The London School of Economics and Political Science

Essays
on the Macroeconomic Impact of Trade
and
Monetary Policy

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others.

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of 29 700 words.

Statement of conjoint work

I confirm that Chapter 2 was jointly co-authored with Professor Francesco Caselli, Miklos Koren and Silvana Tenreyro and I contributed 25% of this work.

Milan Lisicky

Abstract

My thesis consists of three chapters. The aim of the first two chapters is to investigate the linkages between trade and the cross-country comovement and volatility of GDP growth, while the last chapter is an independent study on how the optimal design of monetary policy depends on the share of labour- and capital-intensive sectors.

The first chapter develops a framework to study the effects of international trade on GDP comovement. Using a standard trade-theoretical approach, I first show how the comovement between any pair of countries is linked to shocks affecting both the two countries bilaterally and all other countries. Secondly, I use a calibrated version of the model to assess the importance of the bilateral channel relative to the role of linkages with all other countries.

The second chapter investigates whether and how openness to trade may affect macroeconomic volatility. While greater openness provides a powerful channel for transmission of foreign disturbances, it also lowers the exposure to domestic shocks. My co-authors and I show that as long as the volatility of trading partners and covariance of shocks across countries are not too large, trade can act as a channel for the diversification of country-specific shocks and in that way contribute to lower volatility.

The third chapter examines what is the optimal measure of inflation in a two-sector economy with nominal frictions, where sectors differ in labour intensity. I find that a welfare-oriented central bank should follow more closely the developments in the less labour-intensive sector. The source of this bias is traced back to a greater sensitivity of the marginal product of labour in that sector, so that output dispersion caused by nominal rigidities generates higher efficiency losses where labour is relatively less abundant.

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Trade and Comovement

1.1 Introduction

Does trade enhance comovement? This question naturally arises when looking back at the long synchronised upswings and several downswings posted by most developed economies since the beginning of 1990s. For almost simultaneously, the world has been experiencing an unprecedented expansion of global trade, on the back of the increased integration of China, South-East Asia and, somewhat later, Central and Eastern Europe in global production chains. While the jury is still out on whether and in which sense exactly have business cycles become more correlated[1], it is a well established fact that global merchandise trade grew almost three times as fast as global GDP from the 1960's and even faster in the past twenty or thirty years, until the current crisis.

We look at the relationship between across-the-border trade in goods and comovement in real GDP growth through the lens of a standard general equilibrium framework that conceptualises the demand and supply linkages among countries. The model is a variant of Eaton and Kortum (2002) and Alvarez

[1] See, for instance, Cerqueira and Martins (2009) or Bower and Guillemineau (2006) for an affirmative answer, or Heathcote and Perri (2004) and Doyle and Faust (2002) for a more cautious view. Kose, Otrok and Prasad (2012) coin the phrase 'decoupling' for the observed decrease in business cycle correlations between the blocks of industrial and emerging countries but convergence within each block.

and Lucas (2007) with stochastic country-specific shocks and pair-specific trade shocks. With this model, we set out to investigate whether greater openness to trade affects cross-country correlations and, if so, to examine what determines whether two countries' growth rates will be positively correlated.

The relationship between trade and comovement has been the topic of several theoretical and a long list of empirical studies. To exemplify the former stream of literature, Kose and Yi (2006) study the implications of trade on comovement within a canonical international business cycle model. They conclude, on the basis of empirical work of their own and others, that a standard version of the model fails to replicate the patterns they see in data, for which they coin the term trade-comovement puzzle. In an attempt to resolve the puzzle, Arkolakis and Ramanarayanan (2009) use a framework similar to ours but augmented for vertical specialisation to show that experiments with trade intensity can generate moderate increases in cross-country comovement.

Clearly, multi-country macroeconomic models have predictions about the relationship between trade and comovement so it would be desirable to have a corresponding 'stylized fact' to provide a testing ground for competing theories. A large body of empirical work has already been devoted to the study of effects of trade on comovement, and this is the main stream of literature to which our results speak to. The seminal paper in this field is Frankel and Rose (1998), who estimate regressions of the following form

$$Corr(\hat{y}_{i\tau}, \hat{y}_{j\tau}) = \alpha + \beta\, Trade_{ij\tau} + \epsilon_{ij\tau}$$

where $\hat{y}$ denotes a measure of fluctuations in economic activity in period τ and $Trade_{ij\tau}$ is a measure of trade intensity between countries i and j in the same period. Their estimation uses instrumental variables to control for a bias that arises, as they argue, because countries are likely to stabilise their exchange rates with respect to their main trading partners. The instruments they use are exogenous *bilateral* characteristics that capture the distance of the two countries, existence of a common border etc. Other studies using a similar estimation framework include Baxter and Kouparitsas (2005), Calderon, Chong and Stein

(2007) and others.[2] Our reading of the literature is that it typically seeks for measures and/or estimation techniques that would capture omitted variables – *bilaterally* relevant for i and j – that they consider important in driving the bias in the equation above.

To preview our results, we derive closed-form formulas for bilateral correlations consistent with our model and find that direct trade between i and j does expose country i to shocks that affect its trade partner j, buttressing thus their comovement. So far, this is consistent with the estimation strategies above. However, we also find that the direct trade empirically accounts for less than 10% of the observed comovement. What matters more, according to our model, is that country i trades with one or more other countries k. Then, if k tends to receive similar shocks as j, countries i and j are likely to post high bilateral correlations – independently of whether they trade with each other or not. This 'indirect-trade' effect, as we call it, accounts for the bulk of observed correlations, because the relative strength of the 'direct' trade exposure to j is typically substantially smaller than the exposure to all other trade partners. To relate this finding to the above empirical works, our results support the search for identification strategies that control for the exposure to coincident shocks, for instance through the similarity of the industrial structure. However, our results also suggest that the search should focus on the similarity between i's *other trade partners* and j, and vice versa, between j's *other trade partners* and i, rather than on the similarity between i and j bilaterally.

[2]Calderon, Chong and Stein (2007) and also Gruben, Koo, Millis (2002) base their regressions on intra-industry trade to capture the role of industry-specific shocks. Also Fidrmuc (2001) provides evidence that using the total volume of trade is misguided and confine the role of trade in transmitting shocks to intra-industry trade only. Other relevant works include Kose, Prasad and Terrones (2003), Clark and van Wincoop (2001), Otto, Voss and Willard (2001), Anderson, Kwark and Vahid (1999).

1.2 Model

The model we use here, as well as in Chapter 2, is a version of Eaton-Kortum (2002) and Alvarez-Lucas (2007)'s model with stochastic parameters. Adhering, where possible, to their original notation, the model can be described as follows.

Let us assume that there is a continuum of goods $\omega \in [0,1]$ that can be produced in all n countries in the model at costs $(x_1, x_2, \ldots, x_n)$. Given this mapping between goods ω and cost combinations $x = (x_1, x_2, \ldots, x_n)$, the model can, without loss of generality, be cast in terms of the cost combinations x. Let $\varphi_i(x_i)$ be the density in country i of cost draw x_i (where i here denotes the i^{th} element of x). Assuming it is exponentially distributed with parameters λ_i, the density of x is

$$\varphi(x) = \prod_{i=1}^{n} \varphi_i(x_i) = \prod_{i=1}^{n} \lambda_i e^{-\lambda_i x_i}$$

Parameter λ_i will play a crucial role in our model; let us therefore state here that higher λ_i increases the density of lower cost draws, which amounts to a positive productivity shock.

Let $q_{f,i}(x)$ denote the per-capita quantity of individual goods x that are bought by consumers for final consumption, and let $q_{f,i}$ denote the bundle of all goods that enters utility

$$q_{f,i} = \left(\int_0^\infty q_{f,i}(x)^{\frac{\eta-1}{\eta}} \varphi(x) dx \right)^{\frac{\eta}{\eta-1}}$$

Assuming utility is linear in $q_{f,i}$ and taking domestic prices of good x, $p_i(x)$, as given (there is perfect competition in all markets), cost minimisation implies the following demand functions and price index:

$$q_{f,i}(x) = \left(\frac{p_i}{p_i(x)} \right)^{\eta} q_{f,i}$$

$$p_i = \left(\int_0^\infty p_i(x)^{1-\eta} \varphi(x) dx \right)^{\frac{1}{1-\eta}} \tag{1.1}$$

Let individual purchases of goods $q_i(x_i)$ be produced in i using equipped labour, of which there is L_i in total, and all other goods bundled together in the same CES aggregator as given above. We assume that any individual good can be either directly consumed or used as an intermediate input.

The production function is Cobb-Douglas with β, the share of the unproduced input, being common across countries.

$$q_i\left(x_i\right) = {x_i}^{-\theta} L_i(x_i)^{\beta}\, q_{m,i}\left(x_i\right)^{1-\beta}$$

where $q_{m,i}\left(x_i\right)$ denotes the quantity of the bundle of intermediate goods that it takes to produce a good with cost level x_i. Notice that cost shocks enter the production function as a productivity parameter ${x_i}^{-\theta}$ and recall that shocks are exponentially distributed. As explained in Eaton and Kortum (2002), the transformation ${x_i}^{-\theta}$ makes productivity shocks distributed according to the Frechet distribution, where θ governs the variance of shocks. With higher θ, productivity shocks become more volatile.

Perfect competition and constant returns imply that the price of each individual intermediate good *potentially* produced in the domestic economy at productivity ${x_i}^{-\theta}$ would be

$$p_i(x_i) = B x_i^{\theta} w_i^{\beta} p_i^{1-\beta}$$

where constant $B = \beta^{\beta}(1-\beta)^{1-\beta}$ and w_i is the pay of the unproduced input. When producers have the choice of importing intermediates from abroad, each good $x = (x_1, x_2, \ldots, x_n)$ will be priced at the level of the cheapest supplier

$$p_i\left(x\right) = B \min_j \left(\frac{{x_j}^{\theta} {w_j}^{\beta} {p_j}^{1-\beta}}{\kappa_{ij}}\right)$$

where $1/\kappa$ represents physical trade costs. For one unit to arrive from j to i, $1/\kappa_{ij}$ must be shipped and paid for. We assume $0 < \kappa \leq 1$ and that it is always cheaper to transport goods directly than through a third country ($\kappa_{ij} \geq \kappa_{ik}\kappa_{kj}$). Domestic trade is costless, $\kappa_{ii} = 1$ and we further assume that there are no money tariffs.

Given that domestic prices of goods x are a function of stochastic vector x, it is possible to work out the price index p_i as given by (1.1). Using the properties of the Frechet distribution (following Eaton and Kortum, 2002), we have

$$p_i = A\, B \left(\sum_{j=1}^{n} \left(\frac{{w_j}^{\beta} {p_j}^{1-\beta}}{\kappa_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j\right)^{-\theta} \tag{1.2}$$

which is a set of non-linear equations solvable for p as a function of vector w. Constant A is defined as $A = \Gamma\left(1+\theta\left(1-\eta\right)\right)^{\frac{1}{1-\eta}}$, which implies a parameter restriction $\theta \leq 1/(\eta-1)$, assuming $\eta > 1$).

With a continuum of goods x, the fraction of goods that will be purchased in i from country j is given by the probability that, for each good x country j is the minimum-price supplier in country i. Denoting this fraction d_{ij}, we have

$$d_{ij} = Prob\left(\frac{B{x_j}^{\theta}{w_j}^{\beta}{p_j}^{1-\beta}}{\kappa_{ij}} \leq \min_k\left(\frac{B{x_k}^{\theta}{w_k}^{\beta}{p_k}^{1-\beta}}{\kappa_{ik}}\right)\right)$$

for each $k \neq j$. Exploiting the properties of Frechet distribution further, one can show that this is

$$d_{ij} = (AB)^{-\frac{1}{\theta}} \frac{\left(w_j^{\beta} p_j^{1-\beta} \big/ \kappa_{ij}\right)^{-\frac{1}{\theta}} \lambda_j}{\sum_{k=1}^{n} \left(w_k^{\beta} p_k^{1-\beta} \big/ \kappa_{ik}\right)^{-\frac{1}{\theta}} \lambda_k}$$

Notice that with prices being functions of the wage of the unproduced input, d_{ij} becomes also a function of wages.

Across-the-border linkages are constrained in the model by imposing that spending on goods (which are all potentially tradable) in country i, $L_i p_i q_i$, is equal to total spending of all countries (including i itself) on goods produced in i. Each country spends fraction d_{ji} of $L_j p_j q_j$ on goods produced in i. With eventual imbalances captured by S_i, we have

$$L_i p_i q_i + S_i = \sum_{j=1}^{n} L_j p_j q_j d_{ji} \tag{1.3}$$

With exogenous L_i and S_i, and prices and trade shares determined by wages, as shown above, this equation implies that per-capita spending q_i can also be determined as a function of wages.

Domestically, the model is closed by imposing, first, the national accounting identity that states that total income of labour must be equal to total final expenditure: $L_i w_i = L_i p_i q_{f,i} + S_i$, which determines total spending of households on final goods $L_i p_i q_{f,i}$. Furthermore, the Cobb-Douglas production function stated above implies that the fraction of each good used as an intermediate input is $q_{m,i}(x) = (1-\beta) q_i(x)$. Multiplying both sides by $L_i p_i(x)$ and integrating over

x, the right-hand side gives total spending of firms on intermediate goods, which can be also written as a fraction of total spending in the economy (incl. from abroad): $(1-\beta)(L_i p_i q_i + S_i)$. Adding the expenditure on final and intermediate goods and making them equal to total expenditure on goods $L_i p_i q_i$, gives the relationship

$$L_i w_i = \beta(L_i p_i q_i + S_i) \tag{1.4}$$

which completes the description of the model and can be used to determine the remaining endogenous variable in the model (wages).

For ease of notation, let us make the following substitutions. First, let us denote Z_i the following transformation of the aggregate productivity parameter λ_i:

$$Z_i \equiv L_i^{\frac{\beta}{\theta}} \lambda_i$$

and denote $Lw_i = L_i w_i$. What we gain by these transformations is that we avoid the need to work with parameter L_i, that does not have a self-evident counterpart in data (recall that L_i is the unproduced input in the economy – equipped labour – rather than a simple head count). Lw_i, instead, is total income of the factor, which can be easily mapped to national accounts.

Using this notation, we can now collect the key equations. Substituting from (1.4) for $L_i p_i q_i$ and ignoring trade imbalances, as we do in all subsequent derivations, the trade identity (1.3) is:

$$Lw_i = \sum_{j=1}^{n} Lw_j d_{ji} \tag{1.5}$$

which jointly with the definition of d_{ij} with our transformed variables

$$d_{ij} = (AB)^{-\frac{1}{\theta}} \frac{\left(Lw_j^{\beta} p_j^{1-\beta} \Big/ \kappa_{ij}\right)^{-\frac{1}{\theta}} Z_j}{\sum_{k=1}^{n} \left(Lw_k^{\beta} p_k^{1-\beta} \Big/ \kappa_{ik}\right)^{-\frac{1}{\theta}} Z_k} \tag{1.6}$$

gives a set of equations in prices and aggregate nominal income only. Rewriting the price equation (1.2) with the new notation, it also becomes a function of prices and aggregate income. Jointly, these two form a set of $2n$ equations in $2n$ variables: aggregate nominal income Lw_i and its price level p_i. As shown in

Appendix 1.A.1, trade identities (1.3) are not linearly independent, so we will be able to solve only for relative prices subject to a numeraire. The key relationships, which are used to solved the model, for both algebraic and numerical purposes, then are

$$Lw_i = \sum_{j=1}^{n} Lw_j \frac{\left(Lw_i^{\beta} p_i^{1-\beta} \big/ \kappa_{ji}\right)^{-\frac{1}{\theta}} Z_i}{\sum_{k=1}^{n} \left(Lw_k^{\beta} p_k^{1-\beta} \big/ \kappa_{jk}\right)^{-\frac{1}{\theta}} Z_k}$$
$$p_i = AB \left(\sum_{k=1}^{n} \left(Lw_k^{\beta} p_k^{1-\beta} \big/ \kappa_{ik}\right)^{-\frac{1}{\theta}} Z_k \right)^{-\theta}$$

1.2.1 Closed-form solution

The model has a closed-form solution only in the special cases of autarky and zero trade costs. While these are useful to understand the basic relationship between trade and comovement, they are not helpful in terms of pointing at the key channels through which trade impacts comovement in the data. As we show in section 1.4.1, trade costs are far away from the free-trade assumption among most countries. We therefore proceed to log-linearise the model and describe its solution in terms of log-deviations of shocks around their steady-state values. By doing so, we are able to derive an insightful closed-form solution for covariances and eventually also for correlations between GDP growth rates.

We seek a solution in terms of real GDP $Y_i = Lw_i/p_i$, let us therefore transform the last two equations from the previous section once again, to obtain relationships in terms of Y_i and p_i:

$$Y_i \, p_i = \sum_{j=1}^{n} Y_j \, p_j \frac{\left(Y_i^{\beta} p_i \big/ \kappa_{ji}\right)^{-\frac{1}{\theta}} Z_i}{\sum_{k=1}^{n} \left(Y_k^{\beta} p_k \big/ \kappa_{jk}\right)^{-\frac{1}{\theta}} Z_k} \tag{1.7}$$

$$p_i = AB \left(\sum_{k=1}^{n} \left(Y_k^{\beta} p_k \big/ \kappa_{ik}\right)^{-\frac{1}{\theta}} Z_k \right)^{-\theta} \tag{1.8}$$

Log-linearising both equations and using steady-state trade shares $\overline{d}_{ij}$ as defined in (1.6), we obtain the following relationships[3]:

$$\overline{Y}_i\,\overline{p}_i\,(\hat{y}_i+\hat{p}_i) = \sum_{j=1}^{n}\overline{Y}_j\,\overline{p}_j\,\overline{d}_{ji}$$
$$\left(\hat{y}_j+\hat{p}_j-\frac{\beta\hat{y}_i+\hat{p}_i-\hat{\kappa}_{ji}}{\theta}+\hat{z}_i-\sum_{k=1}^{n}\overline{d}_{jk}\left(-\frac{\beta\hat{y}_k+\hat{p}_k-\hat{\kappa}_{jk}}{\theta}+\hat{z}_k\right)\right)$$
$$\hat{p}_i = \sum_{k=1}^{n}\overline{d}_{ik}\,(\beta\hat{y}_k+\hat{p}_k-\hat{\kappa}_{ik}-\theta\hat{z}_k)$$

Notice that the latter equation implicitly defines a weighted average of relative prices $\sum_{k=1}^{n}\overline{d}_{ik}\,(\hat{p}_k-\hat{p}_i)$, which could help us eliminate relative prices from the former equation. However, there is an important caveat. For each country i, all terms in the price equation are summed over k, which is the second subscript in d_{ik}, while the first summation in the trade identity above is over the first subscript of d_{ji}. Nevertheless, there is a useful relationship that will allow us to proceed with solving both log-linearised equations simultaneously. Namely, let us impose

$$Lw_i d_{ij} = Lw_j d_{ji} \quad \text{or, equivalently} \quad Y_i\,p_i\,d_{ij} = Y_j\,p_j\,d_{ji} \tag{1.9}$$

which states that the value of goods produced by country j used in country i's production equals the value of goods produced by i used in j's output.

The relationship follows from the assumption of symmetric trade costs: $\kappa_{ij} = \kappa_{ji}$, which is also the cornerstone of the strategy we follow to identify trade costs from data; see section 1.4.1 below. Let us explain why (1.9) holds because it is not straightforward. Notice that without making any assumptions on trade costs κ, trade shares (1.6) can be used to write

$$\frac{d_{ij}}{d_{ji}}\frac{d_{jk}}{d_{kj}} = \left(\frac{Lw_j^{\beta}p_j^{2-\beta}\kappa_{ji}}{Lw_i^{\beta}p_i^{2-\beta}\kappa_{ij}}\right)^{-\frac{1}{\theta}}\frac{Z_j}{Z_i}\left(\frac{Lw_k^{\beta}p_k^{2-\beta}\kappa_{kj}}{Lw_j^{\beta}p_j^{2-\beta}\kappa_{jk}}\right)^{-\frac{1}{\theta}}\frac{Z_k}{Z_j}$$

Now, assuming symmetry, $\kappa_{ij} = \kappa_{ji}$ and $\kappa_{jk} = \kappa_{kj}$ gives

$$\frac{d_{ij}}{d_{ji}}\frac{d_{jk}}{d_{kj}} = \left(\frac{Lw_k^{\beta}p_k^{2-\beta}}{Lw_i^{\beta}p_i^{2-\beta}}\right)^{-\frac{1}{\theta}}\frac{Z_k}{Z_i}$$

[3] In what follows, variables with a bar denote steady-state values, while a hat indicates a log-deviation from the steady state.

which, for $\kappa_{ik} = \kappa_{ki}$, equals d_{ik}/d_{ki}. We therefore obtain the relationship

$$\frac{d_{ij}d_{jk}}{d_{ji}d_{kj}} = \frac{d_{ik}}{d_{ki}}$$

which can be arbitrarily expanded, as long the number of countries allows, according to the following the pattern. Note that country labels a, b, etc. can represent any country; what matters is that the circular link $d_{*f}\, d_{f*}$ between any two countries is obeyed.

$$\frac{d_{\mathbf{a}b}\, d_{bc}\, d_{c*} \ldots d_{*y}\, d_{y\mathbf{z}}}{d_{ba}\, d_{cb}\, d_{*c} \ldots d_{y*}\, d_{zy}} = \frac{d_{\mathbf{az}}}{d_{za}}$$

This powerful condition says that the share of country z goods in a's output relative to the share of a's goods in z's output holds for trade that takes place directly between a and z, the right-hand side of the expression, as well as for the relative contents of goods in any other country's (or countries') exports to the pair, i.e. the left-hand side. If, for instance, trade costs and the country-specific shocks are such that country z's share in a's output is five times larger than a's share in z's output, the content of z's good in a's imports from a third country j will be also five times larger than the content of a's goods in z' exports from j. This is because costs of trade via j: $\kappa_{aj}\kappa_{jz}$ and $\kappa_{zj}\kappa_{ja}$, are identical for both countries. When trade costs are symmetric, there is no reason why the relative shares of goods traded directly or indirectly should differ. Naturally, the amount of goods traded between both countries directly and indirectly, via output of other countries, will be very different, depending on how costly trade with j is for both countries and how big j is (both relative to all other countries).

What ever parameter values drive the fact that a's and z's goods account for the relative share $\frac{d_{az}}{d_{za}}$ in each other's production, given the universal symmetry and zero trade imbalances, the same drivers will then substantiate that z's nominal output will be $\frac{d_{az}}{d_{za}}$ times larger than that of a. Notice that equation (1.9) is stricter than the trade identity (1.5) but it is consistent with it (to see it, sum over j, making use of $\sum_{j=1}^{n} d_{ij} = 1$). Mathematically, the derivation of (1.9) uses $(n-1)$ independent equations from the trade identity (1.5), successively eliminating trade shares of other countries.

Having established that $Y_i\, p_i\, d_{ij} = Y_j\, p_j\, d_{ji}$, our log-linearised model becomes

$$\hat{y}_i + \hat{p}_i = \sum_{j=1}^{n} \overline{d}_{ij} \left(\hat{y}_j + \hat{p}_j - \frac{\beta \hat{y}_i + \hat{p}_i - \hat{\kappa}_{ji}}{\theta} + \hat{z}_i - \sum_{k=1}^{n} \overline{d}_{jk} \left(- \frac{\beta \hat{y}_k + \hat{p}_k - \hat{\kappa}_{jk}}{\theta} + \hat{z}_k \right) \right)$$

$$\hat{p}_i = \sum_{k=1}^{n} \overline{d}_{ik} \left(\beta \hat{y}_k + \hat{p}_k - \hat{\kappa}_{ik} - \theta \hat{z}_k \right)$$

Substituting $\sum_{k=1}^{n} \overline{d}_{ik} \left(\hat{p}_k - \hat{p}_i \right)$ from the price equation to corresponding expressions in the trade equation and simple algebra then lead to the following expression for GDP growth rates:

$$\left(\beta + \theta\right) \hat{y}_i = \theta \hat{z}_i + \sum_{j=1}^{n} \overline{d}_{ij} \left(\left(2 + \theta\right) \hat{\kappa}_{ij} + \left(1 + \theta\right) \theta \hat{z}_j \right) + \left(\theta - \beta \left(1 + \theta\right)\right) \sum_{j=1}^{n} \overline{d}_{ij} \hat{y}_j \tag{1.10}$$

where we separate the sums on the right-hand side to motivate the next step, which is to solve the set of i linear equations above by means of matrix algebra. Let us define the following matrices:

$$\begin{aligned} &\mathbf{I} \ldots i \,\mathrm{x}\, i \text{ identity matrix} \\ &\mathbf{y} \ldots i \,\mathrm{x}\, 1 \text{ vector of } \hat{y}_i \\ &\mathbf{D} \ldots i \,\mathrm{x}\, i \text{ matrix of } \overline{d}_{ij} \\ &\mathbf{K} \ldots i \,\mathrm{x}\, i \text{ matrix of } \hat{\kappa}_{ij} \\ &\mathbf{z} \ldots i \,\mathrm{x}\, 1 \text{ vector of } \hat{z}_i \end{aligned}$$

With this notation, the equation becomes

$$\left(\beta + \theta\right) \left(\mathbf{I} - \frac{\theta - \beta \left(1 + \theta\right)}{\beta + \theta} \mathbf{D} \right) \mathbf{y} = \left(2 + \theta\right) \sum_{j=1}^{n} \left(\overline{d} \circ \hat{\kappa} \right)_{ij} + \theta \left(\mathbf{I} + \left(1 + \theta\right) \mathbf{D} \right) \mathbf{z}$$

where $\left(\overline{d} \circ \hat{\kappa} \right)_{ij}$ stands for the ij element of the Hadamard (element-by-element) product of $\overline{d}_{ij}$ and $\hat{\kappa}_{ij}$. For square matrices and symmetric $\mathbf{K}$, the following expression holds (Styan, 1973):

$$\sum_{j=1}^{n} \left(\overline{d} \circ \hat{\kappa} \right)_{ij} = \left(\mathbf{D}\, \mathbf{K} \right)_{ii}$$

which states that the row-sum of an element-by-element product of two square matrices equals the diagonal of their matrix product. Alternatively, the same result can be achieved by $\mathbf{D_L}\,\mathrm{vec}(\mathbf{K})$, where $\mathbf{D_L}$ represents an n-by-n^2 matrix, which is created by stacking n matrices each of the size of $\mathbf{D}$ next to each other (row-wise), where the r^{th} submatrix carries the r^{th} row from the original matrix $\mathbf{D}$ and has all other elements equal to zero. For illustration, for $n = 3$, it would have the following shape

$$\mathbf{D_L} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{21} & d_{22} & d_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{31} & d_{32} & d_{33} \end{bmatrix}$$

The *vec* operator reshapes an n-by-n matrix into an n^2-by-1 column vector. Because of the symmetry in trade costs, we do not need to transpose matrix $\mathbf{K}$ beforehand.

With these preliminary definitions, and denoting $\mathbf{M}(\mathbf{D})$ the invertible matrix

$$\mathbf{M}(\mathbf{D}) = \mathbf{I} - \frac{\theta - \beta\,(1+\theta)}{\beta+\theta}\mathbf{D}$$

the solution to the log-linearised model is given by

$$\mathbf{y} = \frac{1}{\beta+\theta}\,\mathbf{M}(\mathbf{D})^{-1}\left[(2+\theta)\,(\mathbf{D}\,\mathbf{K})_{ii} + \theta\,\left(\mathbf{I} + (1+\theta)\,\mathbf{D}\right)\mathbf{z}\right]$$

or alternatively

$$\mathbf{y} = \frac{1}{\beta+\theta}\,\mathbf{M}(\mathbf{D})^{-1}\left[(2+\theta)\,\mathbf{D_L}\,\mathrm{vec}(\mathbf{K}) + \theta\,\left(\mathbf{I} + (1+\theta)\,\mathbf{D}\right)\mathbf{z}\right] \tag{1.11}$$

which defines the log-deviations of real GDP as a function of shocks that affect bilateral trade costs and country-specific productivity. The expression is sufficiently simple to derive the moments of real GDP growth rate based on the properties of exogenous shocks. Notice the way shocks are transmitted across countries – the strength of cross-country relationships is governed by the matrix of steady-state trade-shares.

What can we say about the impact of shocks on GDP growth rates? Country specific shocks $\hat{z}_i$ affect GDP growth rates in two ways.[4] First, directly through

[4] Let us note that the individual channels we describe here are only hypothetical because, in equilibrium, all of them will work simultaneously, reinforcing or counterbalancing each other. We do so, nonetheless, in belief that this explanation will ease the understanding of the model.

$\mathbf{I}\,\mathbf{z}$ and secondly indirectly through $(1+\theta)\,\mathbf{Dz}$. Recall that higher $\hat{z}_i$ (a positive productivity shock to i), on average, increases productivities of i's individual firms. Other things equal, other countries therefore end up purchasing a wider range of goods from i, which makes i richer. This is where the direct effect comes from. The indirect effect then reflects the role of relative prices on the growth rate of i's real GDP, where prices of i are always measured relative to the weighted average of prices of all countries (weighted by the trade shares $\mathbf{D}$). They play a double role; first, they matter because nominal GDP is divided by the price level and, secondly, they reinforce demand of other countries for i's goods. Regarding the latter, it carries weight θ because, in our model, higher θ raises the variance of individual productivity draws, making higher productivity shocks more likely.[5].

Next, shocks to trade costs κ, say a positive shock that makes i's goods cheaper in other countries through higher $\hat{\kappa}_{ji}$, affect GDP in a similar way as the country-specific shocks. First, there is the direct demand effect – all countries buy more from i when trade costs are lower. Secondly, lower trade costs also make i's prices relatively cheaper, which again works through the $(1+\theta)$ effect described in the previous paragraph. Since trade costs are always relative, the trade-share matrix $\mathbf{D}$ now premultiplies both terms. Should trade costs not be symmetric, the former demand channel would depend on how costly it would be for other countries to import goods from i ($\hat{\kappa}_{ji}$) while the relative-price channel on i's cost of importing goods from countries j ($\hat{\kappa}_{ij}$). However, trade costs are symmetric in our model and therefore both effects are indistinguishable.

Finally, let us reflect on the role of the $\mathbf{M}(\mathbf{D})$ inverse in the solution. The purpose of the (slightly cumbersome) notation we introduced above though which we eliminated the coefficients at the identity matrix was to motivate the following step. Namely, writing $\mathbf{M}$ as a difference $\mathbf{I} - c_m\mathbf{D}$ allows using the power

[5]The distribution of productivity shocks is positively skewed

series expansion for approximating the inverse as follows[6]

$$(\mathbf{I} - c_m\mathbf{D})^{-1} \simeq \sum_{k=0}^{\infty} c_m^k \mathbf{D}^k = \mathbf{I} + c_m\mathbf{D} + c_m^2\mathbf{D}^2 + \ldots$$

Intuitively, what this operation does is that it takes into account the direct impact of shocks described above (through the identity matrix) as well as the indirect impact that carries through via trade. To see it, note that the i^{th} element of the product $\mathbf{M}(\mathbf{D})^{-1}\mathbf{z}$ would be approximately equal to

$$\hat{z}_i + c_m \sum_{a=1}^{n} \overline{d}_{ia}\hat{z}_a + c_m^2 \sum_{a=1}^{n}\sum_{b=1}^{n} \overline{d}_{ib}\overline{d}_{ba}\hat{z}_a + c_m^3 \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} \overline{d}_{ic}\overline{d}_{cb}\overline{d}_{ba}\hat{z}_a + \ldots$$

where the first element is the direct impact of own productivity shock; the second element captures shocks to all countries a (including i itself) as carried through trading with countries a; the third element captures the impact in i of country a's shocks on i's direct trade partner b (where the degree with which it is felt in i depends on i's openness to b and b's openness to a); the fourth element is again the impact in i of a's shocks on i's direct trade partner c as transmitted through c's trade with b (where the impact depends on $\overline{d}_{ic}\overline{d}_{cb}\overline{d}_{ba}$), etc. etc. The inverse therefore takes care of transmitting the shocks via both direct- and indirect-trade channels so that all shocks are felt in each country via all thinkable trade relationships that exist in the equilibrium.

