The London School of Economics
and Political Science

Essays on Monetary and Exchange Rate Policy in Financially Fragile Economies

Luca Fornaro

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

Statement of conjoint work

I confirm that Chapter 4 was jointly co-authored with Dr Gianluca Benigno and I contributed 50% of this work.

Abstract

In my thesis I study policy interventions, with particular attention to monetary and exchange rate policy, in financially fragile economies. The thesis is composed of four chapters, and each chapter deals with different forms of policy interventions and different dimensions of financial fragility. However, the four chapters share a common message: appropriately designed policies can play a key role in improving macroeconomic performance in economies vulnerable to the risk of financial crises.

In the first chapter I consider the role of the exchange rate regime in determining the adjustment to episodes of global deleveraging. To achieve this goal, I develop a framework for understanding the international dimensions of episodes of debt deleveraging. During an episode of international deleveraging world consumption demand is depressed and the world interest rate is low, reflecting a high propensity to save. If exchange rates are allowed to float, deleveraging countries can depreciate their nominal exchange rate to increase production and mitigate the fall in consumption associated with debt reduction. The key insight is that in a monetary union this channel of adjustment is shut off, and therefore the falls in consumption demand and in the world interest rate are amplified. Hence, monetary unions are especially prone to hit the zero lower bound on the nominal interest rate and enter a liquidity trap during deleveraging. In a liquidity trap deleveraging gives rise to a union-wide recession, which is particularly severe in high-debt countries. The model suggests several policy interventions that mitigate the negative impact of deleveraging on output in monetary unions.

In the second chapter, I consider another policy that can be useful in managing episodes of debt deleveraging: debt relief. As illustrated by the analysis in the first chapter, deleveraging can push the economy into a liquidity trap characterized by involuntary unemployment and low inflation. A debt relief policy, captured by a transfer of wealth from creditors to debtors, increases aggregate demand, employment and output. Debt relief may benefit creditors as well as debtors and lead to a Pareto improvement in welfare. The benefits from a policy of debt relief are greater the more the central bank is concerned with stabilizing inflation.

The third chapter considers the role of exchange rate policy in economies in which financial fragility arises because the value of collateral is determined by asset prices. The dependence of collateral on asset prices introduces pecuniary externalities that create scope for policy interventions. In this case, a fundamental trade-off between financial

and price stability arises, because the central bank has an incentive to deviate from its traditional objective of granting price stability in order to manipulate asset prices and collateral. The main result is thus that the presence of pecuniary externalities in the credit markets makes a narrow focus on price stability sub-optimal.

The fourth chapter, joint with Gianluca Benigno, considers the role of foreign reserves in emerging economies characterized by growth externalities and the risk of sudden stops on capital inflows. We present a model that reproduces two salient facts characterizing the international monetary system: Fast growing emerging countries i) Run current account surpluses, ii) Accumulate international reserves and receive net private inflows. We study a two-sector, tradable and non-tradable, small open economy. There is a growth externality in the tradable sector and agents have imperfect access to international financial markets. By accumulating foreign reserves, the government induces a real exchange rate depreciation and a reallocation of production towards the tradable sector that boosts growth. Financial frictions generate imperfect substitutability between private and public debt flows so that private agents do not perfectly offset the government policy. The possibility of using reserves to provide liquidity during crises amplifies the positive impact of reserve accumulation on growth. The optimal reserve management entails a fast rate of reserve accumulation, as well as higher growth and larger current account surpluses compared to the economy with no policy intervention. The model is also consistent with the negative relationship between inflows of foreign aid and growth observed in low-income countries.

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Contents

List of Figures

List of Tables

Chapter 1

International Debt Deleveraging

1.1 Introduction

Episodes of global debt deleveraging are rare, but when they occur they come with deep recessions and destabilize the international monetary system. In the Great Depression of the 1930s the world entered a period of global debt reduction and experienced the most severe recession in modern history. The cornerstone of the international monetary system, the Gold Standard, came under stress and was abandoned in 1936, when the remaining countries belonging to the Gold Block gave up their exchange rate pegs against gold. Almost 80 years later, history seems to be repeating itself. Following the 2007-2008 turmoil in financial markets several advanced economies started a process of private debt deleveraging accompanied by a deep economic downturn, the Great Recession. Once again, the status quo in the international monetary system is challenged, and this time the survival of the Eurozone is called into question. These events might suggest that fixed exchange arrangements, such as monetary unions, are hard to maintain during times of global debt deleveraging, but more research is needed to understand exactly why this chain of events is set in motion during deleveraging episodes.

My objective in this paper is to develop a framework for the study of the implications of debt deleveraging in a group of financially integrated countries. During an episode of international deleveraging world demand for consumption is depressed and the world interest rate is low, reflecting a high propensity to save. If exchange rates are allowed to float, deleveraging countries can rely on a depreciation to increase production and mitigate the fall in consumption associated with debt reduction. The key insight of the paper is that in a monetary union this channel of adjustment is shut off, because high-debt countries cannot depreciate against the other countries in the monetary union, and therefore the falls in the demand for consumption and in the interest rate are amplified. Hence, during an episode of deleveraging monetary unions are especially prone to hit the zero lower bound on the nominal interest rate and enter a liquidity trap. In a liquidity trap standard monetary policy tools are ineffective and deleveraging gives rise to a deflationary recession. This effect contributes to explain why episodes of debt deleveraging are particularly painful for monetary unions.

The model features a continuum of small open economies trading with each other. Each economy is inhabited by households which borrow and lend to smooth the impact of temporary, country-specific, productivity shocks on consumption, in the spirit of the Bewley (1977) closed economy model. Foreign borrowing and lending arise endogenously as households use the international credit market to insure against country-specific productivity shocks.

Each household is subject to an exogenous borrowing limit. I study an episode of deleveraging triggered by a tightening of the borrowing limit, which I call a *deleveraging shock*. To isolate the role of the exchange rate regime in shaping the response to a deleveraging shock I compare the adjustment under two different versions of the model. I start by considering a model without nominal rigidities. I then analyze the case of a monetary union with nominal wage rigidities.

In both versions of the model, the process of debt reduction generates a fall in the world interest rate, which overshoots its long run value. This is due to two different effects. On the one hand, the countries starting with a relatively high stock of debt are forced to reduce it by the tightening of the borrowing limit. On the other hand, the countries starting with a low stock of debt, as well as those starting with a positive stock of foreign assets, want to increase precautionary savings as a buffer against the risk of hitting the borrowing limit in the future. Both effects lower consumption demand and generate a rise in the propensity to save. As a result, the world interest rate must fall to guarantee that the rest of the world absorbs the forced savings of high-debt borrowing-constrained economies.

In a world without nominal rigidities the deleveraging process also entails a rise in production in high-debt economies. Households can repay their debts not only by cutting consumption, but also by working more to increase their labor income. Thus, households living in high-debt countries increase their labor supply in response to the deleveraging shock. If wages are flexible, this generates a drop in real wages and a rise in employment and output in high-debt countries.

A large body of evidence, reviewed below, suggests that nominal wages adjust slowly to shocks. In particular nominal wages do not fall much during deep recessions, in spite of sharp rises in unemployment. With nominal wage rigidities I show that nominal exchange rate flexibility can substitute for nominal wage flexibility. But in a monetary union exchange rates between members are fixed and the adjustment in real wages cannot be achieved through movements in the nominal exchange rate. I focus on this case in the main part of the paper.

The combination of nominal wage rigidities and fixed exchange rates prevents any increase in employment and production in high-debt economies in response to the deleveraging shock. Households living in the high-debt countries of the monetary union have to reduce their debt solely by decreasing consumption. The deep fall in consumption demand coming from high-debt countries amplifies the increase in the propensity to save and the fall in the world interest rate during deleveraging. Because of this effect, the chances that an episode of deleveraging gives rise to a liquidity trap are particularly high for monetary unions.

When the central bank of the monetary union is constrained by the zero lower bound on the nominal interest rate, deleveraging gives rise to a deflationary union-wide recession. Because the interest rate cannot fall enough to guarantee market clearing, firms decrease prices in order to eliminate excess supply. Given the sticky nominal wages, the fall in prices translates into a rise in real wages that reduces employment and production. Moreover, if debt is denominated in nominal terms deflation causes a redistribution of wealth from debtor to creditor countries that further reduces consumption demand and production.[1] The recession hits high-debt countries particularly hard, but the economic downturn also spreads to the countries that are not financially constrained, because the common interest rate and trade linkages tie all the countries of the union together.

Finally, I discuss policy interventions that mitigate the recession during deleveraging in monetary unions. First, I show that if the central bank of the monetary union has a higher inflation target the fall in output during deleveraging is smaller. When the nominal interest rate hits the zero bound the real interest rate is equal to the inverse of expected inflation, and so a higher inflation target implies a lower real interest rate, which stimulates consumption demand and production. Second, I consider a policy that slows down the tightening of the borrowing constraint, giving more time to agents to adjust to the new credit conditions. This policy dampens the rise in the propensity to save during the early phases of the deleveraging episode, stimulating consumption and limiting the drop in output.

This paper is related to several strands of the literature. First, the paper is about liquidity traps. Early works studying liquidity traps in micro-founded models, such as Krugman (1998), Eggertsson and Woodford (2003) and Svensson (2003), were motivated by the weak economic performance of Japan during the 1990s, occurring in the context of low inflation and nominal interest rates stuck at zero. The precipitous fall in policy rates experienced by advanced

[1] This is the debt-deflation effect emphasized by Fisher (1933) in the context of the Great Depression.

economies during the current crisis has renewed the interest in liquidity traps.[2] While traditionally the literature has relied on preference shocks to generate liquidity traps, recently a few contributions have drawn the connection between deleveraging and drops in the interest rate. Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) study the impact of deleveraging shocks on the interest rate in closed economies, while Pierpaolo Benigno and Romei (2012) consider deleveraging in a two-country model. My paper contributes to this literature by demonstrating that monetary unions are more likely to enter a liquidity trap during deleveraging.

A key feature of the model I propose is the presence of nominal wage rigidities. There is extensive evidence in support of the existence of nominal wage rigidities, both at the macro and at the micro level. From a macro perspective, there is evidence that wage contracts are set on average once a year in OECD countries. This observation has been used by Olivei and Tenreyro (2007, 2010) to show empirically that nominal wage rigidities play a key role in transmitting monetary policy shocks to the real economy.[3] There is also evidence suggesting that nominal wages adjust slowly to changes in prices and unemployment during deep recessions. In their empirical studies, Eichengreen and Sachs (1985) and Bernanke and Carey (1996) find that nominal wage rigidities contributed substantially to the fall in output during the Great Depression, in particular among countries belonging to the Gold Block.[4] More recently, Schmitt-Grohé and Uribe (2011) have documented the importance of nominal wage rigidities in the context of the 2001 Argentine crisis and of the Great Recession in countries at the Eurozone periphery.[5] Another strand of the literature shows the relevance of nominal wage rigidities using micro data. For example, Fehr and Goette (2005), Gottschalk (2005) and Barattieri et al. (2010) use worker-level data to show that changes in nominal wages, especially downward, happen infrequently. Fabiani et al. (2010) obtain similar results using firm-level data from several European countries.

