The Balassa-Samuelson Relationship

Theory, Evidence and Implications

Qi Zhang

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

Balassa and Samuelson showed that as we move towards richer countries the measured price level becomes higher. Their proposed explanation was to appeal to the presence of a service element in most goods.

In this thesis, I begin by introducing an exploring of an alternative candidate explanation for the B-S relationship. This explanation is based on an appeal to mismeasured quality. In the model developed in Chapter 2, the well-known difficulties surrounding the problem of making a full and appropriate adjustment for differing quality levels will mean that when the average quality level consumed is higher in richer countries, this will show up in the data as spurious difference in price levels, which will imply the B-S relationship. More interestingly, it also leads to a second testable prediction that is not a prediction of the classic B-S explanation. This second prediction is tested directly at the end of Chapter 2. In testing this prediction, we are led naturally to explore the foundation of the B-S relationship at a disaggregate level.

In Chapter 3, we take a purely statistical approach in asking the question: what is the best statistical description of wealth versus price level relationship for individual products? We arrive at a characterization of the best statistical description which suggests a natural way of ordering products relative to the form of this relationship. A striking pattern emerges, according to which products at the one end of the spectrum are almost all manufactured goods (designated the 'M-group'), while products at the other end of the spectrum are almost all pure services (designated the 'S-group').

In Chapter 4 and 5, we return to theory. We propose a separate model for the S-group in Chapter 4. In Chapter 5 we return to the analysis of Chapter 2, but now we apply the analysis to the M-group only.

Chapter 6 is devoted to exploring the macroeconomic implications of the B-S relationship. The key idea is that a (fast) growing economy will exhibit a (substantial) temporary episode of inflation, as measured by conventional price indices.

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This work would not have been possible without the support and love of my parents, Jingfu and Xiuping. I dedicate this thesis to them.

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Chapter 1

Introduction: the Balassa-Samuelson Relationship

The Balassa-Samuelson relationship, first introduced in 1964[1], links the per capita income level of a country to a broad price index. Balassa and Samuelson showed that as we move towards richer countries the measured price level becomes higher. This represents an apparent violation of Purchasing Power Parity (PPP). Their proposed explanation was to appeal to the presence of a service element in most goods. In other words, there are always local costs of processing, distributing etc., which will reflect local wage rates, leading to higher prices in richer countries.

In this thesis, I begin by introducing an exploring of an alternative candidate explanation for the B-S relationship. This explanation is based on an appeal to mismeasured quality. This is an old theme in the industrial organization literature, which can be traced back to the early hedonic prices literature (Griliches, 1961), and which has been revived as a focus of interest in recent work by Pakes (2003, 2005). In the present setting, the well-known difficulties surrounding the problem of making a full and appropriate adjustment for differing quality levels will mean that when the average quality level consumed is higher in richer countries, this will show up in the data as spurious difference in price levels.

In the model developed in Chapter 2, it is shown that the mismeasured quality model will imply the B-S relationship. More interestingly, it also

[1]It was developed by Balassa (1964) and Samuelson (1964)

leads to a second testable prediction that is not a prediction of the classic B-S explanation. This second prediction is tested directly at the end of Chapter 2. In testing this prediction, we are led naturally to explore the foundation of the B-S relationship at a disaggregate level. This suggests some considerations which lead to the investigation of Chapter 3.

In Chapter 3, we take a purely statistical approach in asking the question: what is the best statistical description of wealth versus price level relationship for individual products? We arrive at a characterization of the best statistical description which suggests a natural way of ordering products relative to the form of this relationship. A striking pattern emerges, according to which products at the one end of the spectrum are almost all manufactured goods, while products at the other end of the spectrum are almost all pure services. This suggests that it might be appropriate to think in terms of modelling this 'services' group (designated the 'S-group') separately from the manufactures group (designated the 'M-group').

In Chapter 4 and 5, we return to theory. We propose a separate model for the S-group in Chapter 4. In Chapter 5 we return to the analysis of Chapter 2, but now we apply the analysis to the M-group only.

Chapter 6 is devoted to exploring the macroeconomic implications of the B-S relationship. The key idea is that a (fast) growing economy will exhibit a (substantial) temporary episode of inflation, as measured by conventional price indices.

We begin in this chapter with a review of the literature surrounding the B-S relationship.

1.1 Literature Review

When countries' price levels are translated to dollars at prevailing nominal exchange rates, rich countries tend to have higher price levels than poor countries. This is known as the 'Penn effect'. The Balassa-Samuelson model, as developed by Balassa (1964) and Samuelson (1964), argues that this relationship reflects the fact that rich countries are relatively more productive in the traded goods sector. Higher productivity in the traded goods sector implies higher wages in the traded goods sector. Since the domestic price level of traded goods is equal to the world price level, nontraded goods producers must raise their prices to provide the higher wages. With constant prices of traded goods and higher prices of nontraded goods, the overall price level must be higher. Empirical tests of the Balassa-Samuelson model have not led to any consensus on the issue. There is empirical support for the model when comparisons are made between the set of 'all poor countries' and 'all rich countries'. However, this effect is not statistically significant within either the poor countries group or the rich countries group (Rogoff (1996), see Figure 1.1).

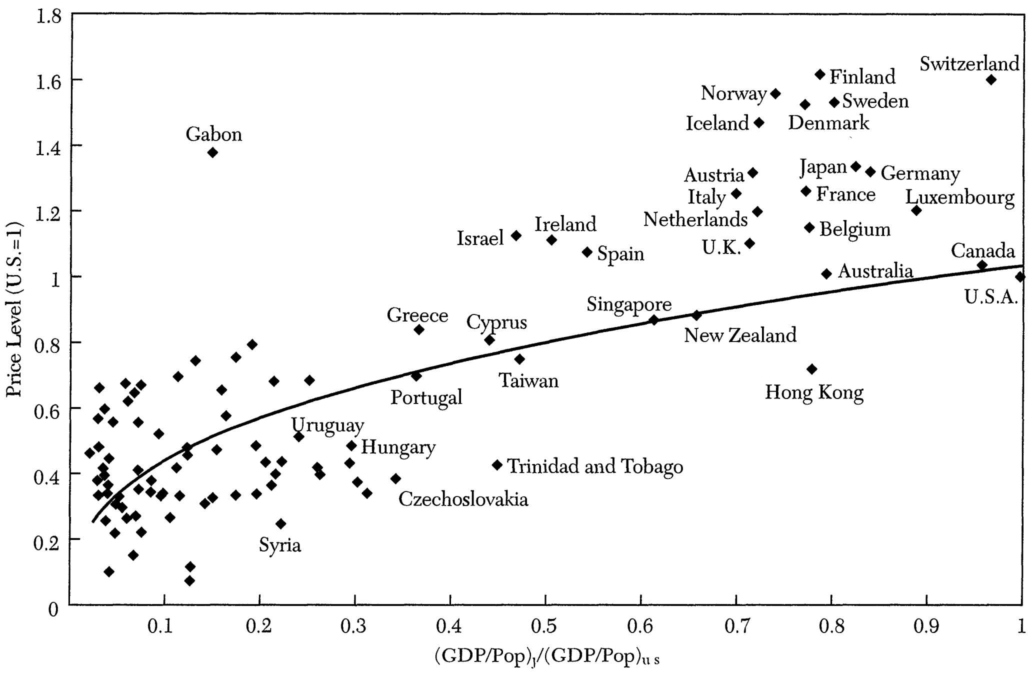


Figure 1.1: Price Level versus GDP per capita in 1990 (U.S.=1)

Notes: Source: The Penn World Table 1994

Using cross-country evidence, in addition to the Penn effect, I find that, controlling for per capita income, income inequality is also correlated with the national price level. Within countries with lower per capita income, income inequality is positively correlated with the national price level, while within countries with higher per capita income, the correlation is negative. The Balassa-Samuelson model is not able to explain this additional fact. Therefore, we need a new model to provide a full explanation of the fact that both per capita income and income inequality matter for the national price level. Chapter 2 offers a new type of explanation for the Penn effect, and for related regularities linking income inequality with the national price level.

I build a hedonic pricing model to model explicitly the link between income distribution and choice of product quality within each country. The central feature of the model is closely analogous to the feature identified by Pakes (2003) in a micro context: quality cannot be perfectly controlled in the price index. I link this idea to the fact that income elasticity of quality is non-negligible and tends to be higher for nontraded goods. Once these two ideas are combined, the model predicts that per capita income has a positive impact on the national price level (the Penn effect). Controlling for per capita income, income inequality has a positive impact on the national price level within countries with lower per capita income. While within countries with higher per capita income, the impact is negative. Or in other words, the effect of income inequality on the national price level is decreasing in per capita income. Therefore, these model predictions are consistent with the empirical evidence mentioned previously.

To understand the intuition, it is important to first realize that although households with higher incomes tend to spend more on all consumption categories[2], consumption categories differ in their income elasticities of quan-

[2]The consumption categories are the 2-digit COICOP (Classification of Individual

tity and quality. Here, quantity refers to the number of units consumed by the households, while quality refers to the desirable characteristic within each unit of consumption goods, which is reflected in the unit price.[3] Empirical evidence such as in Bils and Klenow (2001) combined with the complementary evidence provided in Chapter 2 shows that: For some categories, such as food and housing, households with higher income tend to keep the quantity of the goods they buy constant but buy goods with higher quality. Thus, the income elasticity of quality is high for these goods while that of quantity is low. For other categories, such as clothing and footwear, households with higher income tend to purchase a larger quantity of the goods with a constant level of quality. Therefore, their income elasticity of quantity is high relative to that of quality. Moreover, the goods with relatively high income elasticities of quantity, such as clothing and footwear, are more likely to be traded goods, while those with relatively high income elasticities of quality, such as food and housing, are more likely to be nontraded goods. The focus of Chapter 2 is to investigate this correlation and make a sharp contrast between the roles played by the goods with high tradability and high income elasticity of quantity and the goods with low tradability and low income elasticity of quantity. As a result, in the model it is assumed that households choose one unit of nontradable vertically differentiated goods with varying quality, which are priced locally by a hedonic price

Consumption according to Purpose) divisions, which are Food and non-alcoholic beverages; Alcoholic beverages, tobacco and narcotics; Clothing and footwear; Housing, water, electricity, gas and other fuels; Furnishings, household equipment and routine household maintenance; Health; Transport; Communication; Recreation and culture; Education; Restaurants and hotels; Miscellaneous goods and services.

[3]For example, buying the same meal twice doubles the total expenditure on food. Thus, the number of meals is the quantity of food. In contrast, a meal with organic ingredients is more expensive than one with non-organic ingredients. Hence, the ingredients of a meal count as the quality of food. As for housing, two houses with the same characteristics worth twice as much as one, while two identical houses that only differ in their locations have different unit prices. Therefore, the number of houses is quantity and the location of a house is quality. Similarly, the number of clothes is quantity while whether the clothes are of a high street brand or a designer brand is quality.

function; households also choose the quantity of a tradable homogeneous goods, the price of which is a constant unit price across countries.

The intuition for the Penn effect is that since there is no quality adjustment in the price index, the price level of nontraded goods is just equal to the average expenditure on the one unit of nontraded goods. Moreover, given that consumers in the countries with higher per capita income tend to spend more on nontraded goods, this will imply a higher price level of nontraded goods. With constant prices of traded goods, the national price level will be higher in richer countries, which explains the Penn effect.

To understand how income inequality affects the national price level, it is necessary to know how the national price level is constructed in practice: In the Penn World Table, which is commonly used for the purpose of cross-country price comparisons, suppose we want to construct the national price level for the UK, the first step is to find a base country, say the US. Then we construct the UK's bilateral Laspeyres and Paasche price index relative to the base country. The Laspeyres index and the Paasche index are both the weighted average of the price ratios of traded goods and nontraded goods of the UK relative to the US. The weights in the Laspeyres index are given by the expenditure share of the base country, the US, while the weights in the Paasche index are given by the expenditure share of the UK. The two indices will be further used to construct the national price level of the UK using the Geary-Khamis (GK) method as shown in Deaton and Heston (2010). However, strictly following this method will make the theoretical model intractable. Instead, the geometric mean of the two indices will be used as the UK's national price level to mimic the national price level in the Penn World Table. This is because Deaton and Heston (2010) has shown that the national price level in the Penn World Table can be very well approximated by the geometric mean of the bilateral Laspeyres index and the Paasche index.

The intuition for the impact of income inequality on the national price level is that income inequality can affect the price level of nontraded goods by changing the expenditure share on it. This is because according to the standard IO literature, the price function of vertically differentiated goods is generally nonlinear and is jointly determined by the distribution of consumers' attributes and cost function parameters. Keeping per capita income constant, a higher income inequality implies a more convex price function of nontraded goods. If the elasticity of substitution between traded goods and nontraded goods is high, then this will lead to a smaller expenditure share on nontraded goods. Since in practice, the quality of nontraded goods cannot be perfectly controlled, the lower expenditure share on nontraded goods will imply a lower price level of nontraded goods. The Laspeyres index, which is the average price ratios of traded goods and nontraded goods relative to the U.S. weighted by the expenditure shares of the U.S., will be lower, since the price ratio of nontraded good is lower and there is no change in the price ratio of traded good and the weights. Thus, income inequality has a negative impact on the Laspeyres index. However, the impact of income inequality on the Paasche index, which is the average price ratios of traded good and nontraded goods relative to the U.S. weighted by the country's expenditure shares, will depend on the country's per capita income relative to the US. With a low enough per capita income, the price ratio of nontraded good relative to the U.S. will be lower than the price ratio of traded good relative to the U.S., which is always equal to 1, so a lower expenditure share on nontraded goods will increase the Paasche index. With a high per capita income, the relative price ratio of nontraded goods will be higher than the relative price ratio of traded good, hence the lower expenditure share on nontraded goods will reduce the Paasche index. Given that in the model the geometric mean of these two indices is used as a proxy for the national price level, with a low enough per capita income, income inequality will have a positive impact on the national price level, while with a high per capita income, the impact is negative.

Therefore, per capita income affects the national price level by changing the price level of the nontraded goods and income inequality affects the national price level mainly through its impact on the expenditure share of nontraded goods. Since the product of the expenditure share and the average price level of nontraded goods enters the national price level, a higher per capita income, which increases the average price level of nontraded goods, will strengthen the effect of income inequality, while a lower income inequality, which increases the expenditure share of nontraded goods, strengthens the effect of per capita income. Hence the effect of per capita income is decreasing in income inequality and the effect of income inequality is decreasing in per capita income. Chapter 2 uses disaggregate prices and expenditure shares at the basic heading level from the International Comparison Program (ICP) to show that the intuition provided is consistent with empirical evidence.

Moreover, since the model predicts that the difference in income inequality between two countries determines the slope of the relative price schedule of vertically differentiated goods, the model can generate the implications for how price differentials of the same good between two countries change with quality. One important explanation of price differentials of the same traded good across countries is proposed by Krugman (1987), who refers to it as "pricing to market". Since international arbitrage for many types of goods is difficult or impossible, producers can price discriminate across different international markets. Due to different price elasticities of demand in different countries, profit-maximizing international firms may set a country-specific markup. Hence, even prices of the same traded goods can be different across countries. However, in the "pricing to market" literature, only monopolistic competition models are used. This implies that there are only horizontally differentiated goods in the economy, i.e., all goods have the same quality. Hence, the price ratio of any good between two countries must be the same regardless of the quality of that good. However, empirically the price ratio is not a constant and varies with quality. Chapter 2 tries to address vertically differentiated goods and shows how income distribution affects their relative price schedules across countries. More specifically, it predicts that the difference in income inequality between two countries determines the slope of the price schedule.

Chapter 2

A 'Mismeasured Quality' Interpretation

2.1 The Model

The model is a hedonic pricing model *à la* Rosen (1974), in which consumers and firms choose their optimal positions along an equilibrium price schedule $p(z)$, where z is a vector of characteristics of the product in question.

The focus of the analysis lies in establishing a relationship between a country's level of income, and – more importantly – the form of income distribution in the country, and the pattern of demand for both 'quality' goods and 'commodity' goods.

The novel prediction of the model is that controlling for per capita income, inequality is correlated with the national price level. The basic intuition is: suppose a country, whose income distribution is made up of three income groups with equal population, has a perfectly equal income distribution, i.e. all individuals in the top, middle and bottom income groups have an income level of 100. Given the same Cobb-Douglas utility function, every one spends a same fraction θ of his/her income on good of the same quality z. The implied price of the product will be given by the average expenditure on it, which is equal to 100θ. Now consider a mean-preserving spread of income distribution, under which the top, middle and bottom income groups have income levels of 50, 100 and 150 respectively. The income redistribution has two effects on the demand of the quality goods. First, it requires producers to increase the range of qualities to meet the needs of the newly created rich and poor individuals. Second, it increases the quantity demanded of the existing top and bottom quality products. As the cost function of the quality product is convex, the second impact will lead to higher prices for the existing top and bottom quality products, which will result in a more convex price function. With a more convex price function for the quality product and the unit elasticity of substitution of the Cobb-Douglas utility function, all individuals will respond in this new situation by spending a smaller fraction θ' ($< \theta$) of income on the quality product. As a result, its price level, i.e. the average expenditure on the quality product, is now equal to $(50\theta' + 100\theta' + 150\theta')/3 = 100\theta'$, which is less than before. This is the mechanism through which income inequality affects the measured national price level in the present model.

2.1.1 *The Consumer's Problem*

There is a unit mass of consumers indexed by individual income level c. The income distribution is assumed (conventionally) to follow a Pareto distribution characterized by two parameters k_c and c_m, where c_m is the lower bound of c and k_c is the shape parameter. Hence the probability density function of income is $f(c) = k_c \frac{c_m^{k_c}}{c^{k_c+1}}, k_c > 0, c \in [c_m, \infty)$.

If we decompose the income elasticity of consumption expenditure into an income elasticity of quality and an income elasticity of quantity, it will be shown empirically in what follows that goods differ substantially in their income elasticities of quality and quantity. We will divide goods into three types based on the magnitudes of these two elasticities. The first type of goods, which we call x goods, have zero income elasticity of quality and a non-zero income elasticity of quantity. The second type, which we call z goods, have zero income elasticity of quantity and a non-zero income elasticity of quality. For the third type, both elasticities are non-zero. The fact that some of these elasticities are (close to) zero simply reflects the physical nature of the goods, and so we incorporate these features as given parameters of the model which follows. We begin with a setting where there are just two types of goods, x goods and z goods.[1] We begin from the idea that the x goods, which we may think of as simple 'commodities', are traded internationally at a single price. In other words, we assume purchasing power parity holds for these goods. We simplify notation by choosing the x goods as a numeraire and normalizing their prices to be 1.

We assume the consumer purchases exactly 1 unit of the quality good. Subject to this, consumer preferences are given by a standard Cobb-Douglas utility function $v(x, z) = x^{\alpha} z^{\beta}$, $\alpha + \beta = 1$, where z is the quality level of the quality good consumed. Maximizing utility subject to the budget constraint $c = x + p(z)$ yields the consumer's problem

$$\max_{x,z} v(x, z) = x^{\alpha} z^{\beta}$$

$$s.t. \; c = x + p(z)$$

Given the homogeneous feature of x goods and its normalized price, the total expenditure on x goods is given by the product of quantity consumed x and its unit price 1. The rest of consumption expenditure will be spent on z goods, which is assumed to be a nonlinear function of quality z.

The Lagrangian is given by $L = x^{\alpha} z^{\beta} + \lambda(c - x - p(z))$. First-order necessary conditions imply that

$$\frac{v_z}{v_x} = p'(z)$$

[1]The third type of good, which has nonzero income elasticities of both quality and quantity, can be thought of as a combination of two components, an x component and a z component.

hence

$$\frac{\beta}{\alpha}\frac{x}{z} = p'(z)$$

since $x = c - p(z)$, we have

$$c = \frac{\alpha}{\beta} z p'(z) + p(z)$$

which implies that a consumer chooses a vertically differentiated good with quality z, then his/her income must be equal to $\frac{\alpha}{\beta} z p'(z) + p(z)$. We denote the consumer's income conditional on choosing quality z by $h(z)$. Recalled that $f(c)$ denotes the pdf of income c, and that $c = h(z)$ whence $z = h^{-1}(c)$. From this we can write down the pdf of z, which we denote as $\phi(z)$, as follows:

$$\phi(z) = f(h(z)) \left| \frac{\partial h(z)}{\partial z} \right| \tag{2.1}$$

where $|\cdot|$ denotes the absolute value and is used to ensure that $\phi(z)$ is always positive even if $\partial h(z)/\partial z < 0$.

The mapping from the pdf of c to the pdf of z can be illustrated in Figure 2.1.

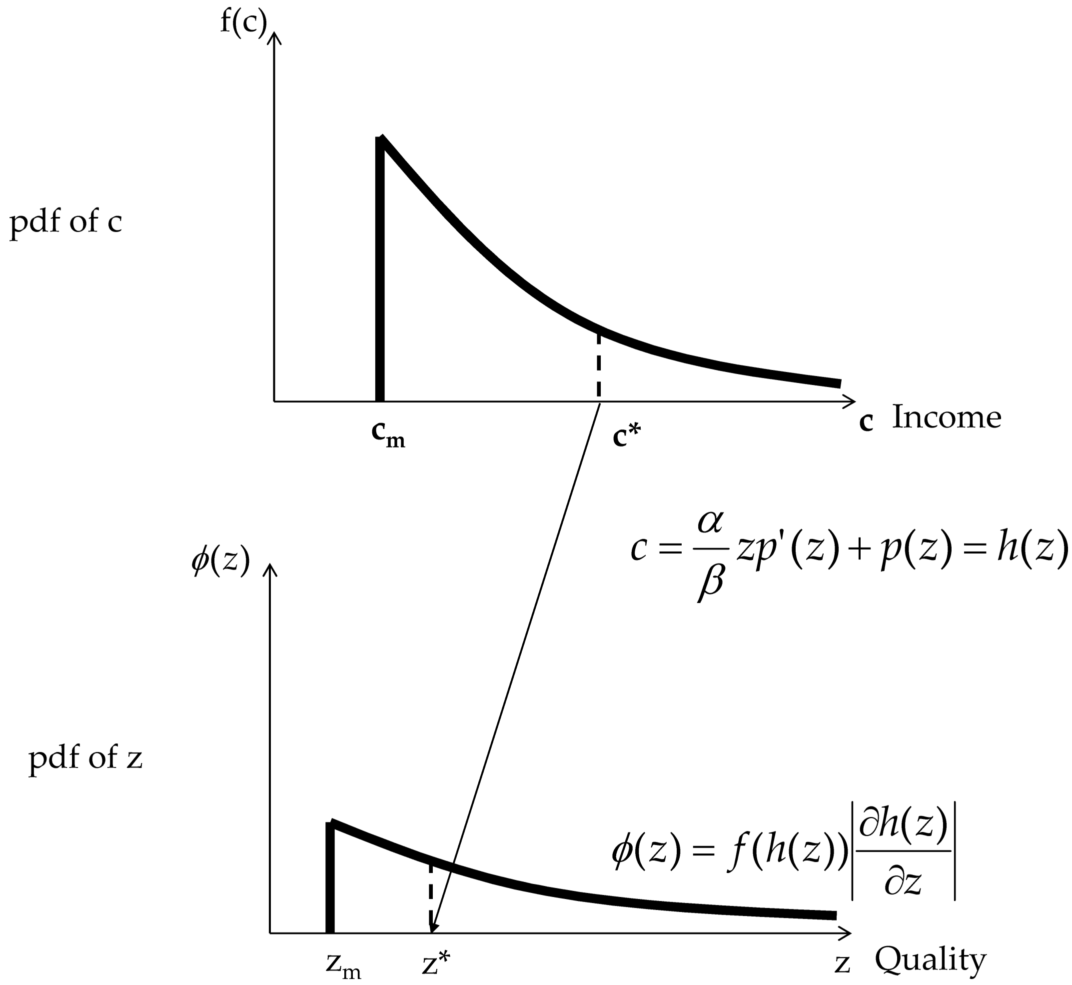


Figure 2.1: Mapping From Income Distribution to Distribution of Quality Demanded

Substituting for $h(z)$ and $f(\cdot)$ in (2.1) yields

$$\begin{aligned}
\phi(z) &= f(\frac{\alpha}{\beta} z p'(z) + p(z)) \left| \frac{\partial(\frac{\alpha}{\beta} z p'(z) + p(z))}{\partial z} \right| \\
&= f(\frac{\alpha}{\beta} z p'(z) + p(z)) \left| \frac{\alpha}{\beta}(p'(z) + z p''(z)) + p'(z) \right| \\
&= k_c c_m^{k_c} (\frac{\alpha}{\beta} z p'(z) + p(z))^{-(k_c+1)} \left| \frac{\alpha}{\beta}(p'(z) + z p''(z)) + p'(z) \right|
\end{aligned}$$

If we denote the quantity demanded for the good with quality z by $Q^d(z)$, then the market demand in a small interval dz near quality z is given by the product of the pdf of quality around z, $\phi(z)$, and the length of the interval:

$$Q^d(z)dz = k_c c_m^{k_c}(\frac{\alpha}{\beta}zp'(z) + p(z))^{-(k_c+1)} \left|\frac{\alpha}{\beta}(p'(z) + zp''(z)) + p'(z)\right| dz$$

2.1.2 The Producer's Problem

On the supply side, there is a unit mass of firms producing vertically differentiated goods indexed by their product quality z. The distribution of the firms is assumed to be the Pareto distribution characterized by two parameters k_z and z_m, where z_m is the lower bound of z and k_z is the shape parameter.[2] The pdf of z is assumed to take the form:

$$g(z) = k_z \frac{z_m^{k_z}}{z^{k_z+1}}, k_z > 0, z \in [z_m, \infty)$$

Producers in all countries are assumed to have the same cost function $\Delta(M, z) = A_z M^\tau z^\gamma$, $\tau > 1, \gamma > 1$, where A_z is the productivity parameter and M denotes the number of units of the product with quality z that the firm produces. We assume $\tau > 1$ and $\gamma > 1$, which ensures that total cost is a convex function in M and z.

The producers are price takers. Furthermore, it is assumed that the producers can vary M but not z. (i.e. a producer's quality is a given parameter in the short run). Therefore, the producer's problem is to maximize profit by choosing its output level M of the quality good:

[2]The reasons for using the Pareto distribution are not only that we can get a closed form solution but also that this assumption is consistent with empirical evidence. Gaffeo et al. (2003) analyze the average size distribution of a pool of the G7 group firms over the period 1987-2000. They find that the empirical distributions are all consistent with the power law. In our model, the quality of a firm is a power transformation of the size of the firm, so it is reasonable to assume quality also follows the Pareto distribution as the Pareto distribution is close under power transformation.

$$\max_{M} Mp(z) - \Delta(M, z)$$

The first-order conditions imply that

$$p(z) = \frac{\partial \Delta}{\partial M} = A_z \tau M^{\tau - 1} z^{\gamma}$$

$$\text{Thus, } M(z) = \left(\frac{p(z)}{A_z \tau z^{\gamma}}\right)^{\frac{1}{\tau-1}}$$

If we denote the firm's output of the quality good as $Q^s(z)$, then the market supply in a small interval dz near quality z is given by the product of the pdf of firms around z, the quantity supplied by each z firm and the length of the interval:

$$Q^s(z)dz = g(z)M(z)dz$$

$$Q^s(z)dz = k_z \frac{z_m^{k_z}}{z^{k_z+1}} \left(\frac{p(z)}{A_z \tau z^{\gamma}}\right)^{\frac{1}{\tau-1}} dz$$

Figure 2.2 shows the relationship between the firms' qualities z and the output level they produce, M.

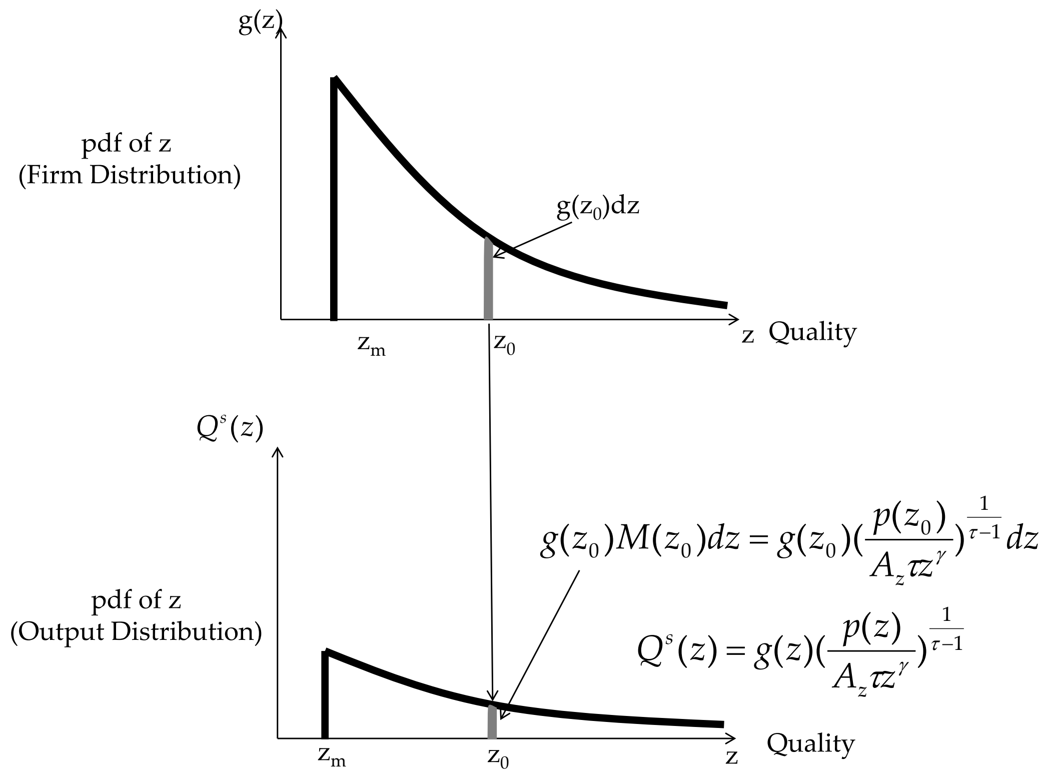


Figure 2.2: Mapping From Firm Distribution to Distribution of Quality Supplied

2.1.3 *Market Equilibrium*

An equilibrium is defined as a triple $\{z(c), M(z), p(z)\}$, where $z(c)$ is the policy functions for consumers, and $M(z)$ for producers, and the price schedule $p(z)$ such that:

(1) $z(c)$ solves the consumer's utility maximization problem taking $p(z)$ as given.

