LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Essays on Monetary Policy, Sovereign Debt and Financial Conditions

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A minha avó Meinha, minha maior fonte de inspiração e exemplo de vida. Benção, vó!

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 29,199 words.

Statement of Conjoint Work

I confirm that Chapter 2 is jointly co-authored with Fernando Eguren-Martin, Sevim Kösem and Andrej Sokol, and I contributed 25% of this work.

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Abstract

The first chapter studies the optimal maturity policy of sovereign debt within the framework of long-scale asset purchase (LSAP) programmes. It presents a model wherein the fiscal authority must navigate interest rate risk alongside a central bank engaging in LSAPs. The model predicts that levels of agreement and coordination between the fiscal authority and the central bank's optimal policies vary depending on the macroeconomic conditions that prompt LSAPs. These predictions find support in empirical evidence from the US, indicating that the Treasury has adjusted its response to the Fed's maturity extraction policies based on the prevailing macroeconomic environment.

The second chapter proposes a novel approach to extract factors from large data sets that maximise covariation with the quantiles of a target distribution of interest. From the data underlying the Chicago Fed's National Financial Conditions Index, we build targeted financial conditions indices for quantiles of future US GDP growth. We show that our indices yield considerably better out-of-sample density forecasts than competing models, as well as insights on the importance of individual financial series for different quantiles. Notably, leverage indicators co-move more with the median of the predictive distribution, while credit and risk indicators are more informative about downside risks.

The third chapter studies bank lending decision when banks play a central role in deposit and money creation while being subject to balance sheet constraints. It analyses how bank lending is affected by the banks' balance sheet dynamics in a low interest rates environment. In addition, it replicates a liquidity shock such as the one that hit the U.S. Treasuries market in March 2020, finding that capital requirements may limit banks' activities in bond markets following shocks like this. Finally, it shows that when banks' leverage reaches high levels, QE can transform liquidity crises into credit crises, worsening banks' situation.

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1.1 Introduction

In many countries, the management of government debt is typically considered separate from monetary policy decisions. Traditionally, the fiscal authority oversees governnment debt management, while the monetary authority (central bank) focuses on conducting monetary policy. However, with the implementation of large-scale asset purchases (LSAPs) programmes by central banks in recent decades, the distinction between the roles and tools of these two entities has become increasingly blurred.

LSAPs fundamentally involve the central bank selling short-term and low-duration sovereign-backed liabilities, such as central bank reserves (as seen in Quantitative Easing programmes) or short-term government bonds (as in Operation Twist), while simultaneously purchasing long-term and high-duration assets, typically long-term government bonds which are also sovereign-backed liabilities. Consequently, LSAPs directly impact the net stock of sovereign debt held by the public and the amount of interest rate risk borne by the government on a consolidated basis[1]. In this respect, LSAPs can be viewed as akin to debt management operations that are conducted by the central bank[2]. This raises questions about whether these programmes align with the prescribed fiscal authority's optimal debt management policy and what factors drive the agreement or conflict between central banks and fiscal authorities in these instances.

To address these questions, this paper investigates the factors influencing the optimal

[1]As discussed in detail in subsequent sections, balance sheets of both fiscal and monetary authorities are here considered part of the consolidated public sector balance sheet.

[2]Central banks typically strive to steer clear of this perspective, aiming to avoid being perceived as monetising deficits or interfering with the mandates of fiscal authorities. Bateman (2023) delves into the fiscal implications of certain policies enacted by the US Federal Reserve, shedding light on the institution's apprehensions regarding public perception.

maturity policy of sovereign debt in contexts where the central bank is implementing LSAPs. For this purpose, I present a model in which the fiscal authority faces interest rate risk and must determine the sovereign debt optimal maturity structure conditional on shocks that prompt the central bank to engage in LSAPs. I find that the level of coordination between the central bank and the fiscal authority policies is conditional on the type of LSAP programme and on the market expectations of future interest rates. Next, I employ US data to examine whether there is any coordination between the Federal Reserve and the Treasury during periods of LSAPs, and assess if the model's predictions align with the empirical evidence. The results demonstrate that the Treasury takes into consideration the consolidated sovereign balance sheet and reacts to the Fed's LSAPs accordingly. Moreover, they suggest that the Treasury's strategy for managing the debt maturity structure during LSAPs can vary, ranging from a less proactive to a more proactive approach, depending on the anticipated trajectory of the interest rates.

The analysis starts with the key insight that what matters for public debt sustainability is not the profile of the total outstanding stock of government securities, but rather the profile of the consolidated stock of sovereign debt, i.e. the privately-held stock of sovereign-backed liabilities. Given the significant impact of LSAPs on the latter, it becomes imperative for the fiscal authority to consider the impact these programmes might have on the debt profile when formulating its debt management policy. This consideration constitutes a crucial dimension of the model introduced later in the paper. I then present empirical evidence from the US indicating that the difference between the maturity profiles of the distinct debt stocks mentioned, referred to here as the maturity gap, was close to zero before 2008. However, this gap has substantially increased and has been fluctuating between 1 and 2 years since then. Thus, the focus on the privately-held stock of sovereign debt is not only theoretically important but also quantitatively relevant in the current economic environment.

In this context, I develop a model in which there is interest rate uncertainty, and both the fiscal authority and central bank can use interventions in the sovereign debt market as a policy tool. The fiscal authority is responsible for determining the optimal maturity structure based on private investors' demand, while the central bank may engage in the purchase or sale of long-term bonds in exchange for short-term interest-bearing reserves, following an exogenous rule. I demonstrate that the focus on the privately-held stock of sovereign debt and the consolidated balance sheet of the fiscal authority and central bank arises endogenously in this framework due to the transfer of central bank profits and losses to the fiscal authority. The optimal maturity structure reflects the tradeoff faced by a risk-averse fiscal authority in terms of a cheaper but riskier debt profile versus a costlier but safer one.

The model predicts that, in the absence of shocks, LSAPs have no direct impact on the fiscal authority's optimal choice of maturity for the consolidated sovereign debt. In contrast, a decrease in the investors' risk-bearing capacity or an increase in their demand for liquidity services prompts the fiscal authority to target a shorter maturity structure. This change in policy aligns with the implementation of LSAPs by the central bank, as they reduce the average maturity of the privately-held stock of debt. Conversely, a decrease in expected future interest rates results in a lengthened optimal maturity structure, leading to a divergence from the effect produced by LSAPs. In this scenario, both the central bank and the fiscal authority employ the same policy instrument but pursue conflicting objectives.

Next, I analyse US data on Treasury securities issuance and on the Fed's holdings and liabilities to evaluate whether the empirical evidence is in line with the predictions of the model. For this, I construct a maturity equivalent metric that enables the measurement of the maturity extraction carried out by the Fed in the sovereign debt market over time. This metric accounts for both balance sheet expansion (increase in the size of holdings) and maturity expansion (increase in the average maturity of holdings).

The findings indicate that the US fiscal authority has been cognisant of the impact of LSAPs on the debt stock held by the public, and its reactions to Fed policies have broadly conformed to the model's predictions. Specifically, the Treasury has reduced the maturity of its issuances during periods such as the onset of the Great Financial Crisis (GFC) and of the Covid outbreak, when investors' risk-bearing capacity was reduced and demand for liquidity was extremely high. In contrast, it has extended the maturity of its issuances in response to lower expected future interest rates and heightened maturity extraction by the Fed. Moreover, the Treasury's response to maturity extraction during LSAP programmes has been consistent with its response during non-LSAP periods.

When the empirical evidence is analysed within the developed theoretical framework, it further suggests that the degree of coordination between the Fed and the Treasury during LSAPs may depend on the behaviour of the expected future interest rates. In periods when expected future rates decrease, the Treasury has a greater incentive to take a proactive stance and lengthen the average maturity of its issuances, moving in the opposite direction and, at least partially, offsetting the Fed's maturity extraction policy. Conversely, when rates are stable, this incentive diminishes, and the Treasury may adopt a more passive approach by allowing the Fed to pursue its goals before responding optimally.

Literature Review

This paper contributes to the literature on optimal public debt policy by introducing a theoretical framework to study sovereign maturity choice under uncertainty in interest rates within an environment of large central bank balance sheets. The foundation of this literature traces back to Barro (1974), whose work established the Ricardian equivalance result under non-distortionary taxes. Subsequent studies by Barro (1979) and Aiyagari et al. (2002) have shown that distortionary sources of revenue undermine Ricardian equivalence, motivating the search for an optimal debt profile or composition.

Several papers have explored the topic of optimal maturity structure under various settings. For instance, Broner, Lorenzoni and Schmukler (2013) and Beetsma et al. (2021) investigate maturity choice in the presence of default risk for emerging markets and monetary unions, respectively. While this paper builds on their theoretical models, it diverges by emphasising refinancing risk, akin to the approaches of Arellano and Ramanarayanan (2012), and by excluding considerations of default. Additionally, this paper incorporates features such as the liquidity provision of safe short-term debt, inspired by Guibaud, Nosbusch and Vayanos (2013) and Greenwood, Hanson and Stein (2015), as well as the price impact of changes in long-term bond supply, as explored by Greenwood and Vayanos (2013). In contrast to these papers, however, I explicitly model the relationship between the fiscal authority and central bank and emphasise the role of their consolidated balance sheet.

This paper is also related to the literature that studies public debt sustainability under stochastic dynamics, which includes works such as Garcia and Rigobon (2004) and Debrun, Jarmuzek and Shabunina (2020). I present empirical evidence highlighting the importance of considering the consolidated balance sheet when assessing interest rate exposure and sustainability of the debt. This becomes especially relevant in a low interest rate environment, such as in Blanchard (2019) and Furman and Summers (2020), since LSAP programmes can significantly reduce the average maturity and duration of the consolidated sovereign debt relatively to the outstanding stock of government securities.

The impacts of central bank balance sheet policies have been extensively studied in the past years. Krishnamurty and Vissing-Jorgensen (2011, 2013) have offered insights into the effects of LSAPs on bond yields, delving into the channels through which these policies operate and their implications for monetary policy. Ray (2019) and Ray, Droste and Gordnichenko (2023) have analysed how quantitative easing (QE) affects output and inflation dynamics. Additionally, Gourinchas, Ray and Vayanos (2022) documented the substantial international spillover effects of LSAPs.

This paper aligns closely with the perspectives in Zampolli (2012) and Chadha, Turner and Zampolli (2013), which view LSAPs as akin to debt management operations. However, it distinguishes itself by emphasising the perspective of the fiscal authority rather than the central bank. In this context, the primary contribution lies in presenting a theoretical framework that tries to elucidate the impact of LSAPs on the debt stock and on the debt issuance policy.

Finally, this paper also contributes to the literature that studies institutional interactions between fiscal authorities and central banks. Orphanides (2016) and Goncharov, Ioannidou and Schmalz (2022) analyse the tensions arising from central bank balance sheet policies and the potential agency problems that they create. Reis (2017b) shows how QE can be a useful tool during a fiscal crisis, while Del Negro and Sims (2015) argue that central banks with large balance sheets require support from the fiscal authority to maintain control of the price level. This paper is closest to Greenwood et al. (2015a), who provide a discussion on conflicts between the Fed and Treasury in the US and the potential for coordination. I build on that discussion from the perspective of debt management and present theoretical intuition as well as empirical evidence for the drivers of cooperation or disagreement between both institutions.

Outline

The paper is structured as follows: Section 1.2 presents a brief discussion on the relationship between fiscal authorities and central banks and some stylised facts on the US consolidated sovereign debt. Section 1.3 develops and solves a model focusing on a risk-averse fiscal authority's optimal sovereign debt maturity problem in a context in which it faces interest rate risk and the central bank implements LSAPs. Section 1.4 describes the data and the construction of the maturity equivalent extraction measure and presents the empirical results, linking them with the model predictions. Finally, Section 1.5 provides the final remarks.

1.2 Institutional Details and Stylised Facts

Central banks are not always officially part of the public sector, but even when they are entirely independent institutions, their liabilities are still backed by the sovereign state.

Money and bank reserves, for instance, serve as universal mediums of exchange because society considers them legal tender, thus trusting that the sovereign government will always accept them. Additionally, any losses or profits incurred by a central bank are eventually transferred, either directly or indirectly, to the general government[3].

In this context, relying only on the traditional metric of general government debt, which focuses solely on securities issued by the fiscal authority and ignores the central bank's assets and liabilities, does not provide a comprehensive assessment of the sovereign's ability to repay its obligations. As discussed in Maia, Garcia and Maia (2022), it is essential to consider the consolidated balance sheet of the general government and the central bank, excluding intra-government holdings. In other words, emphasis should be placed on the stock of sovereign-backed liabilities (or debt) held by the private sector.

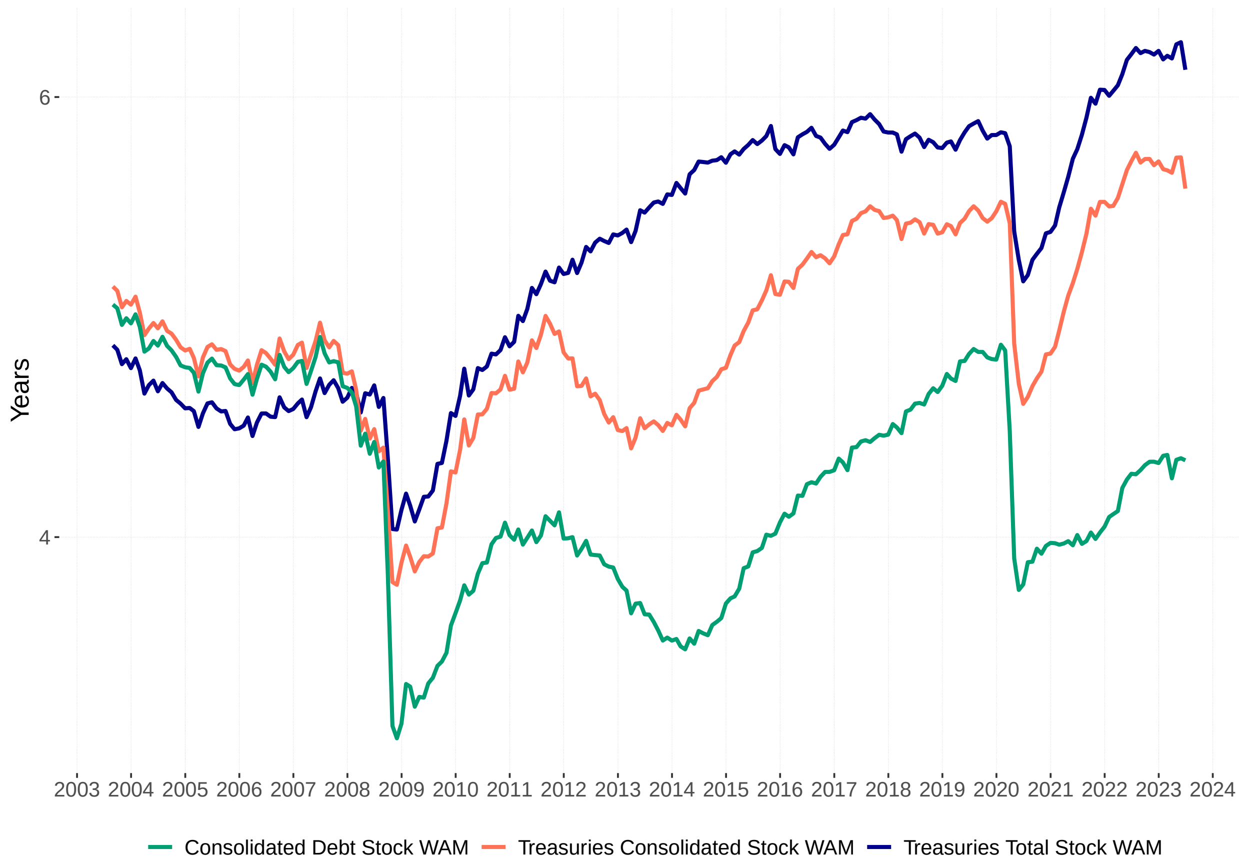
Before the GFC in 2008, central bank balance sheets were relatively small in most countries, making the stock of government securities a relatively reliable proxy for total consolidated sovereign debt. However, in recent years, central banks worldwide have implemented large-scale asset purchases programmes involving substantial injections of overnight interest-bearing reserves and extractions of long term government securities from the market. While these programmes did not directly alter the total outstanding stock of government securities, they significantly increased the consolidated government's exposure to interest rate risk by reducing the average maturity and duration of the privately-held debt.

As emphasised by Blanchard (2019), understanding the maturity profile and interest rate exposure of the outstanding stock is crucial for analysing sovereign debt sustainability and to prevent explosive paths. This underscores the necessity of distinguishing between the stock of government securities and the stock of privately-held sovereign debt. Figure 1.1 illustrates this distinction by using the US as an example[4]. It depicts the evolution of the weighted

[3]Reis (2017a) offers an in-depth discussion on the resource flows from the central bank to the fiscal authorities

[4]Maia, Garcia and Maia (2022) provide evidence for several other countries, encompassing both developed and emerging economies, underscoring that this is not a country specific issue.

Figure 1.1: Weighted Average Maturity Across Different Government Debt Stock Measures



Note: The chart compares the weighted average maturity of three different debt stock measures: "Treasuries Total Stock" series accounts for all securities issued by the U.S. Treasury, "Treasuries Consolidated Stock" refers to all privately-held Treasury securities and "Consolidated Debt Stock" represents all privately-held Treasury securities combined with central bank reserves.

average maturity of three series: i) the total outstanding stock of US Treasury securities; ii) the consolidated (privately-held) stock of US Treasury securities; and iii) the consolidated (privately-held) stock of US sovereign debt, which includes central bank reserves. Since 2007, these trajectories have diverged, resulting in substantial gaps between the maturity series. For instance, an observer focusing solely on the total outstanding stock of Treasury securities and disregarding the Fed's balance sheet might erroneously conclude that US sovereign debt had a higher weighted average maturity in 2011 compared to 2007. In reality, when accounting for Treasury securities held by the Fed, the weighted average maturity remained at the same level and was substantially lower when also considering the Fed's interest-bearing liabilities.

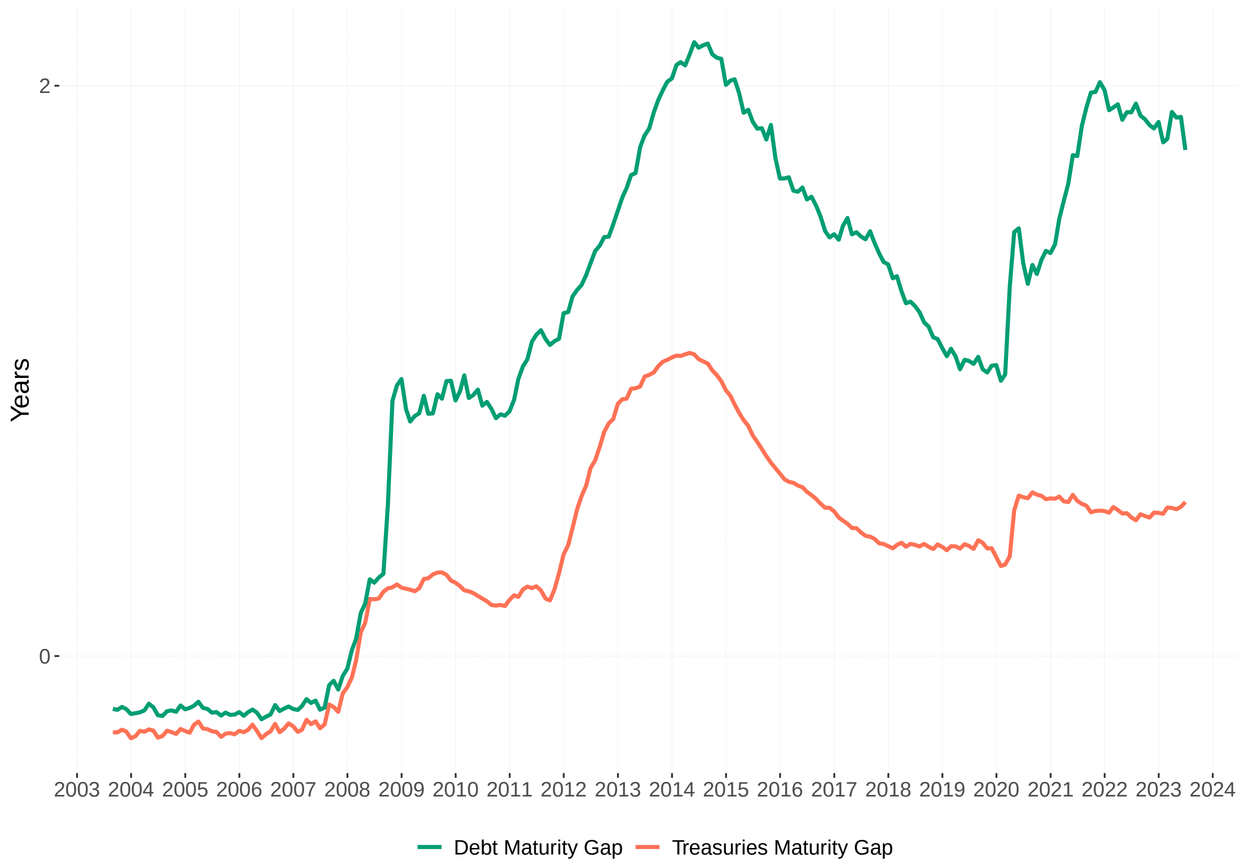
These "maturity gaps" are visually represented in Figure 1.2. The Treasuries Maturity Gap curve depicts the difference between the weighted average maturities of the privately-held stock and of the total outstanding stock of Treasury securities, while the Debt Maturity Gap curve illustrates the difference between the weighted average maturities of the privately-held stock of sovereign debt and of the total outstanding stock of Treasuries. Notably, since the start of LSAPs programmes in 2008, the effective weighted average maturity of the US sovereign debt, when accounting for both Fed's interest-bearing assets and liabilities, has been consistently and significantly lower than the weighted average maturity of the outstanding Treasuries.

While debt management is typically not within the purview of a central bank's responsibilities and objectives, LSAPs enables it to utilise the privately-held sovereign debt as a tool for achieving its monetary policy and financial stability goals. Within this framework, the evidence presented here demonstrates that the authority responsible for debt management (hereafter referred to as the fiscal authority) must consider the impacts of LSAPs when formulating its optimal maturity structure policy. By altering the demand and supply of short and long-term sovereign-backed liabilities, these programmes directly influence the cost and risk profile of the sovereign debt.

1.3 Model

In this section, I present a theoretical model of the fiscal authority's choice of sovereign debt maturity. The model builds on Broner, Lorenzoni and Schmukler (2013) and Beetsma et al. (2021), extended to allow for central bank interventions in the government bonds market (LSAPs) and to introduce refinancing risk due to future interest rate uncertainty. Unlike these papers, that focus on emerging markets or monetary union areas, I abstract

Figure 1.2: Maturity Gaps



Note: The chart illustrates the gap between the "Treasuries Total Stock WAM" series versus the "Consolidated Debt Stock WAM" (Debt Maturity Gap) and the "Tresuries Consolidated Stock WAM" (Treasuries Maturity Gap).

from default risk and rollover crises.

In the model, similar to the framework of Greenwood, Hanson and Stein (2015)[5], investors derive disutility from the liquidity costs associated with holding long-term assets, which makes issuing short-term debt cheaper for the fiscal authority. On the other hand, issuing more short-term debt leaves the government more vulnerable to interest rate fluctuations and thus increases its risk of having to make costly fiscal adjustments in the future.

[5]In Greenwood, Hanson and Stein (2015), investors derive utility from the monetary services provided by short-term bonds. In our setting, from the fiscal authority's perspective, this is exactly equivalent to deriving disutility from holding long-term bonds

The model focuses on the choice of public debt maturity by a fiscal authority that faces this tradeoff and studies how the optimal policy is affected by shocks like the ones that can lead the central bank to engage in LSAP programmes. Since LSAPs alter the proportion of short and long-term debt held by private investors, I use the model to generate predictions about the level of alignment of the fiscal authority and central bank policies.

Time is discrete, there are three periods $t = 0, 1, 2$ and three assets: short (one-period) and long-term (two-periods) bonds and central bank reserves. Interest rates are exogenous and, at $t = 0$, there is uncertainty about the $t = 1$ interest rate.

1.3.1 Agents

Fiscal Authority — At $t = 0$, the fiscal authority starts with a pre-existing stock of short and long-term bonds, $\hat{b}_{0,1}$ and $\hat{b}_{0,2}$, that are due in periods 1 and 2 respectively. Its problem consists in choosing a new debt maturity structure and a future consumption path[6] in order to maximise its expected utility.

The fiscal authority has two sources of revenues: an exogenous risk-free cash flow $\bar{y}$ collected at $t = 2$ and dividends from the central bank[7] d_t, paid every period. All its income is used to consume or repay the debt. Therefore, debt is used as an instrument to move consumption across periods, enabling the optimal consumption path.