The paper also relates to the literature studying precautionary savings in incomplete-market economies with idiosyncratic shocks. The literature includes the seminal works of Bewley

[2]See Robert Hall's presidential address at the 2011 AEA meeting (Hall, 2011). See also Jeanne (2009) and Cook and Devereux (2011), who use a two-country model to study a global liquidity trap.

[3]A similar conclusion is reached by Christiano et al. (2005) using an estimated medium scale DSGE model of the US economy.

[4]The importance of nominal wage rigidities in the US during the Great Depression is discussed in more detail in Bordo et al. (2000).

[5]In addition, several authors, including Shimer (2010), Hall (2011) and Midrigan and Philippon (2011), have emphasized the key role of real wage rigidities in rationalizing the 2008-2009 recession following the turmoil in financial markets. More broadly, Michaillat (2012) shows that real wage rigidities are important in explaining unemployment during recessions in the US. In this paper real wage rigidities arise from the combination of nominal wage rigidities and fixed exchange rates.

(1977), Deaton (1991), Huggett (1993), Aiyagari (1994) and Carroll (1997), who consider closed economies in which consumers borrow and lend to self-insure against idiosyncratic income shocks.[6] Guerrieri and Lorenzoni (2011) use a Bewley model to study the impact of deleveraging on the interest rate in a closed economy. My paper shares with their work the focus on precautionary savings. Starting from Clarida (1990), some authors have used multi-country models with idiosyncratic shocks and incomplete markets to study international capital flows. Examples are Castro (2005), Bai and Zhang (2010) and Chang et al. (2009). This is the first paper that employs a multi-country Bewley model to study the interactions between deleveraging, the exchange rate regime and liquidity traps.

The current events in the Eurozone have revived the literature on the macroeconomic management of monetary unions. Recent contributions build on the multi-country framework developed by Gali and Monacelli (2008).[7] Their framework abstracts from financial frictions, a key element in my analysis. Another recent work that relates to the Eurozone crisis is Schmitt-Grohé and Uribe (2011). The authors highlight how the combination of downward nominal wage rigidities in the non-tradable sector and fixed exchange rates can generate involuntary unemployment and recessions in small open economies. Their focus is on a single small open economy that takes the world interest rate as given, while in my paper the endogenous determination of the world interest rate is crucial.

From an empirical perspective, this paper is linked to the work of Lane and Milesi-Ferretti (2012), who look at the adjustment in the current account balances during the Great Recession. They find that the compression in the current account deficits was larger for those countries that were relying more heavily on external financing before the crisis. Moreover, they find that most of the adjustment passed through a compression in domestic demand, contributing to the severity of the crisis in deficit countries. My model rationalizes these facts.[8]

This paper also speaks to the empirical findings of Mian et al. (2011) and Mian and Sufi (2012). These authors find that the fall in consumption and employment in the US during the 2008-2009 recession was stronger in those counties where the pre-crisis expansion in credit driven by the rise in house prices was more pronounced. This evidence is consistent with the results of my paper, if the monetary union version of the model is interpreted as a large country

[6]There is also a literature relating precautionary savings and the business cycle. The classic contribution is Krusell and Smith (1998), while recent works are Guerrieri and Lorenzoni (2012) and Challe and Ragot (2012). Rather than focusing on business cycles, this paper considers the response of precautionary savings to a large financial shock.

[7]Examples are Werning and Farhi (2012), who look at the optimal management of fiscal policy in a monetary union, and Farhi et al. (2011), who derive a set of fiscal measures able to substitute for exchange rate flexibility inside a currency union. Instead, Pierpaolo Benigno (2004) uses a two-country model to study monetary unions.

[8]The paper is also related to the empirical literature on the rise of precautionary savings during the Great Recession. See Carroll et al. (2012) and Mody et al. (2012).

composed of many different regions. Midrigan and Philippon (2011) also address this evidence using an approach complementary to mine. They look at a cash-in-advance model in which credit can be used as a substitute for fiat money. In their model, the fall in consumption is generated by a decrease in the provision of private credit that tightens households' cash-in-advance constraints, while here the emphasis is on intertemporal debt and liquidity traps. Another empirical work that relates to this paper is Nakamura and Steinsson (2011). Their results on fiscal stimulus across US states lend support to models of monetary unions in which aggregate demand has an impact on production.

The rest of the paper is structured as follows. Section 2 introduces the model and briefly analyzes the steady state. Section 3 considers the adjustment following a deleveraging shock in a world with flexible wages. Section 4 shows that the depressive impact of deleveraging on the interest rate is stronger in a monetary union with nominal wage rigidities. Section 5 describes the role of the zero lower bound in translating a deleveraging episode into a recession. Section 6 introduces a version of the model parameterized at quarterly frequency and performs policy experiments. Section 7 concludes.

1.2 Model

Consider a world composed of a continuum of measure one of small open economies. Each economy can be thought of as a country.[9] Time is discrete and indexed by t. Each country is populated by a continuum of measure one of identical infinitely lived households and by a large number of firms. All economies produce two consumption goods: a homogeneous tradable good and a non-tradable good. Countries face idiosyncratic shocks in their production technologies, while the world economy has no aggregate uncertainty. Households borrow and lend on the international credit markets in order to smooth the impact of productivity shocks on consumption. There is an exogenous limit on how much each household can borrow. I start by analyzing the steady state of the model, in which the borrowing limit is held constant. The next section studies the transition after an unexpected shock that tightens the borrowing limit.

Households. Households derive utility from the consumption of a tradable good C^T and of a non-tradable good C^N and experience disutility from labor effort L. The expected lifetime

[9]Another possibility is to think of an economy as a region inside a large country, for example a US state or county.

utility of the representative household in a generic country i is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{i,t}^T, C_{i,t}^N, L_{i,t}\right)\right]. \tag{1.1}$$

In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time t and β is the subjective discount factor. The period utility function $U(\cdot)$ is assumed to be increasing in the first two arguments, decreasing in the third argument, strictly concave and twice continuously differentiable.

Each household can trade in one period risk-free bonds. Bonds are denominated in units of the tradable consumption good and pay the gross interest rate R_t. The interest rate is common across countries and can be interpreted as the world interest rate.

There are no trade frictions and the price of the tradable good is the same in every country. Normalizing the price of the traded good to 1, the household budget constraint expressed in units of the tradable good is

$$C_{i,t}^T + p_{i,t}^N C_{i,t}^N + \frac{B_{i,t+1}}{R_t} = w_{i,t}L_{i,t} + B_{i,t} + \Pi_{i,t}^T + \Pi_{i,t}^N. \tag{1.2}$$

The left-hand side of this expression represents the household's expenditure. p_i^N denotes the price of a unit of non-tradable good in terms of the tradable good in country i.[10] Hence, the term $C_i^T + p_i^N C_i^N$ is the total expenditure of the household in consumption expressed in units of the tradable good. $B_{i,t+1}$ denotes the purchase of bonds made by the household at time t at price $1/R_t$. If $B_{i,t+1} < 0$ the household is a borrower.

The right-hand side captures the household's income. $w_i L_i$ is the household's labor income. Labor is immobile across countries and hence the wage w_i is country-specific. $B_{i,t}$ is the gross return on investment in bonds made at time $t-1$. Finally, Π_i^T and Π_i^N are the profits received from firms operating respectively in the tradable and in the non-tradable sector. All domestic firms are wholly owned by domestic households and equity holdings within these firms are evenly divided among them.

There is a limit on how much each household is able to borrow. In particular, debt repayment cannot exceed the exogenous limit κ, so that the bond position has to satisfy[11]

$$B_{i,t+1} \geq -\kappa. \tag{1.3}$$

[10] p_i^N is not necessarily equalized across countries because the non-traded good is, by definition, not traded internationally.

[11] Throughout the analysis I assume that the exogenous borrowing limit κ is tighter than the natural borrowing limit.

This constraint captures in a simple form a case in which a household cannot credibly commit in period t to repay more than κ units of the tradable good to its creditors in period $t+1$.[12]

The household's optimization problem is to choose $C_{i,t}^T$, $C_{i,t}^N$, $L_{i,t}$ and $B_{i,t+1}$ to maximize the expected present discounted value of utility (4.1), subject to the budget constraint (4.3) and the borrowing limit (4.9), taking the initial bond holdings $B_{i,0}$ and prices R_t, $p_{i,t}^N$, $w_{i,t}$ as given. The household's first-order conditions can be written as

$$p_{i,t}^N = \frac{U_{C_{i,t}^N}}{U_{C_{i,t}^T}} \tag{1.4}$$

$$-U_{L_{i,t}} = w_{i,t} U_{C_{i,t}^T} \tag{1.5}$$

$$\frac{U_{C_{i,t}^T}}{R_t} = \beta E_t \left[U_{C_{i,t+1}^T} \right] + \mu_{i,t} \tag{1.6}$$

$$B_{i,t+1} \geq -\kappa, \quad \text{with equality if } \mu_{i,t} > 0, \tag{1.7}$$

where U_x denotes the first derivative of the utility function with respect to x and μ_i is the non-negative Lagrange multiplier associated with the borrowing limit. The optimality condition (1.4) equates the marginal rate of substitution of the two consumption goods, tradables and non-tradables, to their relative price. Equation (3.3) is the optimality condition for labor supply. Equation (2.4) is the Euler equation for bonds. The binding borrowing constraint generates a wedge between the marginal utility from consuming in the present and the marginal utility from consuming next period, given by the shadow price of relaxing the borrowing constraint μ_i. Finally, equation (1.7) is the complementary slackness condition associated with the borrowing limit.

Firms. Firms rent labor from households and produce both consumption goods, taking prices as given. A typical firm in the tradable sector in country i maximizes profits

$$\Pi_{i,t}^T = Y_{i,t}^T - w_{i,t} L_{i,t}^T,$$

where Y_i^T is the output of tradable good and L_i^T is the amount of labor employed by the firm. The production function is

$$Y_{i,t}^T = A_{i,t}^T \left(L_{i,t}^T \right)^{\alpha_T},$$

where $0 < \alpha_T < 1$.[13] A_i^T is a productivity shock affecting all firms in the tradable sector

[12] In reality tight access to credit may manifest itself through high interest rates, rather than through a quantity restriction on borrowing. In appendix A.2 I show that it is possible to recast the borrowing limit (4.9) in terms of positive spreads over the world interest rate without changing any of the results.

[13] To introduce constant returns-to-scale in production we can assume a production function of the form

in country i. This is the source of idiosyncratic uncertainty that gives rise to cross-country financial flows in steady state. Profit maximization implies

$$\alpha_T A_{i,t}^T \left(L_{i,t}^T\right)^{\alpha_T - 1} = w_{i,t}.$$

This expression says that at the optimum firms equalize the marginal profit from an increase in labor, the left-hand side of the expression, to the marginal cost, the right-hand side.