(2) $M(z)$ solves the producer's profit maximization problem taking $p(z)$ as given.

(3) Market clears: demand is equal to supply for z goods, i.e., $Q^s(z) = Q^d(z)$ for all z.

2.1.4 Solving Equilibrium

In equilibrium, we must have the market clearing condition $Q^s(z)dz = Q^d(z)dz$. Therefore,

$$k_z \frac{z_m^{k_z}}{z^{k_z+1}} \left(\frac{p(z)}{A_z \tau z^{\gamma}}\right)^{\frac{1}{\tau-1}} dz$$
$$= k_c c_m^{k_c} \left(\frac{\alpha}{\beta} z p'(z) + p(z)\right)^{-(k_c+1)} \left| \frac{\alpha}{\beta}(p'(z) + z p''(z)) + p'(z) \right| dz$$

$$k_z z_m^{k_z} \left(\frac{1}{A_z \tau}\right)^{\frac{1}{\tau-1}} z^{-(k_z+1)-\gamma\frac{1}{\tau-1}} p(z)^{\frac{1}{\tau-1}}$$
$$= k_c c_m^{k_c} \left(\frac{\alpha}{\beta} z p'(z) + p(z)\right)^{-(k_c+1)} \left| \frac{\alpha}{\beta}(p'(z) + z p''(z)) + p'(z) \right| \tag{2.2}$$

This is a second-order nonlinear non-autonomous differential equation defining $p(z), z \in [\overline{z}_m, \infty)$. We impose a boundary condition:

$$c_m = \frac{\alpha}{\beta} \overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m)$$

Here $\overline{z}_m$ is the lowest quality that is viable in the equilibrium, which is determined by the lowest income c_m and the equilibrium price function $p(z)$.

There is no general procedure to obtain the solution of this class of differential equations, so we adopt the standard method of undetermined coefficients, to find a particular solution. We postulate the equilibrium price function is of the form $p(z) = bz^d$, with $d > 0$. We then substitute this form of solution into the market clearing condition to solve for the values of the parameter b and d.

The first and second derivatives of the postulated price function form are

$$p'(z) = bdz^{d-1}, p''(z) = bd(d-1)z^{d-2} \tag{2.3}$$

Substituting (2.3) into (2.2) yields

$$k_z z_m^{k_z} (\tfrac{b}{A_z \tau})^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})}$$
$$= k_c c_m^{k_c} (b(\tfrac{\alpha}{\beta} d + 1))^{-(k_c+1)} (\tfrac{\alpha}{\beta} d^2 + d) b z^{-d(k_c+1)+d-1} \quad (2.4)$$

Since (2.4) holds for all z and both LHS and RHS are power functions of z, it must be true that the two parameters of the power functions on both sides are equal

$$k_z z_m^{k_z} (\frac{b}{A_z \tau})^{\frac{1}{\tau-1}} = k_c c_m^{k_c} (b(\frac{\alpha}{\beta} d + 1))^{-(k_c+1)} (\frac{\alpha}{\beta} d^2 + d) b \quad (2.5)$$

$$-(k_z + 1) + (d - \gamma)\frac{1}{\tau - 1} = -d(k_c + 1) + d - 1 \quad (2.6)$$

From (2.6):

$$d = \frac{k_z + \gamma \left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}} \quad (2.7)$$

Given Equation (2.7), the expenditure share on z goods can be obtained by

$$\frac{p(z)}{c} = \frac{p(z)}{\frac{\alpha}{\beta} z p'(z) + p(z)} = \frac{b z^d}{\frac{\alpha}{\beta} b d z^d + b z^d} = \frac{1}{\frac{\alpha}{\beta} d + 1} \quad (2.8)$$

From (2.7), (2.8) and the fact that $Gini = \frac{1}{2k_c - 1}$ for the Pareto distribution, we can derive how the Gini coefficient affects the convexity of the price function and hence the expenditure share, which is stated in Proposition 1.

Proposition 1 *(Income Distribution and Expenditure Share) Income inequality has a positive impact on the expenditure share of x goods and a negative impact on the expenditure share of z goods. Per capita income has no impact on expenditure shares.*

The intuition behind the proposition is that an increase in the Gini coefficient implies an increase in d and hence a more convex price function. Since the price of x is 1 and z has a non-linear price function, an increase in the convexity of the price function of z will make people spend less fraction of their expenditure on z and more on x due to the high substitutability between the two goods. This mechanism about how income inequality affects expenditure share is crucial in determining how income inequality influences the price level, which will be provided in the next section.

Substituting (2.7) into (2.5) can solve for the other parameter b in the price function:

$$b = \left(\frac{k_c c_m^{k_c} \left(\frac{\alpha}{\beta}\frac{k_z+\gamma\left(\frac{1}{\tau-1}\right)}{k_c+\frac{1}{\tau-1}} + 1\right)^{-k_c \frac{k_z+\gamma\left(\frac{1}{\tau-1}\right)}{k_c+\frac{1}{\tau-1}}}}{k_z z_m^{k_z} \left(\frac{1}{A_z\tau}\right)^{\frac{1}{\tau-1}}}\right)^{\frac{1}{k_c+\frac{1}{\tau-1}}} \tag{2.9}$$

Therefore, one solution to the differential equation is

$$p(z) = bz^d = \left(\frac{k_c c_m^{k_c} \left(\frac{\alpha}{\beta}\frac{k_z+\gamma\left(\frac{1}{\tau-1}\right)}{k_c+\frac{1}{\tau-1}} + 1\right)^{-k_c \frac{k_z+\gamma\left(\frac{1}{\tau-1}\right)}{k_c+\frac{1}{\tau-1}}}}{k_z z_m^{k_z} \left(\frac{1}{A_z\tau}\right)^{\frac{1}{\tau-1}}}\right)^{\frac{1}{k_c+\frac{1}{\tau-1}}} z^{\frac{k_z+\gamma\left(\frac{1}{\tau-1}\right)}{k_c+\frac{1}{\tau-1}}}, z \in [\bar{z}_m, \infty)$$

where $\bar{z}_m$ satisfies

$$c_m = \frac{\alpha}{\beta}\bar{z}_m p'(\bar{z}_m) + p(\bar{z}_m)$$

After obtaining the equilibrium price function, before aggregating it and analysing how income distribution influences the national price level, we can first investigate how income distribution affects prices at product level, i.e. how the difference in income distribution affects the relative price of a product with a particular quality z_0 between two countries.

Suppose the hedonic price functions in country i and country j are $p_i(z) = b_i z^{d_i}$ and $p_j(z) = b_j z^{d_j}$, where b_i, b_j, d_i and d_j are determined as in the equilibrium price function. Then the price ratio of a product with quality z_0

between the two countries is

$$\frac{p_i(z_0)}{p_j(z_{0)}} = \frac{b_i}{b_j} z_0^{(d_i - d_j)}$$

If we keep the income distribution of country j constant, and increase the per capita income of country i while keeping its Gini coefficient constant, this will imply an increase in b_i and hence an increase in the price ratio for all values of z_0. If we keep the income distribution of country j constant, and increase the Gini coefficient of country i while keeping its per capita income coefficient constant, this will imply a decrease in b_i and an increase in d_i. The increase in d_i will imply a higher convexity of the price ratio function. When there are changes in both per capita income and the Gini coefficient, the direction of the change in b will depend on the values of the parameters while the positive relationship between d and the Gini coefficient still hold.

The above results can be formalized as follows:

Proposition 2 *If the price ratio of the same good between two countries i and j* $\frac{p_i(z_0)}{p_j(z_0)}$ *is a function of the quality of that good* z_0*, then the difference in income inequality between the two countries determines the power of the price ratio function. Specifically, if the Gini coefficient of country i is higher (lower) than that of country j, then the price ratio function is upward (downward) sloping.*

We now explore the implications of Proposition 1 and 2 for the B-S relationship, in two alternative settings:

(a) Perfect quality measurement.

(b) A setting where quality is not measured as in Pakes (2003).

The B-S relationship is a relationship between a country's level of income and its national price level of final goods. As consumption bundles consist of the x goods and the z goods in our model, to measure the national price level, ideally we want to use observed data to reveal the unit price of the x

goods and the price schedule of the z goods $p(z)$. Then the price schedule $p(z)$ can be used to construct a price index of the z goods. Finally, the unit price of the x goods and the price index of the z goods are aggregated into a national price level. The above procedures of measurement and aggregation face, however, a prominent practical issue. The issue is that quality cannot be perfectly controlled for the z goods. Pakes (2003) shows how to use hedonics to adjust quality biases in the price indexes of quality goods due to the introduction of new goods. The adjustment procedures require a complete dataset on the characteristics of the goods, which is impossible in reality. Without the level of quality being observed, the observed prices of the z goods cannot tell us anything about the price level of the z goods, as their prices depend on both the level of quality z and the parameters b and d in the price function. The measurement issue at the data collecting stage will also contaminate the aggregation procedure. Without knowing the underlying price schedule $p(z)$, the common practice of constructing the price index of the z goods is to use the simple average of the observed prices as its price index. As a result, a higher price index of the z goods could be either due to higher values of b and d in the price function or simply due to the fact that the prices of higher quality goods have been observed.

Suppose quality can be properly measured, which means that we are able to reveal the underlying price function $p(z)$, then equation (2.9) implies that there still exists the B-S relationship. To see this, suppose we keep a country's Gini index constant and increase its per capita income, this implies a constant k_c but a higher c_m in the Pareto distribution. From (2.7) and (2.9), d will stay constant as before but b will go up, resulting in an upward shift of the price schedule $p(z)$. Hence, for any quality goods, the price will be higher than before. This is because a higher level of per capita income will increase the demand for the higher quality goods. The resulting higher output will increase their prices as the marginal cost is increasing in output. The elasticity of b with respect to c_m is equal to $\frac{k_c}{k_c+\frac{1}{\tau-1}}$. Given reasonable values of the parameters, the elasticity is quantitatively small.

If quality cannot be adjusted as in Pakes (2003), we have to use the simple average of the observed prices of the z goods as its price index. Suppose we again keep a country's Gini index constant and increase its per capita income, now the price index of the z goods not only reflects a higher b as in the previous situation but also reflects the fact that now the country will consume goods with higher qualities than before. This will cause an upward bias in the price index of the z goods and hence in the national price level. Moreover, as income inequality can affect the convexity of the price function and the expenditure share on z goods as shown in Proposition 1 and 2, it will have an impact on the price index of the z goods and the national price level. The details are shown in the next section.

2.1.5 *Aggregate Price Level*

Given the market equilibrium price schedule $p(z)$, we can calculate the average price level of z goods $\overline{p}$, which is the total expenditure on z goods divided by the total number of units.

$$\overline{p} = \frac{\int_{\overline{z}_m}^{\infty} p(z) Q^s(z) dz}{\int_{\overline{z}_m}^{\infty} Q^s(z) dz}$$

$$= \frac{\int_{\overline{z}_m}^{\infty} b z^d k_z z_m^{k_z} \left(\frac{b}{A_z \tau}\right)^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz}{\int_{\overline{z}_m}^{\infty} k_z z_m^{k_z} \left(\frac{b}{A_z \tau}\right)^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz}$$

$$= b \frac{\int_{\overline{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d} dz}{\int_{\overline{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz}$$

$$= b\frac{\frac{z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1}}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1}\Big|_{\overline{z}_m}^{\infty}}{\frac{z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}\Big|_{\overline{z}_m}^{\infty}}$$

If we assume $-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1<0$, then

$$\overline{p} = \frac{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1} b\overline{z}_m{}^d$$

Since $c_m = \frac{\alpha}{\beta}\overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m)$ and $p(z) = bz^d$, $b\overline{z}_m^d = \frac{\beta}{\alpha d+\beta}c_m$, we have

$$\overline{p} = \frac{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1}\frac{\beta}{\alpha d+\beta}c_m \tag{2.10}$$

Substituting (2.7) into (2.10), we have

$$\overline{p} = \frac{k_c}{k_c-1}c_m\frac{\beta}{\alpha\frac{k_z+\gamma(\frac{1}{\tau-1})}{k_c+\frac{1}{\tau-1}}+\beta}$$

Since the Gini coefficient and the mean of the Pareto income distribution are equal to $\frac{1}{2k_c-1}$ and $\frac{k_c c_m}{k_c-1}$, we can express the average price level in terms of the mean and the Gini coefficient

$$\overline{p} = \mu\frac{\beta}{\alpha\frac{k_z+\gamma(\frac{1}{\tau-1})}{\frac{\frac{1}{Gini}+1}{2}+\frac{1}{\tau-1}}+\beta} \tag{2.11}$$

where $\mu = \frac{k_c}{k_c-1}c_m$ and *Gini* are the mean and the Gini coefficient of the income distribution. This equation tells us the effects of per capita income and income inequality on the average price level of z goods, which is summarized in Proposition 3.

Proposition 3 *(Income Distribution and the Disaggregate Price Level) Per capita income has a positive impact on the average price level of z goods, whereas income inequality has a negative impact. Therefore, the elasticity of the average price of* *z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative.*

$Proof$: See Appendix 2.1.

Equation (2.11) also shows that the effect of per capita income on the average price level of z goods depends on income inequality and the effect of income inequality depends on per capita income.

Proposition 4 *The effect of income inequality on the average price level of z goods (the absolute value of $\frac{\partial \overline{p}}{\partial Gini}$) is increasing in per capita income μ, while the effect of per capita income on the average price level of z goods ($\frac{\partial \overline{p}}{\partial \mu}$) is decreasing in income inequality.*

$Proof$: See Appendix 2.2.

To investigate the implications of income distribution for the national price level, we need to construct an aggregate price index.

Although the vertically differentiated goods are produced by local firms, the homogeneous goods are tradable goods with their price level equalized across countries. Therefore, the cross-countries price comparison is still meaningful as we can compare the national price level using the price level of the homogeneous goods as an anchor or a numeraire.

Here for simplicity and in order to derive analytical results, we define the aggregate price level as the average price of x goods and z goods weighted by expenditure shares as in the Laspeyres or Paasche index. In Proposition 5, the results regarding how income distribution affects the aggregate price level are shown.

To make the results comparable with the empirical evidence, the results regarding how income distribution affects the log of the aggregate price level are also shown.

Proposition 5

(a) Income Distribution and the Paasche Index: If we define the aggregate price as the Paasche index

$$P_P = \frac{1}{1}share_x + \frac{\overline{p}}{\overline{p}_0}share_z = 1\frac{\alpha d}{\alpha d + \beta} + \frac{\overline{p}}{\overline{p}_0}\frac{\beta}{\alpha d + \beta},$$

where zero is used in the subscript to denote the variables from the base country, i.e. the U.S., then per capita income has a positive impact on the aggregate price level, or the elasticity of the aggregate price level with respect to per capita income ($e_{P_P,\mu} \equiv \frac{\partial P_P}{\partial \mu}\frac{\mu}{P_P}$) is positive. Moreover, the impact of income inequality on the aggregate price level and the semi-elasticity of the aggregate price level with respect to income inequality ($e_{P_P,Gini} \equiv \frac{\partial P_P}{\partial Gini}\frac{1}{P_P}$) depend on the per capita income relative to the U.S.. They are both positive when per capita income is low enough relative to the U.S. while they are both negative when per capita income is high.

(b) Income Distribution and the Laspeyres Index: If we define the aggregate price as the Laspeyres index

$$P_L = \frac{1}{1}share_{x,0} + \frac{\overline{p}}{\overline{p}_0}share_{z,0} = 1\frac{\alpha d_0}{\alpha d_0 + \beta} + \frac{\overline{p}}{\overline{p}_0}\frac{\beta}{\alpha d_0 + \beta}.$$

Then per capita income has a positive impact on the aggregate price level, i.e. the elasticity of the aggregate price level with respect to per capita income ($e_{P_L,\mu} \equiv \frac{\partial P_L}{\partial \mu}\frac{\mu}{P_L}$) is positive, whereas income inequality has a negative impact, or the semi-elasticity of the aggregate price level with respect to income inequality ($e_{P_L,Gini} \equiv \frac{\partial P_L}{\partial Gini}\frac{1}{P_L}$) is negative.

(c) No matter whether the aggregate price level is defined as the Laspeyres index or the Paasche index, the effect of per capita income on the aggregate price level ($\frac{\partial P_P}{\partial \mu}$ and $\frac{\partial P_L}{\partial \mu}$) is decreasing in income inequality and the effect of income inequality ($\frac{\partial P_P}{\partial Gini}$ and $\frac{\partial P_L}{\partial Gini}$) is decreasing in per capita income μ. Moreover, the elasticity of the aggregate price level with respect to per capita income $e_{P_P,\mu}$ and $e_{P_L,\mu}$ is decreasing in income inequality whereas the semi-elasticity of the aggregate price level with

respect to income inequality $e_{P_P,Gini}$ and $e_{P_L,Gini}$ is decreasing in per capita income μ.

Proof: See Appendix 2.3.

In practice, the way to construct the multilateral price index as in the International Comparison Program (ICP) is different. However, as shown in Deaton and Heston (2010), it can be approximated very well by the bilateral Fisher index, i.e., a geometric mean of the Laspeyres and the Paasche index. Therefore, the results in Proposition 5 can be used to show how income distribution affects the bilateral Fisher index or the national price level.

No matter whether the aggregate index is defined as the Laspeyres index or the Paasche index, the elasticity of the national price level with respect to per capita income is always positive and it is decreasing in income inequality. Since the elasticity of the Laspeyres index with respect to income inequality is negative and the elasticity of the Paasche index with respect to income inequality is decreasing in per capita income, with a low enough per capita income, the elasticity of the bilateral Fisher index could be positive while it is negative with a high per capita income. This is confirmed in Figure 2.3, where the derivatives of the bilateral Fisher index with respect to income inequality $\frac{\partial P_F}{\partial Gini}$ for different combinations of per capita income and income inequality are plotted. For a lower level of per capita income $\frac{\partial P_F}{\partial Gini}$ is positive, while it is negative when per capita income is high.

The intuition behind the proposition is: with higher per capita income, a country will spend a larger amount of its income on z goods, which implies a higher average price of z goods due to the imperfect control over quality. Therefore, with a constant price of x goods, higher per capita income implies a higher aggregate price level. However, with a higher Gini coefficient, a country will spend a smaller fraction of income on z goods. Since in practice, the quality of z goods cannot be easily controlled, the lower expen-

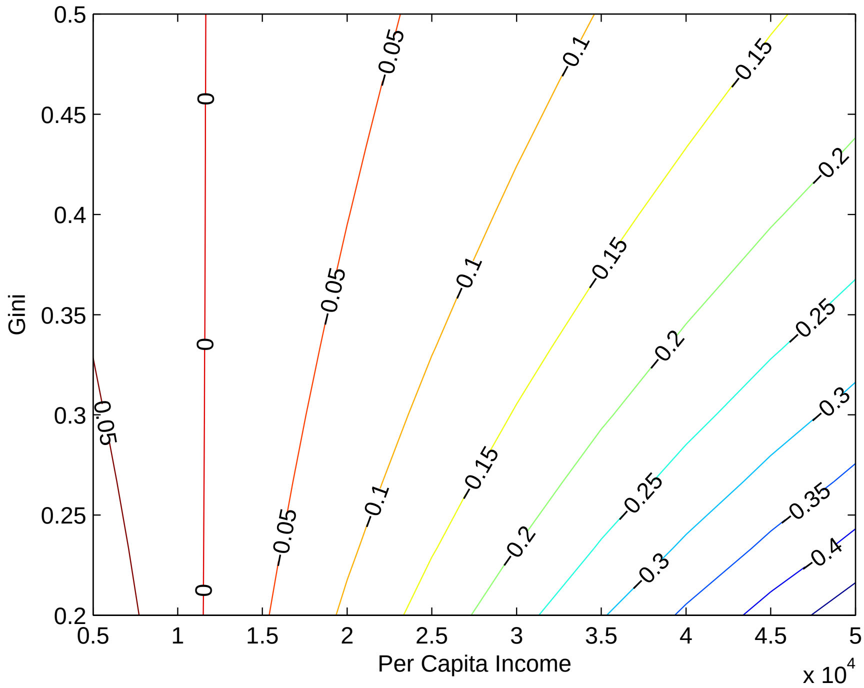


Figure 2.3: The Contour of the Effect of Income Inequality on the Bilateral Fisher Index ($\frac{\partial P_F}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality.

Notes: The base country income distribution is calibrated using U.S. data in 2003.

diture share on z goods will imply a lower measured price level for z goods. The Laspeyres index, which is the average price of x and z relative to the U.S. weighted by the expenditure shares of the U.S., will be lower, since the relative price of z is lower and there is no change in the relative price of x and the weights. However, the impact on the Paasche index, which is the average price of x and z relative to the U.S. weighted by the country's expenditure shares, will depend on the country's per capita income relative to the United States. With a low enough per capita income, the price of x relative to the U.S. will be higher than the price of z relative to the U.S.. So a lower expenditure share on z goods will increase the measured aggregate price level. With a higher per capita income, the relative price of z goods will be comparable with or higher than the relative price of x goods, so the lower expenditure share on z goods will reduce the aggregate price level. Since the product of the expenditure share and the average price of z enters the aggregate price level, a higher per capita income, which increases the average price of z, will strengthen the effect of income inequality, while a lower income inequality, which increases the expenditure share of z, strengthens the effect of per capita income. Hence the effect of per capita income must be decreasing in income inequality and the effect of income inequality must be decreasing in per capita income.

Next, we ask: what is the key feature of the present model that leads to Proposition 5. To address this question, we develop in Appendix 2.4 an alternative model based on the classic model of vertical product differentiation.

As quality is not controlled for in the price index of quality products, the price index of quality products is measured as the average expenditure on quality products. Keeping per capita income constant, the price index of quality products will only depend on its expenditure share. Therefore, whether income inequality can affect the price index of quality products crucially depends on whether income inequality can affect the expenditure shares.

In both the classic model of vertical product differentiation and the model in Chapter 2, the expenditure share of the quality products crucially depends on the convexity of the price schedule of quality products. In the former model, for the sake of illustration, the price schedule is exogenously given, so the expenditure share is constant and cannot be affected by income inequality. This closes off the relation between the price level and income inequality. In the latter model, the price schedule is endogenously determined by the firm distribution and income distribution. As a result, income inequality can affect the convexity of the price schedule and hence the expenditure share. Therefore, the price index of quality products will be affected by income inequality. Since the national price level is a weighted average of the price levels of the quality products and commodity goods, the national price level will also be affected by income inequality.

2.2 *Empirical Tests I: Income Distribution and the Aggregate Price Level*

This model predicts the B-S relationship, but it also predicts a new relationship between income inequality and the national price level, which is summarized in Proposition 5 above. The new relationship is as follows:

Controlling for per capita income, income inequality is correlated with the national price level: within countries with lower per capita income, income inequality is positively correlated with the national price level, while within countries with higher per capita income, the correlation is negative.

In this section, we investigate this prediction directly. In the next section, we investigate some additional predictions of the model that follow from Proposition 1 and 3.

To show that not only per capita income but also income inequality is important in determining the aggregate price level, we extend the regression in Rogoff (1996) by adding the Gini coefficient as an extra regressor to investigate if the Gini coefficient helps to explain national price differentials.

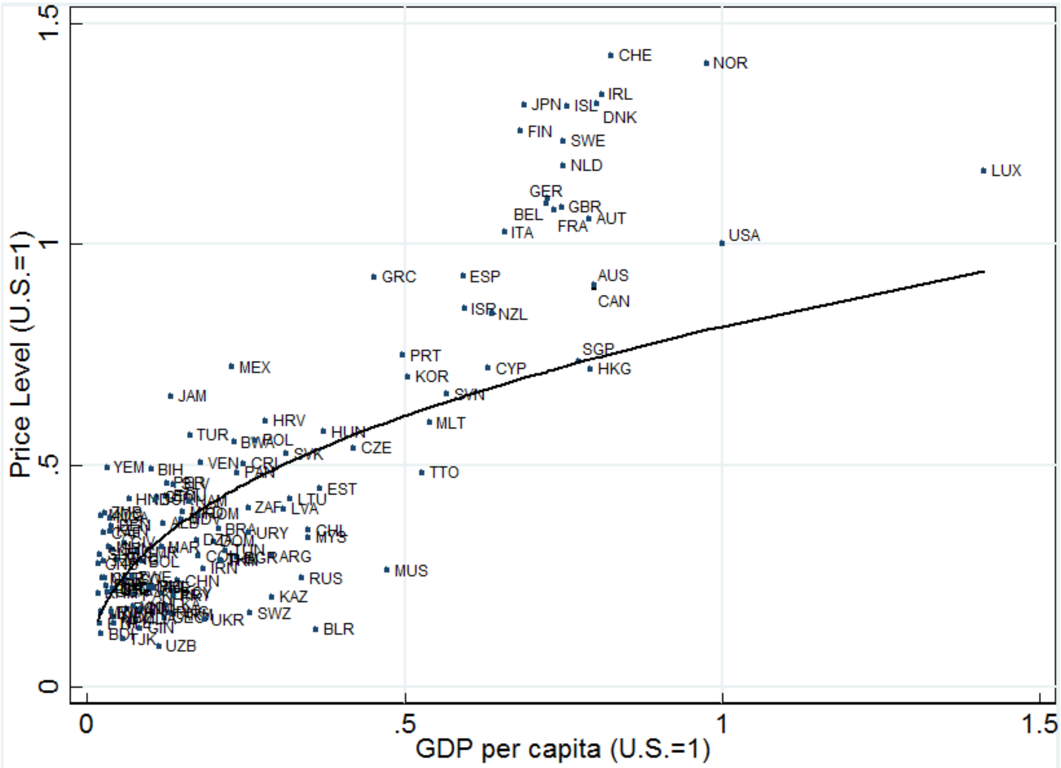


Figure 2.4: Price Level versus GDP per capita in 2003 (U.S.=1)

Notes: Source: The Penn World Table 6.2

First, the figure in Rogoff (1996) is reproduced in Figure 2.4 using 2003 data.[3] The data on prices and income are from Penn World Table PWT 6.2. The data on the Gini coefficients are taken from the World Bank: World Development Indicators 2007. Figure 2.4 shows that the problem with the Balassa-Samuelson hypothesis still persists; it performs well for the whole sample, but does not perform well either within poor countries or within rich countries.

[3]Data from 2003 is used because cross-sectional Gini coefficients from World Development Indicators 2007 is computed not using the same year data for each country, but with most of them observed around the year 2003. Since Gini coefficient is relatively stable within a couple of years, Gini coefficients from WDI 2007 are taken as the Gini coefficients of 2003 for each country. The year 2003 is also chosen as the sample year for other variables in the regression.

Table 2.1: The Effects of the Gini Coefficient in 2003

Independent Variable	Relative Price Level $\log(P_j/P_{U.S.})$ (1)	(2)	(3)	(4)	(5)
Constant	-0.204***	-0.186	1.187***	1.462***	1.168***
	(0.077)	(0.180)	(0.323)	(0.413)	(0.323)
$\log(Y_j/Y_{U.S.})$	0.413***	0.408***	1.180***	1.242***	1.160***
	(0.036)	(0.040)	(0.160)	(0.170)	(0.161)
Gini Index	-	-0.069	-3.766***	-4.115***	-3.678***
		(0.456)	(0.857)	(0.927)	(0.858)
Gini Index $\times \log(Y_j/Y_{U.S.})$	-	-	-1.969***	-2.121***	-1.921***
			(0.399)	(0.427)	(0.400)
VAT	-	-	-	-0.00899	-
				(0.00842)	
Population	-	-	-	-	-3.03e-07
					(2.56e-07)
Observations	130	124	124	118	124
R-squared	0.512	0.499	0.584	0.584	0.589

Note: Data on price and income are taken from the Penn World Tables 6.2. Data on the Gini Index are taken from World Bank: World Development Indicators 2007. VAT data is from International VAT and IPT Service. Population data is from WRDS. ***, ** and * indicate statistically significant different from zero at 1%, 5% and 10% level respectively.

The regressions with the Gini coefficients for the year 2003 are shown in Table 2.1. Results from Regressions (1) to (5) are consistent with the fact that countries with higher income tend to have higher price levels as the estimated coefficients of relative income are all significantly positive. Moreover, the estimated coefficient of per capita income is 0.413 in Regression (1), which is similar to Rogoff's estimate using the 1990 data. However, the Gini coefficient in Regression (2) is not significant, while when the product of the Gini index and relative income is included as an interaction term in Regression (3), both the Gini index and the interaction term become significantly negative. This implies a negative relationship between income inequality and the national price level if country j's per capita income is similar to that of the U.S., and this effect is decreasing in per capita income. This also explains why the Gini index in Regression (2) is not significantly negative. This is because Regression (2) fails to include the interaction term which has significant explanatory power, the estimated coefficient of the Gini index will be the sum of the estimated coefficient of the Gini index in Regression (3) and the product of the estimated coefficient of the interaction term in Regression (3) and the relative income. Since in the sample, most of the relative incomes in logarithm are negative, when they are multiplied with the negative coefficient of the interaction terms, they reduce the magnitude of the negative coefficient of the Gini index and make it insignificant in Regression (2). Therefore, the results show that per capita income has a positive impact on the aggregate price level, i.e. the Penn effect, while income inequality also has a significant impact on the aggregate price level and the impact is decreasing in per capita income. These are consistent with the model predictions in Proposition 5 that per capita income has a positive impact on the national price level and the impact of income inequality on the national price level is decreasing in per capita income. Regressions (4) and (5) control for VAT and population, with the latter being a proxy for market size. The estimation results show that the inclusion of these two control variables does not change the estimation result in (3). Moreover, both control variables are not significant at the 10% level.

