The London School of Economics and Political Science

Essays on Market Microstructure

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

This thesis contains three essays on market microstructure.

Chapter 1 studies how endogenous information acquisition affects financial markets by modelling potentially informed traders who optimally acquire variable information at increasing cost. Prices affect the informed trading by providing incentives for acquiring information. Endogenous information acquisition explains the stylised facts that informed trading and transaction volume spike after informational events and fall over time. My model also tells a cautionary tale for interpreting measures of informed trading. Three common empirical proxies derived under the exogenous assumption (spreads, Easley O'Hara's PIN and blockholder interest) do not agree with each other in my setup.

Chapter 2 develops a more general framework with endogenous information acquisition which I use to examine the behaviour of an optimal monopolistic market maker. Unlike a competitive market maker, he sets prices to increase information revelation which is valuable to him. I characterise market information structure by whether narrower or wider spreads increase the information revealed by trades. An optimal monopolistic market maker may behave differently from the standard exogenous information benchmark. He may set narrower spreads in early periods. On average, spreads may widen over time. The different results arise from the interaction of a monopolistic market maker with endogenous information acquisition.

Chapter 3 studies the impact of confidential treatment requests made by institutional investors to the Securities and Exchange Commission (SEC) to delay disclosure of their holdings. The SEC requires the manager to present a coherent on-going trading program in his request for confidential treatment. If granted, he is restricted to trade in a manner consistent with his reported forecast in the subsequent period. Under the restriction, the manager earns higher expected profits by applying for con-

fidential treatment only if his probability of success exceeds a threshold. The model predicts that the price impact of a disclosed trade due to a confidential treatment request denial is greater than that of a disclosed trade where there is no request.

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Chapter 1

Endogenous Information Acquisition with Sequential Trade

1.1 Introduction

The theoretical market microstructure literature generally assumes that some subset of traders in financial markets are exogenously informed: they know the true value of the asset before arriving in the market. While some authors[1] have considered endogenous information acquisition, the empirical literature on informed trading is still based on the exogenous assumption. For example, Dennis and Weston (2001) identify four commonly used measures[2] to capture informed trading all of which are derived from structural models with exogenously informed traders.

However, investors such as mutual funds, hedge funds and investment banks, clearly rely on costly research to inform their trading decisions. There is also growing evidence to suggest that traders need to exert effort to learn about the effect of news

[1] Starting with Grossman and Stiglitz (1980), see Section 1.2 for a more comprehensive review

[2] 1) bid-ask spread based on Glosten and Milgrom (1985), Glosten and Harris (1988) and Amihud and Mendelson (1985); 2) adverse selection component of spread based on Huang and Stoll (1997); 3) price impact of trade based on Kyle (1985), Foster and Viswanathan (1993) and Hasbrouck (1991); and 4) probability of informed trading based on Easley et al. (1996).

on asset values[3]. To capture this feature, Peng (2005)[4] considers traders who allocate their limited attention between different sources of risk.

I take a more classical approach by modelling endogenously informed traders who can acquire costly information. My model can explain various stylized features of intraday markets which exogenous models cannot. My results also suggest caution when interpreting empirical measures based on the exogenous information assumption.

Building on the sequential trade framework of Glosten and Milgrom (1985), I replace exogenously informed traders with 'potentially informed' ones. These new traders choose how much information to acquire as a function of expected speculative profits from trading, which depend on posted prices. They learn the true value of the asset with a higher probability if they acquire more information and submit a trade only if they successfully learn it, doing nothing otherwise. This setup is different from the approach of Grossman and Stiglitz (1980) who only consider acquiring a fixed amount of information at a fixed cost.

In this chapter, I study a competitive market maker which corresponds well to many different financial markets. Under a general specification for information acquisition, I derive conditions for the existence of interior prices. Unlike with exogenous information, prices and beliefs do not always converge to the true value in the steady state. If the cost function is discontinuous, potentially informed traders eventually stop acquiring information. Trades stop revealing information and market participants then stop updating beliefs, an event I call 'information stoppage'.

I then study a special case with a quadratic information acquisition cost function which lets me characterise costs with a single parameter and solve for prices in closed form. This setup generates three main results[5].

[3]E.g. Hong et al. (2007), Hou and Moskowitz (2005) and Corwin and Coughenour (2008)

[4]Also Peng and Xiong (2006) and VanNieuwerburgh and Veldkamp (2010)

[5]Although these results quantitatively depend on the quadratic specification, the qualitative features would obtain under other cost functions.

First, my model can capture dynamics for informed trading and transaction volume which exogenous models cannot. At an intraday level, real markets exhibit higher volume and more informed trading after an informational event. An event is a shock which causes market prices to deviate from their true value. After an event, potentially informed traders can make speculative profits if they learn the true value of the asset. Therefore they acquire more information and trade more. Another stylized fact is that informed trading and volume fall over time. In my model, as prices converge to the true value, there are lower speculative profits to incentivise potentially informed traders. Therefore they acquire less information and trade less. These dynamics are driven by endogenous information acquisition. Exogenous models assume informed trading and volume are constant throughout the day.

Empirically, there is also significant variation in informed trading and volume between different days. In my model, this corresponds to variations in the size of shocks. Large shocks lead to large price deviations which give potentially informed traders more incentive to acquire information and trade. In exogenous models, prices never affect the behaviour of informed traders. To capture the effect, authors introduce ad hoc variations in the arrival rate of informed traders. For example, Easley and O'Hara (1992) assume event days when volume and informed trading is high, and non event days when they are low. This yields two regimes but also requires the strong assumption that the market maker is uncertain about whether an event has occurred[6]. My model can explain greater variation without the need for event uncertainty.

Second, I find deviations in three common proxies for informed trading: bid ask spreads, Easley et al. (1996)'s PIN and the proportion of hedge fund or block holder interest. In the competitive framework, prices are set as the expected asset value

[6]Easley et al. (2008) suggest a GARCH process for the arrival of informed traders. Easley et al. (2012) scale arrival rate by volume.

conditional on a trade so spreads reveal the amount of updating after a trade. Easley et al. (1996)'s PIN is a structural estimator for the probability of an informed trade. Finally, hedge funds or block holders can be considered informed so their participation is an indicator of informative trading. With exogenous information arrival, the three measures always agree. When there are more informed traders, the probability of an informed trade is high, trades reveal more information and spreads are always wider. However, the three proxies do not always agree with endogenous information acquisition.

In my model, spreads and the probability of an informed trade may diverge with respect to an increase in the proportion of potentially informed traders in the market. As there are more potentially informed traders, they individually acquire less information because there are lower informational rents available from noise traders. At some point, increasing their proportion actually leads to a fall in aggregate information acquisition. My result is also in contrast to Grossman-Stilgitz in which traders can only acquire a fixed quantity of information. They find that more potentially informed traders always leas to a higher probability of informed trade.

On the other hand, spreads are monotonically increasing in the proportion of potentially informed traders as in the standard case. Spreads are determined by the ratio of informed to noise trades. While informed trades fall, noise trades are also falling. Thus, spreads can be wide while the probability of an informed trade is low. Trades can be very informative but occur infrequently.

The three proxies suffer from different deficiencies and should not be used interchangeably. Spreads do not capture the frequency of trading which is an important component of information. The PIN estimator tries to address this but with endogenous information acquisition, it is misspecified. Finally, proxies for potentially informed traders are dangerous because the probability of an informed trade is not monotonic in the proportion of the potentially informed.

My third main result is that in numerical simulations, prices converge more slowly to the true value under endogenous information acquisition compared to the exogenous case. It takes longer for information to enter the market because informed trading falls over time. As expected, the convergence rate is decreasing in the information acquisition cost and increasing in the proportion of potentially informed traders.

The rest of this chapter proceeds as follows: In Section 1.2, I present some related literature. In Section 1.3, I introduce the model with a general cost distribution and show some comparative statics. In Section 1.4, I develop the model with a uniform cost distribution, derive comparative statics and numerical dynamic results. In Section 1.5, I discuss a new structural estimator. In Section 1.6, I conclude. Proofs are in the Appendix.

1.2 Related Literature

1.2.1 Exogenous Information Acquisition

Amongst the standard exogenous information acquisition literature, my model is most closely related to Glosten and Milgrom (1985). I share their defining features: sequential trade, a unit of asset traded each period,a competitive market maker and price inelastic noise traders drawn from a continuum of two types of traders. This is the exogenous information benchmark against which I set my results.

Another basis for comparison is Easley and O'Hara (1992),the theoretical foundation for Easley et al. (1996)'s PIN measure, one of the most widely used empirical estimators for informed trading. They augment the sequential framework of Glosten-Milgrom with a continuous time arrival process for traders to yield a mixed model which can be estimated using maximum likelihood from transactions data. PIN is of interest because it is widely used in the empirical literature as a measure for informed trading. The similarity of their setup to mine means that my results also affect their measure.

1.2.2 Endogenous Information Acquisition

The benchmark for endogenous information models is Grossman and Stiglitz (1980). The defining feature of their model is that traders can choose to observe a signal about the return of the risky asset at a constant cost, either becoming informed or staying uninformed. In equilibrium, both traders have the same expected utility. When more traders become informed, the price system becomes more informative and reveals more information to uninformed traders. My model preserves much of the intuition from Grossman-Stiglitz within a sequential trade framework. A drawback of their setup is that traders are homogenous and receive the same signal at a fixed cost.

Verrecchia (1982) considers heterogenous traders who can acquire variable information whose quality is increasing in its cost. In this setup, prices perform an extra role in aggregating heterogenous information. The information acquisition decision of a trader depends on how much information is revealed through prices in equilibrium. My model also features a transmission mechanism from prices to the amount of information acquired although it operates through ex ante expected profits instead of information revelation.

Litvinova and Hui (2003) add variable cost and precision to Grossman-Stiglitz. They find that some of the original results fail to hold with a different form of endogenous information acquisition. Traders exert less effort to acquire information when more of them do so. Thus, the equilibrium price system is not necessarily more informative when more traders acquire information. I also find this feature in my model. Furthermore, they find that more traders may acquire information even if the cost of acquiring information increases and equilibria do not always exist. They conclude that endogenous information acquisition should be taken seriously in the context of asymmetric information models.

Ko and Huang (2007) focus on overconfident traders who face variable information acquisition costs in a Grossman (1976) setup. They find that overconfidence generally improves market efficiency by driving prices closer to true values. While behavioural agents are their main concern, their results rely crucially on the form of endogenous information acquisition. In contrast, Garcia et al. (2007) look at overconfidence with traders who pay fixed costs for information. They reach the opposite conclusion that overconfidence has no effect on market efficiency and prices. These contrasting findings underline the importance of how we model endogenous information acquisition.

My model is also related to Peng (2005) and Peng and Xiong (2006) in which traders have capacity constraints on their ability to process information and face multiple sources of uncertainty. In equilibrium, they endogenously allocate their capacity to learn about these different sources to minimize wealth uncertainty and make intertemporal consumption decisions. This mechanism captures a similar intuition to my model in which rational agents choose the amount of information to acquire at variable cost.

Other notable contributions include Admati and Pfleiderer (1986, 1987, 1988) and Veldkamp (2006). In their series of papers, Admati and Pfleiderer introduce a market for information which is parallel to the standard asset market. They study how prices are set and how traders behave in both marekts. Veldkamp (2006) develops a competitive information production sector that supplies information at an endogenous price. She models information as a non rivalrous good with a novel production technology which increases its output and lowers price following an increase in demand. This setup generates media frenzies and price herding.

1.3 General Information Acquisition Cost Function

1.3.1 Model Description

This section takes the discrete time Glosten-Milgrom trading framework and replaces exogenously informed traders with potentially informed ones who face costly information acquisition. They optimally choose how much information to acquire as a function of their expected profits which depend on prior beliefs and posted ask and bid prices.

There is one traded asset with value $\hat{v}$ which takes two possible terminal values V and 0 where $V > 0$. A unit of the asset is traded every period. At time t the market maker and potentially informed traders have the same prior belief that $\hat{v} = 0$ with probability μ_t, and $\hat{v} = V$ with probability $1 - \mu_t$. The markets contains three agents: the market maker, the potentially informed trader, and the noise trader. The market maker is risk neutral and competitive. He posts ask and bid prices $\{a_t, b_t\}$ from $P \subset \mathbb{R}_+$ which contains the possible values of $\hat{v}$.

A trader is drawn to trade each period from a continuum of traders. A proportion λ of them is potentially informed while the remaining proportion $1 - \lambda$ are noise traders. Noise traders do not maximise profits and trade for exogenous reasons. They submit trades q_t randomly, either a buy, $q_t = +1$, or a sell, $q_t = -1$, with equal probability $\frac{1}{2}$.

Potentially informed traders are constrained in their actions to be either buyers or sellers with equal probability $\frac{1}{2}$. Buyers can only choose to submit a buy trade or no trade, but not a sell, and similarly sellers can only choose to submit a sell trade or no trade, but not a buy. This assumption is nonstandard but does not qualitatively affect any of my results. I also relax it in Chapter 2. I use it here for analytical simplicity because it yields closed form solutions for ask and bid prices.

Like the standard informed trader, potentially informed traders trade for speculative profits. However they learn the true value of the asset $\hat{v}$ with some probability given by the information arrival functions, $X_{a,t}(a_t)$ for buyers and $X_{b,t}(b_t)$ for sellers, defined over prices $a_t \geq b_t$ and beliefs $\mu_t \in [0, 1]$ at time t. The functions are separate for buyers and sellers because buyers only care about the ask price a_t, and sellers the bid price b_t. When potentially informed traders can do both there is only one information arrival function and it depends on both prices.

$X_{a,t}$ and $X_{b,t}$ capture how prices affect the amount of information potentially informed traders acquire. I restrict them to be consistent with this intuition. $X_{a,t}$ is weakly decreasing in the ask price a_t because expected profits decrease in a_t so potentially informed traders acquire less information. Similarly, X_b is weakly increasing in the bid price b_t. This specification nests the standard Glosten-Milgrom version of exogenously informed traders. My model is equivalent to theirs when $X_{a,t}$ and $X_{b,t}$ are unity for all prices and beliefs.

Section 1.4 develops a microfoundation for the information arrival functions $X_{a,t}$ and $X_{b,t}$. In that setup, potentially informed traders see posted prices, a_t and b_t, and choose how much information to acquire at increasing quadratic cost. $X_{a,t}$ and $X_{b,t}$ describe the solutions for the optimal amount of information that potentially informed traders acquire. They are also consistent with other interpretations. For example, potentially informed traders may have private reservation values or be exogenously price elastic. In this chapter, I maintain the information acquisition story although my results in this section only require the weak restrictions described above.

The introduction of potentially informed traders creates a transmission channel from prices to the amount of information traders acquire which drives most of my later results. It is intuitively similar to the information acquisition equilibrating mechanism in Grossman-Stiglitz. In their model, a proportion of traders chooses to become informed while the rest remain uninformed depending on the fixed information acquisition cost. In mine, the proportion of uninformed traders is fixed but the increasing information acquisition cost determines how much information potentially informed traders acquire. My specification incorporates this information acquisition decision into Glosten-Milgrom in a tractable way which can be used to investigate multiperiod dynamics.

The timeline for each period t is as follows: 1) the market maker posts ask and bid prices $\{a_t, b_t\}$ based on prior beliefs μ_t; 2) a trader is drawn from the continuum of traders with unit mass, potentially informed buyers with probability $\frac{1}{2}\lambda$, potentially informed sellers with probability $\frac{1}{2}\lambda$, or noise traders with probability $1 - \lambda$; 3) the trader submits a unit trade, either a sell, a buy or no trade, $q_t \in \{-1, 0, 1\}$; 4) the market maker completes the trade and forms posterior beliefs $\mu_{t+1}(q_t)$ by Bayes' rule.

1.3.2 Solving the Model

The market maker solves for zero profit ask and bid prices taking into account the best response of potentially informed traders. Let $B_{V,t}(a_t)$ be the conditional probability that a trader submits a buy order if $\hat{v} = V$ and $B_{0,t}(a_t)$ if $\hat{v} = 0$:

$$B_{V,t}(a_t) = \frac{1}{2}\lambda X_{a,t}(a_t) + \frac{1}{2}(1 - \lambda) \tag{1.1}$$

$$B_{0,t}(a_t) = \frac{1}{2}(1 - \lambda) \tag{1.2}$$

Here I assume that potentially informed buyers always submit a buy order if they learn the true asset value is high, $\hat{v} = V$. This is optimal as long as $a_t < V$. Furthermore, they do not trade if $\hat{v} = 0$. The assumption is equivalent to enforcing that the market is always 'open'.

Definition 1.1. *The market is open (closed) at time t on the ask side if it allows* *(excludes) profitable informed trade:* $a_t < \theta_2$ *(*$a_t \geq \theta_2$*). The market is open (closed) at time* t *on the bid side if it allows (excludes) profitable informed trade:* $b_t > \theta_t$ *(*$b_t \leq \theta_1$*). The market is open if it is open on at least one side.*

This definition only depends on the participation of informed traders. Noise traders continue to trade even in a 'closed' market. Under a competitive market maker, markets are always open because with price inelastic noise traders, the only way to obtain the zero profit condition is to trade with informed traders. Thus the 'open' assumption holds.

Analogously, let $S_{V,t}(b_t)$ be the conditional probability that a trader submits a sell order if $\hat{v} = V$ and $S_{0,t}(b_t)$ if $\hat{v} = 0$:

$$S_{V,t}(b_t) = \frac{1}{2}(1-\lambda) \tag{1.3}$$

$$S_{0,t}(b_t) = \frac{1}{2}\lambda X_{b,t}(b_t) + \frac{1}{2}(1-\lambda) \tag{1.4}$$

The probability that a buy order is submitted in period t is:

$$\mu_t B_{0,t}(a_t) + (1-\mu_t) B_{V,t}(a_t) \tag{1.5}$$

and the probability that a sell order is submitted in period t is:

$$\mu_t S_{0,t}(b_t) + (1-\mu_t) S_{V,t}(b_t) \tag{1.6}$$

As in Glosten-Milgrom, and shown in Proposition 2.2 for a more general setup, under the zero profit condition, the market maker sets the ask price a_t^c as the expected value of the asset conditional on a buy order $q_t = +1$ and beliefs μ_t:

$$\begin{aligned} a_t^c &= E[v|q_t = +1] \\ &= \frac{(1-\mu_t)B_{V,t}(a_t)V}{\mu_t B_{0,t}(a_t) + (1-\mu_t)B_{V,t}(a_t)} \\ &= \frac{(1-\mu_t)[\lambda X_{a,t}(a_t) + 1 - \lambda]V}{(1-\mu_t)\lambda X_{a,t}(a_t) + 1 - \lambda} \end{aligned} \tag{1.7}$$

Analogously, he sets the bid price b_t^c as the expected value of the asset conditional on a sell order $q_t = -1$ and μ_t:

$$\begin{aligned} b_t^c &= E[v|q_t = -1] \\ &= \frac{(1-\mu_t)S_{V,t}(b_t)V}{\mu_t S_{0,t}(b_t) + (1-\mu_t)S_{V,t}(b_t)} \\ &= \frac{(1-\mu_t)(1-\lambda)V}{\mu_t \lambda X_{b,t}(b_t) + 1 - \lambda} \end{aligned} \tag{1.8}$$

The market maker completes the trade q_t and forms his posterior belief $\mu_{t+1}(q_t)$ using Bayes' rule. His belief after a buy trade is:

$$\begin{aligned} \mu_{t+1}(+1) &\equiv pr(v = 0 \mid q_t = -1, a_t) \\ &= \frac{\mu_t B_{0,t}(a_t)}{\mu_t B_{0,t}(a_t) + (1-\mu_t)B_{V,t}(a_t)} \\ &= \frac{\mu_t(1-\lambda)}{(1-\mu_t)\lambda X_{a,t}(a_t) + 1 - \lambda} \end{aligned} \tag{1.9}$$

After a sell trade, it is:

$$\begin{aligned} \mu_{t+1}(-1) &\equiv pr(v = 0 \mid q_t = +1, b_t) \\ &= \frac{\mu_t S_{0,t}(b_t)}{\mu_t S_{0,t}(b_t) + (1-\mu_t)S_{V,t}(b_t)} \\ &= \frac{(1-\mu_t)(\lambda X_{b,t}(b_t) + 1 - \lambda)}{\mu_t \lambda X_{b,t}(b_t) + 1 - \lambda} \end{aligned} \tag{1.10}$$

Proposition 1.1. *Zero profit ask and bid prices* (a_t^c, b_t^c) *exist in the range* $[0, V]$ *if the information arrival functions* $X_{a,t}(a_t)$ *and* $X_{b,t}(b_t)$ *are continuous over* $a_t \in [0, V]$ *and* $b_t \in [0, V]$ *respectively.*

The continuity of the information arrival functions $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$ is sufficient to obtain a single crossing property which ensures the existence of prices. The most obvious violation is if potentially informed traders face a fixed cost of information acquisition. Depending on prices, either all potentially informed traders acquire information or none do. Zero profit prices do not exist in general. However, if they do not, the market maker can still open the market by making positive profits.

1.3.3 Prices, Convergence and Information Stoppage

This subsection presents some static features and convergence results from my general setup. I find a new feature I call 'information stoppage' which may arise with endogenous information acquisition.

Proposition 1.2. *If zero profit ask price and bid prices* (a_t^c, b_t^c) *exist, they are monotonically decreasing in the prior belief* μ_t. a_t *and* b_t *tend to the true value* $\hat{v}$ *as* μ_t *tends to certainty, i.e.* $\mu_t = 1$ *or* 0.

Proposition 1.2 gives the standard result that zero profit prices a_t^c and b_t^c are a monotonic function of beliefs and converge to the true value as beliefs tend to certainty. The mid point of prices is the expected value of the asset conditional on beliefs at time t.

Corollary 1.1. *If zero profit ask and bid prices* (a_t^c, b_t^c) *exist and potentially informed buyers and sellers acquire information with strictly positive probability, i.e.* $X_{a,t}(a_t) \geq 0$ *and* $X_{b,t}(b_t) \geq 0$ *for all beliefs* μ_t*, then* a_t^c *and* b_t^c *converge to the true value in the steady state.*

Corollary 1.1 obtains the standard convergence result. If zero profit prices exist and potentially informed traders always acquire some information, then trades always reveal information. The market maker can update beliefs after every trade. Over time, in expectation, beliefs update correctly and prices converge to the true value. Prices always converge with exogenous information acquisition. However, with endogenous information acquisition, an information stoppage can occur. The information arrival functions, $X_{a,t}$ and $X_{b,t}$, for potentially informed traders can be 0 and trades stop revealing information.

Definition 1.2. *An 'information stoppage' occurs if the market is 'open' and the probability that a trader submits an informed trade is* 0*.*

Corollary 1.2. *If an information stoppage occurs in any period, prices do not converge to the true value in the steady state.*

If an information stoppage occurs, the market maker stops updating beliefs because he knows that potentially informed traders stop acquiring information. Both prices a_t and b_t are constant until the final period T once a stoppage occurs. By Definition 1.1, the market can still be open because prices are in the interior of possible asset values $(0, V)$. An informed trader could make a profitable trade if he were drawn into the market but potentially informed traders have no incentive to acquire that information.

An information stoppage in my model is similar to the no trade result of Grossman-Stiglitz. In their model, if the information acquisition cost is too high, no traders acquire information and thus there is no trade. In mine, potentially informed traders may choose to acquire no information for a similar reason. However, unlike in their model, noise traders continue to trade and the market remains open. Also, an information stoppage can occur in any period so a market may start off informative but enter an information stoppage later.

It would be difficult to identify information stoppages empirically because I would need to compare the fundamental value of the asset to a steady state price, neither of which are observable. This model best relates to high frequency markets in which informational events occur frequently and prices do not reach a steady state. However, markets do exhibit periods of low transaction volume with trades having a low price impact which are consistent with an information stoppage.

1.3.4 Relative to Exogenous Information Acquisition

This subsection compares prices, information revelation and transaction volume of a market with endogenous information acquisition to the Glosten-Milgrom benchmark with exogenous information acquisition.

Proposition 1.3. *The zero profit spread in a market with potentially informed traders is weakly narrower than in a market with the same proportion of exogenously informed traders.*

The competitive market maker sets zero profit prices by balancing expected profits from noise traders with losses to informed traders. In my model, only a fraction of potentially informed traders acquire information and trade while exogenously informed traders always trade. To meet the zero profit condition, the market maker sets narrower spreads than the exogenous case. Narrower spreads reduce profits from noise traders and increase the participation of potentially informed traders.

With a competitive market makers, spreads measure the information revealed by trades because they are proportional to the change in beliefs conditional on that trade occurring. By the zero profit condition, prices can be written as:

$$a_t^c = (1 - \mu_{t+1}(+1))V \tag{1.11}$$

$$b_t^c = (1 - \mu_{t+1}(-1))V \tag{1.12}$$

Since spreads are narrower, trades reveal less information in a market with potentially informed traders compared to one with the same proportion of exogenously informed traders.

Corollary 1.3. *Equilibrium expected transaction volume in a market with potentially informed traders is weakly lower than in a market with the same proportion of exogenously informed traders.*

With exogenous information acquisition, expected transaction volume is fixed and constant. In my case, potentially informed traders choose how much information to acquire and thus, how often they trade. Since they acquire information with probability weakly less than 1, expected transaction volume must be lower than with exogenous information acquisition. However, a more important feature model of my model is that transaction volume evolves dynamically. I expand on this in the next section.

1.4 Quadratic Information Acquisition Function

This section develops a specific microfoundation for the information arrival function of potentially informed traders $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$. Potentially informed traders choose how much information to acquire at increasing quadratic cost. This setup lets me characterise information acquisition costs with a single parameter and solve for closed form solutions for prices. I then examine the impact of information acquisition cost, beliefs and potentially informed traders on prices, information revealed by trades, expected transaction volume and the behaviour of potentially informed traders.

1.4.1 Setup

The potentially informed trader can learn the true value of the asset $\hat{v}$ with probability ω by paying the cost $\frac{1}{2}C\omega^2$, where C is a positive parameter which scales the cost of information acquisition. As risk neutral, profit maximising agents, they optimally choose the amount of information ω^* to acquire. Before acquiring information, they have the same prior beliefs as the market maker. A potentially informed buyer acquires the optimal amount of information $\omega^*_{a,t}$ by solving:

$$\max_{\omega_{a,t}} (1-\mu_t)\omega_{a,t}(V-a_t) - \frac{1}{2}C\omega^2_{a,t} \tag{1.13}$$

A potentially informed seller acquires $\omega^*_{b,t}$ by solving:

$$\max_{\omega_{b,t}} \mu_t\omega_{b,t}b_t - \frac{1}{2}C\omega^2_{b,t} \tag{1.14}$$

The solutions determine the information arrival functions $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$ for potentially informed buyers and sellers:

$$\omega^*_{a,t} = X_{a,t}(a_t) = \frac{1}{C}(1-\mu)(V-a_t) \tag{1.15}$$
$$\omega^*_{b,t} = X_{b,t}(b_t) = \frac{1}{C}\mu b_t \tag{1.16}$$

After a potentially informed trader acquires information, there is a random draw to determine if he learns the true value. A seller submits a sell trade $q_t = -1$ if he learns that the true value is low, $\hat{v} = 0$, and no trade otherwise. Similarly, a buyer submits a buy trade $q_t = +1$ if he learns that the true value is high, $\hat{v} = V$, and no trade otherwise. From the market maker's perspective, a potentially informed trader submits a trade q_t with probabilities:

$$q_t = \begin{cases} -1 & \text{with probability } \frac{1}{2C}\mu^2 b_t \\ +1 & \text{with probability } \frac{1}{2C}(1-\mu)^2(V-a_t) \\ 0 & \text{with probability } 1-\frac{1}{2C}\mu^2 b-\frac{1}{2C}(1-\mu)^2(V-a_t) \end{cases}$$

The market maker knows the potentially informed traders' best response functions and sets competitive prices accordingly. The assumption that potentially informed traders are either buyers and sellers means that the zero profit conditions for the ask and bid prices can be solved separately.

Proposition 1.4. *If $C \geq \frac{1}{4}V$, a competitive market maker posts unique ask and bid prices, a_t^c and b_t^c, given by:*

$$a_t^c = V - \frac{(1-\lambda)C}{2\lambda(1-\mu_t)^2}\left[\sqrt{1+\frac{4\mu_t(1-\mu_t)^2\lambda V}{(1-\lambda)C}}-1\right] \tag{1.17}$$

$$b_t^c = \frac{(1-\lambda)C}{2\lambda{\mu_t}^2}\left[\sqrt{1+\frac{4\mu_t^2(1-\mu_t)\lambda V}{(1-\lambda)C}}-1\right] \tag{1.18}$$

Proposition 1.4 requires the restriction that the cost of information acquisition C is sufficiently large relative to the maximum value of the asset V. This restriction implies that buyers and sellers acquire information with probabilities weakly less than 1 across the ranges of prior beliefs μ_t and proportions of potentially informed traders λ and thus prices always take the form in the proposition. If the restriction is relaxed, $C < \frac{1}{4}V$, then prices may imply a probability of information acquisition greater than 1. Prices still exist but they are solved like the exogenous case when traders receive information with probability 1. I restrict C since I am interested in cases when potentially informed traders do not acquire full information.

