.17	-0.38	0.09	0.00
Tbill	0.69	0.11	0.00	0.21	0.09	0.03
AAA	-0.02	0.16	0.92	-0.27	0.07	0.00
Tbond	-0.06	0.09	0.46	-0.23	0.02	0.00

Notes: This table reports the Error-on-revision regression results at both the consensus and individual level. For consensus time-series regressions, standard errors are calculated using the Newey-West method, with the automatic bandwidth selection procedure as proposed by Newey and West (1994). For individual-level panel regressions, standard errors are clustered by both the forecaster and time.

where the left hand side is the deviation of forecaster i at time t, $\mathcal{F}_{i,t}w_{t+h}$, from the consensus forecast, $\bar{\mathcal{F}}_t w_{t+h}$, or the disagreement of forecaster i at t from the consensus. The right-hand side is the individual fixed effect which captures the individual fixed heterogeneity ϕ_i.[15] Again, I choose $h = 3$ in this exercise. R^2 measures how much variation of individual deviation from the consensus forecast is due to individual fixed heterogeneity. Another regression to give us a sense of the existence of $\eta_{i,t}$ and ϕ_i is the following:

$$\mathcal{F}_{i,t}w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \hat{\rho}(\mathcal{F}_{i,t-1}w_{t+h-1} - \bar{\mathcal{F}}_{t-1}w_{t+h-1}) + u_{i,t} \tag{1.26}$$

This regression gives us the persistence $\hat{\rho}$ for individual deviation from the consensus forecast. When the individual deviation from consensus is solely due to individual fixed heterogeneity, ϕ_i is time-invariant and $\hat{\rho} = 1$. When the individual deviation from consensus is solely due to idiosyncratic random noise, $\hat{\rho} = 0$ if idiosyncratic random noise is independent across time.[16] And the results are in the table 1.3.

We can see from the table that the individual fixed effect explains a fair amount of the variation in forecast disagreement, between 16% and 24%. In other words, there is significant persistence in forecast disagreement. However, the larger portion of disagreement is due to idiosyncratic random noise, accounting for 76% to 84% of all the variation in individual deviation from consensus. These two empirical exercises tell us that in SPF across different macroeconomic and financial variables,

[15]Strictly speaking, f_i captures the demeaned individual fixed heterogeneity $\phi_i - \frac{\sum_i \phi_i}{N}$. This can be seen from plugging equation 1.12 into $\mathcal{F}_{i,t}w_{t+h}$ on the left-hand-side of equation 1.24:

$$\mathcal{F}_{i,t}w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \rho^{h-1}(\eta_{i,t} - \frac{\sum_i \eta_{i,t}}{N} + \phi_i - \frac{\sum_i \phi_i}{N}) \tag{1.25}$$

[16]In further discussion, I show that actually idiosyncratic random noise is serially correlated.

Table 1.3: R^2 of Individual Fixed Effect and Persistence of Individual Noise

	RGDP	GDP Price Index	NGDP	CPI	Tbill	Tbond	AAA
R^2 of Individual Fixed Effect	0.19	0.16	0.19	0.24	0.18	0.22	0.24
Persistence of Deviation	0.48***	0.47***	0.56***	0.33***	0.59***	0.52***	0.6***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)

Notes: This table reports the R^2 of individual fixed effect in regression 1.24, and estimates of persistence in regression 1.26.

individual fixed effects account for a significant fraction of the disagreement across forecasters. However, the more substantial source of disagreement is idiosyncratic random noise.

1.5.3 Estimation Strategy

There are four parameters to be estimated for each variable in the survey: the persistence of actual realizations of the variable ρ, the standard deviation of the innovations σ_e, the standard deviation of idiosyncratic random noise σ_η, and the degree of over- or underreaction θ. The steps to estimate this set of parameters $\{\rho, \sigma_e, \sigma_\eta, \theta\}$ for each variable are the following:

1. The parameters ρ and σ_e^2 can be obtained by fitting an AR(1) process to the historical time series for each variable.

2. The idiosyncratic noise can be identified as the residual of the regression 1.11. The estimation of σ_η can be obtained by calculating the standard deviation of the residual $\hat{\eta}_{i,t}$, using the following equation:

$$\hat{\sigma}_\eta^2 = \frac{1}{n-k} \sum_{i=1}^{n} \hat{\eta}_{i,t}^2 \tag{1.27}$$

   where k is the total number of free coefficients in regression equation 1.11.

3. After obtaining the estimation for $\{\rho, \sigma_e^2, \sigma_\eta^2\}$, I can estimate θ by matching the model implied β_{OLS}^C and β_{OLS}^I as in equation 1.18 and 1.19 with the estimated $\hat{\beta}^C$ and $\hat{\beta}^I$ from the data. I obtain estimation for θ using two methods:

   (a) The first method, θs are estimated by minimizing the Euclidean distance between the model implied β_{OLS}s and estimated $\hat{\beta}$s from data:

$$(\hat{\beta}^C - \beta_{OLS}^C \quad \hat{\beta}^I - \beta_{OLS}^I) W (\hat{\beta}^C - \beta_{OLS}^C \quad \hat{\beta}^I - \beta_{OLS}^I)^T$$

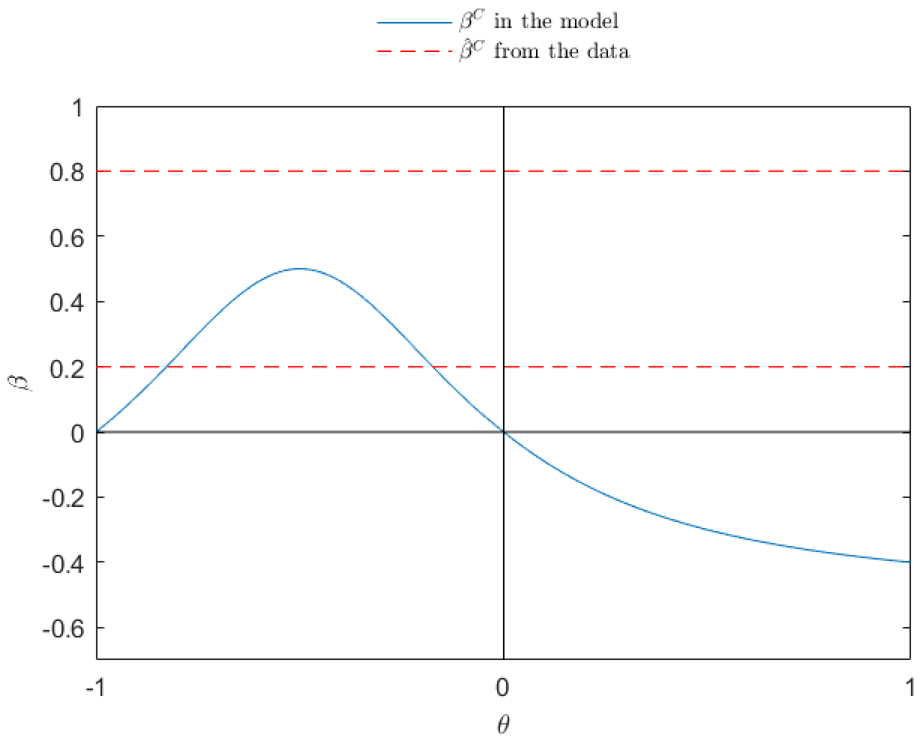
   I evaluate the empirical covariance of $\hat{\beta}^C$ and $\hat{\beta}^I$ by bootstrapping from the panel of forecasters with replacement, and invert it to obtain the optimal weighting matrix W.

(b) The second method, θs are estimated by matching $\hat{\beta}^C$ with β^C_{OLS}, $\hat{\beta}^I$ with β^I_{OLS} separately. This method produces two θs, which should be close to each other.

In the second method, I match βs at the individual and consensus level separately, and obtain two θs. The two θs should be close to each other. Comparing results from the first and second method serves as a robustness check. The grid for θ is restricted on the model implied range: $\theta > -1$. Standard errors for the estimation of θs can be obtained by bootstrapping, during estimation, from the panel of forecasters with replacement

While attempting to match the implied regression coefficients, β_{OLS}, of the model with the estimated regression coefficients, $\hat{\beta}$, derived from the data, two potential issues arise. Firstly, when $\hat{\beta}$ is excessively large, it may not be possible to find a θ value that aligns $\hat{\beta}$ with β_{OLS}. Secondly, even when $\hat{\beta}$ and β_{OLS} can be perfectly aligned, there might be two possible solutions for θ. These issues arise due to the hump-shaped nature of β_{OLS} as a function of θ. Figure 1.2 illustrates the plot of β^C_{OLS} defined in equation 1.18 as a function of θ. In this particular example, ρ is set to 1. Historical time series for variables such as GDP Price Index inflation, 3-month Tbill yield, AAA corporate bond yield, and 10-year Tbond yield exhibit behaviors close to a random walk, with ρ values near 1.

Figure 1.2: β^C_{OLS} as a function of θ



Notes: This figure plots β^C_{OLS} in equation 1.18 as a function of θ in blue line. The red dots are two potential values of empirically estimated $\hat{\beta}^C$.

From the observations in Figure 1.2, it is evident that when $\theta > 0$, β^C_{OLS} decreases with increasing θ, and it remains negative. When $-1 < \theta < 0$, β^C_{OLS} is consistently positive but exhibits a hump-shaped pattern. As θ decreases from 0 to -1, β^C_{OLS} initially increases and then decreases. In other words, a positive β^C_{OLS} indicates an under-reaction to new information, whereas a negative β^C_{OLS} suggests an over-reaction. However, when $\beta^C_{OLS} > 0$, a higher value does not imply a more significant degree of under-reaction to new information. The intuition behind the hump-shaped pattern of β^C_{OLS} within the interval $[-1, 0]$ is as follows: when θ is zero, expectation formation adheres to rational expectations, meaning there is neither over- nor under-reaction to new information, resulting in $\beta^C_{OLS} = 0$. When θ is set to -1, forecasters do not react to new information at all, making forecast revisions uncorrelated with forecast errors. As θ decreases from 0 to -1, the value of β^C_{OLS} is primarily driven by the under-reaction to new information, until a certain threshold is reached, after which the value is determined by the little reaction to new information.

When the empirical value of $\hat{\beta}^C$ exceeds the maximum value of the function β^C_{OLS}, such as in the case of 0.8 in the graph, I identify the corresponding θ as the one yielding the highest β^C_{OLS}. However, if $\hat{\beta}^C$ is positive but not excessively large, as shown by the value of 0.2 in the graph, there are two potential θ values that can match β^C_{OLS} with $\hat{\beta}^C$. In such instances, I report the θ that is closer to 0. By selecting the θ value closer to zero, I adopt a more conservative value regarding the degree of under-reaction observed in the data.

In Chapter 2 of my PhD thesis, I demonstrate that when expectation formation follows the more conventional adaptive expectations framework, a monotonic relationship exists between β^C and the weighting parameter in the adaptive expectations model. And there are no issues in aligning the data-estimated and model-implied β coefficients.

1.5.4 Estimation Results

In the following tables, I report the estimated parameters for each variable: the persistence of the variable ρ, the standard deviation of the innovation σ_e, the standard deviation of the idiosyncratic random noise σ_η, and the degree of under/over-reaction θ.

For the estimation of ρ and σ_e, the fitted results are similar to those in BGMS. The difference is that I have additional 2017-2022 data included in the analysis. The economic disturbances of Covid-19 pandemic reduce the persistence of variables, including real and nominal GDP growth. CPI inflation is much less persistent and more volatile than GDP price index inflation.[17]

[17] CPI inflation is the annualized quarterly growth rate of CPI, while GDP price index inflation is the yearly growth rate of GDP price index. Hence, the persistence and innovation volatility of

Table 1.4: Estimation Results for Model Parameters

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
ρ	0.77	0.98	0.84	0.46	0.99	0.99	0.99
σ_e	1.57	0.5	1.71	2.16	0.57	0.38	0.37
σ_η	1.07	0.85	1.21	0.71	0.46	0.48	0.35

Note: This table reports the estimation results for parameters of the model across different macroeconomic variables: the persistence of the variable ρ, the standard deviation of the innovation σ_e, the standard deviation of the idiosyncratic random noise σ_η.

Table 1.5: Estimation Results for the Degrees of Over/Under-reaction, Method 1

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
θ	-0.06	-0.51^{***}	-0.17^{**}	-0.74^{**}	-0.31^{***}	0.08	0.08^{**}
	(0.07)	(0.02)	(0.07)	(0.34)	(0.04)	(0.08)	(0.04)

Notes: This table reports θ estimation results from method 1. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

I plot the distributions of $\hat{\eta}_{i,t}$ and $\hat{\phi}_i$ for different variables to explore their properties further. In figure 1.3 and 1.4, a fitted normal distribution curve is also included to let us have a sense of the distribution of $\hat{\eta}_{i,t}$ and $\hat{\phi}_i$. The distribution of $\hat{\eta}_{i,t}$ can be fitted reasonably well by a normal distribution centered around zero, except for that $\hat{\eta}_{i,t}$ tends to center around the mean zero much more than a normal distribution. Or $\hat{\eta}_{i,t}$ tends to have a thinner tail. The normal distribution fits not as well for $\hat{\phi}_i$. Considering there are much fewer observations for $\hat{\phi}_i$ compared with $\hat{\eta}_{i,t}$, the poor fit of the normal distribution could also be a result of the limited sample of $\hat{\phi}_i$. The goal of estimating σ_η, $\hat{\eta}_{i,t}$ and $\hat{\phi}_i$ is twofold: first, to have a sense of the distribution and magnitude of idiosyncratic random noise and fixed heterogeneity. Second, the estimated σ_η will be used to see whether it is large enough to reconcile the difference between the gap between $\hat{\beta}^C$ and $\hat{\beta}^I$ as we see in the data.

The estimation yields different outcomes for the estimation of θ. If we look at the estimation results from method 1 in table 1.5, for CPI inflation, GDP price index inflation, and the three-month Tbill yield, θs are significantly negative, around -0.5. There is strong evidence that people are underreacting to new information when

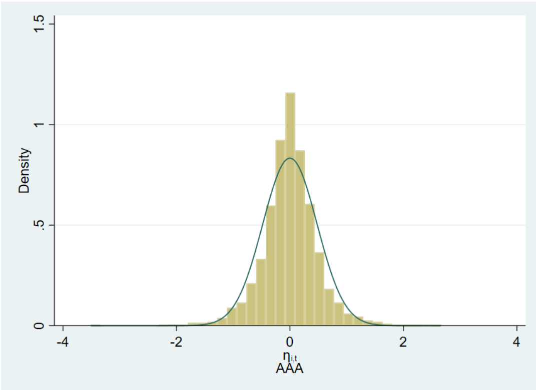
Table 1.6: Estimation Results for the Degrees of Over/Under-reaction, Method 2

		RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
θ	Match β^I	-0.38^{***}	-0.45^{***}	-0.003	0.27	-0.5^{***}	-0.48^{***}	-0.5^{***}
		(0.1)	(0.00)	(0.22)	(0.32)	(0.00)	(0.00)	(0.02)
	Match β^C	-0.13	-0.5^{***}	-0.15^{**}	-0.68^{**}	-0.5^{***}	-0.06	0.05
		(0.09)	(0.00)	(0.07)	(0.22)	(0.00)	(0.08)	(0.04)

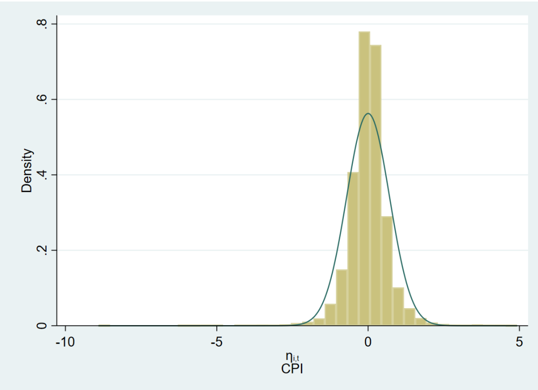
Notes: This table reports θ estimation results from method 2. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

the two variables differ. This arises from the fact that, in SPF, the forecast about CPI inflation is the annualized quarterly growth rate, while the forecast about GDP Price Index is the level. When calculating the inflation from the GDP price index, I calculate the yearly growth rate.

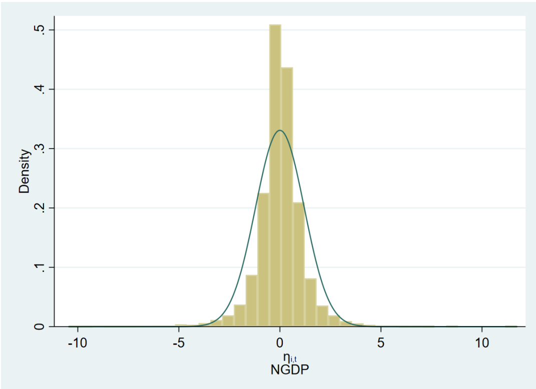
Figure 1.3: Distribution of $\eta_{i,t}$ for Different Variables



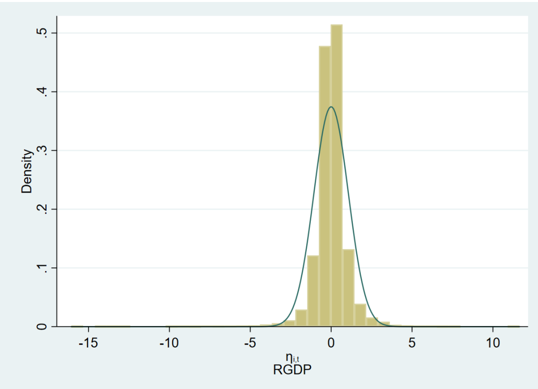
(a) AAA



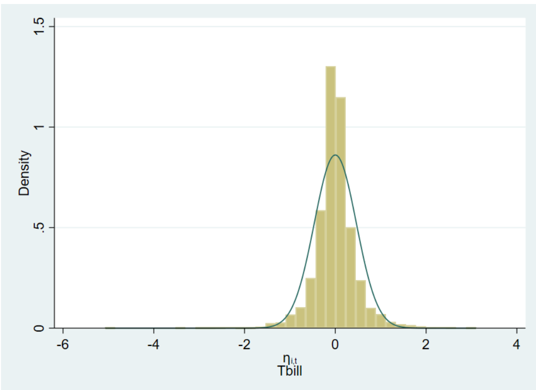
(b) CPI



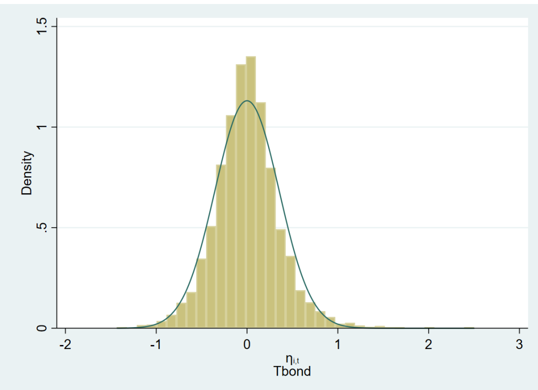
(c) NGDP



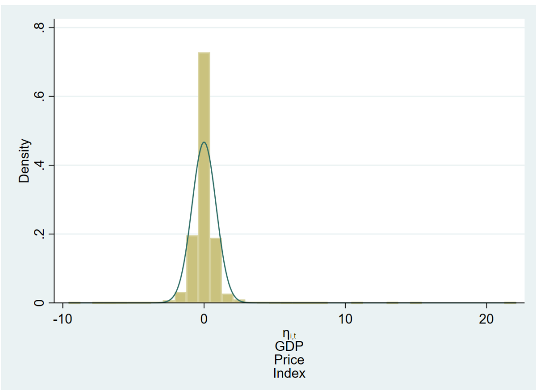
(d) RGDP



(e) Tbill



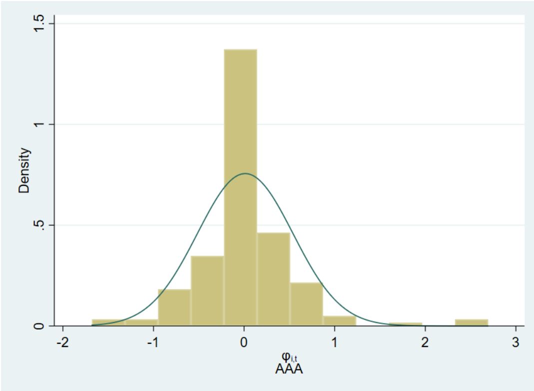
(f) Tbond



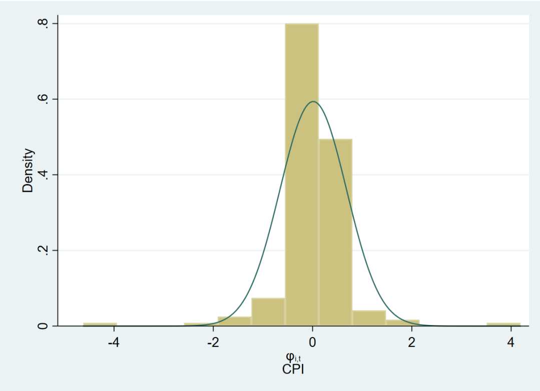
(g) GDP Price Index

Notes: This figure plots for each variable the distribution of empirically estimated idiosyncratic noise $\hat{\eta}_{i,t}$.

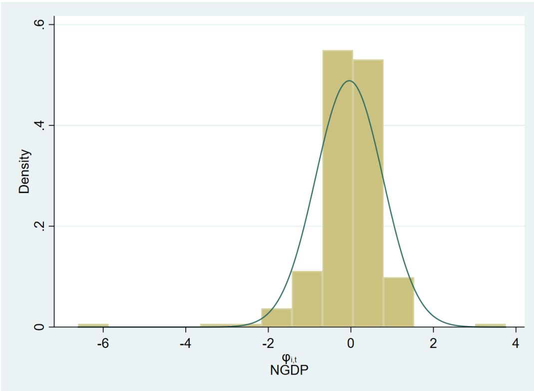
Figure 1.4: Distribution of ϕ_i for Different Variables



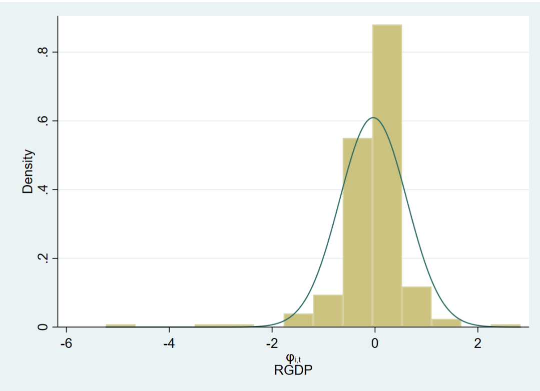
(a) AAA



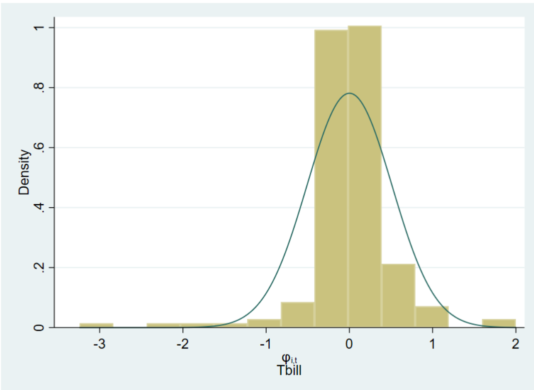
(b) CPI



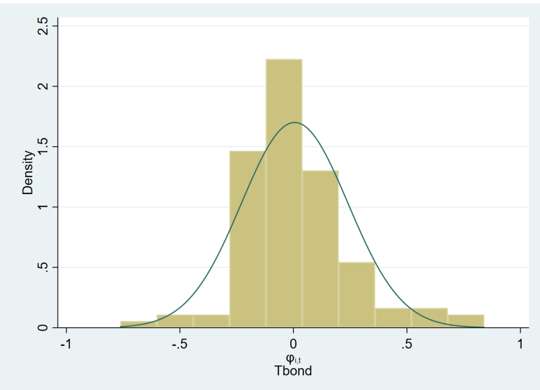
(c) NGDP



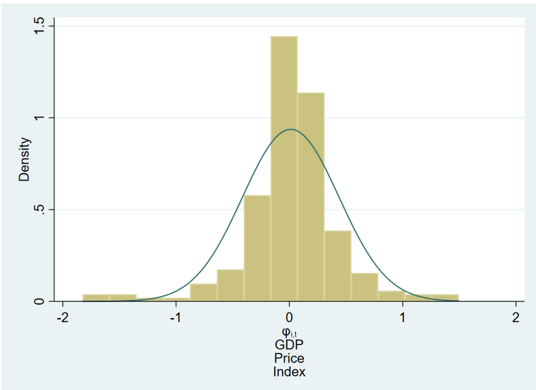
(d) RGDP



(e) Tbill



(f) Tbond



(g) GDP Price Index

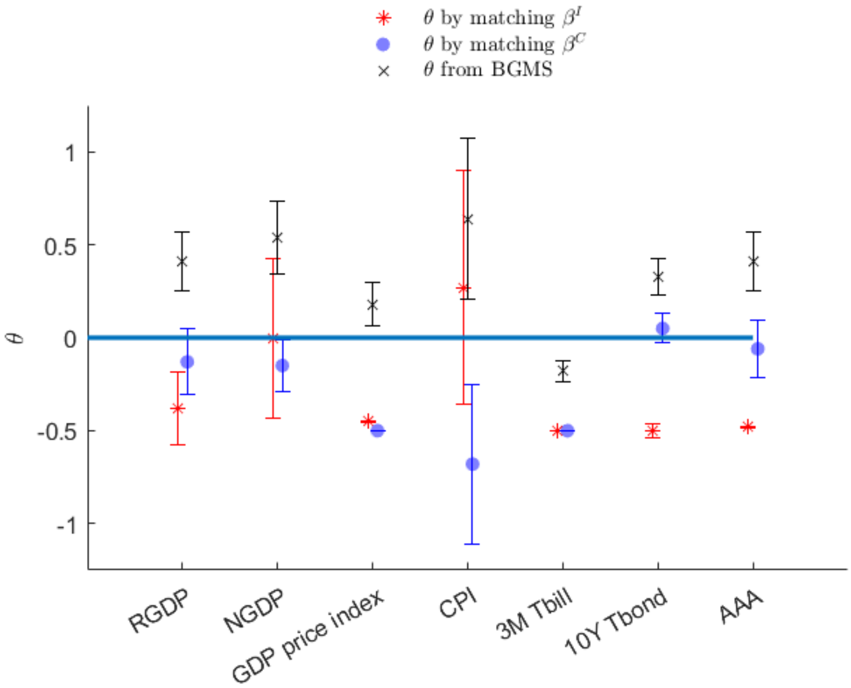
Notes: This figure plots for each variable the distribution of empirically estimated individual fixed heterogeneity $\hat{\phi}_i$.

forming expectations about inflation. For real GDP, nominal GDP, θs are closer to zero but still negative. The evidence for underreaction is not as strong as for the previous three variables. For the ten-year Tbond and AAA corporate bond, θs are around zero. So there is no significant evidence of overreaction nor underreaction for those two variables. The estimation results of θs are not surprising given our previous analysis: $\beta^C_{OLS} > 0$ is the better statistics to make an inference on θ, while $\beta^I_{OLS} < 0$ is the pure result of the large magnitude of idiosyncratic random noise. Table 1.6 reports the estimation of θs using method 2. The key takeaway from table 1.6 is that, no matter whether we estimate θs from $\hat{\beta}^C$ or $\hat{\beta}^I$, we almost always obtain negative θs. In other words, once we consider the idiosyncratic random noise in the data, negative $\hat{\beta}^I$s and positive $\hat{\beta}^C$s all imply underreaction to new information in expectation formation. In Table 1.6, there are instances where the estimation results for θ display zero standard errors. As explained in the earlier section on the "Estimation Strategy," this occurs when the empirical value of $\hat{\beta}$ surpasses the maximum value of the functional β^i_{OLS}, where $i \in \{C, I\}$. The standard errors are calculated through bootstrapping, which involves resampling from the panel of forecasters with replacement. However, in nearly all bootstrapped samples, the estimated $\hat{\beta}$ values exceed the maximum functional value of β^i_{OLS}. Consequently, the standard errors in those cases approach zero.

I compare these results with the estimated values for θ if we do not consider the existence of judgment noise. In BGMS, without formally accounting for the role of idiosyncratic noise, β^I_{OLS} is given by equation 1.20. After obtaining the estimation for ρ by fitting an AR(1) process on the actual historical time series of the forecasted variable, the estimation of θ can be obtained by matching equation 1.20 with the estimated $\hat{\beta}^I$ in the data. The estimated θs using this method are plotted as "x" in black in figure 1.5, with 95% confidence intervals. The results are reported in table 1.14 in the appendix. Consistent with the estimation results in BGMS, except for the 3-month Tbill yields, θs for the other variables are positive, interpreted as over-reaction to new information at the individual level. In comparison, I plot the estimation results from table 1.6 in the same figure. θs obtained from matching $\hat{\beta}^I$ with β^I_{OLS} are plotted as the red "*". θs estimated by matching $\hat{\beta}^C$ with β^C_{OLS} are plotted as the blue dots. 95% confidence intervals are included. After taking into account the idiosyncratic noise in the data, θs implied by $\hat{\beta}^C > 0$ and $\hat{\beta}^I < 0$ are very similar in terms of signs: θs across different variables are mostly negative, meaning underreaction to new information.

In table 1.15 and figure 1.9 presented in the appendix, I provide the estimation results for θ when matching β^C between the model and the data. Specifically, I compare the estimation outcomes when considering a finite number of forecasters to the scenario where the number of forecasters is assumed to be infinite. This analysis aims to address the concern highlighted in Corollary 1. The results demonstrate that

Figure 1.5: θ Estimation for Different Variables



Notes: This figure plots the θ estimation from method 2 for each variable. θs estimated by matching β^I from the model and data are plotted as red stars. θs estimated by matching β^C from the model and data are plotted as blue dots. θs estimated by using the methodology from BGMS 2020 are plotted with black crosses. 95% confidence intervals are included.

the difference stemming from the consideration of a finite number of forecasters is not statistically significant.

In table 1.7, I report the model implied β^C_{OLS} and β^I_{OLS}, and the difference between β^C and β^I in both model and data to see whether the variance of idiosyncratic random noise σ_η can reconcile such difference. Here the estimation results are from the first method. In step 3 of the estimation strategy, θ is obtained by matching the model implied and empirically estimated β^C and β^I. However, whether the model can match estimations of β^C and β^I depends on the value of σ_η, estimated beforehand in step 2. If $\sigma_\eta = 0$, $\beta^C_{OLS} = \beta^I_{OLS}$ in the model. Only when the estimated σ_η is large enough in step 2 is it possible for the model to match the gap between $\hat{\beta}^C$ and $\hat{\beta}^I$ in the data. We can see from the second row to the fourth row in table 1.7that the model can match the $\hat{\beta}^I$ and $\hat{\beta}^C$ fairly well. From the last two rows, we can see that the size of idiosyncratic random noise can satisfactorily reconcile the gap between $\hat{\beta}^C$ and $\hat{\beta}^I$. The two variables that the model does not do a good job at matching are GDP price index and the three-month Tbill. There are at least two possible reasons why the model does not always perfectly fit the estimated $\hat{\beta}^C$ and $\hat{\beta}^I$ in the data. First, many of the point estimates for $\hat{\beta}^C$ are insignificant. The model can easily fit the 95% confidence interval of $\hat{\beta}^C$ for those variables. Second, the estimated $\hat{\beta}^C$ and $\hat{\beta}^I$ from the data do not lie in the range of which expression 1.19 and 1.21 can attain for those variables, given the estimated values for $\{\sigma^2_\eta, \rho, \sigma^2_e\}$ and N. In figure 1.12 in the appendix, the relationship between β^C_{OLS}, β^I_{OLS} and θ is plotted. The parameters used in the exercise are those of GDP price index. In the figure, as θ varies, β^C_{OLS} and β^I_{OLS} are varying between $[-0.5, 0.25]$ and $[-0.5, -0.4]$ respectively. However, the estimated $\hat{\beta}^C$ and $\hat{\beta}^I$ for GDP price index inflation are 1.26 and -0.15, which are both out of the range that the model can fit.

Table 1.7: Matched β^C and β^I

	RGDP	GDP Price Index	NGDP	CPI	Tbill	Tbond	AAA
β^I_{OLS}	-0.33	-0.43	-0.3	-0.36	-0.23	-0.33	-0.38
$\hat{\beta}^I$	-0.28	-0.15	-0.32	-0.38	0.21	-0.23	-0.27
β^C_{OLS}	0.12	0.25	0.14	0.92	0.43	-0.05	-0.01
$\hat{\beta}^C$	0.11	1.26	0.14	1.04	0.69	-0.06	-0.02
$\beta^C_{OLS} - \beta^I_{OLS}$	0.45	0.68	0.44	1.28	0.66	0.28	0.37
$\hat{\beta}^C - \hat{\beta}^I$	0.39	1.41	0.46	1.42	0.48	0.17	0.25

Notes: This table reports the data estimated $\hat{\beta}^I$ and $\hat{\beta}^C$, the model implied β^I_{OLS} and β^C_{OLS}. The gap between the data and model is reported in the last two rows.

1.6 Further Discussion

1.6.1 Dispersed Information or Judgment Noise?

A large literature about over-/underreaction to new information is centered around dispersed information models, where agents receive noisy *signals* about the underlying economic variable. In such models, agents underreact to new information on the consensus level since they do not know whether the new information reflects noise or real innovations to the forecasted variables. See Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma and Shleifer (2020), etc. The judgment noise approach proposed in this chapter and dispersed information are two distinct ways to generate disagreement in forecasts. Forecasters in dispersed information models are fed exogenous signals, so they make different forecasts, even though in their minds they are following the same rule. In my model, people have the same information but still make different forecasts due to "noise in judgment", as Kahneman and his coauthors emphasized. Combined with over/under-reaction in expectation formation, both approaches can reconcile the positive correlation coefficients at the consensus level and the negative ones at the individual level. In my model, forecasters underreact to new information. Such new information is common. Positive correlation coefficients at the consensus level reflect such systematic underreaction in expectation formation. The idiosyncratic random noise in expectations pushes the correlation coefficients at the individual level downward. When such noise is large enough, correlation coefficients at the individual level are negative. In AHS and BGMS, forecasters overreact to their private information (their signals), resulting in negative correlation coefficients at the individual level. However, since information is dispersed, forecasters do not respond to each other's information. This leads to underreaction to new information at the consensus level, meaning positive correlation coefficients at the consensus level. Although both approaches can reconcile the crucial correlation coefficients that we are interested in, they yield completely different implications on forecasters' degree of over/under-reaction to new information, or the key structural parameter θ.

In reality, both noisy information and judgment noise are likely to exist. Conceptually, however, several arguments favor judgment noise as the source of disagreement: first, the commonly used data, the Survey of Professional Forecasters and Bluechip Financial Survey in this strand of literature, are both surveys conducted among professional financial forecasters, whose job it is to watch economic statistics and financial markets closely every day and get paid well for it. Information friction concerns, e.g., costs in acquiring the latest information, might not be a serious problem for those professional forecasters. This is compared to surveys filled by other population groups, including households. Secondly, such surveys examine aggregate macroeconomic variables like inflation, GDP growth rate, or index rates like the AAA corporate rate. Financial forecasters are less likely to have private information about those aggregate variables. The argument supporting disagreement resulting from private information is more valid if the survey asks professional forecasters about their forecasts on individual stocks or corporate bond yields. Third, the judgment noise, in Kahneman's words, is an essential feature of all kinds of human judgment processes. This is even for those judgment processes that people expect to be less noisy, e.g., doctors evaluating patients' medical conditions. When presented with the same information, disagreement in judgment can result from completely random factors, mood, or more complex reason: "the persistent personal reactions of particular individuals to a multitude of features, which determine their reactions to specific cases". Kahneman and his coauthors provide an astonishing example of humans' judgment noise:"when the same software developers were asked on two separate days to estimate the completion time for the same task, the hours they projected differed by 71%, on average." Suppose due to the way that our brain works when making judgment, people can disagree with themselves on the same thing by such a large margin on different days. In that case, it is natural to model forecasters' disagreement with each other as judgment noise.

1.6.2 Serially Correlated Idiosyncratic Random Noise

In the baseline framework, I assume for simplicity that the idiosyncratic random noise is i.i.d. across forecasters and time. In the data, such idiosyncratic random noise might also be serially correlated. Factors behind idiosyncratic random noise might be serially correlated over time. For example, when Jack, the forecaster, gets a promotion, he will likely be happier than average for the following months ahead. So even "mood" itself won't be entirely random over time. Now I want to obtain the serial correlation of idiosyncratic random noise:

$$\hat{\eta}_{i,t} = \rho_\eta \hat{\eta}_{i,t-1} + u_{i,t} \tag{1.28}$$

where $\hat{\eta}_{i,t}$ is the idiosyncratic random noise estimated from SPF in section 1.5. ρ_η is the persistence of $\hat{\eta}_{i,t}$ over time. In order to do this, I run the following dynamic panel regression:[18]

$$\mathcal{F}_{i,t}w_{t+h} = \rho_\eta \mathcal{F}_{i,t-1}w_{t-1+h} + f_t + f_i + u_{i,t} \tag{1.30}$$

The regression results are reported in table 1.8. The model is estimated using the Arellano-Bond (1991) estimator with 15 lags. As we can see, there is significant

[18]The regression equation can be obtained by plugging

$$\mathcal{F}_{i,t}w_{t+h} = \alpha_t + \phi_i + \eta_{i,t} \tag{1.29}$$

into equation 1.28

evidence for the persistence of idiosyncratic random noise across all variables. It turns out $\eta_{i,t}$ is not random over time. Does it affect the result in proposition 1 that idiosyncratic random noise is the reason behind the gap between β^C_{OLS} and β^I_{OLS}? In corollary 2, proposition 1 is extended to consider the persistence in idiosyncratic random noise.

Table 1.8: Persistence of Idiosyncratic Random Noise

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
ρ_η	0.15***	0.11**	0.14***	0.04	0.14***	0.15***	0.07*
	(0.05)	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)

*Notes: This table reports the estimates of persistence in regression 1.30. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*

Corollary 2 *Assume the number of forecasters in the sample is large. When $\eta_{i,t}$ is serially correlated over time with persistence ρ_η, the estimated coefficients of consensus level regression and individual level regression with individual fixed effect are given by*

$$\beta^I_{OLS} = \frac{cov(FE_{i,t}, FR_{i,t})}{var(FR_{i,t})} = \frac{-\theta(1+\theta) - \frac{\sigma^2_\eta}{\rho^2\sigma^2_e} + \frac{\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}}{\rho^2\theta^2 + (1+\theta)^2 + (\rho^2+1)\frac{\sigma^2_\eta}{\rho^2\sigma^2_e} - \frac{2\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}} \tag{1.31}$$

$$\beta^C_{OLS} = \frac{cov(w_{t+h} - E^\theta_t w_{t+h}, E^\theta_t w_{t+h} - E^\theta_{t-1} w_{t+h})}{var(E^\theta_t w_{t+h} - E^\theta_{t-1} w_{t+h})} = \frac{-\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2}$$

When $-1 < \theta < 0$ and $-\theta(1+\theta) < \frac{\sigma^2_\eta}{\rho^2\sigma^2_e} - \frac{\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}$, $\beta^I_{OLS} < 0$ and $\beta^C_{OLS} > 0$.

From equation 1.31, the persistence in $\eta_{i,t}$ has two effects. First, the downward force of idiosyncratic random noise in the numerator is attenuated by $\frac{\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}$. Second, the denominator is lowered by the amount $\frac{2\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}$. The intuition for those two effects is the following: if idiosyncratic random noise is serially correlated, the two adjacent idiosyncratic random noises partly offset each other in forecast revision (look at the term $\eta_{i,t} - \rho\eta_{i,t-1}$ in equation 1.14). This serial correlation reduces the impact of σ^2_η on the covariance between forecast error and forecast revision on the individual level. It also reduces the impact of σ^2_η on the variance of forecast revision on the individual level. The first force narrows the gap between β^I_{OLS} and β^C_{OLS}, while the second force is likely to increase the gap between those two coefficients.