Before we proceed further, we briefly note that the solution can be easily specialised for the extreme cases of autarky and cost-less trade. In autarky, trade shares $\mathbf{D}$ is an identity matrix and trade costs $\mathbf{K}$ are zero. Therefore matrix $\mathbf{M}$ becomes a constant $(2+\theta)\,\beta/\,(\beta+\theta)$ and $\mathbf{y}^{Aut} = \frac{\theta}{\beta}\mathbf{z}$ or $\hat{y}_i^{Aut} = \frac{\theta}{\beta}\hat{z}_i$. Of course, when countries are closed and the only source of volatility are own productivity shocks, GDP growth mimics the behaviour of these shocks. A corresponding formula with costless trade, realising that dk_{ij} are still zero, is

[6] The infinite sum converges. Note that c_m can be alternatively written as $-\frac{\theta(\beta-1)+\beta}{\theta+\beta}$, which makes it clear that $|c_m| < 1$ because β, the share of the intermediate input in production, is always $0 < \beta < 1$. Furthermore, $\mathbf{D}$ is a matrix of trade shares (all elements between zero and one), which become smaller after successive multiplications. In the extreme case of autarky, $\mathbf{D}$ is an identity matrix, for which the sum converges because $|c_m| < 1$.

the following:

$$\mathbf{y}^F = \frac{\theta}{\beta+\theta}\mathbf{M}\big(\mathbf{D}^F\big)^{-1}\Big[\mathbf{z} + (1+\theta)\,\mathbf{D}^F\mathbf{z}\Big]$$

Without matrices, the same relationship derived from (1.10) would be

$$\hat{y}_i^F = \frac{\theta}{\beta+\theta}\hat{z}_i + \frac{(1+\theta)\,\theta}{\beta+\theta}\sum_{j=1}^{n}\overline{d}_{*j}\hat{z}_j + \frac{\theta-\beta\,(1+\theta)}{\beta+\theta}\sum_{j=1}^{n}\overline{d}_{*j}\hat{y}_j^F$$

where the notation d_{*j} for an element in the j^{th} column of $\mathbf{D}^F$ refers to the fact that with $\kappa_{ij}^F = 1$ and equal prices everywhere (because of free trade), d_{ij} does not depend on i any longer, i.e. all countries import the same fraction of goods from j. Notice that the last term is common for all countries and functions as a common scaling constant, amplifying or moderating the response of all countries to shocks. We conclude that the dynamics of GDP growth with cost-less trade is determined not only by own productivity shocks $\hat{z}_i$, as was the case in autarky, but also by other countries' productivity shocks, transmitted through trade.[7]

1.3 Sources of comovement

The point of this section is to investigate the mechanics of the model and study how trade linkages across countries induce comovement. We start with stating the general formula for *covariance* between $\hat{y}_i$ and $\hat{y}_j$. Subsequently, we make several simplifying assumptions that allow us to derive an approximate, yet tractable closed-form solution for *correlations* between GDP growth rates of any two countries. Finally, we present several illustrations that document how various linkages in the model drive comovement.

[7] This formula is, in principle, identical to the equation (2.6) we arrive at in Chapter 2. The only difference concerns the constant before the second term on the right-hand side, which is due to a different choice of the numeraire. Unlike here, the price level (recall that prices are equal across countries with free trade) defined in Chapter 2 by (2.4) is a function of all productivity parameters Z_i, which gives rise to the difference in constants.

Let us start with solution (1.11) we obtained in the previous section. The covariance matrix of $\mathbf{y}$, $Cov(\mathbf{y}) = \mathbb{E}\big(\mathbf{y}\mathbf{y}^T\big)$,[8] is given by the following expression

$$\begin{aligned} Cov(\mathbf{y}) = \frac{1}{(\beta+\theta)^2}\,\mathbf{M}(\mathbf{D})^{-1}\Big[&(2+\theta)^2\,\mathbf{D_L}\,\mathbb{E}\Big(\mathrm{vec}(\mathbf{K})\,\mathrm{vec}(\mathbf{K})^T\Big)\,\mathbf{D_L}^T \\ &+\theta\,(2+\theta)\,\mathbf{D_L}\,\mathbb{E}\Big(\mathrm{vec}(\mathbf{K})\,\mathbf{z}^T\Big)\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}^T\Big) \\ &+\theta\,(2+\theta)\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}\Big)\,\mathbb{E}\Big(\mathbf{z}\,\mathrm{vec}(\mathbf{K})^T\Big)\,\mathbf{D_L}^T \\ &+\theta^2\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}\Big)\mathbb{E}\big(\mathbf{z}\,\mathbf{z}^T\big)\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}^T\Big)\Big]\,\big(\mathbf{M}(\mathbf{D})^{-1}\big)^T \end{aligned} \tag{1.12}$$

which relates covariances of GDP growth rates to covariances of trade shocks, covariances of country-specific shocks and their cross-covariances.

1.3.1 Trade and country-specific shocks

For the sake of clarity, let us now abstract from any volatility in trade costs and focus instead on how country-specific shocks are transmitted through trade. While trade shocks, as we document in section 1.4.1, play an important role in cross-country comovement, we are more interested in the role of trade as a channel for transmission of country-specific (or aggregate) shocks, to which the trade shocks add an additional layer of disturbances. Under the assumption of fixed trade costs and a general covariance matrix of $\hat{z}$, which we label $\mathbf{\Omega}_z$, the formula above collapses to

$$Cov(\mathbf{y}) = \frac{\theta^2}{(\beta+\theta)^2}\,\mathbf{M}(\mathbf{D})^{-1}\Big[\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}\Big)\mathbf{\Omega}_z\Big(\mathbf{I}+(1+\theta)\,\mathbf{D}^T\Big)\Big]\,\big(\mathbf{M}(\mathbf{D})^{-1}\big)^T$$

Notice that in autarky[9], this is equal to $\frac{\theta^2}{\beta^2}\mathbf{\Omega}_z$, which is consistent with our solution for $\hat{y}_i^{Aut} = \frac{\theta}{\beta}\hat{z}_i$. Clearly, with no covariance in underlying shocks, there is no mechanism in the model that would make growth rates correlated.

More interestingly, this formula shows that even if shocks are uncorrelated, trade induces non-zero cross-covariances.[10] We illustrate below in detail what

[8]Log-deviations from steady-state values have zero mean.

[9]Matrix $\mathbf{D}$ is an identity matrix and therefore $\mathbf{M} = \frac{\beta(2+\theta)}{\beta+\theta}$.

[10]Replace $\mathbf{\Omega}_z$ with an identity matrix to see that the off-diagonal elements of the covariance matrix are nonzero. Notice that we cannot conclude that the changes in covariances induced

gives rise to non-zero covariances and whether they also translate in non-zero correlations in several specific environments but the general point is that trade exposes each country to shocks that affect its trade partners and possibly also other countries, not directly connected through trade, which may however tend to receive similar shocks as one of the direct trade partners.

To make our point and illustrate how trade generates comovement, let us write the formula above in a more intuitive way. Expanding the terms inside of the brackets, we have

$$\Big[\dots\Big] = \mathbf{\Omega}_z + (1+\theta)\,\mathbf{\Omega}_z\mathbf{D}^T + (1+\theta)\,\mathbf{D}\mathbf{\Omega}_z + (1+\theta)^2\,\mathbf{D}\mathbf{\Omega}_z\mathbf{D}^T$$

Next, approximation of the inverses as shown in the previous section gives

$$\begin{aligned} Cov(\mathbf{y}) &\simeq \frac{\theta^2}{(\beta+\theta)^2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} c_m^{k+l}\mathbf{D}^k\Big[\dots\Big]\left(\mathbf{D}^l\right)^T \\ &\simeq \frac{\theta^2}{(\beta+\theta)^2}\Big[\mathbf{\Omega}_z + (1+c_m+\theta)\left(\mathbf{D}\mathbf{\Omega}_z + \mathbf{\Omega}_z\mathbf{D}^T\right) \\ &+ (1+c_m+\theta)^2\,\mathbf{D}\mathbf{\Omega}_z\mathbf{D}^T + c_m\,(1+c_m+\theta)\left(\mathbf{D}^2\mathbf{\Omega}_z + \mathbf{\Omega}_z\left(\mathbf{D}^T\right)^2\right) + \dots\Big] \end{aligned} \tag{1.13}$$

As said, successive multiplications by $\mathbf{D}$ represent the pass-through of shocks via trade partners of trade partners etc. With realistic values of $\mathbf{D}$, the off-diagonal elements are typically of the order of 1/100, hence their cross-products have a minimal effect on the final outcome, compared to the first-order terms. Let us therefore abstract from the second- and higher order terms and deliver the intuition with a simplified version of the function. A comparison of the effects we obtain in this way with the results generated with a full version of the formula presented in the following section confirms that the loss of generality makes only little difference to the key results.

by trade will be positive for all parameter values and trade shares. While all elements within the square brackets are by definition positive when $\mathbf{\Omega}_z$ is an identity matrix, the same cannot be said about the inverse that pre- and post-multiplies the bracket. Matrix $\mathbf{M}(\mathbf{D})$ is defined as $\mathbf{I} - c_m\mathbf{D}$, where $c_m = \frac{\theta-\beta(1+\theta)}{\beta+\theta}$ can be positive or negative. Its inverse can have both positive and negative elements. Note that even positive definiteness of matrix $\mathbf{M}$, which could be guaranteed under certain parameter restrictions, would not necessarily result in positive elements in the covariance matrix.

The first-order terms in the covariance matrix are given by the middle line in (1.13). The ij^{th} element of the matrix is

$$Cov(\hat{y}_i, \hat{y}_j) \simeq \frac{\theta^2}{(\beta+\theta)^2} \Big[\omega_{ij} + (1 + c_m + \theta) \Big(\sum_{k=1}^{n} \overline{d}_{ik}\, \omega_{kj} + \sum_{k=1}^{n} \overline{d}_{jk}\, \omega_{ik} \Big)\Big]$$

where ω_{ij} is the ij element of $\mathbf{\Omega}_z$, i.e. covariance between countries i and j's shocks. Denoting the constant

$$c_d = 1 + c_m + \theta = 1 + \frac{\theta - \beta\,(1+\theta)}{\beta+\theta} + \theta = \frac{\theta\,(2+\theta)}{\beta+\theta}$$

and extracting relevant covariances from the sums, we have

$$\begin{aligned} Cov(\hat{y}_i, \hat{y}_j) \simeq \frac{\theta^2}{(\beta+\theta)^2} \Bigg\{ & \Big(1 + c_d\,\big(\overline{d}_{ii} + \overline{d}_{jj}\big)\Big)\omega_{ij} \\ & + c_d \Big(\overline{d}_{ij}\, \omega_{jj} + \overline{d}_{ji}\, \omega_{ii}\Big) \\ & + c_d \sum_{k \neq i,j} \Big(\overline{d}_{ik}\, \omega_{kj} + \overline{d}_{jk}\, \omega_{ik}\Big) \Bigg\} \end{aligned}$$

What does the formula tell about the (first-order) effects of trade on covariances? The way we wrote it suggests there are three factors to consider; with the first and second one closely related because they concern the bilateral characteristics of i and j. The first term reflects how the underlying covariance between i and j's shocks decreases when countries open to trade; $\overline{d}_{ii}$ reaches maximum in autarky and falls as country i starts trading with other countries (trade shares sum to 1). Therefore, as both countries become more open, the less it matters how the underlying $i - j$ shocks comove and the greater is the importance of the other terms in the formula. Notice that the term would decrease also if the two countries trade only with each other. Then, and this is the second term in the formula, their covariance would increasingly capture the pass-through of each others' shocks (ω_{jj} stands for variance of j's shocks). If i and j trade only with each other and are equally volatile, this term would dominate the first one because variances ω_{ii} are larger than covariances ω_{ij}.

What if trade costs (and other parameters) are such that there is no direct trade between i and j but both i and j trade with other countries? Then covariance between i and j's growth would become increasingly determined by the last factor in the formula, which describes how exposed country i is – through trade linkages $\overline{d}_{ik}$ – to shocks that tend to affect (assuming $\omega_{ik} > 0$) both i's trade partners and j; and vice versa, how exposed j is – through shares $\overline{d}_{jk}$ – to shocks that affect j's trade partners and i. Together, these are weighted covariances between own trade partners and the other country in the pair. The term will rise when, say, country i trades with countries, whose shocks comove with j. Notice that it does not require that j also trades with these countries, it only considers their underlying covariances. In other words, covariance of GDP growth between i and j depends on how j comoves with countries i trades with and vice versa. If j tends to get similar shocks as a third country m, then through i's exposure to m the shocks i implicitly gets are those that affect j as well.

This result sheds light on the specification strategy for an empirical investigation of trade and comovement. As discussed in the introductory part of the chapter, the typical approach is to relate a measure of comovement between a pair of countries to a set of bilateral characteristics, including a measure bilateral trade intensity. The model, however, suggests that there is a need for controlling for the underlying covariances among *all* countries one of the pair trades with, for otherwise the last term in the equation above generates a systematic 'error' in the association of trade between two countries and their comovement. This error arises because the covariance between i and j's growth is systematically affected by each of the countries' trade with others that comove with its counterpart in the pair. The strength of this link is pair-specific and therefore it will not be treated with country-specific dummies. We will inspect the quantitative significance of this channel in the empirical part of this chapter.

Until now, we have described the channels through which covariances are affected by trade in our model. By doing so, we have implicitly assumed that trade has no effect on countries' volatility, which meant that covariances were directly comparable. However, we show in detail in Chapter 2 that trade can decrease or increase variances, depending on the particular characteristics of a country's trade partners so the changes in covariances induced by trade could, in principle, be amplified or counterbalanced by the effect of trade on volatility. Let us now therefore extend the analysis to a more general case and see what would be the impact of trade on correlations – a dimensionless measure of linear comovement. The results described so far remain relevant because correlations are covariances scaled by the product of standard deviations.

Variance of i's GDP growth in our model is as follows

$$Var(\hat{y}_i) = Cov(\hat{y}_i, \hat{y}_i) \simeq \frac{\theta^2}{(\beta+\theta)^2} \left\{ \omega_{ii} + 2\, c_d \sum_{k=1}^{n} \overline{d}_{ik}\, \omega_{ik} \right\}$$

which is variance of own shocks plus a weighted average of covariances that i imports through trade (including from itself). If i and its trade partners tend to be hit by similar shocks, i's volatility will be higher compared with what it would have been otherwise. This is the (first-order approximation of the) diversification channel we study in Chapter 2. As said, correlation is covariance between i and j over the product of i and j's standard deviations. Notice therefore that the product in the denominator will be high if both i and j are trading with countries that tend to get similar shocks as i and j, respectively, increasing thus their volatility.

Combining the results for covariance and variances (the numerator and denominator of the expression below) gives the following formula for correlation between i and j's growth:

$$Cor(\hat{y}_i, \hat{y}_j) \simeq$$
$$\frac{\Big(1 + c_d\,(\overline{d}_{ii} + \overline{d}_{jj})\Big)\omega_{ij} + c_d\Big(\overline{d}_{ij}\,\omega_{jj} + \overline{d}_{ji}\,\omega_{ii}\Big) + c_d \sum_{k\neq i,j}\Big(\overline{d}_{ik}\,\omega_{kj} + \overline{d}_{jk}\,\omega_{ik}\Big)}{\Big(\omega_{ii} + 2\,c_d \sum_{k=1}^{n} \overline{d}_{ik}\,\omega_{ik}\Big)^{1/2}\Big(\omega_{jj} + 2\,c_d \sum_{k=1}^{n} \overline{d}_{jk}\,\omega_{jk}\Big)^{1/2}}$$

With this formula at hand, we can conjecture about the relative roles of the direct- and indirect-trade channel on correlations. First, let us summarise the results for bilateral trade intensities $\overline{d}_{ij}$ (direct trade between i and j). If i and j trade only with each other, are symmetric in every respect, and $\mathbf{\Omega}_z$ is an identity matrix, correlation between i and j simplifies to

$$\frac{2\,c_d\,\overline{d}_{12}}{1 + 2\,c_d\,(1 - \overline{d}_{12})}$$

The derivative of this expression with respect to $\overline{d}_{12} = \overline{d}_{21}$ is clearly positive. Direct trade, in this stylized setting, increases bilateral correlation because it makes countries exposed to each others' shocks, while simultaneously decreasing their volatility (trade with a country whose shocks are uncorrelated with the domestic ones works as a hedge against own shocks).

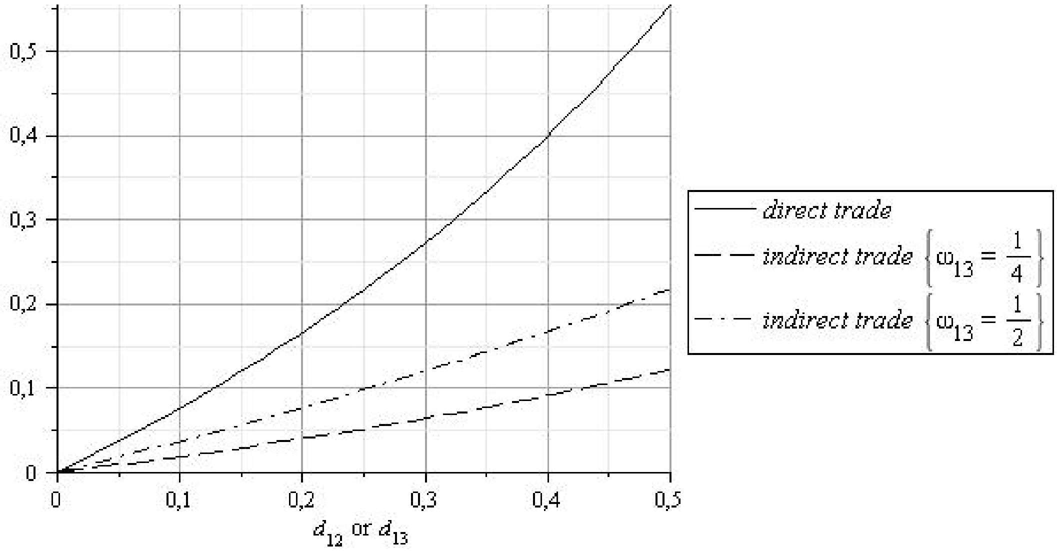
Secondly, consider the case with **no direct trade between** i **and** j but non-zero trade with a third country. Assuming the same kind of symmetry as above, correlation between i and j is

$$\frac{2\, c_d\, \overline{d}_{13}\omega_{13}}{1 + 2\, c_d\, (1 - \overline{d}_{13}) + 2\, c_d\, \overline{d}_{13}\omega_{13}}$$

which shows that trade with the third country matters, to the first-order approximation, only if it has non-zero covariance with one country in the pair. For $\omega_{13} > 0$, there will be a positive effect on i and j's covariances because the country they trade with tends to get similar shocks as the two countries. Variance is affected as well; first because there is lower exposure to domestic shocks and secondly, trade with the third country brings another opportunity for diversification of shocks. With equal variances of shocks, the former will always dominate the latter and variance of GDP growth unambiguously decreases. We have thus established that indirect trade with a country that tends to get similar shocks as i or b gives rise to positive correlation between i and j even if there is no trade between i and j directly. The size of the effect rises with the intensity of trade with the third country and with the magnitude of cross-covariances ω_{13}.

However, the relative force of the third-country channel will, other things equal, be smaller than of the direct-trade channel above because the same factor that drives positive covariances (correlated shocks between i/j and the third country) will, at the same time, limit the possibility for diversification of shocks. Figure 1.1 compares the impact on correlation between i and j when countries move from autarky to a) direct trade (full line) and b) to trade with a third country (dashed lines). As argued, the quantitative impact of direct trade on correlation is larger than the effect of the indirect trade (in this stylised setting).

Figure 1.1: Correlation between i and j's GDP growth: Move from autarky to trade



Note: The figure shows the change in $Cor(\hat{y}_i, \hat{y}_j)$ as countries move from autarky to trade. There are three countries i, j, and k of equal size, for different values of trade intensity. The full line is a case with direct trade between i and j but no trade with k, symmetric trade costs and $\mathbf{\Omega}_z = Cov(\mathbf{Z})$ an identity matrix. The two other lines show the same correlation when there is no direct trade but both i and j trade (symmetrically again) with k. $\mathbf{\Omega}_z$ is a diagonal matrix with nonzero $\omega_{i3} = \omega_{j3}$, taking values as shown in the figure.

However, the latter is by no means insignificant and depends on the magnitude of covariance between shocks.

1.3.2 Sensitivity analysis

We now return to the general formula for covariances (1.12) and present several experiments that assess the robustness of the results we have presented so far to other parameters in the model. In order to make the analysis tractable, we need to make assumptions about the structure of the terms in expected values in (1.12): covariances of trade shocks, covariances of country-specific shocks and their cross covariances.[11]

[11]The matrix of cross-covariances of trade and country-specific shocks is the expression in the second and third line in (1.12). The sum of the two lines is a symmetric square matrix.

Regarding trade shocks, let us denote σ_κ the variance of trade shocks $\hat{\kappa}_{ij}$ and $\sigma_{\kappa\kappa}$ the covariance between trade shocks across pairs. Let these be the same for all country pairs. The symmetry in $\hat{\kappa}$s and the fact that $\kappa_{iit} = 1$ (and therefore $\hat{\kappa}_{iit} = 0$) implies a particular form of the covariance matrix $\mathbf{\Omega}_\kappa$. Appendix 1.A.2 gives an example of how this matrix looks like for three countries. As before, we denote $\mathbf{\Omega}_z$ the covariance matrix of country specific shocks with elements ω_{ij}. Cross-terms $\mathbf{\Omega}_{\kappa z}$ capturing covariances between country-specific shocks and trade shocks are assumed fixed at $\sigma_{\kappa z}$, except for those with $\hat{\kappa}_{iit} = 0$, where they will be zero. With this notation, covariance of GDP growth rates is

$$\begin{aligned}
Cov(\mathbf{y}) = \frac{1}{(\beta+\theta)^2}\,\mathbf{M}(\mathbf{D})^{-1} \Big[& (2+\theta)^2\,\mathbf{D_L}\,\mathbf{\Omega}_\kappa\,\mathbf{D_L}^T \\
& + \theta\ (2+\theta)\,\mathbf{D_L}\,\mathbf{\Omega}_{\kappa z}\Big(\mathbf{I} + (1+\theta)\,\mathbf{D}^T\Big) \\
& + \theta\ (2+\theta)\,\Big(\mathbf{I} + (1+\theta)\,\mathbf{D}\Big)\,\mathbf{\Omega}_{\kappa z}^T\mathbf{D_L}^T \\
& + \theta^2\Big(\mathbf{I} + (1+\theta)\,\mathbf{D}\Big)\mathbf{\Omega}_z\Big(\mathbf{I} + (1+\theta)\,\mathbf{D}^T\Big)\Big]\,\Big(\mathbf{M}(\mathbf{D})^{-1}\Big)^T
\end{aligned}$$

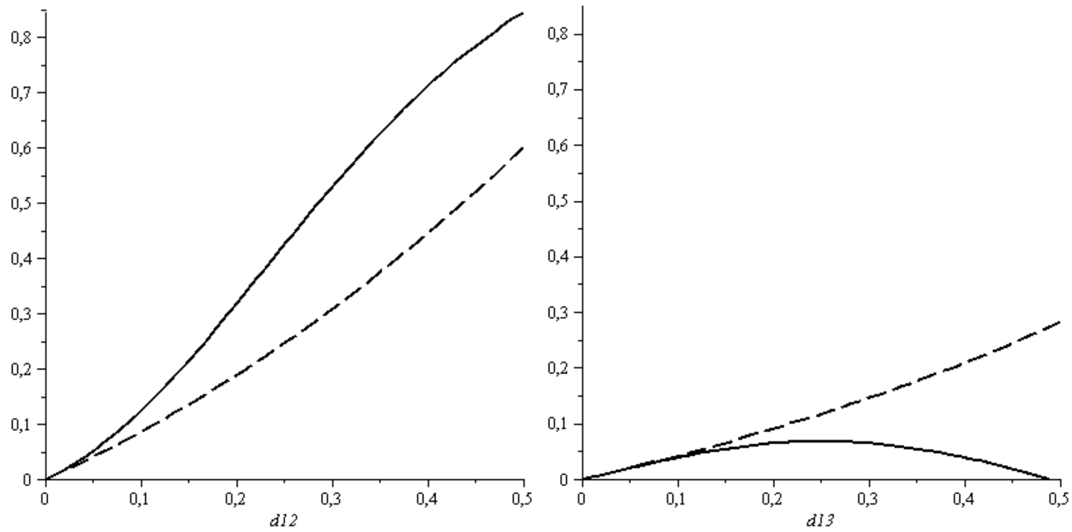
Within this framework, we carry out several simulations:

Volatile trade shocks: When trade shocks are allowed to vary, each country pair receives an additional source of perfectly correlated shocks (correlated within the pair), where the latter follows from our assumption of symmetric trade costs. Not surprisingly, this makes bilateral correlations increase for countries that directly trade with each other. Mathematically, the term $\mathbf{D_L}\,\mathbf{\Omega}_\kappa\,\mathbf{D_L}^T$ within the covariance matrix adds the additional variance (weighted with direct trade shares) and covariances that may affect each of the pair via its trade partners:

$$ij \text{ element of}\,\mathbf{D_L}\,\mathbf{\Omega}_\kappa\,\mathbf{D_L}^T = \overline{d}_{ij}\overline{d}_{ij}\sigma_\kappa + \sum_{k\neq j}\sum_{l\neq i}\overline{d}_{ik}\overline{d}_{jl}\sigma_{\kappa\kappa}$$

If the two countries trade with a third country but not with each other and if covariance among trade shocks are zero across pairs, covariance of their growth rates will not be affected because they do not benefit from the additional source of perfectly correlated shocks, unlike in the case with direct trade. However,

Figure 1.2: Correlation of GDP growth: Volatile trade costs

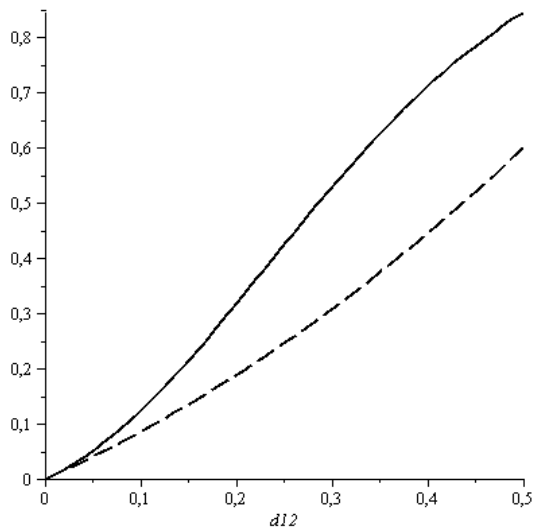


Note: The figure shows the change in $Cor(\hat{y}_1, \hat{y}_2)$ compared with autarky. *Dashed line = Baseline*: $d_{12} = d_{21}$ (left panel) or $d_{13} = d_{23} = d_{31} = d_{13}$ (right panel), no trade otherwise, $\mathbf{\Omega}_z = \mathbf{I}$ (left panel) or $\omega_{13} = \omega_{23} = 1/2$ (right panel), $\mathbf{\Omega}_k = \mathbf{0}$, $\mathbf{\Omega}_{zk} = \mathbf{0}$. *Solid line*: $\mathbf{\Omega}_k$ with $\sigma_\kappa = 1$ and $\sigma_{\kappa\kappa} = 0$.

since each of the countries will now become exposed to this new source of shocks through its trade with the third country, their volatility will rise. Therefore, correlation between their GDP growth rates offsets the indirect-trade channel described above so that, as trade with the third country increases, correlations start falling again. See Figure 1.2 for an illustration.

Volatile, correlated trade shocks: When trade shocks are correlated across country pairs, then it increases comovement because trade partners in the pair become subject to more similar shocks. Naturally, for a pair that trades solely with each other, this makes no difference (the left panel of Figure 1.3 is indistinguishable from the one in 1.2). In the opposite case, for countries that do not trade with each other, the fact that they tend to get coincident trade shocks makes them more correlated. In the formula above, this would be captured with nonzero terms $\sigma_{\kappa\kappa}$.

Figure 1.3: Correlation of GDP growth: Volatile, correlated trade costs



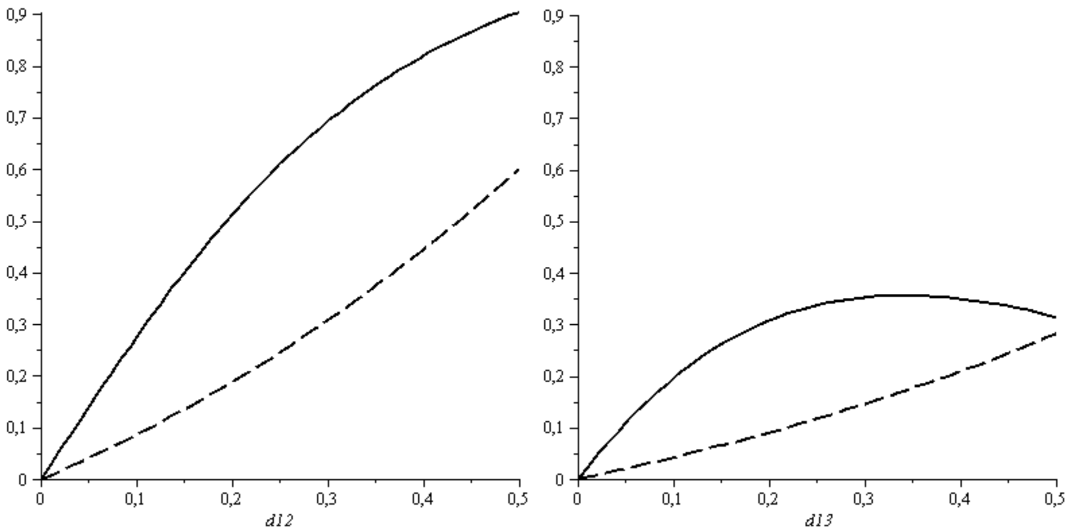
Note: The figure shows the change in $Cor(\hat{y}_1, \hat{y}_2)$ compared with autarky. *Dashed line*: Baseline, see Figure 1.2 for details. *Solid line*: $\mathbf{\Omega}_k$ with $\sigma_\kappa = 1$ and $\sigma_{\kappa\kappa} = 1/2$.

Volatile trade shocks correlated with country-specific shocks: If κs tend to be high (a positive trade shock) when country-specific productivity shocks are high, their joined effect is more pronounced and countries become more correlated. The intuition is similar as for the previous case but the effect is stronger for both scenarios. In the one with direct trade, not only are trade shocks perfectly correlated within the pair (by the symmetry) but they now also tend to comove with both shocks. Because of the symmetry, they amplify bilaterally-good or bilaterally-bad shocks and moderate the opposite ones, which strengthens the diversification nature of trade. In the scenario with indirect trade only, the additional comovement is generated through an identical mechanism but this time with respect to the third country.

Greater θ: The final exercise we present is to verify that our main conclusions presented so far are robust to different values on θ.[12] This parameter determines

[12]We do not report a similar exercise with respect to parameter β because it has a clear counterpart in data. Let us only note that changes of β of the same magnitude as we make to θ produce similar changes to our figures – but of the opposite sign. Recall that β is the

Figure 1.4: Correlation of GDP growth: Volatile trade shocks correlated with $\hat{z}$

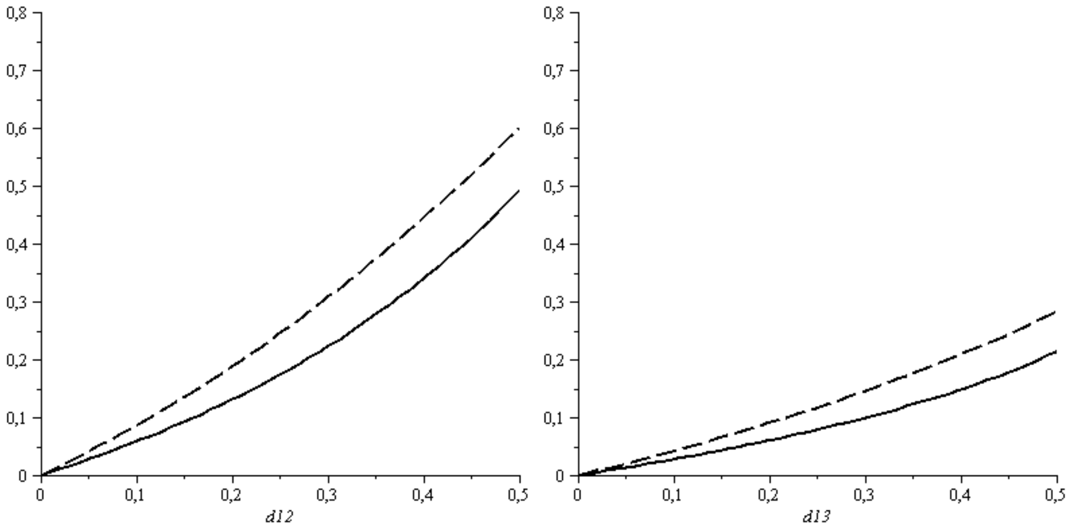


Note: The figure shows the change in $Cor(\hat{y}_1, \hat{y}_2)$ compared with autarky. *Dashed line*: Baseline, see Figure 1.2 for details. *Solid line*: $\mathbf{\Omega}_k$ with $\sigma_\kappa = 1$ and $\sigma_{\kappa z} = 1/2$.

the shape of the distribution from which productivity shocks are drawn. As said, higher θ means that shocks are more volatile. Recall that when we described the solution to our model in section 1.2.1, we argued that shocks affect the model economy through several channels, with one of them operating through changes in relative prices. This mechanism was shown to be positive related to θ in the log-linear approximation of the model. Higher θ therefore makes prices, relative demands for goods and hence also output more sensitive to shocks, increasing thus the potential for exploiting the country's comparative advantage. This is to say that production processes become less similar across countries and therefore their covariance decreases. Likewise, volatility should increase, which leads to an overall fall in correlations and smaller power of trade to generate comovement. Figure 1.5 quantifies this mechanism for the two special cases of openness and for $\theta = 0.5$ versus $\theta = 0.7$. The effect, while significant, is quantitatively smaller than the effects of the above reported exercises and importantly, it does not

share of the unproduced input in our model (equipped labour); higher β therefore decreases the share of traded intermediates, which mutes the response of output to shocks, both foreign and domestic (see (1.11)) and makes countries more correlated.