1.4.2 Spreads

This subsection derives comparative statics for the effect of information acquisition cost C and proportion of potentially informed traders λ on spreads. In any given market, these are fixed exogenous variable, so these statics are for comparisons between different markets with other variables held constant, in particular, prior beliefs μ_t. While μ_t evolves endogenously over time, for now I take them as exogenous.

Proposition 1.5. *The competitive ask price a_t^c is monotonically decreasing, while the bid price b_t^c is monotonically increasing, in the information acquisition cost C. a_t^c and b_t^c tend to the conditional expected asset value $(1-\mu_t)V$ as C tends to infinity.*

By Proposition 1.5, spreads are decreasing in the information acquisition cost C. When C increases, information costs more so, for any set of posted prices, potentially informed traders acquire less. Under the zero profit condition, the market maker sets narrower spreads to give potentially informed traders more incentive to acquire information while reducing expected profits from noise traders.

As the cost of information acquisition C grows to infinity, potentially informed traders stop acquiring information and the model collapses to one without informed traders. Prices are set at the unconditional expected value of the asset. Proposition 1.4 imposes a lower bound for C: $C \geq \frac{1}{4}V$. If I relax the restriction, as C tends to 0, prices converge those from standard Glosten-Milgrom with no information acquisition costs in which potentially informed agents acquire full information.

As described previously, spreads are proportional to how beliefs are updated and thus measure the information revealed by trades. Therefore, following spreads, trades reveal less information as the information cost C rises, in agreement with Grossman-Stiglitz. In their model, a proportion of traders pay the cost to become informed while the rest remain uninformed. When the cost increases, fewer traders become informed so trades are less informative. The information revealed by trades is directly related to the proportion of informed traders. While the intuition is similar, the transmission mechanism in my model is different. The proportion of potentially informed traders is exogenously fixed but trades reveal less information because each trader acquires less information. Thus there is not necessarily a monotonic relationship between the proportion of potentially informed traders and the information revealed by trades.

The information acquisition cost C also affects the expected profits of traders differently in my model compared to Grossman-Stiglitz. In their case, all traders, informed or uninformed, make the same expected profits in equilibrium. A rise in information acquisition cost lowers profits to all traders equally. In my case, similar to Glosten-Milgrom, potentially informed traders make positive expected profits and noise traders make expected losses. Increasing C lowers expected profits to potentially informed traders but also lowers expected losses to noise traders through narrower spreads.

Grossman-Stiglitz has been tested empirically by comparing the performance of passive index mutual funds, as a proxy for uninformed traders, to actively-managed funds, as a proxy for informed traders. Their model predicts that the two should perform similarly. In general, the literature studying mutual fund performance, such as Wermers (2000), Kosowski et al. (2006) and Banegas et al. (2012), find that actively-managed funds out perform the index. While evidence against Grossman-Stiglitz, it is consistent with the form of information acquisition in my model.

Proposition 1.6. *The competitive ask price a_t^c is monotonically increasing, while the bid price b_t^c is monotonically decreasing, in the proportion of potentially informed traders λ. a_t^c and b_t^c tend to the conditional expected asset value $(1-\mu_t)V$ as λ tends to 0. They tend to the V and 0 respectively as λ tends to 1.*

By Proposition 1.6, equilibrium spreads are monotonically increasing in the proportion of potentially informed traders λ. This result is analogous to Glosten-

Milgrom's result that spreads are increasing in the proportion of informed traders and is driven by the zero profit condition of a competitive market maker. When there are more potentially informed traders, the market maker sets wider spreads to decrease expected losses to them and increase expected profits from noise traders.

When there are no potentially informed traders, only noise traders, both prices are the unconditional expected value of the asset. No information is revealed by trades and there is no learning. When there are only potentially informed traders, the market maker sets the maximum spread and the standard no trade result obtains. The market maker closes the market because all trades are with informed traders which entail expected losses.

My model predicts the same relationship between the proportion of potentially informed traders λ and the spread as Glosten-Milgrom. However, the empirical support for this prediction is mixed. For example, Dennis and Weston (2001) find that the size of the spread is negatively related to the amount of institutional ownership, a proxy for informed traders while Heflin and Shaw (2000) find the opposite for block owners.

While many empirical studies use the spread as a measure for information based trading, it also includes other components such as the cost of market making and order processing costs. My model suggests another reason why it might be a poor measure. In standard Glosten-Milgrom, the arrival rate of informed traders is fixed. In my case, potentially informed traders enter at different rates depending on how much information they acquire. This yields another measure for information based trading: the probability of informed trade. The two are not equivalent because a trade may cause a large revision in beliefs but happen with low probability. The spread does not capture this dimension of how information enters the market. In the next subsection, I characterise when the two measures deviate.

1.4.3 Probability of an Informed Trade and Expected Transaction Volume

I define the probability of an informed trade $K_t(a_t, b_t)$ as the unconditional probability that a potentially informed trader, buyer or seller, is drawn into the market and submits an informed trade. It is the sum of the probability of an informed buy trade $G_t(a_t)$ and an informed sell trade $H_t(b_t)$ which are given by:

$$G_t(a_t) = \frac{1}{2}\lambda(1-\mu_t)X_{a,t}(a_t) = \frac{1}{2C}\lambda(1-\mu)^2(V-a_t) \tag{1.19}$$

$$H_t(b_t) = \frac{1}{2}\lambda\mu_t X_{b,t}(b_t) = \frac{1}{2C}\lambda\mu^2 b_t \tag{1.20}$$

The probability of informed trade $K_t(a_t, b_t)$ is empirically relevant because it is analogous to the widely used PIN measure proposed by Easley et al. (1996). PIN is the probability of an informed trade estimated from a structural model with exogenously informed traders. $K_t(a_t, b_t)$ and PIN are the same if in a market with exogenously informed traders but they can differ once I introduce endogenously informed traders.

Proposition 1.7. *In equilibrium, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is at its maximum when beliefs μ_t are weakest, i.e. $\mu_t = \frac{1}{2}$. $K_t(a_t^c, b_t^c)$ tends to 0 as beliefs tend to certainty, i.e. $\mu_t = 1$ or 0.*

By Proposition 1.7, the probability of an informed trade $K_t(a_t^c, b_t^c)$ responds intuitively to beliefs μ_t: it is largest when beliefs are weakest, $\mu_t = \frac{1}{2}$, tending to zero as beliefs tend to certainty, $\mu_t = 0$ or 1. When beliefs are weak, prices are far from their true value. An informed trade yields large speculative profits so potentially informed traders acquire the most information. As beliefs get stronger, prices move toward the true value. Expected profits from trading fall so potentially informed traders acquire less information.

Figure 1.1 shows this result graphically for some choice of model parameters $V =$

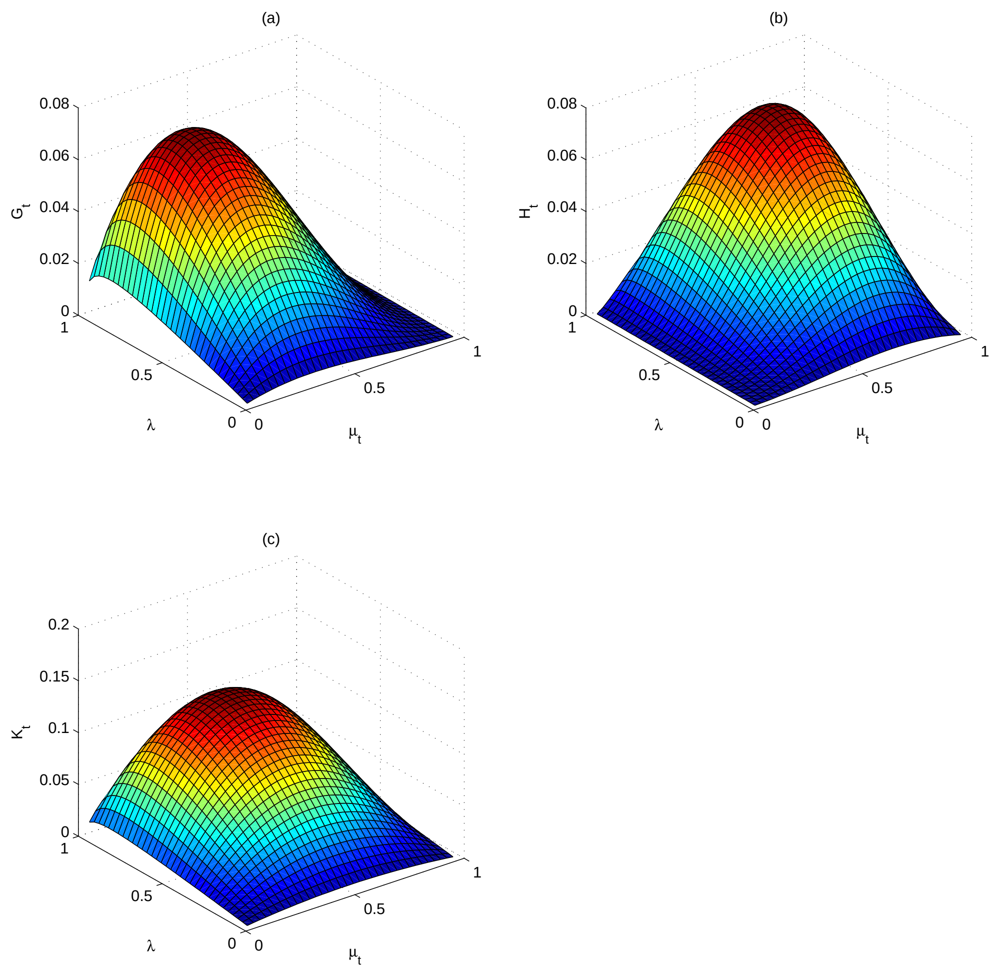

Figure 1.1: Probabilities of an informed: (a) buy trade $G_t(a_t^c)$; (b) sell trade $H_t(b_t^c)$; or (c) trade of either type $K_t(a_t^c, b_t^c)$; against prior beliefs μ_t and the proportion of potentially informed traders λ for model parameters $V = 10$ and $C = 5$.

10 and $C = 5$. Figure 1.1(c) plots the probability of an informed trade $K_t(a_t^c, b_t^c)$ against beliefs μ_t and proportion of potentially informed traders λ. For now, I am interested in μ_t so fix some proportion of λ and look across $K_t(a_t^c, b_t^c)$. The graph is a hump, symmetric about $\mu_t = \frac{1}{2}$. Note that the value of $K_t(a_t^c, b_t^c)$ also depends on λ but across any λ, the shape is the same.

Figures 1.1(a) and 1.1(b) plot the probabilities of an informed buy trade $G_t(a_t^c)$ and sell trade $H_t(b_t^c)$ against μ_t and λ. Unlike the aggregate probability $K_t(a_t^c, b_t^c)$, they are not symmetric about $\mu_t = \frac{1}{2}$. One of the advantages of separating potentially informed buyers and sellers is that I can see their different responses to μ_t. Figure 1.1(a) corresponds to buyers. $G_t(a_t^c)$ is skewed towards $\mu_t = 1$. Potentially informed buyers acquire more information when beliefs tend towards the low asset value because they can make higher profits if they learn that the true value is high. The opposite applies for sellers.

Proposition 1.7 also has implications for the dynamic behaviour of information acquisition. In expectation, beliefs μ_t converge to certainty about the true value over time. If the market starts with uninformative first period beliefs, $\mu_1 = \frac{1}{2}$, then in expectation, μ_t monotonically increases or decreases to 0 or 1 over time. Therefore, the expected probability of an informed trade $K_t(a_t^c, b_t^c)$ also decreases monotonically over time. With exogenously informed traders, it is constant. Note that while the *expected* paths of μ_t and $K_t(a_t^c, b_t^c)$ are monotonic, they need not be for any given realisation. μ_t may fluctuate over time and thus $K_t(a_t^c, b_t^c)$ may rise and fall.

Corollary 1.4. *Expected transaction volume $E_t[|q_t|]$ is at its maximum when beliefs μ_t are weakest, i.e. $\mu_t = \frac{1}{2}$. $E_t[|q_t|]$ tends to $1 - \lambda$ as beliefs tend to certainty, i.e. $\mu_t = 1$ or 0.*

Expected transaction volume $E_t[|q_t|]$ follows the probability of informed trades $K_t(a_t^c, b_t^c)$ in response to changes in prior beliefs μ_t. In my setup, $E_t[|q_t|]$ only depends on the participation of potentially informed traders since noise traders always trade. When potentially informed traders acquire less information, they trade less. Again, Corollary 1.4 also determines the dynamic behaviour of $E_t[|q_t|]$. Starting from an uninformative μ_1, in expectation, $E_t[|q_t|]$ falls over time, in contrast to Glosten-Milgrom.

The dynamic features of the probability of informed trade $K_t(a_t^c, b_t^c)$ and expected transaction volume $E_t[|q_t|]$ with endogenous information acquisition is more empirically appealing than the standard models with exogenously informed traders. A cursory look at high frequency trading data reveals periods of high volume and high participation by institutional traders, often considered informed, which tend to occur after informational events and fall over time. These stylised features are absent from Glosten-Milgrom.

Easley and O'Hara (1992) introduce uncertainty about whether an informational event occurs at the beginning of each trading day. Informed traders only enter the market if it does. This partially accounts for different levels of expected trading volume and informed participation. Easley et al. (1996) then estimate this structural specification. However, this model only allows two trading intensity regimes which last for a whole day. In my model, trading intensity evolves endogenously over time, even within the same day. This seems closer to the stylised features described above. Furthermore, I do not need another dimension of uncertainty. The market maker knows an informational event has occurred. The dynamics are driven by potentially informed traders acquiring different amounts of information over time.

Proposition 1.8. *In equilibrium, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is monotonically decreasing in the information acquisition cost C.*

By Proposition 1.5, spreads are decreasing in the information acquisition cost C. By Proposition 1.8, the probability of an informed trade $K_t(a_t^c, b_t^c)$ responds similarly.

Thus, the market is less informative under both measures as C increases.

Corollary 1.5. *Expected transaction volume $E_t[|q_t|]$ is monotonically decreasing in the information acquisition cost C.*

A higher information cost C leads to lower expected transaction volumes. Like in Corollary 1.4, expected transaction volume $E_t[|q_t|]$ follows the probability of informed trade $K_t(a_t^c, b_t^c)$. This also agrees with Grossman-Stiglitz. Together with Proposition 1.5, Proposition 1.8 and Corollary 1.5 describe all the effects of C in my model.

Fang and Peress (2009) offer some empirical support for Corollary 1.5. They find that media coverage affects the returns of some subset of stocks. If I can interpret media coverage as a proxy for information acquisition costs, because it captures the availability of public information information, then this is in line with my predictions.

Proposition 1.8 offers another testable prediction between information costs and the probability of an informed trade. Ideally I would estimate the probability of an informed trade from a model with endogenous information acquisition and then compare it between assets with different information acquisition costs.

Proposition 1.9. *The probabilities of an informed buy trade G_t^c and sell trade H_t^c have their maximum at $\widetilde{\lambda_t^G} = \frac{Z_t+1}{2Z_t+1}$ and $\widetilde{\lambda_t^H} = \frac{Y_t+1}{2Y_t+1}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$. G_t^c and H_t^c tend to 0 as the proportion of potentially informed traders λ tends to 0 or 1.*

By Proposition 1.9, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is not monotonically increasing in the proportion of potentially informed traders λ. It is decreasing in λ for $\lambda \geq \max\{\frac{Z_t+1}{2+z1}, \frac{Y_t+1}{2Y_t+1}\}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$. In this range, increasing the proportion of potentially informed traders leads to less frequent informed trades.

To see this result graphically, return to Figure 1.1(c) which plots the probability of an informed trade $K_t(a_t^c, b_t^c)$. Fix some prior belief μ_t and look across the proportion of potentially informed traders λ. For $\mu_t = \frac{1}{2}$, $K_t(a_t^c, b_t^c)$ increases with λ until it reaches its maximum at $\widetilde{\lambda_t^K} = \frac{W_t+1}{2W_t+1}$ where $W_t \equiv \frac{1}{2}\sqrt{\frac{1}{2C}V}$. $K_t(a_t^c, b_t^c)$ then falls rapidly to 0 as λ tends to 1. Note that the maximum $\widetilde{\lambda_t^K}$ occurs at larger values as beliefs are more certain, i.e. μ_t closer to 0 or 1.

Proposition 1.10. *The probabilities that a potentially informed buyer or seller is informed, $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$, are monotonically decreasing in the proportion of potentially informed traders λ. $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$ tend to $\frac{1}{C}\mu_t(1-\mu_t)V$ as λ tends to 0. They tend to 0 as λ tends to 1.*

Recall that the probability that a potentially informed buyer or seller is informed, $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$ is given by the amount of they choose to acquire. It starts at $\frac{1}{C}\mu_t(1-\mu_t)V$ and decreases monotonically to 0 with the proportion of potentially informed traders λ. The maximum probability is always weakly less than 1 because of the restriction that $C \geq \frac{1}{4}V$. By Proposition 1.10, increasing λ means each trader acquires less information. However, by Proposition 1.9, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is not monotonic in λ. To understand the two results, see that λ has two effects on $K_t(a_t^c, b_t^c)$: it 1) increases the number of traders who can choose to acquire information; and 2) decreases the amount of information acquired by each trader.

For low λ, the first effect dominates. An increase in λ outweighs the decrease in the information they acquire individually, as measured by $X_{a,t}(a_t^c)$ or $X_{b,t}(b_t^c)$. Thus the probability of an informed trade $K_t(a_t^c, b_t^c)$ increases. There are sufficiently many noise traders yielding expected profits to the market maker to offset losses from more potentially informed trades. However, for λ larger than $\widetilde{\lambda}_t$, the second effect dominates and the relationship reverses. As λ continues to increase, $K_t(a_t^c, b_t^c)$ begins to decrease. The market fills with potentially informed traders and the market maker earns lower profits from noise traders so it can support less information acquisition. In the limit, there are no more noise traders and thus no more profits to offer potentially informed traders. Potentially informed traders stop acquiring information.

A large body of empirical literature studies the effect of institutional or block ownership on asset prices and information revelation. For example, Boehmer and Kelly (2009) look at institutional holdings and informational efficiency of prices, measured by deviations from a random walk. There are various theoretical reasons to examine institutional holdings but the asymmetric information literature, to which my model belongs, interprets them as informed traders. This implies that assets with larger institutional holdings should be more informative. If instead institutions are potentially informed and endogenously acquire costly information, Proposition 1.10 yields conditions when higher institutional holdings leads to less informative markets, as measured by the probability of informed trade. This result might help reconcile the mixed evidence on institutional ownership. It also cautions against using institutional holdings as proxies for informed trading.

Corollary 1.6. *The probability of an informed trade $K_t(a_t^c, b_t^c)$ is decreasing, while the spread is increasing, in the proportion of potentially informed traders λ for $\lambda \geq \max\{\frac{X_t+1}{2X+1}, \frac{Y_t+1}{2Y_t+1}\}$ where $X_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$.*

Corollary 1.6 describes the exact conditions when spreads and the probability of an informed trade $K_t(a_t^c, b_t^c)$ deviate from each other. As noted previously, they need not comove and here I show that they respond differently to changing the proportion of potentially informed traders λ. For λ beyond a certain threshold, increasing it further means that trades cause a larger revisions in belief but occurs less frequently.

The empirical literature uses both spreads and the probability of an informed trade as measures for information revelation. Standard theoretical models suggest they can be used interchangeably. I show when they cannot under the quadratic cost function. While this result is not general to all cost functions, I can show it arises for at least some subset of cost functions.

To better understand the divergence, recall that competitive prices, given by Equations (1.17) and (1.18), are set as the expected value of the asset conditional on a trade. They are proportional to the ratio of expected informed trades to total trades, both informed and noise. Total expected trades or expected transaction volume, denoted $E_t[q_t = +1]$ for buys, $E_t[q_t = -1]$ for sells, and $E_t[|q_t|]$ for all trades, are given by:

$$E_t\left[q_t = +1\right] = (1-\mu_t)G_t(a_t^c) + \frac{1}{2}(1-\lambda) \tag{1.21}$$

$$E_t\left[q_t = -1\right] = \mu_t H_t(b_t^c) + \frac{1}{2}(1-\lambda) \tag{1.22}$$

$$\begin{aligned} E_t\left[|q_t|\right] &= E_t\left[q_t = +1\right] + E_t\left[q_t = -1\right] \\ &= (1-\mu_t)G_t(a_t^c) + \mu_t H_t(b_t^c) + 1 - \lambda \end{aligned} \tag{1.23}$$

In Glosten-Milgrom, the only determinant of spreads is the proportion of informed traders because total expected transaction volume $E_t[|q_t|]$ is constant. In my model, $E_t[|q_t|]$ is endogenous.

Corollary 1.7. *Total expected transaction volume $E_t[|q_t|]$ is decreasing, while spreads are increasing in the proportion of potentially informed traders λ if $\lambda \geq \max\{\frac{Z_t+1}{2Z+1}, \frac{Y_t+1}{2Y_t+1}\}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$.*

By Corollary 1.7, when the probability of an informed trade $K_t(a_t^c, b_t^c)$ is decreasing, expected transaction volume $E_t[|q_t|]$ is also decreasing in λ. This drives the deviation between spreads and $K_t(a_t^c, b_t^c)$ from Corollary 1.6. Although informed trades occur less frequently, they make up a larger proportion of total trades.

Corollary 1.7 also predicts that, like $K_t(a_t^c, b_t^c)$, $E_t[|q_t|]$ is not monotonic in λ. In contrast, $E_t[|q_t|]$ is constant in Glosten-Milgrom. Easley and O'Hara (1992) has two regimes for $E_t[|q_t|]$ but more informed traders still implies higher $E_t[|q_t|]$.

Again the empirical support for the relationship between institutional holdings and transaction volume is mixed. The difficulty for these studies is the endogeneity of holdings. Institutional traders prefer liquid stocks which have higher expected transaction volumes. These stocks then have more institutional investors so it is difficult to determine causality.

1.4.4 Numerical Simulations

This subsection presents some numerical simulations to give a handle on how quickly the market learns when there is endogenous information acquisition and the expected paths of variables such as information revealed by a trade, the probability of an informed trade and expected transaction volume.

Consider a convergence criteria given by some constant $k \in [0, \frac{1}{2}]$. The criterion is met when the absolute difference between the posterior belief of the value of the asset and the true value of the asset is smaller than k: $|E_t[\widehat{v}] - v| < k$. I define convergence speed as the inverse of the unconditional expected number of trading periods needed to meet that criterion. For any set of market parameters, every k is associated with some convergence speed. I say a market 'always converges faster' than another if and only if its convergence speed is weakly greater for all choices of k.

Although I derive analytical results for spreads and the probability of an informed trade, it is harder to pin down convergence speed because it depends on both these variables which are not monotonically related. Furthermore, previous dynamic results appeal to fact that on average, beliefs approach the true value over time. However, convergence speed depends crucially on the exact path of those beliefs, as illustrated in the following example.

Consider two markets with potentially informed traders and some information acquisition cost function. Let it be cheaper to acquire information in market A than market B so trades in A reveal more information for any belief μ_t. In market A, let the first trade be very informative but the second trade a lot less so. This is possible because informativeness depends on the information acquisition cost function. Imagine a cost function with a steep step, like a fixed cost (a discontinuous step violates the assumption for existence of prices by Proposition 1.1). Over the flat part of the cost function, potentially informed traders acquire a lot of information but beyond the step, they acquire much less. Thus the first trade can be very informative while the second is not.

In contrast, in market B, let the first trade be less informative than in market A but the second trade more informative. Consider a market in which it is slightly more expensive to acquire information. For any μ_t, trades in market B are slightly less informative than trades in market A. Starting from the same μ_1, beliefs are updated less after the first trade. If the updated μ_2 does not exceed the step in the cost function after which potentially informed traders acquire less information, the second trade in market B can be more informative relative to market A. Thus, the market maker learns more after two trades in market B than market A despite trades revealing less information in market B for any given μ_t.

I simulate trading in markets with the quadratic cost function under different parameters over 150 periods for 20,000 price paths. I set the high asset value at $V = 10$, the true value of the asset as high, $\widehat{v} = V$, and the starting belief as uninformative, $\mu_1 = \frac{1}{2}$. Figure 1.2 plots the mean and standard deviation of the conditional expected asset value in each period. They represent the expected path at time 0, before any trades, of the mean and standard deviation of the asset value, conditional on the true value of the asset being high. In Figures 1.2(a) and (b), I fix the proportion of potentially informed traders at $\lambda = 0.8$ and plot paths for information acquisition costs $C = \{0, 2, 5, 5, 10\}$. $C = 0$ corresponds to a market with exogenously informed traders. In Figures 1.2(b) and (c), I fix the cost at $C = 2.5$ and plot paths for $\lambda = \{0.2, 0.4, 0.6, 0.8\}$.

In both cases, markets behave as expected. In Figure 1.2(a), the mean expected asset value approaches the true value fastest in the exogenous information acquisition case. Convergence speed is then decreasing in the cost of information acquisition C. In Figure 2(c), convergence speed is increasing in the proportion of potentially informed traders λ. Looking at standard deviations in Figures 1.2(b) and 1.2(d), markets with lower convergence speeds also coincide with higher standard deviations. The initial peak is an artifact of the discrete time setup which artificially restricts the maximum deviation in early periods. After that peak is reached, the standard deviation of expected values falls over time. Standard deviations are persistently higher in markets with lower convergence speed, even after many periods. While the treatment here is cursory, a full numerical survey would support the conclusions that convergence speed is decreasing C and λ.

Next, I study the dynamic behaviour of potentially informed traders. Using the same parameters from before ($V = 10$, $\widehat{v} = V$, $\mu_1 = \frac{1}{2}$) and fixing $\lambda = 0.8$, Figure 1.3 plots mean variables for different information acquisition costs $C = \{0, 2, 5, 5, 10\}$. Again, these represent the expected paths at time 0 for these variables, conditional on the true asset value being high. Figures 1.3(a), (b) and (c) refer to buyers. Figure 1.3(a) plots the difference between the ask price a_t and the conditional expected asset value $E_t[\hat{v}_t]$. This is the change in conditional expectations after a buy trade and represents the amount of information revealed by the trade. Figure 1.3(b) plots the probability that any buyer becomes informed, $G_t(a_t^*)$. Figure 1.3(c) plots the probability that a buy trade is submitted or the expected transaction volume from buyers $E_t[q_t = +1]$. Figures 1.3(d), (e) and (f) plot the same variables for sellers.

Figures 1.3(a) and (b) plot the amount of information revealed by buy and sell trades. All paths fall over time as the expected value of the asset approaches the true value since there is less information to be learned. The paths are not symmetric

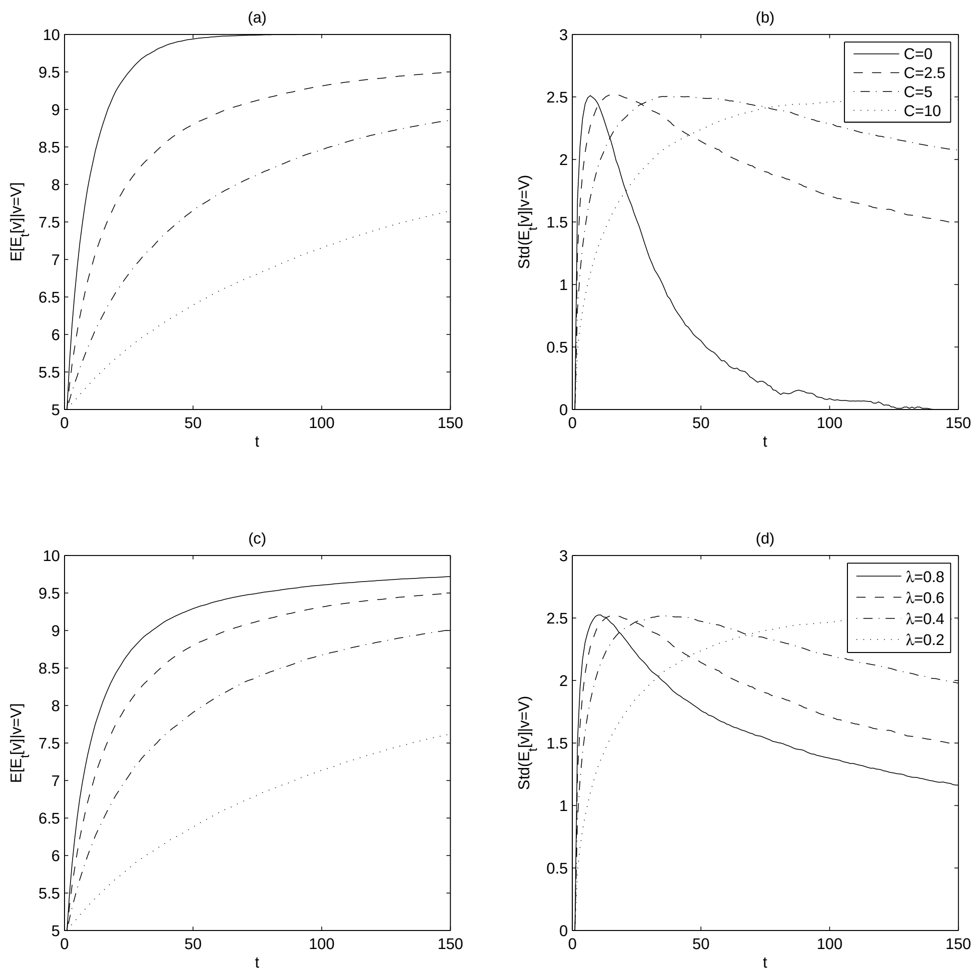

Figure 1.2: Unconditional at time 0 (a) mean and (b) standard deviation of the conditional expected asset value for each period across acquisition costs $C = \{0, 2, 5, 5, 10\}$ holding $\lambda = 0.8$. Unconditional at time 0 (c) mean and (d) standard deviation of each period conditional expected asset value across $\lambda = \{0.2, 0.4, 0.6, 0.8\}$ holding $C = 2.5$.