Similarly, firms in the non-tradable sector maximize profits

$$\Pi_{i,t}^N = p_{i,t}^N Y_{i,t}^N - w_{i,t} L_{i,t}^N,$$

where Y_i^N is the output of non-tradable good and L_i^N is the amount of labor employed in the non-tradable sector. Labor is perfectly mobile across sectors within a country and hence firms in both sectors pay the same wage w_i. The production function available to firms in the non-tradable sector is

$$Y_{i,t}^N = A^N \left(L_{i,t}^N\right)^{\alpha_N},$$

where $0 < \alpha_N < 1$. The term A^N determines the productivity of firms in the non-tradable sector. To reduce the number of state variables and save on computation costs, I assume that A^N is constant and common across all countries.[14] The optimal choice of labor in the non-tradable sector implies

$$p_{i,t}^N \alpha_N A_{i,t}^N \left(L_{i,t}^N\right)^{\alpha_N - 1} = w_{i,t}.$$

Just as firms in the tradable sector, at the optimum firms in the non-tradable sector equalize the marginal benefit from increasing employment to its marginal cost.[15]

Market clearing. Since households inside a country are identical, we can interpret equilibrium quantities as either household or country specific. For instance, the end-of-period net foreign asset position of country i is equal to the end-of-period holdings of bonds of the representative household divided by the world interest rate[16]

$$NFA_{i,t} = \frac{B_{i,t+1}}{R_t}.$$

$Y_{i,t}^T = A_{i,t}^T \left(L_{i,t}^T\right)^{\alpha_T} K^{1-\alpha_T}$, where K is a fixed production factor owned by the firm, for example physical or organizational capital. The production function in the main text corresponds to the normalization $K = 1$.

[14]Empirically, productivity in the non-tradable sectors is much less volatile than in the tradable sectors. For example, see Stockman and Tesar (1995).

[15]Throughout the paper I focus on equilibria in which production always occurs in both sectors. Given the functional forms used in the numerical simulations, it is indeed optimal for firms to always operate in both sectors.

[16]I follow the convention of netting interest payments out of the net foreign asset position.

Market clearing for the non-tradable consumption good requires that in every country consumption is equal to production, that is $C_{i,t}^{N} = Y_{i,t}^{N}$. Moreover, equilibrium on the labor market implies that in every country the labor supplied by the households is equal to the labor demanded by firms, $L_{i,t} = L_{i,t}^{T} + L_{i,t}^{N}$.

These two market clearing conditions, in conjunction with the budget constraint of the household and the expressions for firms' profits, give the market clearing condition for the tradable consumption good in country i

$$C_{i,t}^{T} = Y_{i,t}^{T} + B_{i,t} - \frac{B_{i,t+1}}{R_t}.$$

This expression can be rearranged to obtain the law of motion for the stock of net foreign assets owned by country i, i.e. the current account

$$NFA_{i,t} - NFA_{i,t-1} = CA_{i,t} = Y_{i,t}^{T} - C_{i,t}^{T} + B_{i,t}\left(1 - \frac{1}{R_{t-1}}\right),$$

As usual, the current account is given by the sum of net exports, $Y_{i,t}^{T} - C_{i,t}^{T}$, and net interest payments on the stock of net foreign assets owned by the country at the start of the period, $B_{i,t}(1 - 1/R_{t-1})$.

Finally, in every period the world consumption of the tradable good has to be equal to the world production, $\int_0^1 C_{i,t}^{T}\,\mathrm{d}i = \int_0^1 Y_{i,t}^{T}\,\mathrm{d}i$. This implies that bonds are in zero net supply at the world level, $\int_0^1 B_{i,t+1}\,\mathrm{d}i = 0$.

1.2.1 Equilibrium

Given a sequence of prices $\{R_t, w_{i,t}, p_{i,t}^{N}\}_{t=0}^{\infty}$, define the optimal decisions of the household as $C^T\left(B, A^T\right)$, $C^N\left(B, A^T\right)$ and $L\left(B, A^T\right)$ and the optimal labor demand decisions as $L^T\left(A^T\right)$ and L^N, in a country with bond holdings $B_{it} = B$ and productivity $A_{i,t}^{T} = A^T$. Notice that these decision rules fully determine the transition for bond holdings.

Define $\Psi_t\left(B, A^T\right)$ as the joint distribution of bond holdings and current productivity across countries. The optimal decision rules for bond holdings together with the process for productivity yield a transition probability for the country-specific states $\left(B, A^T\right)$. This transition probability can be used to compute the next period distribution $\Psi_{t+1}\left(B, A^T\right)$, given the current distribution $\Psi_t\left(B, A^T\right)$. We can now define an equilibrium.

Definition 1 *An equilibrium is a sequence of prices* $\{R_t, w_{i,t}, p_{i,t}^{N}\}_{t=0}^{\infty}$*, a sequence of policy rules* $C^T\left(B, A^T\right)$*,* $C^N\left(B, A^T\right)$*,* $L\left(B, A^T\right)$*,* $L^T\left(A^T\right)$*,* L^N *and a sequence of joint distributions for*

Table 1: Parameters

	Value	Source/Target
Risk aversion	$\gamma = 4$	Standard value
Discount factor	$\beta = 0.9756$	$R = 1.025$
Frisch elasticity of labor supply	$1/\psi = 1$	Kimball and Shapiro (2008)
Labor share in tradable sector	$\alpha_T = 0.65$	Standard value
Labor share in non-tradable sector	$\alpha_N = 0.65$	Standard value
Share of tradables in consumption	$\omega = 0.5$	Stockman and Tesar (1995)
TFP process	$\sigma_{A^T} = 0.0194,\ \rho = 0.84$	Benigno and Thoenissen (2008)
Initial borrowing limit	$\kappa = 0.9$	World debt/GDP = 20%

Table 1.1: Parameters - yearly calibration

bond holdings and productivity $\Psi_t\left(B, A^T\right)$*, such that given the initial distribution* $\Psi_0\left(B, A^T\right)$ *in every period* t

- $C^T\left(B, A^T\right), C^N\left(B, A^T\right), L\left(B, A^T\right), L^T\left(A^T\right), L^N$ *are optimal given* $\{R_t, w_{it}, p^N_{it}\}_{t=0}^{\infty}$
- $\Psi_t\left(B, A^T\right)$ *is consistent with the decision rules*
- *Markets for consumption and labor clear in every country* i

$$C^N_{i,t} = Y^N_{i,t}$$

$$C^T_{i,t} = Y^T_{i,t} + B_{i,t} - \frac{B_{i,t+1}}{R_t}$$

$$L_{i,t} = L^T_{i,t} + L^N_{i,t}.$$

- *The market for bonds clears at the world level*

$$\int_0^1 B_{i,t+1}\,\mathrm{d}i = 0.$$

1.2.2 Parameters

The model cannot be solved analytically and I analyze its properties using numerical simulations. I employ a global solution method in order to deal with the nonlinearities involved by a large shock such as the deleveraging shock studied in the next section. Appendix A.1 describes the numerical solution method.

I assume a utility function separable in consumption and labor and a Cobb-Douglas aggregator for consumption

$$U\left(C^T, C^N, L\right) = \frac{C^{1-\gamma}}{1-\gamma} - \frac{L^{1+\psi}}{1+\psi}$$

$$C = \left(C^T\right)^{\omega} \left(C^N\right)^{1-\omega}.$$

A period in the model corresponds to one year.[17] The risk aversion is set to $\gamma = 4$, a standard value. The discount factor is set to $\beta = 0.9756$ in order to match a real interest rate in the initial steady state of 2.5 percent. This is meant to capture the low interest rate environment characterizing the US and the Euro area in the years preceding the start of the 2007 crisis. The Frisch elasticity of labor supply $1/\psi$ is set equal to 1, in line with evidence by Kimball and Shapiro (2008). The labor share in production in both sectors is set to $\alpha_T = \alpha_N = 0.65$, a value in the range of those commonly used in the literature. The share of tradable goods in consumption is set to $\omega = 0.5$, in accordance with the estimates of Stockman and Tesar (1995).

Productivity in the tradable sector A^T follows a normal AR(1) process $A^T_{i,t} = \rho A^T_{i,t-1} + \epsilon_{i,t}$. This process is approximated with the quadrature procedure of Tauchen and Hussey (1991) using 7 nodes.[18] The first order autocorrelation ρ and the standard deviation of the TFP process σ_{A^T} are set respectively to 0.84 and to 0.0194, following the estimates of Gianluca Benigno and Thoenissen (2008).[19]

The borrowing limit in the initial steady state is set to $\kappa = 0.9$ to match a world gross debt-to-GDP ratio of 20 percent. This target corresponds to the sum of the net external debt position of the Euro area debtor countries in 2007, expressed as a fraction of the Euro area GDP.[20]

1.2.3 Steady state

Before proceeding with the analysis of the deleveraging episode, this section briefly describes the steady state policy functions and the stationary distribution of the net foreign asset-to-GDP ratio.

Figure 4.2 displays the optimal choices for the current account and labor as a function of B_t, the stock of wealth at the start of the period, for an economy hit by a good productivity shock, solid lines, and by a bad productivity shock, dashed lines.[21] The left panel shows the current account. As it is standard in models in which the current account is used to smooth consumption over time, a country runs a current account surplus and accumulates foreign assets when

[17] Later, in section 1.6, I will parametrize the model at quarterly frequency to perform policy experiments.

[18] I use the weighting function proposed by Flodén (2008), which delivers a better approximation to high-persistence AR(1) processes than the weighting function originally suggested by Tauchen and Hussey (1991).

[19] These values are in the range of those commonly used in the literature on international risk sharing. See, for example, Corsetti et al. (2008).

[20] The Euro area countries that have a negative net foreign asset position in 2007 are Austria, Finland, Greece, Ireland, Italy, Netherlands, Portugal and Spain. Data are from Lane and Milesi-Ferretti (2007).

[21] Precisely, the high (low) TFP lines refer to economies hit by a productivity shock about two standard deviations above (below) the mean.