Figure 1.5: Correlation of GDP growth: Role of θ



Note: The figure shows the change in $Cor(\hat{y}_1, \hat{y}_2)$ compared with autarky. *Dashed line*: Baseline with $\theta = 0.5$, see Figure 1.2 for details. *Solid line*: $\theta = 0.7$.

change the conclusions qualitatively.

1.4 Quantitative exercises with data

In this section we look at the relationship between trade and GDP correlations, as observed in data, through the lens of the model we studied in the preceding sections. To accommodate the range of parameters we obtain from data, we work with the fully-fledged (not linearised) version of the model and use numerical techniques to compute its solution. The model allows us to investigate various aspects of data to judge the quantitative significance of the competing theoretical factors. Our strategy therefore is to experiment with the pattern of empirical trade linkages, compute alternative GDP growth rates for all countries and years, obtain bilateral correlations Cor_{ij}^{ex} and compare them with those in the baseline. The baseline scenario is based on country-specific shocks $\hat{z}_{it}$ and trade shocks $\hat{\kappa}_{ijt}$ exactly as found in data. Shocks $\hat{z}_{it}$ are taken as exogenous and do not vary across simulations. Trade imbalances are used in the mapping of observables to parameters used in the model but are ignored in the quantitative simulations (they are not explained by the model).

The road map for our work is as follows:

1. We start with a description of our sample and of the data we use in the model. We also provide a check that the baseline, to which we compare all subsequent simulations, matches the correlations found in data.

2. The first question we investigate is the quantitative power of trade to generate comovement. We do so by comparing the results of the model with baseline trade costs and to the volatility that would prevail in autarky.

3. We further examine the properties of trade costs and test whether it is the average level of trade costs, their trend changes over time or their volatility (trade shocks) that drive the results.

4. We then move to study several candidate mechanisms through which trade could play a role as a channel for transmission of country-specific shocks. Namely, we ask:

    - What part of bilateral correlations can be accounted for by *direct* trade within each pair of countries and what is due to the *indirect* trade with all other countries?

    - If the indirect-trade channel turns out important, then which countries matter? Big common trade partners? Big global players overall? Can theory guide us in identifying countries that account for most of the effect of the indirect trade?

1.4.1 Parametrisation of the model

Our sample consists of $n = 69$ countries and an additional aggregate for the rest of the world (ROW). Out of the biggest 30 countries in the world only Russia (together with other post-Soviet republics) is merged in the ROW.[13] The 69 sample countries accounted jointly for 95% world nominal GDP in 2005. Our focus is on annual data from the period 1989-2011; with this choice we mean to take into account the changes in trade linkages brought about by the overall expansion in trade that started to gather pace from 1980s and also take into account the fall of the iron curtain. The length of the sample period is also dictated by sufficient availability of trade data for the large number of country pairs we work with.

There are n^2 parameters in the model we need to identify for every year of data: $n-1$ trade shares d_{ij} for every country i and n productivities Z_i. Starting with trade shares, d_{ij} is the share of goods produced in country j in total demand for goods in country i. Its definition in the model[14] is

$$d_{ij} = \frac{I_{ij}}{L_i p_i q_i} = \frac{I_{ij}}{GNO_i - S_i}$$

with d_{ii} implied from the restriction $\sum_j d_{ij} = 1$. We take imports I_{ij} and trade imbalances S_i from IMF Direction of Trade Statistics (see Appendix 1.A.3 for a more detailed description of all variables). Variable $GNO_i \equiv \int p_i(x) q_i(x) d\Phi(x)$ is the value of total production, or gross output.

Collecting reliable data for gross output is a challenging task because adequately long time series are available only for a handful of countries (more or less those covered in EU KLEMS). We take this route in Chapter 2, where we study volatility of a smaller sample of relatively large countries. However, the study of comovement requires a large sample of countries otherwise the residual ROW

[13] Other important countries (ordered here according to their 2005 nominal GDP in US$) treated as the ROW include Iran, Hong-Kong (China), the Czech Republic, Singapore, Slovakia, Vietnam, Croatia and Libya. See Appendix 1.A.5 for a list of countries and country codes.

[14] Notice that the identification strategy we follow here to back out the parameters of the model is identical to that presented in Chapter 2. However, the choice of variables to which we map our observables, is different.

aggregate quickly starts dominating the comovement patterns for the majority of countries.[15]

We therefore by-pass the need to use (or even estimate) gross output data for a large selection of countries and make a pragmatic assumption that output is a fixed multiple of GDP: $GNO_i = GDP_i/\beta$, where $\beta = 1/2$.[16] This assumption is consistent with our model because gross output *data* (even if statisticians were able to estimate it with a high degree of reliability) are affected by processes that are not captured in the model. Namely, gross output crucially depends on the degree of integration of production processes, which may differ across countries and, more fundamentally for our needs, within a country over time. This is largely relevant for the transition economies included in our sample (most notably China) but it is of some importance also for developed countries, where the organisation of production processes may for instance respond to changes in tax or accounting laws. However, whether a company produces a part of a machine in-house or sublets the task to another domestic company may be irrelevant for the amount of goods the economy trades externally. Yet, it would affect the gross output series and therefore our measures of openness and trade costs. Added to this, the effort statisticians put to constructing a consistent series of gross output is also typically lower than is the case of other key aggregates. Based on this, our trade shares are identified as

$$d_{ij} = \frac{I_{ij}}{GDP_i/\beta - S_i}$$

With a measure of d_{ij} and assuming symmetric trade costs $\kappa_{ij} = \kappa_{ji}$, as we do consistently throughout this chapter, equation (1.6) implies

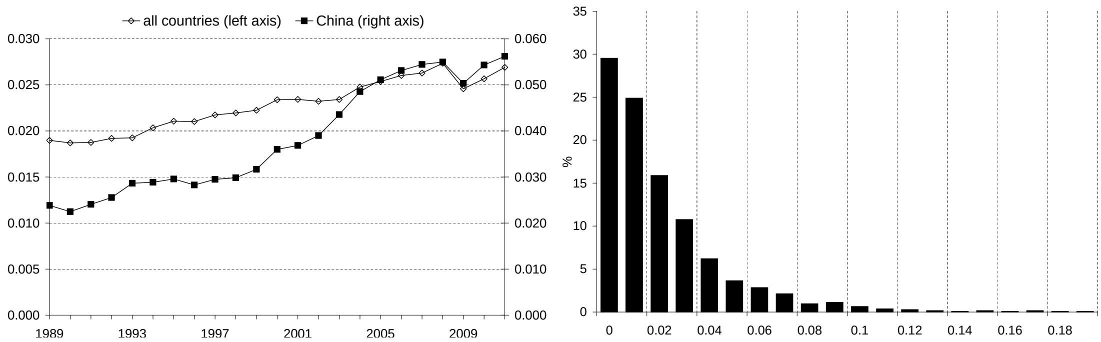
$$\kappa_{ij} = \left(\frac{d_{ij}}{d_{jj}}\frac{d_{ji}}{d_{ii}}\right)^{\theta/2}$$

which means that trade costs are completely determined by the trade shares defined above. This procedure gives a point estimate of trade costs for each of

[15] Even with 69 largest countries in our sample, the ROW still accounts for 5% of world GDP and is the 5^{th} largest economy in the model.

[16] $\beta = 1/2$ corresponds to the average GDP-to-output ratio found in data used in Chapter 2.

Figure 1.6: Average trade costs and distribution of κ_{ijt} in 1989-2011



the more than 2000 country pairs every year. Annual averages of trade costs are shown on the left panel of Figure 1.6. Notice that they are enormous: the average value of κ_{ij} overall is 0.03 (median 0.02), where $\kappa_{ij} = 0$ ensures zero trade between i and j and $\kappa = 1$ is a situation where across-the-border trade is equally costly as domestic trade. Highest values of κ are recorded between Belgium and the Netherlands (0.23), the U.S. and Canada (0.18), and further among Germany and the Netherlands/Belgium/Austria, Belgium and France, and the U.S. and Mexico. A longer list of country pairs with lowest bilateral trade costs is reported in Figure 1.14. Also note that κs grow over time: average κ increase from 0.019 to 0.027 in our sample period, where the trend is much more pronounced for some countries. For instance, average trade costs of all countries with China rise from 0.02 to 0.06 over the same period. The distribution of average trade costs (per country pair over time) is shown on the right panel of Figure 1.6.

Turning to the productivity measure Z_i, equation (1.6) again can be used to express real aggregate GDP as:

$$\frac{Lw_i}{p_i} = (AB)^{-\frac{1}{\beta}} \left(\frac{Z_i}{d_{ii}}\right)^{\theta/\beta}$$

from which we can obtain a measure of shocks Z_i. In 2.A.3 we show that a natural counterpart of the $L_i w_i / p_i$ series in our model is the Penn World Table's series of constant-price GDP in international dollars; we use the data in Chapter 2. This approach, again, is less appropriate when the sample consists of a wider selection of countries because, as mentioned in Deaton and Heston (2010), of the inferior properties of the international-dollar series for some countries;

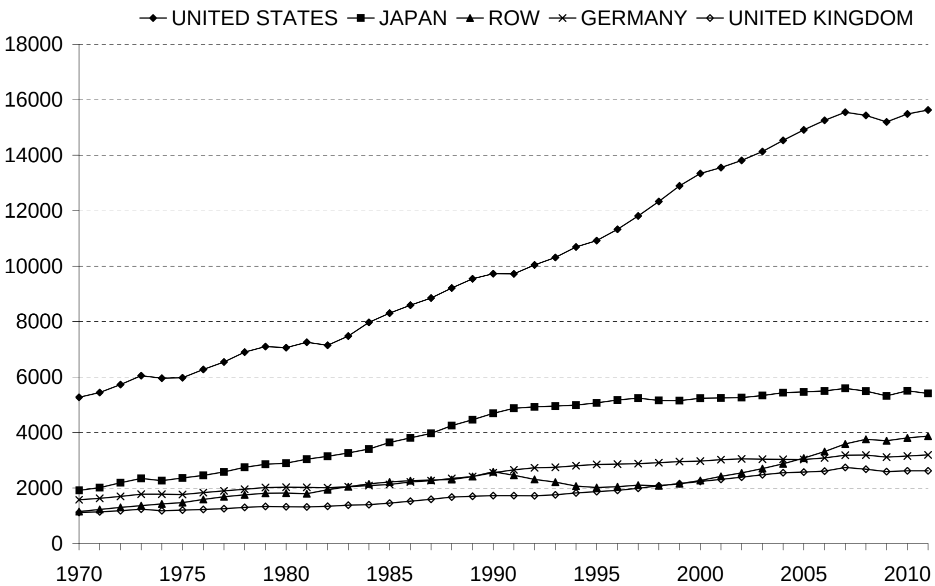
the problem becomes acute when less-developed economies are included in the sample. As the focus of our work is at the comovement of GDP fluctuations, we base our results on constant-price GDP growth rates computed with local-currency GDP data and anchor the series to the 2005 levels of GDP expressed in USD. For the base year, we favour using GDP levels converted by actual exchange rates rather than PPPs because PPPs reflect domestic purchasing power (in comparable terms) rather than actual purchasing power abroad that is key for the transmission of demand shocks. Shocks Z_{it} for 10 biggest countries in the model are shown in Figure 1.7.

Consistently with Chapter 2, where we discuss the choice of θ in bigger detail, we choose $\theta = 1/2$. Simulations reported in the previous section confirmed that both our qualitative conclusions and broadly also the quantitative estimates we make are robust to different values of this parameter.

How good is the fit of our model? We would expect the fit be quite good overall because the same variables that we use to identify the model parameters are those that we expect to get from the model. However, we do not aspire to match individual bilateral correlations because there are two sources of discrepancy that are going to worsen the pair-by-pair fit of the model: the model (1) imposes symmetry on trade costs, and (2) ignores trade imbalances. A comparison of correlation matrices is not straightforward because it involves checking of $\frac{n(n-1)}{2}$ entries; with 70 countries this is 2415 bilateral correlations of GDP growth rates. The first check we do is plotting all bilateral correlations in a scatter plot that shows, pair-by-pair, how successful the model is in matching correlations found in the data, i.e. in generating the same type of comovement. This is the left panel of Figure 1.8. We interpret the figure as giving quite strong support to the model. Obviously, there is a significant amount of mismatch but only few correlations that were significantly positive or negative in the data take the opposite sign in the model. Correlation between the data and model series is positive and high (0.86) and the slope of the trend line we fit through the data is lower but close to the 45° line.

The right panel of the figure report histograms of the bilateral correlations

Figure 1.7: Shocks Z for 10 biggest countries



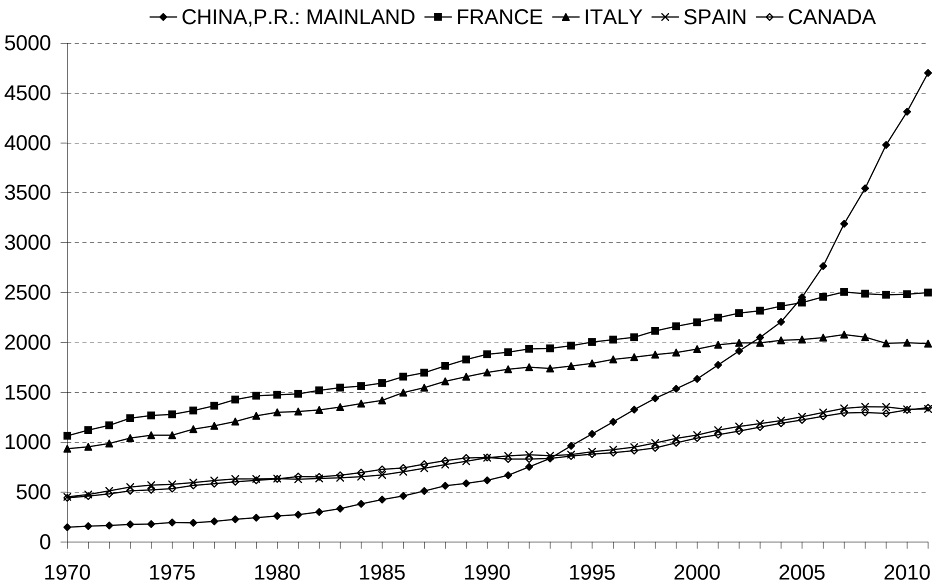
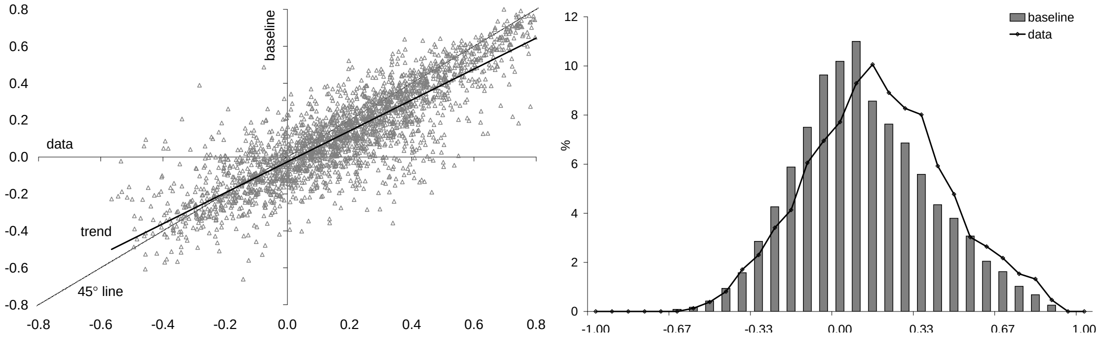


Figure 1.8: Model v Data



Note: The scatter plot shows correlations of GDP growth rates in data (horizontal axis) and as computed by our model and the histogram plots their respective distributions.

from the model (bars) and from the data (line). The point of plotting distributions is to contrast the level of dispersion and other characteristics of both series. In this particular case, the mean of correlations computed from the model is lower than found in the data (0.13 v 0.18) and the difference is more pronounced for the median of the distribution (0.11 v 0.18). To put it differently, the model-generated distribution is more skewed to the right (skewness of 0.27 v 0.09). This is not surprising; the symmetry imposed by the model makes it generate correlations centred closer to zero because trade costs within the pair, averaged by the symmetry, will be generally closer to zero except for the cases when the underlying trade intensities we use to compute κs were high for both countries in the pair. Finally, let us note that there is about the same level of variance in both distributions (0.27 v 0.28). Overall, both the scatter plot and the distributions buttress our confidence that the model is capable to replicate the data we put in in both the qualitative and quantitative sense.

1.4.2 Autarky versus baseline

Having discussed the choice of parameters and the fit of the model, the first question we investigate is the quantitative power of trade to generate comovement. We do so by comparing the baseline results from the model with correlations that we obtain by assuming complete autarky. In this case, $\kappa_{ijt} = 0$ for all $i \neq j$

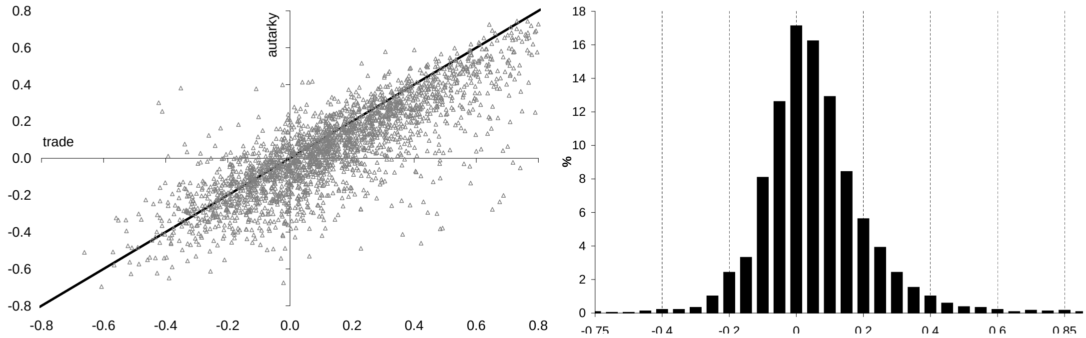
and $\kappa_{ijt} = 1$ for $i = j$. We have shown above that since the only source of volatility in autarky are the country-specific shocks Z_{it}, comovement in GDP growth rates is bound to mimic the correlations in productivity shocks. Should trade have only a marginal effect on correlations, we would expect that baseline correlations be close to those in autarky, i.e. close to the 45° line in the scatter plot Figure 1.9. However, we observe a lot of variation around the diagonal line. Since these two scenarios differ only with κs, we conclude that trade does significantly affect bilateral correlations.

What can we say about the direction of the effect? Observations below the 45° line in the figure are the bilateral correlations that have increased with trade compared with their values in autarky; the strength of the effect rises with the distance to the 45° line. This is the case for 71% pair-wise correlations while for the remaining 29% pairs, trade makes bilateral correlations lower than would be the case in autarky. The histogram of trade-induced changes in correlations (absolute differences to autarky correlations) is depicted on the right panel of the figure. The (unweighted) average of the distribution is 0.08 and median 0.06 but there is a significant amount of variability in the effects on particular correlations (standard deviation is 0.15) and the distribution is strongly skewed to the right. The point we take from this exercise is that trade has a significant effect on bilateral comovement, which is on average positive but the particular outcome depends on how much and with whom a country trades (this is consistent with the theoretical formulas derived in Section 1.3.1).

1.4.3 Role of trade costs

The question is then whether the change in comovement due to trade is driven primarily by the *level* of trade costs or rather by the additional volatility brought about by changes in κ_{ijt}. According to the simulations presented in this section, the answer is that the 'level' effect (of average trade costs) has a relatively small impact on correlations in our sample. The fact that trade costs change over time but also that they are inherently volatile, account for a larger part of the overall

Figure 1.9: Trade v Autarky



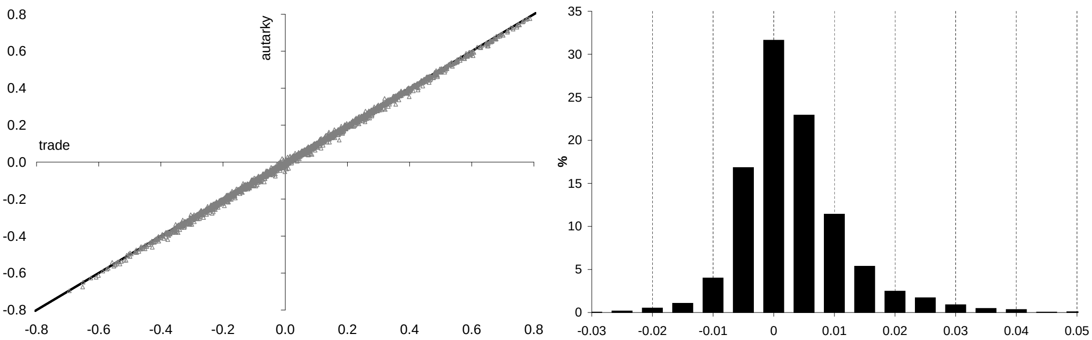
Note: The scatter plot shows bilateral correlations with trade costs as in the data (horizontal axis) against correlations in autarky. The histogram plots the distribution of the absolute change in bilateral correlations induced by trade.

effect.

First, we test the 'level' effect by asking what would the change in bilateral correlations (compared with autarky) be had trade costs been fixed over time at the bilateral averages: $\kappa_{ijt} = \overline{\kappa_{ijt}} \geq 0$. Figure 1.10 shows the scatter plot and histogram analogous to the case with actual κ_{ijt} presented above (notice the difference in scale in the histogram here relative to Figure 1.9). Clearly, correlations differ only marginally from those experienced in autarky; the mean and median change in correlations is 0.01 with standard deviation less than 0.01. Why is it the case?

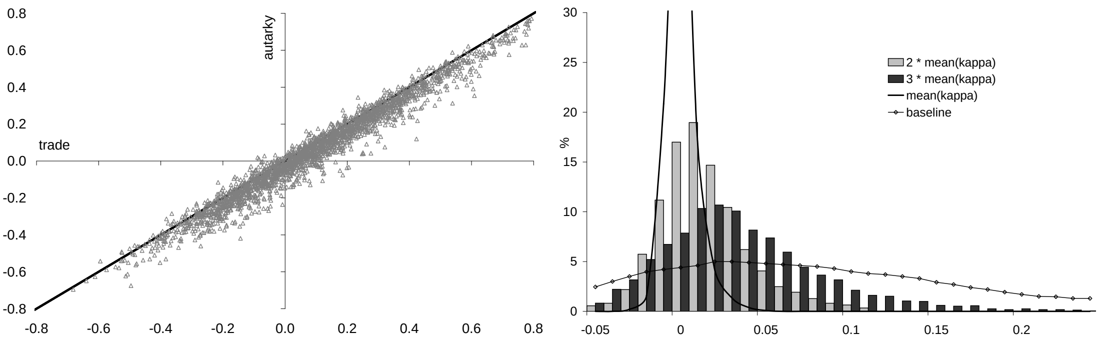
Part of the explanation for the small effect of the *level* of openness is that moves in κ from zero (autarky) to $\overline{\kappa_{ijt}}$ generally do not map one-to-one to changes in correlations; the correlation coefficient between average trade costs, i.e. $\overline{\kappa_{ijt}} - 0$, and the changes in correlations generated by them is only 0.5. However, the small average size of κ also plays a role. The right panel of Figure 1.11 presents the histograms of changes in bilateral correlations (compared with autarky) computed with $\kappa_{ijt}^{*} = 2 \cdot \overline{\kappa_{ijt}}$ and $\kappa_{ijt}^{**} = 3 \cdot \overline{\kappa_{ijt}}$, i.e. with still zero trade shocks but twice and three-times higher levels of κs (smaller trade costs). For the sake of comparison, also shown are the previously discussed distributions obtained with $\overline{\kappa_{ijt}}$ (full line) and the baseline distribution with trade shocks κ_{ijt} (with markers), and a scatter plot of correlations with $\kappa_{ijt}^{**} = 3 \cdot \overline{\kappa_{ijt}}$ against

Figure 1.10: Correlations with trade and in autarky – with constant trade costs



Note: The scatter plot shows bilateral correlations without trade shocks (horizontal axis) against correlations in autarky. The histogram plots the distribution of the absolute change in bilateral correlations induced by the *level*-change in trade costs.

Figure 1.11: Histogram of correlations with constant trade costs – larger κ



Note: The scatter plot shows bilateral correlations with constant $\kappa_{ijt}^{**} = 3 \cdot \overline{\kappa_{ijt}}$ (horizontal axis) against correlations in autarky. The histogram plots the corresponding distributions of changes in bilateral correlations (compared with autarky).

autarky. Both figures suggest that the level of trade costs has, in principle, the power to generate a quantitatively significant change in correlations. However, the average level of trade costs in our sample is too small to do so. On a 0-1 scale, with 0 being autarky and 1 costless trade, the average value of κ_{ij} is only 0.03 (median 0.02) with maximum of 0.23 recorded between Belgium and the Netherlands (see Figure 1.6 for the distribution of κs). When $\overline{\kappa_{ijt}}$ doubles and triples, the mean change in correlations (v autarky) rises from 0.01 to 0.02 (median 0.02) and to 0.04 (median 0.03), respectively.

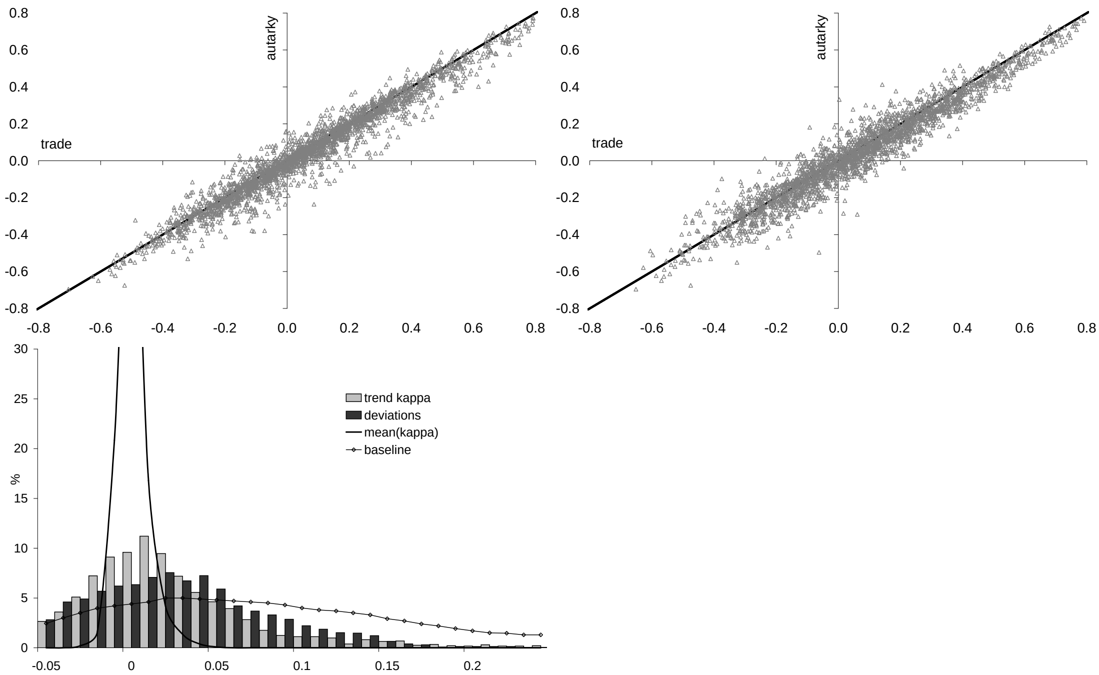
Having discussed why the 'level' effect of lowering trade costs is quantitatively small in our sample, the next question is what other factors are behind the large effects of trade overall, as shown in Figure 1.9. They could be due to strong trends in κ_{ijt}, the volatility around the trends (pure trade shocks), or both. We have illustrated in Figure 1.6 that κs do grow over time and for some countries, especially those that experienced a rapid integration into world trade channels in the sample period, the trend in κs is enormous. Cheaper and faster means of transport make the case of a synchronous downward trend in trade costs intuitively appealing. We therefore ask whether synchronised trends in κ could explain the observed change in bilateral correlations.

We examine this question by using, first, trends extracted from κ_{ijt} with an HP filter[17], and secondly, relative trade shocks applied to the sample and cross-country average $\kappa = 0.03$. Results are presented in Figure 1.12. What we learn from this figure is that both trend changes in κ_{ijt} and shocks to trade costs matter more for comovement than the average level of κ_{ijt}. The distributions of changes in correlations computed in these scenarios are closer to the baseline than those computed with constant κs. Means and medians of both distributions are still small 0.02 (compared with 0.08 in the baseline) but standard deviations rise to 0.06 and 0.07, respective (compared to 0.15 in the baseline).

To summarise the findings of the exercises presented so far, we have shown that trade matters for cross-country correlations. Opening to trade makes correlations between GDP growth rates increase 0.08 points on average and generates a significant level of dispersion (for half of the country pairs the changes in correlations are either negative and lower than -0.01 or more positive and larger than 0.15 – these are the lower and upper quartiles of the distribution). We have further shown that the main driving force behind the effects of trade on correlations seems to be the movements in trade costs over time; both their (possibly synchronised) trends and fluctuations around trends are important factors in accounting for the dispersion and, to a lesser extent, for the magnitude of the effects of trade on comovement. The average level of trade costs has, on the

[17] We use smoothing parameter 6.25, in line with Ravn (2001).

Figure 1.12: Histogram of correlations with smoothed and detrended trade costs



Note: The scatter plots shows bilateral correlations with smoothed κ_{ijt} (top left panel) and detrended κ_{ijt} (top right panel) against correlations in autarky (vertical axis). The histogram plots the corresponding distributions of changes in bilateral correlations (compared with autarky).

other hand, had only a little impact so far but could become more significant in the future, if trade costs continue falling.

1.4.4 Direct versus indirect trade

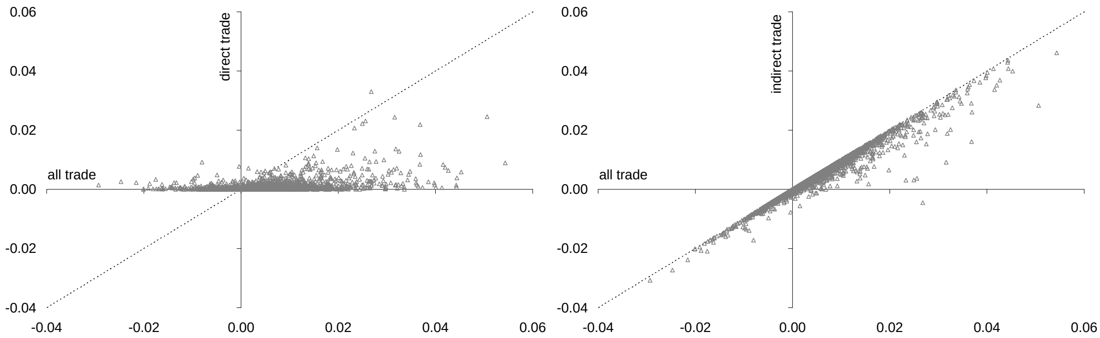
In the exercises that follow, we concentrate on the role of trade in transmitting country-specific shocks. Our aim is to assess the quantitative importance of the two competing channels we have identified in the theoretical section: the role of direct trade linkages between i and j, and the role of indirect trade between i or j with third countries. We have seen above that in a stylised setting with three countries, identical volatilities and full symmetry in trade relationship, the effect of direct trade on bilateral correlations between i and j is relatively larger that that of trade with third countries, in particular if the shocks affecting the third country are not strongly correlated with those hitting i or j. In this section we therefore test to what extent is this stylised setting representative of the patterns in actual data. In order to isolate the transmission role of trade from the effects of falling trade cost or their volatility, we freeze κ_{ij}s at their average pair-specific values.

We proceed as follows: for a country pair a, b, we keep the pair-specific average κ_{ab} as found in data[18] while cancelling all other trade linkages between countries a and b and the rest of the world, as well as among all other countries.[19] In other words, except for trade between a and b there is no other trade taking place globally. With this pattern of trade costs, the model generates a new solution for GDP with a new correlation matrix of growth among all countries. We record the impact on the coefficient of correlation between the concerned country pair and repeat the exercise for another pair. In total, we carry out $n(n-1)/2$ simulations and obtain a matrix of cor^D_{ij} with each element representing correlation between i and j when the only trade these country engage in is trade between them.

[18] For ease of notation, we now ignore bars that in the previous section denoted averages.

[19] To compute cor^D_{ab}, we set $\kappa^D_{ijt} = \kappa_{ijt}$ for $i = a, j = b$ and $i = b, j = a$ (trade costs are symmetric in our model), $\kappa^D_{iit} = 1$ as always for all i, and $\kappa^D_{ijt} = 0$ otherwise.

Figure 1.13: Effects of direct and indirect trade on comovement



Note: The variables shown in these scatter plots measure changes correlations induced by certain type of trade relative to autarky. In the left panel, we compare the scenario with direct (and no other) trade between two countries to the baseline with all trade, while the right panel compares the scenario with no direct trade. Always with no volatility in trade costs.

An alternative way of testing the importance of direct trade linkages is to start from the baseline (when all trade costs are at their averages as measured in data) and cancel only the direct pair-specific links between a and b. Recording the bilateral terms computed in this manner in cor_{ij}^{noD} and repeating the simulations for all country pairs, we obtain bilateral correlations that would prevail without any 'direct' trade between a given pair of countries. Figure 1.13 reports how the scenarios with and without direct trade perform in terms of replicating the changes in correlations obtained with both types of trade (compared always with autarky).