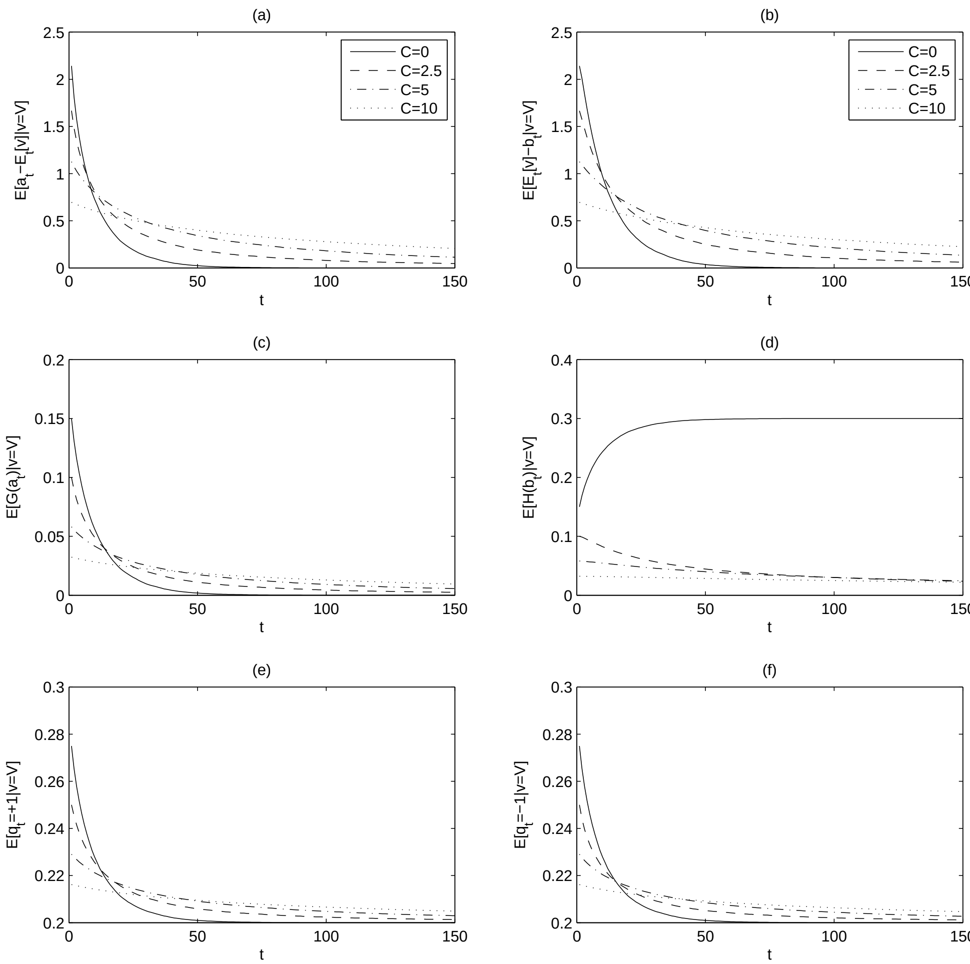

Figure 1.3: Across different information acquisition costs $C = \{0, 2, 5, 5, 10\}$. Absolute difference between conditional expected asset value $E_t[\hat{v}_t]$ and (a) ask price a_t; and (b) bid price b_t. Probability that trader is informed: (c) buyer, $G_t(a_t^*)$; or (d) seller, $H_t(b_t^*)$. Probability that trade is submitted: (e) buy, $E_t[q_t = +1]$; or (f) sell, $E_t[q_t = -1]$.

between buys and sells. In this case, sell trades reveal more information than buys because they provide counter evidence to the prevailing belief that the asset value is high. As expected, information revealed in early periods is decreasing in the information acquisition cost C. In early periods, it falls fastest when C is low. In later periods, trades are actually more informative in markets with a higher C because beliefs are still weak.

Figures 1.3(c) and (d) plot amount of information acquired by buyers, $G_t(a_t^*)$, and sellers, $H_t(b_t^*)$. The big difference is between markets with no information acquisition cost, $C = 0$, and those with positive C. Exogenously informed traders, corresponding to $C = 0$, always acquire information at a constant rate. However, potentially informed traders acquire less information over time. As the expected asset value converges on the true value, there is less incentive for potentially informed traders to acquire information so all the plots fall over time. In early periods, potentially informed traders acquire less information in markets with higher C. Again this reverses in later periods when there is more acquisition despite higher C because of the slower rate of convergence in these markets.

Figures 1.3(e) and (f) plot the expected volume of buys, $E[q_t = +1]$, and sells, $E[q_t = -1]$. The market with exogenously informed traders, $C = 0$, is unique because informed traders always trade. When the asset value is high, all informed buyers submit buy orders. In Figure 1.3(e), as beliefs converge to certainty of the high value, $E[q_t = +1]$ increases until all buyers trade, while in Figure 1.3(f), $E[q_t = -1]$ decreases until only noise traders remain. In contrast, expected transaction volume always falls over time with potentially informed traders. As beliefs converge towards certainty, potentially informed traders acquire less information information and trade less. Again, in early periods, there is higher expected transaction volume in markets with higher information acquisition cost C which can reverse in later periods. For example, in Figure 1.3(e) the paths of $E[q_t = -1]$ for $C = 5$ is higher than for $C = 2.5$

after $t = 20$. In the first case, potentially informed sellers still acquire information because beliefs are still weak.

1.5 A New Structural Estimator

The main empirical implication of my model is that the standard proxies used in the literature do not accurately capture informed trading. In particular, the widely used Easley-O'Hara PIN measure may be misspecified. Fortunately, I can offer a potential alternative.

My original discrete time setup does not lend itself well to empirical estimation. However, following Easley et al. (1996), I can augment it with continuous time Possion arrival processes for traders. The resulting mixed model has a likelihood function which can then be estimated in the same way as PIN. An advantage of my setup is that it nests PIN as a special case when potentially informed traders are not price sensitive and so always acquire information.

Unfortunately, the data requirement for estimating my model is not as straightforward as PIN which only requires the count of buys and sells over each day. With endogenous information acquisition, I would also need the time series of ask and bid quotes, and the prices and times at which trades occur. Furthermore, I would need to impose some structure on the information acquisition cost function. Ideally, it would only take a single parameter, to minimise the dimensionality of the estimation problem. The quadratic cost function might be a suitable candidate.

1.6 Conclusion

This chapter makes three main contributions. First, I offer an intuitive, general extension to Glosten-Milgrom to account for endogenous information acquisition. I model potentially informed traders who face information acquisition costs. They optimally choose how much information to acquire as a function of expected speculative profits. Crucially, their decision depends on posted prices.

Second, my model captures empirical features of real markets which are unexplained in the exogenous information benchmark. An increase in informed trading and transaction volume after a known informational event is driven by higher speculative profits which give potentially informed traders incentives to acquire information. Traders react to different sized shocks so more disruptive events lead to higher activity. The subsequent fall in activity over time occurs because there is less incentive to acquire information as the market learns the true value of the asset.

Third, my model suggests that the empirical proxies used in the literature do not accurately capture informed trading. With endogenous information acquisition, three commonly used proxies (bid ask spreads, proportion of trader types and PIN) do not always agree in cross sectional comparisons. I characterise situations when the spread is wide, there are many potentially informed traders in the market but the probability of an informed trade is low.

The framework I develop lends itself readily to deriving a new structural estimator which can capture the effect of prices on endogenous information acquisition. I intend to pursue this in future work.

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1.8 Appendix

I suppress the time subscript in all proofs. I only present proofs for the ask side. The bid side follows analogously. As shorthand, Z^c refers to any function $Z(a, b)$ which takes arguments (a^c, b^c).

Proof of Proposition 1.1. Let $Z(a) = \frac{(1-\mu)B_V(a)V}{\mu_t B_0(a)+(1-\mu)B_V(a)}$ so that the equilibrium ask price is given by $a^c = Z(a^c)$ from Equation (1.7). $Z(a)$ is given by:

$$Z(a) = \frac{(1-\mu)(\lambda X_a(a) + (1-\lambda))V}{(1-\mu)\lambda X_a(a) + (1-\lambda)}$$

Let X_a be a continuous, monotonic, increasing in a and bounded $[0, 1]$. Therefore the numerator of Z is continuous. The denominator of Z is also continuous and bounded $[0, 1]$ under the same conditions. Therefore Z is also continuous. Taking the first derivative of Z with respect to a yields:

$$\frac{\partial Z}{\partial a} = \frac{\mu(1-\mu)\lambda(1-\lambda)V}{(\mu\lambda X_a + 1 - \lambda)^2}\frac{\partial X_a}{\partial a}$$

$\frac{\partial X_a}{\partial a} \geq 0$ by earlier assumption. Now I need to show that $Z : \mathbb{R} \to [0, V]$. The lower bound of Z is at $X_a = 0$: $Z = (1-\mu)V : \mathbb{R} \to [0,V]$ when $\mu \in [0,1]$. The upper bound of Z is at $F = 1$: $Z = \frac{(1-\mu)V}{1-\mu\lambda} : \mathbb{R} \to [0,V]$ when $\mu \in [0,1]$ and $\lambda \in [0,1]$. Therefore the function $Y = Z(a)$ must cross $Y = a$ once in the range $[0,V]$ and the solution to Equation (1.7) must exist in the range $[0,V]$. For uniqueness, I need to rule out the only other alternative: a continuum of solutions. It is easy to see there are no parameter values such that $a = Z(a)\ \forall\ a$. $a = Z(a)$ occurs at a single crossing.

■

Proof of Proposition 1.2. From Equation (1.7), differentiate a^c implicitly with respect to μ to obtain:

$$\frac{\partial a^c}{\partial \mu} = \frac{-V(1-\lambda)(1-\lambda(1-X_a^c+\mu(1-\mu)(\frac{\partial X_a^c}{\partial \mu}+\frac{\partial a^c}{\partial \mu}\frac{\partial X_a^c}{\partial a^c})))}{(1-\lambda(1-(1-\mu)X_t^c))^2}$$

Rearrange and see that $\frac{\partial a^c}{\partial \mu} \leq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$, $X_a^c \in [0,1]$, $\frac{\partial X_a^c}{\partial \mu} \leq 0$, $\frac{\partial X_a^c}{\partial a^c} \leq 0$ and $a^c \leq V$.

Substituting into Equation (1.7), $a^c(\mu = 0) = V$ and $a^c(\mu = 1) = 0$. ■

Proof of Proposition 1.3. Exogenously informed traders have $X_a = 1$ always. Equilibrium ask price in this market is given by:

$$\bar{a}^c = \frac{(1-\mu)V}{\mu(1-\lambda)+1-\mu}$$

Compare with a^c from Equation (1.7). Let $Z = 1-\lambda(1-g)$. $Z \leq 1$ when $\lambda \in [0,1]$ and $X_a^c : \mathbb{R} \to [0,1]$. Then $\bar{a}^c = \frac{(1-\mu)V}{\mu(1-\lambda)+1-\mu} = \frac{(1-\mu)VX}{\mu(1-\lambda)X+(1-\mu)X} \geq \frac{(1-\mu)VX}{\mu(1-\lambda)+(1-\mu)X} = a^c$ when $Z \leq 1$. ■

Proof of Proposition 1.4. From Equation (1.7) the competitive market maker sets the ask price as the expected value of the asset conditional on a buy occuring:

$$a = \frac{(1-\mu)[\lambda\frac{1}{C}((1-\mu)(V-a)) + 1 - \lambda]V}{(1-\mu)\lambda\frac{1}{C}(1-\mu)(V-a) + 1 - \lambda}$$

The expression is quadratic in a and the two roots are given by:

$$a = V \pm \frac{(1-\lambda)C}{2\lambda(1-\mu)^2}\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right]$$

To obtain a unique solution, one of the roots is ruled out. Under the parameter restrictions, $p \in [0,1]$, $\lambda \in [0,1]$, $V > 0$ and $C > 0$, the expressions $\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}-1$ and $\frac{(1-\lambda)C}{2\lambda(1-\mu_t)^2}$ are both larger than 0 so the first root is larger than V. This is ruled out because the potentially informed trader would always make a loss trading at this price, regardless of the true value.

The remaining root a^c has to satisfy two other restrictions: 1) it implies an information arrival probability, $X_a(a^c)$, which is bounded $[0,1]$, and 2) it is bounded $[0,V]$. For the first part, substitute the root into the expression for X_a:

$$X_a(a^c) = \frac{(1-\lambda)}{2\lambda(1-\mu)}\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right]$$

Under the same parameter assumptions, $X_a(a^c) \geq 0$. It also needs to be shown that $X_a(a^c) \leq 1$. Taking partial derivatives, it can be shown that $\frac{\partial X_a(a^c)}{\partial\lambda} < 0$. Since λ is bounded $[0,1]$, $\arg\max_{\lambda} X_a(a^c) = 0$. By l'Hopital's rule, $\lim_{\lambda\to 0} X(a^c) = \frac{1}{C}\mu(1-\mu)V$. The maximum value this can take is $\frac{V}{4C}$ given $\arg\max_{\mu} \frac{1}{C}\mu(1-\mu)V = \frac{1}{2}$. To satisfy $X_a(a^c) \leq 1$ requires the restriction $C > \frac{1}{4}V$. This completes the first part.

For the second part, it remains to be shown that $a^c \geq 0$. $a^c < 0$ is ruled out because the potentially informed trader would always make a profit trading at this price, regardless of the true value. Taking partial derivatives, $\frac{\partial a^c}{\partial\lambda} > 0$ under the parameter restrictions above. Since λ is bounded $[0,1]$, $\arg\min_{\lambda} a^c = 0$. By l'Hopital's rule, $\lim_{\lambda \to 0} a^c = (1-\mu)V$. The minimum value this can take is 0 given $\arg\min_{\mu}(1-\mu)V = 1$ for $\mu \in [0,1]$. Therefore, a^c is the unique root to Equation (1.7). ■

Proof of Proposition 1.5. Take the derivative of a^c from Equation (1.17) with respect to C and simplify to obtain:

$$\frac{\partial a^c}{\partial C} = -\frac{1-\lambda}{2(1-\mu)^2\lambda}\left[\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} - 1\right]$$

Let $Z = \frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V$. $Z \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $V > 0$. Then $1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V = 1+Z$ and $\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V} = \sqrt{1+2Z}$. Comparing these two expressions in Z, $1+Z \geq \sqrt{1+2Z}$. Therefore $\frac{1+Z}{\sqrt{1+2Z}} > 1$. $\frac{1-\lambda}{2(1-\mu)^2\lambda} \geq 0$ when $\lambda \in [0,1]$. Therefore $\frac{\partial a^c}{\partial C} \leq 0$.

Take the expression for a^c from Equation (1.17). Let $Y(C) = \frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}$. The expression $\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}$ can be written as $(1+Y)^{\frac{1}{2}}$. Using a Taylor expansion: $(1+Y)^{\frac{1}{2}} - 1 \approx (1+\frac{1}{2}Y - \frac{1}{8}Y^2 + ...) - 1 \approx \frac{1}{2}Y - \frac{1}{8}Y^2 + ...$ Also the expression $\frac{(1-\lambda)C}{2\lambda(1-\mu)^2}$ can be written as $\frac{2}{Y}\mu V$. Using this expression with the Taylor expansion, a^c can be written as: $a^c \approx V - \frac{2}{Y}\mu V(\frac{1}{2}Y - \frac{1}{8}Y^2 + ...) \approx V - \mu V + O(Y)$. $\lim_{C\to\infty} Y(C) = 0$. Therefore $a^c \approx (1-\mu)V$. ■

Proof of Proposition 1.6. Take the derivative of a^c from Equation (1.17) with respect to λ and simplify to obtain:

$$\frac{\partial a^c}{\partial \lambda} = \frac{C}{2(1-\mu)^2\lambda^2}\left[\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} - 1\right]$$

$\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} \geq 0$ as shown in the Proof of Proposition 1.5. Also, $\frac{C}{2(1-\mu)^2\lambda^2} \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $C > 0$. Therefore $\frac{\partial a^c}{\partial \lambda} \geq 0$.

Use L'Hopital's rule to evaluate a^c from Equation (1.17) as $\lambda \to 0$. Let $X(\lambda) = (1-\lambda)C\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}-1\right]$ and $Y(\lambda) = 2(1-\mu)^2\lambda$. Then $\frac{\partial X(0)}{\partial \lambda} = -2\mu(1-\mu)^2 V$ and $\frac{\partial Y(0)}{\partial \lambda} = 2(1-\mu)^2$. Therefore $\lim_{\lambda \to 0} \frac{X(\lambda)}{Y(\lambda)} = -\mu V$ and so $\lim_{\lambda \to 0} a^c = (1-\mu)V$. From Equation (1.17) $a^c(\lambda = 1) = V$. ■

Proof of Proposition 1.7. To find the maximum of K with respect to μ, solve the first order condition $\frac{\partial K^c}{\partial \mu} = 0$. Taking the derivative of K^c:

$$\frac{\partial K^c}{\partial \mu} = \frac{\lambda V}{2C}\left[\frac{(1-\mu)(1-3\mu)}{\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}} + \frac{\mu(2-3\mu)}{\sqrt{1+\frac{4\mu^2(1-\mu)\lambda V}{(1-\lambda)C}}}\right]$$

The first order condition is satisfied when $\mu = \frac{1}{2}$.

From Equation (1.19), $G^c(\mu = 1) = 0$ and $G^c(\mu = 0) = 0$. Similarly for $H^c(\mu)$ from Equation (1.20). Combining them obtains the results for $K^c(\mu)$. ■

Proof of Proposition 1.8. Take the derivative of G^c from Equation (1.19) with respect to C and simplify to obtain:

$$\frac{\partial G^c}{\partial C} = -\frac{\mu(1-\mu)^2\lambda V}{2C^2\sqrt{1+\frac{4}{(1-\lambda)C}\mu(1-\mu)^2\lambda V}}$$

$\mu(1-\mu)^2\lambda V \geq 0$ and $1+\frac{4}{(1-\lambda)C}\mu(1-\mu)^2\lambda V \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $V > 0$. Therefore $\frac{\partial G^c}{\partial C} \leq 0$. ■

Proof of Proposition 1.9. Take the derivative of G^c from Equation (1.19) with respect to λ and solve for the first order condition $\frac{\partial G^c}{\partial \lambda} = 0$. Following some tedious algebra, the two roots to the first order condition are given by: $(1-\mu)(1-2\lambda)\sqrt{\frac{1}{C}\mu V}+1-\lambda = 0$ and $(1-\mu)(1-2\lambda)\sqrt{\frac{1}{C}\mu V}-1+p = 0$. These roots are $\lambda = \frac{Z+1}{2Z+1}$ and $\lambda = \frac{Z-1}{2Z-1}$ where $Z = (1-\mu)\sqrt{\frac{1}{C}\mu V}$. $Z \geq 0$ when $\lambda \in [0,1]$, $V \geq 0$ and $C \geq 0$. The second root is ruled out because it implies $\lambda > 1$. Therefore, $\arg\max_{\lambda} G^c = \frac{Z+1}{2Z+1}$.

Substituting into Equation (1.19), $G^c(\lambda = 0) = 0$ and $G^c(\lambda = 1) = 0$. ■

Proof of Proposition 1.10. Take the derivative of X_a^c with respect to λ and simplify to obtain:

$$\frac{\partial X_a^c}{\partial \lambda} = -\frac{1}{2(1-\mu)\lambda^2}\left[\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}\mu^2(1-\mu)\lambda V}} - 1\right]$$

$\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} \geq 1$ as shown in the Proof of Proposition 1.5. $2(1-\mu)\lambda^2 \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$. Therefore $\frac{\partial X_a^c}{\partial \lambda} \leq 0$.

Use L'Hopital's rule to evaluate $X_a^c(\lambda)$ as $\lambda \to 0$. Let $Z(\lambda) = (1-\lambda)\left(\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right)$ and $Y(\lambda) = 2(1-\mu)\lambda$. Then $\frac{\partial Z(0)}{\partial \lambda} = \frac{2}{C}(1-\mu)^2\mu V$ and $\frac{\partial Y(0)}{\partial \lambda} = 2(1-\mu)$. Therefore $\lim_{\lambda \to 0} X_a^c(\lambda) = \frac{1}{C}(1-\mu)\mu V$. Substituting into the expression for X_a^c, $X_a^c(\lambda = 1) = 0$. ■

Chapter 2

Endogenous Information Acquisition and A Monopolistic Market Maker

2.1 Introduction

This chapter examines the impact of endogenous information acquisition by traders on a market with a monopolistic market maker. While the competitive assumption of the previous chapter may apply for the majority of financial markets, there are cases when market makers may have some monopoly power. For example Madhavan and Sofianos (1998) find evidence for it among NYSE specialists and Massa and Simonov (2009) in the Italian interdealer bond market. [1]

[1] Madhavan and Sofianos (1998) look at specialist dealers who are designated as market makers on the NYSE. They find that specialist participation is lower in block trades compared to non block trades, consistent with a market maker doing small trades to acquire information. They also find that participation is higher when bid ask spreads are wide and when previous price movements have been significant, consistent with experimentation. Massa and Simonov (2009) identify individual traders in the Italian interdealer bond market and directly track their behaviour to estimate the beliefs of each trader about how informed the other traders are. They find two types of behaviour: 'hiding' in which they exploit their private information by trading with someone who they believe has less information than them; and 'experimenting' in which they try to learn by trading with

The monopolistic case is also of theoretical interest because it introduces a new intertemporal tradeoff to the market maker. He values information and can influence the information revealed by trades through prices. Therefore, he has the incentive to induce more revelation in early periods which he can exploit in later periods. However, inducing information revelation is costly as it entails trading with informed traders. This is the main insight of Leach and Madhavan (1993) who study the benchmark model with exogenous information acquisition.

I start with the discrete time framework of Glosten and Milgrom (1985) and incorporate price elastic noise traders and a monopolistic market maker from Leach and Madhavan (1993). My innovation is the introduction of potentially informed traders who optimally choose how much information to acquire at variable cost. Similar to Chapter 1, they acquire different amounts of information depending on expected profits from speculative trading which is a function of the prices posted by the market maker.

With my setup, market information structure becomes crucial in determining how prices affect information revelation. Prices now enter both the information acquisition decision of potentially informed traders and the demand of price elastic noise traders. Potentially informed traders consider ex ante expected profits conditional on prior beliefs before acquiring information. A narrower spread increases their expected profits so they acquire more information and trade more frequently. A larger proportion of trades are informed so they reveal more information to the market maker.

Prices also affect price elastic noise traders whose participation depend on their expected losses from trading. A narrower spread lowers expected losses so they trade more frequently. A larger proportion of trades are uninformed so they reveal less information. In the standard case with exogenous information acquisition, this is the only way that prices can affect the information revealed by trades. With endogenous

someone who they believe has more information.

information acquisition, there are two channels which operate in opposite directions.

I characterise market information structure by the effect which dominates. If 'information acquisition dominates', narrower spreads increase the information revealed by trades because prices affect potentially informed traders more than noise traders. If 'noise dominates', wider spreads increase the information revealed by trades. This setup generates two main results.

First, if 'information acquisition dominates', a monopolistic market maker sets narrower spreads in early periods, which are decreasing in the total number of trading periods. He may even set narrower spreads than a competitive market maker, thereby making a loss. In contrast, with exogenously informed traders, he always sets wider spreads in early periods, which are increasing in the total number of trading periods. My result is driven by the new endogenously informed traders. When spreads are narrow, they are incentivised to acquire more information and if this effect dominates, the market maker sets narrower spreads.

If 'noise dominates', the result is reversed and spreads behave as in the benchmark. While narrower spreads increase information acquisition, they also increase noise trader participation. If noise traders are more price sensitive than potentially informed traders, the market maker can increases informativeness of trades by effectively crowding out noise traders with wider spreads. Exogenous information acquisition is a special case this market information structure.

Second, spreads may widen over time on average. In the standard case, spreads always narrow over time on average. This result is driven by the interaction of endogenous information acquisition, which determines how prices affect the information revealed by trades, with a monopolistic market maker, who has an intertemporal trade off between information revelation and short term profits. Trades reveal information and the market maker updates beliefs every period. Starting with an uninformative prior, beliefs grow monotonically more certain on average. With exogenous information, this implies that spreads become monotonically *narrower* on average. In my setup, if information acquisition dominates, they may grow monotonically *wider* on average.

The rest of this chapter proceeds as follows: In Section 2.2, I present some related literature. In Section 2.3, I introduce the model, explain the setup, define market maker objective functions and characterise market information structure. In Section 2.4, I provide a motivating example. In Section 2.5, I show general results for the effect of market structure on bid ask spreads, volumes, and price dynamics over time. I also document information stoppage, the steady state and market failure. In Section 2.6, I conclude. Proofs are in the Appendix.

2.2 Related Literature

See Section 1.2 for a review of related market microstructure literature and endogenous information acquisition. This section presents work specifically looking at monopolistic market makers.

Leach and Madhavan (1993) is the closest paper to mine in structure. They examine price discovery under various market makers in a Glosten-Milgrom framework with elastic noise traders. They show that an optimal monopolistic market maker has an incentive to 'experiment' by setting prices which make trades more informative. They set wider spreads in earlier periods to crowd out elastic noise traders and increase the relative proportion of informed trades. They also find conditions under which having different market makers lead to more robust markets. They present some empirical results to support price experimentation. I generalise their framework so that traders are endogenously informed.

Glosten (1989) is one of the first to analyse the monopolistic market maker. In contrast to Glosten-Milgrom, trades are not restricted to unit amounts and the market maker posts a price schedule over different quantities of the traded asset. They find that if there are not enough noise traders in the market, a competitive market maker is unable to set zero profit prices across all quantities and thus the market breaks down. In contrast, the monopolistic market maker maximises expected profits across quantities so he can subsidise losses from trading in some quantities with profits in others. In some cases, he provides liquidity when a competitive market maker cannot do so. This mechanism is similar to the one in my model except that in my case, the market maker substitutes profits across time instead of quantities.

In general, multi period models with monopolistic market makers are analytically difficult to solve. Das and Magdon-Ismail (2009) approximate beliefs within Glosten-Milgrom by a Gaussian distribution and then solve for the optimal sequential market making algorithm. They find that an optimal monopolistic market maker can provide more liquidity than a perfectly competitive market maker in periods of extreme uncertainty because he is willing to absorb initial losses in order to learn a new valuation rapidly and extract higher profits later. Again, I find a similar intuition.

Madrigal and Scheinkman (1997) considers a market in which traders have private and heterogenous information. The market maker is large and acts strategically because he understands that prices affect first, the information he learns from the order flow, and second, the information he reveals back to other traders. This setup yields a discontinuity in equilibrium prices which Madrigal and Scheinkman interpret as a price crash.

So far market makers have inferred information from anonymous trades. Gammill (1990) lets the market maker learn the identity of traders. He makes small trades with informed traders to extract information and large trades with noise traders to maximise expected profits. This theoretical model finds support in the results of Massa and Simonov (2009) from the Italian interdealer bond market.

2.3 Model

2.3.1 Setup

My model extends Leach-Madhavan by introducing an arrival function to describe potentially informed traders who are sensitive to prices. The basic setup is a Glosten-Milgrom sequential trade model in which noise traders are price elastic. There is one risky asset with stochastic termination value $\widetilde{\theta}$ which takes one of two values, θ_1 and θ_2, where $\theta_2 > \theta_1$. A unit of the asset may be traded each period. There are T trading periods after which the termination value is realised. The market maker posts ask and bid prices, a_t and b_t, at the beginning of each period $t = 1, ..., T$. A trader is drawn at random from a continuum of traders so there is zero probability of the same trader appearing twice. A proportion λ of the population consists of potentially informed traders and $1 - \lambda$ consists of noise traders.

There are three types of agent in this model: the market maker, noise traders and potentially informed traders. The market maker is risk neutral and chooses prices $p_t = (a_t, b_t)$ from $P \subset \mathbb{R}_+$ which contains the possible values of $\widetilde{\theta}$. At the beginning of each period $t = 1, ..., T$ he has prior belief μ_t that the true value of the asset is low, $\widetilde{\theta} = \theta_1$, and belief $1 - \mu_t$ that the true value is high, $\widetilde{\theta} = \theta_2$. He is obliged to trade against any order submitted by a trader.

The noise trader trades for reasons other than speculative profit. In Glosten-Milgrom, noise traders are price inelastic and submit orders randomly. Leach and Madhavan make noise traders price elastic which is essential for the existence of interior prices because it disciplines a monopolistic market maker. In the standard case, he can set maximum spreads, at which informed traders do not participate, and only trade with price inelastic noise traders. Such a market is always closed. However, if noise traders are price elastic, it may be optimal to set interior prices.