Regardless of whether the idiosyncratic noise is serially correlated or not, when we have the value of ρ, β^C_{OLS} serves as the sufficient statistic for identifying θ. Importantly, the serial correlation of the idiosyncratic noise does not affect β^C_{OLS}, as stated in Corollary 2. Consequently, the inference on θ remains unaffected by the serial correlation if we match β^C between the data and the model.

Furthermore, in Table 1.9, I provide the estimation results for θ considering the presence of serial correlation in the idiosyncratic noise using Method 1. As observed, the estimation results are very similar to those presented in Table 1.5. A plot for comparison is provided in figure 1.10 in the appendix. This further supports the notion that the inclusion of serial correlation does not significantly impact the estimation outcomes for θ.

Table 1.9: θ Estimation Results from Method 1, Considering the Serial Correlation of Idiosyncratic Noise

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
θ	-0.06^{*}	-0.5^{***}	-0.15^{**}	-0.71^{***}	-0.29^{***}	0.08	0.09^{**}
	(0.03)	(0.08)	(0.07)	(0.14)	(0.04)	(0.08)	(0.04)

*Notes: This table reports the θ estimates from method 1 after considering the serial correlation of idiosyncratic noise for each variable. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*

1.6.3 Different Time Coverage for Variables

One valid concern regarding the heterogeneous θ values for different variables is the varying time coverage of the variables included in the Survey of Professional Forecasters. As presented in Table 1.1, real GDP growth and nominal GDP growth forecasts have data available from as early as 1968, whereas 10-year Tbond yield forecasts commence from a later period, specifically 1991. Economic conditions in the 1970s, 1980s, and 1990s were notably distinct, characterized by stagflation in the 1970s, a severe recession in the early 1980s, and the longest period of peacetime economic expansion in American history during the 1990s. It is plausible that forecasters' reactions to information were influenced by these varying economic conditions. Hence, it raises the question of whether the heterogeneous $\hat{\beta}$ and θ values for different variables result from the differences in time coverage.

To address this concern, I conduct the Error-on-revision regression separately for each variable using different time periods. For example, for real GDP growth, I performed the regression for the periods 1968-2022, 1982-2022, and 1991-2022. All estimations for θ are obtained by matching β^C between the data and the model. The regression results are reported in table 1.10, and the estimated θ values are visualized in figure 1.6. For a comprehensive overview of the parameter estimations, please refer to table 1.16 in the appendix.

Based on the findings in table 1.10, it is evident that the heterogeneity of $\hat{\beta}$ values across variables is not solely attributed to the different time periods covered by each variable. The significantly positive $\hat{\beta}^I$ for 3-month Tbill yields remains consistent regardless of whether the 1980s are included in the analysis. Similarly, the significantly positive $\hat{\beta}^C$ for GDP Price Index holds across all three sample periods. However, the sign of $\hat{\beta}^C$ for CPI changes after excluding the 1980s, although neither estimate is statistically significant.

Importantly, the implication derived from table 1.10 is that the θ values depicted in figure 1.6 are relatively consistent for each variable across different time periods. This suggests that the heterogeneity observed in $\hat{\beta}$ values is not solely driven by the varying time coverage of the variables.

Table 1.10: Empirical $\hat{\beta}$ from Different Time Periods

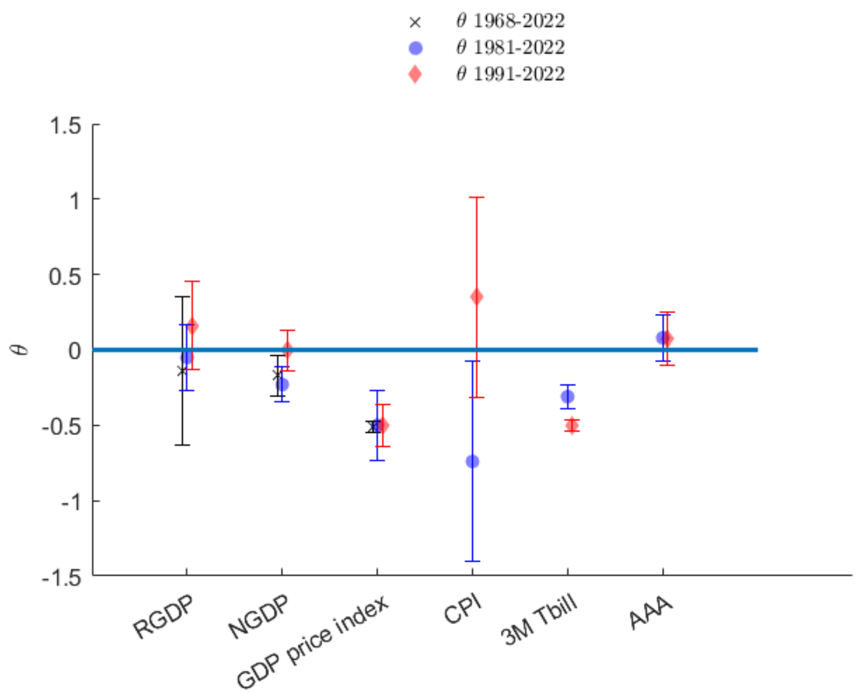
	β^{Census}			$\beta^{Individual}$			Periods
	Point Estimate	SE	p-value	Point Estimate	SE	p-value	
RGDP	0.11	0.31	0.73	-0.28	0.12	0.02	1968-2022
	0.03	0.36	0.93	-0.3	0.23	0.2	1981-2022
	-0.16	0.3	0.59	-0.37	0.26	0.16	1991-2022
GDP Price Index	1.26	0.41	0.00	-0.15	0.07	0.04	1968-2022
	0.54	0.23	0.02	-0.36	0.06	0.00	1981-2022
	0.94	0.41	0.02	-0.3	0.08	0.00	1991-2022
NGDP	0.14	0.25	0.56	-0.32	0.12	0.01	1968-2022
	0.24	0.37	0.51	-0.25	0.19	0.19	1981-2022
	-0.02	0.26	0.94	-0.29	0.25	0.24	1991-2022
CPI	1.04	0.76	0.17	-0.38	0.09	0.00	1981-2022
	-0.29	1.12	0.79	-0.5	0.11	0.00	1991-2022
Tbill	0.69	0.11	0.00	0.21	0.09	0.03	1981-2022
	0.8	0.11	0.00	0.33	0.1	0.00	1991-2022
AAA	-0.02	0.16	0.92	-0.27	0.07	0.00	1981-2022
	-0.01	0.14	0.92	-0.31	0.07	0.00	1991-2022

Notes: This table reports the empirical $\hat{\beta}^C$ and $\hat{\beta}^I$ for different time periods. The standard errors and p-value are reported for each variable.

1.6.4 ECB Survey of Professional Forecasters

Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia is a survey of professional forecasters in the United States. A similar survey was conducted by the European Central Bank (ECB). Since 1999, the ECB has conducted a quarterly survey of expectations for some of the euro area's key macroeconomic variables. It is known as the ECB's Survey of Professional Forecasters. The survey participants are anonymous experts affiliated with financial or non-financial institutions within the European Union. The SPF questionnaire includes questions on HICP inflation, the real GDP growth rate, and the unemployment rate in the euro area over different horizons. The survey currently takes place in the first month of each quarter. The questionnaire is sent out to the participants immediately after the macroeconomic data for the last month of the previous quarter is released. Among all the forecast horizons in the questionnaire, forecasters are requested to provide their expectations for two specific months (quarters for the real GDP growth rate) that are set one and two years ahead of the latest available data for the respective variables. This feature enables us to run the error-on-revision regression using this data. In terms of applying the error-on-revision methodology, there are three main disadvantages of the ECB's SPF compared with SPF from the Philadelphia

Figure 1.6: θs for Different Time Periods



Notes: This figure plots the θ estimates for different time periods. θs estimated from 1968-2022 are plotted as black crosses. θs estimated from 1981-2022 are plotted as blue dots. θs estimated from 1991-2022 are plotted as red squares. 95% *confidence intervals are included.*

Fed: first, it covers a shorter time span. ECB's SPF started in 1999 while the SPF from the Philadelphia Fed started in 1968; second, ECB's SPF has fewer variables; third, ECB's SPF has fewer rolling horizons.[19] That is, ECB's SPF only asks people about their forecast for one year and two years ahead. The Philadelphia Fed's SPF asks about the current quarter, as well as one, two, three, and four quarters ahead. Because of this feature of the ECB's Survey of Professional Forecasters, forecast revision can only be computed from a comparison to the previous year. Yet, the survey of tens of professional forecasters in the European Union over the past 20 years makes the ECB's SPF a valuable source of information about overreaction or underreaction in expectation formation. Does the pattern that we have observed in US's SPF exist in Europe's SPF?

The Error-on-revision regression results are reported in table 1.11. Similar to table 1.2, for real GDP growth rate and inflation, $\hat{\beta}^C$s are positive but $\hat{\beta}^I$s are negative. For unemployment rate, both $\hat{\beta}^C$ and $\hat{\beta}^I$ are positive. However, none of the coeffi-

[19]For HICP inflation, the surveys sent out in the first, second, third, and fourth quarter of the year respectively inquire about the coming December, March, June, and September, within one year and two years ahead. For real GDP growth rate, the surveys sent out in the first, second, third, and fourth quarter of the year respectively inquire about the coming third, fourth, first and second quarter, within one year and two years ahead. For the unemployment rate, the surveys sent out in the first, second, third, and fourth quarter of the year ask about forecast for the coming December, March, June and September, within one year and two years ahead.

cients in table 1.11 is significant. Given our main arguments that β^C_{OLS} is the better statistic for making inferences on the degree of over-/underreaction to new information, expectation formation for inflation exhibits underreaction to new information across different measures of inflation and different surveys. One thing worth noticing is that in table 1.11 we restrict the period to 1999 - 2019, excluding the recent period. The reason for excluding the recent period is that, starting from 2020, there have been many historical episodes of turmoil: the global pandemic, the Russian invasion of Ukraine, etc., which might affect the estimation results of $\hat{\beta}^C$ and $\hat{\beta}^I$. The regression results for the full sample are reported in table 1.17 in the appendix. We can see that $\hat{\beta}^C$ and $\hat{\beta}^I$ of real GDP growth rate change drastically when the periods starting from 2020 are added. While for inflation and unemployment rate, $\hat{\beta}^C$ and $\hat{\beta}^I$ do not exhibit much difference whether the period after 2019 is included.

The respective θ values are plotted in Figure 1.7. Firstly, all three variables yield negative θ values. Secondly, in the case of CPI inflation, θ exhibits a large magnitude, regardless of whether it is estimated from matching β^C or β^I. This observation aligns with the findings obtained from the Fed's Survey of Professional Forecasters, where both the GDP price index and CPI inflation demonstrate significant degrees of under-reaction. The model parameters using the ECB's Survey of Professional Forecasters are reported in table 1.18 in the appendix.

Table 1.11: Error-on-revision Regression Results for ECB's SPF

	$\hat{\beta}^C$			$\hat{\beta}^I$			Periods	Number of Forecasters
	Point Estimate	SE	p-value	Point Estimate	SE	p-value		
Inflation	0.72	0.69	0.30	-0.11	0.15	0.43	1999-2019	44
RGDP	0.09	0.17	0.58	-0.09	0.22	0.67		44
Unemployment	0.12	0.11	0.26	0.02	0.06	0.73		40

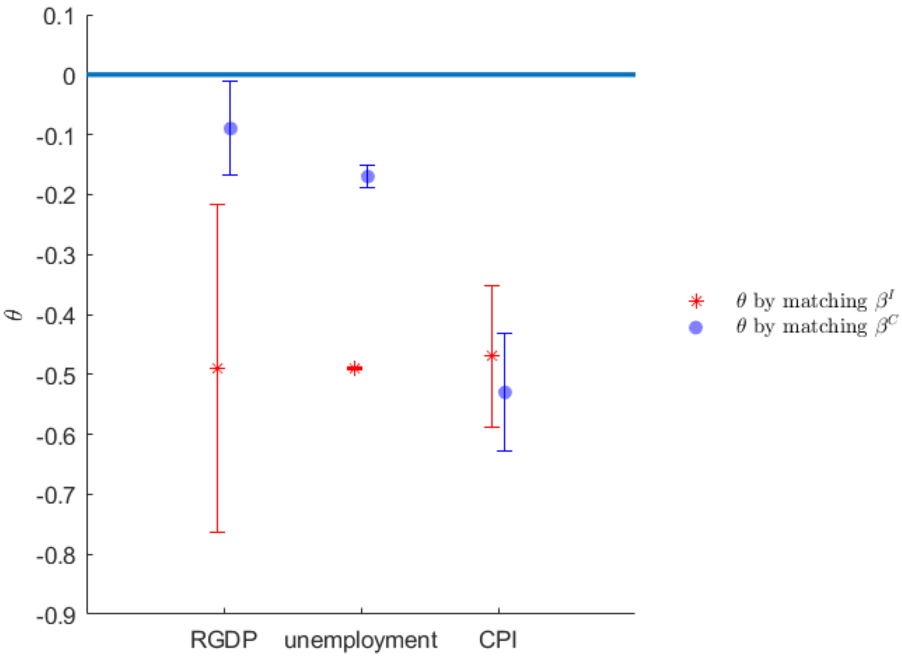
Notes: This table reports the Error-on-revision regression results for ECB's Survey of Professional Forecasters. I report the point estimates, standard errors, p-value. The number of forecasters is the average number of forecasters across all waves of survey.

1.6.5 Expansions and Contractions

In this section, I explore the varying magnitude of idiosyncratic random noise, $\hat{\beta}^C$ and $\hat{\beta}^I$ across expansions and contractions. When applying the forecast error on forecast revision regression, we note that both $\hat{\beta}^C$ and $\hat{\beta}^I$ vary across expansions and contractions: $\hat{\beta}^C$ and $\hat{\beta}^I$ are much more negative in contractions than in expansions. The consensus and individual regressions that we run now are as follows:

$$\begin{aligned} w_{t+1} - \bar{\mathcal{F}}_t w_{t+1} = \beta^C_0 + \beta^C_1 (\bar{\mathcal{F}}_t w_{t+1} - \bar{\mathcal{F}}_{t-1} w_{t+1}) \mathbb{1}_{t \in Bust} \\ + \beta^C_2 (\bar{\mathcal{F}}_t w_{t+1} - \bar{\mathcal{F}}_{t-1} w_{t+1}) + \beta^C_3 \mathbb{1}_{t \in Bust} + u_t \end{aligned}$$

Figure 1.7: θ Estimates for ECB Survey of Professional Forecasters



Notes: This figure plots θ estimates for different variables in ECB's Survey of Professional Forecasters. θs are estimated using method 2. θs estimated by matching β^I are plotted as red stars. θs estimated by matching β^C are plotted as blue dots. 95% confidence intervals are included.

$$
\begin{aligned}
w_{t+1} - \mathcal{F}_{i,t}w_{t+1} = \beta_0^I + \beta_1^I(\mathcal{F}_{i,t}w_{t+1} - \mathcal{F}_{i,t-1}w_{t+1})\mathbb{1}_{t\in Bust} \\
+\beta_2^I(\mathcal{F}_{i,t}w_{t+1} - \mathcal{F}_{i,t-1}w_{t+1}) + \beta_3^I\mathbb{1}_{t\in Bust} + u_{i,t}
\end{aligned}
$$

where $\mathbb{1}_{t\in Bust} = 1$ if t is in an economic contraction. β_1^d, $d \in \{C, I\}$, captures the difference of forecast error and forecast revision correlation coefficient in expansions and contractions in both equations. $\beta_1^d < 0$ means that β^d is lower in contractions than in expansions. Regression results are reported in the following table. The boom and bust are classified according to NBER business cycle expansions and contractions.[20] The forecast in the table is for quarter $t+3$.

For five out of seven variables, $\hat{\beta}_1^C$ and $\hat{\beta}_1^I$ are negative, meaning there is evidence that $\hat{\beta}^C$ and $\hat{\beta}^I$ are more negative in contractions than in expansions. It might mean there is stronger overreaction to information in contractions than expansions. However, when we divide the survey into expansions and contractions, the underlying economic parameters in two subsamples can be very different: the statistical processes generating those state variables, captured by ρ and σ_e, are different in expansions and contractions; the variance of idiosyncratic random noise σ_η^2 is dif-

[20]Since 1968, the NBER contractions include December 1969 to November 1970, November 1973 to March 1975, January 1980 to July 1980, July 1981 to November 1982, July 1990 to March 1991, March 2001 to November 2001, December 2007 to June 2009, February 2020 to April 2020.

Table 1.12: Over-/Underreaction in Expansion and Contraction

	NGDP	RGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
β_1^C	-1.43^{***}	-1.59^{***}	0.37	0.11	-1.3^{***}	-0.56	-1.2^{***}
	(0.51)	(0.39)	(0.47)	(1.39)	(0.5)	(0.41)	(0.44)
β_1^I	-0.51^{***}	-0.21^{**}	0.22	-0.12	-0.59^{***}	-0.34^{*}	-0.45^{***}
	(0.19)	(0.09)	(0.09)	(0.23)	(0.17)	(0.2)	(0.15)
$\Delta\sigma$	45%	33%	31%	71%	63%	-6%	36%

Notes: This table reports the difference in regression coefficients during economic contractions and expansions, as the regression equations in section 1.6.5. Results from both the individual level and consensus level regressions are reported. $\Delta\sigma$ *is the difference of* σ_η *between economic contractions(bust) and expansions(boom),* $\Delta\sigma = \frac{\sigma_\eta^{bust} - \sigma_\eta^{boom}}{\sigma_\eta^{boom}}$. *Standard errors are displayed in parenthesis.* $^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$.

ferent; and of course, it might be due to the different degrees of over-/under-reaction to new information θ in expansions and contractions.

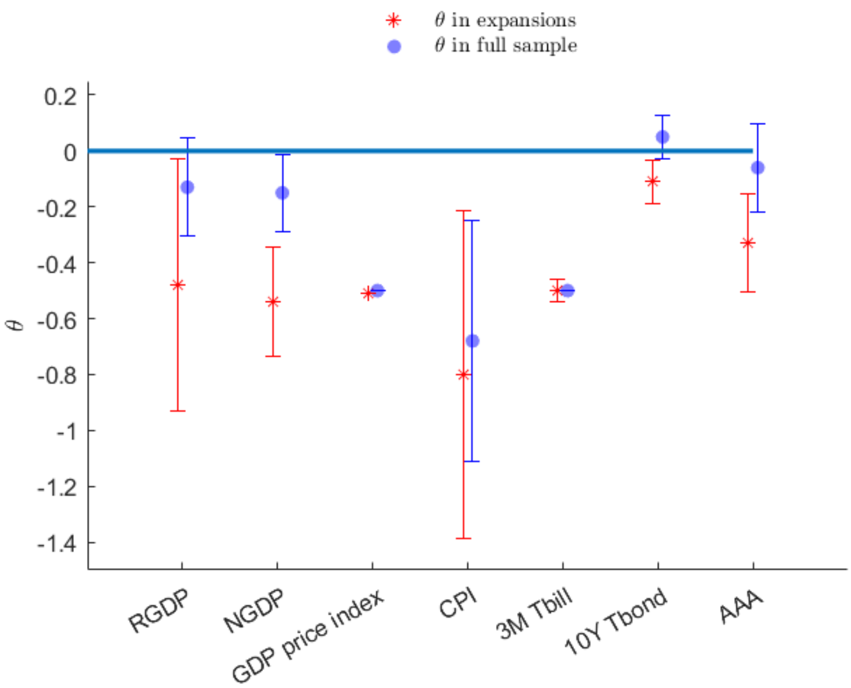
Now that we know the importance of identifying the magnitude of idiosyncratic random noise, we identify this variance as we do in section 1.5, but separately for expansions and contractions. First, idiosyncratic random noise $\hat{\eta}_{i,t}$ is obtained for each variable. Second, I pool $\hat{\eta}_{i,t}$ separately for expansions and contractions to calculate the variance of idiosyncratic random noise. The difference in the standard deviation of idiosyncratic noise between economic expansions and contractions is presented in the third row of Table 1.12.

We can see that, mostly, the variance of idiosyncratic noise in contractions is much more significant than that in expansions, consistent with previous findings that forecast dispersion is higher in crisis periods (e.g., Mankiw, Reis, and Wolfers (2003)).

To compare θ values between economic expansions and contractions, I conducted the Error-on-revision regression solely for economic expansions and estimated θ using β^C values derived from the expansion periods. I then compared these obtained θ values with the θ values estimated from the full sample. The rationale behind focusing on economic expansions only is that these periods have longer time series data, leading to increased statistical power in the regression analysis. Conversely, economic contractions have relatively shorter time series data.

The comparison between θ values from economic expansions and the full sample is illustrated in figure 1.8. Detailed estimates of β^C and θ values are reported in table 1.19 in the appendix. The results indicate some evidence that, for nominal GDP growth rate and 10-year Tbond, θ values tend to be larger in absolute value during economic expansions. However, it is crucial to interpret this finding considering the approach of reporting the θ value closer to zero when there are two θ values (¡0) that can match the β^C values from the data and the model. For GDP price index inflation and 3-month Tbill yield, the θ values are consistently around -0.5. As previously discussed, this indicates that the $\hat{\beta}^C$ values from the data are too large for the model to match accurately. In these cases, I report the θ value that

Figure 1.8: θs in Expansions and the Full Sample



Notes: This table plots the θ estimates from economic expansions, as red stars. θs estimated from the full sample are plotted as blue dots. 95% *confidence intervals are included.*

minimizes the distance between $\hat{\beta}^C$ and β^C_{OLS}.

1.6.6 Additional Test of Coefficient Restrictions

Coibion and Gorodnichenko (2015) conducted several tests regarding the coefficient restrictions in model-implied regression 1.1. Their model suggests three main parameter restrictions, no matter whether under sticky information or noisy information environment: first, the constant term in regression 1.1 should be zero. Second, on the right hand side of the regression equation, the coefficients for the current forecast and forecast made in the last period should be of equal magnitude but possess opposite signs. Third, there should not be any additional variables with predictive power on forecast errors. Instead, forecast revisions should hold all the predictive power over these errors. Their investigations primarily centered around inflation forecasts, with tests for other variables not being conducted. In the case of the inflation rate, the regression specification passed most of the tests for parameter restrictions. In this chapter, I repeat their tests for other variables under consideration. My focus is on the same forecast horizons, $h = 3$, as was done by Coibion and Gorodnichenko (2015). To verify the first coefficient restriction, I present the constant term in equation 1.1. For the second coefficient restriction, a new regression is run as follows:

$$w_{t+3} - \bar{\mathcal{F}}_t w_{t+3} = \beta^C_0 + \beta^C_1 \bar{\mathcal{F}}_t w_{t+3} + \beta^C_2 \bar{\mathcal{F}}_{t-1} w_{t+3} + u_{t,t+3}, \tag{1.32}$$

If the regression in equation 1.1 is the correct specification, estimation results from regression 1.32 should fulfill the condition: $\beta_1^C + \beta_1^C = 0$. To test the third model restriction, a further regression is conducted as follows:

$$w_{t+3} - \bar{\mathcal{F}}_t w_{t+3} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+3} - \bar{\mathcal{F}}_{t-1} w_{t+3}) + \delta z_{t-1} + u_{t,t+3}, \tag{1.33}$$

where z_{t-1} can be any other variables that might have predictive power on forecast errors. Lagged variable is used in the regression, since when forecasters make their forecast at the beginning of a quarter, only the lagged information is fully observed. In my test, I use the lagged quarterly inflation rate and quarterly average 3-month Tbill rate, which are also used in Coibion and Gorodnichenko (2015). As a comparison, I run the following regression:

$$w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \delta z_{t-1} + u_{t,t+h}, \tag{1.34}$$

Tests of coefficient restrictions are conducted at both the consensus and individual levels. The results for the first two tests are reported in table 1.20 and 1.21. The outcomes for the third test are presented in table 1.22 and 1.23. Panel A in both table 1.20 and 1.21 reveals that the intercepts for bond yields, including the 3-month Tbill, 10-year Tbond, and AAA corporate bond, are significantly negative. For other variables, the intercepts are generally not significant. This suggests that forecasters consistently over-predict future yields for bond yields, potentially due to an oversight of the declining trend in yields over the past four decades during their forecasting process.

To evaluate whether the coefficient restriction $\beta_1^C + \beta_1^C = 0$ is rejected or not, we examine the p-value in panel B of table 1.20 and 1.21. This restriction cannot be rejected at the consensus level for the majority of variables. However, it is rejected for CPI inflation and the 3-month Tbill. Conversely, at the individual level, this restriction is rejected for nearly all variables, with the exception of the real GDP growth rate.

Regarding the third coefficient restriction, we turn our attention to δs in table 1.22. Despite the inclusion of forecast revisions on the right-hand side of the regression equation, the lagged inflation rate and 3-month Tbill rate still exhibit significant explanatory power over forecast errors. For instance, the lagged inflation rate significantly negatively predicts forecast errors at the consensus level across all variables. When inflation is high, forecasters tend to make negative forecast errors, indicating an over-prediction of future growth rate, inflation rate, and bond yields based on the last observed quarterly inflation. As expected, without forecast revisions on the right-hand side, the lagged inflation rate and 3-month Tbill rate continue to have significant predictive power on forecast errors, as demonstrated in table 1.23.

1.7 Conclusion

Are people underreacting or overreacting to new information? This chapter revisits two pertinent empirical facts: the correlation coefficients between forecast errors and forecast revisions being positive at the consensus level and negative at the individual level. The literature interprets these facts as overreaction at the individual level and underreaction at the consensus level. I challenge this common interpretation by analyzing how those two coefficients are determined under a parsimonious framework of expectation formation. The expectation formation model deviates from the rational expectation hypothesis in two ways: first, it allows over-/underreaction to new information; second, there are two types of judgment noise, idiosyncratic random noise, and individual fixed heterogeneity. In the model, positive correlation coefficients at the consensus level imply underreactions to new information. However, negative correlation coefficients result from idiosyncratic random noise. This yields an entirely different interpretation of the discrepancies between consensus and individual correlation coefficients. The identified magnitude of idiosyncratic random noise in forecast survey data is large enough to reconcile the gap between those two correlation coefficients. The critical parameter measuring over-/underreaction to new information mostly shows underreaction, different from the results in Bordalo, Gennaioli, Ma, and Shleifer (2020).

There are many more intriguing questions to explore in this strand of literature. For example, what explains the heterogeneous degrees of over-/under-reaction to new information across different variables? This has been noted and investigated by Wang (2021) and d'Arienzo (2020) for bond yields of different maturities. In the data, bonds with shorter maturities exhibit underreactions, while bonds with longer maturities exhibit overreactions. Are there more general patterns that are not specific to the bond yield forecast? What are the possible explanations?

1.8 Appendix

Proof 1 (Proposition 1 and Corollary 1) *To derive the coefficient for individual regression, we can first derive the coefficient for an individual forecaster i:*

$$\beta^i_{OLS} = \frac{cov(e_{t+1} - \rho\theta e_t - \eta_{i,t} - \phi_i, \rho(1+\theta)e_t + \eta_{i,t} + \phi_i - \rho^2\theta e_{t-1} - \rho(\eta_{i,t-1} + \phi_i))}{var(\rho(1+\theta)e_t + \eta_{i,t} + \phi_i - \rho^2\theta e_{t-1} - \rho(\eta_{i,t-1} + \phi_i))}$$
$$= \frac{-(1+\theta)\theta - \frac{\sigma^2_\eta}{\rho^2\sigma^2_e}}{(1+\theta)^2 + \rho^2\theta^2 + (1+\rho^2)\frac{\sigma^2_\eta}{\rho^2\sigma^2_e}} \tag{1.35}$$

Since ϕ_i is constant for individual i, it does not enter into the variance nor covariance term for β^i_{OLS}, so β^i_{OLS} is not affected by σ^2_ϕ. β^i_{OLS} is the same for all i, but the intercepts for different forecasters are distinct (since ϕ_i is unique for different *forecasters). When pooling all forecasters together and running the panel regression with individual fixed effect, I allow different forecasters to have their own intercepts. The regression coefficient is equal to the common coefficient* β^i_{OLS}

$$\beta^I_{OLS} = \beta^i_{OLS}$$

To derive β^C_{OLS}*, first note that*

$$var(\frac{\sum_i^N \eta_{i,t}}{N}) = \frac{\sigma^2_\eta}{N}$$

Then

$$\begin{aligned}
\beta^C_{OLS} &= \frac{cov(w_{t+1} - E^\theta_t w_{t+1}, E^\theta_t w_{t+1} - E^\theta_{t-1} w_{t+1})}{var(E^\theta_t w_{t+1} - E^\theta_{t-1} w_{t+1})} \\
&= \frac{cov(e_{t+1} - \rho\theta e_t - \frac{\sum_i^N \eta_{i,t}}{N} - \frac{\sum_i^N \phi_i}{N}, \rho(1+\theta)e_t + \frac{\sum_i^N \eta_{i,t}}{N} + \frac{\sum_i^N \phi_i}{N} - \rho^2\theta e_{t-1} - \rho(\frac{\sum_i^N \eta_{i,t}}{N} + \frac{\sum_i^N \phi_i}{N})}{var(\rho(1+\theta)e_t + \frac{\sum_i^N \eta_{i,t}}{N} + \frac{\sum_i^N \phi_i}{N} - \rho^2\theta e_{t-1} - \rho(\frac{\sum_i^N \eta_{i,t}}{N} + \frac{\sum_i^N \phi_i}{N}))} \\
&= \frac{-\theta(1+\theta) - \frac{\sigma^2_\eta}{N\rho^2\sigma^2_e}}{(1+\theta)^2 + \rho^2\theta^2 + (1+\rho^2)\frac{\sigma^2_\eta}{N\rho^2\sigma^2_e}} \qquad (1.36)
\end{aligned}$$

When the number of forecasters is large, $\beta^C_{OLS} \xrightarrow{n\to\infty} \frac{-\theta(1+\theta)}{(1+\theta)^2+\rho^2\theta^2}$
Q.E.D.

Proof 2 (Corollary 2) *Similar to proof 1,*

$$\beta^i_{OLS} = \frac{cov(e_{t+1} - \rho\theta e_t - \eta_{i,t} - \phi_i, \rho(1+\theta)e_t + \eta_{i,t} + \phi_i - \rho^2\theta e_{t-1} - \rho(\eta_{i,t-1} + \phi_i))}{var(\rho(1+\theta)e_t + \eta_{i,t} + \phi_i - \rho^2\theta e_{t-1} - \rho(\eta_{i,t-1} + \phi_i))}$$

Now since $\eta_{i,t}$ *is serially correlated as*

$$\eta_{i,t} = \rho_\eta \eta_{i,t-1} + u_{i,t}$$

Since $cov(\eta_{i,t}, \eta_{i,t-1}) = \rho_\eta \sigma^2_\eta$,

$$\beta^i_{OLS} = \frac{-\theta(1+\theta) - \frac{\sigma^2_\eta}{\rho^2\sigma^2_e} + \frac{\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}}{\rho^2\theta^2 + (1+\theta)^2 + (\rho^2+1)\frac{\sigma^2_\eta}{\rho^2\sigma^2_e} - \frac{2\rho_\eta\sigma^2_\eta}{\rho\sigma^2_e}}$$

Similar to proof 1, when pooling all forecasters together and running the panel regression with individual fixed effect, the panel regression coefficient is equal to the common coefficient β^i_{OLS}

$$\beta^I_{OLS} = \beta^i_{OLS}$$

When $N \to \infty$*, judgment noise is averaged out during the calculation of consensus* *forecast, so consensus forecast is given by*

$$\beta^C_{OLS} = \frac{-\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2}$$

Corollary 3 (CG Regression Coefficients under AR(2)) *When the state variable follows an AR(2) process,*

$$w_t = \rho_1 w_{t-1} + \rho_2 w_{t-2} + \epsilon_t$$

and individual forecast is given by

$$E^\theta_{i,t} w_{t+T} = E^\theta_t w_{t+T} + (\eta_{i,t} + \phi_i)\rho^T_\eta$$

where $E^\theta_t w_{t+T}$ is the generalized diagnostic expectation, and $(\eta_{i,t} + \phi_i)\rho^T_\eta$ is the idiosyncratic noise and individual fixed heterogeneity. ρ_η captures the correlation of the noise component across different forecast horizons. With individual fixed effect in the panel regression, the CG regression coefficients at two levels would be given by:

$$\beta^C = \frac{-\theta(1+\theta)}{(1+\theta)^2 + \theta^2[\frac{f_{11}^{(T+1)}}{f_{11}^{(T)}}]^2}$$

$$\beta^I = \frac{-(\theta+1)\theta - [\frac{\rho^T_\eta}{f_{11}^{(T)}}]^2\frac{\sigma^2_\eta}{\sigma^2_\epsilon}}{(1+\theta)^2 + \theta^2[\frac{f_{11}^{(T+1)}}{f_{11}^{(T)}}]^2 + [\frac{\rho^T_\eta}{f_{11}^{(T)}}]^2(1+\rho^2_\eta)\frac{\sigma^2_\eta}{\sigma^2_\epsilon}}$$

$f_{11}^{(j)}$ *is the* $(1,1)$ *element of* F^j, $F = \begin{pmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{pmatrix}$.

Proof 3 (Corollary 3) *Assume the state variable follows the following AR(2) process:*

$$w_t = \rho_1 w_{t-1} + \rho_2 w_{t-2} + \epsilon_t. \tag{1.37}$$

The forecast by the diagnostic expectations with forecast horizon T is given by:

$$E^\theta_t w_{t+T} = E_t w_{t+T} + \theta\left(E_t w_{t+T} - E_{t-1} w_{t+T}\right). \tag{1.38}$$

where

$$E_t w_{t+T} = f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1}.$$

$f_{11}^{(j)}$ *is the* $(1,1)$ *element of* F^j, *where* $F = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$. $f_{12}^{(j)}$ *is the* $(1,2)$ *element of* F^j.

To derive $E^\theta_t w_{t+T}$, first we derive $E_{t-1} w_{t+T}$:

$$E_{t-1} w_{t+T} = f_{11}^{(T+1)} w_{t-1} + f_{12}^{(T+1)} w_{t-2} \tag{1.39}$$

where $f_{11}^{(T+1)} = \rho_1 f_{11}^{(T)} + f_{12}^{(T)}$, $f_{12}^{(T+1)} = \rho_2 f_{11}^{(T)}$.
Now,

$$E_t^{\theta} w_{t+T} = f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} + \theta[f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} - f_{11}^{(T+1)} w_{t-1} - f_{12}^{(T+1)} w_{t-2}] \quad (1.40)$$

It can be verified that the terms in the bracket sum to $f_{11}^{(T)} \epsilon_t$.
So

$$E_t^{\theta} w_{t+T} = f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} + \theta f_{11}^{(T)} \epsilon_t \quad (1.41)$$

$$E_{t-1}^{\theta} w_{t+T} = f_{11}^{(T+1)} w_{t-1} + f_{12}^{(T+1)} w_{t-2} + \theta f_{11}^{(T+1)} \epsilon_{t-1} \quad (1.42)$$

Now we can derive the forecast errors and forecast revisions.

$$\begin{aligned}
&Forecast\ error = w_{t+T} - E_t^{\theta} w_{t+T} \\
&= f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} + \sum_{j=1}^{T} \psi_j \epsilon_{t+j} - f_{11}^{(T)} w_t - f_{12}^{(T)} w_{t-1} - \theta f_{11}^{(T)} \epsilon_t \\
&= \sum_{j=1}^{T} \phi_j \epsilon_{t+j} - \theta f_{11}^{(T)} \epsilon_t
\end{aligned}$$

$$\begin{aligned}
&Forecast\ revision = E_t^{\theta} w_{t+T} - E_{t-1}^{\theta} w_{t+T} = f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} + \theta f_{11}^{(T)} \epsilon_t \\
&- f_{11}^{(T+1)} w_{t-1} - f_{12}^{(T+1)} w_{t-2} - \theta f_{11}^{(T+1)} \epsilon_{t-1} \\
&= f_{11}^{(T)} \left[\rho_1 \cdot w_{t-1} + \rho_2 \cdot w_{t-2} + \epsilon_t\right] + f_{12}^{(T)} w_{t-1} + \theta f_{11}^{(T)} \epsilon_t \\
&- f_{11}^{(T+1)} w_{t-1} - f_{12}^{(T+1)} w_{t-2} - \theta \cdot f_{11}^{(T+1)} \epsilon_{t-1} \\
&= f_{11}^{(T)} \epsilon_t + \theta \cdot f_{11}^{(T)} \epsilon_t - \theta \cdot f_{11}^{(T+1)} \epsilon_{t-1}
\end{aligned}$$

So

$$\begin{aligned}
\beta^c &= \frac{\text{cov}\left(\sum_{j=1}^{T} \psi_j \epsilon_{t+j} - \theta \cdot f_{11}^{(T)} \cdot \epsilon, f_{11}^{(T)} \epsilon_t + \theta \cdot f_{11}^{(T)} \epsilon_t - \theta \cdot f_{11}^{(T+1)} \epsilon_{t-1}\right)}{\text{Var}(FR)} \\
&= \frac{-\left[f_{11}^{(T)}\right]^2 \theta(1+\theta)}{(1+\theta)^2 \left[f_{11}^{(T)}\right]^2 + \theta^2 \left[f_{11}^{(T+1)}\right]^2} = \frac{-\theta(1+\theta)\left[f_{11}^{(T)}\right]^2}{(1+\theta)^2 \left[f_{11}^{(T)}\right]^2 + \theta^2 \left[f_{11}^{(T+1)}\right]^2}
\end{aligned}$$

Now we turn to the derivation of β^I. *Assume the individual level forecast is given by the following:*

$$E_{i,t}^{\theta} w_{t+T} = f_{11}^{(T)} w_t + f_{12}^{(T)} w_{t-1} + \theta \cdot f_{11}^{(T)} \cdot \epsilon_t + \eta_{i,t} \cdot \rho_{\eta}^T, \quad (1.43)$$

where ρ_η *captures the correlation of the noise component across different forecast* *horizons. Similarly, the previous forecast is given by:*

$$E^{\theta}_{i,t-1} w_{t+T} = f_{11}^{(T+1)} w_{t-1} + f_{12}^{(T+1)} w_{t-2} + \theta \cdot f_{11}^{(T+1)} \cdot \epsilon_{t-1} + \eta_{i,t-1} \cdot \rho_{\eta}^{T+1}. \tag{1.44}$$

Similar to the derivation at the consensus level, the forecast error is given by:

$$Forecast\ error = \sum_{j=1}^{T} \psi_j \epsilon_{t+j} - \theta \cdot f_{11}^{(T)} \cdot \epsilon_t - \eta_{i,t} \cdot \rho_{\eta}^{T}. \tag{1.45}$$

The forecast revision is given by:

$$Forecast\ revision = f_{11}^{(T)} \epsilon_t + \theta f_{11}^{(T)} \epsilon_t + \rho_{\eta}^{T} \cdot \eta_{i,t} - \theta \cdot f_{11}^{(T+1)} \epsilon_{t-1} - \rho_{\eta}^{T+1} \cdot \eta_{i,t-1}. \tag{1.46}$$

As a result:

$$\begin{aligned}
\beta^I &= \frac{-\left[f_{11}^{(T)}\right]^2 \theta(1+\theta)\sigma_\epsilon^2 - {\rho_\eta}^{2T}\sigma_\eta^2}{(1+\theta)^2\left[f_{11}^{(T)}\right]^2 \sigma_\epsilon^2 + \sigma_\eta^2\left[{\rho_\eta}^{2T} + \rho_\eta^{2T+2}\right] + \theta^2\left[f_{11}^{(T+1)}\right]^2 \sigma_\epsilon^2} \\
&= \frac{-\left[f_{11}^{(T)}\right]^2 \theta(1+\theta) - \rho_\eta^{2T}\frac{\sigma_\eta^2}{\sigma_\epsilon^2}}{(1+\theta)^2\left[f_{11}^{(T)}\right]^2 + \rho_\eta^{2T}\left(1+\rho_\eta^2\right)\frac{\sigma_\eta^2}{\sigma_\epsilon^2} + \theta^2\left[f_{11}^{(T+1)}\right]^2} \\
&= \frac{-\theta(1+\theta) - \left[\frac{\rho_\eta^T}{f_{11}^{(}T)}\right]^2 \frac{\sigma_\eta^2}{\sigma_\epsilon^2}}{(1+\theta)^2 + \left[\frac{\rho_\eta^T}{f_{11}^{(T)}}\right]^2\left(1+\rho_\eta^2\right)\frac{\sigma_\eta^2}{\sigma_{\varepsilon^2}} + \theta^2\left[\frac{f_{11}^{(T+1)}}{f_{11}^{(T)}}\right]^2}.
\end{aligned}$$

Q.E.D.