We contrast the changes in correlations obtained in both exercises ($cor_{ij}^{D} - cor_{ij}^{A}$ and $cor_{ij}^{noD} - cor_{ij}^{A}$) with those with both types of trade allowed (baseline with fixed trade costs). Visual inspection of scatter plots presented in Figure 1.13 suggests that the direct-trade channel performs relatively poorly compared with the indirect one. With only the direct trade, all changes in correlations are positive, which is in line with the conclusions of our theoretical analysis. However, only in few cases is the direct channel able to move correlations substantially towards those computed with both channels 'on'. Viewed from another perspective, closing the direct channel while keeping all the indirect trade is very close to the solution with both channels 'on'.

To quantify the relative role of the two channels, let us decompose the variance of the contribution of trade to correlations (always with fixed κs) to parts accounted for by direct trade, indirect trade and a residual term, respectively. To be precise, we use the following identity:

$$\left(cor_{ij} - cor_{ij}^{Aut}\right) = \left(cor_{ij}^{D} - cor_{ij}^{Aut}\right) + \left(cor_{ij}^{noD} - cor_{ij}^{Aut}\right) + (\text{residual}_{ij})$$

or with short-hand notation:

$$c_{ij}^{All} = c_{ij}^{D} + c_{ij}^{noD} + r_{ij}$$

As in Fujita and Ramey (2008), we then compute the contributions of the terms on the right-hand side to the variance of the dependent variable as follows:

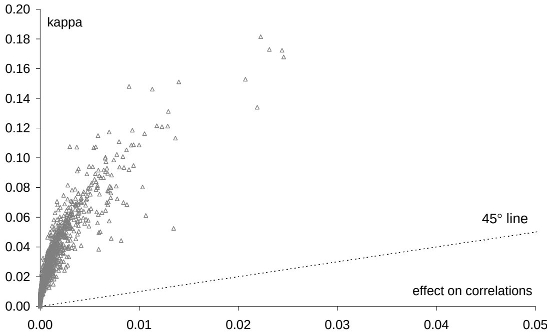
$$Var\left(c_{ij}^{All}\right) = Cov\left(c_{ij}^{All}, c_{ij}^{All}\right) = Cov\left(c_{ij}^{All}, c_{ij}^{D}\right) + Cov\left(c_{ij}^{All}, c_{ij}^{noD}\right) + Cov\left(c_{ij}^{All}, r_{ij}\right)$$

where each of the covariance terms gives the amount of variation in the total contribution of trade to comovement that is generated by the variation in the given variable. Dividing by $Var\left(c_{ij}^{All}\right)$ and multiplying by 100, we obtain the % share of variance explained by each variable. The results are the following: variability in correlations due to direct trade accounts for 9.8% of total variance, variability in correlations due to indirect trade for 90.8% and the residual term for the remaining -0.7%. We therefore conclude that the direct-trade channel tends to explain less than 10% of the 'level' effect of trade on bilateral comovement (keeping in mind that we abstract from any volatility in trade costs).

Not surprisingly, the magnitude of the average trade costs is the key determinant of the strength of the direct-trade channel. See Figure 1.14 for a plot of the relationship and a table of changes in correlations due to the direct channels for countries that encounter large changes in trade costs.

Why is the indirect channel so powerful in data, compared with the analysis in a stylised setting? There are two reasons. First, with many countries, the exposure to the other country in the pair will typically be (much) smaller than the exposure to all the other countries in the world. Also, the underlying

Figure 1.14: Trade costs and the direct-trade channel



		change in cor	kappa
BEL/LUX	NLD	0.033	0.24
CAN	USA	0.022	0.18
NLD	GER	0.023	0.17
BEL/LUX	GER	0.024	0.17
BEL/LUX	FRA	0.025	0.17
AUT	GER	0.021	0.15
MEX	USA	0.014	0.15
IRL	GBR	0.009	0.15
MALAYS.	THA	0.011	0.15
CHE	GER	0.022	0.13
FRA	GER	0.013	0.13

Note: The figure plots bilateral correlations when the only trade any two countries engage in is trade directly between them (horizontal axis) against the level of trade costs κ for the given country-pair (vertical axis). Cases with greatest impact are listed.

shocks are, on average, weakly but positively correlated. We have shown in the theoretical part that the strength of the indirect channel rises with openness to third countries and with the correlation of their shocks with the other country in the pair. Therefore, both these factors work jointly to increase the role of the indirect channel compared with the direct one.

1.4.5 Further thoughts about the indirect-trade channel

Given the significance of the indirect trade, a logical question that arises is what are the important 'third countries' that are responsible for driving the overall effect of the indirect-trade channel. We try to answer this question in three ways. The first and second one are groups of countries that almost automatically suggest them selves given their relative importance in trade of most countries: first, we test the importance for correlations of **big common trade partners** for each pair by pair (keeping all else unchanged); secondly, we test whether the openness or closure of **big global players** matters. To preview the answers, we do not find unambiguous support that one of these groups of countries is the dominant factor behind the 'indirect-trade' channel. Finally, we therefore turn to the theory presented in the previous section to guide us in the identification of the relevant countries.

We do the first two exercises as follows. First, for the n big common trade partners, we start with the baseline with all trade linkages (but average trade costs) and for each pair find the n largest common trade partners based on the geometric average of trade intensities in the baseline and impose prohibitive trade costs between these and i and j. Trade costs for all other countries are as in the baseline. Averaging the d_{ij}s geometrically gives importance to similarity, hence when we then take the first n largest $d_{ik}d_{jk}$, we pick those partners k that are important for both i and j. We do this for all the pairs in the model and record every time the ij element of the correlation matrix. Secondly, for the scenario without the big players, we impose that all countries jointly face prohibitive trade costs against (one country each time): (1) the U.S., then against (2) China, (3) Japan, (4) Germany and (5) the rest of the world aggregate ROW. Finally, we impose prohibitive trade costs against (1-5) jointly. Scatter plots are presented in Appendix 1.A.4.

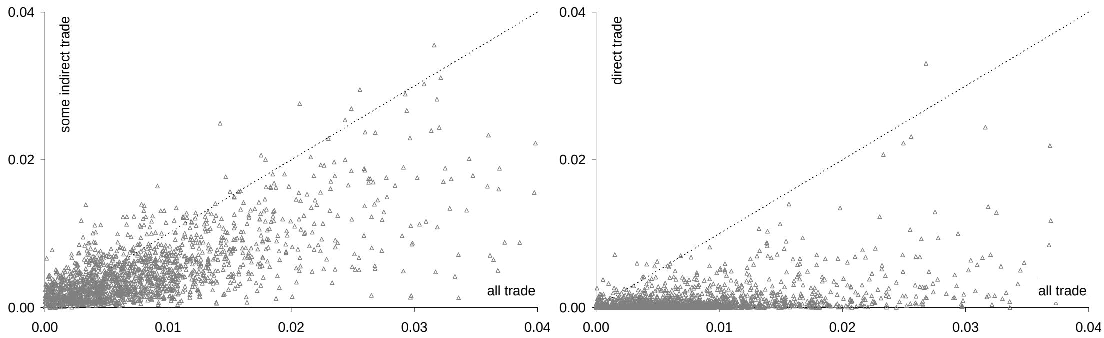
What do we find? Regarding the role of common trade partners, their presence could, in principle, matter for i and j's comovement given that typically they jointly account for a significant share of the pair's trade. Therefore, one could argue, they represent a potentially important channel for the pass-through of shocks from one country in the pair to the other. The more common partners are dropped, each of the pair could start trading with the remaining countries, potentially each with different ones, which could disrupt the overall pattern of correlations. However, Figures 1.16 – 1.20 do not provide convincing answers. The overall effect of dropping common trade partners does not seem to change correlations in a systematic manner. Those that became high (low) with all trade linkages 'on' compared with autarky typically remain high (low) when common trade partners drop. The majority of observations is still reasonably close to the diagonal line (which plots the baseline changes in correlations). The outliers, for which dropping of common trade partners matters significantly, are relatively scarce, although their number rises as the number of dropped common partners goes up.

Regarding the role of big global players, we do this exercise for the U.S., China, Japan, Germany and the rest-of-the-world aggregate (see Figures 1.21 – 1.25). Again, the systematic pattern that was there with all trade seems not to be dramatically affected. We observe the largest effects here in the cases when there is no trade with Germany and the ROW. One could argue that this is because these two countries are strongly integrated (artificially so in the case of the ROW) with a large number of still relatively big countries, playing so the role of a global trade hub. Their collapse (or, better to say, closure) might substantially distort other big countries they trade with, and consequently also the trade patterns of the remaining countries. For Japan and China, the effects are smaller and one could conjecture that this follows from their relatively lower integration with other (larger) countries around the world, so the fact that they disappear from the map of world-wide trade may still disrupt a number of countries (the effect is particularly visible for China), however, the second-order effects are not so damaging as in the case of Germany and the ROW. We would liken them to regional hubs. The U.S. seem to be an intermediate case.

To summarise, the impacts both these groups of countries have on correlations are not as large as one could have expected based on their relative importance (in terms of trade). It seems that the remaining countries, which each pair remains open to, seem powerful enough to deliver a substantial part of correlations between i and j that would be there in the baseline. However, this is not surprising in view of the results we obtained in the theoretical section. We have concluded there that two countries' growth rates will be correlated, among others, if one of the pair trades with another country that tend to be hit by similar shocks as the other country in the pair. To relate this result to the exercises presented here, it suggests that the countries we have dropped (common trade partners or globally big countries) have not fundamentally narrowed the exposure to shocks that *also* affect one of the countries in the pair.

We test whether this conjecture is empirically relevant in the following way. We close all trade (including the direct one between i and j) except five particular trade partners of i and five trade partners of j. We illustrate the principle through which we choose them on one of the pair, say country i. First, we

Figure 1.15: Direct v indirect trade with a small group of countries



Note: The left scatter plot compares changes in correlations induced by indirect trade with 5 selected countries compared to the baseline with all trade. See main text on how the 5 countries are chosen. The right panel plot similar variable for the case of direct trade only.

require that the chosen trade partners of i get hit by shocks that are positively correlated with j. This ensures that a pass-through of shocks to i that affect also j. Secondly, we require that these countries are negatively correlated with i. We do so in order to provide opportunity for the stabilising effect trade has on volatility. If i trades with someone who tends to get good shocks when i receives bad shocks, both countries become less volatile. Because variance is negatively related to correlations, this factor may also relevant for choosing the 'right' countries. Out of the countries, whose shocks are most correlated with j and least correlated with i, we take five biggest ones. We repeat the same procedure for j. For thus the selected countries k we set κ_{ik} and κ_{jk} equal to their respective averages in data and keep all other (off-diagonal) κs zero.

To see how these 5 countries for i and 5 countries for j matter for i and j's comovement, let us compare changes in correlations we obtain in this exercise (for a lack of a better name, we will refer to it as a **model-based exercise** in what follows) with those for the direct- and indirect-trade channel reported in Figure 1.13. For greater clarity, we replicate the left panel (direct trade) of the figure together with results of the model-based case in Figure 1.15, where we change the axis to focus only on cases where trade increases comovement.[20]

[20]Note that by the setup of the exercise, all correlations will be positive, therefore we do have much to say about the cases where direct or indirect trade leads to negative correlations.

Based on the results presented in Figure 1.15, we conclude that the model-based exercise is substantially more successful in matching the baseline correlations than the bilateral channel. The descriptive statistics for the subsample (65% observations) shown in the figure are as follows: the mean change in correlations (trade v autarky) in the baseline is 0.009; direct trade delivers average change only of 0.001 while the indirect-trade case takes us to 0.008; the model-based case gives 0.005. In terms of volatility, the model-based one generates standard deviations of around two-thirds of the those obtained in the baseline and with indirect trade (0.008 and 0.007 respectively, compared with 0.005 in the theory-guided case), while volatility with direct trade is only 0.002. Overall, the figure suggests that a simple model-based selection of countries is capable to generate the same type of a systematic relationship that we observe in the baseline. For the country pairs where trade with all countries results in positive and high comovement, trade with the few countries selected according to our model gives also systematically higher comovement. The slope of a line fitted through the model exercise is 0.52, compared with 0.87 with all indirect trade and 0.13 with direct trade only. This is in a stark contrast to the cases without several common trade partners and big countries, which were not able to generate systematic departures from the baseline.

1.5 Conclusions

This chapter investigates the relationship between across-the-border trade in goods and comovement in real GDP growth. We use data for a large sample of countries that jointly account for 95% of world GDP. The particular questions we ask are: Does openness to trade affect cross-country correlations? If so, what determines whether two countries' growth rates will be positively correlated?

We look at the relationship through the lens of a standard general equilibrium framework that conceptualises the demand and supply linkages among countries, which then work as a network for transmission of shocks. The model gives an affirmative answer to the first question above. Comparing our baseline scenario (parameterised with trade and real GDP data) to autarky, we find that bilateral correlations in the baseline are, on average, 0.08 higher than what they would have been in autarky. However, there is a large amount of dispersion in the effect going in both directions. We therefore conclude that trade matters for comovement but the magnitude and sign of a particular realisation of the effect depends on with whom and how much countries trade.

To answer the question on *how* trade affects comovement, we show that the main driving force seems to have been the movements in trade costs over time; both their (possibly synchronised) trends and fluctuations around trends are important factors in accounting for the effects trade has had comovement. Openness itself, i.e. the average level of trade costs, has had only a limited impact so far but we show that if trade costs continue falling, the level of openness gains on importance in the future.

Finally, we investigate the transmission of country-specific shocks through trade. Deriving a closed-form formula for bilateral correlations, we confirm that trade generates comovement as it exposes countries to shocks that affect their direct trade partners. The role of this 'direct' channel has been a topic of many studies estimating the links between comovement and bilateral trade between i and j. However, we also point to another, less well studied channel that generates comovement among countries irrespectively whether they trade with each other or not. The presence of this 'indirect' (or third-country) channel, if not controlled for, could introduce a systematic bias in the relationship between direct trade and comovement.

The condition for the latter to generate comovement between any two countries is *not*, how one could think, that both of them would have to trade with the same third country, which would than act as an intermediary for transmitting their mutual shocks – while this channel may play a role, it is of second-order importance. Our formulas point to a more straightforward way in which the indirect channel matters: it suffices that countries, one of the pair trades with, tend to receive similar shocks as the other in the pair. In other words, trade between i and k, which exposes i to k's shocks, generates comovement also between

i and j if k and j tend to be hit by similar shocks.

Contrary to the conventional wisdom, we show that trade intensity between any two countries bilaterally matters little for their comovement. In fact, 90% of comovement (with trade costs fixed over time) is accounted for by trade with third countries, i.e. by the indirect channel. Our findings could help shape the identification strategies used in empirical studies because the formulas we have derived suggest what factors need to be controlled for if one wishes to isolate the influence third countries have on bilateral comovement.

1.A Appendix

1.A.1 Proof that trade identities (1.3) are not linearly independent

Summing first (n-1) equations in the system defined by equation (1.3)

$$\sum_{i=1}^{n-1} L_i p_{mi} q_i + \sum_{i=1}^{n-1} S_i = \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n} L_j p_{mj} q_j d_{ji} \right)$$

Using $\sum_{j=1}^{n} S_i = 0$ and rearranging the right-hand side

$$\sum_{i=1}^{n-1} L_i p_{mi} q_i - S_n = \sum_{j=1}^{n} L_j p_{mj} q_j \left(\sum_{i=1}^{n-1} d_{ji} \right)$$

Applying $\sum_{j=1}^{n} d_{ij} = 1$

$$\sum_{i=1}^{n-1} L_i p_{mi} q_i - S_n = \sum_{j=1}^{n} L_j p_{mj} q_j \left(1 - d_{jn}\right)$$

Cancelling terms

$$-S_n = L_n p_{mn} q_n - \sum_{j=1}^{n} L_j p_{mj} q_j d_{jn}$$

Rearranging to get the n-th equation in (1.3)

$$L_n p_{mn} q_n + S_n = \sum_{j=1}^{n} L_j p_{mj} q_j d_{jn}$$

1.A.2 Covariance matrix of $\hat{\kappa}$

The matrix below illustrates the covariance matrix $\mathbf{\Omega}_\kappa = \mathbb{E}\Big(\text{vec}(\mathbf{K})\,\text{vec}(\mathbf{K})^T\Big)$ of vectorised trade shocks $\hat{\kappa}$s. We assume constant variances σ_κ and covariances $\sigma_{\kappa\kappa}$ across pairs of countries $ij - kl$. Notice that because $\kappa_{iit} = 1$ and therefore $\hat{\kappa}_{iit} = 0$, the $(ii - ii)^{th}$, some of the elements in the covariance matrix are zero.

$$\mathbf{\Omega}_\kappa = \begin{array}{cccccccccc}
 & \hat{\kappa}_{11} & \hat{\kappa}_{21} & \hat{\kappa}_{31} & \hat{\kappa}_{12} & \hat{\kappa}_{22} & \hat{\kappa}_{32} & \hat{\kappa}_{13} & \hat{\kappa}_{23} & \hat{\kappa}_{33} \\
\hat{\kappa}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\kappa}_{21} & 0 & \sigma_\kappa & \sigma_{\kappa\kappa} & \sigma_\kappa & 0 & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & 0 \\
\hat{\kappa}_{31} & 0 & \sigma_{\kappa\kappa} & \sigma_\kappa & \sigma_{\kappa\kappa} & 0 & \sigma_{\kappa\kappa} & 1 & \sigma_{\kappa\kappa} & 0 \\
\hat{\kappa}_{12} & 0 & \sigma_\kappa & \sigma_{\kappa\kappa} & \sigma_\kappa & 0 & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & 0 \\
\hat{\kappa}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\kappa}_{32} & 0 & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & 0 & \sigma_\kappa & \sigma_{\kappa\kappa} & \sigma_\kappa & 0 \\
\hat{\kappa}_{13} & 0 & \sigma_{\kappa\kappa} & \sigma_\kappa & \sigma_{\kappa\kappa} & 0 & \sigma_{\kappa\kappa} & \sigma_\kappa & \sigma_{\kappa\kappa} & 0 \\
\hat{\kappa}_{23} & 0 & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & \sigma_{\kappa\kappa} & 0 & \sigma_\kappa & \sigma_{\kappa\kappa} & \sigma_\kappa & 0 \\
\hat{\kappa}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

1.A.3 Data sources

The purpose of this section is to document our data sources and transformations we carry out on the data.

Trade Data: We use US$ bilateral imports data, I_{ij}, from 1989 to 2011 from the IMF's Direction of Trade Statistics (DOTS). Based on the treatment of certain trade unions in DOTS, we merge all data for Belgium and Luxembourg, and South Africa and Botswana into their respective aggregates.

GDP in current prices: We take current-price GDP (valued in US$) for 1989-2010 from World Bank, World Development Indicators, variable NY GDP MKTP CD. Supplementary sources for missing observations are UN Data and New Zealand Statistical Office, growth rates for 2011 (and when missing for 2010) are taken from IMF World Economic Outlook (these may be semi-final estimates or IMF's forecasts).

GDP in constant prices: Constant-price GDP is constructed as follows. First we fix the base year using GDP in 2005 denominated in USD (IMF's World Economic Outlook). Other years are linked to the base year by means of growth rates computed on local-currency constant-price data (the source is IMF World Economic Outlook again).

1.A.4 Figures - scenarios without common / big trade partners

The scatter plots below show changes in correlations induced by certain type of trade relative to autarky. On the horizontal axis (common to all figures) is change in correlations in GDP growth between i and j with all trade costs as in data (average values of trade costs) relative to autarky. Variables shown on the vertical axis differ across figures but always refer to the exercise when i and j stops trading with one or more common trade partners or alternatively, when all countries stop trading with one or more globally big countries, again relative to autarky. Should the dropping of the respective country matter, observations will be further apart from the 45° line. If observations remain close to the diagonal line, trade with the dropped countries is not the key determinant of i and j's mutual correlations.

Figure 1.16: Change in correlations: 1 largest common trade partner stops trading with i and j

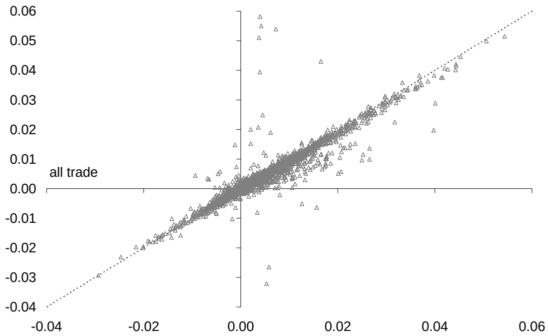


Figure 1.17: Change in correlations: 2 largest common trade partners stop trading with i and j

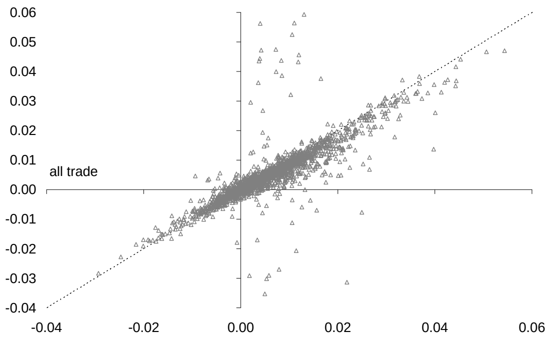


Figure 1.18: Change in correlations: 3 largest common trade partners stop trading with i and j

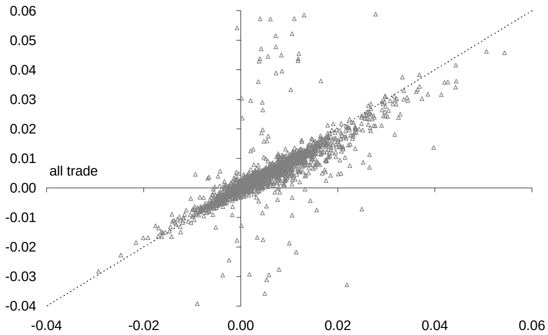


Figure 1.19: Change in correlations: 4 largest common trade partners stop trading with i and j

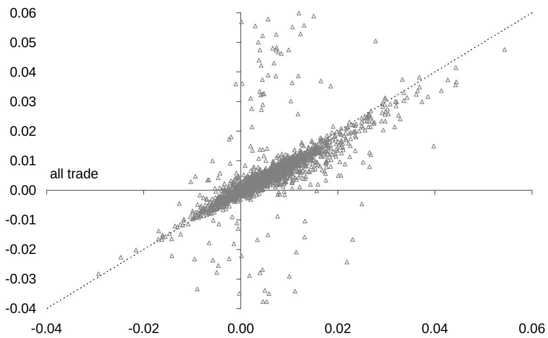


Figure 1.20: Change in correlations: 5 largest common trade partners stop trading with i and j

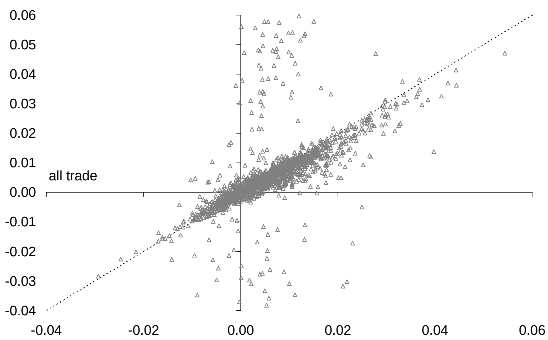


Figure 1.21: Change in correlations: the U.S. stops trading with all

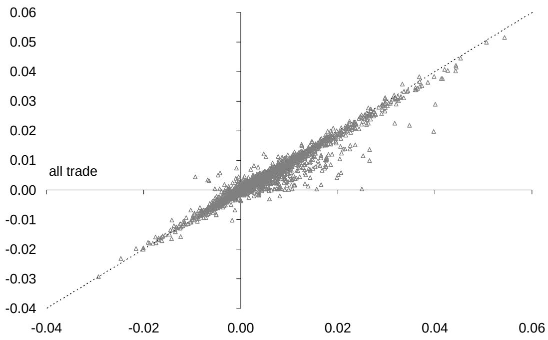


Figure 1.22: Change in correlations: Japan stops trading with all

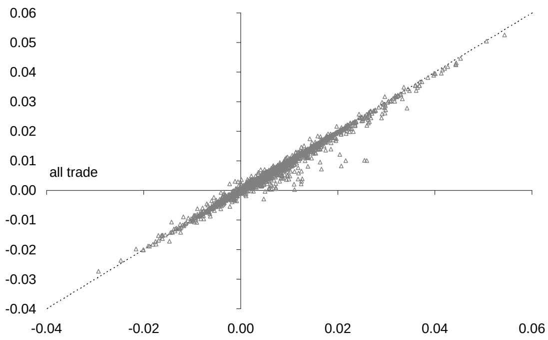


Figure 1.23: Change in correlations: China stops trading with all

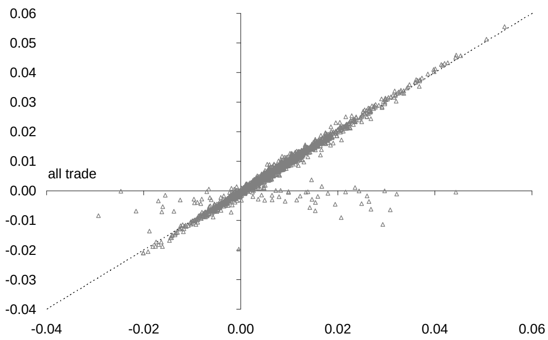


Figure 1.24: Change in correlations: Germany stops trading with all

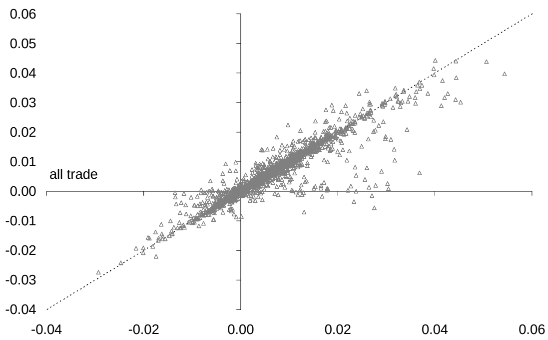


Figure 1.25: Change in correlations: Rest-of-the-world stops trading with all

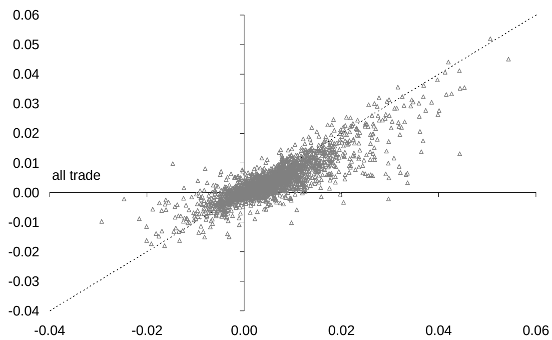
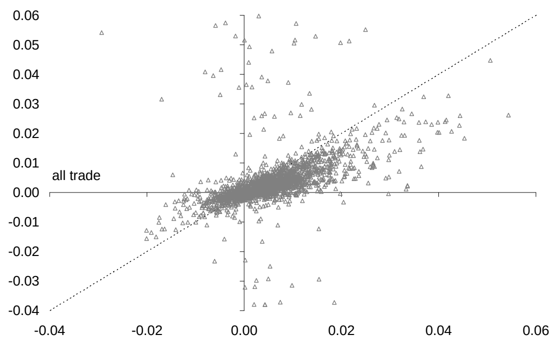


Figure 1.26: Change in correlations: All 5 biggest countries stop trading with all



1.A.5 List of countries

ALGERIA	DZA	KOREA	KOR
ARGENTINA	ARG	KUWAIT	KWT
AUSTRALIA	AUS	MADAGASCAR	MDG
AUSTRIA	AUT	MALAYSIA	MYS
BANGLADESH	BGD	MEXICO	MEX
BELGIUM and LUXEMBOURG	BEL	MOROCCO	MAR
BRAZIL	BRA	NETHERLANDS	NLD
BULGARIA	BGR	NEW ZEALAND	NZL
CANADA	CAN	NIGERIA	NGA
COLOMBIA	COL	NORWAY	NOR
COSTA RICA	CRI	OMAN	OMN
COTE D IVOIRE	CIV	PAKISTAN	PAK
CUBA	CUB	PERU	PER
CYPRUS	CYP	PHILIPPINES	PHL
DENMARK	DNK	POLAND	POL
DOMINICAN REPUBLIC	DOM	PORTUGAL	PRT
ECUADOR	ECU	QATAR	QAT
EGYPT	EGY	ROMANIA	ROM
FINLAND	FIN	SOUTH AFRICA	ZAF
FRANCE	FRA	SAUDI ARABIA	SAU
GERMANY	DEU	SPAIN	ESP
GHANA	GHA	SRI LANKA	LKA
GREECE	GRC	SUDAN	SDN
GUATEMALA	GTM	SWEDEN	SWE
HUNGARY	HUN	SWITZERLAND	CHE
CHILE	CHL	SYRIAN ARAB REPUBLIC	SYR
CHINA (MAINLAND)	CHN	THAILAND	THA
INDIA	IND	TUNISIA	TUN
INDONESIA	IDN	TURKEY	TUR
IRELAND	IRL	UNITED ARAB EMIRATES	ARE
ISRAEL	ISR	UNITED KINGDOM	GBR
ITALY	ITA	UNITED STATES	USA
JAMAICA	JAM	URUGUAY	URY
JAPAN	JPN	VENEZUELA	VEN
KENYA	KEN	ROW	ROW

2

Diversification through Trade

Joint work with

Francesco Caselli, Miklos Koren and Silvana Tenreyro

2.1 Introduction

An important question at the crossroads of macro-development and international economics is whether (and how) openness to trade affects macroeconomic volatility. A widely held view in academic and policy discussions is that international trade leads to higher GDP volatility. The origins of this view are rooted in a large class of theories of international trade predicting that openness to trade increases specialization. Because specialization (or lack of diversification) in production tends to increase a country's exposure to shocks specific to the sectors (or range of products) in which the country specializes, it is generally inferred that trade increases volatility.

This paper revisits the theoretical case for a positive effect of trade on volatility. In particular, it begins by pointing out that the existing wisdom is strongly predicated on the assumption that sector-specific shocks are the dominant source of GDP volatility. Koren and Tenreyro (2007), however, find that country-specific shocks (common to all sectors within a country) are at least as important in shaping volatility patterns in developed countries, and more critically so in developing countries. We argue in this paper that the impact of trade on volatility can be remarkably different if imperfectly correlated country-specific shocks are indeed the dominant source of volatility. Concretely, using one of the canonical models of international trade, we show that openness to trade, by reducing exposure to domestic shocks, can lead to lower GDP volatility; this will be true as long as the volatility of trading partners and covariance of shocks across countries are not too big; in other words, trade can act as a channel for the diversification of country-specific shocks and in that way contribute to lower volatility. More generally, the sign and size of the effect depends critically on the variance-covariance of shocks across countries.

To make our point, we study a model of trade and GDP determination in which shocks are country-specific, affecting all sectors in a country. The model builds on a variation of the Eaton and Kortum (2002) and Alvarez and Lucas (2007)'s model[1], augmented to allow for aggregate shocks. Production combines labour and a variety of tradable inputs that are subject to cost shocks. Some of these shocks are idiosyncratic, as in the original EKAL model, and some are aggregate, affecting all sectors in the country. The model delivers the following predictions. If country-specific shocks are *iid* across countries, a multilateral move from autarky to costless free trade unambiguously reduces volatility in all countries. The reduction in volatility is stronger the smaller the country, ceteris paribus. This is because a smaller country trades relatively more (relative to its GDP) and hence can more easily diversify the exposure to its own-country shocks, both on the demand and supply side. Results can be reversed, however, if the variances of and covariances with trading partners' shocks are high enough. The model also shows that a move from autarky to free trade causes the covariance of growth rates across countries with the rest of the world to increase; this increase is smaller for bigger countries, which, by their sheer size will be relatively less affected by the increase in trade openness. (As is common in Ricardian models, the increase in trade due to lower transaction costs

[1] Henceafter referred to as the EKAL model or each paper separately as EK or AL.

will unambiguously increase the level of output in all countries, but more so in smaller countries).

The model is thus capable (at least qualitatively) to reconcile the substantial and widespread increase in trade flows over the past 30 years, together with the substantial decline in macroeconomic volatility during the same period; it is also consistent with the shoot up in volatility in 2008-2010 and the contraction of trade amidst the crisis. As the model makes clear, however, openness to trade does not always lead to lower volatility: The sign and size of the effect can vary substantially across countries (and, critically, with the set of trading partners). This might explain why direct evidence on the effect of openness on volatility has been ambiguous at best. Some studies find that trade decreases volatility (e.g. Buch, Dropke and Strotmann (2006) for Germany and Burgess and Donaldson (2010)'s for India), while others find that trade increases it (Easterly and Kraay (2000)).

The second part of the paper attempts a quantification of the contribution of trade to the observed changes in volatility since 1970 in a large group of countries. Using a calibrated version of the model developed above, we try to answer the question: How much of the changes of volatility since the 1970s can be attributed to a decline in overall barriers to trade?

The chapter is organized as follows. Section 2.2 presents the model and solves analytically for two special cases, autarky and costless free trade. Section 2.3 presents numerical illustrations. Section 2.4 introduces the data and calibration and, finally, Section 2.5 discusses our quantitative results.