When a noise trader is drawn into the market, he receives a private reservation value drawn from some distribution with cumulative density function $F(r)$ where the average $\int r \, \mathrm{d}F(r)$ is in (θ_1, θ_2). He then submits an order q_t which takes values $\{-1, 1, 0\}$ for a sell, buy or no trade. He submits a sell order if the bid price is higher than his reservation value, a buy order if the ask price is lower than his reservation value, and no trade otherwise:

$$q_t = \begin{cases} -1 & \text{if } b_t > r \\ +1 & \text{if } a_t < r \\ 0 & \text{otherwise} \end{cases}$$

These occur with probabilities:

$$q_t = \begin{cases} -1 & \text{with probability } F(b_t) \\ +1 & \text{with probability } 1 - F(a_t) \\ 0 & \text{with probability } F(a_t) - F(b_t) \end{cases}$$

Like the standard informed trader, potentially informed traders trade for speculative profits. Instead of always knowing the true value of the asset $\widetilde{\theta}$, he learns it with some probability given by the information arrival function $X(a_t, b_t, \mu_t)$ defined over prices $a_t \geq b_t$ and beliefs $\mu_t \in [0, 1]$. The information arrival function X captures the impact of prices on the information acquisition decision of potentially informed traders. They choose an optimal amount of information to acquire depending on expected profits and cost. Section 2.4 develops a motivating example with a full microfoundation for X. In the example, potentially informed traders face a quadratic cost to acquire information of increasing precision. They choose the optimal amount of information to acquire depending on posted prices. Chapter 1 presents an alternative model in which potentially informed traders receive a private information acquisition cost. Then, only some proportion of potentially informed traders acquire information as a function of prices.

For now, I stipulate restrictions to make the information arrival function X consistent with my intuition. X represents a probability so it must be bounded $[0, 1]$. For the potentially informed trader, expected profits from selling the asset increases with the bid price b_t so potentially informed traders acquire more information. Therefore, X should be weakly increasing in b_t: $\frac{\partial X}{\partial b_t} \geq 0 \, \forall \, a_t, b_t, \mu_t$. Conversely, expected profits from buying the asset decrease with the ask price so potentially informed traders acquire less information. X should be weakly decreasing in a_t: $\frac{\partial X}{\partial a_t} < 0 \, \forall \, a_t, b_t, \mu_t$. Note that X is not actually stochastic and merely describes information arrival. Nevertheless, for simplicity I refer to X as if it is the information arrival process.

These restrictions on the information arrival function X are also consistent with other interpretations. For example, potentially informed traders might have private draws of reservation values, like noise traders, corresponding to different realisations of a noisy signal. Under some restrictions, this setup could be made to satisfy the restrictions I describe here. My results would still obtain. However, my intent is to model endogenous information and I will use this interpretation for the rest of the chapter. Note that my general specification nests Leach-Madhavan with exogenously informed traders. The two are equivalent when $X(a_t, b_t, \mu_t) = 1$ for all prices and beliefs.

If the potentially informed trader is drawn and learns the true value of the asset via the information arrival function X, he submits a sell order if the bid price is higher than the true value, a buy order if the ask price is lower than the true value, and no trade otherwise. If he does not learn the true value, he does not trade. He submits a trade:

$$q_t = \begin{cases} -1 & \text{if } b_t > \widetilde{\theta} \\ +1 & \text{if } a_t < \widetilde{\theta} \\ 0 & \text{otherwise} \end{cases}$$

Now I assume that the market maker sets interior prices $p_t \in (\theta_1, \theta_2)$ which implies that potentially informed traders who learn the true value of the asset always trade. I concentrate on this as the leading case for simplicity of notation. At this point, I define when a market is considered 'open' or 'closed'.

Definition 2.1. *The market is open (closed) at time t on the ask side if it allows (excludes) profitable informed trade: $a_t < \theta_2$ ($a_t \geq \theta_2$). The market is open (closed) at time t on the bid side if it allows (excludes) profitable informed trade: $b_t > \theta_t$ ($b_t \leq \theta_1$). The market is open if it is open on at least one side.*

This definition refers only to the participation of informed traders. Noise traders continue to trade even in a 'closed' market. The previous assumption that the market maker sets interior prices prices is equivalent to solving a price setting problem under the constraint that the market is open. In general, the market maker can set prices such that: 1) the market is closed on the buy side, $a_t \leq \theta_1$; 2) the market is closed on the sell side, $b_t \geq \theta_2$; or 3) both of the above. For the unconstrained solution, I need to consider the market maker's objective function in all the constrained cases and choose the optimum.

From the market maker's point of view, a potentially informed trader submits a trade q_t:

$$q_t = \begin{cases} -1 & \text{with probability } \mu_t X(a_t, b_t, \mu_t) \\ +1 & \text{with probability } (1-\mu_t) X(a_t, b_t, \mu_t) \\ 0 & \text{with probability } 1 - X(a_t, b_t, \mu_t) \end{cases}$$

The timeline is as follows: first, the market maker with prior belief μ_t posts ask and bid prices, a_t and b_t. Second, a trader is drawn from the continuum of traders who are potentially informed traders with probability λ or noise traders with probability $1-\lambda$. Third, the drawn trader submits an order q_t based on his objective function. If the trader is potentially informed, he learns the true value of the asset with probability $X(a_t, b_t, \mu_t)$ and submits an order q_t to maximise profits. If the trader is a noise trader, he draws a private reservation value r from a cumulative density function $F(r)$ and submits an order q_t depending on his reservation value. Finally, the market maker fills the order and updates his beliefs given by $\mu_{t+1} \equiv pr(\widetilde{\theta} = \theta_1 \mid q_t, a_t, b_t)$.

The common knowledge of all market participants includes the information arrival function X, the best reply functions of potentially informed traders, the distribution of traders and the history of prices and trades up to time t. These are used to form prior beliefs μ_t. After the market maker completes a trade q_t, he forms posterior beliefs $\mu_{t+1}(q_t)$ using Bayes' rule. After a sell, the posterior belief $\mu_{t+1}(-1)$ is:

$$\begin{aligned} \mu_{t+1}(-1) &\equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = -1, a_t, b_t\right) \\ &= \frac{\mu_t \left[\lambda X(a_t, b_t, \mu_t) + (1-\lambda) F(b_t)\right]}{\mu_t \lambda X(a_t, b_t, \mu_t) + (1-\lambda) F(b_t)} \end{aligned} \tag{2.1}$$

After a buy, it is:

$$\mu_{t+1}(+1) \equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = +1, a_t, b_t\right)$$
$$= \frac{\mu_t(1-\lambda)(1-F(a_t))}{(1-\lambda)(1-F(a_t)) + (1-\mu_t)\lambda X(a_t, b_t, \mu_t)} \tag{2.2}$$

And after no trade:

$$\mu_{t+1}(0) \equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = 0, a_t, b_t\right)$$
$$= \mu_t \tag{2.3}$$

Now I define a measure for how much the market learns from a trade. Let the 'informativeness' of a trade, denoted $N_t(q_t|a_t, b_t, \mu_t)$, be the change in prior belief μ_t after a trade q_t conditional on that prior and a set of posted prices: $N_t(q_t|a_t, b_t, \mu_t) = |\mu_{t+1}(q_t) - \mu_t|$. A trade is more informative if it leads to a larger revision of the prior. A buy always leads to a downward revision, $\mu_{t+1}(+1) \leq \mu_t$, while a sell always leads to an upward revision, $\mu_{t+1}(-1) \geq \mu_t$.

2.3.2 Market Maker Objective Functions

Following Leach and Madhavan, I consider three types of market maker: an optimal monopolistic, a myopic monopolistic, and a competitive market maker. Price discovery under each market maker is determined by their respective objective functions. In all cases, the market maker's one period expected profit is given by:

$$\pi(a_t, b_t; \mu_t) = \lambda X(a_t, b_t, \mu_t)\left[\mu_t(\theta_1 - b_t) + (1-\mu)(a_t - \theta_2)\right]$$
$$+ (1-\lambda)\left[F(b_t)(\theta_2 - b_t) + (1-F(a_t))(a_t - \theta_2)\right] \tag{2.4}$$

An optimal monopolistic market maker maximises profits from trading over every period up to time T. This is not a static problem because the market maker can influence the information revealed by trades. His belief in later periods depends on the prices he posted earlier. He may forgo some profits from earlier periods to increase information revelation and increase expected profits in later periods. His maximisation yields total expected profits given by:

$$V_n^*(\mu_1) = \sup_{\{a_t,b_t\}} E\left[\sum_{t=T-n+1}^{T} \pi(a_t, b_t; \mu_t)\right] \tag{2.5}$$

for n remaining trading rounds. The choice variables are a_t and b_t which are history dependent. The prior belief μ_t evolves by Bayes' rule as described previously. The expectation is taken over all random variables. Current period prices are set to extract information optimally. Equation (2.5) can be written in its Bellman form:

$$V_T^*(\mu_1) = \sup_{\{a_1,b_1\}} \left\{\pi(a_1, b_1; \mu_1) + E\left[V_{T-1}^*(\mu_2(\widetilde{q}))\right]\right\} \tag{2.6}$$

with terminal condition

$$V_1^*(\mu_T) = \sup_{\{a_T,b_T\}} \{\pi(a_T, b_T; \mu_T)\} \tag{2.7}$$

The function $V_n^*(\mu_1)$ is the stochastic dynamic programming problem for the market maker with n periods left to trade before $\widetilde{\theta}$ is revealed. The state variable is the prior belief about the true value of the asset μ_t, the control variables are the ask and bid prices, a_t and b_t, and the transition equation is Bayes' rule.

Proposition 2.1. *For $\mu_t \in (0,1)$ the optimal monopolist's value function $V_{T-1}^*(\mu)$ is convex and nonnegative.*

Nonnegativity is obvious because the market maker can post prices at which no trade would occur. Convexity of the value function is the key property for later results. By the Law of Iterated Expectations, the expected posterior belief under any set of prices must be the prior belief: $E[\mu_{t+1}|\mu_t] = \mu_t$. Together with convexity of the value function, it implies that future information is valuable. For any given prior belief, the market maker expects to be better off in the next period after learning from another trade. Therefore, in non terminal periods, a monopolist market maker never closes the market (by setting $a_t \geq \theta_2$ and $b_t \leq \theta_1$) as trading weakly reveals more information. In non terminal periods, prices which close the market are weakly dominated by those which open it ($a_t < \theta_2$ and $b_t > \theta_1$).

Definition 2.2. *An optimal monopolistic market maker's first period price choice $p_1^* = (a_1^*, b_1^*) \in P$ is the solution to the Bellman equation, Equation (2.6).*

An optimal monopolist market maker recognises that learning is endogenous to the prices he posts. He has the incentive to set prices to encourage learning because he trades over multiple periods. Unlike a competitive market maker, he is not constrained in his ability to set prices. Leach Madhavan call this 'active learning' through 'experimentation'. In contrast, a competitive market maker only learns passively.

Definition 2.3. *A myopic monopolist's first period price choice $p_1^m = (a_1^m, b_1^m) \in P$ is the solution to:*

$$\max \pi(a_1, b_1; \mu_1)$$

A myopic market maker is only concerned with maximising one period profits and does not consider the impact of prices on information revealed by trades. The term may suggest a behavioural story in which the market maker does not recognise the full extent of his actions. However, it is also consistent with a rational market maker facing constraints. Perhaps he only has limited monopoly power because competitors may enter in the next period.

Definition 2.4. *A competitive first period price choice* $p_1^c = (a_1^c, b_1^c) \in P$ *satisfies:*

$$(a_1^c, b_1^c) = \inf\{a - b \in P : \pi(a, b) \geq 0\}$$

In contrast to a monopolist, a competitive market maker cannot trade at all possible prices. Instead, imagine multiple market makers involved in a Bertrand price setting game in which other traders only submit orders at the narrowest spread. Competitive equilibrium prices minimize expected profits subject to nonnegativity.

Proposition 2.2. *In equilibrium, competitive ask and bid prices are given by:*

$$a_t^c = E\left[\widetilde{\theta}\,|q_t = +1, a_t^c, b_t^c; \mu_t\right]$$
$$b_t^c = E\left[\widetilde{\theta}\,|q_t = -1, a_t^c, b_t^c; \mu_t\right]$$

where $a_t^c \geq b_t^c$.

Like Glosten-Milgrom, competitive prices are ex post regret free. The posted ask price is the expected value of the asset conditional on the next trade being a buy while the bid is the expected value conditional on the next trade being a sell. Competition drives expected profits in every period to zero so there is no incentive for the market maker to induce learning. Information still enters the market passively because potentially informed traders acquire information before trading but the value of information does not affect price setting.

2.3.3 Information and Market Structure

There are two transmission channels from prices to the informativeness of trades: the 'information acquisition' and 'noise' channels. I differentiate 'market structure' by whether wider or narrower spreads increase informativeness. Consider the effect of prices on the informativeness of a sell trade $N_t(q_t = -1|a_t, b_t, \mu_t)$.

First, the 'information acquisition' channel captures the impact of prices on the amount of information acquired by potentially informed traders. Consider the posterior belief after a sell trade $\mu_{t+1}(-1)$ given by Equation (2.1). Both ask and bid prices, a_t and b_t, enter the information arrival function $X(a_t, b_t, \mu_t)$. Thus, I further distinguish between a 'direct' and 'indirect' information acquisition channel. The 'direct information acquisition' channel affects a sell trade through the bid price b_t. It has a direct effect because it raises profits from selling the asset when its true value is low. Thus potentially informed traders acquire more information when b_t is high, (recall that $X(a, b, \mu_t)$ is increasing in b_t). This channel makes a sell trade more informative with higher b_t. With the assumption of separate buyers and sellers, this is the only channel which operates because sellers can only trade at b_t.

The 'indirect information acquisition' channel affects a sell trade through the ask price a_t. While it has no impact on profits from selling the asset, it does affect profits from buying the asset when its value is high. Thus, lowering the ask price raises profits and potentially informed traders acquire more information (recall that the information arrival function $X(a_t, b_t, \mu)$ is decreasing in a_t). This is the 'direct information acquisition' channel of a_t on a *buy* trade. However, it also has an indirect effect on a sell trade because when potentially informed traders acquire more information in expectation of the high asset value, they are also more likely to discover that the true value is low.

Second, the 'noise' channel captures the impact of prices on the participation of noise traders. Returning to the expression for the posterior belief after a sell trade $\mu_{t+1}(-1)$ in Equation (2.1), the bid price b_t also enters the density function of noise trader reservation values $F(b_t)$. Raising b_t increases the probability that a noise trader has a reservation value below b_t, the criteria to submit a sell order, $F(b_t)$ is increasing in b_t. Thus, raising b_t decreases the proportion of uninformed sell trades in the market and makes a sell trade more informative. The lower bid price crowds out noise traders so a larger proportion of trades are informed.

The two channels operate symmetrically on the informativeness of a buy trade $N_t(q_t = +1|a_t, b_t, \mu_t)$. First, the direct information acquisition channel makes the informativeness of a buy trade decreasing in the ask price a_t and the indirect channel makes it increasing in the bid price b_t. Second, the noise channel makes it increasing in a_t.

Finally, I characterise market structure by how spreads affect the informativeness of trades. The first market structure is characterised by narrower spreads increasing the informativeness of trades. The information acquisition channel makes narrower spreads increase the informativeness of both buy and sell trades. The noise channel does the opposite, decreasing the informativeness of trades. The aggregate impact on informativeness depends on which channel dominates. When the first effect to be stronger, I get the definition of a market structure in which 'information acquisition dominates informativeness'.

Definition 2.5. *'Information acquisition dominates informativeness' if* $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} > 0$ *and* $\frac{\partial \mu_{t+1}(+1)}{\partial a_t} - \frac{\partial \mu_{t+1}(+1)}{\partial b_t} > 0 \ \forall \ \mu_t \in [0, 1],\ a_t > b_t$.

The first inequality refers to the informativeness a sell trade. The term $\frac{\partial \mu_{t+1}(-1)}{\partial b_t}$ captures both the direct information acquisition and noise channels of the bid price b_t on a sell trade. The information acquisition channel acts to make it positive while the noise channel, negative, so the net sign depends on the relative sizes of the two channels. The other term $\frac{\partial \mu_{t+1}(-1)}{\partial a_t}$ captures the indirect information acquisition channel of the ask price a_t on a sell trade. This term is always negative. In aggregate, I want narrower spreads to increase informativeness of a sell trade, which is equivalent to increasing the posterior after a sell $\mu_{t+1}(-1)$. Hence the inequality: $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} > 0$. The second inequality captures the analogous effect of prices on the informativeness of a buy trade.

The second market structure is characterised by wider spreads increasing the informativeness of trades. Now, the noise channel is stronger than the information acquisition channel, which yields the definition of a market structure in which 'noise dominates informativeness'.

Definition 2.6. *'Noise dominates informativeness' if* $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} < 0$ *and* $\frac{\partial \mu_{t+1}(+1)}{\partial a_t} - \frac{\partial \mu_{t+1}(+1)}{\partial b_t} < 0 \ \forall \ \mu_t \in [0,1],\ a_t > b_t$.

The inequalities are reversed relative to the first market structure. The noise channel must be sufficiently large to overcome both the direct and indirect information acquisition channels. This market information structure nests Leach-Madhavan which corresponds to an information arrival function $X(a, b, \mu)$ of unity. Then, potentially informed traders always receive the true value of the asset and trade, regardless of prices or beliefs. The information acquisition channels do not operate so prices only affect the informativeness of trades through the noise channel. Under my characterisation, noise dominates informativeness and wider spreads increase the informativeness of trades.

Lemma 2.1. *If X and F are differentiable, information acquisition dominates informativeness if* $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t, b_t, \mu_t)\frac{\partial F}{\partial b_t} > 0$, *and* $(1 - F(a_t))(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t, b_t, \mu_t)\frac{\partial F}{\partial a_t} > 0, \ \forall \ \mu_t \in [0,1],\ a_t > b_t$.

Lemma 2.2. *If X and F are differentiable, noise dominates informativeness if* $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t, b_t, \mu_t)\frac{\partial F}{\partial b_t} < 0$ *and* $(1 - F(a_t))(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t, b_t, \mu_t)\frac{\partial F}{\partial a_t} < 0, \ \forall \ \mu_t \in [0,1],\ a_t > b_t$.

The inequalities in Lemmas 2.1 and 2.2 can be separated into the two transmission channels. The first inequality in both Lemmas refer to the informativeness of a sell trade. The term $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t})$ captures the information acquisition channel. It can be further separated into two parts: $F(b_t)\frac{\partial X}{\partial b_t} - F(b_t)\frac{\partial X}{\partial a_t}$. $F(b_t)\frac{\partial X}{\partial b_t}$ captures the direct information acquisition channel of the bid price b_t and $F(b_t)\frac{\partial X}{\partial a_t}$, the indirect information acquisition channel of the ask price a_t. $\frac{\partial X}{\partial b_t}$ is always positive while $\frac{\partial X}{\partial a_t}$ is always negative so the combined term shows the two channels working in the same direction. The level of b_t enters through $F(b_t)$ so the information acquisition channel is stronger when there is greater noise trader participation. Only b_t appears because it has a direct effect on the profitability of a sell trade while a_t has no affect on sells. The term $X(a_t, b_t, \mu_t)\frac{\partial F}{\partial b_t}$ captures the noise channel. $\frac{\partial F}{\partial b_t}$ is always positive so the noise channel always operates counter to the two information acquisition channel. The level of information arrival enters through $X(a_t, b_t, \mu_t)$ so the noise channel is stronger when more potentially informed traders acquire information. Again, the second equalities in the Lemmas refer to the informativeness of a buy trade.

This characterisation of market structure is not exhaustive. I concentrate on market structures which are consistent across all prior beliefs $\mu_t \in [0, 1]$ and prices $a_t > b_t$. In general, it is possible for one channel to dominate informativeness for some range of prior beliefs and prices while the other dominates for a different range. Such markets exhibit changing informational regimes and prices do not have a consistent effect on the informativeness of trades.

2.4 Motivating Examples

This section describes two motivating examples: one for endogenous information acquisition and the other for the benchmark with exogenous information acquisition. The first example follows the quadratic information acquisition cost function described in Chapter 1. These examples are of interest for three reasons. First, the endogenous information acquisition example provides a microfoundation for the information arrival process of potentially informed traders $X(a_t, b_t, \mu_t)$. Second, I can analytically solve the examples for equilibrium prices under my three market maker types for a single period with a straightforward extension to many periods. Finally, they are useful for illustrating the more general results.

For now I only solve the one period version so I drop the time subscript from variables. I consider multiple periods later. The asset $\widetilde{\theta}$ now takes values $\{\theta_1 = 0, \theta_2 = H\}$ where $H > 0$. The setup follows the general model with one alteration: traders are either buyers or sellers with equal probability. They receive their type when they are drawn into the market and can only trade according to type. This assumption is made for tractability and does not affect any qualitative results. Then I specify forms for: 1) the density of noise trader reservation value F; and 2) the information arrival process of potentially informed traders $X(a, b, \mu)$.

First, noise trader reservation value is uniformly distributed between $\underline{\theta}$ and $\overline{\theta}$ where $\underline{\theta}< 0$ and $\overline{\theta} > H$. These bounds mean that noise traders trade even when prices are beyond those at which informed traders stop trading. The uniform distribution of reservation values is equivalent to a linear, downward sloping demand curve for noise buyers facing ask price a, which can be written as $F(b) = \alpha - \beta a$. Analogously, it is equivalent to a linear, upward sloping supply curve for noise sellers facing bid price b, which can be written as $F(a) = \alpha - \beta(H - b)$. I choose the parameters α and β so that probabilities lie within $[0, 1]$. An additional restriction, $\alpha = \frac{1}{2}(\beta V + 1)$, is required for the demand and supply functions to correspond to a uniform distribution of reservation values. For tractability, I solve the model using this formulation instead of the cumulative density function directly.

Second, I provide a microfoundation for endogenous information acquisition. The information arrival process of potentially informed traders X arises from the optimal behaviour of potentially informed traders who face an information acquisition cost which is increasing in the precision of the signal they can acquire. To learn the true value of the asset with probability ω, they must pay the cost $\frac{1}{2}C\omega^2$, where C is a positive parameter. As risk neutral, profit maximising agents, they optimally choose the amount of information to acquire, denoted ω^*. Before acquiring information, they know as much as the market maker, which includes the history of past prices and trades, and so have the same prior beliefs. There is a random draw in which they learn the true value of the asset with probability ω^*. If they learn the true value of the asset they submit a corresponding trade. If not, they have no informational advantage and do not trade. The amount of information acquired by a potentially informed trader is given by ω_p, where p is the price the trader cares about. For a buyer it is the ask price a and for a seller, the bid price b. The objective function for a potentially informed buyer is:

$$(1-\mu)\omega_a(H-a) - \frac{1}{2}C\omega_a^2 \tag{2.8}$$

For a potentially informed seller, it is:

$$\mu\omega_b b - \frac{1}{2}C\omega_b^2 \tag{2.9}$$

Potentially informed traders acquire the optimal amount of information given by:

$$\omega_a^* = \frac{1}{C}(1-\mu)(H-a) \tag{2.10}$$

$$\omega_b^* = \frac{1}{C}\mu b \tag{2.11}$$

If a potentially informed seller learns the true value of the asset, he submits a sell trade $q = -1$. If a potentially informed buyer learns the true value of the asset, he submits a buy trade $q = +1$. Otherwise he does not trade. From the market maker's perspective, a potentially informed trader submits a trade q with probabilities:

$$q = \begin{cases} -1 & \text{with probability } \frac{1}{2C}\mu^2 b \\ +1 & \text{with probability } \frac{1}{2C}(1-\mu)^2(H-a) \\ 0 & \text{with probability } 1 - \frac{1}{2C}\mu^2 b - \frac{1}{2C}(1-\mu)^2(H-a) \end{cases}$$

In the general framework, these probabilities are equivalent to defining a separate information arrival function for buyers and sellers. The new functions would take values $X_a(a,\mu) = \frac{1}{2C}(1-\mu)(H-a)$ for buyers and $X_b(b,\mu) = \frac{1}{C}\mu b$ for sellers.

This setup captures the stylised feature that the amount of information acquired by buyers is decreasing in the ask price and the amount acquired by sellers is increasing in the bid price. Potentially informed traders make speculative profits from the difference between the true value of the asset and the price at which he can buy or sell. He acquires more information when greater profits are available. It motivates the restrictions I impose on the information arrival function of potentially informed traders X in Section 2.4.

The aggregate impact of spreads on the informativeness of trades depends on the transmission channel which dominates. It is easy to show that information acquisition dominates informativeness for this example under the parameter constraint $\alpha > \beta V$. This case is my first motivating example.

Now I describe a version with exogenous information acquisition. Noise traders remain the same as the first example but now potentially informed traders always learn the true value of the asset, similar to Leach-Madhavan. Prices and prior beliefs have no impact on the informativeness of trades. From the market maker's perspective, a potentially informed seller submits a sell trade with probability μ_t and a potentially informed buyer submits a buy trade with probability $1 - \mu_t$ because exogenously informed traders always trade when the market is open. Prices do not affect the information acquired by traders so there is no information acquisition channel. This market structure satisfies the definition that crowding out dominates informativeness. This case is my second motivating example.

It is easiest to see the shape of price schedules graphically. The price schedule refers to the function of equilibrium ask and bid prices, $\{a, b\}$, set by a specific market maker for every possible prior belief. I choose some parameters for which the market is open ($H = 10$, $\lambda = 0.5$, $\beta = \frac{1}{11}$, $\alpha = \frac{1}{2}(\beta H + 1) = \frac{21}{22}$) and plot price schedules of myopic and competitive market makers, denoted $\{a^m, b^m\}$ and $\{a^c, b^c\}$ (see Appendix for details of solving for prices). Figure 2.1(a) shows the benchmark example of exogenous information acquisition and Figure 2.1(b) shows the case with endogenous information acquisition. Solid lines correspond to prices set by the myopic monopolistic market maker $\{a^m, b^m\}$ and dotted lines to those set by a competitive market maker $\{a^c, b^c\}$. The upper of each pair of lines is a and the lower is b. Recall that μ is the prior belief that the true value of the asset is low so all prices are decreasing in μ.

I define a measure for the strength of a prior as $|\mu - \frac{1}{2}|$. This captures how close a prior belief is to either of the extremes. The weakest prior is $\mu = \frac{1}{2}$ when the market is most uncertain about the asset value, assigning each value equal probability. The strongest priors are $\mu = 0$ and $\mu = 1$, when the true asset value is known to be low or high with certainty. I use the terms 'stronger towards' to denote the direction of the prior belief.

Now, I concentrate on the ask price as prior belief μ decreases from 1 to 0. The same intuition applies symmetrically with the bid price as μ goes from 0 to 1. I start with ask prices set by the competitive market maker a^c, denoted by dotted lines in Figure 2.1. At a glance, they behave similarly in both markets. Going from $\mu = 1$ to 0.5, or weakening the prior belief, a^c increases linearly. There is a linear trade off between the participation of potentially informed traders, capturing the amount

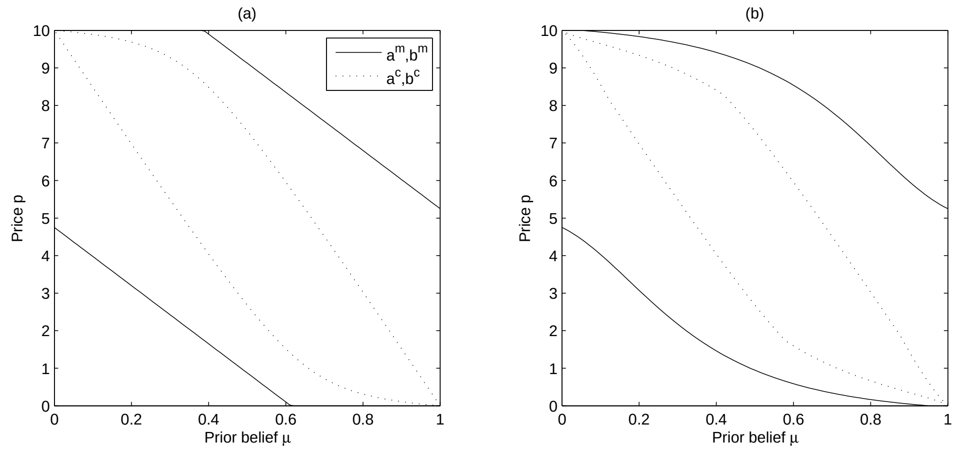

Figure 2.1: Ask and bid prices set by competitive, $\{a^c, b^c\}$, and myopic monopolistic, $\{a^m, b^m\}$, market makers under: (a) exogenous information acquisition; and (b) endogenous information acquisition.

of information acquired, and the participation of noise traders, capturing the price elasticity of their demand. As μ continues to decrease from 0.5 to 0, strengthening the prior belief towards the high asset value, the linear relationship breaks down. a^c increases more slowly in Figure 2.1(b) under endogenous information acquisition than in Figure 2.1(a) under exogenous information acquisition . As μ becomes stronger, a^c approaches the true value of the asset, potentially informed traders earn lower expected profits and thus acquire less information. To maintain the zero profit condition, the market maker must set a lower a^c to incentivise potentially informed traders. This channel does not exist in the exogenous information acquisition example.

