The London School of Economics and Political Science

Essays in Macro-Finance and The Environment

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I declare that my thesis consists of approximately 45 000 words (excluding bibliography).

Statement of co–authored work

I confirm that chapter 1 is adapted from a previous version of a jointly written work with Dr. Ivan Jaccard and Dr. Gauthier Vermandel, where I was primarily responsible for writing different parts of the paper and codes.

Statement of prior publication

A version of chapter 1 was earlier published as an ECB working paper.

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Abstract

In this thesis I investigate the linkages between environmental policy and macro-financial aggregates. The first chapter shows that well-designed environmental policies could lead to lower risk premiums and higher real interest rates. By correcting the externality responsible for climate change, the optimal policy reduces the welfare cost of business cycle fluctuations. This decline in aggregate risk in turn lowers the compensation demanded by investors for holding risky assets as well as the need for precautionary savings. The second chapter expands the scope of the first chapter, where a non-separable externality dis-utility function is introduced, and where I formulate a tractable extension of the model that captures both production damages and market frictions, namely price stickiness. In this chapter I show that utility-based damages allow for better capturing macro-financial dynamics while significantly reducing emissions, in contrast to the production damages specification. Monetary policy is shown to play an important role in emission reduction, however, a trade-off is introduced as central banks work to stabilize interest rates and inflation, or alternatively, allow for higher emission reduction rates. The third chapter builds on the first two chapter and studies the drivers of EU Futures ETS carbon pricing. This chapter uses a macro-finance model to estimate the shock decomposition of the CO_2 price in the data, where we use both a novel data set and strategy to estimate the abatement and policy shocks. This chapter finds that the ETS price is mainly driven by energy, climate sentiment, and abatement shocks. Relative to the social cost of carbon (i.e. the optimal policy), the volatility of the ETS price is 10 times higher. Finally, I show how reducing price uncertainty using a carbon cap rule, can help close the volatility gap with respect to the first best policy.

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Chapter 0

General Introduction

The implementation of a strategy to substantially reduce greenhouse gas (GHG) emissions on a global scale has become a critical priority. Since the 1992 Rio Conference, a debate has persisted among academic and political circles regarding the trade-off between economic growth and the environment. Discussions have focused on finding ways to balance economic activities with environmental concerns, rather than treating them as opposing forces.

On one side, the IPCC projections emphasize the urgent need for ambitious action to reduce greenhouse gas emissions and adapt to the impacts of climate change. Failure to do so will result in significant and possibly irreversible impacts on natural systems and human societies. However, in practice, financial and economic activities on the other side could be subjected to significant mitigation costs, particularly in the short and medium terms.

One of the main ways in which climate change can affect the global economy is through physical risks. These are the direct impacts of climate change, such as damage to property and infrastructure from extreme weather events, loss of crop yields due to drought or flooding, and increased health risks from heat waves and other climate-related hazards. These physical risks can cause significant economic and financial losses for businesses and governments, and can lead to significant financial instability.

In addition to physical risks, climate change also poses transition risks, which are the risks associated with the transition to a low-carbon economy, such as changes in energy prices, carbon taxes, and shifts in consumer preferences. The transition to a low-carbon economy will require significant changes in the way that businesses and governments operate, and this transition could have significant financial impacts.

It is essential to facilitate economic transition by both understanding and designing short-term and medium/long-term policies to bridge the gap between environmental sustainability goals necessary for achieving the net-zero emissions target, and smoothing the impacts of economic costs (due to mitigation policies) while ensuring financial stability (which could be subjected to climate physical risks). As a result, researchers in macro-finance are increasingly investigating these issues.

As part of its financial stability and monetary policy strategy review, the European Central Bank (ECB) has decided to monitor climate risk more closely and its potential consequences for financial stability. Similarly, the Network for Greening the Financial System (NGFS), a group of central banks and financial supervisors, was formed in 2017 to promote the integration of climate-related risks into financial regulation and supervision. However, for now, fiscal policy remains the primary instrument to mitigate present and future climate change damages. Therefore, a successful green transition by 2050 would require a comprehensive understanding of the linkages and interactions of financial markets, the macroeconomy, the climate physical and transition risks, and policy design.

Although pricing the environmental externality and evaluating the global macroeconomic impacts of climate change have been the primary focus of environmental-macroeconomists over the last decade (Schubert 1 [2018]), not much research has been conducted on the inter-linkages between the environmental externality and policy and macro-finance (Annicchiarico et al. [2021]).

First, the potential linkages between i) environmental externality and policy and ii) financial markets could be major concerns. The design of carbon markets is of the highest importance in order to best respond to transition needs while reducing transition mitigation costs. Otherwise, this could hinder financial stability and result in market inefficiencies.

In the following paragraphs of the general introduction, we present the objectives of this thesis, the main results, a snapshot of the related literature (which is later developed within each chapter), as well as the main contributions of the thesis.

The objective of this thesis is to investigate the linkages between climate change physical and transition risk and macro-finance, which has not been significantly addressed in the literature. More precisely, we investigate the following three questions: i) How should we design optimal carbon policy, and what are its impacts on macroeconomic and financial aggregates? ii) What are the interactions between environmental externality, carbon optimal policy, and monetary policy? and iii) What are the drivers of carbon pricing when relying on cap-and-trade markets (the case of the European Union Emissions Trading System–EU ETS)?

The main results of the chapters of this thesis are as follows:

The first chapter finds that well-designed environmental policies could lead to lower risk premiums and higher real interest rates. We obtain this result by introducing an optimal environmental policy into a business cycle model in which finance matters and using higher order non-linear filters to estimate the economy with real US data. By correcting the externality responsible for climate change using a non-separable dis-utility function, the optimal policy reduces the welfare cost of business cycle fluctuations. The compensation demanded by investors for holding risky assets as well as the need for precautionary savings are tightly linked to the presence or absence of carbon mitigation policy.

The second chapter first examines the role of climate damages modeling and then the interaction of environmental policy and monetary policy. Damages are modeled both via a non-separable dis-utility function and a standard production-based climate damages model. In this chapter, I show that utility-based damages, compared to production-based damages, allow for better capture of macro-financial dynamics while significantly reducing emissions. Monetary policy is shown to play an important role in emissions reduction; however, a trade-off is introduced as central banks work to stabilize interest rates and inflation, or alternatively, allow for higher emissions reduction rates.

The third chapter investigates the drivers of cap-and-trade market implicit carbon pricing. This chapter uses a macro-finance model similar to the previous chapters and estimates the economic drivers of the CO_2 price using EU ETS prices, macroeconomic, financial, and environmental data. This chapter finds that the ETS price is mainly driven by energy, climate sentiment, and abatement shocks. Relative to the optimal carbon price, the variations of the EU carbon implicit price are significantly higher. Finally, I show how reducing price uncertainty using a carbon cap rule can help close the volatility gap with respect to the first-best policy.

The related literature (in a nutshell) of this thesis builds on both the macro-finance and climate-economics literatures.

Macro-finance business cycles models: The last century has seen significant developments in the macro/business cycle literature. Since the first business cycle micro-founded framework of Lucas Jr [1978], dynamic stochastic general equilibrium (DSGE) models have become the state-of-the-art framework used by central banks and financial institutions. This has been especially true with the canonical contributions of Smets and Wouters [2007] and Christiano et al. [2005], who established procedures to estimate large models using empirical macro data, thus enabling a large spectrum of policy analyses. In recent years, estimating these frameworks while capturing different state non-linearities has became a priority. This is of the highest importance when thinking of climate change dynamics. We focus on recent developments in non-linear estimations of DSGE models (e.g. Kollmann [2017]) to study climate related questions.

Environmental-macro E-DSGE: Since the early work of William Nordhaus, who aimed to incorporate geophysical and climate change dynamics within standard long-run macroeconomic models (where he developed Integrated Assessment Models (IAMs), e.g. Nordhaus [1992]), most of the research focused on the social cost of carbon, building on his framework and did not investigate the linkages with macro-finance and business cycles implications.

Early work by William Nordhaus (e.g. Nordhaus [1992]) incorporated climate change dynamics within macroeconomic models referred to as Integrated Assessment Models (IAMs). However, most of the research building on Nordhaus early work, focused on the social cost of carbon. Macro-financial implications of climate physical and transition risks remained understudied until very recently as documented by Schubert 1 [2018] and Annicchiarico et al. [2021].

In the main body of this thesis, I build on all these recent macro-finance and environmental-macro literatures[1].

The main contributions of the thesis with respect to the literature are twofold:

i) Theoretically, we propose a general equilibrium macro-finance and climate change frameworks to study the impacts of environmental policy on financial markets. We emphasize the importance of modeling climate dynamics and demonstrate how to achieve a stationary equilibrium in their presence.

ii) Methodologically, all chapters present tractable frameworks that facilitate a more comprehensive approach to studying the linkages between environmental externality and policy, financial markets, and climate. We also provide an approach to estimate macro-financial-climate models using higher-order and non-linear techniques, which is crucial for conducting policy analysis.

In the following, we present the three chapters of the thesis and then conclude.

[1] An extensive literature review is included in each chapter.

Chapter 1

Asset Pricing and Environmental Externality

1.1 Introduction

Current evidence shows that the mean temperature is 1 degree higher than it was in the pre-industrial era. In recent years, this increase in temperature has accelerated and temperatures are currently estimated to rise by about 0.2 degrees per decade.[2] The link between carbon-dioxide emissions (CO2) and climate change is by now clearly established. CO2 emissions are about 20 times higher than they were at the beginning of the 20th century. Moreover, evidence from Antarctic ice cores shows that CO2 emissions have not only risen rapidly, current levels are also the highest in over 400,000 years.[3]

CO2 emissions are not only a low-frequency phenomenon, they also exhibit large cyclical fluctuations. A decomposition between trend and cyclical components reveals that CO2 emissions are procyclical and more volatile than GDP (e.g. Doda [2014]; Heutel [2012]). Against the background of the ongoing debate over emission taxes, these large cyclical fluctuations raise several important questions. In particular, are these strong cyclical fluctuations desirable from a welfare perspective? And how should the optimal carbon tax vary over the business cycle?

This paper addresses these questions by considering the optimal carbon tax in the presence of an environmental externality. The novelty of our approach is to investigate the link between asset-pricing theory —in particular the stochastic discount factor (SDF)— and climate policies. The SDF is a key building block of modern asset-pricing theory (e.g. Cochrane [2011]). Our main contention is that it also has a critical impact on the design of the optimal carbon tax.

Following Stokey [1998], Acemoglu et al. [2012] and Golosov et al. [2014], among others, environmental considerations are captured by introducing an externality into the utility function. Apart from a few exceptions (see for instance Michel and Rotillon [1995]), most papers in this literature use a separable specification that implies no direct link between

[2]Pachauri et al. [2014].

[3]The Economist (2019). "Briefing Climate Change", Sept. 21st-27th.

the environment and the marginal utility of consumption. Our innovation is to study a model in which the presence of an environmental externality raises households' willingness to consume goods.

Our approach can be motivated by the effect of climate change on consumption. As documented by Abel et al. [2018] and Mansur et al. [2008], one perverse effect of climate change is to increase the use of electricity. Higher levels of emissions cause climate change, which in turn increases the need to consume electricity to cool homes. This complementarity between climate change and consumption can be illustrated by the exponential increase in the use of air-conditioning in recent decades.[4] Projections by the International Energy Agency also suggest that this is only the beginning, as the demand for air-conditioning is expected to triple by 2050.[5] This latter result is consistent with the US findings in He et al. [2020]. Using data for a large sample of consumers, they show that pollution, which is highly correlated with CO2 emissions, increases electricity consumption.

Apart from electricity consumption, there is evidence that emissions also increase other types of expenditure. Deschênes et al. [2017] show that air pollution increases the consumption of medical products. There is moreover evidence that emissions raise the demand for goods that are used to mitigate the effect of pollution, such as air purifiers (e.g. Ito and Zhang [2020]). Climate change also increases investment in adaptation measures (e.g. Fried [2019]; Gourio and Fries [2020]). More recently Hsu et al. [2023] show that environmental policy uncertainty is key in explaining asset pricing implications of industrial pollution.

Overall, the evidence therefore suggests the existence of a compensation effect of climate change (e.g. Michel and Rotillon [1995]). As Greenhouse Gas emissions rise, the need to consume electricity as well as other goods to mitigate the effect of climate change becomes more pressing. In other words, the presence of environmental externalities could raise the

[4] The Economist (2018). "Air-conditioners do great good, but at a high environmental cost". August 25th.

[5] International Energy Agency (2018). "Air conditioning use emerges as one of the key drivers of global electricity-demand growth". News, May 15th 2018.

marginal utility of consumption.

From a finance perspective, this non-separability between consumption and the environmental externality has key implications. Indeed, the SDF —the ratio of future to current marginal utility—is at the core of modern asset-pricing theory. Consequently, if environmental factors modify agents' marginal utility of consumption, they will also affect the pricing of risky and safe assets. This compensation effect of climate change therefore implies a potential role for green factors in asset-pricing models.

We model this compensation effect of climate change via an approach similar to that in the seminal contribution of Campbell and Cochrane [1999]. In our case, however, it is the current stock of CO2 emissions rather than past levels of consumption that raises marginal utility. Moreover, following Heutel [2012], the stock of emissions is a slow-moving variable whose level depends on the quantity of emissions. As in Campbell and Cochrane [1999], this specification implies that risk aversion increases as the distance between consumption and the externality, or "surplus consumption" in the case of habits, declines. One advantage of this particular specification is that it will allow us to generate realistic fluctuations in the SDF without introducing too many degrees of freedom.

Relative to the endowment economy approach (e.g. Lucas Jr [1978]), another difference is that we analyze the environmental externality in a production economy, following the seminal contribution of Jermann [1998]. We then derive the optimal tax by comparing the decentralized equilibrium to the planner's problem, as is usually the case in the environmental literature (e.g. Xepapadeas [2005]) or in Ljungqvist and Uhlig [2000] for the case of a consumption externality.

Following Nordhaus [2008] and Heutel [2012], among others, we introduce an abatement technology that firms can use to reduce their carbon footprint. Even when available, firms do not use this technology if emissions are not taxed. The abatement technology diverts resources from production. Consequently, profit-maximizing firms have no incentive to reduce emissions unless they are forced to do so.

Our first main result is that the optimal tax is determined by the shadow value of CO2 emissions. We show that this implicit price can be expressed as the infinite discounted sum of the marginal disutility caused by emissions. This discounted sum is in turn critically affected by the SDF used by agents to price assets. This result therefore highlights the importance of asset-pricing considerations for the design of an optimal environmental tax.

This link between the optimal tax and the SDF breaks the macro-finance separation (e.g. Cochrane [2017]; Tallarini [2000]). The reason is that the model's ability to reproduce basic asset-pricing moments, such as the bond premium for example, has a crucial impact on the SDF. As the optimal tax is in turn determined by the SDF, the model's financial-market implications affect the design of environmental policies, and hence welfare. In contrast, with a separable preference specification we find that the dichotomy between climate policies and finance is close to perfect.

Imposing a tax on emissions restores the first-best allocation by encouraging firms to use the abatement technology. Abating carbon emissions is costly for firms. From the point of view of the social planner, it is therefore optimal to set the cost of abating emissions that firms face to its implicit market price.

Our second main result is that slow movements in the stock of CO2 can have significant financial-market implications. Of particular relevance to Central Banks is the finding that environmental externalities affect the natural rate of interest. Climate change reduces the natural rate of interest.

The intuition behind this result is that the environmental externality generates time-variation in risk aversion, as in a model with external habits. In other words, when firms fail to internalize the damage caused by their emissions, households become more risk-averse. This rise in risk-aversion raises the risk premium demanded by investors, and induces precautionary saving. This stronger precautionary motive in turn explains the effect on the natural rate of interest.

We next show that introducing an optimal environmental tax reduces risk premia and increases the natural rate of interest. Under our baseline scenario, the tax reduces the premium on a long-term bond by half, and increases the natural rate by around 2 percentage points.

This result can be explained by the effect of the optimal policy on risk aversion. A tax on production reduces output, and hence consumption as well as emissions. The key is that the decline in emissions causes a fall in the externality that exceeds the drop in consumption. The resulting increase in this distance between consumption and the externality in turn reduces risk aversion.

Although consumption declines, the optimal tax generates large welfare gains. Under our benchmark calibration, this result is explained by the large fall in emissions induced by the policy. The magnitude of this gain in turn depends on how firms react to the carbon tax. A profit-maximizing firm increases abatement until the marginal cost of abating emissions equals the marginal benefit. Under the optimal policy, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax. This incentive to reduce emissions therefore lies behind the large welfare gain that we obtain.

The effect on welfare critically depends on the efficiency of the abatement technology available in the economy. If the technology is not sufficiently well-developed, the distortion caused by the tax can be sizeable: if firms cannot circumvent the tax by abating emissions, their only choice is to reduce production. In this case, the tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

The effect of the optimal policy on asset prices also depends crucially on the abatement technology. In this model, this can be explained by the impact of the tax on risk aversion. A less-developed technology reduces the decline in the stock of emission induced by the carbon tax. Consequently, a smaller increase in the distance between consumption and the externality can result if the technology is inefficient. This in turn implies a smaller drop in risk aversion, which causes higher risk premia and lower real interest rates.

Our third main result is that the optimal tax is pro-cyclical. As in Ljungqvist and Uhlig [2000], it is therefore optimal to "cool down" the economy during booms and to stimulate it in recessions. Estimating the model using higher-order perturbation methods allows us to estimate the implicit price of carbon. Our approach can therefore be used to provide an estimate of the optimal carbon tax over the business cycle. As illustrated in Figure 1.1, it would have been optimal to progressively increase the tax in the run-up to the financial crisis and to reduce it sharply when the financial shock hit.

The intuition here is that the externality produces excessive fluctuations in risk aversion. As in a model with external habits and time-varying risk aversion (e.g. Campbell and Cochrane [1999]), the externality is beyond the agents' control. By internalizing the effect of emissions on utility, the policy allows the planner to find an optimal trajectory for both consumption and the stock of emissions. Controlling both variables at the same time in turn reduces the variations in "surplus consumption" that are unnecessary from a welfare perspective. These lower fluctuations in turn imply more moderate variations in risk aversion.

During recessions, this optimal trajectory involves lowering the carbon tax. A decline stimulates consumption. This effect helps to reduce risk aversion by increasing the distance between consumption and the externality. The key is that, as in the data, the stock of emissions moves very slowly over time. As the impact of the policy on consumption is more immediate, a tax cut generates a rise in consumption that exceeds the increase in the stock of emissions. The optimal policy therefore allows the planner to mitigate the surge in risk aversion that occurs in recessions.

As pointed out by Bansal et al. [2019] and van den Bremer and van der Ploeg [2019], there is evidence that climate-change risk could already be reflected in current equity prices. In Bansal et al. [2019], this link is explored in a model in which climate change is a source of long-run risk (e.g. Bansal and Yaron [2004]). The long-run risk approach relies on Epstein-Zin-Weil preferences (e.g. Epstein and Zin [1989]; Weil [1989]; Weil [1990]).

The results in Bolton and Kacperczyk [2020] also suggest that exposure to carbon emission is already priced-in by investors. They find that the increase in stock returns caused by higher emissions is economically significant. In Van der Ploeg et al. [2020], the optimal carbon tax is derived in an endogenous-growth model. They also find that the natural rate of interest is lower under laissez-faire.

Bauer and Rudebusch [2020] show that the decline in the natural interest rate observed over the last decade implies a dramatic increase in the social cost of climate change. Our findings are also related to Gollier [2021] who highlights the role of abatement technologies and their efficiency in shaping carbon pricing. Following Piazzesi et al. [2007], we analyze the asset-pricing implications of a nonseparable utility function. Piazzesi et al. [2007] show that variations in the relative share of housing in agents' consumption baskets is a significant source of risk. In our case, it is the slow movements in the environmental externality that affect marginal utility. A review of the macro-financial implications of climate change is provided by Van der Ploeg [2020].

Our approach also builds on Heutel [2012], which is one of the first papers to consider environmental externalities from a business-cycle perspective. Relative to Heutel [2012], the model is estimated and generates a bond premium of about 1 percent. Reproducing a bond premium of this magnitude is a challenge for standard macroeconomic models (e.g. Rudebusch and Swanson [2008]; Rudebusch and Swanson [2012]). Recent improvements in this literature for instance includes the work of Andreasen et al. [2018], which studies feedback effects from long-term bonds to the real economy within a model that matches the level and variability of the term premium.

In our case, environmental factors affect financial markets through the effect of the externality on attitudes towards risk. All else equal, the key is that an increase in the stock of emissions increases risk aversion. While it is difficult to test this hypothesis in the data, recent results in the psychology literature provide some indirect support.

First, in this literature, it is well-established that air pollution tends to increase anxiety. A recent review of the evidence on the link between air pollution and anxiety is provided in Lu [2020]. Air pollution is in turn strongly correlated with CO2 emissions. Second, there is evidence that anxiety and risk aversion are tightly linked. For instance, according to Charpentier et al. [2017], more-anxious individuals exhibit a reduced propensity to take risks. The authors argue that this result is driven by risk aversion, and not loss aversion.

This kind of effect of air pollution on risk aversion is also consistent with the findings in Levy and Yagil [2011] of a negative correlation between air pollution and stock returns. Their interpretation is that air pollution has negative mood effects. As experimental work in Psychology in turn has related bad mood to increased risk aversion, they argue that air pollution could affect stock returns.

1.2 The model

Consider a business-cycle model characterized by discrete time and an infinite-horizon economy populated by *firms* and *households*, which are infinitely-lived and of measure one. In this setup, production by firms produces an environmental externality via emissions, and these latter affect the household welfare by reducing the utility stemming from the consumption of goods. Firms do not internalize the social cost from their emissions of CO2. As such there is market failure, opening the door to optimal policy intervention.

As the contribution of the paper lies in the role of the environmental externality in shaping investors' risk behavior, we start by presenting the accumulation of emissions in the atmosphere. We then explain how this environmental externality affects households' behavior.

1.2.1 Balanced growth

Given that one objective of this paper is to estimate the model, we need to take into account that emissions grow at a different rate from output. In the context of our model, this difference in growth rates can be explained by introducing a rate of Green technological progress.

As is standard in the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This is achieved by introducing labor-augmenting technological progress, denoted by Γ_t. The growth rate of labor-augmenting technological progress is γ^Y, where:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma^Y$$

We denote Green technological progress in the growing economy by Ψ_t. The growth rate of Green progress γ^E is as follows:

$$\frac{\Psi_{t+1}}{\Psi_t} = \gamma^E$$

This trend is necessary to capture the long-term process of the decoupling of output growth from emission growth. As documented by Newell et al. [1999], this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy-intensive technologies in capital goods. An improvement in the technology therefore implies a value for γ^E that is below 1. As in Nordhaus [1991], we assume that this trend is deterministic.

In the following sections, we present the de-trended economy. The detailed derivation of this de-trended economy appears in Appendix C.

1.2.2 Firms and emissions

Following standard integrated assessment models (IAM) (see Nordhaus [1991] and Nordhaus and Yang [1996]), a large part of the accumulation of Carbon Dioxide and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. We therefore employ a similar law of motion as in IAM to describe the concentration process of Carbon Dioxide in the atmosphere:

$$\gamma^X x_{t+1} = \eta x_t + e_t, \tag{1.1}$$

where x_{t+1} is the concentration of gases in the atmosphere, $e_t \geq 0$ the inflow (in kilotons) of Greenhouse Gases at time t, and $0 < \eta < 1$ the linear rate of continuation of CO_2-equivalent emissions that enter the atmosphere on a quarterly basis.[6] Anthropogenic emissions of CO_2 result from both economic production and exogenous technical change:

$$e_t = (1 - \mu_t)\, \varphi_1 y_t^{1-\varphi_2} \varepsilon_t^X. \tag{1.2}$$

Here, the variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t the aggregate production of goods by firms, and variable ε_t^X an AR(1) exogenous shock.

This functional form for emissions allows us to take into account both low- and high-frequency variations in CO_2 emissions. For the high-frequency features of the emissions data, the term $\varphi_1 y_t^{1-\varphi_2}$ denotes the total inflow of pollution resulting from production, prior to abatement. In this expression, φ_1, $\varphi_2 \geq 0$ are two carbon-intensity parameters that respectively pin down the steady-state ratio of emissions-to-output and the elasticity of emissions with respect to output over the last century. While φ_2 is set to 0 in Nordhaus [1991], we follow Heutel [2012] and allow this parameter to be positive to capture potential nonlinearities between output and emissions. Note that for $\varphi_2 < 1$, the emissions function exhibits decreasing returns.

In the de-trended economy, the presence of both Green and labor-augmenting technological progress introduces an adjustment into equation (1.1), where γ^X is given as follows:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2}$$

The remaining set of equations for firms is fairly standard, and similar to Jermann [1998]. In particular, the representative firm seeks to maximize profit by making a trade-off between the desired levels of capital and labor. Output is produced via a Cobb-Douglas production function:

$$y_t = \varepsilon_t^A k_t^\alpha n_t^{1-\alpha}, \tag{1.3}$$

[6] One limitation is that we do not consider emissions from the Rest of the World (ROW). At the same time, US and ROW emissions are strongly correlated at the business-cycle frequency. Moreover, the US accounts for 1/3 of total anthropogenic emissions.

where k_t is the capital stock with an intensity parameter $\alpha \in [0,1]$, n_t is labor, and ε_t^A is a total factor productivity shock that evolves as follows: $\log\left(\varepsilon_t^A\right) = \rho_A \log\left(\varepsilon_{t-1}^A\right) + \eta_t^A$, with $\eta_t^A \sim N(0, \sigma_A^2)$. The capital-share parameter is denoted by α. Firms maximize profits:

$$d_t = y_t - w_t n_t - i_t - f\left(\mu_t\right) y_t - e_t \tau_t \tag{1.4}$$

The real wage is denoted by w_t, $f\left(\mu_t\right)$ is the abatement-cost function, and $\tau_t \geq 0$ a potential tax on GHG emissions introduced by the fiscal authority. Investment is denoted by i_t and the accumulation of physical capital is given by the following law of motion:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon}\left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{1-\epsilon} + \chi_2\right) k_t \tag{1.5}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital and ε_t^I an exogenous shock process, as in Christiano et al. [2014]. This can be interpreted as an investment shock that captures financial frictions associated with asymmetric information or costly monitoring. As in Jermann [1998], χ_1 and χ_2 are two scale parameters that are calibrated to ensure that adjustment costs do not affect the deterministic steady state of the economy. The elasticity parameter $\epsilon > 0$ measures the intensity of adjustment costs.

The abatement-cost function is taken from Nordhaus [2008], where $f\left(\mu_t\right) = \theta_1 \mu_t^{\theta_2}$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement, while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f\left(\mu_t\right)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one.

1.2.3 Households and the environmental externality

We model the representative household via a utility function where the household chooses consumption expenditures as well as its holdings of long-term government bonds. Following Stokey [1998], Acemoglu et al. [2012] and Golosov et al. [2014], we introduce the environmental externality into the utility function. However, instead of considering an additive specification, we assume that the marginal utility of consumption is affected by the externality.

Given our focus on asset prices, we choose a specification similar to that employed in the seminal contribution of Campbell and Cochrane [1999]. As will become clear, adopting this particular specification will dramatically improve the model's ability to generate realistic asset-pricing implications. The difference relative to Campbell and Cochrane [1999] is that it is the disutility caused by pollution rather than past consumption that affects the marginal utility of consumption. As the evolution of x_t is determined by the environmental block of the model (e.g. Nordhaus [1991]), we refer to this preference specification as Campbell and Cochrane/Nordhaus (CCN) preferences.

The utility of the representative agent depends on the distance between consumption and the externality:

$$E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma}, \tag{1.6}$$

where E_0 is the expectations operator conditioned on information at time 0, $\widetilde{\beta}$ the time discount factor adjusted for growth,[7] and $\sigma > 0$ the curvature parameter. The parameter ϕ represents the sensitivity of utility to a rise in CO_2 concentration in the atmosphere, which is denoted by x_t.[8] This can also be interpreted as the proportion of consumers affected by the damage caused by CO2 emissions. Furthermore, the externality is a predetermined variable that moves slowly over time. This is to account for the possible long-term effects of decisions made in the past, which have possibly irreversible future consequences.

[7]Where $\widetilde{\beta} = \beta\gamma^{1-\sigma}$. See Appendix C for a derivation of the effect of growth on the subjective discount factor.

[8]Note that c_t and x_t do not grow at the same rate in the deterministic steady-state of the model. To obtain a stationary utility function, we assume that, in the growing economy, the preference parameter Θ_t is affected by labor-augmenting and Green technological progress. As we show in Appendix C, this implies the following relationship between ϕ and Θ_t :

$$\Theta_t = \phi \frac{(\Gamma_t)^{\varphi_2}}{\Psi_t}$$

.

This assumption has important implications for optimal choices, which we discuss in the following paragraphs.

First, from a consumer's perspective, consumption and the stock of CO2 emissions can be interpreted as complements. As a result, the marginal utility of consumption increases in CO2 concentration, so that households are more willing to consume when GHG concentration is high. This mechanism, pioneered by Michel and Rotillon [1995], is referred to as the *compensation effect*: households consume as a result of the change in marginal utility following an increase in emissions.

Second, this environmental externality in the utility function has important asset-pricing implications. To illustrate, we define, as in Campbell and Cochrane [1999], the consumption surplus ratio, $s_t = (c_t - \phi x_t)/c_t$. When the surplus falls in cyclical downturns, investors require a higher expected return compared to a standard CRRA utility function with $\phi = 0$. Under these preferences, the coefficient of relative risk aversion is given by $-(u_c''/u_c')c_t = \sigma/s_t$. As such, a higher emissions stock reduces the surplus, which in turn increases risk aversion.

The budget constraint of the representative household is as follows:

$$w_t n_t + b_t + d_t = c_t + p_t^B(b_{t+1} - b_t) + t_t \tag{1.7}$$

where the left hand-side refers to the household's different sources of income. Total income is firstly comprised of labor income (with inelastic labor supply n_t). Every period, the agent also receives income from holding a long-term government bond, b_t. As the representative agent owns firms in the corporate sector, there is last dividend income of d_t.

On the expenditure side, the representative household first spends its income on consumption goods, c_t. The price at which newly-issued government bonds are purchased is p_t^B, and the quantity of new government bonds purchased during the period is $b_{t+1} - b_t$. Finally, we assume that the government levies a lump-sum tax of t_t.

1.2.4 Government and market clearing

The government finances its expenditures by issuing a bond and collecting taxes. The government budget constraint is as follows:

$$g_t + b_t = p_t^B(b_{t+1} - b_t) + t_t + \tau_t e_t, \tag{1.8}$$

where public expenditure is denoted by g_t and t_t is a lump-sum tax. The revenue is composed of newly-issued government bonds $b_{t+1} - b_t$ on financial markets to households, while $\tau_t e_t$ denotes the revenues obtained from the implementation of an environmental tax on emissions. In this expression, e_t and τ_t are the level of emissions and the tax, respectively. As in any typical business-cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$, with $\log \varepsilon_t^G = \rho_G \log \varepsilon_{t-1}^G + \eta_t^G$, $\eta_t^G \sim N(0, \sigma_G^2)$, and $\bar{g}$ denoting the steady-state amount of resources that is consumed by the government. This shock accounts for changes in aggregate demand driven by both changes in public spending and the trade balance.

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f\left(\mu_t\right) y_t. \tag{1.9}$$

Finally, for the asset-pricing variables, we calculate the risk-free rate and the conditional risk premium respectively as:

$$1 + r_t^F = \left\{E_t m_{t,t+1}\right\}^{-1}, \tag{1.10}$$

$$E_t(r_{t+1}^B - r_t^F) = E_t((1 + p_{t+1}^B)/p_t^B - (1 + r_t^F)), \tag{1.11}$$

where $m_{t,t+1} = \beta^Y \left\{\lambda_{t+1}/\lambda_t\right\}$ is the stochastic discount factor, and the modified discount factor β^Y is as follows:

$$\beta^Y = \widetilde{\beta}/\gamma^Y$$

1.3 Welfare theorems with environmental preferences

In this section, we derive the optimal tax by comparing the decentralized equilibrium to the planner's problem.

1.3.1 The centralized economy

We start by characterizing the first-best allocation and consider the optimal plan that the benevolent social planner would choose so as to maximize welfare. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 1.3.1 *The optimal policy problem for the social planner is to maximize total welfare in Equation 1.6 by choosing a sequence of allocations for the quantities* $\{c_t, i_t, y_t, \mu_t, e_t, k_{t+1}, x_{t+1}\}$*, for given initial conditions for the two endogenous state variables* k_0 *and* x_0*, that satisfies equations (1.1), (1.2), (1.3), (1.5), and (1.9).*

Define λ_t as the time t marginal utility of consumption, q_t as the shadow value of capital and ϱ_t as the Lagrangian multiplier on the production function (note that both q_t and ϱ_t are expressed in terms of the marginal utility of consumption). The first-order conditions for this problem are as follows:

$$\lambda_t = (c_t - \phi x_t)^{-\sigma}, \tag{1.12}$$

$$1 = \chi_1 \varepsilon_t^I q_t \left(\varepsilon_t^I \frac{i_{t+1}}{k_{t+1}} \right)^{-\epsilon}, \tag{1.13}$$

$$q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right] + \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha \frac{y_{t+1}}{k_{t+1}} \varrho_{t+1} \quad (1.14)$$

where:

$$\beta^Y = \widetilde{\beta} / \gamma^Y$$

Letting v_{Et} denote the Lagrange multiplier (expressed in units of marginal utility of consumption) on equation (1.2), the first-order conditions with respect to the firm's optimal choice of output and abatement are given as follows:

$$\varrho_t + f(\mu_t) + v_{Et}(1-\varphi_2) e_t / y_t = 1, \quad (1.15)$$

$$v_{Et} e_t / (1-\mu_t) = f'(\mu_t) y_t. \quad (1.16)$$

The Lagrange multiplier ϱ_t is usually interpreted as the marginal cost of producing a new good, while v_{Et} is the social planner's value of abatement. Equation (1.15) thus highlights the key role of emissions in shaping price dynamics: the production of one additional unit of goods increases firm profits but is partially compensated by the marginal cost from abating emissions. The planner also takes into account the marginal cost from emitting GHGs in the atmosphere. Notice that if abatement effort is zero, the marginal cost of production is one, as in the standard real business-cycle model. The second equation (1.16) is a standard cost-minimizing condition on abatement: abating CO2 emissions is optimal when the resulting marginal gain (the left-hand side of equation 1.16) is equal to its marginal cost (the right-hand side of the same equation).

Two remaining first-order conditions on each of the environmental variables, namely

x_t and e_t, are necessary to characterize the decision rules of the social planner:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\phi + \eta v_{Xt+1}\right) \tag{1.17}$$

$$v_{Et} = v_{Xt}. \tag{1.18}$$

where:

$$\beta^X = \widetilde{\beta}/\gamma^X$$

Recall that v_{Et} is the Lagrange multiplier on emissions in equation (1.2), while v_{Xt} is the Lagrange multiplier on the law of motion of GHGs in equation (1.1). The variable v_{Xt} can be interpreted as the implicit price of carbon. Equation (1.17) shows that this implicit price can be considered via an asset-pricing formula. The first term- $(\beta^X E_t \frac{\lambda_{t+1}}{\lambda_t}\phi)$ is the discounted utility loss incurred by society from a marginal increase in the stock of emissions in the atmosphere. The second term $(\eta\{E_t \frac{\lambda_{t+1}}{\lambda_t} v^X_{t+1}\})$ is the continuation value of the discounted utility loss caused by emissions, which remain in the atmosphere with probability η. The second equation is the internal cost of GHG emissions for firms, where v_{Et} is the marginal cost for a firm of emitting one kiloton of carbon. In the first-best allocation, this cost must be exactly equal to the price of carbon emissions v_{Xt}.

Definition 1.3.2 *The inefficiency wedge induced by the environmental externality is defined as the gap between the price of carbon emissions and this marginal cost:*

$$\varpi_t = v_{Xt} - v_{Et}. \tag{1.19}$$

When the social cost of carbon is perfectly internalized by society, optimal abatement in (1.18) is such that the marginal cost of emissions equals their price. In this case, it is optimal for firms and society to spend a fraction of resources to reduce CO2 emissions by using the abatement technology $f\left(\mu_t\right)$.