2.2 Model of trade with aggregate shocks

The model is a basic version of EKAL, with aggregate shocks (stochastic λ). There is a continuum of goods $q(x)$ which are produced using equipped labour L (unproduced) and all other produced goods. In particular, each good $q(x)$ is produced by a Cobb–Douglas production function in L and a CES bundle of all the intermediate inputs $q(x)$. Aside from being used in the production of other goods, the $q(x)$s can also be directly consumed. As in EK, the utility derived from consumption takes the same CES form in which the $q(x)$s enter the production function. Notice that the $q(x)$s are therefore both intermediate (when used in producing other $q(x)$s) and final goods (when used in consumption). This is consistent with the national accounts where each sector's output can be both used as intermediate by other sectors and as a final good by consumers. All produced goods $q(x)$ are in principle tradable in international markets (though the cost for some could be very big – so big that they may not end up being traded in equilibrium and only produced domestically).

For the sake of exposition, we first discuss the model in autarky and then allow for international trade. All production is subject to constant returns and we conduct the analysis of the closed economy in units of the economy's endowment L_i. For simplicity, we suppress the subindex i in the description of the closed economy.

2.2.1 Closed economy

Total factor productivity (TFP) varies across intermediate goods; the inverse of TFP levels, x, are modelled as random variables, independent across goods, with common density ϕ. Buyers (who could be final consumers or firms buying intermediate inputs) purchase individual goods $q(x)$ to maximize the CES objective:

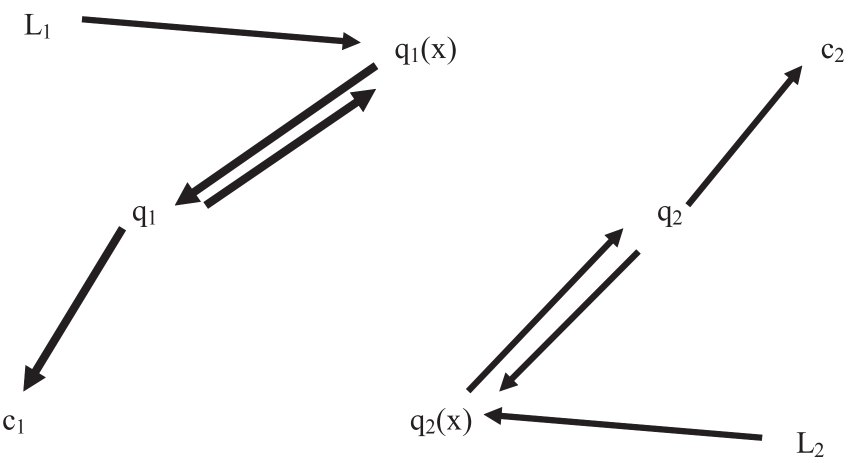
$$q = \left(\int_0^\infty q(x)^{\frac{\eta-1}{\eta}} \phi(x) dx \right)^{\frac{\eta}{\eta-1}}$$

where $\eta > 0$ is the elasticity of substitution across goods. The part of the bundle q that is directly consumed will be denoted c and the part that enters production of $q(x)$ as intermediate inputs q_m. The technology for $q(x)$ is Cobb-Douglas in the effective labour input $s(x)$ and the bundle of intermediate goods q_m defined above:

$$q(x) = x^{-\theta} s(x)^\beta q_m(x)^{1-\beta}$$

The structure of the economy (for two countries that do not trade with each other) is shown in Figure 2.1. The cost draws x are common to all producers in

Figure 2.1: Structure of production in autarky (2 countries)



the economy. Because of constant returns, the number of producers is indeterminate and there is no market power: prices are set at marginal costs; autarky prices of intermediate goods are hence given by:

$$p(x) = Bx^{\theta} w^{\beta} p^{1-\beta}$$

where w is the unit cost of equipped labour, and $B = \beta^{\beta} (1-\beta)^{(1-\beta)}$. Following EKAL, we assume that the density ϕ follows an exponential distribution with parameter λ, $x \sim exp(\lambda)$ and hence the price of q is given by:

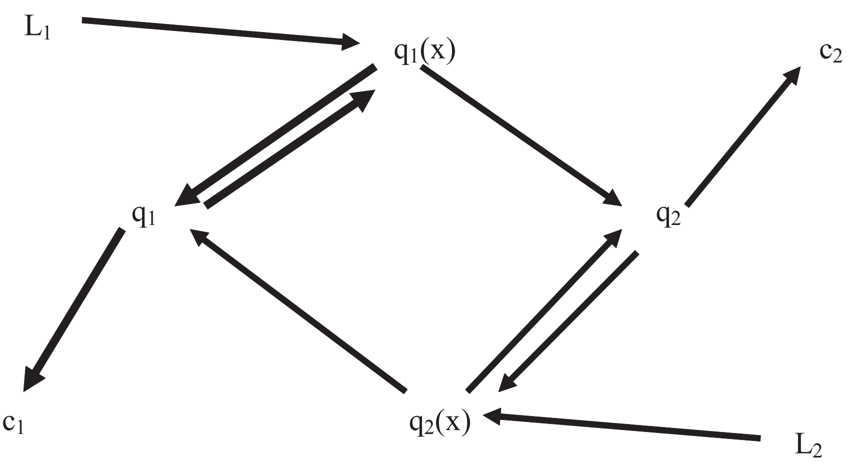
$$p = \left(\lambda \int_0^{\infty} p(x)^{1-\eta} e^{-\lambda x} dx \right)^{\frac{1}{1-\eta}}$$

With some algebra, $p(x)$ and p can be written as multiples of w:

$$p(x) = A^{(1-\beta)/\beta} B^{1/\beta} x^{\theta} \lambda^{\frac{-\theta(1-\beta)}{\beta}} w$$
$$p = (AB)^{1/\beta} \lambda^{-\theta/\beta} w$$

This is a slightly modified version of the EKAL model, which assumes a common distribution of productivity for the whole economy (not just manufacturing, as in EKAL's interpretation); in EKAL, there is a separate non-tradable final good sector (identified with services) with deterministic common technology across all countries. As said, we pose no stark distinction between tradables

Figure 2.2: Structure of production with trade



and non-tradables and rather focus on the average degree of tradability for the whole economy. This modification requires a slightly different interpretation of the empirical counterparts of the model, which we will address at the calibration stage.

2.2.2 International trade

As in EKAL, we assume that intermediate inputs $q(x)$ can be traded internationally; $\phi(x) = \phi(x_1, ..., x_N)$ is now the joint density of goods that have productivity draws $x = (x_1, ..., x_N)$ across countries, where the draws are assumed to be independent across countries: $\phi(x) = (\prod \lambda_i) \exp\left[-\sum \lambda_i x_i\right]$. The structure of production can be then summarized as shown in Figure 2.2.

Delivering a tradable good from country j to country i results in $0 < \kappa_{ij} \leq 1$ goods arriving at j; we assume $\kappa_{ij} \geq \kappa_{ik}\kappa_{kj}$ for all i, k, j and $\kappa_{ii} = 1$. All costs incurred are a net loss. In the calibration, the κs will reflect all costs, including tariffs; so implicitly we adopt the extreme assumption that tariffs are all wasted (perhaps in political elections). The intermediate bundle for use in country i is then:

$$q_i = \left(\int_{\mathbf{R}_+^N} q_i(x)^{\frac{\eta-1}{\eta}} \phi(x) dx \right)^{\frac{\eta}{\eta-1}}$$

where $\phi(x)$ is the probability density function of goods with technology x. The price level in country i is now given by:

$$p_i(w) = AB\left(\sum_{j=1}^{N}\left(\frac{w_j^{\beta}p_j(w)^{1-\beta}}{\kappa_{ij}}\right)^{-1/\theta}\lambda_j\right)^{-\theta}$$

which leads to N equations (p_i) to be solved in terms of $w_i, i = 1, ..., N$. Defining $d_{ij}(w)$ as the fraction of country i's total spending $L_ip_iq_i$ that is spent on goods from country j:

$$d_{ij}(w) = (AB)^{-1/\theta}\left(\frac{w_j^{\beta}p_j(w)^{1-\beta}}{p_i(w)\kappa_{ij}}\right)^{-1/\theta}\lambda_j \tag{2.1}$$

The trade identity requires that dollar payments for goods flowing out of country i to the rest of the world must equal payments flowing in country i from the rest of the world. Allowing for trade imbalance S_i and with $\sum_j d_{ij} = 1$,

$$L_ip_iq_i + S_i = \sum_{j=1}^{N} L_jp_jq_jd_{ji}(w)$$

The Cobb-Douglas assumption and the overall resource constraint for the economy further imply[2]

$$L_iw_i = \beta(L_ip_iq_i + S_i) \tag{2.2}$$

The trade identity therefore simplifies to

$$\frac{L_iw_i}{\beta} = \sum_{j=1}^{N}\left(\frac{L_jw_j}{\beta} - S_j\right)d_{ji}(w) \tag{2.3}$$

In the original EKAL model, the productivity parameters λs are deterministic, so GDP per capita is a deterministic constant for each country j. As said, we assume that λs are subject to shocks. In particular, higher realizations of λ_j lead to stochastically lower costs x in country j and higher GDP$_j$. Stochasticity in λ_j thus imparts stochasticity in GDP$_j$. It is instructive to look at two extreme cases: 1) complete autarky and 2) costless international trade.

[2]Derivation of this equation is shown in the Appendix.

2.2.3 Volatility in autarky

We study the volatility of real GDP, $Y_i = \frac{L_i w_i}{p_i}$, measured as the variance of deviations from mean. Autarkic prices and real GDP are given by:

$$p_i = (AB)^{1/\beta} \lambda_i^{-\theta/\beta} w_i$$
$$Y_i = \frac{L_i w_i}{p_i} = (AB)^{-1/\beta} \lambda_i^{\theta/\beta} L_i$$

Call $Z_i = \lambda_i L_i^{\beta/\theta}$ the weighted productivity of the economy (weighted by its size). Therefore, $Y_i = (AB)^{-1/\beta} Z_i^{\theta/\beta}$; denoting by $\hat{x} \equiv \frac{\Delta \ln x}{\Delta t}$ and evaluating changes around the mean of Z_i, we obtain:

$$\hat{Y}_i = \frac{\theta}{\beta} \hat{Z}_i$$

And hence volatility is given by:

$$Var(\hat{Y}_i) = \left(\frac{\theta}{\beta}\right)^2 Var(\hat{Z}_i)$$

2.2.4 Volatility with costless trade

With no impediments to trade, $\kappa_{ij} = 1$ and trade imbalances zero, we have:

$$p_j = p = (AB)^{1/\beta} \left(\sum_{j=1}^{N} w_j^{-\beta/\theta} \lambda_j \right)^{-\theta/\beta} \tag{2.4}$$

Using this in the formula for trade shares (2.1), we have

$$d_{ji}(w) = w_i^{-\beta/\theta} \lambda_i \left(\sum_{j=1}^{N} w_j^{-\beta/\theta} \lambda_j \right)^{-1}$$

and from the trade identity (2.3) we obtain,

$$w_i = \left(\frac{\lambda_i}{L_i}\right)^{\frac{\theta}{\theta+\beta}} M \tag{2.5}$$

where $M = \left(\sum_{j=1}^{n} \frac{L_j w_j}{\sum_{k=1}^{n} w_k^{-\beta/\theta} \lambda_k} \right)^{\frac{\theta}{\theta+\beta}}$ is common to all countries. Therefore[3]:

$$Y_i = (AB)^{-1/\beta} Z_i^{\frac{\theta}{\beta+\theta}} \left(\sum_{j=1}^{N} Z_j^{\frac{\theta}{\theta+\beta}} \right)^{\theta/\beta}$$

[3] With (2.4) and (2.5), ratio $\frac{w_i}{p_i}$ becomes $(AB)^{-1/\beta} \left(\frac{\lambda_i}{L_i}\right)^{\frac{\theta}{\beta+\theta}} M \left(\sum_{j=1}^{n} \left(\left(\frac{\lambda_j}{Lj}\right)^{\frac{\theta}{\theta+\beta}} M \right)^{-\beta/\theta} \lambda_j \right)^{\theta/\beta}$. Multiplying by L_i and simplifying gives the result.

where $Z_i = \lambda_i L_i^{\beta/\theta}$ as before. The log-linear approximation of this is

$$\hat{Y}_i = \frac{\theta}{\beta+\theta}\hat{Z}_i + \frac{\theta^2}{\beta(\beta+\theta)}\sum_{j=1}^{N}\gamma_j\hat{Z}_j \tag{2.6}$$

where $\gamma_j = \frac{\bar{Z}_j^{\frac{\theta}{\beta+\theta}}}{\sum_{j=1}^{N}\bar{Z}_j^{\frac{\theta}{\beta+\theta}}}$ is the country j's share in the sum of weighted productivities of all countries. Rearranging, we have:

$$\hat{Y}_i = \frac{\theta}{\beta}\left(\frac{\beta+\theta\gamma_i}{\beta+\theta}\right)\hat{Z}_i + \frac{\theta^2}{\beta(\beta+\theta)}\sum_{j\neq i}\gamma_j\hat{Z}_j \tag{2.7}$$

And therefore, volatility in free trade is given by:

$$\begin{aligned} Var(\hat{Y}_i) \quad = \quad & \left(\frac{\theta}{\beta}\right)^2\left\{\left(\frac{\beta+\theta\gamma_i}{\beta+\theta}\right)^2 Var(\hat{Z}_i) + \left[\frac{\theta}{\beta+\theta}\right]^2\sum_{j\neq i}\gamma_j^2 Var(\hat{Z}_j)\right\} \\ & +2\left(\frac{\theta}{\beta(\beta+\theta)}\right)^2\theta\frac{\beta+\theta\gamma_i}{(\beta+\theta)}\sum_{j\neq i}\gamma_j Cov(\hat{Z}_{j,}\hat{Z}_i) \end{aligned} \tag{2.8}$$

Compared with the variance in autarky, $Var(\hat{Y}_i) = \left(\frac{\theta}{\beta}\right)^2 Var(\hat{Z}_i)$, it is clear that the volatility due to domestic productivity fluctuations, $Var(\hat{Z}_i)$, now receives a smaller weight because $\left[\frac{\beta+\theta\gamma_i}{\beta+\theta}\right] < 1$ since $\gamma_i < 1$. The smaller the country in terms of its presence in international trade, the smaller the impact of domestic volatility of shocks, $\hat{Z}_i$, on its GDP, relative to autarky. Openness to trade, however, exposes the country to other countries' productivity shocks and these contribute positively to volatility. The question is then whether the gain in diversification (given by lower exposure to domestic productivity) is bigger than the increased exposure to new shocks. The answer depends on the relative sizes of the countries and the variance-covariance matrix of shocks across them. If all countries have the same variance $Var(\hat{Z}_j) = \sigma$ and the $\hat{Z}_j$ are uncorrelated, the volatility of the country in free trade (2.8) becomes:

$$Var(\hat{Y}_i) = \left(\frac{\theta}{\beta}\right)^2\left\{\left[\frac{\beta+\theta\gamma_i}{\beta+\theta}\right]^2 + \left[\frac{\theta}{\beta+\theta}\right]^2\sum_{j\neq i}\gamma_j^2\right\}\sigma$$

which is lower than the volatility in autarky if an only if:

$$\left[\frac{\beta+\theta\gamma_i}{\beta+\theta}\right]^2 + \left[\frac{\theta}{\beta+\theta}\right]^2\sum_{j\neq i}\gamma_j^2 < 1$$

Or, put differently, iff[4]:

$$2\beta\theta(\gamma_i - 1) + \theta^2 \left[\sum_j \gamma_j^2 - 1 \right] < 0 \tag{2.9}$$

which is always true (recall $\gamma_j < 1$) and $\sum_{j=1}^{N} \gamma_j^2 \leq 1$. Of course, if other countries have higher variances or the covariance terms are important, then the weights countries receive matter and the resulting change in volatility cannot be signed.

2.3 Numerical illustrations

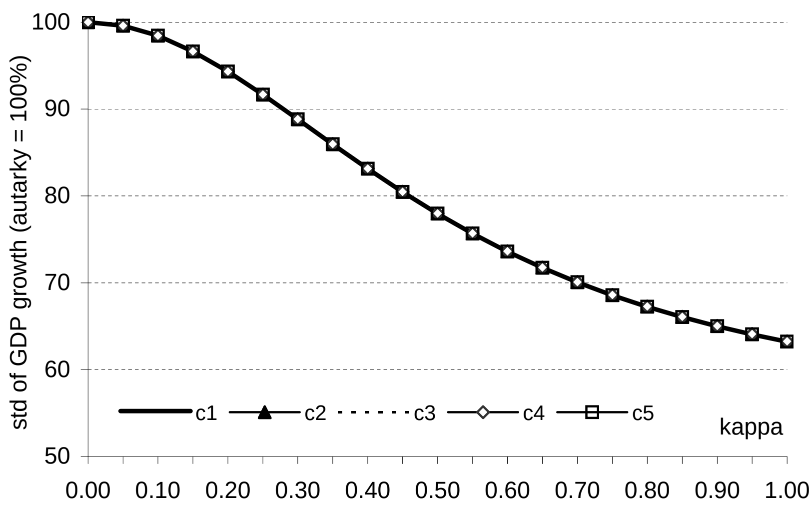
We simulate the model for many periods (or realizations of λ_j) and obtain simulated time series of $\text{GDP}_{t,j}$ for different degrees of openness, gauged by trade costs κ. This exercise is aimed at confirming the intuition on the qualitative mechanism; later on we attempt a more realistic calibration. We then compute volatility of each country's GDP. The qualitative exercise consists of drawing $\lambda = (\lambda_1 ... \lambda_n)$ each period from a normal distribution with fixed mean and std deviation (matching average values in the sample); we choose θ, α, and β as in AL. We then explore the following (qualitative) experiments: 1) Widespread decrease in international trade barriers, 2) A Big Country joins the World, and 3) A crisis hits a big country.

2.3.1 Widespread decrease in international trade barriers

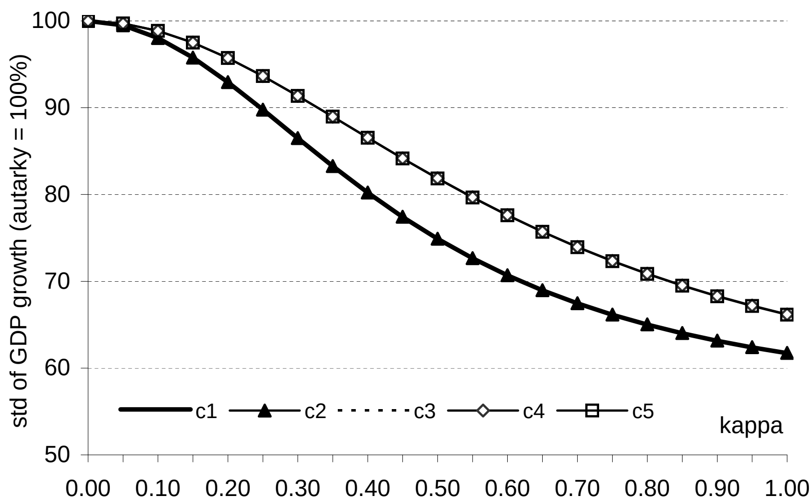
We set $L_n = 1$ and $\kappa_{ijt} = \kappa_t$ increases uniformly over time from the case of autarky ($\kappa = 0$) to free trade ($\kappa = 1$) for $i \neq j$, with $\kappa_{iit} = 1$. The upper panel of Figure 2.3 shows that as κ_t increases, that is, as trading costs decrease, volatility decreases; countries are able to diversify uncorrelated country-specific shocks.

[4] See the Appendix for proof.

Figure 2.3: Volatility: uniform decrease in trade barriers



Note: Figure shows standard deviation of GDP (in log-deviations) relative to standard deviation computed when $\kappa = 0$ (case of autarky).



Note: As above. Countries 4 and 5 are big.

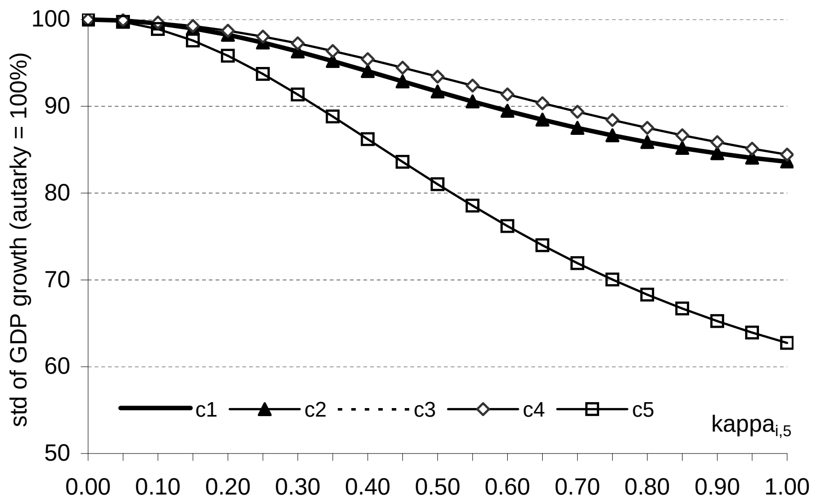
If the size of countries is modified to allow for some big countries, $L_n = (1, 1, 1, 3, 3)$, and all else stays as before, the lower panel of Figure 2.3 clearly show that the decline in volatility is smaller for big countries.

2.3.2 Big country joins the world

We keep all parameters as before, with $L_n = (1, 1, 1, 3, 3)$ but assume that four countries are open to trade with each other (constant $\kappa_{ij} = 0.3$ for $i, j \neq 5$,

$\kappa_{ii} = 1$) and one of the big countries moves from autarky to free trade with the remaining countries ($\kappa_{i5t} = \kappa_{5jt}$ for $i, j \neq 5$ increases uniformly). The country that joins experiences a significant decline in volatility, in line with the conclusions of the above simulations (see Figure 2.4). Other countries also see some decline in volatility as their trading costs against big country 5 fall; the decrease is smaller than in previous simulations because their mutual trading barriers do not change. This simulations suggest that an increase in openness vis-à-vis one big country is also consistent with an overall decrease in volatility.

Figure 2.4: Big country joins the world



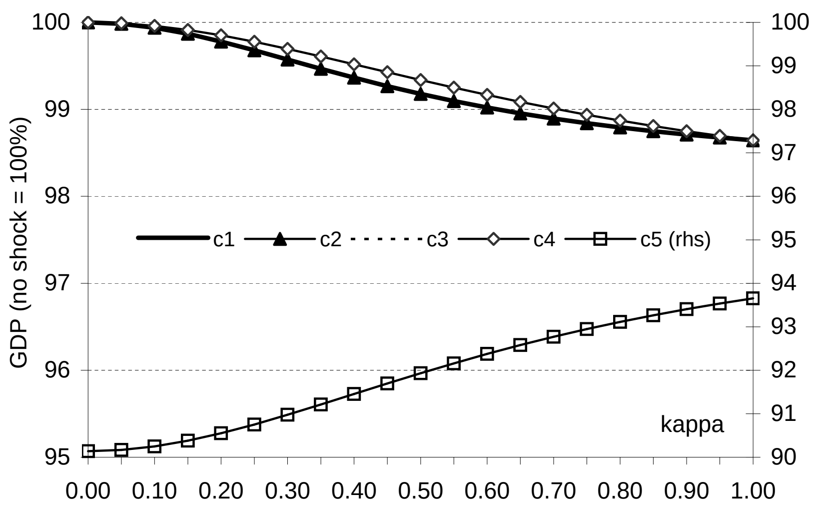
Note: Trade costs are fixed for countries 1-4 but uniformly decrease for their trade with country 5. See Figure 2.3 for further description.

2.3.3 Crisis hits a big country

We keep the parameters as before, $L_n = (1, 1, 1, 3, 3)$, with κ_{ij} increases uniformly over time from autarky to free trade and explore what happens to GDP if one of the big countries (country 5) experiences a 10% fall in λ. The more open to trade countries are, the more the countries that were not hit by the shock suffer the impact the contraction in the big country. (When countries are completely closed, of course they experience no change in GDP). Conversely, for the country that suffered the shock, higher openness helps mitigate the impact.

The more open the country is, the lower the fall in its own GDP. See Figure 2.5 for illustration. The model is therefore consistent with the notion that with greater trade openness, a large shock to a particular country (e.g. US), can be more strongly transmitted to other countries through stronger demand linkages.

Figure 2.5: Shock to big country



Note: Figure shows the % change in GDP that follows after country 5 is hit by a 10% shock to λ. Trade costs decrease uniformly for all countries; countries 4 and 5 are big compared with 1-3.

2.4 Mapping the model into the data

To identify the key variables from our model with their counterparts in data we will stick to the convention introduced earlier in this paper and identify the weighted shocks $Z_i = \lambda_i L_i^{\beta/\theta}$ rather than shocks λ_i and the size of the economy L_i separately. Allowing for this modifications, we get the following modified equilibrium conditions:

$$d_{ij} = (AB)^{-1/\theta} \left(\frac{(L_j w_j)^{\beta} {p_j}^{1-\beta}}{p_i \kappa_{ij}} \right)^{-1/\theta} Z_j \tag{2.10}$$

$$p_i = AB \left(\sum_{j=1}^{n} \left(\frac{(L_j w_j)^{\beta} p_j^{1-\beta}}{\kappa_{ij}} \right)^{-\frac{1}{\theta}} Z_j \right)^{-\theta} \tag{2.11}$$

$$\frac{L_i w_i}{\beta} = \sum_{j=1}^{N} \left(\frac{L_j w_j}{\beta} - S_j \right) d_{ji} \tag{2.12}$$

where $B = \beta^{\beta} (1 - \beta)^{1-\beta}$ and $A = \left[\int_0^{\infty} e^{-z} z^{\theta(1-\eta)} dz \right]^{1/(1-\eta)}$.

It is of some importance to be clear about the meaning of the words 'imports' and 'exports', which will play a key role in our measurement exercise. The quantity flowing from country i to j could be evaluated as the quantity leaving country i, or as the country reaching country j. Similarly, this quantity could be valued at country i prices, or at country j prices. We adopt the convention that 'imports' are quantities arriving evaluated at receiving-country prices, while 'exports' are quantities departing evaluated at sending country prices. With this convention, if $q_{ij}(x)$ is the quantity of good x leaving country j for country i we have

$$I_{ij} = \int p_i(x) \kappa_{ij} q_{ij}(x)$$

whereas the exports from country j to country i are

$$E_{ij} = \int p_j(x) q_{ij}(x)$$

Notice that for a good shipped from j to i we have $p_i(x)\kappa_{ij} = p_j(x)$ so our definitions imply that $I_{ij} = E_{ij}$. This latter point explains why equation (2.12) holds. While the left-hand-side describes production in country i, and the right-hand-side described uses of country i's output, it is not immediately clear why this is written in terms of other country's imports. The answer is that with our convention the value of other countries imports from i equals the value of country i exports to them.

For our purposes, it is important that we interpret q_i not as *a good* but as a shorthand for the value of the bundle of goods $q(x)$ (some produced domestically, some imported) that are used in domestic production or consumed. Further, p_i is a price index for this basket. Note that there are only $N-1$ linearly independent equations in (2.12) so one of the endogenous variables in the system has to be normalized.

2.4.1 Identifying the observables

There are four objects in the model that have a fairly clear mapping into observable data. These are: real GDP (in PPP), gross output, imports, and exports. In turn, these can be combined to compute measures of $L_i p_i q_i$, S_i, d_{ij} and that of β. Starting with real GDP, $L_i w_i$ is the value of payments received by the unproduced input, i.e. nominal GDP. $L_i w_i / p_i$ are nominal payments deflated by the price index, or a measure of real GDP. We show in the Appendix that the PWT series of constant-price GDP expressed in PPP maps well to our measure of $L_i w_i / p_i$.

$L_i p_i q_i$ is the value of all purchases by domestic agents. It is therefore equal to gross output of the economy plus imports minus exports:

$$L_i p_i q_i = GNO_i - S_i$$

S_i is exports minus imports, both evaluated at domestic dollar prices. Formally, this is

$$S_i = \sum_k E_{ki} - \sum_k M_{ik}$$

GNO_i is the value of total production, or gross output. In the model it is the quantity $GNO_i \equiv \int p_i(x) q_i(x) d\Phi(x)$. The countries for which we can construct this series account for 91 percent of world GDP and for 84 percent of world exports in 2000. For countries for which we are unable to find estimates of total gross output we estimate the series using data on gross output in industry, value added, population and year dummies. More details in the Appendix.

d_{ij} is the share of goods produced in country j in total demand for goods in country i. This is defined as

$$d_{ij} = \frac{I_{ij}}{L_i p_i q_i} = \frac{I_{ij}}{GNO_i - S_i}$$

with d_{ii} implied from the restriction $\sum_j d_{ij} = 1$.

The share of unproduced input in the production of intermediates β_i follows from equation (2.2)

$$\beta_i = \frac{L_i w_i}{L_i p_i q_i + S_i} = \frac{GDP_i}{GNO_i}$$

In the exercises we report, we use a constant value $\beta = 0.5$ for all countries and years, which is the average found in data.

Finally, we use a value of $\theta = 0.5$. In the model, higher θ implies higher variance of productivity shocks and increases the potential to exploit comparative advantage of each country. There is no clear empirical counterpart to this in existing empirical work. Typically, that work is based on estimates of the elasticity of trade shares with respect to trading costs, where the latter are proxied as the maximum difference between prices in two countries (see EK). This is not really the case for our model, in which many goods are not traded in equilibrium and for which the difference in trading costs cannot be observed. But along the arguments of Simonovska and Waugh (2009), our point is that existing estimates of trade elasticities in current empirical work, underestimate the θ in our model.

2.4.2 Computing the unobservables

This section discusses our identification strategy regarding trade costs κ and shocks $Z_i = \lambda_i L_i^{\beta/\theta}$. We begin by assuming symmetric trade costs $\kappa_{ij} = \kappa_{ji}$ for all i, j. From equation (2.10), we have

$$\frac{d_{ji}}{d_{ii}} = \left(\frac{p_j \kappa_{ji}}{p_i}\right)^{1/\theta} \quad \text{and} \quad \frac{d_{ij}}{d_{jj}} = \left(\frac{p_i \kappa_{ij}}{p_j}\right)^{1/\theta}$$

Applying $\kappa_{ij} = \kappa_{ji}$, we obtain a formula that relates trade costs *entirely* to the trade shares defined above.

$$\kappa_{ji} = \left(\frac{d_{ij}}{d_{jj}} \frac{d_{ji}}{d_{ii}}\right)^{\theta/2}$$

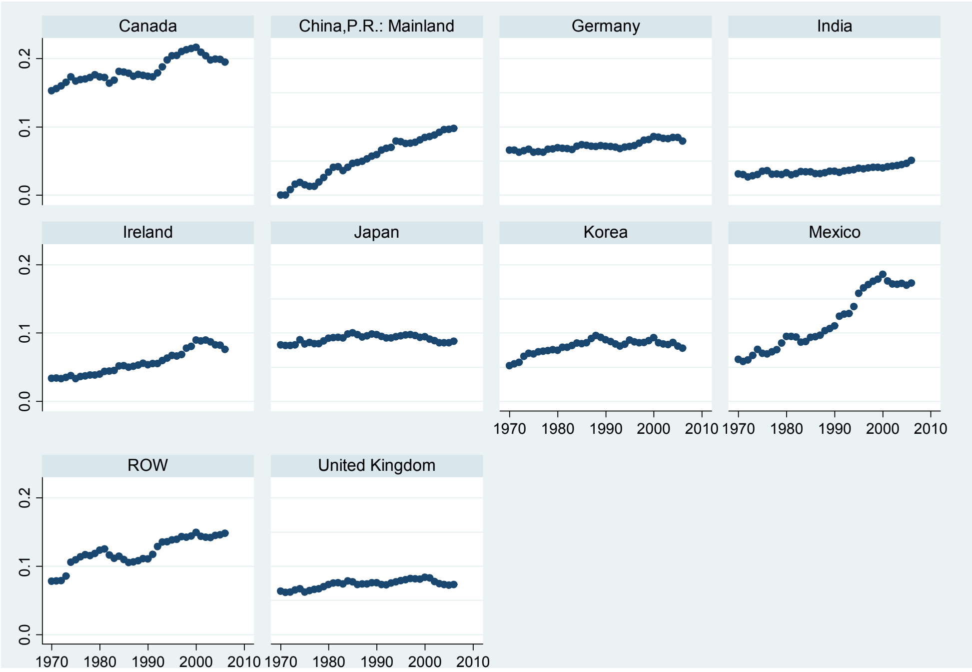
For illustration, Figure 2.6 plots the values of κ_{ijt} for $i = US$ and selected trade partners j.