Next, I look at the prices set by a myopic monopolistic market maker $\{a^m, b^m\}$, denoted by solid lines in Figure 2.1. In one period, this is equivalent to optimal monopolistic prices. The spread is considerably wider than under competition in both markets. A monopolistic market maker maximises profits and wider spreads yield higher expected profits from trades with noise traders and lower expected losses from trades with informed traders. However, he does not set the maximum spread because noise traders are price elastic and their participation falls as the spread widens. Again, I consider the ask price a^m as prior belief μ decreases from 1 to 0. In Figure 2.1(a), with exogenous information acquisition, a^m increases linearly until it reaches the high asset value H, at about $\mu = 0.4$, after which it is constant. After that point, the market maker closes the market to potentially informed buyers and only trades with noise traders. In contrast, in Figure 2.1(b), with endogenous information acquisition, the rate of increase falls and the ask price only reaches H at about $\mu = 0.1$.

The difference between the two examples is due to the price sensitivity of information acquisition by potentially informed traders. Exogenously informed traders always trade if the ask price a^m is below the true value. When the prior belief μ is strong towards the high asset value, the market maker has a high expectation that an informed trader submits a buy trade. He wants to make the least expected losses when trading with these informed traders so he sets a high a^m. This price also reduces the participation of noise traders but the effect on informed traders has a greater impact on the market maker's expected profits. However, endogenously informed traders respond to wider spreads. They make lower profits from trading with wider spreads so they acquire less information and trade less. For a given spread, the market maker expects fewer informed trades under endogenous information acquisition and can therefore set narrower spreads to increase the participation of noise traders.

These examples motivate the two market characterisations from Section 2.3. The first example with endogenous information acquisition satisfies the conditions for information acquisition dominating informativeness from Definition (2.5). The second example with exogenous information acquisition satisfies the conditions for crowding out dominating informativeness from Definition (2.6). I refer to them by these general characterisations in later sections.

2.5 Market Microstructure

2.5.1 Prices and Volumes

The main result in this section is that endogenous information acquisition affects how an optimal monopolistic market maker sets prices. He sets narrower spreads than the myopic monopolist when information acquisition dominates informativeness, and wider spreads when noise dominates informativeness. The inverse relation holds for expected transactions volume. My model nests Leach-Madhavan as a special case of the second market information structure. In the following propositions, the market maker sets prices in the first of T trading periods. I suppress the subscript for $t = 1$ for clarity.

Proposition 2.3. *If markets are open under both competitive and monopolistic market makers and information acquisition dominates informativeness, then spreads are narrower under an optimal monopolist than under a myopic monopolist. They are also narrower under a competitive market maker than under a myopic monopolist. Specifically:* $b^* \geq b^m$, $b^c \geq b^m$ *and* $a^* \leq a^m$, $a^c \leq a^m$.

Proposition 2.4. *If markets are open under both competitive and monopolistic market makers and noise dominates informativeness, then spreads are wider under an optimal monopolist than under a myopic monopolist, which are wider than under a competitive market maker. Specifically:* $b^c \geq b^m \geq b^*$ *and* $a^c \leq a^m \leq a^*$.

The relationship between prices set by a myopic monopolistic market maker and those set by a competitive one is the same in both markets. A myopic monopolistic market maker never sets narrower spreads than a competitive one because by definition, he makes negative expected profits by doing so. However, the prices set by an optimal monopolistic market maker relative to a myopic one depend on the market information structure. If information acquisition dominates informativeness, an optimal monopolist market maker facing multiple trading periods sets narrower spreads while if noise dominates informativeness, he sets wider spreads.

The intuition for this price setting result is that an optimal monopolist market maker who trades over multiple periods has the incentive to increase informativeness of trades in the early periods because stronger beliefs yield higher expected profits in later periods. By Proposition 2.1, the monopolist market maker's value function is convex in the prior belief μ so increasing the change of that prior, increases the expected profit of the market maker. The next insight is that how the market maker can change prices to increase the informativeness of trades depends on the market information structure.

Propositions 2.3 and 2.4 describe the price setting of different market makers *within* a market. However, I cannot use them to compare prices *between* markets because my characterisation of market structure is insufficient to determine price levels. It only describes how price changes affect the informativeness of trades. In general, price levels are determined by the exact relationship between the information arrival function of potentially informed traders X and the density function of noise trader reservation values F.

I turn to my motivating examples to illustrate these results. Figure 2.2 plots first period prices $\{a, b\}$ and expected profits $E[\pi]$ against prior belief μ with four market maker types (competitive, myopic, optimal two period and optimal three period) under my two market information structures.

Figures 2.2(a) and (c) plot the benchmark case in which crowding out dominates informativeness. In Figure 2.2(a), the spread set by an optimal monopolistic market maker is increasing in the number of trading periods. Also, both monopolistic spreads are wider than the competitive one. In turn, the expected first period profits of an optimal monopolistic market maker shown in Figure 2(c) are decreasing in the number

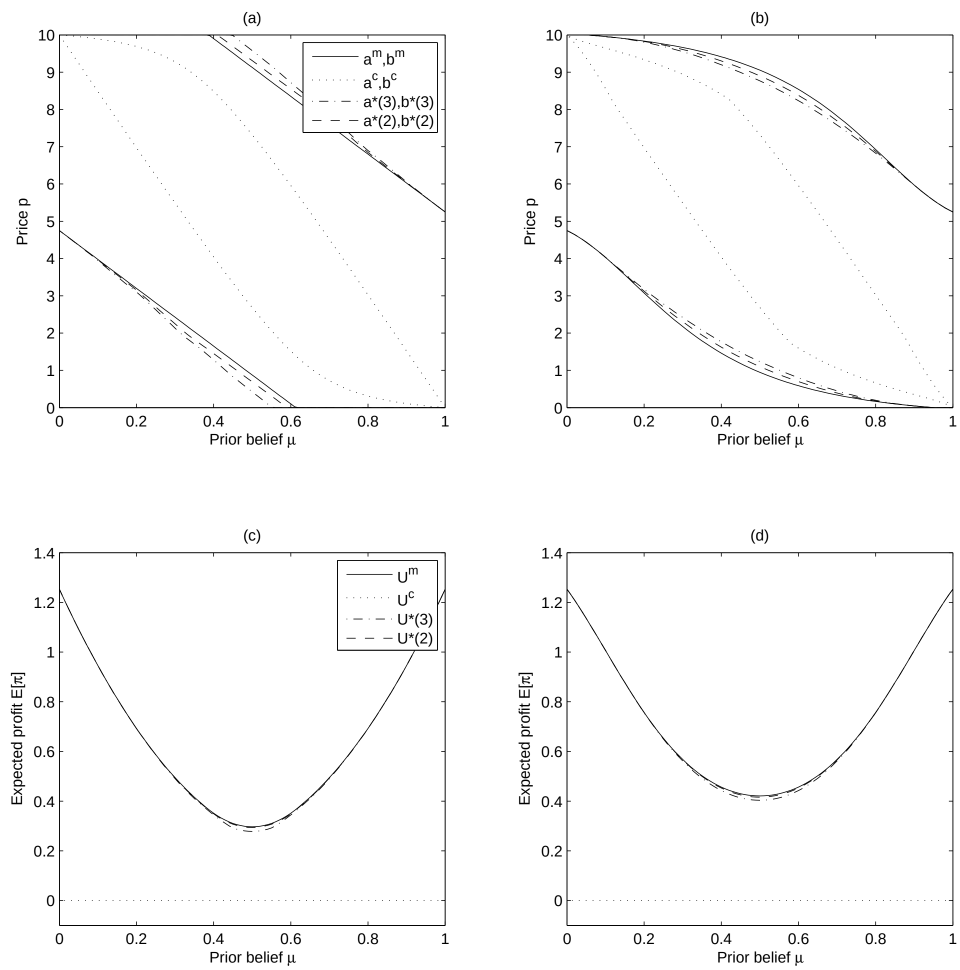

Figure 2.2: Ask and bid prices $\{a, b\}$ set by different market makers when: (a) crowding out dominates informativeness; and (b) information acquisition dominates informativeness. Expected first period profits $E[\pi]$ earned by different market makers under: (c) crowding out dominates informativeness; and (d) information acquisition dominates informativeness. $a^*(T)$ denotes the first period ask price under an optimal monopolistic market maker operating over T periods and so on. Also, $a^m = a^*(1)$.

of trading periods. He takes lower first period profits to increase the informativeness of trades when there are more future periods to exploit that information.

Figures 2.2(b) and (c) plot the case in which information acquisition dominates informativeness. Figure 2.2(b) contrasts with the benchmark in Figure 2.2(a). Now the spread set by an optimal monopolistic market maker is decreasing in the number of trading periods. Note that both monopolists still set wider spreads than a competitive market maker. While this holds in this example, in general the optimal monopolistic market maker can set spreads which are narrower than the competitive one with this market structure. Next, although spreads behave differently, expected first period profits of an optimal monopolistic market maker shown in Figure 2(d) are still decreasing in the number of trading periods. Again, more informative trades come at the expense of first period profits.

Corollary 2.1. *If markets are open and information acquisition dominates informativeness, the first period spreads set by an optimal monopolistic market maker are weakly decreasing in the number of trading periods T.*

Corollary 2.2. *If markets are open and noise dominates informativeness, first period spreads set by an optimal monopolistic market maker are weakly increasing in the number of trading periods T.*

Increasing the number of trading periods increases the incentive for the market maker to increase informativeness in early periods because by Proposition 2.1, the market maker has a convex value function. Then the market maker affects prices through spreads depending on the market information structure as described by Propositions 2.3 and 2.4.

Corollary 2.3. *If markets are open with both competitive and monopolistic market makers and information acquisition dominates informativeness, then expected transaction volume is higher under an optimal monopolist than under a myopic monopolist,* *and it is higher under a competitive market maker than under a myopic monopolist, specifically:* $E[|q^*|] \geq E[|q^m|]$ *and* $E[|q^c|] \geq E[|q^m|]$.

Corollary 2.4. *If markets are open with both competitive and monopolistic market makers and noise dominates informativeness, then expected transaction volume is lower under an optimal monopolist than under a myopic monopolist, which is lower than under a competitive market maker, specifically:* $E[|q^*|] \leq E[|q^m|] \leq E[|q^c|]$.

The relationship between transaction volume and informativeness of trades is not always the intuitive one. By Corollary 2.3, when information acquisition dominates informativeness, higher transaction volume corresponds to more informative trades. However, by Corollary 2.4, when noise dominates informativeness, the opposite holds and lower transaction volume corresponds to more informative trades. With exogenous information acquisition, Leach and Madhavan find only the second case.

Corollary 2.4 for the general model with elastic noise traders and a monopolistic market maker shares the intuition of Corollary 1.3 for the model with inelastic noise traders and a competitive market maker. By Corollary 1.3, for some range of the proportion of potentially informed traders λ, lower expected transaction volume $E[|q|]$ coincides with more informative trades. The two results are driven by the participation of potentially informed traders relative to noise traders. Under the conditions given by the corollaries, trades are more informative despite there being fewer expected trades because a larger proportion of them are informed.

Returning to my motivating examples, Figure 2.3 plots expected transaction volumes $E[|q|]$ against prior belief μ in my examples using the same parameters as before. Figure 2.3(a) plots expected transaction volumes under the price schedules from Figure 2.2(a), when crowding out dominates informativeness. The optimal monopolistic market maker's expected transaction volume $E[|q^*|]$ is decreasing in the number of his trading periods T. Similarly, Figure 2.3(b) corresponds to Figure 2.2(b), when

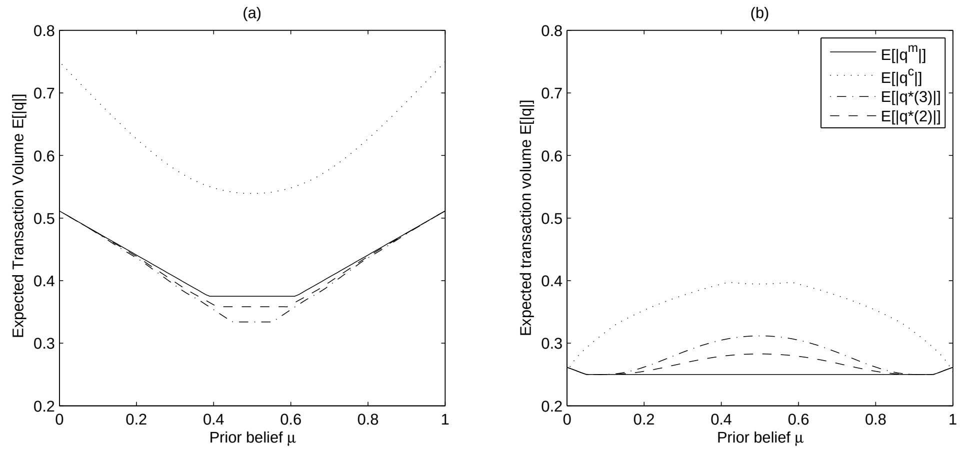

Figure 2.3: Expected transaction volumes $E[|q|]$ facing different market makers when: (a) crowding out dominates informativeness; and (b) information acquisition dominates informativeness. $E[|q^*(T)|]$ denotes the expected transaction volume under an optimal monopolistic market maker operating over T periods. Also $E[|q^m|] = E[|q^*(1)|]$.

information acquisition dominates informativeness. Now, $E[|q^*|]$ is increasing in T. In both examples, there is higher expected transaction volume under a competitive market maker $E[|q^c|]$ because he sets narrower spreads. However, it does not hold in general. In a market where information acquisition dominates informativeness, $E[|q^*|]$ can be larger than $E[|q^c|]$.

Comparing between the two market structures, any given market maker type faces higher expected transaction volume when crowding out dominates informativeness. This relationship is not general to market structure type. There happens to be a direct relationship between the motivating examples because the proportion of traders are the same in both of them while potentially informed traders are either exogenously or endogenously informed. Since exogenously informed traders always trade but potentially informed traders only trade if they successfully acquire information, there must be higher transaction volume in the first case.

2.5.2 Dynamics of Prices

In Glosten-Milgrom, spreads always narrow after a trade which strengthens beliefs and widen after a trade which weakens them. I find conditions under which spreads can behave in the opposite direction, widening after a trade which strengthens beliefs and narrowing after a trade which weakens them.

The price schedule is the function of equilibrium ask and bid prices $\{a_t, b_t\}$ set by a market maker at time t for every possible prior belief μ_t. Price schedules are characterised by whether for a given time t, spreads are symmetric and monotonically narrowing or widening in the strength of beliefs, $|\mu_t - \frac{1}{2}|$. I concentrate on the standard case the price schedules imply spreads which narrow in the strength of beliefs although in general, they need not be. The following results refer to the evolution of spreads over time, from time t to $t+1$.

Corollary 2.5. *If information acquisition dominates informativeness, optimal monopolistic spreads widen after a trade which strengthens beliefs, i.e.* $\mu_t + 1$ *closer to* 0 *or* 1. *If crowding out dominates informativeness, optimal monopolistic spreads may narrow after a trade which weakens beliefs, i.e.* $\mu_t + 1$ *closer to* $\frac{1}{2}$.

Although Corollaries 2.1 and 2.2 describe the relationship between first period spreads and the total number of trading periods, this does not translate into the dynamic behaviour of spreads because beliefs evolve endogenously between periods. By Corollary 2.5, spreads may behave counterintuitively, widening after a trade which strengthens beliefs, or narrowing after a trade which weakens them.

Figure 2.4 shows an example of each case. It plots the spreads $a_t - b_t$ for periods $t = 1$ and $t = 2$ across beliefs μ_t. In Figure 2.4(a), information acquisition dominates informativeness so spreads at $t = 1$, drawn with a solid line, are narrower than at $t = 2$, drawn with a broken line, across beliefs μ_t. Consider an uninformative prior belief in period 1, $\mu_1 = 0.5$ and a sell trade which strengthens beliefs in period 2 to

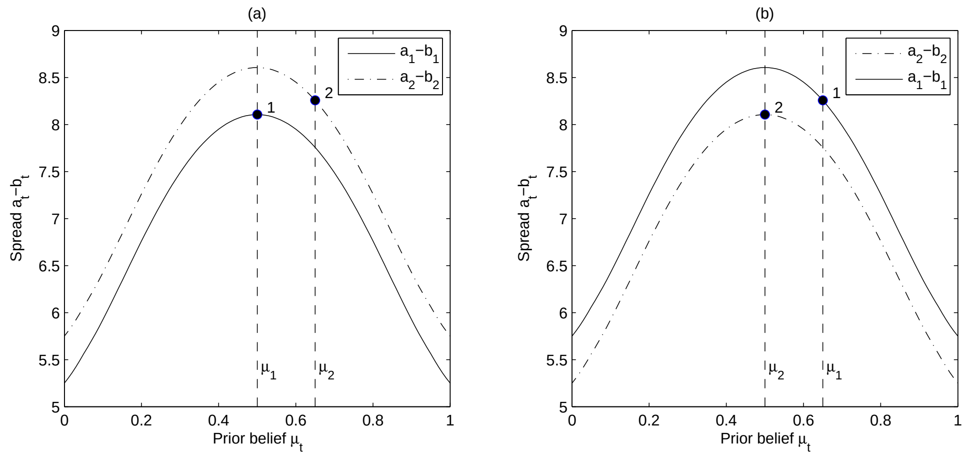

Figure 2.4: Dynamic behaviour of spread $a_t - b_t$ after a trade which (a) strengthens beliefs when information acquisition dominates informativeness; and (b) weakens beliefs when noise dominates informativeness.

$\mu_2 = 0.65$. In this example, the spread in period 2 is wider than in period 1.

In Figure 2.4(b), noise dominates informativeness so spreads at $t = 1$, solid line, are wider than at $t = 2$, broken line, across beliefs μ_t. Now consider a prior belief in period 1 at $\mu_1 = 0.65$ and a buy trade which weakens beliefs in period 2 to $\mu_2 = 0.5$. In this example, the spread in period 2 is narrower than in period 1. Note that neither of these outcomes are necessary. For example, in Figure 2.4(a), a larger μ_2 could mean a narrower spread in period 2.

The counterintuitive spread dynamics are driven by the interaction of an optimal monopolistic market maker and market information structure. The optimal monopolistic market maker sets price schedules which change over time and the market information structure determines the direction of the change. The evolution of spreads then depends on relative effects of the change in beliefs after a trade and the difference in optimal spreads between each period.

While the examples above apply to realised trades over two periods, the results also apply to expected trades over multiple periods. Starting from an uninformed prior $\mu_1 = \frac{1}{2}$, expected beliefs grow monotonically stronger over time. If each period's price schedule implies sufficiently wider spreads each period, it is possible for expected spreads to widen every period from $t = 1$ to T.

2.5.3 Information Stoppage and the Steady State

I find a new phenomenon which I call 'information stoppage' in markets with endogenous information acquisition. It occurs when potentially informed traders choose to stop acquiring information while the market remains open. Unlike in exogenous information acquisition models, such as Glosten-Milgrom and Leach-Madhavan, prices do not always converge to the true value in the steady state.

Definition 2.7. *An 'information stoppage' occurs when a market is open and the probability of an informed trade arriving is* 0*.*

Potentially informed traders learn the true value of the asset through their information arrival function X. This function is bounded $[0, 1]$ so it is possible for it to be 0 for some range of its arguments (a_t, b_t, μ_t). If the prior belief μ_t happens to fall in that range, the market maker can set prices $\{a_t, b_t\}$ such that potentially informed traders do not acquire information and do not trade. Then only noise traders participate.

Corollary 2.6. *If an information stoppage occurs in any period, the informativeness of all subsequent trades is zero.*

Corollary 2.6 follows from Definition 2.7 and the Bayes' rule for updating beliefs given in Equations 2.1 and 2.2. If an information stoppage occurs, all market participants know that potentially trades stop acquiring information. Trades stop revealing any information. The market maker does not update beliefs or change prices. Both are constant until the final period T and thus make up the steady state. By Definition 2.1 the market can still be open because prices are in the interior range of possible asset values (θ_1, θ_2). If a speculative trader who knows the true value of the asset, he could make a profitable trade. However, during an information stoppage, traders choose not to acquire information.

An information stoppage cannot arise with standard exogenously informed traders because they are informed regardless of prices or beliefs. Potentially informed traders endogenously choose to acquire information and can optimally choose not to. In the standard case, prices always converge to the true value in the steady state. In mine, this is not generally true. By Corollary 2.6, prices stop changing after an information stoppage. Those prices make up the steady state and may diverge permanently from the true value.

The possibility of information stoppages is entirely due to endogenous information acquisition. They can occur in either of my market structure categories and under any market maker type. A competitive market maker who sets prices under the zero profit condition may encounter it by chance. A monopolistic market maker may choose to induce an information stoppage because it removes potentially informed traders from the market. If they have a weak demand for information, an information stoppage means they can just trade with noise traders.

To illustrate information stoppage, my first motivating example with endogenous information acquisition needs a slight addition. The original version does not exhibit this feature because expected profits from speculative trades are always positive when the market is open and the cost potentially informed traders pay starts at 0 so they always acquire some amount of information. Therefore, I introduce a new fixed cost of information acquisition D in addition to the variable cost from before. This is a one period case so again I suppress time subscripts t. The objective function of a potentially informed buyer becomes:

$$(1-\mu)\omega_a(H-a) - \frac{1}{2}C\omega_a^2 - D \tag{2.12}$$

and a seller becomes:

$$\mu\omega_b b - \frac{1}{2}C\omega_b^2 - D \tag{2.13}$$

A potentially informed trader acquires the optimal amount of information given by: $\omega_a^* = \frac{1}{C}(1-\mu)(H-a)$ for buyers and $\omega_b^* = \frac{1}{C}\mu b$ for sellers, as before. However, now they also check their expected profit net of information acquisition cost and only acquire information if it is positive. Otherwise, they do not acquire information or trade. Figure 2.5 presents this example together with the originals for comparison. I set parameter $D = 0.005$ for the new case and $D = 0$ refers to the first two. The other parameters are as before. Figure 2.5(a) plots the price schedules $\{a, b\}$ in the different market structures. Figures 2.5(b) and (c) plot expected probabilities of information arrival of potentially informed buyers, given by X_a, and sellers, given by X_b.

Now, I look in detail at potentially informed buyers and ask prices set by a monopolistic market maker a^*. Wide dotted lines represent the benchmark market with fixed cost 0 while solid lines represent the market with fixed cost $D = 0.005$. In Figure 2.5(c), the expected probability of information acquisition by potentially informed buyers X_a are the same for prior beliefs $\mu = 0.24$ to 1, regardless of the fixed cost. However, for $\mu = 0$ to 0.24, the myopic monopolistic market maker induces potentially informed buyers to stop acquiring information in the market with fixed cost D but not in the market with fixed cost 0. The corresponding prices in Figure 2.5(a) show that both markets remain open. Thus there is an information stoppage on the buy side for this range of beliefs. A small price deviation is sufficient to cause the information stoppage when compared to the market without a fixed cost. Note that in this range, only a partial stoppage occurs because it is confined to potentially informed buyers while sellers continue to acquire information. Buy trades become uninformative but the market maker continues to update beliefs following a sell trade. The behaviour of sellers and bid prices follow analogously.

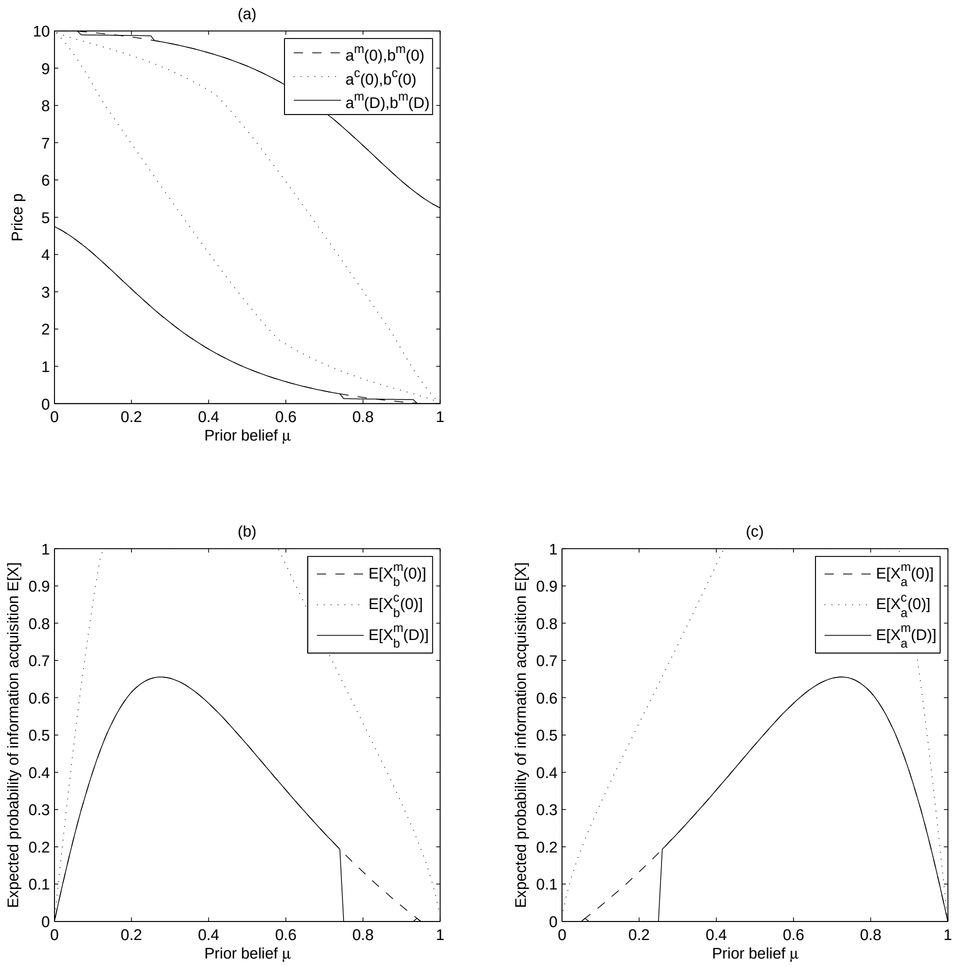

Figure 2.5: (a) Ask and bid prices $\{a, b\}$; (b) expected probability of information acquisition by potentially informed sellers X_a; and (c) expected probability of information acquisition by potentially informed buyers X_b; across markets with different fixed costs and competitive and myopic monopolistic market maker types. $a^c(D)$ denotes the ask price under a competitive market maker in a market with fixed cost $D = 0.005$. $E[X_b^c(0)]$ denotes the expected probability of information arrival by potentially informed sellers under a competitive market maker in a market with fixed cost 0 and so on.

Although this example does not display a full information stoppage when all potentially informed traders stop acquiring information, I can obtain a weaker version of Corollary 2.6: the steady state ask price cannot exceed the one at which a partial buy side stoppage occurs and vice versa for the steady state bid price. The partial stoppage in my example is an artifact of separating buyers and sellers. In the general model, potentially informed traders can both buy and sell so an information stoppage affects both sides simultaneously.

2.5.4 Market Failure

One of Leach and Madhavan's main objectives is to compare the robustness of markets under a monopolistic versus a competitive market maker. Their robustness results extend to my model because they are not affected by the introduction of potentially informed traders. Leach and Madhavan find that with F concentrated in $[\theta_1, \theta_2]$, if the market is open under a competitive market maker, it is also open under a monopolistic one. Furthermore, there are cases when a competitive market maker fails to open a market but an optimal monopolistic one does, so monopolistic markets are more robust. This result is driven by the multiperiod considerations of the monopolistic market maker. If F is not concentrated in $[\theta_1, \theta_2]$, the opposite result obtains that competitive market makers are more robust than monopolistic ones. A monopolistic market maker may set price outside the range (θ_1, θ_2) to close the market and take advantage of noise traders whose reservation values lie there.

2.6 Conclusion

This chapter makes two main contributions. First, I develop a general multiperiod framework for endogenous information acquisition by potentially informed traders who can acquire information at variable cost. Crucially, prices now enter the decision making of potentially informed traders. Thus, they affect the informativeness of trades through two transmission channels: information acquisition and noise trading. The first channel captures the effect of prices on ex ante expected profits of potentially informed traders and thus the amount of information they acquire. The second captures the participation of price elastic noise traders. I characterise markets by the channel which dominates informativeness. If information acquisition dominates, informativeness of trades decrease with spreads while if crowding out dominates, it increases with spreads.

Second, I use my model to analyse the behaviour of a monopolistic market maker. I find that his price setting behaviour depends on the market information structure, setting narrower spreads if the information acquisition channel dominates. In contrast, with exogenous information acquisition, an optimal monopolistic market maker sets wider spreads. I show this is a special case of a market where crowding out dominates informativeness. Transaction volume and price dynamics also depend on market structure and may operate counter to the standard intuition. If information acquisition dominates informativeness, the informativeness of trades increases with expected transaction volume. Furthermore, spreads may widen over time on average.

My results suggest that endogenous information acquisition has a significant effect on market outcomes. If it is indeed a feature of real markets, neglecting it in empirical studies may be detrimental to their results.