At $t = 0$, the fiscal authority can adjust its debt maturity structure by choosing new stocks of short and long-term bonds, $b_{0,1}$ and $b_{0,2}$. In period 1, maturing short-term bonds must be repaid and a new stock of short-term debt $b_{1,2}$ can be issued. Finally, in period 2, all remaining debt must be repaid and the surplus is consumed.

Formally, the fiscal authority's optimisation problem is given by:

[6] For simplicity, I assume $t = 0$ consumption was already realised or decided ex-ante.

[7] Note that this can be negative, which would be equivalent to a transfer from the fiscal authority to the central bank.

$$\begin{aligned}
\max_{\{G_1,G_2,b_{0,1},b_{0,2},b_{1,2}\}} \quad & \mathbb{E}_0\left[u_g(G_1)+u_g(G_2)\right] \\
\text{s.t.} \quad & P_{0,1}\cdot\hat{b}_{0,1}+P_{0,2}\cdot\hat{b}_{0,2}=P_{0,1}\cdot b_{0,1}+P_{0,2}\cdot b_{0,2}+d_0 \\
& G_1=P_{1,2}\cdot b_{1,2}-b_{0,1}+d_1 \\
& G_2=\bar{y}-b_{0,2}-b_{1,2}+d_2,
\end{aligned} \tag{1.1}$$

where $u_g(\cdot)$ is such that $u_g'(\cdot)>0$ and $u_g''(\cdot)<0$, G_t is the government consumption at period t, and $P_{t,t+j}$ is the time t price of bonds that matures j periods ahead.

Central Bank — The central bank starts with a stock $\bar{z}$ of risk-free reserves and $\hat{b}^{cb}_{0,2}$ of government long-term bond holdings. Its initial equity is zero and it must pay dividend d_t to the fiscal authority at the end of each period.

In period 0, the central bank decides whether to buy or sell long-term bonds, choosing a final stock $b^{cb}_{0,2}$ that will be held to maturity. Reserves pay the risk-free short-term interest rate and can be adjusted in periods 0 and 1. Just like the bonds, they must be fully repaid at the end of $t=2$.

The flow budgets of the central bank will be:

$$\begin{aligned}
t=0: \quad & z_0-\bar{z}=P_{0,2}\cdot\left(b^{cb}_{0,2}-\hat{b}^{cb}_{0,2}\right)+d_0 \\
t=1: \quad & z_1-R^f_0\cdot z_0=d_1 \\
t=2: \quad & b^{cb}_{0,2}=R^f_1\cdot z_1+d_2,
\end{aligned} \tag{1.2}$$

where R^f_t is the gross one-period interest rate that holds between periods t and $t+1$

For simplicity, I assume d_t depends of central bank profits based on an exogenous rule. As in Reis (2017b), asset purchases policies consist of changes in the central bank's balance sheet such that changes in the bonds held by the central bank $b^{cb}_{0,2}$ are exactly equal to changes in reserves z_t[8].

Investors — Investors are risk averse and maximise their expected utility function. They start with wealth W_0 and receive no income in the following periods. At each period, they choose how much they want to consume and then allocate the remaining budget into the available financial assets.

In period 0, they can choose between holding reserves and both types of bonds. As mentioned before, long-term bonds generate a disutility $\nu(\cdot)$ for the investors as they are subject to price risk in period 1 and therefore are unable to provide liquidity services in the same way one-period safe assets do. At $t = 1$, the interest rate uncertainty realises, short-term bonds and reserves issued at $t = 0$ are repaid and investors can update their portfolio to achieve the desired consumption path in periods 1 and 2.

The representative investor will thus solve:

$$\begin{aligned} \max_{\{C_0, C_1, C_2, b^i_{0,1}, b^i_{0,2}, b^i_{1,2}, z_0, z_1\}} \quad & C_0 + \mathbb{E}_0\left[\beta \cdot u_i(C_1) + \beta^2 \cdot u_i(C_2)\right] - \nu\left(b^i_{0,2}\right) \\ \text{s.t.} \quad & C_0 = W_0 - P_{0,1} \cdot b^i_{0,1} - P_{0,2} \cdot b^i_{0,2} - z_0 \\ & C_1 = b^i_{0,1} + R^f_0 \cdot z_0 - P_{1,2} \cdot b^i_{1,2} - z_1 \\ & C_2 = b^i_{1,2} + b^i_{0,2} + R^f_1 \cdot z_1, \end{aligned} \tag{1.3}$$

where C_t is consumption at t, $b^i_{t,t+j}$ is the investor's holdings of bonds issued at time t that mature at $t + j$ and $\nu(\cdot)$ is such that $\nu(\cdot)' > 0$ and $\nu(\cdot)'' > 0$.

To simplify the analysis, I assume that the initial risk-free short-term interest rate is zero and that $\mathbb{E}_0[R^f_1] \geq 1$. Since both short-term bonds and reserves are riskless assets, we have that:

[8]For conciseness, we abstract from central bank holdings of short-term bonds. In this configuration, they are precisely equivalent to reserves, rendering QE and Operation Twist programmes identical in effect.

$$P_{0,1} = \frac{1}{R_0^f} = 1. \tag{1.4}$$

Similarly, at period 1, the risk-free short-term interest rate R_1^f is revealed to investors and thus all uncertainty is eliminated so that:

$$P_{1,2} = \frac{1}{R_1^f}, \tag{1.5}$$

for a realisation of R_1^f.

Finally, I follow Broner, Lorenzoni and Schmukler (2013) and further assume that investors demand a constant risk premium to "lock up" their capital in commitment to long-term investment[9]. As a result, one can write the price of the long-term bond as:

$$P_{0,2} = \mathbb{E}_0 \left[P_{1,2}\right] - \kappa - v' \left(b_{0,2}^i\right) = \mathbb{E}_0 \left[\frac{1}{R_1^f}\right] - \kappa - v' \left(b_{0,2}^i\right), \tag{1.6}$$

where $\kappa > 0$ is a constant parameter.

1.3.2 Rewriting the Fiscal Authority's Problem

Following the discussion in Section 1.2, one can define the period t amount of consolidated sovereign debt $B_{t,t+j}$ that is due at $t + j$ as:

$$\begin{aligned} B_{0,1} &\equiv b_{0,1} + z_0 \\ B_{1,2} &\equiv b_{1,2} + R_1^f \cdot z_1 \\ B_{0,2} &\equiv b_{0,2} - b_{0,2}^{cb}. \end{aligned} \tag{1.7}$$

[9]This is equivalent to duration risk in a setting where long-term bonds are traded in period 1.

The short-term consolidated debt stock consists of short-term bonds issued by the fiscal authority as well as reserves issued by the central bank. In our setting, these assets are equivalent: both are one period risk-free government-backed liabilities. At the same time, the stock of long-term consolidated sovereign debt is given by the stock of long-term bonds issued by the fiscal authority net of the central banks' holdings. Note that, in equilibrium, the stock of consolidated debt must be equal to the amount of debt held by the private sector, i.e., $B_{t,t+j} = b^i_{t,t+j}$.

Using the central bank's flow budgets and the definition of consolidated sovereign debt above, one can rewrite the fiscal authority's maximisation problem as:

$$\begin{aligned}
\max_{\{G_1,G_2,b_{0,1},b_{0,2},b_{1,2}\}} \quad & \mathbb{E}_0\left[u_g(G_1) + u_g(G_2)\right] \\
\text{s.t.} \quad & P_{0,1}\cdot \hat{B}_{0,1} + P_{0,2}\cdot \hat{B}_{0,2} = P_{0,1}\cdot B_{0,1} + P_{0,2}\cdot B_{0,2} \\
& G_1 = P_{1,2}\cdot B_{1,2} - B_{0,1} \\
& G_2 = \bar{y} - B_{0,2} - B_{1,2}.
\end{aligned} \tag{1.8}$$

This is exactly the same debt management problem as before, but consolidating the balance sheets of the fiscal authority and the central bank. It becomes evident that what is relevant for the fiscal authority's budget constraints is the stock and the profile of the privately-held sovereign debt. However, the fiscal authority can only control these variables conditional on the central bank's balance sheet.

Lemma 1.1. *LSAP programmes have a direct impact on the fiscal authority's optimal issuance policy by changing the maturity profile of the privately-held stock of sovereign debt.*

In other words, a LSAP programme works like a debt management policy. By swapping a short-term government-backed liability (reserves) by a long-term one (long-term bonds), it shortens the maturity of the consolidated sovereign debt, potentially moving it away from the original fiscal authority's optimal choice. In reaction, even when relative prices are not affected, the fiscal authority may have to alter its issuance policy.

1.3.3 Fiscal Authority's Tradeoff

To solve the model[10], I assume the fiscal authority chooses its optimal maturity policy after the central bank has decided on LSAPs and on its balance sheet[11]. This allows the paper to focus on the drivers of the maturity policy from the fiscal authority's perspective.

First, note that, using the prices previously obtained, one can combine the three fiscal authority's budget constraints to get:

$$G_2 = \bar{y} - B_{0,2} - R_1^f \cdot \left(G_1 + \hat{b}_{0,1} - P_{0,2} \cdot \left(B_{0,2} - \hat{b}_{0,2} \right) \right). \tag{1.9}$$

Solving the fiscal authority's optimisation problem backwards, the first-order condition at period 1 with respect to G_1 is given by:

$$u'_g(G_1) = R_1^f \cdot u'_g(G_2). \tag{1.10}$$

At this point, all uncertainty has been removed and thus the fiscal authority simply chooses its optimal consumption path $\{G_1^*, G_2^*\}$ based on the realised short-term interest rate R_1^f.

Going back to period 0, the fiscal authority has to choose the debt maturity structure

[10] Detailed proofs and derivations are provided in Appendix A.

[11] For simplification, the central bank is assumed here to follow an exogenous rule. In practice, these two agents make decisions dynamically and simultaneously. However, as explained earlier, while LSAPs influence the consolidated debt maturity structure, debt management typically lies outside the central bank's mandate, which focuses on monetary policy and financial stability. Public debt management is generally the responsibility of fiscal authorities, and this paper focuses on their perspective. Introducing a setting where the central bank's decision is endogenous or dynamic would introduce complex and unnecessary feedback effects on the fiscal authority's policy rule. These effects would heavily depend on assumptions about the force and speed of action of each institution, a discussion beyond the scope of this paper.

conditional on the optimal consumption path evaluated in period 1. The first-order condition with respect to $B_{0,2}$ at $t = 0$ is:

$$\mathbb{E}_0 \left[\left(u_g'(G_1^*) + u_g'(G_2^*) \cdot \frac{\partial G_2^*}{\partial G_1} \right) \cdot \frac{\partial G_1^*}{\partial B_{0,2}} + \left(\frac{\partial G_2^*}{\partial B_{0,2}} + \frac{\partial G_2^*}{\partial P_{0,2}} \cdot \frac{\partial P_{0,2}}{\partial B_{0,2}} \right) \cdot u_g'(G_2^*) \right] = 0. \quad (1.11)$$

Using Equations 1.6, 1.9 and 1.10, the result above simplifies to:

$$\mathbb{E}_0 \left[\left(-1 + R_1^f \cdot \left[\left(\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v'\left(B_{0,2}\right) \right) - v''(B_{0,2}) \cdot (B_{0,2} - \hat{B}_{0,2}) \right] \right) \cdot u_g'(G_2^*) \right] = 0. \quad (1.12)$$

Finally, one can rewrite the first-order condition at period 0 to obtain the result expressed in the lemma below:

Lemma 1.2. *The tradeoff faced by a risk averse fiscal authority when choosing its debt maturity structure is characterised by:*

$$\mathbb{E}_0 \left[R_1^f \right] + Cov_0 \left(\frac{u_g'(G_2^*)}{\mathbb{E}_0 \left[u_g'(G_2^*) \right]}, R_1^f \right) = \frac{1}{\left(\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v'\left(B_{0,2}\right) \right) - v''(B_{0,2}) \cdot (B_{0,2} - \hat{B}_{0,2})}. \quad (1.13)$$

Observe that the left-hand side of Equation 1.13 represents the marginal cost of issuing one extra unit of short-term debt (both short-term bonds and reserves), which is given by the expected return plus a covariance term that accounts for the drawback caused by the exposure to interest rate risk, while the right-hand side represents the marginal cost of issuing one extra unit of long-term bonds. As expected, in the optimal point, the fiscal authority chooses a debt maturity structure that equalises these two marginal costs.

1.3.4 Maturity Structure Predictions

Using a first-order Taylor approximation of $u_g'(G_2)$ around the point $R_1^f = \mathbb{E}_0\left[R_1^f\right]$ and assuming CARA utility function for the fiscal authority, one can rewrite Equation 1.13 as:

$$\mathbb{E}_0\left[R_1^f\right] - \alpha \cdot Var_0\left(R_1^f\right) \cdot G_2'\left(\mathbb{E}_0\left[R_1^f\right]\right) = \frac{1}{\left(\mathbb{E}_0\left[\frac{1}{R_1^f}\right] - \kappa - v'\left(B_{0,2}\right)\right) - v''(B_{0,2}) \cdot (B_{0,2} - \hat{B}_{0,2})}, \tag{1.14}$$

where $\alpha > 0$ is a constant that represents the fiscal authority degree of risk aversion.

Equation 1.14 characterises the non-monetary marginal cost associated with issuing short-term debt as a function of the magnitude of the fiscal authority's risk aversion, of the level of risk associated with the future interest rate and of how consumption is affected by a change in the expected future interest rate.

In order to extract some predictions from the model, I use Equation 1.14 to perform comparative static exercises, thus assuming solution to be internal and ensuring positive stocks of both short and long-term debt. In addition, the initial debt maturity structure at period 0 is assumed to be optimal ex-ante, more specifically $\hat{B}_{0,2} = B_{0,2}^*$, and therefore any change in the fiscal authority's maturity choice is due only to the shock in question. As a consequence of this assumption, income effects related to movements in bond prices will be eliminated and the analysis simplified.

Comparative Statics — The first result comes immediately and works as a building block for the subsequent ones as well for interpreting the empirical results:

Lemma 1.3. *In the absence of shocks, the fiscal authority's response to LSAPs is to increase the maturity of newly issued bonds so as to restore the consolidated sovereign debt maturity structure to the optimal one, which remains unaltered.*

In the context of debt management, if no shocks have hit the economy, the optimal maturity structure of the privately-held sovereign debt is still the same and the impact LSAP programmes have is to move the effective maturity away from it, by swapping long-term for short-term debt. Therefore, the fiscal authority will look to lengthen the maturity of its new issues in order to offset LSAPs' impact, thus bringing the maturity of the consolidated debt stock back to the original optimal level.

This result illustrates the non-neutrality of LSAP programmes in relation to debt management. LSAPs impact the optimal debt issuance policy by changing the maturity structure of the consolidated debt. Furthermore, it shows that LSAPs are neutral when one considers only the general government debt and the stock of outstanding bonds. This underlines the importance of consolidating the public sector balance sheet and focusing on the privately-held stock of sovereign debt when analysing maturity structure management.

Using Lemma 1.3 as baseline, the subsequent results explore how the fiscal authority's optimal maturity changes when LSAPs are implemented in response to unanticipated shocks to investor risk-bearing capacity, to demand for liquidity services and to the expected future interest rate.

Starting with the first two shocks,

Proposition 1.1. *A reduction in the risk-bearing capacity of investors (higher κ) or a linear increase in the demand for liquidity services (higher η, where $\nu(\cdot) = \eta \cdot \phi(\cdot)$) leads the fiscal authority to target a shorter maturity structure for the consolidated sovereign debt.*

The intuition behind Proposition 1.1 is that, as investors face reduced risk-bearing capacity or higher liquidity costs associated with holding long-term debt in comparison to short-term debt, the price of long-term bonds goes down, which makes it costlier for the fiscal authority to borrow long-term. As a consequence, the fiscal authority will modify the maturity composition in favor of short-term debt, considering it as a more cost-effective option.

In this setting, one could argue that the fiscal authority optimal policy is in line with the execution of LSAPs by the central bank, as both result in the shortening of the maturity of the consolidated stock of debt. Put it differently, by taking on the role of market maker and reacting to bond disruptions through the implementation of LSAP programmes, the central bank is, at least qualitatively, acting in the same way as the fiscal authority would.

Next, Proposition 1.2 focus on a shock to the period 0 expectations of the future interest rate, $\mathbb{E}_0\left[R_1^f\right]$:

Proposition 1.2. *A reduction in the expected future interest rate (lower* $\mathbb{E}_0\left[R_1^f\right]$*) leads the fiscal authority to target a longer maturity structure for the consolidated public debt.*

There are two channels in place here in Proposition 1.2. First, when expected future interest rate falls, the term premium decreases because the long-term bond price is more sensitive than the short-term price. This effectively reduces the relative price of the long-term bond. Second, and most relevant, when the expected future interest rate $\mathbb{E}_0\left[R_1^f\right]$ falls, while keeping the distribution of the actual R_1^f at period 1 fixed, the risk of a higher than expected realisation increases. As a result, the fiscal authority becomes effectively less willing to take risks and thus has more incentive to smooth the fluctuations in its marginal utility. Therefore, it shifts away from the risky short-term debt and towards the safe long-term debt. These two effects work in the same direction, ultimately resulting in the fiscal authority selecting a longer maturity compared to the original optimal.

Observably, when the central bank engages in LSAPs in an environment in which expected future interest rates are declining, it produces an effect on the debt maturity structure that diverges from the direction predicted by Proposition 2. While LSAPs effectively shorten the maturity of privately-held sovereign debt, the fiscal authority aims to respond to the decline in expected future interest rates by lengthening it. This incongruity between LSAPs and the optimal maturity policy indicates that the central bank and the fiscal authority use the same policy instrument while pursuing conflicting objectives.

1.4 Empirical Results

In this section, I describe the primary sources of data and explain the construction of my measure of consolidated sovereign debt. Subsequently, I use the model predictions to clarify and to provide context for the evidence presented in the empirical tests.

1.4.1 Data and Consolidated Sovereign Debt Measure

I collect panel data on the outstanding Treasury securities, available in the US Treasury Monthly Statement of the Public Debt, and use it to extract information about the individual securities monthly issuance and about the profile of the outstanding stock. In parallel, I gather data on the Fed holdings and purchases of Treasury securities from the Federal Reserve Bank of New York's System Open Market Account (SOMA) database and on the Fed liabilities from the Board of Governors of the Federal Reserve System[12]. The focus on US data is driven by its relevance in terms of central bank balance sheet policies and by availability of data on central bank's individual securities holdings. The sample goes from July 2003 until June 2023.

Using both datasets, it becomes possible to calculate the privately-held stock of Treasury securities by subtracting the Fed holdings from the total outstanding stock. I then use this series to construct the consolidated debt measure by adding two Fed overnight liabilities: bank reserves (calculated as total deposits net of the Treasury General Account balance) and total reverse repurchase agreements. The key point here is that these liabilities currently bear interest[13], thereby contributing to the exposure of the consolidated sovereign portfolio to interest rate risk. Consequently, a metric that overlooks these interest-bearing central

[12]See H.4.1 Statistical Release on factors affecting reserve balances

[13]This has not always been the case, as the Fed only started paying interest on bank reserves in October 2008.

bank liabilities will substantially underestimate the impact of interest rate fluctuations on government budget.

Following the methodology of Greenwood et al. (2015), I proceed to construct a measure of ten-year maturity equivalents. This measure is employed to assess the extent of maturity/duration extracted from the market by the Fed during each period. It enables the capture of two distinct forces driving the maturity extraction performed by the Fed: changes in the amount of holdings for a fixed weighted average maturity (balance sheet expansion) and changes in their weighted average maturity for a fixed stock of holdings (maturity expansion). The measure is calculated as follows:

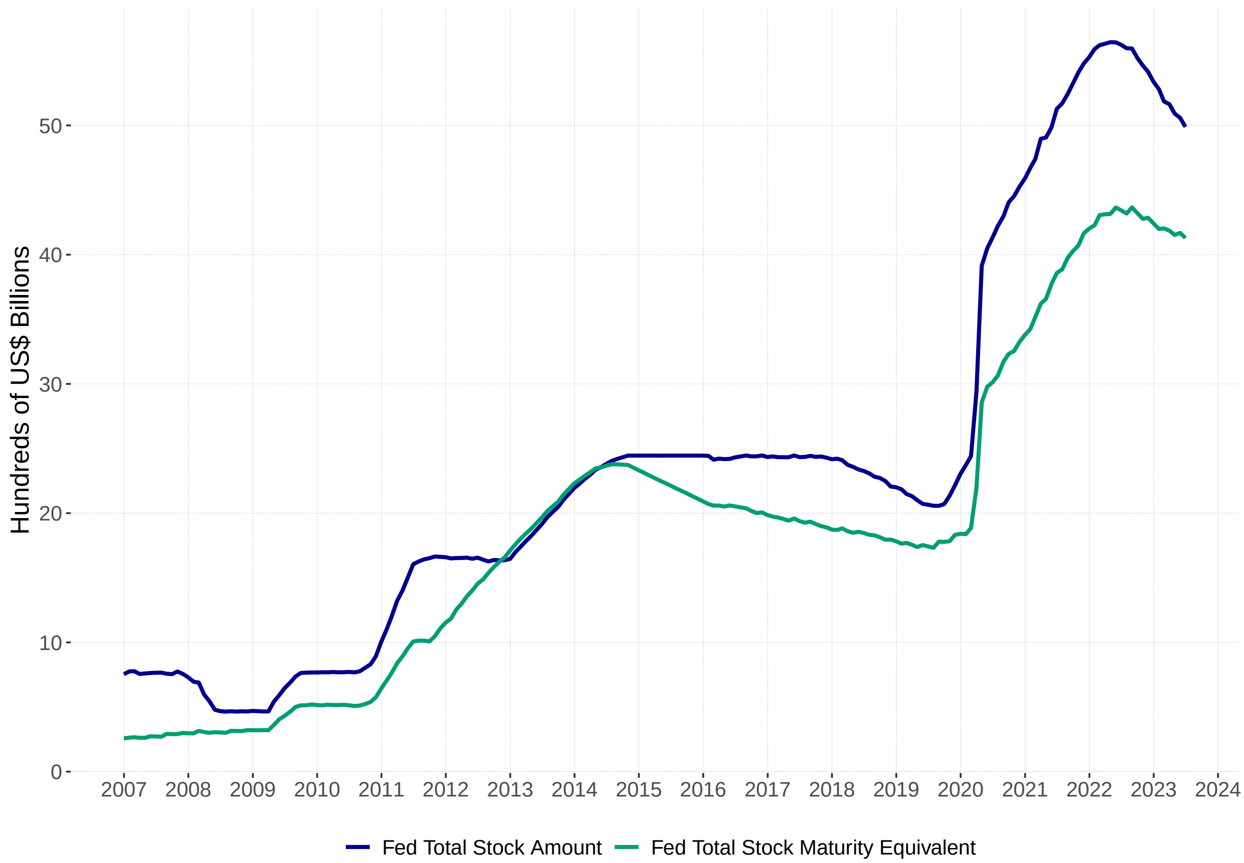
$$\underbrace{\Delta\left(\frac{Holds_t \cdot WAM_t}{Mat^{10-yr}}\right)}_{\text{Maturity Equivalent Extraction}} = \frac{1}{Mat^{10-yr}} \cdot \left(\underbrace{\Delta Holds_t \cdot WAM_{t-1}}_{\text{Balance Sheet Expansion}} + \underbrace{\Delta WAM_t \cdot Holds_t}_{\text{Maturity Expansion}}\right), \quad (1.15)$$

where $Holds_t$ is the stock of Treasury securities held by the Fed at period t, WAM_t is the weighted average maturity of the holdings at t and Mat^{10-yr} is the maturity of a ten-year bond, which is 10[14].

In this context, Figure 1.3 documents the evolution of both the total stock amount of holdings and the total stock of maturity equivalent held by the Fed. It is evident that these two series not only differ over time, but there are periods when they even move in opposite direction. The differences between the two series are explained by Figure 1.4, which displays the weighted average maturity of the Fed's total stock of Treasuries holdings. The large-scale asset purchases programmes led to an increase in the stock of maturity equivalents extracted by the Fed, via both balance sheet and maturity expansions. Thus, focusing on only one

[14]The choice to normalise for a ten-year bond is arbitrary. In this case, the maturity equivalent measures the amount of securities extracted by the Fed in a world in which all securities were ten-year bonds. The idea is just to make securities with different maturities comparable, so one could pick any other maturity as the reference point.