Proposition 1.3.1 *In a centralized equilibrium, the social cost of carbon is perfectly internalized by the planner. The marginal cost of emissions is therefore equal to the price of carbon emissions. This implies (from the previous definition) a first-best allocation with an inefficiency wedge* $\varpi_t = 0$.

The resulting equilibrium is optimal, as the social cost of the externality is perfectly internalized by society. As a consequence, the inefficiency wedge from carbon emissions is zero. In the following section, we show that this optimum is not reached in a *laissez-faire* equilibrium with profit-maximizing firms.

1.3.2 The competitive equilibrium

We now describe the competitive equilibrium resulting from economic decisions taken by households and firms separately, with no centralization. This decentralized economy is also referred to as the competitive or *laissez-faire* equilibrium, where social preferences for carbon are different across firms and households. We propose the following definition to characterize this economy.

Definition 1.3.3 *The laissez-faire equilibrium is defined as a competitive equilibrium in which the environmental tax on carbon emissions* τ_t *is set to 0. Households maximize utility in Equation 1.6 under constraints (1.7) and (1.5). Firms maximize profits (1.4) under constraints (1.2) and (1.3).*

Relative to the efficient equilibrium, the difference here is that firms maximize profits and no longer consider the stock of CO2 emissions as a control variable. This implies that firms and households exhibit different preferences regarding carbon emissions. As a result, the price of carbon for firms differs from that obtained in the centralized economy. Since emissions are costly to abate, and given that firms do not internalize the effect of their emissions on consumers, the cost of carbon emissions for firms is zero. In contrast, the price of carbon for households, which we denote v_{Xt}, is given as follows:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\phi + \eta v_{Xt+1}\right) \tag{1.20}$$

We here have a market failure, as the social value of carbon differs between the emitters of carbon and the agents who experience the social loss.

As emissions are not taxed, the shadow cost for a firm to emit CO2 in the atmosphere is zero:[9]

$$v_{Et} = 0. \tag{1.21}$$

In this setup, firms simply cost-minimize by optimally choosing zero abatement spending: with a cost of releasing CO2 of zero, firms have no incentive to allocate resources to use the abatement technology $f\left(\mu_t\right)$ to reduce emissions. The socially-optimal level of abatement is not implemented, as the equilibrium abatement share is zero in the *laissez-faire* equilibrium:

$$\mu_t = 0. \tag{1.22}$$

Consequently, the marginal cost of production ϱ_t is similar to that obtained in any typical real business-cycle model. In terms of the notation introduced in definition 1.3.3, this produces an environmental inefficiency wedge that differs from zero:

$$\varpi_t = v_{Xt} - v_{Et} = v_{Xt}. \tag{1.23}$$

CO2 emissions therefore create a market failure via an environmental externality. As a result, the first welfare theorem breaks down as the competitive equilibrium does not coincide with the social planner's outcome. The externality, measured by the inefficiency

[9]The optimality conditions corresponding to the *laissez-faire* equilibrium are derived in Appendix D.

wedge ϖ_t, distorts the equilibrium and gives rise to a deadweight loss proportional to v_{Xt}. Note that the first welfare theorem applies only if the environmental policy has no effect on preferences, which is the case only if $\phi = 0$.

1.3.3 Environmental policy

In the presence of the environmental externality reflected in $\varpi_t > 0$, the social value of carbon differs across agents. This market failure opens the door for government policy to address this externality and render the *laissez-faire* allocation the same as that of the social planner. In particular, the government can introduce a tax, τ_t, on GHG emissions to be paid by firms. This policy tool has two interpretations. It first can be considered as a tax on carbon emissions, in the same spirit as a standard Pigouvian tax that aims to force firms to internalize the social cost of carbon emissions on household utility, thereby correcting the market failure (i.e. the negative externality) by setting the tax equal to the price of carbon emissions.

An alternative interpretation is that the government creates a market for carbon emissions (i.e. a carbon-permits market). Here the government regulates the quantity of emissions. The optimal value for this instrument can be directly computed from a Ramsey optimal problem. Comparing the social planner's solution to the competitive equilibrium, we make the following proposition:

Proposition 1.3.2 *The first-best allocation can be attained by using the instrument τ_t in order to close the inefficiency gap (i.e. $\varpi_t = 0$). This condition is achieved by setting the carbon tax such that:*

$$\tau_t = v_{Xt}.$$

As shown in Appendix D, setting the environmental tax to v_{Xt} ensures that the first-order conditions under the competitive and centralized equilibria coincide. This result is fairly intuitive. In the absence of an environmental policy, abatement reduces profits, and firms will not be willing to bear this cost unless an enforcement mechanism is implemented. The government can impose a price on carbon emissions by choosing the optimal tax (either quantity- or price-based, as discussed in Weitzman [1974]) to produce the desired level of abatement. This environmental policy forces firms to internalize the effect of emissions, which in turn leads to a better integration of economic and environmental policies.

Furthermore, as argued in both the public economics and environmental literatures (Goulder [1995]), either a tax or a permit policy would generate revenue that could be used as a “double dividend” to not only correct the externality but also reduce the number of distortions due to the taxation of other inputs, such as labor and capital. Moreover, an equivalence between the tax and permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme (Heutel [2012]).

1.4 Estimation

In this section, we estimate the structural parameters of the model using Bayesian methods. For a presentation of the method, we refer to the canonical papers of An and Schorfheide [2007] and Smets and Wouters [2007]. As the U.S. has not implemented any environmental policy, we propose to estimate the *laissez-faire model*. The following sub-sections discuss the non-linear method employed for the estimation, the data transformation and calibration, the priors and the posteriors.

1.4.1 Solution method

Since we want to accurately measure higher-order effects of environmental preferences (e.g. precautionary saving, utility curvature), we consider a second-order approximation to the decision rules of our model. Taking higher-order approximated models to data remains a challenge as the nonlinear filters that are required to form the likelihood function are computationally expensive. An inversion filter has recently emerged as a computationally-cheap alternative to apply nonlinear models to data (e.g. Guerrieri and Iacoviello [2017], Atkinson et al. [2020]). Initially pioneered by Fair and Taylor [1987], this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g. Kalman or particle),[10] the inversion filter relies on an analytic characterization of the likelihood function. Kollmann [2017] provided the first application of the inversion filter to second- and third-order approximations to the decision rules in a rational-expectations model.[11] To allow the recursion, this filter imposes that the number of fundamental shocks must be equal to the number of observable variables. Note that, for linearized models, this restriction is standard following Smets and Wouters [2007]. For the relative gains of the inversion filter with respect to a particle filter, we refer to Cuba-Borda et al. [2019] and Atkinson et al. [2020].

The model is estimated using four observable macroeconomic time-series, which are jointly replicated by the model through the joint realization of four corresponding innovations. Note that we use the pruning state-space to obtain the matrices of the policy rule using the Dynare package of Adjemian et al. [2011]. From this state-space representation, we reverse the observation equations to obtain the sequence of shocks. Unlike Kollmann [2017] who limits the analysis to a frequentist approach, we augment the likelihood function with prior information in the same spirit as Smets and Wouters [2007]. This method requires a sampler, here Metropolis-Hastings, to draw the parametric uncertainty.

1.4.2 Data

The model is estimated with Bayesian methods on U.S. Quarterly data over the sample time period 1973Q1 to 2018Q4, which are all taken from FRED and the U.S.

[10]For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde et al. [2016].

[11]Kollmann [2017] posits a modified higher-order decision rule in which powers of exogenous innovations are neglected to obtain a straightforward observation equation inversion. In this paper, we include these terms of the decision rule.

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Concerning the transformation of series, the aim is to map non-stationary data to a stationary model (namely, GDP, consumption, investment and CO2 emissions). Following Smets and Wouters [2007], data exhibiting a trend or unit root are rendered stationary in two steps. We first divide the sample by the working-age population. Second, data are taken in logs and we apply a first-difference filter to obtain growth rates. Real variables are deflated by the GDP deflator price index. The measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_A + \Delta \log (\tilde{y}_t) \\ \log \gamma_A + \Delta \log (\tilde{c}_t) \\ \log \gamma_A + \Delta \log (\tilde{\imath}_t) \\ \log \gamma_A^{1-\varphi_2} \gamma_E + \Delta \log (\tilde{e}_t) \end{bmatrix}, \qquad (1.24)$$

where a variable with a tilda, $\tilde{x}_t$, denotes the de-trended version of a level variable, x_t.

1.4.3 Calibration and prior distributions

The calibrated parameters are reported in Table 1.6. The calibration of the parameters related to business-cycle theory is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the Government spending to GDP ratio to 20 percent, and the share of hours worked per day to 20 percent. The environmental component parameters of the models, when not estimated, are set in a similar fashion as Nordhaus [2008] and Heutel [2012]. We set the parameter φ_1 to match an average steady-state of pollution in laissez-faire equilibrium, which corresponds to the 2005 value of atmospheric carbon mass of 800 gigatons. The continuation rate of carbon in the atmosphere, denoted η, is set to match a roughly 139 years half time of atmospheric carbon dioxide, as in Nordhaus [1991].[12] Finally, for the abatement-cost function, we set $\theta_1 = 0.05607$ and $\theta_2 = 2.8$ as in Nordhaus [2008] and Heutel [2012].

For the remaining set of parameters and shocks, we employ Bayesian methods. Table 1.7 summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. Let us first discuss the prior for structural disturbances. The prior information on the persistence of the Markov processes and the standard deviation of innovations are taken from Guerrieri and Iacoviello [2017]. In particular, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks we choose an inverse gamma distribution with mean 0.01 and standard deviation of 1.

For the parameters which have key asset-pricing implications, we translate some bound restrictions from the matching moments exercise of Jermann [1998] into prior distributions. In particular, the elasticity of Tobin's Q to the investment-capital ratio is assumed to follow a Gamma distribution with prior mean of 4 and standard deviation of 1. The latter implies a support for ϵ close to the bound $\epsilon \in [0.16; +\infty]$ of Jermann [1998]. In addition, we set the capital intensity α to follow a Beta distribution with mean of 0.25 and standard deviation 0.02 in order to be close to the value estimated by Jermann [1998]. Note that we set a tight prior on this parameter in order to match the tight interval range of α that replicates the U.S. investment-to-output ratio. Jermann [1998] calibrates the risk aversion coefficient to 5 to be consistent with asset-pricing models. However, a high value for σ typically generates strong consumption-smoothing behavior in the Euler equation that is at odds with the data. Environmental economics typically favors values close to 2, while likelihood-estimated models usually find values below 2 (e.g. Smets and Wouters

[12]Let us assume that each unit of CO2 is subject to an idiosyncratic shock, denoted ω, and that the carbon is reused or sequestered in a carbon sink. This random variable is drawn from a binomial distribution, $\omega \sim B(n, p)$ with n the number of trials and p the probability of success $p = 1 - \tilde{\eta}$. We thus determine the number of trials, n, that are necessary on average for one unit of carbon to be sequestrated. Recall that $E(\omega) = n.p$, by imposing $E(\omega) = 1$ we calculate that the average number of trials necessary for carbon sequestration is $n = 1/(1 - \tilde{\eta})$. On an annual basis, the latter becomes $n = 0.25/(1 - \tilde{\eta})$. Recall that in the balanced growth path the effective continuation rate of carbon is $\tilde{\eta} = \eta\gamma_A\gamma_E^{1-\varphi_2}$. Then imposing an average half time of carbon of 139, we deduce the value of η as $\tilde{\eta} = (1 - 0.25/139)(\gamma_A\gamma_E^{1-\varphi_2})^{-1}$.

[2007]). To reconcile these three literatures, we propose to estimate this key parameter agnostically by imposing a rather diffuse information through a Gamma distribution with a prior mean of 2 and standard deviation of 0.35. This prior allows the parameter to be either high (i.e. close to 5), as in asset-pricing models, or lower (i.e. close to 2), as in the environmental models in Stern [2008] and Weitzman [2007], or low (i.e. equal to one), as in estimated business-cycle models. Unlike Jermann [1998], we cannot directly estimate $\beta\gamma_A^{-\sigma}$, because of weak identification when using full-information methods. We thus follow Smets and Wouters [2007] and estimate instead the term $(1/\beta-1)100$: this allows to easily impose prior information based on a Gamma distribution with a mean of 0.5 and standard deviation 0.25. The resulting prior allows the discount factor to roughly lie between 0.99 and 0.9980.[13]

Regarding the slopes of growth, we discuss first the productivity one (denoted $(\gamma_A - 1) \times 100$) that follows a Gamma distribution with a prior mean of 0.5 and a standard deviation of 0.04 in order to match the average 0.40 percent quarterly growth rate. For the (de)coupling rate (denoted $(\gamma_E - 1) \times 100$), we let the data be fully informative about the slope through a normal distribution with prior mean 0 and standard deviation 0.25. Finally, the last remaining parameter is the utility loss from cumulative CO2 emissions, ϕ. As in Campbell and Cochrane [1999], and given that we have several exogenous shocks, this parameter has to be restricted to ensure that surplus consumption always remains positive. This restriction ensures the non-negativity of the Lagrangian multiplier on the budget constraint (otherwise the budget constraint would not bind). We thus express this parameter in terms of steady-state consumption, $\phi\bar{c}/\bar{x}$, and impose an uninformative prior with an uniform distribution with mean 0.5 and standard deviation 0.285. This prior induces a bound restriction such that $\phi\bar{c}/\bar{x} \in [0;1]$, this is rather conservative as, unlike

[13]Note in addition that our prior mean for $(1/\beta-1)100$ is much higher than that in Smets and Wouters [2007] as our model is non-linear, and thus features the precautionary saving effect that drives down the real rate. With the prior information of Smets and Wouters [2007], we would obtain a real rate below zero; we thus re-adjust the prior information to render our non-linear model consistent with US real rate data.

Beta distributions, it does not favor any particular value within this interval.[14]

1.4.4 Posterior distributions

In addition to prior distributions, Table 1.7 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 50,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

The results of the posterior distributions for each estimated parameter are listed in Table 1.7 and Figure 1.2. It is clear from Figure 1.2 that the data were informative, as the shape of the posterior distributions is different from the priors. Our estimates of the structural parameters that are common with Smets and Wouters [2007] are mostly in line with those they find. The persistence of productivity and spending shocks are, for instance, very similar to theirs. The risk-aversion coefficient σ has a posterior mean of 4.2, which is lower than the value in Jermann [1998]. It is however higher than the values reported in environmental macroeconomic and estimated DSGE models: for example, Smets and Wouters [2007] find a value of 1.38 for this parameter. Another key parameter that determines the consumption surplus is $\phi\bar{c}/\bar{x}$. We find a value of 0.67 which is very close to that estimated by Smets and Wouters [2007] in the case of external consumption habits (0.71). The corresponding value of ϕ, given the steady state ratio $\bar{c}/\bar{x}$, is 0.0004. Regarding the growth rate of productivity, our estimated value, 0.34, is lower than that in Smets and Wouters [2007], but this is unsurprising as economic growth is lower in our sample given that we exclude the 1960s and include the last decade. Regarding the last estimated parameter common with Smets and Wouters [2007], the data suggest a value for capital intensity α close to 0.41, which is higher than the estimated values of Jermann [1998] and Smets and Wouters [2007]. This is important, as estimated DSGE models typically predict

[14] Note that with the bounds $\hat{\phi} = \phi\bar{c}/\bar{x} \in [0; 1)$, the MRS$=\bar{c} - \phi\bar{x} = \bar{c} - \hat{\phi}\bar{c}$, as in any standard model featuring external consumption habits.

very low values for α that are at odds with data on both the capital structure of firms and the investment-to-output ratio. Finally for the discount rate, denoted $100\left(\beta^{-1}-1\right)$, we find a posterior mean of 0.13 that generates a discount factor of 0.9987.

The last remaining parameters are not common with Smets and Wouters [2007]. For the elasticity of Tobin's Q to the investment capital ratio ϵ, we find a posterior mean of 1.44 that is higher than that in Jermann [1998]. The value of the elasticity of emissions to output, φ_2, is 0.36, which is remarkably close to that estimated by Heutel [2012]. Finally, for the decoupling rate we find that energy-saving technological change has caused reductions in CO2 of about 2% annually.

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	Data [5%;95%]	Model	Data [5%;95%]	Model
$100\times\Delta\log\left(y_t\right)$	[0.28;0.50]	0.34	[0.69;0.85]	0.81	[1.00;1.00]	1.00
$100\times\Delta\log\left(c_t\right)$	[0.36;0.55]	0.34	[0.60;0.74]	0.90	[0.54;0.76]	0.58
$100\times\Delta\log\left(i_t\right)$	[0.07;0.68]	0.34	[1.91;2.34]	2.58	[0.61;0.80]	0.72
$100\times\Delta\log\left(e_t\right)$	[-0.53;0.07]	-0.24	[1.88;2.31]	2.12	[-0.01;0.35]	0.24

TABLE 1.1
Data moments vs. model moments (with parameters taken at their posterior means)

To assess the relevance of the estimated model, as in Jermann [1998], we compare the observable moments taken at a 90 percent interval versus the asymptotic moments generated by the model using a second-order approximation to the policy function. Table 1.1 reports the results. We find that our model does a reasonably good job at replicating some salient features of the data, as most of the moments simulated by the estimated model fall within the 95 percent confidence interval of the data.

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables that we introduce. Once the shock process parameters have been estimated, it is possible to simulate the model by drawing shocks from the estimated distribution. As illustrated in Table 1.1, however, this procedure does not ensure that the unconditional standard deviations observed in the data are matched perfectly.

Letting $u(c_t-\mathcal{C}_t)$ denote the utility function, with $\mathcal{C}_t$ the reference variable to calculate

	Standard Cons. habits	Pollution externality
Utility function $u(c_t - \mathcal{C}_t)$	$\mathcal{C}_t = \phi c_{t-1}$	$\mathcal{C}_t = \phi x_t$
Surplus parameter ϕ	0.21	0.67
Prior probability	0.50	0.50
Log marginal data density	2004.77	2045.99
Bayes ratio	1.0000000000	8.02e17
Posterior model probability	0.0000000000	1.0000000000

TABLE 1.2
The comparison of prior and posterior model probabilities in the internal consumption habits and the environmental preferences models (with parameters taken at their posterior mode).

the surplus consumption ratio, a natural question at this stage is how relevant is our specification of environmental preferences with respect to a standard consumption habits model *à la* Jermann [1998]. Using an uninformative prior distribution over models (i.e. 50% prior probability for each model), Table 1.2 shows both the posterior odds ratios and model probabilities taking the consumption habits model $\mathcal{M}$ ($\mathcal{C}_t = \phi c_{t-1}$) as the benchmark model. We examine the hypothesis H_0: $\mathcal{C}_t = \phi c_{t-1}$ against the hypothesis H_1: $\mathcal{C}_t = \phi x_t$. The posterior odds of the null hypothesis of surplus based on lagged consumption is 8e17: 1, which leads us to strongly reject the null. The surplus consumption ratio is therefore more relevant when it is based on the stock of emissions rather than past consumption. This result should however be qualified, as prior distributions were selected here to estimate our model and do not necessarily fit the benchmark model of H_0. This can diminish the empirical performance of the benchmark. The goal of this exercise is not to show that one model outperforms another, but to highlight that our model is least as consistent with the data as the standard habits-type model.

1.5 Results

Our main simulation results appear in Table 1.3 below. The top panel of this table shows the average level of consumption and the stock of CO2 emissions, which are denoted by $E(c_t)$ and $E(x_t)$, respectively. The agent's lifetime utility, $E(\mathcal{W}_t)$, is our measure of welfare. The average tax chosen by the social planner is $E(\tau_t)$.

The asset-pricing implications appear in the middle panel, where $400E(r_t^F)$, $400E\left(r_{t+1}^B - r_t^F\right)$ and $std(\hat{\lambda}_t)$ are the mean real risk-free rate, the mean bond premium, expressed in annualized percent, and the standard deviation of marginal utility respectively. The average coefficient of relative risk aversion is $E(RRA_t)$ and $std(\widehat{rra}_t)$ is a measure of its standard deviation (expressed in log-deviations from the steady state).

The bottom panel of Table 1.3 first lists the share of emissions that firms choose to abate, $E(\mu_t)$. The average cost of abatement is $E(f(\mu_t))$, and $E(\tau_t e_t/y_t)$ is the average cost of the tax borne by firms as a share of GDP.

The first column shows these model implications in the decentralized *laissez-faire* equilibrium with a tax set to zero. Columns (2) to (4) show what happens once the optimal tax is introduced. The optimal-policy results are listed for three different values of the parameter θ_1. This latter measures the efficiency of the abatement technology, with higher θ_1 corresponding to a less-efficient technology. As $\theta_1 = 0.05607$ is the value used in the literature (e.g. Nordhaus [2008]; Heutel [2012]), the results in column (2) correspond to our baseline scenario.

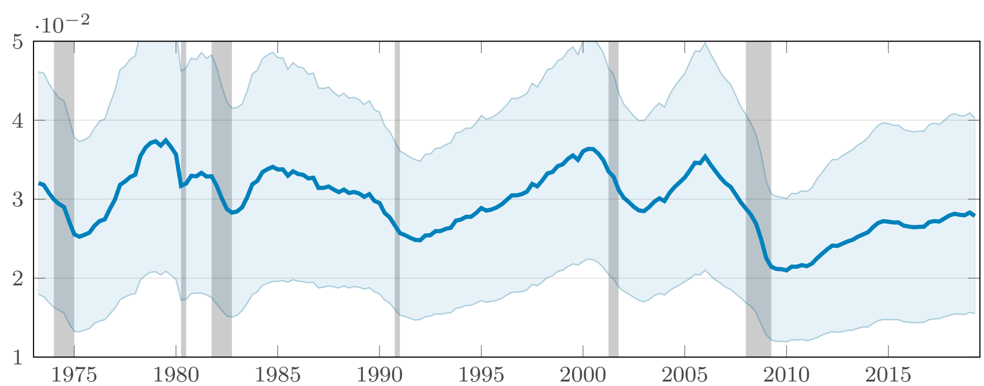
1.5.1 The size and the cyclicality of the optimal tax

The first main takeaway from Table 1.3 is that a small average carbon tax is sufficient to restore the first-best allocation. In our benchmark scenario, which corresponds to $\theta_1 = 0.05607$, the total tax bill is on average around two percent of GDP ($E(\frac{\tau_t e_t}{y_t}) = 0.02$).

As can be seen by comparing the total tax bill across columns 2 to 4, in the worst-case scenario, corresponding to a value for θ_1 implying a very-inefficient abatement technology, the total tax bill rises to 5.7 percent of GDP. In this adverse scenario, firms only manage to abate about 6 percent of all emissions, $E(\mu_t) = 0.0592$, once the tax is introduced.

One advantage of our method is that it can be used to construct counterfactual scenarios. In particular, we can answer the following question: What would the optimal tax τ_t have been in the United States from 1973 to 2018, had this optimal policy been implemented? Figure 1.1 provides the answer. The optimal tax is time-varying, and rises in booms and falls during recessions. The optimal tax is thus strongly pro-cyclical, as illustrated by Figure 1.3, so that the tax bill $\tau_t e_t / y_t$ falls during major recessions, like the global financial crisis.

The optimal tax is pro-cyclical because the externality induces excessive fluctuations in risk aversion. As in a model with external habits and time-varying risk aversion (e.g. Campbell and Cochrane [1999]), agents take the externality as given. As the optimal tax reproduces the first-best allocation, it eliminates this inefficiency by making firms internalize the effect of their production on consumers. Our analysis therefore provides a novel interpretation of the result in Ljungqvist and Uhlig [2000] for the case of habits. As shown in Table 1.2, one motivation for our approach is that our specification is strongly supported by the data, especially relative to habits.



Notes: The simulated path is expressed in levels. The blue shaded area is the parametric uncertainty at 95% confidence level, drawn from 1,000 Metropolis-Hastings random iterations. The blue line represents the mean of these 1,000 simulated paths. The gray shaded areas are NBER-dated recessions in the US.

FIGURE 1.1. Historical variations in the environmental tax

It is important to note that the fluctuations in risk aversion are essentially driven by consumption, not by the externality. In line with the evidence, we assume that the stock of CO2 depreciates very slowly over time. Whereas the flow of emissions can be volatile, the stock of emissions, and hence the externality, moves only very slowly over the business cycle.

1.5.2 The risk premium and the risk-free rate in the laissez-faire equilibrium

As can be seen in column (1), the model generates an average bond premium, i.e. $400E\left(r_{t+1}^B - r_t^F\right)$, of about 1.3 percent. Although small, generating a bond premium of this magnitude remains a challenge for a large class of General-Equilibrium models with production. In our case, this relative success is due to our preference specification, which generates time-variation in risk aversion, as in Campbell and Cochrane [1999].

As in Jermann [1998], the positive bond premium that we obtain is due to interest-rate risk. The price of long-term bonds is determined by the term structure of interest rates. The key is that in this model short- and long-term interest rates are counter-cyclical. With interest rates rising during recessions, bond holders can expect capital losses to occur precisely during periods of low consumption and high marginal utility. Long-term bonds are therefore not good hedges against consumption risk. The positive bond premium is thus a compensation for holding an asset whose price declines during periods of low consumption.

In this model, the mean risk-free rate $400E(r_t^F)$ is critically affected by uncertainty. As in Jermann [1998], a greater variance in marginal utility reduces the unconditional mean risk-free rate. The intuition is that a higher volatility of marginal utility implies more uncertainty about future valuations, and greater uncertainty in turn increases agents' willingness to build precautionary buffers. This effect therefore captures the impact of this precautionary motive on equilibrium interest rates.

1.5.3 Asset prices under the optimal policy

Relative to the *laissez-faire* equilibrium, the optimal tax has a sizeable effect on the mean risk-free rate. In the baseline scenario, under optimal taxation, our model predicts a rise in the average risk-free rate of around 2 percent. This effect on the risk-free rate can be better understood by comparing the volatility of marginal utility $std(\hat{\lambda}_t)$ in the two cases. One main effect of the tax is to reduce the volatility of marginal utility. Fluctuations in marginal utility provide a measure of uncertainty about future valuations. The lower volatility therefore reflects that agents face less uncertainty after the introduction of the tax. The higher mean risk-free rate can therefore be interpreted as reducing agents' precautionary saving motives.

The second effect of the tax is to reduce the risk premium. This can be explained by the effect of the tax on risk aversion. The carbon tax reduces both consumption and the stock of emissions, with the reduction in the latter being larger. The distance between consumption and the externality therefore rises. In this model, a larger gap between consumption and the externality in turn reduces risk aversion.

In contrast to an endowment economy, in our production economy lower risk aversion affects the dynamics of consumption as it implies a higher elasticity of intertemporal substitution (EIS). In other words, agents' consumption-smoothing motives are reduced under the optimal policy. This willingness to tolerate larger fluctuations in consumption has in turn asset-pricing implications. As agents are less reluctant to reduce consumption during recessions, there is less need to insure against such outcomes. Consequently, the premium required to compensate investors for holding an asset the price of which falls in recessions is also lower.

1.5.4 Welfare analysis

To assess the welfare implications of the optimal policy, Table 3 also shows agents' lifetime utility $E\left(\mathcal{W}_t\right)$, where:

$$E(\mathcal{W}_t) = E_0 \left\{ \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma} \right\}$$

As can be seen by comparing the value of $E(\mathcal{W}_t)$ across columns (1) and (2), the policy generates a sizeable rise in welfare. This welfare gain illustrates that the fall in the stock of emissions $E(x_t)$ more than compensates for the lower average consumption the tax produces. This result highlights the importance of the elasticity of emissions to a change in the tax. As this elasticity depends on firms' willingness to reduce emissions, we now discuss the role of the abatement technology.

	Laissez-faire	Optimal policy		
	Estimation (1972-2019)	$\theta_1 = 0.05607$	$\theta_1 = 0.28844$	$\theta_1 = 3.4996$
	(1)	(2)	(3)	(4)
	Business-cycle variables			
$E(c_t)$	0.5502	0.5206	0.5310	0.5409
$E(x_t)$	848.9287	380.1978	632.2172	777.2627
$E(\mathcal{W}_t)$	-206778.4449	-10649.9577	-43607.6821	-124258.5811
$E(\tau_t)$	0.0000	0.0353	0.0390	0.0433
$std(\tau_t)$	0.0000	0.0063	0.0083	0.0101
	Asset-pricing implications			
$400E\left(r_t^F\right)$	3.5870	5.4100	4.7417	4.0028
$400E\left(r_{t+1}^B - r_t^F\right)$	1.1542	0.6432	0.9176	1.1432
$std(\hat{\lambda}_t)$	2.4445	1.2753	1.7525	2.1893
$E(RRA_t)$	32.1922	12.9862	19.6120	27.1408
$std(\widehat{rra}_t)$	0.5837	0.3045	0.4185	0.5228
	Abatement technology			
$E(\mu_t)$	0.0000	0.5269	0.2234	0.0592
$E(f(\mu_t))$	0.0000	0.0094	0.0044	0.0013
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0233	0.0423	0.0566

Notes: The first column is the estimated model under the laissez-faire equilibrium, with no abatement and no environmental tax. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match an abatement share $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4), due to the contribution of future shocks to the asymptotic mean of these variables.

TABLE 1.3

The model simulation results

1.5.5 The role of the abatement technology

The purpose of columns (3) and (4) is to illustrate that the effect of the optimal tax critically depends on the efficiency of the abatement technology. In the laissez-faire equilibrium, the externality not being internalized leads firms to spend nothing on abatement. By forcing firms to internalize the externality, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax.

In our preferred scenario, about 55 percent of emissions are abated once the optimal tax is introduced. As shown in the bottom panel of Table 1.3, when θ_1 is above 0.056, less-efficient technology reduces the share of emissions abated $E(\mu_t)$. Note that as abatement-technology efficiency declines, the planner also chooses to allocate a larger fraction of resources to consumption. This reflects that this model embeds a trade-off between consumption and the abatement technology. The marginal cost of renouncing a unit of consumption should equal the marginal benefit from abating one unit of emissions. Consequently, the planner finds it optimal to allocate more resources to consumption as abatement-technology efficiency falls.

As can be seen by comparing $E(\mathcal{W}_t)$ across columns (2) to (4), the size of the welfare gain depends critically on the abatement technology. This illustrates that the distortion caused by the tax can be sizeable if the technology is not sufficiently well-developed. If emissions are costly to abate, the policy has a stronger negative impact on production, as it is more difficult for firms to circumvent the tax. In this case, the tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

Comparing the effect of the optimal tax on $400E\left(r_t^F\right)$ and $400E\left(r_{t+1}^B - r_t^F\right)$, the effect on asset prices also depends crucially on θ_1. Relative to the first-best scenario, the effect of the tax on the risk premium is more muted when the abatement technology is less efficient.

This illustrates that part of the reduction in uncertainty is due to the additional margin provided by the abatement technology. The effect of θ_1 is therefore akin to the adjustment-cost parameter in Jermann [1998]. The more efficient is the abatement technology, the easier it is for agents to insure against unexpected shocks. This greater flexibility makes the economy less risky from a consumption-smoothing perspective, which reduces the risk premium and increases the risk-free rate.

1.5.6 The coefficient of relative risk aversion

Table 1.3 also lists the average level of risk aversion, where risk aversion is defined as follows:

$$RRA_t = -\frac{u''_c}{u'_c} c_t$$

In the *laissez-faire* equilibrium, this average level is 32. Once the tax is introduced, this falls to around 13. The main effect of the tax is then to increase the distance between consumption and the externality. As in Campbell and Cochrane [1999], risk aversion in our model is determined by "surplus consumption". A greater distance between consumption and the externality therefore implies a lower coefficient of relative risk aversion.

1.5.7 Climate policy and asset prices with standard preferences

In many models, the EIS mainly affects quantities, whereas asset-pricing implications are driven by risk aversion (e.g. Cochrane [2017]; Tallarini [2000]). In contrast, the financial and macroeconomic implications of our model are tightly linked. The specification with CCN preferences creates this interaction between finance and the environmental policy. This point is illustrated in Table 1.8, which repeats the experiment shown in Table 1.3 using a separable specification. We analyze the effect of the optimal policy in a model in which preferences are as follows:

$$\mathcal{W}_t = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\log c_t - \phi \frac{x_t^{\chi}}{\chi} \right)$$

where, following Stokey [1998], χ is set to 1.2. To ensure comparability, the parameter ϕ is calibrated to imply an optimal tax similar to that obtained in the case of CCN preferences.

With constant relative risk-aversion, the model is no longer able to generate a realistic risk premium in the *laissez-faire* equilibrium. Relative to the case of CCN preferences, the risk premium falls from about 1.2 percent to essentially 0. In this case, the dichotomy between climate policies and finance is also close to perfect. Indeed, as illustrated in Table 1.8 the introduction of the optimal tax essentially has no effect on the risk-free rate and risk premium. In a model in which risk plays no role, one may therefore be tempted to conclude that climate risk and environmental policies have a negligible effect on financial markets.

Since we use a log utility specification for consumption, we also tried to increase the curvature coefficient from 1 to 20. We find that increasing curvature has a negligible impact on the risk premium but generates a very large increase in the mean risk-free rate. With a high curvature coefficient, the optimal policy also has no effect on the model's asset-pricing implications. Therefore, the dichotomy between climate policies and finance cannot be broken by a very high value of the curvature coefficient.

1.5.8 The responses to shocks

Figure 1.4 compares the response of consumption c, abatement μ, emissions e and the optimal tax τ following a positive technology shock. As can be seen by comparing the red crosses to the green circles in the upper-left panel, the first key difference is that the response of consumption on impact is stronger under the optimal policy. This can be explained by the lower EIS. In models with habits, relative risk aversion and the EIS are connected. As the tax reduces risk aversion, it also increases the EIS.

As illustrated in the upper-right panel of Figure 1.4, the second key difference is that the quantity of emissions that firms choose to abate increases sharply during boom periods. Once the optimal policy is introduced, firms therefore find it optimal to use the abatement technology to reduce the burden of the tax.

The lower left panel of Figure 1.4 shows that the pro-cyclical response of the abatement technology implies lower emissions under the optimal policy. In contrast to the *laissez-faire* equilibrium, emissions therefore become counter-cyclical once the optimal tax is introduced.

Finally, the lower-right panel of Figure 1.4 depicts the response of the optimal tax, which is constant and equal to zero in the *laissez-faire* equilibrium. As in Ljungqvist and Uhlig [2000], the optimal tax is pro-cyclical when the economy is hit by a technology shock. Relative to the decentralized equilibrium, the planner therefore chooses to cool down the economy during booms.

The response to an investment-specific technology shock is shown in Figure 1.5. This shock generates a negative co-movement between consumption and investment. Relative to the *laissez-faire equilibrium*, the optimal policy attenuates the fall in investment by reducing the tax as well as abatement. Introducing this shock reduces the volatility of investment, which in turn explains the lower value of the adjustment-cost parameter that we find compared to Jermann [1998].

The response to a government spending shock is shown in Figure 1.6. In both cases, a positive government-spending shock reduces consumption. In our model, this can first be explained by the negative wealth effect from the shock. On impact, the shock has no effect on production, but increases the share of output allocated to government spending. On impact, consumption and investment therefore have to fall.

This negative wealth effect is reinforced by a negative substitution effect. As in models with habits and adjustment costs, this reflects the increase in the real interest rate generated by the shock. As agents become more reluctant to save as consumption falls, the real interest rate has to rise to restore equilibrium.

This illustrates the trade-off between environmental protection and macroeconomic stabilization in this model. Whereas emissions decline in the *laissez-faire* case, the social planner chooses to increase the stock of pollution. The social planner internalizes that the shock reduces the resources available for consumption. It is therefore optimal to mitigate the effect of the shock by lowering abatement as well as the tax (see the upper-right and lower-right panels of Figure 1.6). When the consumption cost is too large, environmental policy is used to mitigate the adverse effect of the shock. In this case, the planner chooses macroeconomic stabilization over environmental protection.

Relative to a standard business-cycle model, the main innovation is the introduction of emission shocks. In the *laissez-faire* equilibrium, consumption falls on impact and then increases above its steady-state level (see the upper-left panel of Figure 1.7). As emission shocks do not affect output, their main effect is to reduce "surplus consumption". The only way to mitigate the effect of this rise in the emissions stock is then to increase consumption. The problem is that to do so income has to rise first. The only way of raising income in this model is to accumulate capital. This explains why on impact consumption needs to fall. This fall is necessary to finance an increase in investment, which in turn allows agents to increase output. A few quarters after the shock, as the higher investment raises output, consumption gradually increases. The short-term decline in consumption is therefore compensated by a rise in the medium-term. As illustrated by the red dotted line in the upper-left panel of Figure 1.7, consumption initially declines and then increases above its steady state a few periods after the shock.