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2.8 Appendix

2.8.1 Proofs

Proof of Proposition 2.1. To establish nonnegativity, I use the fact that the market maker can post prices outside of the interval $[\theta_1, \theta_2]$ in all periods and earn zero expected profits. This sets the lower bound on the value function for all beliefs μ. To establish convexity, consider a $\mu' \in [\mu, \mu'']$ for $\mu, \mu'' \in [0,1]$ and $\mu'' \geq \mu$. I can then write:

$$\mu' = \phi\mu + (1-\phi)\mu''$$

where

$$\phi = (\mu'' = \mu')/(\mu'' - \mu)$$

Starting from the prior μ', suppose there is a set of prices that could induce a posterior of μ with probability ϕ and μ'' with probability $1-\phi$. This is more informative than prices that do not change the posterior because it could lead to a revision in beliefs. Since the market maker cannot be worse off on average by learning the outcome from these prices, it must be that:

$$\phi V^*_{T-1}(\mu) + (1-\phi)V^*_{T-1}(\mu'') \geq V^*_{T-1}(\mu')$$

which is the requirement for convexity. ■

Proof of Proposition 2.2. By Bayes' rule:

$$\begin{aligned} E\left[\widetilde{\theta}\,|q=+1;a,b\right] &= \frac{\lambda X(a,b)(1-\mu)\theta_2 + (1-\lambda)(1-F(a))E_\mu[\widetilde{\theta}]}{\lambda X(a,b)(1-\mu) + (1-\lambda)(1-F(a))} \\ E\left[\widetilde{\theta}\,|q=-1;a,b\right] &= \frac{\lambda X(a,b)\mu\theta_1 + (1-\lambda)F(b)E_\mu[\widetilde{\theta}]}{\lambda X(a,b)\mu + (1-\lambda)F(b)} \end{aligned}$$

Solve for $(1-\lambda)(1-F(a))E_{\mu}[\widetilde{\theta}]$ and $(1-\lambda)F(b)E_{\mu}[\widetilde{\theta}]$, substitute into Equation (2.4) and rearrange to obtain:

$$\begin{aligned}\pi(a,b) \quad = \quad & \left(a - E\left[\widetilde{\theta}\,|q=+1;a,b\right]\right)[\lambda X(a,b)(1-\mu)+(1-\lambda)(1-F(a))] + \\ & \left(E\left[\widetilde{\theta}\,|q=-1;a,b\right]-b\right)[\lambda X(a,b)\mu+(1-\lambda)F(b)]\end{aligned}$$

Since $\Pr(q=+1)=\lambda X(a,b)(1-\mu)+(1-\lambda)(1-F(a))$ and $\Pr(q=-1)=\lambda X(a,b)\mu+(1-\lambda)F(b)$, the the market maker's profit in Equation (2.4) can be written as:

$$\pi(a,b)=\Pr(q=+1)\left(a-E\left[\widetilde{\theta}\,|q=+1;a,b\right]\right)+\Pr(q=-1)\left(E\left[\widetilde{\theta}\,|q=-1;a,b\right]-b\right)$$

and the result follows immediately. ■

Proof of Proposition 2.3. This proof closely follows the one for Proposition 3 in Leach and Madhavan. First I show that $a^c < a^m$ with all market structures. Suppose the opposite, $a^c > a^m$. By definition of a^c, the expected profits of a^m must be negative, which contradicts the definition of a^m.

The next results hinge on the impact of prices on beliefs with different market structures. I begin with the case when information acquisition dominates informativeness. Then, by Definition 2.5, $\frac{\partial\mu_2(-1)}{\partial b} > 0$ and $\frac{\partial\mu_2(+1)}{\partial a} > 0$ for all $\mu \in [0,1]$ when markets are open. Now suppose that a^m is the unique solution to the myopic monopolistic market maker's problem and that $a^m < a^*$. By the definition of a^m, it must be that $\pi(a^m,b,\mu) > \pi(a^*,b,\mu)$. Since μ_2 is increasing in a, the expected value functions yield:

$$E\left[V^*_{T-1}(\mu_2(a^m,b,\widetilde{q}(a^m)))\right] \geq E\left[V^*_{T-1}(\mu_2(a^*,b,\widetilde{q}(a^*)))\right]$$

However, this relation contradicts the definition of a^*. Therefore, if information acquisition dominates informativeness, then $a^c \leq a^m$ and $a^* \leq a^m$. The bid side is analogous and spread implications follow immediately. ■

Proof of Proposition 2.4. When crowding out dominates informativeness, by Definition 2.6, $\frac{\partial \mu_2(-1)}{\partial b} < 0$ and $\frac{\partial \mu_2(+1)}{\partial a} < 0$ for all $\mu \in [0,1]$ when markets are open. Fix a b and suppose that $a^c > a^*$. Then $\mu_2(+1, a^*) \geq \mu_2(+1, a^c)$ while $\mu_2(-1)$ is the same for either a. From Proposition 2.1, convexity of the value function implies:

$$E\left[V^*_{T-1}(\mu_2(a^c, b, \widetilde{q}(a^c)))\right] \geq E\left[V^*_{T-1}(\mu_2(a^*, b, \widetilde{q}(a^*)))\right]$$

for any b. To satisfy the definition of a^*, it must also be that $0 \leq \pi(a^c, b, \mu) \leq \pi(a^*, b, \mu)$. But this means that a^* is lower than a^c and yields non negative profits, contradicting the definition that a^c is the lowest ask price yielding non negative profits. Therefore, it must be that $a^* \geq a^c$.

Now suppose that a^m is the unique solution to the myopic monopolistic market maker's problem and that $a^m > a^*$. By the definition of a^m, $\pi(a^m, b, \mu) > \pi(a^*, b, \mu)$. Since μ_2 is decreasing in a the expected value functions yield:

$$E\left[V^*_{T-1}(\mu_2(a^m, b, \widetilde{q}(a^m)))\right] \geq E\left[V^*_{T-1}(\mu_2(a^*, b, \widetilde{q}(a^*)))\right]$$

However this relation contradicts the definition of a^*. Therefore, if crowding out dominates informativeness, then $a^c \leq a^m \leq a^*$. The bid side is analogous and spread implications follow immediately. ■

2.8.2 Motivating Examples

I solve analytically for equilibrium prices in my motivating example with endogenous information acquisition under a myopic market maker. The assumption of separate buyers and sellers means I can consider his objective function for ask and bid prices independently. I concentrate on the ask price. The linear demand function of noise buyers is $\alpha - \beta a$ and the optimal amount of information acquisition by potentially informed buyers is $\omega_a^* = \frac{1}{C}(1-\mu)(V-a)$. I define $B_V(a)$ as the conditional probability of a buy order occurring when $v = V$ and $B_0(a)$, when $v = 0$. Then I have:

$$
\begin{aligned}
B_V(a) &= \frac{1}{2C}\lambda(1-\mu)(V-a) + \frac{1}{2}(1-\lambda)(\alpha - \beta a) \\
B_0(a) &= \frac{1}{2}(1-\lambda)(\alpha - \beta a)
\end{aligned}
$$

The market maker sets the ask price a to maximise expected profits from trade, given prior belief μ:

$$\max_a \mu B_V(a)(a - V) + (1-\mu)B_0 a$$

The unique solution is:

$$a^m = \frac{\frac{1}{C}\mu^2\lambda V + (1-\lambda)(\alpha + \mu\beta V)}{\frac{1}{C}\mu^2\lambda + 2\beta(1-\lambda)}$$

Similarly, for the bid price:

$$b^m = \frac{\mu(1-\lambda)\beta V - (1-\lambda)(\alpha - \beta V)}{\frac{1}{C}(1-\mu)^2\lambda + 2\beta(1-\lambda)}$$

Since this is an interior solution, I need to check that the demand and supply functions of noise traders, and the optimal amount of information acquisition by potentially informed traders are both within $[0, 1]$ when choosing parameters α, β, λ, V and C. If these bounds are exceeded, the solution is a boundary case.

For a myopic market maker with exogenous information acquisition, I follow the same steps except that informed traders always acquire information. This yields prices:

$$
\begin{aligned}
a^m &= \frac{\mu\lambda + 2(1-\lambda)(\alpha + \mu\beta V)}{4(1-\mu)\beta} \\
b^m &= \frac{2(1-\lambda)((1+\mu)\beta V - \alpha) - (1-\mu)\lambda}{4(1-\mu)\beta}
\end{aligned}
$$

Again, I need to check that the demand and supply functions of noise traders are within $[0, 1]$ when choosing parameters α, β, λ and V. If these bounds are exceeded, the solution is a boundary case.

Chapter 3

Confidential Treatment Requests

3.1 Introduction

Regular mandatory disclosure of holdings by institutional investors allows fund investors to better evaluate the performance of the funds and help them in their asset allocation and diversification decision. However, it also has its drawbacks[1]. Specifically, other market participants may copy the trades of the investment managers and thus free-ride on the latter's research expertise. Frank et al. (2004), and Wang and Varbeek (2010) use the term copycat funds to describe these investors[2]. The mimicking trades of these copycats would make it more expensive for the investment managers if they decide to acquire more shares in subsequent quarters. This may have negative consequences on informational efficiency of markets if mandatory disclosure reduces the information acquisition efforts of institutional investors. To balance the

[1]See Wermers (2001) on a discussion of how more frequent mandatory disclosure of mutual funds could potentially reduce their profits.

[2]Frank et al. (2004) provide empirical evidence that after expenses, copycat funds earned statistically indistinguishable and possibly higher returns. They argue that if investors buy actively managed funds to obtain high net-of-expenses returns, then copycat funds could potentially erode their market share by offering comparable returns net of expenses. Wang and Varbeek (2010) show that the relative success of copycat funds have improved after 2004, when the SEC increased the mandatory disclosure frequency to quarterly from semi-annual previously.

competing interests, a provision in Section 13(f) allows them to seek confidential treatment for some of their holdings. If approved by the SEC, these holdings will be disclosed at a later date, usually up to one year.

We show that confidential treatment requests impacts the trading strategy and expected profits of institutional investors and the price informativeness of disclosed trades. In this model, we examine the trading strategy of an informed investment manager when he applies for confidential treatment. We assume that the manager seeks confidential treatment on his initial trade to better exploit his private information on the asset over two trading periods[3]. The manager trades in the first period and applies for confidential treatment on this trade. The SEC decides whether to approve this request before the manager trades in the second period. The model most similar to ours is Huddart et al. (2001). Their model is an extension of Kyle (1985) with mandatory disclosure of trades. A perfectly informed risk-neutral insider's trades include a random noise component to disguise the information-based component of the trades when they are publicly disclosed. This diminishes the market maker's ability to draw inferences on the insider's information from his disclosed trades. The insider therefore does not surrender his entire informational advantage after his first trade is disclosed. The authors term this 'trading strategy dissimulation'. Other theoretical papers with variations of this dissimulation strategy include Zhang (2004), Zhang (2008), Huang (2008) and Buffa (2010).

In our model, it follows that the manager cannot report the true fair value to the SEC and use Huddart et al. (2001)'s dissimulation strategy at the same time. According to current SEC regulatory guidelines on confidential treatment requests, the fund manager needs to detail a specific on-going investment program in his application. The trade that he wants to delay disclosure therefore needs to be coherent with the

[3]There are other possible motives for confidential treatment requests, which are beyond the scope of this chapter. They include manipulation (see Fishman and Hagerty (1995), and John and Narayanan (1997)) and window-dressing (see Musto (1997), and Meier and Schaumburg (2006)).

investment objective he reports to the SEC. For example, suppose the initial price of the asset before he made his first trade is 10 and he reports the true fair value of 30 to the SEC, his first trade needs to be a buy for the investment program to be coherent. Adding a dissimulation noise term in the first trade may result in a sell instead of a buy. This would result in the SEC rejecting the application.

We find that the equilibrium strategy of the manager is to dissimulate his reported estimate of the fair value to the SEC. Back to the above example, it means that he reports to the SEC a noisy signal that is a sum of the true fair value and a random normally distributed noise term. This random noise term is proportional to the unconditional variance of the fair value. Given this reported noisy signal, the manager has an estimate of the fair value, using the projection theorem of normal random variables. In the event that confidential treatment is denied, the random noise term prevents the market-maker from perfectly inferring the true fair value. Similar to Huddart et al. (2001), no invertible trading strategy can be part of a Nash equilibrium if the manager does not add noise to the true fair value. Suppose the manager reports the true fair value to the SEC. The market-maker will set a perfectly elastic price in the event that the application is rejected and the manager's trade is disclosed. The manager thus would have an incentive to deviate from reporting the true fair value, and make infinite trading profits in the second period if his application is rejected.

Besides the initial trade, we also assume that the manager's subsequent trade is coherent with the reported estimate of the fair value of the asset, in the event confidential treatment is granted. Let us suppose that the manager knows that the true fair value is 20, the estimate he reported to the SEC is 30 and the price in the first round of trading is 25. The manager is committed to buy in the second period if he is granted confidential treatment, even though he is expected to make a loss if he does so. We assume that non-compliance of the reported investment program would result in punitive costs in the form of rejections in future applications by the SEC. We believe that this assumption is reasonable as the second trade is also observable by the SEC. In addition, Agarwal et al. (2011) provide empirical evidence that past confidential treatment denial rates is the single most important predictor of future denial rates. Therefore it is important for managers to have a good filing track record as it would affect the probability of success in future applications.

Although the granting of confidential treatment prevents the market-maker from inferring the manager's signal from his trade, the commitment to the reported investment program to the SEC reduces his expected profits. This is because the manager would not be able to fully exploit his knowledge of the true fair value in the event his application for confidential treatment is granted. We find that if the probability of application success is below a certain threshold, the expected profits of the manager is lower than in a scenario where he always discloses his trades, as in Huddart et al. (2001).

To our knowledge, this is the first theoretical model that examines the impact of confidential treatment requests on the trading strategies by informed traders. The empirical literature is also relatively new as databases of institutional holdings like Thomson Reuters Ownership Data generally do not include data on confidential holdings. Agarwal et al. (2011), and Aragon et al. (2011) are two empirical studies that examine confidential treatment filings. Compared to other investment managers, hedge funds are the most aggressive applicants for confidential treatment of their trades. Both papers document that confidential holdings exhibit superior performance. The first paper also finds a significant positive market reaction after the involuntary disclosure of hedge funds' holdings due to quick rejections of confidential treatment requests by the SEC. The authors conclude that the rejections force the revelation of information that has not been reflected in the stock prices, and this may disrupt the funds' stock acquisition strategies. Their findings support the assumption in our model that confidential treatment applications are primarily for protecting private information. This is in contrast to Cao (2011) who finds evidence that investment firms with poor past trading performance use confidential treatment to hide the liquidation of stocks in their portfolio that have performed poorly. Our model assumes that the manager does not have such window-dressing motives.

The rest of this chapter is structured as follows. Section 3.2 discusses the SEC regulatory guidelines on confidential treatment requests. Section 3.3 describes the model under 2 different scenarios. In the first scenario, the SEC restricts the manager's second period trade such that it is consistent with his reported forecast, in the event confidential treatment is granted. We believe that this scenario is the best depiction of current SEC regulations. We also examine the case where there is no restriction on the manager's second period trade. Comparative statics is discussed in Section 3.4, where we compare the model against a two-period Huddart et al. (2001) and a two-period Kyle (1985) model. Section 3.5 concludes.

3.2 SEC Regulatory Guidelines on Confidential Treatment Requests

Section 13(f) of the Securities Exchange Act of 1934 requires investment managers (who manage more than US$100 million in assets) to publicly disclose their portfolio holdings within 45 days after the end of every quarter. Section 13(f) was enacted by Congress in 1975 to allow the public to have access to the information regarding the purchase, sale and holdings of securities by institutional investors. However, the mandatory disclosure of holdings before an ongoing investment program is complete would be detrimental to the interests of the institutional investor and its fund investors. To balance these competing interests, the SEC allows institutional investors to apply for confidential treatment.

Generally, confidential treatment requests are granted if the investment manager can demonstrate that confidential treatment is in the public interest or for the protection of the investors. According to the SEC [4], there are several key criteria that the manager needs to fulfill for his confidential treatment request to be successful. Firstly, the manager needs to detail a specific investment program. He needs to provide the SEC information regarding the program's ultimate objective and describe the measures taken during that quarter toward effectuating the program. He also needs to provide information on the trades that are made in that quarter to support the existence of the program. Secondly, the investment program must be an on-going one that continues through the date of the filing. Thirdly, the manager must show that the disclosure of the fund's holdings would reveal the investment strategy to the public. Lastly, he must demonstrate that failure to grant confidential treatment to the holdings would harm the fund's performance. This would include lost profit opportunities due to mimicking strategies of other copycat investors as well as front-running activities by other market participants. If the manager's application is unsuccessful, he is required to disclose the holdings within 6 business days.

We attempt to explicitly model the above guidelines. We assume that an informed investment manager details a "a specific investment program" by submitting to the SEC his signal of the fair value of the asset. This signal can be interpreted as a target price for the manager. The manager also needs to submit a trade that he has already made in the previous quarter which is consistent with the target price. In the event he is granted confidential treatment, he has to continue trading in the subsequent period in a manner that is consistent with the original target price. This is because the investment program is an "on-going" one.

[4] See http://www.sec.gov/divisions/investment/guidance/13fpt2.htm for a description of the application process for confidential treatment. These rules were introduced in 1998 to prevent investment managers to use confidential treatment requests as a tool to manipulate the market.

The SEC application guidelines for confidential treatment requests imply that the trades are typically large trades[5] that have huge price impact and are done over more than one quarter. The SEC receives about 60 such requests every quarter. A recent example is Berkshire Hathaway's (Warren Buffett's investment holding company) purchase of a 5.5 stake in IBM worth US$10 billion in 2011[6]. The SEC allowed the company to defer disclosure of the IBM trades by a quarter. Without confidential treatment being granted, it is likely that the purchase would be more costly.

It is noted that the granting of confidential treatment by the SEC is not a guaranteed event. In their sample of confidential treatment requests from 1999 to 2007, Agarwal et al. (2011) report that 17.4 were denied by the SEC. Even applications by well-known investors like Warren Buffett's Berkshire Hathaway have previously been rejected[7], with a 72.3 rejection rate from 65 applications. The distribution of rejection rates shows considerable variation across managers.

3.3 Model

3.3.1 Set-up

This Kyle (1985)-type model employs a setting similar to the two-period model in Huddart et al. (2001). There are two trading periods indexed by $n \in \{1, 2\}$. The discount rate is normalised to zero for simplicity. There is one risky asset in the market with a liquidation value of v, where $v \sim N(P_0, \Sigma_0)$. v is realised after the second trading period. There are liquidity traders who summit exogenously generated orders u_n in each trading period, where $u_n \sim N(0, \sigma_u^2)$. We assume that u_1, u_2 and

[5]In Agarwal et al. (2011)'s sample, the average confidential holding represents 1.25 of all the shares outstanding by the issuer compared to the average of 0.68 for disclosed holdings.

[6]http://dealbook.nytimes.com/2011/11/14/one-secret-buffett-gets-to-keep/

[7]See http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aNd_pTpcmBwA &refer=news_index

v are all mutually independent.

A risk-neutral informed investment manager observes v perfectly before trading commences. He decides to apply for confidential treatment for his first period trade before making the trade. He trades x_1 in the first period and declares to the SEC that he has a signal θ of the asset value. Let D denote the event in which the first period trade is disclosed (application is unsuccessful) and N denote the event in which the trade is not disclosed (application is successful)[8]. The application for confidential treatment is successful with a probability of α. The manager trades x_2^N (x_2^D) in the second period if the application is successful (unsuccessful).

There exists a competitive risk neutral market maker who sets prices. He cannot distinguish the trades of the manager from the other uninformed orders of the liquidity traders. He only observes the aggregate order flow y_n in each period and sets the price to be equal to the posterior expectation of v. The price is therefore semi-strong efficient and the market-maker makes zero expected profits due to Bertrand competition with potential rival market-makers. In the event that the manager's first period trade is disclosed, the market-maker updates his expectation of v to P_1^* from the first period price P_1 before trading commences in the second period. Conversely, if there is no disclosure, the market-maker infers that confidential treatment has been granted.

If the manager decides to apply for confidential treatment, we show that an equilibrium exists where he declares to the SEC that he has a signal θ, where $\theta = v + \eta$, $\eta \sim N\left(0, \sigma_\eta^2\right)$, and η is distributed independently of v and u_n. η is the noise term that the manager adds to v when he applies for confidential treatment. Given θ, his reported forecast of v is v'. According to the projection theorem of normal random

[8]Similar to Huddart et al. (2001), since trading occurs only once for every reporting period, the disclosure of holdings is equivalent to the disclosure of trades.

variables,

$$v' = P_0 + \frac{\Sigma_0}{\Sigma_0 + \sigma_\eta^2} (\theta - P_0) \tag{3.1}$$

As mentioned earlier, to stand any chance of getting SEC approval for confidential treatment, the manager needs to report a coherent on-going trading program. This means that his first period trade x_1 must be consistent with v'. If his application is successful, his second period trade also needs to be consistent with v' and not v. Using backward induction, this means that the manager chooses x_2^N to maximise his expected second period profits $E(\pi_2)$ as if his signal is v' instead of v. His maximisation problem is

$$x_2^N \in \arg\max_{x_2^N} E(\pi_2|v') \tag{3.2}$$

Referring to the numerical example described in the introduction, we have $P_0 = 10$, $P_1 = 25$, $v = 20$ and $v' = 30$. The manager is committed to buy in the second period (since $v' > P_1$) even though he would make an expected loss in this trade (since $v < P_1$). If the application is rejected, the informed trader is forced to disclose his first period trade before trading commences in the second period. However, the informed trader is now free to make use of his knowledge of v in his second period trade x_2^D as his trading strategy is now not bounded by the confidential treatment request. In contrast to (3.2), the maximisation problem is now

$$x_2^D \in \arg\max_{x_2^D} E(\pi_2|v) \tag{3.3}$$

We define Σ_1^N and Σ_1^D as the amount of private information that the manager can exploit in the second period of trading, in the event that confidential treatment is granted and not granted respectively

$$\Sigma_1^N = var\left(v'|y_1\right) = var\left(v' - P_1\right) \tag{3.4}$$

$$\Sigma_1^D = var\left(v|x_1\right) = var\left(v - v'\right) \tag{3.5}$$

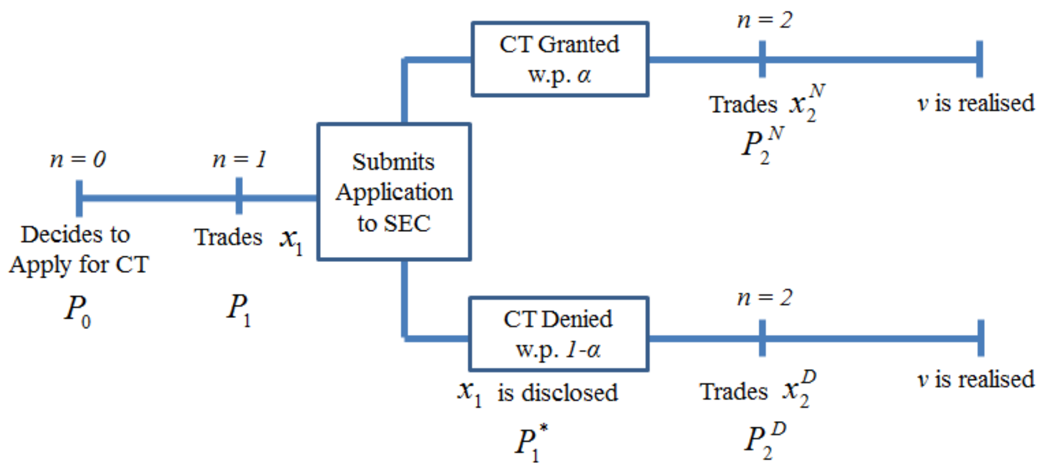

Figure 3.1: Timeline of events of confidential treatment request

Figure 3.1 shows the timeline of the model.

3.3.2 SEC restricts the manager's second period trade after confidential treatment is granted

Proposition 3.1. *If the investment manager applies for confidential treatment and the SEC restricts his second period trade in the event confidential treatment is granted, a subgame perfect linear equilibrium exists in which*

1. *The manager submits his noisy signal θ to the SEC whereby*

$$\theta = v + \eta, \eta \sim N\left(0, \sigma_\eta^2\right) \qquad \sigma_\eta^2 = h\Sigma_0$$

where $0 \leq h \leq 1$ is the only real positive root of the following equation, such that $\lambda_1 > 0$, $\lambda_2^D > 0$, $\lambda_2^N > 0$

$$\left((1-\alpha)^2 - h\right)\sqrt{(1-\alpha)^2 + h} - \alpha\,(1-\alpha)^2\sqrt{h} = 0$$

2. *The manager's trading strategies and expected profits are of the linear form*

$$x_1 = \beta_1 \left(v' - P_0\right)$$

$$\beta_1 = \frac{\sigma_u}{\sqrt{\Sigma_0}} \frac{\sqrt{h(1+h)}}{(1-\alpha)} \qquad \lambda_1 = \frac{\sqrt{\Sigma_0}}{\sigma_u} \frac{(1-\alpha)}{\left((1-\alpha)^2+h\right)} \sqrt{\frac{h}{1+h}}$$

$$x_2^D = \beta_2^D \left(v - v'\right) \qquad \Sigma_1^D = \frac{h}{1+h} \Sigma_0$$

$$x_2^N = \beta_2^N \left(v' - P_1\right) \qquad \Sigma_1^N = \frac{(1-\alpha)^2}{(1+h)\left((1-\alpha)^2+h\right)} \Sigma_0$$

$$\beta_2^D = \frac{\sigma_u}{\sqrt{\Sigma_1^D}} \qquad \lambda_2^D = \frac{\sqrt{\Sigma_1^D}}{2\sigma_u}$$

$$\beta_2^N = \frac{\sigma_u}{\sqrt{\Sigma_1^N}} \qquad \lambda_2^N = \frac{\sqrt{\Sigma_1^N}}{2\sigma_u}$$

$$E\left(\pi_1\right) = \frac{\beta_1(1-\lambda_1\beta_1)\Sigma_0}{1+h}$$

$$E\left(\pi_2^N\right) = \frac{\sigma_u\sqrt{\Sigma_1^N}}{2} \qquad E\left(\pi_2^D\right) = \frac{\sigma_u\sqrt{\Sigma_1^D}}{2}$$

3. *The market-maker's pricing rule is of the linear form*

$$P_1 = P_0 + \lambda_1 y_1$$

$$P_1^* = v'$$

$$P_2^D = v' + \lambda_2^D y_2^D$$

$$P_2^N = P_1 + \lambda_2^N y_2^N$$

Proof: See Appendix

The main intuition of the proof is as follows. After computing x_2^N and x_2^D, by backward induction, we derive the total expected profits in both periods and then take the first order condition with respect to x_1. The first order condition equation will be in terms of $v - P_0$ and x_1. Following from Huddart et al. (2001), for the mixed strategy $\theta = v + \eta$, $\eta \sim N\left(0, \sigma_\eta^2\right)$ to hold in equilibrium, the manager must be different across all values of x_1, as x_1 is a function of θ. The coefficients of $v - P_0$ and x_1 must therefore be zero, resulting in two simultaneous equations. The other parameters can then be solved.

The variance of the noise σ_η^2 that the manager adds to the forecast he submits to the SEC is directly proportional to the unconditional variance of the fair value Σ_0. In the event that confidential treatment is granted, the second period trade

$x_2^N = \beta_2^N (v' - P_1)$ is a linear function of v', in spite of the manager knowing that the true fair value is v. On the other hand, if the confidential treatment request is denied, the manager's second period trade is $x_2^D = \beta_2^D (v - v')$ as the manager is now free to make use of his knowledge of v.

The market-maker is able to infer v' perfectly from x_1 because x_1 is a linear function of $v' - P_0$. He updates his expectation of v to $P_1^* = v'$ from P_1 before trading commences in the second period.