Figure 1.3: Fed Total Stock Amount vs. Extracted Maturity Equivalent

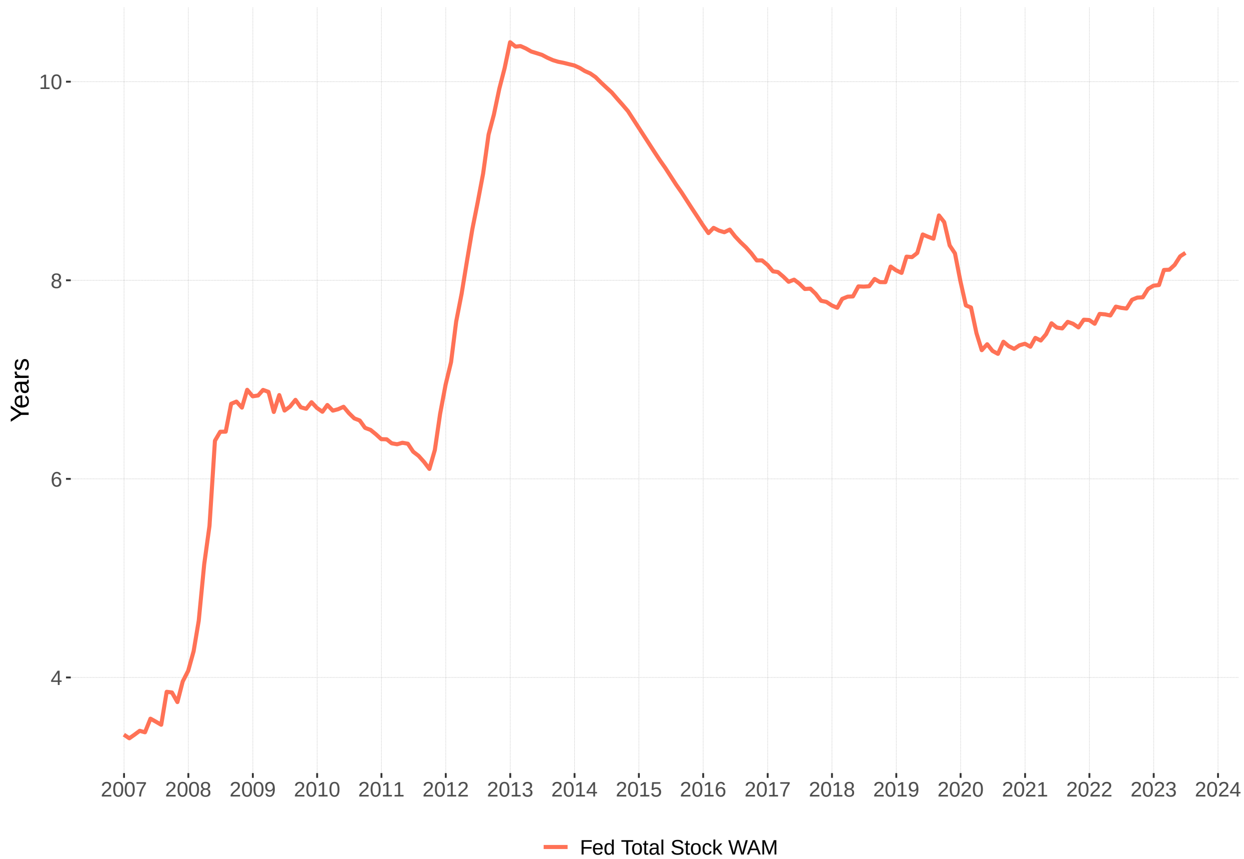


Note: This chart compares the Fed's total holdings of Treasury securities and its stock of maturity equivalents, as calculated using the methodology described in (1.15).

of these drivers would result in a misestimation of the total amount of maturity extracted from the aggregate private portfolio and, as a consequence, of the change in the interest rate risk exposure of the sovereign portfolio. This evidence underscores the importance of constructing the maturity equivalent series in order to accurately assess the impacts of the LSAPs programmes.

In the empirical exercises that follow, two additional sources of data are employed. Firstly, I construct expected future interest rates series using the Kim and Wright (2005) database, accessible on the Federal Reserve Board website. Following their methodology, the expected future short interest rate is defined as the difference between the fitted instantaneous for-

Figure 1.4: Fed Total Treasuries Stock Weighted Average Maturity



Note: This chart displays the weighted average maturity of the stock of Treasuries held by the Fed.

ward rate and the instantaneous forward term premium for that horizon. Secondly, control variables, including the effective Fed Fund rate, GDP growth, CPI, and others, are sourced from the St. Louis Fed's FRED database.

1.4.2 Specifications and Results

The baseline regression equation links the weighted average maturity of newly issued Treasury securities to the expected future interest rate, to the amount of overnight interest-bearing Fed liabilities and to the stock of maturity equivalent extracted from the markets by the Fed:

$$WAM_{t+1} = \alpha + \beta_1 \cdot \mathbb{E}_t\left[i_{t+24}\right] + \beta_2 \cdot Res_t + \beta_3 \cdot Fed_t^{10yEq} + \delta \cdot X_t + \epsilon_{t+1}, \tag{1.16}$$

where WAM_{t+1} is the weighted average maturity of securities issued by the Treasury in the next three months following t, $\mathbb{E}_t\left[i_{t+24}\right]$ is the time t expectation for the Fed Funds rate two years ahead, Res_t is the amount of bank reserves and reverse repo agreements as a share of the total consolidated debt, Fed_t^{10yEq} is the share of the total stock of ten-year maturity equivalents issued by the Treasury that was extracted from the market and is currently held by the Fed, and X_t is the set of controls that includes the current Fed Funds rate, the 2-year and 10-year slopes of the yield curve, the expected inflation two years ahead, GDP growth, quarter dummies to adjust for seasonal fluctuations that are not related to the Treasury policy and time fixed effects to account for the different LSAP periods. The time frames for these periods correspond to the duration of the five main individual LSAP programmes up to this date: QE1, QE2, Operation Twist (OT), QE3 and QE4.

The decision to employ a 3-month average for the variable WAM_{t+1} is informed by the recognition that, particularly for long-term securities, the time between auctions may extend beyond one month, rendering a monthly measure susceptible to excessive noise. Additionally, using lagged explanatory variables mitigates potential feedback effects stemming from the Treasury's maturity choices. For the expected future interest rate and expected inflation, a 2-year horizon is selected as it represents approximately half of the average consolidated debt stock weighted average maturity during the sample period. This choice ensures a balanced consideration of future trends while maintaining relevance to the Treasury refinancing problem. Furthermore, the choice of the variables Res_t and Fed_t^{10yEq} serves to normalise the amount of overnight interest-bearing Fed liabilities and the maturity extraction conducted by the Fed, respectively, relative to the total amount of consolidated sovereign debt and to the total outstanding stock of maturity equivalents issued by the Treasury, thereby accounting for any increasing trend in the size of debt markets.

Estimates for regression 1.16 are presented in Table 1.1. The coefficients for expected future interest rates are consistently negative across all specifications, with statistical significance observed in the majority of cases. This aligns with the model's prediction that the fiscal authority tends to extend the maturity of its securities in periods of lower expected future interest rates. Conversely, the coefficients for the Fed overnight liabilities and for the stock of extracted maturity equivalents are both positive and statistically significant. This finding supports the notion that the consolidated debt is pivotal for sovereign debt management policies. If the Treasury's sole concern were the debt it issues in the form of Treasury securities, one would anticipate no response to the share of Fed overnight liabilities, leading to an insignificant coefficient for Res_t. Similarly, if the Treasury were exclusively focused on the profile of the total outstanding stock of debt, regardless of its holders, one would expect no reaction to the Fed's maturity extraction programmes, resulting in an insignificant coefficient for Fed_t^{10yEq}.

Regarding the control variables, higher current Fed Funds rate and GDP growth are associated with longer maturities. As expected, the coefficients for the slopes in the short and long parts of the yield curve have inverted signs: higher short-term yields or lower long-term yields tend to increase the weighted average maturity of Treasury issuances. Lastly, the coefficient for the expected inflation becomes insignificant once the Fed liabilities and the stock of extracted maturity are included in the regression.

Next, I explore how the Treasury's response to Fed maturity extraction varies across different LSAP programmes. To examine the heterogeneity of the Treasury reaction function, I allow the variable Fed_t^{10yEq} to interact with the time fixed effects for the LSAP periods, as follows:

Table 1.1: Treasury Issuance Policy

	Treasury Issuance WAM					
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[i_{t+24}]$	-0.227^{***}	-0.183	-0.348^{**}	-0.240^{***}	-0.592^{***}	-0.576^{***}
	(0.082)	(0.126)	(0.136)	(0.056)	(0.089)	(0.098)
Res_t				2.708^{***}	4.091^{***}	4.106^{***}
				(0.484)	(0.549)	(0.548)
Fed_t^{10yEq}				3.321^{***}	2.543^{***}	2.581^{***}
				(0.620)	(0.624)	(0.624)
i_t	-0.041	-0.191^{**}	-0.159^{*}	0.098^{***}	0.438^{***}	0.440^{***}
	(0.054)	(0.093)	(0.091)	(0.035)	(0.076)	(0.076)
$2ySlope_t$		0.344	0.272		1.207^{***}	1.224^{***}
		(0.310)	(0.302)		(0.250)	(0.250)
$10ySlope_t$		-0.325^{***}	-0.321^{***}		-0.081^{**}	-0.082^{**}
		(0.048)	(0.048)		(0.041)	(0.041)
$\mathbb{E}_t[\pi_{t+24}]$			0.299^{***}			-0.048
			(0.099)			(0.071)
ΔGDP_t			2.853^{*}			3.253^{***}
			(1.454)			(0.908)
Observations	237	237	234	237	237	234
R^2	0.336	0.501	0.541	0.790	0.811	0.828
LSAPs FE	N	N	N	Y	Y	Y
Quarter FE	N	N	Y	N	N	Y

Note: *p<0.1; **p<0.05; ***p<0.01

$$\begin{aligned}WAM_{t+1} = \alpha + \beta_1 \cdot \mathbb{E}_t\left[i_{t+24}\right] + \beta_2 \cdot Res_t + \beta_3 \cdot Fed_t^{10yEq} + \\ + \sum_i \gamma_i \cdot Fed_t^{10yEq} \cdot D_{t+1}^i + \delta \cdot X_t + \epsilon_{t+1},\end{aligned} \tag{1.17}$$

where D_{t+1}^i is a dummy variable that takes value 1 during LSAP period i and 0 in other periods [15]. Results are presented in Table 1.2.

Each pair of columns in Table 1.2 corresponds to the interaction of Fed_t^{10yEq} with a different set of dummies D_{t+1}^i. In specifications (1) and (2), a single dummy encompasses all LSAP programmes. In (3) and (4), each LSAP period is considered individually, except for OT and QE3 that are combined together due to their overlapping time frames, which could suggest some continuity or coordination of policies between them. Finally, in columns (5) and (6), OT and QE3 are separated from each other and thus all five LSAP programmes are treated individually [16].

In accordance with both the model predictions and the results presented in Table 1.1, the coefficients for the expected future interest rate are negative and statistically significant for all specifications. Similarly, the coefficients for Fed overnight liabilities as a share of the total consolidated debt and for the share of 10-year maturity equivalents extracted by the Fed are consistently positive and statistically meaningful. Additionally, the behaviour of control variables is also consistent with Table 1.1 results and in line with expectations.

When examining columns (1) and (2), the non-significant coefficients for the interaction term suggest that the Treasury's response to the Fed maturity extraction during LSAPs aligns, on average, with its response during non-LSAP periods. This indicates that, control-

[15] To maintain consistency, D_{t+1}^i spans the same 3 month window that defines the WAM_{t+1} dependent variable.

[16] To accurately analyse the heterogeneity of the Treasury's reaction function to the Fed maturity equivalent extraction policy, the LSAP dummies are limited to the phases of the programmes that involved Fed interventions in the Treasury securities market.

Table 1.2: Treasury Issuance Policy during LSAPs

	Treasury Issuance WAM					
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[i_{t+24}]$	-0.522^{***}	-0.485^{***}	-0.611^{***}	-0.613^{***}	-0.595^{***}	-0.610^{***}
	(0.085)	(0.091)	(0.090)	(0.099)	(0.085)	(0.093)
Res_t	4.052^{***}	4.012^{***}	4.300^{***}	4.329^{***}	3.225^{***}	3.158^{***}
	(0.494)	(0.488)	(0.550)	(0.551)	(0.560)	(0.553)
Fed_t^{10yEq}	2.682^{***}	2.822^{***}	1.986^{***}	1.950^{***}	2.514^{***}	2.488^{***}
	(0.592)	(0.581)	(0.669)	(0.672)	(0.641)	(0.632)
Fed_t^{10yEq} * QE's	-0.701	-0.909				
	(0.591)	(0.576)				
Fed_t^{10yEq} * QE1			-1.430	-2.166	0.393	-0.093
			(6.971)	(6.869)	(6.600)	(6.404)
Fed_t^{10yEq} * QE2			-3.604	-2.238	-1.068	-0.096
			(4.209)	(4.195)	(3.990)	(3.886)
Fed_t^{10yEq} * OTQE3			1.256	1.223		
			(0.999)	(0.988)		
Fed_t^{10yEq} * OT					-4.744^{*}	-4.228^{*}
					(2.494)	(2.383)
Fed_t^{10yEq} * QE3					12.832^{***}	13.786^{***}
					(2.436)	(2.353)
Fed_t^{10yEq} * QE4			1.637	1.060	1.969	1.485
			(1.288)	(1.271)	(1.220)	(1.185)
i_t	0.403^{***}	0.398^{***}	0.441^{***}	0.444^{***}	0.364^{***}	0.360^{***}
	(0.073)	(0.072)	(0.077)	(0.076)	(0.074)	(0.073)
$2ySlope_t$	1.013^{***}	1.014^{***}	1.265^{***}	1.268^{***}	1.127^{***}	1.107^{***}
	(0.226)	(0.224)	(0.252)	(0.253)	(0.240)	(0.237)
$10ySlope_t$	-0.023	-0.018	-0.104^{**}	-0.109^{**}	-0.156^{***}	-0.166^{***}
	(0.035)	(0.035)	(0.042)	(0.043)	(0.042)	(0.042)
$\mathbb{E}_t[\pi_{t+24}]$		-0.071		-0.018		0.010
		(0.068)		(0.073)		(0.068)
ΔGDP_t		3.297^{***}		3.133^{***}		2.933^{***}
		(0.926)		(0.918)		(0.855)
Observations	237	234	237	234	237	234
R^2	0.802	0.818	0.814	0.829	0.835	0.854
Quarter FE	N	Y	N	Y	N	Y

Note: *p<0.1; **p<0.05; ***p<0.01

ling for potential different macroeconomic environments, the Treasury maintains a consistent maturity policy for its new issuances, which takes into account the profile of the consolidated debt stock. Such behaviour is in line with the predictions of the model. Moving to columns (3) and (4), the insignificant coefficients for all four interaction terms further bolster this conclusion. They indicate no meaningful change in the Treasury's reaction function during these individual intervals, suggesting that the average response remained consistent across all the four analysed LSAP periods.

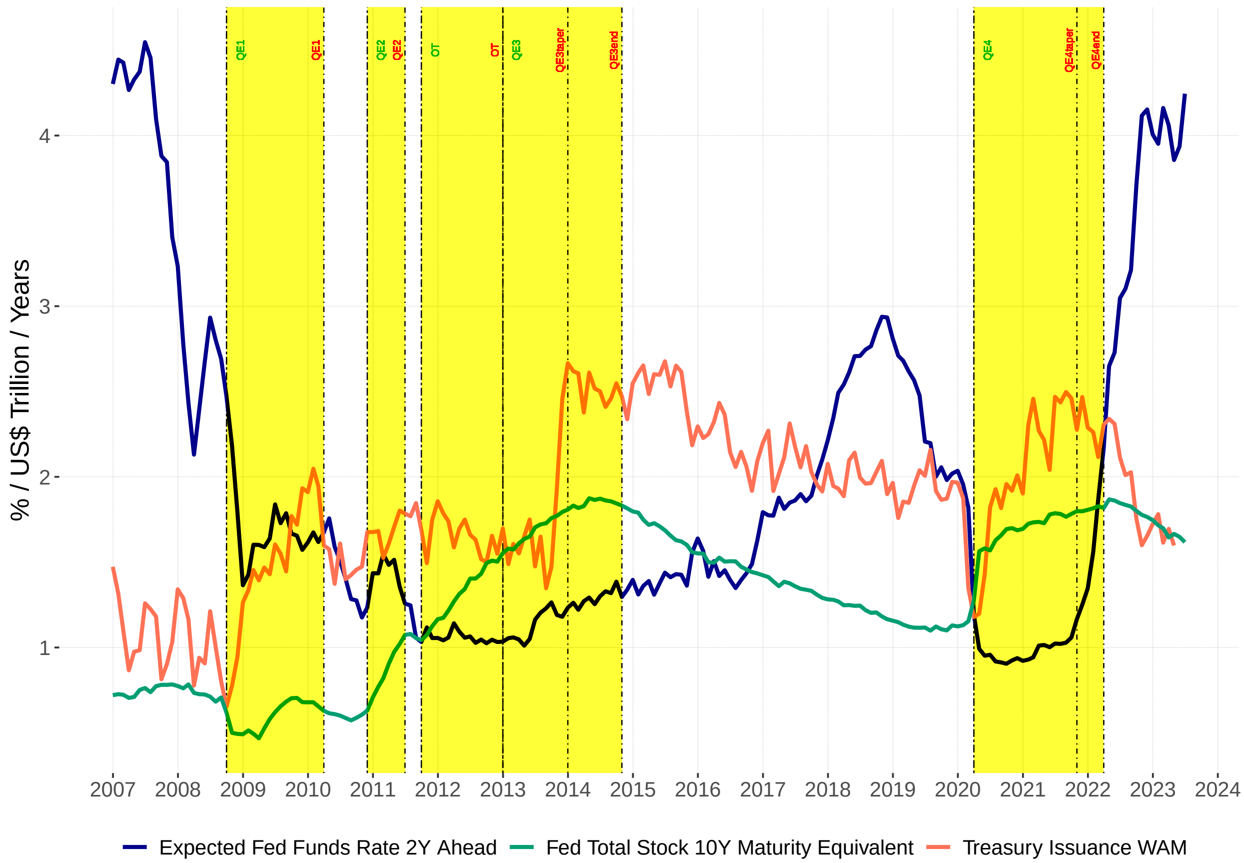
A different pattern emerges when treating OT and QE3 as distinct and separate programmes in columns (5) and (6), though. While the coefficients for QE1, QE2 and QE4 remain statistically non-significant, indicating responses that are similar with those observed during non-LSAP periods, the coefficients for OT and for QE3 become statistically significant and exhibit opposite signs. The negative coefficient for OT suggests a subdued reaction to the Fed's purchases of long-term Treasury securities compared to other periods, whereas the positive coefficient for QE3 indicates a stronger response, with the Treasury injecting relatively more maturity equivalents into the markets per unit extracted by the Fed. One possible interpretation for this finding is that the Treasury actively pursued distinct policies during these two programmes compared to the other three and to moments when no programme was in place. Alternatively, given the immediate succession of QE3 following OT, the overresponse during QE3 may have been a compensation for the underresponse during OT, which could be justified by the unique character of the OT programme. The insignificant coefficients in columns (3) and (4) support this second interpretation, suggesting that the combined effect throughout both initiatives was in line with the behaviour seen during non-LSAP periods, indicating a potential offsetting effect between the two programmes' individual responses.

To delve deeper and potentially gain a more comprehensive understanding of the reasons behind the heterogeneous reaction functions observed during OT and QE3, Figure 1.5 depicts the Treasury issuance 3-month average WAM centered around that month juxtaposed against the expected future interest rate for two years ahead and the share of the total stock of 10-year maturity equivalents held by the Fed. The shaded areas represent the five analysed LSAP programmes. Additionally to the start and end dates of each programme, I have added vertical dotted lines to denote the initiation dates of QE3 and QE4 tapering - the moments in which the Fed reversed its policy and started to significantly reduce the pace of its maturity extraction.

Figure 1.5 reveals additional variations in patterns observed in the Treasury issuance WAM during LSAPs. In both QE1 and QE4, there was an initial reduction followed by a sustained increase, ultimately reaching levels significantly higher than those before each respective LSAP programme. The subsequent declines following the conclusion of QE1 and start of QE4 tapering also suggest adjustments by the Treasury in response to the Fed's policy reversal. QE2 had a relatively shorter duration and, unlike QE1 and QE4, no initial reduction in the Treasury issuance WAM. However, as indicated by the results in Table 1.2, the more modest maturity lengthening implemented in QE2 resulted in a net average Treasury policy similar to the one observed in QE1 and QE4, when also considering the macroeconomic environment. Lastly, from the initiation of OT until the start of QE3 tapering, the Treasury issuance WAM followed a declining path despite the Fed's increasing maturity extraction. Nevertheless, after the onset of QE3 tapering, the Treasury promptly and significantly raised the average maturity of its issuances, nearly doubling the WAM of the securities auctioned compared to the previous periods.

Consideration of the expected future interest rate behaviour can provide further insight into the findings illustrated in Figure 1.5, aligning them with the model predictions. The model posits that during periods where investors have reduced risk-bearing capacity or increased demand for liquidity, such as those following the Global Financial Crisis (GFC) and Covid-19 shocks, the fiscal authority will look to shorten the maturity structure of the debt, in line with what was observed during early stages of QE1 and QE4. Conversely and also consistent with Figure 1.5 evidence, as markets stabilise and initial shocks subside, the model

Figure 1.5: Treasury WAM vs. Expected Future Interest Rates vs. Fed Maturity Extraction



Note: The chart shows the weighted average maturity of newly issued Treasury securities in the next 3 months compared to the expected Fed Funds rate two years ahead, along with the amount of maturity equivalents extracted from the market by the Fed. Yellow shaded areas indicate the windows of LSAPs, with top green labels marking the beginning and top red labels indicating the tapering or end of each programme.

predicts that, all else equal, the Treasury would extend the average maturity of its issuances in response to the maturity extraction performed by the Fed and to the considerably lower expected future interest rates during QE1 and QE4 compared to preceding periods.

The model also predicts that the relatively stable behaviour of the expected future interest rate during QE2, OT and QE3 would result in less pronounced adjustments from the Treasury. While the model still anticipates some degree of maturity extension in response to the Fed purchases, the Treasury's incentives to lengthen the maturity of its issuances during these programmes were lower compared to QE1 and QE4, periods of significant reduction in expected future interest rates. While no causality can be claimed here, the swift and substantial policy reversal following the start of QE3 tapering appears to support this interpretation: with expected future interest rates not driving maturity extension, the Treasury seems to have adopted a more passive approach during OT and the initial stages of QE3, refraining from interfering with the Fed's maturity extraction policy. However, as soon as QE3 tapering began, signaling the reversal of the Fed's policy, the Treasury promptly took an active stance and adjusted its issuance policy accordingly: it significantly intensified the injection of maturity equivalents into the market, thereby compensating for the underreaction in the previous period.

These observations seem to corroborate the model's proposition that the degree of coordination between the central bank and the fiscal authority during LSAPs is contingent upon the behaviour of the expected future interest rate. During periods of reduced expected future rates, the fiscal authority's incentives for active debt management are stronger, leading to policies that may offset, at least partially, the maturity extraction associated with LSAP programmes. Conversely, when expected future rates remain stable, the fiscal authority may adopt a more passive stance in managing the debt maturity structure, waiting for the central bank to achieve its monetary policy or financial stability goals before taking the actions prescribed by the optimal debt management policy.

1.5 Conclusion

This paper studies the factors influencing the optimal maturity policy of sovereign debt in the context of central banks implementing large-scale asset purchases programmes. The paper's first contribution is to underscore the importance of focusing on the consolidated sovereign debt when analysing debt management policies. In this regard, it is argued that that LSAPs shorten the maturity structure of privately-held sovereign debt, a change that may not align with the fiscal authority's optimal debt management policy.

To provide a formal analysis of the trade-offs faced by the fiscal authorities, I develop a model wherein the fiscal authority faces interest rate risk and must determine the optimal maturity structure conditional on shocks that prompt the central bank to engage in LSAPs. The model predicts that the level of coordination between fiscal authorities and central banks' optimal policies depends on the macroeconomic environments that lead to the LSAPs. During periods of lowered investors' risk-bearing capacity or increased demand for liquidity, such as those following financial stability shocks, the optimal debt management policy is to shorten the debt maturity, aligning the fiscal authority and central bank in the same direction. Conversely, lower expected future interest rates incentivise the fiscal authority to lengthen the debt maturity, conflicting with the central bank's LSAP policy.

Lastly, I provide empirical evidence indicating that the US Treasury's maturity policy during LSAP programmes has mostly been in line with expectations and with the model predictions. The findings highlight that the Treasury has been attentive to the consolidated sovereign balance sheet and to the Fed's maturity extraction policies. Additionally, they suggest that the Treasury may decide to take a more active or passive stance on the management of the debt maturity structure during LSAPs depending on the behaviour of the expected future interest rates.