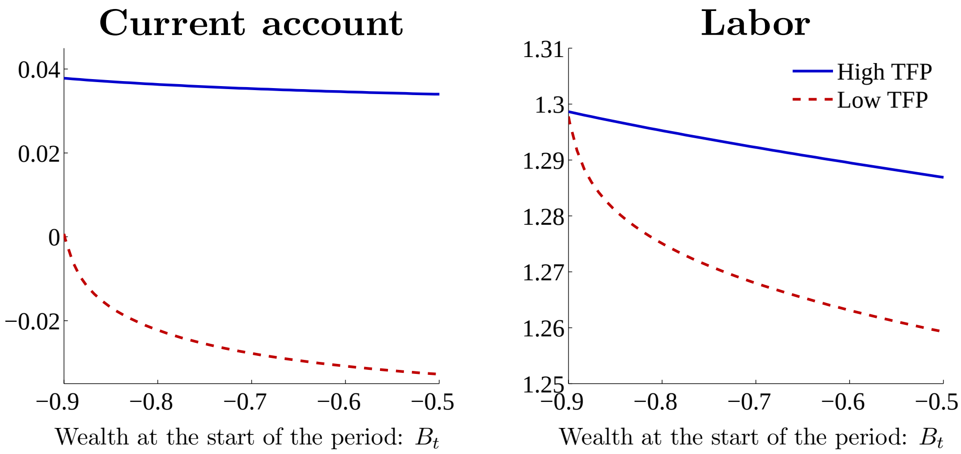


Figure 1.1: Policy functions in steady state.

productivity is high, while it runs a current account deficit and reduces its stock of foreign assets when productivity is low. This allows households to mitigate the impact of temporary productivity shocks on consumption. However, the borrowing limit interferes with consumption smoothing. To see this point, notice that the decrease in net foreign assets following a bad productivity shock gets smaller as the start-of-period wealth falls. This happens because households, as they approach the constraint, reduce the accumulation of debt in response to bad productivity shocks for fear of ending up against the borrowing limit.[22]

The right panel illustrates the optimal choice of labor. In general, equilibrium labor is higher when productivity is high. Intuitively, when productivity is higher firms are able to pay higher wages and this induces households to supply more labor. However, as the start-of-period wealth decreases the distance between the two lines tends to fade away. In fact, households that start the period with a high stock of debt are willing to work more for a given wage, since the borrowing limit interferes with their ability to further accumulate debt in order to smooth the impact of productivity shocks on consumption.

Figure 1.2 shows the steady state distribution of the net foreign asset-to-GDP ratio. The distribution is truncated and skewed toward the left. Both of these features are due to the borrowing limit. In fact, while there is no limit to the positive stock of net foreign assets that a country can accumulate, the borrowing constraint imposes a bound on the negative net foreign asset position that a country can reach. In particular, the largest net foreign debt position-to-GDP ratio that a country can reach in the initial steady state is close to 65 percent.

[22]Indeed, when the borrowing limit is hit the country can no longer use the current account to smooth consumption and the change in net foreign assets following a bad productivity shock is equal to zero.

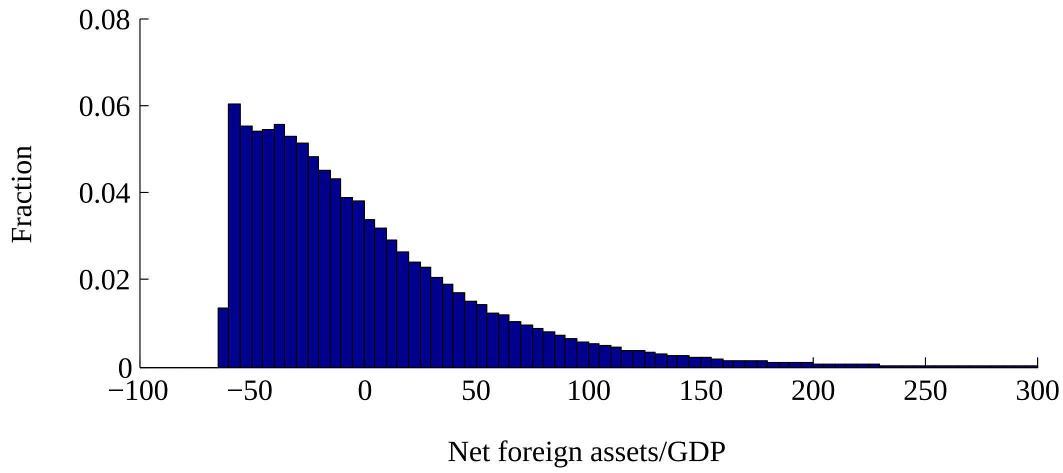


Figure 1.2: Steady state distribution of net foreign assets/GDP.

1.3 Deleveraging with flexible wages

This section analyzes the transition during a deleveraging episode induced by a tightening of the borrowing limit. I consider a world economy that starts in steady state with $\kappa = 0.9$. In period $t = 0$ there is an unexpected and permanent fall in the borrowing limit which goes to $\kappa' = 0.675$, so that the new borrowing limit is equal to 75 percent of the initial one.[23] This generates a reduction in the steady state world gross debt-to-GDP ratio of about 5 percent.[24]

Figure 4.4 displays the transitional dynamics of the world economy following the shock to the borrowing limit. The figure shows the path for the exogenous borrowing limit and the response of the world gross debt-to-GDP ratio, the world interest rate and the world production of tradable and non-tradable goods.

The tightening of the borrowing limit triggers a decrease in the foreign debt position of highly indebted countries. At the same time, surplus countries are forced to reduce their positive net foreign asset position, which is the counterpart of foreign debt in indebted countries. The result is a progressive compression of the net foreign asset distribution. As showed by the the top right panel of figure 4.4, on impact the world debt-to-GDP ratio falls by almost 1 percent. Afterward, the world slowly transits toward the new steady state debt distribution, in which the world debt-to-GDP ratio is equal to 15 percent.

The world interest rate drops sharply after the shock and overshoots its value in the new

[23] For simplicity, I consider an exogenous drop in the borrowing limit. See Perri and Quadrini (2011) for a model in which changes in the borrowing limit are the result of self-fulfilling expectations.

[24] This number is not an unreasonable estimate of the adjustment that the Eurozone may undergo during the next years. For instance, the deviation from a linear trend, computed using data for the period 1980 − 2007, of the net external debt position of the Euro area debtor countries in 2007, expressed as a fraction of the Euro area GDP, is close to 5 percentage points. This suggests that the ratio of the net external debt position of Eurozone debtor countries to Euro area GDP should fall by 5 percent in order to go back to trend.

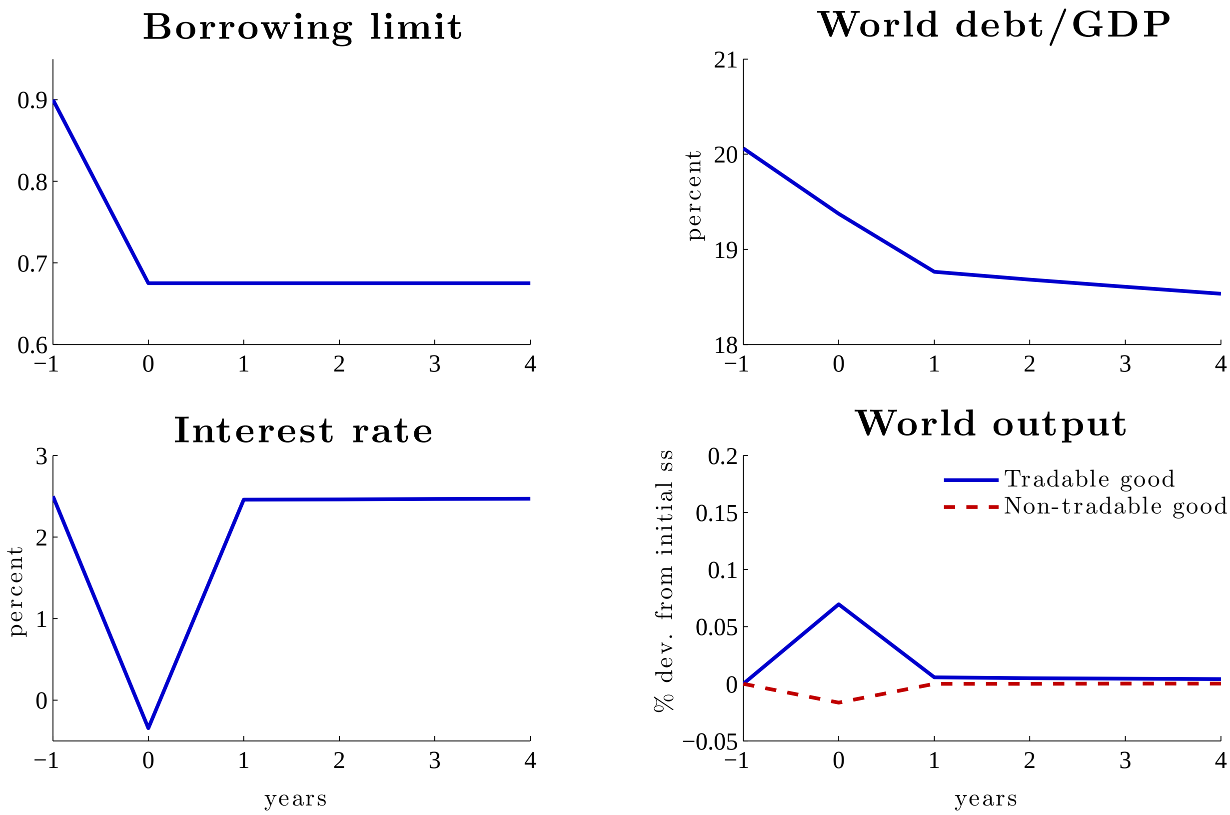


Figure 1.3: Response to deleveraging shock - flexible wages.

steady state. This result is reminiscent of the findings of Guerrieri and Lorenzoni (2011) in closed economies. The fall in the interest rate signals an increase in the desire to save, or equivalently a fall in the desire to consume. This is due to two distinct effects. First, countries that start with a high level of foreign debt, more precisely countries that start with a stock of bonds $-\kappa \leq B_{i,0} < -\kappa'$, are forced to reduce their foreign debt position. This corresponds to a forced increase in savings that depresses the demand for consumption in high-debt countries.[25] Second, even the countries that are not directly affected by the tightening of the constraint experience an increase in the propensity to save. In fact, unconstrained countries want to accumulate precautionary savings to self-insure against the risk of hitting the now-tighter borrowing limit in the future, following a sequence of bad realizations of the productivity shock. Both these effects imply an increase in the propensity to save at the world level. In order to reach equilibrium on the bond market the interest rate has to fall, so as to induce the unconstrained countries to consume more and reduce their demand for saving instruments. This explains the fall in the world interest rate.

Concerning output, there is not much action going on at the world level. On impact, the world output of the tradable good increases by little more than 0.05 percentage points above

[25] This effect is also present in Eggertsson and Krugman (2012) and in Pierpaolo Benigno and Romei (2012).

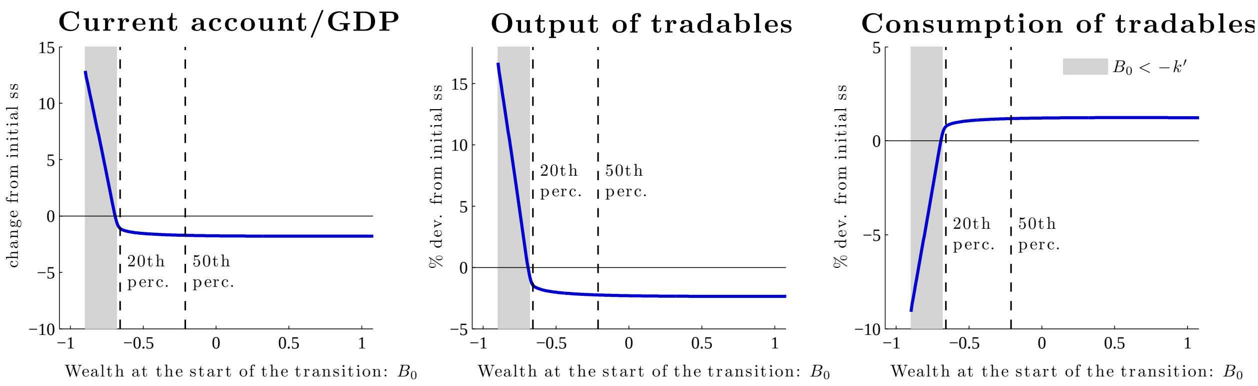


Figure 1.4: Impact responses to deleveraging shock across the NFA distribution - flexible wages.

its value in the initial steady state, while there is an almost imperceptible fall in the world output of non-tradable goods. However, the lack of aggregate movements in world output masks important country-level composition effects, to which we turn next.