3.3.3 SEC does not restrict the manager's second period trade

In the next proposition, we will examine the manager's equilibrium trading strategy if the SEC does not restrict his second period trade when confidential treatment is granted. The manager is free to use his knowledge of v in his second period trade. We add an upper hat to the endogenous parameters in this equilibrium to distinguish them from those in Proposition 3.1. Therefore in contrast to (3.2), the manager's maximisation problem in the second period when confidential treatment is granted is

$$\widehat{x}_2^N \in \arg\max_{\widehat{x}_2^N} E\left(\widehat{\pi}_2 | v\right) \tag{3.6}$$

Proposition 3.2. *If the investment manager applies for confidential treatment and the SEC does not restrict his second period trade, a subgame perfect linear equilibrium exists in which*

1. *The manager submits his noisy forecast $\widehat{\theta}$ to the SEC whereby*

$$\widehat{\theta} = v + \widehat{\eta}, \widehat{\eta} \sim N\left(0, \widehat{\sigma}_\eta^2\right) \qquad \widehat{\sigma}_\eta^2 = g\Sigma_0$$

where $0 \leq g \leq 1$ is the only real positive root of the following equation, such that $\widehat{\lambda}_1 > 0$, $\widehat{\lambda}_2^D > 0$, $\widehat{\lambda}_2^N > 0$

$$\alpha\sqrt{\frac{g}{g+1}}-(1-\alpha)\left(g^{2/3}(1-\alpha)^{-4/3}-1\right)\sqrt{\frac{\frac{g^{2/3}}{1+g}(1-\alpha)^{2/3}+1}{g^{-1/3}(1-\alpha)^{2/3}+1}}=0$$

2. *The manager's trading strategies and expected profits are of the linear form*

$$\widehat{x}_1=\widehat{\beta}_1\left(\widehat{v}'-P_0\right)$$

$$\widehat{\beta}_1=\frac{\sigma_u}{\sqrt{\Sigma_0}}\left(\frac{1-\alpha}{\sqrt{g}}\right)^{1/3}\sqrt{1+g} \qquad \widehat{\lambda}_1=\frac{\sqrt{\Sigma_0}(1-\alpha)}{\sigma_u\left(\frac{1-\alpha}{\sqrt{g}}\right)^{1/3}\left(g^{1/3}(1-\alpha)^{1/3}+1-\alpha\right)\sqrt{1+g}}$$

$$\widehat{x}_2^D=\widehat{\beta}_2^D\left(v-\widehat{v}'\right) \qquad \widehat{\Sigma}_1^D=\frac{g}{1+g}\Sigma_0$$

$$\widehat{x}_2^N=\widehat{\beta}_2^N\left(v-\widehat{P}_1\right) \qquad \widehat{\Sigma}_1^N=\frac{\frac{g}{1+g}\left(\frac{1-\alpha}{\sqrt{g}}\right)^{2/3}+1}{\left(\frac{1-\alpha}{\sqrt{g}}\right)^{2/3}+1}\Sigma_0$$

$$\widehat{\beta}_2^D=\frac{\sigma_u}{\sqrt{\widehat{\Sigma}_1^D}} \qquad \widehat{\lambda}_2^D=\frac{\sqrt{\widehat{\Sigma}_1^D}}{2\sigma_u}$$

$$\widehat{\beta}_2^N=\frac{\sigma_u}{\sqrt{\widehat{\Sigma}_1^N}} \qquad \widehat{\lambda}_2^N=\frac{\sqrt{\widehat{\Sigma}_1^N}}{2\sigma_u}$$

$$E\left(\widehat{\pi}_1\right)=\frac{\widehat{\beta}_1\left(1-\widehat{\lambda}_1\widehat{\beta}_1\right)\Sigma_0}{1+g}$$

$$E\left(\widehat{\pi}_2^N\right)=\frac{\sigma_u\sqrt{\widehat{\Sigma}_1^N}}{2} \qquad E\left(\widehat{\pi}_2^D\right)=\frac{\sigma_u\sqrt{\widehat{\Sigma}_1^D}}{2}$$

3. *The market-maker's pricing rule is of the linear form*

$$\widehat{P}_1=P_0+\widehat{\lambda}_1\widehat{y}_1$$

$$\widehat{P}_1^*=\widehat{v}'$$

$$\widehat{P}_2^D=\widehat{v}'+\widehat{\lambda}_2^D\widehat{y}_2^D$$

$$\widehat{P}_2^N=\widehat{P}_1+\widehat{\lambda}_2^N\widehat{y}_2^N$$

Proof: See Appendix

Since the manager is free to use his knowledge of v, his second period trade given confidential treatment is $\widehat{x}_2^N=\widehat{\beta}_2^N\left(v-\widehat{P}_1\right)$ instead of $\widehat{\beta}_2^N\left(v'-\widehat{P}_1\right)$. Similar to the result in Proposition 3.1, the variance of the noise $\widehat{\sigma}_\eta^2$ that the manager adds to the forecast he submits to the SEC is also directly proportional to the unconditional variance of the fair value Σ_0.

Corollary 3.1. *Under both scenarios in Propositions 3.1 and 3.2, a) if $\alpha=0$, the equilibrium is equivalent to a two-period Huddart et al. (2001) model; b) if $\alpha=1$, the equilibrium is equivalent to a two-period Kyle (1985) model.*

If $\alpha=0$, the manager has no chance of getting confidential treatment. Therefore he always discloses his first period trade and this is equivalent to a two-period Huddart et. al (2001) model. The manager adds η to v when he reports his signal to the SEC, where $\sigma_\eta^2 = \Sigma_0$. The manager's first period of trade has the same amount of dissimulation as in a two-period Huddart et al. (2001) model[9]. Similarly, if $\alpha = 1$, the manager is always successful in getting confidential treatment. His first period trade is $x_1 = \beta_1 (v - P_0)$ and he reports $\theta = v$ to the SEC. His second period is $x_2 = \beta_2^N (v - P_1)$ as this is consistent with his reported signal v to the SEC. This scenario is thus equivalent to a two-period Kyle (1985) model.

3.4 Comparative Statics

In this section, we will focus on analysing the parameters in Proposition 3.1 and 3.2. We first compare the total expected profits against those that the manager is expected to receive if he always discloses his initial trade.

3.4.1 Manager's Profits

Proposition 3.3. *Compared with the expected profits where the manager always discloses his initial trade (as in Huddart et al. (2001)), a) if the SEC restricts the second period trade in the event confidential treatment is granted, the manager's expected profits will be lower if* $0 \leq \alpha \leq \alpha^*$*, where* $\alpha^* \approx 0.361$*; b) if the SEC does not restrict the second period trade, the manager's expected profits will be always higher for* $0 \leq \alpha \leq 1$

Proof: See Appendix

[9]The first period trade in a two-period Huddart et al. (2001) model is $\overline{x}_1 = \overline{\beta}_1 (v - P_0) + \overline{z}_1$, where $\overline{z}_1$ is the dissimulation term that has a variance of $\frac{\sigma_u^2}{2}$. In Proposition 3.1, the first period trade can be expressed as $x_1 = \frac{\beta_1 \Sigma_0}{\Sigma_0 + \sigma_\eta^2} (v - P_0) + \frac{\beta_1 \Sigma_0}{\Sigma_0 + \sigma_\eta^2} \eta$. It follows that if $\sigma_\eta^2 = \Sigma_0$, the equilibrium in Proposition 3.1 is equivalent to Huddart et al. (2001)'s. The same applies for Proposition 3.2 too.

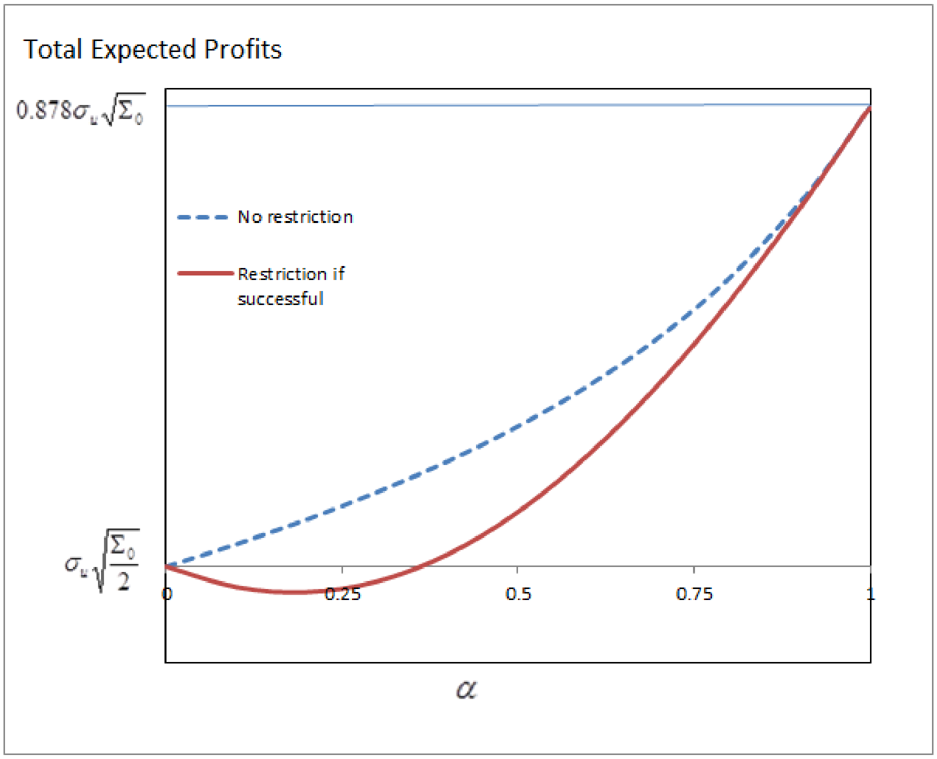

Figure 3.2: Total expected profits of manager under the 2 different assumptions

Fig 3.2 shows the total expected profits (over the two periods) of the manager when he applies for confidential treatment, under the scenarios in Propositions 3.1 and 3.2. The total expected profits under the two-period Huddart et al. (2001) equilibrium is $\sigma_u\sqrt{\frac{\Sigma_0}{2}}$, while those of a two-period Kyle (1985)[10] is approximately $0.878\sigma_u\sqrt{\Sigma_0}$. As discussed in Corollary 3.1, the equilibrium under both scenarios is equivalent to a two-period Huddart et al. (2001) model if $\alpha = 0$, and a two-period Kyle (1985) model if $\alpha = 1$. For all values of α between 0 and 1, the total expected profits in the equilibrium with no second period trade restriction is higher than $\sigma_u\sqrt{\frac{\Sigma_0}{2}}$. On the other hand, in the equilibrium with the second period trade restriction, the total expected profits are lower than $\sigma_u\sqrt{\frac{\Sigma_0}{2}}$ for $0 \leq \alpha \leq \alpha^*$.

[10]See Huddart et al. (2001). The paper's Proposition 2 shows the expected profits of a two-period Huddart et al. (2001) dissimulation equilibrium, while Proposition 1 shows the expected profits in a two-period Kyle (1985) model. Note that there is a typo in Proposition 1: $E(\pi_1) = \frac{\sqrt{2K(K-1)}}{4K-1}\sigma_u\sqrt{\Sigma_0}$ instead of $E(\pi_1) = \frac{2K(K-1)}{(4K-1)^2}\sigma_u\sqrt{\Sigma_0}$.

	Restriction in second period trade if confidential treatment is granted	No restriction in second period trade
$E(\pi_1)$	Always Higher	Always Higher
$E(\pi_2)$	Lower for $0 \leq \alpha \leq 0.854$	Always Higher
$E(\pi_2^N)$	Lower for $0 \leq \alpha \leq 0.485$	Always Higher
$E(\pi_2^D)$	Always Lower	Always Lower
$E(\pi_1)+E(\pi_2)$	Lower for $0 \leq \alpha \leq 0.361$	Always Higher

Figure 3.3: Comparison of expected profits with tTwo-period Huddart et al. (2001) model

To understand why the manager might have lower expected profits if he applies for confidential treatment in the scenario in Proposition 3.1, let us examine the expected profits in both periods separately. Figure 3.3 shows the comparison of the expected profits of the manager in the scenarios of Proposition 3.1 and 3.2 against those of a two-period Huddart et al. (2001) model, where the insider always discloses his first trade. In their model, the informed insider earns the same expected profits $\frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$ in both periods. In our model under both scenarios, the manager always earns higher expected profits in the first period, i.e. $E(\pi_1) \geq \frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$ and $E(\widehat{\pi}_1) \geq \frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$. This is because both σ_η^2 and $\widehat{\sigma}_\eta^2$ are less than Σ_0, implying that the manager is more aggressive in exploiting his information in the first period. In the second period, in the event that confidential treatment is denied, the disclosure of the first period trade results in both $E\left(\pi_2^D\right)$ and $E\left(\widehat{\pi}_2^D\right)$ to be lower than $\frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$. This is because the market-maker updates the price to reflect the information contained in the disclosed trade, reducing the information advantage that the manager can exploit in the second period.

The comparison results diverge in the event that confidential treatment is granted. We find that $E\left(\widehat{\pi}_2^N\right) \geq \frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$for all values of α between 0 and 1, while $E\left(\pi_2^N\right) \leq \frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$ for $0 \leq \alpha \leq 0.485$. Under the scenario in Proposition 3.1, the manager is only able to trade based on his knowledge of v' instead of v. His information advantage in

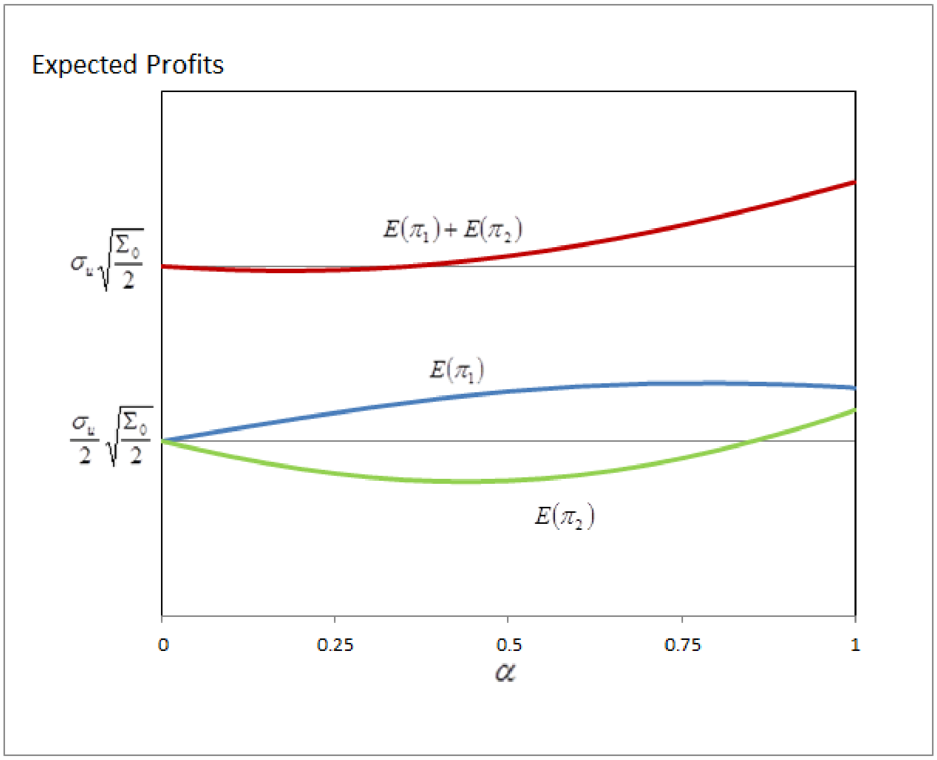

Figure 3.4: Expected profits of manager in the 2 trading periods under the assumption that the SEC restricts the second period trade if confidential treatment request is successful

the second period is therefore reduced with this restriction. The reduction in expected profits in $E\left(\pi_2^N\right)$ causes $E\left(\pi_2\right) \leq \frac{\sigma_u}{2}\sqrt{\frac{\Sigma_0}{2}}$ for $0 \leq \alpha \leq 0.854$. Figure 3.4 shows the breakdown in the expected profits of the manager in Proposition 3.1 graphically.

As discussed earlier, Σ_1^D and Σ_1^N measure the amount of private information that the manager can exploit in the second period of trading. These parameters are related to the second period expected profits since $E\left(\pi_2^D\right) = \frac{\sigma_u\sqrt{\Sigma_1^D}}{2}$ and $E\left(\pi_2^N\right) = \frac{\sigma_u\sqrt{\Sigma_1^N}}{2}$. It appears that Σ_1^N should always be greater than Σ_1^D since disclosing the first period trade will result in a loss in the information advantage of the manager. However, if confidential treatment is not granted, the manager can make use of his knowledge of v, while if it is granted, he can only exploit his knowledge of v'. Figure 3.5 shows the relationship between Σ_1^D, Σ_1^N and $E\left(\Sigma_1\right) = \alpha\Sigma_1^N + \left(1-\alpha\right)\Sigma_1^D$ with α. Interestingly, we find that $\Sigma_1^D > \Sigma_1^N$ for $0 \leq \alpha \leq 0.209$. In contrast, in the scenario where the SEC does not restrict the manager's second period trade, we find that $\widehat{\Sigma}_1^D < \widehat{\Sigma}_1^N$ for all values of α between 0 and 1. This is shown in Figure 3.5.

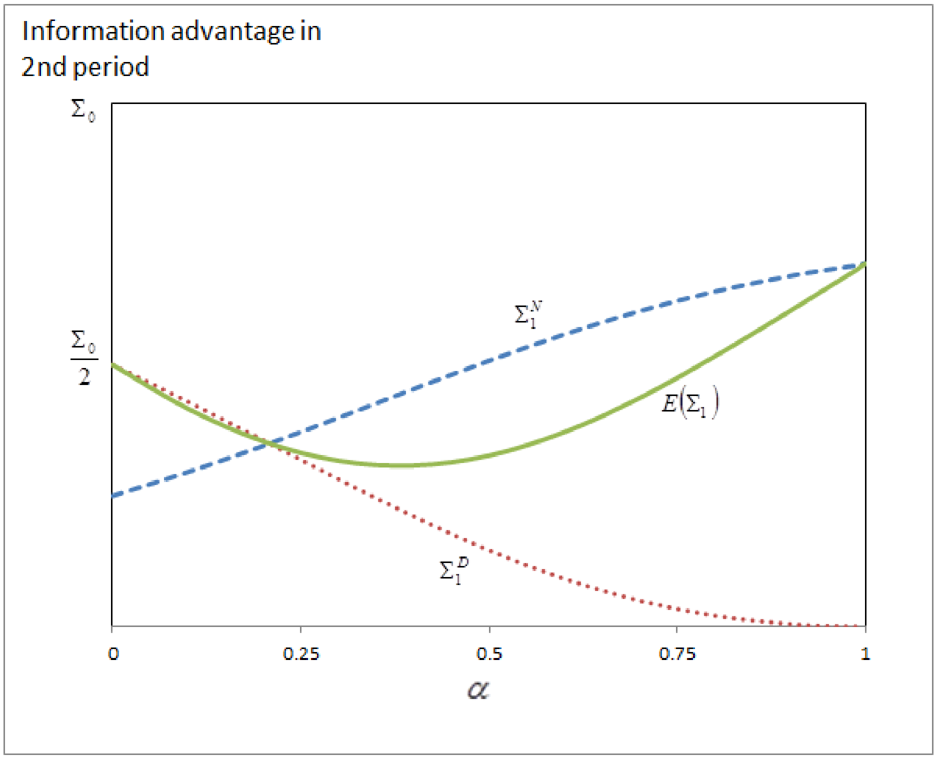

Figure 3.5: Information advantage of manager in the 2nd period under the assumption that the SEC restricts the second period trade if confidential treatment request is successful

3.4.2 Noise Added to Reported Forecast to the SEC

Corollary 3.2. *a) Under both scenarios in Propositions 3.1 and 3.2, the manager adds less noise to his reported forecast to the SEC as α increases. b) The manager adds less noise in the equilibrium in Proposition 3.1 compared to that in Proposition 3.2.*

Figure 3.7 shows the relationship between α and the noise that the manager adds to the forecast that he submits to the SEC. As α increases, the manager adds less noise to the forecast, i.e. both $\frac{d\sigma_\eta^2}{d\alpha}$ and $\frac{d\widehat{\sigma}_\eta^2}{d\alpha}$ are negative. This is because adding more

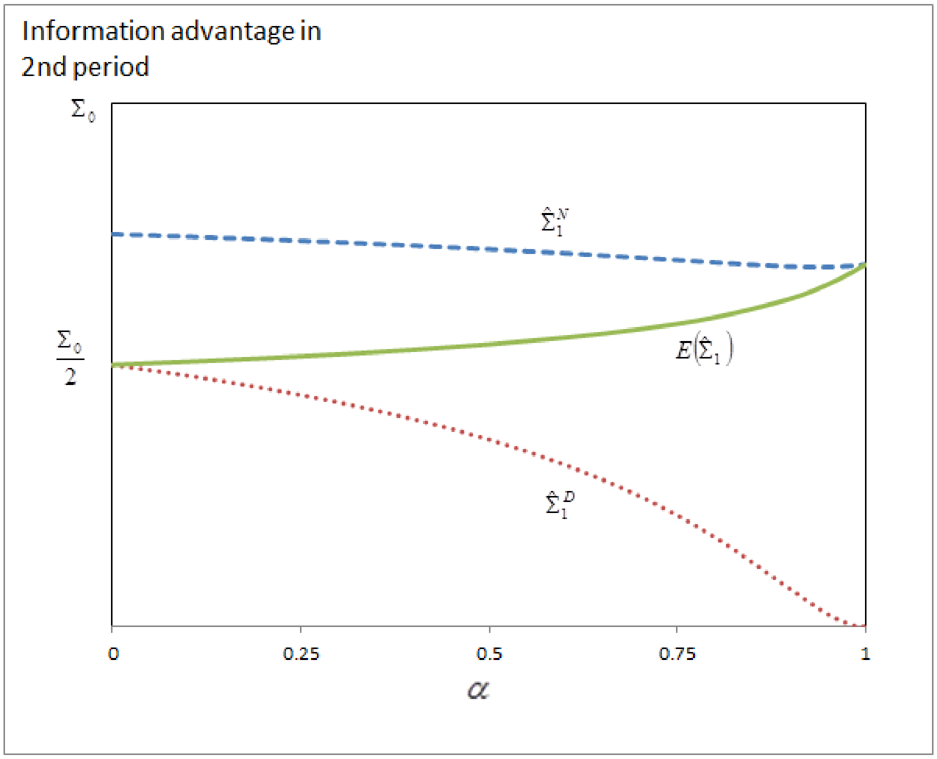

Figure 3.6: Information advantage of manager in the 2nd period under the assumption that the SEC does not restrict the second period trade

noise in the forecast would be more beneficial to the manager ex-post, in the event that his application is rejected. If $\alpha = 1$, the equilibrium is a two-period Kyle (1985) model where there is no noise (the manager reports the true fair value of v to the SEC), while if $\alpha = 0$, the equilibrium is a two-period Huddart et al. (2001) model where the noise term is Σ_0 . In addition, we note that $\sigma_\eta^2 \leq \widehat{\sigma}_\eta^2$ for all values of α between 0 and 1. Adding more noise to the forecast would result in a v' that varies more from the true fair value v. If the SEC forces the manager to trade based on the reported v' in the event that confidential treatment is granted, the manager would forgo substantial trading profits if he adds too much noise in his application in the first period. The restriction on the second period trade therefore forces the manager to be more truthful in the forecast that he submits to the SEC.

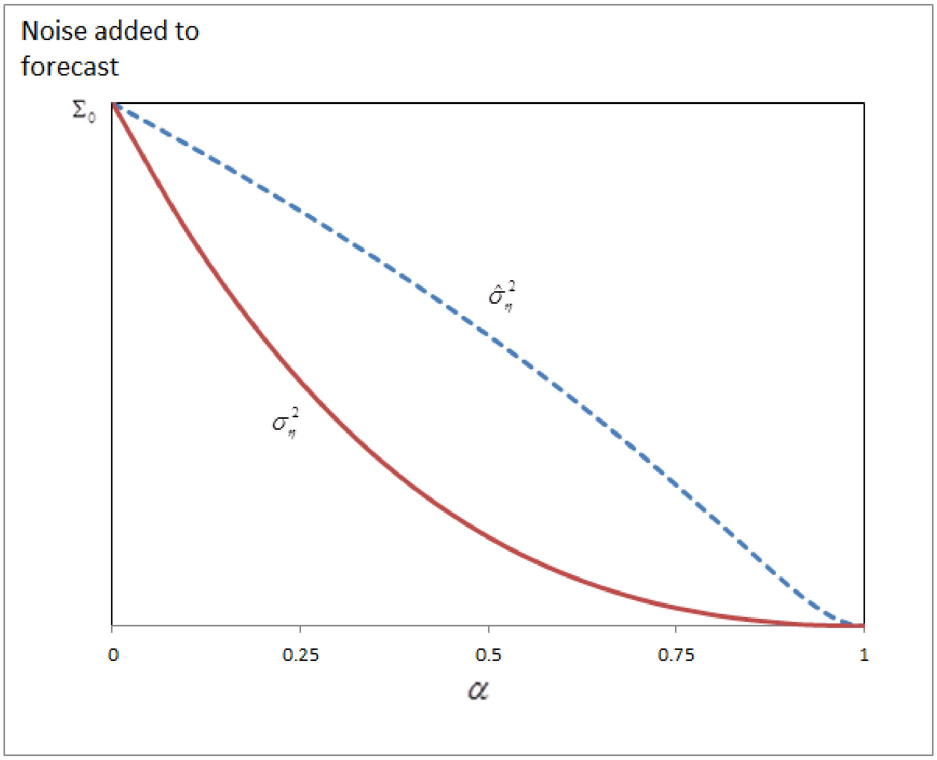

Figure 3.7: Noise added to the forecast by manager in his confidential treatment request under the 2 different assumptions

3.4.3 Price Impact of Disclosed Trade

Upon facing a rejection of the confidential treatment request, the manager needs to disclose his first period trade. The market-maker updates the price from P_1 to $P_1^* = v'$ before trading commences in the second period. The price impact of the disclosed trade is

$$E\left(\frac{v' - P_1}{x_1}\right) = \frac{1}{\beta_1} - \lambda_1 \tag{3.7}$$

The first period trade x_1 thus has a price impact of λ_1 on P_1 and another price impact of $\frac{1}{\beta_1} - \lambda_1$ when it is disclosed. Following from Proposition 2 in Huddart et al. (2001), if the manager does not apply for confidential treatment, the corresponding price impact of the disclosed trade is $\frac{1}{2\sigma_u}\sqrt{\frac{\Sigma_0}{2}}$.

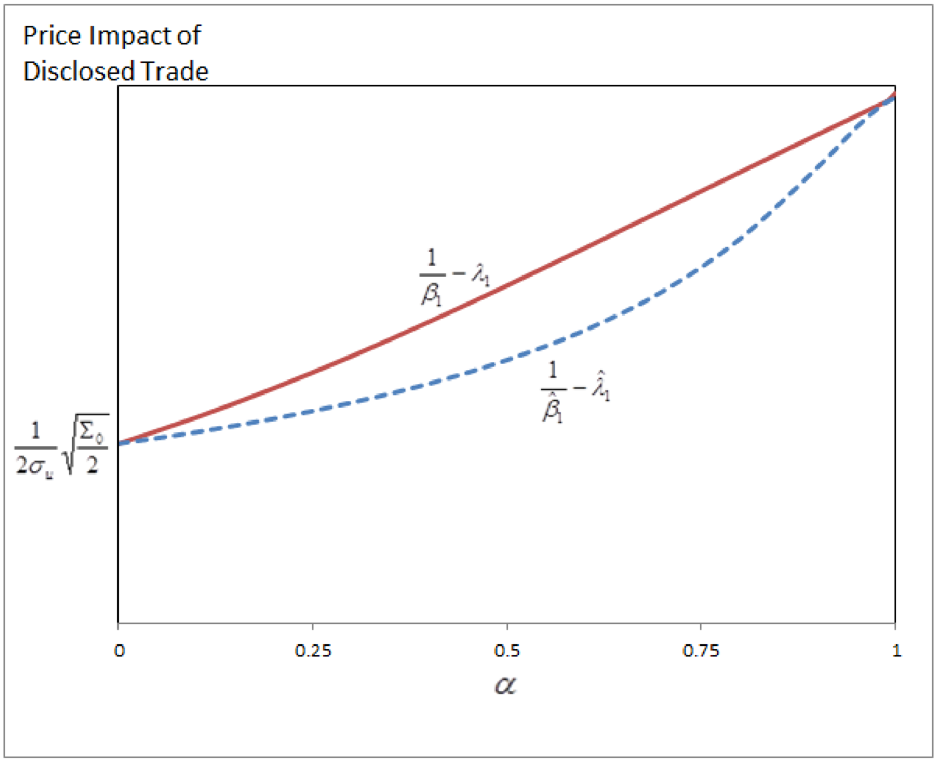

Figure 3.8: Price impact if manager's trade is disclosed due to unsuccessful confidential treatment request under the 2 different assumptions

Figure 3.8 depicts the positive relationship between the price impact of the disclosed trade and α. The price impact due to a confidential treatment request denial is greater than that of a voluntarily disclosed trade (where $\alpha = 0$). If managers with a better market reputation of uncovering the fair value of stocks like Warren Buffett are assigned a higher α, then it follows that their disclosed trades due to confidential treatment denials will result in a larger price impact. In addition, we note that the price impact under the scenario where the SEC restricts the second period trade is greater than the price impact under the scenario where there are no restrictions, i.e. $\frac{1}{\beta_1} - \lambda_1 \geq \frac{1}{\widehat{\beta}_1} - \widehat{\lambda}_1$. This follows from Figure 3.7, as the manager adds less noise under the first scenario and therefore the disclosed trade is more informative.

Agarwal et al. (2011) document a significant positive market reaction associated with involuntary disclosure of positions due to relatively quick confidential treatment denials[11] by the SEC. The authors attribute the market reaction as evidence supporting the private information motive of confidential treatment requests. The results of our model imply that the market reaction would be greater for managers with higher α.

3.4.4 Liquidity

We next examine the welfare implications of liquidity traders if the manager applies for confidential treatment. Compared to the case where the manager always discloses his initial trade, confidential treatment implies greater information asymmetry between the manager and the market maker. We would expect greater transaction costs for liquidity traders as market depth decreases. Figure 3.9 depicts the relationship between α and the market-maker's liquidity parameters in Proposition 3.1. In the two-period Huddart et al. (2001) model, $\overline{\lambda}_1 = \overline{\lambda}_2 = \frac{1}{2\sigma_u}\sqrt{\frac{\Sigma_0}{2}}$. Since the liquidity parameters in the second period λ_2^N and λ_2^D are different and liquidity traders by definition cannot choose when they can trade, we compute the expected value of the liquidity parameter in the second period: $E(\lambda_2) = \alpha\lambda_2^N + (1-\alpha)\lambda_2^D$. It can be seen that $\lambda_1 \geq \frac{1}{2\sigma_u}\sqrt{\frac{\Sigma_0}{2}}$ for all values of α, while that is not true for $E(\lambda_2)$. However the average liquidity parameter $\frac{\lambda_1 + E(\lambda_2)}{2}$ over the two periods is greater than $\frac{1}{2\sigma_u}\sqrt{\frac{\Sigma_0}{2}}$ for $\alpha \geq \alpha^*$. We therefore conclude that liquidity traders are worse off if the investment manager applies for confidential treatment. We also arrive at the same conclusion when there is no restriction in the second period trade by the SEC, as shown in Figure 3.10. In this scenario, even $E\left(\widehat{\lambda}_2\right)$ is greater than $\frac{1}{2\sigma_u}\sqrt{\frac{\Sigma_0}{2}}$.