Next, for $i = j$, equation (2.10) can again be used to write real aggregate GDP as:

$$\frac{w_i L_i}{p_i} = (AB)^{-\frac{1}{\beta}} \left(\frac{Z_i}{d_{ii}}\right)^{\theta/\beta} \tag{2.13}$$

Therefore, with a measure of $\frac{w_i L_i}{p_i}$, we can retrieve the exogenous process Z_i. Selected series of Z_{it} are reported in Figure 2.7. As we show in the Appendix, the

Figure 2.6: Trade costs of USA and selected trade partners

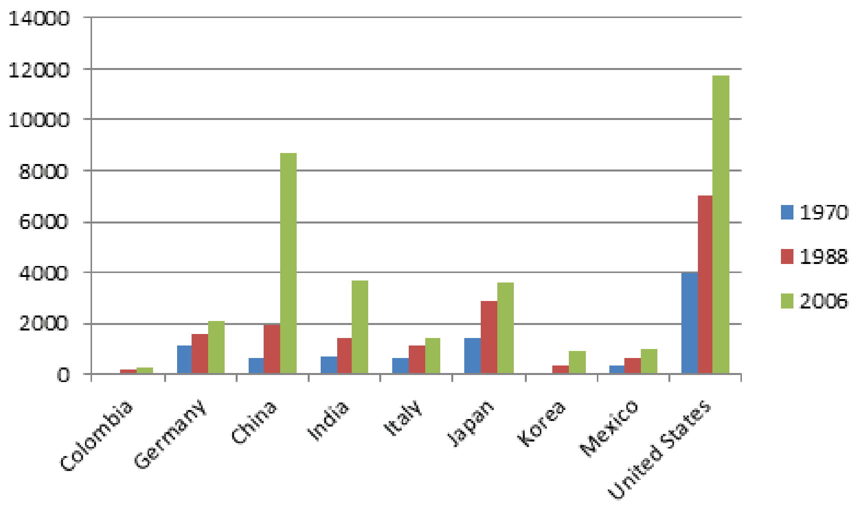


measure of constant-price GDP in international dollars of the PWT corresponds in our model to the quantity $\mu \frac{w_{i,t} L_{i,t}}{p_{i,t}}$ so using this in the above expression we are able to retrieve the composite measure of shocks up to a positive constant μ common across countries and periods. Once we have the values for Z_i and κ_{ij}, we can solve the model and we can then ask what fraction of the decline in volatility can be attributed to openness to trade or the process for Z_i. We give a preliminary answer to the question in the following section, where we remain agnostic about the properties of trade costs κ. Results using a full parameterisation of the model are then presented in section 2.5.

2.4.3 Minimalist counterfactual

Having identified real GDP, $Y_i = L_i w_i / p_i$, and trade shares d_{ij}, we can use the equilibrium equation (2.13) in a logarithmic form to get a sense of the contribution of trade to the change in volatility. Let us denote by z_{it} the natural logarithm of shocks Z_{it} and y_{it} the log of total (not per capita) real GDP of

Figure 2.7: Shocks Z for selected countries and years



country i in year t:

$$y_{it} = \text{const} + \frac{\theta}{\beta}\left(z_{it} - \ln d_{ii,t}\right)$$

We can then decompose GDP volatility as

$$\underbrace{\text{Var}\left(\tilde{y}_i\right)}_{(2)} = \left(\frac{\theta}{\beta}\right)^2 \left[\underbrace{\text{Var}(\tilde{z}_i)}_{(3)} + \underbrace{\text{Var}(\ln \tilde{d}_{ii})}_{(4)} \underbrace{-2\,\text{Cov}(\tilde{z}_i, \ln \tilde{d}_{ii})}_{(5)}\right] \tag{2.14}$$

where the tildes indicate growth rates and the numbers below the expressions link each term with the corresponding column in Table 2.1.

Trade policy can change the last two terms in the brackets, but not the first (at least not directly). We estimate each of the three terms before and after the mid 1980s, and study how they contributed to the decline in volatility in different countries. This is a decomposition, so all volatility will be accounted for – the residual $\text{Var}(z_i)$ will pick up all the slack. Table 2.1 summarizes the results. The last column of the table gives the relative importance of the joint contribution of the change in $Var(\ln \tilde{d}_{ii})$ and $Cov(\tilde{z}_i, \ln \tilde{d}_{ii})$ in the total change in $Var(\tilde{y}_i)$.

There are two lessons to take from this exercise. First, the change in volatility of variables associated with trade has in most cases contributed to greater stability of economic output. Secondly, the impact has varied widely among countries

Table 2.1: Minimalist counterfactual: Change in volatility from 1970-1984

	% change in Std(y_i)	Absolute Difference				% share of (2) accounted for by (4+5)
		Var(yi)	Var(zi)	Var(dii)	Cov	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(4+5)/(2)x100**
Australia	-34	-3.74	-6.09	-0.49	2.84	-63
Austria	-61	-8.80	-6.14	-0.12	-2.55	30
Belgium	-47	-6.25	-1.58	4.59	-9.26	75
Canada	-25	-3.28	0.00	-0.11	-3.17	100
China	-20	-3.23	-2.39	0.42	-1.26	26
Colombia	-15	-1.37	-1.26	0.25	-0.36	8
Denmark	-29	-4.78	-5.76	0.05	0.93	-21
Finland	37	8.78	13.58	0.62	-5.42	-55
France	-34	-3.39	-2.36	-0.42	-0.60	30
Germany	-26	-2.41	-2.58	0.89	-0.72	-7
Greece	-55	-21.45	-23.08	0.62	1.01	-8
India	-6	-0.84	-2.17	0.25	1.08	-160
Ireland	-8	-2.38	1.21	-1.26	-2.33	151
Italy	-60	-10.10	-4.63	-0.54	-4.93	54
Japan	-8	-0.96	-2.74	-0.32	2.11	-185
Korea	14	7.07	7.38	-0.57	0.25	-4
Mexico	-22	-9.48	-1.81	1.02	-8.69	81
Netherlands	-29	-2.44	1.35	3.22	-7.01	155
Norway	-1	-0.09	0.46	-1.10	0.55	605
Portugal	-55	-31.05	-27.14	-0.63	-3.28	13
Spain	-35	-5.52	-4.64	-0.05	-0.82	16
Sweden	12	0.98	2.17	0.58	-1.77	-122
UK	-38	-4.66	-3.45	-0.76	-0.46	26
USA	-55	-8.82	-8.56	-0.08	-0.18	3

Note: The table presents a decomposition of the volatility of GDP growth into terms related to unobservable shocks and terms related to trade policy. It shows how the respective terms changed between two periods 1970-1984 and 1985-2006. The last column gives the share of the change in volatility that can broadly be accounted for by terms related to trade. See equation (2.14).

and has been especially strong in small open economies like Belgium, Ireland and the Netherlands. Large developed countries, with the exception of Japan, have benefited less because their reliance on trade is substantially smaller. We will seek to confirm these preliminary findings in the following section.

2.5 Counterfactual simulations

Suppose the level of openness from 1970-1984 had not changed in the post 1985 period. How would have volatility changed, given the lower degree of openness in the latter period? In this exercise, we use the series of shocks Z_{it} and trade costs κ_{ijt} as measured above and simulate two scenarios. In the baseline, we let the properties of shocks and the level of openness to evolve as in the data while in the counterfactual exercise the level of openness stays at the pre-1984 level (shocks are as in the baseline).

In order for our results not to be driven by a particular realization of shocks we compute this exercise with artificially generated series of shocks and do so many times (5000). Disturbances Z_{it} are modelled as an AR(1) process in log deviations around country-specific trends (HP trends). The latter are taken as given in all simulations. What differs across simulations are the stationary innovations around trends, which are bootstrapped from Z_{it} computed in the previous section. We thus preserve the stochastic properties of our detrended series of Z_{it} in each period (1970-1984 and 1985-2006).[5] We have experimented with preserving the contemporaneous covariance structure in shocks across countries but this distinction has not proved quantitatively important.

Our trade costs κ are derived from bilateral trade data. Since our point is to show how a general increase in trade openness could have affected volatility of GDP growth, we abstract from the observed volatility in the series of κ and take a representative value for each pair ij and each period (1970 for the first and 2000 for the latter) and keep these values constant within periods. In the counterfactual exercise we keep the 1970 value constant both within and between periods. When the 1970 value of trade costs was missing for a particular pair of countries because of the lack of bilateral trade data, we used the earliest recorded

[5] The problem of initial values was addressed by simulating long series for each period and removing the redundant years at the beginning of the series.

value instead. Trade imbalances are treated as exogenous in the original EKAL model and we therefore ignore them in the simulations below.

With a newly generated series of Z_{it} and the representative values of κ_{ijt} we solve the model and compute the new series for GDP, detrend it by using the HP filter (separately for each period) and compute the relative change in volatility between the first and second period.

Table 2.2 summarizes our results. The first column in the table shows the change in volatility that would have prevailed under the counterfactual exercise (trade costs are kept at the 1970 level) and the second column reports the results of the baseline exercise, when the representative value of trade costs changes between periods. For illustration, these values are also shown in Figure 2.8. A comparison of these two exercises shown in the last two columns of the table gives the contribution of trade costs (the only variable that differs between the two reported scenarios) to volatility. The main finding is that in *all* countries lower trade costs, i.e. increased trade, contributed to lower volatility than it would have been otherwise.

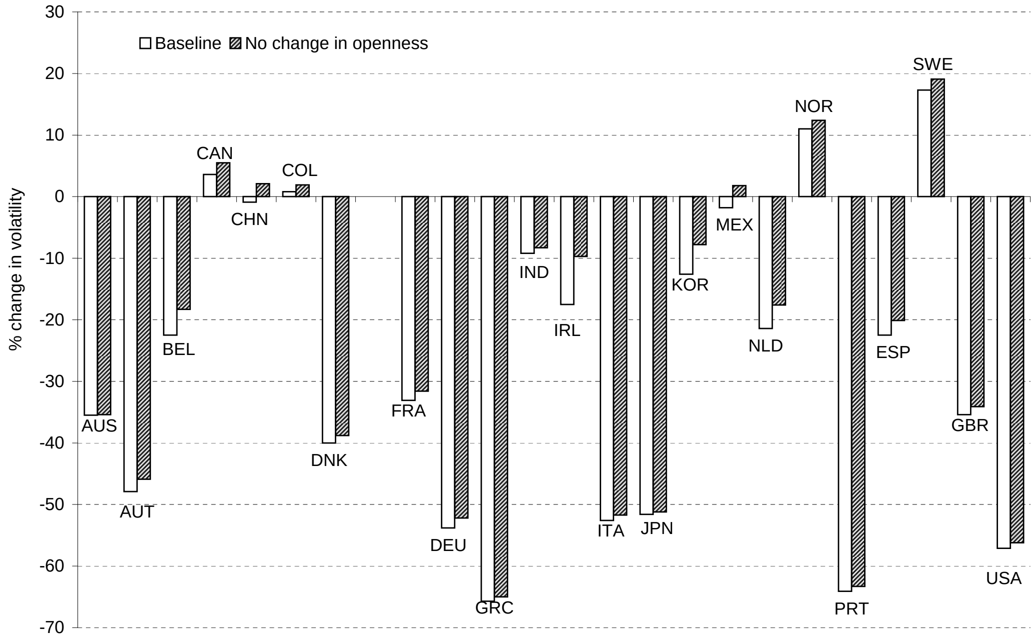
Even though the quantitative significance of the diversification channel seems to be small, averaging to about 3 percentage points in fall in volatility, there are large difference across countries. The countries that seem to have benefited most from greater openness were, in that order, Ireland, Belgium, Korea and the Netherlands – all small open economies. At the other end of the spectrum there were larger or less diversified countries Australia, Colombia, India and Japan.

Comparing the relative contribution of openness to volatility, there is strong correlation (0.70) between the contribution of trade to the change in volatility (column $|(1-2)|$) of Table 2.2 and the change in average trade costs for each country[6], confirming our intuition that countries where trade expanded most have experienced greatest decreases in volatility.

Returning to the minimalist counterfactual introduced in the previous section, we find negative correlation (-0.44) between the sum of the two channels we ascribed to trade policy (columns 4 and 5 in Table 2.1) and the change in volatil-

[6] Average trade costs of country i are computed as averages of κ_{ijt} over j in each period.

Figure 2.8: Change in volatility 70-84 v 85-06, counterfactual and baseline



Note: The figure shows the change in volatility of GDP growth rate between the two periods with (baseline) and without (counterfactual) changes in trade costs between periods. Country codes refer to countries listed in Table 2.2. Finland is not shown due to a different scale.

ity computed in this exercise, which we take as a sign of consistency between the volatility decomposition exercise and the numerical simulations presented here.

2.6 Conclusions

This chapter revisits a question that keeps coming up in policy discussions of the pros and cons of trade liberalizations, particularly in low income economies: How does openness to trade affect GDP volatility? We develop a general equilibrium quantitative framework to formalise the diversification channel in which trade acts as a hedge against shocks to individual suppliers. The logic of the mechanism we study is as follows. When the production process relies on different inputs that can be sourced from different countries, a shock to a particular supplier (a domestic or foreign one) is easier to accommodate because the pool of potential suppliers is wider and the potential for diversification of cost shocks is greater. The channel is the stronger the lower are trade costs that agents face when trading goods across countries.

We derive formulas for the variance of GDP growth in autarky and free trade and show, first, that trade directly decreases volatility because domestic productivity fluctuations receive smaller weight with free trade than in autarky. Secondly, we show that trade exposes the country to other countries' productivity shocks and these contribute positively to volatility. The overall effect on volatility therefore depends on the relative sizes of the countries and the variance-covariance matrix of shocks across them.

Using data on international trade, GDP and gross output we use the model to quantify the contribution of trade to the changes in volatility since 1970 and find that in all countries lower trade costs generated lower volatility than it would have been otherwise. The quantitative significance of the diversification channel seems to be small on average but its role rises in small open economies, in line with the qualitative predictions of the model. One reason why the channel does not find larger support in data is that costs of across-the-border trade we are able to identify are huge compared with cost-less domestic trade. Stylised illustrations presented in section 2.3 indicate that greatest gains (in terms of lower volatility) from openness accrue only when trade openness reaches much larger a degree than is currently the case for most economies.

The framework we study in this paper investigates one of the two main mechanisms that can mediate the relationship between trade and volatility. The other mechanism emphasises the role of sectoral shocks and supports the view that as countries becomes specialised in sectors according to their comparative advantage, they become increasingly vulnerable to shocks in that particular sectors. In our future research, we plan to nest the two mechanisms in the framework used here and assess their quantitative importance.

Table 2.2: Change in volatility 70-84 v 85-06, counterfactual and baseline

	% change in volatility		absolute difference	relative to baseline (%)
	counterfact.	baseline		
	(1)	(2)	|(1-2)|	|(1-2)/(2)|
Australia	-35.4	-35.5	**0.1**	**0.2**
Austria	-45.9	-47.9	**1.9**	**4.1**
Belgium	-18.3	-22.5	**4.2**	**18.6**
Canada	5.5	3.6	**1.8**	**51.1**
China	2.1	-0.9	**3.0**	**330.8**
Colombia	1.9	0.8	**1.1**	**137.3**
Denmark	-38.8	-40.0	**1.3**	**3.2**
Finland	67.5	62.8	**4.7**	**7.5**
France	-31.6	-33.1	**1.5**	**4.6**
Germany	-52.2	-53.8	**1.6**	**3.0**
Greece	-65.0	-65.7	**0.7**	**1.1**
India	-8.3	-9.2	**0.8**	**9.0**
Ireland	-9.7	-17.5	**7.8**	**44.7**
Italy	-51.7	-52.6	**0.9**	**1.7**
Japan	-51.2	-51.6	**0.4**	**0.8**
Korea	-7.8	-12.6	**4.7**	**37.7**
Mexico	1.8	-1.8	**3.7**	**198.7**
Netherlands	-17.6	-21.4	**3.9**	**18.0**
Norway	12.4	11.0	**1.4**	**12.6**
Portugal	-63.3	-64.1	**0.8**	**1.3**
Spain	-20.1	-22.5	**2.3**	**10.4**
Sweden	19.1	17.3	**1.8**	**10.2**
UK	-34.1	-35.4	**1.3**	**3.6**
USA	-56.2	-57.1	**0.9**	**1.6**

Note: The table shows changes in volatility of GDP growth rates between the two periods. In the baseline, trade costs κ are allowed to change between periods (they take fixed values in each period), while in the other scenario ('counterfactual') trade costs in 1985-2006 are kept at their 1970-1984 values.

2.A Appendix

2.A.1 Derivation of equation (2.2)

Equation (2.2), stating that $L_i w_i = \beta(L_i p_i q_i + S_i)$ can more intuitively be expressed as follows (add $L_i p_i q_i$ and S_i to both sides and rearrange)

$$L_i p_i q_i = (1-\beta)\left(L_i p_i q_i + S_i\right) + L_i w_i - S_i$$

where on the left hand side is the total value of domestic spending on goods, which are partly expended on intermediates and partly in the form of final demand for goods. To add intuition to the first term on the right-hand side (at the cost of loose notation), notice that the total payments to domestic producers of individual goods originate either from domestic or foreign sources. In *per capita* terms we have

$$\int p(x)\,q(x)\,d\Phi(x) = \int\limits_{\text{sold domestically}} p(x)\,q(x)\,d\Phi(x) + \int\limits_{\text{exported}} p(x)\,q(x)\,d\Phi(x)$$

Next, the *per capita* spending on goods $p_i q_i$ accrues partly to domestic producers and partly to foreigners:

$$p_i q_i = \int\limits_{\text{bought domestically}} p(x)\,q(x)\,d\Phi(x) + \int\limits_{\text{imported}} p(x)\,q(x)\,d\Phi(x)$$

Now, obviously, the value of goods sold and bought domestically will be identical in the equilibrium so combining these two lines we arrive in

$$\int p(x)\,q(x)\,d\Phi(x) = p_i q_i + \int\limits_{\text{exported}} p(x)\,q(x)\,d\Phi(x) - \int\limits_{\text{imported}} p(x)\,q(x)\,d\Phi(x)$$

Finally, perfect competition and the Cobb-Douglas formulation implies that $1-\beta$ of this expression accrues to the produced input, i.e. to intermediates. In aggregate terms this becomes $(1-\beta)\left(L_i p_i q_i + S_i\right)$.

2.A.2 Derivation of equation (2.9): volatility with free trade

Start with the original condition that shows that GDP under costless trade less is volatile than under autarky.

$$\frac{(\beta+\theta\gamma_i)^2+\theta^2\sum_{j\neq i}^{N}\gamma_j^2}{(\beta+\theta)^2}<1$$

The following steps, first, expand the numerator and adds terms; the second line completes the square and collect several terms. Finally, the last line moves $(\theta\gamma_i)^2$ to the expression in square brackets (note the change of the index under the summation sign) and cancels common terms. This inequality holds since $\gamma_i < 1$ for all i.

$$\begin{aligned}
\frac{\beta^2+(\theta\gamma_i)^2+2\beta\theta\gamma_i+\theta^2-\theta^2+2\beta\theta-2\beta\theta+\theta^2\sum_{j\neq i}^{N}\gamma_j^2}{(\beta+\theta)^2} &< 1\\
\frac{(\beta+\theta)^2+(\theta\gamma_i)^2+2\beta\theta(\gamma_i-1)+\theta^2\left[\sum_{j\neq i}^{N}\gamma_j^2-1\right]}{(\beta+\theta)^2} &< 1\\
2\beta\theta(\gamma_i-1)+\theta^2\left[\sum_{j=i}^{N}\gamma_j^2-1\right] &< 0
\end{aligned}$$

2.A.3 Proof that L_iw_i/p_i maps to constant-price GDP in PPP

It is instructive to start with variable P_i that in the Penn World Tables denotes the price level of GDP, or more precisely the USD value of local expenditures over expenditures evaluated in international prices. While the PWT variables are originally defined (and computed) in terms of expenditures and relative prices, it is possible to cast them in terms of prices and quantities as follows:

$$P_i=\frac{\sum_g p_{g,i}\,q_{g,i}}{\sum_g p_g\,q_{g,i}}$$

with $p_{g,i}$ and $q_{q,i}$ represent the USD price and quantity of good g respectively and p_g is the price of the same good in an international currency. Index g represents spending groups (basic headings in the PWT terminology), which are constructed in a way that the sum of these expenditure groups adds to total GDP. One of these groups are net exports, valuation of which follows the assumption that

$$p_{nx,i}\, q_{nx,i} = p_{nx}\, q_{nx,i} = S_i$$

where S_i is in USD.

In our model, consumers buy all individual goods $q(x)$ and bundle them using the CES aggregator in a final good q_f. Hence, a PWT statistician would be able to sample only from this one final good in each country and the quantity P_i measured becomes

$$P_{i,t} = \frac{p_{i,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}}{p_t\, q_{f,i,t}\, L_{i,t} + S_{i,t}}$$

Setting $P_{US,t} = 100$ as is the case in the PWT implies $p_t = p_{US,t}/100$ for all t. The denominator of $P_{i,t}$ is the current-price GDP in international prices

$$CGDP_{i,t} = p_{US,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}$$

and the real-price (Laspeyres) GDP in international prices is defined as

$$RGDP_{i,t} = p_{US,T}\, q_{f,i,t}\, L_{i,t} + S_{i,t}^{T}$$

where the last term captures real net exports in year t valued at prices from base year T. Using the income-expenditure identity $L_{i,t}\, w_{i,t} = p_{i,t}\, q_{f,i,t}\, L_{i,t} + S_{i,t}$ and simple algebra we get

$$\begin{aligned}
RGDP_{i,t} &= p_{US,T}\, \frac{(L_{i,t}\, w_{i,t} - S_{i,t})}{p_{i,t}} + S_{i,t}^{T} \\
&= p_{US,T}\, \frac{L_{i,t}\, w_{i,t}}{p_{i,t}} - \frac{p_{US,T}}{p_{i,t}} S_{i,t} + S_{i,t}^{T} \\
&= p_{US,T}\, \frac{L_{i,t}\, w_{i,t}}{p_{i,t}} - \frac{p_{US,T}}{p_{i,T}} \frac{p_{i,T}}{p_{i,t}} S_{i,t} + S_{i,t}^{T} \\
&= p_{US,T}\, \frac{L_{i,t}\, w_{i,t}}{p_{i,t}} + S_{i,t}^{T} \left(1 - \frac{p_{US,T}}{p_{i,T}}\right) \\
&\approx \mu \frac{L_{i,t}\, w_{i,t}}{p_{i,t}}
\end{aligned}$$

The last equality follows the PWT convention of valuing net exports by the price index of domestic absorption for years other than the base year. By dropping the last term in the approximation we assume that changes in *real* net exports are small for most countries relative to domestic absorption. Given the weight attached to $S^T_{i,t}$ this assumption will be of importance only for countries with price level far off the US one in the base year.

This equation allows us to identify real GDP computed from our model with variable $RGDP_{i,t}$ as measured by the PWT, up to a constant common to all countries and all years.

2.A.4 Data description

Our sample consists from 24 countries, which we call the core countries, for which we were able to collect a sufficient amount of data with none or very little estimation. Other countries, for which less data are available and more estimation was needed, form the rest of the world (ROW). The choice of the core countries was dictated mainly by the availability of data for total gross output; they include: the U.S., Mexico and Canada, Australia, Asia is represented by China, Japan, Korea and India, South America by Colombia, and the rest are advanced European countries: the U.K., a composite of France and its oversee departments, Germany, Italy, Spain, Portugal, a composite of Belgium and Luxembourg, the Netherlands, Finland, Sweden, Norway, Denmark, Greece, Austria and Ireland. While some important countries appear only in our ROW variable (most notably Brazil, Russia, Turkey, Indonesia, Malaysia and oil exporters), the selection of core countries is sufficiently representative in terms of geographic location and the share in the world trade and GDP. The time period we study covers years from 1970 to 2006. We focus on annual data.

The strategy regarding the rest of the world was to use the GDP and population data for those for which we were able to find a full series, look for their individual total output, estimate it when missing and subsequently aggregate. Due to trade data availability, the following groups of countries were merged into a single entity each: former Soviet Union, countries forming the South African Common Customs Area and former Czechoslovakia.

To identify variables in the model three main groups of data were needed. First, we use the PWT variable RGDP to identify real GDP. The series is in international dollars and is available for most countries in the world. Next, we use gross output data, obtained from the EU KLEMS database, the UN database and other sources. Finally, the basis for our trade data is the IMF DOTS database. The rest of the section describes our data sources and estimation methods.

Real GDP: Source is PWT 6.3, variable RGDPL, GDP per capita, international prices, constant prices of 2005, Laspeyres index. Aggregate GDP is a product of RGDPL and variable POP defined below. Real GDP for former USSR and Czechoslovakia required special attention:

- Former Czechoslovakia: for 1990-06 the source is PWT 6.3, sum of the GDP series for the Slovak and Czech Republics; for 1970-89 data are from PWT 5.6 (the growth rate of the data from PWT 5.6 was applied starting with the overlapping year 1990).

- Former USSR: for 1994-2006 the source is PWT 6.3, sum of the GDP series for individual post-soviet republics; for 1989/90-93 when data in PWT 6.3 are missing, the growth rate of individual countries from the World Bank, WDI (April 2010), GDP in constant 2005 international dollar was used; in 1989 for 5 republics neither the WB data were available so the growth rate of Russia was applied; for 1970-1988 the growth rate from PWT 5.6 was used starting in the overlapping year 1989.

Gross Output: With the exception of India and China, the sources of data for total gross output in core countries are the same that were use to construct output in industry and are defined below. Total output of India (1970-1998) and China (whole series) is not available. We use the available data for output in industry and estimate the missing part, output in services, by regressing output in services for the remaining core countries on their GDP, output in industry, population, CGDP from the PWT, value added in services and a set of year dummies. Output in services and value added in services was obtained as a difference between the respective values for the aggregate economy and industry. The estimation technique was a Poisson regression adapted from Silva and Tenreyro (2006). For India, the missing years were generated using the growth rate of the estimated series.

Gross output data for the rest of the world come from UN Data. Missing values were generated using the growth rate of estimated output (a Poisson regression of total output on GDP and population). Individual country data (after conversion to USD) were then aggregated to the ROW. The series we obtain has a well behaved output/GDP ratio for all years.

Trade Data: We use bilateral imports and exports from 1970 to 2006 from the IMF's Direction of Trade Statistics kindly provided by Julian Di Giovanni. The DOTS reports bilateral gross trade flows. An import data point is I_{ij}, or the dollar value of imports by country i from country j, at country i prices.

There are minor discrepancies between the data and the conventions adopted in the paper, which we do not address. One problem is that imports are evaluated gross of transport costs but not gross of tariffs. Hence we underestimate the quantity $\int p_i(x)q_{ij}(x)d\Phi(x)$ for every $j \neq i$. Another possible problem is that the import data contains re-imports and the export data re-exports.

Auxiliary Data:

- CGDP: GDP per capita, international prices, current prices, PWT 6.3. Converted to aggregate GDP by multiplying by total population.

- GDP in local currency: World Bank, World Development Indicators (April 2010), variable GDP in current LCU. Data for the former Soviet Union and Czechoslovakia come from the UN National Account Main Aggregates Database. Data are available for the currently dissolved entities until 1990 and for their successors states from 1990 onwards. Year 1990 is available for both series. The post-1990 values were computed as a sum of GDP in USD of the successor states and the pre-1990 totals were scaled to match the composite 1990 value.

- Population: PWT 6.3, variable POP.

- Exchange rate: World Bank, World Development Indicators (April 2010), variable Official Exchange Rate defined as LCU per US$, period average. This series was used to convert total output and GDP in local currency units to USD. When currency reported by the WB was not consistent with the series used in the sample, the PWT exchange rate was used.

- Value added in industry and total value added is primarily derived from the EU KLEMS database (November 2009 and March 2008 edition). Industry covers the same sectors as defined in output in industry. When unavailable, other sources were used and linked to the main series by means of growth rates: UN Data (India, Mexico, Norway and Colombia), OECD STAN (Japan), Canadian Statistical Office's, Statistical Yearbooks of China.

- Output in industry is defined as the sum of output in agriculture, hunting, forestry and fishing, mining and quarrying, and manufacturing and is measured in units of local currency. For most countries, the source is the EU KLEMS database (November 2009 and March 2008 editions), variable gross output at current basic prices. When missing, the following sources were used: UN Data (Norway and Colombia), OECD STAN (Japan), Canadian Statistical Office's (Canada), Statistical Yearbooks of China, Statistical Office of India, INEGI (Mexico). Two remarks are due with respect to China and India.

  - Regarding Chinese data, the primary concern was the methodological change initiated around 1998, when China stopped reporting *total* industrial output and limited the coverage to industrial output of firms with annual sales above 5m yuan (USD 625 000). The sectoral coverage remained the same in both series. There were 5 years of overlapping data of both series over which the share of the 5m+ firms on total output decreased from 66 to 57 percent. The chosen approach to align both series was to take the levels of output from the pre-1999 series (output of all firms) and apply the growth rate of output of 5m+ firms in the post-1999 period. This procedure probably exaggerates the level of output in the last seven years and leads to an enormous increase in the output/GDP in industry ratio (from 3.5 in 1999 to 6.0 in 2006). Our conjecture is that the ratio would be less steep if the denominator was value added in industry (unavailable on a comparable basis) because the GDP figure includes net taxes, which might take large negative values. Output in industry of all firms reflects the 1995 adjustment with the latest economic census.

– The Statistical Office of India reports years 1999-2006 on the SNA93 basis. Earlier years were obtained using growth rates of sectoral output as defined in their 'Back Series' database. The main issue with India was the large share of 'unregistered' manufacturing that is reported in the SNA93 series but missing in the pre-1999 data. The 'unregistered' manufacturing covers firms employing less than 10 workers and is also referred to as the informal or unorganized sector. We reconstructed the total manufacturing output using the assumption that the share of registered manufacturing output in total manufacturing output mirrors the share of value added of the registered manufacturing sector in total value added in manufacturing (available from the 'Back Series' database).

3

Optimal Monetary Policy with Industry and Services

3.1 Introduction

This chapter investigates what measure of inflation should a welfare-minded central bank target in an economy with nominal price and wage rigidities and two sectors that differ in the share of labour used in production. The welfare-theoretic analysis of monetary policy had traditionally focused at reducing the 'shoe-leather' costs associated with the opportunity costs of holding money. Following the publication of Rotemberg and Woodford (1997), researchers started to study also other sources of frictions, most notably those brought about by delays in the adjustment of nominal prices contracts. For instance, Woodford (2003) presents a framework in which asynchronous price adjustment that leads to discrepancies between relative prices causes an inefficient allocation of resources. A major advantage of this approach compared to the traditional one is that it provides specific guidelines to the central bank regarding the measure of inflation and/or output gap to target in order to maximise utility of economic agents.

The issue of an optimal target of a monetary authority has received a considerable interest in the literature. In a multi-sector or multi-country setting, Aoki (2001) proved that for an economy where one sector exhibits nominal frictions while the other sector's prices are perfectly flexible the optimal policy is to stabilise inflation in the sticky-price sector. In this particular framework, the monetary policy that follows only one of the two sectors also helps establish the efficient outcome for the entire economy. Aoki's work justifies the tendency of central banks to target the "core" inflation (an index that excludes goods whose prices change frequently) instead of targeting an overall measure of prices. Benigno (2004) extends this result to the open-economy setting and concludes that if the central bank can commit itself to inflation-targeting policies, it is optimal to give higher weight to the country/sector with greater nominal rigidities (see Proposition 4 in Benigno, 2004).

Erceg, Henderson and Levin (2000) and Erceg and Levin (2006) study the implications for monetary policy of nominal frictions in the labour market coupled with differences in the durability of output. The former paper shows that staggered price and wage contracts in a one-sector economy imply a trade-off between the goals of inflation and output gap stabilisation. Erceg and Levin (2006), which is the work most related to this chapter, then implement the same features in a two-sector setting. The key lesson that emerges from their study is that factors unrelated to the source of nominal frictions (durability of output) can affect the design of optimal policy. The monetary authority in their model prefers to give unequal weight to the two sectors even though the nominal frictions are equally severe across sectors. In particular, the authors show that the optimal inflation target in this economy is biased toward the durable goods sector.

The aim of this chapter is to investigate the monetary-policy implications of another important and highly realistic facet of the economic landscape, namely of the differences in the relative use of labour in the production process. My model economy is closed and consists of two sectors. For the sake of illustration, the relatively labour-intensive one is labelled 'services' while the relatively capital-intensive one is labelled 'manufacturing'. I abstract from all other differences between the two sectors. I show that a central bank operating under the inflation-targeting regime finds it optimal in this specific environment to target an inflation index that puts systematically higher weight on inflation in manufacturing. The result originates in different slopes of the marginal product of labour in the two sectors. The marginal product of labour is steeper in the capital-intensive sector, implying that a given change in output would require a greater percentage change in the labour input compared with the labour-intensive one.

The key assumptions that underlie this result are imperfect competition in the goods market and nominal price rigidities. Imperfect competition makes firms supply differentiated goods that are all purchased in the equilibrium. Nominal rigidities then create a non-degenerate dispersion in individual prices and outputs in both sectors and lead to inefficiencies that can be addressed by the toolkit of the monetary authority. Assuming goods weigh equally in terms of utility, note that a fictitious social planner entrusted by the task to produce a given level of the consumption basket would spread the production of individual goods equally among all firms. However, such an outcome is infeasible in a market economy if prices of the individual goods differ from each other. Limited substitutability then implies that there is more output (and therefore labour) needed to generate the same amount of utility compared with the equilibrium of the social planner. The faster the marginal product falls with the increase in output the more of the labour input is needed to produce the additional level of output and the greater are welfare losses compared with the efficient allocation. A welfare-maximizing central bank therefore strives to limit the dispersion in output of individual goods in manufacturing more emphatically than in services. It does so by reducing the incentives to alter prices after a shock hits the manufacturing sector, which substantiates greater weight of manufacturing inflation in the optimal inflation index.