As can be seen by comparing the red-dotted and green-circled line, the response of consumption and emissions is very different under the optimal policy. The planner chooses to allocate a large fraction of resources to the abatement technology. It is therefore optimal to reduce consumption and investment to finance abatement to prevent emissions from rising.

As illustrated in the lower-right panel, the social planner also chooses to reduce the tax. The tax reduction helps to mitigate the fall in consumption and investment that is necessary to finance abatement.

1.6 Robustness checks

This section discusses two robustness checks. First, asset-pricing models are not only evaluated in terms of their ability to match asset market facts. Reproducing the volatility of macroeconomic aggregates, such as consumption, is also an important test for this class of models. Second, since we use a solution method that is relatively novel, we compare it to other nonlinear methods that are more widely-used in the literature.

1.6.1 The volatility of consumption

As discussed in subsection 1.4.4, the model overstates the volatility of consumption when simulated. Using consumption as an observable variable ensures that the model can perfectly reproduce the historical path of consumption growth over the estimation period. However, when simulated using the estimated values for the shock parameters, and as shown in Table 1.1, we obtain that consumption is more volatile than output, which does not fit the facts. This naturally raises the concern that our model's ability to generate realistic asset-pricing facts comes at the cost of implausibly-large fluctuations in consumption growth.

This section shows that this counterfactual implication does not affect the main message of the paper. To illustrate, we consider a simplified version of the model in Section 2 in which technology shocks are the only source of business-cycle fluctuations and where all variables grow at the same rate. Then, following the analysis in Jermann [1998], we calibrate the main model parameters to maximize its ability to match a set of moments that includes the volatility of consumption.

To ease the comparison with Jermann [1998], we target the same stylized facts, with one exception, and calibrate a similar set of parameters using the simulated method of moments. In our case, the five parameters are: (i) the adjustment-cost parameter, ϵ; (ii) the marginal-damage parameter, ϕ; (iii) the subjective discount factor, β; (iv) the technology-shock standard deviation, σ_A; and (v) the shock-persistence parameter, ρ_A. The first four moments to match are the standard deviations of output, consumption and investment, and the mean risk-free rate. Since the model in Jermann [1998] tends to generate excessive risk-free rate variations, we target a risk-free rate standard deviation of 5 percent instead of a 6.18 percent risk premium. The loss function is minimized for the following combination of parameter values:

ϵ	ϕ	β^Y	σ_A	ρ_A
0.36	0.0028	0.993	0.01	0.96

All other parameter values are kept at their estimated values. The moments corresponding to the laissez-faire economy appear in the first column of Table 1.4. Compared to Jermann [1998], the model generates a lower risk-free rate standard deviation and is still able to reproduce the low mean risk-free rate as well as the volatility of macroeconomic aggregates. As regards the moments that were not targeted, shown in the last two rows of Table 1.4, the model generates a bond premium of 3.4 percent. As the carbon tax is zero in the laissez-faire economy, the abatement chosen by firms is constant at a value of zero.

The second column of Table 1.4 lists the simulated moments when the optimal tax is introduced. As in the previous section, we first consider a scenario in which firms are able to abate around 50 percent of all emissions under the tax. The moments in this scenario appear under the column $\mu = 0.5$. Comparing the laissez-faire economy to the optimal-tax case, the risk-free rate rises by about one percentage point, and the risk premium falls under the optimal policy. The effect on the risk premium is particularly large, as the tax generates a fall of about 2.4 percentage points. Moreover, relative to the analysis from the previous section, this sizeable effect is obtained in a model with one single source of shocks. To sum up, the optimal tax also has sizeable asset-pricing implications in a version of the model that reproduces the fact that consumption is half as volatile as output.

The second main takeaway is that this robustness analysis confirms the importance of the abatement technology in our results. If firms can only abate 10 percent of emissions

	Laissez-faire Economy	Optimal Policy	
	$\mu = 0.0$	$\mu = 0.5$	$\mu = 0.1$
$std(y)$	1.0	1.0	1.0
$std(c)$	0.5	0.3	0.8
$std(i)$	2.5	0.7	1.5
$400E(r_t^F)$	0.9	1.9	0.8
$400std(r_t^F)$	5.2	2.3	4.8
Non-targeted moments			
$400E(r_{t+1}^B - r_t^F)$	3.4	1.0	3.2
$E(\mu)$	0	0.5	0.1

TABLE 1.4
Laissez-faire vs. Optimal Policy

following the tax, the effect on the risk-free rate and the bond premium becomes negligible. This scenario corresponds to the case with an inefficient abatement technology.

This result also confirms that the asset-pricing effect that we obtain depends critically on the additional margin that is activated by the optimal policy. Once the optimal environmental tax is introduced, the abatement technology is used to reduce the amount of consumption risk in the economy. If sufficiently flexible, this margin helps agents to smooth consumption, which in this class of models not only reduces precautionary savings but also the compensation for holding a risky asset such as a long-term bond.

1.6.2 Comparison with the particle filter

In this section, we investigate whether our results continue to hold with alternative filtering methods other than the inversion filter. In the asset-pricing literature, the natural benchmark for non-linear models is particle filtering, as the latter allows likelihood-based inference of nonlinear and/or non-normal macroeconomic models (e.g. van Binsbergen et al. [2012]; Andreasen [2012]). The inversion and particle filters are algorithms that recursively update and estimate the state and find the innovations driving a stochastic process given a set of observations.

The inversion filter does so by inverting the model's recursion rule, while the particle filter uses a sequential Monte Carlo method. Both estimation methods require the use of numerical approximation techniques that introduce error between the "true" value of the parameter and its estimate.

In the implementation of the particle filter, it is common to posit that the data-generating process (DGP) includes measurement errors. As underlined by Cuba-Borda et al. [2019], the presence of measurement error may seem to be an innocuous way of getting around degeneracy issues when choosing a computationally-manageable number of particles. As the number of innovations must be the same as the number of observables, the inversion filter may exhibit misspecification errors if measurement errors are part of the DGP. It is nonetheless standard to assume no measurement errors for linearized models, following Smets and Wouters [2007].

Sample:	Historical Data		Artificial Data
Filter:	(1) Particle	(2) Inversion	(3) Inversion
Estimated Parameters			
Productivity AR(1)	0.9714 *[0.9459;0.9851]*	0.9727	0.9632
Productivity std	0.0074 *[0.0067;0.0080]*	0.0076	0.0075
Premium			
Premium laissez-faire	0.7500 *[0.6230;0.9118]*	0.8412	0.7867
Premium tax policy	0.3516 *[0.2851;0.4232]*	0.3774	0.3759

Notes: 25,000 iterations of the random-walk Metropolis-Hastings algorithm are drawn for the posterior uncertainty for each model. The maximization of the mode is carried out via simplex optimization routines. The confidence intervals in column(1) are drawn from the posterior uncertainty from 1,000 draws from the Metropolis-Hastings algorithm. The artificial data in column (1) are obtained from 1,000 simulations of the estimated model with the particle-filtering method.

TABLE 1.5
Outcomes from the particle vs. inversion filters under historical and simulated data

To gauge how much our results are robust to misspecification errors, we estimate our model solved up to the second order with innovations to productivity estimated with output growth as an observable variable. We limit ourselves to productivity shocks as these are the main driver of the risk premium. The rest of the parameters are set to the posterior mean taken from the previous estimation in Table 1.7. We consider three situations: (1) the particle filter algorithm as described in Fernández-Villaverde and Rubio-Ramírez [2007] estimated on US data;[15] (2) the inversion filter estimated on US data; and (3) the inversion filter estimated on 1,000 simulated output-growth data from the particle filter from column (1) that includes measurements error. The latter allows us to see whether measurement errors affect the inference of structural parameters when using the inversion filter. Table 5 shows the results.

The comparison of columns (1) and (2) shows whether the inversion filter and particle filter outcomes differ. The two filters provide a very similar measure of the likelihood function, as the differences in the inference of structural parameters are only minor. In particular, the outcome from the inversion filter always lies in the confidence interval of that from the particle filter, both for the estimated structural parameters and the premium effects. The fact that the lower risk premium from environmental policy is very similar across estimation methods is also reassuring, and suggests that our results may remain similar under alternative filtering methods.

To make sure that the robustness of our results to measurement errors holds unconditionally in larger samples, we follow Fernández-Villaverde and Rubio-Ramírez [2005] and simulate 1,000 output-growth data from the model in column (1). We estimate the model on this artificial data using the inversion filter and list the outcomes in column (3). The inversion filter infers a value that is close to the true parameter values, despite the presence of measurement errors.

[15] We use 10,000 particles to approximate the likelihood, and set the variance of the measurement errors to 10% of the sample variance of the observables to help estimation. These values are very standard in the literature.

1.7 Conclusion

Drawing from the macroeconomic, financial, and environmental literatures, this paper introduces an environmental externality into the neoclassical growth model. Our first main takeaway is that the optimal carbon tax is determined by the implicit price of CO2 emissions. We then show how to use asset-pricing theory to estimate the optimal carbon tax over the business cycle.

In our economy, risk aversion is higher when firms do not internalize the damage caused by emissions. We show that this higher risk aversion in turn raises risk premia and lowers the natural rate of interest by increasing precautionary saving. In the *laissez-faire* equilibrium, the key is that a fraction of these variations in risk aversion are excessive. The optimal policy therefore eliminates inefficient fluctuations in risk aversion.

The main policy implication is that the effectiveness of the policy critically depends on the abatement technology, so that policy success may depend on the timing of implementation. Clearly, improving the existing emission-abatement technology should come first. Once available, an efficient technology would help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

As our study focuses primarily on tax policy, future research could investigate how a permits market could affect asset prices and welfare, either by considering the case of asymmetric information,[16] or by developing a framework where both households and firms are affected by the externality. This type of framework would allow for multi-policy evaluation, such as the comparison of tax and cap-and-trade policies.

Another important limitation of our analysis is that the deterministic growth rate of the economy is given exogenously. On the contrary, abatement choice is endogenously determined, and as we are primarily interested in the cyclicality of the carbon tax, our analysis focuses on business-cycle frequency. Addressing this question in a unified framework in which long-term growth and business cycle fluctuations can be jointly analyzed

[16] Asymmetric information breaks the equivalence between the tax and the permit policy (Heutel [2012]).

would be a major step forward.

We also restrict our analysis to the case of fiscal policy, and do not study the interaction between the carbon tax and other policy instruments. Understanding how the optimal carbon tax will affect the conduct of monetary and macro-prudential policies is another important avenue for further research (e.g. Benmir and Roman [2020]).

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Appendices

1.A Appendix - A: tables

Model counterpart	Name	Values
$\bar{N}$	Labor supply	0.20
δ_K	Depreciation rate of capital	0.025
$\bar{g}/\bar{y}$	Public spending share in output	0.20
$\bar{x}$	Atmospheric carbon (gigatons) in laissez-faire	800
$[4(1-\gamma_A\gamma_E^{1-\varphi_2}\eta)]^{-1}$	Half-life of CO2 in years	139
θ_1	Abatement cost	0.05607
θ_2	Curvature abattement cost	2.8

TABLE 1.6
Calibrated parameter values (Quarterly basis)

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes:					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.01	1	0.008 *[0.007;0.009]*
Std. spending	σ_G	$\mathcal{IG}_1$	0.01	1	0.035 *[0.032;0.039]*
Std. abatement	σ_X	$\mathcal{IG}_1$	0.01	1	0.020 *[0.018;0.022]*
Std. investment	σ_I	$\mathcal{IG}_1$	0.01	1	0.014 *[0.012;0.016]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.944 *[0.930;0.955]*
AR(1) spending	ρ_G	$\mathcal{B}$	0.50	0.20	0.953 *[0.932;0.967]*
AR(1) abatement	ρ_X	$\mathcal{B}$	0.50	0.20	0.896 *[0.828;0.947]*
AR(1) investment	ρ_I	$\mathcal{B}$	0.50	0.20	0.998 *[0.998;0.999]*
Structural parameters:					
Productivity growth rate	$(\gamma_A - 1) \times 100$	$\mathcal{G}$	0.50	0.04	0.340 *[0.301;0.387]*
Output-CO2 (de)coupling rate	$(\gamma_E - 1) \times 100$	$\mathcal{N}$	0	0.25	-0.45 *[-0.538;-0.346]*
Discount rate	$(\beta^{-1} - 1) \times 100$	$\mathcal{G}$	0.50	0.25	0.139 *[0.051;0.343]*
Capital intensity	α	$\mathcal{B}$	0.25	0.02	0.412 *[0.374;0.453]*
Capital-cost elasticity	ϵ	$\mathcal{G}$	4	1	1.448 *[1.029;2.038]*
Utility loss on emissions	$\phi \times \bar{c}/\bar{x}$	$\mathcal{U}$	0.50	0.285	0.677 *[0.611;0.730]*
Relative risk aversion	σ	$\mathcal{G}$	2.00	0.35	4.198 *[3.681;4.740]*
Output-CO2 elasticity	φ_2	$\mathcal{B}$	0.50	0.20	0.367 *[0.138;0.633]*
Log-marginal data density					-2124.0769

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{U}$ the uniform distribution.

TABLE 1.7
Prior and Posterior distributions of structural parameters

The log utiliy case ($u = \log c_t - \phi \frac{x_t^\chi}{\chi}$)

	Laissez-faire	Optimal policy $\theta_1 = 0.05607$	$\theta_1 = 0.48164$	$\theta_1 = 6.4039$
	(1)	(2)	(3)	(4)
	Business-cycle variables			
$E(c_t)$	0.5274	0.5136	0.5178	0.5210
$E(x_t)$	804.3029	348.5493	629.2131	745.8746
$E(\mathcal{W}_t)$	-1102.7147	-673.4293	-921.8315	-1043.4189
$E(\tau_t)$	0.0000	0.0389	0.0530	0.0581
	Asset-pricing implications			
$400E\left(r_t^F\right)$	6.2415	6.2430	6.2425	6.2421
$400E\left(r_{t+1}^B - r_t^F\right)$	0.0715	0.0696	0.0704	0.0709
$std(\hat{\lambda}_t)$	0.2194	0.2135	0.2130	0.2134
$E(RRA_t)$	1.0000	1.0000	1.0000	1.0000
$std(\widehat{rra}_t)$	0.0000	0.0000	0.0000	0.0000
	Abatement technology			
$E(\mu_t)$	0.0000	0.5563	0.1999	0.0500
$E(f(\mu_t))$	0.0000	0.0111	0.0055	0.0015
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0239	0.0597	0.0781

Notes: The first column shows the results in the laissez-faire (counter-factual) equilibrium, where we use the estimated values obtained for non-separable utility. We calibrate $\phi = 5.7105e-05$ in order to match the optimal tax obtained in the case of non-separable utility. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match abatement shares of $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4) due to the contribution of future shocks to the asymptotic mean of these variables.

TABLE 1.8

Counter factual robustness check – The case of separable utility.

1.B Appendix - B: figures

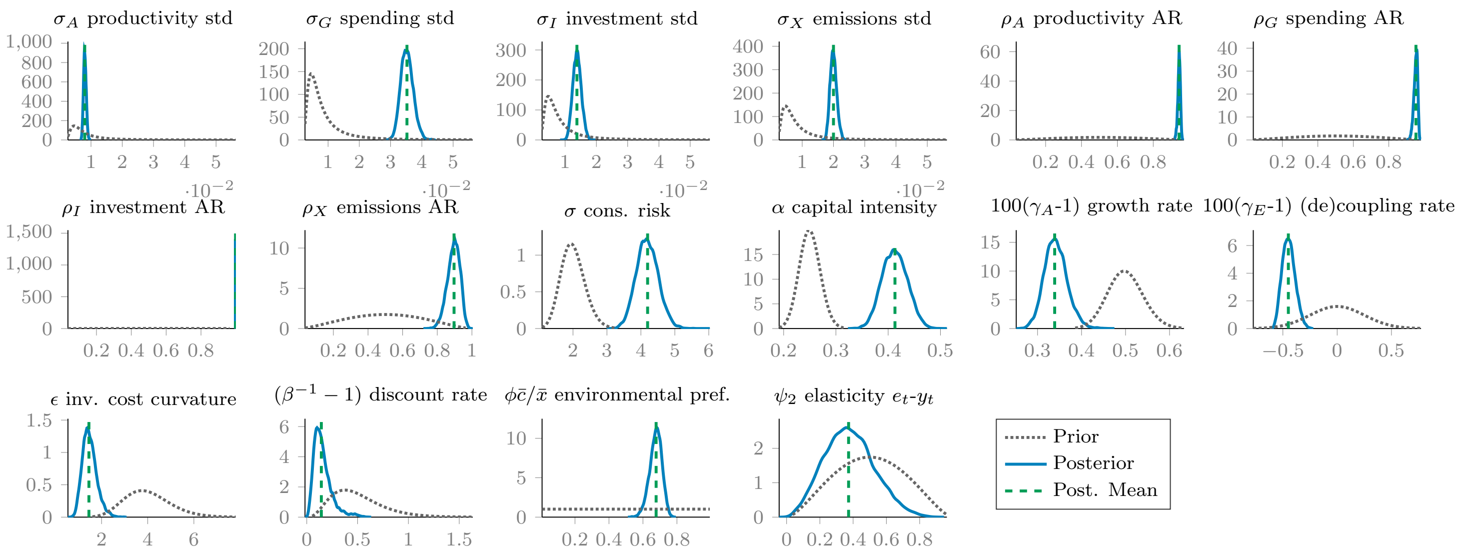
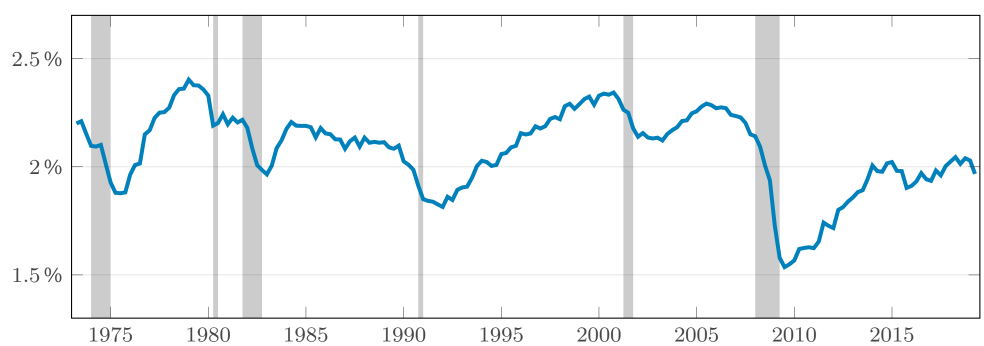
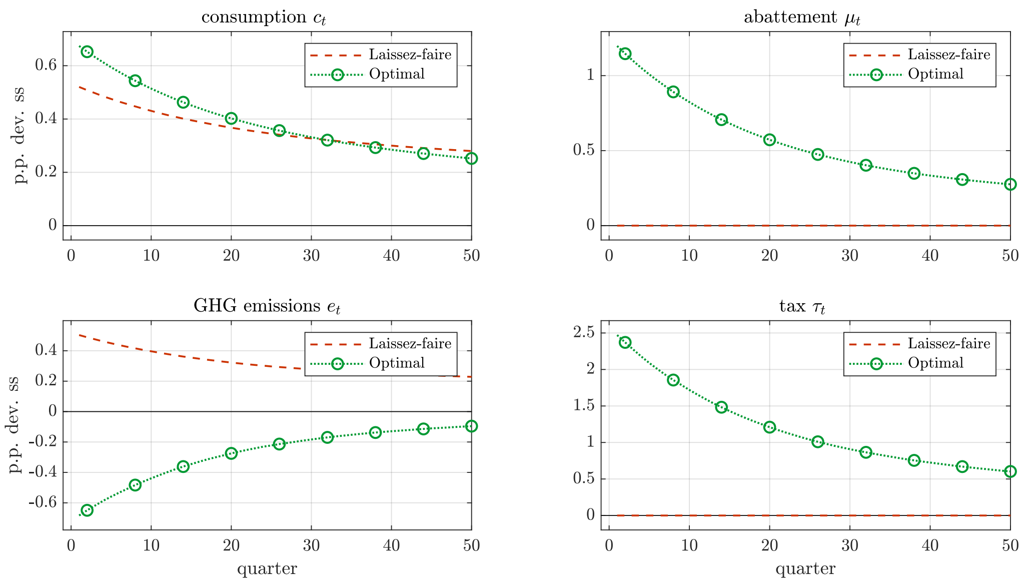


FIGURE 1.2. Prior and posterior distributions of the estimated parameters



Notes: The simulated path is expressed in levels. The blue line represents the mean of 1,000 simulated paths of Metropolis-Hastings random iterations. The gray shaded areas are NBER-dated recessions in the US.

FIGURE 1.3. Historical variations in the tax bill in % of GDP, $\tau_t e_t / y_t$



Notes: The IRFs are generated using a second-order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

FIGURE 1.4. Impulse responses from an estimated TFP shock

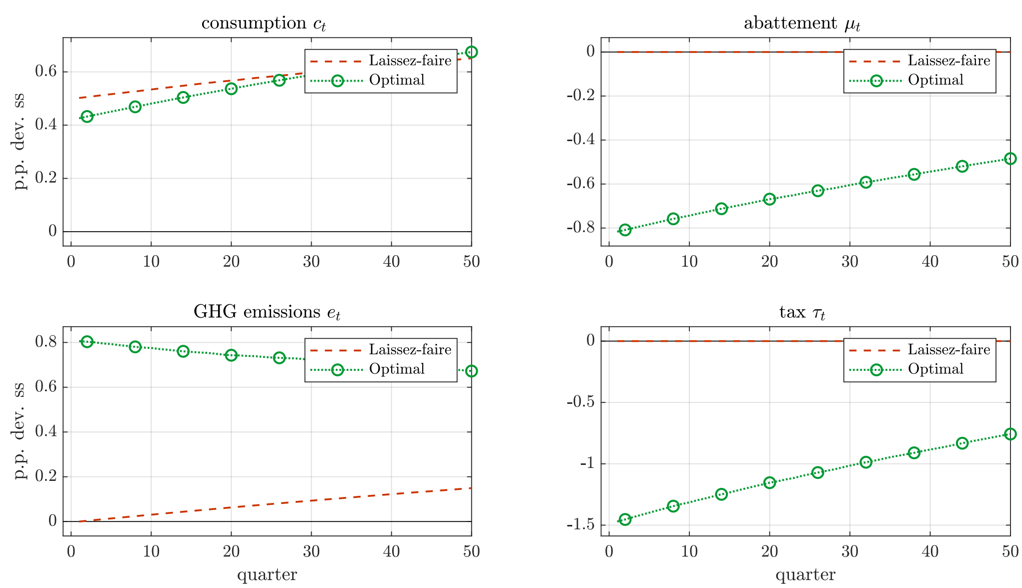


FIGURE 1.5. Impulse responses from an investment-specific technology shock

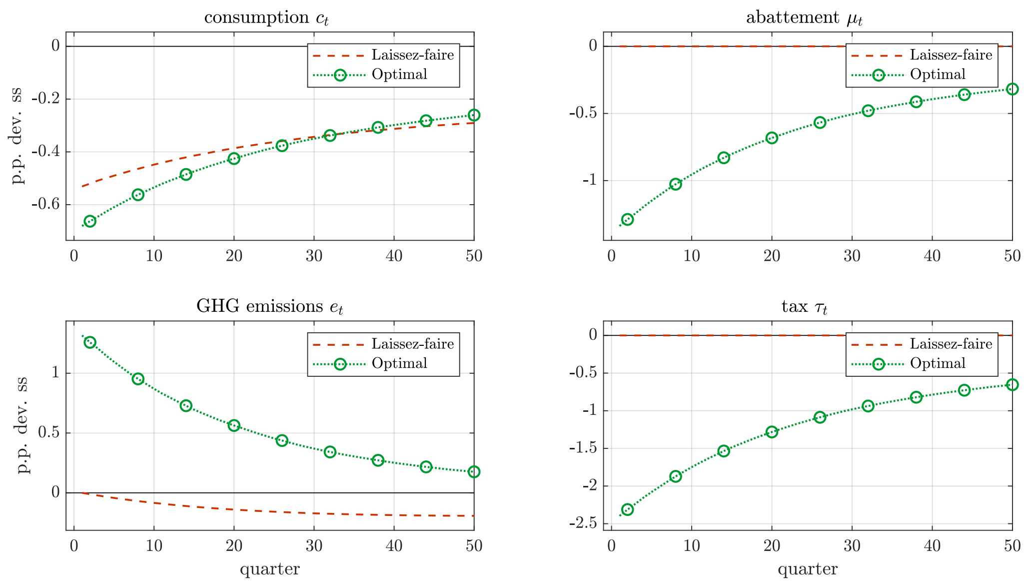


FIGURE 1.6. Impulse responses from a government-spending shock

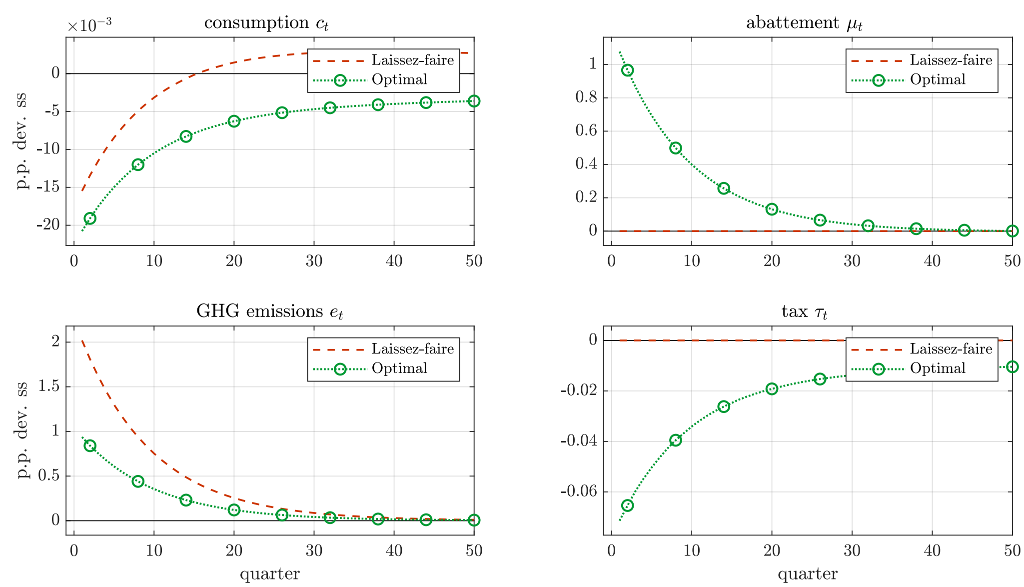


FIGURE 1.7. Impulse responses from an emissions shock

1.C Appendix - C: Balanced growth (not for publication)

Labor-augmenting technological progress is denoted by Γ_t. The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ^Y, where:

$$\Gamma_{t+1} = \gamma^Y \Gamma_t \tag{1.25}$$

Stationary variables are denoted by small caps, whereas variables that are growing are denoted by capital letters. For example, in the growing economy output is denoted by Y_t. De-trended output is thus obtained by dividing output in the growing economy by the level of labor-augmenting technological progress:

$$y_t = \frac{Y_t}{\Gamma_t} \tag{1.26}$$

The production function of emissions is also subject to technological progress. We denote the level of Green technological progress by Ψ_t. The growth rate of Green technological progress is γ^E.

$$\Psi_{t+1} = \gamma^E \Psi_t \tag{1.27}$$

Note that an improvement in the Green technology implies a value for γ^E that is below one.

1.C.1 The de-trended economy

In the growing economy, with labor-augmenting technological progress, the production function is as follows:

$$Y_t = \varepsilon_t^A K_t^\alpha (\Gamma_t n_t)^{1-\alpha} \tag{1.28}$$

where hours worked n_t and the technology shock ε_t^A are stationary variables.

In the de-trended economy, we have that:

$$y_t = \varepsilon_t^A k_t^\alpha {n_t}^{1-\alpha} \tag{1.29}$$

Moreover, the economy's resource constraint is:

$$y_t = c_t + i_t + f(\mu_t) y_t \tag{1.30}$$

where the share of abated emissions μ_t is a stationary variable between 0 and 1. The capital-accumulation equation in the growing economy is:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{1.31}$$

In the de-trended economy, we thus have that:

$$\gamma^Y k_{t+1} = (1-\delta) k_t + i_t \tag{1.32}$$

Emissions, which we denote by E_t, in the growing economy are given as follows:

$$E_t = (1-\mu_t) \varphi_1 Y_t^{1-\varphi_2} \Psi_t \tag{1.33}$$

where φ_1 and φ_2 are parameters.

In the de-trended economy, we have that:

$$e_t = (1-\mu_t) \varphi_1 y_t^{1-\varphi_2} \tag{1.34}$$

where:

$$e_t = \frac{E_t}{\Psi_t \left(\Gamma_t\right)^{1-\varphi_2}} \tag{1.35}$$

In the growing economy, the stock of emissions in the atmosphere is denoted by X_t. The accumulation of emissions in turn depends on the level of new emissions E_t :

$$X_{t+1} = \eta X_t + E_t \tag{1.36}$$

where η is the fraction of the stock of emissions that remains in the atmosphere.

In the de-trended economy, we have that:

$$\gamma^X x_{t+1} = \eta x_t + e_t \tag{1.37}$$

where, to simplify notation, we define γ^X as follows:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2} \tag{1.38}$$

In the growing economy, the utility function is as follows:

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \Theta_t X_t\right)^{1-\sigma}}{1-\sigma} \tag{1.39}$$

where C_t is consumption, β the subjective discount factor, σ the curvature parameter, and Θ_t a preference parameter that measures the disutility caused by the stock of emissions.

The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{\left(c_t - \phi x_t\right)^{1-\sigma}}{1-\sigma} \tag{1.40}$$

where, to simplify notation, we define $\widetilde{\beta}$ as follows:

$$\widetilde{\beta} = \beta \gamma^{1-\sigma} \tag{1.41}$$

A stationary utility function is obtained by assuming that the preference parameter Θ_t has a trend. In the de-trended economy, the preference parameter is constant, which implies the following relationship between Θ_t and ϕ.

$$\Theta_t = \phi \frac{(\Gamma_t)^{\varphi_2}}{\Psi_t} \tag{1.42}$$

1.D Appendix - D: The optimal tax (not for publication)

1.D.1 Centralized problem

We characterize here the first-best equilibrium. A social planner maximizes welfare, which leads producers to internalize the social cost of emissions. The problem for the social planner reads as follows:

$$\begin{aligned}
\mathcal{L} = E_0 \Bigg\{ & \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma} \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \left[y_t - c_t - i_t - g_t - f(\mu_t) y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t q_t \left[(1-\delta) k_t + \left[\frac{\chi_1}{1-\epsilon} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right] k_t - \gamma^Y k_{t+1} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \varrho_t \left[\varepsilon_t^A k_t^{\alpha} n_t^{1-\alpha} - y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \upsilon_{Xt} \left[\gamma^X x_{t+1} - \eta x_t - e_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \upsilon_{Et} \left[e_t - (1-\mu_t) \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2} \right] \Bigg\}
\end{aligned}$$

The marginal utility of consumption c_t is:

$$\lambda_t = (c_t - \phi x_t)^{-\sigma} \tag{1.43}$$

Optimal investment i_t is given by:

$$1 = \varepsilon_t^I q_t \chi_1 \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{-\epsilon} \tag{1.44}$$

The optimal capital supply is given by:

$$q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right) + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\}$$

where:

$$\beta^Y = \widetilde{\beta} / \gamma^Y$$

The first-order condition on output y_t is:

$$[1 - f(\mu_t)] - \varrho_t - \upsilon_{Et} (1 - \varphi_2) \frac{e_t}{y_t} = 0$$

The optimal fraction of abatement μ_t is given by:

$$f'(\mu_t) y_t = \upsilon_{Et} \frac{e_t}{(1 - \mu_t)} = 0 \tag{1.45}$$

The optimal quantity of emissions e_t per quarter reads as follows:

$$\upsilon_{Et} = \upsilon_{Xt} \tag{1.46}$$

While the shadow value of pollution is:

$$\lambda_t \upsilon_{Xt} = \beta^X E_t \phi (c_{t+1} - \phi x_{t+1})^{-\sigma} + \eta \beta^X E_t \lambda_{t+1} \upsilon_{Xt+1} \tag{1.47}$$

where:

$$\beta^X = \widetilde{\beta}/\gamma^X \tag{1.48}$$

1.D.2 Laissez-faire equilibrium

Assume the following functional form for $f(\mu_t)$:

$$f(\mu_t) = \theta_1 \mu_t^{\theta_2} \tag{1.49}$$

Firms are profit-maximizing:

$$\max_{k_t, n_t, \mu_t, e_t} d_t = y_t - w_t n_t - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t$$

Subject to the capital-accumulation constraint:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It} \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t \tag{1.50}$$

Subject to the emission law of motion:

$$e_t = \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2} \tag{1.51}$$

And subject to the supply curve:

$$y_t = \varepsilon_{At} k_t^{\alpha} n^{1-\alpha} \tag{1.52}$$

The Lagrangian reads as follows:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{\lambda_t}{\lambda_0} \left\{ \begin{array}{c} y_t - w_t n - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t \\ +v_{Et} \left[e_t - \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2} \right] \\ +\varrho_t \left[\varepsilon_{At} k_t^{\alpha} n^{1-\alpha} - y_t \right] \\ +q_t \left[(1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It} \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t - \gamma^Y k_{t+1} \right] \end{array} \right\}$$

The first-order condition on emissions e_t is given by:

$$v_{Et} = \tau_t \tag{1.53}$$

Optimal minimization of labor inputs N_t reads as:

$$w_t = \varrho_t (1-\alpha) \frac{y_t}{n_t} \tag{1.54}$$

The optimal quantity of physical capital k_{t+1}:

$$\lambda_t q_t = \beta^Y E_t \lambda_{t+1} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right] + \beta^Y E_t \lambda_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \varrho_{t+1}$$

The marginal profit for an additional unit produced is:

$$\varrho_t = 1 - \theta_1 \mu_t^{\theta_2} - v_{Et}(1-\varphi_2) \frac{e_t}{y_t} \tag{1.55}$$

Optimal abatement μ_t is given by:

$$v_{Et} \frac{e_t}{1-\mu_t} = \theta_1 \theta_2 \mu_t^{\theta_2 - 1} y_t \tag{1.56}$$

In the laissez-faire economy, there is no environmental policy:

$$\tau_t = 0$$

Recall that firms do not consider the stock of emissions x_t as a state variable. In equilibrium the cost of carbon v_{Xt}, as considered by firms, is 0 because they do not internalize the effects of emissions on households. As a result, since in the laissez-faire equilibrium τ_t is set to 0, the first-order conditions with respect to emissions imply that $v_{Et} = 0$. From the first-order conditions with respect to μ_t and y_t, this in turn implies $\mu_t = 0$ and $\varrho_t = 1$.

1.D.3 Competitive equilibrium under optimal policy

The first-best equilibrium that corresponds to the problem of the social planner can be attained by setting the tax τ_t equal to the price of carbon. In the centralized equilibrium, the price of carbon is determined by the optimality condition with respect to x_t. The optimal tax is therefore:

$$\tau_t = v_{Xt} \tag{1.57}$$

Once the optimal tax is implemented, in the laissez-faire equilibrium, equation (1.53) then implies that:

$$v_{Et} = v_{Xt} \tag{1.58}$$

The optimality condition shown in equation (1.46) is therefore satisfied, as the cost of abating emissions is exactly equal to the social cost of emissions.

Chapter 2

Macro-Finance and Climate Change

2.1 Introduction

Climate change has become one of the most significant and complex challenges currently facing our planet. Furthermore as climate change has the potential to significantly impact the well-being of households and the production decisions of firms, the possible macroeconomic and financial implications in the near and distant future could be substantial.

Recent work by Benmir and Roman [2020] argues that monetary and financial instruments could play an important role in climate mitigation efforts, especially as it poses a risk to financial stability and monetary policy conduction.