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A Appendix - Chapter 1

A.1 Bond Prices

The investors' problem given by (1.3) can be solved backwards. At $t = 1$, all uncertainty is eliminated, so investors solve:

$$\begin{aligned} \max_{\{C_1, C_2, b^i_{1,2}, z_1\}} \quad & u_i(C_1) + \beta \cdot u_i(C_2) \\ \text{s.t.} \quad & C_1 = b^i_{0,1} + R^f_0 \cdot z_0 - P_{1,2} \cdot b^i_{1,2} - z_1 \\ & C_2 = b^i_{1,2} + b^i_{0,2} + R^f_1 \cdot z_1. \end{aligned} \tag{A.1}$$

Assuming internal solution in short-term bonds and reserves markets, we have:

$$P_{1,2} = \beta \cdot \frac{u_i'(C_2)}{u_i'(C_1)} = \frac{1}{R^f_1}. \tag{A.2}$$

Going back to the original problem at $t = 0$ and again assuming internal solution in short-term bonds and reserves markets, one must have:

$$P_{0,1} = \mathbb{E}_0 \left[\beta \cdot u_i'(C_1)\right] = \frac{1}{R^f_0}. \tag{A.3}$$

Further solving the first-order condition with respect to $B_{0,2}$ one can obtain:

$$\begin{aligned} P_{0,2} &= \mathbb{E}_0 \left[\beta^2 \cdot u_i'(C_2)\right] - v'(b^i_{0,2}) \\ &= \mathbb{E}_0 \left[\beta \cdot u_i'(C_1)\right] \cdot \mathbb{E}_0 \left[\beta \cdot \frac{u_i'(C_2)}{u_i'(C_1)}\right] + Cov\left(\beta \cdot u_i'(C_1), \beta \cdot \frac{u_i'(C_2)}{u_i'(C_1)}\right) - v'(b^i_{0,2}) \\ &= \mathbb{E}_0 \left[\frac{1}{R^f_1}\right] - \kappa - v'(b^i_{0,2}), \end{aligned} \tag{A.4}$$

where $\kappa = -Cov\left(\beta \cdot u_i'(C_1), \beta \cdot \frac{u_i'(C_2)}{u_i'(C_1)}\right)$ is assumed to be constant.

A.2 Lemma 1.2

The $t = 2$ fiscal authority's consumption can be rewritten as:

$$\begin{aligned} G_2 &= \bar{y} - B_{0,2} + \left(\frac{G_1 + B_{0,1}}{P_{1,2}} \right) \\ &= \bar{y} - B_{0,2} - R_1^f \cdot (G_1 + B_{0,1}). \end{aligned} \tag{A.5}$$

Using this, one can rewrite the fiscal authority's problem at $t = 1$ as:

$$\begin{aligned} \max_{\{G_1, G_2\}} \quad & u_g(G_1) + u_g(G_2) \\ \text{s.t.} \quad & G_2 = \bar{y} - B_{0,2} - R_1^f \cdot (G_1 + B_{0,1}). \end{aligned} \tag{A.6}$$

The first-order condition with respect to G_1 is given by:

$$u_g'(G_1) = R_1^f \cdot u_g'(G_2). \tag{A.7}$$

At the same time, using the $t = 0$ budget constraint and the prices obtained in equations (A.3) and (A.4), G_2 will be such that

$$G_2 = \bar{y} - B_{0,2} + R_1^f \cdot \left(G_1 + \hat{B}_{0,1} - \left[\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v'(b_{0,2}^i) \right] \cdot \left(B_{0,2} - \hat{B}_{0,2} \right) \right). \tag{A.8}$$

One can then write the fiscal authority's expected utility function at $t = 0$ as:

$$\begin{aligned} U^* &= \mathbb{E}_0 \left[u_g(G_1^*) + u_g(G_2^*) \right] \\ &= \mathbb{E}_0 \left[u_g(G_1^*) + u_g \left(\bar{y} - B_{0,2} + R_1^f \cdot \left(G_1^* + \hat{B}_{0,1} - \left[\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v'(b_{0,2}^i) \right] \cdot \left(B_{0,2} - \hat{B}_{0,2} \right) \right) \right) \right], \end{aligned} \tag{A.9}$$

where * denotes the optimum as evaluated at $t = 1$.

Differentiating U^* with respect to $B_{0,2}$ yields equation (1.11). Then, combining this result with (A.4), (A.7) and (A.8) gives us equation (1.12). Next, define $\chi \equiv \left(\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v'(B_{0,2}) \right) - v''(B_{0,2}) \cdot (B_{0,2} - \hat{B}_{0,2})$. By rearranging the terms in (1.12) we obtain:

$$\chi \cdot \left(-\mathbb{E}_0 \left[u_g'(G_2^*) \cdot \frac{1}{\chi} \right] + \mathbb{E}_0 \left[u_g'(G_2^*) \cdot R_1^f \right] \right) = 0$$

$$\chi \cdot \left(\mathbb{E}_0 \left[R_1^f \right] + Cov_0 \left(\frac{u_g'(G_2^*)}{\mathbb{E}_0 \left[u_g'(G_2^*) \right]}, R_1^f \right) \right) = 1 \tag{A.10}$$

$$\mathbb{E}_0 \left[R_1^f \right] + Cov_0 \left(\frac{u_g'(G_2^*)}{\mathbb{E}_0 \left[u_g'(G_2^*) \right]}, R_1^f \right) = \frac{1}{\chi},$$

which characterises the tradeoff faced by the fiscal authority as exposed in (1.13).

□

A.3 Lemma 1.3

Assume the initial debt maturity structure at $t = 0$ is such that $\hat{B}_{0,2} = B_{0,2}^*$. Then, at the optimal, the fiscal authority's tradeoff described in equation (1.14) will be such that:

$$\mathbb{E}_0 \left[R_1^f \right] - \alpha \cdot Var_0 \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) = \frac{1}{\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v' \left(B_{0,2} \right)}. \tag{A.11}$$

Next, using (A.7) in the fiscal authority's $t = 2$ budget constraint (A.8) and differentiating with respect to R_1^f, one can obtain:

$$G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) = -\frac{1}{\left(1 + \mathbb{E}_0 \left[R_1^f \right] \right)^2}.$$

$$\cdot \left[\bar{y} - B_{0,2} + B_{0,1}^* - P_{0,2} \cdot \left(B_{0,2} - B_{0,2}^* \right) - \frac{\left(log \left(\mathbb{E}_0 \left[R_1^f \right] \right) + \left(1 + \mathbb{E}_0 \left[R_1^f \right] \right) \right)}{\alpha} \right]. \tag{A.12}$$

Thus, at the optimal $B_{0,2} = B_{0,2}^*$:

$$\frac{dG_2'\left(\mathbb{E}_0\left[R_1^f\right]\right)}{dB_{0,2}} = \frac{1}{\left(1+\mathbb{E}_0\left[R_1^f\right]\right)^2} \cdot \left(1+P_{0,2}\right) > 0. \tag{A.13}$$

Note that, in the absence of shocks, any LSAP programme will move $B_{0,2}$ away from the optimal $B_{0,2}^*$ and lead to a shorter maturity structure than the one desired by the fiscal authority. Equation (A.11) will not hold anymore as the marginal cost of issuing one long-term bond, on the right-hand side, will now be lower than the marginal cost of issuing one extra short-term bond, on the left-hand side. As a consequence, the fiscal authority's optimal response would be to take advantage of that and issue more long-term debt, until the maturity structure goes back to the optimal original point.

□

A.4 Proposition 1.1

First, differentiating both sides of (1.14) with respect to κ at the optimal $B_{0,2} = B_{0,2}^*$ and using the envelope theorem gives us:

$$\begin{aligned} 0 = &-\left[\mathbb{E}_0\left[R_1^f\right] - \alpha \cdot Var\left(R_1^f\right) \cdot G_2'\left(\mathbb{E}_0\left[R_1^f\right]\right)\right] d\kappa \\ &-\left[2 \cdot v''\left(B_{0,2}^*\right) \cdot \left(\mathbb{E}_0\left[R_1^f\right] - \alpha \cdot Var\left(R_1^f\right) \cdot G_2'\left(\mathbb{E}_0\left[R_1^f\right]\right)\right)\right] dB_{0,2} \\ &-\left[\left(\mathbb{E}_0\left[\frac{1}{R_1^f}\right] - \kappa - v'\left(B_{0,2}^*\right)\right) \cdot \left(\alpha \cdot Var\left(R_1^f\right) \cdot \frac{dG_2'\left(\mathbb{E}_0\left[R_1^f\right]\right)}{dB_{0,2}}\right)\right] dB_{0,2}. \end{aligned} \tag{A.14}$$

Rearranging the terms above while using (A.13) and the fact that $v''(\cdot) > 0$ one can get:

$$\frac{dB_{0,2}}{d\kappa} < 0. \tag{A.15}$$

Next, doing the same for η, where $v(\cdot) = \eta \cdot \phi(\cdot)$ yields:

$$
\begin{aligned}
0 = &- \left[\phi'\left(B^*_{0,2}\right) \cdot \left(\mathbb{E}_0\left[R^f_1\right] - \alpha \cdot Var\left(R^f_1\right) \cdot G'_2\left(\mathbb{E}_0\left[R^f_1\right]\right)\right)\right] d\eta \\
&- \left[2 \cdot \eta \cdot \phi''\left(B^*_{0,2}\right) \cdot \left(\mathbb{E}_0\left[R^f_1\right] - \alpha \cdot Var\left(R^f_1\right) \cdot G'_2\left(\mathbb{E}_0\left[R^f_1\right]\right)\right)\right] dB_{0,2} \\
&- \left[\left(\mathbb{E}_0\left[\frac{1}{R^f_1}\right] - \kappa - v'\left(B^*_{0,2}\right)\right) \cdot \left(\alpha \cdot Var\left(R^f_1\right) \cdot \frac{dG'_2\left(\mathbb{E}_0\left[R^f_1\right]\right)}{dB_{0,2}}\right)\right] dB_{0,2}.
\end{aligned}
\tag{A.16}
$$

Since $\phi'(\cdot) > 0$ and $\phi''(\cdot) > 0$, then

$$
\frac{dB_{0,2}}{d\eta} < 0. \tag{A.17}
$$

□

A.5 Proposition 1.2

By differentiating both sides of (1.14) with respect to $\mathbb{E}_0\left[R^f_1\right]$ at the optimal while keeping the distribution of R^f_1 unchanged, one can obtain:

$$
\begin{aligned}
0 = & \left[\underbrace{\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} \cdot \left(\mathbb{E}_0 \left[R_1^f \right] - \alpha \cdot Var \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \right)}_{a_1} \right] d\mathbb{E}_0 \left[R_1^f \right] \\
& + \left[\underbrace{P_{0,2} \cdot \left(1 - \alpha \cdot Var \left(R_1^f \right) \cdot \frac{dG_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right)}{d\mathbb{E}_0 \left[R_1^f \right]} \right)}_{a_2} \right] d\mathbb{E}_0 \left[R_1^f \right] \\
& - \left[\underbrace{2 \cdot v'' \left(B_{0,2}^* \right) \cdot \left(\mathbb{E}_0 \left[R_1^f \right] - \alpha \cdot Var \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \right)}_{a_3} \right] dB_{0,2} \\
& - \left[\underbrace{P_{0,2} \cdot \left(\alpha \cdot Var \left(R_1^f \right) \cdot \frac{dG_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right)}{dB_{0,2}} \right)}_{a_4} \right] dB_{0,2}.
\end{aligned}
\tag{A.18}
$$

We already know that $a_3 > 0$ and $a_4 > 0$, so $a_3 + a_4 > 0$. Therefore, we need to find the sign of $a_1 + a_2$:

$$a_1 + a_2 = \mathbb{E}_0 \left[\frac{1}{R_1^f} \right] \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} \cdot \left(\mathbb{E}_0 \left[R_1^f \right] - \alpha \cdot Var \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \right) +$$

$$+ P_{0,2} \cdot \left(1 - \alpha \cdot Var \left(R_1^f \right) \cdot \frac{dG_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right)}{d\mathbb{E}_0 \left[R_1^f \right]} \right)$$

$$a_1 + a_2 = \mathbb{E}_0 \left[\frac{1}{R_1^f} \right] \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} \cdot \left(\mathbb{E}_0 \left[R_1^f \right] - \alpha \cdot Var \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \right) +$$

$$+ \left(\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v' \left(B_{0,2}^* \right) \right) \cdot \left(1 - \frac{Var \left(R_1^f \right)}{1 + \mathbb{E}_0 \left[R_1^f \right]} \cdot \left(\frac{1}{\mathbb{E}_0 \left[R_1^f \right]} - 2 \cdot \alpha \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \right) \right)$$

$$a_1 + a_2 = \alpha \cdot Var \left(R_1^f \right) \cdot G_2' \left(\mathbb{E}_0 \left[R_1^f \right] \right) \cdot \left[\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} + \frac{2}{1 + \mathbb{E}_0 \left[R_1^f \right]} \right] -$$

$$- \left(\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] - \kappa - v' \left(B_{0,2}^* \right) \right) \cdot \frac{Var \left(R_1^f \right)}{1 + \mathbb{E}_0 \left[R_1^f \right]} \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} - \kappa - v' \left(B_{0,2}^* \right). \tag{A.19}$$

Using equation (A.11), one can simplify (A.19) to:

$$\begin{aligned} a_1 + a_2 = {} & -P_{0,2} \cdot \frac{Var \left(R_1^f \right)}{1 + \mathbb{E}_0 \left[R_1^f \right]} - \kappa - v' \left(B_{0,2}^* \right) - \\ & - \left(\frac{1}{P_{0,2}} - \mathbb{E}_0 \left[R_1^f \right] \right) \cdot \left[\mathbb{E}_0 \left[\frac{1}{R_1^f} \right] \cdot \frac{1}{\mathbb{E}_0 \left[R_1^f \right]} + \frac{2}{1 + \mathbb{E}_0 \left[R_1^f \right]} \right]. \end{aligned} \tag{A.20}$$

Thus, as long as the term premium $\frac{1}{P_{0,2}} - \mathbb{E}_0 \left[R_1^f \right]$ is bounded from below at a not too negative value, we will have $a_1 + a_2 < 0$. Assume this is the case.

Then, going back to equation (A.18):

$$(a_1 + a_2)\, d\mathbb{E}_0 \left[R_1^f \right] = (a_3 + a_4)\, dB_{0,2}. \tag{A.21}$$

Hence, it must be that:

$$\frac{dB_{0,2}}{d\mathbb{E}_0 \left[R_1^f \right]} < 0. \tag{A.22}$$

$\square$

2 Targeted Financial Conditions Indices and Growth-at-Risk

2.1 Introduction

The importance of the financial system in both pricing and influencing macroeconomic developments is widely recognised. One expression of that is the widespread use of financial variables for macroeconomic modelling, both theoretical and empirical (see Bernanke et al. (1999), Gertler and Kiyotaki (2010), Christiano et al. (2014) and Gilchrist and Zakrajsek (2012), among many others). A frequent challenge is the wide range of candidate variables for such modeling efforts. The development of so-called financial conditions indices, which summarise information contained in a large number of variables, is a response to that problem (see Arrigoni et al. (2022)). Financial conditions indices have been monitored in their own right, but also used for forecasting and other analytical purposes. Among these, "at-risk" modelling, which seeks to explain the occurrence and assess the likelihood of tail events for a range of variables, has emerged as a very popular application, starting with Adrian et al. (2019).

The standard approach for modelling such outcomes typically entails using a pre-existing measure of financial conditions, usually designed with the sole aim of capturing common variation across a wide range of financial variables[17]. However, that may lead to a disconnect between the way financial conditions indices are constructed and their subsequent use. Starting from this observation, we propose a method to extract financial conditions indices that are specifically tailored to explain or forecast any part of the distribution of a variable of interest. That is, we devise a methodology to estimate *targeted* financial conditions indices

[17] With the notable exception of Giglio et al. (2016), which we discuss in detail below.

(TFCIs), notably for "at-risk" modelling applications.

Our approach works by rotating an initial orthogonalisation of a panel of financial indicators – in our case, the components of the Chicago Fed's National Financial Conditions Index (NFCI, Brave and Butters (2011)) – in order for one or more of the resulting components to maximise covariation with individual quantiles of a target of interest – in our application, future US GDP growth. We show that this yields indices that are economically intuitive, smoother when computed in real time, and with better out-of-sample forecasting power than existing alternatives.

Specifically, we show that our model delivers statistically and economically significant gains in terms of probability scores and better-calibrated densities than a number of alternatives, including the NFCI itself. Moreover, we also find that compared to a more 'traditional' index based on principal component analysis (PCA), a TFCI optimised to forecast the left tail of GDP growth one year ahead tends to put more emphasis on developments in credit and risk rather than leverage. The opposite is true for a TFCI for the conditional median.

Related Literature

Our paper is related to several literature strands. Most directly, it complements papers that develop financial conditions indices (Hatzius et al. (2010), Brave and Butters (2012), Kremer et al. (2012), Arregui et al. (2018) and Arrigoni et al. (2022)), and those that make use of such indices to model tail risk in macroeconomic and financial variables (Adrian et al. (2019), Adams et al. (2021), Chari et al. (2020), Figueres and Jarocinski (2020), Eguren-Martin et al. (2021), Eguren-Martin and Sokol (2022), Gelos et al. (2022), Amburgey and McCracken (2023), among others). By performing both steps jointly, our paper seeks to link the two.

Both the "at-risk" papers cited above and our own approach can be categorised as quantile regression models with factor-augmented predictors (Ando and Tsay (2011)). But while the focus of that paper was on developing methods for selecting the optimal number of principal components of a data set to include in a model, we go one step further by allowing for targeted factor extraction based on a variable of interest.

Giglio et al. (2016) is, to our knowledge, the only paper that shares our goal, namely to extract information from a panel of variables based on covariation with the quantiles of a target variable of interest. But one reason to develop our own approach is that with a large panel such as the one underlying the NFCI, we have found their method to be wanting in terms of out-of-sample performance[18]. We conjecture that this is due to the ability of our approach to remove some idiosyncratic variation from the financial series before focusing on fitting the quantiles of the target variable.

Finally, it is worth distinguishing our approach from so-called 'quantile factor models' (Ando and Bai (2020), Chen et al. (2021)). The aim of those approaches is to uncover factors driving quantile covariation across a panel of variables. While our TFCIs can sometimes exhibit meaningful covariation with some of the quantiles of the underlying financial series – for example, tail outcomes in credit spreads coinciding with spikes in the TFCI – that relationship is entirely subordinate to the aim of delivering covariation between the TFCI and a specific quantile of the target variable.

Paper Structure

The rest of the paper is organised as follows: in Section 2.2 we introduce our approach to targeted factor extraction. In Section 2.3 we apply our method to US financial conditions and GDP growth and revisit the drivers underlying growth-at-risk. In Section 2.4 we compare out-of-sample performance to available alternatives, and in 2.5 we conclude.

[18]Giglio et al. (2016) illustrate their approach on a panel of 19 variables, roughly a fifth of the size of our panel.

2.2 *Targeted* Factor Extraction

In this section we outline a novel approach to factor extraction, the main contribution of our paper. Our objective is to extract one or more common factors from a potentially very large set of variables. The crucial restriction is that the factors are required to maximise the forecasting power of our model for a specific quantile and horizon of a target variable. In our application, the set of variables from which factors are extracted are the series underlying the Chicago Fed's National Financial Conditions Index (NFCI), and the target variable is US GDP growth.

In a nutshell, our approach uses orthonormal rotations to re-orient an initial factor decomposition of the underlying (financial) variables so as to maximise their explanatory power for a given quantile and horizon of our target variable (GDP growth). Let z_t be an observation from a panel of n series that have mean zero and (for simplicity) unit variance. Let $\mathbb{Z}$ stack the T observations z_t', and $\mathbb{F}$, which stacks f_t', be any factor decomposition of $\mathbb{Z}$, for example the full set of (standardised) PCA scores. Then

$$z_t = \Lambda f_t, \tag{2.1}$$

where Λ is a $n \times n$ matrix of factor loadings. Any orthonormal rotation of Λ and $\mathbb{F}$ will also yield an admissible factor decomposition of $\mathbb{Z}$. Thus, let $G(\theta)$ be a $n \times n$ rotation matrix parametrised by the vector of angles θ. A set of new factors $\tilde{f}_t(\theta)$ can be recovered by simply rotating the original factors (or equivalently, loadings), because

$$z_t = \Lambda f_t = \Lambda G(\theta) G'(\theta) f_t \equiv \tilde{\Lambda}(\theta) \tilde{f}_t(\theta). \tag{2.2}$$

$G(\theta)$ is constructed in a similar fashion as in Haberis and Sokol (2014), namely as the product of suitably chosen Givens matrices:

$$G(\theta) = \prod_{i=1}^{\min(s,n-1)} \prod_{j=i+1}^{r} G_{i,j}(\theta_{i,j}), \tag{2.3}$$

where the only non-zero elements of $G_{i,j}(\theta_{i,j})$ are $g_{kk} = 1,\ k \neq i,j,\ g_{kk} = \cos\theta_{i,j},\ k = i,j$ and $g_{ji} = -g_{ij} = -\sin\theta_{i,j}$. The parameter $r \leq n$ determines the dimension of the column (sub-) space of Λ that is rotated by $G(\theta)$, while $s < r$ controls the number of factors included in the regression models (see below). As further discussed in Section 2.3, we choose r, the dimension of the column space to be rotated, dynamically for each vintage, quantile and horizon from a grid, based on local fit adjusted for degrees of freedom ($R^1(\tau)$), defined as:

$$R^1(\tau) = 1 - \frac{\hat{V}(\tau)}{\tilde{V}(\tau)} \frac{T-1}{T-p}, \tag{2.4}$$

where $\hat{V}(\tau)$ denotes the sum of weighted absolute residuals of a candidate model, $\tilde{V}(\tau)$ the sum of weighted absolute residuals of a model consisting only of a constant and p is the total number of parameters, including the angles in θ[19].

Now consider the following specification of the conditional quantile function of response variable y_{t+h} for quantile τ:

[19]Since $\hat{V}(\tau)$ is also the key ingredient of the likelihood of a linear quantile regression model, this is essentially a shortcut to the likelihood ratio test proposed by Koenker and Machado (1999), the only difference being the absence of an adjustment for curvature.

$$
\begin{aligned}
Q\left(y_{t+h}|w_t,\ \tilde{f}_t\left(\theta_\tau\right),\tau\right) &= \alpha_\tau' w_t + \gamma_\tau'\left(\theta_\tau\right) s_\tau \tilde{f}_t\left(\theta_\tau\right) \\
&= \begin{bmatrix} \alpha_\tau' & \gamma_\tau'\left(\theta_\tau\right) \end{bmatrix} \begin{bmatrix} w_t \\ s_\tau \tilde{f}_t\left(\theta_\tau\right) \end{bmatrix} \\
&\equiv \beta_\tau'\left(\theta_\tau\right) x_t\left(\theta_\tau\right).
\end{aligned} \tag{2.5}
$$

Here w_t captures any explanatory variables not included in z_t, such as deterministic terms or lagged values of y_t, and s_τ is an $s \times n$ matrix that selects the first $s \leq n$ elements of $\tilde{f}_t\left(\theta_\tau\right)$; the parameter s controls the number of factors included in the regression and determines the length of $\gamma_\tau\left(\theta_\tau\right)$. We limit ourselves to $s = 1$; that is, we stick to a single factor to be used for the modelling of our variable of interest. This is motivated by the objective to have a single financial conditions index, both for ease of tracking its time variation and for comparison with existing methods[20].

For a given rotation of the original factors θ_τ, $\hat{\beta}_\tau\left(\theta_\tau\right)$ solves the quantile regression problem:

$$
\hat{\beta}_\tau\left(\theta_\tau\right) = \arg\min_{\beta_\tau\left(\theta_\tau\right)} \frac{1}{T}\sum_{t=1}^{T} \rho_\tau\left(y_t - \beta_\tau'\left(\theta_\tau\right) x_t\left(\theta_\tau\right)\right), \tag{2.6}
$$

where $\rho_\tau\left(u\right) = u\left(\tau - \mathbb{I}\left(u < 0\right)\right)$ is the check function.

Our object of interest is θ_τ^*, the set of angles, and therefore rotated factors, that, given a choice of r and s, maximises the fit of the model:

[20] Ando and Tsay (2011) investigate the choice of s in the context of choosing the optimal number of PCA scores to include in a factor-augmented quantile regression. Their methods are not directly portable to our setting, while our approach for choosing r based on R^1, or the likelihood ratio test in Koenker and Machado (1999) can be easily extended to the choice of s. However, to avoid over-fitting, $s << r$, that is, only a small subset of the rotated factors will enter the regression.

$$\theta_\tau^* = \arg\min_{\theta_\tau} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau \left(y_t - \hat{\beta}_\tau' (\theta_\tau) \, x_t (\theta_\tau) \right). \tag{2.7}$$

θ_τ^* is not available in closed form, but can be recovered by numerical optimisation[21].

To summarise, in our application we have, for each horizon and quantile of interest, a fitted model of the following form:

$$\hat{Q} \left(\Delta gdp_{t+h,t} | x_t (\theta_\tau^*), \tau \right) = \beta_\tau' (\theta_\tau^*) \begin{bmatrix} 1 \\ \Delta gdp_{t,t-h} \\ \tilde{f}_t (\theta_\tau^*), \end{bmatrix} \tag{2.8}$$

where $\Delta gdp_{t+h}, t$ denotes cumulative GDP growth between periods t and $t+h$, 1 multiplies a (quantile- and horizon-specific) constant and $\tilde{f}_t (\theta_\tau^*)$ is our targeted factor, which is also quantile- and horizon-specific.