In terms of the modelling approach, this paper builds on the analysis of Erceg et al. (2000), who extend the welfare-theoretical approach to monetary policy by allowing for the presence of two production factors, and generalizes it to consider the implications of different labour intensities. Another (and rather technical) point of departure from Erceg et al. (2000) is that the analysis in the present paper is embedded in a framework with a utility function that is not separable in sectoral bundles; as was shown in Aoki (2001) and Benigno (2004) this feature is not only more realistic but also allows for richer implications for the conduct of monetary policy.

The chapter is organized as follows. Section 3.2 introduces the economy and defines the equilibrium. Section 3.3 presents log-linear versions of the key model equations and solves for the equilibrium under flexible prices and wages. Finally, section 3.4 derives the social welfare function and discusses the optimal structure of the inflation index.

3.2 Model

The model is a variant of Rotemberg and Woodford (1997) with imperfect competition and Calvo-style (random duration) price contracts in goods markets. It follows Erceg and Levin's (2000) strategy to incorporate random-duration nominal wage contracts and production factors into the model. On top of that, factor intensities are allowed to differ across sectors. The utility function is such that the structural equations of the model depend on relative prices.

The economy consists of two sectors that differ in their relative use of labour and capital. Manufacturing is assumed to be the relatively capital-intensive sector and services the relatively labour-intensive one (note that this is only a rough description of reality, because a heavy use of capital is typical for some services too, for example in telecommunications). Both factors are free to move within a sector but are sufficiently specialized so that they cannot be used interchangeably in both sectors. Similarly as Erceg et al. (2000) and Erceg and Levin (2006), this model abstracts from endogenous capital accumulation. The following part of the section describes the behaviour of households and firms, respectively, in the model.

3.2.1 Households

The decision makers in this economy are two sets of identical infinitely-lived and fully forward-looking households, each of measure one, that supply their work in the labour markets in one of the two sectors. They consume an index of consumption goods produced in both sectors, which is represented by the Cobb-Douglas aggregator (3.1). The aggregator exhibits unitary elasticity of substitution between the sectoral bundles and the shares of sectoral consumption bundles are given by ψ_m and ψ_s, which add to one. Households allocate their expenditure in a cost-minimizing way according to prices of the respective sectoral bundles (3.2) so that the aggregate price index corresponding to one unit of their aggregate consumption is given by (3.3). Note that the household-specific subscripts are suppressed from reasons that will shortly become apparent.

$$C_t \equiv \frac{C_{m,t}^{\psi_m} C_{s,t}^{\psi_s}}{(\psi_m)^{\psi_m} (\psi_s)^{\psi_s}} \tag{3.1}$$

$$C_{j,t} = \left[\frac{P_{j,t}}{P_t}\right]^{-1} \psi_j \, C_t, \quad j \in \{m, s\} \tag{3.2}$$

$$P_t = P_{m,t}^{\psi_m} P_{s,t}^{\psi_s} \tag{3.3}$$

At the sectoral level, households consume all the differentiated goods produced in each sector but the degree of substitution between them, θ, is higher than the one between goods produced in different sectors. Their preferences over the continuum of goods are captured by the Dixit-Stiglitz utility function (3.4), in which each good carries equal weight. Similarly as above, optimality conditions imply demand functions (3.5) and the price index of the sectoral basket (3.6).

$$C_{j,t} \equiv \left[\int_0^1 C_{j,t}(f)^{\frac{\theta-1}{\theta}} \, df\right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \tag{3.4}$$

$$C_{j,t}(f) = \left[\frac{P_{j,t}(f)}{P_{j,t}}\right]^{-\theta} C_{j,t} \tag{3.5}$$

$$P_{j,t} = \left[\int_0^1 P_{j,t}(f)^{1-\theta} \, df\right]^{\frac{1}{1-\theta}} \tag{3.6}$$

Apart from consumption, each household has to determine the amount of work it wishes to supply in the labour market, over which it is endowed with monopolistic power; hours worked by different households are therefore imperfect substitutes. Each firm in the production process employs a labour index $L_{j,t}$ that is composed of hours worked by all households in the sector. It is useful to think of the labour aggregation process as if it was carried out by a fictitious labour agency that observes all the wage contracts in force and allocates workers to the composite labour index in a cost-minimizing fashion described by the demand functions (3.8). Subsequently, the agency passes the index at cost $W_{j,t}$, defined in (3.9), to all firms operating in the sector. The agency allows workers to adjust the wage contracts only after passing of a random interval of time (Calvo-style wage contacts).

$$L_{j,t} \equiv \left[\int_0^1 L_{j,t}\left(h\right)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}, \quad \phi > 1 \tag{3.7}$$

$$L_{j,t}\left(h\right) = \left[\frac{W_{j,t}\left(h\right)}{W_{j,t}}\right]^{-\phi} L_{j,t} \tag{3.8}$$

$$W_{j,t} = \left[\int_0^1 W_{j,t}\left(h\right)^{1-\phi} dh\right]^{\frac{1}{1-\phi}} \tag{3.9}$$

With these preliminary definitions in mind we are now ready to define the households' decision problem. Their objective at time t is to maximise a time-separable utility function composed of the consumption index (3.1), the hours worked $L_{j,t}$ and money balances $M_{j,t}$, which provide liquidity services. The objective function takes the following form

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \mathrm{U}\left[C_{t+i}\left(h\right)\right] + \mathrm{V}\left[L_{j,t+i}\left(h\right)\right] + \mathrm{M}\left[\frac{M_{j,t+i}\left(h\right)}{P_{t+i}}\right]\right\} \tag{3.10}$$

where the operator E_t denotes expectations over all possible future states of nature conditional on the information available in t and β is a discount factor. Apart from nominal money balances, one-period state-contingent bonds $B_{t,s}$ are traded in the economy. They do not enter utility but serve as a risk sharing device. The price of a financial claim to one unit of nominal income in a particular state of nature s in the next period is $\delta_{t,t+1,s}$, which will be henceforth referred to as the stochastic discount factor. If the central bank in this economy supplies risk-less one-period bonds then it follows that the gross nominal interest rate on these assets at t satisfies $R_t = \left[\mathrm{E}_t \delta_{t,t+1}\right]^{-1}$.

The period budget constraint then imposes that household's h labour income and wealth at the beginning of period t must be spent either on consumption, lump-sum taxes or purchases of financial assets.

$$\begin{aligned} P_t C_t(h) + M_{j,t+1}(h) + \int_s \delta_{t,t+1,s} B_{t+1,s}(h) \\ \leq (1+g_w)\, W_{j,t}(h)\, L_{j,t}(h) + M_{j,t}(h) + B_t(h) + H_t(h) - T_t(h) \end{aligned} \quad (3.11)$$

where $H_t\,(h)$ denotes household's aliquot share in firms' profits and also includes the income from administering the fixed level of capital in that sector. Note that the role of government in this economy is limited to the collection of lump-sum taxes that are spent on price-subsidies to firms g_p and wage subsidies to households g_w. The idea behind these subsidies is to make the central bank solely responsible for distortions that originates from nominal rigidities, while the government applies transfer schemes to eliminate the inefficiencies related to monopolistic markets.

The first-order conditions of this problem with respect to consumption and bonds are given by

$$\mathrm{U_C}\left[C_t\,(h)\right] = \Lambda_t P_t \quad (3.12)$$

$$R_t^{-1} = \beta \mathrm{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \quad (3.13)$$

Assuming that the central bank carries out its monetary policy by setting the nominal interest rate, Rotemberg and Woodford (1997) show that it is safe to neglect the optimality condition for real money balances. The additional first-order condition would only pin down the nominal level of money. Due to the assumption of complete markets, the marginal utility of income, Λ_t, is common to all households so that they choose an identical level of consumption.

Finally, let us turn attention to the decision problem of households adjusting their wage contracts in period t. Since the labour agency allows to re-optimise the nominal wage only to a fraction $(1-\eta)$ of randomly selected workers each period, an individual worker therefore takes into account that he or she might not be able to change the wage in the subsequent periods. The worker will thus maximise the utility function (3.10) with a discount factor $(\beta\eta)$, which reflects the expected length of the new wage contract, subject to the budget constraints (3.11) and the labour demands (3.8), which are taken into account because the worker is a monopolistic provider of $L(h)$. Assuming that the government provides the worker with subsidy g_w that eliminates the monopolistic distortions, the first order condition with respect to wages is given by

$$\mathrm{E_t}\sum_{i=0}^{\infty}(\beta\,\eta)^i\,W_{j,t+i}^{\phi}L_{j,t+i}\left\{\mathrm{V_L}\left[\left(\frac{W_{j,t}(h)}{W_{j,t+i}}\right)^{-\phi}L_{j,t+i}\right]+\Lambda_{t+i}W_{j,t}(h)\right\}=0 \quad (3.14)$$

Because this equation is independent of all worker-specific variables but wages it follows that all workers adjusting their wage in a given period will set them at an identical level, $W_{j,t}^{*}$, which greatly simplifies the sectoral wage index (3.9). It can be shown (as in Calvo, 1983) that it becomes

$$W_{j,t}^{1-\phi}=\eta W_{j,t-1}^{1-\phi}+(1-\eta)\,W_{j,t}^{*\,1-\phi} \quad (3.15)$$

Workers, who are unable to adjust wages in the given period, commit themselves in the wage contract to provide their variety of labour at the wage rate set in preceding periods according to the labour demands (3.8). Note that under flexible prices, the optimality conditions for consumption (3.12) and labour (3.14) imply that wages must equal to the marginal rate of substitution, which is defined here for future reference:

$$MRS_{j,t}=-\frac{V_L(L_{j,t})}{U_C(C_t)} \quad (3.16)$$

3.2.2 Firms

Each individual firm is a monopolistic supplier of the differentiated good $Y_{j,t}(f)$, which it produces by means of the Cobb-Douglas technology (3.17) using capital services and the index of labour hours described above. In order to investigate the implications of different labour intensity for the conduct of monetary policy, labour intensity is allowed to differ across sectors with $\alpha_m \geq \alpha_s$. I further assume that all firms operating in a sector are subject to common shocks that jointly affect the productivity of both production factors in the sector. The production function is defined by

$$Y_{j,t}(f) = A_{j,t} K_{j,t}(f)^{\alpha_j} L_{j,t}(f)^{1-\alpha_j} \tag{3.17}$$

Similarly to households, firms sign Calvo-style nominal price contracts that allow them to change their prices only at exogenous random intervals. Specifically, a fraction $(1-\epsilon)$ of firms gets to choose a new price at the beginning of a quarter. The probability of re-adjusting the price in any given period is independent of time that has elapsed since the firm set its price the last time. The decision problem of firm f therefore consists of maximizing the present discounted value of expected profits achieved under the assumption that the price set in t would, with probability ϵ, apply also in the future, subject to the demand functions for its particular variety given by (3.2) and (3.5). Taking into account the market clearing conditions $Y_{j,t} = 2\,C_{j,t}$ and assuming that the government applies production subsidy g_p to offset monopolistic mark-ups, the first-order condition of this problem is

$$\mathrm{E_t} \sum_{i=0}^{\infty} \epsilon^i \delta_{t,t+i} \left(\frac{P_{j,t}(f)}{P_{j,t+i}} \right)^{-\theta} \left(\frac{P_{j,t+i}}{P_{t+i}} \right)^{-1} Y_{t+i} \left(P_{j,t}(f) - MC_{j,t+i} \right) = 0 \tag{3.18}$$

Note that in the limiting case of fully flexible prices when $\epsilon = 0$ this equation imposes that prices equal the cost $MC_{j,t}$ arising from production of one unit of $Y_{j,t}(f)$.

Firms not able to re-set their prices in the period solve the following minimisation problem

$$\min_{K,L} \; P^k_{j,t} K_{j,t}(f) + W_{j,t} L_{j,t}(f) \quad \text{s.t.} \quad A_{j,t} K_{j,t}(f)^{\alpha_j} L_{j,t}(f)^{1-\alpha_j} \geq 1 \tag{3.19}$$

the first-order conditions of which imply, first, that all firms in a sector employ identical capital-labour ratios and, secondly, that marginal costs can be expressed as a weighted average of factor prices divided by the productivity shock $A_{j,t}$ or, alternatively, as a ratio of wages and the marginal product of labour.

$$MC_{j,t} = \frac{P^{k,\alpha_j}_{j,t} W^{1-\alpha_j}_{j,t}}{A_{j,t}\, \alpha_j^{\alpha_j} \left(1-\alpha_j\right)^{1-\alpha_j}} = \frac{W_{j,t}}{\left(1-\alpha_j\right) A_{j,t} K_j^{\alpha_j} L_{j,t}^{-\alpha_j}} = \frac{W_{j,t}}{MPL_{j,t}} \tag{3.20}$$

Writing marginal costs in terms of wages and the marginal product of labour simplifies the solution of the model because it allows to abstract from computing the price of capital (note that the sectoral level of capital is fixed). Since marginal costs are independent of firm-specific variables, it follows from the price setting equation (3.18) that all firms adjusting their prices in a given period will choose the same price $P^*_{j,t}$. The sectoral price index (3.6) therefore simplifies to

$$P^{1-\theta}_{j,t} = \epsilon P^{1-\theta}_{j,t-1} + (1-\epsilon)\, P^{*\,1-\theta}_{j,t} \tag{3.21}$$

The last equation states that prices in force in period t can be decomposed into a fraction ϵ of price contracts passed from the previous period and a fraction $(1-\epsilon)$ of newly adjusted prices. To complete the description of the block of firms it remains to define the sectoral level of output that corresponds to the sectoral labour index $L_{j,t}$, used in the equations above, and the fixed level of capital in a sector. Canzoneri, Cumby and Diba (2005) show that it is given by

$$\begin{aligned} Y_{j,t} &= A_{j,t} K_j^{\alpha_j} L_{j,t}^{1-\alpha_j} \left(DP_{j,t}\right)^{-1} \\ DP_{j,t} &\equiv \int_0^1 \left[\frac{P_{j,t}(f)}{P_{j,t}}\right]^{-\theta} df \end{aligned} \tag{3.22}$$

where the last term reflects price dispersion in sector j – a term, that will play a crucial role in the analysis below. Its inverse relationship to sectoral output aptly illustrates how price dispersion decreases the amount of output available for consumption for a given level of the labour index. Canzoneri at al. (2005) relate the price dispersion to welfare losses by observing that the social planner, in order to maximise consumption, would allocate production of differentiated goods equally to all firms (because their weights in the consumption index are equal). But since firms, due to nominal price rigidities, charge different prices in the equilibrium, the Pareto efficient allocation is infeasible. Lower demands for one good must be more than compensated by higher demands for other goods (due to their imperfect substitutability) to achieve a given level of utility. Higher sectoral output then implies higher demand for labour and correspondingly greater disutility from labour. Analogously to the price and wage indices defined above, it can be shown that the price dispersion can be restated in a computationally simpler form of

$$DP_{j,t} = \epsilon \left(\frac{P_{j,t-1}}{P_{j,t}} \right)^{-\theta} DP_{j,t-1} + (1-\epsilon) \left(\frac{P^*_{j,t}}{P_{j,t}} \right)^{-\theta} \tag{3.23}$$

3.2.3 Equilibrium and parameterisation

The preceding subsections described the optimal behaviour of households and firms and their mutual relationships. We are now ready to define the equilibrium for the whole economy. It is defined as a set of allocations that includes the aggregate and sectoral consumption bundles C_t and $C_{j,t}$ and the labour indices $L_{j,t}$, and a set of prices that includes the aggregate and sectoral price indices P_t, $P_{j,t}$, wage indices $W_{j,t}$, prices and wages of those who can adjust them in the given period, $P^*_{j,t}$ and $W^*_{j,t}$, and the marginal costs $MC_{j,t}$ so that, for a given level of the nominal interest rate R_t set by the central bank and the stochastic productivity processes (defined below), the following conditions hold for all $t \geq 0$:

1. households maximise consumption over time according to (3.12) and (3.13) and set wages according to (3.14)
2. firms set prices according to (3.18) and (3.20)
3. the price and wage indices satisfy (3.3), (3.21) and (3.15)
4. sectoral outputs are given by the demand functions (3.2)
5. sectoral labour indexes follow from the aggregate resource constraints as stated in (3.22) and (3.23)
6. markets clear, so that we have $Y_{j,t} = 2\,C_{j,t}$ for both sectors and $Y_t = 2\,C_t$ at the aggregate level.

This system does not have a closed-form solution, nonetheless it can be identified by perturbing the deterministic equilibrium, in which the price and wage inflation is zero and all variables take constant values over time. Description of how this system behaves in a vicinity of the deterministic equilibrium is a topic of section 3.3.2. Before turning to the log-linear analysis though, a description of the functional forms and parameter values used in the computations is due.

The following functions for the period utility functions of consumption and leisure are assumed in the model:

$$\mathrm{U}(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$
$$\mathrm{V}_j(L_{j,t}) = \kappa_j \frac{(1-L_{j,t})^{1-\chi}}{1-\chi}$$

Allowing the leisure preference parameter κ_j to differ across sectors, as e.g. in Erceg and Levin (2006), greatly facilitates the derivation of the social welfare function because it permits to consider the steady-state elasticity of substitution in leisure identical across sectors. Different elasticities would bring about an additional source of dissimilarity, which would obscure the results presented in this chapter. For the sake of comparability, the above-referred study is also the source of most of the parameters used here, in particular of σ, χ, θ and ϕ. The structural parameters in the utility functions are set to $\sigma = 2$ and $\chi = 3$ and the shares of labour hours to leisure in the steady state are equal to 1/2 in both sectors, which calibrates the weights of leisure in the utility function κ_j. Next, in order to simplify interpretation of optimal targeting rules, the shares of sectors in the aggregate output are equal to $\psi_m = \psi_s = 0.5$. The discount rate β equals 0.99 implying that the steady-state real interest rate is 1.01% on a quarterly basis or roughly 4% annually. Parameters $\theta = \phi = 4$ so that steady state mark-ups are equal to 33%. Finally, the expected contractual duration of prices and wages is four quarters (the parameters ϵ and η are both equal to 0.75, as in Erceg et al., 2000).

What remains to specify are the properties of stochastic innovations. Erceg and Levin (2006) characterise them by means of bivariate AR(1) processes, which allows the coefficients and standard errors to differ in both sectors. However, since the strategy followed in this paper is to isolate the effects of different labour intensities, the properties of shocks are assumed to be symmetric here. They follow a bivariate first-order stochastic process $A_t = 0.95\, A_{t-1} + e_t$, where e_t is an *i.i.d.* process with variances $\sigma_m^2 = \sigma_s^2 = 0.0086^2$, in line with the traditional RBC literature, and $corr\,(e_m, e_s) = 0.29$, as in Erceg and Levin (2006). The computational approach followed here is based on the estimation of variances of certain variables included in the model, so only the properties of stochastic processes matter, the particular draws of shocks are not relevant for the results.

3.3 Equilibrium of the log-linearised model

The linear-quadratic approach of Rotemberg and Woodford (1997) to welfare evaluation relies on the second-order Taylor-series approximation to the social welfare function and first-order approximations to the structural equations of the model that were derived in the previous section. This section outlines the structural equations in their log-linear form while the derivation of the social welfare function is postponed to section 3.4. The structural equations shown here define how the system responds to *small* shocks that perturb the non-stochastic equilibrium (small in the sense that first-order Taylor series expansion still provides an accurate description of the system).

This section is divided into two parts: the first solves for the equilibrium of the log-linear model in the case of fully flexible wages and prices, which represents the Pareto efficient allocation in this model. The other section derives the equations in the presence of nominal price and wage contracts.[1]

[1]In what follows, all nominal variables are rendered stationary by suitable transformations so that wages and prices set in a given period are standardized by the corresponding sectoral indices. The sectoral price and wage aggregates and the nominal interest rate are divided by the aggregate price level. A percentage deviation of a variable X_t from its steady state value $\overline{X}$ will be denoted with lower-case letters, e.g. x_t, and represent a first-order approximation to $ln\left(X_t/\overline{X}\right)$. Superscripted variables, x^n, denote the value of the variable x in the efficient equilibrium and, finally, $\hat{x}$ stands for the gap between the actual level of a variable and its value in the efficient equilibrium, $x - x^n$.

3.3.1 Flexible price and wage equilibrium

Let us start with defining the two fundamental parameters of the log-linear model: elasticity of marginal utility of consumption and elasticity of marginal disutility of labour. The former is defined as $\rho = -\frac{U_{CC}}{U_C}\overline{C}$ and the latter as $\omega = \frac{V_{LL}}{V_L}\overline{L}$. With the functional forms specified above it is evident that these parameters are common to both sectors and are equal to σ and $\chi/2$, respectively. The weighted average of shocks in the economy is $a_t^w = \psi_m a_{m,t} + \psi_s a_{s,t}$ and, finally, the common denominator of the equations that follow, Λ, is equal to $\omega + \rho + (1-\rho)(\psi_m \alpha_m + \psi_s \alpha_s)$. With these definitions in mind, we can now proceed to characterise the solution of the efficient equilibrium.

The real interest rate, as defined by the Euler equation (3.12, 3.13), is given by $i_t^n = \mathrm{E}_t \rho \left(y_{t+1}^n - y_t^n\right)$. Socially optimal allocation requires that the impacts of productivity shocks are spread equally among households in both sectors, so that their consumption (and the aggregate output) corresponds to

$$y_t^n = \left((1+\omega)/\Lambda\right) a_t^w \tag{3.24}$$

Similarly, the social planner then distributes the given change in production equally among all workers, therefore also the labour indices (and individual hours of work) respond only to the economy-wide average of shocks

$$l_{j,t}^n = \left((1-\rho)/\Lambda\right) a_t^w$$

These two equations imply that the marginal rate of substitution is equalized across sectors, hence also real wages (normalized by the aggregate price level) respond identically: $mrs_{j,t}^n = w_{j,t}^n = \left((\omega+\rho)/\Lambda\right)\, a_t^w$. However, the social planner recognizes that the marginal product of labour, defined in (3.20), is steeper in the capital-intensive sector and allocates the sectoral outputs accordingly.

$$mp\,l_{j,t}^n = a_{j,t} - \alpha_j \left((1-\rho)/\Lambda\right) a_t^w \tag{3.25}$$

$$y_{j,t}^n = (1-\alpha_j)\left((1-\rho)/\Lambda\right)\, a_t^w + a_{j,t} \tag{3.26}$$

At this place it is appropriate to foreshadow that the steeper slope of the marginal product in manufacturing will, with rigid prices, motivate the desire of the central bank to fight inflation in the manufacturing sector more emphatically than in services because a given change of output will require greater response of the labour index there. We will return to this point in section 3.4. For future reference, relative prices are defined as $p_{rel,t} = p_{m,t} - p_{s,t}$ and reflect the different response of sectoral outputs:

$$p^n_{rel,t} = (\alpha_m - \alpha_s)\,((1-\rho)/\Lambda)\ a^w_t - (a_{m,t} - a_{s,t}) \tag{3.27}$$

This concludes the description of the efficient equilibrium of the model and we now turn to the case of asynchronous price and wage adjustments.

3.3.2 Equilibrium with nominal rigidities

The demand side of the model is characterized by the inter-temporal IS equation

$$\hat{y}_t = \mathrm{E}_t\hat{y}_{t+1} - \frac{1}{\rho}\,(i_t - \mathrm{E}_t\pi_{t+1} - i^n_t) \tag{3.28}$$

derived from the first-order conditions of the household problem. This equation embodies the negative dependence of the output gap, $\hat{y}$ on the gap between the real interest rate and its counterpart in the efficient equilibrium, where the latter summarizes how current and expected future productivity shocks affect the course of the economy. Some authors refer to it from this reason as the "Wicksellian" natural rate of interest (e.g., Woodford, 2003). By solving the equation forward one can further show that the central bank can stabilise the *aggregate* output gap by adjusting the expected path of the real interest rate along the natural interest rate.

The supply block of the dynamic model consists of two sets of behavioural equations that characterise the price setting and wage setting processes, respectively, and several definitions describing the linkages between sectoral price and wage indices.

First, log-linearising the firms' price setting equation, (3.18), together with the sectoral price index, (3.21), yields

$$\pi_{j,t} = \mu_p\,(w_{j,t} - mp\,l_{j,t} - p_{j,t}) + \mathrm{E}_t\beta\pi_{j,t+1} \tag{3.29}$$

where the parameter $\mu_p = \frac{1-\epsilon}{\epsilon}(1-\epsilon\beta)$ determines the degree of inflation persistence. The marginal product of labour follows from the definition in (3.20), where the labour index is replaced using the sectoral resource constraint (3.22). Substituting in the solutions from the efficient equilibrium and re-arranging (as in Erceg et al., 2000), it is possible to show that the marginal product is negatively related to the sectoral output gap (the last term at the right-hand side). To conserve space, the following expression uses signed shares of sectoral consumption bundles in the consumption aggregator $\widetilde{\psi}_m = \psi_m$ and $\widetilde{\psi}_s = -\psi_s$.

$$mpl_{j,t} = w^n_{j,t} - p^n_{j,t} - \frac{\alpha_j}{1-\alpha_j}\left(\hat{y}_t - \widetilde{\psi}_j\,\hat{p}_{rel,t}\right) \tag{3.30}$$

By following a similar strategy, from the wage-setting relationship (3.14) and the wage index (3.15) it is possible to derive a dynamic equation that determines sectoral wage inflation as a function of expected wage inflation and the deviations of the marginal rate of substitution from the real wage.

$$\pi^w_{j,t} = \mu_w\left(mrs_{j,t} - w_{j,t}\right) + \mathrm{E}_t\beta\pi^w_{j,t+1} \tag{3.31}$$

where the coefficient of inertia in wage inflation $\mu_w = \frac{1-\eta}{\eta}(1+\omega\phi)(1-\eta\beta)$ plays a similar role as μ_p above. The marginal rate of substitution is obtained by log-linearising (3.16)

$$mrs_{j,t} = w^n_{j,t} + \left(\frac{\omega}{1-\alpha_j} + \rho\right)\hat{y}_t - \widetilde{\psi}_j\left(\frac{\omega}{1-\alpha_j}\right)\hat{p}_{rel,t} \tag{3.32}$$

where one can observe that lower substitutability in consumption (higher ρ) makes the marginal rate of substitution relatively more responsive to the aggregate output gap than to changes in relative prices. This will become important in section 3.4, where it will counteract the central bank's willingness to allow extensive deviations in outputs across sectors.

Finally, the following relationships between prices, wages and the corresponding inflation rates close the system of log-linear equations:

$$\pi_{j,t} = p_{j,t} + \pi_t - p_{j,t-1} \tag{3.33}$$

$$\pi^w_{j,t} = w_{j,t} + \pi_t - w_{j,t-1} \tag{3.34}$$

$$\hat{p}_{rel,t} = p_{m,t} - p_{s,t} - p^n_{rel,t} \tag{3.35}$$

$$0 = \psi_m p_{m,t} + \psi_s p_{s,t} \tag{3.36}$$

where the first two equations simply follow from the definitions of inflation rates (normalized by the aggregate price index), the third equation defines the relative price gap and the final relationship is a log-linear version of the aggregate price index (3.3). Assuming that the path of the nominal interest rate is exogenously determined by the central bank, the equations reported in this section form a block of fourteen linear equations that fully describe the dynamic response of all endogenous variables to productivity shocks.

3.4 Nominal rigidities and welfare

This section turns to the main question addressed in this paper: what is the optimal policy of the central bank when sectors do not employ identical factor proportions and how can it be implemented? To answer the first question, a social welfare function is derived to see what variables should the central bank stabilize, while the issue of its successful implementation will be discussed subsequently by means of numerical simulations.

I follow here the work of Rotemberg and Woodford (1997), who showed how to derive a social welfare function from microeconomic foundations and how to compare it to its ideal value achievable in an economy without nominal frictions. Furthermore, the derivation of the welfare criterion consistent with this particular model makes use of strategies outlined in Aoki (2001), Benigno (2004) and Erceg et al. (2000). To start with, the social welfare is defined as an unconditional expected value of the sum of households' discounted utility. Taking unconditional expectations is needed in order to take into account all possible histories of shocks that may have occurred prior to date t. Before taking the unconditional expectations, the sum of discounted (*time t-conditional*) utilities is denoted SW_t

$$SW_t = E_t \sum_{i=0}^{\infty} \beta^i \left[2\, U(C_{t+i}) + \int_0^1 V(L_{m,t+i}\,(h))\, dh + \int_0^1 V(L_{s,t+i}\,(h))\, dh \right] \quad (3.37)$$

The criterion that the monetary authority wishes to maximise is defined as $SW = E\,(SW_t - SW_t^n)$. The law of iterated expectations then implies that this problem is equivalent to minimizing the *period* average utility losses from deviating from the efficient equilibrium.

The Rotemberg and Woodford's method proceeds by taking second-order Taylor expansions of each element in (3.37), which allows one to describe the welfare criterion exclusively as a function of second moments of the aggregate and sectoral output gaps and the price and wage inflation rates in each sector.[2]

$$\begin{aligned} E\,\widehat{SW} = {} & \gamma_y\,(1-\rho)\, var\,\hat{y} - \psi_m\,\gamma_y\,\frac{1+\omega}{1-\alpha_m}\, var\,\hat{y}_m - \psi_s\,\gamma_y\,\frac{1+\omega}{1-\alpha_s}\, var\,\hat{y}_s \\ & - \frac{\gamma_p}{1-\alpha_m}\, var\,\pi_m - \frac{\gamma_p}{1-\alpha_s}\, var\,\pi_s - \gamma_w\, var\,\pi_m^w - \gamma_w\, var\,\pi_s^w \end{aligned} \quad (3.38)$$

plus terms independent of policy and the third- and higher-order terms. This welfare function implies that there are two kinds of variables to be taken into account by the social planner – the gaps between the actual and the optimal levels of output and sectoral measures of nominal variability – both of which are rooted in the presence of nominal rigidities but hint at different aspects of welfare losses.[3]

The former term, variance of output gaps, reflects the costs of output deviations from its efficient level and points to the objective of the social planner to respond to sector-specific shocks so that resources are allocated efficiently across sectors. Note that the presence of output *gaps* indicates that a complete

[2]The reader is kindly referred to the Appendix for details of the derivation. The coefficients in (3.38) are defined as follows: $\gamma_y = U_C\overline{C}$, $\gamma_p = -V_L\overline{L}\,\frac{\epsilon\,\theta}{2(1-\epsilon)^2}$ and $\gamma_w = -V_L\overline{L}\,\frac{\eta\,\phi\,(1+\omega\,\phi)}{2\,(1-\eta)^2}$. Note that all these terms are positive and ρ is larger than one, so that all terms in (3.38) enter with a negative sign.

[3]For an excellent exposition of the role played by each element of welfare functions of this sort see Woodford (2003).

stabilisation of the level of output is undesirable; the central bank aims, instead, at establishing such changes in output that would prevail under flexible prices (recall that nominal rigidities are the only source of distortions in this economy).

The other set of variables that appear in (3.38) is related to the price and wage dispersion, which are caused by asynchronous nominal adjustment. These terms give rise to welfare losses even if the central bank manages to close the output gap completely because the lags in nominal adjustment generate inefficient dispersion of demands for individual goods and workers. Because goods and workers are imperfect substitutes, as can be seen from the aggregators (3.4) and (3.7), disproportional consumption of the individual goods results in spending more resources to produce a given level of the consumption bundle (and similarly for workers). To see this, notice that all the goods carry equal weight in the aggregator, hence the optimal allocation would require each firm to produce an equal amount of its own good.

3.4.1 Role of sectoral labour intensities

The first question addressed in this paper – Should the central bank respond to differences in labour intensities across sector? – can be answered by inspecting the social welfare function (3.38). The answer is "Yes", labour intensity affects the weights of some, not all though, of the sector-specific variables in the welfare function. The weight of sector j variables is inversely related to labour intensity $1 - \alpha_j$, which means that the social planner can, other things equal, achieve a greater increase in the average utility by pursuing a policy that is biased towards lowering the variance in the capital-intensive sector. In order to understand the nature of the task faced by the central bank, let us inspect the origin of the sector-specific weights: the marginal product of labour.

As was already indicated in section 3.3.1, the slope (in absolute value) of the marginal product increases with capital intensity (falls with labour intensity), which implies that production of an additional unit of output makes capital-intensive firms hire more labour than labour-intensive firms. When demand for output increases, a capital-intensive firm needs to hire relatively more workers compared with the labour-intensive firm, where the marginal product is flatter, since the marginal product decreases faster with each additional worker hired.

This logic carries through also to the sectoral level. Rewriting the production function of an individual firm using the sectoral labour demand $L_j = \int L_j(f)\,df$ as $Y_j(f) = A_j(K_j/L_j)^{\alpha_j} L_j(f)$, it is possible to show that the sectoral labour demand is given by[4]

$$l_j = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\,\theta(1-\alpha_j)} var_f y_j(f) \tag{3.39}$$

The first term on the right-hand side reflects the average amount of the labour index employed by firms in a sector, which is also the origin of the sectoral output-gap coefficients in the social welfare function, as shown in the Appendix.

The other term on the right-hand side represents the extra labour cost caused by asynchronous price adjustment and imperfect substitutability among goods: if the demands for individual goods differ across firms then the total amount of output produced by individual firms (hence the total amount of labour) has to increase in order to meet a given demand for the sectoral output. The variance term can be directly related to price dispersion if one recalls that the demand for an individual good is given by (3.5), which implies that $var_f y_j(f) = \theta^2 var_f p_j(f)$. Erceg et al. (2000) then show that

$$E\, var_f p_j(f) = \frac{\epsilon}{(1-\epsilon)^2}\, var\, \pi_j$$

which gives rise to the price-inflation term in the social welfare function (3.38).