As highlighted by the European Central Bank President Christine Lagarde, and the findings of Benmir et al. [2020], central banks and financial authorities will increasingly need to incorporate climate variables in their macroeconomic models, as CO_2 impacts monetary aggregates, namely the natural interest rates. Specifically, central bank models should be expanded to include short-term and long-term effects of climate change. The interactions between financial and macroeconomic climate shocks is an equally important source of risk for the future conduct of fiscal and monetary policy.

Applying macroeconomic and financial theoretical tools to address issues at the intersection of climate change economics, monetary economics, and financial economics, allows for investigating one of the main questions asked in policy circles, which is: 'How should fiscal and monetary policies respond to climate change?' The lack of research on monetary policy and its role in climate mitigation, serves as the backdrop, from which this paper seeks to contribute to i) improve the understanding of the complex interactions between the climate, macroeconomics, and financial systems, which feeds into central banks current debates; and ii) enhance the ways by which the adoption of sustainable principles in central banks' investment approaches could help foster the transition to a clean economy.

The academic literature is very limited in terms of that which addresses the interactions between climate change and monetary policies in a unified framework using macro-finance modeling and which also investigates the implications of the former with regards to the latter. Environmental issues occupy a very small place in macro-financial models, and their study remains largely the prerogative of microeconomics and public economics. Schubert 1 [2018] competently exposes the limitations of the recent literature and advocates for more research that accurately integrates environmental issues into macroeconomics. The literature is mainly split between the long-term macroeconomic framework, on the one hand, which investigates the long-term climate dynamics and different macroeconomic aggregates, through the work of Xepapadeas [2005], Nordhaus [2008], Stern [2008], Acemoglu et al. [2012], and Golosov et al. [2014] among many others, and, on the other hand, the literature that investigates the impact of climate related risks on financial aggregate such as Bansal et al. [2019], Derwall et al. [2005], and van den Bremer and van der Ploeg [2019], among others. This latter strand of literature however tends to overlook macro dynamics and macro-finance linkages.

There is a clear dichotomy between these literatures and that working on linking climate dynamics, macroeconomic aggregates, and financial components, although only very recently that macro-financial models start incorporating climate dynamics. Among the first to investigate the linkages between monetary and climate dynamics, Annicchiarico and Di Dio [2015] study how different environmental policies interact with the economy's response to various types of shocks (e.g. productivity shocks, public consumption shocks, monetary policy shocks). Recently, Benmir and Roman [2020] built a full-fledged model linking the financial friction macro literature to the climate macro literature. Benmir and Roman [2020] investigate the role of macroprudential policy and quantitative easing in the climate mitigation efforts. Both Carattini et al. [2021] and Diluiso et al. [2020] build on Benmir and Roman [2020] to further investigate macro-financial policies and climate mitigation. Similare approaches have been also used to study the long-run impacts of climate change or quantify the role of uncertainty by using DSGE frameworks (e.g. van der Ploeg et al. [2020] and Barnett et al. [2020]).

Expending on the RBC model in Benmir et al. [2020], I investigate the effects of different fiscal policies (i.e. consumption based, production based, optimal, and both a fixed tax rate and a fixed cap target) and monetary policies (namely, interest rate setting) on observable economic, environmental, and financial aggregates (eg. GDP, consumption, CO2 emissions, temperature, investment, interest rates, and risk premia). Specifically, I expand the model by including pricing frictions, production damages to the non-separable disutility stemming from the CO_2 externality, and long-term dynamics. This allows for studying the interaction between fiscal and monetary policies, as well as examining how the pricing of carbon varies depending on the modeling approach. I am then able to perform long-term simulations under different climate scenarios (as in Golosov et al. [2014]), taking into account the shift in the financial structure over time. Both on the production side, and consumer side, the model introduces an environmental component affecting the household marginal consumption as in Benmir et al. [2020], and production output via non-linear damages as in Dietz et al. [2020], this facilitates an investigate of how the dynamics of the economy depends on the negative environmental externality, namely carbon emissions, which represent the climate risk.

While the starting point of the entire modeled economy is calibrated using both economic and financial data estimates from the US[17] in order to match a wide range of characteristics of the financial system as well as results from the Green Asset Pricing paper cited above, all the shocks however are estimated using Bayesian estimation techniques on US quarterly data. Regarding the simulated path of the economy, I base the scenarios on Representative Concentration Pathway (RCPs) from the The Intergovernmental Panel on Climate Change (IPCC) report.

Whereas macro-financial models typically focus on the very short-term monetary policies, I expand the scope and use of this framework to investigate the medium/business cycle

[17] In this paper, I consider the US to be a closed economy and that the rest of the world is not increasing their emissions (i.e. a cooperation scenario with respect to carbon pricing).

trends,[18] thus performing analyses that allow for investigating the role of such policies on emissions reduction and other environmental and economic variables.

The first main result highlights the key role of damages stemming from climate risk modeling with respect to the emission reduction targets and the macro-financial aggregates. The dynamics of real rates and inflation exhibit opposite dynamics when climate damages are modeled through the utility or production function. In addition, depending on the modeling choice, the optimal policy restoring the first best allocation is sufficient to decrease emissions by a significant amount in the case of non-separable dis-utility modeling, whereas the same policy instrument fails to achieve the Paris Agreement target under the production damages modeling. In the latter case, a second best instrument (i.e. a fixed policy rate) is necessary to achieve the targeted emissions reduction.

Second, pricing carbon following a non-separable dis-utility allows for keeping emissions under control, even in the worse case RCP 8.5 scenario, where it is more difficult to do so under the production damages specification.

Another important finding is that the monetary authority plays major role in emissions reduction when setting its policy rates. Relying on a Taylor rule or following a Ramsey optimal policy when setting its rates, central banks under the presence of the externality, face a trade-off between emission reduction and real rate/inflation targeting.

Finally, the interaction between different monetary and fiscal policies highlights the difficulty and inability thus far, to achieve both significant emissions reduction and welfare gain, and to keep macro-finance aggregates under targeted levels. This last finding, is inline with Benmir and Roman [2020], where they suggest macro-prudential and unconventional monetary policy, whose role is to reduce the welfare inefficiency and financial volatility when targeting high levels of emissions reduction.

The paper is organized as follows: i) I present the model, ii) I describe the model's fiscal and monetary policies, iii) I show the calibration, the data, and the estimation posterior

[18] To perform long term (over 100 years) scenario analysis, one should drop the new keynsian part as it is not adapted for such long-run horizon.

results, iv) I outline the results of the model, and finally v) I conclude and offer some suggestions for future work in this vein.

2.2 The model

I model a discrete-time, infinite-horizon economy. The latter is composed of *firms*, *households* (which are infinitely lived and of measure one), *government*, and a *central banking authority*. In this setup, production by firms induces an environmental externality through emissions, while households experience a direct disutility stemming from CO_2. I then consider three different cases: i) the emissions stemming from the production side affects output (through damages due to raising emissions), ii) emissions impacts the welfare of households by decreasing the utility stemming from consuming goods, or iii) both.[19]

I start by presenting the firms' final goods, then I move to the dynamics of the environmental externality within the intermediate firm goods, before focusing on the household problem, and finally I present the government and central bank policy setting.

2.2.1 Firms and the environmental externality

2.2.1.1 The final firms

The production sector is comprised of final firms and intermediate firms. The representative final firms produce a final good y_t in a competitive economy. Using no more than capital and labor to produce the intermediate good $y_{j,t}$ (where $j \in (0, 1)$ is the continuum of intermediate goods firms), intermediate firms supply the final sector. In other words, the "bundling" of intermediate goods leads to a final good.

The final firms in the model are looking for profit maximization (in nominal terms), at

[19] Firms do not internalize the social cost from their emissions of CO2. This gives rise to a market failure that opens the door for optimal policy intervention.

a given price p_t subject to the intermediate goods j at prices $p_{j,t}$:

$$\max_{y_{j,t}} \Pi_t^{\text{Final}} = p_t y_t - \int_0^1 p_{j,t} y_{j,t} dj, \tag{2.1}$$

where the aggregation of the final firms reads as:

$$y_t = \int_0^1 \left(y_{j,t}^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}. \tag{2.2}$$

where θ is the constant elasticity of substitution between the differentiated intermediate goods.

The first order condition for the final firm profit maximization problem yields:

$$y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\theta} y_t. \tag{2.3}$$

Under perfect competition and free entry, the price of final goods is denoted p_t, while the price $p_{j,t}$ is the price charged by the intermediate firm j.

The price of final aggregate goods is given by:

$$p_t = \left(\int_0^1 p_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{2.4}$$

2.2.1.2 The intermediate firms

Turning now to our intermediate representative firms j, who seek profit maximization by making a trade-off, on the one hand between the desired level of capital and labor, and on the other hand, the level of investment in abatement technology and the cost of the environmental policy, both in order to maximize profit. As presented in Heutel [2012], the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and the stock of pollutant alters production possibilities of firms. However, I differ from Dietz and Venmans [2019] insofar as this model incorporates the damages from the cumulative emissions x_t as follows:

$$y_{j,t} = (1 - d(x_t))\varepsilon_t^A k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha},\ \alpha \in (0, 1), \tag{2.5}$$

where $d(x_t)$ is a convex polynomial function of order 2 displaying the stock of emissions level ($d(x_t) = a + bx_t + cx_t^2$), with $(a,b,c) \in \mathbb{R}^3$, which is fitted to replicate damages as specified by Nordhaus and Moffat [2017]. A sensitivity analysis is run using Dietz and Venmans [2019] specification.

Furthermore, global temperature t_t^{Temp} is linearly proportional to the level of cumulative emissions as argued by Dietz and Venmans [2019]:

$$t_t^{Temp} = \upsilon_1^{Temp}(\upsilon_2^{Temp} x_{t-1} - t_{t-1}^{Temp}) + t_{t-1}^{Temp}, \tag{2.6}$$

with υ_1^{Temp} and υ_2^{Temp} chosen following Dietz and Venmans [2019].

In addition, α is the classical elasticity of output with respect to capital, and ε_t^A is a technology shock that follows an $AR(1)$ process: $\varepsilon_t^A = \rho_A \varepsilon_{t-1}^A + \sigma_A \eta_t^A$, with $\eta_t^A \sim \mathcal{N}(0, 1)$. Furthermore, the carbon emissions stock x_t follows a law of motion:

$$x_t = (1 - \gamma_d)x_{t-1} + e_{j,t} + e^*, \tag{2.7}$$

where $e_{j,t}$ is the flow of emissions in each intermediate firm at time t and γ_d the decay rate.[20] e^* represents the rest of the world's emissions.

The emissions level is modeled by a nonlinear technology (i.e. abatement technology μ) that allows for reducing the inflow of emissions:

$$e_{j,t} = (1 - \mu_{j,t})\, \varphi_1 y_{j,t}^{1-\varphi_2} \varepsilon_t^X \Psi_t. \tag{2.8}$$

[20] While Dietz and Venmans [2019] argue that $\gamma_d = 0$, we take γ_d close enough to zero but different than 0, in order to allow for stationarity with the law of motion of cumulative emissions.

As in Heutel [2012] $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t is the aggregate production of goods from firms, and the variable ε_t^X is an AR(1) exogenous shock on the carbon intensity of firms and Ψ_t is a technical change trend in carbon intensity.[21]

As highlighted in Benmir et al. [2020] the functional form of emissions allows for taking into account both low and high frequency variations in CO_2 emissions, where the term $\varphi_1 y_t^{1-\varphi_2}$ represents the high frequency features of the emissions data. The parameters φ_1, $\varphi_2 \geq 0$ represent the carbon intensity parameters, which allow for pining down the steady state ratio of emissions intensity.

Furthermore, firm j incurs a cost $z_{j,t}$ for every emission unit abated, where $\mu_{j,t}$ is the abatement level.

Following Heutel [2012], abatement costs reads as follows:

$$z_{j,t} = f(\mu_{j,t}) y_{j,t}, \tag{2.9}$$

where

$$f(\mu_{j,t}) = \theta_1 \mu_{j,t}^{\theta_2}, \ \theta_1 > 0, \, \theta_2 > 1, \tag{2.10}$$

with θ_1 and θ_2 representing the cost efficiency of abatement parameters for each sector.

Thus the profits of the representative intermediate firm $\Pi_{j,t}$ will be impacted by the presence of the environmental externality. The revenues are the real value of intermediate goods $y_{j,t}$, while the costs arise from wages w_t (paid to the labor force $n_{j,t}$[22]), investment in capital $k_{j,t}$ (with returns r_t^k), abatement $\mu_{j,t}$ (the firms are enduring), and any environmental damages captured by emissions $e_{j,t}$ (environmental taxes).

$$\begin{aligned} \Pi_{j,t} &= p_{j,t} y_{j,t} - w_t N - r_t^k k_{j,t} - f\left(\mu_{j,t}\right) y_{j,t} - \tau_t e_{j,t} \\ &= \left(p_{j,t} - mc_{j,t}\right) y_{j,t}, \end{aligned} \tag{2.11}$$

As firms are not free to update prices each period, they first choose inputs so as to

[21] For simplicity, we assume that the exogenous trend Ψ is not affected by abatement μ.

[22] Which we assume to be fixed in our setup $n_{j,t} = N$

minimize cost, given a price, subject to the demand constraint. The cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, costs of abatement, and output:

$$\varrho_{j,t} = \varrho_t = \frac{r_t^k k_t}{\alpha y_t}, \tag{2.12}$$

$$\tau_t = \frac{\theta_1 \theta_2}{\varphi_1} \mu_{j,t}^{\theta_2 - 1} y_{j,t}^{\varphi_2}, \tag{2.13}$$

$$mc_{j,t} = mc_t = \varrho_t + \theta_1 \mu_t^{\theta_2} + (1 - \varphi_2)\tau_t (1 - \mu_t)\varphi_1 y_t^{-\varphi_2}, \tag{2.14}$$

where $\varrho_{j,t} = \varrho_{t,k}$ is the marginal cost component related to the same capital demand all firms choose (2.12). This marginal cost component is common to all intermediate firms.

Equation (2.13) is the optimal condition for abatement: abating CO_2 emissions is optimal when marginal gain equal marginal cost. As in Benmir et al. [2020] this highlights the key role of emissions in shaping price dynamics.

In addition, abatement effort μ_t is common to all intermediate firms, as the environmental cost is constant across intermediate firms.

Furthermore, as the impact of the environmental externality is not internalized by the firms (i.e. they take x_t as given), the shadow value of the environmental externality is zero.

The total marginal cost captures both abatement and emissions costs as shown above in equation (2.14). Also, in the case of the laissez-faire scenario, $mc_t = \varrho_t$ as the firms are not subject to emissions and abatement constraints.

In addition, the monopolistic firms engage in price setting à la Rotemberg [1982]. Thus, the profit maximization of intermediate firms reads as follows:

$$\max_{p_{it}} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda_{t+\tau}^c}{\lambda_t^c} \left[\frac{p_{j,t+\tau}}{p_{t+\tau}} \left(\frac{p_{j,t+\tau}}{p_{t+\tau}} \right)^{-\epsilon} - mc_{t+\tau} \left(\frac{p_{j,t+\tau}}{p_{t+\tau}} \right)^{-\epsilon} - \frac{\chi}{2} \left(\frac{p_{j,t+\tau}}{p_{j,t-1+\tau}} - 1 \right)^2 \right] y_{t+\tau}$$

First order condition, assuming symmetry reads as:

$$\chi\pi_t\left(\pi_t-1\right)=(1-\epsilon)+\epsilon mc_t+\left\{\Lambda_{t,t+1}\frac{y_{t+1}}{y_t}\chi\pi_{t+1}\left(\pi_{t+1}-1\right)\right\}. \tag{2.15}$$

This equation represents the inflation dynamic, in other words the New Keynesian Philips Curve.

2.2.2 Households and the environmental externality

The representative household problem is approached using a CRRA utility function, whereby household chooses consumption expenditures, as well as investment and its holding of long-term government bonds. Building on Benmir et al. [2020] who draws on Stokey [1998], Acemoglu et al. [2012] and Golosov et al. [2014], the environmental externality is housed in a non-separable fashion into the utility function.

This functional choice is important insofar as the dynamics related to asset prices facilitate the capture of the interactions between the climate externality and macro-finance.

The marginal utility of the representative agent, thus depends on the disutility stemming from the externality stock:

$$\max_{\{c_t,k_{t+1},i_t,b_{t+1}\}} E_0\sum_{t=0}^{\infty}\beta^t\frac{\left(c_t-\phi_t x_t\right)^{1-\sigma}}{1-\sigma}, \tag{2.16}$$

where $\beta\in[0,1]$ is the time discount factor, and $\sigma>0$ the curvature parameter, while the parameter ϕ_t represents the sensitivity of utility to a rise in CO_2 concentration in the atmosphere x_t, and is chosen following Benmir et al. [2020].

The representative household budget constraint reads:

$$c_t+i_t+\frac{p_t^B(b_{t+1}-b_t)}{p_t}+T_t=w_tN+r_tk_t+\frac{b_t}{p_t}, \tag{2.17}$$

where c_t and i_t are household choices of consumption and investment, respectively. b_t is the holding of long-term government nominal bonds at price p_t and with returns p_t^B. N is the inelastic labor supply. The capital stock rented to firms is denoted by k_t, where r_t is the rental rate of capital. Finally, the government levies a lump-sum tax, which is denoted by T_t.

The physical capital follows the following law of motion:

$$k_{t+1} = (1-\delta)k_t + \psi\left(\varepsilon_t^I \frac{i_t}{k_t}\right) k_t, \tag{2.18}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital, $\psi(\bullet)$ is an adjustment cost function on investment, and ε_t^I is an exogenous shock process following Christiano et al. [2014]. As in Benmir et al. [2020] this investment shock captures financial frictions associated with asymmetric information or costly monitoring.

2.2.3 Public authorities

2.2.3.1 Government

The issuing of bonds and collection of taxes allows the government to finance its expenditures as follows:

$$g_t + \frac{b_t}{p_t} = \frac{p_t^B(b_{t+1} - b_t)}{p_t} + T_t + \tau_t e_t, \tag{2.19}$$

where g_t refers to the public expenditures, T_t the lump-sum tax, and $\tau_t e_t$ the revenues raised from the environmental policy when conducted. As it is in a standard business cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$ with $\log\varepsilon_t^G = \rho_G \log\varepsilon_{t-1}^G + \eta_t^G$, $\eta_t^G \sim N(0,\sigma_G^2)$ and $\bar{g}$ denotes the steady state amount of resources that is consumed by the government.

2.2.3.2 Central bank

The central bank follows a standard Taylor [1993] rule to set the interest rate:

$$r_t - \bar{r} = \rho_r \left(r_{t-1} - \bar{r}\right) + \left(1 - \rho_r\right) \left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(y_t - \bar{y}\right)\right] + log(\varepsilon_t^R), \tag{2.20}$$

where $\bar{r}$ is the steady state of the nominal rate r_t, $\rho_r \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, and ϕ_y is the output gap stance penalizing deviations of output from its steady state. As for the case of the technology, investment, and government spending, ε_t^R is a sepcfic monetary shock, which also follows an AR(1) process: $\log \varepsilon_t^R = \rho_R \log \varepsilon_{t-1}^R + \eta_t^R$, $\eta_t^R \sim N(0, \sigma_G^2)$.

Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$r_t = r_t^F E_t \left\{\pi_{t+1}\right\}. \tag{2.21}$$

As the aim is to replicate the current economic conditions as closely as possible, I calibrate the model such that the nominal rate is extremely low by historical standards (1 percent at the steady state). This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero (i.e. the zero lower bound (ZLB)).

2.2.3.3 Market clearing

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f\left(\mu_t\right) y_t + \frac{\chi}{2}(\pi - \bar{\pi})^2 y_t. \tag{2.22}$$

2.3 The social cost of carbon and the optimal tax

In this section, I derive the social cost of carbon to which the the optimal tax would be set, by comparing the decentralized equilibrium with respect to the problem of the social planner[23].

Definition 2.3.1 *The social planner maximizes welfare subject to the budget constraints and the presence of the externality. The social cost of carbon under the presence of both non-separable dis-utility and production damages reads as follows:*

$$SCC_t = E_t \left\{ m_{t,t+1}(\phi_{t+1} + (1-\delta_d)SCC_{t+1} + (b + 2cx_{t+1})(1 - d(x_{t+1}))^{-1}\varrho_{t+1}y_{t+1}) \right\} \tag{2.23}$$

The optimal tax is comprised of two parts:

- The damages of climate risk stemming from the dis-utility of consumers: ϕ_{t+1}.
- The damages of climate risk stemming from the production side: $(b + 2cx_{t+1})(1 - d(x_{t+1}))^{-1}\varrho_{t+1}y_{t+1}$.

Proposition 2.3.1 *As a first best allocation, the social planner would set the optimal policy equal to the social cost of carbon in order to maximize the welfare:*

$$\tau_t = SCC_t \tag{2.24}$$

However, the regulator might decide to target a specific price level:

Proposition 2.3.2 *The regulator decides to set a fixed carbon tax in order to achieve a specific emission reduction objective. In this case the policy maker could decide to monitor the price for carbon exogenously by setting the tax rate in each period:*

$$\tau_t = \textit{Fixed Carbon Tax} \tag{2.25}$$

[23] Please refer to the appendix for the full derivations performed following Ljungqvist and Uhlig [2000]

Similarly, the regulator decides to set a quantity objective. This is equivalent to a targeted emission level such as that which the European Trading Scheme utilizes (and which is outlined in the following proposition).

Proposition 2.3.3 *The regulator decides to set a fixed emissions cap in order to achieve a specific emission rate:*

$$e_t = \textit{Fixed Carbon Cap} \tag{2.26}$$

2.4 The Ramsey optimal monetary policy

The central bank could either chose to follow a standard Taylor monetary rule as the one shown in section 2.2.3.2, or set an optimal monetary policy, which maximizes welfare. The Ramsey optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy.

Definition 2.4.1 *From a time-less perspective, the social planner will maximize household's lifetime wealth subject to the economy constraints.*
Let $\lambda_{j,t}$, where $j \in \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$ represents the sequences of Lagrange multipliers corresponding to the set of allocations and prices $\{c_t, k_t, i_t, y_t, g_t, x_t, e_t, r_t^F, p_t^B, r_t^k, p_t, \tau_t, \upsilon_{Xt}, \upsilon_{Et}, T_t, mc_t, \mu_t, \pi_t, \varrho_t\}$ defining the sequences of constraints and first order conditions (2.5,2.6,2.7,2.8,2.12,2.13,2.14,2.15,2.18,2.19,2.21,2.22,2.29,2.30,2.31, 2.37,2.38) and given one of the environmental policies chosen by the environmental regulator (2.24, 2.25, 2.26), as well as a set of stochastic processes $\{\epsilon_t^A, \epsilon_t^X, \epsilon_t^I, \epsilon_t^G\}$ and a given B_0 plans for the control variables $\{x_{t-1}, k_{t-1}\}$.

Proposition 2.4.1 *To maximize the welfare, the central bank sets its interest rates following the Ramsey optimal rule derived from the problem set above following Schmitt-Grohé and Uribe [2007], given any chosen carbon policy. In this case the optimal first best coincide when both carbon pricing follows the SCC and monetary policy is set following the*

Ramsey planner. The four equations setting the Ramsey monetary policy as presented in the appendix are: 2.65, 2.66, 2.67, *and* 2.68.

Cental banks, however, do not always chose to optimally set their monetary policy, and instead target specific inflation and output gap levels. Using empirical data on the US, Taylor suggests a specific monetary rule that reacts to the output gap, the inflation gap, and the past nominal rate.

Proposition 2.4.2 *If the central bank decides to set the monetary policy following a standard Taylor rule such as the one specified in* 2.20 *and not follow an optimal Ramsey policy, then nominal rates would be set as following:*

$$r_t - \bar{r} = \rho_r \left(r_{t-1} - \bar{r}\right) + \left(1 - \rho_r\right) \left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(y_t - \bar{y}\right)\right] + log(\varepsilon_t^R) \tag{2.27}$$

2.5 Data and calibration

2.5.1 Data

I estimate the five shocks persistence and standard deviation using Bayesian estimation methods (An and Schorfheide [2007]) on U.S. quarterly data over the sample time period 1973Q1 to 2018Q4, which are all taken from FRED, the U.S. Energy Information Administration and from estimates of the shadow rate performed by Wu and Xia [2016]. I use the shadow rate for the periods where the interest rates were at the zero lower bound, in order to capture some of the effects of unconventional monetary policy.

As data exhibits trends, unit roots, and/or seasonality, I follow Smets and Wouters [2007], to first deseasonalize and detrend the data to allow for a stationnary mapping of the data to model variables (namely, GDP, consumption, investment and CO2 emissions, as well as the adjusted federal fund rate (where I use the shadow rates for periods with very low funds rate)). I then divide the sample by the working-age population, except for the funds rates. Next, all data but the adjusted fed fund rates are taken in logs and I apply a first-difference filter to obtain growth rates. For the adjusted fed fund rates, I take the growth rate as a level first difference over the mean. Real variables are deflated by the GDP deflator price index. The measurement equations mapping the model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Adjusted Fed Fund Rate Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_A + \Delta \log (\tilde{y}_t) \\ \log \gamma_A + \Delta \log (\tilde{c}_t) \\ \log \gamma_A + \Delta \log (\tilde{\imath}_t) \\ \log \gamma_A^{1-\varphi_2} \gamma_E + \Delta \log (\tilde{e}_t) \\ \frac{\Delta(\tilde{r}_t)}{4\bar{r}} \end{bmatrix}, \qquad (2.28)$$

where a variable with a tilda, $\tilde{x}_t$, denotes the de-trended version of a level variable, x_t.

2.5.2 Calibration and prior distributions

All calibrated parameters are reported in Table 1.6. As in Benmir et al. [2020], the calibration of the parameters related to the macro business-cycle literature is standard and remains unchanged.[24] All calibrated parameters are set at a quarterly frequency, with the depreciation rate of physical capital set equal to 2.5 percent, the ratio of public spending to output set at 20 percent, the endogenously determined hours worked at the steady state set at .2, and the consumption habits set equal to 0.8 as in Jermann [1998]. As for the investment cost curvature, present in the capital adjustment quadratic cost function ϵ_k, the risk aversion σ, the discount rate parameter b, the productivity growth rate $(\gamma_A - 1)100$, I use the estimated values of Benmir et al. [2020].

Turning now to the environmental parameters, I follow a similar strategy, and calibrate the structural parameters, such as the abatement cost parameters $\theta_1 = .05607$ and $\theta_2 = 2.8$ following Heutel [2012] and Nordhaus [2008], and the temperature reaction parameters

[24] Please refer to the appendix for more details.

$\upsilon_1^{Temp} = .5$ as well as $\upsilon_2^{Temp} = .00125$ following Dietz and Venmans [2019] in order to retrieve the actual temperature levels at the steady state, which correspond to present cumulative emissions $\bar{x} = 840$ GTCO. The production damage function parameters ($d0$, $d1$, $d2$, and $d3$) are set to fit the projected damages outlined in Dietz and Venmans [2019].[25] For the remaining environmental parameters (namely the decay rate η, the de-coupling rate stemming from the negative trend on emissions γ_E, and the emission-to-output intensity φ_2), I use the values estimated in Benmir et al. [2020].

Moving the the New Keynesian part, I set the imperfect substitution between goods ϵ to 6 as in Smets and Wouters [2007], the Rotemberg adjustment pricing cost χ to 100, and the monetary Taylor rule smoothing parameters ρ_c, ϕ_π, and ϕ_y to 0.8, 5.8, and 0.1125, respectively, as in Smets and Wouters [2007] again. For the smoothing parameter that I introduce with respect to climate change reaction ϕ_x, I set its value to 2.1 and perform sensitivity analysis.

For the remaining set of parameters and shocks, I employ Bayesian methods. More specifically I use the Kalman filter to perform the estimations. Table 1.7 summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. I follow closely Guerrieri and Iacoviello [2017] to set the distributions of the prior information on the persistence of the Markov processes and the standard deviation of innovations. As in Benmir et al. [2020] the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks I choose an inverse gamma distribution with mean 0.01 and standard deviation of 1.

2.5.3 Posterior distributions

In addition to prior distributions, Table 1.7 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 20,000

[25] The damages parameters are calibrated to match the US economy.

iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

The results of the posterior distributions for each estimated parameter are listed in Table 1.7. It is clear from Table 1.7 that the data were informative, as the shape of the posterior distributions is different from the priors. The estimates of the structural shocks parameters that are common with Smets and Wouters [2007] are mostly in line with those they find and the findings in Benmir et al. [2020]. This is particularly reassuring and further reinforces the results of Benmir et al. [2020].

Although, the estimation does not ensure perfect matching between model moments and the unconditional standard deviations observed in the data, Bayesian estimation allows replication of the historical path of the estimated observable variables. Using the the estimated values for the shocks parameters, I simulate the model by drawing shocks from the estimated distribution. As in Jermann [1998], Table 1.1 summarizes the observable moments taken at a 90 percent interval versus the asymptotic moments generated by the model using a second-order approximation to the policy function. The model is able to replicate reasonably well the main moments as most of the simulated estimates fall within the 95 percent confidence interval of the data.

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	Data [5%;95%]	Model	Data [5%;95%]	Model
$100 \times \Delta \log(y_t)$	[0.27;0.50]	0.34	[0.7;0.86]	0.77	[1.0;1.0]	1.00
$100 \times \Delta \log(c_t)$	[0.36;0.55]	0.34	[0.60;0.75]	1.13	[0.54;0.76]	0.75
$100 \times \Delta \log(i_t)$	[0.06;0.69]	0.34	[1.93;2.38]	1.45	[0.61;0.80]	0.75
$100 \times \Delta \log(e_t)$	[-0.52;0.09]	-0.24	[1.88;2.31]	2.11	[-0.01;0.36]	0.23
$100 \times \Delta \log(r_t)$	[-0.01;0.01]	-0.00	[0.04;0.05]	0.09	[-0.2681;0.1120]	-0.48

TABLE 2.1
Data moments vs. model moments (with parameters taken at their posterior means)

2.6 Results

2.6.1 General model dynamics

The main simulation results appear in Table 2.2 and Table 2.4 below. The two tables highlight the mean level of the following aggregates for the case of non-separable dis-utility and the case of production damages: consumption $E(c_t)$, stock of CO2 emissions $E(x_t)$, the welfare $E(\mathcal{W}_t)$, the planner optimal tax rate $E(\tau_t)$, the mean risk-free rate $400E(r_t^F)$, the mean bond premium $400E\left(r_{t+1}^B - r_t\right)$, the expected mean inflation $E(\pi_{t+1})$, the standard deviation of marginal utility $std(\hat{\lambda}_t)$, the average coefficient and standard deviation of relative risk aversion $E(RRA_t)$ and $std(\widehat{rra}_t)$, the mean abatement level, $E(\mu_t)$, the average cost of abatement $E(f(\mu_t))$, and finally the tax as a percentage of GDP $E(\tau_t e_t/y_t)$.

As in Benmir et al. [2020] the first column represents the model implications under the no policy scenario (i.e. the *laissez-faire* equilibrium). The columns (2) to (4) summarize the results of the sensitivity analysis under the optimal policy scenarios and under different levels of abatement efficiency θ_1.

The results with respect to the business-cycle components highlight a key difference between non-separable specification and production damages specification. Consumption *increases* under an optimal policy, when emissions impact the production side, stemming from both i) a low tax level (.09%) and ii) a higher net disposable income (the low value of the tax is proportionally lower than the production gains from emissions reduction). Whereas, consumption *decreases* when the emissions stock impacts household marginal consumption, as both i) the tax level required to restore the first best allocation is significantly higher (3%) and ii) the disposable income level is lower between the laissez-faire and optimal policy scenario (in the latter case, the cost of the tax reduces the disposable income of the representative household).

Furthermore, although consumption increases following the tax introduction under the production damages scenario compared to the dis-utility case, the lifetime welfare is significantly higher under the non-separable dis-utility case than in the production damages case, where the welfare gains are marginal. The impact of the tax policy is also significantly different between the two cases, as emissions are reduced considerably at a higher rate when modeling the damages through consumer dis-utility than when modeling through the production side.

Turning to the asset pricing implications of the model, introducing pricing frictions improves the bond premium levels with respect to the RBC finding in Benmir et al. [2020]. However, the main finding of the present paper is that there are different (opposite) dynamics of the macro-financial aggregates with respect to the two modeling cases. On one hand, under the non-separable dis-utility, the stochastic discount factor is directly impacted by the externality, and therefore it is impacted by the environmental policy, which in turn impacts the real rates, bond premium, inflation, and risk aversion. On the other hand, under the production damages case, the stochastic discount factor is not directly impacted by the externality, thus the environmental policy implementation doesn't have a significant impact on the macro-finance aggregates. More specifically, under the production damages case, the introduction of the environmental policy introduces uncertainly as the real rate and bond premium decreases and increases respectively (although it is significantly a very small change), while under the non-separable utility case, the tax allows for reducing the premium by more than 60% and significantly increases the real rate, thus eliminating the uncertainty. The key is that in this model short and long-term interest rates are counter-cyclical. In one case, the increase in output and consumption through the tax introduction decreases the short-term interest rates, while under the other case it raises the short-term risk-free rates as consumption and output fall.

The main difference between the two cases is that a higher volatility of marginal utility implies more uncertainty about future valuations, and greater uncertainty in turn increases agents' willingness to build precautionary savings. The marginal utility is impacted differently between the two cases. When using a non-separable utility specification, we allow for capturing direct effect of climate risk within the consumer investments/savings choice, while we fail to directly captured this effect through the production damages specification.

Moving to the environmental aggregates, the key component allowing for emissions reduction is abatement. Firms equate their marginal abatement costs to the tax level in order to determine how much abatement to set, as shown in Equation 2.13. The high tax rate generated under the non-separable case allows for a significantly higher abatement levels (55%) as compared to the case of production damages, where the low tax rate generates very little abatement, thus not allowing for a substantial emission reduction.

			Optimal policy					
	Laissez-faire		$\theta_1 = 0.05607$		$\theta_1 = 0.31555$		$\theta_1 = 3.8288$	
	(1)		(2)		(3)		(4)	
	Damages Utility	Damages TFP	Damages Utility	Damages TFP	Damages Utility	Damages TFP	Damages Utility	Damage TFP
Business-cycle								
$E\left(c_t\right)$	0.5214	0.4911	0.4858	0.4919	0.4972	0.4914	0.5085	0.4911
$E\left(x_t\right)$	851.9093	851.2194	356.6578	785.4637	628.6346	824.8087	775.3981	844.1437
$E(\mathcal{W}_t)$	-180640.1953	-2439.5549	-9732.7209	-2427.6617	-41169.2816	-2435.8798	-113569.6318	-2440.1267
$E(\tau_t)$	0.0000	0.0000	0.0340	0.0009	0.0381	0.0010	0.0428	0.0010
Asset-pricing								
$400E\left(r_t^F\right)$	3.2642	5.8685	5.4705	5.8683	4.6478	5.8683	3.7356	5.8683
$400E\left(r_{t+1}^B - r_t^F\right)$	1.2216	0.3264	0.5030	0.3267	0.8456	0.3269	1.1612	0.3270
$E\left(\pi_{t+1}\right)$	0.9867	0.9982	0.9967	0.9982	0.9935	0.9982	0.9899	0.9982
$std(\hat{\lambda}_t)$	1.9083	0.6403	0.9202	0.6408	1.3292	0.6403	1.6944	0.6401
$E(RRA_t)$	31.8004	8.7429	13.1821	8.7292	20.1447	8.7387	27.3793	8.7436
$std(\widehat{rra}_t)$	0.4545	0.1525	0.2192	0.1526	0.3166	0.1525	0.4036	0.1525
Environment								
$E\left(\mu_t\right)$	0.0000	0.0000	0.5530	0.0773	0.2248	0.0307	0.0599	0.0078
$E\left(f(\mu_t)\right)$	0.0000	0.0000	0.0108	0.0000	0.0049	0.0000	0.0014	0.0000
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0000	0.0240	0.0015	0.0467	0.0016	0.0633	0.0017

Notes: The first column (1) shows the results in the laissez-faire (counter-factual) equilibrium under both the case of non-separable disutility and production damages, where we use the estimated values obtained for non-separable utility. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match abatement shares of $\bar{\mu}$ of 20% and 5% for the standard case of non-separable utility. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4) due to the contribution of future shocks to the asymptotic mean of these variables.