Table 1.13: Error-on-revision Regression Results for Different Time Periods

	$\hat{\beta}^C$			$\hat{\beta}^I$			Periods
	Point Estimate	SE	p-value	Point Estimate	SE	p-value	
RGDP	0.11	0.31	0.73	-0.28	0.12	0.02	1968-2022
	0.03	0.36	0.93	-0.3	0.23	0.2	1981-2022
	-0.16	0.3	0.59	-0.37	0.26	0.16	1991-2022
GDP Price Index	1.26	0.41	0.00	-0.15	0.07	0.04	1968-2022
	0.54	0.23	0.02	-0.36	0.06	0.00	1981-2022
	0.94	0.41	0.02	-0.3	0.08	0.00	1991-2022
NGDP	0.14	0.25	0.56	-0.32	0.12	0.01	1968-2022
	0.24	0.37	0.51	-0.25	0.19	0.19	1981-2022
	-0.02	0.26	0.94	-0.29	0.25	0.24	1991-2022
CPI	1.04	0.76	0.17	-0.38	0.09	0.00	1981-2022
	-0.29	1.12	0.79	-0.5	0.11	0.00	1991-2022
Tbill	0.69	0.11	0.00	0.21	0.09	0.03	1981-2022
	0.8	0.11	0.00	0.33	0.1	0.00	1991-2022
AAA	-0.02	0.16	0.92	-0.27	0.07	0.00	1981-2022
	-0.01	0.14	0.92	-0.31	0.07	0.00	1991-2022

Notes: This table reports the robustness check for whether $\hat{\beta}^C$ and $\hat{\beta}^I$ vary across different time periods. For consensus time series regressions, standard errors are Newey-West (1994) with automatic bandwidth selection. For individual panel regressions, standard errors are clustered by forecaster and time.

Table 1.14: Inference on θ without Considering Idiosyncratic Noise

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
θ from matching β^I	0.41^{***}	0.18^{***}	0.54^{***}	0.64^{***}	-0.18^{***}	0.41^{***}	0.33^{***}
	(0.08)	(0.06)	(0.1)	(0.22)	(0.03)	(0.08)	(0.05)

*Notes: This table displays the estimated θs from matching β^I_{BGMS} with empirically estimated $\hat{\beta}^I$ as in BGMS. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*

Table 1.15: Inference on θ from $\hat{\beta}^C$ with Finite and Infinite N

		RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
θ frommatching β^C	Infinite N	-0.1	-0.5^{***}	-0.11^{*}	-0.62^{**}	-0.5^{***}	0.01	0.06
		(0.08)	(0.00)	(0.07)	(0.23)	(0.01)	(0.07)	(0.04)
	Finite N	-0.13	-0.5^{***}	-0.15^{**}	-0.68^{**}	-0.5^{***}	-0.06	0.05
		(0.09)	(0.00)	(0.07)	(0.22)	(0.00)	(0.08)	(0.04)
Number of Forecasters		36	36	36	33	35	30	35

*Notes: This table displays the estimated θs from matching β^C_{OLS} with empirically estimated $\hat{\beta}^C$, with and without considering the finite number of forecasters N. The number of forecasters N for each variable is also reported. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*

Table 1.16: Estimation Results for Each Variable in Different Time Periods

	ρ	σ_e	σ_η	θ	Periods
RGDP	0.77	1.57	1.05	-0.14	1968-2022
				(0.25)	
	0.69	1.55	0.74	-0.05	1981-2022
				(0.11)	
	0.64	1.66	0.69	0.16	1991-2022
				(0.15)	
NGDP	0.84	1.71	1.24	-0.17^{**}	1968-2022
				(0.07)	
	0.77	1.79	1.01	-0.23^{***}	1981-2022
				(0.06)	
	0.69	1.89	0.91	-0.003	1991-2022
				(0.07)	
GDP Price Index	0.98	0.5	0.88	-0.51^{***}	1968-2022
				(0.02)	
	0.99	0.42	0.64	-0.5^{***}	1981-2022
				(0.12)	
	0.98	0.39	0.53	-0.5^{***}	1991-2022
				(0.07)	
CPI	0.46	2.16	0.73	-0.74^{**}	1981-2022
				(0.34)	
	0.3	2.07	0.61	0.35	1991-2022
				(0.34)	
Tbill	0.99	0.57	0.48	-0.31^{***}	1981-2022
				(0.04)	
	0.99	0.38	0.36	-0.5^{***}	1991-2022
				(0.02)	
AAA	0.99	0.38	0.48	0.08	1981-2022
				(0.08)	
	0.99	0.28	0.5	0.07	1991-2022
				(0.09)	

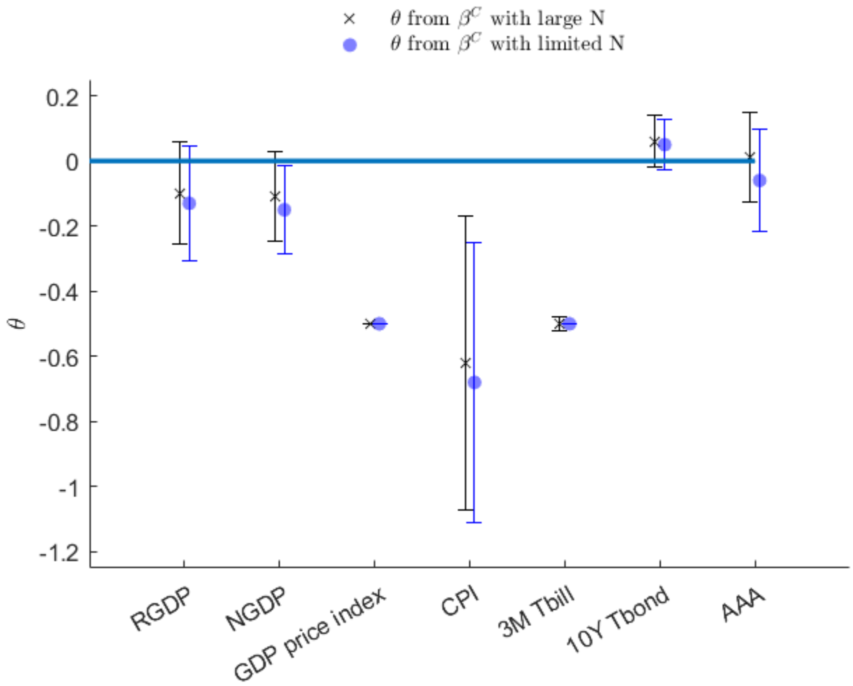
Notes: This table displays the estimation results for parameters covering different time periods.

Table 1.17: Error-on-revision Regression Results for ECB's SPF Full Sample

	$\hat{\beta}^C$			$\hat{\beta}^I$			Periods	Number ofForecasters
	Point Estimate	SE	p-value	Point Estimate	SE	p-value		
Inflation	1.71	1.18	0.15	0.30	0.33	0.38	1999-2021	44
RGDP	-0.69	0.23	0.00	-0.66	0.16	0.00		44
Unemployment	0.02	0.10	0.81	-0.07	0.06	0.30		40

Notes: This table reports the Error-on-revision regression results when, additionally, the pandemic period is also included.

Figure 1.9: Inference on θ from $\hat{\beta}^C$ with Finite and Infinite N



Notes: This figure plots θs estimation from table 1.15. θs estimated with $N \to \infty$ are plotted as black crosses. θs estimated with the actual number of forecasters are plotted as blue dots. 95% confidence intervals are included.

Table 1.18: Model Parameters Using ECB's SPF

	Inflation	RGDP	Unemployment
ρ	0.88	0.86	0.99
σ_e	0.42	0.73	0.23
σ_η	0.27	0.37	0.3

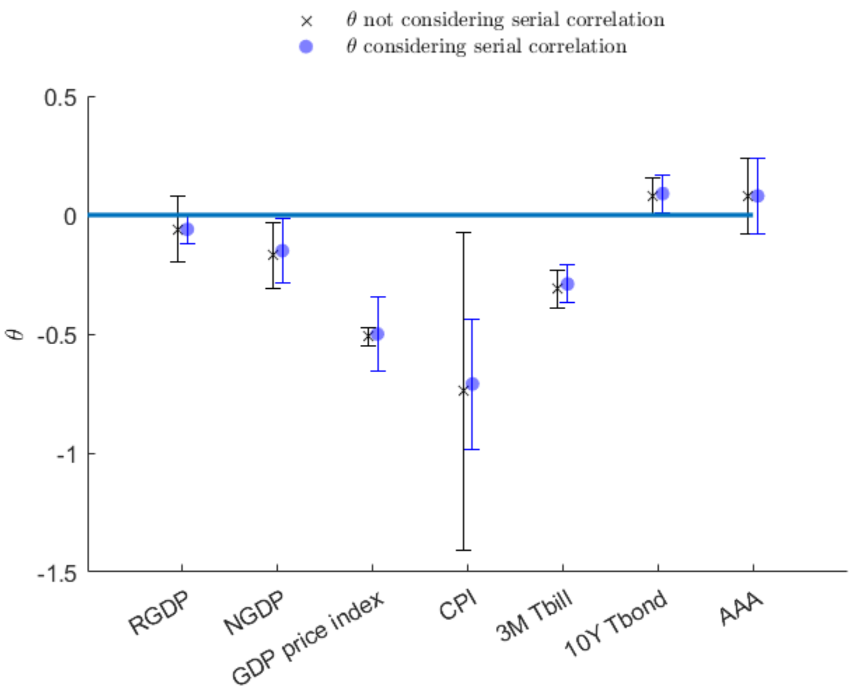
Notes: This table reports the model parameters using ECB's Survey of Professional Forecasters. ρ is the persistence of the variables as AR(1) processes. σ_e is the standard deviation of the innovations. σ_η is the standard deviation of the idiosyncratic noise.

Table 1.19: β^C and θ in Economic Expansions

	NGDP	RGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
β^C	0.79*	0.53*	1.08***	0.78	0.66***	0.08	0.25
	(0.41)	(0.31)	(0.24)	(0.63)	(0.22)	(0.25)	(0.21)
θ	−0.54***	−0.48**	−0.51***	−0.8**	−0.5***	−0.11***	−0.33***
	(0.1)	(0.23)	(0.00)	(0.3)	(0.02)	(0.04)	(0.09)

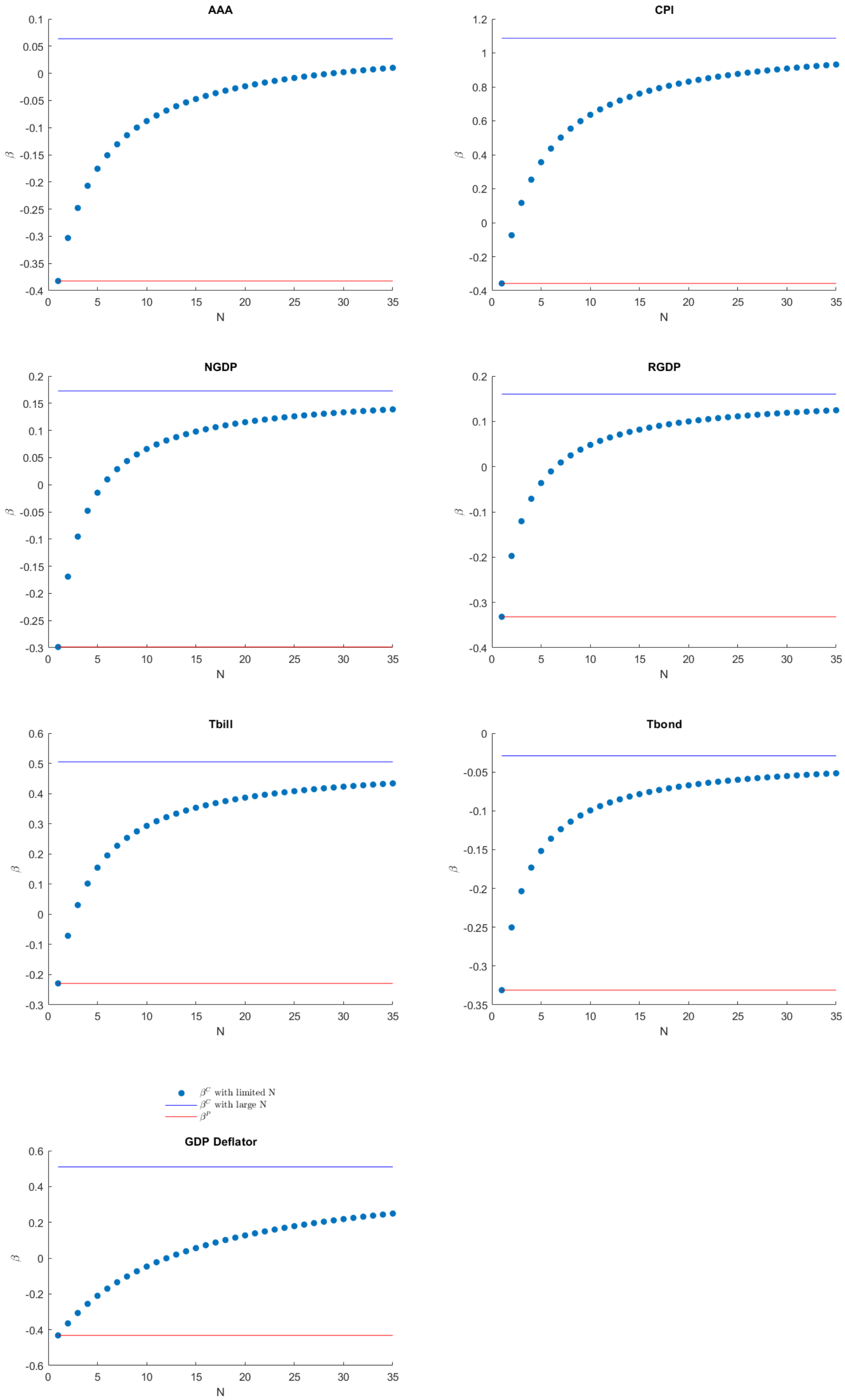
*Notes: This table reports β^Cs estimated using economic expansions and the respective θs. Standard errors are displayed in parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*

Figure 1.10: Inference on θ from Method 1 Considering and Not Considering Serial Correlation in Idiosyncratic Noise



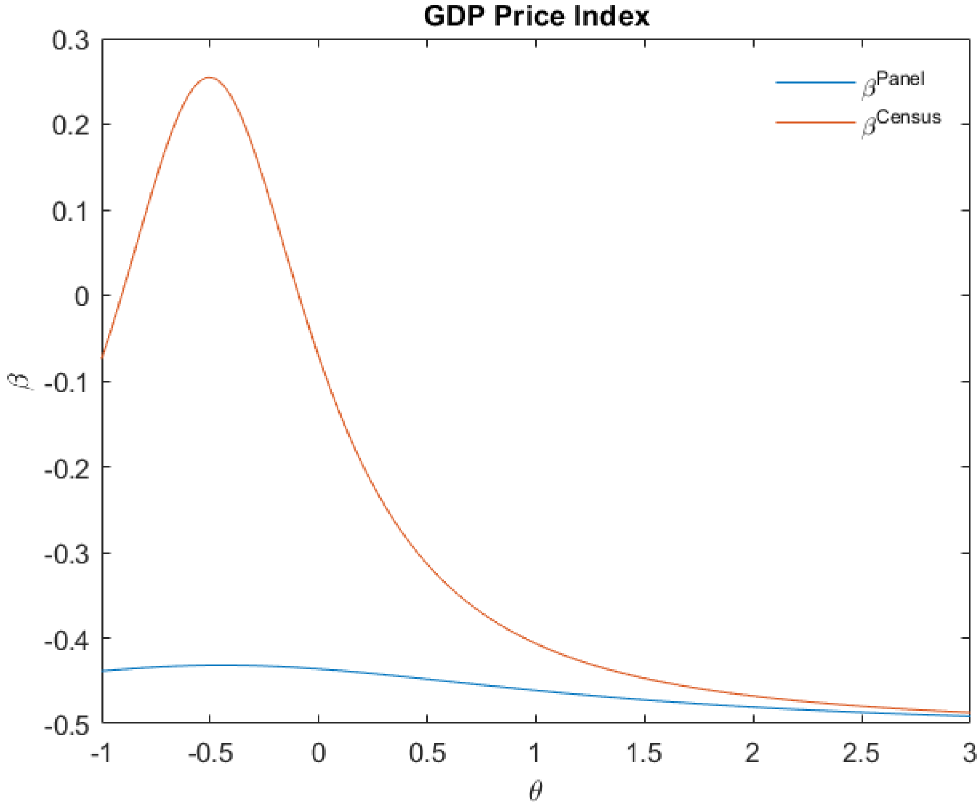
Notes: This figure plots θs estimation from table 1.9 considering serial correlation in idiosyncratic noise, alongside with results from table 1.5, without considering serial correlation in idiosyncratic noise, as comparison. 95% confidence intervals are included.

Figure 1.11: β^C_{OLS} varies with N



Note: This figure reports how β^C_{OLS} in equation 1.21 changes while the number of forecasters N increases.

Figure 1.12: The effect of θ on β^C_{OLS} and β^I_{OLS}



Notes: This figure plots how β^I_{OLS} and β^C_{OLS} change while θ changes using the parameters of GDP price index inflation.

Table 1.20: Two Coefficient Restriction Tests at the Individual Level

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: $w_{t+h} - \mathcal{F}_{i,t}w_{t+h} = \beta_0^I + \beta^I(\mathcal{F}_{i,t}w_{t+h} - \mathcal{F}_{i,t-1}w_{t+h}) + u_{i,t,t+h}$							
Constant	-0.33^{**}	-0.25^{*}	0.07	-0.21	-0.45^{***}	-0.56^{***}	-0.52^{***}
	(0.14)	(0.14)	(0.08)	(0.19)	(0.07)	(0.06)	(0.06)
Obs.	4504	5619	5576	4124	3935	3228	3348
R^2	0.1	0.09	0.17	0.05	0.06	0.08	0.13
Panel B: $w_{t+h} - \mathcal{F}_{i,t}w_{t+h} = \beta_0^I + \beta_1^I\mathcal{F}_{i,t}w_{t+h} + \beta_2^I\mathcal{F}_{i,t-1}w_{t+h} + u_{i,t,t+h}$							
Constant	0.09	1.91^{***}	1.38^{***}	2.05^{***}	0.34^{***}	0.15	1.24^{***}
	(0.39)	(0.58)	(0.27)	(0.42)	(0.11)	(0.2)	(0.27)
β_1^I	-0.32^{**}	-0.44^{***}	-0.35^{***}	-0.81^{***}	0.13	-0.29^{***}	-0.4^{***}
	(0.15)	(0.12)	(0.1)	(0.1)	(0.1)	(0.09)	(0.07)
β_2^I	0.18^{*}	0.09	-0.06	-0.004	-0.35^{***}	0.12	0.15^{**}
	(0.11)	(0.08)	(0.07)	(0.08)	(0.09)	(0.09)	(0.06)
p-value	0.21	0.00	0.00	0.00	0.00	0.00	0.00
Obs.	4504	5619	5576	4124	3935	3228	3348
R^2	0.11	0.15	0.27	0.12	0.18	0.13	0.25

Notes: This table reports the regression results of the first two tests of coefficient restrictions at the individual level. Panel A reports the constant term in Error-on-revision regression. Panel B reports regression results on testing $\beta_1^I + \beta_2^I = 0$.

Table 1.21: Two Coefficient Restriction Tests at the Consensus Level

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: $w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+h} - \bar{\mathcal{F}}_{t-1}w_{t+h}) + u_{t,t+h}$							
Constant	0.23	-0.22	-0.02	-0.13	-0.38^{***}	-0.52^{***}	-0.5^{***}
	(0.21)	(0.17)	(0.12)	(0.23)	(0.08)	(0.09)	(0.09)
Obs.	203	205	203	159	154	114	156
R^2	0.001	0.004	0.24	0.02	0.09	0.0001	0.0001
Panel B: $w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \beta_1^C\bar{\mathcal{F}}_t w_{t+h} + \beta_2^C\bar{\mathcal{F}}_{t-1}w_{t+h} + u_{t,t+h}$							
Constant	-0.53	-0.28	0.06	1.18^{*}	-0.08	-0.46^{**}	0.002
	(0.89)	(0.8)	(0.21)	(0.62)	(0.12)	(0.2)	(0.31)
β_1^C	0.02	0.17	1.2^{***}	0.11	0.53^{***}	-0.06	-0.03
	(0.4)	(0.3)	(0.44)	(0.43)	(0.19)	(0.12)	(0.16)
β_2^C	0.08	-0.16	-1.22^{***}	-0.56	-0.61^{***}	0.05	-0.04
	(0.6)	(0.39)	(0.44)	(0.43)	(0.19)	(0.12)	(0.16)
p-value	0.69	0.93	0.60	0.01	0.00	0.71	0.05
Obs.	203	205	203	159	154	115	156
R^2	0.003	0.005	0.2	0.06	0.12	0.002	0.05

Notes: This table reports the regression results of the first two tests of coefficient restrictions at the consensus level. Panel A reports the constant term in Error-on-revision regression. Panel B reports regression results on testing $\beta_1^C + \beta_2^C = 0$.

Table 1.22: Coefficient Restriction Test: the Predictive Power of Variables other than Forecast Revisions

		RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: $w_{t+h} - \mathcal{F}_{i,t}w_{t+h} = \beta_0^I + \beta^I(\mathcal{F}_{i,t}w_{t+h} - \mathcal{F}_{i,t-1}w_{t+h}) + \delta z_{t-1} + u_{i,t,t+h}$								
inflation	β^I	-0.35	-0.13	-0.38^{***}	-0.32^{***}	0.23^{**}	-0.19^{**}	-0.26^{***}
		(0.22)	(0.13)	(0.07)	(0.07)	(0.09)	(0.08)	(0.07)
	δ	-0.23^{***}	-0.25^{***}	-0.01	-0.15	-0.06^{*}	-0.07^{***}	-0.05^{**}
		(0.08)	(0.1)	(0.04)	(0.1)	(0.03)	(0.02)	(0.02)
R^2		0.14	0.09	0.24	0.07	0.07	0.12	0.14
3M T-bill rate	β^I	-0.18^{***}	-0.18^{**}	-0.17^{**}	-0.38^{***}	0.23^{**}	-0.23^{***}	-0.27^{***}
		(0.06)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.07)
	δ	-0.18^{***}	-0.22^{***}	-0.02	-0.12	-0.19^{***}	-0.001	-0.01
		(0.06)	(0.06)	(0.04)	(0.09)	(0.04)	(0.03)	(0.03)
R^2		0.12	0.11	0.18	0.06	0.16	0.08	0.12
Panel B: $w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+h} - \bar{\mathcal{F}}_{t-1}w_{t+h}) + \delta z_{t-1} + u_{t,t+h}$								
inflation	β^C	-0.07	0.21	0.47^{*}	1.47^{**}	0.65^{***}	0.05	0.05
		(0.33)	(0.29)	(0.28)	(0.68)	(0.15)	(0.16)	(0.18)
	δ	-0.23^{***}	-0.32^{***}	-0.09^{***}	-0.26^{***}	-0.07^{***}	-0.08^{***}	-0.07^{***}
		(0.07)	(0.08)	(0.03)	(0.08)	(0.02)	(0.03)	(0.03)
R^2		0.08	0.12	0.09	0.08	0.11	0.04	0.04
3M T-bill rate	β^C	0.04	-0.01	1.24^{***}	3.41^{***}	0.62^{***}	-0.04	-0.01
		(0.31)	(0.13)	(0.45)	(1.00)	(0.16)	(0.09)	(0.16)
	δ	-0.04	-0.07	-0.02	-0.36^{***}	-0.08^{***}	0.02	-0.03
		(0.05)	(0.07)	(0.03)	(0.12)	(0.02)	(0.02)	(0.03)
R^2		0.01	0.01	0.23	0.49	0.14	0.004	0.01

Notes: This table reports the predictive power of lagged inflation rate, 3-month Tbill rate and forecast revisions on forecast errors. Inflation rate is the lagged quarterly CPI inflation rate. 3M Tbill rate is the lagged quarterly average 3-month Tbill rate.

Table 1.23: Predictive Power of Lagged Inflation Rate and 3-Month Tbill Rate on Forecast Errors

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: $w_{t+h} - \mathcal{F}_{i,t}w_{t+h} = \beta_0^C + \delta z_{t-1} + u_{i,t,t+h}$							
inflation	-0.23^{***}	-0.26^{***}	-0.02	-0.16	-0.08^{**}	-0.08^{***}	-0.06^{**}
	(0.06)	(0.09)	(0.04)	(0.1)	(0.04)	(0.02)	(0.02)
R^2	0.11	0.08	0.19	0.07	0.04	0.09	0.11
3M T-bill rate	-0.16^{***}	-0.21^{***}	-0.03	-0.13	-0.21^{***}	-0.006	-0.02
	(0.06)	(0.06)	(0.04)	(0.09)	(0.04)	(0.03)	(0.03)
R^2	0.08	0.08	0.17	0.05	0.14	0.05	0.1
Panel B: $w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \delta z_{t-1} + u_{t,t+h}$							
inflation	-0.22^{***}	-0.31^{***}	-0.09^{**}	-0.22^{***}	-0.07^{***}	-0.08^{***}	-0.06^{***}
	(0.07)	(0.1)	(0.04)	(0.07)	(0.02)	(0.02)	(0.02)
R^2	0.08	0.11	0.06	0.05	0.02	0.04	0.03
3M T-bill rate	-0.05	-0.07	-0.01	-0.34^{**}	-0.08^{***}	0.02	-0.02
	(0.05)	(0.07)	(0.05)	(0.14)	(0.02)	(0.02)	(0.03)
R^2	0.01	0.01	0.001	0.3	0.06	0.004	0.007

Notes: Inflation rate is the lagged quarterly CPI inflation rate. 3M Tbill rate is the lagged quarterly average 3-month Tbill rate.

1.9 Data Appendix

1.9.1 Construction of Variables

For the construction of forecast errors and forecast revisions using variables from SPF and the real time macro data, over the forecast horizon of one year, the method is similar to that of BGMS (2020). Here I include the instructions for the set of variables used in this chapter for the completeness of the chapter. The time of survey is included. For some variables, SPF asks forecasters to forecast in terms of levels of those variables. In such cases, I transform levels into growth rates.

1. NGDP

   - Time: During the third week of the second month of the quarter.
   - Question: The level of the current quarter's nominal GDP and for the upcoming four quarters.
   - Forecast: The nominal GDP growth from the end of quarter $t-1$ to the end of quarter $t+3$ is given by $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - 1$. In this formula, t represents the quarter for which the forecast is being made, and x is the GDP level for a specified quarter. The x_{t-1} term refers to the initial release of the actual value from quarter $t-1$, which is accessible by the time of the forecast in quarter t.
   - Revision: $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - \frac{\mathcal{F}_{t-1} x_{t+3}}{\mathcal{F}_{t-1} x_{t-1}}$
   - Actual forecast: $\frac{x_{t+3}}{x_{t-1}} - 1$, utilizing real-time macroeconomic data released during quarter (t+4).

2. RGDP

   - Time: During the third week of the second month of the quarter.
   - Question: The level of the current quarter's real GDP and for the upcoming four quarters.
   - Forecast: The real GDP growth from the end of quarter $t-1$ to the end of quarter $t+3$ is given by $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - 1$. In this formula, t represents the quarter for which the forecast is being made, and x is the GDP level for a specified quarter. The x_{t-1} term refers to the initial release of the actual value from quarter $t-1$, which is accessible by the time of the forecast in quarter t.
   - Revision: $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - \frac{\mathcal{F}_{t-1} x_{t+3}}{\mathcal{F}_{t-1} x_{t-1}}$
   - Actual forecast: $\frac{x_{t+3}}{x_{t-1}} - 1$, utilizing real-time macroeconomic data released during quarter (t+4).

3. GDP Price Index

- Time: During the third week of the second month of the quarter.
- Question: The level of the current quarter's GDP price index and for the upcoming four quarters.
- Forecast: The real GDP growth from the end of quarter $t-1$ to the end of quarter $t+3$ is given by $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - 1$. In this formula, t represents the quarter for which the forecast is being made, and x is the GDP level for a specified quarter. The x_{t-1} term refers to the initial release of the actual value from quarter $t-1$, which is accessible by the time of the forecast in quarter t.
- Revision: $\frac{\mathcal{F}_t x_{t+3}}{x_{t-1}} - \frac{\mathcal{F}_{t-1} x_{t+3}}{\mathcal{F}_{t-1} x_{t-1}}$
- Actual forecast: $\frac{x_{t+3}}{x_{t-1}} - 1$, utilizing real-time macroeconomic data released during quarter (t+4).

4. CPI

- Time: During the third week of the second month of the quarter.
- Question: Current quarter's CPI growth rate and for the upcoming four quarters.
- Forecast: $\mathcal{F}_t x_{t+3}$, where t is the quarter of forecast and x is the CPI growth rate.
- Revision: $\mathcal{F}_t x_{t+3} - \mathcal{F}_{t-1} x_{t+3}$.
- Actual forecast: x_{t+3}.

5. AAA

- Time: During the third week of the second month of the quarter.
- Question: Current quarter's AAA corporate bond yield and for the upcoming four quarters.
- Forecast: $\mathcal{F}_t x_{t+3}$, where t is the quarter of forecast and x is the level of AAA corporate bond yield.
- Revision: $\mathcal{F}_t x_{t+3} - \mathcal{F}_{t-1} x_{t+3}$.
- Actual forecast: x_{t+3}.

6. TBILL

- Time: Around the 3rd week of the middle month in the quarter.
- Question: Current quarter's 3-month treasury yield and for the upcoming four quarters.
- Forecast: $\mathcal{F}_t x_{t+3}$, where t is the quarter of forecast and x is the level of 3-month treasury yield.

- Revision: $\mathcal{F}_t x_{t+3} - \mathcal{F}_{t-1} x_{t+3}$.
- Actual forecast: x_{t+3}.

7. TBOND

- Time: Around the 3rd week of the middle month in the quarter.
- Question: Current quarter's 10-year treasury yield and for the upcoming four quarters.
- Forecast: $\mathcal{F}_t x_{t+3}$, where t is the quarter of forecast and x is the level of 10-year treasury yield.
- Revision: $\mathcal{F}_t x_{t+3} - \mathcal{F}_{t-1} x_{t+3}$.
- Actual forecast: x_{t+3}.

Chapter 2

Adaptive Expectations and Over-/Under-reaction to New Information

2.1 Introduction

The Rational Expectations Hypothesis has been the workhorse model of macroeconomic expectations. Nevertheless, contemporary studies, rich in empirical evidence drawn from diverse survey data, have challenged its supremacy. A line of work reveals the predictability of forecast errors using different forecast surveys (Coibion and Gorodnichenko 2012, 2015; Fuhrer 2019; Bordalo, Gennaioli, Ma and Shleifer 2020; Adam, Marcet and Beutel 2017, etc). This surge in data has given birth to innovative models of expectation formation, aiming to better align theories with observed empirical patterns. Among those models, diagnostic expectations model has gathered substantial attention among researchers. Grounded in psychological principles, the diagnostic expectations model is able to explain the predictability of forecast errors in the survey data, and elucidate other empirical puzzles in finance and macro, such as the excess volatility of stock prices, credit, and investment (Gennaioli, Ma and Shleifer 2016; Bordalo, Gennaioli and Shleifer 2018; Bordalo, Gennaioli, La Porta and Shleifer 2019).

This chapter underscores a limitation of the generalized diagnostic expectations model: its inability to capture the stickiness exhibited by certain variables in survey data, notably the inflation rate and the 3-month Tbill. Stickiness here is quantified using the regression coefficients of Error-on-revision regressions (commonly known as the CG regression) introduced by Coibion and Gorodnichenko (2015). This method is the standard approach for determining over- or under-reactions to new information in survey-based observations. In contrast to the diagnostic expectations model's shortcomings, this research demonstrates the adaptive expectations model's versatility in aligning with this dimension of forecast surveys. Additionally, I also illustrate that adaptive expectations' backward-looking feature can explain two other phenomena discussed in the literature: horizon-increasing over-reaction (Afrouzi et al. 2021; Bordalo et al. 2019; Bouchaud et al. 2019; Giglio and Kelly 2018), and dynamic responses of outcomes and forecasts: forecasts initially under-react to shocks compared with outcome but over-shoot later on (Angeletos, Huo and Sastry 2021).

Starting from the 1950s, the adaptive expectations model (Cagan 1956; Friedman 1957) became the standard approach to model macroeconomic expectations. Adaptive expectations are backward-looking, positing that the current forecast is a weighted average of the current observation and the previous forecast. Over time, it faced criticism primarily because of its inability to respond quickly to clearly understood regime shifts, a shortcoming highlighted by Lucas (1976).[1] Consequently, the adaptive expectations model was gradually overshadowed by the Rational Expectations Hypothesis. Yet, this chapter endeavors to argue that, despite its criticisms, the inherent backward-looking nature of adaptive expectations can effectively explain several patterns presented in survey data, where the rational expectations model clearly can not.

Central to this chapter is an exploration of how the CG regression coefficient behaves under the adaptive expectations framework. If we assume the underlying variable follows an AR(1) process, then the CG regression coefficients are determined by the persistence of the underlying variable and the weighting parameter under adaptive expectations. The weighting parameter gauges the weight put on the current observation. The CG coefficient is decreasing in the weighting parameter and increasing in the persistence of the underlying variable. In other words, the higher the weighting parameter, the lower the persistence, the more overreaction there is in expectations formation. Interestingly, with a sufficiently low weighting parameter, the adaptive expectations model can achieve arbitrary levels of forecast stickiness. This high degree of flexibility is absent in the diagnostic expectations model. The reason is that, under diagnostic expectations model, the forecasts are the weighted average of two adjacent rational forecasts, which limits the stickiness.

The previous insights apply to the consensus level regressions without considering idiosyncratic noise.[2]Relying on the same argument in chapter 1, the gap between the consensus and individual level regression coefficients can be explained by the presence of idiosyncratic noise in individual forecasts. The CG equation features the current forecast on both its left and right sides, but with contrasting signs. Any idiosyncratic noise in individual forecasts, regardless of their cause, explain the difference between the consensus and individual regression coefficients.

[1]For example, when the central bank adopts a more aggressive inflation-fighting monetary policy after a prolonged period of high inflation, the inflation expectation of the public could very well fall quickly, whereas under adaptive expectations, inflation expectations only fall gradually.

[2]In this chapter, I assume the number of forecasters are large, so idiosyncratic noise does not impact the consensus level regression coefficients.

I match the model implied regression coefficients with the empirical counterpart, separately for the consensus and individual level. The adaptive expectations model performs commendably, matching twelve out of fourteen coefficient estimates, as opposed to the generalized diagnostic expectations model which can only reconcile six. The weighting parameter is heterogeneous across different variables. Variables with higher consensus level CG coefficients, including the CPI, the GDP Price Index inflation and the 3-month Tbill yield, exhibit lower weighting parameter, meaning forecasts react less to the current observation.

Further examination reveals the adaptive expectations model's ability to account for two additional sets of empirical evidence. First, I study whether the CG coefficients decrease over the forecast horizon under adaptive expectations, meaning there is more over-reaction as the forecast horizon extends. Under the assumption of horizontal term structure of forecasts, the CG coefficients are indeed decreasing over horizon under adaptive expectations. The reason is that forecasters do not take into account that the impact of shocks diminishes over time. Second, I investigate the implications of adaptive expectations on the dynamic response of forecast errors. It turns out that the initial under-reaction and later over-shooting documented in Angeletos, Huo and Sastry (2021) is a natural implication of adaptive expectations model. Upon a shock, forecasts adjust in the direction of the actual outcome. But forecasts can never adjust too much, as they are always anchored by the previous forecasts.

This chapter is an extension of chapter one, focusing on adaptive expectations model, which were overshadowed by the rational expectations hypothesis. A significant critique against adaptive expectations, termed the Lucas Critique, arises due to its exogenously set weighting parameter, which remains fixed regardless of policy shifts. However, Evans and Ramey (2006) show that, there are regimes where the weighting parameter is invariant. To further advocate for the adaptive expectations model, this chapter shows that the model's backward-looking feature provides valuable insights in understanding empirical puzzles in forecast surveys.

2.2 Adaptive Expectations

The adaptive expectations hypothesis can be traced back to 1930s, and was formally introduced in the 1950s (see Cagan (1956), Friedman (1957)). Adaptive expectations were widely used in macroeconomics in the 1960s and 1970s. For example, inflation expectations were usually modeled as adaptive expectations.[3] Denote the variable of interest as w_t, and people's period t forecast for the period $t+1$ outcome as

[3] See Evans and Honkaphohja (2001).

$\mathcal{F}_t w_{t+1}$. The adaptive expectations forecast is given by

$$\mathcal{F}_t w_{t+1} = \gamma w_t + (1-\gamma)\mathcal{F}_{t-1} w_t \tag{2.1}$$

The current forecast of future variable w_{t+1} is a weighted average of the current observation and the previous forecast. γ is the (adaptive) weighting parameter. According to adaptive expectations, people form expectations of the future according to what they observe in the current period and past expectations. Since past expectations also depend on observations in the further past, current expectations depends on all the historical observations. This can be illustrated by the following equation, which is obtained by iterating equation (2.1) backward.

$$\mathcal{F}_t w_{t+1} = \gamma \sum_{i=0}^{\infty} (1-\gamma)^i w_{t-1-i} \tag{2.2}$$

By the equation above, γ has to be between $(0, 2)$ so that $\mathcal{F}_t w_{t+1}$ is not explosive. For values of γ within the interval [0,1], equation (2.2) can be interpreted in the following manner: the current expectation is a weighted average of past observations, with forecasters assigning lower weights to observations that are further in the past.

2.3 CG Regressions

Since the seminal work of Coibion and Gorodnichenko (2015), many researchers have run regressions with forecast errors as dependent variables, and forecast revisions as the explanatory variables to detect the existence of over-/under-reaction in expectations formation. Denote the macroeconomic variable as w_t, e.g., inflation. The following two equations are the "forecast errors on forecast revisions", henceforth Error-on-revision, or CG, regressions:

$$w_{t+1} - \bar{\mathcal{F}}_t w_{t+1} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+1} - \bar{\mathcal{F}}_{t-1} w_{t+1}) + u_{t,t+1}, \tag{2.3}$$

$$w_{t+1} - \mathcal{F}_{i,t} w_{t+1} = \beta_0^I + \beta^I(\mathcal{F}_{i,t} w_{t+1} - \mathcal{F}_{i,t-1} w_{t+1}) + u_{i,t,t+1}. \tag{2.4}$$

The consensus level regression is given by equation (2.3), while the individual level regression is represented by equation (2.4). A key distinction between these two equations is that in equation (2.3), $\bar{\mathcal{F}}_t w_{t+1}$ is the mean forecast across all forecasters, whereas in Equation (2.4), $\mathcal{F}_{i,t} w_{t+1}$ denotes an individual's forecast. Both equations (2.3) and (2.4) have forecast errors as the dependent variable. The right-hand sides of the equations correspond to forecast revisions at the consensus and individual levels. A positive β^C is interpreted as under-reaction in the literature, meaning that when a forecast revision is positive, the subsequent forecast error also tends to be positive. This implies that the upward forecast revision is insufficient compared to the true realization of w_{t+1}. Conversely, a negative β^C indicates over-reaction, as a positive forecast revision generally leads to a negative forecast error, suggesting that the upward revision is excessive compared to the true realization of w_{t+1}. Similar interpretations apply to $\beta^I < 0$ and $\beta^I > 0$, which are viewed as over-reaction and under-reaction at the individual level, respectively. In most empirical studies, it is found that $\hat{\beta}^C > 0$ and $\hat{\beta}^I < 0$ (Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma and Shleifer (2020), Fuhrer (2018)).