2.3 Targeted Financial Conditions Indices and US Growth-at-Risk

In order to showcase the main advantages of our approach, we model the predictive distribution of US GDP growth, the chosen target variable of several recent contributions to the "at-risk" literature (Giglio et al. (2016), Adrian et al. (2019), Adams et al. (2021), Plagborg-Moller et al. (2020), among others). We first describe the construction of our targeted financial conditions indices (including the underlying data used), then discuss their main features and differences with respect to existing approaches.

[21] We provide replication codes in MATLAB for the purpose.

2.3.1 Data and Index Construction

There is a tradition of papers extracting information from financial variables for monitoring and forecasting purposes (see Section 2.1). Due to its popularity, we take the Chicago Fed's NFCI (Brave and Butters (2011)) as our starting point.

Specifically, we focus on the more than 100 series comprising the NFCI, at monthly frequency and over the 1973 - 2019 sample[22]. The authors group the series into three categories: leverage, credit and risk; we follow that categorisation in our color-coding in subsequent charts[23]. We start by standardising the underlying contributions to then extract principal components, which we use both as a benchmark and as the initial orthogonalisation to initialise our method (see Section 2.2)[24].

We focus on a range of quantiles of GDP growth both 1 quarter and 4 quarters ahead, in line with the literature[25]. For each, the specification of our model is laid out in equation (2.8). Given our focus on delivering a single targeted financial conditions index for each quantile and horizon, we use a single factor for our forecasts (that is, we set $s = 1$). Moreover, we choose r (which determines the number of standardised PCA scores to be rotated) from a dynamic grid capped at 15% of the number of available indicators in each vintage, in order to avoid overfitting[26].

[22]We downloaded the underlying contributions to the NFCI from Bloomberg, using **ALLX NFCI <GO>**. The same contributions, from 2008 onwards, are also available on the Chicago Fed website here.

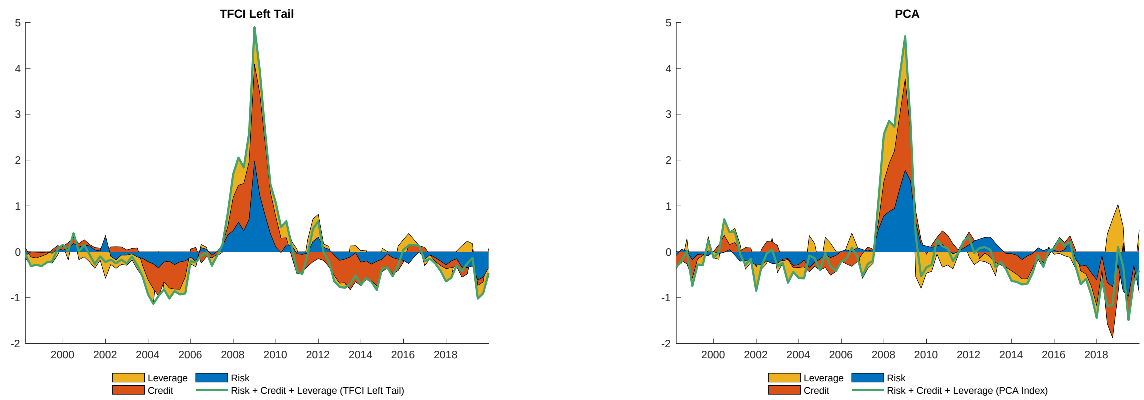
[23]See Appendix for a full list of the series included. Although some series already start in 1971, we follow Brave and Butters (2011) in considering data form 1973 onwards, which is when at least 25% of the series comprising the final dataset are available.

[24]While the NFCI is based on a dynamic factor model, the correlation between the extracted factor and the first principal component of the underlying data is 0.97 over our forecast evaluation sample (see next Section).

[25]1 quarter ahead, the left-hand side variable is the seasonally-adjusted QoQ annualised growth rate; 1 year ahead, it is the YoY growth rate.

[26]We conjecture that our ability to set the parameter r is one reason for the better out-of-sample performance of our approach compared to Giglio et al. (2016), as setting $r << n$ allows to filter out some idiosyncratic variation from the financial variables before focusing on covariation with individual quantiles of our target.

Figure 2.1: Left tail TFCI and PCA index - 1 Year Ahead



Note: The figures plot the real-time (ex ante) time series of the a) Left Tail TFCI (5th Percentile) and b) PCA Index, when forecasting 1 year ahead. The indices comprise three subgroups: leverage (yellow), credit (red) and risk (blue). Both indices have been standardised.

2.3.2 Financial Conditions and US GDP-at-Risk Over Time

In this subsection we focus on the 5^{th} percentile of the distribution of US GDP growth four quarters ahead, one possible definition of growth-at-risk (see, for example, IMF (2017)).

Figure 2.1 shows the real-time[27] evolution of the TFCI that results from targeting the 5^{th} percentile of GDP growth four quarters ahead, decomposed into the contributions of each financial category. The right panel shows an analogous figure for a PCA-based version, where the index in each period is the last observation of the first principal component of the underlying series available at the time[28]. Higher values indicate tighter financial conditions. We show the PCA-based version as a benchmark for two reasons. First, over the sample shown in Figure 2.1, the first principal component of the most recent data vintage (2019Q4) correlates almost perfectly with the corresponding NFCI vintage. And second, PCA scores are the starting point of our approach, so differences in loadings between our index and the

[27]Or ex-ante, as opposed to the ex post series fitting the realised data that is used at the estimation step.

[28]For each period, the contributions to the index are obtained by inverting $\tilde{\Lambda}\left(\theta_{\tau}^{*}\right)$ and multiplying the elements of the first row by the original (standardised) underlying series.

first principal component are an object of interest in its own right.

There are a few points worth highlighting. First, and most notably, both indices increase sharply as the global financial crisis (GFC) starts unfolding in 2007, with similar dynamics and relative contributions from the three groups of variables. However, apart from that period, the two indices exhibit more heterogeneous behaviour: our index is smoother, with contributions from each variable category building up and retracing over time. In contrast, the PCA-based version is significantly more volatile, both in terms of the aggregate index and the contributions of the various groups. Another feature that stands out is that in the run-up to the GFC, our TFCI for the left tail pointed to a more protracted period of loose financial conditions.

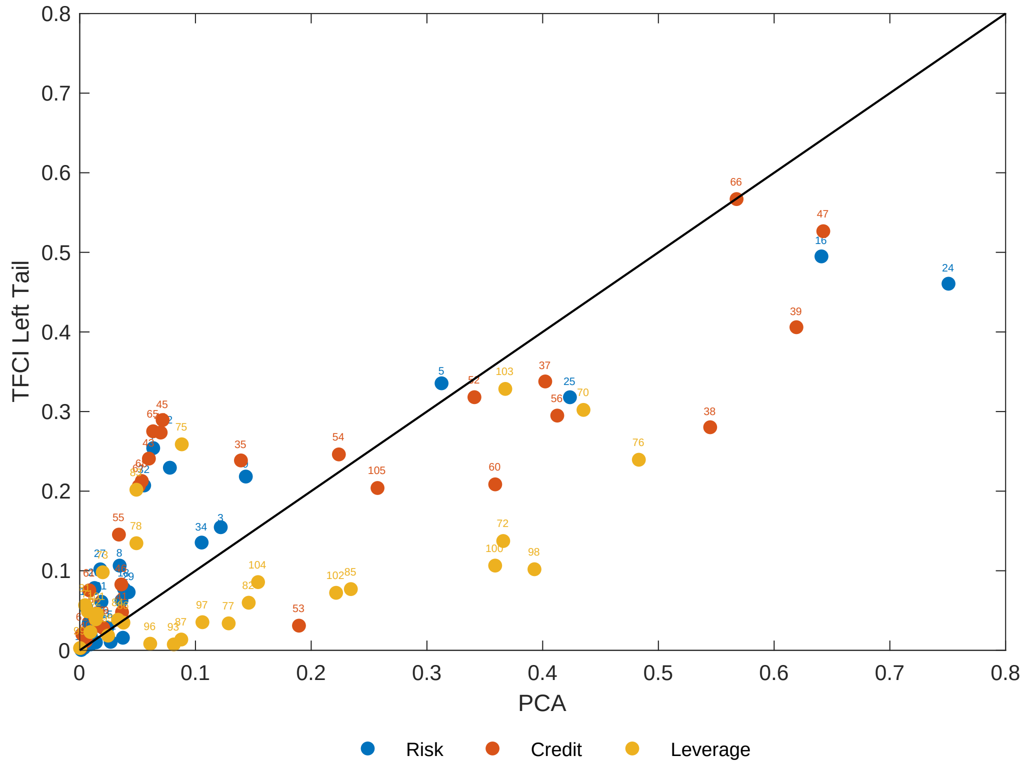
Our framework also offers insights into which financial variables are associated most strongly with the dynamics of a quantile of interest. While this of course does not imply causation, it is nevertheless suggestive and can be associated to existing narratives about the links between the financial sector and the macro economy. Figure 2.2 thus compares the squared loadings of the components of our TFCI with the loadings on the first principal component, both averaged across all vintages in our sample[29]. Each dot corresponds to one of the component series, colour-coded according to the three broad groups they belong to[30]. Series close to the diagonal behave similarly in the TFCI and a in PCA-based index, while series above it tend to comove more closely with our TFCI than with the first principal component.

Two features of Figure 2.2 stand out. First, our index appears less clearly associated with the developments of a small set of variables compared to PCA. This can be inferred from the fact that most of the series with squared loadings above 0.3 lie below the 45-degree line. Second, compared to PCA our index appears to co-move less strongly with indicators of

[29] In each vintage, squared loadings correspond to the share of variance of each indicator explained by the targeted index.

[30] A table matching indicator numbers to each one of the series can be found in the Appendix.

Figure 2.2: Average Real-Time Squared Loadings, 1 Year Ahead

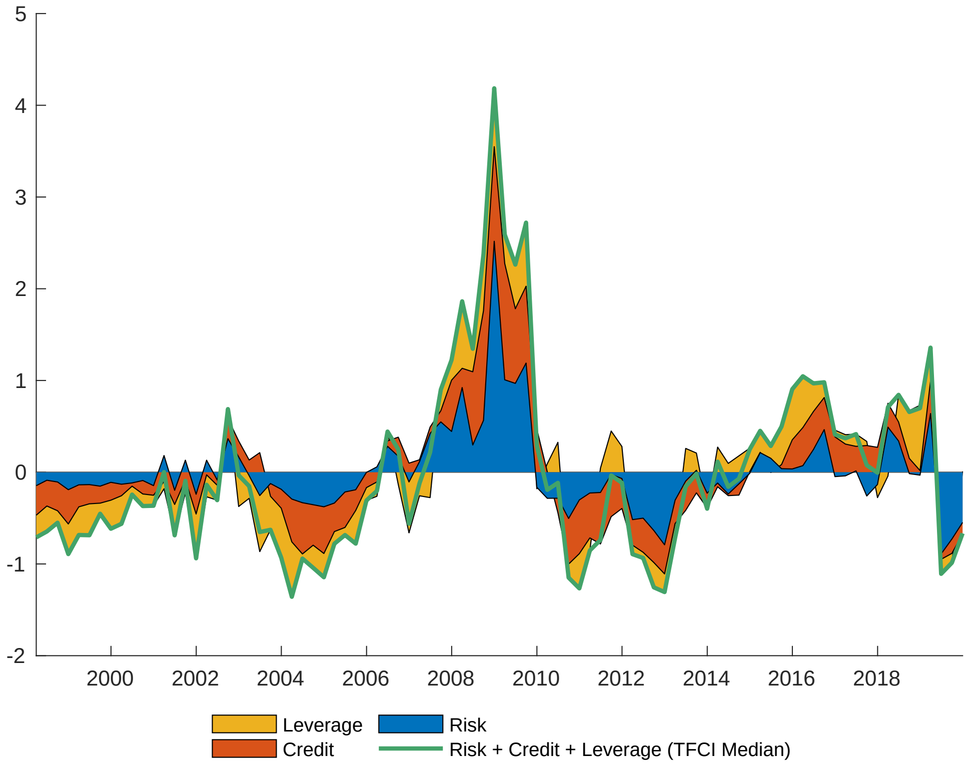


Note: Sample averages of real-time (ex ante) squared loadings for the PCA vs. Left Tail TFCI indices, when forecasting 1 year ahead. Each dot corresponds to one component series. See Appendix for the series' legends.

leverage, and more strongly with credit and risk indicators, corroborating earlier findings on the role of credit for predicting crisis-type events (Schularick and Taylor (2012)). That said, the variables displaying the highest squared loadings are similar across approaches: some credit-related variables, mostly survey-based measures of access to credit for consumers and small firms (from NFIB and FRB Senior Loan Officer surveys), and a few risk-related ones, such as the slope of the US Treasury yield curve and interbank deposit spreads.

Finally, Figure 2.3 shows a version of our TFCI that targets the median of the distribution of US GDP growth one year ahead, rather than its left tail. Compared to the one

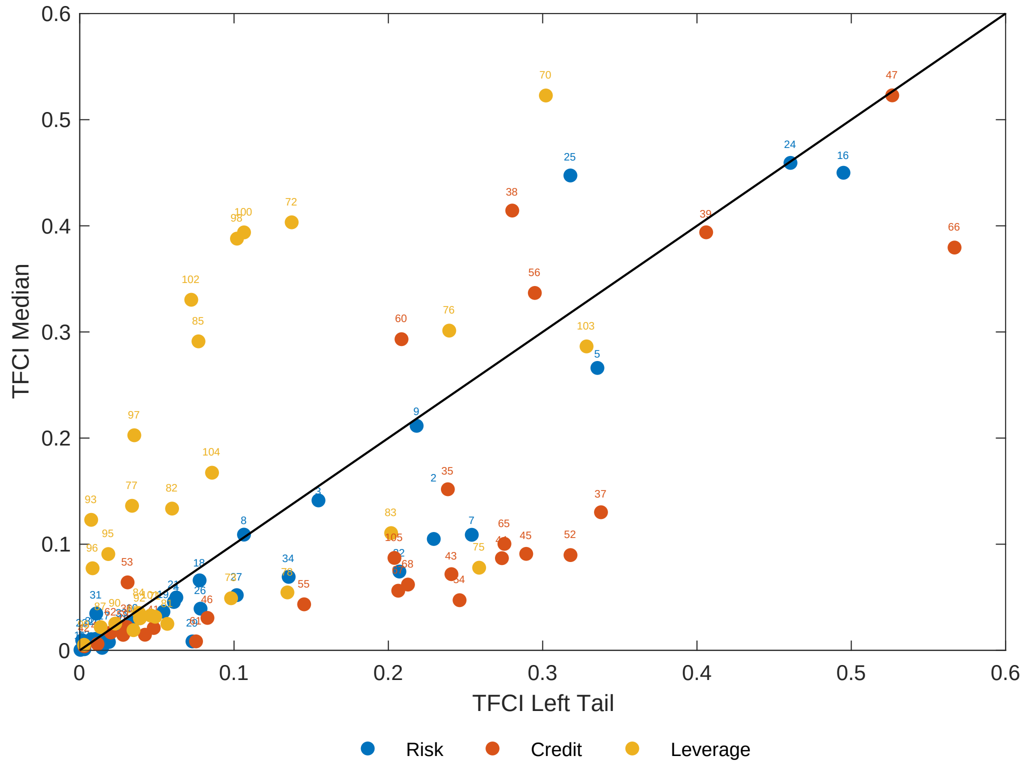
Figure 2.3: Real-time Median TFCI, 1 Year Ahead



Note: Real-time (ex ante) Median TFCI time series and the contributions of each subgroup.

targeting the 5^{th} percentile (shown in Figure 2.1), this index tracks more closely business cycle developments, and displays more even contributions from the three types of components. This is corroborated by Figure 2.4, which compares the squared loadings of each series on the median-based TFCI with its left-tail counterpart. The former explains a higher share of the variance of several indicators of leverage, and a lower share of the variance of a number of credit indicators. The stronger correlation of leverage-type variables with indices that target business cycle-type variation can also be understood in terms of earlier findings, notably related to the relevance of financial accelerator-type dynamics in explaining economic activity (see Bernanke et al. (1999), among many others).

Figure 2.4: Average Real-Time Squared Loadings, 1 Year Ahead



Note: Sample averages of real-time (ex ante) squared loadings for the Left Tail TFCI vs. Median TFCI, when forecasting 1 year ahead. Each dot corresponds to one component series. See Appendix for the series' legends.

In sum, this section shows that financial conditions indices based on common variation across financial variables tend to be associated with a small number of indicators, and that this does not match variation extracted *optimally* for forecasting individual quantiles of the distribution of GDP growth. Our targeted financial conditions indices, when applied separately to the left tail and median of future GDP growth, show that different types of financial variables contain relevant information for each. Leverage-type variables are more important for forecasting the centre of the predictive distribution, while risk and credit variables are more important for forecasting left-tail events. After revisiting this narrative evidence, we next turn to the out-of-sample forecasting performance of our TFCIs.

2.4 Out-of-Sample Performance

In this section we evaluate the out-of-sample forecasting performance of our method for US GDP growth, considering 1 and 4 quarters ahead forecasts. We compare our approach to three alternatives: a model that relies on the Chicago Fed's NFCI, that is, the specification used by Adrian et al. (2019) and Adams et al. (2021); an even simpler variant that uses the first principal component (PCA) of the series underlying the NFCI; and a model that uses quantile-specific indices constructed with Giglio et al. (2016)'s partial quantile regression method (GKP)[31]. In all cases, we include lagged GDP growth as an additional regressor, as well as a constant. Overall, we find that our model performs considerably better than the alternatives across the entire predictive distribution on a number of established metrics.

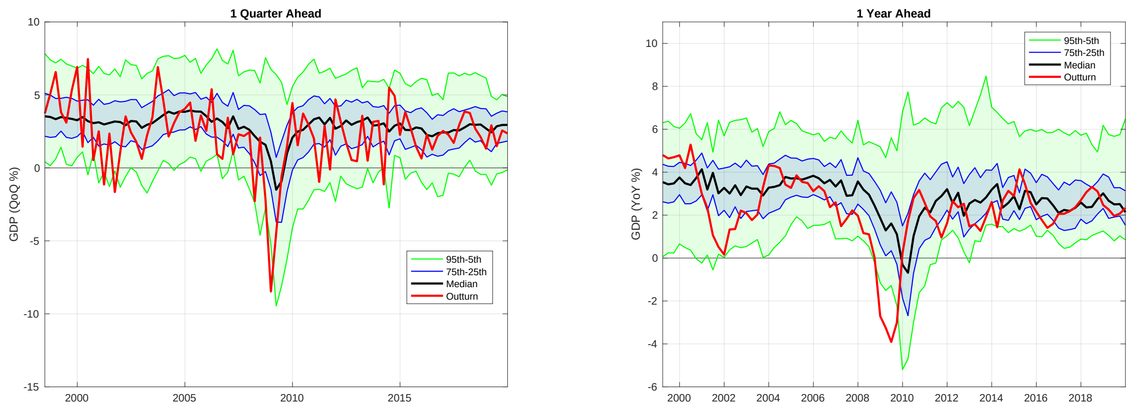
Our evaluation sample spans 1999Q1:2019Q4, and therefore includes both the early 2000s recession and the Great Financial Crisis[32]. To avoid look-ahead bias in the construction of the indices, we compute our TFCI, as well as the PCA and GKP indices recursively from the underlying financial data, using only information that was available in real time. This means fewer financial variables enter the indices in earlier samples than in later ones[33]. For the NFCI, we currently use the 2019Q4 vintage up to the forecast date in each vintage; this benchmark therefore suffers from look-ahead bias and puts it at an advantage relative to the

[31]Giglio et al. (2016) propose an approach, called partial quantile regression, which seeks to summarise the cross section of (financial) predictors according to their covariation with a chosen quantile of the target. In spirit, it is therefore the approach closest to ours. In practice, however, we show that on our dataset at least, their approach is not competitive with any of the alternatives, including our model.

[32]Although the overall results are robust to it, we chose not to include the pandemic period in our evaluation sample, as the GDP dynamics during that period were not matched by developments in financial conditions that could have helped predicting them. Results are available upon request.

[33]For each vintage, we keep only variables that were available for at least 50% of the sample up until the forecast date. This avoids distortions by ensuring that only indicators with sufficient variation over the estimation sample are considered in each vintage.

Figure 2.5: TFCI Forecasts vs. Outturns



Note: Selected predictive quantiles based on TFCIs over time against outturns, a) 1 quarter and b) 1 year ahead. The QoQ growth rate is seasonally-adjusted and annualised.

other models. As Adams et al. (2021), we abstract from the issue of GDP revisions and simply use the 2019Q4 vintage consistently across models.

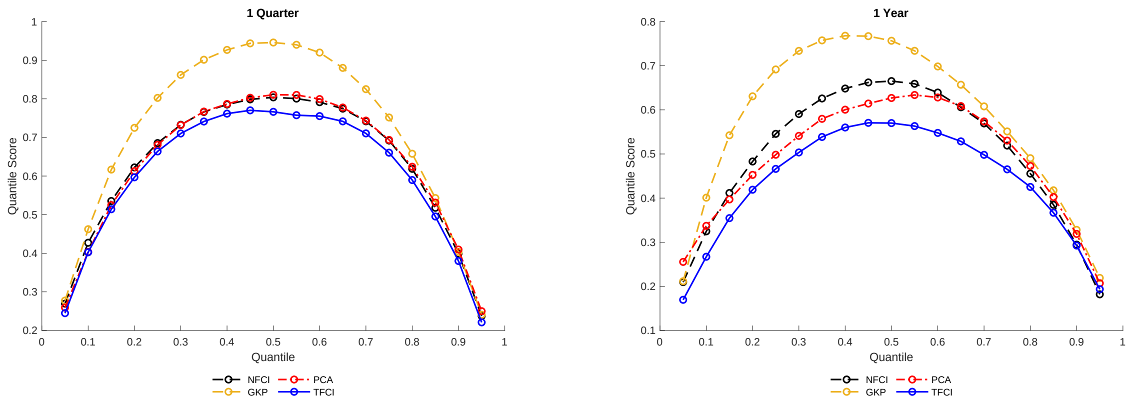
Figure 2.5 shows data outturns against estimated predictive densities constructed using our approach, and we provide similar pictures for the other models in the Appendix. To compare the performance of these predictive densities across models, we focus on three evaluation metrics: (i) quantile scores, (ii) a set of quantile-weighted scores proposed by Gneiting and Ranjan (2011), and (iii) probability integral transforms (PITs).

The quantile score (or tick loss function, see Giacomini and Komunjer (2005)) is defined as:

$$QS_{v,\tau,h} = \rho_\tau \left(y_{t_v+h} - \hat{P}^{-1}_{v,h}(\tau) \right). \tag{2.9}$$

The score penalises outturns that are more extreme (i.e. fall further in the corresponding tail) than the predictive quantile $\hat{P}^{-1}_{v,h}(\tau)$. It stands in the same relation to the loss function used in quantile regression (Koenker and Bassett (1978)) as the squared forecast error to

Figure 2.6: Average Quantile Scores



Note: Average quantile scores for all models, a) 1 quarter and b) 1 year ahead. Lower values represent better performance.

OLS regression.

We follow Gneiting and Ranjan (2011) and plot average quantile scores for all models over our evaluation sample in Figure 2.6. For both the 1 quarter and 4 quarters ahead horizons, our model forecasts yield average quantile scores lower or equal to those from all other models across quantiles. This indicates that our method is competitive across the entire distribution, and that the summary scores discussed next are not driven by its performance in a specific region of the predictive distribution.

Gneiting and Ranjan (2011) also propose a set of quantile-weighted versions of continuously ranked probability scores to assess forecasting performance in specific regions of the predictive distribution. The general form of their scores is

$$GR_{v,\tau,h} = \int_0^1 QS_{v,\tau,h} w\left(\tau\right) d\tau, \tag{2.10}$$

where w are non-negative weight functions on the real line. GR scores are essentially variously-weighted sums of the quantiles scores discussed above and are therefore useful summary statistics for comparisons and formal testing[34].

Table 2.1 shows the average GR Scores for our model and the ratios of the corresponding scores of the other three models to ours, 1 and 4 quarters ahead. TFCI forecasts generally outperform all alternatives for all weighting functions and both horizons. For 1 quarter ahead forecasts, the gains relative to PCA and the NFCI, the main focus of the "at-risk" literature, are generally around 5%. GKP performs considerably worse, notably in the centre and left tail of the predictive distribution, where the losses are around 20%. The same pattern persists for 1 year ahead forecasts, but in this case the performance gains of our forecasts compared to the alternative models are larger: the gains in the left tail are 17% and 40% relative to the NFCI and GKP, respectively[35]. For all ratios reported in Table 2.1, we test for equal forecast performance (as in Diebold and Mariano (1995), Amisano and Giacomini (2007)), relying on the asymptotic normality of the GR scores, and find that with very few exceptions, the differences we report are statistically significant at the 10% level or better.