Figure 1.4 illustrates how the response to the deleveraging shock in period $t = 0$ varies across the initial distribution of net foreign assets.[26] The figure shows the response, that is the change with respect to the initial steady state value, of the current account-to-GDP ratio, the output of the traded good and the consumption of the traded good. To ease interpretation the figure also displays the position of the 20^{th} and the 50^{th} percentile of the bond distribution.[27] The shaded areas denote the countries that start the transition with $B_{i,0} < -\kappa'$ and hence are forced to improve their bond position by the tightening of the constraint. They represent roughly 20 percent of the countries in the world.

The figure indicates that the sign of the response to the deleveraging shock essentially depends on whether the country is forced to reduce its stock of debt by the tightening of the constraint or not. This happens because constrained countries are directly affected by the tightening of the constraint, while the response of the rest of the world is mainly dictated by the fall in the interest rate.

The left panel of figure 1.4 shows that the tightening of the constraint forces high-debt countries to improve their foreign asset position by increasing their current account balances. To understand the macroeconomic implications, it is useful to go back to the equation describing the current account

$$CA_{i,t} = Y_{i,t}^T - C_{i,t}^T + B_{i,t}\left(1 - \frac{1}{R_{t-1}}\right).$$

[26] To construct this figure, I first computed the response in period $t = 0$ to the deleveraging shock for every possible realization of the state variables $\{A_0^T, B_0\}$. Then I computed an aggregate response as a function of B_0 by taking the weighted average of the single country responses. The weights are given by the fraction of countries having a given realization of A_0^T conditional on B_0.

[27] To improve readability, the figure is truncated at the 90^{th} percentile of the bond distribution.

This expression makes clear that an economy can improve its current account by increasing its output of the tradable good, by decreasing the consumption of the tradable good or through a combination of both. The middle and right panels of figure 1.4 show that constrained countries adjust both through the output and the consumption margin.[28] Hence, in the absence of nominal rigidities, a decrease in capital inflows due to a tightening of the borrowing constraint has an expansionary impact on the production of the traded good in high-debt countries.[29] Later, we will see that the combination of nominal wage rigidities and fixed exchange rates overturns this counterfactual implication of the model.

The countries that are not directly affected by the tightening of the constraint follow an opposite adjustment pattern. The sharp decrease in the world interest rate induces the unconstrained countries to reduce their stock of foreign assets by running current account deficits. The deficits in the current account are achieved trough a combination of lower production of the tradable good and higher consumption. Hence, following a deleveraging shock the model without nominal rigidities displays a shift of production of tradable goods from wealthy countries toward high-debt countries.[30]

The response of output to the shock to the borrowing limit is associated with changes in real wages. To see this point, it is useful to rearrange the optimality condition for firms in the tradable sector to obtain

$$L_{i,t}^T = \left(\frac{\alpha_T A_{i,t}}{w_{i,t}}\right)^{\frac{1}{1-\alpha_T}}.$$

This expression implies that, given values for the parameters α_T and $A_{i,t}^T$, an increase in employment in the tradable sector in country i has to come with a decrease in the real wage $w_{i,t}$.[31]

Following the deleveraging shock, households in high-debt countries increase labor supply to boost labor income and to repay debts without cutting consumption too severely. The increase in labor supply translates into a fall in real wages, which represent the cost of labor in terms of the tradable consumption good. In turn, the fall in real wages makes more profitable for firms

[28] Quantitatively, the increase in production of the tradable good dominates the fall in consumption.

[29] See Chari et al. (2005) for a discussion of this feature of the frictionless neoclassical model.

[30] The figure also highlights the importance of nonlinearities. In fact, while the response of unconstrained countries does not depend much on their initial stock of assets, the initial debt position has a strong impact on the response of constrained countries.

[31] More precisely, given that the production function is Cobb-Douglas we can write the elasticity of real wages with respect to employment in the tradable sector as

$$\frac{\partial w_{i,t}}{\partial L_{i,t}^T}\frac{L_{i,t}^T}{w_{i,t}} = \alpha_T - 1.$$

Given that $\alpha_T = 0.65$, a one percent increase in employment in the tradable sector entails a 0.35 percent decrease in the real wage.

in the tradable sector to employ labor. This effect leads to an increase in employment and output in the tradable sector in high-debt economies. Hence, the fall in real wages in high-debt countries plays a key role in shaping the adjustment to the deleveraging shock.

The empirical evidence reviewed in the introduction suggests that nominal wages adjust sluggishly to shocks. In particular, a recurrent pattern in severe recessions is that nominal wages do not fall much, even in the face of large rises in unemployment. It is then difficult to imagine that the adjustment in real wages required by the deleveraging shock could come from an adjustment in nominal wages.

In a world in which exchange rates are allowed to float, nominal exchange rate flexibility may substitute for the lack of nominal wage flexibility. The intuition can be gained using a simple partial equilibrium approach. Suppose that there is an international currency in which the tradable good is priced. Let P^T denote the price of the tradable good expressed in units of the international currency. Given the absence of trade frictions, the law of one price holds and the price of the tradable good in terms of the domestic currency is given by

$$P^T_{i,t} = S_{i,t} P^T_t,$$

where $S_{i,t}$ denotes the nominal exchange rate of country i's against the key currency, i.e. the units of country i currency needed to buy one unit of the key currency.

The real wage, that is the nominal wage divided by the price of the tradable good, is now given by

$$w_{i,t} = \frac{W_{i,t}}{P^T_{i,t}} = \frac{W_{i,t}}{S_{i,t} P^T_t},$$

where $W_{i,t}$ denotes the nominal wage in country i. This expression shows that, given P^T_t and $W_{i,t}$, a reduction in the real wage can come through a nominal exchange rate depreciation against the key currency, that is an increase in S_i. It follows that to mimic the response to the deleveraging shock under flexible wages, despite the presence of nominal wage rigidities, high-debt countries should let their exchange rate depreciate against the key currency, while low-debt countries should let their nominal exchange rate appreciate. Indeed, from the point of view of a single country replicating the flexible wage equilibrium through movements in the nominal exchange rate corresponds to the optimal policy.

Proposition 1 *From the perspective of a single country the flexible wage equilibrium attains the first best.*

Proof. See appendix A.3. ■

Looking at the current events affecting the Euro area, many commentators have argued that the combination of rigidities in wage setting and fixed exchange rates has contributed to the severity of the crisis in deleveraging countries.[32] A point that is often overlooked is that in a financially integrated world all the countries are tied together by the world interest rate, and that the exchange rate regime can have an important role in shaping the behavior of the world interest rate during an episode of global debt deleveraging. The next section introduces a model of a monetary union and shows that important insights can be gained from adopting a general equilibrium approach and taking into account the interactions across countries inside a monetary union.

1.4 Deleveraging in a monetary union with nominal wage rigidities

This section focuses on the impact of deleveraging in a monetary union with nominal wage rigidities. To consider the case of a monetary union we have to modify the model introduced in the previous section in a few dimensions. In particular, the model presented in this section explicitly considers nominal, in addition to real, variables.

In a monetary union there is a single currency that is used for transactions in all the participating countries. For simplicity, I will consider a world in which every country belongs to the monetary union. From now on, I will then use the words monetary union and world interchangeably.

The household's budget constraint in units of currency is

$$P_t^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N + \frac{B_{i,t+1}}{R_t^N} = W_{i,t} L_{i,t} + B_{i,t} + \Pi_{i,t}^T + \Pi_{i,t}^N.$$

In this expression, P^T denotes the price of a unit of tradable consumption good in terms of currency. Since the tradable good is homogenous and there are no trade frictions, its price is common across all the countries. P_i^N is the nominal price of a unit of non-tradable consumption good, and it is country specific. Realistically, bonds are denominated in units of currency and R^N denotes the gross nominal interest rate. W_i is the nominal wage in country i. Finally, Π_i^T and Π_i^N are now the profits of the firms expressed in nominal terms.

For consistency with the model outlined in the previous section, I assume that the borrowing constraint limits the amount of tradable goods that a household can commit to repay during

[32] For example, see Feldstein (2010) and Krugman (2010).

the following period. Formally, for every household the end-of-period bond position has to satisfy

$$\frac{B_{i,t+1}}{P^T_{t+1}} \geq -\kappa.$$

There is a single central bank that uses the nominal interest rate R^N as its policy instrument. I start by considering the case of a central bank that targets inflation in the tradable sector. This policy captures in a simple way the objective of stabilizing prices across all the countries in the union, usually characterizing central banks in monetary unions. Moreover, this policy allows for a clean comparison with the flexible wage economy described in the previous sections. In fact, as long as the central bank avoids unexpected movements in the price of the tradable good, nominal bonds and bonds denominated in units of tradables are perfect substitutes.[33] To simplify the exposition, I start by focusing on a central bank that strictly targets inflation in the traded sector, and hence sets $P^T_t = P^T_{t-1}$ in every period t.

To capture the sluggish adjustment of nominal wages typical of deep recessions, while keeping the intuition underlying the main result of the paper transparent, I start by considering a very simple form of nominal wage rigidities. I assume that wages are completely rigid during the first period in which the unexpected shock to the borrowing limit hits the economy, period $t = 0$, while they become fully flexible thereafter.[34] Once wages are set, workers stand ready to supply the labor demanded by firms. Moreover, I assume that nominal wages in $t = 0$ are set after the uncertainty about the idiosyncratic productivity shocks is resolved, but before the shock to the borrowing limit hits the economy. These assumptions about wage setting isolate the role of wage rigidities in shaping the adjustment to the deleveraging shock, abstracting from the impact of wage rigidities on normal business cycle fluctuations, captured by the idiosyncratic productivity shocks.

More precisely, the timing during period $t = 0$ is the following:

1. At the start of the period countries are hit by their idiosyncratic productivity shocks.

[33] To see this point, consider that the Euler equation for bonds denominated in units of currency is

$$\frac{U_{C^T_{i,t}}}{R^N_t P^T_t} = \beta E_t \left[\frac{1}{P^T_{t+1}} \left(U_{C^T_{i,t+1}} + \mu_{i,t} \right) \right],$$

while the Euler equation for bonds denominated in units of tradables is

$$\frac{U_{C^T_{i,t}}}{R_t} = \beta E_t \left[U_{C^T_{i,t+1}} \right] + \mu_{i,t}.$$

In absence of unexpected movements in the price of the tradable good we can write $R_t = R^N_t P^T_t / P^T_{t+1}$ and verify that the two Euler equations are identical and the two assets are perfect substitutes.