To compute forecast revisions in CG regressions, we require two periods ahead forecasts, denoted by $\mathcal{F}_t w_{t+2}$. However, adaptive expectations do not provide an explicit structure for $\mathcal{F}_t w_{t+2}$ or forecasts of varying horizons. In the primary exercise, I assume that the term structure of adaptive expectations is horizontal, as made explicit in Assumption 1.

Assumption 1 *Throughout the analysis below, I assume the term structure of expectations is horizontal:*

$$\mathcal{F}_t w_{t+1} = \mathcal{F}_t w_{t+i}, i \geq 2 \tag{2.5}$$

This assumption can be justified in two ways. First, if a "Law of Iterated Expectation" for adaptive expectations were to exist, assumption 1 would be an immediate implication.[4] Second, when γ remains consistent across different forecast horizons, Assumption 1 holds true. This can be observed from equation (2.2).

Assumption 1 is used in the calculation of forecast revisions in CG regressions using the simulated data, which will be discussed later. To test the assumption's validity, we substitute $\bar{\mathcal{F}}_{t-1} w_{t+1}$ and $\mathcal{F}_{i,t-1} w_{t+1}$ in equations (2.3) and (2.4) with $\bar{\mathcal{F}}_{t-1} w_t$ and $\mathcal{F}_{i,t-1} w_t$, respectively, and then compare the resulting regression coefficients. That is, we simply calculate the forecast revision as the revision in forecasts of the same forecast horizon. The robustness check in section 2.8.5 reveals that the regression coefficients are quantitatively quite similar, even when we compute the forecast revision as $\mathcal{F}_{i,t} w_{t+1} - \mathcal{F}_{i,t-1} w_t$ instead of $\mathcal{F}_{i,t} w_{t+1} - \mathcal{F}_{i,t-1} w_{t+1}$ in the regression. This holds true for the consensus level regression as well. Hence, assumption 1 is not critical to our analysis of CG regression coefficients under adaptive expectations

2.4 Adaptive Expectations and Over-/Under-reaction

Do adaptive expectations result in over- or under-reaction to new information, as indicated by the Error-on-revision regression coefficients? To address this question,

[4] Assume there is a version of "Law of Iterated Expectations" under adaptive expectations,

$$\mathcal{F}_t w_{t+2} = \mathcal{F}_t(\mathcal{F}_{t+1} w_{t+2}) = \mathcal{F}_t(\gamma w_{t+1} + (1-\gamma)\mathcal{F}_t w_{t+1}) = \mathcal{F}_t w_{t+1},$$

where the first equation follows from the assumed "Law of Iterated Expectations" under adaptive expectations.

let's start with a straightforward example. Suppose the variable follows an AR(1) process:

$$w_t = \rho w_{t-1} + e_t, \tag{2.6}$$

where e_t follows a normal distribution, $e_t \sim N(0, \sigma_e^2)$. This section will demonstrate that whether adaptive expectations result in over- or under-reaction to new information depends on the relative magnitude of the adaptive parameter, γ, and the persistence of the AR(1) process, ρ. With a fixed γ, a higher persistence in ρ yields stronger under-reaction to new information; conversely, a lower persistence in ρ prompts stronger over-reaction.

2.4.1 The Case of Single Transitory Shock

In this section, I depict the impulse response function of the variable, along with forecasts formulated using adaptive expectations, the associated forecast errors, and forecast revisions, all in response to a single shock to the variable at period 1. Subsequently, I will discuss the determinants of over- or under-reaction under adaptive expectations.

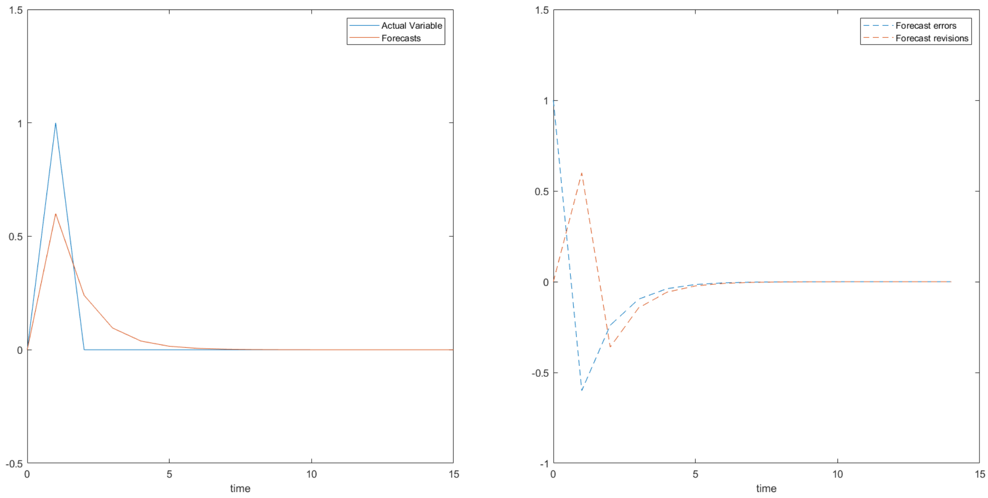
When $\rho = 0$

When $\rho = 0$, the variable has zero persistence, meaning that the influence of a shock does not carry forward into subsequent periods. In this case, regardless of the specific value of γ in the range of $(0, 2)$, expectation formation will exhibit over-reaction to new information. This is because any adjustment of forecasts in response to the shock is an overreaction, given that the shock does not affect future periods. This concept is illustrated in Figure 2.1, where γ is set to be $\frac{3}{5}$. In this section, while plotting the impulse response functions, the timing of variables is as follows: For period t, I plot the actual variable w_t alongside the forecasts for the next period, represented as $\mathcal{F}_t w_{t+1}$. The forecast errors at period t are computed as $w_{t+1} - \mathcal{F}_t w_{t+1}$, and the forecast revisions at period t are calculated as $\mathcal{F}_t w_{t+1} - \mathcal{F}_{t-1} w_t$.

The left panel of the figure shows that the forecasts rise in response to the shock. The increase in the forecasts is not as substantial as the actual shock, but the effect on the forecasts persists. Despite the fact that the shock is a one-time event, the forecasts decrease gradually over time due to the inherent stickiness in the adaptive expectations framework. This means that even though the actual variable has returned to its steady state level (which is zero in this numerical example), the forecasts still remain positive.

In the right panel, it's shown that forecast revisions and forecast errors are negatively correlated for the first three periods following the shock, but then they start moving together. When we run an Error-on-revision regression, the initial period of negative correlation is what dominates the results. As shown in Lemma

Figure 2.1: IRFs of the Actual Variable, Forecasts, Forecast Errors and Forecast Revisions



Notes: This figure illustrates the impulse response function of the actual variable, forecasts, forecast errors, and forecast revisions in response to a single shock. The forecasts at period t are designated for the next period and represented as $\mathcal{F}_t w_{t+1}$. The forecast errors at period t are calculated as $w_{t+1} - \mathcal{F}_t w_{t+1}$. The forecast revisions at period t are computed as $\mathcal{F}_t w_{t+1} - \mathcal{F}_{t-1} w_t$. Here, the parameter γ is set to $\frac{3}{5}$. The persistence of the AR(1) process is zero.

2, β is indeed negative, and is not affected by the value of γ.[5] The proof of Lemma 2 is in the appendix.

Lemma 2 *If $t = 0, 1, 2, ...,$ and the time series are sufficiently long, and given that the persistence of the actual variable is zero (i.e., $\rho = 0$), then in the case of a one-time shock to the actual variable at period 1, the coefficient β obtained from running the Error-on-revision regression is equal to $-1/2$ for any value of γ within the range (0,2).*

The negative correlation between forecast errors and revisions indeed indicates an over-reaction to new information under adaptive expectations. The intuition behind this over-reaction is as follows: when there is a one-time shock that only affects the first period, any adjustment of expectations in this period can be deemed as an over-reaction, because the shock does not impact the actual variable in subsequent periods. However, agents who form expectations adaptively are not aware that the shock's influence is limited to the first period only, and as a result, they overreact to this shock.

[5]The Error-on-revision regression in this scenario, featuring a single shock and a long time series, results in many forecast errors and forecast revisions being close to zero. This means that the regression coefficient is predominantly determined by the observations from the initial few periods following the shock. During these initial periods, we see a strong negative correlation between forecast errors and forecast revisions. This leads to the conclusion of over-reaction in the adaptive expectations model in the face of a single, non-persistent shock.

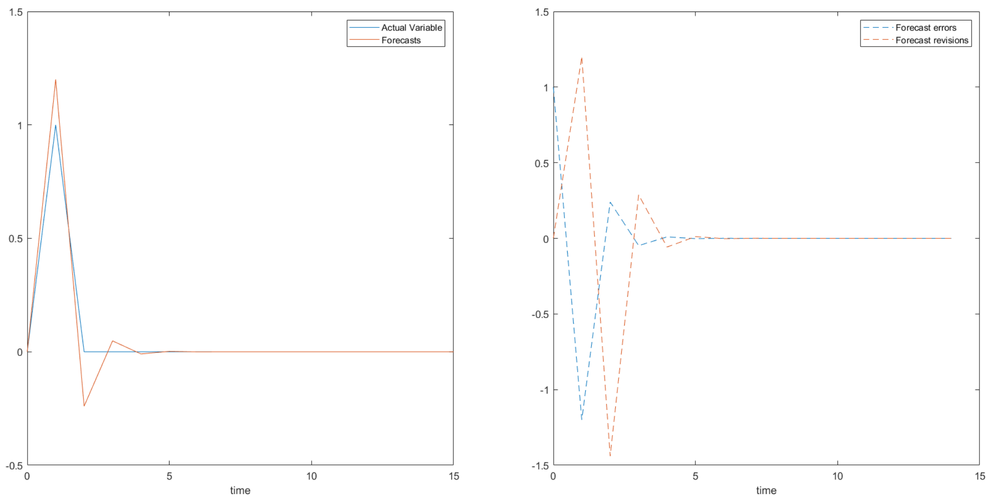
In Figure 2.2, when γ is in the interval $(1, 2)$ (specifically, $\gamma = 1.2$), we observe that the forecasts rise more than the magnitude of the shock itself. This pattern exhibits a form of "extrapolation," as characterized in works such as Barberis, Greenwood, Jin, and Shleifer (2018), Liao, Peng, and Zhu (2022).

What is particularly interesting is the oscillatory behavior of the forecasts. This occurs because, when $\gamma > 1$, forecasts tend to over-adjust in the direction of past forecast errors. Essentially, forecasters are attributing too much weight to recent changes, causing them to overshoot the actual variable in their forecasts. This leads to corrections in subsequent periods, resulting in a pattern of oscillation around the steady state. This is easier to see by rewriting equation (2.1) as follows:

$$\mathcal{F}_t w_{t+1} = w_t + \alpha(w_t - \mathcal{F}_{t-1} w_t), \tag{2.7}$$

where $\gamma = 1 + \alpha$. The forecast about next period's outcome is equal to the current observation adjusted in the direction of the last forecast error.

Figure 2.2: IRFs of the Actual Variable, Forecasts, Forecast Errors and Forecast Revisions



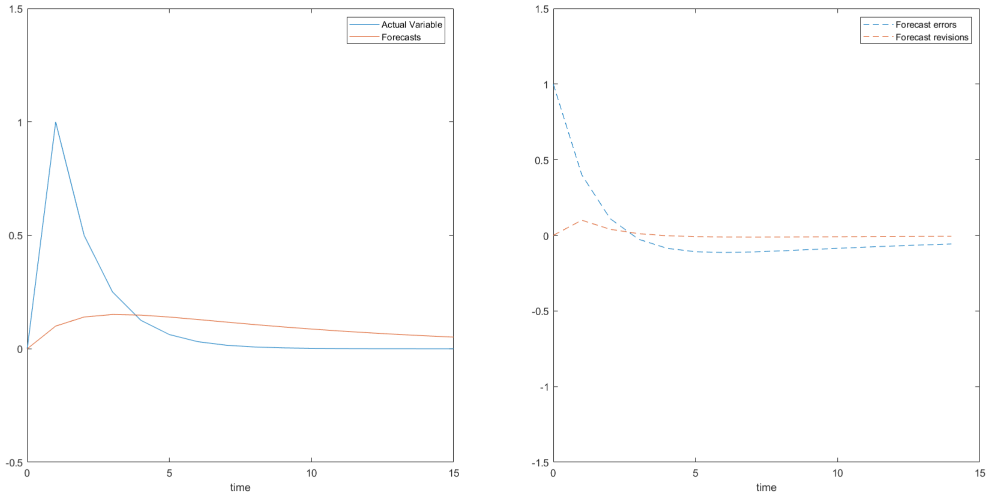
Notes: This figure illustrates the impulse response function of the actual variable, forecasts, forecast errors, and forecast revisions in response to a single shock. The forecasts at period t are designated for the next period and represented as $\mathcal{F}_t w_{t+1}$. The forecast errors at period t are calculated as $w_{t+1} - \mathcal{F}_t w_{t+1}$. The forecast revisions at period t are computed as $\mathcal{F}_t w_{t+1} - \mathcal{F}_{t-1} w_t$. Here, the parameter γ is set to 1.2.

When $\rho > 0$

When the variable of interest exhibits persistence, i.e., $\rho > 0$, the behavior of adaptive expectations — whether they exhibit over- or under-reaction to new information — depends on the relative magnitudes of the adaptive parameter γ and the persistence parameter ρ. If we fix γ, the following patterns emerge: As ρ increases, there is stronger evidence of under-reaction. As ρ decreases, there is stronger evidence of over-reaction. Similarly, if we fix ρ: As γ increases, there is stronger evidence of over-reaction. As γ decreases, there is stronger evidence of under-reaction.

Figure 2.3 illustrates a scenario with $\gamma = 0.1$ and $\rho = 0.5$. Here, the implied value of β is equal to 4.1. By contrast, Figure 2.4 depicts a situation where $\gamma = 0.9$ and $\rho = 0.5$. In this instance, the implied value of β is -0.19. A comparison of these two graphs unveils some insightful dynamics. When γ is large, forecast errors and forecast revisions exhibit stronger negative correlation in the first few periods: when forecasters see a large w_t, they aggressively revise their forecasts upward, at the same time making the most negative forecast errors. This strong negative correlation in the first few periods determines the negative β when γ is large.

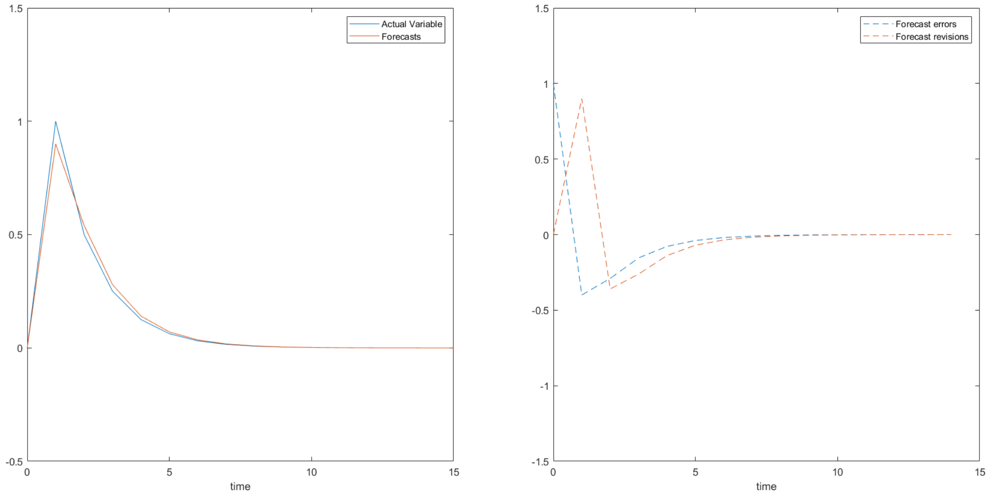
Figure 2.3: IRFs of the Actual Variable, Forecasts, Forecast Errors and Forecast Revisions



Notes: This figure illustrates the impulse response function of the actual variable, forecasts, forecast errors, and forecast revisions in response to a single shock. The forecasts at period t are designated for the next period and represented as $\mathcal{F}_t w_{t+1}$. The forecast errors at period t are calculated as $w_{t+1} - \mathcal{F}_t w_{t+1}$. The forecast revisions at period t are computed as $\mathcal{F}_t w_{t+1} - \mathcal{F}_{t-1} w_t$. Here, the parameter γ is set to $\frac{1}{10}$. ρ is set to $\frac{1}{2}$.

To understand why a lower ρ value, given a fixed γ, tends to imply over-reaction, we need to draw a parallel with the scenario where $\rho = 0$. In this situation, people generally fail to recognize that a shock has only a transitory impact on future periods. As a consequence, they place excessive weight on their current observations when formulating forecasts, leading to over-reaction. Conversely, when ρ is large, the shock in period one exerts a more prolonged influence on the actual variable in subsequent periods. In response to this, individuals relatively underweight their current observations when making forecasts. This tendency results in under-reaction to new information, explaining why a large ρ can induce under-reaction. This same line of reasoning can help clarify why a larger γ is likely to lead to over-reaction. An

Figure 2.4: IRFs of the Actual Variable, Forecasts, Forecast Errors and Forecast Revisions



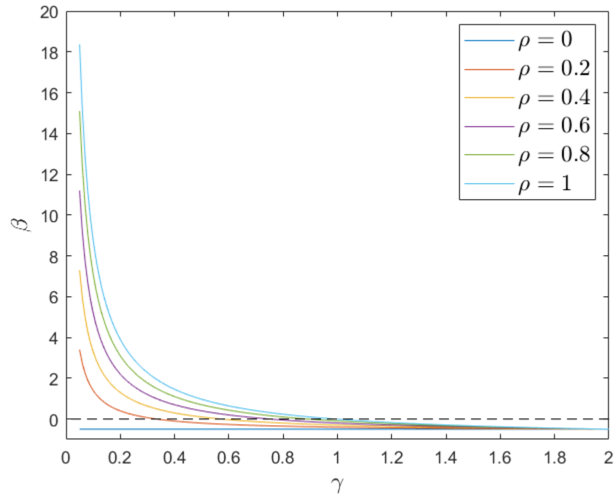
Notes: This figure illustrates the impulse response function of the actual variable, forecasts, forecast errors, and forecast revisions in response to a single shock. The forecasts at period t are designated for the next period and represented as $\mathcal{F}_t w_{t+1}$. The forecast errors at period t are calculated as $w_{t+1} - \mathcal{F}_t w_{t+1}$. The forecast revisions at period t are computed as $\mathcal{F}_t w_{t+1} - \mathcal{F}_{t-1} w_t$. Here, the parameter γ is set to $\frac{9}{10}$. ρ is set to $\frac{1}{2}$.

increased γ value essentially implies that individuals put a higher weight on their most recent observation while adjusting their forecasts. As a result, they are more reactive to changes, often leading to over-adjustment of their forecasts in response to new information, hence the over-reaction.

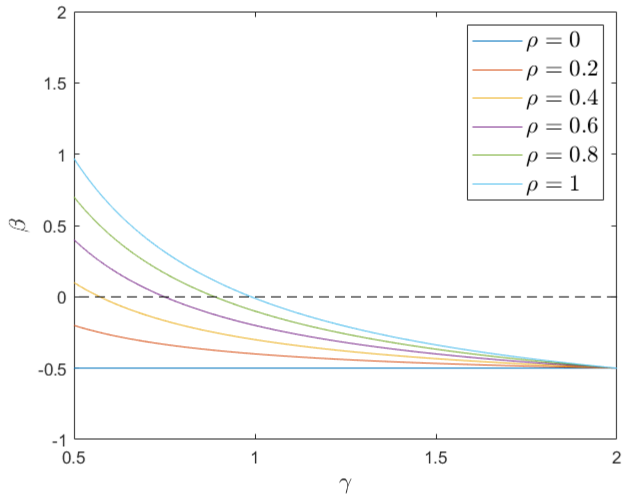
Figure 2.5 displays the relationship between β and γ, while also considering the effect of ρ. In panel (a), the plot covers the range $(0, 2)$ for γ. Panel (b), on the other hand, offers a zoomed-in version around the range $(0.5, 2)$ for γ. The plots indicate that β is a downward sloping function of γ. Keeping γ constant, β is increasing in ρ. As γ approaches zero, β tends to infinity. Conversely, when γ gets closer to 2, β descends towards $-\frac{1}{2}$. Importantly, note that the value of β is not influenced by the value of ρ when $\rho = 2$. That is, ρ is equal to $-\frac{1}{2}$ no matter the value of ρ. Therefore, the range for β when $0 < \rho \leq 1$ and $0 < \gamma \leq 2$ is always $[-\frac{1}{2}, +\infty)$. The reason why β tends towards positive infinity as γ approaches zero is that when γ is very small, forecasts become exceedingly sticky. This leads to minimal forecast revisions in terms of magnitude. Given that forecast revisions form the right-hand side of the regression equations, the regression coefficient tends to be large under these circumstances.

The key findings of this section are summarized in the following lemma and conjecture. The proof of Lemma 3 is in the appendix. Conjecture 1 has not been proved analytically due to the complex expression of β in terms of ρ and γ, but it is confirmed in figure 2.6 under various values of ρ and γ.

Figure 2.5: β as a Function of γ. One single shock.



(a)



(b)

Notes: This figure depicts β as a function of γ under adaptive expectations, specifically in the context of a single shock, for varying values of ρ. Panel (b) offers a more detailed view around $\gamma = 1$, zooming in on the region in panel (a).

Lemma 3 *Let $t = 0, 1, 2, \dots,$ and assume the time series is sufficiently long. The persistence of the actual variable is positive, denoted by $\rho > 0$. When there is a one-time shock to the variable at period 1, β resulted from running the Error-on-revision regression is characterized by:*

- $\lim_{\gamma \to 2} \beta = -\frac{1}{2}$.
- $\lim_{\gamma \to 0} \beta = +\infty$.

Conjecture 1 *The β obtained in Lemma 3 is monotonically decreasing in γ and monotonically increasing in ρ.*

2.4.2 The Case of A Series of Shocks

The case of a series of shocks closely resembles that of a single shock. For a given set of values for ρ and γ, I simulate the actual variables and forecasts following adaptive expectations for $20,000$ periods, and obtain the CG regression coefficients using the simulated data. As depicted in Panel (a) figure 2.6, the relationship between β and γ mirrors the relationship observed in the case of a single shock (as shown in figure 2.5). β decreases with increasing γ. When γ approaches zero, β becomes exceedingly large, whereas when γ approaches 2, β nears $-\frac{1}{2}$. In Panel (b), I plot the relationship between β and ρ. When γ is fixed, β is increasing in ρ. The crucial takeaway is that when $\{\rho, \gamma\}$ is within the range $[0,1]x(0,2]$, β has a lower limit of $-\frac{1}{2}$ but can attain substantially high values, particularly if γ is sufficiently small and ρ is not too low.

Next, I present two examples that demonstrate the evolution of the actual variable and forecasts when γ assumes different values. In figure 2.7, I chart the time series of the actual outcomes and forecasts according to adaptive expectations, using γ values of $\frac{1}{2}$ and $\frac{3}{2}$, respectively. When γ equals $\frac{1}{2}$, as in Panel (a), the forecasts appear less volatile, or more persistent, when compared to the actual realizations. Conversely, when γ equals $\frac{3}{2}$, as in Panel (b), the forecasts are more volatile, exhibiting stronger extrapolative behavior, compared to the actual variables.

2.4.3 Individual Level Regression Coefficients

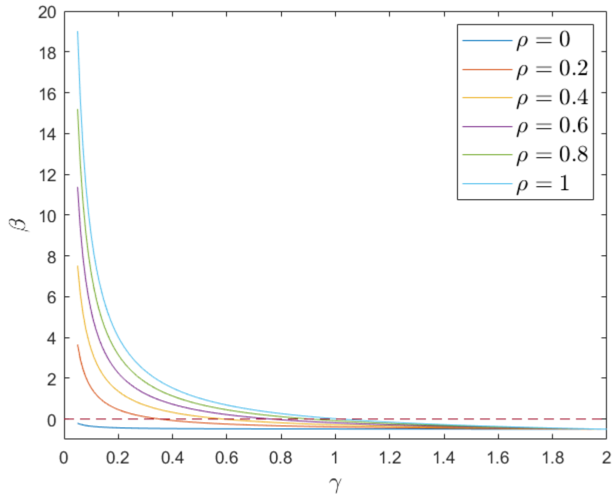
In this section, I investigate how the individual level regression coefficient is different from the consensus level regression coefficient. As argued in chapter one, idiosyncratic noise accounts for the gap between the regression coefficients at the consensus level and individual level from an econometric perspective, no matter what the reason is behind such idiosyncratic noise. In the sections above, I have unpacked how CG regression coefficients are determined under adaptive expectations model without noise. Here I provide numerical examples of how the individual level regression coefficients are determined considering the existence of idiosyncratic noise.

I simulate 30 forecasters indexed by i using the following forecasting rule for 10000 periods:

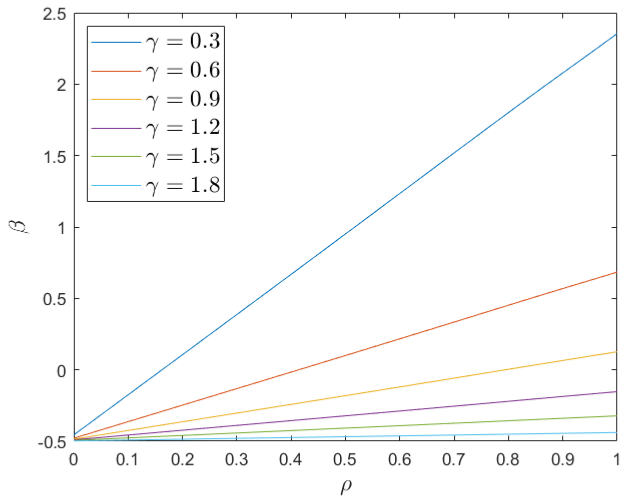
$$\mathcal{F}_{i,t}w_{t+1} = \gamma w_t + (1-\gamma)\mathcal{F}_{i,t-1}w_t + \eta_{i,t} \tag{2.8}$$

where $\eta_{i,t} \sim N(0, \sigma_\eta^2)$, and is i.i.d across forecasters and time. Each period, on top of the usual adaptive expectations rule, forecasts are subject to random idiosyncratic noise, which can result from lots of reasons: mood, different forecasting models, statistical measure errors, judgment noise in the sense of Sunstein, Kahneman and Sibony (2021). Here it is a reduced way of modeling such noise, no matter what the reason is behind such noise. By the nature of the adaptive expectations model, the impact of the idiosyncratic noise on forecasts is not transitory. The impact of

Figure 2.6: β as a Function of γ. A series of shocks.



(a)

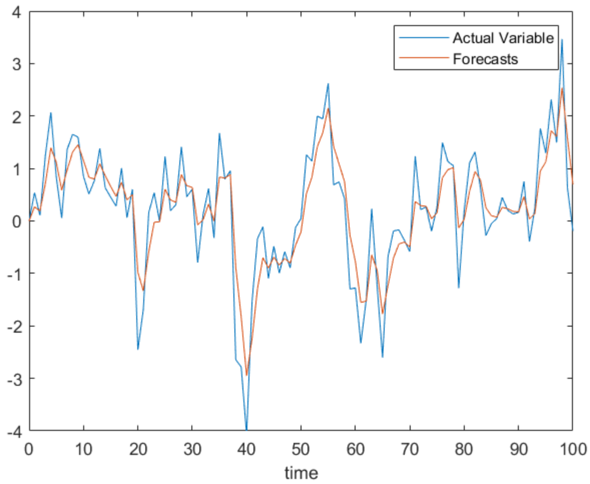


(b)

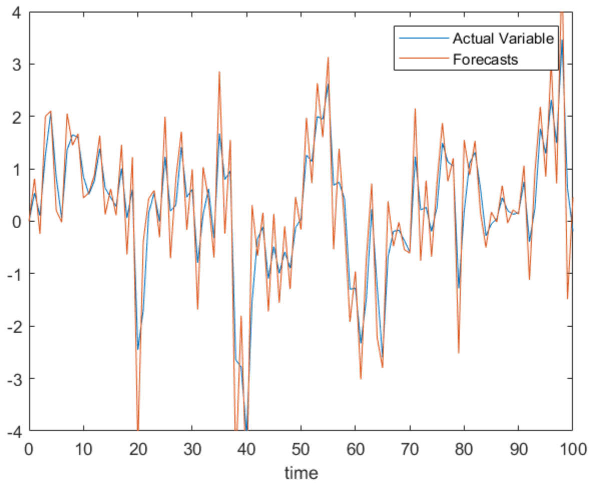
Notes: Panel (a) depicts β as a function of γ under adaptive expectations, specifically in the context of a series of shocks, for varying values of ρ. Panel (b) depicts β as a function of ρ under adaptive expectations, specifically in the context of a series of shocks, for varying values of γ.

noise on forecasts persists into the future through the backward-looking feature of adaptive expectations. In Figure 2.8, we plot β as a function of γ, with different variances of idiosyncratic noise, σ_e. First, it is noticeable that β no longer decreases monotonically with γ. Rather, β demonstrates a hump-shaped relationship with increasing γ. The downward-sloping segment shares the same reasoning as the case without idiosyncratic noise: higher γ values lead forecasters to extrapolate current observations into the future, resulting in more significant over-reaction to new information. However, when γ is small and approaches zero, β is primarily driven downwards by the idiosyncratic noise term. When $\gamma = 1$, the OLS estimator of β^I ceases to be a consistent estimator. This can be demonstrated by rearranging

Figure 2.7: Time Series of Actual Variable and Forecasts.



(a)



(b)

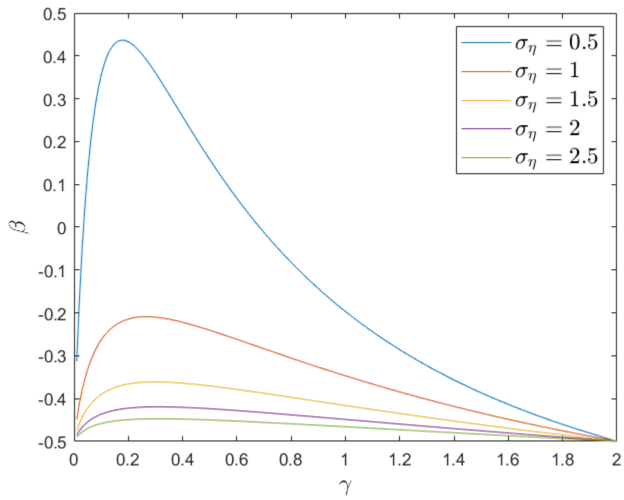
Notes: This figure illustrates the time series of the actual variable, following an AR(1) process, and forecasts, following adaptive expectations. The persistence parameter of the AR(1) process is set at $\rho = 0.6$. The weighting parameter in adaptive expectations is $\gamma = 0.5$ in Panel (a), and $\gamma = 1.5$ in Panel (b).

equation (2.8) and representing the forecast revision as follows:

$$\mathcal{F}_{i,t}w_{t+1} - \mathcal{F}_{i,t-1}w_t = \gamma w_t - \gamma \mathcal{F}_{i,t-1}w_t + \eta_{i,t} \tag{2.9}$$

when γ is close to zero, the first two terms on the right-hand-side is negligible and forecast revisions mainly capture the idiosyncratic noise term. As argued in chapter one, this idiosyncratic noise term pushes β towards negative. This is why there is a steep drop of β when γ is close to zero. Another pattern, which is consistent with our prior, is that β decreases as the variance of the idiosyncratic noise goes up.

Figure 2.8: β^I as a function of γ. $\rho = 1$. $\sigma_e = 1$.



Notes: This figure plots individual level CG coefficients under adaptive expectations, with varying variance of the idiosyncratic noise σ_η^2. In this numerical example, the AR(1) process for the actual variable has persistence $\rho = 1$, and innovation standard deviation of $\sigma_e = 1$. I simulate forecasts for 30 forecasters over a span of 10000 periods.

2.5 Diagnostic Expectations and CG Coefficients: A Comparison

In their 2018 study, Bordalo, Gennaioli, and Shleifer demonstrated that when the actual variable w_t follows an AR(1) process with normally distributed innovations, the diagnostic expectations forecast is given by:

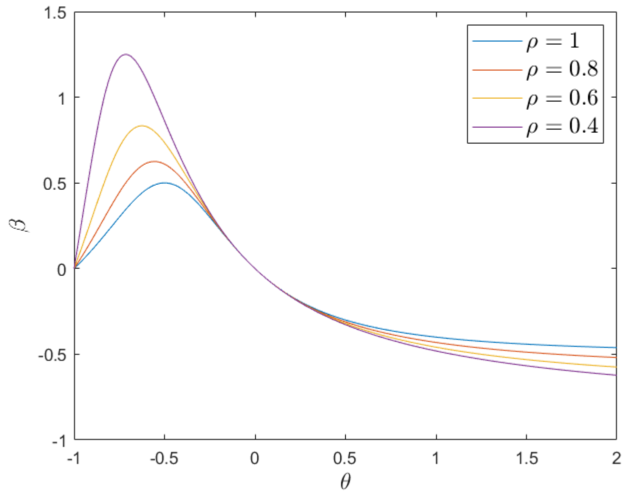
$$E^\theta w_{t+1} = E_t w_{t+1} + \theta(E_t w_{t+1} - E_{t-1} w_{t+1}) = \rho w_t + \rho\theta e_t \tag{2.10}$$

The "representativeness heuristic" leads to the derivation of the expression for $E^\theta w_{t+1}$, which incorporates the kernel of truth logic: relative to rational expectation, individuals tend to over-react to new information they encounter in period t by a factor of $\theta(E_t w_{t+1} - E_{t-1} w_{t+1})$. The degree of this overreaction is measured by the parameter θ. Diagnostic expectations is a model of over-reaction with $\theta > 0$. However, I allow θ to be from the domain $[-1, +\infty)$ so that over-reaction, under-reaction and rational expectations are all allowed.

Adaptive expectations yields a wider range of model-implied β, compared with diagnostic expectations. The reason is that when γ gets close to zero, forecasts according to adaptive expectations can reach any level of stickiness, resulting in β of arbitrary magnitude. In the extreme case of $\gamma = 0$, where there is no adjustment of forecasts and forecasts stay constant over time, forecast revisions are almost zero. Forecast revisions being on the right hand side of the equation and close to zero can lead to arbitrarily large β.

However, the generalized diagnostic expectations does not allow for such high degrees of stickiness in the forecast data. As we can see from figure 2.9, under diagnostic expectations, β is not monotonic in θ. It is hump-shaped with the maximum value obtained for a value of θ between -1 and 0.

Figure 2.9: β as a function of θ.



Notes: This figure plots β as a function of the diagnosticity parameter θ from the diagnostic expectations model, for varying values of rho. ρ is the persistence of the AR(1) process of the actual variable.

The reason for the hump shape can be better explained by looking at the equations of forecast errors and forecast revisions. Under generalized diagnostic expectations model, the forecast error is given by:

$$w_{t+1} - E_t^\theta w_{t+1} = e_{t+1} - \rho\theta e_t. \tag{2.11}$$

The forecast revision is

$$E_t^\theta w_{t+1} - E_{t-1}^\theta w_{t+1} = \rho(\theta + 1)e_t - \rho^2\theta e_{t-1}. \tag{2.12}$$

The rationale behind the hump-shaped relationship can be understood as follows: The ability to predict forecast errors using forecast revisions arises from the fact that the current news, e_t, plays a role in both forecast errors and forecast revisions. When $\theta = 0$, the formation of expectations aligns with rational expectations. The forecast errors in equation (2.11) are dictated solely by future innovations, e_{t+1}. In this context, forecast revisions—which comprise current and past innovations—are unable to predict forecast errors, leading to $\beta = 0$. However, when θ decreases, there is an under-reaction to the new information, which makes β turn positive. Mathematically speaking, this transition stems from the fact that e_t begins to emerge in equation (2.11). As θ continues to decrease, the magnitude of β increases because e_t increasingly influences forecast errors. Nevertheless, as θ further decreases, the forecast begins to respond less to the current news, e_t. This causes e_t to impact forecast revisions to a lesser degree, thereby reducing the predictability of forecast errors based on forecast revisions. In the extreme case where $\theta = -1$, forecast revisions do not incorporate e_t at all, making it impossible to predict forecast errors from forecast revisions. From figure 2.9, the range β under diagnostic expectations can cover varies with ρ. The lower the ρ, the wider the range for β.

Why does the adaptive expectations model yield a monotonic relationship between β and γ, while diagnostic expectations lead to a hump-shaped relationship between β and θ? The primary reason lies in the difference in the degree of "stickiness" permitted by the two expectation models. Examining the adaptive expectations equation (2.1), we find that when $\gamma = 0$, forecasts based on adaptive expectations remain at the initial value of forecasts, $\mathcal{F}_0 w_1$. As previously argued, β can reach arbitrarily high values when we adjust the γ parameter. However, the generalized diagnostic expectations model does not allow for this level of stickiness in forecasts. Upon rewriting equation (2.10) as follows:

$$E^{\theta} w_{t+1} = (1+\theta) E_t w_{t+1} - \theta E_{t-1} w_{t+1}, \tag{2.13}$$

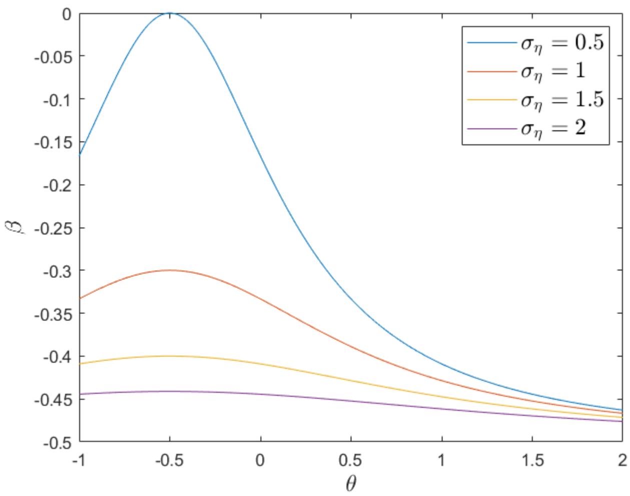
we observe that, when θ is within the range (0,1), the generalized diagnostic expectations model forms a weighted average of the current rational expectations and the previous rational expectations, limiting the degree of stickiness. Since rational expectations are never sticky, a weighted average of the current and previous rational expectations also cannot be sticky. Thus, the difference in the degree of stickiness allowed in the two models accounts for the contrasting behavior of β in relation to γ in adaptive expectations and θ in diagnostic expectations.

In the pursuit of matching negative β, the contrast between adaptive expectations and diagnostic expectations is somewhat nuanced. As θ approaches positive infinity, β tends toward $-\frac{1}{1+\rho^2}$. On the other hand, as γ approaches 2, β tends toward $-\frac{1}{2}$.[6] Given $\rho < 1$, $-\frac{1}{1+\rho^2}$ is less than $-\frac{1}{2}$. Thus, compared to adaptive expectations, diagnostic expectations can match a broader range of negative β values. However, the possibility of γ exceeding 2 allows adaptive expectations to match an even wider range of negative β, as β can fall below $-\frac{1}{2}$. This greater flexibility comes with the trade-off of potentially creating explosive trajectories for forecasts.

In Figure 2.10, I illustrate β as a function of θ given different variances of idiosyncratic noise, denoted as σ_η^2. As is evident, idiosyncratic noise applies a downward pressure on β^I.

[6] Again γ is restricted to be below 2 to ensure that the forecasts are not explosive over time.