Table 2.2: Different Behaviors of Income Distributions in Different Samples in 2003

	Relative Price Level $\log(P_j/P_{U.S.})$							
	Threshold=0.33 of $Y_{U.S}$				Threshold=0.60 of $Y_{U.S}$			
	Poor Countries		Rich Countries		Poor Countries		Rich Countries	
Independent Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	-0.899***	-1.376***	0.256***	0.999***	-0.713***	-0.978***	0.154**	1.066***
	(0.124)	(0.223)	(0.085)	(0.224)	(0.107)	(0.204)	(0.074)	(0.168)
$\log(Y_j/Y_{U.S.})$	0.157***	0.159***	0.915***	0.910***	0.223***	0.234***	0.361	0.378*
	(0.049)	(0.048)	(0.181)	(0.145)	(0.044)	(0.046)	(0.230)	(0.205)
Gini Index	-	1.113**	-	-2.125***	-	0.684	-	-2.721***
		(0.440)		(0.632)		(0.441)		(0.477)
Observations	96	95	34	29	105	102	25	22
R-squared	0.097	0.154	0.444	0.642	0.196	0.211	0.097	0.660

Note: Data on price and income are taken from the Penn World Tables 6.2. Data on the Gini Index are taken from World Bank: World Development Indicators 2007. ***, ** and * indicate statistically significant different from zero at 1%, 5% and 10% level respectively.

Given the significance of the interaction term, to further understand how income distribution affects the national price level in poor and rich countries, in Table 2.2 the whole sample is split into two subsamples according to the relative income level, and the above regressions are run for the two subsamples. First, the threshold is set to 33% of the per capita GDP of the U.S.. The Penn effect is confirmed in Regressions (1) to (4) as the estimated coefficients are all significantly positive. However, within poor countries, income inequality has a positive impact on the national price level; whereas within rich countries, the impact is negative. This is consistent with the negative coefficient of the interaction term in Table 2.1, i.e. the impact of income inequality on the national price level is decreasing in per capita income. Moreover, this is also consistent with the model prediction shown in Figure 2.3 that within countries with lower per capita income, income inequality has a positive impact on the national price level, while within countries with higher per capita income, the impact is negative. When we increase the threshold to 60%, Regressions (5) to (8) again confirm the Penn effect. Given the higher threshold and the fact that the impact of income inequality is decreasing in per capita income, within poor countries, the impact of income inequality is positive but not significant while within rich countries, the impact of income inequality becomes more negative. In terms of R^2, it can be seen that within poor countries, the inclusion of the Gini coefficient increases the R^2 marginally, while within rich countries the inclusion of the Gini coefficient increases the R^2 significantly. These results have confirmed that the relationship between per capita income and national price level is far less impressive both within poor countries and within rich countries. Moreover, income inequality plays an important role in explaining national price differentials, especially within rich countries. As for the quantitative impact of income distribution, the estimated coefficients in Regression (8) imply that aggregate price level increases by about 0.38 percentage points with the 95% confidence interval being $[-0.051, 0.807]$ in response to a one percentage point increase in per capita income, whereas the price level decreases by about 2.72 percent with the 95% confidence interval being [1.723, 3.719] in response to a one hundred basis points increase in the Gini coefficient. Moreover, as the Gini coefficients are usually measured with large errors, the magnitude of the estimate is probably biased downwards.

As have been shown in the model, the positive relationship between per capita income and the national price level in Table 2.1 is due to the fact that quality is not controlled for. For example, in Bils and Klenow (2001), they use the U.S. data to show that quality growth of 66 durable goods causes an over-estimation of inflation by 2.2%. If quality cannot be controlled for, then it will show up in the price index. Moreover, due to the fact that income elasticity of quality for many consumption goods are non-negligible and tend to be higher for nontraded goods, which are priced in a non-linear way, income distribution will matter for people's choice of quality and will affect the national price level through the price of nontraded goods. This is why income inequality also affects the measured national price level.

2.3 *Empirical Tests II: Income Distribution, Disaggregate Price levels and Expenditure Shares*

In this section, we examine the further predictions of the model that follow from Proposition 1 and 3. For convenience, we repeat the statements of these propositions as follows:

Proposition 1 *(Income Distribution and Expenditure Share) Income inequality has a positive impact on the expenditure share of x goods and a negative impact on the expenditure share of z goods. Per capita income has no impact on expenditure shares.*

Proposition 3 *(Income Distribution and the Disaggregate Price Level) Per capita* *income has a positive impact on the average price level of z goods, whereas income inequality has a negative impact. Therefore, the elasticity of the average price of z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative.*

These propositions can be tested using a 2-step procedure, as follows:

Step 1: Using consumption expenditure data, we can identify which good in the consumption bundle is more like the x goods and which good is more like the z goods.

Step 2: To test if the empirical effects of income distribution on the price level and expenditure share of the x and z goods are the same as predicted in Proposition 1 and 3.

Since the aggregate price level is an average price level of consumption weighted by expenditure shares, we have to understand the aggregation methods used in practice in order to show that both the assumptions in the model and the model mechanism are consistent with the data. In the construction of both national price indices such as the CPI and multilateral price indices in the Penn World Table, the first step is to construct the sub-indices for different components of consumer expenditure. Then, expenditure data from each country's national account is used to construct the weights for different components and all the sub-indices are aggregated into an aggregate price index using these weights. However, some aspects of this aggregation method can have important consequences.

As it will be shown in Section 2.3.1, for consumption goods such as food and housing, the income elasticity of quantity is close to zero, while for consumption goods such as clothes, the income elasticity of quality is close to zero. Based on whether income elasticity of quality is zero or income elasticity of quantity is zero or both are nonzero, we can identify three types of goods. We call the first type x goods and the second type z goods. This observation combined with the aggregation method can have two consequences. Firstly, due to the lack of data on the characteristics of goods, the aggregation method is not able to control quality, hence the higher quality of z goods will be translated into a higher price. Secondly, as has already been shown in the model, income inequality affects the price function and the expenditure share of z goods, and hence the aggregate price level.

Guided by the dichotomy of x and z goods, to understand how income distribution influences the aggregate price level, Section 2.3.2 investigates how disaggregate prices and expenditure shares change with income distribution, which can be used to show that the mechanism of the model is consistent with the data.

2.3.1 *Identification of x goods and z goods*

In the traditional literature, prices usually do not play a role and consumption (physical quantity) is equivalent to consumption expenditure given that the price function is linear and unit price is constant. Hence the income elasticity for one good is the income elasticity of consumption (or consumption expenditure) for that good.

However, in this chapter, because a type z good is priced in a non-linear way, the equivalence between consumption and consumption expenditure is broken.

In general, since expenditure is the product of quantity and unit price, which depends on the quality of the good, income elasticity of expenditure can be decomposed into income elasticity of quantity and income elasticity of quality (unit price) as follows:

$$\begin{aligned} e_{expenditure} &= \frac{dlog(consumption\ expenditure)}{dlog(income)} \\ &= \frac{dlog(quantity \times unit\ price)}{dlog(income)} \\ &= \frac{d[log(quantity) + log(unit\ price)]}{dlog(income)} \\ &= \frac{dlog(quantity)}{dlog(income)} + \frac{dlog(unit\ price)}{dlog(income)} \\ &= e_{quantity} + e_{quality} \end{aligned}$$

Moreover, consumption goods differ in their income elasticities of quantity and quality. Here we try to divide all the consumption goods into two groups according to their relative magnitudes of these two elasticities. The first type of goods which we call x goods has very low income elasticity of quality but high income elasticity of quantity. Given this fact, it is assumed that consumers can only change the quantity of x goods but not the quality. The other type of goods which we call z goods has very high income elasticity of quality but low income elasticity of quantity. Similarly, it is assumed that consumers can only change the quality of z goods but not the quantity.

In this subsection, expenditure and quantity data from the U.S. are used to identify which category a particular consumption good belongs to. This subsection focuses on four categories of consumption goods, namely food, housing, clothes and vehicles. It is shown that the income elasticities of quantity of food and housing are close to zero and the income elasticity of quality of clothes is close to zero, while both elasticities are nonzero for vehicles. Table 2.3 also shows that these four categories plus hotels and restaurants account for on average more than 60% of the total expenditure within OECD countries, and the expenditure on z goods constitute on average around 70% of the total expenditure of these five categories, hence it is important to incorporate the dichotomy of x goods and z goods into the model due to its significant expenditure share.[4]

Table 2.3: Expenditure Shares of Food, Housing, Apparel, Transportation and Restaurants and Hotels

Country	Share of the Five Categories	Share of z goods	Share of z goods within the Five Categories
Max	0.751	0.508	0.805
Min	0.466	0.314	0.632
Average	0.606	0.426	0.703
Standard Deviation	0.051	0.043	0.042

Food

As has been documented in the literature, calorie intake does not vary with permanent income across households. Specifically, Aguiar and Hurst (2005) find that employed household heads with a higher income consume similar amounts of calories as employed household heads with a lower income. However, conditional on log calories, they find that the income elasticities of vitamin A and vitamin C are over 0.30 and the income elasticities of vitamin E and calcium are 0.17 and 0.08, respectively. In addition, the income elasticity of cholesterol is negative.

The results suggest that the income elasticity of quantity for food is very close to zero. Households with higher income do not consume a larger quantity of food than households with lower income. Instead, they consume higher quality foods, such as those rich in vitamin and calcium. On the other hand, low income households consume cheaper calories by having a higher composition of fat and cholesterol in their diets.

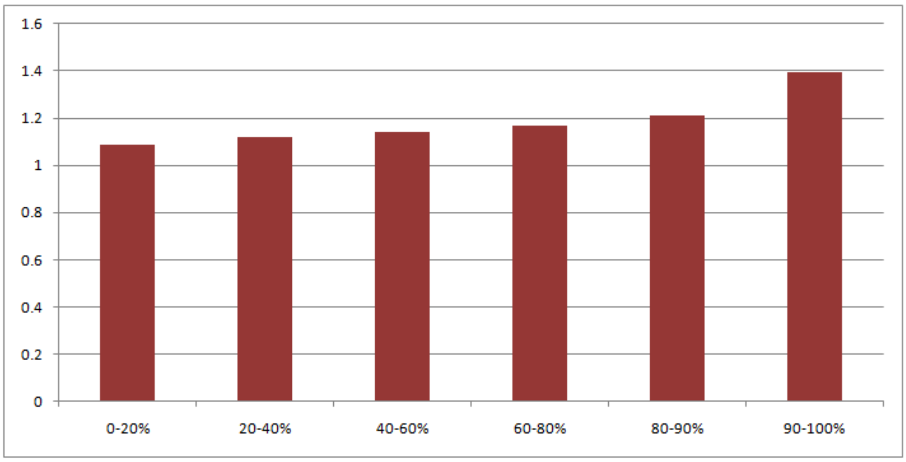
[4]Since food at restaurant is a substitute of food at home and staying at a hotel is a substitute of staying at home, given the z goods feature of food and housing, hotels and restaurants should be z goods as well.

Housing

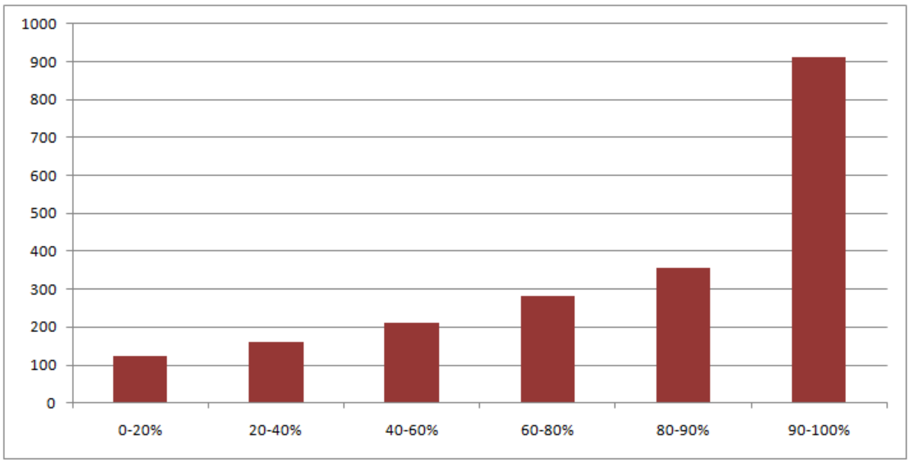
As has been defined previously, the number of houses counts as quantity whereas other characteristics of a house, such as square meters, all count as quality. Micro-evidence has shown that the price function of housing is nonlinear. For example, Anderson (1985) estimates the hedonic price function of housing, i.e. regressing the housing price on characteristics of the house which include structural characteristics of the house, improvements to the house, physical characteristics of the lot, neighbourhood characteristics, etc. He shows that the price function is convex. Even if we define housing by square meters, the price function is still estimated as a convex function. For example, Coulson (1992) estimates a nonparametric response of housing price to floorspace size. The marginal price is estimated to be increasing, which implies a convex price function. Mason and Quigley (1996) estimate the hedonic price indices for downtown Los Angeles and they find the price function is convex in size (1000 sq ft). Also, Bao and Wan (2004) find that the sale price per square foot is increasing in gross area controlling for other characteristics using Hong Kong data.

We use data from the SCF (Survey of Consumer Finances) to compute the number of residences, total value of residences and value per residence by percentile of income. Figure 2.5 plots the results for the year 2004.

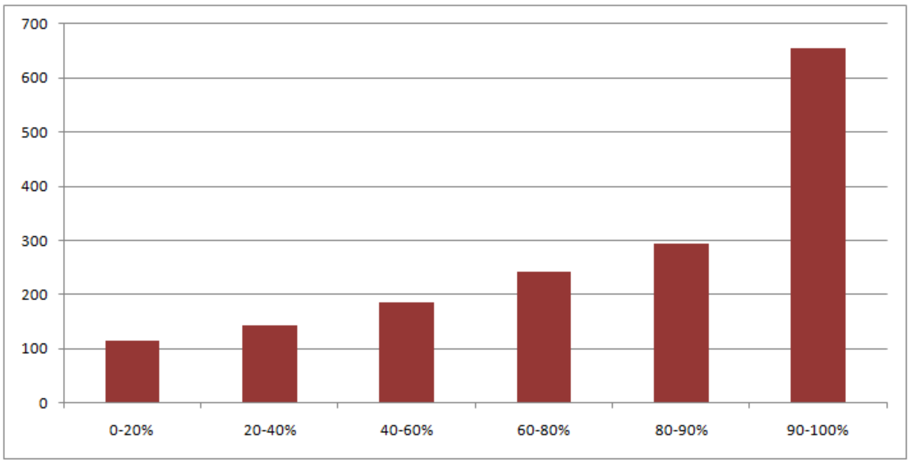
By inspection, one can see that as we move from the low percentile of income to the high percentile of income, the number of residences only changes modestly and almost all the variation in total value of residence is due to the variation in value per residence. This implies a very low income elasticity of quantity.



(a) Number of Residences



(b) Total Value of Residences (thousands of dollars)



(c) Value Per Residence (thousands of dollars)

Figure 2.5: Housing Quantity and Quality by Percentile of Income (SCF 2004)

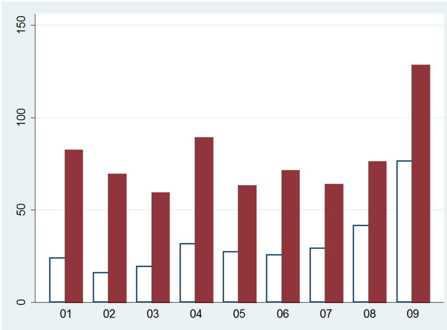
Clothes

The detailed expenditure data on clothes are extracted from the raw data files of the Consumer Expenditure Survey (CEX), which includes both the expenditure on and quantity of clothes. The total expenditure on a certain type of clothes is divided by the number of clothes to get the unit price. Figure 2.6 plots the total expenditure and the unit price for different types of clothes across the nine income classes in the CEX. The white bars denote the total expenditure and the black bars denote the unit price. It can be seen that for all types of clothes, unit prices are almost the same for all income classes, which suggests a very low income elasticity of quality.

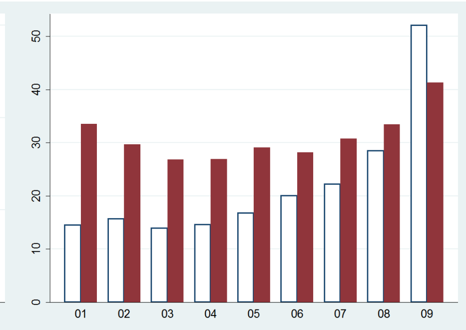
Vehicle

Data on the quantity and expenditure of vehicles is also available from the CEX. As has been done for housing, Figure 2.7 plots the number of vehicles, total value of vehicle purchases and unit value per vehicle across income classes. It shows that neither income elasticity of quantity nor income elasticity of quality is zero. As we move from lower income groups to higher income groups, both quantity and quality increase significantly.

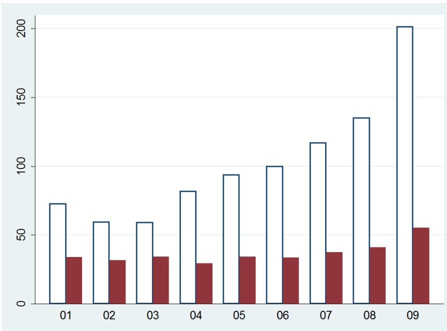
In addition to the four consumption goods noted above, Bils and Klenow (2001) also document the relative importance of income elasticity of quality and quantity for 66 durable goods in the CEX. Although they assume that the hedonic price function is linear, their results are consistent with some of the above evidence. For example, the income elasticity of quality of clothes is very low and the income elasticities of quantity and quality of vehicles are of the same magnitude.



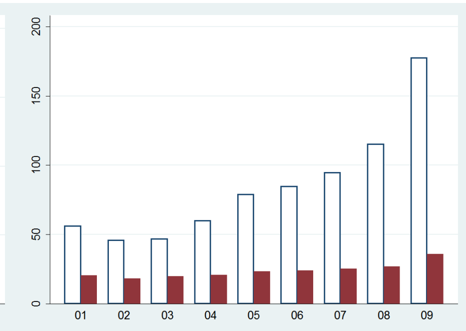
(a) Coats



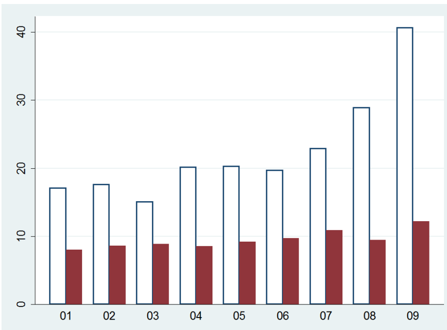
(b) Sweaters



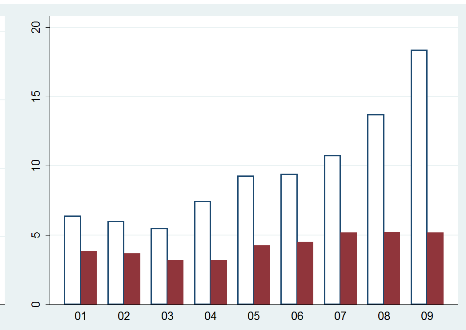
(c) Pants



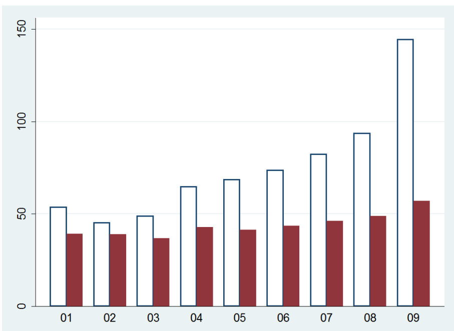
(d) Shirts



(e) Undergarments



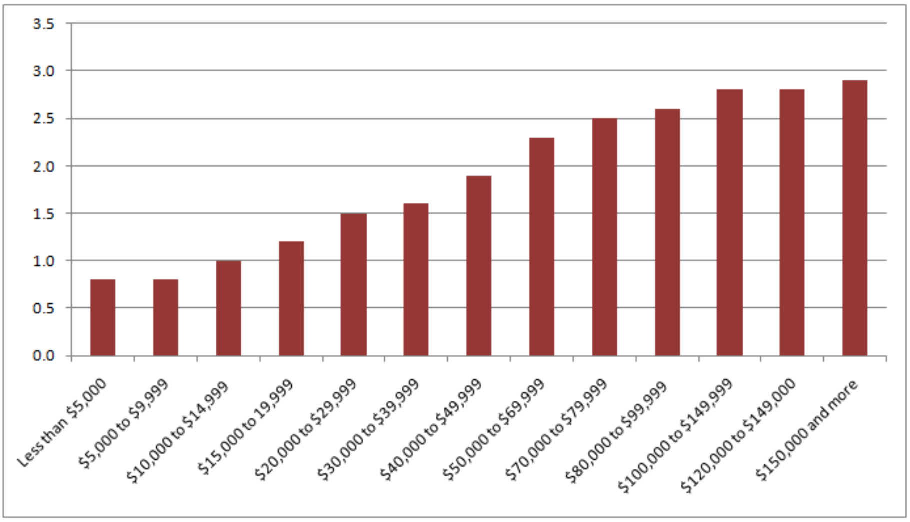
(f) Hosiery



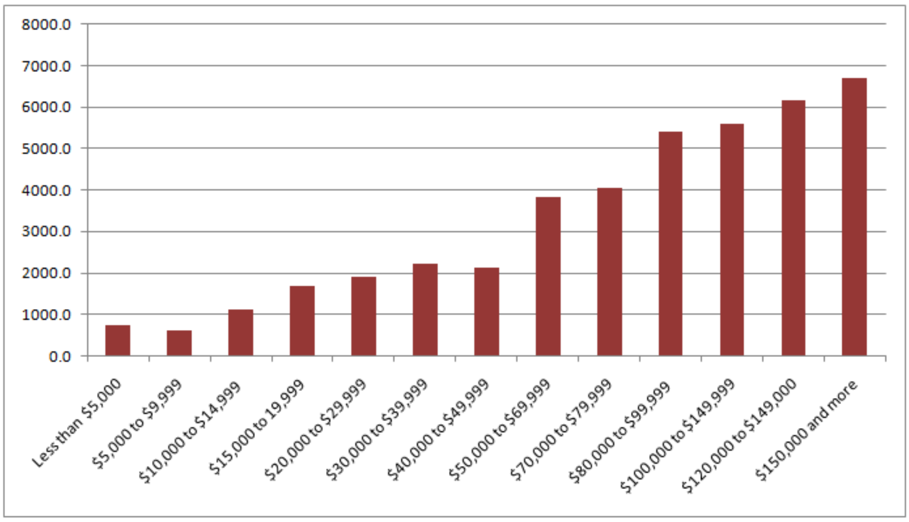
(g) Footwear

Figure 2.6: Total Expenditure and Unit Price for Different Types of Clothes in Dollars (CEX 2003).

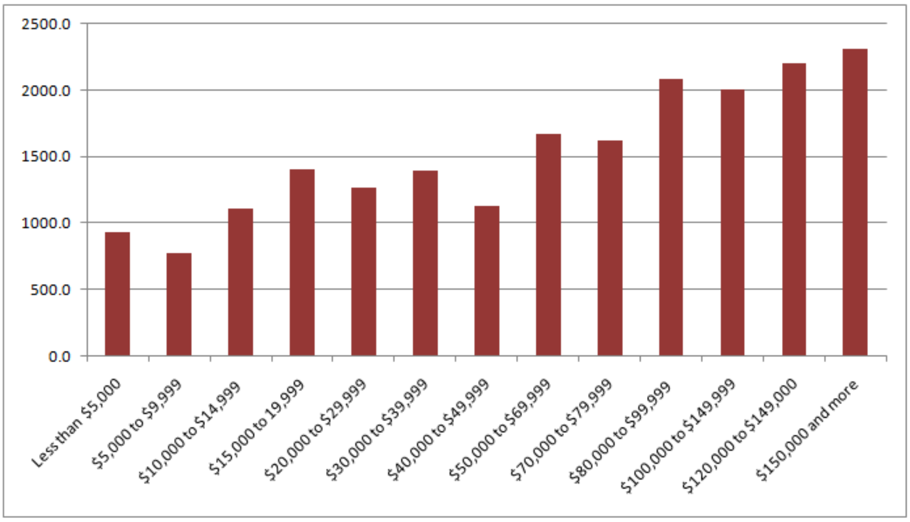
Notes: White bars denote total expenditure and dark bars denote unit price.



(a) Number of Vehicles



(b) Total Value of Vehicles (dollars)



(c) Value Per Vehicle (dollars)

Figure 2.7: Vehicle Quantity and Quality by Percentile of Income (CEX 2007)

In summary, the evidence presented shows that the income elasticities of quantity of food and housing are close to zero, the income elasticity of quality of clothes is close to zero and both are nonzero for vehicles. Some may argue that the observed two elasticities are equilibrium outcomes, which are endogenous. Hence, the observed patterns cannot be taken as primitives in the model. However, the observed elasticities, especially the zero income elasticity of quantity of food and housing, are not due to equilibrium outcomes, but instead, are due to the nature of the goods. For example, the daily calorie intake has to be within a certain range regardless of income and for convenience, a household usually has one primary residence. Finally, given the fact that the tradability of food and housing is generally lower than that of clothes and vehicles, it is reasonable to assume that x goods are tradable with the price normalized to 1 and z goods are non-tradable with a non-linear price function.

2.3.2 *The Impact of Income Distribution on the Price Levels of Individual Product Groups and Expenditure Shares*

Since the national price level is an average price level of disaggregate price levels weighted by expenditure shares. By examining how disaggregate price levels and expenditure shares vary with income distribution, we can trace out the main drivers of the positive relationship between per capita income and the national price level, and, more importantly, the relationship between income inequality and the national price level.

In this subsection, we investigate empirically how income distribution affects disaggregate price levels and expenditure shares. This leads to direct tests of the model's predictions in Proposition 1 and 3.

In the model, whether one good belongs to x or z will crucially determine how income distribution affects its price level and expenditure share.

However, in practice, few goods are pure x or pure z. Nevertheless, we can quantify the degree to which a good belongs to x or z based on the features of these two types of goods. The prices of the x goods are assumed to be equalized across countries and the prices of the z goods are locally determined and related to local per capita income. We can therefore use the elasticity of a product's price with respect to per capita income, which is designated here as the quality index of the product, to measure whether the product is more like a x good or a z good.

As the most disaggregate level of the PPP data from the ICP is the basic heading level, we first compute the quality index for each basic heading by running cross-country regressions, by regressing the log of the price levels of one product on the log of countries' per capita income.

Then we regress the disaggregate price level on per capita income, the Gin coefficient and the product of per capita income and the Gini coefficient for each basic heading using the underlying PPP data from the ICP[5] and show how the estimation results vary from the basic headings with high quality index to the basic headings with low quality index.

$$
\begin{aligned}
log(Price_i) &= \beta_0 + \beta_1\, log(Per\ Capita\ Income) + \beta_2\, Gini \\
&\quad + \beta_3 log(Per\ Capita\ Income) \cdot Gini + \epsilon_i
\end{aligned}
$$

To do so, we plot the estimated coefficient from the above regression against the quality index to see how the latter affects the coefficient of per capita income, the Gini coefficient and the interaction term. Panel (a), (b)

[5]The dataset used here is from the ICP benchmark 2005, which provides disaggregate price indices and expenditure data at the basic heading level. They are the underlying data behind the national price level in the Penn World Table. The basic headings which are classified as government consumption or investment are excluded from this study since the consumption of these categories is due to other reasons that are not supposed to be captured by this chapter.

and (c) in Figure 2.8 plot these estimated coefficients, i.e. $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$, against the quality index.

(a) $\hat{\beta}_1$ vs Quality index (b) $\hat{\beta}_2$ vs Quality index

(c) $\hat{\beta}_3$ vs Quality index (d) $Corr(Expenditure\ Share, Gini)$ vs Quality Index

Figure 2.8: How the Quality Index Affects the Impact of Income Distribution on the Price Level of Individual Product Groups and Expenditure Share (Source: ICP 2005)

Notes: The size of markers in the above scatter plots is proportional to the average expenditure share of each basic heading over all the countries in the ICP program.

[redacted]

[redacted]

[redacted]

[redacted]

Thus, the empirical evidence at the disaggregate level is consistent with the model's mechanisms, through which income distribution affects the national price level.

Chapter 3

An Examination of Product Level Data

3.1 Introduction

Chapter 2 is devoted to exploring the Balassa-Samuelson relationship at the aggregate level. To further understand the sources of the aggregate relationship at the disaggregate level, in this chapter, we take a purely statistical approach in asking the question: what is the best statistical description of the relationship between wealth and the price levels of individual products.

The motivation of doing so is that the huge variations in the aggregate B-S relationship across countries with different levels of income imply that using the B-S hypothesis as the single general theory to explain national price levels is far from satisfactory. The driving forces of the huge variations are crucial to understand the aggregate price level. For example, Rogoff (1996) has shown that there is empirical support for it when comparisons are made between the set of poor countries and the set of rich countries. However, its explanatory powers are far less impressive within either the poor countries group or the rich countries group. As shown in Chapter 2, regressing the national price level on per capita GDP generates an R^2 of 0.51 in the whole sample. When the whole sample is split according to per capita income with 60 percent of the US GDP per capita as the threshold, the R^2s within the poor countries and the rich countries become 0.20 and 0.10 respectively. In addition to R^2, the two countries groups also differ in the elasticity of the national price level with respect to per capita income. Within the poor countries, the elasticity of the national price level with respect to per capita income is around 0.22, which is far lower than the elasticity of 0.36 within the rich countries. The above two results are robust to the choice of threshold. For example, using one third of the US GDP per capita as the threshold can only affect the results quantitatively but not qualitatively.

Since the national price level is an average of disaggregate price levels weighted by expenditure shares, we can use the underlying disaggregate price levels and expenditure shares to dig out the sources of the variations in the B-S relationship. Our goal in this chapter is to find the cleanest statistical description of the relationship between wealth and the price levels of individual products. However, testing the Balassa-Samuelson hypothesis using disaggregate price levels is not new in the literature. For example, Heston et al. (1994) use disaggregate price levels to test an intermediate prediction of the hypothesis: the price ratio of tradable to nontradable is decreasing in income. Similar tests can be found in Kravis and Lipsey (1988). These tests did provide empirical support for the hypothesis, but they did not address the variations of the B-S effects across countries and products.