TABLE 2.2

The main model simulation results

2.6.2 The Paris Agreement target

Table 2.3 below, compares the main model optimal policy findings with a targeted policy that aims for a 55% emissions ($E(e_t)$) reduction by 2030, which corresponds to the net zero 2050 objectives. To be able to compare a fixed targeted policy (i.e. 55% emissions reduction)[26] and an optimal policy, I first run the model simulations for both specifications (non-separable dis-utility and production damages) under the laissez-faire scenario to retrieve the steady state levels of emissions. Then, I set the fixed policy rate in order to retrieve a 55% emissions reduction. The value corresponding to a 55% emissions reduction under both damages modeling specifications is reported under $E(e_t)$ and is .85 and .86 for the non-separable dis-utility and production damages cases, respectively.

From the simulation results, it is clear that the optimal policy under the non-separable dis-utility is aligned with the Paris Agreement and the net zero objectives (as $0.63 < 0.85$). This means that the optimal policy is able to reduce emissions by more than 60%[27] by generating about 15% higher levels of abatement compared to the fixed policy scenario. It also allows for significant welfare improvement and higher uncertainty reduction as the premium is 10 basis point lower, due to the precautionary savings dynamics triggered by a higher tax level allowing for more emissions reduction.

The production damages case, however, shows that the optimal policy is not enough to achieve a 55% percent emissions reduction ($1.40 > 0.86$), and that a fixed tax rate is needed to achieve such objectives. Setting a fixed tax rate targeting a 55% emissions decrease, by allowing for about 40% abatement levels as compared to the laissez-faire scenario, is however welfare distortionary as it further reduces consumption and output ($E(y_t)$), as then firms have to pay higher abatement costs (0.5%), whereas under the optimal policy, the abatement costs were significantly low.

[26] This within the range of Biden's pledge of 50-52% reduction below 2005 level by 2030.

[27] As 0.85 represents 55% emissions reduction, and 0.63 allows for more than 60% emissions reduction compared to the laissez-faire scenario.

	55% emission reduction target			
	Dis-utility		Production damages	
	(1)		(2)	
	Fixed Tax or Cap	Optimal Policy	Fixed Tax or Cap	Optimal Policy
Business-cycle				
$E\left(e_t\right)$	0.8537	0.6399	0.8698	1.4093
$E\left(y_t\right)$	0.9152	0.9018	0.8948	0.9018
$E\left(c_t\right)$	0.4943	0.4858	0.4878	0.4919
$E\left(\lambda_t\right)$	278.6948	154.9828	25.3388	24.3606
$E\left(x_t\right)$	475.8092	356.6578	484.7548	785.4637
$E(\mathcal{W}_t)$	-15338.7441	-9732.7209	-2500.4497	-2427.6617
$E(\tau_t)$	0.0200	0.0340	0.0200	0.0009
Asset-pricing				
$400E\left(r_t^F\right)$	5.2596	5.4705	5.8631	5.8683
$400E\left(r_{t+1}^B - r_t^F\right)$	0.6215	0.5030	0.3304	0.3267
$E\left(\pi_{t+1}\right)$	0.9956	0.9967	0.9981	0.9982
$E(RRA_t)$	15.2119	13.1821	8.8065	8.7292
Environment				
$E\left(\mu_t\right)$	0.4125	0.5530	0.4284	0.0773
$E\left(f(\mu_t)\right)$	0.0047	0.0108	0.0052	0.0000
$E(\frac{\tau_t e_t}{y_t})$	0.0187	0.0240	0.0195	0.0015

Notes: The first column (1) shows the results of both the fixed carbon price or fixed emissions cap and the optimal policy under the case of non-separable dis-utility, where I use the estimated values obtained for non-separable utility. Column (2) is the equilibrium under the two sets of environmental policies with production damages. All cases are simulated using $\theta_1 = .05$.

TABLE 2.3

Emissions reduction under an optimal policy versus a target fixed policy

2.6.3 RCPs scenarios analysis

In this section, I highlight the mean model dynamics under both the non-separable specification and production damages, following the main three RCP scenarios and an environmental policy, namely, RCP2.6, RCP 3, RCP 6, and RCP 8.5. In order to replicate the expected levels of CO_2 highlighted in the IPCC report, I introduce an exogenous growth rate of GDP that would allow for the following levels of CO_2 stock:

- RCP 2.6 is the baseline of the current state of the world, i.e. 840 GTCO,
- RCP 3 expects a mean level of CO_2 stock to reach 530 ppm by 2100, i.e. 1123.6 GTCO,
- RCP 6 expects a mean level of CO_2 stock to reach 620 ppm by 2100, i.e. 1314.4 GTCO,
- RCP 8.5 expects a mean level of CO_2 stock to reach 950 ppm by 2100, i.e. 2014 GTCO.

Table 2.4 and Table 2.5 summarize the results of the RCP projections for both the case of non-separable dis-utility and the production damages, respectively.

First focusing on the case of non-separable dis-utility (i.e. Table 2.4), I calibrate the exogenous growth rate of GDP to 0.45%, 0.7%, 1.3%, which enables the matching of RCP 2.6, RCP 3, RCP 6, and RCP 8.5, respectively.

Comparing the optimal environmental policy scenario to the laissez-faire under all RCPs shows a significant welfare improvement both due, on one hand, to the higher levels of GDP, and, on the other hand, to the positive impact of the tax on the marginal utility of consumption. The optimal policy under the non-separable dis-utility is able to significantly reduce the emissions levels and even keep the stock of emissions at the current levels under the worse scenario (i.e. RCP 8.5) with an economy growing at about 1.3% yearly. Furthermore, under the laissez-faire scenario a rising stock of emissions has a net positive impact on the bond premium as the risk aversion levels decrease with higher emissions stock, since uncertainty increases with respect to an increase of climate risk. The real rate decreases, thus highlighting the role climate risk could play in the falling of r^* (Bauer and Rudebusch [2020]). The policy implementation is able to revert the dynamics of r^* as shown and explained in the general model dynamics. This finding is of high importance as it highlights the potential linkages between climate risks and the monetary aggregates.

Turning now to the case of production damages (i.e. Table 2.5), I calibrate the exogenous growth rate of GDP slightly differently to be able to match the same stock of emissions as the one in the non-separable dis-utility, as the steady state levels of both specifications are different by nature. Thus, I set the GDP growth rate to 0.47%, 0.73%, 1.43%, which enables the matching of RCP 2.6, RCP 3, RCP 6, and RCP 8.5, respectively.

Under both the laissez-faire and optimal policy scenarios, the rise in emissions stock, following the exogenous GDP growth, improves the welfare, as in the case of non-separable utility. This is due both to the positive income effect stemming from the growth perspective and the reduction in emissions levels (in the case of the optimal policy scenario). However, two main differences from the non-separable utility case are: i) the ability to significantly keep the emissions levels under the ratified Paris Agreement levels and ii) the behavior of monetary aggregates, namely the real rate and bond premium. For the first, under the production damages case, emission reduction is twice as low as the non-separable dis-utility, thus confirming the findings of the previous section. Second, under the laissez-faire case, the real rate increases, while the premium decreases. This finding is coherent with respect to the model specification as the stochastic discount factor, or in other words the marginal utility, fails to capture the climate risk and the impacts emissions stock could have on investment and consumption decisions of households. However, this results is hard to reconcile with the empirical literature attempting to explain the falling of r^*. Moreover, the introduction of an optimal policy as compared to the laissez-faire scenario raises the risk premium, which suggest a distortion and inefficiency from a monetary policy perspective.

	RCP Scenarios							
	RCP 2.6		RCP 3		RCP 6		RCP 8.5	
	(1)		(2)		(3)		(4)	
	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy
Business-cycle								
$E\left(y_t\right)$	0.9925	0.9018	1.5444	1.4042	1.9841	1.8046	3.9205	3.5722
$E\left(c_t\right)$	0.5214	0.4858	0.7907	0.7442	0.9879	0.9399	1.5768	1.6356
$E\left(\lambda_t\right)$	6946.6156	154.9828	1189.9725	25.7155	454.6404	9.5520	41.2614	0.7727
$E\left(x_t\right)$	851.1118	356.3239	1123.1986	470.3754	1314.3224	550.5436	2014.8995	844.8422
$E(\mathcal{W}_t)$	-180640.1942	-9732.7209	-48394.7269	-2496.5152	-23850.3960	-1183.5858	-4341.6658	-183.4188
$E(\tau_t)$	0.0000	0.0341	0.0000	0.0402	0.0000	0.0441	0.0000	0.0569
Asset-pricing								
$400E\left(r_t^F\right)$	3.2642	5.4705	3.2567	5.4748	3.2510	5.4782	3.2277	5.4933
$400E\left(r_{t+1}^B - r_t^F\right)$	1.2216	0.5030	1.2324	0.5005	1.2409	0.4985	1.2762	0.4899
$E\left(\pi_{t+1}\right)$	0.9867	0.9967	0.9815	0.9955	0.9774	0.9945	0.9599	0.9904
$E(RRA_t)$	31.8004	13.1821	21.7820	8.6570	18.0597	6.8940	12.9284	4.0310
Environment								
$E\left(\mu_t\right)$	0.0000	0.5530	0.0000	0.5530	0.0000	0.5530	0.0000	0.5531
$E\left(f\left(\mu_t\right)\right)$	0.0000	0.0108	0.0000	0.0108	0.0000	0.0108	0.0000	0.0108
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0240	0.0000	0.0240	0.0000	0.0240	0.0000	0.0240

Notes: Each column (1), (2), (3), and (4) shows the results of the four different RCPs both for thelaissez-faire and the oprimal policy (counter-factual) equilibrium under both the case of non-separable dis-utility and production damages, where we use the estimated values obtained for non-separable utility.

TABLE 2.4

RCP scenarios simulation results under non-separable dis-utility

	RCP Scenarios							
	RCP 2.6		RCP 3		RCP 6		RCP 8.5	
	(1)		(2)		(3)		(4)	
	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy	Laissez-faire	Optimal Policy
Business-cycle								
$E\left(y_t\right)$	0.9008	0.9017	1.4375	1.4424	1.8660	1.8777	3.7655	3.8708
$E\left(c_t\right)$	0.4912	0.4919	0.7716	0.7747	0.9847	0.9913	1.7554	1.7860
$E\left(\lambda_t\right)$	24.5171	24.3753	3.6020	3.5449	1.2617	1.2281	0.0867	0.0789
$E\left(x_t\right)$	850.9184	790.7758	1123.7717	1010.6681	1314.6911	1155.3967	2014.5926	1627.9942
$E(\mathcal{W}_t)$	-2439.6266	-2428.7533	-568.1892	-561.2570	-256.5978	-251.3565	-34.3506	-32.0386
$E(\tau_t)$	0.0000	0.0009	0.0000	0.0021	0.0000	0.0033	0.0000	0.0102
Asset-pricing								
$400E\left(r_t^F\right)$	5.8685	5.8683	5.8736	5.8731	5.8778	5.8770	5.8976	5.8946
$400E\left(r_{t+1}^B - r_t^F\right)$	0.3264	0.3267	0.3232	0.3236	0.3205	0.3211	0.3077	0.3097
$E\left(\pi_{t+1}\right)$	0.9982	0.9982	0.9975	0.9974	0.9969	0.9968	0.9949	0.9943
$E(RRA_t)$	8.7429	8.7303	5.5574	5.5356	4.3467	4.3184	2.3747	2.3279
Environment								
$E\left(\mu_t\right)$	0.0000	0.0751	0.0000	0.1064	0.0000	0.1284	0.0000	0.2091
$E\left(f\left(\mu_t\right)\right)$	0.0000	0.0000	0.0000	0.0001	0.0000	0.0002	0.0000	0.0007
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0014	0.0000	0.0025	0.0000	0.0034	0.0000	0.0074

Notes: Each column (1), (2), (3), and (4) shows the results of the four different RCPs both for thelaissez-faire and the oprimal policy (counter-factual) equilibrium under both the case of non-separable dis-utility and production damages, where we use the estimated values obtained for non-separable utility.

TABLE 2.5
RCP scenarios simulation results under production damages

2.6.4 Transition pathway scenario analysis

Turning to the transition pathways, I investigate the behavior of the model's policies under, on one hand, the three different environmental policies, and, on the other hand, the two monetary policies. As I consider the pathways of different monetary and financial aggregates, I limit my analysis to a 10 year horizon. As for the RCP scenarios, I use an exogenous growth rate of GDP in order to simulate economic growth. I first compare the laissez-faire scenario with the environmental policy scenario, before primarily focusing on the differences between the environmental policy scenarios and the interaction with different monetary policy rules.

Figure 2.1 and Figure 2.2 highlight the key role of the damage specification with respect to emission stock control as well as with respect to household lifetime welfare under the presence of the environmental externality. The main finding is that the non-separable dis-utility allows for capturing the benefits of the tax on the consumption levels of households, while the production damages case fails to capture the benefits of the tax as it is a burden on household disposable income. Furthermore, the non-separable dis-utility specification allows for capturing the impacts of fiscal policy on monetary aggregates, confirming findings of Bauer and Rudebusch [2020], while the optimal tax on the case of production damages doesn't affect the monetary aggregates. This finding is of high importance, as monetary policy intervention in mitigation efforts hinges on linkages between climate risks and macro-finance stability indicators, which makes it within the mandate of central banks to intervene.

More specifically, there is a clear trade-off between the two modeling specifications. On one hand, under the non-separable dis-utility case, emissions are reduced at a higher rate but require higher disposable income costs. On the other hand, under the production damages case, disposable income is higher; however, emission reduction is significantly lower. Also, the optimal environmental policy has a clear impact on the real rate and inflation, and allows for a substantial increase of these macro-financial aggregates as compared to the laissez-faire under the non-separable utility case. Meanwhile, under the production damages specification, there is almost no policy impact as the stochastic discount factor allowing for interest rates to vary is not directly impacted by the climate risk nor climate policies.

Figure 2.3, Figure 2.4, Figure 2.5, and Figure 2.6, show the key role monetary policy plays in climate mitigation. When central banks follow a Taylor rule under the presence of the climate externality (Figure 2.3), emissions are reduced at a significantly higher rate than under an optimal Ramsey rule (Figure 2.4). However, this higher emission reduction requires a low level of inflation, while under the Ramsey policy, central banks are able to raise inflation keeping interest rates stable. Monetary authorities clearly introduce a trade-off between higher inflation targets and stable real rates, on one hand, and emissions reduction, on the other hand, when they set their interest rate targets. Higher inflation rates and higher real rates crowd out investment and thus reduce abatement levels, which trigger lower levels of emission reduction. This is an important finding as it clearly shows the impact of monetary policy on climate aggregates.

Finally, Figure 2.7, Figure 2.8, Figure 2.9, and Figure 2.10 focus on the dynamics of the economy pathways under the three different environmental policy scenarios adopting a Taylor and Ramsey monetary rule, respectively. In the case of non-separable dis-utility and under a Taylor monetary rule, the optimal environmental policy is the best instrument insofar as its capacity to efficiently meet emissions reduction targets without major sacrifices in household lifetime wealth. The optimal policy also allows for slightly higher interest rates and inflation rates than the carbon fixed rate policy and carbon cap and trade policy, which are both unable to trigger sufficient levels of abatement in order to efficiently mitigate CO_2 emissions. However, under the production damages case, both the fixed carbon rate and cap and trade policies allow for higher emissions reductions, thus requiring the social planner to use a second best instrument as the optimal policy emission reduction results are not enough to match the desired Paris Agreement levels. Turning now to a central bank that sets its policy following an optimal Ramsey rule, it is clear that real rates as well as inflation are at desired levels under all three environmental policy scenarios. However, the trade-off between keeping interest rates stable and achieving positive higher inflation, on one hand, and lower emissions, on the other hand, fails to materialize under both the optimal policy case and the fixed tax rate case. Abatement investment decreases, thus driving emissions upward. The most efficient policy from an emission reduction perspective in this case is the fixed cap policy as it allows for higher abatement levels in order to keep emissions under a specific targeted level. However, the welfare cost is far more important than the two other environmental policies.

The interaction between different monetary and fiscal policies highlights the difficulty and inability, on one hand, to achieve a significant emission reduction and welfare gain, and on the other hand, to keep macro-finance aggregates under desired targeted levels. This confirms the findings of Benmir and Roman [2020], thus, suggesting the need to further investigate the efficacy of different macro-prudential and/or unconventional monetary policy that could help dampen the negative welfare impacts of second best fiscal policy or optimal monetary policy. Benmir and Roman [2020] show how each one of these two policies is able to reduce the inefficiency wedge on welfare as well as on financial volatility.

2.7 Conclusion

Drawing from the macro-finance and climate literatures, this paper investigates how modeling climate damages either through a utility function or through production function under the presence of monetary policy, impact emissions reduction targets, carbon pricing, and macro-financial aggregate moments and pathways.

In the modeled economy, I show how modeling damages, stemming from climate risk either through the utility function or production function, is fundamental in determining both CO_2 market price and the dynamics of the macro-finance aggregates. Furthermore, the modeling choice plays a central role in triggering high or low levels of abatement, thus achieving or not the emissions reduction targets aligned with the Paris Agreement. I show that depending on the modeling choice, the optimal policy restoring the first best allocation is sufficient to decrease emissions by a significant amount in the case of non-separable disutility modeling, whereas the same policy instrument fails to achieve the Paris Agreement target under the production damages modeling. In this case, a second best instrument (i.e. a fixed policy rate) is necessary to achieve the targeted emissions reduction.

From a macro-finance perspective, real rates and inflation have opposite dynamics when climate damages are modeled through the utility function or through the production function.

Another important result is the identification of the major role that the monetary authority plays in emissions reduction when setting its policy rates. Relying on a Taylor rule or a following a Ramsey optimal policy when setting its rates under the presence of the externality, central banks face a trade-off between emission reduction and real rate/inflation targeting.

While this paper focuses primarily on fiscal and monetary policies, future research could investigate how the climate externality affects the natural rate. For example, one could introduce financial intermediaries, and estimating an economy where different frictions are present and then using the estimates in a friction-less RBC framework to investigate the dynamics of the natural real rates.

Another limitation of this paper is that the households are not heterogeneous, thus the welfare analysis could further benefit from the introduction of heterogeneous households à la Ben Moll in order to address the different welfare impacts following different household distributions using a Heterogeneous Agent New Keynesian (HANK) model.

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Appendices

2.A Appendix - A: tables

Model counterpart	Name	Values
Standard Model Parameters		
h	Consumption habits	0.8
$\bar{N}$	Labor supply	0.20
δ_K	Depreciation rate of capital	0.025
$\bar{g}/\bar{y}$	Public spending share in output	0.20
ϵ_k	Investment cost curvature	1.45151
ϕ	Share of the externality in utility flow	0.67756
σ	Risk aversion	4.19826
b	Discount rate parameter	0.14331
$(\gamma_A - 1) \times 100$	Productivity growth rate	0.340
Environmental Parameters		
$\bar{x}$	Atmospheric carbon (gigatons) in laissez-faire	840
$[4(1 - \gamma_A \gamma_E^{1-\varphi_2} \eta)]^{-1}$	Half-life of CO2 in years	139
θ_1	Abatement cost	0.05607
θ_2	Curvature abattement cost	2.8
υ_1^{Temp}	Temperature reaction parameter	0.5
υ_2^{Temp}	Temperature reaction parameter	0.00125
d_0	Damage function parameter	1.3950e-3
d_1	Damage function parameter	-6.6722e-6
d_2	Damage function parameter	1.4647e-8
d_3	Damage function parameter	1
$(\gamma_E - 1) \times 100$	Output-CO2 (de)coupling rate	-0.45
φ_2	Output-CO2 elasticity	0.367
New Keynesian Parameters		
ϵ	Imperfect substitution between goods	6
χ	Rotemberg adjustment cost	100
ρ_r	Monetary policy smoothing	.8
ϕ_π	Reaction central bank to inflation	5.8
ϕ_y	Reaction central bank to output gap	.1125
ϕ_x	Reaction central bank to stock of emissions	2.1

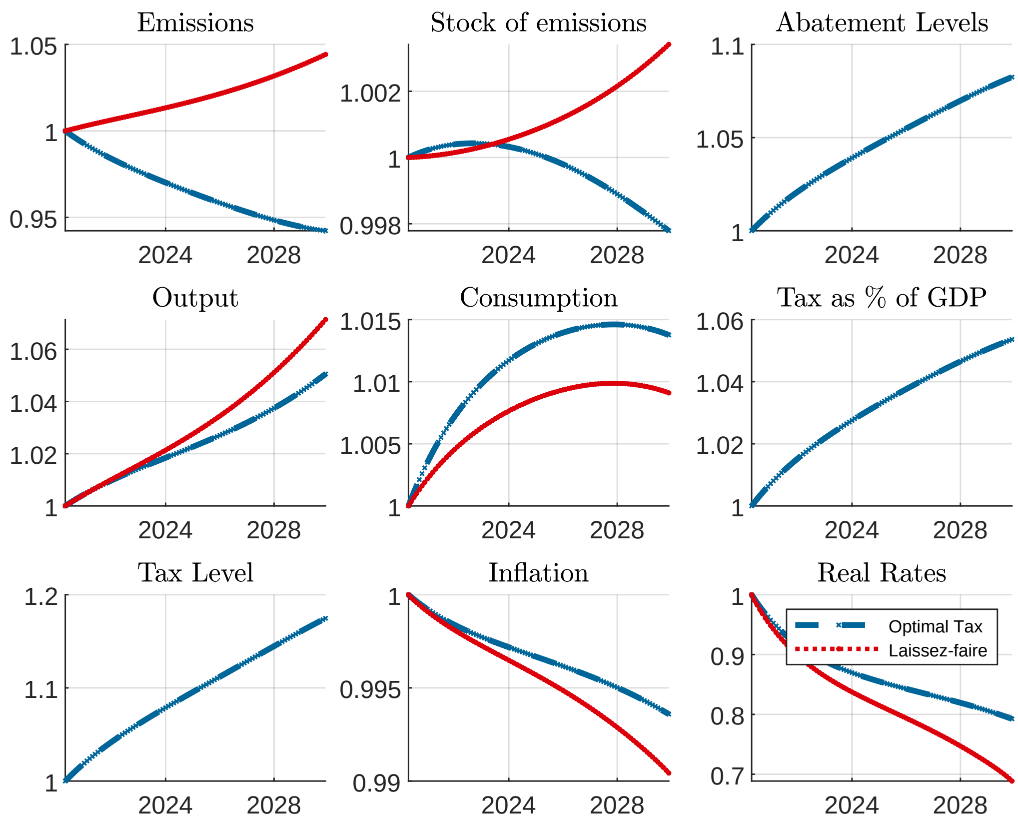
TABLE 2.6
Calibrated parameter values (Quarterly basis)

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes:					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.01	1	0.0077 *[0.0070;0.0084]*
Std. spending	σ_G	$\mathcal{IG}_1$	0.01	1	0.0258 *[0.0235;0.0281]*
Std. abatement	σ_X	$\mathcal{IG}_1$	0.01	1	0.020 *[0.0183;0.0218]*
Std. investment	σ_I	$\mathcal{IG}_1$	0.01	1	0.0135 *[0.0117;0.0152]*
Std. investment	σ_R	$\mathcal{IG}_1$	0.01	1	0.0046 *[0.0042;0.0051]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.9705 *[0.9661;0.9744]*
AR(1) spending	ρ_G	$\mathcal{B}$	0.50	0.20	0.9668 *[0.9606;0.9732]*
AR(1) abatement	ρ_X	$\mathcal{B}$	0.50	0.20	0.8939 *[0.8447;0.9453]*
AR(1) investment	ρ_I	$\mathcal{B}$	0.50	0.20	0.9965 *[0.9940;0.9990]*
AR(1) investment	ρ_R	$\mathcal{B}$	0.50	0.20	0.9662 *[0.9467;0.9887]*
Log-marginal data density					3170.493970

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{U}$ the uniform distribution.

TABLE 2.7
Prior and Posterior distributions of structural parameters

2.B Appendix - B: Figures



Notes: The pathways are generated using perfect foresight planner solution. I include both a GDP exogenous growth and a tax growth rate for the case when the tax is set as a fixed rate. I also exclude the first period and then normalize to one as the initial steady states are different. The y-axis represents deviation from the initial steady state.

FIGURE 2.1. The main economy variables pathways under the Laissez-faire versus the Optimal Policy: The case of non-separable dis-utility

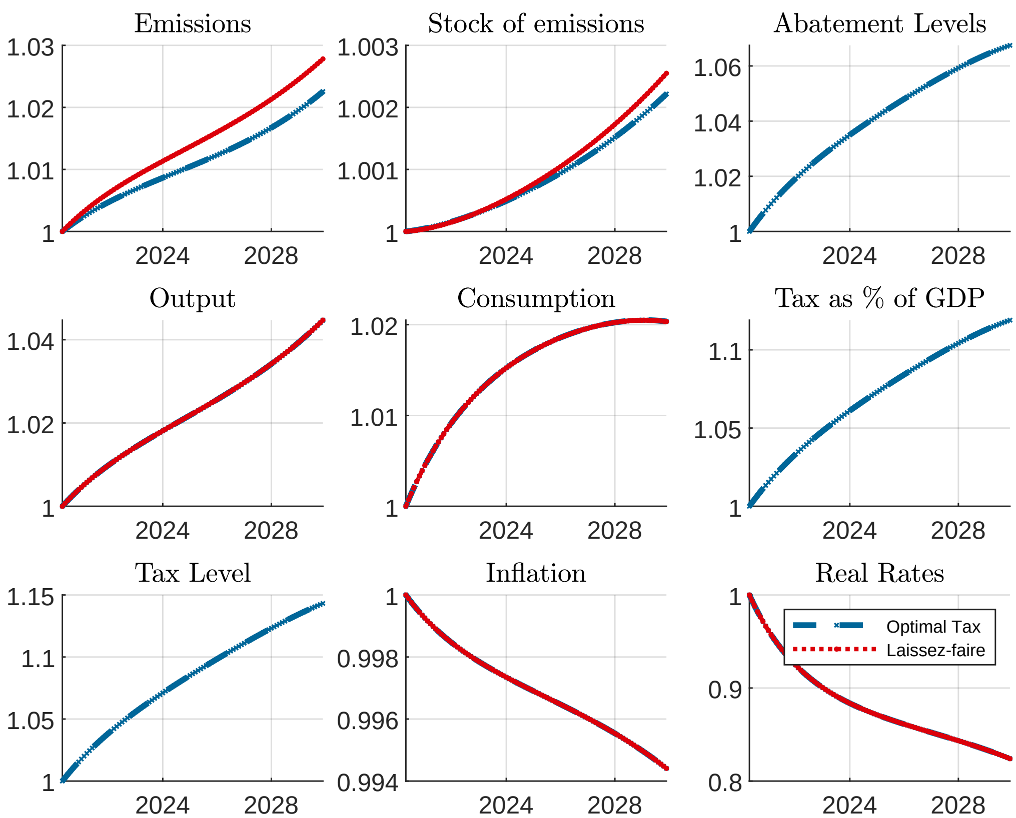


FIGURE 2.2. The main economy variables pathways under the Laissez-faire versus the Optimal Policy: The case of production damages

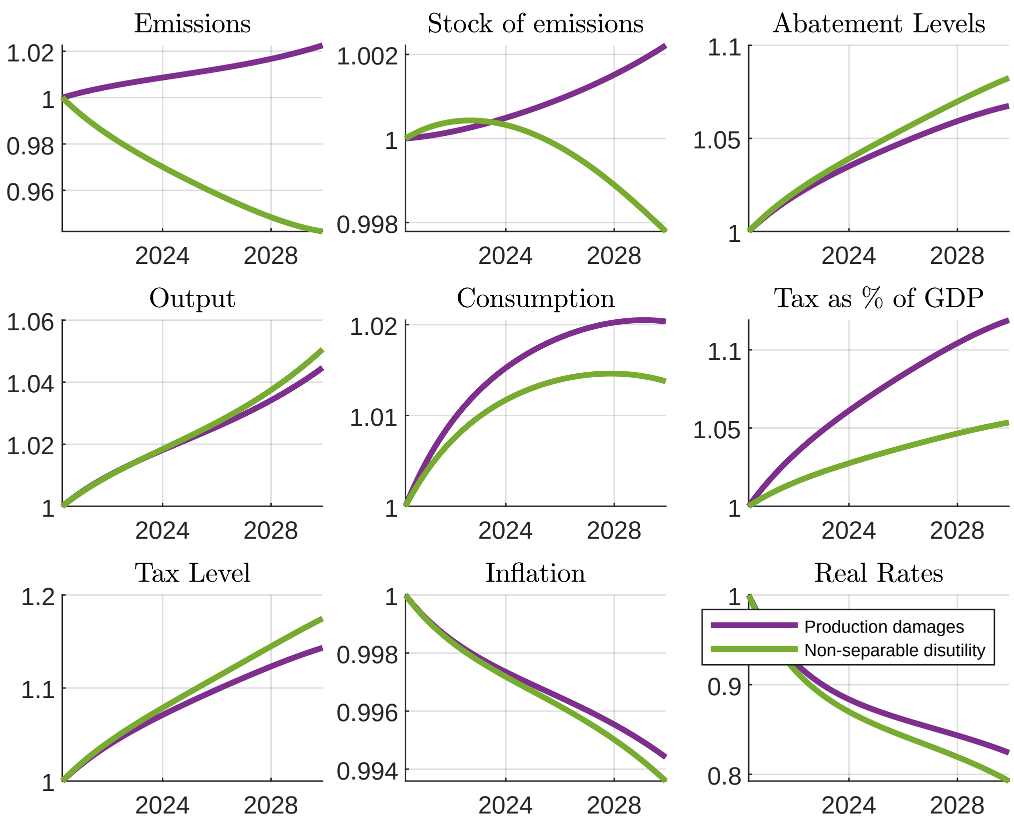


FIGURE 2.3. The main economy variables pathways under a Taylor Optimal Monetary Rule: The case of non-separable dis-utility versus production damages

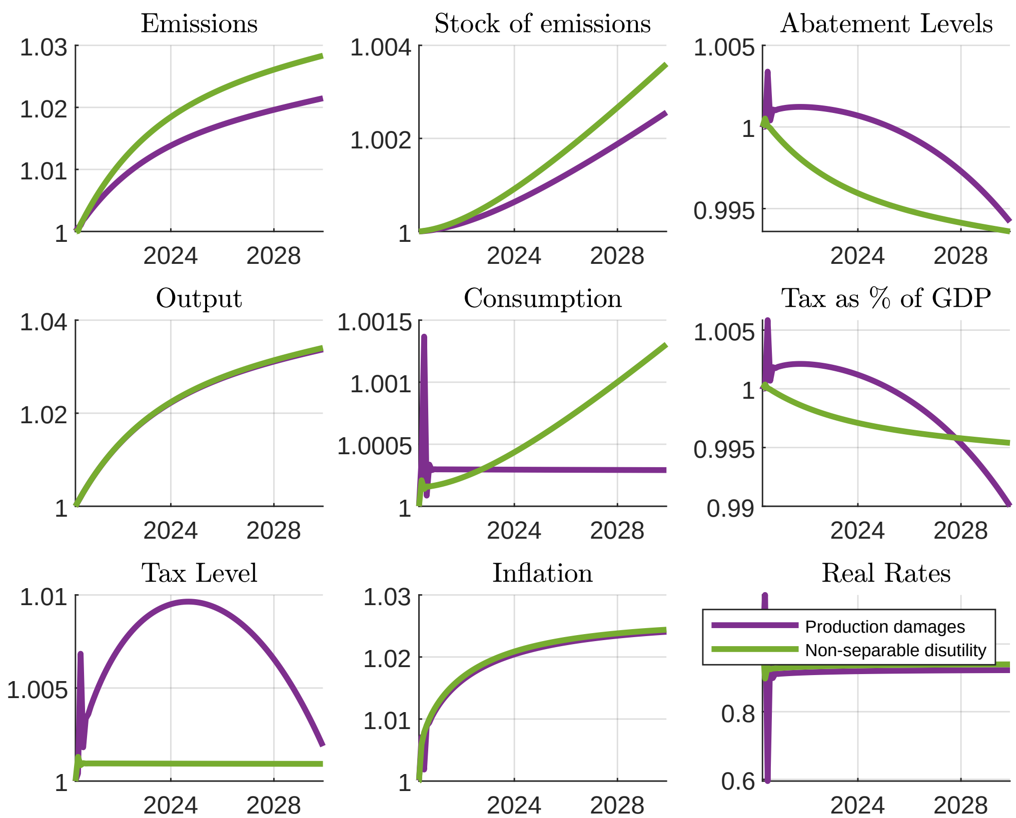


FIGURE 2.4. The main economy variables pathways under a Ramsey Optimal Monetary Rule: The case of non-separable dis-utility versus production damages

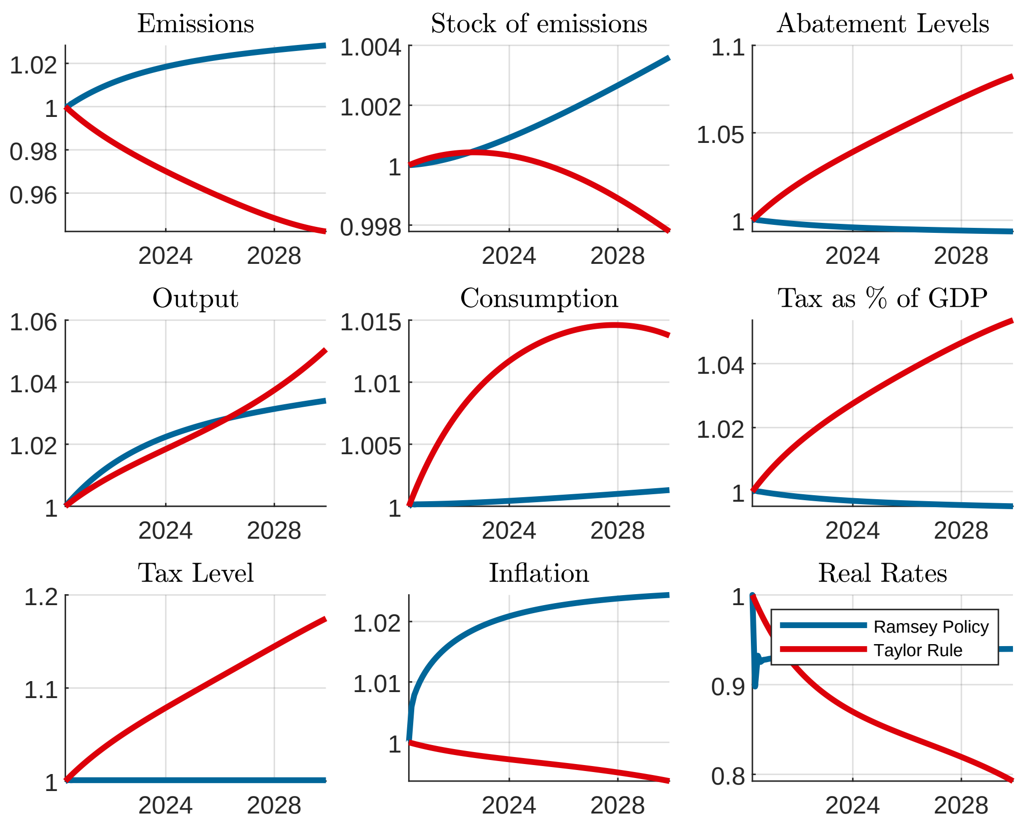


FIGURE 2.5. The main economy variables pathways under a Taylor rule and a Ramsey Optimal Monetary Rule: The case of non-separable dis-utility

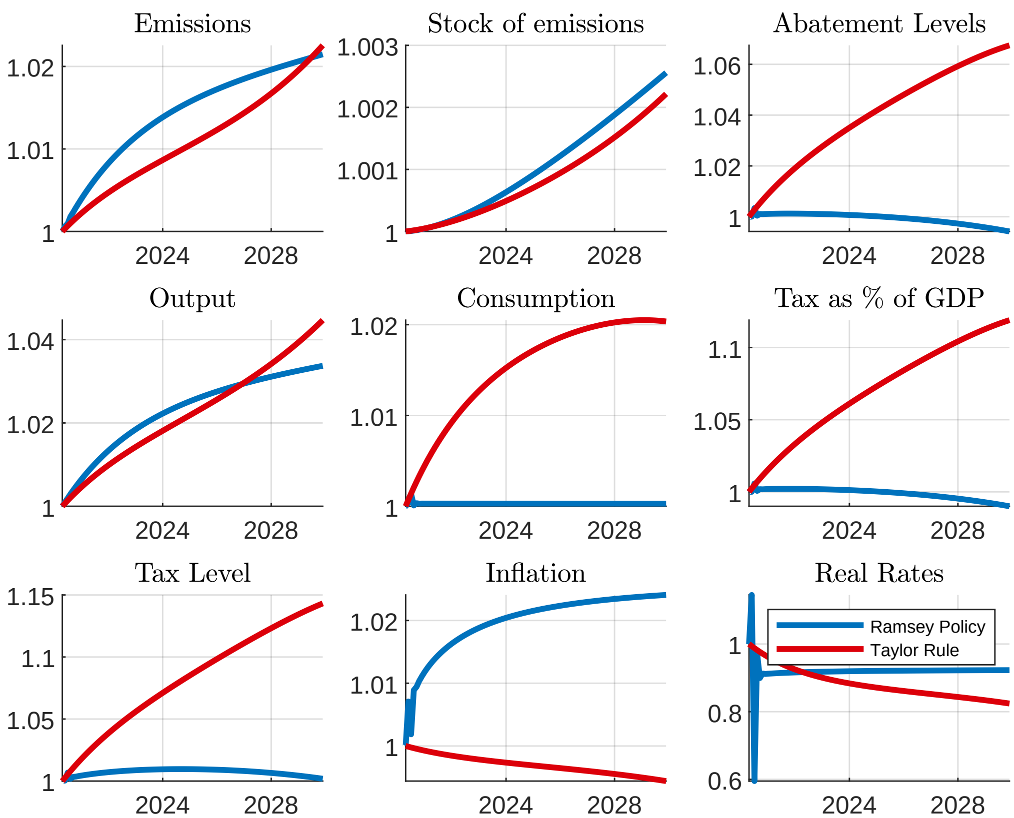


FIGURE 2.6. The main economy variables pathways under a Taylor rule and a Ramsey Optimal Monetary Rule: The case of production damages

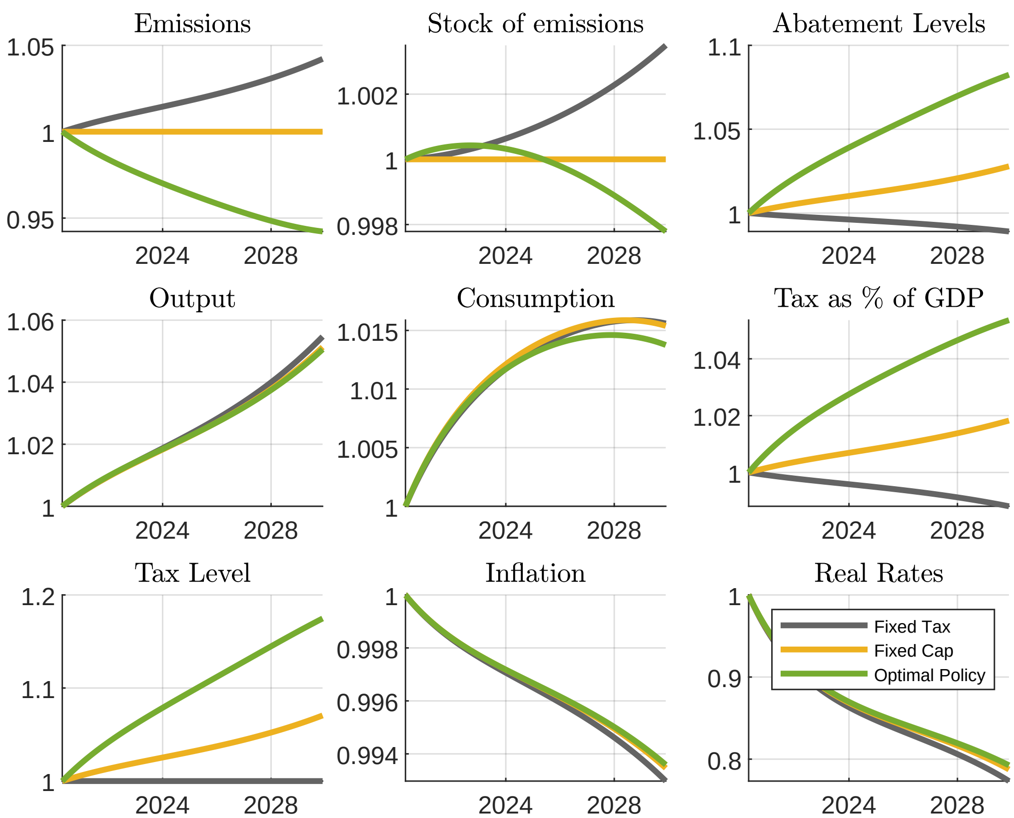


FIGURE 2.7. The main economy variables pathways: The non-separable dis-utility case under the three environmental policies and Taylor monetary rule.