Figure 2.10: β^I as a function of θ



Notes: This figure plots β^I as a function of the diagnosticity parameter θ from the diagnostic expectations model, for varying values of σ_η. The persistence of the AR(1) process is $\rho = 1$, and the standard deviation of the innovation is $\sigma_e = 1$.

2.6 Data

The main data set used in this chapter is the Survey of Professional Forecasters, the same as the one used in Chapter 1. For completeness, a brief introduction of this data set is included here. The Survey of Professional Forecasters (SPF), facilitated by the Federal Reserve Bank of Philadelphia, represents a robust source of expert predictions pertaining to a range of macroeconomic and financial indicators, with data available from 1968 onwards. Administered on a quarterly basis, the survey is typically conducted near the conclusion of each quarter's second month. This scheduling enables forecasters to account for the most recent developments in macroeconomic variables up to and including quarter $t-1$ when making predictions for quarter t.

It is crucial to note that not all variables possess historical records dating back to the survey's inception in 1968. This study does not incorporate every variable available within the SPF dataset. Instead, the present analysis centers on seven key variables, namely: real GDP growth rate (RGDP), nominal GDP growth rate (NGDP), GDP price index inflation (GDP Price Index), Consumer Price Index inflation (CPI), 3-month Treasury bill yield (Tbill), AAA corporate bond yield (AAA), and Treasury bond corporate bond yield (Tbond). Forecasts are formulated for both the current and subsequent four quarters. To maintain anonymity, the SPF assigns unique identification codes to each participating forecaster.[7] In each survey wave,

[7]However, the forecaster identification is not entirely accurate. In the documentation of SPF, it mentions: "In these surveys, we have noticed some occurrences in which an individual participates, suddenly drops out of the panel for a large number of periods, and suddenly re-enters, suggesting that the same identifier might have been assigned to different forecasters," and "it can be difficult to assign an identification number to an individual who changes his place of employment but

the mean number of participating forecasters varies between 27 and 36, contingent upon the specific variable under examination. The aggregate count of forecasters contributing to the survey ranges from 446 to 448. On average, individual panelists partake in approximately 23 waves of survey. As such, the Survey of Professional Forecasters constitutes an unbalanced panel dataset.

Owing to the potential subsequent revisions in the dissemination of macroeconomic statistics, this study employs vintage data to represent the observed realizations of economic variables. This approach ensures alignment with the forecasters' observations of economic indicators at the time of formulating their predictions. The initial release of macroeconomic statistics is sourced from the real-time dataset provided by the Federal Reserve Bank of Philadelphia. In contrast, financial variables such as bond yields are not subject to revisions. Historical data on bond yields can be obtained from the Federal Reserve Bank of St. Louis.

The Survey of Professional Forecasters predominantly emphasizes the levels of macroeconomic variables rather than growth rates. All levels are transformed into growth rates for the purposes of this study (except for bond yields). The majority of empirical analyses in this chapter is on a forecast horizon extending three quarters ahead. As such, the growth rate represents the annual growth rate between quarter $t-1$ and quarter $t+3$. For instance, when determining the actual growth rate of real GDP from quarter $t-1$ to quarter $t+3$, real GDP data for the corresponding quarters is derived from the vintage data released during quarter $t+4$. To compute the real GDP growth rate forecasts from $t-1$ to $t+3$, the forecast generated at t and the initial release of real GDP at $t-1$, published at t, are utilized. Forecasters with fewer than ten appearances in the survey are excluded. Those steps are mostly the same as BGMS (2020) in order to replicate their results. The summary statistics on individual forecast errors and forecast revisions for $t+3$ are presented in Table 2.1.

Table 2.1 presents the standard deviation, which is defined as the standard deviation of individual forecast errors or revisions when pooled across quarters and forecasters. The average standard deviation is derived by initially calculating the standard deviation for each quarter, followed by computing the average across quarters. The summary statistics reveal a substantial number of observations for individual forecast errors and revisions across each variable, attributable to the survey's extensive time series and panel structure.

The higher quantity of observations for forecast errors compared to forecast revisions can be explained as follows: to compute forecast revisions, both the $t+3$ and the lagged $t+4$ forecasts are required. However, within the SPF, certain forecasters offer forecasts for $t+3$ without providing corresponding $t+4$ forecasts, resulting in a greater number of empty cells for $t+4$ relative to $t+3$. Another notable pattern

remains in the survey."

Table 2.1: Summary Statistics

	RGDP	NGDP	GDP Price Index	CPI	Tbill	AAA	Tbond
Individual Forecast Error of $t+3$							
Number of Obs.	7505	7523	7455	5258	5054	4305	4039
Mean	-0.37	-0.24	0.12	-0.22	-0.56	-0.48	-0.53
Standard Dev.	2.31	2.48	1.59	2.41	1.18	0.97	0.81
Average Standard Dev.	1	1.19	0.77	0.73	0.48	0.53	0.4
Individual Forecast Revision of $t+3$							
Number of Obs.	5696	5710	5712	4224	4055	3409	3311
Mean	-0.14	-0.11	0.03	-0.07	-0.2	-0.13	-0.14
Standard Dev.	1.37	1.52	0.98	0.77	0.67	0.6	0.51
Average Standard Dev.	0.91	1.09	0.72	0.66	0.44	0.48	0.36
Number of Forecasters	36	36	36	33	32	27	35
Time Periods	1968-2022			1981-2022			1991-2022

Notes: This table reports the summary statistics of individual forecast errors and forecast revisions. Standard Dev. is the standard deviation after pooling all the observations. Average Standard Dev. is obtained by first calculating the standard deviation for each quarter and then averaging across quarters. Number of forecasters is the average number of forecasters across different waves of survey. All forecast errors and revisions are calculated at horizon $t+3$.

is that the mean forecast errors and revisions are mostly indistinguishable from zero.[8] This absence of discernible discrepancy indicates that there is no evidence of systematic bias in forecasts or asymmetry in forecast revisions.

The average standard deviation highlights the presence of systematic disagreement among forecasters, which aligns with the concept of judgment noise as described by Kahneman et al. (2021). The periods covered for each variable differ: for RGDP, NGDP, and GDP price index inflation, the survey commenced as early as 1968, whereas the survey for Tbond began more recently in 1991.

2.7 Estimation

In this section, I first present the Error-on-revision regression results. Second, I report the evidence on the existence of two types of judgment noise. After that, the estimation strategy and estimation results are discussed.

2.7.1 Error-on-revision Regression Results

In the "forecast error on forecast revision" benchmark regression, I employ a $t+3$ forecast horizon to align with the BGMS study, enabling a comparison of regression outcomes. Table 2.2 presents both consensus and individual level regression results. Consensus time series regression standard errors are adjusted using Newey-West (1994) automatic bandwidth selection, while individual panel regression standard

[8] The mean is calculated across forecasters and across time.

errors are clustered by forecaster and time. Most β^I estimations are significantly negative, except for the three-month T-bill yield, which is significantly positive. The β^C point estimations differ across variables. For long-maturity bonds, such as the ten-year T-bond and the AAA corporate bond yield, $\hat{\beta}^C$ values are insignificant. In contrast, for inflation and the T-bill yield, $\hat{\beta}^C$ is significantly positive. Nominal and real GDP growth exhibit positive but insignificant $\hat{\beta}^C$ values. The regression outcomes mostly resemble those in the BGMS study, with minor deviations. For instance, BGMS reports significant $\hat{\beta}^C$ values for both real and nominal GDP. This discrepancy stems from the additional six years of data in this study, extending to 2022, while BGMS data ends in 2016. Regressions using the same data periods yield nearly identical results to BGMS. To maintain consistency with the literature, this chapter focuses on the general case where $\hat{\beta}^C > 0$ and $\hat{\beta}^I < 0$.

Under the Full Information Rational Expectation (FIRE) hypothesis, both β^C and β^I are expected to be zero. However, evidence of $\beta^C > 0$ and $\beta^I < 0$ contradicts the FIRE hypothesis. BGMS suggests that $\beta^C > 0$ may result from a combination of an information friction and a rationality violation, while $\beta^I < 0$ implies overreaction to information during individual expectation formation.

Table 2.2: Error-on-revision Regression Coefficients

	$\hat{\beta}^C$			$\hat{\beta}^I$		
	Point Estimate	SE	p-value	Point Estimate	SE	p-value
RGDP	0.11	0.31	0.73	-0.28	0.12	0.02
GDP Price Index	1.26	0.41	0.00	-0.15	0.07	0.04
NGDP	0.14	0.25	0.56	-0.32	0.12	0.01
CPI	1.04	0.76	0.17	-0.38	0.09	0.00
Tbill	0.69	0.11	0.00	0.21	0.09	0.03
AAA	-0.02	0.16	0.92	-0.27	0.07	0.00
Tbond	-0.06	0.09	0.46	-0.23	0.02	0.00

Notes: This table reports the Error-on-revision regression results at both the consensus and individual level. For consensus time-series regressions, standard errors are calculated using the Newey-West method, with the automatic bandwidth selection procedure as proposed by Newey and West (1994). For individual-level panel regressions, standard errors are clustered by both the forecaster and time.

2.7.2 Estimation Strategy for Model with Adaptive Expectations

The goal of this estimation exercise is to estimate the crucial adaptive weighting parameter γ within the adaptive expectations model. For this task, I employ the Simulated Method of Moments (SMM) estimation methodology. The estimation of γ is achieved by aligning the model-implied regression coefficients with the data-estimated regression coefficients. Specifically, I match the regression coefficients at the consensus level and individual level separately to derive two distinct sets of estimations for γ. Other parameters to be estimated are: ρ, $\frac{\sigma_\eta}{\sigma_e}$. The steps to obtain the estimation for this set of parameters for each variable are as follows:

- Fit an AR(1) process to the historical time series to obtain estimates of ρ and σ_e for each variable.

- $\{\sigma_\eta\}$ is obtained by running the following regression:

$$\mathcal{F}_{i,t}w_{t+3} = \alpha_t + \phi_i + \eta_{i,t} \tag{2.14}$$

  where α_t captures the consensus opinion on forecasting and ϕ_i captures the individual heterogeneity. The estimation of σ_η, the standard deviation of these noise terms, can be achieved by calculating the standard deviation of these residuals.

- With the given γ, and utilizing previously estimated values of ρ and σ_e, the time series for the actual variable can be simulated over 10,000 periods, using an AR(1) process. We then simulate individual-level forecasts for 30 forecasters, adhering to equation (2.9). By averaging the forecasts across these individual forecasters, we are able to compute the consensus-level adaptive expectations forecasts.

- After simulating the data, we conduct a CG regression and calculate the model-implied β^C_{OLS} and β^I_{OLS}. We then separately identify the γ value that aligns with β^C_{OLS} and β^I_{OLS}. In instances where the model fails to match the data, we report the γ value which results in a model-implied β_{OLS} closest to the data. We calculate standard errors via bootstrap: conducting random resampling from the panel of forecasters with replacement.

2.7.3 Estimation Results

Table 2.3 provides the estimation results, adhering to the aforementioned procedure.[9] Firstly, the estimated γ value differs across distinct variables. Inflation rates for the GDP price index and CPI, as well as the 3-month T-bill yield, are associated with lower γ values, while the growth rates for real and nominal GDP, AAA corporate bond yield, and the 10-year T-bond yield exhibit higher γ values. This implies that the forecasts for GDP price index inflation, CPI inflation, and the 3-month T-bill yield are stickier, and the forecasts for the other variables are more prone to extrapolation. This aligns with the findings from the first chapter that the former

[9] During the estimation of γ^I, each value of β^I_{data} might correspond to two values of γ that align the model with the data. In the primary body of this chapter, I concentrate on the higher value. In figure 2.8 in the appendix, I report the point estimate for the lower value.

three variables display a stronger under-reaction to new information, while the latter three show more over-reaction. It might be inferred that forecasters employing adaptive expectations have identified the optimal adaptive parameter for different variables. Secondly, the γ^C and γ^I values do not consistently correspond with each other in most instances. This is not unexpected. Considering the simplicity of the current model, it is unrealistic to expect it to perfectly fit the data.

Table 2.3: Estimation Results for γ.

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
γ^C	0.78***	0.43***	0.79***	0.26***	0.59***	1***	1.06***
	(0.08)	(0.03)	(0.06)	(0.08)	(0.03)	(0.07)	(0.04)
γ^I	0.98***	0.16***	1.12***	1.26***	0.08***	0.3***	0.65***
	(0.11)	(0.00)	(0.11)	(0.29)	(0.00)	(0.08)	(0.08)
$\gamma^I_{noiseless}$	1.28***	1.17***	1.4***	1.3***	0.82***	1.37***	1.29***
	(0.43)	(0.39)	(0.47)	(0.44)	(0.25)	(0.46)	(0.43)

*Notes: This table reports the γ estimation results. The first row presents γ^C by matching consensus level regression coefficients. The second row shows γ^I by matching individual level regression coefficients. The third row reports $\gamma^I_{noiseless}$ by matching individual level regression coefficients without considering the idiosyncratic noise component. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

The third row of the table reports the results of the γ estimation, achieved by aligning the model-implied and data-estimated β^I without accounting for the idiosyncratic noise component. Omitting the idiosyncratic noise component, we consistently obtain a higher estimate for the γ parameter. For variables such as bond yields and the GDP price index, this discrepancy is considerable. This observation aligns with the point made in chapter one: drawing conclusions about forecasters' overreaction or underreaction (in this case, the γ parameter) from the individual level regression coefficients can be misleading.

Table 2.4 presents how well the model can replicate the two regression coefficients $\hat{\beta}^C$ and $\hat{\beta}^I$. As we can see, the model nearly perfectly matches 12 out of the 14 point estimates of β^C or β^I. However, the model fails to perfectly match two of them: β^I for the GDP price index and the 3-month T-bill yield. For the latter, although the model doesn't perfectly align with the data, β^I_{model} does fall within the 95% confidence interval. For the GDP price index, β^I_{model} is not within the 95% confidence interval of β^I_{data}. However, this current estimation exercise holds $\rho, \frac{\sigma_\eta}{\sigma_e}, \theta$ constant, thereby ignoring the sampling variation in these estimates. If we simultaneously match all moments, we will have more degrees of freedom to align with β^I.

2.7.4 Graphic Illustration

Figures 2.11 to 2.13 illustrate how βs are matched between the model and data under adaptive expectations, while figures 2.14 and 2.15 do the same for diagnostic

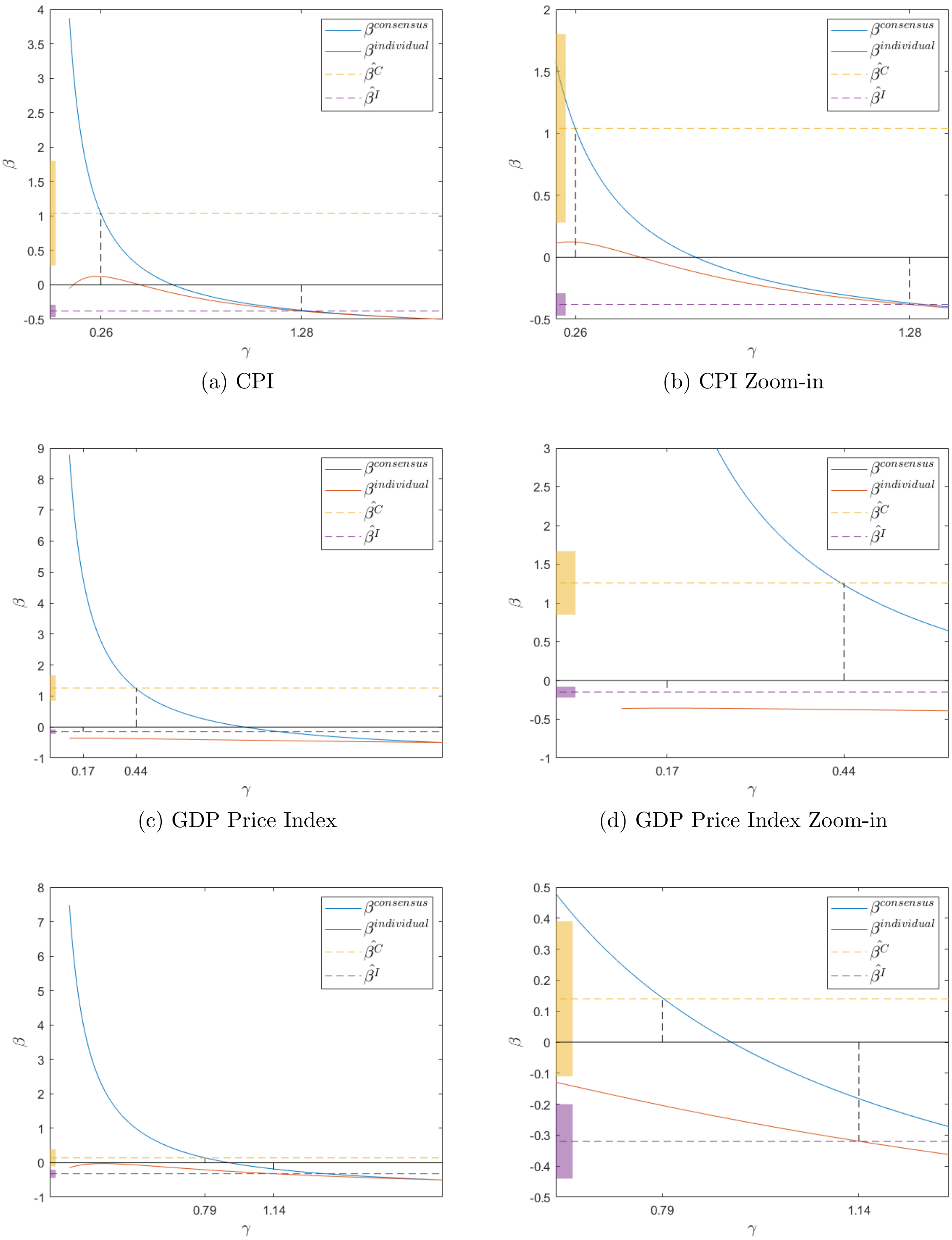
Table 2.4: Model Regression Coefficients and Data Estimated Regression Coefficients

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
β^C_{model}	0.1101	1.2575	0.1403	1.0397	0.6899	-0.021	-0.058
β^C_{data}	0.11	1.26	0.14	1.04	0.69	-0.02	-0.06
β^I_{model}	-0.2802	-0.3616	-0.32	-0.3801	0.13	-0.27	-0.23
β^I_{data}	-0.28	-0.15	-0.32	-0.38	0.21	-0.27	-0.23

Notes: This table reports the β values from the data for each variable, as well as the corresponding β values from the model, at both the individual and consensus levels.

expectations. In these figures, β^C and β^I from the model are plotted as functions of the adaptive parameter γ for adaptive expectations. For diagnostic expectations, β^C and β^I from the model are plotted as functions of the diagnosticity parameter θ. Assuming a large number of forecasters, for the model-implied β^C, the impact of idiosyncratic noise is averaged out. The point estimates of empirical β^C and β^I ($\hat{\beta}^C$ and $\hat{\beta}^I$) are depicted as horizontal dotted lines. To the left of the figures, the one standard error bands of $\hat{\beta}^C$ and $\hat{\beta}^I$ are provided for reference, these standard errors are derived from the OLS estimators of $\hat{\beta}^C$ and $\hat{\beta}^I$. There are several key takeaways from this exercise: First, it's clear that the adaptive expectations model offers more flexibility in matching the data and the model, specifically for β^C. For instance, with the GDP price index inflation, β^C under diagnostic expectations (shown in figure 2.14) is hump-shaped and fails to match the empirical $\hat{\beta}^C$ at its maximum value. Conversely, under adaptive expectations, the model can accommodate arbitrarily large β^C provided the γ parameter is sufficiently small. This arises from the ability of the adaptive expectations model to accommodate arbitrary degrees of stickiness, an attribute lacking in the diagnostic expectations model. Second, due to the broader value coverage of β^C under adaptive expectations, β^I under adaptive expectations also spans a wider range of values. This is clearly evidenced when comparing β^I of AAA corporate bond yield, GDP Price Index inflation, and 10-year Tbond yield under both adaptive expectations and diagnostic expectations. In summary, the adaptive expectations model, by allowing for larger degrees of stickiness, is better equipped to match the empirically estimated $\hat{\beta}^C$ and $\hat{\beta}^I$.

Figure 2.11: Adaptive Expectation β from the Model and Data



(a) CPI

(b) CPI Zoom-in

(c) GDP Price Index

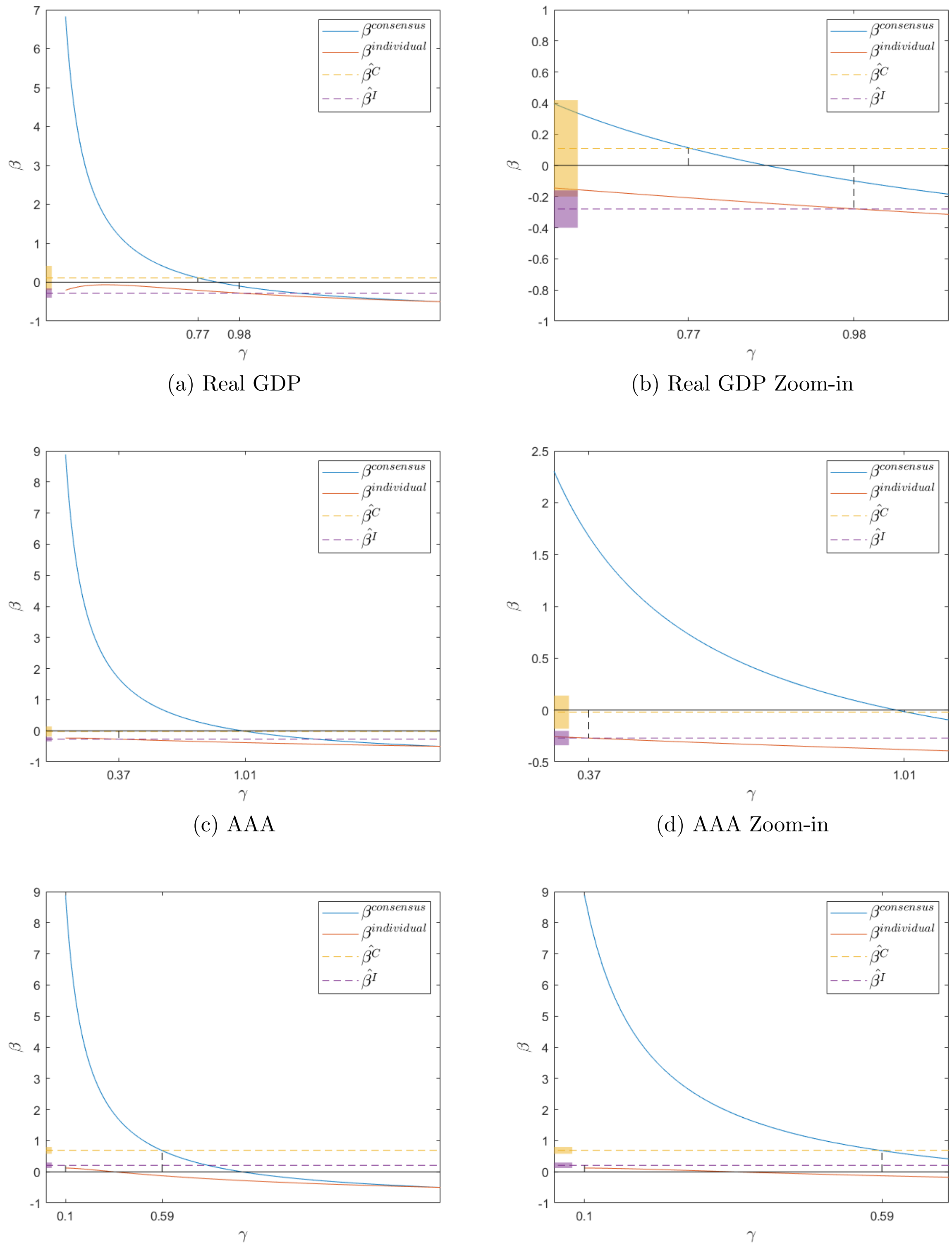
(d) GDP Price Index Zoom-in

(e) Nominal GDP

(f) Nominal GDP Zoom-in

Notes: This figure graphically illustrates the matching of βs between the model and data under adaptive expectations. Variables under consideration are CPI inflation, GDP price index inflation, nominal GDP growth rate. The dotted lines in the figure represent the empirically estimated values of $\hat{\beta}^C$ and $\hat{\beta}^I$. To the left of the figure, the yellow and purple rectangles indicate the one standard error band of the OLS estimations for $\hat{\beta}^C$ and $\hat{\beta}^I$, respectively.

Figure 2.12: Adaptive Expectation β from the Model and Data Part 2



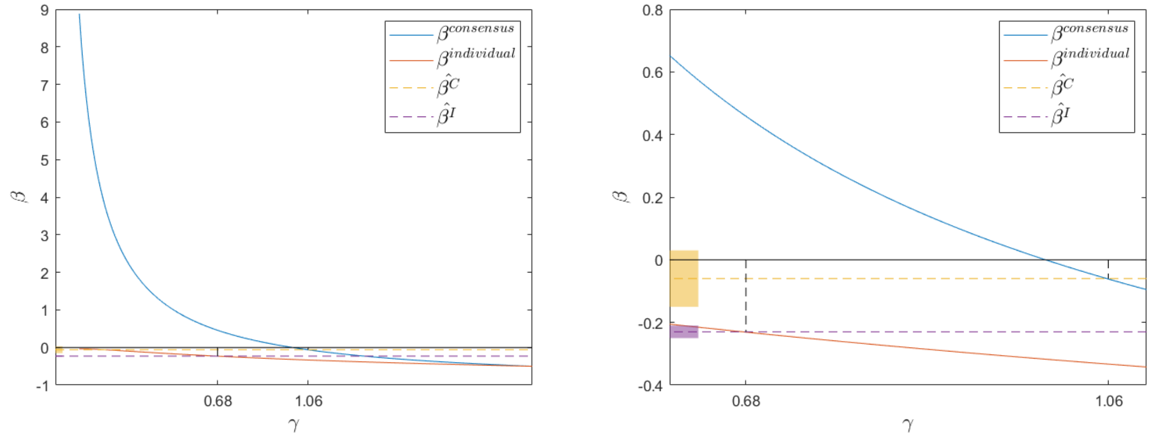
(a) Real GDP (b) Real GDP Zoom-in

(c) AAA (d) AAA Zoom-in

(e) Tbill (f) Tbill Zoom-in

Notes: This figure graphically illustrates the matching of βs between the model and data under adaptive expectations. Variables under consideration are real GDP growth rate, AAA corporate bond yield and 3-month Tbill yield. The dotted lines in the figure represent the empirically estimated values of $\hat{\beta}^{C}$ and $\hat{\beta}^{I}$. To the left of the figure, the yellow and purple rectangles indicate the one standard error band of the OLS estimations for $\hat{\beta}^{C}$ and $\hat{\beta}^{I}$, respectively.

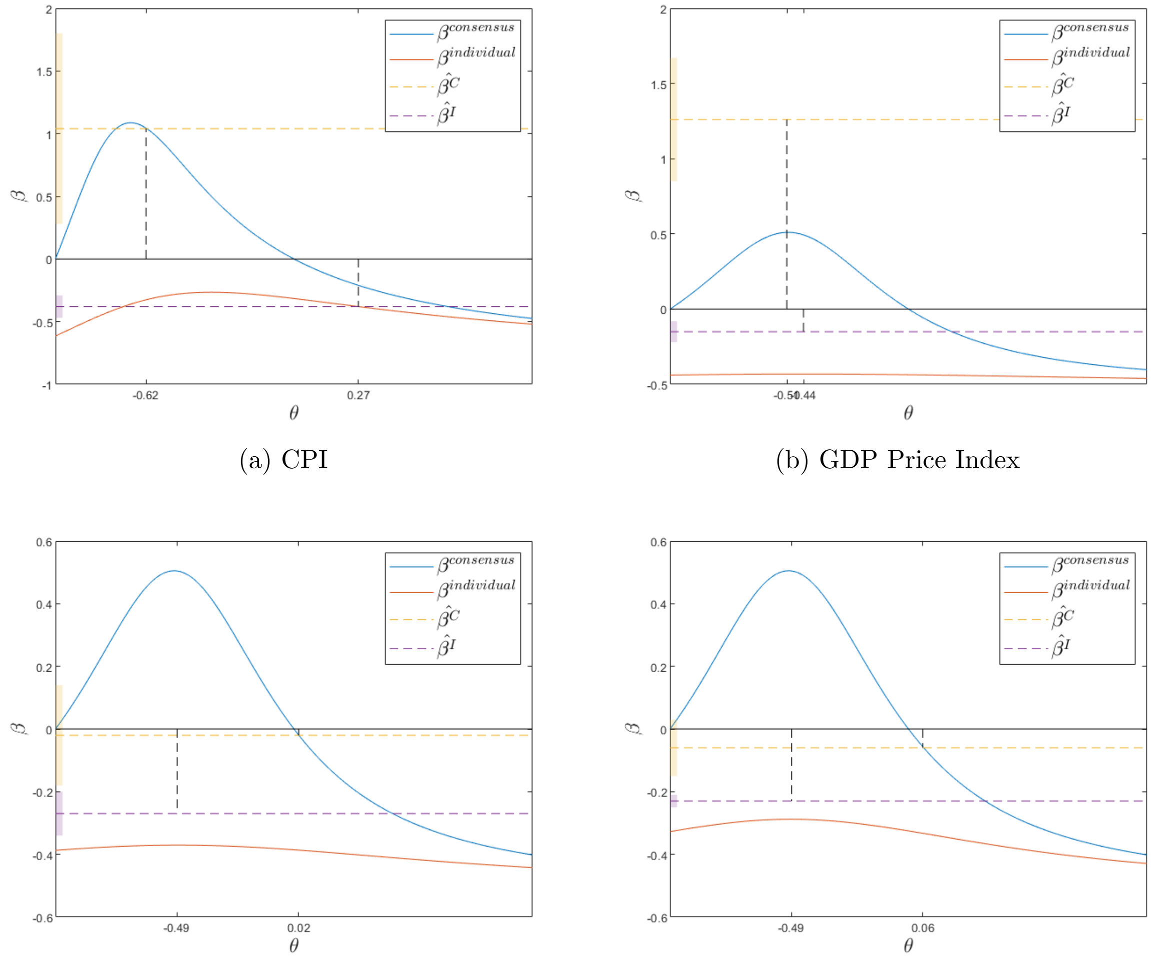
Figure 2.13: Adaptive Expectation β from the Model and Data Part 3



(a) Tbond (b) Tbond Zoom-in

Notes: This figure graphically illustrates the matching of βs between the model and data under adaptive expectations. Variable under consideration is 10-year Tbond yield. The dotted lines in the figure represent the empirically estimated values of $\hat{\beta}^C$ and $\hat{\beta}^I$. To the left of the figure, the yellow and purple rectangles indicate the one standard error band of the OLS estimations for $\hat{\beta}^C$ and $\hat{\beta}^I$, respectively.

Figure 2.14: Diagnostic Expectation β from the Model and Data

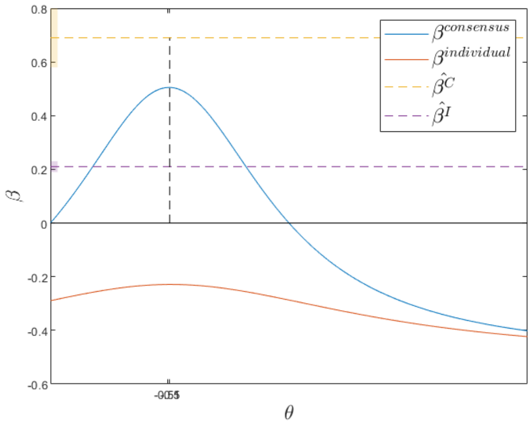


(a) CPI (b) GDP Price Index

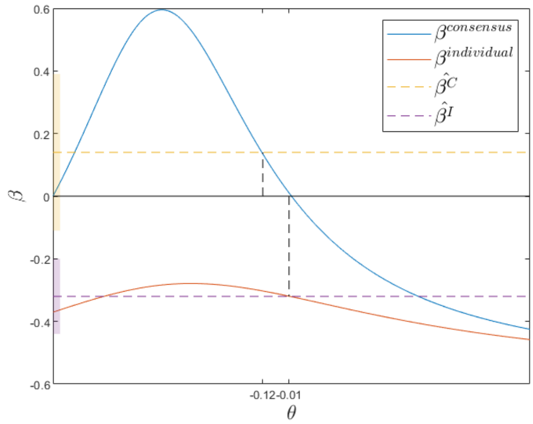
(c) AAA (d) Tbond

Notes: This figure graphically illustrate the matching of βs between the model and data under diagnostic expectations. Variables under consideration are CPI and GDP price index inflation, AAA corporate bond yield and 10-year Tbond yield. The dotted lines in the figure represent the empirically estimated values of $\hat{\beta}^C$ and $\hat{\beta}^I$. To the left of the figure, the yellow and purple rectangles indicate the one standard error band of the OLS estimations for $\hat{\beta}^C$ and $\hat{\beta}^I$, respectively.

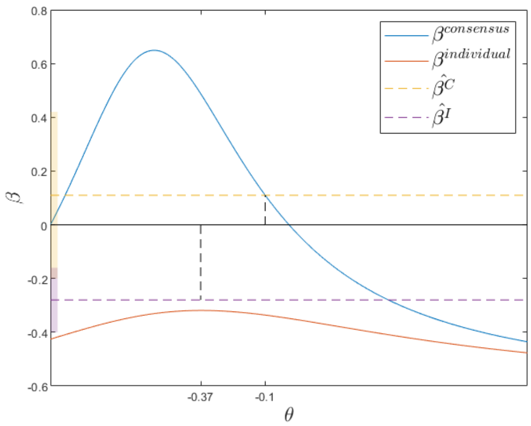
Figure 2.15: Diagnostic Expectation β from the Model and Data Part 2



(a) Tbill



(b) Nominal GDP



(c) Real GDP

Notes: This figure graphically illustrate the matching of βs between the model and data under diagnostic expectations. Variables under consideration are 3-month Tbill, nominal and real GDP growth rate. The dotted lines in the figure represent the empirically estimated values of $\hat{\beta}^C$ and $\hat{\beta}^I$. To the left of the figure, the yellow and purple rectangles indicate the one standard error band of the OLS estimations for $\hat{\beta}^C$ and $\hat{\beta}^I$, respectively.

2.8 Further Discussion

In this section, I will undertake several analyses related to the adaptive expectations model.

2.8.1 Over-/Under-reaction to New Information across Forecast Horizons

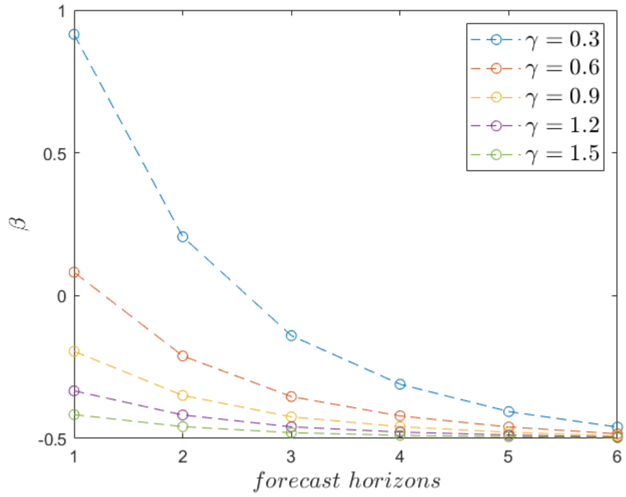
Recent literature provides evidence suggesting that the degree of over-/under-reaction varies across forecast horizons: the longer the forecast horizons, the stronger the degree of over-reaction. This pattern has been documented in experimental evidence such as Afrouzi et al. (2021), equity analysts' forecasts of long-term and short-term earnings growth as seen in Bordalo et al. (2019) and Bouchaud et al. (2019), as well as in asset prices as per Giglio and Kelly (2018).

In this section, I explore the implications of adaptive expectations on Error-on-revision regression coefficients across different forecast horizons. Under Assumption 1, I obtain Error-on-revision regression coefficients under the adaptive expectations model by running simulations. I simulate a representative agent forming adaptive expectations, devoid of idiosyncratic noise, across various forecast horizons for 10,000 periods. I use various weighting parameters, γs, in the simulation and calculate the Error-on-revision regression coefficients. The underlying variable follows an AR(1) process with a persistence of $\rho = 0.5$. Figure 2.16 plots βs for various forecast horizons under different weighting parameters. From Figure 2.16, we observe that the further into the future the forecasts are made, the more negative the βs become, indicating stronger evidence of over-reaction. When the underlying variable follows an AR(1) process, the impact of shocks diminishes over time. However, under adaptive expectations, forecasters do not account for this diminishing impact while making forecasts. This neglect of the diminishing impact of shocks is partly due to Assumption 1: the term structure of forecasts is horizontal even though the underlying variable follows an AR(1) process.

2.8.2 Dynamic Responses of Outcomes and Forecasts

Beyond the Error-on-revision regressions, researchers have also applied regression models to forecast errors based on current observational levels, as suggested by Kohlhas and Walther (2021). Angeletos, Huo, and Sastry (2021), in their efforts to characterize forecasters' errors in a more dynamic manner, document a novel set of facts. They note that, in response to two key shocks—one that accounts for most business-cycle variations in unemployment and other macroeconomic quantities, and another primarily responsible for the business-cycle variations in inflation—the impulse response functions of average unemployment and inflation forecasts initially

Figure 2.16: β for Different Forecast Horizons and Various γs. $\rho = 0.5$.



Notes: This figure plots βs for different forecast horizons and various weighting parameter γs. The persistence of the underlying variable is set to be 0.5.

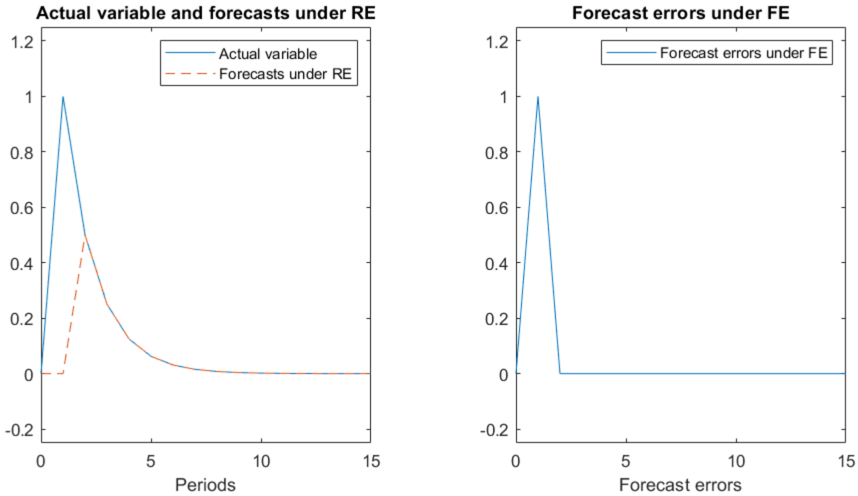
display an under-reaction, but subsequently overshoot.

The researchers account for this empirical phenomenon by integrating the elements of noisy information and over-extrapolation into their expectations framework. At the onset of a shock, noisy information takes precedence, causing forecasters to under-react since they lack full confidence in the information they observe. However, over time, over-extrapolation comes into play, leading to an overshooting of forecasts.

However, such an impulse response function could be a natural outcome of adaptive expectations, a framework that is considerably simpler than the one proposed by Angeletos, Huo, and Sastry (2021). In figures 2.17 and 2.18, I have plotted the impulse response functions of both the actual outcome and the forecasts in response to an unexpected shock occurring in the first period, under both rational and adaptive expectations. To maintain consistency with the approach of Angeletos, Huo, and Sastry (2021), the forecasts utilize the value from the previous period, $E_{t-1}w_t$ under rational expectations and $\mathcal{F}_{t-1}w_t$ under adaptive expectations. This approach to timing forecasts is distinct from the one employed in previous figures, specifically from figure 2.1 to 2.4.