Our empirical strategy is, instead of assuming a stable relationship between the price levels of individual products and per capita income across countries, to adopt a more agnostic approach by allowing for more flexibility in the parametric relationship between the price level and per capita income to accommodate the large variations in the relationship. This approach turns out to enable us to identify a clear and striking empirical pattern: for some products, the B-S relationship is weak and disperse. While for other products, the B-S relationship is highly nonlinear, which is best described as a spline relationship: within low- and middle- income countries, the relationship between per capita income and the price level is weakly positive, while within high-income countries, there is a sudden increase in the slope of the positive relationship.

[redacted]

(a) [redacted] (b) [redacted]

Figure 3.1: Two Archetypes of Price-Wealth Relationships

We illustrate the two types of relationship in Figure 3.1. [redacted]

The details about how to quantify the B-S relationship at product level and their theoretical implications will be provided in the next section.

3.2 Empirical Evidence: Characterizing the B-S Relationship at Product Level

The disaggregate price levels used in this chapter are from the ICP 2005 benchmark dataset, which includes all the price levels at the basic heading level. Basic headings are defined as the most disaggregate price level, at which there exists matching expenditure data from national accounts. These price levels of basic heading are just the disaggregate price levels underlying the national price level in the Penn World Table.

To get an idea of how the B-S effect varies across products and countries, we first plot the scatter diagram of the price level against GDP per capita (in logarithm) relative to the US for each basic heading in Figure 3.2. On inspection, we can identify a clear and striking empirical pattern: for some products, the B-S relationship is weak and disperse. For example, in the case of [redacted] While for other products, the B-S relationship is highly nonlinear, which is best described as a spline relationship: within low- and middle- income countries, the relationship between per capita income and the price level is weakly positive, while within high income countries, there is a sudden increase in the slope of the positive relationship. For example, in the case of [redacted]

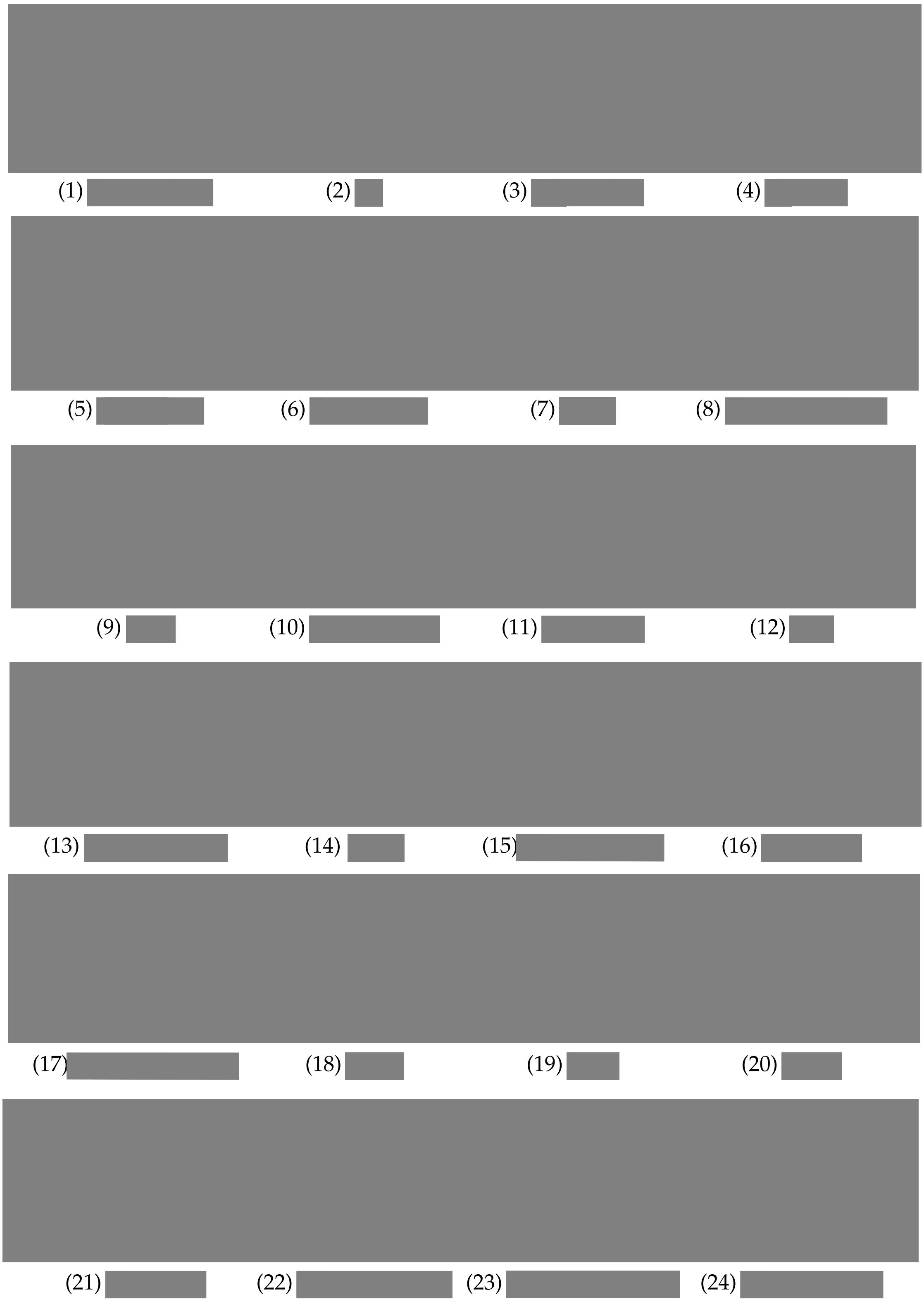


Figure 3.2: Disaggregate Price Level of Basic Headings and GDP Per capita

[redacted]

(25) [redacted] (26) [redacted] (27) [redacted] (28) [redacted]

[redacted]

(29) [redacted] (30) [redacted] (31) [redacted] (32) [redacted]

[redacted]

(33) [redacted] (34) [redacted] (35) [redacted] (36) [redacted]

[redacted]

(37) [redacted] (38) [redacted] (39) [redacted] (40) [redacted]

[redacted]

(41) [redacted] (42) [redacted] (43) [redacted] (44) [redacted]

[redacted]

(45) [redacted] (46) [redacted] (47) [redacted] (48) [redacted]

Figure 3.2 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita

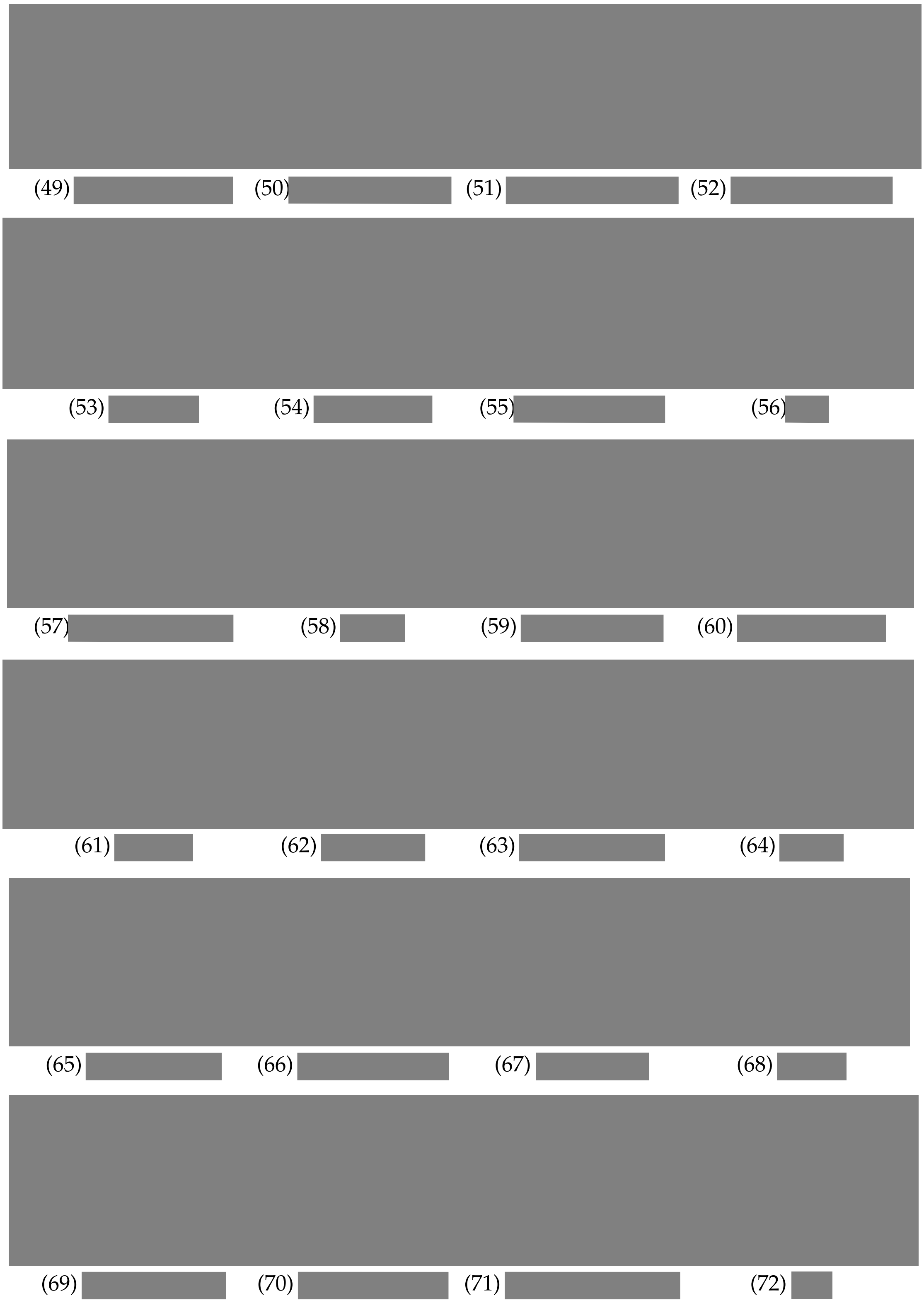


Figure 3.2 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita

[redacted]

(73) [redacted] (74) [redacted] (75) [redacted] (76) [redacted]

[redacted]

(77) [redacted] (78) [redacted] (79) [redacted] (80) [redacted]

[redacted]

(81) [redacted] (82) [redacted] (83) [redacted] (84) [redacted]

[redacted]

(85) [redacted] (86) [redacted] (87) [redacted] (88) [redacted]

[redacted]

(89) [redacted] (90) [redacted] (91) [redacted] (92) [redacted]

[redacted]

(93) [redacted] (94) [redacted] (95) [redacted] (96) [redacted]

Figure 3.2 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita

(97) [redacted] (98) [redacted] (99) [redacted] (100) [redacted]

(101) [redacted] (102) [redacted] (103) [redacted] (104) [redacted]

(105) [redacted] (106) [redacted] (107) [redacted] (108) [redacted]

(109) [redacted] (110) [redacted] (111) [redacted] (112) [redacted]

(113) [redacted] (114) [redacted] (115) [redacted] (116) [redacted]

(117) [redacted] (118) [redacted] (119) [redacted] (120) [redacted]

Figure 3.2 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita

[redacted]

(121) [redacted] (122) [redacted] (123) [redacted] (124) [redacted]

[redacted]

(125) [redacted] (126) [redacted] (127) [redacted]

Figure 3.2 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita

To facilitate the test of our later hypotheses, it is better to quantify the observed patterns. However, it is not clear a priori how best to characterize these relationships. We therefore adopt a number of different approaches.

Approach 1 – a quadratic fit: we first use a quadratic function, i.e. a second-order polynomial to fit the data. The R-squared from the quadratic estimation is then chosen as the summary statistic of the degree to which each scatter plot is a 'spline' relationship. The disadvantage of this approach is that it leads to occasional spurious results. For example, as shown in panel (a) of Figure 3.3, in the case of [redacted]

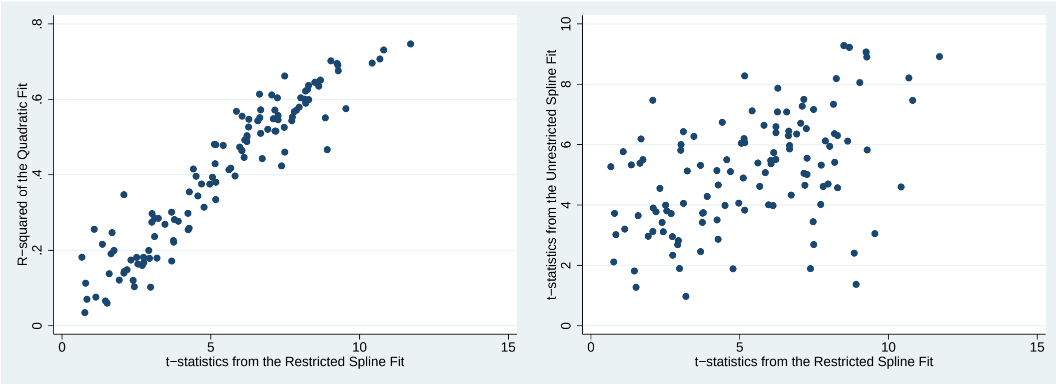
Approach 2 – an unrestricted spline: we then try an unrestricted spline, i.e. a piecewise linear function with two segments. The two segments are defined by intercept and slope parameters. Maximum likelihood estimation is used to determine the four parameters for each basic heading. The t-statistic of the slope coefficient of the second segment is used as the summary statistic for each scatter plot to indicate the degree of a 'spline' relationship. However, this second approach cannot overcome the shortcomings of Approach 1. For example, in panel (b) of Figure 3.3 the unrestricted spline again yields a downward-sloping part that carries a large standard error, and is probably spurious.

(a) Quadratic Fit (b) Unrestricted Spline Fit (c) Restricted Spline Fit

Figure 3.3: Comparison of the Three Approaches for the Case of [redacted]

Approach 3 – a restricted spline: to avoid the arguably spurious results in the previous two approaches, we modify the second approach by restricting the slope of the first segment to be zero. Now the spline is determined by three parameters: the vertical position of the segment on the left, the horizontal position of the intersection (break point) and the slope of the segment on the right. Maximum likelihood estimation is used to estimate the three parameters. The t-statistic of the slope coefficient is used to measure the degree of a 'spline' type relationship, as in the second approach.

We could also use other specifications to identify the spline relationship. However, the three approaches all work in the sense that the summary statistics do a good job in identifying the 'spline' relationship, which can be summarized by one robust measure. In the panel (a) of Figure 3.4 we plot the summary statistics from Approach 1 against those from Approach 3. In panel (b), the summary statistics from Approach 2 are plotted against those from Approach 3. Both two plots show a positive relationship, which implies that the results from all the three approaches are consistent. However, as Approach 3 avoids the probably spurious result of a falling segment, we use this as our preferred approach for the rest of the chapter.



(a) Approach 1 vs Approach 3 (b) Approach 2 vs Approach 3

Figure 3.4: Comparison of Summary Statistics of Three Approaches

We now proceed to list all products ordered by our spline measure in Table 3.1. All basic headings are classified into ND (non-durable), SD (semi-durable), S (service), IS (individual service), CS (collective service) and IG (investment goods) by nature of its output. The detailed methodology about this classification can be found in Organisation for Economic Co-operation and Development (2006).

The ranking of the slope coefficient is shown in Table 3.1. This gives the magnitude of the elasticity of the price level with respect to per capita income, which is a candidate measure of the spline relationship. On inspection, one can see that there is no positive or negative relationship between the two ranks. As the measures from the above three approaches are broadly consistent, the slope coefficient does not emerge as a robust measure of the spline relationship, and we prefer to use the t-statistic of the slope coefficient.

Table 3.1: The Ranking of Basic Headings by the Degree of Spline Relationship

Rank	Basic Heading Name	Classification	Slope Rank
1	[redacted]	S	1
2	[redacted]	D	16
3	[redacted]	D	6
4	[redacted]	ND	18
5	[redacted]	S	14
6	[redacted]	IG	5
7	[redacted]	ND	57
8	[redacted]	SD	4
9	[redacted]	ND	49
10	[redacted]	ND	89
11	[redacted]	ND	48
12	[redacted]	ND	52
13	[redacted]	ND	51
14	[redacted]	ND	37
15	[redacted]	ND	47
16	[redacted]	ND	62
17	[redacted]	D	25
18	[redacted]	D	71
19	[redacted]	ND	90
20	[redacted]	ND	103
21	[redacted]	ND	87
22	[redacted]	IG	22
23	[redacted]	ND	79

24	D	10
25	SD	12
26	ND	126
27	S	99
28	ND	15
29	S	97
30	ND	35
31	D	98
32	SD	45
33	D	7
34	ND	43
35	ND	58
36	S	123
37	ND	121
38	D	17
39	ND	110
40	ND	120
41	ND	44
42	D	8
43	ND	9
44	ND	73
45	ND	88
46	ND	46

47		ND	122
48		ND	23
49		D	13
50		S	113
51		S	114
52		S	80
53		ND	63
54		S	101
55		S	27
56		ND	108
57		S	129
58		SD	86
59		S	84
60		S	112
61		IG	21
62		S	128
63		S	95
64		D	66
65		IG	42
66		S	111

67	S	72
68	S	77
69	CS	38
70	IS	39
71	IS	40
72	ND	118
73	ND	65
74	S	107
75	ND	67
76	S	60
77	ND	119
78	S	85
79	S	116
80	ND	100
81	S	83
82	S	115
83	SD	53
84	S	117
85	SD	19
86	IG	78
87	S	109
88	ND	69
89	ND	96

90	ND	30
91	SD	36
92	S	91
93	S	93
94	IG	64
95	SD	11
96	D	26
97	S	94
98	SD	29
99	SD	31
100	S	33
101	S	61
102		34
103	ND	50
104	S	102
105	ND	127
106	IS	76
107	CS	32
108	S	92
109	IS	68
110	ND	56
111	IS	74
112	IS	75
113	S	124

114		IS	82
115		IS	81
116		S	28
117		ND	20
118		S	125
119		CS	105
120		CS	104
121		IS	54
122		S	41
123		SD	24
124		S	59
125		CS	106
126		S	55
127		IS	70

An examination of this list suggests a pattern: manufactured products lie towards the top of the list, while pure services are at the bottom of the list. For example, the basic headings of services, including S (service), IS (individual service) and CS (collective service), mostly appear in the latter half of the table. Especially they dominate the bottom of it. Among the last 25 basic headings, there are only five that are not services. In other words, the basic headings of services tend to display a 'spline' pattern in their B-S Price Wealth relationships. This suggests looking for a further breakdown of all basic headings by our spline measure and by nature of its output (services or non-services) to examine the importance of this mechanism.

To do so, we first discretize the B-S price wealth relationship by choosing a threshold for the measure of the spline relationship, i.e. the t-statistic of the slope coefficient: all the basic headings with a t-statistic less than 2.5 belong to type I, such as jam, while all those with a t-statistic greater than or equal to 2.5 belong to type II. Then, depending on whether the output is services or non-services, and on whether its price-wealth relationship is Type I or Type II, we allocate all the basic headings into a 2×2 table. This is shown in Figure 3.5.

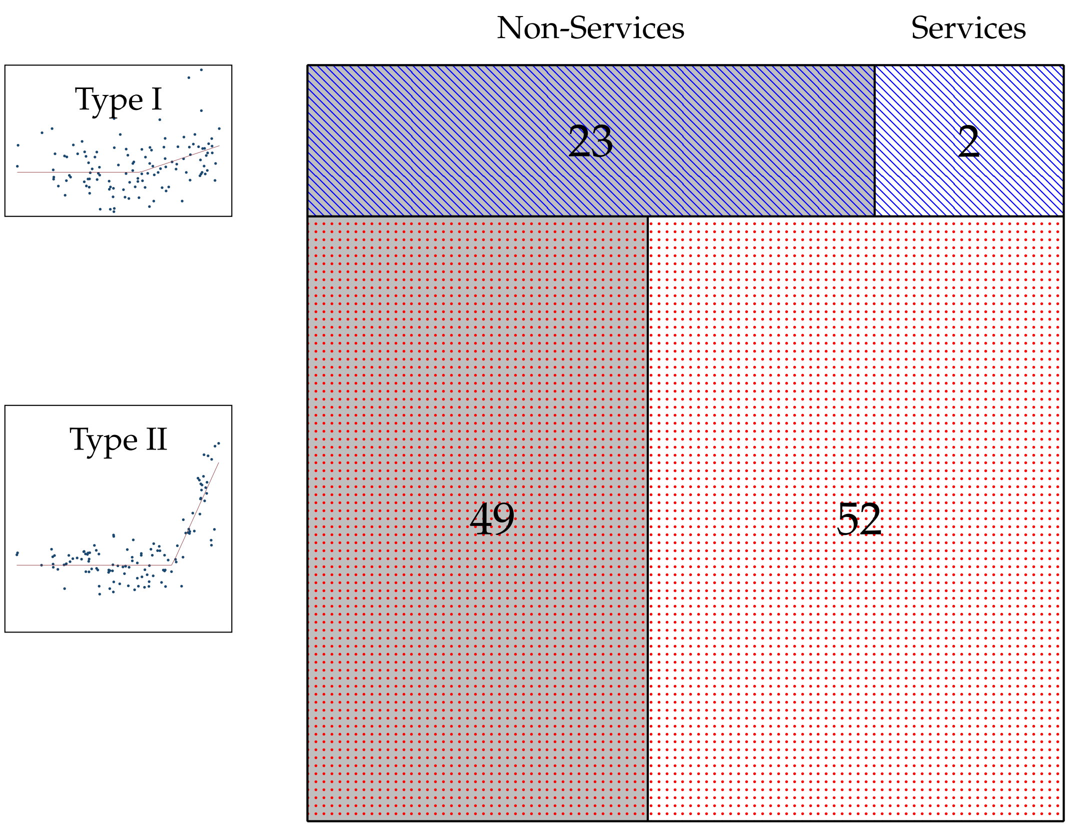


Figure 3.5: The Distribution of Basic Headings by Nature of Output (Services or Non-Services) and Price-Wealth Relationship

Notes: The basic headings with a price-income relationship of type I are represented by rectangles filled with north east lines. Those with a type II relationship are represented by those filled with dots. The nature of output of basic headings is indicated by the background color: services are marked by a white colour while non-services are marked by a gray one.

On inspection, we find few services basic headings displaying a Type I pattern except for the two in the top right corner. The top left rectangle contains 23 non-services basic headings, the price levels of which do not vary much with GDP per capita. This can be seen as empirical support for the Purchasing Power Parity (PPP) proposition: as most of non-services products are tradable, international arbitrage of the tradable non-services products can eliminate any price differentials across countries. However, the PPP proposition only holds for a small fraction of all basic headings. There are overall 101 basic headings in the bottom two rectangles with their price wealth relationship displaying the spline shape, which cannot be explained by the traditional theories of the national price level. In particular, 49 of them are non-services. This is in sharp contrast to the PPP proposition. These empirical results suggest a theoretical strategy: it may be best to proceed by looking for separate theoretical foundations for the (spline) relationship for non-services and for the (spline) relationship for services. We will pursue this approach in the next two chapters.

Chapter 4

Analysing S-group Products

4.1 Introduction

In Chapter 3, we have studied the Balassa-Samuelson Price Wealth relationship, i.e. the relationship between the national price level and GDP per capita, at the basic heading level. Plotting the price level against GDP per capita[1], we found that the relationships vary substantially across basic headings. But we can identify two archetypes, denoted by type I and type II. Type I displays a weak and disperse relationship between the price level and GDP per capita. Type II exhibits a highly nonlinear pattern: within low- and middle- income countries, the B-S Price Wealth relationship is weakly positive, while the slope of the positive relationship suddenly increases when we move to high-income countries. The substantial variations in B-S Price Wealth relationship across products and countries suggest that the traditional explanation of national price levels, the Balassa-Samuelson hypothesis among others, is not able to provide satisfactory answers to national price differentials at both aggregate and disaggregate levels. To seek other explanations, we investigated how the various B-S Price Wealth relationships are related to the characteristics of each basic heading. Our strategy is to quantify the B-S Price Wealth relationship by measuring the degree of belongingness (membership) of each basic heading to these two archetypes. The steps to compute this measure are given as follow: we first adopted a restricted spline function, which can nest the two archetypes, to fit the scatter

[1] The logarithm of GDP per capita

plots of the B-S Price Wealth relationship. More specifically, it is a piecewise linear function with two segments. The segment on the left hand side is horizontal and the one on the right hand side is upward-sloping. The t-statistics of the slope coefficient of the upward-sloping segment is then used as the measure of a basic heading's membership of type II.

As shown in Table 3.1, we rank basic headings by their type II memberships in an ascending order. All basic headings are classified into ND (non-durable), SD (semi-durable), S (service), IS (individual service), CS (collective service) and IG (investment goods) by nature of its output. The detailed methodology about this classification can be found in Organisation for Economic Co-operation and Development (2006). Going through the long table suggests that special attentions are needed for services, as their basic headings are not randomly distributed in the table. The basic headings of services, including S (service), IS (individual service) and CS (collective service), mostly appear in the latter half of the table. Especially they dominate the bottom of it. For example, among the last 25 basic headings, there are only five that are not services. In other words, the basic headings of services tend to display a 'spline' pattern in their B-S Price Wealth relationships. This implies that the 'spline' relationship may be caused by some unique features of services and we need to further explore in this direction.

This suggests that it might be appropriate to think in terms of modelling the 'service' group (designated the 'S-group') separately from the manufactures group (designated the 'M-group').

Enlightened by the high likelihood of a 'spline' relationship among the basic headings of the S-group products, in this chapter we focus on explaining this phenomenon. As the B-S Price Wealth relationship of the S-group basic heading is a result of its service nature and other idiosyncratic factors. Therefore, in order to identify the common statistical property of the

B-S Price Wealth relationship of the S-group products and cancel out the idiosyncrasy of each basic heading, instead of working at the basic heading level as in Chapter 3, we will work at a more aggregate level: the S-groups and the M-group basic headings. We will construct two aggregate price indexes for the two groups and study the B-S Price Wealth relationship at the group level. By this method, not only can we easily extract the common statistical properties of the two groups, we can also contrast their statistical properties and infer the distinctive features of each group. As it will become clear below, the above strategy greatly facilitates us in finding the empirical facts and proposing hypotheses accordingly.

In this chapter, our focus will be the basic headings of the S-group. As the nontradable basic headings consist of only the basic headings of services and construction, we will be literally looking into the rectangle in the bottom right corner in Figure 3.5, the largest one in the table, which contains 52 services basic headings displaying the 'spline' pattern in their B-S Price Wealth relationships. However, there is a distinctive feature about services that distinguishes them from other products with low tradability, which is usually caused by the high transportation cost relative to the unit value: labour is the major input for producing services and hence local wages play a crucial role in determining its price level. This feature turns out to play an important role in explaining the price level of the S-group products.

The rest of the chapter is organized as follows: Section 2 provides the empirical evidence about how the aggregate price levels of the S-group products and the M-group products change with GDP per capita and shows that their B-S Price Wealth relationship are statistically different. Section 3 develops a general model to investigate the theoretical possibilities of how the service wage changes with the average wage or GDP per capita, as the service wage is a crucial determinant of the price level of services. In addition, sectoral wage data are used to test the model's predictions. Section 4 summarizes the main findings of the chapter and concludes.

4.2 *The Aggregate Price Levels of the S-group products and the M-group products*

To set the stage for the theoretical analysis in later sections, this section provides the empirical evidence on the B-S Price Wealth relationship of the S-group products. As the data on the price level are available only at the basic heading level and our focus is the common property of the B-S Price Wealth relationship among the S-group basic headings, the different levels of aggregation in the data and in our goal require us to first construct the aggregate price index of the S-group products. We therefore divide all the basic headings into two groups: the S-group and the M-group. Then two aggregate price indexes are constructed for the two groups using the EKS method, a standard aggregation methodology in the common practice of international price comparisons. The two aggregate price indexes are used to investigate the B-S Price Wealth relationship of the S-group products and the M-group products. The advantage of studying the price levels at the group level is that it can cancel out the effects of each basic heading's idiosyncratic characteristics and make it easier for us to identify the common statistical properties of each group. In addition, this method can also contrast the two aggregate price indexes and identify how the special features of the S-group products distinguish its B-S Price Wealth relationship from that of the M-group products.

Figure 4.1 and Figure 4.2 are the scatter plots of the two price indexes against the log of GDP per capita. As the B-S Price Wealth relationships of service basic headings usually display the nonlinear 'spline' pattern in

Chapter 3, it is no wonder that the aggregate price wealth relationship of the S-group products also displays a similar nonlinear relationship: the relationship is slightly positive within poor- and middle- income countries while there is a sudden increase in the slope of the positive relationship among rich countries. However, in Figure 4.2 the aggregate price wealth relationship of the M-group products displays a different pattern: for a group of very rich countries the relationship is significantly positive, while for other countries, the log of a country's GDP per capita has little predictive power for its M-group price level. In other words, even for countries with very similar levels of GDP per capita there are still huge variations in their price levels of the M-group products. In terms of the magnitude of price dispersion, the S-group products and the M-group products are quite similar.

Further regression analysis also shows that the B-S Price Wealth relationships of the two groups are significantly different. The overall explanatory power of GDP per capita for the price level of the S-group products is much higher than for that of the M-group products. Regressing the price level of the S-group products on the log of GDP per capita generates a R^2 of 0.553, while using the price level of the M-group products yields a R^2 of 0.159. This result is robust to alternative specifications. For example, regressing the log of the price level of the S-group products on the log of GDP per capita generates a R^2 of 0.593, while the R^2 is only 0.120 for the case of the M-group products.

Therefore, we can see that contrasting the two aggregate B-S Price Wealth relationships enables us to identify the common statistical property shared by all the basic headings of the S-group products, which is not possible if we study the relationships at the basic heading level. In addition, the different patterns observed in the B-S Price Wealth relationships of services and non-services require us to provide different explanations for the two sectors. We start with services in the next section.