Finally, we also compute probability integral transforms (PITs), defined as:

$$u_{v,h} = \int_{-\infty}^{y_{t_v+h}} \hat{p}_{v,h}(x)dx \equiv \hat{P}_{v,h}(y_{t_v+h}), \tag{2.11}$$

where $\hat{p}_{v,h}(\cdot)$ is the predictive density function estimated in vintage v for forecast horizon h, and $\hat{P}_{v,h}(\cdot)$ the corresponding cumulative distribution function. An ideally-calibrated model should deliver a sequence of predictive distributions whose PITs are distributed uniformly over the unit interval, that is, should lie on the diagonal of each panel of Figure 2.7.

[34]Unlike weighted versions of the traditional log score (Amisano and Giacomini (2007)), Gneiting and Ranjan (2011) scores retain propriety (see also Diks et al. (2011)) and are amenable to standard statistical testing techniques.

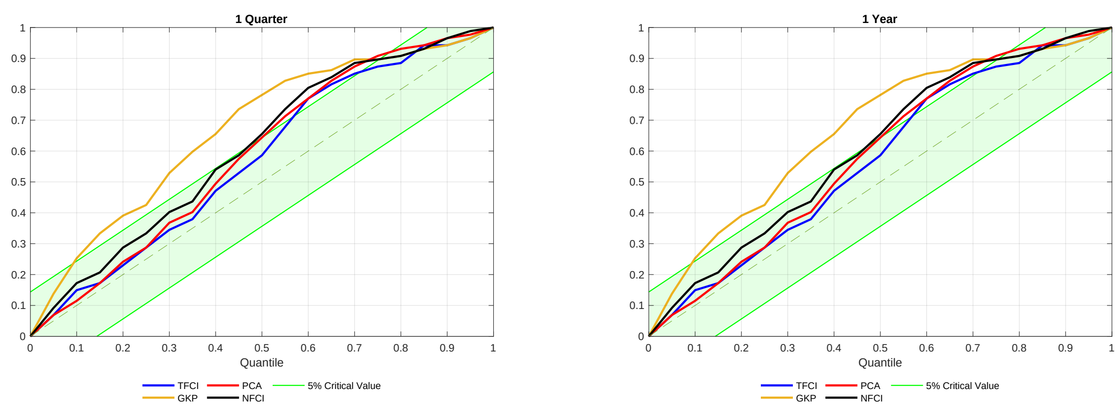
[35]This chimes with the notion that 1 year ahead forecasts exploiting financial information are less noisy than those for shorter horizons, or put differently, that the information extracted from our data is more relevant for longer horizons.

Table 2.1: Average GR Scores and GR Scores Ratios

	1 Quarter Ahead				1 Year Ahead			
	TFCI	GKP	PCA	NFCI	TFCI	GKP	PCA	NFCI
Uniform (w_0)	0.58	**1.19**	**1.05**	**1.05**	0.42	**1.32**	**1.12**	**1.14**
Center (w_1)	0.11	**1.20**	**1.05**	**1.04**	0.08	**1.33**	**1.11**	**1.15**
Tails (w_2)	0.13	**1.16**	**1.05**	**1.06**	0.10	**1.29**	**1.16**	1.12
Right Tail (w_3)	0.18	**1.16**	**1.06**	**1.05**	0.13	**1.23**	**1.11**	**1.10**
Left Tail (w_4)	0.18	**1.21**	1.04	1.05	0.13	**1.40**	**1.15**	**1.17**

Note: The table shows average Gneiting and Ranjan (2011) scores for our model (TFCI) and for different weighting functions: $w_0 = 1$; $w_1(\tau) = \tau(1-\tau)$; $w_2(\tau) = (2\tau - 1)^2$; $w_3(\tau) = \tau^2$; $w_4(\tau) = (1-\tau)^2$. Scores for the remaining models are reported as ratios to the respective TFCI score. A ratio > 1 indicates that a model performs worse than the TFCI, and numbers in bold denote statistically significant differences at the 10% confidence level or better using the same testing strategy as Diebold and Mariano (1995), Amisano and Giacomini (2007).

Figure 2.7: Probability Integral Transforms (PITs)



Note: The charts show the probability integral transforms (PITs) for each model and for both predictive horizons. The green band represents the 10% critical region, as in Rossi and Sekhposyan (2014). An ideally-calibrated model lies on the diagonal throughout the quantiles, so the closer to it, the better.

For 1 quarter ahead forecasts (left panel), the NFCI, PCA and our model mostly fall within the 10% critical region of test proposed by Rossi and Sekhposyan (2014), while GKP displays a clear tendency to over-predict GDP growth (too many outcomes fall in the lower tail). The tendency to over-predict GDP growth 1 year ahead (right panel) is more pronounced for all models, but even where our model lies outside of the critical region, it still tends to be the better-calibrated one.

2.5 Conclusion

We propose a novel approach to extract factors from large data sets that maximises covariation with the quantiles of a target distribution of interest. We showcase our methodology by constructing *targeted* financial conditions indices for US GDP-at-risk (and other portions of the predictive distribution, such as the median). We show that this yields targeted financial conditions indices that are economically intuitive, smoother when computed in real time, and with superior out-of-sample forecasting power compared to existing alternatives.

While the application to financial conditions and GDP-at-risk is of special interest due to the existing literature on the subject and its continued policy relevance, our method is general and flexible and could be easily applied to other problems as well.

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B Appendix - Chapter 2

Table B.1: List of Indicators comprising the NFCI, by Group

Risk Indicators

1) 1-mo. Asset-backed/Financial commercial paper spread
2) BofAML Home Equity ABS/MBS yield spread
3) 3-mo. Financial commercial paper/Treasury bill spread
4) Commercial Paper Outstanding
5) BofAML 3-5 yr AAA CMBS OAS spread
6) Counterparty Risk Index (formerly maintained by Credit Derivatives Research)
7) ICE BofAML ABS/5-yr Treasury yield spread
8) 3-mo./1-wk AA Financial commercial paper spread
9) ICE BofAML Financial/Corporate Credit bond spread
10) ICE BofAML Mortgage Master MBS/10-year Treasury yield spread
11) Treasury Repo Delivery Fails Rate
12) Agency Repo Delivery Failures Rate
13) Corporate Securities Repo Delivery Failures Rate
14) Agency MBS Repo Delivery Failures Rate
15) FDIC Volatile Bank Liabilities
16) 3-mo. Interbank Deposit Spread (OBFR/LIBID-Treasury)
17) On-the-run vs. Off-the-run 10-yr Treasury liquidity premium
18) Total Money Market Mutual Fund Assets/Total Long-term Fund Assets
19) Fed Funds/Overnight Treasury Repo rate spread
20) Fed Funds/Overnight Agency Repo rate spread
21) Repo Market Volume (Repurchases+Reverse Repurchases of primary dealers)
22) Fed Funds/Overnight MBS Repo rate spread
23) 3-mo./1-wk Treasury Repo spread
24) 10-yr/2-yr Treasury yield spread
25) 2-yr/3-mo. Treasury yield spread
26) 10-yr Interest Rate Swap/Treasury yield spread
27) 2-yr Interest Rate Swap/Treasury yield spread
28) 3-mo. Overnight Indexed Swap (OIS)/Treasury yield spread
29) 3-mo. LIBOR/CME Term SOFR-Treasury spread
30) 1-yr./1-mo. LIBOR/CME Term SOFR spread
31) Advanced Foreign Economies Trade-weighted US Dollar Value Index
32) CBOE Market Volatility Index VIX
33) 1-mo. BofAML Option Volatility Estimate Index
34) 3-mo. BofAML Swaption Volatility Estimate Index

Credit Indicators

35) 1-mo. Nonfinancial commercial paper A2P2/AA credit spread
36) Moody's Baa corporate bond/10-yr Treasury yield spread
37) UM Household Survey: Auto Credit Conditions Good/Bad spread
38) Commercial Bank 48-mo. New Car Loan/2-yr Treasury yield spread
39) Commercial Bank 24-mo. Personal Loan/2-yr Treasury yield spread
40) S&P US Bankcard Credit Card: 3-mo. Delinquency Rate
41) Consumer Credit Outstanding
42) S&P US Bankcard Credit Card: Excess Rate Spread
43) FRB Senior Loan Officer Survey: Tightening Standards on Large C&I Loans
44) FRB Senior Loan Officer Survey: Tightening Standards on Small C&I Loans
45) FRB Senior Loan Officer Survey: Tightening Standards on CRE Loans
46) S&P US Bankcard Credit Card: Receivables Outstanding
47) FRB Senior Loan Officer Survey: Willingness to Lend to Consumers
48) NY Fed Consumer Credit Panel: Loan Delinquency Status: Non-current (Percent of Total Balance)
49) American Bankers Association Value of Delinquent Consumer Loans/ Total Loans
50) American Bankers Association Value of Delinquent Home Equity Loans/ Total Loans
51) American Bankers Association Value of Delinquent Credit Card Loans/ Total Loans
52) UM Household Survey: Durable Goods Credit Conditions Good/Bad spread
53) Finance Company Owned & Managed Receivables
54) UM Household Survey: Mortgage Credit Conditions Good/Bad spread
55) BofAML High Yield/Moody's Baa corporate bond yield spread
56) 30-yr Jumbo/Conforming fixed rate mortgage spread
57) Markit High Yield (HY) 5-yr Senior CDS Index
58) Markit Investment Grade (IG) 5-yr Senior CDS Index
59) MBA Serious Delinquencies
60) Money Stock: MZM
61) 30-yr Conforming Mortgage/10-yr Treasury yield spread
62) Bond Market Association Municipal Swap/State & Local Government 20-yr GO bond spread
63) NACM Survey of Credit Managers: Credit Manager's Index
64) Commercial Bank Noncurrent/Total Loans
65) FRB Senior Loan Officer Survey: Tightening Standards on RRE Loans
66) NFIB Survey: Credit Harder to Get
67) FRB Senior Loan Officer Survey: Increasing spreads on Large C&I Loans
68) FRB Senior Loan Officer Survey: Increasing spreads on Small C&I Loans
105) CBOE Crude Oil Volatility Index, OVX

Leverage Indicators

69) Nonmortgage ABS Issuance (Relative to 12-mo. MA)
70) Broker-dealer Debit Balances in Margin Accounts
71) New US Corporate Debt Issuance (Relative to 12-mo. MA)
72) Commercial Bank C&I Loans/Total Assets
73) CMBS Issuance (Relative to 12-mo. MA)
74) COMEX Gold/NYMEX WTI Futures Market Depth
75) Commercial Bank Consumer Loans/Total Assets
76) FRB Commercial Property Price Index
77) 10-yr Constant Maturity Treasury yield
78) Commercial Bank Total Unused C&I Loan Commitments/Total Assets
79) Net Notional Value of Credit Derivatives
80) CME E-mini S&P Futures Market Depth
81) Total Assets of Finance Companies/GDP
82) Total Assets of Funding Corporations/GDP
83) S&P 500 Financials/S&P 500 Price Index (Relative to 2-yr MA)
84) Total Agency and GSE Assets/GDP
85) Total Assets of Insurance Companies/GDP
86) Fed funds and Reverse Repurchase Agreements/Total Assets of Commercial Banks
87) CoreLogic National House Price Index
88) New State & Local Government Debt Issues (Relative to 12-mo.h MA)
89) Total MBS Issuance (Relative to 12-mo. MA)
90) S&P 500, NASDAQ, and NYSE Market Capitalization/GDP
91) S&P 500, S&P 500 mini, NASDAQ 100, NASDAQ mini Open Interest
92) 3-mo. Eurodollar, 10-yr/3-mo. swap, 2-yr and 10-yr Treasury Open Interest
93) Total Assets of Pension Funds/GDP
94) CME Eurodollar/CBOT T-Note Futures Market Depth
95) Total REIT Assets/GDP
96) Commercial Bank Real Estate Loans/Total Assets
97) Total Assets of Broker-dealers/GDP
98) Commercial Bank Securities in Bank Credit/Total Assets
99) New US Corporate Equity Issuance (Relative to 12-mo. MA)
100) Federal, state, and local debt outstanding/GDP
101) Total Assets of ABS issuers/GDP
102) Wilshire 5000 Stock Price Index
103) Household debt outstanding/PCE Durables and Residential Investment
104) Nonfinancial business debt outstanding/GDP

Figure B.1: Model Forecasts vs. Outturns, 1 Quarter Ahead

Note: Predictive densities time series for each model compared to the outturn one quarter ahead. The QoQ growth rate is seasonally-adjusted and annualised.

Figure B.2: Model Forecasts vs. Outturns, 1 Year Ahead

Note: Predictive densities time series for each model compared to the outturn one year ahead.

3 Bank Lending, Deposit Creation and Balance Sheet Constraints

3.1 Introduction

The Basel III regulatory framework was introduced after the 2008 Great Financial Crisis (GFC) with the aim of limiting the risks taken by banks and promoting greater stability in the financial system. One of its key aspects was the introduction of different balance sheet constraints that require banks to maintain minimum liquidity and capital cushions. Even though these requirements have been successful in mitigating systemic risks and strengthening the banking sector, they have made bank balance sheet management considerably more complex.

When setting up their investment and strategies, banks not only need to take into account their internal controls and stress tests, but also need to make sure they are meeting the several requirements imposed by regulators. In this context, fully understanding the dynamics of banks' balance sheets becomes critical to assess the impacts of these constraints on banking activities such as lending and market making.

Traditional microeconomic modelling of banks has been built on the "financial intermediation theory". Typically, in these models (e.g., Klein (1971), Diamond and Dybvig (1983), Diamond (1984)), in order to lend, banks need to first raise funds through deposits from households or companies that want to save. The idea behind this is simple: it should not be possible to lend something that the lender does not have under its custody.

In modern economies, however, a bank does not need to raise money before lending. Unlike other financial intermediaries, banks can simply create by themselves the deposits that will fund these loans. This happens because bank deposits are considered to be a type of money so when a loan is agreed between a bank and a non-bank costumer, the bank instantaneously creates a deposit in the borrower's account[36]. From the bank's balance sheet perspective, before any other transaction is made, the increase in assets due to the new loan is accompanied by an increase in liabilities due to the new deposit.

This raises the question: Does it matter whether banks raise deposits first or lend first? If banks lend first and then create deposits, lending expands their balance sheets. Conversely, if deposits come first and lending is simply an asset reallocation, then lending does not lead to a balance sheet expansion. These two lending dynamics result in different balance sheet outcomes and may lead to different behaviours by banks that must comply with regulatory requirements. This distinction is particularly significant in a low interest rate environment. In normal times, even if lending expands its balance sheet, a bank can shrink it back to its original size by reducing deposit rates and thus sending deposits away. However, when rates are close to the lower bound, banks cannot cut deposit rates further and thus lack the flexibility to adjust their balance sheet size through liabilities after granting a loan. Consequently, in a setting where banks are constrained by regulatory limits on balance sheet size, whether lending expands the balance sheet becomes crucial for their lending decisions.

Considering all this, I study in this paper how balance sheet regulation limits banks' activities in an environment of low interest rates and unconventional monetary policy. For this, I develop a model in which banks create deposits when lending, are subject to Basel III requirements and face endogenous deposit and loan demands from the real sector of the economy.

The first main contribution of the paper is to show that bank lending decision depends on the interaction between banks' balance sheet dynamics resulting from deposit creation and regulatory constraints. In this context, I find that lower interest rates create additional challenges for bank lending under Basel III framework. The model shows that, when the economy is close to its effective lower bound, banks have one less margin to adjust their

[36]McLeay et al. (2014) provide a comprehensive discussion on the real mechanics of modern bank lending. For a model that incorporates this mechanism, refer to Bianchi and Bigio (2022).

balance sheet size. If they are unable to cut deposit interest rates, then they cannot shrink their balance sheet by reducing liabilities. As a consequence, leverage constrained banks will have no other choice rather than to lend less in order to comply with regulation requirements.

Next, motivated by the Covid-19 crisis, I investigate how balance sheet constraints may limit banks activity in bond markets following a liquidity shock. In March 2020, an unprecedented flood of sales hit the U.S. Treasuries market as agents reacted to pandemic-related news. At the same time, banks, which are responsible for most of the dealer activity in this market, could not handle this massive trading volume and soon stress levels peaked with investors unable to sell their bonds. As shown by Duffie (2020), the market meltdown only stopped when the Fed decided to step in by doing a quantitative easing (QE) to buy these Treasuries.

Extending the model to include both a liquidity shock and secondary bond market allows for the formalisation of the conditions under which banks are unable to absorb investor selling demand. Banks will stop making market if their balance sheets are inflated and if additional capital commitments are too high. Then, investors will not be able to sell their bonds, which can potentially trigger a liquidity crisis such as the one seen in March 2020.

Finally, the model predicts that the impact of unconventional monetary policy on bank lending depends not only on who the central bank buys from in an asset purchase programme but also on banks' balance sheet capacity. In this context, and in line with some of the Fed's expressed concerns (See Federal Reserve Board (2020)), it is shown that central banks may transform a liquidity crisis into a credit crisis if they use QE to flood the market with liquidity when banks are unable to absorb this money into their balance sheets due to leverage constraints.

Literature Review

In the early 20th century, the understanding of leading economists (Wicksell (1907), Hahn (1920)) was that banks can create the deposits they need to fund loans and that bank money creation expands the supply of credit. Using Schumpeter's (1934) words, the belief was that: "*credit is essentially the creation of purchasing power for the purpose of transferring it to the entrepreneur, but not simply the transfer of existing purchasing power*".

However, starting with Gurley and Shaw (1955, 1956) and Tobin (1963), the thought that banks are most like any other financial intermediary started to gain traction and became dominant in literature. Tobin (1963) argues that, apart from special financial regulation constraints that banks may be subject to, the difference between banks and other financial intermediaries is "*superficial and irrelevant*" and that they are limited by the same kinds of economic pressures so they should be treated equally. Since then, the "financial intermediation theory" framework where banks are only able to lend if they raise funds in advance was adopted by an extensive list of papers including Diamond and Dybvig (1983), Bernanke (1993), Allen and Santomero (2001), Diamond and Rajan (2001), Gertler and Kiyotaki (2011).

More recently, after the GFC exposed regulation problems with respect to credit conceded by banks, some central banks (McLeay et al. (2014), Deutsche Bundesbank (2017), Jakab and Kumhof (2019)) pointed out that incorporating banks' ability to create deposits is fundamental to better understanding and designing mechanisms to supervise modern financial systems. In this same context, Donaldson et al. (2018), in order to investigate how banks create funding liquidity, develop a model in which banks make loans even if they have no initial deposits to lend out. In this paper, I build on this literature and explore the interaction between the deposit creation feature with financial regulation constraints. I argue that, especially in a low interest rates environment, the creation of deposits when lending influences banks' balance sheet management and thus banks behave differently than other financial intermediaries.

Since the Basel III regulatory framework was introduced, banks have been following stricter rules in terms of leverage, risk-taking and liquidity. Gambacorta and Karmakar (2016) show that leverage and risk-sensitive capital requirements complement each other, with the former being tighter during booms and the latter being tighter in a bust. Using a different perspective, my model shows that in the case of a negative liquidity shock, it's the leverage constraint that tends to bind.

Gropp et al. (2018) provide evidence that banks increase their capital ratios by reducing their risk-adjusted assets, especially loans, instead of raising their level of equity. Keister (2019) argues that new liquidity constraints lead banks to hold large amounts of high-quality liquid assets (HQLA) in the form of excess reserves which allow them to keep flexibility in managing their balance sheets. In addition, Brooke et al. (2015) provide a survey about capital regulation with evidence pointing to its impact in terms of reducing the likelihood and severity of potential crises but also reducing banks' capacity to make loans and then stimulate aggregate output. This study contributes to this literature in a complementary way, also investigating the role of balance sheet constraints in limiting bank lending but adding banks' deposit creation feature and focusing on its interaction with the zero lower bound.

The impact of banks' balance sheets on monetary policy transmission through the "balance sheet" and "bank lending" channels is well-documented in the literature (Bernanke and Blinder (1988), Bernanke and Gertler (1995)). Work by Disyatat (2010) also argues that the impact of monetary policy on banks' balance sheet strength and risk perception leads the banking sector to act as an absorber or amplifier of shocks originating in the financial system. In line with this literature, my model predicts that banks' balance sheet play a central role in channeling shocks and monetary policy into the real economy and in enhancing the financial system stability.

In recent years, a lot of attention has been devoted to a low interest rates environment and its direct impact on bank lending. Balloch and Koby (2020) provide evidence that low rates result in significantly lower loan growth and bank profitability. Heider et al. (2019) show that banks are reluctant to pass on negative rates to depositors which may lead to limited transmission of policy rates and, as consequence, reduced stimulus and financial instability. With results that point in a similar direction, Brunnermeier and Koby (2019) and Eggertsson et al. (2019) investigate the existence of a reversal interest rate, a rate at which accommodative monetary policy reverses and becomes contractionary for lending. I build on the evidence provided by this literature on the existence of a lower bound for the deposit interest rate but, unlike these papers, I focus on the indirect impact of the lower bound on bank's ability to manage their balance sheet and thus to comply with regulation.

Finally, this paper contributes for the early literature that studies the Covid-19 crisis by helping to understand and modelling what motivated banks' behaviour before and after the QE done to stabilise the U.S. Treasuries market in early 2020. Along the same lines, Duffie (2020) and Vissing-Jorgensen (2021) show that the Fed rescued the Treasury market during the Covid crisis by providing liquidity when they removed bonds from the market, while Koont and Walz (2021) provide evidence that the relaxation of the SLR (leverage) constraint after the Fed intervention was effective in preventing larger credit contractions.

Outline

The rest of this paper is structured as follows. In Section 3.2, I present a brief summary of modern banks' role in lending and money creation. In Section 3.3, I present a model with endogenous bank deposit and loan demand, focusing on banks' profit maximisation problem subject to financial regulation constraints. In Section 3.4, I introduce the low interest rate setting and discuss the effects of the interaction between leverage constraint and the zero lower bound in banks' lending decision. Then, in Section 3.5, the model is extended with the introduction of a liquidity shock and unconventional monetary policy. Lastly, Section 3.6 concludes with the final remarks.

3.2 How do Modern Banks Operate?

In modern economies, bank deposits are by far the most important form of money — approximately 97% of the total amount in circulation (McLeay et al. (2014)). In this context, households' saving decision is not as crucial for bank lending as financial intermediation theory predicts. When households decide to consume instead of saving, they will use their money – usually in the form of bank deposits – to pay for the goods and services provided by sellers. There is a change in the holding of the funds but the total amount of deposits in the banking system is still the same. Thus, the households' decision of saving by itself does not increase the aggregate amount of deposits available for banks to lend.

Today, when a bank makes a loan, it does so by creating an additional deposit in the name of the borrower instead of handing him loads of cash that were raised in advance from a saver. Of course, bank deposits are fully convertible into cash and thus borrowers can still withdraw these funds, zeroing their accounts. However, when they use this money to pay for transactions, whoever is receiving it will most probably deposit this amount in a bank either due to safety or inflation reasons. Therefore, even if the money moves to a different bank and the borrowers's bank deposits stay on the same level, the aggregate amount of deposits in the financial system increases with the new loan.

It is important to note that, in the same way a new loan leads to creation of deposits (and money), the repayment of an existing loan by a non-banking agent leads to the destruction of deposits. Money can also be destroyed when banks issue other liabilities and when they sell assets to the non-banking sector. This dynamic of creation and destruction of deposits, and consequently money, is relevant because it impacts banks' balance sheets as well as asset prices.

Even though banks can make loans without needing to raise the funds in advance, they cannot extend an unlimited amount of credit just because they want to. In fact, banks face limits imposed by demand, competition, regulatory requirements and monetary policy decisions. In the following sections, these restrictions are incorporated to banks' optimisation problem in the model and one can observe how bank lending is impacted and limited by each one of them.

3.3 Model

In this section, I present a model featuring a continuum of identical banks that provide credit to the real sector of the economy by creating new deposits. Firms use that money to pay wages for heterogeneous households, who need to save through cash, bank deposits or government bonds. If they choose bank deposits, there will be more deposits in the economy after the loans. Conversely, if they choose cash or bonds, the money will flow into the government's account at the central bank, which is kept separate from the private banking system[37]. In this scenario, the amount of deposits available to banks will remain unchanged, as the new deposits created by the loans will be immediately destroyed when used to pay for cash or bonds.

Time is discrete and there are three periods $t = 0, 1, 2$. All financial and productive decisions are taken at $t = 0$. After that, at $t = 1$, the economy is hit by a liquidity shock θ that alters the value of holding bonds for households, who then can engage with banks in a secondary bond market that is monitored by the central bank. At $t = 2$, assets' payoffs and consumption realise. There are four agents: households, firms, banks and the public sector (which includes the central bank). Hereinafter in this section I describe these agents, how they participate in the economy and the choices they make at $t = 0$. The developments of the model after $t = 0$ are explored in section 3.5.