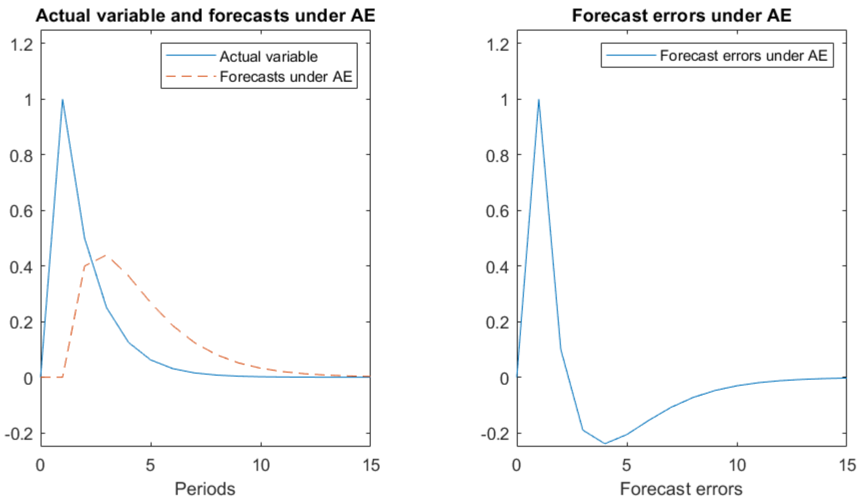
Under rational expectations, forecasters do not react to an unexpected shock upon its occurrence, as they are not aware of it. However, after the shock is recognized, forecasters generate accurate predictions since rational forecasters understand the shock's impact on the underlying variable. In contrast, under adaptive expectations, forecasts initially under-react and subsequently overshoot. This can be explained by the fact that when $\gamma < 1$, forecasts tend to adjust in the direction of the observed outcome from previous forecasts, yet they never fully catch up. Consequently, when a shock occurs and the observed outcome surpasses the previous forecast, forecasts rise but are marked by an initial under-reaction. As the shock fades and the observed outcome drops below the previous forecast, forecasts decrease, but they tend to overshoot.

Figure 2.17: Impulse Response Function of Outcomes and Forecasts



Notes: This figure presents the impulse response function of the actual variable and forecasts in reaction to an unexpected shock, when forecasts follow rational expectations. The actual variable is denoted by its period t value, w_t. Forecasts under the rational expectations model utilize the value from the preceding period, $E_{t-1}w_t$. The forecast errors are computed as the difference between the actual variable and the previous period's forecast, represented by $w_t - E_{t-1}w_t$.

Figure 2.18: Impulse Response Function of Outcomes and Forecasts



Notes: This figure presents the impulse response function of the actual variable and forecasts in reaction to an unexpected shock, when forecasts follow adaptive expectations. The actual variable is denoted by its period t value, w_t. Forecasts under the adaptive expectations model utilize the value from the preceding period, $\mathcal{F}_{t-1}w_t$. The forecast errors are computed as the difference between the actual variable and the previous period's forecast, represented by $w_t - \mathcal{F}_{t-1}w_t$.

2.8.3 Model Implied Regression

Coibion and Gorodnichenko (2015) proposed the Error-on-revision regression to test the implication of noisy information and sticky information frameworks. This chapter, however, concentrates on studying the implications of adaptive expectations on Error-on-revision regression coefficients. I estimate the weighting coefficient γ by matching the regression coefficients derived from the model under adaptive expectations, and from the data. Instead of matching the regression coefficients from Error-on-revision regression, I run the following regression equation implied by adaptive expectations model. Let's start by examining adaptive expectations across different forecast horizons:

$$\mathcal{F}_t w_{t+h} = \gamma_h w_t + (1-\gamma_h)\mathcal{F}_{t-1} w_{t+h-1}, h = 0, 1, 2, 3,$$

where the current forecast of quarter h in the future is formed as a weighted average of current observation and past forecasts. Notably, when forecasters complete the survey, they can only access data from the previous quarter. Rather than observing w_t, forecasters observe w_{t-1}. As a result, the actual model under consideration is the following:

$$\mathcal{F}_t w_{t+h} = \gamma_h w_{t-1} + (1-\gamma_h)\mathcal{F}_{t-1} w_{t+h-1}, h = 0, 1, 2, 3, \tag{2.15}$$

After rearranging the terms, we derive the following regression equation implied by the model:

$$\mathcal{F}_t w_{t+h} - \mathcal{F}_{t-1} w_{t+h-1} = \gamma_{h,0} + \gamma_h (w_{t-1} - \mathcal{F}_{t-1} w_{t+h-1}) + \epsilon_{h,t}, h = 0, 1, 2, 3. \tag{2.16}$$

I run the regression at both the individual and consensus levels, with the results presented in tables 2.5 and 2.6. The individual level regression should be estimated with Arellano-Bond (1991) estimator. When γ is close to 0, forecasts are sticky—forecasters do not significantly adjust their forecasts according to their most recent observations. As γ increases, forecasters place more weight on the most recent observations while making their forecasts.

There are several patterns discernible from tables 2.5 and 2.6. First, the estimates for γ are predominantly positive and range between 0 and 1, regardless of whether they're at the consensus or individual level and irrespective of the forecast horizons. This corroborates the assumption that γ is between 0 and 1, and the forecast is a weighted average of the latest observation and the last forecast.

Second, γ decreases as forecast horizons extend. The shorter the forecast horizon, the greater the weight placed on the most recent observation of the respective variable. As the forecast horizon lengthens, the forecasts become stickier, with forecasters making fewer adjustments when they observe the latest data. This contrasts with the previously mentioned evidence in the literature, which shows that the degree of over-reaction increases with longer forecast horizons.

Third, γ varies across different variables. For the GDP Price Index and the CPI inflation rate, γ is relatively lower, particularly from the perspective of the consensus-level regression. For the CPI inflation, γ is close to zero, regardless of the forecast horizons, and whether it's considered at the consensus or individual level. For the 3-month Tbill rate, real GDP growth rate, and nominal GDP growth rate, γ is quite large with shorter forecast horizons, implying that forecasters put all the weight on current observations or even extrapolate based on current observations. However, with larger forecast horizons, all weight shifts to past forecasts, implying a high degree of stickiness in forecasts.

The patterns outlined above are qualitatively consistent at both the consensus and individual levels, albeit with some quantitative differences. For instance, at the individual level, γ for the 3-month Tbill yield is smaller in magnitude at horizon t, but larger in magnitude at longer horizons, compared to the consensus level. Robustness checks utilizing w_t instead of w_{t-1} are reported in table 2.9 and 2.10 in the appendix. The majority of patterns noted earlier regarding the model-implied regression remain consistent, irrespective of whether we use w_t or w_{t-1}. The only difference observed in tables 2.9 and 2.10 is that for the 3-month Tbill yield, 10-year Tbond yield, and AAA corporate bond yield, the γ value are much more uniform across forecast horizons.

Comparing the γ estimation results from Table 2.3 (where γ is estimated by matching the empirically observed regression coefficients with those implied by the model) reveals differences in γ estimation between the two methods. Specifically, when making these comparisons, we should focus on the forecast horizon $t+3$ presented in Tables 2.5 and 2.6. Estimations from Table 2.3 are predominantly larger than those in Tables 2.5 and 2.6. One potential explanation for this discrepancy could be that the benchmark adaptive expectations model doesn't perfectly capture the true process of expectation formations.

Table 2.5: Adaptive Expectations Model Implied Regression Results: Consensus Level

		RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
	t	1.07***	1.1***	0.35***	0.16	2.11***	1.81***	1.1***
		(0.33)	(0.3)	(0.08)	0.11	(0.24)	(0.18)	(0.13)
γ_h	$t+1$	0.42**	0.28	0.14***	0.01	0.03	1.32***	0.84***
		(0.17)	(0.21)	(0.05)	(0.03)	(0.3)	(0.25)	(0.19)
	$t+2$	0.12*	−0.01	0.07*	0.04**	−0.09	0.96***	0.55***
		(0.07)	(0.05)	(0.04)	(0.02)	(0.2)	(0.21)	(0.15)
	$t+3$	0.01	−0.09***	0.06*	0.07***	−0.05	0.65***	0.35***
		(0.08)	(0.02)	(0.03)	(0.01)	(0.12)	(0.16)	(0.11)

Notes: This table reports the regression results from equation (2.16) at the consensus level, for different variables and various forecast horizons. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.6: Adaptive Expectations Model Implied Regression Results: Individual Level

		RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
	t	1.03***	1.05***	0.7***	0.42***	1.40***	1.34***	0.92***
		(0.34)	(0.3)	(0.05)	(0.07)	(0.11)	(0.09)	(0.06)
γ_h	$t+1$	0.26	0.44***	0.47***	0.1***	0.66***	0.99***	0.75***
		(0.23)	(0.13)	(0.05)	(0.03)	(0.10)	(0.07)	(0.05)
	$t+2$	-0.03	0.17***	0.34***	0.1***	0.43***	0.75***	0.61***
		(0.08)	(0.06)	(0.04)	(0.03)	(0.08)	(0.06)	(0.05)
	$t+3$	-0.1**	0.07	0.29***	0.12***	0.32***	0.6***	0.49***
		(0.04)	(0.07)	(0.04)	(0.02)	(0.06)	(0.06)	(0.04)

Notes: This table reports the regression results from equation (2.16) at the individual level, for different variables and various forecast horizons. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.

2.8.4 Test on Parameter Restrictions

There are certain parameter restrictions imposed on the benchmark adaptive expectations model that may not necessarily hold when compared with empirical data. To characterize actual expectation formation processes more closely, I conduct tests on two parameter restrictions related to the benchmark adaptive expectations model using the following regression equation:

$$\mathcal{F}_t w_{t+h} - \mathcal{F}_{t-1} w_{t+h-1} = \alpha_{h,0} + \alpha_{h,1} w_{t-1} + \alpha_{h,2} \mathcal{F}_{t-1} w_{t+h-1} + \epsilon_{h,t}, h = 0, 1, 2, 3. \quad (2.17)$$

Instead of following the original CG regression, this equation originates from the benchmark adaptive expectations model as described in the section above. The first parameter restriction is, the benchmark adaptive expectations model assumes a zero constant term, $\alpha_{h,0} = 0$. In other words, there is no constant component beyond the latest observation and past forecasts. The second restriction is, the coefficients in front of the latest observation and past forecasts have the same magnitude but with opposite signs, $\alpha_{h,1} + \alpha_{h,2} = 0$. I present the regression results for various forecast horizons and variables in tables 2.11 and 2.12.

Let's start by analyzing the test at the consensus level. Firstly, in most instances, the constant term is significantly positive. In only three out of the 28 variable-horizon pairs is the constant term not significant at the 90% confidence level. Secondly, the p-value for the test $\alpha_{h,1} + \alpha_{h,2} = 0$ is generally small, implying that the parameter restriction in the benchmark adaptive expectations model is largely refuted by the data. In 20 out of the 28 variable-horizon pairs, the parameter restriction is rejected at the 95% confidence level. The coefficient preceding the latest observation $\alpha_{h,1}$ decreases over forecast horizons, while the coefficient in front of the last forecasts $\alpha_{h,2}$ increases over these horizons. As the forecast horizon extends, the revision in forecast becomes less influenced by the most recent observation. Instead, forecasts become increasingly sticky, and depend more heavily on past forecasts. In some instances, for example Panel A in table 2.11 for the 3-month Tbill yield, 10-year Tbond yield and AAA corporate bond yield, $\alpha_{h,1} > 1$ and $\gamma_{h,2} < -1$. This suggests that in these scenarios, forecasters over-extrapolate from their latest observation, and adjust in the opposite direction of past forecasts. Thirdly, the R^2 value decreases over forecast horizon. This is consistent with the finding that forecasts become stickier when forecast horizon extends. The right-hand-side variables have smaller explanatory power for the revision in forecasts. The empirical patterns at the consensus level are also consistent at the individual level, as shown in table 2.12. The individual level regression should be estimated with Arellano-Bond (1991) estimator.

I conduct the robustness test in which forecasters update their forecasts according to w_t instead of w_{t-1}. I present the regression results for various forecast horizons and variables in tables 2.13 and 2.14. The results generally hold with some quantitative difference. For example, the R^2 is higher when w_t is used instead of w_{t-1}. This is intuitive since w_t should have more explanatory power than w_{t-1} for current forecasts.

2.8.5 A Robustness Test on Regression Specification

Throughout the prior computations of Error-on-revision regression coefficients using the simulated data, forecast revisions are determined under Assumption 1. This assumption was primarily introduced for simplification purposes. Nonetheless, it may raise concerns that our analysis of Error-on-revision regression coefficients under adaptive expectations could be overly reliant on this assumption. To determine whether the prior results rely on this assumption, we can easily test its influence. For this, I conduct the following regression:

$$w_{t+h} - \bar{\mathcal{F}}_t w_{t+h} = \beta_0^C + \beta^C(\bar{\mathcal{F}}_t w_{t+h} - \bar{\mathcal{F}}_{t-1} w_{t+h-1}) + u_{t,t+h}, \tag{2.18}$$

$$w_{t+h} - \mathcal{F}_{i,t} w_{t+h} = \beta_0^I + \beta^I(\mathcal{F}_{i,t} w_{t+h} - \mathcal{F}_{i,t-1} w_{t+h-1}) + u_{i,t,t+h}. \tag{2.19}$$

Here, the concept of forecast revision is slightly altered compared to its traditional definition. The "modified forecast revision" refers to the adjustment between two consecutive one-period forecasts, replacing $\mathcal{F}_{i,t-1} w_{t+h}$ with $\mathcal{F}_{i,t-1} w_{t+h-1}$. If the regression coefficients estimated from equations (2.18) and (2.19) align with those from the original regression, it would suggest that Assumption 1 does not significantly influence our results. In other words, it does not substantially affect our findings regarding the implications of adaptive expectations on Error-on-revision regression coefficients.

In Table 2.7, the results of the regression are presented. The column titled "CG" reports the original Error-on-revision regression coefficients, while the "Modified" column shows the adjusted regression coefficients. Regardless of whether $\mathcal{F}_{i,t-1}w_{t+h}$ or $\mathcal{F}_{i,t-1}w_{t+h-1}$ is employed, the quantitative difference between the regression coefficients is negligible. There isn't a single case where the coefficients from the two regression specifications are statistically distinguishable. Hence, the Error-on-revision regression coefficients remain robust, irrespective of how the forecast revisions are calculated. As such, Assumption 1 is not important with respect to the Error-on-revision regression coefficients.

Table 2.7: Modified Error-on-revision Regression Results

	β^{Census}				$\beta^{Individual}$			
	CG	R^2	Modified	R^2	CG	Adj R^2	Modified	Adj R^2
RGDP	0.11	0.0009	0.17	0.012	-0.28^{***}	0.098	-0.08	0.086
	(0.31)		(0.1)		(0.12)		(0.18)	
GDP Price Index	1.26^{***}	0.24	0.93^{***}	0.103	-0.15^{***}	0.169	-0.26^{***}	0.183
	(0.41)		(0.33)		(0.07)		(0.06)	
NGDP	0.14	0.004	0.19^{***}	0.015	-0.32^{***}	0.094	-0.14	0.066
	(0.25)		(0.07)		(0.12)		(0.21)	
CPI	1.04	0.018	0.69	0.009	-0.38^{***}	0.054	-0.41^{***}	0.057
	(0.76)		(0.72)		(0.09)		(0.08)	
Tbill	0.69^{***}	0.098	0.73^{***}	0.111	0.21^{***}	0.064	0.17^{*}	0.059
	(0.11)		(0.16)		(0.09)		(0.1)	
Tbond	-0.06	0.0001	-0.03	0.0002	-0.23^{***}	0.078	-0.29^{***}	0.086
	(0.09)		(0.09)		(0.02)		(0.09)	
AAA	-0.02	0.0001	0.003	0.0000	-0.27^{***}	0.125	-0.34^{***}	0.140
	(0.16)		(0.15)		(0.07)		(0.06)	

Notes: This table reports the regression results from equation (2.18) and (2.19). Column "CG" reports the original Error-on-revision regression coefficients. Column "Modified" reports the modified regression coefficients. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.

2.9 Conclusion

While the Rational Expectations Hypothesis has long been the cornerstone of modeling macroeconomic expectations, contemporary empirical data from forecast surveys challenges its dominance. The diagnostic expectations model, when generalized to allow for under-reaction to new information, struggles to capture the observed stickiness in specific variables. Conversely, the adaptive expectations model, rooted in mid-20th century thought, displays the ability to account for recently documented empirical patterns in survey data, particularly in terms of matching the CG regression coefficients. This chapter shows that the model's backward-looking feature is useful in understanding forecast survey data. Thus, the adaptive expectations model should still be a valuable tool in the evolving landscape of modeling macroeconomic expectations.

2.10 Appendix

2.10.1 Proofs

Proof 4 (Lemma 1) *When there is a one-time shock at period 1, the time series for the actual variable for* $t = 0, 1, 2, 3..., T...$ *is given by*

$$0, \sigma_e, 0, 0... \tag{2.20}$$

The time series of forecasts is given by

$$0, \gamma\sigma_e, (1-\gamma)\gamma\sigma_e, (1-\gamma)^2\gamma\sigma_e..., (1-\gamma)^{T-1}\gamma\sigma_e... \tag{2.21}$$

From the time series above, we see that $\gamma < 2$ *is required so that forecasts eventually converge to zero. The time series of forecast errors is given by*

$$\sigma_e, -\gamma\sigma_e, -(1-\gamma)\gamma\sigma_e, -(1-\gamma)^2\gamma\sigma_e..., -(1-\gamma)^{T-1}\gamma\sigma_e... \tag{2.22}$$

The time series of forecast revisions is given by

$$0, \gamma\sigma_e, -\gamma^2\sigma_e, -(1-\gamma)\gamma^2\sigma_e..., -(1-\gamma)^{T-2}\gamma^2\sigma_e... \tag{2.23}$$

The Error-on-revision regression coefficient is given by

$$\beta = \frac{\sum_t (FE_t - \overline{FE_t})(FR_t - \overline{FR_t})}{\sum_t (FR_t - \overline{FR_t})^2} \tag{2.24}$$

when T *is large enough, mean forecast revisions are given by the following*

$$\begin{aligned}\overline{FR_t} &= \lim_{T\to\infty} \frac{1}{T}(\gamma\sigma_e - \gamma^2\sigma_e - (1-\gamma)\gamma^2\sigma_e - ... - (1-\gamma)^{T-2}\gamma^2\sigma_e - ...) \\ &= \lim_{T\to\infty} \frac{1}{T}(\gamma\sigma_e - \gamma^2\sigma_e \frac{1-(1-\gamma)^T}{1-(1-\gamma)}) = \lim_{T\to\infty} \frac{1}{T}(\gamma\sigma_e - \gamma\sigma_e) = 0\end{aligned}$$

Similarly, we can show that $\overline{FE_t} = 0$*. Now the denominator is given by*

$$\begin{aligned}\sum_t (FR_t - \overline{FR_t})^2 &= \gamma^2\sigma_e^2 + \gamma^4\sigma_e^2 + (1-\gamma)^2\gamma^4\sigma_e^2 + ... + (1-\gamma)^{2T-4}\gamma^4\sigma_e^2 + ... \\ &= \gamma^2\sigma_e^2 + \gamma^4\sigma_e^2 \frac{1-(1-\gamma)^{2T}}{1-(1-\gamma)^2}\end{aligned}$$

when T *is large, denominator is given by* $\gamma^2\sigma_e^2 + \frac{\gamma^4\sigma_e^2}{1-(1-\gamma)^2}$ *Similarly, the numerator is given by*

$$\sum_t (FE_t - \overline{FE_t})(FR_t - \overline{FR_t}) = -\gamma^2\sigma_e^2 + \frac{(1-\gamma)\gamma^3\sigma_e^2}{1-(1-\gamma)^2} \tag{2.25}$$

After some tedious algebra, we arrive at the expression $\beta = -\frac{1}{2}$ *Q.E.D.*

Proof 5 (Lemma 2) *When there is a one-time shock at period 1, the time series for the actual variable for* $t = 0, 1, 2, 3..., T...$ *is given by*

$$0, \sigma_e, \rho\sigma_e, \rho^2\sigma_e... \tag{2.26}$$

The time series of forecasts is given by

$$0, \gamma\sigma_e, (1-\gamma+\rho)\gamma\sigma_e, [(1-\gamma)^2+\rho^2+\rho(1-\gamma)]\gamma\sigma_e..., \gamma\sigma_e\frac{\rho^T-(1-\gamma)^T}{\rho-1+\gamma}... \tag{2.27}$$

From the time series above, we see that $\gamma < 2$ *is required so that forecasts do not explode. The time series of forecast errors is given by*

$$\sigma_e, \sigma_e(\rho-\gamma), \sigma_e[\rho^2-\gamma(1-\gamma+\rho)], ..., \frac{\sigma_e}{\rho-1+\gamma}[(\rho-1)\rho^T+\gamma(1-\gamma)^T], ... \tag{2.28}$$

The time series of forecast revisions is given by

$$0, \gamma\sigma_e, \gamma\sigma_e(\rho-\gamma), ..., \frac{\gamma\sigma_e}{\rho-1+\gamma}[\rho^{T-1}(\rho-1)+(1-\gamma)^{T-1}\gamma], ... \tag{2.29}$$

The Error-on-revision regression coefficient is given by

$$\beta = \frac{\sum_t(FE_t-\overline{FE_t})(FR_t-\overline{FR_t})}{\sum_t(FR_t-\overline{FR_t})^2} \tag{2.30}$$

when T *is large enough, mean forecast revisions are given by the following*

$$\overline{FR_t} = \lim_{T\to\infty}\frac{1}{T}(\mathcal{F}_1-\mathcal{F}_0+\mathcal{F}_2-\mathcal{F}_1+...+\mathcal{F}_T-\mathcal{F}_{T-1}) = \lim_{T\to\infty}\frac{1}{T}\gamma\sigma_e\frac{\rho^T-(1-\gamma)^T}{\rho-1+\gamma} = 0$$

Similarly, we can show that $\overline{FE_t} = 0$. *Now when* T *is large enough, after some algebra the denominator is given by*

$$\sum_t(FR_t-\overline{FR_t})^2 = \frac{\gamma^2\sigma_e^2}{(\rho-1+\gamma)^2}[\frac{(\rho-1)^2}{1-\rho^2}+\frac{\gamma^2}{1-(1-\gamma)^2}+\frac{2(\rho-1)\gamma}{1-\rho(1-\gamma)}]$$

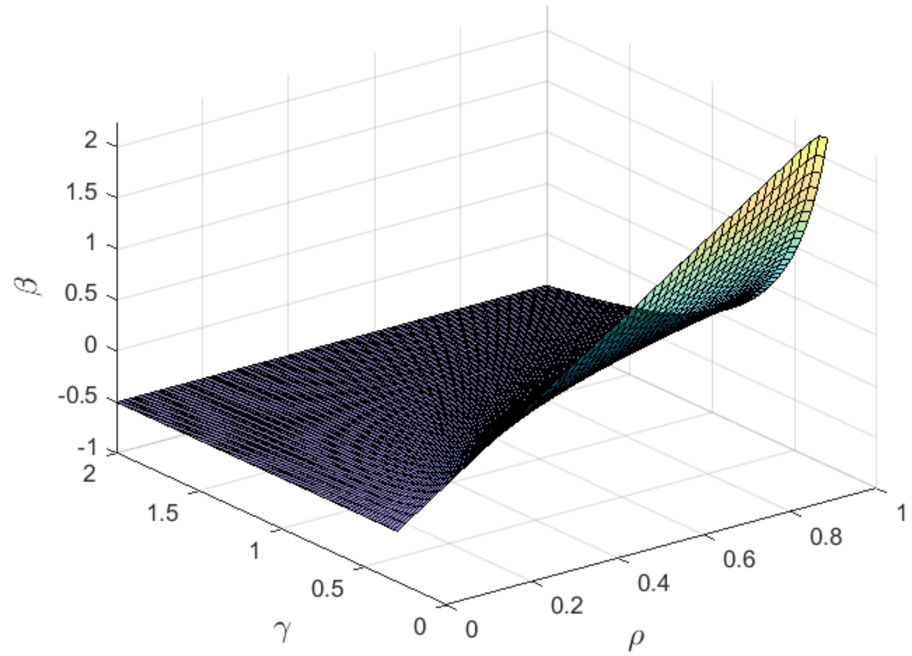
Similarly, when T *is large enough, the numerator is given by*

$$\sum_t(FE_t-\overline{FE_t})(FR_t-\overline{FR_t}) = \frac{\gamma\sigma_e^2}{(\rho-1+\gamma)^2}[(\rho-1)\frac{\rho}{-1-\rho}+\frac{(\rho-1)\gamma(\rho+1-\gamma)}{1-(1-\gamma)\rho}+\frac{\gamma^2(1-\gamma)}{1-(1-\gamma)^2}] \tag{2.31}$$

It is straightforward to check that $\lim_{\gamma\to 0}\beta = +\infty$.
By L'Hôpital's rule, $\lim_{\gamma\to 2}\beta = -\frac{1}{2}$.
Q.E.D.

2.10.2 Extra Tables and Figures

Figure 2.19: β as a Function of γ and ρ.



Notes: This figure plots β as a function of ρ and γ. The persistence parameter ρ is within $[0,1]$. The adaptive parameter γ is within $[0.3,2]$.

Table 2.8: Lower Values of γ Estimation.

	RGDP	GDP Price Index	NGDP	CPI	Tbill	AAA	Tbond
γ^I	0.07	only one	0.038	0.02	only one	0.05	0.01

Notes: This table presents the lower value of γ when two solutions for γ are available in matching β^I between the model and data.

Table 2.9: Robustness Test: Adaptive Expectations Model Implied Regression Results, Consensus Level

		RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
	$t+1$	1.29***	1.3***	0.53***	0.51***	0.83***	0.77***	0.7***
		(0.16)	(0.14)	(0.09)	(0.03)	(0.02)	(0.01)	(0.02)
γ_h	$t+2$	0.76***	0.76***	0.28***	0.11***	0.93***	0.75***	0.73***
		(0.28)	(0.25)	(0.05)	(0.02)	(0.05)	(0.02)	(0.05)
	$t+3$	0.33	0.37**	0.18*	0.06***	0.85***	0.69***	0.66***
		(0.23)	(0.18)	(0.03)	(0.02)	(0.06)	(0.03)	(0.06)
	$t+4$	0.14	0.19	0.14*	0.04**	0.72***	0.61***	0.57***
		(0.16)	(0.12)	(0.02)	(0.02)	(0.04)	(0.04)	(0.05)

Notes: This table reports the regression results from equation (2.16) at the consensus level, with w_t instead of w_{t-1}, for different variables and various forecast horizons. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.

Table 2.10: Robustness Test: Adaptive Expectations Model Implied Regression Results, Individual Level

		RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
	$t+1$	1.27***	1.26***	0.76***	0.57***	0.83***	0.78***	0.74***
		(0.21)	(0.21)	(0.05)	(0.03)	(0.03)	(0.09)	(0.03)
γ_h	$t+2$	0.82***	0.81***	0.52***	0.16***	0.90***	0.75***	0.71***
		(0.22)	(0.2)	(0.05)	(0.02)	(0.03)	(0.03)	(0.03)
	$t+3$	0.46***	0.48***	0.39***	0.11***	0.79***	0.67***	0.64***
		(0.13)	(0.12)	(0.04)	(0.02)	(0.04)	(0.03)	(0.04)
	$t+4$	0.26***	0.3***	0.32***	0.08***	0.65***	0.59***	0.55***
		(0.08)	(0.08)	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)

*Notes: This table reports the regression results from equation (2.16) at the individual level, with w_t instead of w_{t-1}, for different variables and various forecast horizons. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

Table 2.11: Consensus Level Coefficient Restriction Test Equation (2.17)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Forecast horizon: t							
α_0	1.55^{***}	2.9^{**}	0.28^{***}	1.29^{***}	0.18^{***}	0.19^{***}	-0.03^{***}
	(0.47)	(0.81)	(0.1)	(0.17)	(0.05)	(0.03)	(0.07)
α_1	0.39^{***}	0.63^{***}	0.38^{***}	0.3^{***}	2.23^{***}	1.8^{***}	1.17^{***}
	(0.11)	(0.1)	(0.06)	(0.05)	(0.29)	(0.18)	(0.15)
α_2	-1.11^{***}	-1.16^{***}	-0.46^{***}	-0.78^{***}	-2.26^{***}	-1.83^{***}	-1.16^{***}
	(0.23)	(0.21)	(0.08)	(0.07)	(0.27)	(0.18)	(0.15)
p-value	0.00	0.00	0.00	0.00	0.03	0.002	0.26
Obs.	210	212	211	162	157	117	159
R^2	0.49	0.24	0.16	0.3	0.54	0.65	0.41
Forecast horizon: $t+1$							
α_0	1.52^{**}	2.51^{**}	0.18^{***}	0.34^{***}	0.13^{*}	0.41^{***}	0.24^{***}
	(0.71)	(1.25)	(0.07)	(0.06)	(0.07)	(0.08)	(0.07)
α_1	0.01	0.2^{**}	0.19^{***}	0.01	0.39	1.38^{***}	0.83^{***}
	(0.05)	(0.1)	(0.06)	(0.03)	(0.28)	(0.28)	(0.2)
α_2	-0.62^{*}	-0.62^{**}	-0.25^{**}	-0.14^{***}	-0.43	-1.42^{***}	-0.86^{***}
	(0.32)	(0.31)	(0.07)	(0.03)	(0.28)	(0.28)	(0.2)
p-value	0.02	0.05	0.02	0.00	0.01	0.0003	0.004
Obs.	210	212	211	162	157	117	159
R^2	0.3	0.24	0.08	0.12	0.07	0.52	0.27
Forecast horizon: $t+2$							
α_0	1.19^{***}	1.59^{*}	0.12^{**}	0.17^{***}	0.13	0.55^{***}	0.34^{***}
	(0.64)	(0.95)	(0.05)	(0.04)	(0.1)	(0.14)	(0.1)
α_1	-0.11^{**}	-0.00	0.11^{**}	0.04^{**}	0.24	1.09^{***}	0.61^{***}
	(0.05)	(0.05)	(0.04)	(0.02)	(0.24)	(0.27)	(0.17)
α_2	-0.35	-0.26	-0.14^{**}	-0.1^{***}	-0.28	-1.16^{***}	-0.65^{***}
	(0.26)	(0.2)	(0.05)	(0.03)	(0.25)	(0.29)	(0.17)
p-value	0.05	0.11	0.06	0.00	0.02	0.002	0.004
Obs.	210	212	211	162	157	117	159
R^2	0.23	0.13	0.04	0.13	0.06	0.45	0.21
Forecast horizon: $t+3$							
α_0	0.96^{*}	1.00	0.10^{*}	0.17^{***}	0.12	0.61^{***}	0.32^{***}
	(0.56)	(0.63)	(0.05)	(0.06)	(0.1)	(0.2)	(0.1)
α_1	-0.13^{***}	-0.08^{***}	0.08^{**}	0.06^{***}	0.13	0.83^{***}	0.43^{***}
	(0.04)	(0.03)	(0.04)	(0.01)	(0.16)	(0.25)	(0.11)
α_2	-0.24	-0.09	-0.11^{**}	-0.12^{***}	-0.17	-0.91^{***}	-0.47^{***}
	(0.21)	(0.11)	(0.05)	(0.02)	(0.17)	(0.28)	(0.12)
p-value	0.07	0.12	0.13	0.01	0.02	0.01	0.005
Obs.	210	212	211	162	157	117	159
R^2	0.21	0.1	0.04	0.28	0.05	0.36	0.15

*Notes: This table reports the regression results from equation (2.17) at the consensus level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

Table 2.12: Individual Level Coefficient Restriction Test Equation (2.17)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Forecast horizon: t							
α_0	1.78**	4.46***	1.02***	1.7***	0.16***	0.15***	0.1
	(0.72)	(1.45)	(0.19)	(0.19)	(0.05)	(0.05)	(0.09)
α_1	0.36**	0.32**	0.39***	0.36***	0.63***	1.32***	0.9***
	(0.14)	(0.15)	(0.06)	(0.05)	(0.09)	(0.09)	(0.06)
α_2	−1.1***	−1.13***	−0.82***	−1.04***	−0.68***	−1.35***	−0.92***
	(0.22)	(0.19)	(0.04)	(0.06)	(0.09)	(0.09)	(0.06)
p-value	0.006	0.00	0.00	0.00	0.02	0.06	0.35
Obs.	4962	4971	4954	4327	4116	3440	3473
R^2	0.47	0.49	0.37	0.44	0.18	0.53	0.4
Forecast horizon: $t+1$							
α_0	1.81***	4.22***	0.85***	1.41***	0.2***	0.29***	0.34***
	(0.73)	(1.52)	(0.14)	(0.10)	(0.05)	(0.06)	(0.1)
α_1	0.04	−0.05	0.36***	0.06***	0.44***	0.97***	0.71***
	(0.98)	(0.10)	(0.04)	(0.02)	(0.07)	(0.07)	(0.05)
α_2	−0.72***	−0.77***	−0.64***	−0.61***	−0.49***	−1.01***	−0.76***
	(0.2)	(0.18)	(0.04)	(0.04)	(0.07)	(0.07)	(0.05)
p-value	0.005	0.00	0.00	0.00	0.003	0.0054	0.006
Obs.	4960	4973	4953	4329	4116	3441	3475
R^2	0.32	0.35	0.29	0.3	0.15	0.41	0.33
Forecast horizon: $t+2$							
α_0	1.59***	3.55***	0.73***	1.29***	0.25***	0.39***	0.48***
	(0.59)	(1.26)	(0.14)	(0.15)	(0.05)	(0.07)	(0.13)
α_1	−0.06	−0.05	0.28***	0.06***	0.32***	0.73***	0.57***
	(0.98)	(0.10)	(0.03)	(0.02)	(0.06)	(0.06)	(0.04)
α_2	−0.52***	−0.55***	−0.51***	−0.55***	−0.38***	−0.78***	−0.63***
	(0.14)	(0.13)	(0.04)	(0.05)	(0.06)	(0.06)	(0.05)
p-value	0.003	0.00	0.00	0.00	0.003	0.0008	0.001
Obs.	4953	4961	4949	4331	4116	3439	3471
R^2	0.26	0.27	0.23	0.27	0.14	0.33	0.28
Forecast horizon: $t+3$							
α_0	1.43***	3.02***	0.67***	1.18***	0.25***	0.46***	0.55***
	(0.48)	(1.00)	(0.12)	(0.14)	(0.05)	(0.08)	(0.11)
α_1	−0.09	−0.07	0.23***	0.08***	0.32***	0.58***	0.45***
	(0.08)	(0.09)	(0.03)	(0.01)	(0.06)	(0.06)	(0.04)
α_2	−0.43***	−0.43***	−0.45***	−0.51***	−0.38***	−0.64***	−0.52***
	(0.11)	(0.1)	(0.04)	(0.05)	(0.06)	(0.06)	(0.04)
p-value	0.001	0.00	0.00	0.00	0.0004	0.0001	0.0001
Obs.	4923	4932	4918	4296	4116	3408	3453
R^2	0.23	0.22	0.21	0.29	0.14	0.28	0.23

*Notes: This table reports the regression results from equation (2.17) at the individual level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

Table 2.13: Robustness: Consensus Level Coefficient Restriction Test Equation (2.17)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Forecast horizon: t							
α_0	0.29	0.97	0.29**	0.33***	0.05***	0.04***	−0.07
	(0.47)	(0.62)	(0.12)	(0.09)	(0.01)	(0.01)	(0.06)
α_1	1.12***	1.14***	0.53***	0.48***	0.82***	0.77***	0.74***
	(0.25)	(0.2)	(0.1)	(0.03)	(0.02)	(0.01)	(0.03)
α_2	−1.33***	−1.36***	−0.62***	−0.61***	−0.83***	−0.77***	−0.72***
	(0.14)	(0.13)	(0.12)	(0.03)	(0.02)	(0.01)	(0.02)
p-value	0.14	0.03	0.00	0.00	0.47	0.46	0.02
Obs.	209	212	209	162	157	117	159
R^2	0.7	0.72	0.29	0.78	0.95	0.88	0.8
Forecast horizon: $t+1$							
α_0	1.1**	1.99***	0.21***	0.26***	0.16***	0.17***	0.13**
	(0.44)	(0.73)	(0.08)	(0.05)	(0.04)	(0.03)	(0.05)
α_1	0.59***	0.69***	0.32***	0.1***	0.92***	0.74***	0.73***
	(0.19)	(0.16)	(0.06)	(0.02)	(0.06)	(0.02)	(0.06)
α_2	−1.01***	−1.02***	−0.38***	−0.2***	−0.93***	−0.75***	−0.73***
	(0.31)	(0.27)	(0.07)	(0.02)	(0.05)	(0.02)	(0.06)
p-value	0.002	0.01	0.00	0.00	0.04	0.04	0.41
Obs.	209	212	209	162	157	117	159
R^2	0.5	0.53	0.21	0.33	0.88	0.83	0.72
Forecast horizon: $t+2$							
α_0	1.12*	1.63*	0.15**	0.12***	0.24***	0.29***	0.26***
	(0.58)	(0.85)	(0.06)	(0.04)	(0.06)	(0.04)	(0.07)
α_1	0.28	0.37**	0.2***	0.06*	0.84***	0.69***	0.65***
	(0.18)	(0.16)	(0.03)	(0.01)	(0.07)	(0.03)	(0.06)
α_2	−0.68*	−0.63**	−0.25***	−0.11***	−0.86***	−0.71***	−0.68***
	(0.36)	(0.3)	(0.04)	(0.02)	(0.07)	(0.02)	(0.06)
p-value	0.03	0.07	0.01	0.00	0.02	0.003	0.04
Obs.	209	212	209	162	157	117	159
R^2	0.31	0.31	0.15	0.24	0.79	0.79	0.65
Forecast horizon: $t+3$							
α_0	1.03*	1.16	0.13**	0.15***	0.32***	0.4***	0.32***
	(0.61)	(0.71)	(0.05)	(0.04)	(0.07)	(0.06)	(0.08)
α_1	0.14	0.2	0.16***	0.04*	0.71***	0.62***	0.57***
	(0.14)	(0.13)	(0.03)	(0.02)	(0.04)	(0.04)	(0.05)
α_2	−0.5	−0.37	−0.19***	−0.09***	−0.74***	−0.66***	−0.6***
	(0.34)	(0.24)	(0.04)	(0.02)	(0.05)	(0.06)	(0.05)
p-value	0.07	0.11	0.02	0.01	0.01	0.001	0.03
Obs.	209	212	209	162	157	117	159
R^2	0.21	0.17	0.13	0.14	0.69	0.76	0.58

*Notes: This table reports the regression results from equation (2.17) at the consensus level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. In this test, forecasters update their forecasts according to w_t instead of w_{t-1}. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

Table 2.14: Robustness: Individual Level Coefficient Restriction Test Equation (2.17)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Forecast horizon: t							
α_0	0.31	0.88	0.89***	0.76***	0.00***	0.02	−0.07
	(0.89)	(2.04)	(0.16)	(0.13)	(0.01)	(0.03)	(0.07)
α_1	1.14***	1.1***	0.56***	0.46***	0.84***	0.79***	0.76***
	(0.39)	(0.41)	(0.06)	(0.03)	(0.02)	(0.03)	(0.03)
α_2	−1.29***	−1.28***	−0.86***	−0.78***	−0.83***	−0.78***	−0.74***
	(0.19)	(0.19)	(0.05)	(0.03)	(0.03)	(0.03)	(0.03)
p-value	0.62	0.58	0.00	0.00	0.01	0.19	0.08
Obs.	4929	4971	4895	4327	4144	3440	3473
R^2	0.65	0.66	0.41	0.64	0.86	0.76	0.62
Forecast horizon: $t+1$							
α_0	1.2**	2.58**	0.78***	1.28***	0.08***	0.13***	0.12
	(0.54)	(1.19)	(0.12)	(0.1)	(0.02)	(0.04)	(0.08)
α_1	0.59***	0.58***	0.41***	0.1***	0.91***	0.74***	0.71***
	(0.19)	(0.19)	(0.04)	(0.02)	(0.03)	(0.03)	(0.04)
α_2	−1.01***	−1.01***	−0.67***	−0.6***	−0.9***	−0.74***	−0.71***
	(0.23)	(0.22)	(0.05)	(0.04)	(0.03)	(0.03)	(0.03)
p-value	0.02	0.02	0.00	0.00	0.42	0.77	0.57
Obs.	4926	4973	4894	4329	4146	3441	3475
R^2	0.49	0.51	0.33	0.33	0.74	0.66	0.54
Forecast horizon: $t+2$							
α_0	1.34***	2.68**	0.63***	1.24***	0.16***	0.24***	0.27***
	(0.5)	(1.05)	(0.11)	(0.15)	(0.03)	(0.05)	(0.1)
α_1	0.32***	0.33***	0.26***	0.06***	0.78***	0.66***	0.62***
	(0.12)	(0.12)	(0.03)	(0.01)	(0.04)	(0.03)	(0.04)
α_2	−0.77***	−0.77***	−0.47***	−0.53***	−0.79***	−0.68***	−0.64***
	(0.19)	(0.18)	(0.04)	(0.05)	(0.04)	(0.03)	(0.04)
p-value	0.005	0.01	0.00	0.00	0.76	0.15	0.1
Obs.	4919	4961	4859	4331	4141	3439	3471
R^2	0.36	0.37	0.24	0.27	0.6	0.57	0.46
Forecast horizon: $t+3$							
α_0	1.32***	2.48***	0.63***	1.23***	0.25***	0.33***	0.36***
	(0.45)	(0.9)	(0.11)	(0.16)	(0.04)	(0.05)	(0.09)
α_1	0.19**	0.2**	0.26***	0.03**	0.63***	0.58***	0.52***
	(0.08)	(0.09)	(0.03)	(0.02)	(0.04)	(0.03)	(0.03)
α_2	−0.63***	−0.6***	−0.47***	−0.49***	−0.65***	−0.61***	−0.56***
	(0.16)	(0.14)	(0.04)	(0.05)	(0.04)	(0.03)	(0.03)
p-value	0.002	0.01	0.00	0.00	0.04	0.01	0.01
Obs.	4889	4932	4859	4296	4116	3408	3453
R^2	0.28	0.28	0.24	0.26	0.49	0.49	0.38

*Notes: This table reports the regression results from equation (2.17) at the individual level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. In this test, forecasters update their forecasts according to w_t instead of w_{t-1}. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

2.10.3 Extra Test of Parameter Restriction

To characterize actual expectation formation processes more closely, I conduct tests on two parameter restrictions related to the benchmark adaptive expectations model using the following regression equation:

$$\mathcal{F}_t w_{t+h} = \gamma_{0,h} + \gamma_{1,h} w_{t-1} + \gamma_{2,h} \mathcal{F}_{t-1} w_{t+h-1} + u_{t,h}, h = 0, 1, 2, 3, \tag{2.32}$$

The first parameter restriction is, the benchmark adaptive expectations model assumes a zero constant term, $\gamma_{0,h} = 0$. In other words, there is no constant component beyond the latest observation and past forecasts. The second restriction is, the model assumes the current forecast is the weighted average of the latest observation and past forecasts, with the sum of these two coefficients equating to one, $\gamma_{1,h} + \gamma_{2,h} = 1$. I present the regression results for various forecast horizons and variables in tables 2.15 and 2.16.