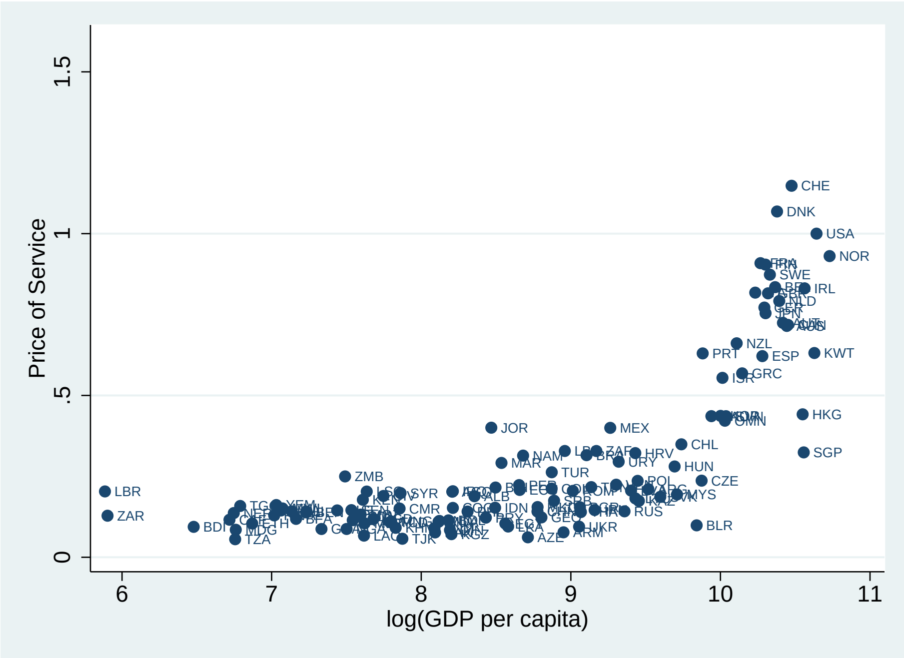


Figure 4.1: Price of S-group vs log(GDP per capita)

4.3 *The Service Sector*

To explain the common statistical property of the B-S Price Wealth relationship shared by the basic headings of services, we have to begin with the fundamental features of services. In the context of our research, we will focus on two key ideas:

(a) Many services, and especially personal services, involve labour as the dominant input, and local labour cost may account for almost all of total cost. Imported inputs such as raw materials, machines and equipment play a relatively very small role in producing services. For example, the price of 'hairdressing services' is determined almost wholly by the wage level of hairdressers. In other words, these products correspond to a polar case in which the price level rises in direct proportion to GDP per capita.

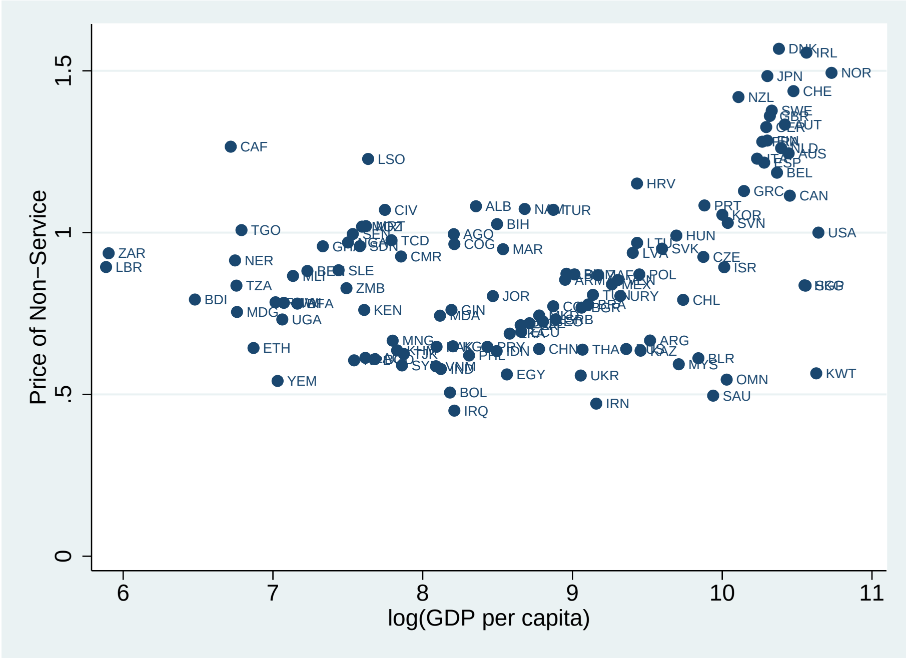


Figure 4.2: Price of M-group products vs log(GDP per capita)

This is also true for the prices of non-market services, i.e. those services that are not sold on markets and are predominantly provided by governments. These include collective government consumption (such as police, defense, fire-fighting and general administration), health and education. As market prices are not available, statisticians working on international comparisons have resorted to the use of inputs into the production of non-market services as proxies for output. Since the input costs are mainly made up of the wage costs of the employees involved in producing services, the wage rate in the service sector plays a decisive role in determining its price level. If this is true, then in Figure 4.1 we have in effect plotted the wage in the service sector against GDP per capita, which is closely linked with the average wage. In fact, the conventional semi-log form of plot, in which we plot the absolute price level against the log of GDP per capita, is not the natural choice of specification. It is appropriate for these industries, to use logs either on both axes, or on neither. Taking a semi-log specification introduces a distortion, so that a plot corresponding to a ray through the origin becomes a convex curve on the semi-log diagram. In other words, the 'spline' relation might be just be a spurious effect arising from the inappropriate specification. We return to this point later.

(b) There is, however, a complicating factor that might in principle modify this 'average wage versus GDP per capita' interpretation in a fundamental way. In terms of the 'hairdresser' example, the question is whether hairdressers occupy the same position in a country's occupational wage distribution as we go from poor countries to rich countries. More generally, the wage in the service sector may not be proportional to the overall average wage in the economy, as wage rates vary across sectors. For example, the employees in the manufacturing sector usually enjoy the highest wages as their highly skilled labour can be further augmented by the advanced production technology adopted in the sector. While in the agricultural sector, the least skilled labour combined with the least technology entails the lowest wage. The service sector is somewhere between the above two cases, so its wage rate is generally higher than the agricultural wage but lower than the manufacturing wage. In addition to the sectoral wage difference, the employment shares of the above three sectors also depend on each country's level of development. For example, Kuznets (1966) and Maddison (1980) documented that the agricultural employment share tends to decline and the manufacturing share and the service share tend to rise as a country develops. Therefore, the way in which the service wage changes with the average wage or GDP per capita is a matter of importance. Given the tight link between the service wage and its price level, the relationship between the service wage and the average wage may be reflected in the scatter plot in Figure 4.1 and could lead to a 'spline' pattern. Therefore, we need to incorporate the two facts into a theoretical model to analyze their potential effects on the B-S Price Wealth relationship.

4.3.1 *A Distributional Bias Hypothesis*

Due to the different wages across sectors, the service wage is in general not proportional to the average wage. To understand how the service wage is related to the average wage, we incorporate the facts that wages vary across sectors and sectoral employment shares depend on a country's level of development into a theoretical model. The features of the model are the heterogeneity in individual's ability and sectoral productivity and using the manufacturing employment share as an exogenous proxy for the level of development.

The economy consists of a continuum of agents indexed by its ability θ, which is assumed to follow a uniform distribution between 0 and 1, i.e., its pdf is given by $f(\theta) = 1, 0 \leq \theta \leq 1$. There are three sectors in the economy: agriculture, manufacturing and service, which all adopt linear production technologies. One unit of labour input with ability θ yields $\sigma\theta$, $\sigma^2\theta$ and $\sigma^3\theta$ units of output in the agriculture, service and manufacturing sectors respectively, where $\sigma > 1$. It is assumed that there is a perfect competition in labour markets, so wages are equal to the marginal product of labour in the three sectors.

In this chapter, we focus on only the efficient outcomes by assuming that there is a perfect sorting in allocating individuals with different abilities to the three sectors: a fraction μ of total population with the highest ability will be working in the manufacturing sector; individuals at the lower end of the ability distribution will take the employment in the agriculture sector and the rest will go to the service sector. μ is assumed to be exogenous. In addition, the number of services each individual needs to consume in a fixed period of time, say one year, is assumed to be one. Moreover, we assume each employee in the service sector can provide k services in a year. Hence, the employment share of the service sector is also exogenously determined, which is equal to $\frac{1}{k}$. The ability window for the three sectors is shown in Figure 4.3.

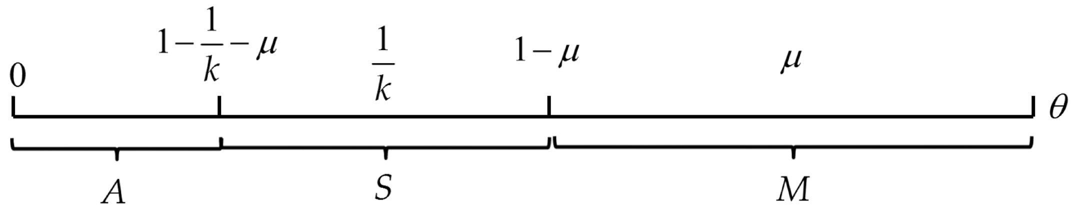


Figure 4.3: Locating the Three Sectors on the Ability Distribution

We can now compute the average wages in the three sectors. Firstly, the typical wage in the agriculture sector is

$$W_A = \sigma\theta,\ \theta \in [0, 1 - \mu - \frac{1}{k}]$$

Hence, given the uniform distribution of ability, the average wage is equal to

$$\bar{W}_A = \frac{1 - \mu - \frac{1}{k}}{2}\sigma$$

Secondly, the wage in the service sector is

$$W_S = \sigma^2\theta,\ \theta \in [1 - \mu - \frac{1}{k}, 1 - \mu]$$

and hence the average wage in this sector is

$$\bar{W}_S = \frac{1 - \mu - \frac{1}{k} + 1 - \mu}{2}\sigma^2$$

Similarly, we can obtain the typical wage and the average wage in the manufacturing sector respectively:

$$W_M = \sigma^3\theta,\ \theta \in [1 - \mu, 1]$$

$$\bar{W}_M = \frac{1+(1-\mu)}{2}\sigma^3$$

Given the above wages in the three sectors, we can compute the average wage in the whole economy, which is equal to the weighted-average of the wages in the three sectors. The weights are given by employment shares.

$$\begin{aligned}\bar{W}_{All} &= \bar{W}_A(1-\mu-\frac{1}{k}) + \bar{W}_S(\frac{1}{k}) + \bar{W}_M\mu \\ &= \frac{(1-\mu-\frac{1}{k})^2}{2}\sigma + \frac{2-2\mu-\frac{1}{k}}{2}\frac{1}{k}\sigma^2 + \frac{2-\mu}{2}\mu\sigma^3\end{aligned}$$

After obtaining the wages, we can now plot the average wages and the ratios of the sectoral wage to the overall average wage against the exogenous parameter μ in Figure 4.4 and 4.5 respectively to study how these wages or wage ratios change with level of development. We assume $k = 2$ and $\sigma = 1.5$ in the two figures.

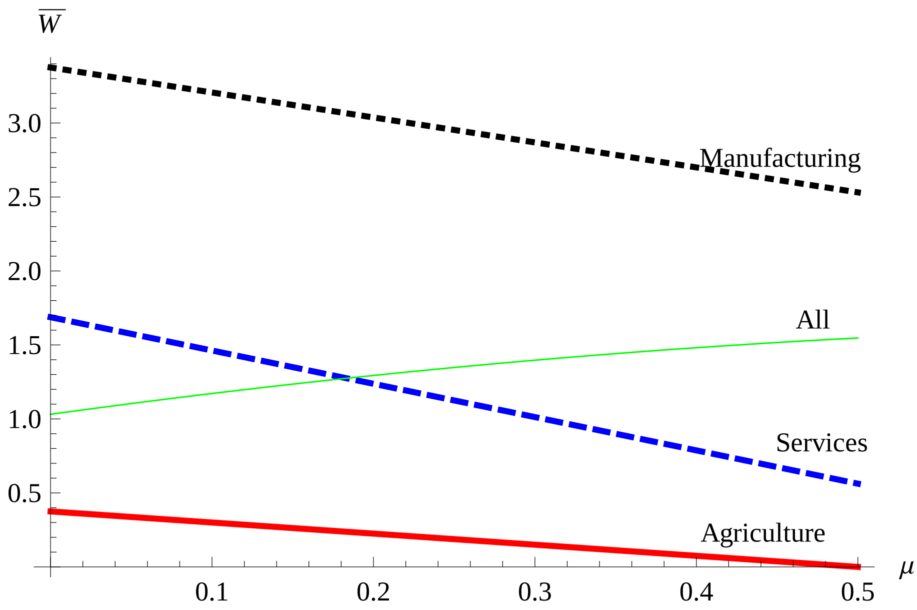


Figure 4.4: Average Wages

In Figure 4.4, as μ increases from 0 to $\frac{1}{k}$, more and more individuals with lower abilities enter the manufacturing sector and dilute the average ability in the sector. Therefore, the average wage in the sector is decreasing in μ.

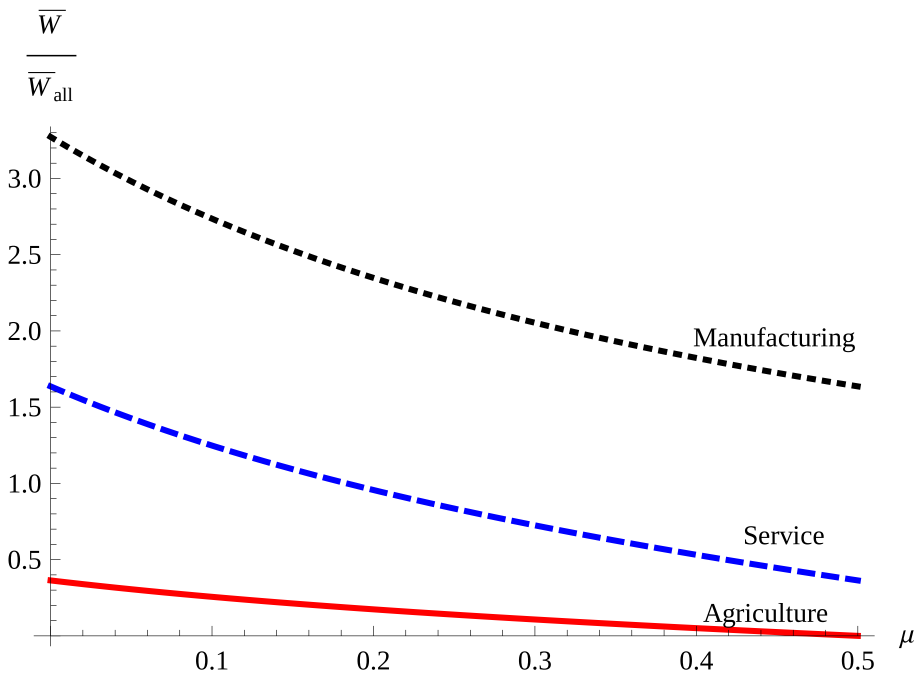


Figure 4.5: Wage Ratios

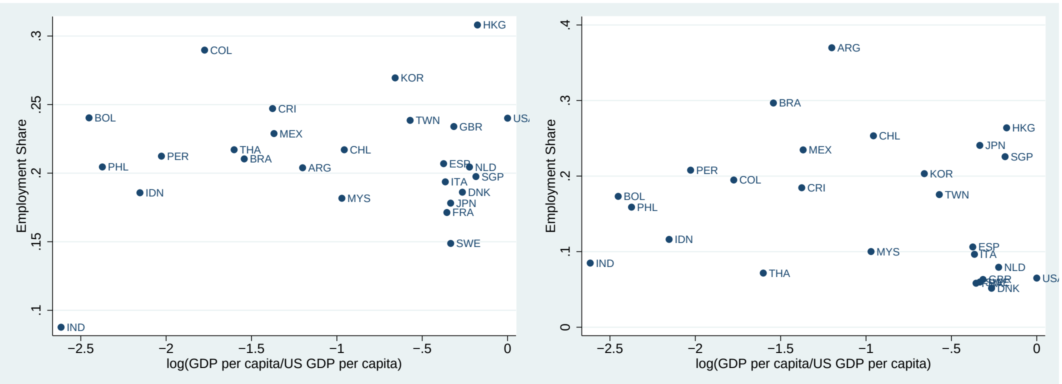
In addition, the increase in μ pushes the ability windows of the agriculture sector and service sector to the lower end of the ability distribution. Hence, the average wages in the two sectors are also decreasing in μ. Although all the sectoral wages are decreasing in μ, given that the wages in the manufacturing sector is on average higher than those in the other two sectors, an increase in μ, i.e. a higher employment share of manufacturing sector, can offset the decreasing sectoral wages and increase the overall average wage in the economy. Since the sectoral wages are downward sloping and the average wage is upward sloping in Figure 4.4, the ratios of the sectoral wage to the average wage must be decreasing in μ, as shown in Figure 4.5. However, the slope of the service wage ratio curve depends on the value of σ, a parameter controlling the productivity differences across the three sectors. The higher the value of σ, the larger the discrepancies there will be in the sectoral wages. As a result, the slope of the service wage ratio curve will be more negative. On the contrary, given a very low value of σ, the slope of the service wage ratio will become very flat.

Therefore, as the model predicts, if we allow for different sectoral wages and the dependence of sectoral employment shares on a country's level of development, the ratio of the service wage to the average wage will depend on productivity difference between the three sectors. Given very small productivity difference, the service wage will be nearly proportional to the average wage. We will test these predictions of the theoretical model in the next subsection.

4.3.2 Sectoral Wages and Employment Shares: Empirical Evidence

To test the hypothesis proposed in Section 4.3.1, we use the 10-Sector Database from the International Comparisons of Output and Productivity by Industry (ICOP), which collects the data on the value added and employment of 10 sectors for about 30 countries in Asia, East and West Europe, and North and South America. Although the database does not contain direct measures of sectoral wages, we divide the total value added of each sector by its employment to get a proxy for the sectoral wage. The year 2003 is chosen as our sample year as the data are only available for a small number of countries after that year.

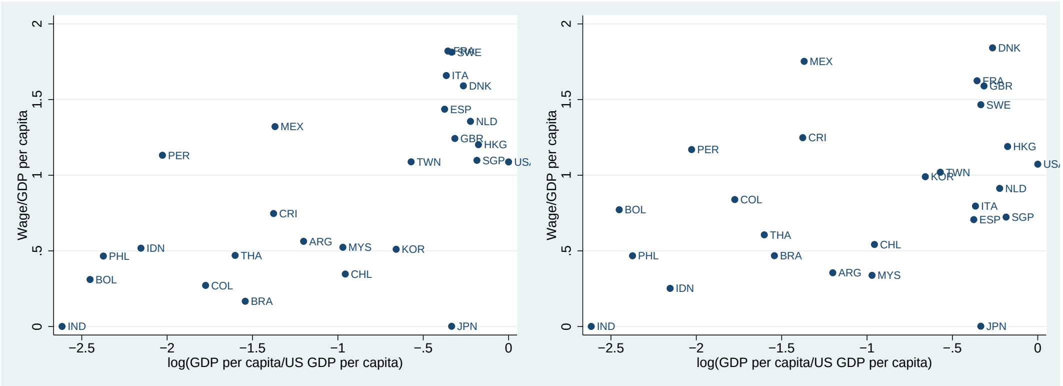
In order to find the wages in producing services, we need to identify whether the output of the 10 sectors are services or non-services. However, the classification of the 10 sectors is a classification of productive activity but not a classification of goods and services. It is based on the International Standard Industrial Classification of All Economic Activities (ISIC) Rev. 2. In ISIC Rev.2, service sectors are defined as the following four sectors among the 10: Wholesale and retail trade and restaurants and hotels; Transport, storage and communication; Financial, insurance and real estate and business services; Community, social and personal services. Although these sectors are classified into the service sector, some of their outputs have characteristics of goods. For example, the product of the sector of Transport, storage and communication may be classified either as goods or services depending on the medium by which these output are supplied. For instance, on the one hand transportation services provide services to the general public, while on the other hand, the infrastructure of transportation is often considered as manufactured goods. The same rule also applied to the sector of Financial, insurance, real estate and business services. As the outputs of these two sectors are mixtures of goods and services, we will focus on the two uncontroversial service sectors to test our hypothesis. One is Wholesale and Retail Trade, Hotels and Restaurants. The other is Community, Social and Personal Services.



(a) Wholesale and Retail Trade, Hotels and Restaurants

(b) Community, Social and Personal Services

Figure 4.6: Employment Shares of Service Sectors



(a) Wholesale and Retail Trade, Hotels and Restaurants (b) Community, Social and Personal Services

Figure 4.7: Ratios of Service Wage to GDP per capita

We first plot the employment shares of the two sector against the log of GDP per capita in Figure 4.6. On inspection, we can find that there are no relationship between the employment share and GDP per capita for the two sectors. This is consistent with our assumption in the model that the service employment share is exogenously given and does not vary with GDP per capita. Figure 4.7 are the scatter plots of the sectoral wage relative to GDP per capita against the log of GDP per capita for the two service sectors. This can be considered as a direct test of the service wage ratio curve in Figure 4.5. Both the two scatter plots display a weakly positive relationship, which suggests that the distributional bias predicted in the model, i.e. the service wage does not change proportionally with the average wage, is not quantitatively important in the data. Therefore, the hypothesis that the service wage is proportional to the average wage holds well in reality. If this is the case, plotting the price level of services against the log of GDP per capita in Figure 4.1 should display an exponential relationship. This is confirmed in Figure 4.8, where the log of the price level of service is plotted against the log of GDP per capita. The linear fit in the figure suggests the relationship between the log of the price level and the log of GDP per capita is well captured by a linear function. In other words, there is a constant elasticity of the price level of services with respect to GDP per capita. This suggests that the previous figures aiming at showing the price wealth relationship of services may be subject to misspecification errors. If the service wage is indeed proportional to the average wage or GDP per capita, plotting the price level of services against the log of GDP per capita will generate an exponential curve, which usually has a J-like shape. This could explain why we found many nonlinear 'spline' shapes in Chapter 3 in the basic headings of services. As shown in Figure 4.8, a logarithm specification generates a very good fit, which suggests a constant elasticity of the price level with respect to GDP per capita. As a result, the nonlinear 'spline' effect disappears in Figure 4.8. In addition, the point estimate of the elasticity is 0.489 with a standard error of 0.037. Therefore, the hypothesis that the price level of services changes proportionally with GDP per capita should be rejected, although the service wage changes proportionally with GDP per capita as the wage data suggested. One candidate explanation for this could be that the costs of other inputs in the service sector do not change much with GDP per capita, such as the highly tradable raw materials.

Therefore, we now know that the 'spline' pattern observed in the B-S Price Wealth relationship of services is primarily due to the specification we adopted in the scatter plot of Figure 4.1. Given the large elasticity of the price level of services with respect to GDP per capita as a result of the tight link between the price level and wage, the scatter plot in the Figure 4.1 should have been fitted by an exponential curve instead of a restricted spline function. This is confirmed in Figure 4.8 as a linear fitting can well capture the relationship between the log of the price level of services and the log of GDP per capita.

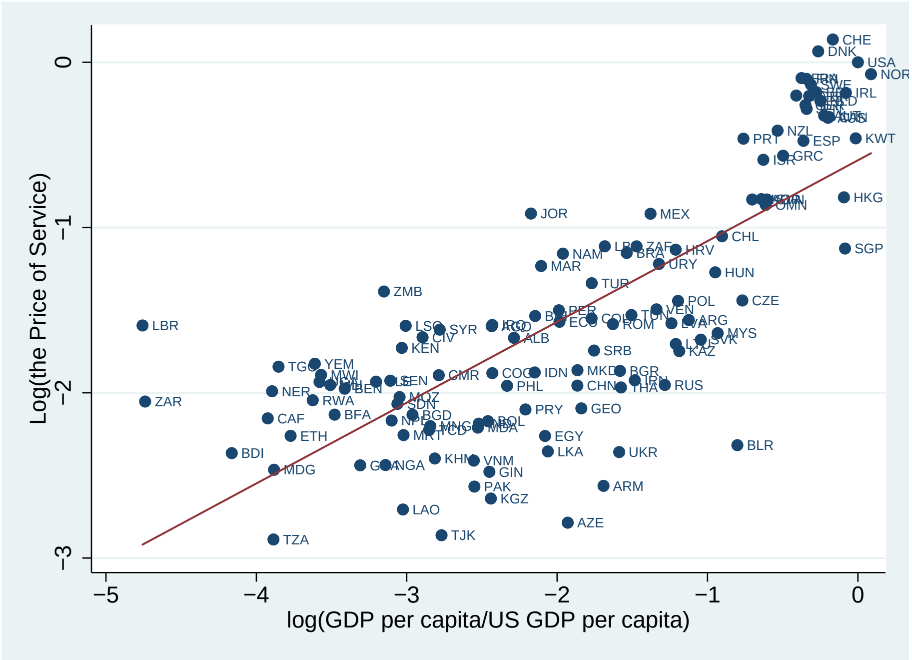


Figure 4.8: Log of Price of S-group products vs log of GDP per capita

4.4 Conclusion

As the spline pattern observed in the B-S Price Wealth relationship in Chapter 3 appears a lot of times in the basic headings of services, it suggests that there is a need to study them as a group.

To explain the empirical pattern for S-group products, we have built a theoretical model to analyze how the service wage changes with the average wage, which is closely linked with GDP per capita. We then use the data on sectoral wages and employment shares in the 10-Sector Database of ICOP to test the model. The empirical evidence suggests a constant elasticity of the price level of the S-group products with respect to GDP per capita. In other words, there is no significant bias arising from the position of "hairdressers" in the national wage distribution. It follows that the scatter plot of the price level of the S-group products against the log of GDP per capita should display an exponential form. This explains why we find a nonlinear 'spline' relationship in Figure 4.1. Moreover, this can be confirmed in Figure 4.8: plotting the log of the price level against the log of GDP per capita eliminates the nonlinearity.

However, the above explanation does not apply to M-group products. Plotting the log of the price level of M-group products against the log of GDP per capita does not eliminate the nonlinearity found in Figure 4.2. This suggests a different explanation for the price level of the M-group products. We return to this in the next chapter.

Chapter 5

Analysing M-group Products

5.1 Introduction

Given the argument of the preceding chapter, that the S-group products demand a different type of model, it is natural to return to the analysis of Chapter 2, based on the mismeasured quality model, and to re-estimate the results, comparing the (already reported) results for the full set of indexes with the results for a modified price index based on M-group products only.

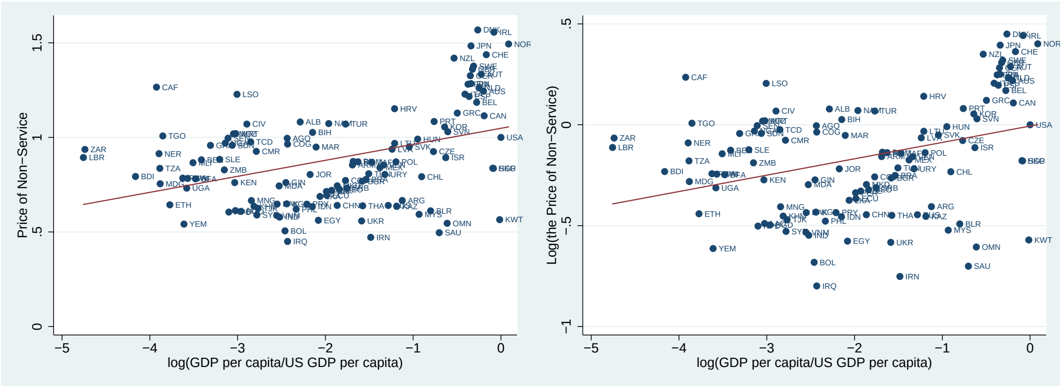
The model in Chapter 2 assumes the consumption bundle is made up of two types of goods, tradable homogeneous goods x and nontradable vertically differentiated goods z. It is assumed that every country has local firms to product the second type of z goods, whose prices are affected by the local distribution of income. This establishes a distinction between the goods that are priced internationally and the goods that are priced locally. The prices of internationally priced goods tend to be equalized across countries, while the prices of locally priced goods are affected by the local distribution of income.

In general, nontradable goods are priced locally. Some tradable goods are priced internationally and some are priced locally. In Chapter 2, we sharpen the contrast between the locally priced nontradable goods and the internationally priced tradable goods by abstracting from the locally priced tradable goods.

In Chapter 5, we make a contrast between the locally priced and internationally priced tradable goods. It would be of interest to extend the model to incorporate all the three types of goods in a multi-country model, so that the internationally priced tradable goods are produced by any country in the world, the locally priced nontradable goods are produced by a group of local firms, and the locally priced tradable goods are produced by a group of international firms.

5.2 *Some Preliminary Observations*

To analyse the M-group products, the scatter plot for the B-S price wealth relationship using an index based on M-group products only is shown in Panel (a) of Figure 5.1, where the price level of the M-group products is plotted against the log of GDP per capita. In Panel (b) of Figure 5.1, the log of the price level of the M-group products is plotted against the log of GDP per capita. Two properties are worth noting. The first property is nonlinearity. As shown in Figure 5.1, plotting the log of the price level of non-services against the log of GDP per capita in (b) cannot eliminate the nonlinearity in (a). There are still obvious variations in the price wealth relationship across countries: within low- and middle- income countries, the relationship is weakly positive; while within rich countries, the slope of the relationship is large and significant. The second property is dispersion. Regressing the price level of the M-group products on the log of GDP per capita generates an R^2 of 0.159. This can be compared to the R^2 for the S-group products, shown in Figure 4.1, of 0.553. The low value of R^2 is robust to alternative specifications: instead of using the semi-log specification, regressing the log of the price level of the M-group products on the log of GDP per capita generates an R^2 of 0.119, while we would get a much higher 0.594 in the case of the S-group products.