[34] Section 1.6 introduces a model in which wage rigidities last longer than a single period.

2. Nominal wages are set so that the pattern of production characterizing the flexible wage equilibrium is replicated as long as the central bank sticks to the inflation target, that is if $P_0^T = E_{-1}\left[P_0^T\right]$.

3. The shock to the borrowing limit is revealed to agents.

Afterward, in periods $t > 0$, wages become again fully flexible.

To understand the implications of this form of nominal wage rigidities, denote by $\hat{L}_{i,0}^T$ the notional equilibrium labor in the traded sector that would prevail in country i and period $t = 0$ in the absence of the shock to the borrowing limit, that is in the initial steady state. Wages are then set according to

$$W_{i,0} = \alpha_T A_{i,0}^T E_{-1}\left[P_0^T\right]\left(\hat{L}_{i,0}^T\right)^{\alpha_T - 1}.$$

Once wages are set, equilibrium labor is determined by firms' labor demand. Combining the expression for wages and firms' labor demand gives

$$L_{i,0}^T = \left(\frac{P_0^T}{E_{-1}\left[P_0^T\right]}\right)^{\frac{1}{1-\alpha_T}} \hat{L}_{i,0}^T.$$

Hence, the assumptions about wage setting imply that on impact the shock to the borrowing limit affects equilibrium labor in the tradable sector only if it induces unexpected movements in the nominal price of the traded good.

Figure 1.5 shows how the monetary union with nominal wage rigidities responds to a tightening of the borrowing limit. As in the previous section, in period $t = 0$ the union is subject to an unexpected permanent drop in the borrowing limit, such that the final borrowing limit is equal to 75 percent of the initial one. This triggers a process of deleveraging that leads to a progressive reduction in the world debt-to-GDP ratio, as shown by the top-right panel of figure 1.5.

The bottom-left panel of the figure shows the response of the interest rate.[35] For ease of comparison, the figure shows both the path of the interest rate in the economy with flexible wages, the solid line, as well as the response of the interest rate in the monetary union with nominal wage rigidities, the dashed line. As it happened with flexible wages, deleveraging triggers a fall in the world interest rate. However, quantitatively the fall in the interest rate is much larger in a monetary union. In fact, in the model with flexible wages the interest rate

[35] Notice that, since inflation in the tradable sector is zero, the interest rate displayed in figure 1.5 can be interpreted both as the nominal rate or as the real rate, defined as the nominal rate deflated by inflation in the tradable sector.

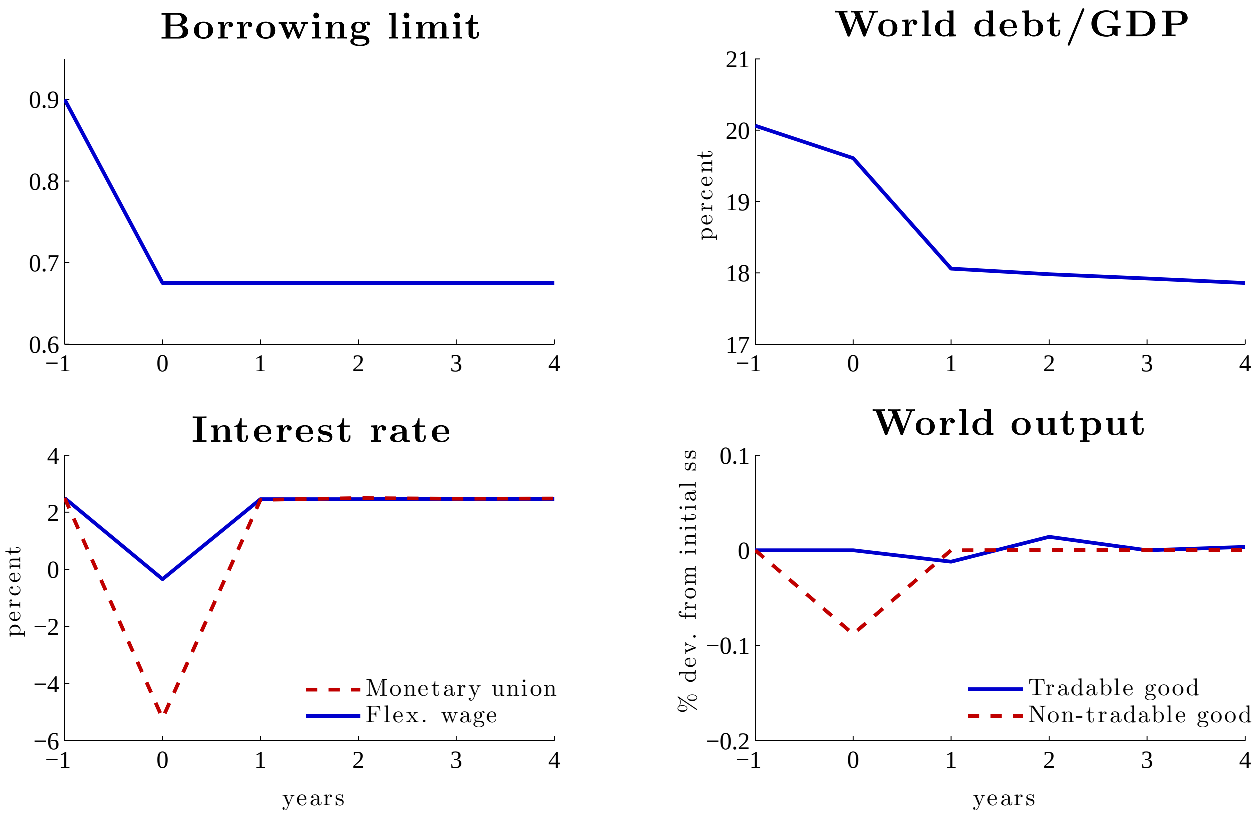


Figure 1.5: Response to deleveraging shock - monetary union with nominal wage rigidities.

falls on impact by around 2.5 percentage points. Instead, in a monetary union with nominal wage rigidities the fall in the interest rate is three times larger, since it goes from 2.5 percent to around −5 percent. Hence, in a monetary union the combination of nominal wage rigidities and fixed exchange rates amplifies the fall in the interest rate following a deleveraging shock.

To gain intuition about this effect, it is useful to look at the behavior of high-debt borrowing-constrained countries. Figure 1.6 displays the impact responses of the current account-to-GDP ratio, the output and consumption of the traded good and the output of the non-traded good across the initial distribution of net foreign assets. As it happened in the previous section, high-debt countries are forced to improve their current account by the tightening of the constraint. However, in a monetary union with nominal wage rigidities improving the current account through an increase in the production of the traded good is no longer an option, In fact, given that nominal wages do not adjust, this would require a nominal exchange rate depreciation, which is ruled out by the participation in the monetary union. This is illustrated by the top-right panel of figure 1.6, which shows that the combination of nominal wage rigidities and fixed exchange rates shuts down the response of the output of tradable goods to the deleveraging shock.[36]

[36]Precisely, this happens because the central bank hits the inflation target, so $P_0^T = E_{-1}\left[P_0^T\right]$ and the pattern of production during period $t = 0$ is the same as the one in the initial steady state.

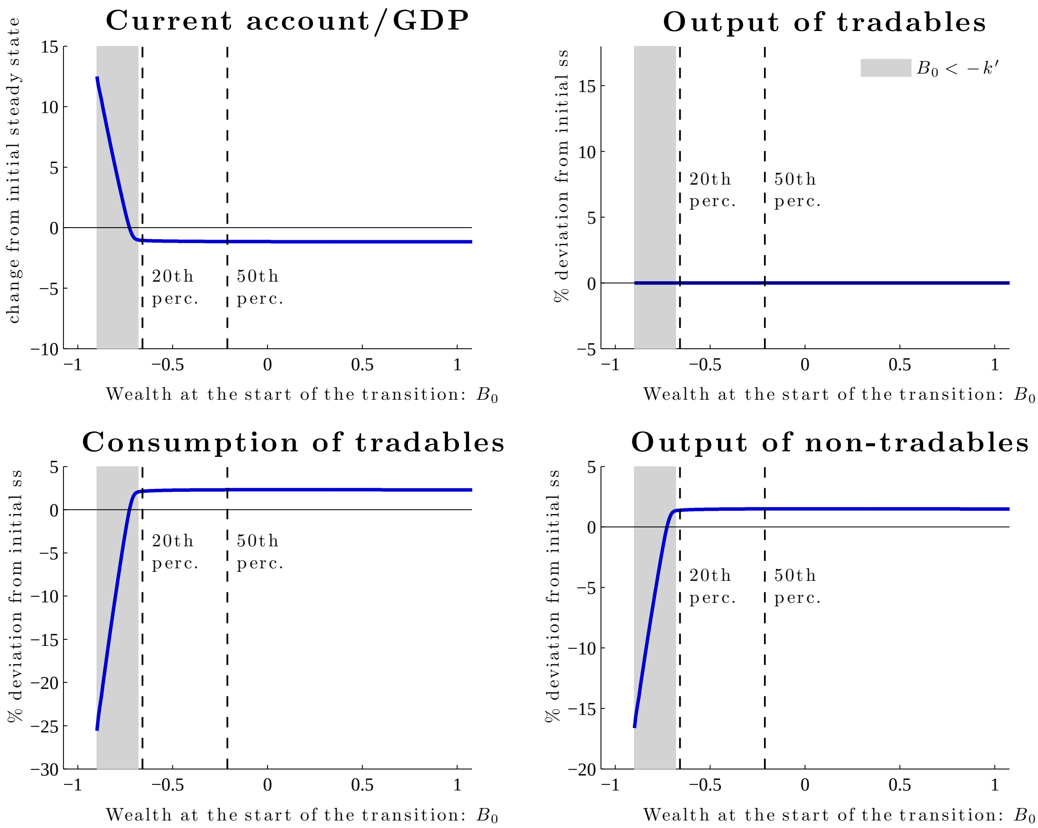


Figure 1.6: Impact responses to deleveraging shock across the NFA distribution - monetary union with nominal wage rigidities.

It follows that the improvement in the current account in high-debt countries has to come solely through a cut in the consumption of the tradable good. In fact, the bottom-left panel of figure 1.6 shows that high-debt economies adjust through deep cuts in the consumption of the traded good. The fact that constrained countries have to adjust exclusively through a cut in consumption implies a bigger fall in the demand for consumption compared to the world with flexible wages. In turn, the interest rate has to fall by more to induce unconstrained countries to increase consumption and pick up the slack left by constrained economies. Hence, the chances that a deleveraging shock pushes the world into a liquidity trap, that is a situation in which the nominal interest rate hits the zero lower bound, are higher if countries are part of a monetary union.