There are several patterns in table 2.15. Firstly, in most instances, the constant term is significantly positive. In only four out of the 28 variable-horizon pairs is the constant term not significant at the 90% confidence level. Secondly, the p-value for the test $\gamma_{1,h} + \gamma_{2,h} = 1$ is generally small, implying that the parameter restriction in the benchmark adaptive expectations model is largely refuted by the data. In 23 out of the 28 variable-horizon pairs, the parameter restriction is rejected at the 95% confidence level. The coefficient preceding the latest observation γ_1 decreases over forecast horizons, while the coefficient in front of the last forecasts γ_2 increases over these horizons. As the forecast horizon expands, forecasters assign less weight to the latest observation. Instead, forecasts become increasingly sticky, and depend more heavily on past forecasts. In some instances, for example Panel A in table 2.15 for the 3-month Tbill yield and the 10-year Tbond yield, $\gamma_1 > 1$ and $\gamma_2 < 0$. This suggests that in these scenarios, forecasters over-extrapolate from their latest observation, and adjust in the opposite direction of past forecasts. Thirdly, the R^2 value is substantial in many cases. For instance, for the 3-month Tbill yield, 10-year Tbond yield, and AAA corporate bond yield, the R^2 value approaches 1. For real and nominal GDP growth rates, and the CPI inflation rate, R^2 is small at forecast horizon t, but increases over forecast horizons. The R^2 value of the CPI inflation rate doubles from 0.46 at forecast horizon t to 0.95 at forecast horizon $t+3$.

The patterns outlined above are qualitatively consistent at both the consensus and individual levels, albeit with some quantitative differences. Robustness tests using w_t as the latest observation instead of w_{t-1} are reported in table 2.17 and 2.18. Most of the patterns described above are robust, with some quantitative difference. For example, for 3-month Tbill yield and 10-year Tbond yield, at the consensus level, $\gamma_1 > 1$ and $\gamma_2 < 0$ are not robust depending on whether w_t or w_{t-1} is used as the latest observation.

Table 2.15: Consensus Level Coefficient Restriction Test Equation (2.32)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: Forecast horizon: t							
γ_0	1.56***	2.87***	0.29***	1.25***	0.18***	0.2***	−0.04
	(0.48)	(0.81)	(0.1)	(0.17)	(0.05)	(0.03)	(0.07)
γ_1	0.39***	0.62***	0.37***	0.29***	2.31***	1.83***	1.2***
	(0.11)	(0.1)	(0.06)	(0.04)	(0.26)	(0.17)	(0.15)
γ_2	−0.1	−0.15	0.55***	0.25***	−1.34***	−0.86***	−0.19
	(0.23)	(0.21)	(0.08)	(0.07)	(0.25)	(0.17)	(0.14)
p-value	0.00	0.0008	0.003	0.00	0.03	0.001	0.22
Obs.	210	212	211	162	157	117	159
R^2	0.05	0.16	0.87	0.46	0.99	0.98	0.99
Panel B: Forecast horizon: $t+1$							
γ_0	1.51**	2.47**	0.19***	0.34***	0.13*	0.42***	0.24***
	(0.67)	(1.21)	(0.07)	(0.06)	(0.07)	(0.08)	(0.06)
γ_1	0.01	0.18*	0.19***	0.01	0.43	1.41***	0.86***
	(0.07)	(0.09)	(0.06)	(0.03)	(0.28)	(0.27)	(0.2)
γ_2	0.39	0.4	0.75***	0.86***	0.53*	−0.46*	0.11
	(0.3)	(0.3)	(0.07)	(0.04)	(0.29)	(0.28)	(0.2)
p-value	0.02	0.05	0.01	0.00	0.01	0.0002	0.002
Obs.	210	212	211	162	157	117	159
R^2	0.15	0.33	0.93	0.85	0.97	0.98	0.98
Panel C: Forecast horizon: $t+2$							
γ_0	1.19**	1.57*	0.13**	0.23***	0.13	0.57***	0.33***
	(0.6)	(0.91)	(0.05)	(0.06)	(0.1)	(0.14)	(0.09)
γ_1	−0.11**	−0.01	0.11**	0.03	0.25	1.1***	0.64***
	(0.04)	(0.04)	(0.05)	(0.02)	(0.27)	(0.27)	(0.17)
γ_2	0.64***	0.76***	0.85***	0.89***	0.7**	−0.18	0.32*
	(0.6)	(0.19)	(0.05)	(0.03)	(0.28)	(0.29)	(0.18)
p-value	0.03	0.1	0.04	0.0002	0.01	0.001	0.001
Obs.	210	212	211	162	157	117	159
R^2	0.33	0.55	0.95	0.93	0.97	0.97	0.98
Panel D: Forecast horizon: $t+3$							
γ_0	0.98*	0.97	0.12**	0.2***	0.13	0.64***	0.35***
	(0.51)	(0.6)	(0.05)	(0.06)	(0.11)	(0.2)	(0.09)
γ_1	−0.12***	−0.08***	0.09**	0.06***	0.16	0.84***	0.48***
	(0.03)	(0.02)	(0.04)	(0.007)	(0.19)	(0.25)	(0.12)
γ_2	0.75***	0.92***	0.88***	0.87***	0.8***	0.07	0.48***
	(0.18)	(0.1)	(0.05)	(0.02)	(0.2)	(0.27)	(0.13)
p-value	0.04	0.11	0.07	0.002	0.01	0.01	0.001
Obs.	210	212	211	162	157	117	159
R^2	0.43	0.71	0.96	0.95	0.97	0.97	0.98

Notes: This table reports the regression results from equation (2.32) at the consensus level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$*. Standard errors are displayed in parenthesis. ** $p < 0.1$*, *** $p < 0.05$*, ****$p < 0.01$*.*

Table 2.16: Individual Level Coefficient Restriction Test Equation (2.32)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: Forecast horizon: t							
γ_0	1.78**	4.46***	0.85***	1.7***	0.09**	0.15**	0.1
	(0.72)	(1.45)	(0.14)	(0.19)	(0.04)	(0.05)	(0.09)
γ_1	0.36**	0.32**	0.36***	0.36***	1.44***	1.32***	0.9***
	(0.14)	(0.15)	(0.04)	(0.05)	(0.12)	(0.09)	(0.06)
γ_2	−0.1	−0.13	0.36***	−0.04	−0.46***	−0.35***	0.08
	(0.22)	(0.19)	(0.04)	(0.06)	(0.11)	(0.09)	(0.06)
p-value	0.0006	0.0005	0.00	0.00	0.17	0.06	0.35
Obs.	4962	4971	4953	4327	4144	3440	3473
R^2	0.04	0.17	0.78	0.35	0.98	0.97	0.98
Panel B: Forecast horizon: $t+1$							
γ_0	1.81**	4.22**	0.73***	1.41***	0.16***	0.29***	0.34***
	(0.73)	(1.52)	(0.14)	(0.1)	(0.05)	(0.06)	(0.1)
γ_1	0.04	0.05	0.28***	0.06***	0.63***	0.97***	0.71***
	(0.09)	(0.10)	(0.03)	(0.02)	(0.09)	(0.07)	(0.05)
γ_2	0.28	0.23	0.49***	0.39***	0.32***	−0.006	0.24***
	(0.2)	(0.18)	(0.04)	(0.04)	(0.09)	(0.07)	(0.05)
p-value	0.005	0.004	0.00	0.00	0.02	0.005	0.006
Obs.	4960	4973	4949	4329	4146	3441	3475
R^2	0.14	0.35	0.82	0.55	0.96	0.95	0.97
Panel C: Forecast horizon: $t+2$							
γ_0	1.59***	3.55***	0.73***	1.29***	0.2***	0.39***	0.48***
	(0.59)	(1.26)	(0.14)	(0.15)	(0.05)	(0.07)	(0.13)
γ_1	−0.06	−0.05	0.28***	0.06***	0.43***	0.73***	0.57***
	(0.09)	(0.10)	(0.03)	(0.02)	(0.07)	(0.06)	(0.04)
γ_2	0.48***	0.45***	0.49***	0.45***	0.51***	0.22***	0.37***
	(0.14)	(0.13)	(0.04)	(0.05)	(0.07)	(0.06)	(0.05)
p-value	0.003	0.004	0.00	0.00	0.003	0.0008	0.001
Obs.	4963	4961	4949	4331	4141	3439	3471
R^2	0.26	0.51	0.82	0.63	0.96	0.94	0.96
Panel D: Forecast horizon: $t+3$							
γ_0	1.43***	3.02***	0.67***	1.18***	0.25***	0.46***	0.55***
	(0.48)	(1.00)	(0.12)	(0.14)	(0.05)	(0.08)	(0.11)
γ_1	−0.09	−0.07	0.23***	0.08***	0.32***	0.58***	0.45***
	(0.08)	(0.09)	(0.03)	(0.01)	(0.06)	(0.06)	(0.04)
γ_2	0.57***	0.57***	0.55***	0.49***	0.62***	0.36***	0.48***
	(0.11)	(0.1)	(0.04)	(0.05)	(0.06)	(0.06)	(0.04)
p-value	0.001	0.002	0.00	0.00	0.0004	0.0001	0.0001
Obs.	4923	4932	4918	4296	4116	3408	3453
R^2	0.35	0.62	0.83	0.69	0.95	0.93	0.96

Notes: This table reports the regression results from equation (2.32) at the individual level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. *Standard errors are displayed in parenthesis.* * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.

Table 2.17: Robustness Check: Consensus Level Coefficient Restriction Test Equation (2.32)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: Forecast horizon: t							
γ_0	0.3	0.94*	0.3**	0.3***	0.04***	0.05***	−0.08
	(0.5)	(0.57)	(0.12)	(0.07)	(0.01)	(0.01)	(0.06)
γ_1	1.12***	1.13***	0.5***	0.47***	0.82***	0.77***	0.76***
	(0.27)	(0.19)	(0.09)	(0.03)	(0.02)	(0.01)	(0.03)
γ_2	−0.33**	−0.34***	0.41***	0.41***	0.17***	0.23***	0.26***
	(0.15)	(0.12)	(0.11)	(0.02)	(0.02)	(0.01)	(0.02)
p-value	0.15	0.02	0.003	0.001	0.59	0.88	0.01
Obs.	209	212	209	162	157	117	159
R^2	0.44	0.53	0.89	0.85	0.99	0.99	0.99
Panel B: Forecast horizon: $t+1$							
γ_0	1.1***	1.96***	0.22***	0.26***	0.15***	0.18***	0.13**
	(0.4)	(0.7)	(0.08)	(0.04)	(0.04)	(0.03)	(0.05)
γ_1	0.58***	0.67***	0.3***	0.1***	0.92***	0.75***	0.74***
	(0.18)	(0.15)	(0.05)	(0.02)	(0.06)	(0.02)	(0.05)
γ_2	−0.002	−0.007	0.63***	0.8***	0.07	0.24***	0.25***
	(0.29)	(0.26)	(0.08)	(0.02)	(0.06)	(0.02)	(0.05)
p-value	0.0005	0.006	0.002	0.00	0.03	0.01	0.38
Obs.	209	212	209	162	157	117	159
R^2	0.4	0.58	0.94	0.89	0.97	0.99	0.99
Panel C: Forecast horizon: $t+2$							
γ_0	1.12**	1.6**	0.16***	0.18***	0.24***	0.31***	0.25***
	(0.53)	(0.82)	(0.06)	(0.05)	(0.06)	(0.04)	(0.06)
γ_1	0.27	0.36**	0.19***	0.05***	0.85***	0.69***	0.68***
	(0.17)	(0.15)	(0.03)	(0.01)	(0.07)	(0.03)	(0.06)
γ_2	0.32	0.38	0.76***	0.88***	0.13*	0.28***	0.3***
	(0.33)	(0.28)	(0.04)	(0.02)	(0.07)	(0.02)	(0.06)
p-value	0.02	0.06	0.003	0.0001	0.02	0.002	0.04
Obs.	209	212	209	162	157	117	159
R^2	0.4	0.64	0.96	0.94	0.99	0.99	0.99
Panel D: Forecast horizon: $t+3$							
γ_0	1.05*	1.12*	0.14***	0.18***	0.33***	0.41***	0.33***
	(0.55)	(0.68)	(0.05)	(0.04)	(0.08)	(0.06)	(0.07)
γ_1	0.14	0.19	0.15***	0.03***	0.72***	0.62***	0.61***
	(0.13)	(0.12)	(0.03)	(0.01)	(0.04)	(0.04)	(0.06)
γ_2	0.49	0.63***	0.81***	0.9***	0.25***	0.34***	0.37***
	(0.31)	(0.23)	(0.04)	(0.02)	(0.04)	(0.04)	(0.06)
p-value	0.04	0.11	0.01	0.0001	0.01	0.0006	0.009
Obs.	209	212	209	162	157	117	159
R^2	0.44	0.73	0.97	0.94	0.99	0.99	0.99

*Notes: This table reports the regression results from equation (2.32) at the consensus level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. In this test, forecasters update their forecasts according to w_t instead of w_{t-1}. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

Table 2.18: Robustness Check: Individual Level Coefficient Restriction Test Equation (2.32)

	RGDP	NGDP	GDP Price Index	CPI	Tbill	Tbond	AAA
Panel A: Forecast horizon: t							
γ_0	0.31	0.88	0.89***	0.76***	0.00	0.02	-0.07
	(0.89)	(2.04)	(0.16)	(0.13)	(0.01)	(0.03)	(0.07)
γ_1	1.14***	1.1***	0.56***	0.46***	0.84***	0.78***	0.76***
	(0.39)	(0.41)	(0.06)	(0.03)	(0.02)	(0.03)	(0.03)
γ_2	-0.29	-0.28	0.14***	0.22***	0.17***	0.22***	0.26***
	(0.19)	(0.19)	(0.05)	(0.03)	(0.03)	(0.03)	(0.03)
p-value	0.62	0.58	0.00	0.00	0.01	0.19	0.08
Obs.	4929	4971	4895	4327	4144	3440	3473
R^2	0.37	0.44	0.67	0.58	0.99	0.98	0.99
Panel B: Forecast horizon: $t+1$							
γ_0	1.2**	2.57**	0.78***	1.28***	0.08***	0.13***	0.13
	(0.54)	(1.19)	(0.12)	(0.1)	(0.02)	(0.04)	(0.08)
γ_1	0.59**	0.58***	0.41***	0.1***	0.91***	0.74***	0.71***
	(0.19)	(0.19)	(0.04)	(0.02)	(0.03)	(0.03)	(0.04)
γ_2	-0.006	-0.01	0.33***	0.4***	0.1***	0.25***	0.29***
	(0.23)	(0.22)	(0.05)	(0.04)	(0.03)	(0.03)	(0.03)
p-value	0.02	0.02	0.00	0.00	0.42	0.77	0.57
Obs.	4926	4973	4894	4329	4146	3441	3475
R^2	0.36	0.51	0.79	0.57	0.99	0.97	0.98
Panel C: Forecast horizon: $t+2$							
γ_0	1.34***	2.68**	0.68***	1.24***	0.16***	0.24***	0.27***
	(0.5)	(1.05)	(0.13)	(0.15)	(0.03)	(0.05)	(0.1)
γ_1	0.32***	0.33***	0.32***	0.06***	0.78***	0.66***	0.62***
	(0.12)	(0.12)	(0.03)	(0.01)	(0.04)	(0.03)	(0.04)
γ_2	0.23	0.23	0.46***	0.47***	0.21***	0.24***	0.36***
	(0.19)	(0.18)	(0.05)	(0.05)	(0.04)	(0.05)	(0.04)
p-value	0.005	0.009	0.00	0.00	0.76	0.15	0.1
Obs.	4919	4961	4890	4331	4141	3439	3471
R^2	0.36	0.58	0.83	0.63	0.98	0.96	0.97
Panel D: Forecast horizon: $t+3$							
γ_0	1.32***	2.48***	0.63***	1.23***	0.25***	0.33***	0.36***
	(0.45)	(0.9)	(0.11)	(0.16)	(0.04)	(0.05)	(0.09)
γ_1	0.19**	0.2**	0.26***	0.03**	0.63***	0.58***	0.52***
	(0.08)	(0.09)	(0.03)	(0.02)	(0.04)	(0.03)	(0.03)
γ_2	0.37**	0.4***	0.53***	0.51***	0.35***	0.39***	0.44***
	(0.16)	(0.14)	(0.04)	(0.05)	(0.04)	(0.03)	(0.03)
p-value	0.002	0.005	0.00	0.00	0.04	0.01	0.01
Obs.	4889	4932	4859	4296	4116	3408	3453
R^2	0.39	0.65	0.84	0.67	0.97	0.95	0.97

*Notes: This table reports the regression results from equation (2.32) at the individual level for each variable and for multiple forecast horizons. The p-value is from the Wald test of the coefficient restriction $\gamma_1 + \gamma_2 = 1$. In this test, forecasters update their forecasts according to w_t instead of w_{t-1}. Standard errors are displayed in parenthesis. * $p < 0.1$, ** $p < 0.05$, ***$p < 0.01$.*

2.11 Data Appendix

2.11.1 Construction of Variables

For the construction of forecast errors and forecast revisions using variables from SPF and the real time macro data, over the forecast horizon of one year, the method is similar to that of BGMS (2020). The instructions are documented in the data appendix of chapter 1. In section 2.8.3, various forecast horizons are used. The construction of forecasts varies slightly depending on how forecasts are reported in SPF. The details are documented as following:

1. NGDP:
   - Forecast for $t+h(h \in \{1,2,3,4\})$:
   - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t-1$.

2. RGDP:
   - Forecast for $t+h(h \in \{1,2,3,4\})$: $(\frac{\mathcal{F}_t x_{t+h-1}}{x_{t-1}} - 1) * \frac{4}{h}$. The forecast on level is transformed into the growth rate, and the factor $\frac{4}{h}$ is to annualize the growth rate.
   - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t-1$.

3. GDP Price Index:
   - Forecast for $t+h(h \in \{1,2,3,4\})$:
   - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t-1$.

4. CPI:
   - Forecast for $t+h(h \in \{1,2,3,4\})$: $\mathcal{F}_t x_{t+h-1}$, where t is the quarter of forecast and x is the CPI inflation.
   - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t-1$.

5. AAA:
   - Forecast for $t+h(h \in \{1,2,3,4\})$: $\mathcal{F}_t x_{t+h-1}$, where t is the quarter of forecast and x is the level of AAA corporate bond yield.
   - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t-1$.

6. TBILL:

    - Forecast for $t + h(h \in \{1, 2, 3, 4\})$: $\mathcal{F}_t x_{t+h-1}$, where t is the quarter of forecast and x is the level of 3-month treasury yield.
    - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t - 1$.

7. TBOND:

    - Forecast for $t + h(h \in \{1, 2, 3, 4\})$: $\mathcal{F}_t x_{t+h-1}$, where t is the quarter of forecast and x is the level of 10-year treasury yield.
    - Current observation: x_{t-1}, when forecasters making forecasts in period t, they can only observe the macroeconomic data released in $t - 1$.

Chapter 3

Consequences of Financial Constraints on Economic Growth and Employment: the Role of the Zero Lower Bound

3.1 Introduction

What impact do financial frictions have on economic growth and employment? The literature largely agrees that increased financial frictions negatively affect both. This position is fortified by theoretical models from Aghion, Angeletos, Banerjee and Manova (2010), Moll (2014), Midrigan and Xu (2014), and empirical findings from Siemer (2019), Duygan-Bump, Levkov and Montoriol-Garriga (2015), Chodorow-Reich (2014), Benmelech, Frydman and Papanikolaou (2019), and Duval, Hong and Timmer (2020).

This chapter contributes to this discussion by examining the role of the nominal interest rate, particularly when it hits the zero lower bound. Our findings offer a surprising contrast to conventional wisdom, suggesting that, at the zero lower bound, tightening funding constraints can potentially increase employment without negatively impacting growth.

This research builds upon the innovative model by Benigno and Fornaro (2018) that combines endogenous growth and nominal rigidities, resulting in two possible steady states. The "good" steady state exhibits full employment, a high growth rate and a positive nominal interest rate. The "bad" steady state, called the stagnation trap, has low employment, slow growth, and the nominal interest rate is at the zero lower bound.

I introduce an earnings-based funding constraint into this model, exploring its impact on the economic steady states. This specification of financial frictions departs from the collateral-based constraint traditionally discussed in literature (Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999)), instead aligning with recent research demonstrating evidence of earnings-based constraints (Lian and Ma (2021), Drechsel (2023)).

My analysis focuses on steady states, and then studies how a tightening of the constraint affects the steady state outcome. As the constraint tightens, it first impacts the full employment state, limiting the potential for investment in innovations and subsequently leading to slower growth and reduced aggregate demand. This decline necessitates a lower nominal interest rate to align with decreased employment, thus creating what I refer to as the "constrained high growth" steady state. Further tightening of the constraint will at some point also affect the "stagnation trap", resulting in a "constrained low growth" steady state.

Interestingly, when the nominal interest rate hits the zero lower bound, continued tightening of the constraint does not decrease the growth rate, but instead leads to an increase in employment. The reason is that, at the zero lower bound the credit constraint can be relaxed if the production activity, or employment, increases. Since the zero lower bound is binding, the expansion will not be undone by an increase in the nominal interest rate.

However, too much constraint tightness is untenable, at which point no steady state can be maintained due to insufficient funds for maintaining the minimum growth rate dictated by the zero lower bound.

Literature

This chapter integrates two primary threads of literature. The first strand concerns the impact of financial constraints on economic outcomes. Traditionally, the discourse over the last two decades has revolved around asset-based borrowing constraints as posited by Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Hart and Moore (1994). However, recent micro-level data supports the concept of earnings-based borrowing constraints, challenging the conventional asset-based perspective. Lian and Ma (2021) delineate that in U.S. non-financial firms, only 20% of debt value relies on physical assets, while a significant 80% hinges on cash flows from firms' operations. Complementarily, Drechsel (2023) illustrates that the response of credit to investment shocks at both aggregate and firm levels aligns with the predictions of earnings-based constraints. This chapter studies a funding constraint following the earnings-based borrowing constraint literature.

The second strand of the literature this chapter engages with the interaction between financial frictions and economic growth, focusing on productivity growth and employment. Empirical evidence from Duval, Hong and Timmer (2020) establishes that firms with weaker balance sheets prior to the Global Financial Crisis (GFC) experienced a larger decline in total factor productivity post-GFC, particularly those confronting severe credit condition tightening. This drop was amplified by a reduction in innovation activities, a crucial pathway through which financial frictions undermined productivity growth post-GFC.

Employment also bears the cost of financial frictions. Benmelech, Frydman and Papanikolaou (2019) provide empirical evidence demonstrating a significant and negative causal impact of financing frictions on firm employment during the Great Depression. This pattern is echoed in more recent financial crises: Siemer (2019), Duygan-Bump, Levkov and Montoriol-Garriga (2015), and Chodorow-Reich (2014) all highlight the role of financial constraints in exacerbating employment losses. A theoretical model from Aghion, Angeletos, Banerjee and Manova (2010) reinforces this negative link, suggesting that tighter credit can elevate economic volatility while depressing growth. Other theoretical work discussing the negative impact of credit frictions on productivity and economic growth include Moll (2014), Hsieh and Klenow (2009), etc.

The remainder of this chapter is organized as follows, first, I describe the model environment and the funding constraint. Second, I characterize the steady states when the constraint is (not) binding both graphically and analytically. I also summarize how the steady states change when the constraint keeps tightening.

3.2 Model

This model's environment is based on the structure presented by Benigno and Fornaro (2018), and a brief outline is offered here. The aim of this study is to integrate a funding constraint into the model and evaluate its impact on the economy. For a more comprehensive understanding of the model, please refer to the detailed explanation in Benigno and Fornaro (2018). An illustrative graph summarizing the economy is provided in figure 3.1.

The households' utility function is given by the standard CRRA utility:

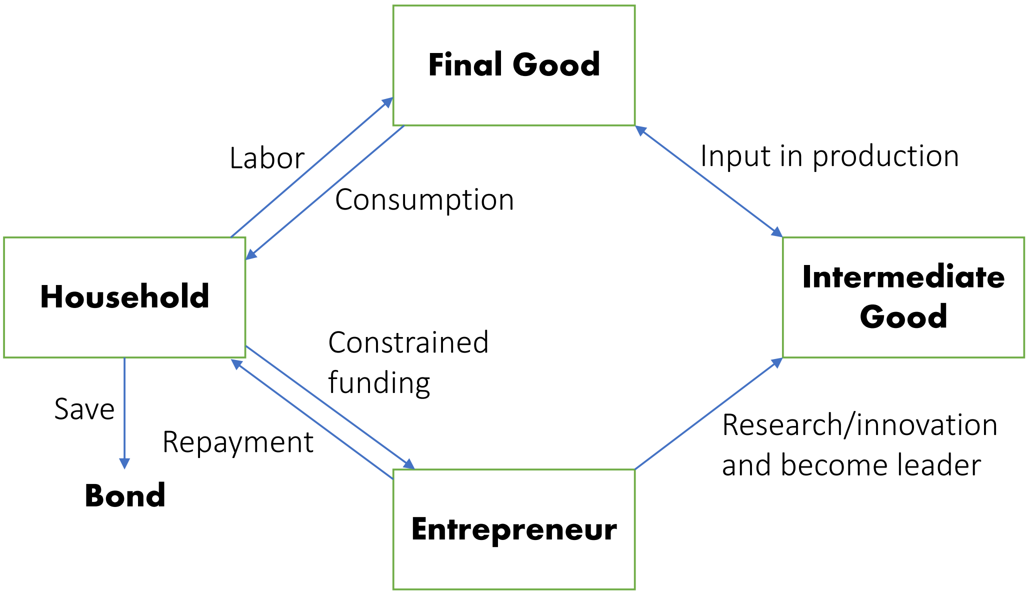
$$E_0 \left[\sum_{t=0} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) \right], \tag{3.1}$$

where β is the discount factor, C_t is the households' consumption and σ measures the risk aversion. The budget constraint of the households is given by

$$P_t C_t + \frac{b_{t+1}}{1+i_t} = w_t L_t + b_t + d_t, \tag{3.2}$$

where P_t is the price level of the final good, i_t is the nominal interest rate, b_t is the risk-free bond, $w_t L_t$ represents labor income, d_t comprises repayments and dividends from firms. If we denote the Lagrange multiplier of the budget constraint as λ_t, then the first-order conditions with respect to the choice of consumption C_t and b_{t+1} are

Figure 3.1: An Illustration of the Model



given by

$$\lambda_t = \frac{C_t^{-\sigma}}{P_t}, \tag{3.3}$$

$$\lambda_t = \beta \left(1 + i_t\right) E_t \left[\lambda_{t+1}\right]. \tag{3.4}$$

The final good Y_t is produced with intermediate goods and labor according to the following production function:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{jt}^{\alpha} dj. \tag{3.5}$$

There is a continuum of intermediate good producers, indexed by $j \in [0, 1]$. A_{jt} is the quality of the intermediate good produced by producer j, x_{jt} is the amount of intermediate good j used in production, and $0 < \alpha < 1$. The final good sector is competitive. The optimal conditions for choosing labor, L_t, and intermediate good x_{jt} are given by

$$P_t(1-\alpha)L_t^{-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{jt}^{\alpha} dj = W_t, \tag{3.6}$$

$$P_t \alpha L_t^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha-1} = P_{jt}, \tag{3.7}$$

where P_{jt} is the price of intermediate good j.

One unit of the intermediate good is produced using one unit of the final good. Intermediate good producers in each industry j compete in an oligopolistic market. In each industry j, there is a leader with product quality A_{jt}. Each follower possesses a product quality of $\frac{A_{jt}}{\gamma}$, where γ quantifies the gap between the leader and the followers with $\gamma > 1$. Under this market structure, the optimal price-setting rule for the leader in industry j is given by

$$P_{jt} = \xi P_t, \text{ where } \xi = \min\left\{\gamma^{1-\alpha}, 1/\alpha\right\} > 1. \tag{3.8}$$

The proof of the price-setting rule can be found in the appendix. Combining equations 3.7 and 3.8, the demand for intermediate input j is given by

$$x_{jt} = \left(\frac{\alpha}{\xi}\right)^{\frac{1}{1-\alpha}} A_{jt} L_t. \tag{3.9}$$

After substituting equation 3.9 into the production function, we get

$$Y_t = \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t, \tag{3.10}$$

where $A_t \equiv \int_0^1 A_{jt} dj$ is the average quality of the intermediate inputs. The profit of the leader in industry j is given by

$$P_{jt} x_{jt} - P_t x_{jt} = P_t \bar{\omega} A_{jt} L_t, \tag{3.11}$$

where $\bar{\omega} \equiv (\xi - 1)(\alpha/\xi)^{1/(1-\alpha)}$.

The R&D activities undertaken by entrepreneurs in the intermediate goods sector lead to productivity growth. With probability μ_{jt}, the entrepreneur in industry j succeeds in R&D and improves the quality from A_{jt} to γA_{jt}. μ_{jt} is given by:

$$\mu_{jt} = \min\left(\frac{\chi I_{jt}}{A_{jt}}, 1\right), \tag{3.12}$$

where χ measures the difficulty of innovations, I_{jt} is the investment into innovations. The successful entrepreneur becomes the leader in the industry for the next period. Entrepreneurs obtain funds for the investment from households, and repay all the profits to households. The value of becoming the leader in the next period is given by:

$$V_t\left(\gamma A_{jt}\right) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \bar{\omega} \gamma A_{jt} L_{t+1}\right], \tag{3.13}$$

where the right hand side is the expected profit discounted using households' discount factor $\beta\lambda_{t+1}/\lambda_t$. Assuming there is a large number of entrepreneurs, the free-entry condition for entrepreneurs is equal to

$$P_t I_{jt} \geq \frac{\chi I_{jt}}{A_{jt}} V_t\left(\gamma A_{jt}\right), \tag{3.14}$$

where the left hand side is the innovation investment cost and the right hand side is the expected payoff. We can substitute 3.13 into the inequality above, and summarize the R&D decision with the following complementary slackness condition:

$$\mu_t \left(\frac{P_t}{\chi} - \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \gamma \bar{\omega} L_{t+1} \right] \right) = 0. \tag{3.15}$$

This condition ensures that the expected payoff of research investment is non-negative, otherwise the research investment is zero.

Benigno and Fornaro (2018) assumes an exogenous growth rate for wages, specifically,

$$W_t = \bar{\pi}^w W_{t-1}, \tag{3.16}$$

where $\bar{\pi}^w$ is constant over time. By combining equation 3.6 and equation 3.9, we get the following expression for the price level,

$$P_t = \frac{1}{1-\alpha} \left(\frac{\xi}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \frac{W_t}{A_t}. \tag{3.17}$$

By the law of large number, aggregate productivity evolves as follows:

$$A_{t+1} = \mu_t \gamma A_t + (1 - \mu_t) A_t \tag{3.18}$$

The rate of productivity growth is given by

$$g_{t+1} = \frac{A_{t+1}}{A_t} = \mu_t(\gamma - 1) + 1. \tag{3.19}$$

By combining equations 3.16 and 3.17, we get that the inflation rate is given by

$$\pi_t = \frac{\bar{\pi}^w}{g_t}. \tag{3.20}$$

Higher productivity growth corresponds to lower inflation. The central bank sets the nominal interest rate as follows:

$$1 + i_t = \max \left((1 + \bar{i}) L_t^{\phi}, 1 \right), \tag{3.21}$$

where $\bar{i} \geq 0$ and $\phi > 0$. The central bank aims to stabilize output under this interest rate rule.

Funding Constraint

The above is a brief description of the environment in Benigno and Fornaro (2018). In Benigno and Fornaro's 2018 study, entrepreneurs face no financial friction when attempting to secure funds. As described below, I add an earnings-based funding constraint on entrepreneurs' investment in R&D. The existing literature has underscored the significance of financial constraints from both theoretical and empirical perspectives. Over the past two decades, the focus has largely been on "asset-based" constraints, wherein the borrowing capacity of agents is restricted by the liquidation value of physical assets they can offer as collateral (Hart and Moore (1994); Kiyotaki and Moore (1997); Bernanke, Gertler, and Gilchrist (1999)).

Recently, however, microeconomic data has indicated a widespread adoption of loan covenants that curtail a company's ability to secure financing when its current earnings are low. More specifically, Lian and Ma (2021) discovered that for non-financial U.S. firms, 20% of debt by value is based on assets, whereas 80% is cash flow-based lending. Furthermore, Drechsel (2023) has found that the earnings-based constraint holds more relevance in both macro and micro data.

In the study by Benigno and Fornaro (2018), the financing structure is based on an equity contract between entrepreneurs and households. I retain this structure in my model, but introduce an additional earnings-based funding constraint to the financing process. This approach is adopted to maintain the simplicity inherent in their model. In the spirit of Lian and Ma (2021) and Drechsel (2023), I consider a funding constraint which applies when entrepreneurs are attempting to secure funds from households. This constraint is based on the profits for the industry leader in the current period:

$$P_t I_{jt} \leq \eta P_t \bar{\omega} A_{jt} L_t, \tag{3.22}$$

where η is the stringency coefficient and $P_t \bar{\omega} A_{jt} L_t$ is the profit made by the leader in the intermediate sector in that period. The amount of funds that entrepreneurs can obtain is constrained by firms' profits in the intermediate sector in the current period. There are two primary reasons I've chosen the earnings-based funding constraint over the more conventional asset-based constraint. Firstly, research investment in this model doesn't use physical capital, and the model lacks capital as collateral. Secondly, there is likely a strong correlation between current profit and firm value. Thus, outcomes under earnings-based or asset-based constraints might closely align. Exploring this similarity is a potential avenue for future research. By the equation above, we get

$$\mu_{jt} \leq \eta \chi \bar{\omega} L_t. \tag{3.23}$$

By the Law of Large Numbers, on aggregate, we get the following expression for the growth rate:

$$g_{t+1} = A_{t+1}/A_t = \mu_t \gamma + 1 - \mu_t = \mu_t(\gamma - 1) + 1 \leq \chi \eta \bar{\omega} L_t(\gamma - 1) + 1. \tag{3.24}$$

This equation indicates that, for a specified level of employment, the growth rate is bound by the funding constraint. In this model, the driver of growth stems from productivity innovation. Therefore, limiting investments in innovation equates to constraining the growth rate. Given a certain employment level, the growth rate cap will be higher if either the productivity from research χ is higher, the funding constraint η is less stringent, or the increase in productivity for each innovation γ is greater.

When the funding constraint is not binding, the optimal investment decision is dictated by equation 3.15. Under this condition, entrepreneurs have ample access to funds, enabling them to continue investing until they can no longer profit from such investments. Conversely, when the funding constraint becomes so severe that entrepreneurs cannot secure the funds as suggested by equation 3.15, then the funds invested is defined by the binding funding constraint 3.22. In situations where they can't obtain the optimal amount of funds, entrepreneurs resort to obtaining the maximum amount possible.

Aggregation and Market Clearing

For simplicity, I assume that entrepreneurs in the intermediate goods sector distribute all their income back to households, including both funds repayments and earnings. Market clearing for the final good implies:

$$Y_t - \int_0^1 x_{jt} dj = C_t + \int_0^1 I_{jt} dj, \tag{3.25}$$

Using equations 3.9 and 3.10, we can write Y_t as:

$$Y_t - \int_0^1 x_{jt} dj = \Psi A_t L_t, \tag{3.26}$$

where $\Psi = (\alpha/\xi)^{\frac{\alpha}{1-\alpha}}(1-\alpha/\xi)$.

3.3 Equilibrium

The equilibrium of this model can be summarized by four equations and one funding constraint. In describing the steady state equilibrium, the funding constraint is omitted if it isn't restrictive. However, when it is binding, it replaces the equation that defines optimal investment in scenarios devoid of financial friction.