While the inefficiencies generated by contractual price setting depend on the relative labour-intensity of production, it is not the case with dispersion in individual labour hours that arises from wage rigidities. The key relationship in this regard is the log-linearised version of the sectoral labour aggregate (3.7), by means of which the fictitious labour agency aggregates the individual hours of work into the composite requested by firms. Relating it to the aggregate amount

[4] This is equation (3.53) in the Appendix.

of hours supplied by households, $N_j = \int L_j(h)\, dh$, it is possible to derive the following relationship[5]

$$n_j = l_j + \frac{1}{2\,\phi}\, var_h l_j(h) \tag{3.40}$$

which shows that in order to produce a given amount of the labour index l_j households have to provide more labour hours in total when the individual labour supplies differ from each other. The non-degenerate dispersion in individual labour hours is, similarly to the price rigidities described above, caused by asynchronous wage adjustment and by the fact that individual workers are imperfect substitutes to each other (with greater elasticity of substitution between individual workers, ϕ, the inefficiencies decrease). With regard to the questions addressed in this paper, it is important to notice that these inefficiencies are independent of the labour-intensity because the way workers are assembled to the final labour composite is common to both sectors.

3.4.2 Role of aggregate output gap

The results obtained so far imply that the output gap and the price inflation in manufacturing receive a higher weight in the social welfare function (3.38) compared with their counterparts in services. However, the welfare function also shows that the central bank faces a trade-off when it tries to stabilise one sector (around its efficient level) more intensively at the expense of greater volatility in the other sector. Mathematically, the trade-off is represented by the appearance of the aggregate output gap (square) in the welfare function (before taking unconditional expectations the variance term is in fact equal to $\hat{y}^2$). Since the weights of sectoral outputs in the aggregate output correspond to ψ_m and ψ_s and, most notably, are not affected by the respective labour intensities, minimizing this term would require to minimise both sectoral output gaps according to the weights they receive in the aggregate.

Intuitively, this term originates in households' relative willingness to adjust their labour supply and to substitute in consumption between the sectoral bun-

[5]This is equation (3.48) in the Appendix.

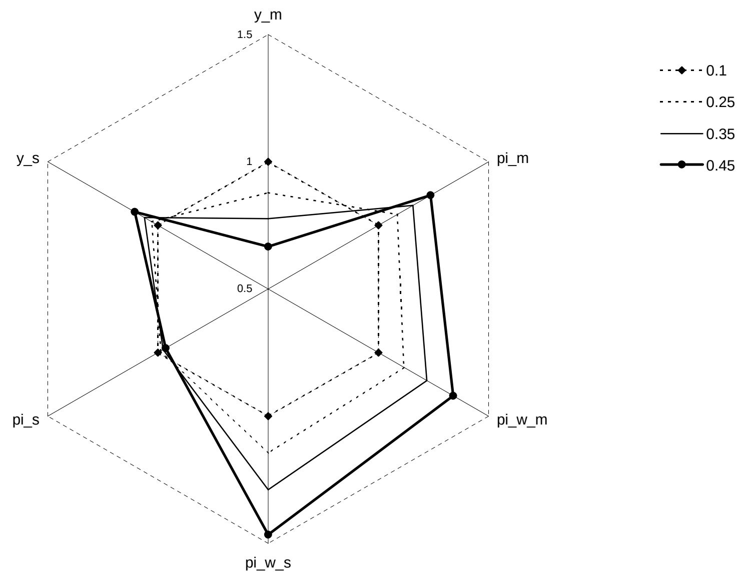
dles. While it may be optimal for the central bank to push manufacturing always closer to its efficient level (to reduce the inefficiencies resulting from output variability), these efforts are limited on the consumers' side by the elasticity of their preferences. The latter is dictated by parameters ρ and ω, which are inversely related to elasticities of utility from consumption and labour, respectively. In particular, if ρ is relatively high (consumption is inelastic), consumers would prefer the central bank to respond equivalently to shock in both sectors to achieve a more balanced path of the aggregate consumption. Similarly, if ω is relatively low (labour supply is elastic), the fact that output dispersion requires greater shifts in the labour index hurts workers relatively less and thus the central bank is motivated to reduce output losses in both sectors in a more balanced way.

The desire to smooth the relative discrepancies between sectoral outputs is also magnified by the presence of wage rigidities. When workers are contractually committed to provide labour hours at the existing wage rate, adjustment to the efficient level of output lasts longer since the pull of the wealth effect, which would otherwise push demands in desired direction, is mitigated. Thus the welfare maximizing central bank has an additional incentive to curb the output gaps equally in both sectors to accelerate the aggregate output gap adjustment (by making the covariance between the output gaps more negative).

3.4.3 Optimal monetary policy

To solve for the optimal policy I follow the Lagrangean approach of Woodford (2003) which consists in maximizing the social welfare function (3.38), the exact form of which is given in the Appendix, subject to the behavioural constraints presented in section 3.3.2. Next, I use the Anderson-Moore (1983) algorithm to map the system of the dynamic first-order conditions into an auto-regressive form, the stationarity of which then allows to iterate the system to obtain variances of all variables and, in the final step, to evaluate the social welfare function. Solving for an explicit policy rule, such as the Taylor rule, is not feasible here, similarly as in Benigno (2004).

Figure 3.1: Variances of sectoral variables in the optimum: $\alpha_s = 0.10$, α_m varies



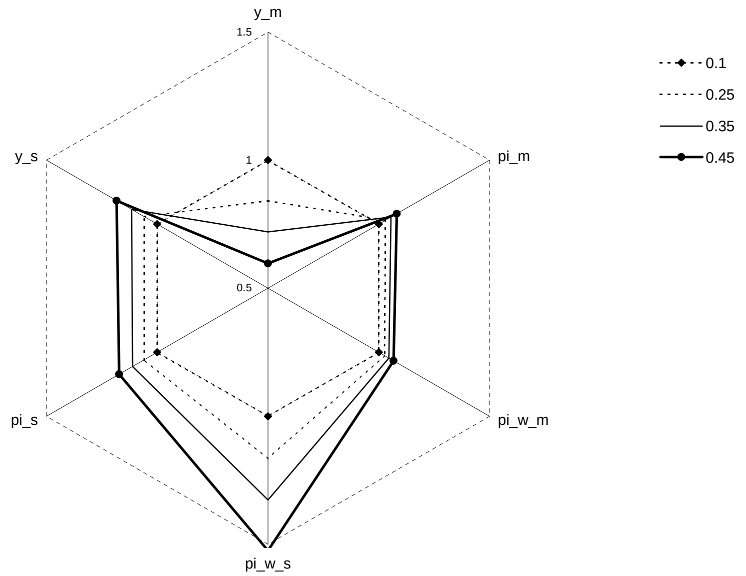
Note: The figure shows socially-optimal variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of capital intensity in manufacturing, α_m. All variances are expressed relative to those achieved when $\alpha_m = \alpha_s = 0.1$.

The logic behind the optimal response of the central bank is demonstrated in Figures 3.1 and 3.2. The figures show the socially-optimal variances of sectoral variables (output gaps and inflation) that feature in the social welfare function. Figure 3.1 refers to the baseline scenario with an equal degree of price and wage rigidities, while Figure 3.2 shows the outcome with relatively low wage rigidity. All variances are expressed relative to the case when labour intensity is equal across sectors.

Both figures confirm the intuition described above: as capital intensity in manufacturing increases (labour intensity falls) it is optimal to reduce variance of the output gap in that sector (denoted as `y_m`) at the cost of higher variability of the output gap in services, `y_s`.

With relatively flexible wages, one can observe that the central bank is able to respond more emphatically to shocks in manufacturing to a much greater extent

Figure 3.2: Variances of sectoral variables in the optimum with less rigid wages



Note: The figure shows socially-optimal variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of capital intensity in manufacturing, α_m. All variances are expressed relative to those achieved when $\alpha_m = \alpha_s = 0.1$.

than the in the baseline scenario because the adjustment is faster overall. Hence, the trade-off between reducing the inefficient output dispersion in manufacturing and increasing the output gap overall (due to lower covariance between the two sectors) is less difficult.

As indicated above, the optimal plan is defined only implicitly as an autoregressive stochastic process of the endogenous variables (and of the corresponding Lagrange multipliers). Such a prescription is, however, of limited value for the central bank that seeks to communicate its policy to general public in the hope to influence its inflation expectations. In the next section I will therefore analyze the optimal actions of the monetary authority assuming that it can commit itself to a credible inflation target.

3.4.4 Implications for inflation targeting

Having explored the socially optimal plan for this economy, we will now investigate how the optimal plan can be implemented by means of inflation targeting. In particular, the question is: What is the optimal inflation target in an economy with nominal frictions and two sectors that differ in the extent of capital and labour intensity?

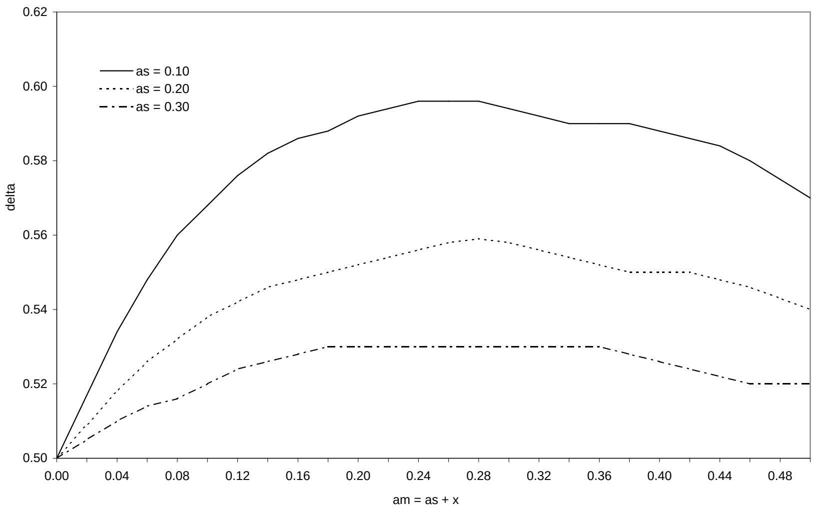
The choice of inflation targeting is grounded in the fact that rules based particularly on inflation do not require the knowledge of the efficient levels of output or relative prices, as would be the case if one wanted to target measures including also output *gaps*. In particular, we will inspect the performance of following inflation targeting rule

$$\delta\,\pi_m + (1-\delta)\,\pi_s = 0$$

where we are interested in finding the optimal value of δ that allows the central bank to come as close as possible to the optimal plan. This rule can be understood as an implicit form of an interest-rate rule that requires the central bank to increase the interest rate when the weighted index of inflation increases above zero and vice versa, to decrease the nominal interest rate if the index drops below zero. To the extent that price inflation reflects output variability, as explained in section 3.4.1, it is also an indirect measure of shocks that hit the given sector. If prices and wages were fully flexible, the relative prices would immediately adjust to reflect the relative productivities in both sectors and the resulting demand shift would bring about the efficient structure (and level) of production.

With rigid prices (and wages), the central bank can, by adjusting the nominal interest rate, attempt to stimulate aggregate demand in order to mimic the efficient relative price adjustment so that the economy starts shifting toward the efficient level of output. Furthermore, by adjusting the sectoral weights in the targeted inflation index it is able to systematically assign higher or lower priority to shocks (and output gaps) in a particular sector. Complete stabilisation of

Figure 3.3: Optimal weight of manufacturing (δ) in the inflation target



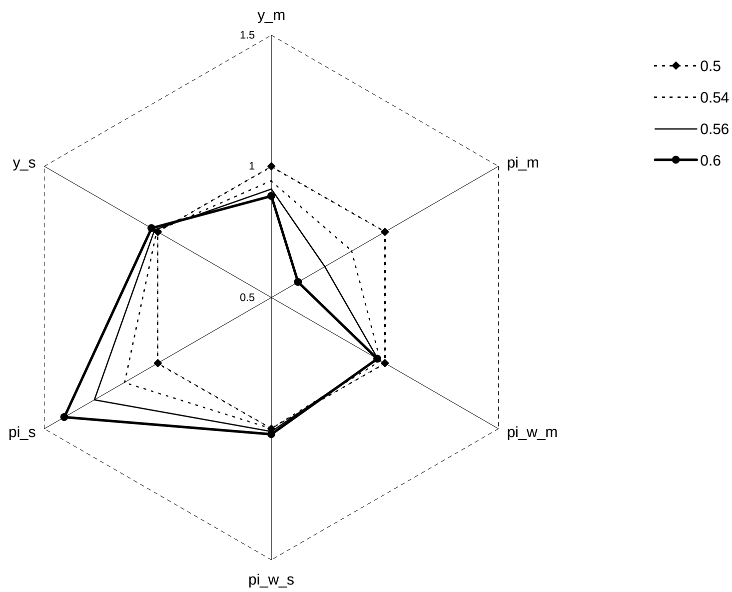
Note: The figure shows the weight of inflation in the capital-intensive sector in the central bank's optimal inflation target. Optimal weights are reported for selected values of capital intensity in services `as` (different lines) and corresponding ranges of capital intensity in manufacturing `am` (along the horizontal axis).

both output gaps in an economy, where both sectors exhibit nominal rigidities, is nevertheless infeasible (as shown in Benigno, 2004).

Figure 3.3 reports the optimal weights of manufacturing, δ, for selected values of capital intensity in services ($\alpha_s \in \{0.1, 0.2, 0.3\}$) and corresponding ranges of capital intensities in manufacturing defined as $\alpha_m \in (\alpha_s, \alpha_s + 0.5)$.

The key observation that emerges from the figure is that, as the capital intensity in manufacturing increases, the optimal inflation index assigns higher weight to manufacturing, which effectively means that the central bank prioritizes closing of the output gap in that sector. The intuition for this result goes back to the discussion in the section 3.4.1: the central bank tries to reduce inefficiencies resulting from output dispersion, the extent of which is greater in the relatively more capital-intensive sector. Note that if sectors do not differ in factor intensities, the figure shows that their respective weights in the inflation index exactly correspond to their weights in the consumption basket: $\psi_m = \psi_s = 0.5$ (for comparative purposes I will henceforward refer to such a policy as a *symmetric* targeting).

Figure 3.4: Elements of welfare on the way to the optimal policy



Note: The figure shows variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of the share δ of the capital-intensive sector in central bank's inflation target. Capital intensities are $\alpha_s = 0.10$ and $\alpha_m = 0.25$, for which the optimal weight of manufacturing is 0.6. All variances are expressed relative to those achieved under a symmetric inflation target ($\delta = 0.5$).

The outcome from following the optimal targeting policy on the variables included in the social welfare function is demonstrated in Figure 3.4, which confirms that, as the central bank starts moving away from the symmetric targeting rule ($\delta = \psi_m$) toward the optimal one, the output dispersion (i.e., price inflation) in manufacturing decreases and the output of this sectors follows its efficient path more closely. On the other hand, the output gap and, most significantly, the dispersion of individual outputs (price inflation) in services takes the opposite course.

Two other results follow from Figure 3.3. First, as the capital intensity of services increases as well, the extent of the optimal bias toward manufacturing decreases (this is represented by a shift to a lower curve in the figure).[6] With

[6]Mathematically, this can be seen from the relative weights of $\hat{y}_m$ and $\hat{y}_s$ in the objective

greater use of capital in the economy overall, the marginal product of labour becomes steeper in both sectors and the preference to fight inefficiencies in only one of the sectors becomes less obvious (notice that the relationship between output dispersion and its cost in terms of the additional increase in the labour index in (3.39) is non-linear as well).

Finally, one can observe that the size of the optimal bias increases in a concave fashion and, in fact, as the *difference* in the capital intensity across sectors becomes large it eventually reverses its trend. This finding can be understood in the light of the central bank's trade-off indicated in section 3.4.2. As capital-intensity in manufacturing increases the central bank is tempted to close the manufacturing output gap with greater force than in services. However, by doing so the central bank allows the productivity shocks in services to take their natural course, which would eventually result in larger volatility of consumption and output overall. This can be tolerated only to the extent given by consumers' preferences. Should it become excessively volatile, the welfare maximizing central bank starts responding to the sectoral shocks in a more balanced way.

This trade-off also stands behind the poor performance of the inflation targeting rule in the quantitative sense. Table 3.1 compares the performance of the optimal inflation target to the symmetric one. The measure of performance compares the size of dead-weight losses eliminated under the targeting policy with optimal weights δ^* to the improvement achievable by implementing the fully optimal plan. Following Benigno (2004), it is constructed as follows

$$\frac{\widehat{SW}\left(\delta^*\right) - \widehat{SW}\left(\delta = 1/2\right)}{\widehat{SW}^* - \widehat{SW}\left(\delta = 1/2\right)}$$

It turns out that while targeting the optimal inflation measure represents an improvement compared to the symmetric target, the magnitude of welfare gains is rather small compared to that achievable under the fully optimal policy. This finding accords with conclusions of Erceg and Levin (2006), who inspect the performance of inflation targeting in a two-sectors economy, where sectors differ in

function: $\psi_m \gamma_y \frac{1+\omega}{1-\alpha_m} / \psi_s \gamma_y \frac{1+\omega}{1-\alpha_s} = \frac{1-\alpha_s}{1-\alpha_m}$. Higher α_s decreases the relative weight of the output gap in manufacturing.

Table 3.1: Welfare improvement with the optimal inflation target

α_s	α_m	δ^*	Performance (%)
0.10	0.30	0.58	2.7
0.10	0.45	0.59	2.8
0.20	0.40	0.54	1.0
0.20	0.55	0.55	1.1
0.30	0.50	0.53	0.3
0.30	0.65	0.53	0.3

the durability of their products. Neither in their model nor in the present paper is the inflation targeting policy able to achieve a substantial welfare improvement because it fails to move aggregate output sufficiently close to the efficient outcome.

3.5 Conclusions

The issue of optimal inflation targeting has received considerable interest in recent literature. Continuing in the research agenda of Rotemberg and Woodford (1997), who first implemented a micro-founded social welfare function to monetary theory, I study the optimal behaviour of the central bank in a two-sector economy with nominal rigidities, where sectors differ in the extent of their labour intensity. By deriving the social welfare function, I find that the optimal inflation to target in such an economy is systematically biased toward the capital-intensive sector.

The source of this bias is traced back to differences in the slope of the marginal product of labour in the two sectors. Nominal rigidities and limited substitutability across individual goods generate dispersion in the equilibrium amounts of output and labour, which is a source of welfare losses compared with the efficient outcome. Steeper marginal product of labour in manufacturing gives rise to higher efficiency costs of the fluctuations in the labour input in this sector. These findings complement those of Benigno (2004) and Erceg and Levin (2006) who conclude that the optimal measure of inflation to target should be biased toward the sector that exhibits a higher degree of nominal rigidities and a relatively more durable output, respectively. This paper focuses at highly realistic differences on the production side of the economy and examines their implications for the design of the optimal monetary policy.

3.A Appendix

This section shows how to derive the social welfare function (3.38). It builds on strategies suggested in Aoki (2001), Benigno (2004) and Erceg et al. (2000), whose features the present model makes use of. Let us start with the average welfare of households in period t (compare to 3.37), where the time subscripts are dropped for ease of notation.

$$SW = 2\,U(C) + \int_0^1 V(L_m\,(h))\,dh + \int_0^1 V(L_s\,(h))\,dh \tag{3.41}$$

Implicit in the equation is the assumption of complete markets, which allows households to choose identical level of consumption in all states of nature. I will now take a second-order Taylor expansion of each element in the welfare criterion as is shown by the following preliminary example. The utility function can be approximated up to the second order by

$$U(C) = U(\overline{C}) + U_C\left(C - \overline{C}\right) + \frac{1}{2}U_{CC}\left(C - \overline{C}\right)^2 + o\left(\|\xi\|^3\right)$$

Noting that C can be written as $\overline{C}e^c$, where $c = ln\left(C/\overline{C}\right)$ and approximating it up to the second order by $\overline{C}\left(1 + c + \frac{1}{2}c^2\right) + o\left(\|\xi\|^3\right)$, we can rewrite (3.A) as

$$U(C) = U(\overline{C}) + U_C\overline{C}\left(c + \frac{1}{2}c^2\right) + \frac{1}{2}U_{CC}\overline{C}^2c^2 + o\left(\|\xi\|^3\right)$$

or, equivalently

$$U(C) = U_C\overline{C}\left(c + \frac{1}{2}c^2\right) + \frac{1}{2}U_{CC}\overline{C}^2c^2 + t.i.p. + o\left(\|\xi\|^3\right)$$

where the term $t.i.p.$ refers to terms independent of policy and the last term on the right-hand side includes all terms of order three and higher (henceforth I will neglect both of them). With these steps in mind and recalling that aggregate consumption ($C = Y/2$) is related to sectoral outputs according to (3.1), which I repeat here for convenience:

$$C = \frac{C_m^{\psi_m}C_s^{\psi_s}}{(\psi_m)^{\psi_m}\,(\psi_s)^{\psi_s}} \tag{3.42}$$

we can approximate the utility from consumption as follows.

$$
\begin{aligned}
U(Y/2) = {} & U_{Y_m}\overline{Y}_m\left(y_m + \frac{1}{2}y_m^2\right) + \frac{1}{2}U_{Y_mY_m}\overline{Y}_m^2 y_m^2 \\
& + U_{Y_s}\overline{Y}_s\left(y_s + \frac{1}{2}y_s^2\right) + \frac{1}{2}U_{Y_sY_s}\overline{Y}_s^2 y_s^2 \\
& + U_{Y_mY_s}\overline{Y}_m\overline{Y}_s y_m y_s
\end{aligned}
\tag{3.43}
$$

Furthermore, by differentiating (3.42) and setting $\rho = -U_{CC}\overline{C}/U_C$ one can prove the following relationships

$$
\begin{aligned}
U_{Y_j}\overline{Y}_j &= \psi_j U_C\overline{C} \\
U_{Y_mY_m}\overline{Y}_m^2 &= -\psi_m U_C\overline{C}\left(\rho\psi_m + \psi_s\right) \\
U_{Y_sY_s}\overline{Y}_s^2 &= -\psi_s U_C\overline{C}\left(\rho\psi_s + \psi_m\right) \\
U_{Y_mY_s}\overline{Y}_m\overline{Y}_s &= \psi_m\psi_s U_C\overline{C}\left(1-\rho\right)
\end{aligned}
\tag{3.44}
$$

that will be used shortly.

The second order approximation of each of the average labour supplies in (3.41) takes the form of

$$
E_h V\left(L_j\left(h\right)\right) = V_L\overline{L}\left(E_h l_j\left(h\right) + \frac{1}{2}E_h l_j\left(h\right)^2\right) + \frac{1}{2}V_{LL}\overline{L}^2 E_h l_j\left(h\right)^2 \tag{3.45}
$$

where E_h denotes an average over households in a sector. This step makes use of the fact that steady-state labour supply is equal in both sectors. To eliminate the quadratic terms in l_h we can use the definition of variance,

$$
E_h l_j\left(h\right)^2 = var_h l_j\left(h\right) + \left(E_h l_j\left(h\right)\right)^2 \tag{3.46}
$$

and the terms in expectations can be obtained from the equality of the aggregate labour hours supplied by workers to the employment agency and the aggregate demand by firms. The former is given by the labour index (3.7)

$$
L_j \equiv \left[\int_0^1 L_j\left(h\right)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}
$$

which in the second order approximation reads as

$$
l_j = E_h l_j\left(h\right) + \frac{1}{2}\frac{\phi-1}{\phi} var_h l_j\left(h\right) \tag{3.47}
$$

Denoting N_j the total amount of hours actually supplied by households, $N_j = \int L_j(h)\, dh$, which can be approximated as $n_j = E_h l_j(h) + \frac{1}{2} var_h l_j(h)$, the above equation can be equivalently restated as

$$n_j = l_j + \frac{1}{2\,\phi} var_h l_j(h) \tag{3.48}$$

which is equation (3.40) in the main text. The left hand side of (3.47), the percentage change of the sectoral labour supply, must be in the equilibrium equal to the percentage change of the sectoral labour demand, which aggregates the labour demands of individual firms: $L_j = \int L_j(f)\, df$, or in the linear-quadratic approximation,

$$l_j = E_f l_j(f) + \frac{1}{2} var_f l_j(f) \tag{3.49}$$

Next, I make use of the production function to eliminate $E_f l_j(f)$. Recall that all firms in a sector employ identical capital-labour ratio, so that we can write $Y_j(f) = A_j(K_j/L_j)^{\alpha_j} L_j(f)$ or in the log-linear form: $y_j(f) = a_j - \alpha_j l_j + l_j(f)$ because the sectoral level of capital is fixed. From here we have the following relationships

$$E_f l_j(f) = E_f y_j(f) - a_j + \alpha_j l_j \tag{3.50}$$

$$var_f l_j(f) = var_f y_j(f) \tag{3.51}$$

Next, we employ the definition of the sectoral output, (3.4),

$$Y_j \equiv \left[\int Y_j(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$

which can be approximated by

$$y_j = E_f y_j(f) + \frac{1}{2}\frac{\theta-1}{\theta} var_f y_j(f) \tag{3.52}$$

Solving for $E_f y_j(f)$ from (3.52) and substituting it in (3.50), and then using it together with (3.51) in the sectoral labour demand equation, (3.49), we have equation (3.39), which appears in the main text:

$$l_j = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\,\theta\,(1-\alpha_j)} var_f y_j(f) \tag{3.53}$$

Finally, equalizing the sectoral labour supply (3.47) and labour demand (3.53) we can solve for $E_h l_j(h)$

$$E_h l_j(h) = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\theta(1-\alpha_j)} var_f y_j(f) - \frac{1}{2}\frac{\phi - 1}{\phi} var_h l_j(h) \quad (3.54)$$

We are now ready to return to the average disutility from labour. Substitute first the first-order term of (3.54) in (3.46) and then use the result together with (3.54) again in (3.45) and rearrange the terms to get

$$\begin{aligned} E_h V(L_j(h)) = V_L \overline{L}\frac{y_j - a_j}{1-\alpha_j} + \frac{1}{2}\Bigg(& \frac{V_L \overline{L}}{\theta(1-\alpha_j)} var_f y_j(f) \\ & + \left(\frac{V_L\overline{L}}{\phi} + V_{LL}\overline{L}^2\right) var_h l_j(h) + \left(V_L\overline{L} + V_{LL}\overline{L}^2\right)\left(\frac{y_j - a_j}{1-\alpha_j}\right)^2 \Bigg) \end{aligned} \quad (3.55)$$

Before we proceed further it is useful to factor out $V_L\overline{L}$ from the second line and use the elasticity $\omega = V_{LL}\overline{L}/V_L$. Furthermore, the term $V_L\overline{L}/(1-\alpha_j)$ can be then replaced by $-2\,U_{Y_j}\,\overline{Y}_j$. This relationship follows from equalizing the supply of sectoral output (in the steady-state), as given in (3.22), to the demand for sectoral output from (3.2), where I made use of (3.20) to substitute for p_j. Solving for wages and substituting the resulting expression into the marginal rate of substitution, (3.16), gives the desired relationship.

The next step consists in employing the definitions from (3.44) to simplify (3.55) and (3.44) so that, after we substitute them back in (3.41), we have

$$\begin{aligned} \frac{SW}{U_C\overline{C}} = & \psi_m y_m^2 - \psi_m(\rho\psi_m + \psi_s) y_m^2 + \psi_s y_s^2 - \psi_s(\rho\psi_s + \psi_m) y_s^2 \\ & + 2\psi_m\psi_s(1-\rho) y_m y_s + 2\psi_m a_m + 2\psi_s a_s \\ & - \psi_m \frac{1+\omega}{1-\alpha_m}(y_m - a_m)^2 - \psi_s\frac{1+\omega}{1-\alpha_s}(y_s - a_s)^2 \\ & - \frac{1}{\theta}\left(\psi_m var_f y_m(f) + \psi_s var_f y_s(f)\right) \\ & - \left(\frac{1}{\phi} + \omega\right)\left((1-\alpha_m)\psi_m var_h l_m(h) + (1-\alpha_s)\psi_s var_h l_s(h)\right) \end{aligned} \quad (3.56)$$

With some algebra and using the definition of the aggregate output from (3.42), which in the log-linear form reads as $y = \psi_m y_m + \psi_s y_s$, the first three lines of (3.56) can be simplified as follows

$$\frac{SW_{1-3}}{U_C\overline{C}} = (1-\rho) y^2 - \psi_m\frac{1+\omega}{1-\alpha_m}(y_m - a_m)^2 - \psi_s\frac{1+\omega}{1-\alpha_s}(y_s - a_s)^2 \quad (3.57)$$

where I have neglected terms independent of output. Notice that an equivalent relationship can be obtained for the case without nominal rigidities. Subtracting the efficient solution from (3.57), expanding the squares, substituting for a_j from (3.26) and subsequently for a^w from (3.24), this equation simplifies in

$$\frac{\widehat{SW}_{1-3}}{U_C\overline{C}} = (1-\rho)\,\hat{y}^2 - \psi_m \frac{1+\omega}{1-\alpha_m}\hat{y}_m^2 - \psi_s \frac{1+\omega}{1-\alpha_s}\hat{y}_s^2 \tag{3.58}$$

where the terms with hat refer to output gaps, e.g. $\hat{y} = y - y^n$. Lastly, from the definition of the aggregate output and the demand curves (3.2) it is straightforward to derive the following relationships between the aggregate output gap and the relative price gap (defined as $\hat{p}_{rel} = p_{rel} - p_{rel}^n$): $\hat{y} = \hat{y}_m + \psi_s \hat{p}_{rel}$ and $\hat{y} = \hat{y}_s - \psi_m \hat{p}_{rel}$. Substituting these relationships in the equation above, it can be equivalently expressed as

$$\frac{\widehat{SW}_{1-3}}{U_C\overline{C}} = c_g\,\hat{y}^2 + 2\,\psi_m\psi_s\left(\frac{1+\omega}{1-\alpha_m} - \frac{1+\omega}{1-\alpha_s}\right)\hat{y}\,\hat{p}_{rel} - c_t\,\hat{p}_{rel}^2 \tag{3.59}$$

where

$$\begin{aligned} c_g &= (1-\rho) - (1+\omega)\left(\frac{\psi_m}{1-\alpha_m} + \frac{\psi_s}{1-\alpha_s}\right) \\ c_t &= \psi_m\psi_s\,(1+\omega)\left(\frac{\psi_m}{1-\alpha_s} + \frac{\psi_s}{1-\alpha_m}\right) \end{aligned} \tag{3.60}$$

Turning now to the last two rows in (3.56) we can rewrite the variance terms as follows. First, from the demand functions (3.5) and (3.8) we have that $var_h l_j\,(h) = \phi^2 var_h w_j\,(h)$ and $var_f y_j\,(f) = \theta^2 var_f p_j\,(f)$. Erceg et al. (2000) show that

$$E\,var_h w_j\,(h) = \frac{\eta}{(1-\eta)^2}\,var\,\pi_j^w \tag{3.61}$$

$$E\,var_f p_j\,(f) = \frac{\epsilon}{(1-\epsilon)^2}\,var\,\pi_j \tag{3.62}$$

Collecting the results of equation (3.58), taking unconditional expectations (the proof that $E\,\hat{y}^2 = var\,\hat{y}$ can be found in Erceg at al., 2000), and adding the variance terms from (3.61) and (3.62), the social welfare function becomes

$$\begin{aligned}\frac{E\,\widehat{SW}}{U_C\overline{C}} &= (1-\rho)\,var\,\hat{y} - \psi_m \frac{1+\omega}{1-\alpha_m}\,var\,\hat{y}_m - \psi_s \frac{1+\omega}{1-\alpha_s}\,var\,\hat{y}_s \\ &- \frac{\epsilon\theta}{(1-\epsilon)^2}\left(\psi_m\,var\,\pi_m + \psi_s\,var\,\pi_s\right) \\ &- \frac{\eta\phi\,(1+\omega\phi)}{(1-\eta)^2}\left(\psi_m\,(1-\alpha_m)\;var\,\pi_m^w + \psi_s\,(1-\alpha_s)\;var\,\pi_s^w\right)\end{aligned} \tag{3.63}$$

or alternatively, which is equation (3.38) in the main text,

$$\begin{aligned}E\,\widehat{SW} &= \gamma_y\;(1-\rho)\,var\,\hat{y} - \psi_m\,\gamma_y\,\frac{1+\omega}{1-\alpha_m}\,var\,\hat{y}_m - \psi_s\,\gamma_y\,\frac{1+\omega}{1-\alpha_s}\,var\,\hat{y}_s \\ &- \frac{\gamma_p}{1-\alpha_m}\,var\,\pi_m - \frac{\gamma_p}{1-\alpha_s}\,var\,\pi_s - \gamma_w\,var\,\pi_m^w - \gamma_w\,var\,\pi_s^w\end{aligned} \tag{3.64}$$

where the coefficients are defined as follows

$$\gamma_y = U_C\overline{C} \qquad \gamma_p = -V_L\overline{L}\,\frac{\epsilon\,\theta}{2\,(1-\epsilon)^2} \qquad \gamma_w = -V_L\overline{L}\,\frac{\eta\,\phi\;(1+\omega\,\phi)}{2\;(1-\eta)^2} \tag{3.65}$$

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