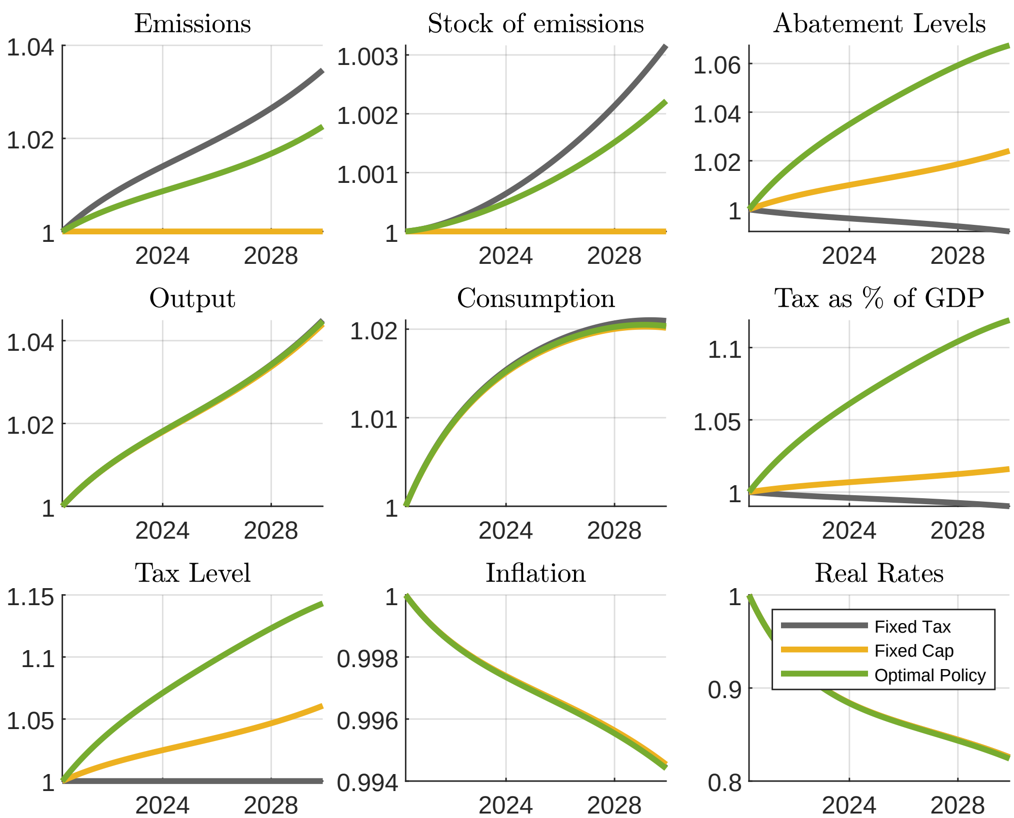


FIGURE 2.8. The main economy variables pathways: The production damages case under the three environmental policies and Taylor monetary rule.

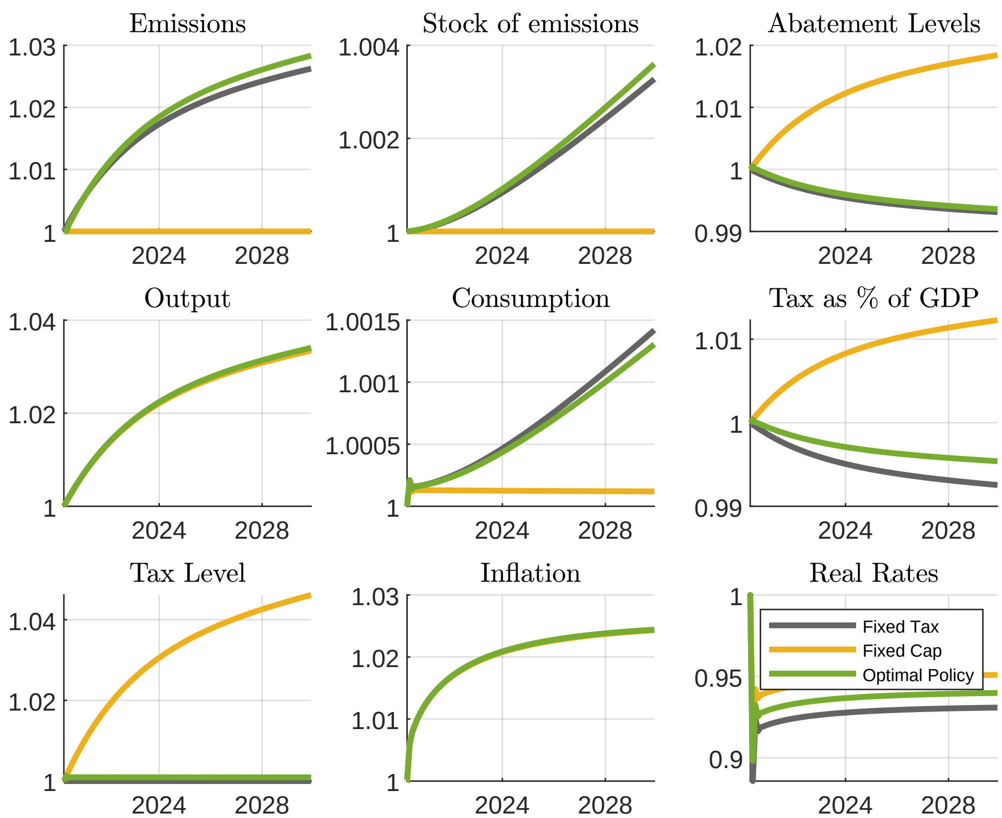


FIGURE 2.9. The main economy variables pathways: The non-separable dis-utility case under the three environmental policies and Ramsey monetary rule.

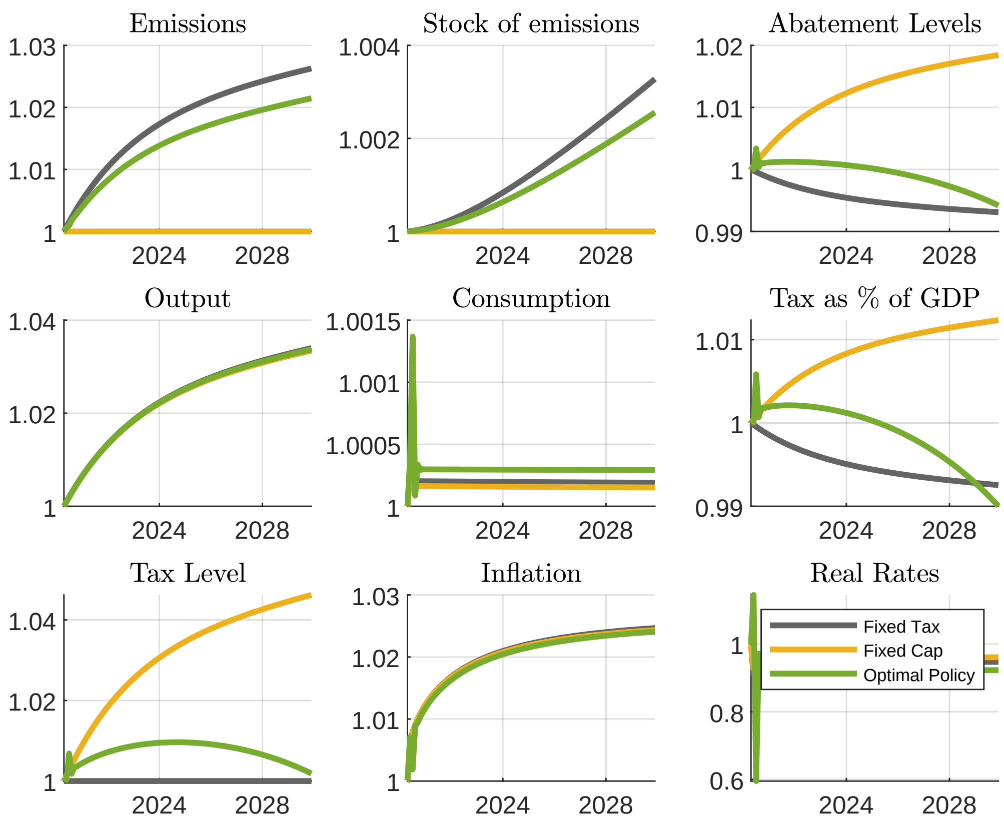


FIGURE 2.10. The main economy variables pathways: The production damages case under the three environmental policies and Ramsey monetary rule.

2.C Appendix - C: The optimal tax

2.C.1 Centralized problem: the household problem

The first best equilibrium under a balanced growth path is characterized as follows:

$$
\begin{aligned}
\mathcal{L} = E_0 \Bigg\{ & \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma} \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \left[p_t y_t - c_t - i_t - g_t - f(\mu_t) y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t q_t \left[(1-\delta) k_t + \left[\frac{\chi_1}{1-\epsilon_k} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon_k} + \chi_2 \right] k_t - \gamma^Y k_{t+1} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t \varrho_t \left[\varepsilon_t^A (1 - d(x_t)) k_t^{\alpha} n_t^{1-\alpha} - y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t v_{Xt} \left[\gamma^X x_{t+1} - \eta x_t - e_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_t v_{Et} \left[e_t - (1-\mu_t) \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2} \right] \Bigg\}
\end{aligned}
$$

The marginal utility of consumption c_t is:

$$
\lambda_t = (c_t - \phi x_t)^{-\sigma} \tag{2.29}
$$

Optimal investment i_t is given by:

$$
1 = \varepsilon_t^I q_t \chi_1 \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{-\epsilon_k} \tag{2.30}
$$

The optimal capital supply is given by:

$$
q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{\chi_1}{1-\epsilon_k} \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon_k} + \chi_2 - \chi_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon_k} \right) + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\} \tag{2.31}
$$

where:

$$\beta^Y = \widetilde{\beta}/\gamma^Y$$

The first-order condition for output y_t is:

$$[mc_t - f(\mu_t)] - \varrho_t - v_{Et}(1-\varphi_2)\frac{e_t}{y_t} = 0 \tag{2.32}$$

The optimal fraction of abatement μ_t is given by:

$$f'(\mu_t)\, y_t = v_{Et}\frac{e_t}{(1-\mu_t)} \tag{2.33}$$

The optimal quantity of emissions e_t per quarter reads as:

$$v_{Et} = v_{Xt} \tag{2.34}$$

The social cost of carbon ($SCC_t = v_{Xt}$) reads as:

$$\lambda_t v_{Xt} = \beta^X E_t \phi\,(c_{t+1} - \phi x_{t+1})^{-\sigma} + \eta\beta^X E_t\lambda_{t+1}v_{Xt+1} + \beta^X E_t\lambda_{t+1}(b+2cx_{t+1})(1-d(x_{t+1}))^{-1}\varrho_{t+1}y_{t+1} \tag{2.35}$$

where:

$$\beta^X = \widetilde{\beta}/\gamma^X \tag{2.36}$$

Finally, the first order conditions with respect to bonds holding reads as follows:

$$(1+r_t^F)^{-1} = \widetilde{\beta}\frac{\lambda_{t+1}}{\lambda_t} \tag{2.37}$$

$$E_t\frac{p_t^B}{(p_{t+1}^B+1)} = \widetilde{\beta}\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}} \tag{2.38}$$

2.C.2 Intermediate firms problem

First the intermediate firms decide on their demand for inputs (labor and capital) by minimizing their costs:

$$\min_{k_{j,t-1},\mu_{j,t},e_{j,t}} r_t^k k_{j,t-1} + f(\mu_{j,t})y_{j,t} + \tau_t e_{j,t} \tag{2.39}$$

s.t.

$$(1-d(x_t))\epsilon_t^A k_{j,t-1}^{\alpha}(n_{j,t})^{1-\alpha} \geq y_{j,t} \tag{2.40}$$

$$e_{j,t} = (1-\mu_{j,t})\,\varphi_1 y_{j,t}^{1-\varphi_2} \tag{2.41}$$

The minimization problem reads:

$$\begin{aligned}\mathcal{L} = E_0 \Bigg\{ & \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{\lambda_t}{\lambda_0} \left(r_t^k k_{j,t-1} + f(\mu_{j,t})y_{j,t} + \tau_t (1-\mu_{j,t})\,\varphi_1 y_{j,t}^{1-\varphi_2} \right) \\ & + \sum_{t=0}^{\infty} \widetilde{\beta}^t \varrho_{j,t} \left[(1-d(x_t))\epsilon_t^A k_{j,t-1}^{\alpha}(n_{j,t})^{1-\alpha} - y_{j,t} \right] \\ & + \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{\lambda_t}{\lambda_0} v_{Et} \left[(e_{j,t} - (1-\mu_{j,t})\,\varphi_1 y_{j,t}^{1-\varphi_2}) \right] \Bigg\}\end{aligned}$$

The first-order condition with respect to the firm's optimal choice of output, abatement, and emissions are as follows:

$$r_t^k = \alpha \varrho_{j,t}(1-d(x_t))\epsilon_t^A k_{j,t-1}^{\alpha-1}(n_{j,t})^{1-\alpha} \tag{2.42}$$

$$v_{Et} = \frac{\theta_1\theta_2}{\varphi_1}\mu_{j,t}^{\theta_2-1} y_{j,t}^{\varphi_2} \tag{2.43}$$

$$v_{Et} = \tau_t \tag{2.44}$$

where $\varrho_{j,t}$ is nominal marginal cost expression of firm j. Since firms face the same factor prices, the marginal cost is equal across intermediate firms ($\varrho_{j,t} = \varrho_t$).

Rewriting the first order conditions in terms of the real marginal cost, denoted as mc_t, the marginal cost reads (FOC with respect to $y_{j,t}$):

$$mc_{j,t}(=mc_t) = \varrho_t + \theta_1 \mu_t^{\theta_2} + (1-\varphi_2)\tau_t(1-\mu_t)\varphi_1 y_t^{-\varphi_2} \tag{2.45}$$

Thus, the profits for intermediate firms reads:

$$\Pi_{j,t} = \left(\frac{p_{j,t}}{p_t} - mc_t\right) y_{it} \tag{2.46}$$

In addition, the monopolistic firms engage in price setting à la Rotemberg, where price updates are subject to adjustment costs given by $\Delta_{it} = 0.5\theta \left(p_{j,t}/p_{j,t-1} - 1\right)^2$.

Thus, the profit maximization of intermediate firms reads as follows:

$$\max_{\{p_{j,t}\}} \sum_{\tau=0}^{\infty} m_{t,t+\tau} \left(\left[\frac{p_{j,t+\tau}}{p_{t+\tau}} - mc_{t+\tau}\right] y_{j,t+\tau} - \Delta_{j,t+\tau} y_{t+\tau} \right) \tag{2.47}$$

where $p_{j,t}$ is the optimal selling price for firms and $m_{t,t+\tau}$ is the stochastic discount factor taken from the household problem, and $y_{j,t+\tau} = \left(\frac{p_{j,t+\tau}}{p_{t+\tau}}\right)^{-\epsilon} y_{t+\tau}$.

The first order condition with respect to price setting $p_{j,t}$ reads as follows:

$$\chi\pi_t\left(\pi_t - 1\right) = (1-\epsilon) + \epsilon mc_t + E\left\{\beta^Y \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \chi\pi_{t+1}\left(\pi_{t+1} - 1\right)\right\} \tag{2.48}$$

2.C.3 Laissez-faire equilibrium

In the laissez-faire economy, there is no environmental policy, and thus the tax is set equal to zero and firms are not engaging in abatement efforts:

$$\tau_t = 0 \tag{2.49}$$

$$\mu_t = 0 \tag{2.50}$$

2.C.4 Competitive equilibrium under optimal policy

The first-best equilibrium that corresponds to the problem of the social planner can be attained by setting the tax τ_t equal to the price of carbon. In the centralized equilibrium, the price of carbon is determined by the optimality condition with respect to x_t. The optimal tax is therefore:

$$\tau_t = v_{Xt} \tag{2.51}$$

Once the optimal tax is implemented, in the laissez-faire equilibrium implies that:

$$v_{Et} = v_{Xt} \tag{2.52}$$

The optimality condition shown in equation (2.34) is therefore satisfied, as the cost of abating emissions is exactly equal to the social cost of emissions.

2.C.5 Fixed tax policy (price instrument) or a fixed cap policy (quantity instrument)

A second best equilibrium involves setting setting the carbon tax to a fixed rate or setting emissions according to fixed cap, which would facilitate reaching a specific emissions target.

2.C.5.1 Fixed tax policy

$$\tau_t = \text{Fixed Carbon Tax} \tag{2.53}$$

2.C.5.2 Fixed cap policy

$$e_t = \text{Fixed Carbon Cap} \tag{2.54}$$

2.D Appendix - D: The Ramsey monetary policy

$$
\begin{aligned}
\mathcal{L} = E_0 & \left\{ \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma} \right. \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{1,t} \left[p_t y_t - c_t - i_t - g_t - f(\mu_t) y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{2,t} \left[(1-\delta) k_t + \left[\frac{\chi_1}{1-\epsilon} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon_k} + \chi_2 \right] k_t - \gamma^Y k_{t+1} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{3,t} \left[\varepsilon_t^A (1 - d(x_t)) k_t^\alpha n_t^{1-\alpha} - y_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{4,t} \left[\gamma^X x_{t+1} - \eta x_t - e_t \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{5,t} \left[e_t - (1-\mu_t) \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{6,t} \left[\lambda_t - (c_t - \phi x_t)^{-\sigma} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{7,t} \left[1 - \varepsilon_t^I q_t \chi_1 \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{-\epsilon_k} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{8,t} \left[q_t - \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{\chi_1}{1-\epsilon_k} \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon_k} + \chi_2 - \chi_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon_k} \right) - \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{9,t} \left[\left[mc_t - f(\mu_t) \right] - \varrho_t - v_{Et} (1-\varphi_2) \frac{e_t}{y_t} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{10,t} \left[f'(\mu_t) y_t - v_{Et} \frac{e_t}{(1-\mu_t)} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{12,t} \left[\lambda_t v_{Xt} - \beta^X E_t \phi (c_{t+1} - \phi x_{t+1})^{-\sigma} - \eta \beta^X E_t \lambda_{t+1} v_{Xt+1} - \beta^X E_t \lambda_{t+1} (b + 2c x_{t+1}) (1 - d(x_{t+1}))^{-1} \varrho_{t+1} y_{t+1} \right] \\
& + \sum_{t=0}^{\infty} \widetilde{\beta}^t \lambda_{13,t} \left[r_t^k - \alpha \varrho_{j,t} (1 - d(x_t)) \epsilon_t^A k_{j,t}^{\alpha-1} (n_{j,t})^{1-\alpha} \right]
\end{aligned}
$$

$$+\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\lambda_{14,t}\left[\chi\pi_{t}\left(\pi_{t}-1\right)-(1-\epsilon)-\epsilon mc_{t}-E\left\{\beta^{Y}\frac{\lambda_{t+1}}{\lambda_{t}}\frac{y_{t+1}}{y_{t}}\chi\pi_{t+1}\left(\pi_{t+1}-1\right)\right\}\right]$$
$$+\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\lambda_{15,t}\left[(1+r_{t}^{F})^{-1}-\widetilde{\beta}\frac{\lambda_{t+1}}{\lambda_{t}}\right]$$
$$+\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\lambda_{16,t}\left[r_{t}-E_{t}\pi_{t+1}r_{t}^{F}\right]$$
$$+\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\lambda_{17,t}\left[E_{t}\frac{p_{t}^{B}}{(p_{t+1}^{B}+1)}-\widetilde{\beta}\frac{\lambda_{t+1}}{\lambda_{t}}\frac{1}{\pi_{t+1}}\right]$$
$$+\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\lambda_{18,t}\left[\text{The Chosen Environmental Policy}\right]\Bigg\}$$

The first order conditions with respect to the set of the defined variables are outlined below.

FOC with respect to c_t:

$$(c_t-\phi x_t)^{\sigma}=\lambda_{1,t}-\lambda_{6,t}\sigma(c_t-\phi x_t)^{-\sigma-1} \tag{2.55}$$

FOC with respect to x_{t+1}:

$$\gamma^{X}\lambda_{4,t}-\widetilde{\beta}\phi(c_{t+1}-\phi x_{t+1})^{\sigma}-\widetilde{\beta}\lambda_{3,t+1}(2cx_{t+1}+bx_{t+1})\frac{y_{t+1}}{(1-d(x_{t+1})}-\eta\widetilde{\beta}\lambda_{4,t+1} \tag{2.56}$$
$$-\widetilde{\beta}\lambda_{6,t+1}\sigma\phi(c_{t+1}-\phi x_{t+1})^{-\sigma-1}-\beta^{X}\lambda_{12,t}E_t\phi^{2}\left(c_{t+1}-\phi x_{t+1}\right)^{-\sigma-1}-\beta^{X}\lambda_{12,t}E_t\lambda_{t+1}2c\varrho_{t+1}\epsilon_t^{A}k_t^{\alpha}n_t^{1-\alpha}=0$$

FOC with respect to y_t:

$$\lambda_{1,t}(mc_t-f(\mu_t))-\lambda_{3,t}-\lambda_{5,t}(1-\mu_t)\epsilon_t^{X}\varphi_1 y_t^{-\varphi_2}+\lambda_{9,t}\varphi_2 v_{Et}(1-\varphi_2)(1-\mu_t)y_t^{-\varphi_2-1}+\lambda_{10,t}f'(\mu_t) \tag{2.57}$$
$$+\lambda_{14,t}E_t\left\{\beta^{Y}\frac{\lambda_{t+1}}{\lambda_t}\frac{y_{t+1}}{y_t^{2}}\chi\pi_{t+1}\left(\pi_{t+1}-1\right)\right\}-\widetilde{\beta}^{-1}\lambda_{8,t-1}(\varrho_t\alpha\frac{1}{k_t})=0$$

FOC with respect to i_t:

$$\begin{aligned}&-\lambda_{1,t}-\lambda_{2,t}(\chi_1(\epsilon_t^I\frac{i_t}{k_t})^{-\epsilon^k})+\lambda_{7,t}(q_t\epsilon_t^I\epsilon_k\chi_1\frac{i_t}{k_t})^{-\epsilon^k-1})\\&-\widetilde{\beta}^{-1}\lambda_{8,t-1}\beta^Y\frac{\lambda_t}{\lambda_{t-1}}q_t((\chi_1(1-\epsilon^k)-\chi_1(1-\epsilon^k)(\epsilon_t^I\frac{i_t}{k_t})^{-\epsilon^k})=0\end{aligned} \tag{2.58}$$

FOC with respect to k_t:

$$\begin{aligned}&\lambda_{2,t}\left(((1-\delta)+\Psi_{k_t}(.))k_t+(1-\delta)k_t+\left[\frac{\chi_1}{1-\epsilon}\left(\varepsilon_t^I\frac{i_t}{k_t}\right)^{1-\epsilon_k}+\chi_2\right]\right)-\lambda_{2,t-1}\widetilde{\beta}^{-1}\beta^Y\\&\lambda_{2,t}\alpha\frac{y_t}{k_t}-\lambda_{2,t}\epsilon_t^Iq_t\chi_1\Psi_{k_t}-\lambda_{13,t}\alpha(\alpha-1)\frac{y_t}{k_t^2}\\&-\widetilde{\beta}^{-1}\lambda_{8,t-1}\left(\beta^YE_t\frac{\lambda_t}{\lambda_{t-1}}q_t\Psi_{k_t}+\varrho_t\alpha\frac{y_t}{k_t^2}\right)=0\end{aligned} \tag{2.59}$$

FOC with respect to μ_t:

$$-\lambda_{1,t}f'(\mu_t)y_t+\lambda_{5,t}\epsilon_t^X\varphi_1y_t^{1-\varphi_2}+\lambda_{9,t}(-f'(\mu_t)+\upsilon_{Et}(1-\varphi_2)y_t^{-\varphi_2})+\lambda_{10,t}\left(f''(\mu_t)-\upsilon_{Et}\frac{e_t}{(1-\mu_t)^2}\right)=0 \tag{2.60}$$

FOC with respect to e_t (in case of a cap and trade policy, the last term must be added, otherwise, the term disappears):

$$-\lambda_{4,t}+\lambda_{5,t}-\lambda_{10,t}\frac{\upsilon_{Et}}{1-\mu_t}+\lambda_{18,t}\text{Cap Policy}=0 \tag{2.61}$$

FOC with respect to τ_t:

$$\lambda_{18,t}=0 \tag{2.62}$$

FOC with respect to υ_{Et}: (when the environmental policy is set optimally, the last term needs to be added)

$$-\lambda_{9,t}\left((1-\varphi_2)\frac{e_t}{y_t}\right)-\lambda_{10,t}\left(\frac{e_t}{(1-\mu_t)}\right)-\lambda_{18,t}=0 \tag{2.63}$$

FOC with respect to v_{Xt}:

$$\lambda_{12,t}\lambda_t-\eta\beta^X\widetilde{\beta}^{-1}\lambda_t\lambda_{12,t-1}=0 \tag{2.64}$$

FOC with respect to r_t:

$$\lambda_{16,t}=0 \tag{2.65}$$

FOC with respect to r_t^F:

$$-(1+r_t^F)^{-2}\lambda_{15,t}-\lambda_{16,t}E_t\pi_{t+1}=0 \tag{2.66}$$

FOC with respect to p_t^B:

$$\lambda_{17,t}E_t\frac{1}{p_{t+1}^B+1}-\widetilde{\beta}^{-1}\lambda_{17,t-1}\frac{p_{t-1}^B}{(1+p_t^B)^{-2}}=0 \tag{2.67}$$

FOC with respect to π_t:

$$\begin{aligned}&\lambda_{14,t}\left(\chi(\pi_t-1)+\chi\pi_t\right)+\widetilde{\beta}^{-1}\lambda_{14,t-1}\left(\beta^Y\frac{\lambda_t}{\lambda_{t-1}}\frac{y_t}{y_{t-1}}\left(\chi(\pi_t-1)+\chi\pi_t\right)\right)\\&-\widetilde{\beta}^{-1}\lambda_{16,t-1}r_{t-1}^F+\widetilde{\beta}^{-1}\lambda_{17,t-1}\beta^Y\frac{\lambda_t}{\lambda_{t-1}}\frac{1}{\pi_t}=0\end{aligned} \tag{2.68}$$

The simulation are then run using dynare Ramsey algorithm (Adjemian et al. [2011])

Chapter 3

Weitzman Meets Taylor: ETS Futures Drivers and Carbon Cap Rules

3.1 Introduction

In this paper, we estimate the drivers of the European Union Emission Trading System (EU ETS) cap-and-trade implicit carbon price, and elucidate the roles of different shocks in steering the carbon price over the studied period. To this end, we develop a macro-finance model that encompasses two sectors: i) an energy sector; and ii) a non-energy final sector, and accounts for climate dynamics. We first estimate our model using Bayesian techniques and retrieve the shock decomposition of the implicit carbon price. Then we compare the theoretical social cost of carbon to the estimated EU ETS implicit carbon price. Finally, we propose a new systematic approach to set the supply of carbon allowances that we call carbon cap rule. It allows for closing part of the gap with respect to the first best optimal carbon price and could be implemented by policy makers in practice.

Our main finding is that EU ETS carbon cap policy generates higher levels of price volatility, which are mainly driven by both abatement cost shocks and climate sentiment shocks, compared to the optimal SCC. We then demonstrate that reducing price uncertainty can help close the gap with respect to the optimal policy. To achieve this, we develop an innovative method to infer abatement shock series, using information contained in the market price of carbon, and propose a new carbon cap rule that allows for achieving a lower level of price volatility.

To internalize the impacts of the carbon externality, public economists have long argued that setting a carbon price equal to the social cost of carbon "SCC" (the shadow value of CO_2) would set the economy in its welfare enhancing trajectory while providing an incentive for emissions reduction. However, the level of the SCC has been the subject of major debates (e.g. Stern [2008] and Nordhaus [2008]). It remains very uncertain what is the correct level of the SCC as it hinges over the level of climate damages, the climate dynamics, and discount rate.

To address this challenge, many governments and public authorities from both advanced and developing economies have implemented either a carbon tax or a cap-and-trade system. The aim is to reduce emissions by either directly imposing a price on carbon or letting the price on carbon be determined by market participants trading carbon allowances. However, these policies cannot guarantee that the actual carbon price would lead to a first best optimal allocation, and may introduce market frictions due to the policy and market designs (Goulder [2013] and Jenkins [2014]).

The challenge that fiscal and public authorities face in setting a carbon policy, and thus an implicit carbon price, is similar to the challenges faced by monetary and financial policy authorities. Monetary authorities set interest rates according to specific rules, such as the Taylor [1993] rule, rather than directly following the natural interest rate which cannot be observed. A similar parallel can be drawn with the SCC, which as previously mentioned, is subject to various sources of uncertainty and is difficult to estimate and track over time. To address this issue, Grosjean et al. [2016] propose the idea of a central bank of carbon, which would function similarly to a monetary central bank. The regulator would set the carbon price and monitor the implicit carbon price, accounting for business cycle fluctuations that are argued to be important in Benmir et al. [2020]. Implementing such a system could help mitigate some of the market frictions associated with cap-and-trade market designs, and allow for closer alignment with the SCC.

Theory predicts that the allowance price should reflect market fundamentals related to the marginal costs of emissions abatement (e.g. Montgomery [1972] and Rubin [1996]).[28] Shifts in business-as-usual emissions, determined by changes in demand for allowances (e.g. weather, economic activity, and energy intensity of their products), and shifts in the supply of abatement (e.g. supply of fossil fuels, the response of consumers to fuel prices, and the cost of new technologies for production), change expectations about market fundamentals. Most existing cap-and-trade programs cover the largest domestic energy-intensive industries (e.g. electricity and heat production, cement manufacture, iron, and

[28]For a recent survey of permit pricing theory, see Weitzman [1974], Hoel and Karp [2002], Newell and Pizer [2003], Wood and Jotzo [2011], and Karp and Traeger [2018].

steel production), and changes in the demand for emissions and supply of abatement are likely to be the greatest source of uncertainty in the allowance market.

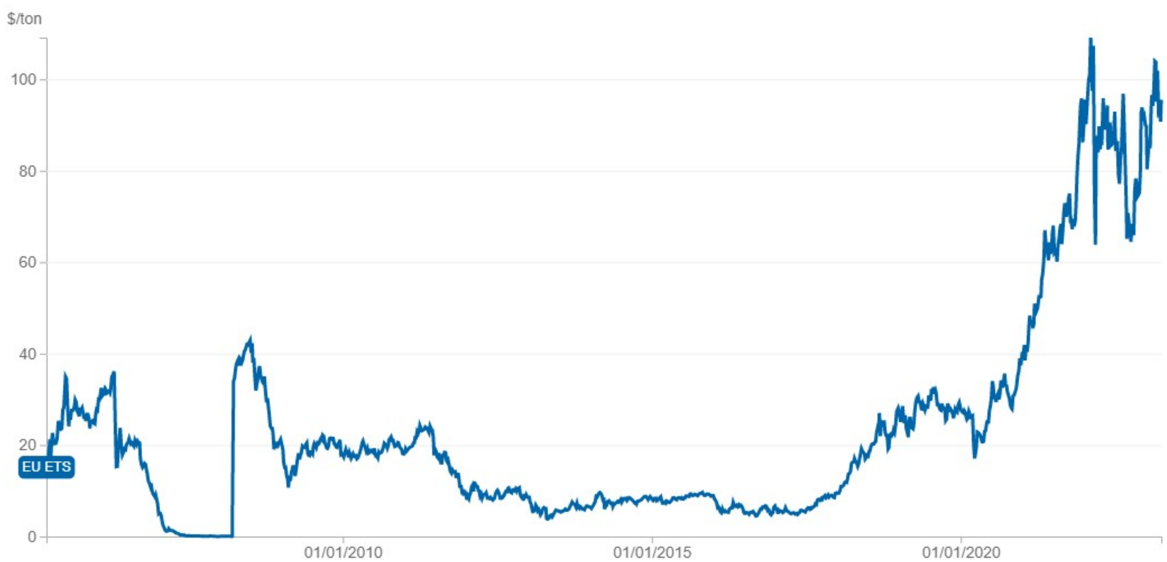
A series of studies empirically analyzes the relevance of the theoretically motivated allowance price drivers in the California cap-and-trade program (e.g. Borenstein et al. [2019]) and the EU ETS (e.g. Hintermann et al. [2016] and Friedrich et al. [2020]). On the demand side, the common finding is that fossil fuels played a major role. In particular, while most of the papers find that oil and gas play a significant role, coal seems not to be a relevant factor. Across most studies, economic activity and growth announcements are clear price drivers too. On the supply side, a challenge for empirical studies is that many price drivers are *not directly observable*. For example, while the supply of fossil fuels is observable, technological development and innovation, and expectations about them, are *unobservable*. Hence changes in costs of the abatement technology have been hardly considered in empirical studies, despite their relevance in the theoretical prediction of allowance prices.