If we combine the two household optimality conditions 3.3 and 3.4, with $\frac{A_{t+1}}{A_t} = g_{t+1}$ and $\pi_{t+1} = \bar{\pi}^w / g_{t+1}$, we can obtain the following Euler equation, which is the first key relationship:

$$c_t^{\sigma} = \frac{g_{t+1}^{\sigma-1} \bar{\pi}^w}{\beta (1+i_t) E_t \left[c_{t+1}^{-\sigma} \right]}, \tag{3.27}$$

where we define $c_t = C_t / A_t$. Assuming that $\sigma > 1$, a positive correlation exists between productivity growth and current consumption. This arises because the wealth effect, spurred by increased income, outweighs the substitution effect that would prompt deferred consumption. The second key relationship is obtained by combining equations 3.3 and 3.15:

$$(g_{t+1} - 1)\left(1 - \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{-\sigma}\chi\gamma\bar{\omega}L_{t+1}\right]\right) = 0. \tag{3.28}$$

When $g_{t+1} > 1$, there is a positive correlation between expected productivity growth and expected employment. The rationale is that increased employment boosts profits in the intermediate sector. This, in turn, drives greater investment in R&D, leading to enhanced productivity growth. The third key equation combines market clearing condition equation 3.25, output equation 3.26 and the fact that $\int_0^1 I_{jt}dj = A_t\left(g_{t+1} - 1\right)/(\chi(\gamma - 1))$:

$$c_t = \Psi L_t - \frac{g_{t+1} - 1}{\chi(\gamma - 1)}. \tag{3.29}$$

Household consumption is calculated as GDP minus the investment in innovations. The fourth key equation is the monetary policy rule:

$$1 + i_t = \max\left((1 + \bar{i})L_t^{\phi}, 1\right). \tag{3.30}$$

And finally the funding constraint, which is equivalent to a constraint on growth rate:

$$g_{t+1} \leq \chi\eta\bar{\omega}L_t(\gamma - 1) + 1. \tag{3.31}$$

The funding constraint sets a cap on the growth rate relative to a given employment level. As financial conditions deteriorate (with a smaller η), the growth rate decreases. When the constraint 3.31 is not binding, the investment in innovations is dictated by the complementary slackness condition, equation 3.28. When the constraint is binding, the investment in innovations is determined by the binding constraint 3.31. The equilibrium in this model is that a set of processes $\{g_{t+1}, L_t, c_t, i_t\}_{t=0}^{+\infty}$ satisfying equations 3.27 - 3.29 and the binding constraint, that is, 3.31 when the equality holds, as well as $L_t \leq 1$.

3.4 Non-stochastic Steady States

The work of Benigno and Fornaro (2018) suggests the existence of two non-stochastic steady states, when the funding constraint is not a restriction. The first, referred to here as the "unconstrained full employment steady state," is characterized by full employment $L_t = 1$, a high growth rate, and a positive interest rate. The second, termed the "unconstrained stagnation trap," is marked by a slower growth rate, heightened unemployment, and a nominal interest rate that hits the zero lower bound. When the funding constraint is sufficiently stringent, it starts to influence these two steady states. The two steady states under these stringent funding conditions are called the "constrained high growth steady state" and "constrained low growth steady state". The result is that the effect of a stricter funding constraint on economic results varies drastically depending on whether the steady state nominal interest rate has hit the zero lower bound or not.

The non-stochastic steady state equilibria are characterized by the following equations with constant values of growth rate g, normalized consumption c, employment L and nominal interest rate i.

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}^w} \tag{3.32}$$

$$g^{\sigma} = \max(\beta\chi\gamma\bar{\omega}L, 1) \tag{3.33}$$

$$c = \Psi L - \frac{g-1}{\chi(\gamma-1)} \tag{3.34}$$

$$1 + i = \max\left((1+\bar{i})L^{\phi}, 1\right) \tag{3.35}$$

$$g \leq \chi\eta\bar{\omega}L(\gamma-1) + 1 \tag{3.36}$$

When the funding constraint is slack enough, the financial friction does not impact the investment decision of the entrepreneurs. In this situation, the steady state is characterized by equation 3.32 to equation 3.35. When the funding constraint is tight enough and prevents the entrepreneurs from investing at the level according to equation 3.33, investment decision is dictated by the binding constraint 3.36. In the following part of this section, I characterize the four different types of steady states mentioned above.

3.4.1 Graphic Illustration

The model's steady states can be depicted graphically through the intersection of two curves on the $\{L, g\}$ plane. The assumptions underlying the slopes of displayed relationships are described in section 3.5. The first curve, GG curve, corresponds to the growth equation 3.33. The "G" stands for "growth". As displayed in figure 3.2, the horizontal section of GG curve represents the scenario where research investment is not profitable, thus no investment in research takes place. The upwardly sloping section illustrates that, when L is sufficiently large, increased employment generates more profit through the standard market size effect. This in turn motivates entrepreneurs to invest in research, resulting in a higher growth rate. The second curve, referred to as the AD curve, corresponds to the combination of the Euler Equation 3.32 and the interest rate rule 3.35. AD curve characterizes the aggregate demand of the economy. That is,

$$g^{\sigma-1} = \frac{\beta}{\bar{\pi}^w} \max\left((1+\bar{\imath})L^{\phi}, 1\right). \tag{3.37}$$

The horizontal section of the AD curve corresponds to the employment level at which the zero lower bound is binding. The growth rate maintains constant in consistency with the Euler Equation. The upward sloping part represents the employment level where the zero lower bound isn't binding. In these cases, the central bank raises the interest rate in response to increased employment levels, causing the growth rate to rise, as determined by the Euler Equation. The CC line, meanwhile, represents the funding constraint when it is binding. Its expression is given by

$$g = \chi\eta\bar{\omega}L(\gamma - 1) + 1. \tag{3.38}$$

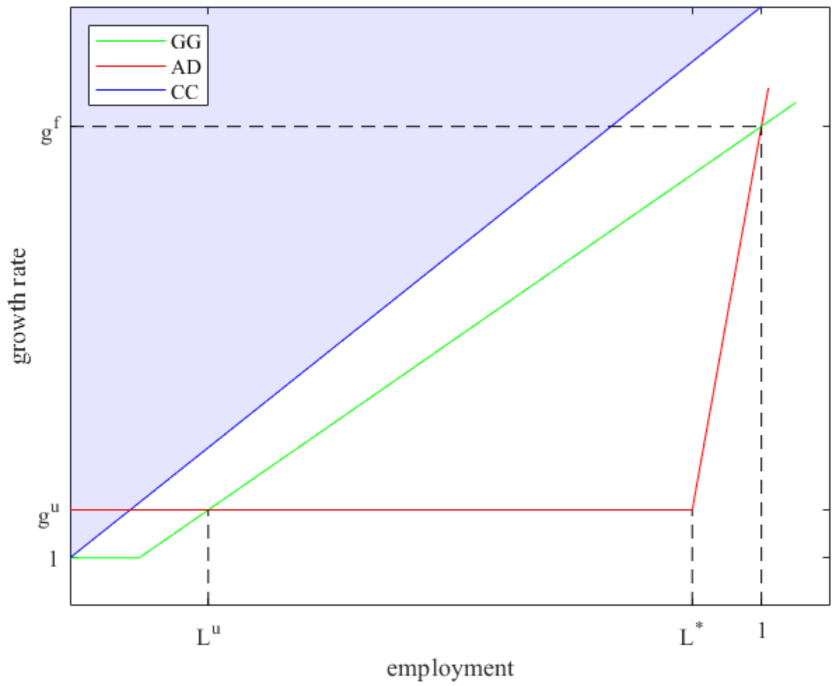
The CC line delineates the upper limit of the growth rate for a given level of employment. Any combination of $L - g$ above the CC line is unattainable, as entrepreneurs face constraints based on their current profit when they seek to obtain funds for innovation investments. Please note that the upward sloping portions of both the GG and AD lines aren't technically straight lines. However, for the sake of simplicity, we represent them as straight lines in the graph instead of curves.

Case 1: When the Funding Constraint Is Not Binding

Figure 3.2 illustrates a scenario where the funding constraint is sufficiently relaxed, allowing for the existence of both a full employment and a liquidity trap steady states as outlined in Benigno and Fornaro (2018).[1] The shaded area represents the $\{L, g\}$ combinations that are excluded by the constraint. For a given level of employment, the growth rate within the shaded area is unachievable as entrepreneurs cannot obtain sufficient funds for investment in the research sector. In figure 3.2, the available funds for entrepreneurs permit the occurrence of both the full employment steady state and the stagnation trap. In figure 3.2, the GG and AD curves intersect at two points. The intersection with a higher growth rate and employment level represents the full employment steady state. Conversely, the intersection with a lower growth rate and employment level corresponds to the stagnation trap. This coexistence of the two steady states in Benigno and Fornaro (2018) necessitates a sufficiently large stringency coefficient for the funding constraint.

[1] In this particular scenario, assumptions 2, 3, 4, and 5 to be described in section 3.5 hold true.

Figure 3.2: Both Full Employment and Liquidity Trap Steady States as in Benigno and Fornaro (2018) Exist



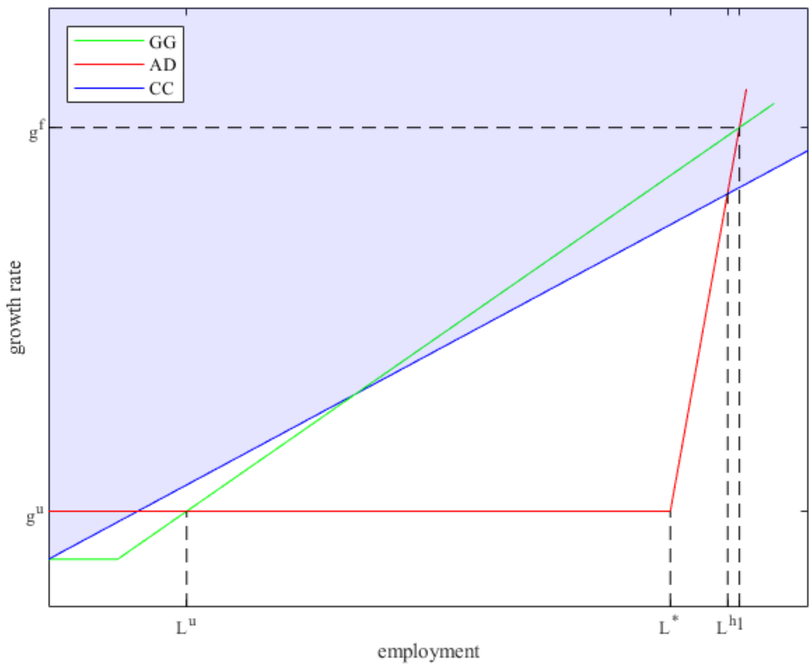
Case 2: When the Funding Constraint Rules Out the Full Employment Steady State

Figure 3.3 presents a scenario where the stringency coefficient is significantly lower than in figure 3.2, resulting in insufficient funds for entrepreneurs to support the full employment steady state, as illustrated in figure 3.3.[2] We maintain the same parameters as in figure 3.2 but decrease η. Consequently, the GG and AD curves remain unchanged, while the slope of the CC line diminishes. In this scenario, entrepreneurs lack the necessary funds for research investment to reach the full employment steady state. The intersection of the AD curve and CC line now denotes the high growth steady state. The stagnation trap steady state remains unaffected by the tightened funding constraint, meaning entrepreneurs still have access to the funds needed to sustain this steady state. The high growth steady state, characterized by L^h and g^h, exhibits growth and employment levels that sit between the full employment steady state and stagnation trap ($L^u < L^h < 1$, $g^u < g^h < g^f$). The more stringent the constraint, or equivalently, the lower η is, the slower the growth and lower the employment levels in the economy. The underlying rationale is that less available funds lead to decreased investment in innovations, which in turn results in a lower growth rate. This reduced growth rate translates to a lower nominal interest rate, as dictated by the Euler equation. Consequently, the employment level must also decrease, as per the monetary policy rule. This underlines the crucial role of suffi-

[2]In this case, assumptions 2, 4, and 7 in section 3.5 are satisfied, while assumption 8 is strictly violated.

cient funding source for entrepreneurs in this economy: ample funding source makes full employment steady state attainable. When the full employment steady state is out of reach, the growth rate and employment level in the high growth steady state are determined by the binding funding constraint.

Figure 3.3: Full Employment Steady State in Benigno and Fornaro (2018) Is Ruled Out



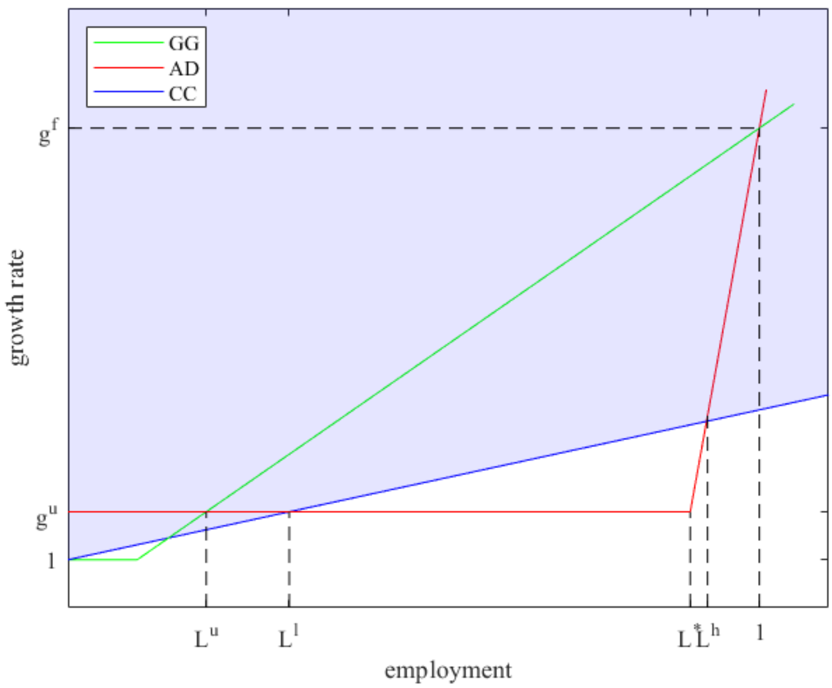
Case 3: When the Funding Constraint Rules Out the Full Employment Steady State and Affects the Stagnation Trap

When assumption 8 is met, the funding constraint becomes so stringent that it rules out both the full employment steady state and affects the stagnation trap.[3] This is illustrated in figure 3.4. However, the constraint determines two new steady states: the high growth steady state and the low growth steady state. As the constraint continues to tighten, both the growth rate and employment level in the high growth steady state decline. The intersection of the horizontal part of the AD curve and the CC line represents the low growth steady state. As the constraint intensifies, the growth rate remains constant. This is because the zero interest rate anchors the fixed growth rate g^u through the Euler Equation. Interestingly, as the constraint tightens, employment levels actually increase. The reason is that at the zero lower bound, the constraint can be relaxed if real activity, or employment, increases. And this won't be undone by the increase in nominal interest rate due to the binding zero lower bound.

[3]The exact condition is given in assumption 8.

Whenever there are two possible steady state equilibria, one with high growth and another with low growth, the determination of which particular steady state the economy settles into is dictated by the expectations of the agents. As discussed in section 3.5.2, in this economy, expectations of economic growth are self-fulfilling.

Figure 3.4: Both Full Employment and Stagnation Trap Steady States in Benigno and Fornaro (2018) Are Ruled Out



Case 4: When the Funding Constraint Rules Out Any Steady States

Finally, at some point, the value of η is so low that entrepreneurs can not obtain enough funds to even sustain the lowest possible growth rate g^u.[4] g^u is the growth rate corresponding to the zero lower bound of nominal interest rate. In figure 3.5, there is no intersection between the CC line and AD curve.

Effects of the Funding Constraint η

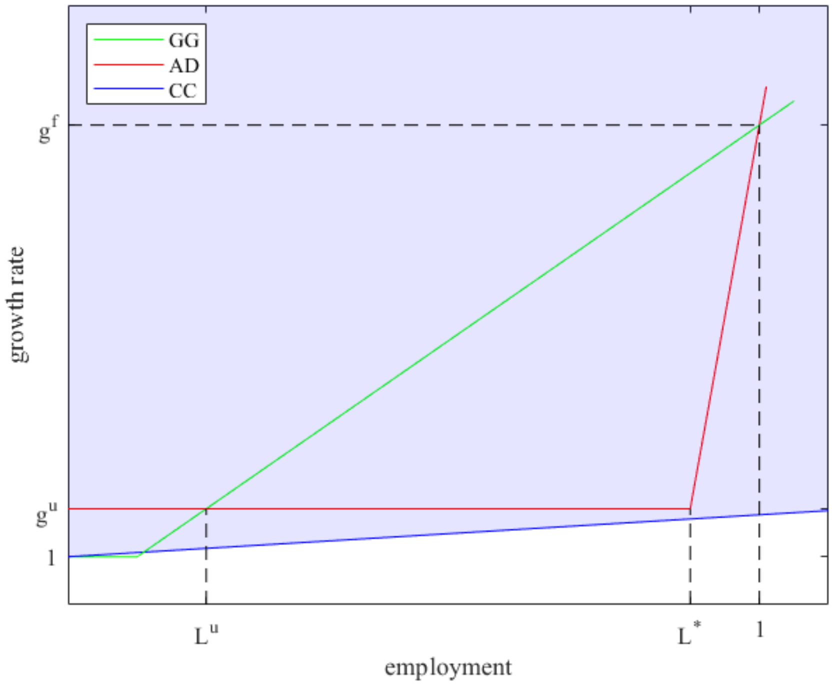
Next, I provide a summary of how the change in funding constraint parameter η affects the steady state growth rate and employment level, as illustrated in figures 3.6 and 3.7. For simplicity of notation, let

$$\eta_1 = \frac{(\beta\chi\gamma\bar{\omega})^{1/\sigma} - 1}{\chi\bar{\omega}(\gamma - 1)},$$

$$\eta_2 = \frac{\left[\frac{\bar{\pi}^w}{\beta} - \frac{\bar{\pi}^w}{\beta}\frac{\sigma}{\sigma-1}\right]\beta\gamma}{\gamma - 1}, \textit{ and}$$

[4]The exact condition is given in assumption 9.

Figure 3.5: When No Steady State Exists



$$\eta_3 = \frac{(\beta\chi\gamma\bar{w})^{\frac{1}{\sigma}} - 1}{\chi\bar{w}(\gamma-1)(\frac{\beta}{\bar{\pi}^w})^{\frac{1}{\phi}}(\beta\chi\gamma\bar{w})^{\frac{1-\sigma}{\phi\sigma}}}.$$

The determination of η_1, η_2 and η_3 is discussed in section 3.5. When the funding constraints are sufficiently relaxed, specifically when $\eta > \eta_1$, the constraint does not affect the full employment steady state and stagnation trap steady state: the funds are ample to maintain both steady states. In this scenario, the economy behaves as if there are no financial frictions at all.

As the funding restrictions become more strict, and the value of η drops, the growth rate in the high growth rate steady state starts to decrease. η_1 is the constraint parameter that just supports the full employment steady state in Benigno and Fornaro (2018). If η falls below η_1, in the full employment steady state, investors can not obtain enough funds to invest in innovations, leading to a direct negative impact the growth rate of the economy. g in the good steady state decreases as η decreases. As a result, Euler equation 3.32 indicates a decrease in i, consequently implying a decline in L in the good steady state.

η_2 is the constraint parameter that just sustains the stagnation trap steady state in Benigno and Fornaro (2018). When $\eta_2 < \eta < \eta_1$, η is still large enough to support the stagnation trap steady state. However, if η drops below η_2, the financial friction in the economy is so severe that the stagnation trap steady state can not be maintained either. Yet, the growth rate within this steady state must remain constant to align with the zero nominal interest rate, as expressed through the Euler equation 3.32. In order to achieve the same growth rate amidst deteriorating financial conditions, it becomes necessary to boost employment.

Finally, when η drops below η_3, the financial restrictions become so severe that they can't even support the lowest possible steady state growth rate. Under this condition, no steady state can exist any longer.

Figure 3.6: Effects of η on Steady State Productivity Growth Rate g

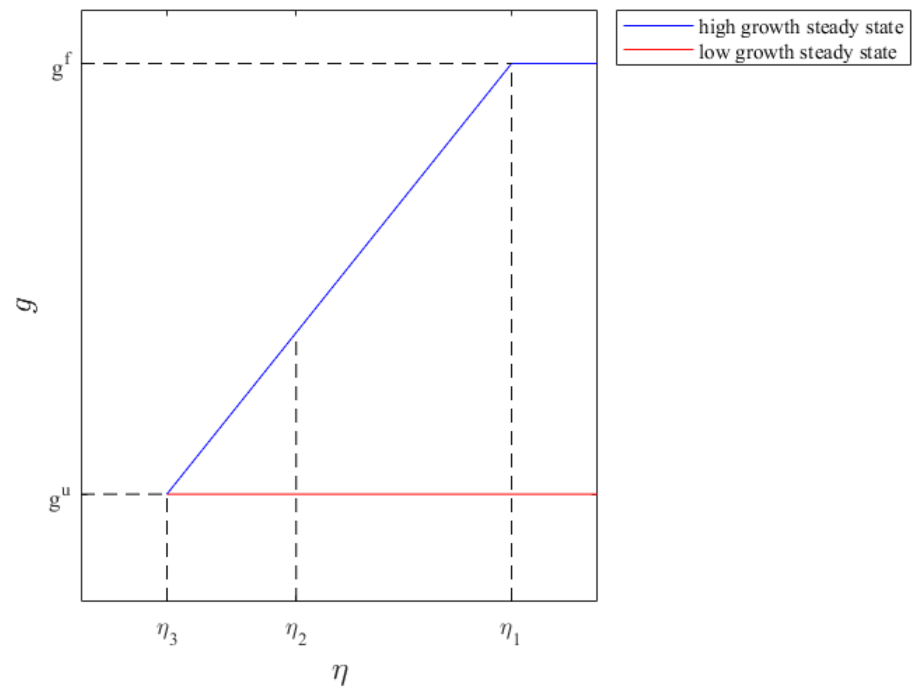
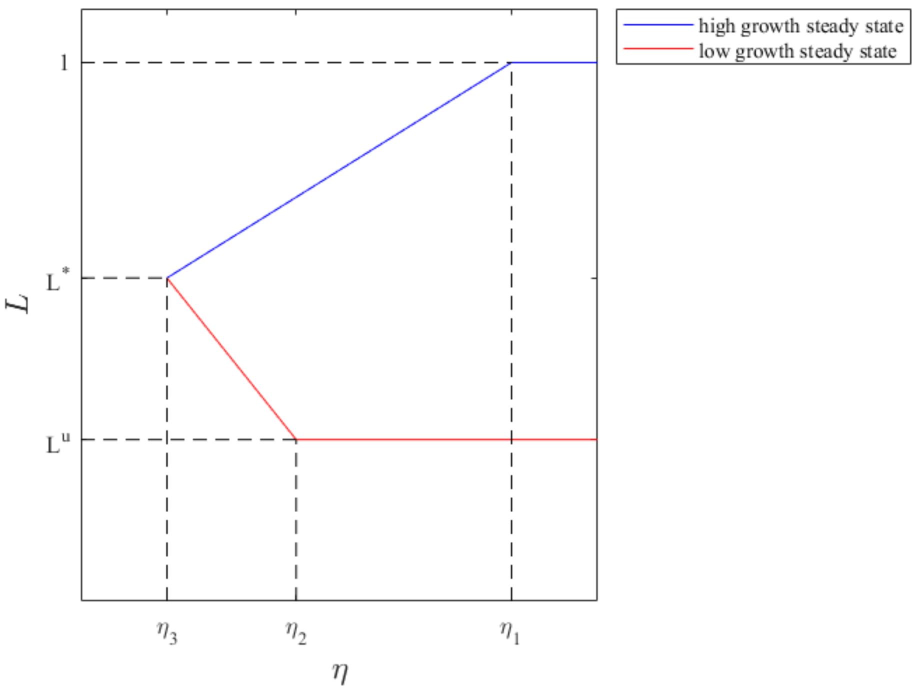


Figure 3.7: Effects of η on Steady State Productivity Growth Rate L



3.5 Analytical Characterisation

In this section, I characterize different steady states shown in the section above analytically.

3.5.1 Unconstrained Full Employment Steady State

When the funding constraint is sufficiently lax, or in other terms, when the coefficient η is significantly large, the full employment steady state as outlined in Benigno and Fornaro (2018) can be achieved. Let's represent the variables at the full employment steady state with the superscript f. Let $L^f = 1$, plug it into equation 3.33 above, we can get:

$$g_f = \max\left((\beta\chi\gamma\bar{\omega})^{\frac{1}{\sigma}}, 1\right) \tag{3.39}$$

We plug in $g = g^f$ in equation 3.32 to get the nominal interest rate in the full employment steady state $i^f = \frac{\left(g^f\right)^{\sigma-1}\bar{\pi}^w}{\beta} - 1$. By equation 3.35 we can see that, $\bar{i} = i^f$. Finally we can calculate steady state consumption $c^f = \Psi - \frac{g^f-1}{\chi(\gamma-1)}$ by setting $L = 1$ and $g = g^f$ in equation 3.34.

Assumption 2 *The parameters satisfy:*

$$\sigma > 1 \tag{3.40}$$

$$\bar{i} = \frac{(\beta\chi\gamma\bar{\omega})^{1-\frac{1}{\sigma}}\bar{\pi}^w}{\beta} - 1 > 0 \tag{3.41}$$

$$1 < (\beta\chi\gamma\bar{\omega})^{\frac{1}{\sigma}} < \min(1 + \Psi\chi(\gamma-1), \gamma) \tag{3.42}$$

Assumption 3 *The parameters satisfy:*

$$\eta \geq \frac{(\beta\chi\gamma\bar{\omega})^{1/\sigma} - 1}{\chi\bar{\omega}(\gamma-1)} \tag{3.43}$$

Here is a proposition similar to proposition 1 in Benigno and Fornaro (2018).

Proposition 2 *Suppose assumption 2 and 3 hold. There exists a unique full employment steady state with $L^f = 1$. The full employment steady state is characterized by positive growth $g^f > 1$ and by a positive nominal interest rate $i^f > 0$.*

Proof 6 *When assumption 3 is met, we have $\eta\chi\bar{\omega}(\gamma-1) + 1 \geq (\beta\chi\gamma\omega)^{1/\sigma} = g^f$. This implies that if the constraint is relaxed enough, the full employment steady state with $L^f = 1$ is attainable. If the growth rate at the full employment steady state does not surpass the growth rate limit required by the constraint, the full employment steady state remains unaffected by the constraint. In such a scenario, the proof of* *the existence and uniqueness of the full employment steady state mirrors exactly the proof provided in Benigno and Fornaro (2018).*

Inequality 3.40, meaning low levels of intertemporal substitution, ensures the positive relationship between growth rate and interest rate in the Euler Equation. Under equation 3.41, the nominal interest rate at the full employment steady state is positive. Under condition 3.42, households' steady state consumption is positive and the innovation probability μ ranges between 0 and 1.

Intuitively, if the stringency coefficient of the constraint exceeds the right-hand side value in inequality 3.43, the amount of money that entrepreneurs can obtain (and consequently invest in research) is sufficient to sustain the growth rate at the full employment steady state.

3.5.2 Unconstrained Stagnation Trap

Under slack enough funding constraints, let's examine the unemployment steady state, or stagnation trap, denoted by the superscript u. Assume that η is substantial enough to satisfy assumption 5 below. Consider the situation when the nominal interest rate is at the zero lower bound $i = 0$. By equation 3.32, we get $g^u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}$. Since at the full employment steady state $i^f = \bar{i} > 0$, we deduce $g^u < g^f$ from equation 3.32. The real interest rate $(1+i)/\pi = g^\sigma/\beta$ is increasing in the growth rate g, hence the real interest rate is lower in the stagnation trap. In the stagnation trap, labor supply $L^u < 1$ is also less than in the full employment steady state. We summarize our findings about the stagnation trap in proposition 3.

Assumption 4 *The parameters satisfy:*

$$1 < \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} \tag{3.44}$$

$$\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} < 1 + \frac{\frac{\xi}{\alpha}-1}{\xi-1}\left(\frac{\beta}{(\bar{\pi}^w)^\sigma}\right)^{\frac{1}{\sigma-1}}\frac{\gamma-1}{\gamma} \tag{3.45}$$

$$\phi > 1 - \frac{1}{\sigma} \tag{3.46}$$

Assumption 5 *The parameters satisfy that:*

$$\frac{\left[\frac{\bar{\pi}^w}{\beta} - \frac{\bar{\pi}^w}{\beta}\frac{\sigma}{\sigma-1}\right]\beta\gamma}{\gamma-1} \leq \eta \tag{3.47}$$

Please note that when assumptions 2 and 3 are satisfied, then assumption 5 is also fulfilled. To put it differently, when assumption 2 holds, we observe:[5]

$$\frac{\left[\frac{\bar{\pi}^w}{\beta} - \frac{\bar{\pi}^w}{\beta}\frac{\sigma}{\sigma-1}\right]\beta\gamma}{\gamma-1} < \frac{(\beta\chi\gamma\bar{\omega})^{1/\sigma}-1}{\chi\bar{\omega}(\gamma-1)} \tag{3.48}$$

Proposition 3 *Under assumptions 2, 4, 5, there exists a unique stagnation trap. At the stagnation trap, the economy is in a liquidity trap ($i^u = 0$), $L^u < 1$, as well as a growth trap $g^u < g^f$. The real interest rate is also lower than in the full employment steady state $1/\pi^u < \left(1 + i^f\right)/\pi^f$.*

Proof 7 *By integrating $g^u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}$ with equation 3.33, we derive the stagnation trap's employment level as $L^u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{\sigma}{\sigma-1}}/(\beta\chi\gamma\bar{\omega})$. Upon fulfilling assumption 5, it results in $\eta\chi\bar{\omega}L^u(\gamma-1)+1 \geq \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} = g^u$. Thus, under the condition of assumption 5, the growth rate at the stagnation trap's steady state does not go beyond the growth rate threshold dictated by the funding limitation. Consequently, the constraint doesn't impact the stagnation trap's steady state. In this scenario, the verification of existence and uniqueness of the stagnation trap steady state is exactly the same as the proof provided in Benigno and Fornaro (2018).*

Inequality 3.44 guarantees that g^u remains positive in the stagnation trap. Given that patents do not depreciate in this economy, growth rate has a lower bound of zero. Inequality 3.45 assures that consumption maintains positive. Inequality 3.46 rules out the existence of unemployment steady states with positive nominal interest rate.

The intuition of proposition 3 is similar to that of proposition 2: the constraint needs to be slack enough to support the existence of the stagnation trap steady state. The difference is that for stagnation trap steady state to exist, the threshold of η is lower. Compared with the stagnation trap, the full employment steady state requires better financial condition to exist.

The way in which agents settle into one of the two steady states described above is driven by their expectations. If agents hold an optimistic outlook about future economic growth, this stimulates aggregate demand. A positive nominal interest rate aligns with high employment. High aggregate demand and employment bolsters production profits, which then results in further investment into innovations, thereby validating the initial high economic growth expectations.

Conversely, if agents are pessimistic about future economic growth, this curtails aggregate demand and results in low employment. The nominal interest rate hits the zero lower bound due to this low level of employment. Consequently, profits derived

[5] It's straightforward to use inequality 3.41 to prove it.

from the production process diminish, leading to a reduced level of investment in innovations. This sequence of events justifies the initial expectations of low economic growth.

3.5.3 Constrained High Growth Steady State

The preceding two sections demonstrate that given sufficiently lax constraints, the existence of the two steady states - full employment and stagnation trap, as per Benigno and Fornaro (2018), is not impacted by the constraint. The full employment steady state demands better financial circumstances relative to the stagnation trap. However, if assumption 3 is violated, a steady state exists with a growth rate and employment level surpassing that of the stagnation trap, alongside a positive nominal interest rate. But the growth rate, employment level and the nominal interest rate are all lower than the levels in the unconstrained full employment steady state. This particular steady state is referred to as the high growth steady state, denoted by superscript h.

Assumption 6

$$\phi > \sigma - 1 \tag{3.49}$$

Assumption 7 *The parameters satisfy:*

$$\frac{(\beta\chi\gamma\bar{w})^{\frac{1}{\sigma}} - 1}{\chi\bar{w}(\gamma-1)(\frac{\beta}{\bar{\pi}^w})^{\frac{1}{\phi}}(\beta\chi\gamma\bar{w})^{\frac{1-\sigma}{\phi\sigma}}} < \eta < \frac{(\beta\chi\gamma\bar{\omega})^{1/\sigma} - 1}{\chi\bar{\omega}(\gamma-1)} \tag{3.50}$$

Proposition 4 *Suppose assumption 2, 4, 6 and 7 hold. Then, there exists a unique constrained high growth steady state with $L^u < L^h < 1$. The constrained high growth steady state is characterized by positive growth $g^u < g^h < g^f$ and by a positive nominal interest rate $i^h < i^f$.*

The proof can be found in the appendix. Assumption 6 ensures the uniqueness of the high growth steady state given a fixed η. Assumption 7 eliminates the possibility of the full employment steady state while permitting enough funds to support the high growth steady state. The finding from proposition 4 is expected: the constraint hinders investment into innovations, which drive growth. The more constrained entrepreneurs are, the less they invest in innovations, leading to slower growth. This lower growth aligns with a lower nominal interest rate via the Euler Equation. And this lower nominal interest rate, in turn, results in lower employment levels.

3.5.4 Constrained Low Growth Steady State

Similar to the constrained high growth steady state, when assumption 5 is breached, there exists a low growth steady state denoted by the superscript l, characterized in the following proposition:

Assumption 8 *The parameters satisfy:*

$$\frac{(\beta\chi\gamma\bar{w})^{\frac{1}{\sigma}}-1}{\chi\bar{w}(\gamma-1)(\frac{\beta}{\bar{\pi}^w})^{\frac{1}{\phi}}(\beta\chi\gamma\bar{w})^{\frac{1-\sigma}{\phi\sigma}}}<\eta<\frac{\left[\frac{\bar{\pi}^w}{\beta}-\frac{\bar{\pi}^w}{\beta}\frac{\sigma}{\sigma-1}\right]\beta\gamma}{\gamma-1} \tag{3.51}$$

Proposition 5 *Suppose assumption 2, 4 and 8 hold. There exists a unique low growth steady state with $L^u < L^l < L^h$. The constrained low growth steady state is characterized by positive growth $g^u = g^l < g^h$ and by a zero nominal interest rate.*

The proof can be found in the appendix. At the low growth steady state, the nominal interest rate is zero, leading to a growth rate fixed at the level in the stagnation trap $g^l = g^u$. Intriguingly, in order to maintain the same growth rate under deteriorated funding conditions (lower η), employment in the economy should rise to relax the credit constraint so that entrepreneurs can obtain the funds to sustain g^u. Consequently, the impact of a lower η is counterbalanced by increased employment. Since the zero lower bound is binding, the expansion will not be undone by an increase in the nominal interest rate. Thus, poorer funding conditions contribute to a rise in the employment level in the economy.

3.5.5 No Steady State Exists

In the two sections above, we see that when the constraint is stringent enough, it rules out the original steady states in Benigno and Fornaro (2018), while giving rise to two constrained steady states, one with high growth and one with low growth. However, when the constraint keeps tightening, there are scenarios where no steady state exists any more.

Assumption 9 *The parameters satisfy:*

$$\eta<\frac{(\beta\chi\gamma\bar{w})^{\frac{1}{\sigma}}-1}{\chi\bar{w}(\gamma-1)(\frac{\beta}{\bar{\pi}^w})^{\frac{1}{\phi}}(\beta\chi\gamma\bar{w})^{\frac{1-\sigma}{\phi\sigma}}} \tag{3.52}$$

Proposition 6 *Suppose assumption 2, 4 and 9 hold. Then, no steady state exists.*

When the funding environment is extremely bad, it becomes impossible for entrepreneurs to secure sufficient funds needed to maintain the corresponding growth rate for a given level of employment. Thus no steady state exists.

3.6 Conclusion

Built on Benigno and Fornaro (2018), this chapter explores the influence of an earnings-based funding constraint on productivity growth and employment. Our model integrates elements of endogenous growth, nominal rigidities, and the zero lower bound on nominal interest rates. The study reveals that the effects of a tightened constraint differ based on the status of the nominal interest rate's zero lower bound. If the steady state nominal interest rate is positive and the constraint is active, a tighter constraint negatively affects both economic growth and employment. This outcome arises as increased financial frictions inhibit investment in innovation, thereby reducing the growth rate. The subsequent decline in aggregate demand leads to a lower employment level. Conversely, when the steady state nominal interest rate is bound at zero and the constraint is in effect, tightening the constraint does not influence the economy's growth rate, yet it prompts an increase in employment levels. This counter-intuitive result stems from the need for a constant growth rate to align with the fixed nominal interest rate. Therefore, the level of employment rises to ensure that entrepreneurs can obtain sufficient funds to maintain the steady growth rate. In summary, the implications of financial constraints are multifaceted and closely intertwined with the conditions of the nominal interest rate and the broader economy.

3.7 Appendix

Price-setting Rule of Oligopolists

The following is a proof for the price-setting rule of oligopolists in the intermediate goods sector:

Proof 8 *Suppose there are no other competitors, price-setting oligopolists choose prices to solve the following profit maximization problem:*

$$\max_{P_{jt}} (P_{jt} - P_t) x_{jt} \tag{3.53}$$

subject to the demand function 3.7. The demand function can be rearranged as

$$x_{jt} = \frac{P_{jt}}{P_t}^{\frac{1}{\alpha-1}} M \tag{3.54}$$

where M is a function of the parameter α, variables L_t and A_{jt}. Plug x_{jt} into the profit function and take first order condition, we get the optimal price as

$$P_{jt} = \frac{P_t}{\alpha}. \tag{3.55}$$

Now consider the competitors in intermediate goods sector j with productivity $\frac{A_{jt}}{\gamma}$. If the leader charges more than $\gamma^{1-\alpha} P_t$, then the competitors can capture the whole market by charging price P_t. So leaders never charge more than $\gamma^{1-\alpha} P_t$.

Proof for Proposition 4

Proof 9 *To prove the existence and uniqueness of the high growth steady state, it is equivalent to prove that the CC line intersects with the upward sloping part of the AD curve once and only once. And we need to check that at the intersection, the consumption is positive. Denote the employment level at the intersection between the horizontal part and upward sloping part of AD curve as L^*. When assumption 7 is satisfied, the CC line is above g^u at $L = L^*$ and lower than g^f at $L = 1$. Since both AD curve and CC line are continuous, there must be at least one intersection. Under assumption 6, the gradient of the AD curve is always higher than the slope of the CC line in the $[L^*, 1]$ interval. It is straightforward to check that, under assumption 6, the gradient of AD curve is the lowest at L^*, and it is larger than the gradient of the largest possible gradient for the CC line, the one that goes through $(1, g^f)$. As a result, there is at most one intersection between CC line and AD curve when L is in the interval $[L^*, 1]$. In summary, there is one and only one intersection between the CC line and AD curve at the interval $[L^*, 1]$. Now we need to check whether the consumption at those high growth steady states is positive or not. By the market clearing condition 3.34, the consumption at the steady state with L^*, denoted as c^*, should be higher than the consumption at the stagnation trap c^u. The reason is that consumption is increasing in L. Thus $c^* > 0$. At the same time, the consumption at the full employment steady state c^f is also positive. If we plug the AD curve 3.37 into the consumption function 3.34, and derive the derivative of c with respect to L:*

$$\frac{\partial c}{\partial L} = \Psi - \frac{\frac{\beta}{\bar{\pi}^w}(1+\bar{i})\frac{\phi}{\sigma-1}L^{\frac{\phi}{\sigma-1}-1}}{\chi(\gamma-1)} \tag{3.56}$$

Since $\phi > \sigma - 1$, $\frac{\partial c}{\partial L}$ decreases in L. It means that when L increases from L^* to 1, consumption either first increases then decreases, or keeps increasing. Combined with the fact that both c^* and c^u are positive, consumption at steady states along $[L^*, 1]$ must be positive.

Proof for Proposition 5

Proof 10 *To prove the existence and uniqueness of the low growth steady state, we need to demonstrate that the CC line intersects the horizontal segment of the AD curve precisely once. Under assumption 8, it's easy to see that at L^u the CC line is below the horizontal AD curve, while at L^* the CC line is above the horizontal AD curve. Given that both the CC line and AD curve in this region are continuous straight lines, for any η that satisfies assumption 8, there must be exactly one intersection between the CC line and the horizontal AD curve. According to the market clearing condition 3.34, consumption is a function that increases with employment L. Compared to the stagnation trap, the other low growth steady states maintain the* *same growth rate but exhibit higher employment. Therefore, consumption at these low growth steady states c^l, must exceed c^u, and consequently, it must be positive.*

Proof for Proposition 6

Proof 11 *To demonstrate that no steady state exists in the economy, we need to show that there's no intersection between the CC line and the AD curve. When η equals the right-hand side of inequality 3.52, the CC line passes through the point (L^*, g^u). If η decreases even further, the CC line doesn't intersect either the horizontal or the upward sloping part of the AD curve. Consequently, there's no steady state solution.*

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