Moreover, while the deleveraging shock had an expansionary effect on output in high-debt countries in the absence of nominal rigidities, this is no longer the case when wages are rigid and the nominal exchange rates cannot adjust. Indeed, on impact the deleveraging shock generates a drop in the production of non-traded goods in high-debt countries, as highlighted by the bottom-right panel of figure 1.6. To understand why this happens, consider that labor demand from firms in the non-traded sector is given by

$$L_{i,t}^N = \left(\alpha_N A^N \frac{P_{i,t}^N}{W_{i,t}}\right)^{\frac{1}{1-\alpha_N}},$$

while households' optimality conditions give an expression for the nominal price of the non-tradable good

$$P_{i,t}^N = \frac{1-\omega}{\omega}\frac{C_{i,t}^T}{C_{i,t}^N}P_t^T. \tag{1.8}$$

The drop in the consumption of the traded good experienced by high-debt countries generates a real exchange rate depreciation, that is a fall in the relative price of non-tradables. Since the central bank strictly targets inflation in the traded sector and nominal exchange rates are fixed, the real exchange rate depreciation translates into a fall in the nominal price of the non-tradable good. Given the fixed nominal wages, this implies that employing labor in the non-traded sector becomes less profitable and firms in high-debt countries are pushed to reduce their labor demand and lower the production of the non-traded good.

The interaction between nominal wage rigidities and fixed exchange rates generates a recession in the countries that end up being financially constrained following the deleveraging shock. The next section shows how the recession can spread to unconstrained countries if the deleveraging shock pushes the union into a liquidity trap.

1.5 The role of the zero lower bound

The previous section considered a central bank freely able to set the nominal interest rate in order to hit the inflation target. In reality, nominal interest rates cannot fall below zero. This section considers explicitly the role of the zero lower bound on the nominal interest rate and shows that deleveraging can generate a union-wide recession if the zero lower bound on the interest rate becomes binding.

Define $\hat{R}_t^N$ as the nominal interest rate consistent with the central bank's inflation target. In this section the focus is on a central bank that sets the interest rate according to $R_t^N = \max\left(\hat{R}_t^N, 1\right)$. This rule implies that the central bank sticks to the inflation target as long as this does not imply a negative nominal rate, otherwise it sets the nominal interest rate to zero.[37]

From the analysis in the previous section we know that in $t = 0$, the first period in which the borrowing limit gets tighter, the nominal interest rate consistent with zero inflation in the

[37] Remember that R^N denotes the gross nominal interest rate.

price of the tradable good is negative. Hence, the central bank sets $R_0^N = 1$. However, at this interest rate the market for consumption of the traded good does not clear, since demand for consumption is too weak to absorb the whole production of tradables. Excess supply induces firms to cut the nominal price of the traded good until equilibrium on the traded good market is restored. We then have that the unexpected deleveraging shock triggers an unexpected fall in the nominal price of the traded good, so that $P_0^T < E_{-1}\left[P_0^T\right]$.

The unexpected fall in the nominal price of the traded good has two distinct effects. On the one hand, the fall in the price of the traded good reduces the profitability of employing labor in the traded sector. This leads to a fall in the world production of tradables. Indeed, this is the mechanism through which deflation in the traded sector restores equality between the demand and the supply of the traded good.

On the other hand, since bonds are denominated in units of currency, the fall in the nominal price of the traded good increases the debt burden of debtor countries in terms of the tradable consumption good, giving rise to an effect akin to Fisher's debt deflation. This unexpected wealth redistribution from debtor to creditor countries further depresses aggregate demand inside the monetary union. The result is that once the zero lower bound on the nominal interest rate is taken into account, deleveraging can push the whole monetary union into a recession.

Figure 2.4 illustrates this result by plotting the response of the union to the deleveraging shock in the case in which the central bank is constrained by the zero bound on the interest rate. The tightening of the borrowing limit has a depressive effect on the interest rate, which on impact hits the zero lower bound. This induces a fall in the nominal price of the tradable good. In turn, the combination of nominal wage rigidities and deflation reduces the profitability of employing labor in the tradable sector. This explains the union-wide drop in the output of traded goods, which falls by almost 3 percentage points below its value in the initial steady state. Moreover, deflation in the tradable sector puts downward pressure on the nominal prices of the non-traded goods, as shown by equation (4.6). Deflation in the price of the non-traded good pushes firms in the non-traded sector to cut employment and production. Because of this effect also the aggregate production of non-traded goods falls.

To see how the recession affects differently the countries depending on their initial debt positions, it is useful to look at figure 1.8. Both high-debt and low-debt countries experience a similar fall in the output of the tradable good. This happens because the demand for the traded good, and so its price, depends on the demand from all the countries in the union.

The consumption of the traded good exhibits a different pattern. In fact, the countries

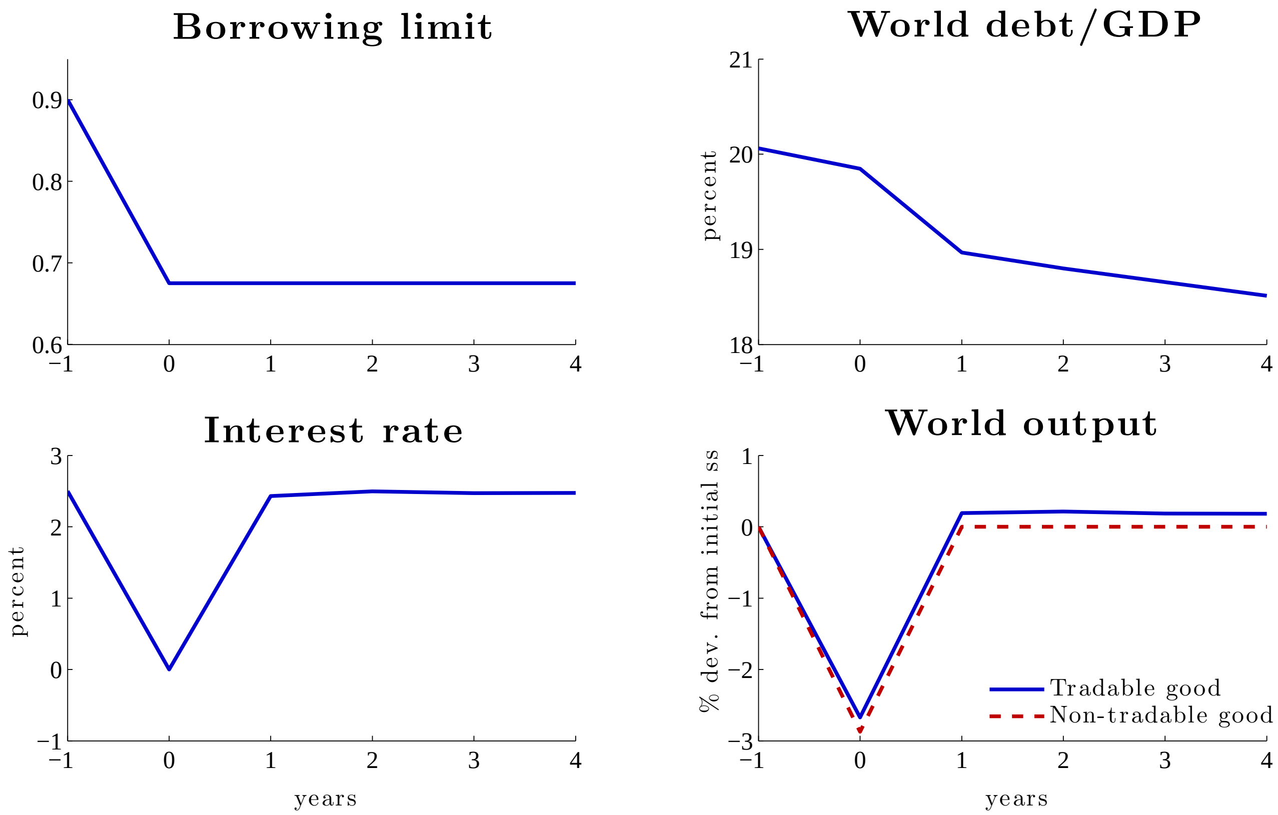


Figure 1.7: Response to deleveraging shock - liquidity trap in a monetary union.

featuring a high initial debt experience deep falls in the consumption of the traded good, much larger than the one experienced in the absence of the zero lower bound. This happens because constrained countries have a high propensity to consume out of current income. Hence, the fall in the production of tradables directly translates into a fall in consumption. In addition, deflation increases the initial debt position of debtor countries, the Fisher's debt deflation effect, and this further depresses their consumption of tradable goods.

Concerning the production of non-tradables, figure 1.8 shows that high-debt countries exhibit deep falls in employment and output in the non-traded sector. As before, this happens because the fall in the consumption of the traded good generates a real exchange rate depreciation. Since the nominal exchange rate cannot adjust, the real depreciation results in a fall in the nominal price of non-tradables. Given the fixed wages, deflation in the non-traded sector induces a fall in employment. The result is that the whole union enters a recession, but the crisis hits particularly hard the non-traded sectors in high-debt countries.

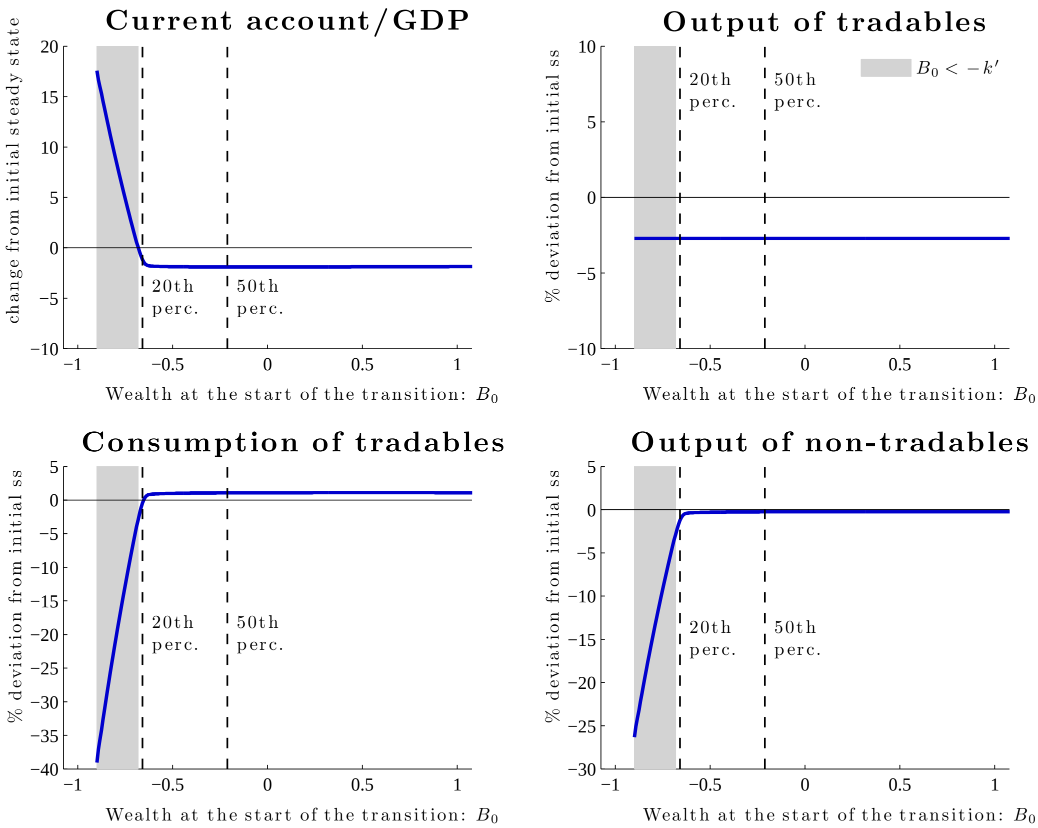


Figure 1.8: Impact responses to deleveraging shock across the NFA distribution - liquidity trap in a monetary union.