[11] They classify these quick denials as filings that are denied within 45-180 days after the quarter-end portfolio date.

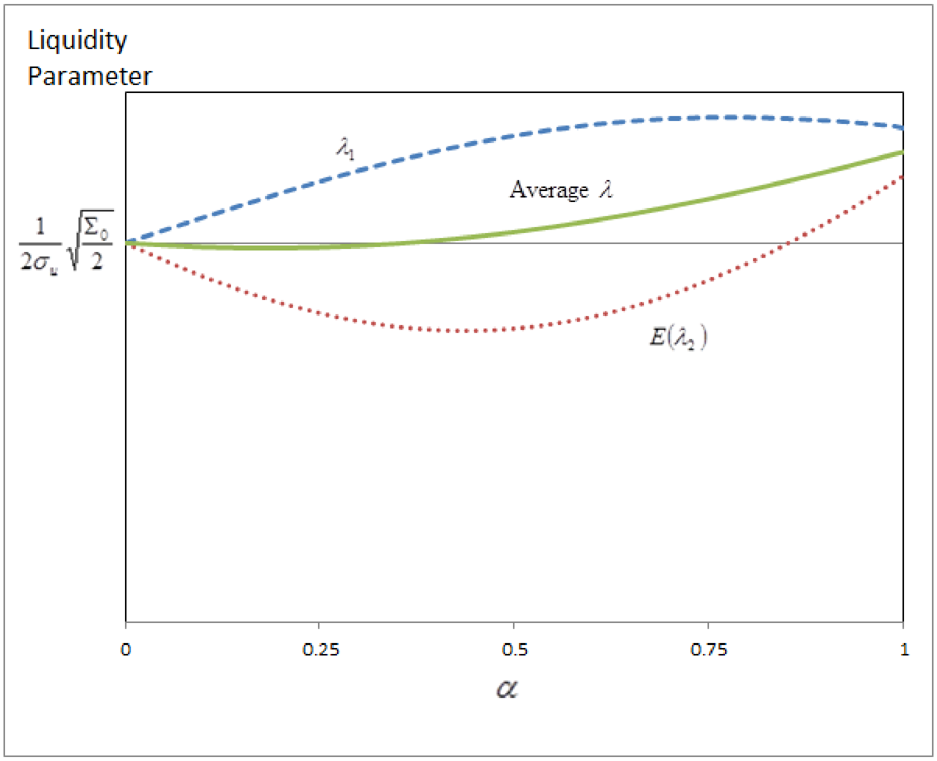

Figure 3.9: Liquidity parameter under the assumption that the SEC restricts the second period trade if confidential treatment request is successful

3.4.5 Potential Policy Change

Under current SEC policy, the manager needs to make the initial trade before he submits his confidential treatment request, to prove that the trade is part of an ongoing trading program. As discussed earlier, the manager faces the risk that the application is rejected and the trade is disclosed. A potential policy change that increases the manager's welfare would be for him to apply for confidential treatment and the SEC making the decision on the request before trading commences. Similar to the scenario in Proposition 3.2 where there is no restriction on the manager's second period trade, he would always apply for confidential treatment. The manager would be in a two-period Kyle (1985) equilibrium with probability α, and Huddart et. al (2001) equilibrium with probability $1-\alpha$. The manager's profit functions under both scenarios in Propositions 3.1 and 3.2 are convex in α (see Figure 3.2 for $0 \leq \alpha \leq 1$.

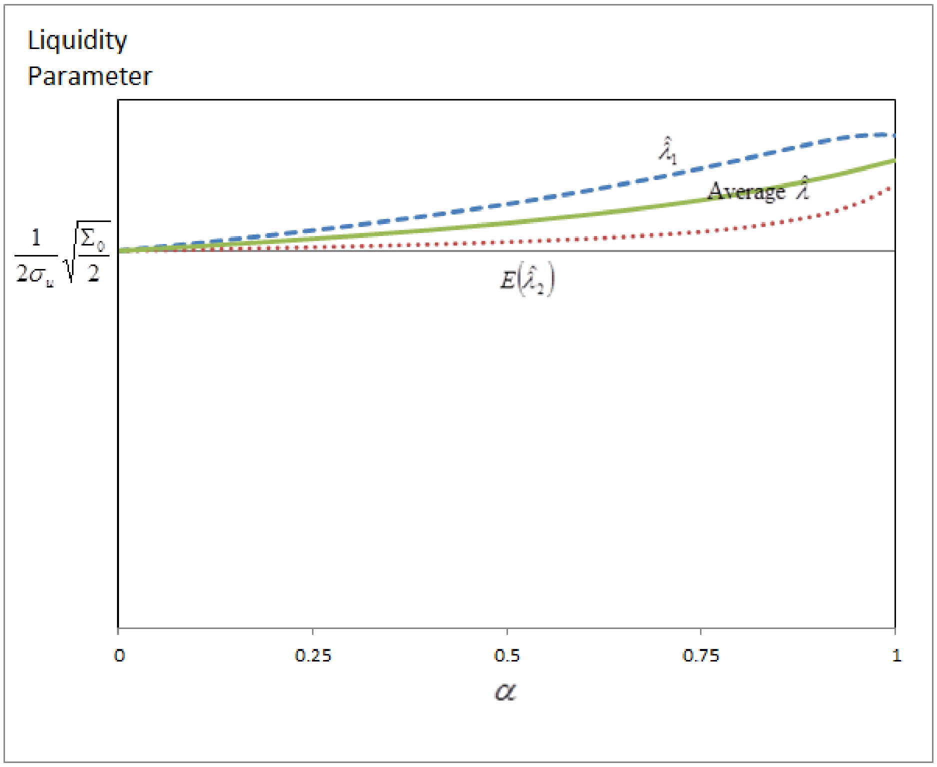

Figure 3.10: Liquidity parameter under the assumption that the SEC does not restrict the second period trade

This is because in the event of a successful application, he does not forgo any expected profits by adding noise in the initial trade, unlike the earlier scenarios. Therefore the manager would be better off with this change in policy. Correspondingly, expected liquidity falls and noise traders are worse off.

3.5 Conclusion

Our primary contribution is a theoretical model which describes market microstructure with confidential treatment requests of trades by investment managers. These trades are typically large ones that have huge price impact and are done over more than one quarter. The key feature we capture is that the SEC requires the manager to present a coherent on-going trading program in his application for confidential treatment. In the event his confidential treatment request is granted, he has to trade in a manner consistent with his reported forecast in the subsequent period. We assume that failure to do so would result in future rejections by the SEC and model this as an exogenous restriction in the manager's second period trade. Analogous to Huddart et al. (2001)'s dissimulation trading strategy, in equilibrium, the manager adds noise to the forecast that he reports to the SEC.

Our model explains various stylized facts described in the empirical literature. Although all investors can apply for confidential treatment, not everybody does. Furthermore, when they do apply, they are not always successful. Our model predicts that with the SEC restriction in the second period, managers only earn higher expected profits if their probability of successful application is higher than a certain threshold. If there is no such restriction, expected profits would always be higher. This is consistent with managers having heterogeneous probabilities of success. For instance, funds that employ quantitative and statistical arbitrage trading strategies involving multiple assets may find it more difficult to convince the SEC that disclosure would reveal the trading strategy to the public and harm its performance[12]. This is because the SEC will only grant confidential treatment on a position-by-position basis. In addition, Agarwal et al. (2011) report that hedge funds with higher past rejection rates are more likely to be rejected again in future applications which supports the assertion that the probability of success is a fund characteristic.

Aragon et al. (2011) and Agarwal et al. (2011) both find confidential holdings of hedge funds yield superior performance. In our model, trading after a successful application has higher expected profits whenever managers find it ex ante optimal to apply. Agarwal et al. (2011) further report a significant positive market reaction after

[12]See http://sec.gov/rules/other/34-52134.pdf. It is a rejection letter issued by the SEC on Two Sigma Investments LLC confidential treatment request in 2005. The fund uses trading strategies based on statistical models. In another case, D.E. Shaw & Company, a large quant-oriented hedge fund manager filed for confidential treatment for its entire second quarter portfolio in 2007. Their request was rejected and they were forced to disclose their whole portfolio valued at US$79 billion.

the involuntary disclosure of hedge funds' trades following rejections of confidential treatment requests. We also find that in our model. The noise that the manager adds to the first period trade successfully obscures some of his private information which can be exploited in the second period. However, a failed application reveals this information and prices react accordingly.

Finally, we examine the impact of confidential treatment provisions on market liquidity and the welfare of liquidity traders. We find that market depth is lower when the manager applies for confidential treatment. Liquidity traders will be worse off.

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3.7 Appendix

Proof of Proposition 3.1. If the application is not successful, his first period trade will be disclosed. The market-maker observes x_1 and is able to infer v' perfectly. The price of asset will be adjusted to v' before the second round of trading commences. Assume that

$$x_2^D = \beta_2^D (v - v') \tag{3.8}$$

$$P_2^D = v' + \lambda_2^D y_2^D$$

If the application is successful, his first period trade will not be disclosed. Assume that

$$x_2^N = \beta_2^N (v' - P_1) \tag{3.9}$$

$$P_2^N = P_1 + \lambda_2^N y_2^N$$

The model is solved by backward induction. Let us first analyse the scenario in which the application is not successful and the informed trader is forced to disclose his first period trade. The informed trader maximises second period profits

$$E\left[\left(v - P_2^D\right) x_2^D | v\right] = E\left[\left(v - v' - \lambda_2^D x_2^D\right) x_2^D\right]$$

Taking first order condition with respect to x_2^D results in the following equations

$$\begin{aligned} x_2^D &= \frac{1}{2\lambda_2^D}\left(v - v'\right) \\ \beta_2^D &= \frac{1}{2\lambda_2^D} \\ E\left[\pi_2^D\left(v', v\right)\right] &= \frac{1}{4\lambda_2^D}\left(v - v'\right)^2 \end{aligned} \tag{3.10}$$

In the event that the application is successful, the informed trader has to choose x_2^N that is coherent with v'. This means that x_2^N is chosen such that it maximises second period profits as if the informed trader has a signal v'.

$$E\left[\left(v - P_2^N\right) x_2^N | v'\right] = E\left[\left(v' - P_1 - \lambda_2^N x_2^N\right) x_2^N\right]$$

Taking first order condition with respect to x_2^N

$$\begin{aligned} x_2^N &= \frac{1}{2\lambda_2^N}\left(v' - P_1\right) \\ \beta_2^N &= \frac{1}{2\lambda_2^N} \end{aligned} \tag{3.11}$$

Since the informed trader knows v instead of v', the expected profits in the second period when confidential treatment is granted is

$$E\left[\pi_2^N\left(P_1,v'\right)|v\right]=E\left[\left(v-P_2^N\right)x_2^N|v\right]=\frac{1}{2\lambda_2^N}\left(v-\frac{v'}{2}-\frac{P_1}{2}\right)\left(v'-P_1\right)$$

Stepping back to the first period, the total expected profits in both periods is

$$\begin{aligned}&E\left[\left(v-P_1\right)x_1+\left(1-\alpha\right)\pi_2^D\left(v',v\right)+\alpha\pi_2^N\left(P_1,v'\right)|v\right]\\ =\ &E\left[\begin{array}{c}\left(v-P_0-\lambda_1x_1\right)x_1+\frac{1-\alpha}{4\lambda_2^D}\left(v-P_0-\frac{x_1}{\beta_1}\right)^2\\ +\frac{\alpha}{2\lambda_2^N}\left(v-P_0-\frac{x_1}{2\beta_1}-\frac{\lambda_1x_1}{2}\right)\left(\frac{x_1}{\beta_1}-\lambda_1x_1\right)\end{array}\right]\end{aligned}$$

Taking first order condition with respect to x_1

$$\left(v-P_0\right)\left(1-\frac{1-\alpha}{2\lambda_2^D\beta_1}+\frac{\alpha}{2\lambda_2^N}\left(\frac{1}{\beta_1}-\lambda_1\right)\right)+x_1\left(-2\lambda_1+\frac{1-\alpha}{2\lambda_2^D\beta_1^2}-\frac{\alpha}{2\lambda_2^N}\left(\frac{1}{\beta_1^2}-\lambda_1^2\right)\right)=0$$

The second-order condition is

$$-2\lambda_1+\frac{1-\alpha}{2\lambda_2^D\beta_1^2}-\frac{\alpha}{2\lambda_2^N}\left(\frac{1}{\beta_1^2}-\lambda_1^2\right)\leq 0$$

Following from Huddart et al. (2001), for the mixed strategy $\theta=v+\eta$, $\eta\sim N\left(0,\sigma_\eta^2\right)$ to hold in equilibrium, the manager must be indifferent across all values of x_1, as x_1 is a function of θ. We seek positive values of λ_1, λ_2^D and λ_2^N such that

$$1-\frac{1-\alpha}{2\lambda_2^D\beta_1}+\frac{\alpha}{2\lambda_2^N}\left(\frac{1}{\beta_1}-\lambda_1\right)=0$$

and

$$-2\lambda_1+\frac{1-\alpha}{2\lambda_2^D\beta_1^2}-\frac{\alpha}{2\lambda_2^N}\left(\frac{1}{\beta_1^2}-\lambda_1^2\right)=0$$

Re-arranging terms,

$$\beta_1=\frac{1}{\lambda_1}-\frac{1-\alpha}{2\lambda_2^D}\tag{3.12}$$

and

$$\beta_1 = \frac{2\lambda_2^N - \alpha\lambda_1}{\lambda_1\left(4\lambda_2^N - \alpha\lambda_1\right)} \tag{3.13}$$

Using the projection theorem of normal random variables on y_1, y_2^N and y_2^D, we obtain

$$\lambda_1 = \frac{\frac{\beta_1\Sigma_0^2}{\Sigma_0+\sigma_\eta^2}}{\frac{\beta_1^2\Sigma_0^2}{\Sigma_0+\sigma_\eta^2} + \sigma_u^2} \tag{3.14}$$

$$\Sigma_1^D = \frac{\sigma_\eta^2}{\Sigma_0 + \sigma_\eta^2}\Sigma_0 \tag{3.15}$$

$$\Sigma_1^N = \frac{\Sigma_0^2}{\Sigma_0 + \sigma_\eta^2} - \frac{\left(\frac{\beta_1\Sigma_0^2}{\Sigma_0+\sigma_\eta^2}\right)^2}{\frac{\beta_1^2\Sigma_0^2}{\Sigma_0+\sigma_\eta^2} + \sigma_u^2} \tag{3.16}$$

$$\lambda_2^D = \frac{\beta_2^D\Sigma_1^D}{\beta_2^{D2}\Sigma_1^D + \sigma_u^2} \tag{3.17}$$

$$\lambda_2^N = \frac{\beta_2^N\Sigma_1^N}{\beta_2^{N2}\Sigma_1^N + \sigma_u^2} \tag{3.18}$$

(3.10) and (3.17) imply

$$\beta_2^D = \frac{\sigma_u}{\sqrt{\Sigma_1^D}} \tag{3.19}$$

$$\lambda_2^D = \frac{\sqrt{\Sigma_1^D}}{2\sigma_u} \tag{3.20}$$

while (3.11) and (3.18) imply

$$\beta_2^N = \frac{\sigma_u}{\sqrt{\Sigma_1^N}} \tag{3.21}$$

$$\lambda_2^N = \frac{\sqrt{\Sigma_1^N}}{2\sigma_u} \tag{3.22}$$

Substituting (3.14), (3.15) and (3.20) into (3.12) gives us

$$\beta_1 = \frac{\sigma_u \sigma_\eta}{(1-\alpha)\,\Sigma_0}\sqrt{\frac{\Sigma_0 + \sigma_\eta^2}{\Sigma_0}} \tag{3.23}$$

$$\lambda_1 = \frac{(1-\alpha)\,\Sigma_0 \sigma_\eta}{\sigma_u\left(\sigma_\eta^2 + (1-\alpha)^2\,\Sigma_0\right)}\sqrt{\frac{\Sigma_0}{\Sigma_0 + \sigma_\eta^2}} \tag{3.24}$$

Substituting (3.16), (3.22), (3.23) and (3.24) into (3.13) results in the following equation for σ_η^2

$$\left((1-\alpha)^2 - h\right)\sqrt{h + (1-\alpha)^2} - \alpha\,(1-\alpha)^2\sqrt{h} = 0 \tag{3.25}$$

where $\sigma_\eta^2 = h\Sigma_0$

Expected profits in first period

$$E\,(\pi_1) = E\,[(v - P_1)\,x_1|v]$$
$$= E\,[(v - P_0 - \lambda_1\beta_1\,(v' - P_0))\,\beta_1\,(v' - P_0)]$$
$$= \tfrac{\beta_1(1-\lambda_1\beta_1)\Sigma_0}{1+h}$$

Expected profits in second period with successful application

$$E\left(\pi_2^N\right) = E\left[\left(v - P_2^N\right)x_2^N|v\right]$$
$$= E\left[\left(v - v' + \tfrac{1}{2}\,(v' - P_1)\right)\beta_2^N\,(v' - P_1)\right]$$
$$= \tfrac{\beta_2^N\Sigma_1^N}{2}$$

Expected profits in second period with unsuccessful application

$$E\left(\pi_2^D\right) = E\left[\left(v - P_2^D\right)x_2^D|v\right]$$
$$= E\left[\tfrac{1}{2}\beta_2^D\,(v - v')^2\right]$$
$$= \tfrac{\beta_2^D\Sigma_1^D}{2}$$

■

Proof of Proposition 3.2. If the manager's second period trade is not enforced by the SEC in the event he is granted confidential treatment, he is free to use v instead of $\widehat{v}'$. Therefore we have

$$\widehat{x}_2^N = \widehat{\beta}_2^N \left(v - \widehat{P}_1\right) \tag{3.26}$$

$$E\left[\widehat{\pi}_2^N\left(\widehat{P}_1, v\right)|v\right] = E\left[\left(v - \widehat{P}_2^N\right)\widehat{x}_2^N|v\right] = \frac{1}{4\widehat{\lambda}_2^N}\left(v - \widehat{P}_1\right)^2$$

Similar to the proof in Proposition 3.1, we obtain

$$\widehat{\beta}_2^N = \frac{1}{2\widehat{\lambda}_2^N} \tag{3.27}$$

$$\widehat{x}_2^D = \widehat{\beta}_2^D\left(v - \widehat{v}'\right) \tag{3.28}$$

$$\widehat{\beta}_2^D = \frac{1}{2\widehat{\lambda}_2^D} \tag{3.29}$$

Stepping back to the first period, the total expected profits in both periods is

$$\begin{aligned} &E\left[\left(v - \widehat{P}_1\right)\widehat{x}_1 + (1-\alpha)\,\widehat{\pi}_2^D\left(\widehat{v}', v\right) + \alpha\widehat{\pi}_2^N\left(\widehat{P}_1, v\right)|v\right] \\ &= E\left[\left(v - P_0 - \widehat{\lambda}_1\widehat{x}_1\right)\widehat{x}_1 + \tfrac{1-\alpha}{4\widehat{\lambda}_2^D}\left(v - P_0 - \tfrac{\widehat{x}_1}{\widehat{\beta}_1}\right)^2 + \tfrac{\alpha}{4\widehat{\lambda}_2^N}\left(v - P_0 - \widehat{\lambda}_1\widehat{x}_1\right)^2\right] \end{aligned}$$

Taking first order condition with respect to x_1

$$(v - P_0)\left(1 - \frac{1-\alpha}{2\widehat{\lambda}_2^D\widehat{\beta}_1} - \frac{\alpha\widehat{\lambda}_1}{2\widehat{\lambda}_2^N}\right) + \widehat{x}_1\left(-2\widehat{\lambda}_1 + \frac{1-\alpha}{2\widehat{\lambda}_2^D\widehat{\beta}_1^2} + \frac{\alpha\widehat{\lambda}_1^2}{2\widehat{\lambda}_2^N}\right) = 0$$

The second-order condition is

$$-2\widehat{\lambda}_1 + \frac{1-\alpha}{2\widehat{\lambda}_2^D\widehat{\beta}_1^2} + \frac{\alpha\widehat{\lambda}_1^2}{2\widehat{\lambda}_2^N} \le 0$$

For the mixed strategy $\theta = v + \eta$, $\eta \sim N\left(0, \widehat{\sigma}_\eta^2\right)$ to hold in equilibrium, the manager must be different across all values of $\widehat{x}_1$, as $\widehat{x}_1$ is a function of θ. We seek positive values of $\widehat{\lambda}_1$, $\widehat{\lambda}_2^D$ and $\widehat{\lambda}_2^N$ such that

$$1-\frac{1-\alpha}{2\widehat{\lambda}_2^D\widehat{\beta}_1}-\frac{\alpha\widehat{\lambda}_1}{2\widehat{\lambda}_2^N}=0$$

and

$$-2\widehat{\lambda}_1+\frac{1-\alpha}{2\widehat{\lambda}_2^D\widehat{\beta}_1^2}+\frac{\alpha\widehat{\lambda}_1^2}{2\widehat{\lambda}_2^N}=0$$

Re-arranging terms

$$\widehat{\beta}_1=\frac{2\widehat{\lambda}_2^N-\alpha\widehat{\lambda}_1}{\widehat{\lambda}_1\left(4\widehat{\lambda}_2^N-\alpha\widehat{\lambda}_1\right)} \tag{3.30}$$

$$\widehat{\lambda}_1=\frac{1-\alpha}{\widehat{\beta}_1\left(2\widehat{\lambda}_2^D\widehat{\beta}_1+1-\alpha\right)} \tag{3.31}$$

Using the projection theorem of normal random variables on $\widehat{y}_1$, $\widehat{y}_2^N$ and $\widehat{y}_2^D$, we obtain

$$\widehat{\lambda}_1=\frac{\frac{\widehat{\beta}_1\Sigma_0^2}{\Sigma_0+\widehat{\sigma}_\eta^2}}{\frac{\widehat{\beta}_1^2\Sigma_0^2}{\Sigma_0+\widehat{\sigma}_\eta^2}+\sigma_u^2} \tag{3.32}$$

$$\widehat{\Sigma}_1^D=\frac{\widehat{\sigma}_\eta^2}{\Sigma_0+\widehat{\sigma}_\eta^2}\Sigma_0 \tag{3.33}$$

$$\widehat{\Sigma}_1^N=\Sigma_0-\frac{\left(\frac{\widehat{\beta}_1\Sigma_0^2}{\Sigma_0+\widehat{\sigma}_\eta^2}\right)^2}{\frac{\widehat{\beta}_1^2\Sigma_0^2}{\Sigma_0+\widehat{\sigma}_\eta^2}+\widehat{\sigma}_u^2} \tag{3.34}$$

$$\widehat{\lambda}_2^D=\frac{\widehat{\beta}_2^D\widehat{\Sigma}_1^D}{\widehat{\beta}_2^{D2}\widehat{\Sigma}_1^D+\sigma_u^2} \tag{3.35}$$

$$\widehat{\lambda}_2^N=\frac{\widehat{\beta}_2^N\widehat{\Sigma}_1^N}{\widehat{\beta}_2^{N2}\widehat{\Sigma}_1^N+\sigma_u^2} \tag{3.36}$$

(3.29) and (3.35) imply

$$\widehat{\beta}_2^D=\frac{\sigma_u}{\sqrt{\widehat{\Sigma}_1^D}} \tag{3.37}$$

$$\widehat{\lambda}_2^D = \frac{\sqrt{\widehat{\Sigma}_1^D}}{2\sigma_u} \tag{3.38}$$

while (3.27) and (3.36) imply

$$\widehat{\beta}_2^N = \frac{\sigma_u}{\sqrt{\widehat{\Sigma}_1^N}} \tag{3.39}$$

$$\widehat{\lambda}_2^N = \frac{\sqrt{\widehat{\Sigma}_1^N}}{2\sigma_u} \tag{3.40}$$

Substituting (3.32), (3.33) and (3.38) into (3.31) gives us

$$\widehat{\beta}_1 = \sigma_u \left(\frac{1-\alpha}{\Sigma_0 \widehat{\sigma}_\eta}\right)^{1/3} \sqrt{\frac{\Sigma_0 + \widehat{\sigma}_\eta^2}{\Sigma_0}} \tag{3.41}$$

$$\widehat{\lambda}_1 = \frac{1-\alpha}{\sigma_u \left(\frac{1-\alpha}{\Sigma_0 \widehat{\sigma}_\eta}\right)^{1/3} \left(\widehat{\sigma}_\eta \left(\frac{1-\alpha}{\Sigma_0 \widehat{\sigma}_\eta}\right)^{1/3} + 1 - \alpha\right)} \sqrt{\frac{\Sigma_0}{\Sigma_0 + \widehat{\sigma}_\eta^2}} \tag{3.42}$$

Substituting (3.34), (3.40), (3.41) and (3.42) into (3.30) results in the following equation for $\widehat{\sigma}_\eta^2$

$$\alpha \sqrt{\frac{g}{g+1}} - (1-\alpha)\left(g^{2/3}(1-\alpha)^{-4/3} - 1\right) \sqrt{\frac{\frac{g^{2/3}}{1+g}(1-\alpha)^{2/3} + 1}{g^{-1/3}(1-\alpha)^{2/3} + 1}} = 0 \tag{3.43}$$

where $\widehat{\sigma}_\eta^2 = g\Sigma_0$

Expected profits in first period

$$\begin{aligned} E(\widehat{\pi}_1) &= E\left[\left(v - \widehat{P}_1\right)\widehat{x}_1 | v\right] \\ &= E\left[\left(v - P_0 - \widehat{\lambda}_1 \widehat{\beta}_1 (\widehat{v}' - P_0)\right) \widehat{\beta}_1 (\widehat{v}' - P_0)\right] \\ &= \frac{\widehat{\beta}_1 \left(1 - \widehat{\lambda}_1 \widehat{\beta}_1\right) \Sigma_0}{1+g} \end{aligned}$$

Expected profits in second period with successful application

$$
\begin{aligned}
E\left(\widehat{\pi}_2^N\right) &= E\left[\left(v-\widehat{P}_2^N\right)\widehat{x}_2^N|v\right]\\
&= E\left[\tfrac{1}{2}\widehat{\beta}_2^N\left(v-\widehat{P}_1\right)^2\right]\\
&= \tfrac{\widehat{\beta}_2^N\widehat{\Sigma}_1^N}{2}
\end{aligned}
$$

Expected profits in second period with unsuccessful application

$$
\begin{aligned}
E\left(\widehat{\pi}_2^D\right) &= E\left[\left(v-\widehat{P}_2^D\right)\widehat{x}_2^D|v\right]\\
&= E\left[\tfrac{1}{2}\widehat{\beta}_2^D\left(v-\widehat{v}'\right)^2\right]\\
&= \tfrac{\widehat{\beta}_2^D\widehat{\Sigma}_1^D}{2}
\end{aligned}
$$

■

Proof of Proposition 3.3. If the SEC constraints the manager's second period trade, the manager's total profits is lower than those obtained from a trading strategy of disclosure as in Huddart et al. (2001) if

$$E\left(\pi_1\right)+\alpha E\left(\pi_2^N\right)+\left(1-\alpha\right)E\left(\pi_2^D\right)\le\sigma_u\sqrt{\frac{\Sigma_0}{2}} \tag{3.44}$$

From the plot of the expected profit function in Figure 3.2, there is a threshold value of α which we will call α^*, below which total expected profits from application are lower than with disclosure. α^* satisfies the equality

$$E\left(\pi_1\right)+\alpha E\left(\pi_2^N\right)+\left(1-\alpha\right)E\left(\pi_2^D\right)=\sigma_u\sqrt{\frac{\Sigma_0}{2}} \tag{3.45}$$

Substituting the profit functions in Proposition 3.1 into (3.45)

$$\frac{1-\alpha}{\sqrt{1+h}}\left[\frac{2\sqrt{h}}{h+\left(1-\alpha\right)^2}+\frac{\alpha}{\sqrt{h+\left(1-\alpha\right)^2}}+\sqrt{h}\right]-\sqrt{2}=0 \tag{3.46}$$

Notice that the exogenous parameters σ_u and Σ_0 are not present in (3.46). From (3.46) and (3.25), we obtain numerically to 3 decimal places:

$$\alpha^* \approx 0.361$$

On the other hand, if the SEC does not restrict his second period trade, we find that

$$E\left(\widehat{\pi}_1\right) + \alpha E\left(\widehat{\pi}_2^N\right) + (1-\alpha) E\left(\widehat{\pi}_2^D\right) \geq \sigma_u \sqrt{\frac{\Sigma_0}{2}} \tag{3.47}$$

This means the manager's expected profits will always be higher than in the Huddart et al. (2001) case. ■