Operational experience with cap-and-trade programs highlights that the allowance supply schedule is not static but subject to potential policy revisions. The California proposed regulatory amendments[29] in 2013 and the EU decision in 2021 on 2030 targets, are examples of an adjustment of the medium-term cap as part of the periodic updating of the long-term cap. At the same time, due to the inflexibility of most cap-and-trade designs to adjust the legislated caps within each commitment period to current contingencies (e.g. severe economic shocks), supply management mechanisms that make the cap endogenous have been discussed and, in some cases, introduced. A prime example of such market intervention is the so-called EU Market Stability Reserve. While interventions in the policy program are observable, shifts in the policy (shocks) are not, which leads to policy uncertainty. The European cap-and-trade program is particularly well suited for the analysis of the role of policy uncertainty. In response to severe demand shocks during the third

[29] The proposed revisions cover several areas of the regulation, including allocation and distribution of allowances, see: https://www.edf.org/sites/default/files/content/carbon-market-california-year_two.pdf

phase of the EU ETS, a series of proposals and decisions have been announced with the objective of restoring the stringency of the EU ETS cap. This provides us with a unique period characterized by significant policy events and associated carbon price volatility, as shown in Figure 3.1.

FIGURE 3.1. EU ETS Carbon Prices



Note: The figure presents the carbon prices in the EU ETS and is constructed using data from the International Carbon Action Partnership: https://icapcarbonaction.com.

Our approach builds upon these theoretical and empirical findings and answers a distinct research question from a large body of the environmental economics literature, both theoretical and empirical. Previous studies such as Fowlie [2010], Acemoglu et al. [2012], Fowlie and Perloff [2013], Aghion et al. [2016], Pommeret and Schubert [2018], and Acemoglu et al. [2019] have examined how allowance prices in the EU ETS impact macro-financial aggregates like clean technology investment. However, very little research has been conducted on the underlying factors driving the implicit carbon price in the EU ETS market.[30]

Our contribution to the literature is twofold. Firstly, we propose a novel strategy to estimate and decompose the drivers of the EU ETS. Secondly, we introduce a carbon cap rule that smooths business cycle fluctuations with respect to the EU ETS estimated policy, and get closer to the social cost of carbon (SCC), which represents the first-best optimal policy.

Building on the existing empirical literature summarized earlier, we examine the relationship between the EU-wide allowance price and a set of observable determinants that capture changes in market fundamentals of key regulated sectors and changes in economic activity across the EU ETS countries, which allows us to better micro-found our macro-finance framework.[31]

To this end, we estimate a panel Vector Autoregression (VAR) to examine how the EU allowance price responds to key macroeconomic, demand, and price aggregate shocks, and show that energy is an important component to consider when analyzing carbon prices. Most of early the business cycle environmental-macro models referred to as E-DSGE (e.g., Fischer and Springborn [2011] and Heutel [2012]) that investigate the linkages between environmental policy and macroeconomic aggregates do not model explicitly energy production as a intermediary input.[32] As such, investigating whether energy inputs and prices had an impact on ETS prices allows for better micro-founding our framework. Second, we build a macro-finance model with energy inputs and prices and rely on a novel estimation strategy to investigate the drivers of inherent market volatility within the implicit carbon prices, which is highly important for business cycle welfare costs.

[30] The major focus of environmental and climate economists over the last decade, as summarized in the literature review conducted by Schubert 1 [2018] has been the pricing of the environmental externality and the global macroeconomic impacts of climate change. Not much research has investigated the linkages between macro-finance and environmental policy frameworks such as the interactions between carbon markets (e.g., EU ETS) and macro-financial aggregates.

[31] The EU ETS currently operates in 30 countries: the 27 EU member states plus Iceland, Liechtenstein, and Norway. The United Kingdom left the EU on 31 January 2020 but remained subject to EU rules until 31 December 2020. In our analysis, we considered the 27 EU member states and the United Kingdom.

[32] We note however, that recently business cycle environmental-macro frameworks started including energy as an input.

3.2 Empirical Results

3.2.1 Data

The data set used[33] in this section was obtained from the ECB Statistical Data Warehouse, Eurostat database, and Bloomberg.

The data set includes series from all 28 EU countries, spanning from January 2013 to December 2019[34], which corresponds to the third phase of the ETS implementation. Our analysis focuses mainly on the third phase of the ETS, as the first two phases were trial periods and carbon price levels were extremely low.

To allow for a refined analysis we use monthly frequency as the benchmark. For our study, we use ETS futures prices, consumption preference index, industrial production index, energy production and prices (for oil brunt, gas, and coal). Specifically, we use EU consumer and industrial production index surveys, both of which are available at monthly frequencies, to capture the shocks to the EU ETS carbon price stemming from consumption preferences and TFP shocks.

3.2.2 The EU Panel VAR

In this section, we examine the relationship between the EU-wide emission permit price and the set of *observable* determinants highlighted above[35] Specifically, we employ a panel VAR and derive the impulse responses of the variables of interest following different macroeconomic, demand, and price aggregate shocks.

Drawing on the existing literature, we consider the same set of shocks that have been previously reviewed: consumption index, industrial production index, inflation, oil supply,

[33] All data were either extracted directly on a monthly basis or transformed from a weekly frequency (case of ETS futures) to a monthly frequency.

[34] We exclude the covid-19 period from our study.

[35] The EU ETS currently operates in 30 countries: the 27 EU member states plus Iceland, Liechtenstein, and Norway. The United Kingdom left the EU on 31 January 2020 but remained subject to EU rules until 31 December 2020. In our analysis, we consider the 27 EU member states and the United Kingdom.

coal supply, gas supply, oil price, coal price, and gas price. Let Y_t denote the corresponding 9×1 vector of observables. We assume that the dynamics of these observables are described by a system of linear simultaneous equations:

$$Y_t = \sum_{j=1}^{p} A_j Y_{t-j} + \eta_t, \tag{3.1}$$

where η_t is a vector of reduced-form VAR innovations. We can re-write the reduced form innovations as a vector of structural shocks ϵ_t,

$$\eta_t = \Gamma \epsilon_t, \tag{3.2}$$

where Γ is a non-singular 9 x 9 matrix.

3.2.3 The ETS Futures Prices Responses To Macro and Energy Price Aggregates: Results

Turning to the results of our panel-VAR model, figure 3.2 presents the standard inference results. Specifically, it shows the cumulative impulse responses (IRFs) of the ETS future carbon prices to different macro and energy price shocks. The solid black lines represent the estimated paths, while the shaded blue areas denote the 68 and 95 percent confidence bands.

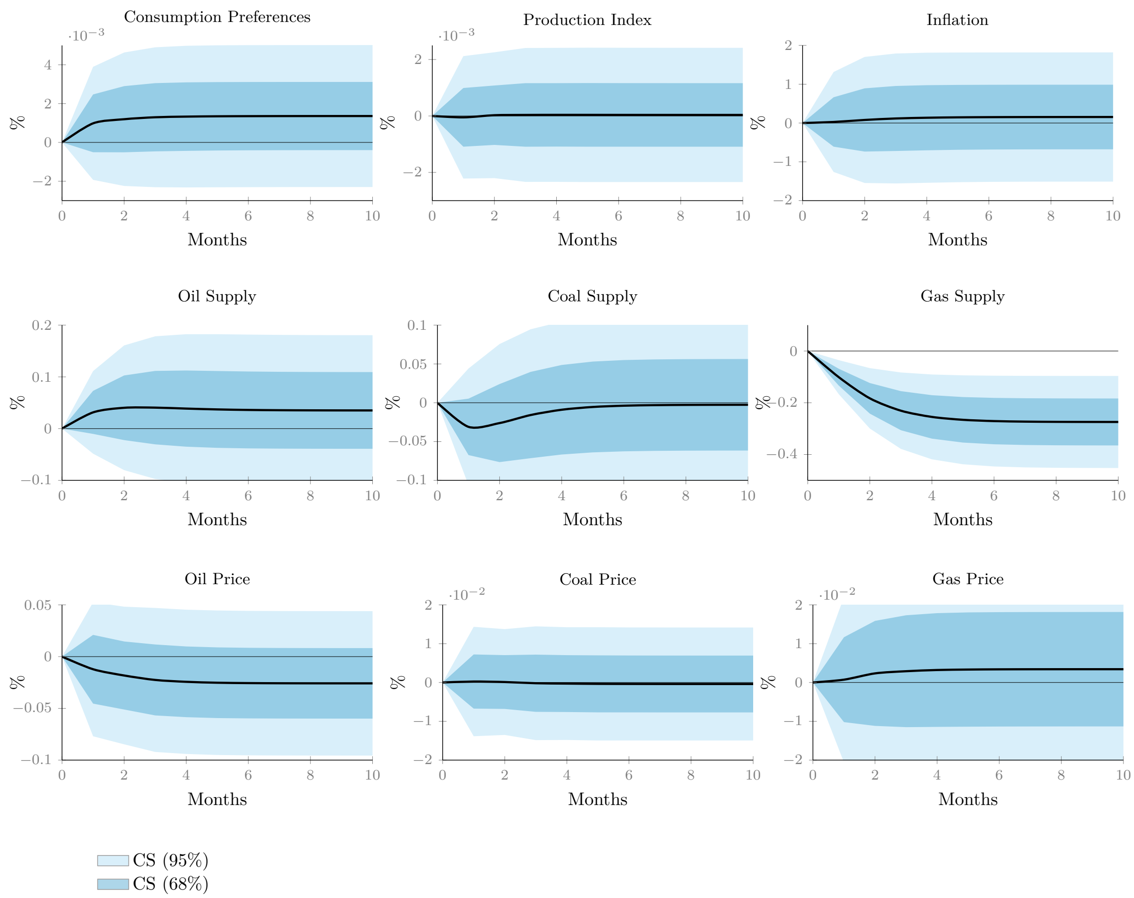


FIGURE 3.2. ETS Futures Response To Macro and Price Aggregates

Notes: The figure shows ETS price cumulative impulse responses to different macro and price aggregates over monthly periods.

The results indicate that a consumption preference shock leads to a persistent increase in ETS prices, as the demand for goods goes up. Although not statistically significant, the industrial production shock tends to decrease the ETS futures prices in the short run, but it has a positive impact on prices over the long run. Similarly, inflation does not appear to play an important role in ETS price variations over the studied period.

Regarding the energy drivers, we find that oil and gas play significant and important roles in determining ETS price levels. Following an oil supply shock, the ETS futures price increases persistently, while it decreases following a gas supply shock. This is consistent with Mansanet-Bataller et al. [2007] and Alberola et al. [2008] findings, as gas energy-based supply is less emission-intensive compared to oil-based energy supply. A similar pattern is observed when the ETS prices are subjected to gas and oil price shocks, where the futures prices tend to persistently increase following a gas price shock and decrease following an oil price shock. These last two results could be further confirmed if data on renewable energy were available for all 28 EU countries during the studied period. Finally, the results are inconclusive when examining coal energy supply and prices.

Although the panel VAR model enables us to determine the significant role of the energy sectors in shaping the ETS future prices, it is unable to capture how policy and abatement shocks interact. This is primarily due to the lack of data on the latter. Therefore, to estimate the impact of these two shocks, we need to use a structural model.

3.3 The model

Consider a business-cycle model characterized by discrete time and an infinite-horizon economy populated by three types of agents: energy firms, final firms, and households, which are infinitely-lived and account for a measure of one. In this setup, energy producers create an environmental externality via CO2 emissions, which affect final firms through a damage function to their productivity, and subsequently alter the welfare of the representative households. As energy producers do not internalize the social cost of their CO2 emissions, a market failure arises.

We begin by presenting the climate dynamics, followed by a description of the energy firms and non-energy firms' problems, and finally, the role of households in the model.

3.3.1 Climate dynamics

Following standard integrated assessment models (IAMs) (see Nordhaus [1991] and Nordhaus and Yang [1996]), with cast envrionmental externality within a macro-finance framework. A large part of the accumulation of Carbon Dioxide and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. As such, in the spirit of Dietz and Venmans [2019], we describe the temperature and concentration process of Carbon Dioxide in the atmosphere as follows. First, the global temperature T_t^o is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions as argued by Dietz and Venmans [2019]:[36,37]

$$T_{t+1}^o = \zeta_1^o(\zeta_2^o X_t - T_t^o) + T_t^o, \tag{3.3}$$

with ζ_1^o and ζ_2^o chosen following Dietz and Venmans [2019].

Furthermore, the cumulative carbon emissions X_t follows a law of motion:

$$X_{t+1} = \eta X_t + E_t + E_t^*, \tag{3.4}$$

where x_{t+1} is the concentration of gases in the atmosphere, $E_t \geq 0$ the inflow of Greenhouse Gases at time t, E_t^* the inflow of non-anthropogenic emissions,[38] and $0 < \eta < 1$ the linear rate of continuation of CO_2-chosen very close to 1 as argued by Dietz and Venmans [2019].[39] Anthropogenic emissions of CO_2 result from energy production Y_t^n and are subject to an exogenous trend Γ_t^E which captures the decoupling between CO_2 emissions and production:

$$E_t = (1 - \mu_t)\,\varphi Y_t^n \Gamma_t^E, \tag{3.5}$$

[36] To allow for convergence in the auto-regressive law of motion for the stock of emissions process, we slightly depart from the transient climate response to cumulative carbon emissions theory and set $\eta \neq 1$. However, we choose η sufficiently close to one such that $X_t \approx X_0 + \sum_{i=0}^{t}(E_i + E_i^*)$.

[37] We note that while differences on climate dynamics and damages modeling over the long horizon (whether à la Golosov et al. [2014], à la Nordhaus [2017], or à la Dietz and Venmans [2019], among others) induce consequent impacts on macroeconomic aggregate equilibriums, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not induce significant impacts on macroeconomic aggregate equilibriums.

[38] In the absence of anthropogenic emissions, $E = 0$, and the cumulative carbon emissions converges to its pre-industrial value, i.e. 545 Gigatonnes. E_t^* is set to match the actual level of pollution stock today.

[39] We consider that emissions from the Rest of the World (ROW) E_t^* are growing at the same rate as the EU economy. (The EU and the ROW emissions were strongly correlated at the business-cycle frequency over the studied period.)

Here, the variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by energy firms Y_t^n.

This functional form for emissions allows us to take into account both low- and high-frequency variations in CO_2 emissions. For the high-frequency features of the emissions data, the term Y_t^n denotes the total inflow of pollution resulting from production, prior to abatement. In this expression, $\varphi \geq 0$ is carbon-intensity parameter that pin down the steady-state ratio of emissions-to-output.

3.3.2 Energy Firms

The energy producers seek to maximize profit by making a trade-off between the desired levels of capital and labor, subject to the energy price. Energy is produced via a Cobb-Douglas production function:

$$Y_t^n = \varepsilon_t^{A_n} A_t^n (K_t^n)^{\alpha_n} (\Gamma_t^Y l_t^n)^{1-\alpha_n}, \tag{3.6}$$

where K_{t^n} is the capital stock used by the energy firms with an intensity parameter $\alpha_n \in [0,1]$, l_t^n is labor, $A_t^n > 0$ is the productivity level, and $\varepsilon_t^{A_n}$ is a total energy productivity shock that evolves as follows: $\log\left(\varepsilon_t^{A_n}\right) = \rho_{A_n} \log\left(\varepsilon_{t-1}^{A_n}\right) + \eta_t^{A_n}$, with $\eta_t^{A_n} \sim N(0, \sigma_{A_n}^2)$. The capital-share parameter is denoted by α_n. The energy sector grows at the economy growth rate $\gamma^y = \frac{\Gamma_t^Y}{\Gamma_{t-1}^Y}$.

Energy producers maximize profits:

$$\Pi_t^E = p_t^n Y_t^n - w_t^n l_t^n - I_t^n - f\left(\mu_t\right) Y_t^n - \tau_t E_t. \tag{3.7}$$

The energy relative price and real wage are denoted by p_t^n and w_t^n, while $f\left(\mu_t\right)$ represents the abatement-cost function, and $\tau_t \geq 0$ a price (i.e. the carbon policy in place) on GHG emissions introduced by the fiscal authority. Investment is denoted by I_t^n and the accumulation of physical capital is given by the following law of motion:

$$K_{t+1}^{n} = (1-\delta)K_t^n + I_t^n, \tag{3.8}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital.

The abatement-cost function is taken from Nordhaus [2008], where $f(\mu_t) = \theta_1 \mu_t^{\theta_2} \varepsilon_t^Z$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement, while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f(\mu_t)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one. Finally, ε_t^Z represents an AR(1) shock to the abatement cost, which captures market uncertainties about both abatement investment cost and technology.

3.3.3 Final goods firms

Final firms seek to maximize profit by making a trade-off between the desired levels of capital, energy used, and labor. Output is produced via a Cobb-Douglas production function:

$$Y_t^y = \varepsilon_t^{A_y} A_t^y d(T_t^o)(K_t)^{\alpha_1}(Y_t^n)^{\alpha_2}(\Gamma_t^Y l_t^y)^{1-\alpha_1-\alpha_2}, \tag{3.9}$$

where K_t is the capital stock used by the final firms with an intensity parameter $\alpha_1 \in [0,1]$, Y_t^n is the energy with an intensity parameter α_2, l_t^n is labor, $A_t^y > 0$ is the productivity level, and ε_t^A is a total factor productivity shock that evolves as follows: $\log\left(\varepsilon_t^{A_y}\right) = \rho_{A_y} \log\left(\varepsilon_{t-1}^{A_y}\right) + \eta_t^{A_y}$, with $\eta_t^{A_y} \sim N(0, \sigma_{A_y}^2)$. $d(T_t^o)$ represents a convex function relating the temperature level to a deterioration in output ($d(T_t^o) = ae^{-b_t T_t^{o2}}$). The damage sensitivity $b_t = \frac{b}{\Gamma_t^{Y2}}$ is adjusted with the economy growth rate with $(a,b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. As highlighted by Benmir and Roman [2020], the business cycle literature typically features preferences and/or production functions with $\Gamma_t^Y = 1$ for all t, as people assume no long-run growth. However, as we are also interested in estimating historical series of shocks, our economy features a growth trend Γ_t^Y different than 1 in hours worked. Therefore, we introduce ${\Gamma_t^Y}^2$ to the damage sensitivity parameter b, such that $d(t_t^o) = ae^{-\frac{b}{{\Gamma_t^Y}^2} {T_t^o}^2} = ae^{-b{t_t^o}^2}$. The goal is to ensure the existence of a balanced growth path without a loss of generality, as over the studied period $b_t \approx \frac{b}{(\Gamma_t^Y)^2}$. Energy producers maximize profits:

$$\Pi_t^F = Y_t^y - w_t^y l_t^y - I_t^y - p_t^n Y_t^n. \tag{3.10}$$

where the real wage is denoted by w_t^y and capital investment by I_t^y. The accumulation of physical capital is given by a similar law of motion to the energy firms:

$$K_{t+1}^y = (1-\delta) K_t^y + I_t^y, \tag{3.11}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital.

3.3.4 Households

We model the representative household via a utility function where the household chooses consumption expenditures and its holdings of long-term government bonds.

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u\left(C_t\right), \tag{3.12}$$

where E_0 is the expectations operator conditioned on information at time 0, β the time discount factor, C_t consumption, and ε_t^B is an AR(1) preference shock, with $\log \varepsilon_t^B = \rho_B \log \varepsilon_{t-1}^B + \eta_t^B$, $\eta_t^B \sim N(0, \sigma_B^2)$. The law of motion for the habit stock is set following Campbell and Cochrane [1999] ($C_{t-1} = hc_{t-1}$).

The budget constraint of the representative household is as follows:

$$w_t^y l_t^y + w_t^n l_t^n + r_t B_t + \Pi_t^E + \Pi_t^F - T_t = C_t + B_{t+1} \tag{3.13}$$

where the left-hand side refers to the household's different sources of income. Total income is first comprised of labor income. Every period, the agent also receives income from holding a long-term government bond, B_t at a return r_t. As the representative agent owns firms in the corporate sector, there is last a dividend income from both the energy firms and final firms Π_t^E and Π_t^F

On the expenditure side, the representative household first spends its income on consumption goods, C_t. Finally, we assume that the government levies a lump-sum tax of t_t.

3.3.5 Government and market clearing

The government finances its expenditures by collecting taxes. The government budget constraint is as follows:

$$G_t = T_t + \tau_t E_t \tag{3.14}$$

where public expenditure is denoted by G_t and T_t is a lump-sum tax.

The revenue is composed of $\tau_t E_t$ which represents the revenues obtained from the implementation of an environmental policy. In this expression, E_t and τ_t are the level of emissions and the carbon price, respectively. As in any typical business-cycle model, government spending is a fraction of output.

The resource constraint of the economy reads as follows:

$$Y_t^y = C_t + I_t^y + I_t^n + G_t + f(\mu_t) Y_t^n. \tag{3.15}$$

3.3.6 The environmental policy

As the implementation of the social cost of carbon[40] (i.e. the central planner optimal shadow value of carbon price) is usually very difficult given the number of aggregates the government will need to closely monitor very frequently over the business cycle, most

[40]For the full derivation please refer to the technical Appendix A.

public authorities (e.g. the EU, California in the U.S., the Québec province in Canada, among others) opted for a cap-and-trade market as it is the case for the ETS market. Our cap policy then reads as:

$$E_t = \varepsilon_t^{CS} \text{Cap Level}_t. \tag{3.16}$$

where Cap Level is the desired level emissions level and ε_t^{CS} an AR(1) climate sentiment shock which mainly captures the policy uncertainty over cap target (or equivalently the quotas policy availability).

3.3.7 Balanced growth

As the focus of the paper is to estimate the drivers of the ETS market, we derive the de-trended model over its balanced growth path. We also take into account that emissions grow at a different rate from output as in Benmir et al. [2020]. In the context of our model, this difference in growth rates can be explained by introducing a rate of *green* technological progress.

As is standard in the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This is achieved by the labor-augmenting technological progress, denoted by Γ_t^Y. The growth rate of labor-augmenting technological progress is γ^y, where:

$$\frac{\Gamma_{t+1}^Y}{\Gamma_t^Y} = \gamma^y.$$

We denote green technological progress in the growing economy by Γ_t^E. The growth rate of green progress γ^e is as follows:

$$\frac{\Gamma_{t+1}^E}{\Gamma_t^E} = \gamma^e.$$

This trend is necessary to capture the long-term process of the decoupling of output growth from emission growth. As documented by Newell et al. [1999], this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy-intensive technologies in capital goods. An improvement in the technology therefore implies a value for γ^e that is below 1. As in Nordhaus [1991], we assume that this trend is deterministic.

In the appendix sections, we present the de-trended economy. The detailed derivation of this de-trended economy as well as the social planner solution and decentralized solutions.

3.4 Estimation

The aim of the estimation is to identify series of shocks that can account for the variability in the carbon price. However, this is a challenging task, particularly when estimating the model at a monthly frequency. In our case, since we want to infer a series of abatement costs while lacking relevant data, we must develop an innovative method. In this section, we outline our approach and data sources, before briefly presenting the estimation results.

3.4.1 Strategy

To analyze the factors driving EU ETS futures, we need to estimate the model at a monthly frequency while accounting for various factors that may affect carbon quota demand and supply. However, some relevant series may not be available at a monthly frequency or may be nonexistent. Specifically, the abatement cost faced by firms cannot be directly observed at the macro level.

To account for possible changes in the regulatory framework and changes in firms' perception of policy stringency, we estimate a series of climate sentiment shocks. We leverage the fact that the ETS is designed to make emissions fall at a constant rate during the study period. By feeding a stationary emissions series into the (de-trended) model, we are able to retrieve the climate sentiment series.

For goods productivity and consumption shocks, we rely on Eurostat surveys. For energy productivity shocks, we use net energy generation. Finally, we use ETS futures prices to infer the series of abatement cost shocks. Given that our other shock series can explain the main drivers of carbon allowance demand and supply, the remaining volatility observed in the ETS market can only be explained by abatement shocks. Our model's structural character links the marginal cost of abatement for firms to the carbon price, allowing us to infer a series of abatement cost shocks that could not be estimated otherwise. To the best of our knowledge, this paper is the first to propose a method for estimating abatement cost dynamics, connecting environmental economics theory to empirical evidence.

3.4.2 Data

We estimate our model using Bayesian methods on monthly EU data from January 2013 to December 2018, corresponding to the third phase of the EU Emissions Trading System (ETS). We source our data from Eurostat, Bloomberg, and the Edgar database.

To map our model to the data, we augment our equilibrium equations with observation equations as follows:

To map our model to data, we augment our equilibrium equations with a set of observation equations defined as follows:

$$\begin{bmatrix} \text{Production Index Growth} \\ \text{Consumption Index Growth} \\ \text{Per Capita Emissions Growth} \\ \text{Per Capita Energy Production Growth} \\ \text{Real } CO_2 \text{ Price Growth} \end{bmatrix} = \begin{bmatrix} (\gamma^y y_t - y_{t-1})/y_{t-1} \\ (\gamma^y c_t - c_{t-1})/c_{t-1} \\ \log \gamma^x + \Delta \log (e_t) \\ \log \gamma^y + \Delta \log (y_t^n) \\ \Delta \log (\tau_t) \end{bmatrix}, \tag{3.17}$$

where γ^x the trend in emissions and γ^y the trend growth rate of the economy. Since the model is stationary, we need to transform data (namely, production, consumption, CO_2 emissions, energy supply, and the EU ETS future prices) accordingly. Following the seminal contribution of Smets and Wouters [2007], data exhibiting a unit root are made stationary by taking the log difference when needed.

3.4.3 Calibration

Before estimating shock processes and innovations, we calibrate most of the parameters to match key aggregates in the European Union. The full list of calibrated parameters and targeted moments is available in table 3.2 and table 3.3, respectively.

3.4.4 Estimated parameters

We estimate our model's parameters using the Metropolis Hastings algorithm to sample from the distribution. We use four chains of 50,000 draws each. Figure 3.3 displays prior and posterior densities for the parameters we estimate, as well as their posterior mean. In addition, table 3.4 summarizes the results of the estimation.

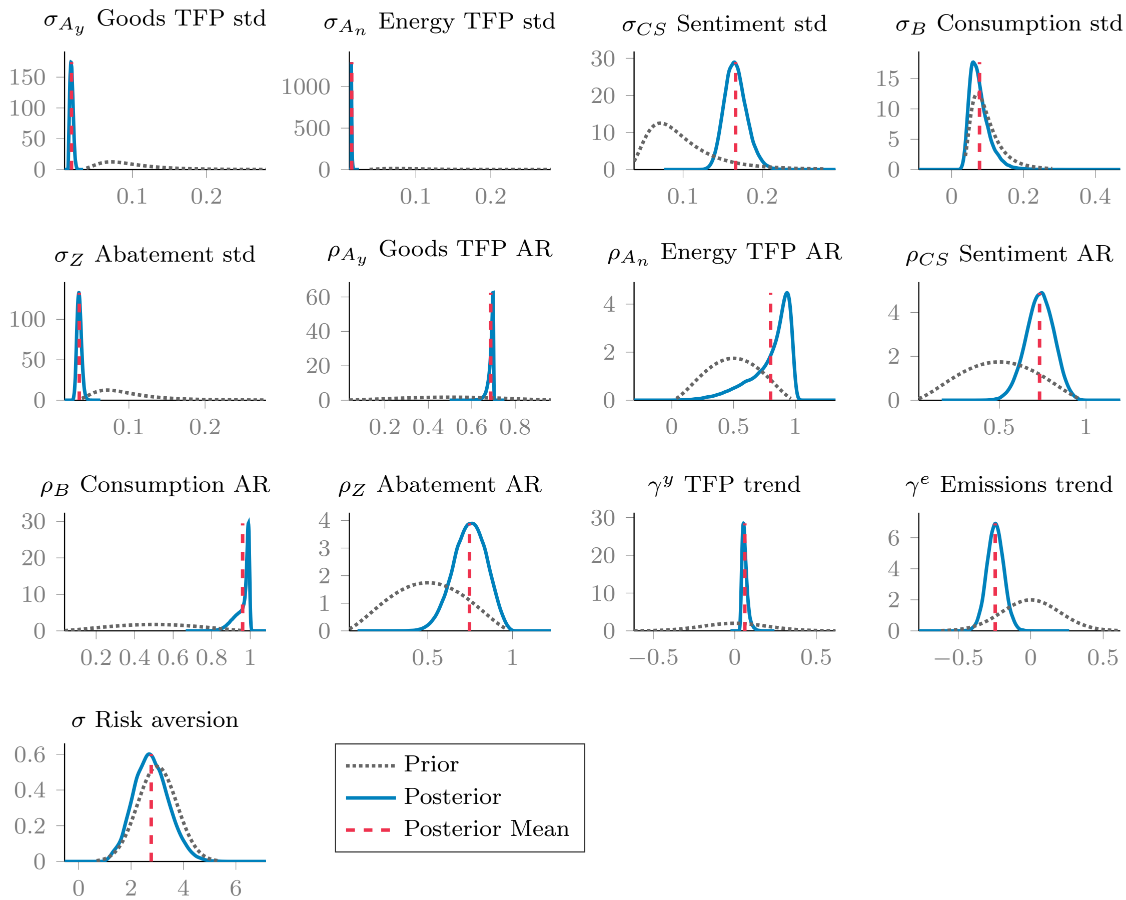


FIGURE 3.3. Priors, posteriors, and posterior means

Notes: The figure shows prior and posteriors densities, as well as the posterior mean of our estimated parameters.

Figure 3.3 displays prior and posterior densities for the estimated parameters, as well as their posterior means. Most of the parameters are well identified, indicating that the data used is informative. However, the risk aversion and the standard deviation of the consumption shock are known to be hard to estimate, and estimation does not provide a lot of information on them. Despite providing very little information on their potential value, trends on emission and output are well identified. Our model's estimation confirms the presence of a decoupling between output and emissions, with a negative value for γ^e and a positive value for γ^y, even with normal priors centered around 0.

3.5 ETS Futures Drivers and Optimal Policy

In this section, we investigate the main drivers of the EU ETS futures market, leveraging our estimated parameters and shock series. Furthermore, we compare our estimated model with a counterfactual economy, where the regulator sets the carbon price to the social cost of carbon, to assess the potential benefits of an optimal carbon pricing policy. These two analyses highlight the significant volatility induced by the ETS market over the studied period.

3.5.1 Uncovering Drivers in the ETS Futures Market

We start by investigating the primary drivers of EU ETS futures using our estimated model parameters and shocks series. To dissect the influence of various shocks and their relative significance on the trajectory of EU carbon allowances futures prices, we undertake a historical decomposition of the carbon price in our estimated model. Energy TFP shocks impact energy generation, and thus, the equilibrium energy price. Goods TFP shocks represent standard productivity shocks for final goods firms, whereas consumption shocks affect consumer preferences. Abatement shocks denote unexpected alterations in energy companies' ability to abate part of their emissions. A positive abatement shock could refer to either innovation in green technologies or an increase in the adoption of existing technologies. Lastly, climate sentiment shocks take into account any other forces driving the ETS futures price that would not be captured by the other shocks. We posit that these residual movements in the carbon price are mainly due to policy surprises.

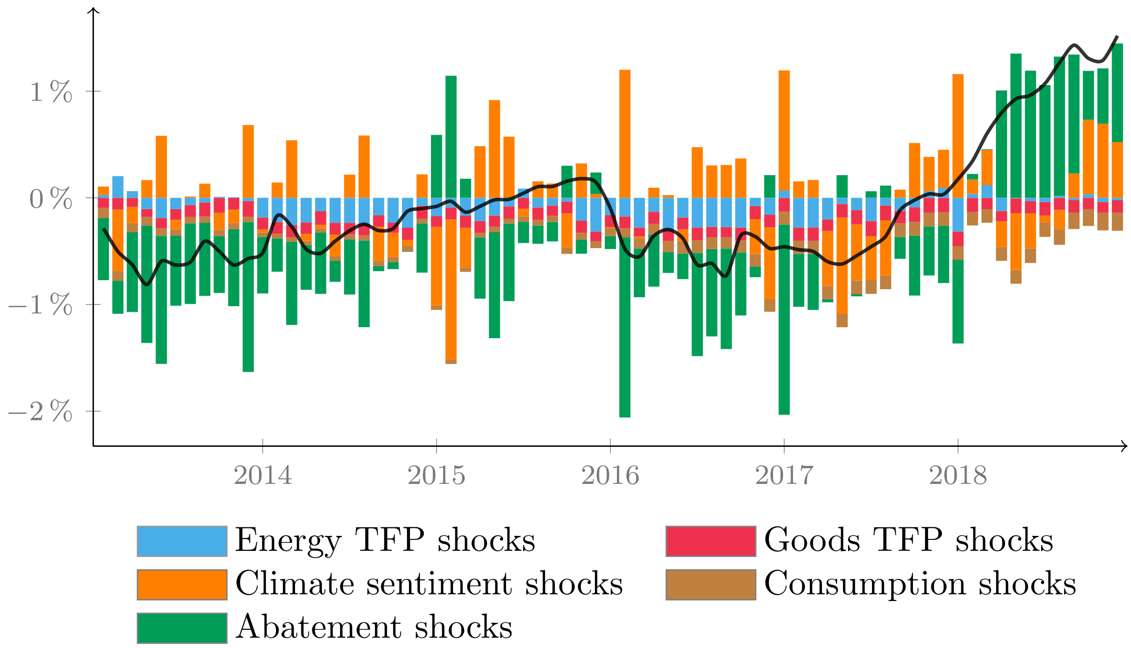


FIGURE 3.4. ETS Futures Historical Decomposition

Notes: The figure shows the path of the ETS carbon price (black line) decomposed into various drivers over the estimated period (2013 – 2019).

Figure 3.4 displays the path of the (de-trended) ETS carbon price divided into different drivers over the 2013-2019 period. The main takeaway of this analysis is that ETS futures have been primarily driven by three forces: abatement, climate sentiment, and to a lesser extent, energy shocks, during the period under scrutiny. Among the factors that influence firms' demand and supply of allowances, we observe that firms tend to react mainly to alterations in their abatement technology and changing conditions in the energy market. The former aligns with environmental economics theory, while the latter has been documented in empirical research and points to interconnections between the ETS market and energy markets. On the whole, this result reconciles empirical evidence with carbon pricing theory. The other two factors that could impact firms' demand for quotas (goods supply and consumer demand shocks) do not generate as much volatility, which is not surprising since these are indirect effects that impact energy firms' production and not emissions directly. Regarding the role of public authorities, our decomposition shows that uncertainty over the supply of quotas also accounts for a significant portion of the variance in the carbon price. This finding aligns with Känzig [2021] research, indicating that political uncertainty may have generated increased volatility on the ETS futures market.

Figure 3.5 shows the contribution of each driver to the variance of the carbon price over different horizons, shedding light on the shocks that have a lasting impact on the carbon allowance market.