(a) Price of M-group Products vs log of GDP per capita

(b) Log of Price of M-group Products vs log of GDP per capita

Figure 5.1: Balassa-Samuelson Price Wealth Relationship for M-group Products

5.3 *Re-estimating the Relationship*

To formally show if income inequality can help explain the price level of M-group products, we replicate the regressing of the national price level on income distribution in Table 2.1 of Chapter 2, but now we do so for the price level of S-group products, M-group products and the national price level for the year of 2005. The year 2005 is chosen as the sample year because the disaggregate price levels used to construct the price indices of the S-group products and the M-group products are from the ICP Benchmark Dataset 2005.

All the estimation results are shown in Table 5.1. In the first three regressions, the relationships between the national price level and income distribution are consistent with what we have found in Chapter 2. The slope coefficient in Regression (1), i.e. the elasticity of the national price level with respect to per capita income is significantly positive. The point estimate is 0.354, which is of the same magnitude as in Chapter 2. Including the Gini index as an additional regressor in Regression (2) only changes the results slightly. However, in Regression (3), when we include an interaction term, defined as the product of the Gini index and per capita income, both the Gini index and the interaction term become significantly negative. As argued in Chapter 2, this is due to the fact that the national price level depends both on per capita income and income inequality, and on their product. The lack of significance of the Gini index in Regression (2) is due to misspecification errors. The preferred specification of Regression (3) raises R^2 from 0.465 in Regression (1) to 0.554.

In Regression (4)-(9), we run the same regressions as above but using the disaggregate price levels of S-group products and M-group products. This can help us identify the sources of the results in Regression (1)-(3). On inspection, we can find that all the above qualitative results in Regression (1)-(3) in terms of the significance of and the sign of estimated coefficients and the improvement in R^2 also hold for the case of S-group products and M-group products. In Regression (4) and (7), per capita income has significantly positive impact on both the two price levels. Including the Gini index and the interaction term as additional regressors makes all estimated coefficients in Regression (6) and Regression (9) significant. The additional explanatory powers of the latter two regressors also increase R^2.

However, the magnitude of the results varies substantially between S-group products and M-group products. Firstly, the elasticity of the price level of S-group products with respect to per capita income, i.e. the coefficients of $\log(Y_j/Y_{U.S.})$ in Regression (4)-(6), are much higher than those of M-group products in Regression (7)- (9). For example, in Regression (4) the estimated elasticity is 0.489, which is six times the estimate in Regression (7). This is consistent with the fact that labour is the most important input in producing services as well as the fact that the service wage is nearly proportional to the average wage or GDP per capita. For M-group products, labour input plays less important role, so its price level will be less sensitive to the average wage.

Secondly, the R^2 in Regression (4) is 0.594. In Regression (6), adding the Gini index and the interaction term only increases the R^2 by 0.007. This again shows the crucial role of GDP per capita in explaining the price level of S-group products. The Gini index can only provide a slight increase in explanatory power. However, a comparison of R^2s between Regression (7) and (9) shows a substantially different picture. In Regression (7), given the small slope coefficient, the R^2 is only 0.119, but including per capita income and the Gini index increases R^2 to 0.370 in Regression (9). This implies that, compared with per capita income, income inequality matters much more for the price level of M-group products than that of S-group products. In other words, the non-service component of the national price level is more sensitive to income inequality than the service component. As the M-group basic headings are mostly tradable, the above results suggest that tradable vertically differentiated goods are the more source through which income inequality influences the national price level.

Thus the estimation results in Table 5.1 suggest that income inequality is an important factor influencing the price level of M-group products. It can explain a large fraction of the variation in its price level that cannot be explained using per capita income only. The large improvement in R^2 that occurs when we introduce the income inequality variable can partially explain one property implied by Figure 5.1: the high level of dispersion in the B-S Price Wealth relationship may reflect a failure to take income inequality into account. To investigate this, we plot the residuals from Regression (7) and (9) against the log of GDP per capita in Figure 5.2. We can see that the nonlinearity implied by Figure 5.1 is obvious in the residual plot as shown in Panel (a) of Figure 5.2. Including the Gini index and the interaction term in Regression (9) can effectively eliminate the nonlinearity. As evident in

Table 5.1: Income Distribution and the National Price Level: S-group products and M-group products

	Aggregate: $\log(P_j/P_{U.S.})$			S-group: $\log(P_{j,S}/P_{U.S.,S})$			M-group: $\log(P_{j,NS}/P_{U.S.,NS})$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\log(Y_j/Y_{U.S.})$	0.354***	0.361***	1.210***	0.489***	0.503***	1.464***	0.0811***	0.0886***	0.621***
	(0.0350)	(0.0390)	(0.188)	(0.0372)	(0.0416)	(0.199)	(0.0203)	(0.0207)	(0.0962)
Gini Index		0.0219	-3.96***		0.665	-3.84***		-0.298	-2.80***
		(0.544)	(1.00)		(0.581)	(1.06)		(0.289)	(0.511)
Gini Index $\times \log(Y_j/Y_{U.S.})$			-2.21***			-2.50***			-1.39***
			(0.482)			(0.509)			(0.246)
Constant	-0.152*	-0.149	1.293***	-0.593***	-0.834***	0.798**	-0.00628	0.144	1.049***
	(0.0787)	(0.205)	(0.366)	(0.0839)	(0.219)	(0.387)	(0.0458)	(0.109)	(0.187)
Observations	120	114	114	120	114	114	120	114	114
R^2	0.465	0.468	0.554	0.594	0.586	0.661	0.119	0.187	0.370

Standard errors in parentheses
$^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Panel (b), there is little nonlinearity left. The improvement in R^2 is evident from inspection of the scatters: the variance of the residuals shrinks.

Hence, the inclusion of the income inequality variable helps to resolve the puzzles in Figure 5.1. The interpretation of these empirical results is left for the next section.

(a) Residuals from Regression (7)

(b) Residuals from Regression (9)

Figure 5.2: Model Selection for the Price Level of M-group products

5.4 *Reconciliation of the New Results and Chapter 2*

Now our problems hinge on explaining two discrepancies between the model in Chapter 2 and empirical evidence in this chapter. On the one hand, income inequality can provide only a small amount of additional explanatory power for the price level of the S-group products on top of GDP per capita, while the model in Chapter 2 predicts that the impact of income inequality on the national price level should be mainly through its impact on the price level of nontradable vertically differentiated goods. On the other hand, income inequality is an important explanatory variable for the price level of the M-group products despite that the M-group products comprise mostly tradable goods, whose prices tend to be equalized across countries according the PPP proposition. We will reconcile these discrepancies in turn.

5.4.1 *Income Distribution and the Price Level of the S-group Products*

Firstly, the reconciliation of the discrepancy between the model and the empirical evidence in terms of the ability of income inequality in explaining the price level of the S-group products is related to the way the price level of services is measured in practice. Services, according to whether the market price is available, can be divided into market services and non-market services. The non-market services are predominantly provided by governments, which include collective government consumption (such as police, defence, fire-fighting and general government administration), health and education. Due to the fact that the bulk of these output is not sold on markets. So market prices are not available. And without prices, these outputs cannot be valued or compared satisfactorily. In the absence of measures of prices and output for these sectors, statisticians working on international comparisons - as well as national income accountants - have resorted to the use of inputs into the production of non-market services as proxies for output. Input costs are often available. The inputs include the sum of the wage costs of the employee involved in producing the services; the intermediate consumption of goods and services (materials used and rents, for example) and the services rendered by capital during the production process. As labour is the major input in producing non-market services, wage plays an essential role in determining the total cost or the price level reported by the International Comparison Program. Moreover, given the service wage changes proportionally with the average wage and the large share of non-market services in total services, GDP per capita should be a fairly good predictor for the input cost (or the price level) of services. The model in Chapter 2, however, examines the relationship between income distribution and the price level, but not the input costs, of nontradable vertically differentiated goods and predicts that income inequality should affect the price level of services. As the price level of services is largely unobservable, due to non-market services, the empirical result regarding the impact of income inequality on the price level of services in this chapter is not a direct test of the model in Chapter 2. It only tests how income inequality affects the input cost instead of the price level.

5.4.2 Income Distribution and the Price Level of the M-group Products

Secondly, to explain why income inequality has significant impacts on the price level of the M-group products, we will show below that the model's mechanism for nontradable vertically differentiated goods to be affected by income inequality in the model can also be applied to tradable vertically differentiated goods. Although these goods are tradable in the traditional sense, i.e. transportation cost is low compared with unit values, we cannot simply apply the PPP proposition and claim that their price levels must be equalized across countries. This is because once tradable goods are vertically differentiated, the way tradable goods are priced and the way their price level is compiled will be changed.

Once tradable goods become vertically differentiated or they can be produced at different levels of quality, a natural outcome will be each country's comparative advantage in producing a product with a certain level of quality: rich countries may be more dominant in the market of high quality products due to their advanced technology, while lower income countries may have a large market share in the low-end markets because of their cost advantage. The theoretical foundation and empirical supports was introduced in Sutton and Trefler (2011). The above specialization implies that these tradable vertically differentiated goods will be supplied by a group of producers, each of which is specialized to produce a product with a particular quality. Thanks to their comparative advantages, these producers can price discriminate against each country's market to the extent that the international price differential can make international arbitrage profitable even after paying trade costs. Hence, to demonstrate how these producers price the tradable vertically differentiated goods according to each country's demand condition, we need to introduce quality to a model of 'Pricing to Market', which was introduced by Krugman (1987) to describe the practice of price discrimination across countries when international arbitrage is difficult or impossible. Our method is, therefore, to apply hedonic price model to an international context. This is very similar to the model in Chapter 2, but now we only need to replace nontradable vertically differentiated goods in the model by tradable vertically differentiated goods. As has been explained above, once tradable goods are differentiated by quality, their price levels are not determined internationally but will be linked with the local income distribution.

After the price function is determined, how their prices are compiled into a price index is also an important issue. Quality as a complicating factor for the B-S Price Wealth relationship has been studied in the literature. As quality cannot be perfectly controlled in compiling price indexes, the price index is just a simple average of individual prices without eliminating the impact of quality. Therefore, a higher quality will show up as a higher price level. For example, Schott (2004) showed that even within a Harmonized System (HS) - 10 category, quality is still an important explanatory variable for the U.S. important price. Empirical supports for the role of quality in the B-S Price Wealth relationship appeared in Goldberg and Verboven (2001), Hummels and Skiba (2004), Hallak (2006), Choi et al. (2009) and Imbs et al.

(2010).

Given the above implications of vertical product differentiation, the next question is what mechanisms in the model cause the 'spline' pattern observed in the B-S Price Wealth relationship of the M-group products. As shown in Figure 5.1, using a linear fitted line, i.e. assuming a constant elasticity of price level with respect to GDP per capita, significantly underpredicts the price level of the M-group products for rich countries.

We can illustrate this by applying the model in Chapter 2 to the M-group products. Instead of assuming the consumption bundle is made up of both nontradable and tradable goods, it is now assumed that the consumption bundle includes only the M-group products, which consists of two types of goods: homogeneous goods and vertically differentiated goods, both of which are tradable. It is assumed that the consumption decision of each individual is to choose the quantity of the homogeneous goods and the quality of the vertically differentiated goods.

As the quality is constant for the homogeneous goods, there is no quality bias in its price index. However, the quality control problem in constructing the price level of vertically differentiated good is a practical issue that has yet been solved satisfactorily. As a result, quality can hardly be controlled in the price index and higher quality products imply higher prices. In addition, as implied by the standard hedonic price model in the literature such as in Rosen (1974) and Berry et al. (1995), the distribution of quality depends on the distribution of consumers' attributes, such as income distribution. Therefore, it means that in addition to per capita income, income inequality will matter for the aggregate price level of vertically differentiated goods by affecting its quality distribution. Similar to the model in Chapter 2, the new model will predict that a higher income inequality will imply a more convex price function of the vertically differentiated goods. Given a high enough elasticity of substitution between the homogeneous goods and the differentiated goods, consumers will respond to the change in income inequality by lowering the expenditure share on differentiated goods. As its quality cannot be controlled, the lower expenditure will be translating to a lower price level. With a constant price level of the homogeneous goods, the aggregate price level of the M-group products will be lower. Therefore, the low goodness-of-fit in Figure 5.1 is actually due to the failure in taking into account the impact of income inequality. As income inequality is usually lower in rich countries than the rest of the world and the coefficient of the Gini index is estimated to be negative in Regression (6), the misspecification of only using GDP per capita in fitting the price level of the M-group products will cause the underprediction for rich countries in Figure 5.1. Moreover, the aggregate price level of the M-group products is an weighted average of the price levels of homogeneous goods and vertically differentiated goods, so the product of the expenditure share and the price level of differentiated goods is a crucial component in the formula of the aggregate price level. The higher per capita income in rich countries, which implies a higher price level of differentiated goods, can interact with income inequality and magnify the negative impact of income inequality on the aggregate price level. This can be empirically supported by the significantly negative coefficient of the interaction term in Regression (6) of Table 5.1. In other words, the underprediction caused by failing to use income inequality is especially severe in rich countries. Hence, if we take into account the impact of income inequality, we can explain the spline pattern in Figure 5.1.

This can be confirmed by Figure 5.2, where we plot the residuals from Regression (4) and (6) against the log of GDP per capita. In panel (a), we can see that the linear model is not able to capture the nonlinearity between the log of the price level and the log of GDP per capita, as there is some heteroskedasticity showing up in the residuals. However, as shown in Panel (b) Regression (6) does a good job in eliminating the heteroskedasticity. This implies that the model implied by Regression (6) is a better model for the price level of the M-group products. The spline pattern we have found in Figure 5.1 is actually an outcome of missing an important explanatory variable.

5.5 Conclusion

Plotting the log of the price level of the M-group products against the log of GDP per capita is not able to eliminate the nonlinearity in the semi-log scatter plot. In addition, the B-S Price Wealth relationship is also disperse. These two puzzling properties suggest a different explanation other than the one for the case of the S-group products. We try to explain these puzzles by focusing on one type of deviations from the PPP due to the vertical product differentiation of tradable goods. The vertical product differentiation combined with comparative advantages in producing these goods implies that these goods will be supplied by a group of producers, each of which is specialized to produce a particular quality product. Due to their comparative advantage, they will adopt the 'Pricing to Market' practice to price discriminate against each country. To analyze how income distribution affects the price level through quality, we study the hedonic pricing model in an international context. We therefore revisit the model in Chapter 2, but only apply it for the price level of the M-group products. We find that the nonlinear pattern found in Figure 5.1 is mainly due to the fact that we fail to take into account the impact of income inequality. As empirically the income inequality has a negative impact on the aggregate price level of the M-group products, this means that the linear fitted line will underpredict the price level for rich countries, whose Gini indexes are in general lower than the rest of the world. This is why in Figure 5.1 a linear fit fails and the scatter plot displays a nonlinear ‘spline’ pattern.

Chapter 6

Macroeconomics Implications

6.1 Introduction

In Chapter 2, we have used empirical evidence and a theoretical model to show that if product quality cannot be perfectly controlled in the price level, income distribution, i.e. both per capita income and income inequality, matters for the national price level. However, in addition to international price comparisons, the quality control issue also applies to the price index within one country. Pakes (2003) shows how to use hedonics to ameliorate quality bias in price indexes if we can collect the complete dataset on products' characteristics, which is impossible in reality. Therefore, in practice hedonic quality adjustment is not widely adopted for the price indexes. For example, there is a very limited set of CPI items that utilize hedonic quality adjustment, which includes only clothes, major appliances, television and other video equipment. Hence, the mechanism of how income distribution affects the national price level is likely to be relevant within each country, i.e. the evolution of income distribution may influence the change of domestic price index or inflation.

In this chapter we first extend the static model to a dynamic one and solve the market equilibrium. Using the market equilibrium, we can obtain the theoretical prediction about how growth, income inequality and inflation co-evolve over time. Then we investigate the empirical implications of the dynamic model. First, we use China as an example to compare the actual price index and the price index corrected for the B-S effect in order to show

how the quality issue affects the price index within one country. Second, in an appendix of this chapter, we compare the predictions by the dynamic model, i.e. how income distribution and inflation co-evolve over time, with the empirical evidence from the US, the UK, Australia and Sweden.

6.2 The Dynamic Model

This section extends the static model in Chapter 2 to a dynamic one. Recall that the model in Chapter 2 is a hedonic pricing model, in which consumers and firms choose their optimal positions along an equilibrium price $p(z)$, where z is the characteristics of the product in question and $p(z)$ is determined by the interaction between suppliers and consumers of that product.

On the demand side, there is a unit mass of consumers indexed by individual income level c. The income distribution is assumed to be exogenous and follow a Pareto distribution. We also assume the consumer purchases exactly one unit of the quality good z and spends the rest of his/her income on a homogeneous good x.

On the supply side, there is a unit mass of firms producing vertically differentiated goods. The distribution of the firms is also assumed to be exogenous and follow the Pareto distribution.

To solve the market equilibrium, we first convert the income distribution to the distribution of quality demanded using the first order conditions of the consumer's problem. We then convert the firm distribution to the distribution of quality supplied using the first order conditions of the producer's problem. Finally, we use the market clearing conditions for the quality product to solve for the equilibrium price schedule $p(z)$.

In the dynamic model, however, the income distribution is not exogenously given anymore. The evolution of the income distribution depends on the initial income distribution and the evolution of each individual's optimal choice. On the other hand, although the initial firm distribution is exogenously given, the evolution of firm distribution is endogenously determined. Therefore, we cannot solve $p(z)$ in the dynamic model period by period as in the static model. The time path of $p(z)$ for every time period has to be solved simultaneously.

Firstly, we solve the individual optimization problem for a typical individual with initial choices of $x(0)$, $z(0)$ and an initial capital stock $k(0)$ taking the price function of the vertically differentiated goods, i.e. the time-varying paths of $b(t)$ and $d(t)$ in the price function $p(z(t), b(t), d(t)) = b(t)z(t)^{d(t)}$, as given. This is because the dynamic price function is jointly determined period by period, as in the static model, by the interactions between the producers and consumers of the quality product. As a result, the price function is out of the reach of each individual and has to be taken as given. Given the individual optimal path of $z(t)$ and the initial distribution of $z(0)$ across individuals. We can obtain, for each time t, the distribution of quantity demanded for quality z – $Q^d(z, t)$.

On the other hand, we solve the profit maximization problem of each individual producer and obtain the distribution of quantity supplied for quality z – $Q^s(z, t)$. Finally, as the market for quality goods z needs to clear in each period, we equalize $Q^d(z, t)$ and $Q^s(z, t)$ for all t and solve for the paths of $b(t)$ and $d(t)$. Using the paths of $b(t)$ and $d(t)$, we can obtain the evolution of other variables in the model, such as the distribution of $x(t)$ and $z(t)$.

6.2.1 The Consumer's Problem: An AK Model with Two Goods

In this subsection, the consumer's problem in the static model is extended to a dynamic framework. As in the static model, each individual chooses a vertically differentiated good by choosing its quality level z and spends the rest of his/her total consumption expenditure on a homogeneous good by choosing the quantity x. The price function of the vertically differentiated goods is taken as given for each individual as it is jointly determined by the interactions between all the producers and consumers. For the moment, we assume the path of the price function is characterized by the paths of $b(t)$ and $d(t)$. The income of each individual is the capital income generated by investing capital in an AK technology. This assumption is based on two considerations. This chapter focuses on how the evolution of consumption distribution affects the price function of vertically differentiated goods and hence the allocation of consumption expenditure between the two goods. Therefore, we try to keep the production side as simple as possible. In addition, as will be shown below, the simplification also enables us to obtain closed form solutions for consumption distribution: consumption distribution always follows the Pareto distribution. Therefore, given the paths of $b(t)$ and $d(t)$, $t \geq 0$, the optimization problem for an individual, whose initial consumption choices and capital are $x(0)$, $z(0)$ and $k(0)$ respectively, is:

$$\max_{x(t),z(t),k(t)} \int_0^\infty e^{-\rho t} \frac{[x(t)^\alpha z(t)^\beta]^{1-\phi}}{1-\phi} dt$$

$$s.t.\ k'(t) = Ak(t) - x(t) - b(t)z(t)^{d(t)}$$

where ρ is the time preference parameter and ϕ is the constant coefficient of relative risk aversion. Here the instantaneous utility function is a monotonic transformation of the Cobb-Douglas utility used in the static model. We use the transformed utility function to make the dynamic model comparable with the standard growth model with one good.

The Hamiltonian of the problem is

$$H = e^{-\rho t}\frac{[x(t)^{\alpha}z(t)^{\beta}]^{1-\phi}}{1-\phi} + \lambda(t)[Ak(t) - x(t) - b(t)z(t)^{d(t)}]$$

FOC with respect to $x(t)$ implies:

$$\frac{\partial H}{\partial x(t)} = 0$$

$$e^{\rho t}\alpha x(t)^{\alpha-1}z(t)^{\beta}[x(t)^{\alpha}z(t)^{\beta}]^{-\phi} = \lambda(t) \tag{6.1}$$

FOC with respect to $z(t)$ implies:

$$\frac{\partial H}{\partial z(t)} = 0$$

$$e^{\rho t}\beta x(t)^{\alpha}z(t)^{\beta-1}[x(t)^{\alpha}z(t)^{\beta}]^{-\phi} = b(t)d(t)z(t)^{d(t)-1}\lambda(t) \tag{6.2}$$

FOC with respect to $k(t)$ implies:

$$\frac{\partial H}{\partial k(t)} = -\lambda'(t)$$

$$A\lambda(t) = -\lambda'(t) \tag{6.3}$$

Equation (6.3) can be solved to obtain

$$\lambda(t) = C_1 e^{-At} \tag{6.4}$$

where C_1 is a constant to be determined by initial conditions.

Eliminating $\lambda(t)$ from Equation (6.1) and Equation (6.2) and expressing $x(t)$ in terms of $z(t)$ yields:

$$x(t) = \frac{\alpha b(t)d(t)z(t)^{d(t)}}{\beta} \tag{6.5}$$

Substituting Equation (6.4) and Equation (6.5) into Equation (6.1) and solving for $z(t)$, we can express $z(t)$ as a function of $b(t)$ and $d(t)$:

$$z(t) = e^{N[d(t)]t} M[b(t), d(t)] \{\frac{z(0)}{M[b(0), d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}} \tag{6.6}$$

where $N[d(t)]$ is function of $d(t)$ and $M[b(t), d(t)]$ is a function of $b(t)$ and $d(t)$.

Substituting Equation (6.6) into the price function $p[z(t), b(t), d(t)] = b(t)z(t)^{d(t)}$ yields

$$p[z(t), b(t), d(t)] = b(t)e^{N[d(t)]d(t)t} M[b(t), d(t)]^{d(t)} \{\frac{z(0)}{M[b(0), d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}d(t)} \tag{6.7}$$

We can also substitute Equation (6.6) into Equation (6.5) to get the expenditure on the homogeneous good $x(t)$:

$$x(t) = \frac{\alpha}{\beta} d(t)b(t)e^{N[d(t)]d(t)t} M[b(t), d(t)]^{d(t)} \{\frac{z(0)}{M[b(0), d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}d(t)} \tag{6.8}$$

Dividing Equation (6.7) by the sum of Equation (6.7) and Equation (6.8) yields the expenditure share of z

$$s_z(t) = \frac{\beta}{\beta + \alpha d(t)} \tag{6.9}$$

Next, using Equation (6.6) we can show that if the initial distribution of z follows the Pareto distribution, then it will follow the Pareto distribution thereafter. We first introduce the following lemma:

Lemma 1 *If a random variable x follows the Pareto distribution with pdf $f(x) = k_x \frac{x_m^{k_x}}{x^{k_x+1}}$, then a power transformation of x, i.e. $y = ax^b$, also follows the Pareto distribution with pdf $f(y) = k_y \frac{y_m^{k_y}}{y^{k_y+1}}$ with $y_m = ax_m^b$ and $k_y = \frac{k_x}{b}$.*

Proof: See Appendix 6.1

From Equation (6.6), we know that for any $t > 0$, $z(t)$ is a power function of $z(0)$. Therefore, every individual's choice of z at time t $z(t)$ will be a power transformation of its initial choice $z(0)$, and the power transformation is the same for everyone. Hence, if the initial distribution of $z(0)$ follows the Pareto distribution, according to Lemma 1 the distribution of $z(t)$ will follow the Pareto distribution thereafter. Suppose the initial Pareto distribution of $z(0)$ is characterized by the two parameters: the lower bound $z_m(0)$ and the shape parameter $k_z(0)$, then by Equation (6.6) and Lemma 1, the distribution of $z(t)$ is characterized by

$$k_z(t) = k_z(0)\frac{N[d(0)]}{N[d(t)]} \tag{6.10}$$

$$z_m(t) = e^{N[d(t)]t}M[b(t),d(t)]\{\frac{z_m(0)}{M[b(0),d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}} \tag{6.11}$$

Therefore, the distribution of quantity demanded for quality z can be obtained by plugging Equation (6.10) and (6.11) into the pdf of $z(t)$:

$$Q^d(z,t) = k_z(t)\frac{z_m(t)^{k_z(t)}}{z^{k_z(t)+1}}$$

6.2.2 *The Producer's Problem*

The producer's problem in the dynamic model is similar to the one in the static model. Each producer only lives for one period. The distribution of the firm is also assumed to be the Pareto distribution, characterized by a constant parameter $\mathbb{Z}_m$ and a time-varying parameter $k_{\mathbb{Z}}(t)$. Therefore, its pdf is:

$$g[t,z(t)] = k_{\mathbb{Z}}(t)\frac{\mathbb{Z}_m^{k_{\mathbb{Z}}(t)}}{z(t)^{k_{\mathbb{Z}}(t)+1}}$$

As the producers have no market power and take the price function $b(t)$ and $d(t)$ as given, each producer's problem is to maximize its profit by choosing the quantity:

$$\max_{M} \; Mp[z(t), b(t), d(t)] - A_z M^{\tau}[z(t)]^{\gamma}$$

FOC with respect to M implies that

$$M[z(t)] = [\frac{b(t)z(t)^{d(t)}}{A_z \tau z(t)^{\gamma}}]^{\frac{1}{\tau-1}}$$

Therefore, the distribution of quantity supplied for quality z is given by

$$Q^s(z,t) = g[t, z(t)]M(z(t)) \quad (6.12)$$

$$= k_{\mathbb{Z}}(t)\frac{\mathbb{Z}_m^{k_{\mathbb{Z}}(t)}}{z(t)^{k_{\mathbb{Z}}(t)+1}}[\frac{b(t)z(t)^{d(t)}}{A_z \tau z(t)^{\gamma}}]^{\frac{1}{\tau-1}} \quad (6.13)$$

However, although all firms at equilibrium have the same linear technology, it can be shown that the total profit for the firm producing the good with quality $z(t)$ is equal to $b(t)^{\frac{\tau}{\tau-1}} A_z^{\frac{-1}{\tau-1}} \tau^{\frac{-\tau}{\tau-1}} (\tau-1) z^{\frac{d(t)\tau-\gamma}{\tau-1}}$. It implies that if $\frac{d(t)\tau-\gamma}{\tau-1} > 0$, which is reasonable to assume, then the firm producing a higher quality good has a bigger size and hence a higher profit. Therefore, the firm with lower profits has an incentive to learn the technology of producing higher quality goods in the long run, although it will incur some costs by doing this. The value of $\frac{d(t)\tau-\gamma}{\tau-1}$ is hence an indication of the level of incentive for firms to change their technology. This chapter assumes that this value must be equal to some constant at steady state. Any deviation from this value will lead to changes in the firm distribution, which is due to the reallocation of firms on the quality dimension driven by higher profits. It is assumed that the firm's distribution dynamics is governed by the following equation:

$$k'_{\mathbb{Z}}(t) = -\xi(\frac{d(t)\tau-\gamma}{\tau-1} - \omega) \quad (6.14)$$

where ω is the steady state value of $\frac{d(t)\tau-\gamma}{\tau-1}$ and ξ is the speed of adjustment. The implication of this differential equation is that at steady state the power of z in the profit function is equal to the steady state value, so the cost of changing quality is equal to the benefit and hence the firm distribution will stay constant. If the power is greater than the steady state value, some lower quality good producing firms will have incentives to upgrade their quality, which results in a lower $k_{\mathbb{Z}}$.

6.2.3 Solving the Market Equilibrium

In the equilibrium, we must have the market clearing condition:

$$Q^s(z(t),t) = Q^d(z(t),t)$$

$$k_{\mathbb{Z}}(t)\frac{\mathbb{Z}_m^{k_{\mathbb{Z}}(t)}}{z(t)^{k_{\mathbb{Z}}(t)+1}}[\frac{b(t)z(t)^{d(t)}}{A_z\tau z(t)^{\gamma}}]^{\frac{1}{\tau-1}} = k_z(t)\frac{z_m(t)^{k_z(t)}}{z^{k_z(t)+1}}$$

The LHS can be simplified as

$$LHS = k_{\mathbb{Z}}(t)\mathbb{Z}_m^{k_{\mathbb{Z}}(t)}[\frac{b(t)}{A_z\tau}]^{\frac{1}{\tau-1}}z(t)^{-[k_{\mathbb{Z}(t)}(t)+1]+[d(t)-\gamma](\frac{1}{\tau-1})}$$

Plugging Equation (6.10) and (6.11) into the RHS yields:

$$RHS =$$
$$k_z(0)\frac{N[d(0)]}{N[d(t)]}\{e^{N[d(t)]t}M[b(t),d(t)]\{\frac{z_m(0)}{M[b(0),d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}}\}^{k_z(0)\frac{N[d(0)]}{N[d(t)]}}z(t)^{-k_z(0)\frac{N[d(0)]}{N[d(t)]}-1}$$

As the power of $z(t)$ on both sides must be the same, we have

$$k_{\mathbb{Z}}(t) = [d(t)-\gamma](\frac{1}{\tau-1}) + k_z(0)\frac{N[d(0)]}{N[d(t)]} \tag{6.15}$$

In addition, the constant terms in the LHS and the RHS must be the same:

$$k_{\mathbb{Z}}(t)\mathbb{Z}_m^{k_{\mathbb{Z}}(t)}[\frac{b(t)}{A_z\tau}]^{\frac{1}{\tau-1}}$$
$$= k_z(0)^{\frac{N[d(0)]}{N[d(t)]}}\{e^{N[d(t)]t}M[b(t),d(t)]\{\frac{z_m(0)}{M[b(0),d(0)]}\}^{\frac{N[d(t)]}{N[d(0)]}}\}^{k_z(0)\frac{N[d(0)]}{N[d(t)]}} \quad (6.16)$$

Therefore, Equation (6.15), (6.16) and (6.14) combined with the initial values $k_{\mathbb{Z}}(0)$, $k_z(0)$ and $z_m(0)$ can determine the paths of $b(t)$, $d(t)$, $k_{\mathbb{Z}}(t)$ and $k_z(t)$.