[37]This money may be reinserted into the banking system when the cash is deposited at a bank or when the government decides to spend. Until then, however, it remains outside the banking system and is not accounted for in banks' balance sheets.

3.3.1 Households

A continuum of heterogeneous households chooses consumption c and labour h to maximise their expected utility, which takes the form, for household j:

$$U_{j,0} = \mathbb{E}_0 \left[\frac{c_j^{1-\sigma}}{1-\sigma} - \frac{h_j^{1+\eta}}{1+\eta} \right], \qquad \sigma < 1, \qquad \eta > 0. \tag{3.1}$$

Households work and receive wages w at $t = 0$. They are risk-averse and derive disutility from working. With $\sigma < 1$, households increase their labour supply when facing higher wages, with a Frisch elasticity of $\frac{1}{\eta}$. Consumption only takes place at the final period, $t = 2$. In this context, household j can invest his wages in cash $m_{j,0}$, bank deposits $d_{j,0}$ or in bonds $b_{j,0}$ to transfer wealth from $t = 0$ to $t = 2$ in order to finance consumption. Household j budget constraint at $t = 0$ then is given by:

$$d_{j,0} + b_{j,0} + m_{j,0} \leq w \cdot h_j. \tag{3.2}$$

At $t = 2$, cash and bank deposits are paid back and households receive interest rate i_M for the former and i_D for the latter. Bonds are also repaid, with interest i_B, but households are heterogeneous in terms of bond payoff. In addition to the interest rate i_B, they receive a premium $\psi_j \sim U[-\kappa, \kappa]$ for holding the bond. One could think of ψ_j as an individual convenience yield. After all decisions are taken, households face a common liquidity shock θ at $t = 1$, for which they assign zero probability of $\theta \neq 1$ ex-ante. As such, household j budget constraint at $t = 2$ reads:

$$c_j \leq (1 + i_D) \cdot d_{j,0} + \theta \cdot (1 + i_B + \psi_j) \cdot b_{j,0} + (1 + i_M) \cdot m_{j,0}. \tag{3.3}$$

Let $i_M = 0$. If $i_D < 0$, then $d_{j,0} = 0$ for all j and the bank is all-equity. I assume this is never optimal for the banks[38] and therefore they set $i_D \geq i_M = 0$ and no household wants to hold cash. In this case, households will only invest in deposits or bonds.

Note that, for household j to be indifferent between deposits and bonds, it must be the case that $\psi_j = \bar{\psi_0} \equiv i_D - i_B$. Households with $\psi_j < \bar{\psi_0}$ will only invest in deposits and those with with $\psi_j > i_D - i_B$ will only invest in bonds. Therefore, households' first order conditions imply that labour is supplied according to:

$$h_j = \begin{cases} [(1+i_D)\cdot w]^{\frac{1-\sigma}{\eta+\sigma}}, & \text{if } \psi_j \leq i_D - i_B \\ [(1+i_B+\psi_j)\cdot w]^{\frac{1-\sigma}{\eta+\sigma}}, & \text{if } \psi_j > i_D - i_B. \end{cases} \tag{3.4}$$

3.3.2 Firms

The firms start with no equity and thus need to borrow from banks at $t = 0$ to finance their operations. Labour is the only input in their production function so the amount borrowed l is entirely used to pay wages and is immediately transferred to households. Once goods are sold at $t = 2$, the revenue is used to repay the loans and any remaining profit is consumed by the firms.

All firms have access to the same technology and production function:

$$F(h) = A \cdot h^{\alpha}, \qquad 0 < \alpha < 1. \tag{3.5}$$

Additionally, they all face the same loan interest rate i_L, which is set by banks. Firms

[38]When the bank operates with an all-equity structure, no balance sheet constraint is binding. Therefore, the marginal cost of setting $i_D = 0$ (and holding marginal deposits) is exactly zero. As long as banks have any asset available with non-negative expected return, they will always prefer to set $i_D = 0$ and hold an additional asset on their balance sheet, rather than setting $i_D < 0$ and remaining all-equity. To keep the analysis relevant and because the paper focus on how balance sheet constraints interacts with bank lending and deposit creation, I will focus on the case where $i_D \geq 0$ from this point onward.

maximise their expected profits by choosing how much labour to employ at the given wage level w. Formally, the firms' problem is:

$$\begin{aligned} \max_{h_F} \quad & \Pi_F = F(h_F) - (1+i_L)\cdot l \\ \text{s.t.} \quad & l = w \cdot h_F. \end{aligned} \tag{3.6}$$

Solving the problem above, the firms' demand for labour is given by:

$$h_F = \left[\frac{\alpha A}{(1+i_L)\cdot w}\right]^{\frac{1}{1-\alpha}}. \tag{3.7}$$

Labour Market Equilibrium

Considering the solution for both households' and firms' problems, one can find that the wage w in equilibrium is:

$$w = \left[\left(\frac{\alpha A}{(1+i_L)}\right)^{\frac{\eta+\sigma}{\alpha-1}} \cdot g\left(i_D\right)\right]^{\frac{\alpha-1}{1+\eta-\alpha(1-\sigma)}}, \tag{3.8}$$

where $g\left(\cdot\right)$ is differentiable and $g'(\cdot) > 0$ [39].

Under this wage level, the amount of labour employed h in goods' production is given by:

$$h = \left[\left(\frac{\alpha A}{(1+i_L)}\right)^{\frac{1-\sigma}{\eta+\sigma}} \cdot g\left(i_D\right)\right]^{\frac{\eta+\sigma}{1+\eta-\alpha(1-\sigma)}}. \tag{3.9}$$

Finally, the firms' financing needs will lead them to demand the following amout of loans:

[39] For full expression of $g\left(\cdot\right)$, see Appendix C.1.

$$l = w \cdot h = \left[\left(\frac{\alpha A}{(1+i_L)} \right)^{\frac{1+\eta}{\alpha\cdot(\eta+\sigma)}} \cdot g\left(i_D\right) \right]^{\frac{\alpha\cdot(\eta+\sigma)}{1+\eta-\alpha(1-\sigma)}} \equiv L\left(i_D, i_L\right). \tag{3.10}$$

Hence, the loan demand function $L(\cdot)$ is decreasing in the loan interest rate i_L, the borrowing cost, and increasing in the deposit interest rate i_D, which is the revenue generated by depositing the borrowed funds in bank accounts.

The deposit and bond demands will be such that:

$$d_0 = w \cdot \int_{-\kappa}^{i_D - i_B} \left[(1+i_D)\cdot w\right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i \equiv D(i_D, i_L) \tag{3.11}$$

and

$$b_{0,h} = w \cdot \int_{i_D - i_B}^{\kappa} \left[(1+i_B+\psi_i)\cdot w\right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i \equiv B(i_D, i_L). \tag{3.12}$$

It is shown in Appendix C.1 that $B(\cdot)$ is decreasing in both i_D and i_L, while $D(\cdot)$ is increasing in i_D and decreasing in i_L. The intuition is that a higher i_D leads more households to save in deposits rather than in bonds, while a higher i_L reduces the amount of loan demand from the firms, which results in lower aggregate households savings and thus in less deposits and bonds.

3.3.3 Banks

Banks' Assets and Liabilities

Banks may hold three types of assets in their balance sheets: reserves, bonds and loans. First, since the amount of reserves in the economy is determined by the central bank and banks are the only private agents allowed to hold them, banks' reserves allocation r is exogenously determined and is taken as given by them[40]. Second, banks face a perfectly elastic supply when choosing how much to invest in bonds $b_{0,B}$, and unlike households, they are not subject to the liquidity shock θ on bonds' payoff. Third, given the loan demand $L(i_D, i_L)$, banks set the lending interest rate i_L and elastically supply loans for the productive sector of the economy.

On the liabilities side, banks' start with initial equity n_0. Besides that, banks set the deposit interest rate i_D, at which level they elastically create deposits to satisfy demand $D(i_D, i_L)$.

Regulatory Financial Constraints

Following the Basel III regulatory framework, the banking sector faces, at any point in time, three financial constraints in the model. First, in the spirit of the Liquidity Coverage Ratio (LCR), a liquidity constraint of the form:

$$r + b_B \geq \phi_{LCR} \cdot d, \tag{3.13}$$

where ϕ_{LCR} is the deposits' run-off rate. This constraint aims to promote the resilience of the

[40]Since the model focuses on the aggregate banking sector capacity to provide credit and intermediation services rather than individual bank decisions, I assume banks are homogeneous for simplification. When banks are heterogeneous, different banks might choose varying levels of reserves but the overall reserves allocation for the aggregate banking sector is still be determined exogenously by the Central Bank.

Assets	Liabilities
Loans (l)	Deposits (d)
Reserves (r)	
	Equity (n)
Bonds (b_B)	

Figure B.1: Bank's balance sheet

liquidity risk profile of banks by ensuring that they hold enough high-quality liquid assets (reserves and bonds) to cover for their expected net cash outflows in the short term.

Second, banks face a capital constraint of the form:

$$n \geq \phi_{RWA}^{L} \cdot l + \phi_{RWA}^{R} \cdot r + \phi_{RWA}^{B} \cdot b_B, \tag{3.14}$$

where ϕ_{RWA}^{L}, ϕ_{RWA}^{R} and ϕ_{RWA}^{B} are risk weights for loans, reserves and bonds, respectively. This constraint establishes the minimum capital requirements for the amount of risk-adjusted assets held by a bank. Since both reserves and bonds are considered riskless in Basel-III regulatory framework, I assume in the model $\phi_{RWA}^{R} = \phi_{RWA}^{B} = 0$.

Finally, representing the Supplementary Leverage Ratio (SLR), banks are subject to a leverage constraint that takes the form of:

$$n \geq \phi_{SLR} \cdot (r + b_B + l), \tag{3.15}$$

where ϕ_{SLR} denotes the amount of capital that a bank must hold for each unit of asset in its balance sheet. Note that, in contrast to the capital constraint, the leverage constraint does not distinguish between safer or riskier assets and treats all of them equally.

Banks' Problem

In this context, at $t = 0$, banks maximise their expected net worth n subject to the financial regulation constraints described above and to the balance sheet identity as follows:

$$\begin{aligned}
\max_{\{i_L, i_D, b_{0,B}\}} \quad & n = \mathbb{E}_0[(1+i_L)\cdot L(i_D,i_L) + (1+i_R)\cdot r + (1+i_B)\cdot b_{0,B} \\
& \qquad - (1+i_D)\cdot D(i_D,i_L)] \\
\text{s.t.} \quad & r + b_{0,B} \geq \phi_{LCR}\cdot D(i_D,i_L) \\
& n \geq \phi^{L}_{RWA}\cdot L(i_D,i_L) \\
& n \geq \phi_{SLR}\cdot (r + b_{0,B} + L(i_D,i_L)) \\
& r + b_{0,B} + L(i_D,i_L) = D(i_D,i_L) + n_0 \\
& i_D \geq i_M = 0.
\end{aligned} \tag{3.16}$$

where i_R is the interest rate paid on reserves exogenously set by the central bank.

Banks' Optimal Choice

Banks' first order conditions provide the following rate-setting rule for loans [41]:

[41] The rate-setting rule for deposits can be find in Appendix A.1

$$i_L = [(1-\delta_L)\cdot i_B + \delta_L \cdot i_D] + \frac{1}{\epsilon_L} +$$

$$+ \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot \{\lambda_{LCR}\cdot[(1-\delta_L)+\phi_{LCR}\cdot\delta_L]\} + \tag{3.17}$$

$$+ \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot \left\{\lambda_{SLR}\cdot\phi_{SLR}\cdot\delta_L + \lambda_{RWA}\cdot\phi^L_{RWA}\right\},$$

where $\delta_L \equiv \frac{\frac{\partial D(\cdot)}{\partial i_L}}{\frac{\partial L(\cdot)}{\partial i_L}} \in (0,1)$ measures how deposits change compared to loans following a change in loan interest rate, ϵ_L is defined as the semi-elasticity of loan demand function with respect to the loan interest rate and λ_j is the Lagrange multiplier for constraint j.

The first term in brackets represents the return households get for borrowing \$1 and investing it from $t = 0$ until $t = 2$. A share δ_L of the loans will be kept in the form of deposits, for which the banks will have to pay i_D. The remaining $(1-\delta_L)$ share will be invested in bonds that will generate revenue i_B.

In addition, the term associated with λ_{LCR} shows how liquidity concerns may constrain bank lending activity. A loan that is fully invested in bonds will reduce the amount of HQLA held by the bank and tighten the LCR constraint by a factor of 1 — reduction in $r+b_{0,B}$. The same loan, if fully invested in deposits, will not change the bank's HQLA balance but will increase deposits, which also tightens the LCR constraint, but by a factor ϕ_{LCR} — increase in $\phi_{LCR}(\theta)\cdot D(i_D, i_L)$. Hence, regardless of how the money is invested by households, a loan always have a negative impact on banks' liquidity.

In terms of leverage, lending only has an impact on banks' balance sheets when is internally retained in the form of deposits. In this case, the new asset is matched with a new liability and banks' leverage automatically increases. This tightening in the SLR constraint — an increase in $\phi_{SLR}\cdot(r + b_{0,B} + L(i_D, i_L))$ — is captured by the term associated with

λ_{SLR} and does not occur when the money is taken out of households' accounts since in this case the bank's balance sheet size does not change — the increase in $L(i_D, i_L)$ is completely offset by a reduction in $r + b_{0,B}$. Finally, the last term $\lambda_{RWA} \cdot \phi^L_{RWA}$ indicates the costs associated with increasing the amount of risky assets when granting a loan. This reflects on the tightening of the RWA constraint and does not depend on households' choices.

The terms associated with regulatory constraints are irrelevant when these constraints are not binding as their Lagrange multipliers are zero. However, whenever these constraints bind, the optimal rates will be constrained such that banks' balance sheets satisfy regulation requirements.

3.3.4 Public Sector

The government supplies bonds elastically at $t = 0$ and repays them at $t = 2$ through some technology $y(x) = (1 + i_B) \cdot x$. The central bank sets the interest rate paid on reserves i_R, chooses the reserves level r in the financial system and then takes reserves deposits or lends reserves to banks in order to match the target r.

3.4 Bank Lending and the Zero Lower Bound

As previously mentioned, a growing body of empirical works have identified particular effects of low-rates environment (negative or positive and close to zero rates) on banks activities. Evidences point to reduced banks' spreads and ultimately, profitability, which has the potential to become a concern for monetary authorities (Coeure (2016), Lane (2016)). As shown by Heider et al. (2019), one of the reasons for these particular effects of low rates is that banks seem to be reluctant to pass on negative rates to depositors due to fears of them running to withdraw their money. This not only increases banks' funding costs when policy rates are negative but also creates additional restrictions to their lending activity.

In the model, if the zero lower bound constraint is not binding, we have the Lagrange multiplier $\zeta_{ZLB} = 0$ and $i_D > 0$. Then, using the deposit rate setting rule obtained in the banks' problem first order conditions, it is possible to simplify the loan interest rate setting rule to:

Lemma 3.1 (Lending away from the ZLB)**.**

$$i_L = i_B + \frac{1}{1 - \delta_D \cdot \delta_L} \cdot \left[\frac{1}{\epsilon_L} - \delta_L \cdot \frac{1}{\epsilon_D} \right] + \\ + \frac{1}{1 + \lambda_{RWA} + \lambda_{SLR}} \cdot \left\{ \lambda_{LCR} + \lambda_{RWA} \cdot \phi^L_{RWA} \right\}, \tag{3.18}$$

where $\delta_D \equiv \frac{\frac{\partial L(\cdot)}{\partial i_D}}{\frac{\partial D(\cdot)}{\partial i_D}} \in (0, 1)$ measures how loans change compared to deposits following a change in deposit interest rate.

One can see that, in this case, when we account for the optimal i_D there is no term associated with the SLR constraint in i_L setting rule anymore. This happens because banks have the flexibility to lend as much as they want while managing their balance sheet size through their liabilities. If they want to lend more and additional deposit creation may become an issue, they are able to freely lower their deposit interest rate and avoid an increase in their leverage. Hence, any required adjustment to SLR constraint is made by changing the level of i_D instead of i_L. However, this is only possible when the economy is away from the zero lower bound so banks can freely reduce i_D whenever they want.

When the economy finds itself in the ZLB, banks would ideally like to set a negative deposit interest rate but this is not possible without depositors running away. In the model, this will be reflected by $\zeta_{ZLB} > 0$ and $i_D = 0$. Thus, the loan interest rate will be set according to:

Lemma 3.2 (Lending at the ZLB).

$$
\begin{aligned}
i_L = \; & (1-\delta_L)\cdot i_B + \frac{1}{\epsilon_L} + \\
& + \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot \{\lambda_{LCR}\cdot[(1-\delta_L)+\phi_{LCR}\cdot\delta_L]\} + \\
& + \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot \left\{\lambda_{SLR}\cdot\phi_{SLR}\cdot\delta_L + \lambda_{RWA}\cdot\phi^{L}_{RWA}\right\}.
\end{aligned}
\tag{3.19}
$$

Unlike in the previous case, a bank with leverage concerns in the ZLB cannot reduce its deposit interest rate to prevent its balance sheet from increasing with lending. In practice, this means that banks will necessarily keep a fraction δ_L of the newly created deposit as liabilities on their balance sheet, which leads to a tightening of the leverage constraint. Therefore, as one can see in (3.19), banks facing a binding SLR constraint will set higher loan interest rates and lend less than they would otherwise.

The comparison between setting rules in (3.18) and (3.19) can be summarised in the following result:

Proposition 3.1 (SLR and ZLB interaction). *Bank lending is not necessarily constrained by leverage concerns when the economy is away from the lower bound. Close to it, however, liabilities adjustments are not possible and thus leverage constrained banks must lend less.*

This result sheds light on how the impact of leverage requirements on bank lending is contingent upon the flexibility banks have in adjusting their balance sheets. When banks face limitations in calibrating the size of their liabilities, such as in the ZLB case, credit availability in the economy may decrease. Banks that are concerned with their leverage will respond by raising loan rates to discourage borrowers, thereby avoiding further balance sheet expansion beyond the regulatory limits. Conversely, when banks enjoy complete flexibility and can freely adjust their liabilities, leverage requirements do not significantly constrain lending.

3.5 Liquidity Shock and QE

In this section, assuming an economy close to the zero lower bound, I introduce a liquidity shock such as the one caused by the Covid-19 crisis on the U.S. Treasury market into the model and investigate what effects it has on banks' activities. As pointed out by Duffie (2020), the Covid-19 shock triggered heavy investor trade demands that overwhelmed the dealers' capacity to make market. One of the reasons this happened was the fact that capital requirements increase the banks' balance sheet costs associated with intermediation activities, limiting their ability to play the role of market makers. Since the bulk of Treasuries transactions is intermediated by bank-affiliated securities dealers, this restriction has important implications in terms of market stability and policy design which will be explored in this section.

3.5.1 Liquidity Shock and the Bond Market

Model Extension

At $t = 1$, the unexpected shock θ realises. Assume that the aggregate level of reserves r set by the central bank is kept constant and, at this point, the production decisions are final so they cannot be changed anymore. Liquid assets can still be traded so households and banks can engage in a secondary bond market if they wish to.

Households — After becoming aware of the value of θ at $t = 1$, households reoptimise their portfolios of bonds and deposits by choosing $b_{j,1}$ and $d_{j,1}$ to maximise consumption c_j, given their initial $t = 0$ choice of labour h_j, bonds $b_{j,0}$ and deposits $d_{j,0}$. For a given bond equilibrium price p in the secondary market, the household i who will be indifferent between holding bonds and deposits at $t = 1$ is such that: $\psi_i = \bar{\psi_1} \equiv \frac{p}{\theta} \cdot (1+i_D) - (1+i_B)$. Therefore, if $p > \theta$, households with $\psi_i \in \left[i_D - i_B, \bar{\psi_1}\right]$ will enter the market to sell bonds. Conversely, if $p < \theta$, those with $\psi_i \in \left[\bar{\psi_1}, i_D - i_B\right]$ will enter the market to buy bonds.

Banks — Banks' payoffs are not directly affected by the shock, but they may benefit from households' increased willingness to buy or sell bonds. At this point, they simply assess whether it's worth making market or not, subject to regulation constraints and given their choices at $t = 0$. If a bank decides to buy bonds, it must pay households by creating new deposits. On the other hand, when selling bonds, banks receive the payment in the form of households' deposits. Since deposits are banks' own liabilities, they are destroyed as soon as the ownership is transferred to banks. At $t = 1$, banks will solve:

$$\max_{\{b_{1,B}\}} \quad (1+i_B) \cdot b_{1,B} - (1+i_D) \cdot [p \cdot (b_{1,B} - b_{0,B}) + d_0]$$

$$\begin{aligned} \text{s.t.} \quad & r + b_{1,B} \geq \phi_{LCR} \cdot [(p \cdot (b_{1,B} - b_{0,B}) + d_0)] \\ & n_1 \geq \phi^L_{RWA} \cdot l \\ & n_1 \geq \phi_{SLR} \cdot (r + p \cdot b_{1,B} + l) \\ & r + p \cdot b_{1,B} + l = [p \cdot (b_{1,B} - b_{0,B}) + d_0] + n_0 \\ & 0 \leq b_{1,B} \leq b_{0,B} + b_{0,H}. \end{aligned} \tag{3.20}$$

Unconstrained Equilibrium

From the banks' problem, one can see that when $p \leq \frac{1+i_B}{1+i_D}$, banks will be willing to buy as many bonds as there are available. On the other hand, if $p \geq \frac{1+i_B}{1+i_D}$, banks will be willing to sell all of their bonds. Under these circumstances, banks can always make a profit out of trading and thus, as long as they are not constrained by regulation, they will always be willing to make market and take the opposite position of households.

Hence, when balance sheet constraints are not a concern for banks, the secondary bond market always clears and there is no need for any public policy intervention.

Constrained Equilibrium

What if banks must comply with balance sheet requirements and Basel III constraints are potentially binding? In this case, households' choices remain the same while banks may not be able to demand or supply as much bonds as they would like to in an unconstrained setting. Depending on the role banks intend to play in the secondary bond market and the amount to be traded, different regulatory constraints may bind and thus different market equilibria may arise after the liquidity shock.

(a) $\theta > 1$

In this case, households receive a higher payoff from bonds so some of them will be willing to use their deposits to buy banks' bonds. At the same time, as long as $p \geq \frac{1+i_B}{1+i_D}$, banks want to sell their bonds and ideally minimise $b_{1,B}$. Then, from the regulatory constraints and banks choices at $t = 0$, it follows that:

Lemma 3.3 (Constrained Eq. with $\theta > 1$)**.** *Following a positive liquidity shock, the SLR and RWA constraints will never bind. The LCR, however, may bind if there are not enough* *reserves:*

$$\phi_{LCR} \cdot [d_0 - p \cdot b_{0,B}] > r. \tag{3.21}$$

Selling bonds at the price $p > \frac{1+i_B}{1+i_D}$ increases the banks' expected net worth while reducing banks' leverage and keeping constant the amount of risky assets they hold. Hence, SLR and RWA would slacken here.

In contrast, LCR would tighten and potentially bind under these circumstances. The sale of bonds decreases banks' HQLA holdings so if reserves are not enough to fully account for liquidity requirements, banks will not be able to sell as much bonds as they would like to. In the end, there would be an excess demand for bonds in the market.

(b) $\theta < 1$

A low realisation of θ will reduce the households' expected payoff from holding bonds. As a consequence, some of them will be willing to sell the bonds they have in exchange for deposits. Meanwhile, as long as $p \leq \frac{1+i_B}{1+i_D}$ banks would like to buy all available bonds and ideally maximise $b_{1,B}$. In this context, it can be shown that:

Lemma 3.4 (Constrained Eq. with $\theta < 1$)**.** *Following a negative liquidity shock, the LCR and RWA constraints will never bind. The SLR, however, may bind if the bank does not have enough balance sheet space and if the spread earned buying bonds is lower than the increase in capital requirements induced by the trade:*

$$b_{1,B} > \left| \frac{\Xi_{SLR}}{\Gamma_{SLR}} \right| \tag{3.22}$$

and

$$(1 + i_B) - p \cdot (1 + i_D) < \phi_{SLR}, \tag{3.23}$$

where $\Xi_{SLR} \equiv (1+i_D)\cdot[d_0 - p\cdot b_{0,B}] - (1+i_L-\phi_{SLR})\cdot l - (1+i_R-\phi_{SLR})\cdot r$ *and* $\Gamma_{SLR} \equiv (1+i_B-\phi_{SLR}) - p\cdot(1+i_D)$

Buying bonds at the price $p \leq \frac{1+i_B}{1+i_D}$ increases the banks' expected net worth while increasing banks' HQLA holdings and keeping constant the amount of risky assets they hold. This leads to a slackening of the LCR and RWA constraints.