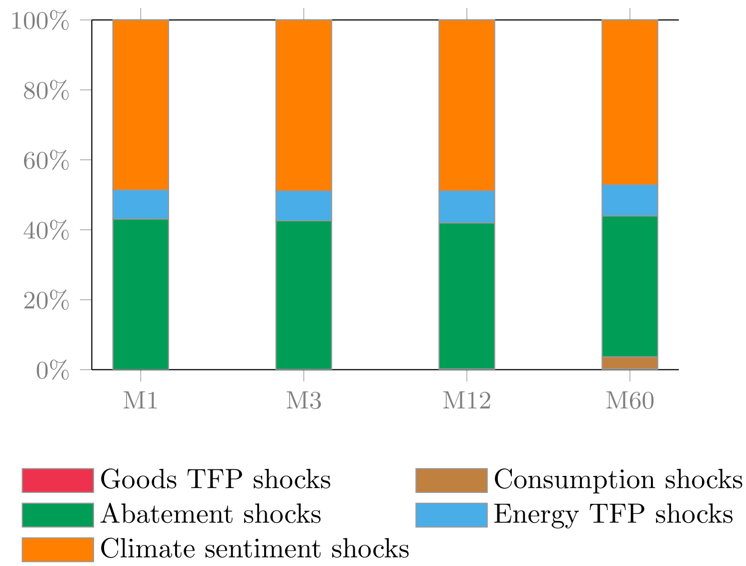


FIGURE 3.5. ETS Futures Variance Decomposition

Notes: The figure shows the ETS price variance decomposition conditional on different horizons: one month, three months, one year, and five years. This is the theoretical variance decomposition of the carbon price, taking into account the estimated variances of shocks.

The three main drivers that influence the path of the carbon price, namely climate sentiment, energy TFP, and abatement shocks, explain a relatively constant proportion of the variance across horizons. However, in the long run, the importance of climate sentiment shocks tends to decrease, while consumption shocks become more relevant. This suggests that changes in the marginal utility of consumption, which affect the stochastic discount factor of firms, have lasting effects on the path of the carbon price, despite explaining a relatively small share of the variance.

3.5.2 ETS and Optimal Policy

We proceed to compare the estimated carbon price with an optimal benchmark, which assumes that a social planner would set a tax to the social cost of carbon. To simulate the optimal scenario, we use the estimated parameters and shock series and replace our carbon price equation with the social cost of carbon. We also eliminate the climate sentiment shock since there is no uncertainty about the joint path of carbon price and emissions in this scenario.

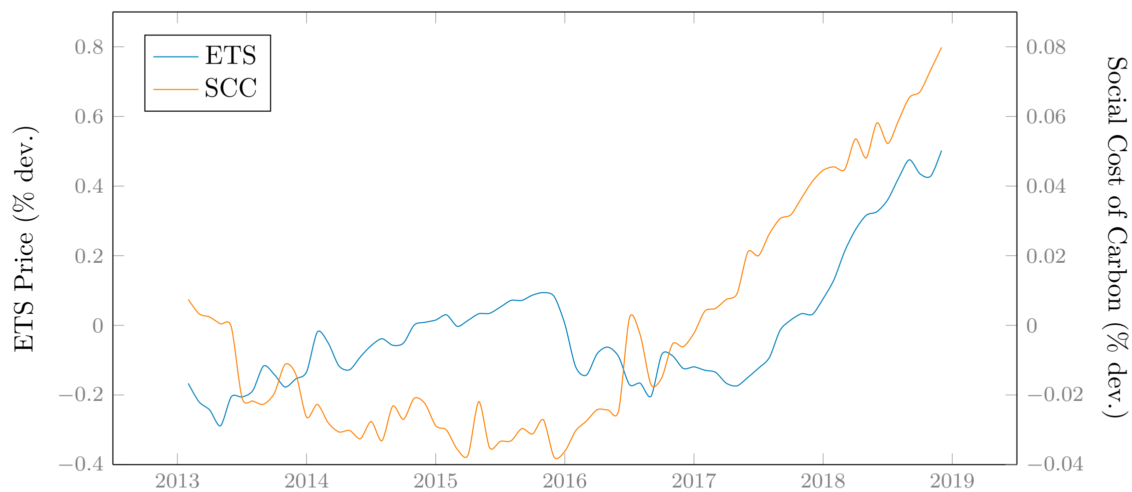


FIGURE 3.6. ETS Price vs SCC Variations

Notes: The figure shows the deviations of the estimated ETS price and the counterfactual SCC in percentage deviations from their respective steady states.

Figure 3.6 displays percentage deviations from the steady-state for both the social cost of carbon and the estimated ETS price. The graph reveals some correlation towards the end of the sample, indicating that the rise in the ETS price in recent years is consistent with the optimal policy. Nonetheless, the SCC scale is ten times smaller than that of the ETS price, implying that the social planner would have preferred a smoother path for the carbon price. Table 3.1 compares several moments in the estimated ETS cap policy case and in the counterfactual optimal case. Lower volatility in the carbon price results in more stable abatement and marginal costs for firms at the expense of slightly bigger variations in emissions. The higher carbon price would also generate welfare gains for households. However, due to the significant uncertainty surrounding the estimation of the social cost of carbon (see *e.g.* Cai and Lontzek [2019] or Barnett et al. [2020]), we only report these welfare gains for completeness, and do not draw any conclusion on the level of welfare gains or losses.

Given the uncertainty around the estimation of the optimal carbon price, it seems unlikely that a regulator would base policy solely on one particular estimation of the social cost of carbon. Instead, a more realistic approach would be to agree on a specific path for emissions through a cap-and-trade market (or equivalently a specific path for the carbon price) and then adjust quotas based on shocks to the economy. In the next section, we will investigate the potential of such smart carbon cap rules. To provide an analogy, just as a central bank's role is to contain inflation using its main discount rate, while minimizing the secondary effects on the financial system that could ultimately affect households, a carbon central bank could be tasked with controlling emissions while minimizing side effects on businesses that could ultimately impact households.

3.6 Carbon Cap Rules

As shown in the previous section, the implementation of the EU ETS during phase 3 has led to increased volatility in the price of carbon compared to the optimal policy. This excess uncertainty has implications for firms, as shocks to the carbon price can lead to variations in their marginal cost and impact financial markets through firms' risk premia, as highlighted by Benmir and Roman [2020].

To address this issue, we explore the potential of a carbon cap rule (CCR) and evaluate how it would have performed over the period studied. A CCR involves an institution monitoring the carbon allowance market and adjusting quotas' supply at a monthly frequency to steer the carbon price faced by firms towards a target. In essence, a European Carbon Central Bank could use the Market Stability Reserve to buy allowances on the market to strengthen the price or sell allowances on the market to cool down the price, similarly to how the European Central Bank uses open market operations to steer the financing conditions of banks.

We propose a carbon cap rule that can be considered the equivalent of a Taylor rule for environmental policy. Using the information in columns (1) and (2) of table 3.1, we assume that the CCR should react to deviations in both the abatement cost and emissions levels to minimize the standard deviation of the resulting carbon price. In our model, the de-trended carbon cap is no longer fixed and can deviate slightly from the value consistent with the Paris Agreement in the short run. The equation for the carbon cap rule becomes:

$$\text{Cap Level}_t = \overline{\text{Cap Level}} + \phi_e * 100(e_t - \bar{e}) + \phi_z * 100(z_t - \bar{z}),$$

where $\bar{e}$ and $\bar{z}$ are the de-trended steady-state emissions and abatement cost, respectively. Note that reacting to deviations from the de-trended steady state ensures that the resulting emissions path is consistent with the EU's emission reduction goal. To conduct our counterfactual exercise and compute relevant statistics, we use our estimated parameters and series of shocks and simulate a model with the cap on emissions set according to the CCR.[41] We then find the parameters that minimize the standard deviation of the carbon price over the sample period using a quasi-Newton method to update our initial guess. The path of the economy is simulated at the second order for each pair of parameter in the carbon cap rule, until the algorithm converges.

[41] Note that we keep the climate sentiment shock active, although one could argue that if a carbon cap rule was implemented, climate sentiment would not impact the path of emissions. Our counterfactual can thus be seen as a conservative case where some unexplained volatility on emission remains.

	ETS Cap Policy Estimated Column (1)	Social Cost of Carbon Optimal Column (2)	Carbon Cap Rule $\phi_z = 13.11$ and $\phi_e = .15$ Column (3)
Welfare (% change w.r.t. SCC)	-1.74 %	0 %	-1.74 %
Welfare (Std. Dev.)	1.03 %	1.03 %	1.02 %
Emissions (Std. Dev.)	3.18 %	6.48 %	4.54 %
Abatement Cost (Std. Dev.)	19.13 %	11.88 %	11.94 %
Marginal Abatement Cost (Std. Dev.)	21.85 %	15.95 %	15.54 %
Carbon Price (in euros)	17.49	29.12	18.07
Carbon Price (Std. Dev.)	18.66 %	2.96 %	4.24 %

TABLE 3.1
Policy Scenarios Estimated Second Moments

Notes: The table reports various moments under a set of scenarios. The first column corresponds to the estimated model, the second column corresponds to the counterfactual optimal case, and the third column corresponds to the counterfactual carbon cap rule. The carbon cap rule is $\text{Cap Level}_t = \overline{\text{Cap Level}} + \phi_e * 100(e_t - \bar{e}) + \phi_z * 100(z_t - \bar{z})$.

The performance of our carbon cap rule is shown in column (3) of table 3.1. Reacting strongly to the abatement cost shock, but relatively little to emission shocks, the rule is able to achieve results close to the optimal policy. Interestingly, the coefficient on the reaction to emission shocks is positive, indicating that the regulator would slightly increase the cap following a positive shock to emissions. Overall, the standard deviation of the carbon price under the carbon cap rule is only 1.3 points higher than that of the optimal policy. The abatement cost volatility is nearly in line with the value from the SCC scenario, and the carbon cap rule reduces the volatility of emissions even further. Figure 3.7 depicts the path of the carbon price under the carbon cap rule, which exhibits muted variations compared to the ETS case, but amplified compared to the SCC case. Notably, the sharp increase in the price in recent years confirms that the carbon cap rule reacts consistently with the social cost of carbon.

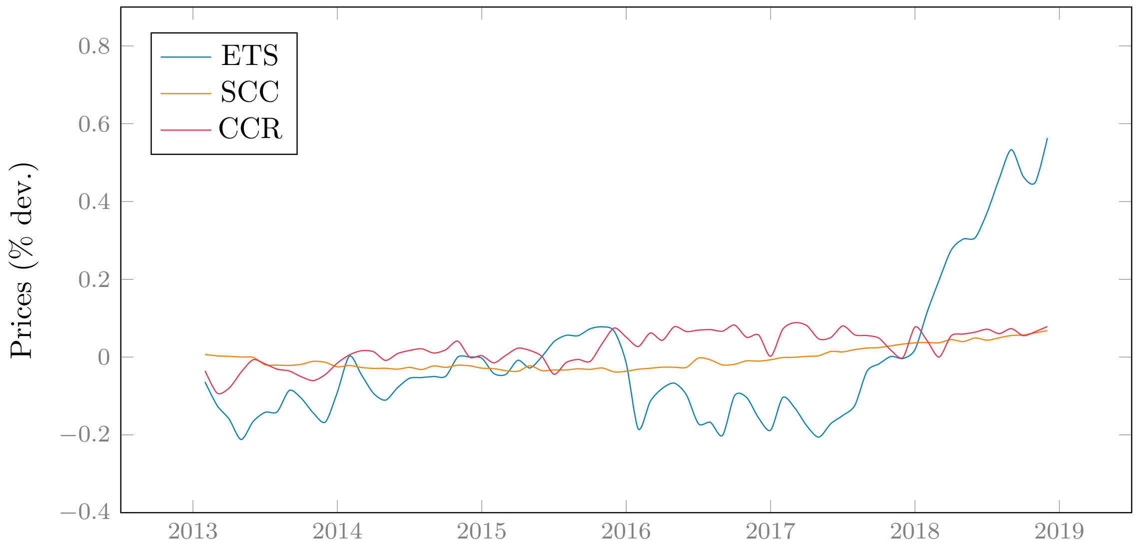


FIGURE 3.7. ETS vs SCC vs CCR Variations

Notes: The figure shows the deviations of the estimated ETS price, the counterfactual SCC, and the counterfactual CCR in percentage deviations from their respective steady states.

In summary, our analysis demonstrates that a carbon cap rule can minimize the volatility of the carbon price. While our specification does not reach the level implied by the social cost of carbon, it significantly reduces the uncertainty related to abatement costs for firms. The regulator can adjust the cap level using both abatement cost and emission shock series, thereby reducing price uncertainty. Our innovative strategy enables the implementation of this type of policy even in the absence of structured abatement cost data at the macro level. Therefore, the flexibility of the carbon cap rule explored in this paper can be adopted with the currently available data.

3.7 Conclusion

This article provides a comprehensive analysis of the drivers of carbon pricing in the EU ETS market, using a macro-finance model. Our results highlight that abatement cost shocks, climate sentiment shocks, and energy shocks are the main factors driving carbon pricing. We also demonstrate that reducing price uncertainty can help close the gap with respect to the optimal policy. To achieve this, we developed an innovative method to infer abatement shock series, using information contained in the market price of carbon.

These findings have important implications for policymakers. Our insights into the factors influencing carbon pricing can inform the design of more effective policies for reducing greenhouse gas emissions and mitigating climate change. In particular, our study introduces the concept of carbon cap rules and suggests that a carbon central bank could have achieved the emissions reduction witnessed during the study period, while limiting carbon price volatility.

Overall, we believe that this analysis advances our understanding of the EU ETS market and can inform future research on carbon pricing and climate policy. By improving our understanding of the dynamics underlying carbon pricing, we can move closer to achieving the goals of the Paris Agreement and mitigating the impact of climate change.

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Appendices

3.A Model Calibration and Estimation

TABLE 3.2
Calibration

Parameter	β	α_1	α_2	α_n	δ	$\frac{g}{y}$	a	b	φ	η	ζ_1^o	ζ_2^o	θ_1	θ_2
Value	0.999	0.333	0.040	0.333	0.008	0.220	1.000	0.040	0.830	0.002	0.500	0.001	0.100	2.700

TABLE 3.3
Moments matching

Variable	Label	Model Steady-State	Model Conditional Mean	Data	Source
ETS Mean Carbon Price	$E(\tau)$	7.39	18.31	7.54	World Bank
Emission to Output Ratio	$E\left(\frac{E}{Y}\right)$	0.24	0.20	0.24	Authors' Calculations
Share of Energy in Output	$E\left(\frac{p^n Y^n}{Y}\right)$	0.04	0.04	0.04	Authors' Calculations
Temperature	$E(T^o)$	1.00	1.00	1.00	NOAA
Cumulative Emission	$E(X)$	801	803	800	Copernicus (EC)

TABLE 3.4
Estimated Parameters

		Prior Distributions			Posterior Distributions	
		Distribution	Mean	Std. Dev.	Mean	[0.05 ; 0.95]
Shock processes:						
Std. Dev. Goods Productivity	σ_A	$\mathcal{IG}_2$	0.10	0.05	0.018	[0.015 ; 0.022]
Std. Dev. Energy Productivity	σ_{A_n}	$\mathcal{IG}_2$	0.10	0.05	0.014	[0.014 ; 0.015]
Std. Dev. Climate Sentiment	σ_{CS}	$\mathcal{IG}_2$	0.10	0.05	0.035	[0.030 ; 0.040]
Std. Dev. Consumption	σ_B	$\mathcal{IG}_2$	0.10	0.05	0.073	[0.036 ; 0.109]
Std. Dev. Abatement Cost	σ_Z	$\mathcal{IG}_2$	0.10	0.05	0.167	[0.144 ; 0.190]
AR(1) Goods Productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.687	[0.671 ; 0.700]
AR(1) Energy Productivity	ρ_{A_n}	$\mathcal{B}$	0.50	0.20	0.795	[0.542 ; 0.991]
AR(1) Climate Sentiment	ρ_{CS}	$\mathcal{B}$	0.50	0.20	0.748	[0.596 ; 0.915]
AR(1) Consumption	ρ_C	$\mathcal{B}$	0.50	0.20	0.958	[0.899 ; 0.997]
AR(1) Abatement Cost	ρ_Z	$\mathcal{B}$	0.50	0.20	0.734	[0.602 ; 0.867]
Structural Parameters:						
TFP Trend	$(\gamma^y - 1) \times 100$	$\mathcal{N}$	0.00	0.20	0.063	[0.037 ; 0.087]
Emissions Trend	$(\gamma^e - 1) \times 100$	$\mathcal{N}$	0.00	0.20	-0.245	[-0.342 ; -0.148]
Risk Aversion	σ	$\mathcal{N}$	3.00	0.75	2.675	[1.624 ; 3.626]

Notes: $\mathcal{IG}_2$ denotes the Inverse Gamma distribution (type 2), $\mathcal{B}$ the Beta distribution, and $\mathcal{N}$ the Gaussian distribution.

3.B Balanced Growth Path

In order to perform our structural parameters estimation through Bayesian estimation, we first need to specify the de-trended economy over its balanced growth path.

The growth rate of Γ_t^Y determines the growth rate of the economy along the balanced growth path.[42] This growth rate is denoted by γ^Y, where:

$$\Gamma_t^Y = \gamma^Y \Gamma_{t-1}^Y \tag{3.18}$$

Stationary variables are denoted by lower case letters, whereas variables that are growing are denoted by capital letters. For example, in the growing economy, output in the energy and non-energy sectors are denoted by Y_t^y and Y_t^n De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

$$y_t^y = \frac{Y_t^y}{\Gamma_t^Y} \tag{3.19}$$

$$y_t^n = \frac{Y_t^n}{\Gamma_t^Y} \tag{3.20}$$

Energy sectoral emissions, which we denote by E_t in the growing economy are given as follows:

$$E_t = (1 - \mu_t)\varphi Y_t^n \Gamma_t^E \tag{3.21}$$

where Γ_t^E represents the decoupling of CO_2 emissions with respect to output trend.

Thus, in the de-trended economy, CO_2 emissions law of motion reads as follows:

$$e_t = (1 - \mu_t)\varphi y_t^n \tag{3.22}$$

where:

[42] In our setup both sectors grow at the same rate Γ_t^Y.

$$e_t = \frac{E_t}{\Gamma_t^Y \Gamma_t^E} \tag{3.23}$$

The abatement cost in the growing economy is:

$$Z_t = f(\mu_t) Y_t^n \tag{3.24}$$

Thus, in the de-trended economy, the abatement cost reads as follows:[43]

$$z_t = f(\mu_t) y_t^n \tag{3.25}$$

The stock of emissions in the atmosphere is denoted by X_t, while the temperature is called T_t^o in the growing economy:

$$X_{t+1} = \eta X_t + E_t + E_t^* \tag{3.26}$$

$$T_{t+1}^o = \zeta_1(\zeta_2 X_t - T_t^o) + T_t^o \tag{3.27}$$

The de-trended X_t and T_t^o read as follows:

$$\gamma^x x_{t+1} = \eta x_t + e_t + e^* \tag{3.28}$$

$$\gamma^x t_{t+1}^o = \zeta_1(\zeta_2 X_t - T_t^o) + T_t^o \tag{3.29}$$

where:

$$x_t = \frac{X_t}{\Gamma_t^Y \Gamma_t^E} \tag{3.30}$$

$$t_t^o = \frac{T_t^o}{\Gamma_t^Y \Gamma_t^E} \tag{3.31}$$

with $\gamma^x = \gamma^y \gamma^e$.

[43] Please note that μ_t is stationary.

In the growing economy, with the above growth progress, the production function for energy and non-energy is defined as follows:

$$Y_t^n = \varepsilon_t^{A^n} A_t^n (K_t^n)^{\alpha_n} (\Gamma_t^y l_t^n)^{1-\alpha_n} \tag{3.32}$$

$$Y_t^y = \varepsilon_t^A A_t^y d(T_t^o)(K_t^y)^{\alpha_1}(Y_t^n)^{\alpha_2}(\Gamma_t^y l_t^y)^{1-\alpha_1-\alpha_2} \tag{3.33}$$

where per energy and non-energy labor l_t^y, l_t^y, the technology shocks $\varepsilon_t^{A^n}, \varepsilon_t^A$ and the TFP levels A_t^n, A_t^y are all stationary variables. Furthermore, the climate damage function captures the growth rate Γ_t^y such that $d(T_t^o) = ae^{-b_t(T_t^o)^2} = e^{-\frac{b}{\Gamma_t^2}(T_t^o)^2}$. Capturing the growth rate of the economy within the damage function allows us to simplify the de-trended form of the damage function without a loss of generality as over the studied period (a 10-15 year horizon).

De-trending the production functions gives the following:

$$y_t^n = \varepsilon_t^{A^n} A_t^n (k_t^n)^{\alpha_n} (l_t^n)^{1-\alpha_n} \tag{3.34}$$

$$y_t^y = \varepsilon_t^A A_t^y d(t_t^o)(k_t^y)^{\alpha_1}(y_t^n)^{\alpha_2}(l_t^y)^{1-\alpha_1-\alpha_2} \tag{3.35}$$

The capital-accumulation equations for both the energy and non-energy sectors in the growing economy read as:

$$K_{t+1}^n = (1-\delta)K_t^n + I_t^n \tag{3.36}$$

$$K_{t+1}^y = (1-\delta)K_t^y + I_t^y \tag{3.37}$$

In the de-trended economy, we thus have:

$$\gamma^y k_{t+1}^n = (1-\delta)k_t^n + i_t^n \tag{3.38}$$

$$\gamma^y k_{t+1}^y = (1-\delta)k_t^y + i_t^y \tag{3.39}$$

with both capital and investment de-trended variables reading as: $k_t^y = \frac{K_t^y}{\Gamma_t^y}$ and $i_t^y = \frac{I_t^y}{\Gamma_t^y}$, respectively.

Moreover, the economy's resource constraint is:

$$y_t^y = c_t + g_t + f(\mu_t)y_t^n - p_t^n y_t^n \tag{3.40}$$

Finally, in the growing economy, the utility function is as follow:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} \right) \tag{3.41}$$

where C_t is consumption, β the subjective discount factor, and σ the curvature parameter. The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - \tilde{h}c_{t-1})^{1-\sigma}}{1-\sigma} \right) \tag{3.42}$$

where we denote $\tilde{\beta} = \beta(\gamma^y)^{1-\sigma}$ and $\tilde{h} = h(\gamma_y)^{-1}$.

3.C The Social Planner Equilibrium: Centralized Economy

The benevolent social planner optimal allocation and optimal plan would choose to maximize welfare by choosing a sequence of allocations, for given initial conditions for the endogenous state variables, that satisfies the economy constraints.[44]

[44] This equilibrium will provide a benchmark solution, which we use to compare with the allocation obtained in the decentralized economy for the carbon policy.

The planners' problem reads as follows:

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Bigg(& \frac{(c_t - \tilde{h}c_{t-1})^{1-\sigma}}{1-\sigma} \\
& + \lambda_t (y_t + p_t^n y_t^n - c_t - i_t^y - i_t^n - g_t - f(\mu_t) y_t^n) \\
& + \lambda_t q_t^y ((1-\delta) k_t^y + i_t^y - \gamma_y k_{t+1}^y) \\
& + \lambda_t q_t^n ((1-\delta) k_t^n + i_t^n - \gamma_y k_{t+1}^n) \\
& + \lambda_t \Psi_t^y (\varepsilon_t^{A_y} e^{-b(t_t^o)^2} A^y (k_t^y)^{\alpha_1} (y_t^n)^{\alpha_2} (l_t^y)^{1-\alpha_1-\alpha_2} - y_t^y) \\
& + \lambda_t \Psi_t^n (\varepsilon_t^{A_n} A^n (k_t^n)^{\alpha_n} (l_t^n)^{1-\alpha_n} - y_t^n) \\
& + \lambda_t V_t^X (\gamma^x x_{t+1} - \eta x_t - e_t - e^*) \\
& + \lambda_t V_t^T (\gamma^x t_{t+1}^o - \upsilon_1^o (\upsilon_2^o x_t - t_t^o) - t_t^o) \\
& + \lambda_t V_t^E (e_t - (1-\mu_t) \varphi y_t^n) \Bigg)
\end{aligned}
$$

where as we will show below the Social Cost of Carbon SCC_t is the shadow value with respect to the temperature damages $\S_T^t$.

The first order conditions determining the SCC_t are the ones with respect to t_t^o, x_t, while the FOCs with respect to e_t, μ_t determine the level of abatement needed:

$$\gamma^x V_t^T = \tilde{\beta} E_t \{ \Lambda_{t+1} ((1-\zeta_1) V_{t+1}^T - (-2bt_{t+1}^o) \Psi_{t+1}^y y_{t+1}^y) \} \tag{3.43}$$

$$\gamma^x V_t^X = \tilde{\beta} E_t \{ \Lambda_{t+1} (\zeta_1 \zeta_2 V_{t+1}^T + \eta V_{t+1}^X) \tag{3.44}$$

$$V_t^E = V_t^X \tag{3.45}$$

$$f'(\mu_t) = \varphi V_t^E \tag{3.46}$$

3.D The Decentralized Economy

3.D.1 Households

Households maximize utility over consumption and leisure subject to their budget constraint. They choose consumption expenditures and holdings of government bonds, and receive transfers as well as dividends from owned firms.

$$\max_{\{c_t, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - \tilde{h} c_{t-1})^{1-\sigma}}{1-\sigma} \right) \tag{3.47}$$

s.t.

$$c_t + b_{t+1} = w_t l_t^y + w_t^n l_t^n + r_t b_t + t_t + \Pi_t^y + \Pi_t^n$$

From the FOCs, we get:

$$\lambda_t = \varepsilon_t^B \left(c_t - \tilde{h} c_{t-1} \right)^{-\sigma} - \varepsilon_{t+1}^B \tilde{\beta} \tilde{h} \left(c_{t+1} - \tilde{h} c_t \right)^{-\sigma}$$

$$\tilde{\beta} r_t \Lambda_{t+1} = 1$$

where we note $\Lambda_t = \frac{\lambda_t}{\lambda_{t-1}}$ and $\tilde{h} = \frac{h}{\gamma^y}$.

3.D.2 Energy Firms

Energy producers maximize profits choosing capital investment and labour wages, as well as the investment in abatement as the regulator imposes a carbon price on their level of emissions. The production technology is a cobb-douglas while the abatement investment is a convex function on abatement levels. Capital depreciate and follow a standard law of motion.

The firms problem reads:

$$\max_{\{y_t^n, i_t^n, \mu_t\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Lambda_{t+1} \Pi_t^n$$

where $\Pi_t^n = p_t^n y_t^n - w_t^n l_t^n - i_t^n - f(\mu_t)\, y_t^n - e_t \tau_t$

s.t.

$$y_t^n = \varepsilon_t^{A_n} A^n (k_t^n)^{\alpha_n} (l_t^n)^{1-\alpha_n}$$
$$e_t = (1-\mu_t)\varphi y_t^n$$
$$\gamma^y k_{t+1}^n = (1-\delta) k_t^n + i_t^n$$

The FOCs with respect to capital, investment, labour, abatement, and energy output read as:

$$q_t^n \gamma^y = \tilde{\beta} \Lambda_{t+1} q_{t+1}^n \left((1-\delta) + \alpha_n \Psi_{t+1}^n \frac{y_{t+1}^n}{k_{t+1}^n} \right)$$
$$q_t^n = 1$$
$$w_t^n = (1-\alpha_n) \frac{y_t^n}{l_t^n}$$
$$f'(\mu_t) = \varphi \tau_t$$
$$\Psi_t^n = p_t^n - (\theta_1 \mu_t^{\theta_2} + \tau_t (1-\mu_t)\varphi)$$

where we denote Ψ_t^n and q_t^n the Lagrange multipliers associated with production inputs and investment.

3.D.3 Non-energy final firms

Non-energy producers maximize profits:

$$\Pi_t^y = y_t^y - w_t^y l_t - i_t^y - p^n y_t^n.$$

s.t.

$$y_t^y = \varepsilon_t^{A_y} A^y (k_t^y)^{\alpha_1} (y_t^n)^{\alpha_2} (l_t^y)^{1-\alpha_1-\alpha_2}$$
$$\gamma^y k_{t+1}^y = (1-\delta) k_t^y + i_t^y$$

The FOCs with respect to capital, investment, labour, and energy yield the factor prices:

$$q_t^y \gamma^y = \tilde{\beta} \Lambda_{t+1} q_{t+1}^y \left((1-\delta) + \alpha_y \Psi_{t+1}^y \frac{y_{t+1}^y}{k_{t+1}^y} \right)$$
$$q_t^y = 1$$
$$w_t^y = (1-\alpha_1-\alpha_2) \Psi_t^y \frac{y_t^y}{l_t^y}$$
$$p_t^n = \alpha_2 \Psi_t^y \frac{y_t^y}{y_t^n}$$

where we denote Ψ_t^y and q_t^y the Lagrange multipliers associated with production inputs and investment.

We can also easly check that $\Psi_t^y = 1$ as we are in an RBC case.

3.D.4 Environmental Policy

When the regulator sets optimally the environmental policy, he or she sets the carbon price equal to the social cost of carbon as shown in the case of the social planner:

$$\tau_t = V_t^X$$

Otherwise, the regulator could decide to set an emission cap or a fixed price such that:

$$\tau_t = \text{Fixed Price}$$

or

$$e_t = \text{Cap Level}$$

3.E Set of Equilibrium Conditions

3.E.1 Model Equations

Households

$$\lambda_t = \varepsilon_t^B \left(c_t - \frac{h}{\gamma^y} c_{t-1} \right)^{-\sigma} - \varepsilon_{t+1}^B \tilde{\beta} \frac{h}{\gamma^y} \left(c_{t+1} - \frac{h}{\gamma^y} c_t \right)^{-\sigma}$$

$$\tilde{\beta} r_t \Lambda_{t+1} = 1$$

$$\Lambda_t = \frac{\lambda_t}{\lambda_{t-1}}$$

Climate Dynamics

$$\gamma^x t_{t+1}^o = \upsilon_1^o (\upsilon_2^o x_t - t_t^o) + t_t^o$$

$$\gamma^x x_{t+1} = \eta x_t + e_t + e^*$$

$$\Theta_t = a e^{-b (t_t^o)^2}$$

Non-Energy Production

$$y_t^y = \varepsilon_t^{A_y} \Theta_t A^y (k_t^y)^{\alpha_1} (y_t^n)^{\alpha_2} (l_t^y)^{1-\alpha_1-\alpha_2}$$

$$\gamma^y k_{t+1}^y = i_t^y + (1-\delta) k_t^y$$

$$\gamma^y = \tilde{\beta} \Lambda_{t+1} \left((1-\delta) + \alpha_1 \Psi_{t+1}^y \frac{y_{t+1}}{k_{t+1}^y} \right)$$

$$w_t^y = (1 - \alpha_1 - \alpha_2) \Psi_t^y \frac{y_t}{l_t^y}$$

$$p_t^n = \frac{\alpha_2 \Psi_t^y y_t}{y_t^n}$$

Energy Production

$$y_t^n = \varepsilon_t^{A_n} A^n (k_t^n)^{\alpha_n} (l_t^n)^{1-\alpha_n}$$

$$e_t = (1-\mu_t)\varphi y_t^n$$

$$\gamma^y = \tilde{\beta}\Lambda_{t+1}\left((1-\delta) + \alpha_n \Psi_{t+1}^n \frac{y_{t+1}^n}{k_{t+1}^n}\right)$$

$$w_t^n = (1-\alpha_n)\Psi_t^n \frac{y_t^n}{l_t^n}$$

$$\gamma^y k_{t+1}^n = i_t^n + (1-\delta)k_t^n$$

$$p_t^n = \Psi_t^n + \theta_1 \mu_t^{\theta_2} + p_t^e (1-\mu_t)\varphi$$

$$z_t = \varepsilon_t^z \theta_1 \mu_t^{\theta_2} y_t^n$$

$$\hat{z}_t = \varepsilon_t^z \frac{\theta_2 \theta_1 \mu_t^{\theta_2 - 1}}{\mu_t}$$

$$p_t^e = \varepsilon_t^z \frac{\theta_1 \theta_2 \mu_t^{\theta_2 - 1}}{\varphi}$$

Government

$$g_t = \frac{\bar{g}}{\bar{y}} y_t$$

$$t_t + p_t^e e_t = g_t$$

Environmental Policy (All different cases that we consider in the paper)

$$e_t = \varepsilon_t^{cap} \text{cap}$$

$$\tau_t = \text{fixed price}$$

$$\tau_t = V_t^X$$

where the Social Cost of Carbon V_t^X is characterised by the following two equations:

$$\gamma^x V_t^T = \tilde{\beta}\Lambda_{t+1}((1-\zeta_1)V_{t+1}^T - (-2bt_{t+1}^o)\Psi_{t+1}^y y_{t+1}^y)$$
$$\gamma^x V_t^X = \tilde{\beta}\Lambda_{t+1}(\zeta_1\zeta_2 V_{t+1}^T + \eta V_{t+1}^X)$$

Aggregate Resource Constraint

$$y_t = c_t + i_t^y + i_t^n + z_t + g_t - p_t^n y_t^n$$

Shock law of motions

$$\log\left(\varepsilon_t^B\right) = \rho_B \log\left(\varepsilon_{t-1}^B\right) + \eta_t^B, \text{ with } \eta_t^B \sim N(0, \sigma_B^2)$$
$$\log\left(\varepsilon_t^{A_y}\right) = \rho_{A_y} \log\left(\varepsilon_{t-1}^{A_y}\right) + \eta_t^{A_y}, \text{ with } \eta_t^{A_y} \sim N(0, \sigma_{A_y}^2)$$
$$\log\left(\varepsilon_t^{A_n}\right) = \rho_{A_n} \log\left(\varepsilon_{t-1}^{A_n}\right) + \eta_t^{A_n}, \text{ with } \eta_t^{A_n} \sim N(0, \sigma_{A_n}^2)$$
$$\log\left(\varepsilon_t^{p_n}\right) = \rho_{p_n} \log\left(\varepsilon_{t-1}^{p_n}\right) + \eta_t^{p_n}, \text{ with } \eta_t^{p_n} \sim N(0, \sigma_{p_n}^2)$$
$$\log\left(\varepsilon_t^{z}\right) = \rho_{z} \log\left(\varepsilon_{t-1}^{z}\right) + \eta_t^{z}, \text{ with } \eta_t^{z} \sim N(0, \sigma_{z}^2)$$
$$\log\left(\varepsilon_t^{cap}\right) = \rho_{cap} \log\left(\varepsilon_{t-1}^{cap}\right) + \eta_t^{cap}, \text{ with } \eta_t^{cap} \sim N(0, \sigma_{cap}^2)$$

Chapter 4

General Conclusion

In this thesis we examine the climate physical and transition risk within estimated macro-financial frameworks.

In the first chapter, we assesses the marco-financial implications of setting an optimal carbon price. To do so, we introduce the environmental externality into the neoclassical growth model and show that the optimal carbon price is determined by the implicit price of CO2 emissions. The study uses asset-pricing theory to estimate the optimal carbon price over the business cycle and shows that risk aversion is higher when firms do not internalize the damage caused by emissions, which raises risk premia and lowers the natural rate of interest by increasing precautionary saving. The main policy implication is that the effectiveness of the policy depends on the abatement technology and the timing of implementation. The study also suggests that future research could investigate the interaction between the carbon tax and other policy instruments, such as monetary and macro-prudential policies, and compare tax and cap-and-trade policies.

In the second chapter, the scope of analysis is expanded. The linkages and interactions of different environmental policies (such as cap-and-trade, fix tax rates, optimal SCC) and monetary policy are investigated. This chapter, began by investigating how modeling climate damages through a utility function or a production function affects emissions reduction targets, carbon pricing, and macro-financial aggregates under the presence of monetary policy. The choice of modeling damages is crucial in determining CO_2 market prices, achieving emissions reduction targets, and macro-finance dynamics. The optimal policy instrument varies depending on the modeling choice, with a fixed policy rate necessary to achieve the Paris Agreement target under production damages modeling. The monetary authority faces a trade-off between emission reduction and real rate/inflation targeting.

In the third chapter, we investigate the drivers of carbon pricing in the EU ETS market using a macro-finance model and identifies abatement cost shocks, climate sentiment shocks, and energy shocks as the main factors driving carbon pricing. We find that reducing price uncertainty can help close the gap with respect to the optimal policy. We also introduce the concept of carbon cap rules and show how it helps eliminate some of the unintended volatility. The insights into the factors influencing carbon pricing can inform the design of more effective policies for reducing greenhouse gas emissions and mitigating climate change. The analysis advances our understanding of the EU ETS market and could inform future research on carbon pricing and climate policy.