Firstly, setting $t = 0$ in Equation (6.15) yields:

$$k_{\mathbb{Z}}(0) = [d(0) - \gamma](\frac{1}{\tau - 1}) + k_z(0) \quad (6.17)$$

Setting $t = 0$ in Equation (6.16) implies that

$$k_{\mathbb{Z}}(0)\mathbb{Z}_m^{k_{\mathbb{Z}}(0)}[\frac{b(0)}{A_z\tau}]^{\frac{1}{\tau-1}} = k_z(0)[z_m(0)]^{k_z(0)}$$

from which we can obtain the initial value $b(0)$.

Given the initial values $k_{\mathbb{Z}}(0)$ and $k_z(0)$, $d(0)$ can be obtained by substituting these values into Equation (6.17). Using $d(0)$ and Equation (6.15) and (6.14), we can solve the paths of $k_{\mathbb{Z}}(t)$ and $d(t)$. Substituting the path of $d(t)$ into Equation (6.10) yields the path of $k_z(t)$. Finally, using the paths of $k_{\mathbb{Z}}(t)$, $d(t)$ and Equation (6.16), we can solve the path $b(t)$.

6.2.4 *The Isomorphism between the Model and an AK Model with Time-varying Preference*

This subsection shows that the AK model with two goods but with fixed preference parameters is isomorphic to an AK model with a one-good time-varying preference. The individual optimization problem in the one-good model is

$$\max \int_0^\infty e^{-\rho t} u_t(c(t))dt$$

$$s.t.\ k'(t) = Ak(t) - c(t)$$

where $u_t(c(t)) = \eta(t)\frac{c(t)^{1-\theta(t)}}{1-\theta(t)}$; $\theta(t)$ *and* $\eta(t)$ are exogenous; A is the parameter of the linear AK technology; $\theta(t) = 1 - (\alpha + \frac{\beta}{d(t)})(1-\phi)$ and $\eta(t) = ((\frac{\alpha}{\beta}d(t))^{\alpha} b(t)^{-\frac{\beta}{d(t)}} (\frac{\alpha}{\beta}d(t)+1)^{-(\alpha+\frac{\beta}{d(t)})})^{(1-\phi)}(\alpha + \frac{\beta}{d(t)})$.

The transversality condition for the above dynamic problem is given by

$$\lim_{t\to\infty} k(t)exp(-At) = 0$$

The Hamiltonian of this problem is:

$$H = e^{-\rho t} u_t(c(t)) + \lambda(t)[Ak(t) - c(t)]$$

FOC with respect to $c(t)$ implies

$$\frac{\partial H}{\partial c(t)} = 0$$

$$e^{-\rho t} u_t'(c(t)) = \lambda(t) \tag{6.18}$$

FOC with respect to $k(t)$ implies

$$\frac{\partial H}{\partial k(t)} = -\dot{\lambda}$$

or

$$A = -\frac{\dot{\lambda(t)}}{\lambda(t)} \tag{6.19}$$

As $c(t) = p[z(t), b(t), d(t)] + x(t)$ and $p[z(t), b(t), d(t)] = s_z(t)c(t)$,

$$\begin{aligned} u_x[x(t), z(t)] &= \alpha x(t)^{\alpha-1-\alpha\phi} z(t)^{\beta-\beta\phi} \\ &= \alpha[c(t)(1-s_z(t))]^{\alpha-1-\alpha\phi}[\frac{c(t)s_z(t)}{b(t)}]^{\frac{\beta-\beta\phi}{d(t)}} \end{aligned}$$

Using the expression of $\eta(t)$ and $\theta(t)$, it can be shown that

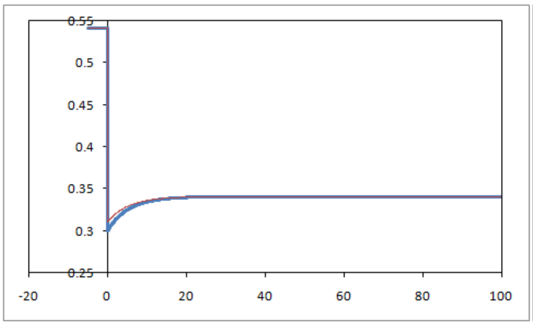
$$u_x(x(t), z(t)) = \eta(t)c(t)^{-\theta(t)} = u_t'(c(t))$$

Therefore, equation (6.18) and (6.19) are exactly the same as the two FOCs derived when using the direct utility function, i.e. Equation (6.1) and (6.4). The same FOCs will generate the same aggregate dynamics, which proves the isomorphism between the two models.

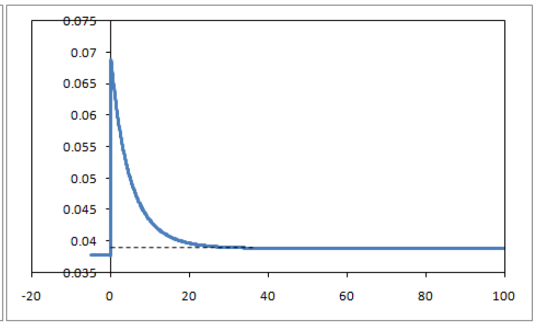
6.2.5 Transitional Dynamics

The theoretical prediction about inequality, growth and inflation can be investigated by studying the transitional dynamics. All the parameters apart from ρ, A, ϕ, ω and ξ are the same as in the static model. There is no direct evidence on the choices of the four parameters mentioned above. Reasonable values are chosen for them to reflect the main qualitative behavior of the model.

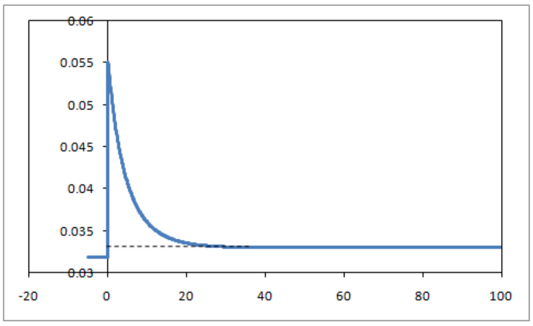
In addition, for the dynamics of consumption and capital not to explode, we have to find the initial values of $k_{\mathbb{Z}}(0)$, $k_z(0)$, $z_m(0)$ such that $k(t)$ converges to its steady state value as $t \to \infty$. Our method is to first find the steady values for $k(t)$ and then use it as the initial condition to solve the capital accumulation equation backwards in time to obtain the path of $k(t)$. Using the above initial values and parameters, the result from transitional dynamics are plotted in Figure 6.1.



(a) Gini of Consumption and Income



(b) Consumption Growth



(c) Inflation

Figure 6.1: Dynamics of the Aggregate Economy

As a negative shock to consumption inequality or income inequality will change the price function of the differentiated goods and the profit function for firms, the growth rate of consumption is higher during transition than at steady state. The intuition is as follows: from inspection of Figure 6.1, it can be seen that a negative shock on consumption inequality or income inequality will change the price function of the differentiated goods, making the convexity of the price function less than its steady state value. As a result, the marginal utility of a fixed amount of consumption is increasing during the transition. Due to the intertemporal substitution, the growth rate of consumption must be higher following a shock than its steady state value. Hence inflation in the prices of z goods and in aggregate will be higher since the quality of z goods is not controlled for in the construction of the inflation index. Therefore, the aggregate dynamics generated by the model shows that a lower degree of income inequality is associated with higher growth and inflation during the transition.

6.3 *The Implications: The Mismeasured Quality Corrected for the Price Level in China*

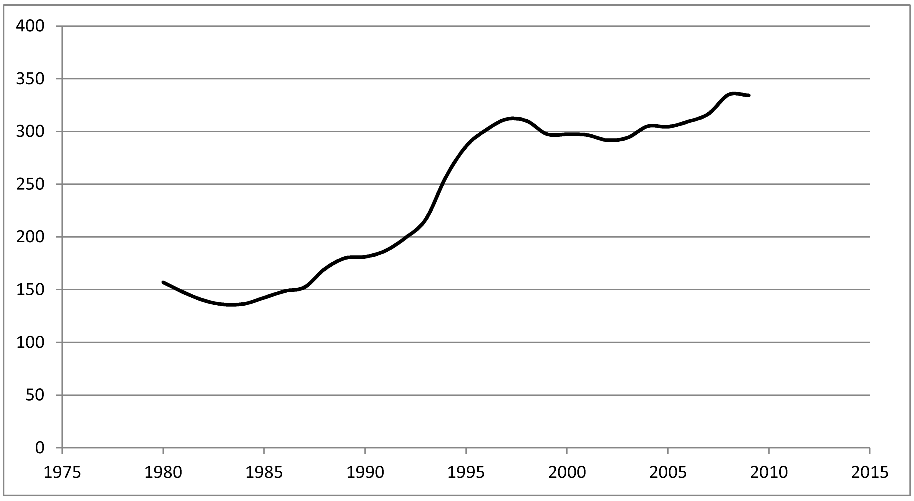
As one of the implications of the dynamic model is that if quality cannot be perfectly controlled, higher price levels of GDP per capita will be translating to higher price indexes, the section uses China as an example to show its empirical relevance. To do so, we compare the actual price index of China with the price index corrected for the B-S effect. Panel (a) of Figure 6.2 shows the price index in terms of RMB, which is increasing over time from 1980 to 2009. The time series of RMB exchange rate is shown in Panel (b). In Panel (c), the actual price index relative the US is plotted, which is decreasing. This is mainly due to the depreciation of RMB during this period.

To implement the correction, we first estimate the B-S effect each period, i.e. the elasticity of the price index with respect to GDP per capita. Then we deduct from the actual price index the B-S effect implied by the increase in GDP per capita. As the B-S effect is time-varying, we have two versions of corrections. In the first version of correction, we deduct from the actual price index the time-varying B-S effect. In the second version, we deduct the average B-S effect. The comparison of the actual price index and the corrected price index for China from 1980 to 2009 are shown in Figure 6.3.

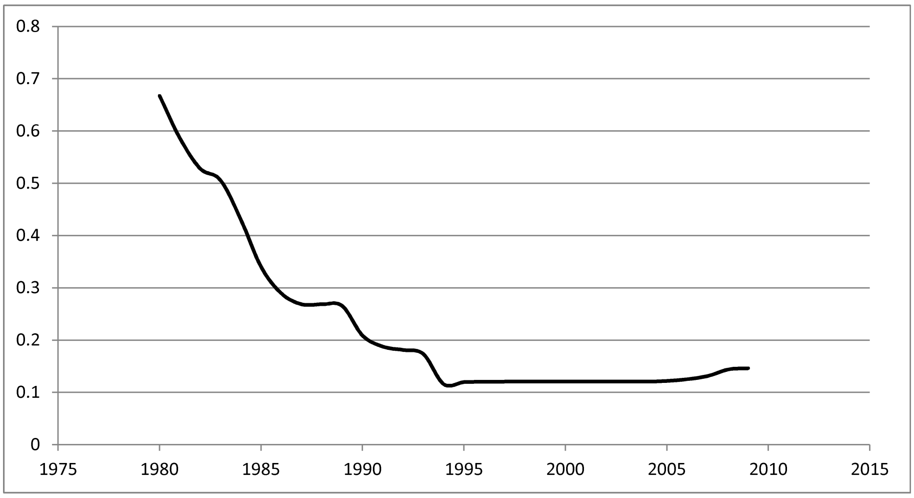
On inspection, one can see that quality control problem is a serious problem as there is a large difference between the actual price index and the adjusted price index. For example, in Panel (b) the price index adjusted for the time-varying B-S effect is only 75% of the actual price index in 2009. In Panel (c), the price index adjusted for the average B-S effect is around 90% of the actual price index in 2009, as the B-S effect is increasing over time after 1980.

In terms of the changes in the price indexes, of the 64.2% rise in the price index from 1994 to 2010, 35.0% can be attributed to the bias associated with the B-S effect in Panel (b) and 5.7% can be attributed to the bias associated with the B-S effect in Panel (c). Although the price index here is the international price index, given that the quality control problems also persist in domestic price indexes, it is likely that there may be a large component in the domestic price index that is needed to be corrected.

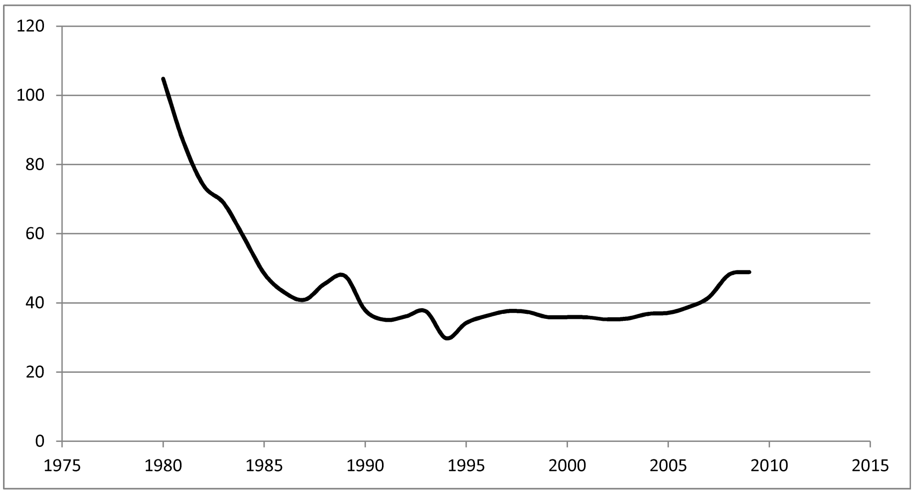
We note a second implication related to the comovement of income distribution and inflation. But many factors that are difficult to control for affect these variables, so a simple comparison cannot constitute a valid test. For the sake of completeness, however, we return these comparisons to Appendix 6.2.



(a) Actual Price Index in RMB

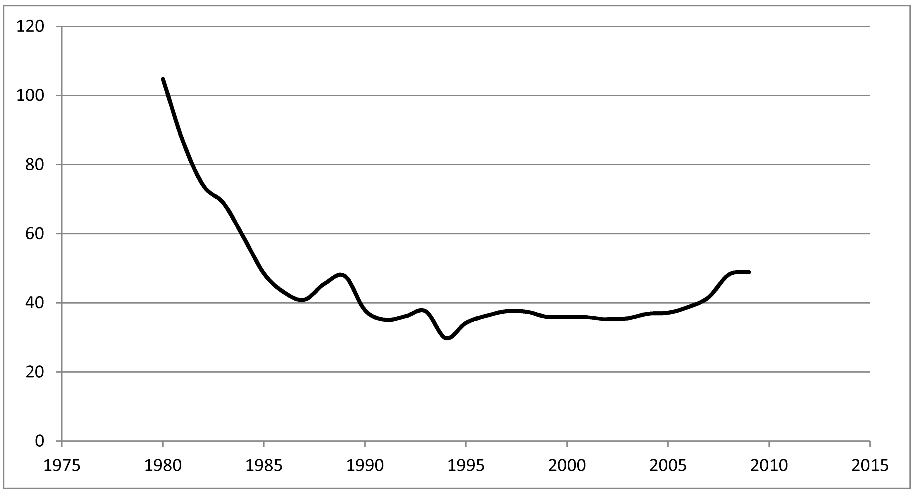


(b) Exchange Rate

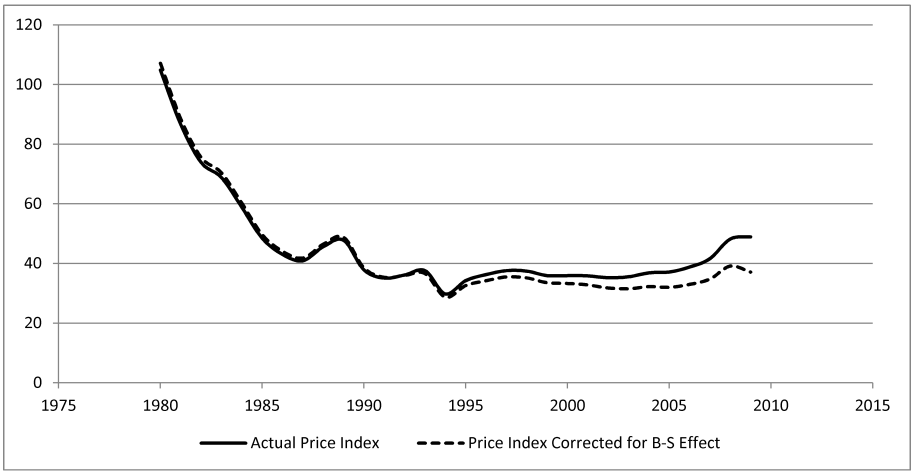


(c) Actual Price Index in US$

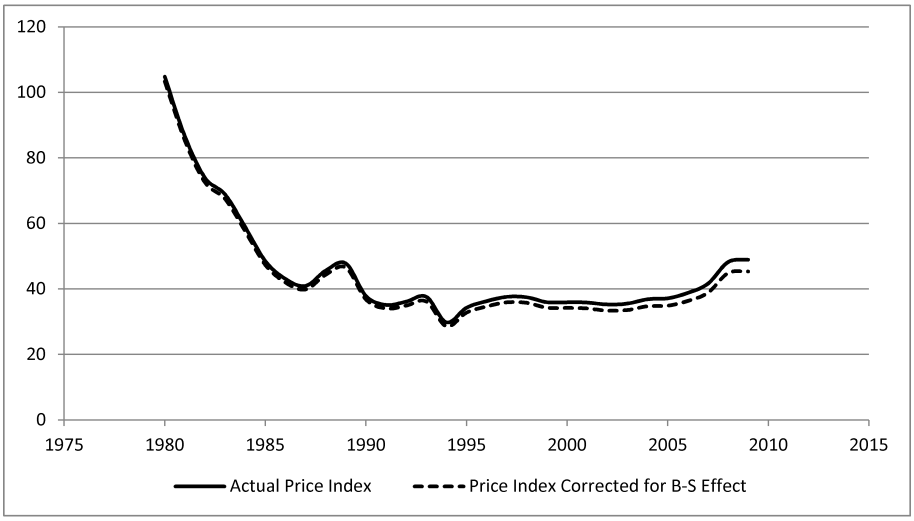
Figure 6.2: Actual Price Index: the Case of China



(a) Actual Price Index in US$



(b) Actual Price Index vs. Price Index Adjusted for Time-Varying B-S Effect



(c) Actual Price Index vs. Price Index Adjusted for Average B-S Effect

Figure 6.3: Actual Price Index vs. Adjusted Price Index: the Case of China

6.4 Conclusion

As an extension of Chapter 2, this chapter tries to address the question of how income distribution affects the price level or inflation with one country. This static model in Chapter 2 is extended to a dynamic one to investigate the macroeconomic implications of its mechanism. As the quality control difficulties persist in both international price indexes and domestic price indexes, just as income distribution matters for the national price level, income distribution can also affect domestic price index and inflation. For example, in the case of China, there is a large difference between the actual price index and the price index corrected for the B-S effect, especially after 1980s. Moreover, empirical evidence from four countries shows that inflation is positively correlated with the growth of GDP and is negatively correlated with income inequality. These empirical findings are consistent with the theoretical prediction from the dynamic model.

In the literature, inflation models are usually based on the Phillips curve specification, i.e. a positive relationship between inflation and real activity. The use of aggregate variable on real activity in the Phillips curve specifications implicitly assume a representative agent framework, i.e. the first moments of these aggregate variables contain sufficient information about inflation. However, this chapter suggests that other moments of income distribution, such as income inequality, may contain additional information about inflation.

Chapter 7

Summary and Conclusion

It has been shown in Chapter 6 that another macroeconomic effect related to an induced bias in measured price indices. For example, in the case of China, of the 64.2% rise in the price index from 1994 to 2010, up to 35.0% can be attributed to the bias associated with the B-S effect.

The focus of this thesis, however, lies with the microeconomics of the B-S effect. The traditional argument was that the effect was because of the 'service content' of traded goods. Here, an alternative view has been developed: it has been argued that there is a natural split between two groups of products used in the index: M-group products and S-group products. The classification is based on nature of output. All products are classified into ND (non-durable), SD (semi-durable), S (service), IS (individual service), CS (collective service) and IG (investment goods). The detailed methodology about this classification can be found in Organisation for Economic Co-operation and Development (2006). Here, S-group products include S (service), IS (individual service) and CS (collective service). ND (non-durable), SD (semi-durable) and IG (investment goods) are classified as M-group products.

It has been argued that a different type of model is appropriate in the two cases. For S-group products a model was proposed in Chapter 4 that reflects the fact that these are almost pure labour services. A fundamental implication of this model is that the conventional semi-log representation of the B-S relationship is inappropriate for these goods.

In Chapter 2, and in Chapter 5, a new model appropriate to the M-goods was developed. The central idea is that of 'mismeasured quality', and this can be seen as a contribution to the new strand of literature on IO that focuses on the unavoidable shortcoming of hedonic indices.

The novel implication of this new model is that the form of income distribution, as measured by the Gini coefficient, should be correlated the national price level, controlling for per capita income. This implication is consistent with the evidence provided in Chapter 2 and 5.

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Appendix 2

2.1 Proof of Proposition 3

Since $\tau > 1$,

$$\frac{\partial \overline{p}}{\partial \mu} = \frac{\beta}{\alpha \frac{k_z + \gamma \frac{1}{\tau - 1}}{\frac{\frac{1}{Gini} + 1}{2} + \frac{1}{\tau - 1}} + \beta} > 0$$

$$\frac{\partial \overline{p}}{\partial Gini} = \mu \frac{-\alpha\beta}{(\alpha \frac{k_z + \gamma \frac{1}{\tau - 1}}{\frac{\frac{1}{Gini} + 1}{2} + \frac{1}{\tau - 1}} + \beta)^2} \frac{k_z + \gamma \frac{1}{\tau - 1}}{(\frac{\frac{1}{Gini} + 1}{2} + \frac{1}{\tau - 1})^2} \frac{1}{2(Gini)^2} < 0$$

Therefore,

$$e_{\overline{p},\mu} = \frac{\partial \overline{p}}{\partial \mu} \frac{\mu}{\overline{p}} > 0$$

$$e_{\overline{p},Gini} = \frac{\frac{\partial \overline{p}}{\overline{p}}}{\partial Gini} = \frac{\partial \overline{p}}{\partial Gini} \frac{1}{\overline{p}} < 0$$

That is the elasticity of the average price level of the z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative. Q.E.D.

2.2 Proof of Proposition 4

$$\frac{\partial \overline{p}}{\partial \mu \partial Gini} = \frac{-\alpha\beta}{(\alpha \frac{k_z+\gamma\frac{1}{\tau-1}}{\frac{\frac{1}{Gini}+1}{2}+\frac{1}{\tau-1}}+\beta)^2} \frac{k_z+\gamma\frac{1}{\tau-1}}{(\frac{\frac{1}{Gini}+1}{2}+\frac{1}{\tau-1})^2} \frac{1}{2(Gini)^2} < 0$$

According to Young's theorem, $\frac{\partial \overline{p}}{\partial \mu \partial Gini} = \frac{\partial \overline{p}}{\partial Gini \partial \mu}$. Therefore, the absolute value of $\frac{\partial \overline{p}}{\partial Gini}$ is increasing in μ. And $\frac{\partial \overline{p}}{\partial \mu}$ is decreasing in $Gini$. Q.E.D.

2.3 Proof of Proposition 5

Given the definition of the aggregate price level as the Paasche index,

$$P_P = 1\frac{\alpha d}{\alpha d+\beta} + \frac{\overline{p}}{\overline{p}_0}\frac{\beta}{\alpha d+\beta}.$$

$$= (1-\frac{\beta}{\alpha d+\beta}) + \frac{\mu\frac{\beta}{\alpha d+\beta}}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\frac{\beta}{\alpha d+\beta}. \tag{A.2.1}$$

we have

$$\frac{\partial P_P}{\partial \mu} = (\frac{\beta}{\alpha d+\beta})^2 \frac{1}{\mu_0\frac{\beta}{\alpha d_0+\beta}} > 0 \tag{A.2.2}$$

$$\frac{\partial P_P}{\partial Gini} = \frac{\beta}{(\alpha d+\beta)^2}\alpha\frac{\partial d}{\partial Gini} + \frac{\mu}{\mu_0\frac{\beta}{ad_0+\beta}}\beta^2(-2)(\alpha d+\beta)^{-3}\alpha\frac{\partial d}{\partial Gini}$$

$$= [\frac{\alpha\beta}{(\alpha d+\beta)^2} - 2\frac{\mu}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\alpha\beta^2(\alpha d+\beta)^{-3}]\frac{\partial d}{\partial Gini}$$

$$= [1-2\frac{\mu\frac{\beta}{\alpha d+\beta}}{\mu_0\frac{\beta}{\alpha d_0+\beta}}]\frac{\alpha\beta}{(\alpha d+\beta)^2}\frac{\partial d}{\partial Gini} \tag{A.2.3}$$

Hence, the impact of per capita income on the Paasche index is positive while the impact of income inequality on the Paasche index depends on the country's income distribution relative to the base country, i.e. the U.S.. $\frac{\partial P_P}{\partial Gini}$ for different combinations of per capita income and income inequality using the value of parameters from Table A.2.1 is plotted in Figure A.2.1. It can be seen that the sign of $\frac{\partial P_P}{\partial Gini}$ crucially depends on the level of per capita income. With low per capita income, income inequality has a positive impact on the national price level. On the other hand, with high per capita income, income inequality will have a negative impact on the national price level.

Table A.2.1: Calibration

Baseline specification	
$\alpha = 0.1$	share of homogeneous goods
$\beta = 0.9$	share of differentiated goods
$\epsilon = 1$	elasticity of substitution
$A_z = 1$	productivity parameter
$\tau = 2$	quantity elasticity of production cost
$\gamma = 2$	quality elasticity of production cost
$k_c = 2.1667$	power parameter in Pareto distribution of income
$c_m = 24621$	minimum income
$k_z = 2$	power parameter in Pareto distribution of quality
$z_m = \frac{\sqrt{2}}{2}$	minimum quality

Moreover,

$$\frac{\partial P_P}{\partial \mu \partial Gini} = -2\frac{1}{\mu_0 \frac{\beta}{\alpha d_0 + \beta}} \alpha \beta^2 (\alpha d + \beta)^{-3} \frac{\partial d}{\partial Gini} < 0. \tag{A.2.4}$$

Thus, the effect of per capita income on the aggregate price level ($\frac{\partial P_P}{\partial \mu}$) is decreasing in income inequality and the effect of income inequality ($\frac{\partial P_P}{\partial Gini}$) is decreasing in per capita income μ.

Since the elasticity of the aggregate price level with respect to per capita income $e_{P_P,\mu} \equiv \frac{\partial P_P}{\partial \mu} \frac{\mu}{P_P}$ has the same sign as $\frac{\partial P_P}{\partial \mu}$ and the semi-elasticity of the aggregate price level with respect to income inequality $e_{P_P,Gini} \equiv \frac{\partial P_P}{\partial Gini} \frac{1}{P_P}$ has the same sign as $\frac{\partial P_P}{\partial Gini}$, it is easy to show that $e_{P_P,\mu}$ is positive and the sign

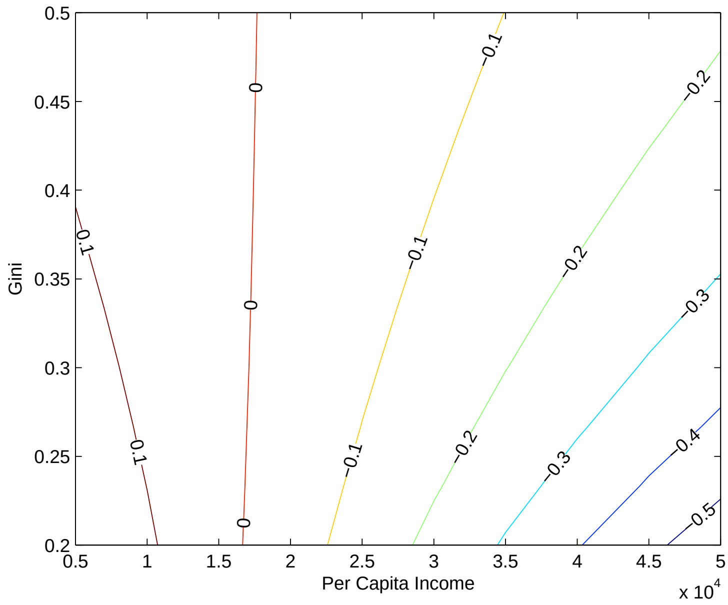


Figure A.2.1: The Contour of the Effect of Income Inequality on the Paasche Index ($\frac{\partial P_P}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality

Notes: The base country income distribution is calibrated using U.S. data in 2003.

of $e_{P_P,Gini}$ depends on per capita income of the country in question relative to the U.S.. With a low enough per capita income $e_{P_P,Gini}$ is positive, while $e_{P_P,Gini}$ is negative with a high level of per capita income.