Besides increasing their expected equity value, this purchase also increases banks' leverage since it is financed by the creation of new deposits. Hence, there are two opposing forces slackening and tightening the SLR constraint at the same time. If the balance sheet cost of creating deposits is higher than the incremental net worth and if, in addition, there is not enough balance sheet space available to absorb this difference, then banks cannot buy all the bonds supplied by households. In this case, households will not be able to sell all of their bonds and consequently the bond market will not clear.

These findings highlight how bank regulatory constraints impact market-making ability by restricting banks' capacity to hold specific assets. Each constraint responds differently to varying circumstances. In terms of government bonds, the most relevant Basel III constraints are the SLR and the LCR, and they operate in opposing directions. Government bonds are counted positively as HQLAs, thus contributing to a more relaxed LCR. However, they are weighed negatively against leverage requirements, effectively tightening the SLR. Consequently, banks may face challenges in making markets: SLR concerns may hinder bond purchases, while LCR concerns may limit bond sales. Given the crucial role of banks as market makers, addressing these limitations may require policy adjustments or interventions to ensure the smooth functioning of markets.

3.5.2 QE as a Tool for Financial Stability

As previously mentioned, following the Covid-19 shock in March 2020, banks were not able to absorb the huge quantities of bonds put up for sale by investors. As described in the Fed's 2020 Financial Stability Report, while investors sold Treasuries, dealers took in large amounts of these securities onto their balance sheets and eventually reached their capacity to absorb these sales, leading to high levels of stress in the Treasuries market.

The deterioration seen in the Treasuries market in this episode is a reflection of the limits imposed by the leverage constraint on banks, as described in Lemma 3.4. As banks' balance sheet became inflated, the potential profits of intermediation were not enough to account for the massive capital commitments needed to comply with requirements such as the SLR and therefore banks stopped providing liquidity for investors who were trying to sell their bonds.

To prevent the market from collapsing, the Fed decided to intervene by promoting QE in which it purchased over $1 trillion of Treasuries, helping to restore market liquidity. The option of using QE to promote market stability, however, is not one without consequences for the banking system. When QE takes place, bonds bought by the central bank are taken out of the market and reserves are injected in the banking system as mean of payment for the asset purchases. This holds even if the seller is not a bank, due to the fact that non-bank agents cannot hold central bank reserves accounts and thus the transactions settlement has to go through banks.

This banks' unique feature of being able to hold reserves deposits in the central bank has an important ramification that is reported in Lemma 3.5.

Lemma 3.5 (QE and Balance Sheets)**.** *The impact of a QE on banks' balance sheets depends on whom the central bank buys from. When buying from non-banks, a QE increases the banking sector overall leverage. In contrast, this does not happen when it buys from banks.*

The intuition behind this lemma is the following: if the central bank buys bonds from a bank, there is a reduction in the amount of bonds and an increase in the amount of reserves in bank's possession. Nevertheless, there is no change in the size of the bank's balance sheet.

If the central bank buys bonds from a non-bank agent, it reduces the agent's holding of bonds and delivers reserves as payment for the transaction. However, non-bank agents cannot hold central bank reserves accounts which means that the payment has to go through the banking system. Thus, the central bank credits reserves for a bank in which the agent has account and then the bank credits a deposit in the corresponding amount in the name of the agent. In this case, even if the bank does not take part in the transaction, QE increases the amount of reserves and deposits in the bank's balance sheet.

Therefore, when the central bank buys bonds from non-banks, it automatically increases banks' leverage, while when it buys directly from banks, there is only an exchange of assets in banks' possession. This is relevant because, as we have seen, leverage constraints are one of the main limiting factors for banks' activity in intermediating the bond market. By using QE to provide liquidity to non-bank investors and solve the market instability created by the banking system lack of balance sheet space, a central bank may indeed worsen the banks' problem. This leads us to the main result exposed in Proposition 3.2:

Proposition 3.2 (Exchanging a Liquidity Crisis for a Credit Crisis)**.** *Central banks may solve a liquidity crisis by creating a credit contraction (and possibly a crisis) if they do QE to provide liquidity for non-bank investors in a market where the banks do not have enough free balance sheet space.*

As explained in Lemma 3.5, a QE targeting non-bank sellers increases the overall leverage of the banking sector due to the enlargement of banks' balance sheets. In this case, if the SLR is already a concern for banks in this economy, QE will tighten it even more and may lead to violations of the constraint. Moving forward, banks would have to deal with the higher levels of reserves $\hat{r} > r$ when complying with the SLR regulation which will leave less balance sheet space for alternative investments.

Equation (3.15) reveals that in a scenario where the SLR is binding, any exogenous increase in reserves must be accompanied by either an increase in equity or a reduction in bonds or loans. However, raising equity can be a lengthy process, and if there are no buyers for bonds in the private market, banks with limited balance sheet capacity and a need to deleverage have no alternative but to reduce the credit extended to the economy. This situation becomes critical, especially during a period when markets are still recovering from a liquidity shock. A significant reduction in aggregate bank lending could potentially trigger an even more severe crisis—this time, a credit crisis.

This was, in fact, a major concern for banks and the Fed after the massive purchases of Treasuries in March 2020. On April 1st, the Fed announced the temporarily easing of its leverage rules for large banks, exempting any holdings in reserves or Treasuries from the SLR calculations. This measure was aimed at giving banks' an extra cushion in terms of lending capacity to allow for the smooth functioning of markets during the pandemic recovery. Koont and Walz (2021) provide early evidence in favour of the relaxation of the SLR during this episode. In line with the predictions of my model and with the Fed expectations, omitting reserves and Treasuries in the SLR calculations increased bank repo intermediation and allowed for a strong expansion of traditional bank credit.

3.6 Conclusion

Modern banks are not only financial intermediaries, but also deposit (and money) creators. When banks lend, they do it by creating deposits which are cash equivalent liabilities issued by themselves. Taking this into account is fundamental to understand how banks' regulatory requirements limit their activities. In this paper, I show that low rates environments accentuate the power of leverage constraints in limiting bank lending. This happens because, at the zero lower bound, when banks lend they are not able to avoid holding the newly created deposits in their balance sheets by reducing i_D. Thus, they cannot adjust their balance sheet size and lending necessarily leads to increased leverage.

I also show that leverage concerns may lead banks to stop intermediating bond markets following a liquidity shock such as the one that hit the Treasuries market in the 2020 Covid-19 Crisis. In a situation like that, banks will not be able to meet investor trade demands if required capital commitments are high and there is not enough free balance sheet space. Importantly and in line with some of the Fed's concerns, the model also predicts that while solving a liquidity crisis, a QE may lead to a contraction in aggregate bank lending and, as a consequence, to a credit crisis.

Results here presented contribute to the relevant discussion on how to think about the business of banking, especially in the post-Basel III world where regulations became more strict and complex. Going forward, it remains important to better understand bank lending funding and balance sheet management, as well as the way banks adapt to low-rates environments and unconventional monetary policy.

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C Appendix - Chapter 3

C.1 Labour Market Equilibrium

When labour market is in equilibrium, it must be the case that the amount of labour supplied by households equals the amount demanded by firms:

$$\begin{aligned} h_H &= \int_{-\kappa}^{i_D - i_B} \left[(1+i_D)\cdot w\right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i + \int_{i_D-i_B}^{\kappa} \left[(1+i_B+\psi_i)\cdot w\right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i \\ &= w^{\frac{1-\sigma}{\eta+\sigma}} \cdot \left[\underbrace{(1+i_D)^{\frac{1-\sigma}{\eta+\sigma}} \cdot (i_D - i_B + \kappa) + \frac{\eta+\sigma}{1+\eta} \cdot \left((1+i_B+\kappa)^{\frac{1+\eta}{\eta+\sigma}} - (1+i_D)^{\frac{1+\eta}{\eta+\sigma}} \right)}_{g(i_D)} \right] \\ &= h_F = \left(\frac{\alpha A}{(1+i_L)\cdot w} \right). \end{aligned} \tag{C.1}$$

Rearranging the terms above gives us:

$$w = \left[\left(\frac{\alpha A}{(1+i_L)} \right)^{\frac{\eta+\sigma}{\alpha-1}} \cdot g\left(i_D\right) \right]^{\frac{\alpha-1}{1+\eta-\alpha(1-\sigma)}}. \tag{C.2}$$

Additionally, from (C.1), one can see that:

$$\frac{dg}{di_D} = \frac{1-\sigma}{\eta+\sigma} \cdot (1+i_D)^{\frac{1-\eta-2\sigma}{\eta+\sigma}} \cdot (i_D - i_B + \kappa) > 0. \tag{C.3}$$

□

C.2 Households' Total Demand for Bonds and Deposits

All investors below the cutoff $\bar{\psi}_0 = i_D - i_B$ will invest their income in deposits while those above $\bar{\psi}_0$ will invest in bonds. Therefore, the households' total demand for bonds will be:

$$b_{0,H} = w \cdot \int_{i_D - i_B}^{\kappa} \left[(1 + i_B + \psi_i) \cdot w \right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i$$
$$= w^{\frac{1+\eta}{\eta+\sigma}} \cdot \frac{\eta+\sigma}{1+\eta} \cdot \left[(1 + i_B + \kappa)^{\frac{1+\eta}{\eta+\sigma}} - (1 + i_D)^{\frac{1+\eta}{\eta+\sigma}} \right] = B\left(i_D, i_L\right). \tag{C.4}$$

From (C.2) and (C.3), it is immediate that $\frac{dw}{di_D} < 0$ and $\frac{dw}{di_L} < 0$. Hence, it must be the case that $\frac{dB}{di_D} < 0$ and $\frac{dB}{di_L} < 0$.

Similarly, the households' total demand for deposits will be:

$$d_0 = w \cdot \int_{-\kappa}^{i_D - i_B} \left[(1 + i_D) \cdot w \right]^{\frac{1-\sigma}{\eta+\sigma}} d\psi_i$$
$$= w^{\frac{1+\eta}{\eta+\sigma}} \cdot (1 + i_D)^{\frac{1-\sigma}{\eta+\sigma}} \cdot (i_D - i_B + \kappa) = D\left(i_D, i_L\right). \tag{C.5}$$

Again, since $\frac{dw}{di_L} < 0$, it is immediate that $\frac{dD}{di_L} < 0$.

At the same time, by the aggregate resources constraint, $D\left(i_D, i_L\right) + B\left(i_D, i_L\right) = L\left(i_D, i_L\right)$. Taking derivatives with respect to i_D on both sides of this equation gives us that $\frac{dD}{di_D} > 0$.

□

C.3 Rate Setting Rules for Loans and Deposits

Banks solve the maximisation problem given by (3.15). The Lagrangian of this problem is given by:

$$\begin{aligned} \mathcal{L} = n &- \lambda_{LCR} \cdot \left[\phi_{LCR} \cdot D(i_D, i_L) - r - b_{0,B} \right] - \lambda_{RWA} \cdot \left[\phi_{RWA}^{L} \cdot L(i_D, i_L) - n \right] \\ &- \lambda_{SLR} \cdot \left[\phi_{SLR} \cdot (r + b_{0,B} + L(i_D, i_L)) - n \right] - \mu_{BSI} \cdot \left[r + b_{0,B} + L(i_D, i_L) - D(i_D, i_L) - n_0 \right] \\ &+ \zeta_{ZLB} \cdot i_D, \end{aligned} \tag{C.6}$$

where $n = (1 + i_L) \cdot L(i_D, i_L) + (1 + i_R) \cdot r + (1 + i_B) \cdot b_{0,B} - (1 + i_D) \cdot D(i_D, i_L)$. In addition, λ_i, μ_{BSI} and ζ_{ZLB} are the Lagrange multiplier of regulatory constraint i, of the balance sheet identity and of the zero lower bound condition, respectively.

Defining $\frac{\partial L / \partial i_L}{L} = -\epsilon_L$ and $\frac{\partial D / \partial i_D}{D} = \epsilon_D$, one can obtain the following first-order conditions for i_L, i_D and $b_{0,B}$ respectively:

$$(1+i_L) = \delta_L \cdot (1+i_D) + \frac{1}{\epsilon_L} + \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot$$
$$\cdot \left[\lambda_{LCR} \cdot \phi_{LCR} \cdot \delta_L + \lambda_{RWA} \cdot \phi^L_{RWA} + \lambda_{SLR} \cdot \phi_{SLR} + \mu_{BSI} \cdot (1-\delta_L)\right], \tag{C.7}$$

$$(1+i_D) = \delta_D \cdot (1+i_L) - \frac{1}{\epsilon_D} + \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot$$
$$\cdot \left[-\lambda_{LCR} \cdot \phi_{LCR} - \lambda_{RWA} \cdot \phi^L_{RWA} \cdot \delta_D - \lambda_{SLR} \cdot \phi_{SLR} \cdot \delta_D + \mu_{BSI} \cdot (1-\delta_D)\right], \tag{C.8}$$

and

$$\mu_{BSI} = \lambda_{LCR} - \lambda_{SLR} \cdot \phi_{SLR} + [1+\lambda_{RWA}+\lambda_{SLR}] \cdot (1+i_B), \tag{C.9}$$

where $\frac{\partial D/\partial i_L}{\partial L/\partial i_L} \equiv \delta_L$ and $\frac{\partial L/\partial i_D}{\partial D/\partial i_D} \equiv \delta_D$.

Then, using (C.9) in (C.7) and (C.8), respectively gives us the rate-setting rule for loans in (3.17) and the following rule for deposits when $\zeta_{ZLB} = 0$:

$$i_D = \delta_D \cdot i_L + (1-\delta_D) \cdot i_B - \frac{1}{\epsilon_D} +$$
$$+ \frac{1}{1+\lambda_{RWA}+\lambda_{SLR}} \cdot \left[\lambda_{LCR} \cdot (1-\phi_{LCR}-\delta_D) - \lambda_{SLR} \cdot \phi_{SLR} - \lambda_{RWA} \cdot \phi^L_{RWA} \cdot \delta_D\right]. \tag{C.10}$$

□

C.4 Lemmas 3.1 and 3.2

Away from the ZLB, we know that $\zeta_{ZLB} = 0$ and i_D follows the rule set in (C.10). Then, we can substitute (C.10) in the loan rate equation (3.17) to get (3.18), which defines Lemma 3.1.

When at the ZLB, we know that $\zeta_{ZLB} > 0$ and $i_D = 0$. Then, we can substitute $i_D = 0$ in the loan rate equation (3.17) to get (3.19) which defines Lemma 3.2.

□

C.5 Proposition 3.1

As one can see in (3.18) and (C.10), when the economy is away from the lower bound:

$$\frac{\partial i_L}{\partial \lambda_{SLR}} = 0 \tag{C.11}$$

and

$$\frac{\partial i_D}{\partial \lambda_{SLR}} = \phi_{SLR} \cdot \frac{1}{1 + \lambda_{RWA} + \lambda_{SLR}} > 0. \tag{C.12}$$

Then, in this setting, all the adjustments in the balance sheet due to leverage concerns are made through i_D. This happens because, independent of how much the bank decides to lend, it can always reduce i_D enough to keep households away from deposits and thus prevent its balance sheet from increasing.

On the other hand, when the economy is at the lower bound, $i_D = 0$ and (3.19) tells us that:

$$\frac{\partial i_L}{\partial \lambda_{SLR}} = \phi_{SLR} \cdot \delta_L \cdot \frac{1}{1 + \lambda_{RWA} + \lambda_{SLR}} > 0. \tag{C.13}$$

Hence, at the lower bound, any loan necessarily increases banks liabilities (and balance sheet) by a share δ_L of the total amount lent since banks cannot cut i_D to keep households away from deposits. Thus, the only way banks have to delever is by increasing i_L and, as a consequence, reducing lending.

□

C.6 Lemmas 3.3 and 3.4

At $t = 1$, the bank will solve the problem specified in equation (3.20). First, note that given choices at $t = 0$, the LCR, RWA and SLR constraint gives us the following condition for $b_{1,B}$, respectively:

$$b_{1,B} \geq \frac{1}{1 - p \cdot \phi_{LCR}} \cdot [\phi_{LCR} \cdot (d_0 - p \cdot b_{0,B}) - r] \equiv \kappa_{LCR}, \tag{C.14}$$

$$b_{1,B} \cdot \overbrace{[(1+i_B) - p \cdot (1+i_D)]}^{\Gamma_{RWA}} \geq (1+i_D) \cdot (d_0 - p \cdot b_{0,B}) - \\ \underbrace{-(1+i_R) \cdot r - (1+i_L - \phi_{RWA}^L) \cdot l}_{\Xi_{RWA}}, \tag{C.15}$$

and

$$b_{1,B} \cdot \overbrace{[(1+i_B - \phi_{SLR}) - p \cdot (1+i_D)]}^{\Gamma_{SLR}} \geq (1+i_D) \cdot (d_0 - p \cdot b_{0,B}) - \\ \underbrace{-(1+i_R - \phi_{SLR}) \cdot r - (1+i_L - \phi_{SLR}) \cdot l}_{\Xi_{SLR}}. \tag{C.16}$$

Lemma 3.3 - $\theta \geq 1$

When $\theta \geq 1$, and as long as $p \geq \frac{1+i_B}{1+i_D}$, banks will act as market makers and sell as many bonds as possible. In this case, each Basel III constraint will move as follows:

1. LCR

   To comply with the LCR, banks must set $b_{1,B} = \max\left\{0, \kappa_{LCR}\right\}$. Then, one can see that

$$\kappa_{LCR} \leq 0 \Leftrightarrow \phi_{LCR} \cdot (d_0 - p \cdot b_{0,B}) \leq r. \tag{C.17}$$

   If (C.17) holds, $b_{1,B} = 0$ and banks are able to sell as much bonds they want. Otherwise, they must keep at least $\kappa_{LCR} > 0$ bonds to ensure their HQLA holdings are enough to comply with the LCR.

$\square$

2. RWA

   First, note that, because the bank must comply with the RWA regulation at $t = 0$, we know that:

$$(1 + i_R) \cdot r + (1 + i_B) \cdot b_{0,B} + (1 + i_L) \cdot l - (1 + i_D) \cdot d_0 \geq \phi^L_{RWA} \cdot l. \tag{C.18}$$

Thus, when $p \geq \frac{1+i_B}{1+i_D}$, it must be the case that:

$$(1 + i_R) \cdot r + p \cdot (1 + i_D) \cdot b_{0,B} + (1 + i_L - \phi^L_{RWA}) \cdot l - (1 + i_D) \cdot d_0 \geq 0. \tag{C.19}$$

Hence, $\Xi_{RWA} \leq 0$.

Then, one can see that RWA will not constrain the banks in this situation. Since $\Gamma_{RWA} \leq 0$, then $0 \geq b_{1,B} \leq \frac{\Xi_{RWA}}{\Gamma_{RWA}}$, which means that RWA bounds $b_{1,B}$ from above, which will not restrict the banks in this case since they are trying to sell their holdings instead of buying more.

□

3. SLR

This follows the same logic as in the RWA proof above. SLR regulation at $t = 0$ and $p \geq \frac{1+i_B}{1+i_D}$ imply that:

$$(1+i_R-\phi_{SLR}) \cdot r+p \cdot (1+i_D) \cdot b_{0,B}+(1+i_L-\phi_{SLR}) \cdot l-(1+i_D) \cdot d_0 \geq \phi_{SLR} \cdot b_{0,B} \geq 0. \tag{C.20}$$

Hence, $\Xi_{SLR} \leq 0$.

Then, one can see that SLR will not constrain the banks in this situation. Since $\Gamma_{SLR} < 0$, then $0 \geq b_{1,B} < \frac{\Xi_{SLR}}{\Gamma_{SLR}}$, which means that SLR bounds $b_{1,B}$ from above, which will not restrict the banks in this case since they are trying to sell their holdings instead of buying more.

□

Lemma 3.4 - $\theta < 1$

If $\theta < 1$, then as long as $p \leq \frac{1+i_B}{1+i_D}$, banks would like to buy as much bonds as they can. Then, each Basel III constraint will move as follows:

1. LCR

Note that one can rewrite (C.14) as:

$$\Delta b_{1,B} \geq \frac{1}{1 - p \cdot \phi_{LCR}} \cdot [\phi_{LCR} \cdot d_0 - r - b_{0,B}] \,. \tag{C.21}$$

Because banks satisfied the LCR regulation at $t = 0$, we know that the right-hand side of (C.21) is negative. Because banks are looking to set $\Delta b_{1,B} \geq 0$ and adding HQLA assets to their portfolios can only improve their situations, one can readily observe that LCR will not bind in this case.

□

2. RWA

Note that when $p \leq \frac{1+i_B}{1+i_D}$, $\Gamma_{RWA} \geq 0$. Additionally, one can rewrite (C.15) as:

$$\begin{aligned} \Delta b_{1,B} \cdot \Gamma_{RWA} \geq {} & (1 + i_D) \cdot d_0 - (1 + i_B) \cdot b_{0,B} \\ & - (1 + i_R) \cdot r - (1 + i_L - \phi^L_{RWA}) \cdot l. \end{aligned} \tag{C.22}$$

Because banks satisfied the RWA constraint at $t = 0$, the right-hand side of (C.22) must be negative. This implies that the RWA constraint bounds $\Delta b_{1,B}$ from below at a non negative number. Since banks are trying to buy bonds, we know that $\Delta b_{1,B} \geq 0$ and therefore RWA cannot bind.

□

3. SLR

In a similar way, one can rewrite (C.16) as:

$$\begin{aligned} \Delta b_{1,B} \cdot \Gamma_{SLR} \geq {} & (1 + i_D) \cdot d_0 - (1 + i_B - \phi_{SLR}) \cdot b_{0,B} \\ & - (1 + i_R - \phi_{SLR}) \cdot r - (1 + i_L - \phi_{SLR}) \cdot l. \end{aligned} \tag{C.23}$$

Again, because banks satisfied the SLR constraint at $t = 0$, the right-hand side of (C.23) must be negative. Then,

(a) If $\Gamma_{SLR} = (1 + i_B - \phi_{SLR}) - p \cdot (1 + i_D) \geq 0$, SLR bounds $b_{1,B}$ from below at a negative number. Since banks are trying to buy bonds and thus set $\Delta_{1,B} \geq 0$, RWA will not bind under this circumstances.

(b) If $\Gamma_{SLR} = (1 + i_B - \phi_{SLR}) - p \cdot (1 + i_D) < 0$, SLR bounds $\Delta b_{1,B}$ from above at a positive number.

This means that banks will set $b_{1,B} = \min \left\{ b_{0,B} + b_{0,H}, \left| \frac{\Xi_{SLR}}{SLR} \right| \right\}$ and the SLR will bind if $\left| \frac{\Xi_{SLR}}{\Gamma_{SLR}} \right| < b_{0,B} + b_{0,H}$.

$\square$

C.7 Lemma 3.5

1. If the central bank buys bonds from a bank:

   Central Bank Assets: Bonds $\Uparrow$

   Central Bank Liabilities: Reserves $\Uparrow$

   Total Central Bank Balance Sheet Size $\Uparrow$

   Bank Assets: Bonds $\Downarrow$, Reserves $\Uparrow$

   Bank Liabilities: No change

   Total Bank Balance Sheet Size: No change

2. If the central bank buys bonds from a non-bank agent:

   Central Bank Assets: Bonds $\Uparrow$

   Central Bank Liabilities: Reserves $\Uparrow$

   Total Central Bank Balance Sheet Size $\Uparrow$

   Non-Bank Agent Assets: Bonds $\Downarrow$, Bank Deposits $\Uparrow$

   Non-Bank Agent Liabilities: No change

   Total Non-Bank Agent Balance Sheet Size: No change

Bank Assets: Reserves $\Uparrow$

Bank Liabilities: Non-Loan Related Deposits $\Uparrow$

Total Bank Balance Sheet Size $\Uparrow$

Hence, a QE buying from non-banks increases banks' leverage while a QE buying from banks does not. Therefore, from whom the central bank buys in a QE matters for the overall leverage of the banking system.

From Lemma (3.2), we know that leverage is a relevant factor in setting loan interest rate and the amount of credit extended by banks. As a consequence, from whom the central bank buys in a QE matters for bank lending.

$\square$

C.8 Proposition 3.2

Following a negative liquidity shock in the bond market, if the central bank decides to provide liquidity to the market by promoting a QE targeting non-banks sellers, it will increase the overall leverage of the banking sector as shown in Lemma 3.5.

Immediately after the QE:

$$\text{QE} \quad \Rightarrow \quad r\uparrow, d\uparrow \;\; \Rightarrow \quad \text{SLR tightens}$$

If SLR was already binding as suggests the fact that banks stopped their dealers activities, then banks' leverage shadow cost λ_{SLR} will increase after the QE. Hence, by equation (C.13) in the proof of Proposition 3.1, banks will increase the loan interest rate in the next time they are able to set it. As a consequence, there will be a reduction in bank lending, which could lead to a credit crisis if this effect is strong enough.

$$\lambda_{SLR}\uparrow \;\; \Rightarrow \quad i_L\uparrow \;\; \Rightarrow \quad L\downarrow \;\; \Rightarrow \quad \text{Credit Crisis}$$

$\square$