Furthermore,

$$\frac{\partial e_{P_P,\mu}}{\partial Gini} = \frac{\partial[\frac{\partial P_P}{\partial \mu}\frac{\mu}{P_P}]}{\partial Gini}$$

$$= \frac{\partial P_P}{\partial \mu \partial Gini}\frac{\mu}{P_P} + \frac{\partial P_P}{\partial \mu}\frac{\mu}{-P_P^2}\frac{\partial P_P}{\partial Gini} \tag{A.2.5}$$

Substituting Equation (A.2.1), (A.2.2), (A.2.3) and (A.2.4) into Equation (A.2.5), we can obtain

$$\frac{\partial e_{P_P,\mu}}{\partial Gini} = \frac{\mu}{P_P}\frac{\partial d}{\partial Gini}(\alpha d+\beta)^{-3}\frac{1}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\alpha\beta^2\{-2-\frac{\beta}{\alpha d+\beta}[1-2\frac{\overline{P}}{\overline{P}_0}]\frac{1}{P_P}\}$$

Therefore, the condition for $\frac{\partial e_{P_P,\mu}}{\partial Gini} < 0$ is

$$\frac{\beta}{\alpha d + \beta}(2\frac{\overline{P}}{\overline{P}_0} - 1)\frac{1}{P_P} < 2$$

Intuitively, we can notice that this condition can be satisfied as long as the income distribution is not too far away from that of the U.S.. For example, when the income distribution of the country in question is similar to that of the base country, the U.S. , in which case both $\frac{\overline{P}}{\overline{P}_0}$ and P_P are around 1, the left hand side will be around $\frac{\beta}{\alpha d+\beta}$, which is much less than 2.

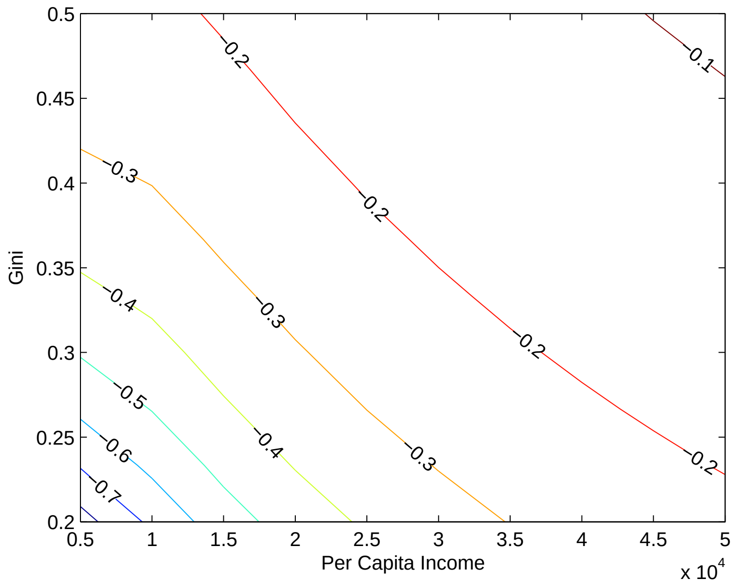


Figure A.2.2: The Contour of the Effect of Income Inequality on $e_{P_P,\mu}$ ($\frac{\partial e_{P_P,\mu}}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality

Notes: The base country income distribution is calibrated using U.S. data in 2003.

To check the sign of $\frac{\partial e_{P_P,\mu}}{\partial Gini}$ more generally, its value for different combinations of per capita income and income inequality is plotted in Figure A.2.2. The Figure shows that $\frac{\partial e_{P_P,\mu}}{\partial Gini}$ is negative for all possible combinations of per capita income and income inequality. Hence, the elasticity of the aggregate price level with respect to per capita income is decreasing in income inequality. Similarly,

$$\frac{\partial e_{P_P,Gini}}{\partial \mu} = \frac{\partial[\frac{\partial P_P}{\partial Gini}\frac{1}{P_P}]}{\partial \mu}$$

$$= \frac{\partial P_P}{\partial Gini \partial \mu}\frac{1}{P_P} + \frac{\partial P_P}{\partial Gini}\frac{-1}{P_P^2}\frac{\partial P_P}{\mu}$$

$$= \frac{1}{P_P}[\frac{\partial P_P}{\partial Gini \partial \mu} - \frac{\partial P_P}{\partial Gini}\frac{1}{P_P}\frac{\partial P_P}{\mu}] = \mu\frac{\partial e_{P_P,\mu}}{\partial Gini}$$

Since $\frac{\partial e_{P_P,Gini}}{\partial \mu}$ has the same sign as $\frac{\partial e_{P_P,\mu}}{\partial Gini}$, the semi-elasticity of the aggregate price level with respect to income inequality is decreasing in per capita income.

If we define the aggregate price level as the Laspeyres index, then

$$P_L = (1 - \frac{\beta}{\alpha d_0 + \beta}) + \frac{\mu \frac{\beta}{\alpha d + \beta}}{\mu_0 \frac{\beta}{\alpha d_0 + \beta}} \frac{\beta}{\alpha d_0 + \beta}$$

$$= (1 - \frac{\beta}{\alpha d_0 + \beta}) + \frac{\mu}{\mu_0}\frac{\beta}{\alpha d + \beta} \quad \text{(A.2.6)}$$

Therefore,

$$\frac{\partial P_L}{\partial \mu} = \frac{\beta}{\mu_0(\alpha d + \beta)} > 0 \quad \text{(A.2.7)}$$

$$\frac{\partial P_L}{\partial Gini} = \frac{\mu}{\mu_0}\frac{-\beta}{(\alpha d + \beta)^2}\alpha\frac{\partial d}{\partial Gini} < 0 \quad \text{(A.2.8)}$$

$$\frac{\partial P_L}{\partial \mu \partial Gini} = \frac{1}{\mu_0}\frac{-\beta}{(\alpha d + \beta)^2}\alpha\frac{\partial d}{\partial Gini} < 0 \quad \text{(A.2.9)}$$

Hence, the impact of per capita income on the Laspeyres index is positive while the impact of income inequality on the Laspeyres index is negative. Moreover, the impact of per capita income on the Laspeyres index is decreasing in income inequality and the impact of income inequality on the Laspeyres index is decreasing in per capita income.

Since the elasticity of the aggregate price level with respect to per capita income $e_{P_L,\mu} \equiv \frac{\partial P_L}{\partial \mu}\frac{\mu}{P_L}$ has the same sign as $\frac{\partial P_L}{\partial \mu}$ and the semi-elasticity of the aggregate price level with respect to income inequality $e_{P_L,Gini} \equiv \frac{\partial P_L}{\partial Gini}\frac{1}{P_L}$ has the same sign as $\frac{\partial P_L}{\partial Gini}$, it is easy to show that $e_{P_L,\mu}$ is positive and the sign of $e_{P_L,Gini}$ is negative.

Furthermore,

$$\frac{\partial e_{P_L,\mu}}{\partial Gini} = \frac{\partial[\frac{\partial P_L}{\partial \mu}\frac{\mu}{P_L}]}{\partial Gini}$$

$$= \frac{\partial P_L}{\partial \mu \partial Gini}\frac{\mu}{P_L} + \frac{\partial P_L}{\partial \mu}\frac{\mu}{-P_L^2}\frac{\partial P_L}{\partial Gini}$$

$$= \frac{\mu}{P_L}[\frac{\partial P_L}{\partial \mu \partial Gini} - \frac{\partial P_L}{\partial \mu}\frac{1}{P_L}\frac{\partial P_L}{\partial Gini}] \tag{A.2.10}$$

Substituting Equation (A.2.6), (A.2.7), (A.2.8) and (A.2.9) into Equation (A.2.10), we can obtain

$$\frac{\partial e_{P_L,\mu}}{\partial Gini} = \frac{\mu}{P_L}\frac{1}{\mu_0}\frac{\alpha\beta}{(\alpha d + \beta)^2}\frac{\partial d}{\partial Gini}[-1 + \frac{\beta}{\alpha d + \beta}\frac{\mu}{\mu_0}\frac{1}{P_L}]$$

Therefore, the condition for $\frac{\partial e_{P_L,\mu}}{\partial Gini} < 0$ is

$$\frac{\beta}{\alpha d + \beta}\frac{\mu}{\mu_0}\frac{1}{P_L} < 1$$

Intuitively, this condition can be satisfied as long as the income distribution of the country in question is not far away from that of the base country the U.S.. For example, when the income distribution is similar to that of the U.S. , in which case both $\frac{\mu}{\mu_0}$ and P_L are around 1, the left hand side will be around $\frac{\beta}{\alpha d+\beta}$, which is less than 1.

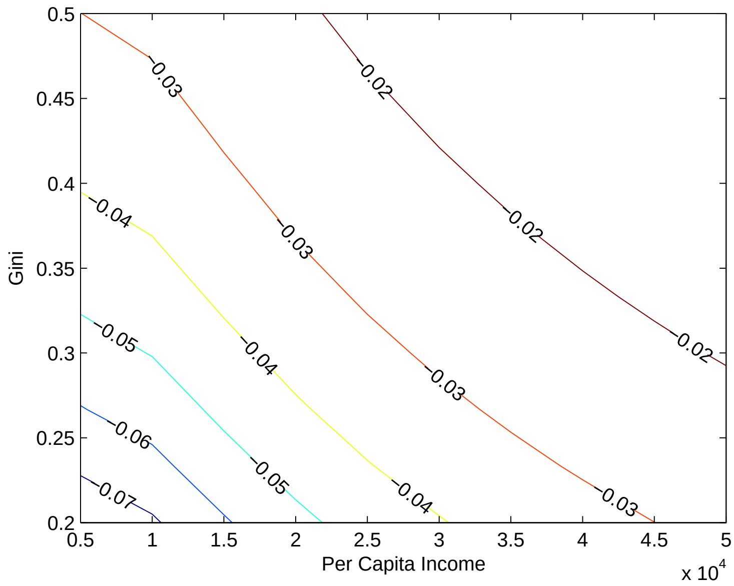


Figure A.2.3: The Contour of Effect of Income Inequality on $e_{P_L,\mu}$ ($\frac{\partial e_{P_L,\mu}}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality

Notes: The base country income distribution is calibrated using U.S. data in 2003.

To check the sign of $\frac{\partial e_{P_L,\mu}}{\partial Gini}$ more generally, its value for different combinations of per capita income and income inequality is plotted Figure A.2.3. The Figure shows that $\frac{\partial e_{P_L,\mu}}{\partial Gini}$ is negative for all possible combinations of per capita income and income inequality. Hence, the elasticity of the aggregate price level with respect to per capita income is decreasing in income inequality. Similarly,

$$\frac{\partial e_{P_L,Gini}}{\partial \mu} = \frac{\partial[\frac{\partial P_L}{\partial Gini}\frac{1}{P_L}]}{\partial \mu}$$

$$= \frac{\partial P_L}{\partial Gini \partial \mu}\frac{1}{P_L} + \frac{\partial P_L}{\partial Gini}\frac{-1}{P_L^2}\frac{\partial P_L}{\partial \mu}$$

$$= \frac{1}{P_L}[\frac{\partial P_L}{\partial Gini \partial \mu} - \frac{\partial P_L}{\partial Gini}\frac{1}{P_L}\frac{\partial P_L}{\partial \mu}]$$

$$= \frac{1}{\mu}\frac{\partial e_{P_L,\mu}}{\partial Gini}$$

Since $\frac{\partial e_{P_L,Gini}}{\partial \mu}$ has the same sign as $\frac{\partial e_{P_L,\mu}}{\partial Gini}$, the semi-elasticity of the aggregate price level with respect to income inequality is decreasing in per capita income. Q.E.D.

2.4 Income Inequality and the National Price Level in the Classic Model of Vertical Product Differentiation

2.4.1 Model

In the model, the world consists of countries which only differ in their distribution of disutility from labour. Each country comprises a continuum of consumers, indexed by ψ, a parameter characterizing the level of disutility from labour. A consumer uses his/her labour income c to consume a vertically differentiated/quality product, the price schedule of which is taken as given for each individual. The rest of the income is spent on a commodity good. The utility maximization problem is given by

$$\max_{l,u} U = u(\frac{c-p}{p_x}) - \psi l^2$$

$$s.t.\ c = l \cdot w$$

where u is the quality level of the vertically differentiated product and p is the price level; p_x is the unit price of the commodity good; w is the wage rate, l is the labour input.

It is assumed that the commodity good is produced with a simple fixed proportion technology: one unit of labour produces one unit of output. Assuming both the commodity good market and the labour market are competitive, the price of the commodity good p_x is given by marginal cost, which is equal to the wage rate w. Hence, we can take $w = p_x$ as the numeraire and set them equal to 1. The above problem now becomes

$$\max_{l,u} U = u(c - p) - \psi l^2$$

$$s.t.\ c = l$$

Since c is the gross consumption of commodity i.e. l

$$U = u(l - p) - \psi l^2$$

$$\frac{dU}{dl} = u - 2\psi l = 0 \text{ or } l = \frac{u}{2\psi} \tag{A.2.11}$$

The utility score of the consumer is

$$\begin{aligned} U &= u(l - p) - \psi l^2 \\ &= u(\frac{u}{2\psi} - p) - \psi(\frac{u}{2\psi})^2 \\ &= \frac{u^2}{2\psi} - up - \frac{u^2}{4\psi} \\ &= \frac{u^2}{4\psi} - up \end{aligned}$$

which gives the criterion for choosing a (u, p) offer. The consumer choose the (u, p) offer that maximizes

$$\frac{u^2}{4\psi} - up$$

subject to this being greater than $\frac{1}{4\psi}$ (from not buying any quality good).

Now suppose that the lowest quality is 1 and the product can be produced from one unit of commodity good without any additional cost. Moreover, it is assumed that all qualities are available at the price schedule $p(u) =$

u^n. An individual consumer will choose u^* to maximize the utility score:

$$\frac{u^2}{4\psi} - u^{n+1}$$

FOC implies that

$$\frac{u^*}{2\psi} = (n+1)u^{*n}$$

$$u^* = [2(n+1)]^{\frac{-1}{n-1}} \psi^{\frac{-1}{n-1}} \quad \text{(A.2.12)}$$

Hence, the price of the optimal choice of product is given by

$$p(u^*) = [2(n+1)]^{\frac{-n}{n-1}} \psi^{\frac{-n}{n-1}} \quad \text{(A.2.13)}$$

and by (A.2.11) the corresponding income level is equal to

$$l = \frac{u^*}{2\psi} \quad \text{(A.2.14)}$$

$$= [2(n+1)]^{\frac{-1}{n-1}} \frac{1}{2} \psi^{\frac{-n}{n-1}} \quad \text{(A.2.15)}$$

Dividing (A.2.13) by (A.2.15) gives the expenditure share on the quality good:

$$\frac{p(u^*)}{l} \quad \text{(A.2.16)}$$

$$= \frac{[2(n+1)]^{\frac{-n}{n-1}}}{\frac{1}{2}[2(n+1)]^{\frac{-1}{n-1}}} \quad \text{(A.2.17)}$$

$$= \frac{1}{n+1} \quad \text{(A.2.18)}$$

2.4.2 *Income Distribution, Expenditure Share and the Disaggregate Price Level*

Proposition 6 *(Income Distribution and Expenditure Share) Neither per capita income nor income inequality can affect the expenditure shares of the commodity good and the quality product.*

Proof: The expenditure share of the quality product is equal to $\frac{1}{n+1}$, where n is a parameter in the exogenously given price schedule of the quality product $p(u) = u^n$. Therefore, neither per capita income nor income inequality can affect the expenditure share. QED

Proposition 7 *(Income Distribution and the Disaggregate Price level) Per capita income has a positive impact on the average price level of the quality product. Keeping per capita income constant, income inequality has no impact the price level of the quality product.*

Proof: The pdf of the income distribution is assumed to be $f(l)$, $\underline{l} \leq l \leq \bar{l}$, where $\underline{l}$ and $\bar{l}$ are the lower and upper bounds of the income distribution. The per capita income of the income distribution is denoted μ_l, which is equal to

$$\int_{\underline{l}}^{\bar{l}} l \cdot f(l)dl$$

If quality is not controlled for, then the price level of the quality products $\overline{p}$ is computed as the total expenditure on the quality products divided by the total number of units:

$$\overline{p} = \frac{\int_{\underline{l}}^{\bar{l}} l \cdot sh(l) \cdot f(l)dl}{\int_{\underline{l}}^{\bar{l}} f(l)dl}$$

From the individual optimization problem, the expenditure share on the quality product is the same for every individual, which is equal to $\frac{1}{n+1}$.

Therefore, the total expenditure on the quality products is equal to

$$\int_{\underline{l}}^{\overline{l}} l \cdot \frac{1}{n+1} \cdot f(l)dl.$$

In addition, as there is a unit mass of consumers, if every individual buys one unit of the quality product, the total number of units is equal to

$$\int_{\underline{l}}^{\overline{l}} f(l)dl = 1$$

Therefore, the price level of the quality products is

$$\overline{p} = \frac{\int_{\underline{l}}^{\overline{l}} l \frac{1}{n+1} f(l)dl}{\int_{\underline{l}}^{\overline{l}} f(l)dl} = \frac{1}{n+1} \int_{\underline{l}}^{\overline{l}} l f(l)dl = \frac{1}{n+1} \mu_l \qquad \text{(A.2.19)}$$

Hence, the price level of the quality products is increasing in per capita income. Moreover, keeping per capita income constant, changes in income inequality cannot affect the price level of the quality products. QED

2.5 Income Distribution and the Aggregate Price Level

Model prediction: This model predicts the B-S relationship, i.e. a positive relationship between per capita income and the national price level. Controlling for per capita income, income inequality has no impact on the national price level. These are summarized in Proposition 8.

Proposition 8 *(Income Distribution and the National Price level) Per capita income has a positive impact on the national price level. Keeping per capita income constant, changes in income inequality have no impact on the national price level.*

Proof: The national price level $\mathbb{P}$ is the expenditure-share-weighted average of the price levels of the commodity good and the quality products:

$$\mathbb{P} = 1 \cdot (1 - sh) + \overline{p} \cdot sh \tag{A.2.20}$$

Substituting $sh = \frac{1}{n+1}$ and (A.2.19) into (A.2.20) yields:

$$\mathbb{P} = \frac{n}{n+1} + \frac{1}{(n+1)^2}\mu_l$$

Therefore, the national price level is increasing in per capita income μ_l. Keeping per capita income constant, changes in income inequality have no impact on the national price level. QED

The reader who is familiar with the I.O. literature on vertical product differentiation, in which the number of products is finite, may ask: would the results be different if the number of products is finite, rather than a continuum. Intuitively, if seems clear that this would not change the present results. Suppose now only a finite number of qualities is available. If each individual's optimal choice of quality u^* as shown in (A.2.12) is available, he/she will choose that quality and spend $1/(n+1)$ of expenditure on it. If that quality is not available, then the individual's choice will be a quality product as close to u^* as available and the expenditure share will be as close to $1/(n+1)$ as possible. On average, there is no systematic deviation of the expenditure share from $1/(n+1)$, which is a constant independent of income inequality.

Appendix 6

6.1 Proof of Lemma 1

$$y = ax^b$$

$$x = (\frac{y}{a})^{\frac{1}{b}} = y^{\frac{1}{b}} a^{\frac{-1}{b}}$$

Since the pdf of x is $f(x)$, the pdf of y is

$$g(y) = f((\frac{y}{a})^{\frac{1}{b}}) | \frac{\partial (\frac{y}{a})^{\frac{1}{b}}}{\partial y} |$$

$$= k_x \frac{x_m^{k_x}}{(y^{\frac{1}{b}} a^{\frac{-1}{b}})^{k_x+1}} (\frac{1}{b} y^{\frac{1}{b}-1} a^{\frac{-1}{b}})$$

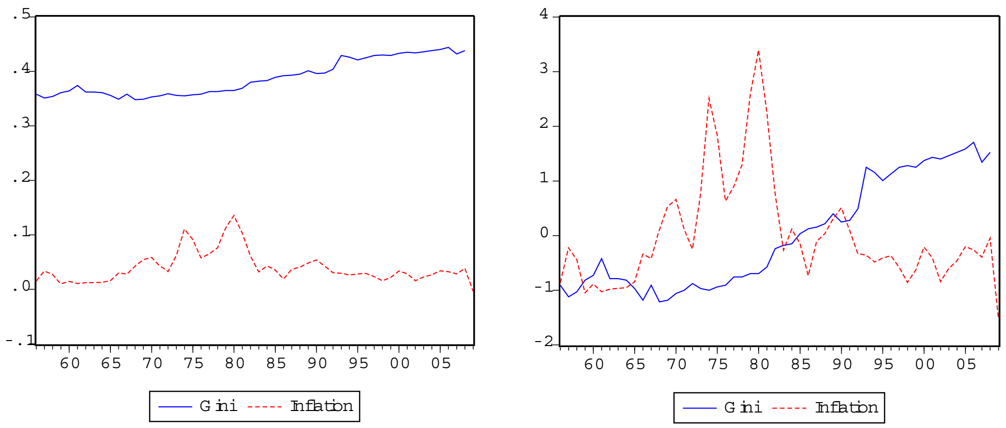
$$= \frac{k_x}{b} \frac{(a x_m^b)^{\frac{k_x}{b}}}{y^{\frac{k_x}{b}+1}}$$

which is also a form of the Pareto distribution $g(y) = k_y \frac{y_m^{k_y}}{y^{k_y+1}}$ with $y_m = a x_m^b$ and $k_y = \frac{k_x}{b}$. Q.E.D.

6.2 Implication 2: The Comovement of Income Distribution and Inflation

This section uses the data from four countries, the U.S., the U.K., Australia and Sweden to investigate the relationship between inequality, growth and inflation to see if the predictions of the dynamic model are consistent with empirical evidence.

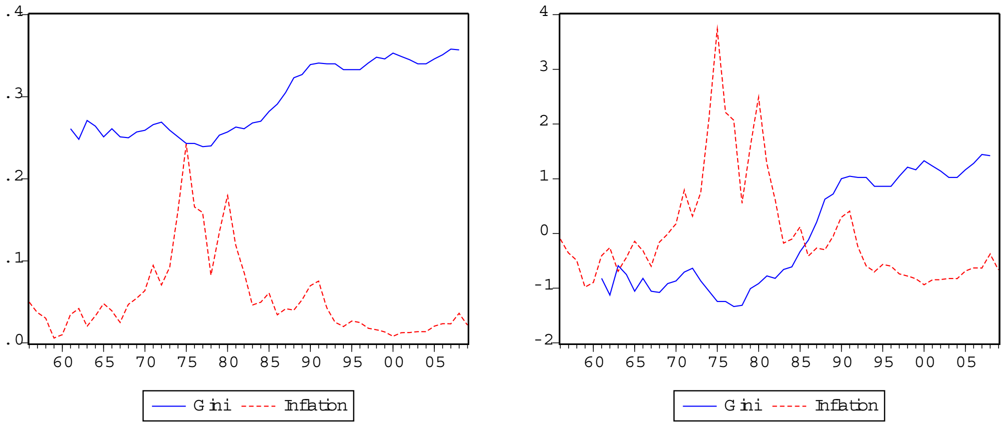
The measure of income inequality is usually sporadic for many countries; a long term and consistent measure of income inequality is only available in few countries, such as the U.S., the U.K., Australia and Sweden. Figure A.6.4, A.6.5, A.6.6 and A.6.7 plot the time series of the income Gini index and inflation for the four countries.[1] In order to show the relationship between inflation and inequality more clearly, normalized data is also plotted in Panel (b) of each figure. From these figures, we can notice that there is a striking negative relationship between inflation and income inequality in all four countries. This observation is confirmed by the simple correlation between inflation and income inequality as shown in Table A.6.2, where all the correlations are significantly negative.



(a) Linear Scaling (b) Normalized Scaling

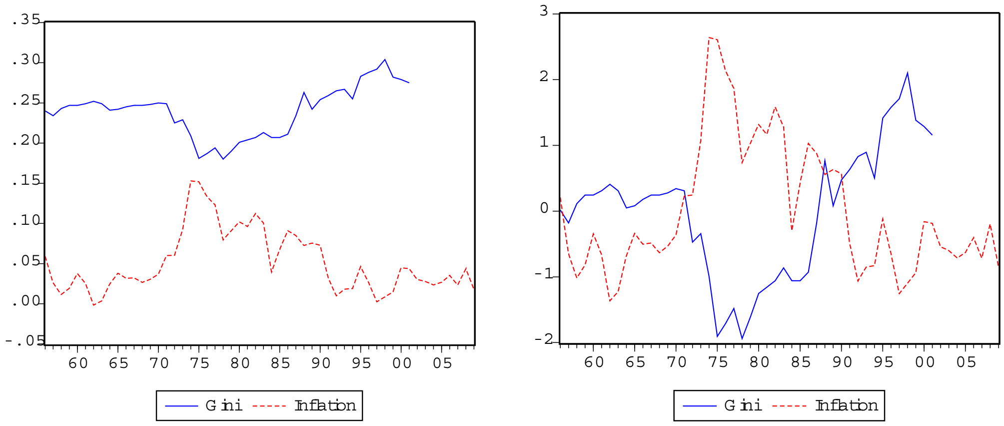
Figure A.6.4: Inflation and Inequality: US 1956-2008

[1]The U.S. Gini is the family income Gini coefficients from the Current Population Survey (CPS). The Gini coefficient of the U.K. is the household income Gini from the Institute For Fiscal Studies (IFS) spreadsheet. The Australian income Gini is based on the income Gini in Leigh (2005). The Swedish income Gini is household income Gini from Statistics Sweden.



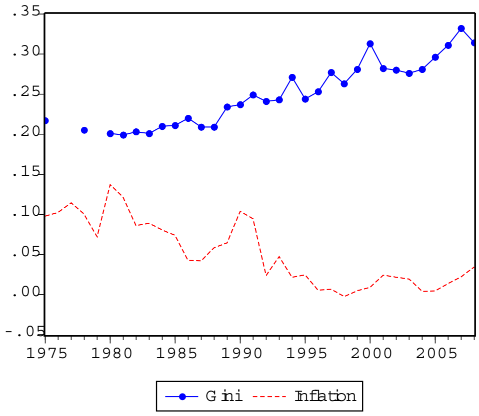
(a) Linear Scaling (b) Normalized Scaling

Figure A.6.5: Inflation and Inequality: UK 1956-2008

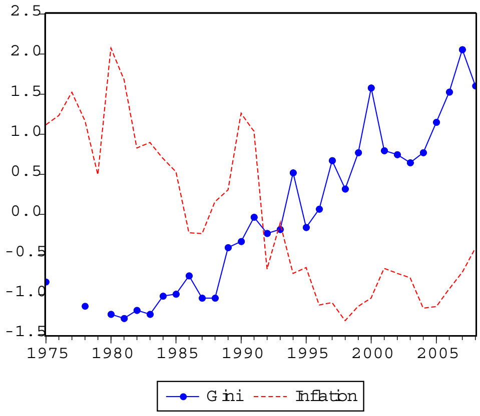


(a) Linear Scaling (b) Normalized Scaling

Figure A.6.6: Inflation and Inequality: Australia 1956-2008



(a) Linear Scaling

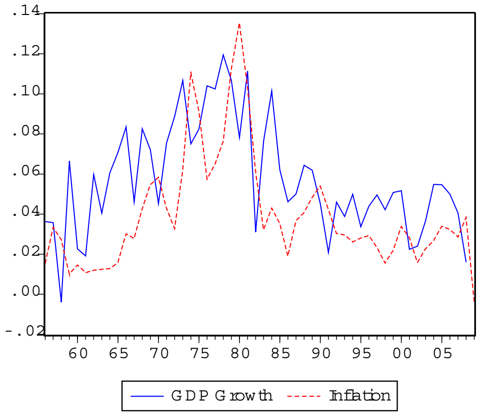


(b) Normalized Scaling

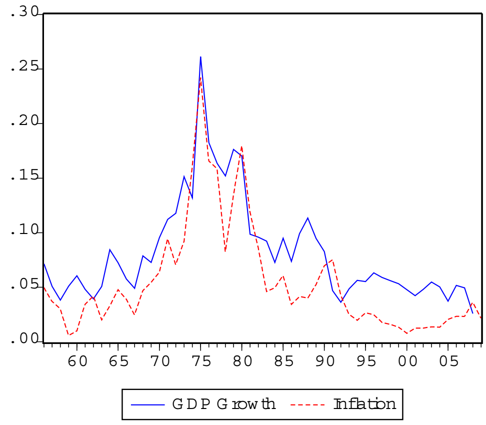
Figure A.6.7: Inflation and Inequality: Sweden 1975-2008

Table A.6.2: Correlation between Inflation and the Gini Index in the Four Countries

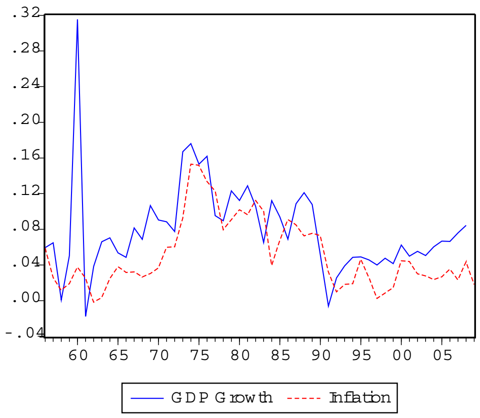
	US	UK	Australia	Sweden
Correlation	-0.318	-0.617	-0.755	-0.751
P-value	0.021	0.000	0.000	0.000



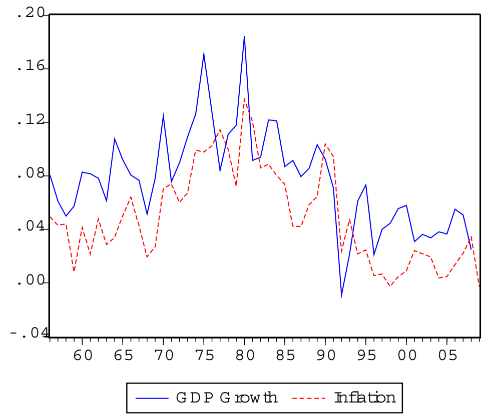
(a) US



(b) UK



(c) Australia



(d) Sweden

Figure A.6.8: Inflation and the GDP Growth: 1956-2008

In addition, Figure A.6.8 plots the time series of the GDP growth and inflation for the four countries. On inspection, one can observe that in all the countries, inflation is positively correlated with the growth of GDP, which is again confirmed by significantly positive correlation in Table A.6.3

Table A.6.3: Correlation between Inflation and the GDP Growth in the Four Countries

	US	UK	Australia	Sweden
Correlation	0.5669	0.8985	0.5582	0.7553
P-value	0.000	0.000	0.000	0.000

Therefore, the above evidence from the four countries shows that inflation is negatively correlated with income inequality and positively correlated with the growth of GDP, which is consistent with the predictions of the dynamic model.