1.6 Policy implications

1.6.1 A model with multi-period wage rigidities

Which policy interventions can mitigate the recession associated with deleveraging inside a monetary union? I address this question using the model as a laboratory to perform policy experiments. This section considers a version of the model parameterized at quarterly frequency in which the adjustment to the deleveraging shock lasts more than one period and dynamic effects take the center stage. Indeed, whenever a liquidity trap lasts more than one period strong amplification effects are set in motion, so a quarterly parametrization is better suited to capture the quantitative implications of the model.

As a first step, I introduce a dynamic process of wage adjustment in which nominal rigidities last more than a single period. As in the previous section, I still assume than in the first period in which the borrowing limit gets tighter, $t = 0$, nominal wages are fully rigid. As in the previous section, in $t = 0$ nominal wages are set after the realization of the idiosyncratic productivity shocks, but before the shock to the borrowing limit is revealed to agents. The

Table 2: Parameters (quarterly)

	Value	Source/Target
Discount factor	$\beta = 0.9938$	$R = 1.025$ (annualized)
TFP process	$\sigma_{A^T} = 0.0106$, $\rho = 0.9573$	Benigno and Thoenissen (2008)
Initial borrowing limit	$\kappa = 3.24$	World debt/GDP = 80%
Final borrowing limit	$\kappa' = 2.43$	World debt/GDP = 60%
Target for trad. inflation	$\pi = 2\%$	Standard value (annualized)

Table 1.2: Parameters - quarterly calibration

difference is that now in $t > 0$ wages are no longer fully flexible, but evolve according to

$$W_{i,t} = \left(W_{i,0}\pi^t\right)^{\phi_t}\left(W_{i,t}^{flex}\right)^{1-\phi_t}.$$

This expression implies that the nominal wage in period t in country i is a weighted average of the nominal wage in country i in period $t = 0$, $W_{i,0}$, and of the wage that would clear the market for labor, $W_{i,t}^{flex}$.[38] This reduced form captures in a simple way a case in which every period only part of the wages are adjusted. The fraction of the wages that do not adjust are indexed on the inflation target in the tradable sector, π. The weights given to rigid wages, ϕ_t, declines linearly over time

$$\phi_t = \max\{0, \phi_0 - \Delta t\},$$

so that in the long run wages become fully flexible.[39] The parameter Δ is set so that complete wage flexibility is reached after two years, or eight quarters.

To focus on dynamics effects, the version of the model presented in this section is parameterized at quarterly frequency. Table 2 displays the value of the parameters that change with respect to the annual parametrization used in the previous sections. The discount factor β is adjusted so as to target an annualized real interest rate in the initial steady state of 2.5 percent. Also, the parameters governing the TFP process are adjusted so that TFP in the tradable sector exhibits the same persistence and standard deviation as in the annual parametrization, once it is aggregated annually.[40]

[38] Formally, $W_{i,t}^{flex}$ is defined as the wage that equates the marginal disutility of labor to the marginal benefit that the household gets from working more

$$W_{i,t}^{flex} = -\frac{U_{L_{i,t}}}{U_{C_{i,t}^T}} P_t^T.$$

[39] This assumption is akin to abstracting from the impact of wage rigidities on normal business cycles, driven by the productivity shocks, in order to fully concentrate on their interaction with the deleveraging shock.

[40] To convert the parameters from annual to quarterly frequency I use the following formulas

$$\rho = \rho_a^{\frac{1}{4}}$$

The borrowing limit in the initial steady state is set to $\kappa = 3.24$ to target a world gross debt-to-GDP ratio of 80 percent. This is the same target as in the annual calibration, taking into account the fact that now GDP in each period corresponds to quarterly GDP in the data. Accordingly, the borrowing limit in the final steady state is set to $\kappa' = 2.43$ to target a ratio of gross debt-to-GDP in the final steady state of 60 percent. Finally, the inflation target in the tradable sector is set to 2 percent per year, in line with the definition of price stability given by the FED and the ECB.

In this section, consistent with the model with annual parametrization, the borrowing limit takes one year to reach its lower value in the new steady state, κ'.[41] In particular, I assume that from period $t = 0$ on the borrowing limit follows the linear adjustment path

$$\kappa = \max\{\kappa', \kappa - \Delta_\kappa t\}.$$

The parameter Δ_κ is chosen so that it takes four quarters, or one year, for the borrowing limit to reach its new steady state value. As before, the initial fall in the borrowing limit happening in $t = 0$ is not anticipated by agents, while from period $t = 0$ on agents correctly anticipate the path of adjustment of the borrowing limit.

Figure 1.9 illustrates the transitional dynamics after a shock to the borrowing limit for the model with multi-period wage rigidities. In period $t = 0$, the monetary union is hit by an unexpected tightening of the borrowing limit that reaches its new steady state value in four quarters. The deleveraging shock induces agents to increase savings and reduce the demand for consumption, driving down the interest rate. In response, the central bank lowers the nominal interest rate to zero in an attempt to hit the inflation target and the economy enters a liquidity trap that starts in period $t = 0$ and lasts four quarters. Since the nominal interest rate cannot go low enough to guarantee market clearing, prices fall to restore equality between demand and supply. This is illustrated by the bottom-left panel of figure 1.9, which shows the path of the monetary union aggregate consumer price index (CPI).[42] Deflation leads to an increase in

$$\sigma^2_{A^T} = \frac{8\left(1-\rho^2\right)}{2+3\rho+2\rho^2+\rho^3}\frac{\sigma^2_{A^T,a}}{1-\rho_a^2},$$

where the a subscript denotes annual parameters.

[41] In addition to comparability with the results presented in the previous sections, one reason to consider a gradual adjustment of the borrowing limit is the fact that the model features only debt contracts that last one period, that is one quarter. In reality, debt can take maturities that are longer than one quarter. Considering a gradual adjustment in the borrowing limit is a simple way of capturing the fact that long term debt allows agents to adjust gradually to the new, tighter, credit conditions.

[42] Formally, the CPI in a generic country i is defined as the minimum price of a unit of the consumption basket C_i

$$CPI_{i,t} = \omega^{-\omega}\left(1-\omega\right)^{\omega-1}\left(P_t^T\right)^{\omega}\left(P_{i,t}^N\right)^{1-\omega}.$$

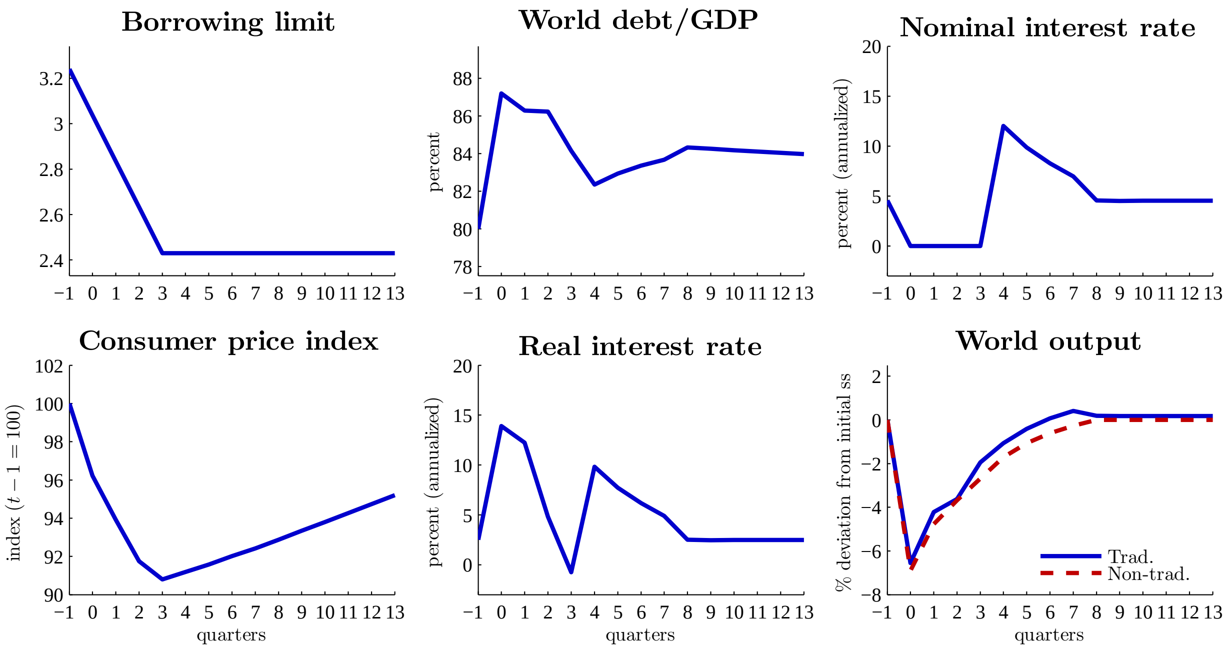


Figure 1.9: Response to deleveraging shock - multi-period liquidity trap.

the world real interest rate that further depresses consumption demand leading to even more deflation.[43] This amplification effect, which is not present when the liquidity trap lasts just one period, sharpens the recession. In fact, the fall in nominal prices is not matched by an equivalent fall in nominal wages. This reduces profits and induces firms to cut employment and production both in the tradable and in the non-tradable sector.

The result is a prolonged recession that affects all the countries belonging to the monetary union. Quantitatively, the recession is particularly severe during the first year following the deleveraging shock. In fact, on impact world output of the traded good falls by almost 7 percentage points below its value in the initial steady state, and after one year, in period $t = 3$, it is still more than 3 percentage points below trend. Also the world output of non-tradables exhibits a large fall during the first year of deleveraging. In addition, the deleveraging process does not follow a monotonic pattern, as it happened before. Instead, initially the debt-to-GDP ratio rises due to the sharp fall in GDP. Only starting from the second quarter the ratio of gross world debt-to-GDP declines.[44]

Once the liquidity trap is over, in period $t = 4$, the central bank raises the nominal interest rate above its value in the new steady state. This happens because the fall in prices coupled

The aggregate CPI of the monetary union is defined as $CPI_t = \int_0^1 CPI_{i,t}\,\mathrm{d}i$.

[43]For consistency with the previous sections, the real rate is defined as the nominal rate deflated by inflation in the traded sector. However, quantitatively the difference between inflation in the traded sector and CPI inflation are negligible.

[44]This is consistent with the path of the private debt-to-GDP ratio observed during several deleveraging episodes. See McKinsey (2010, 2012).

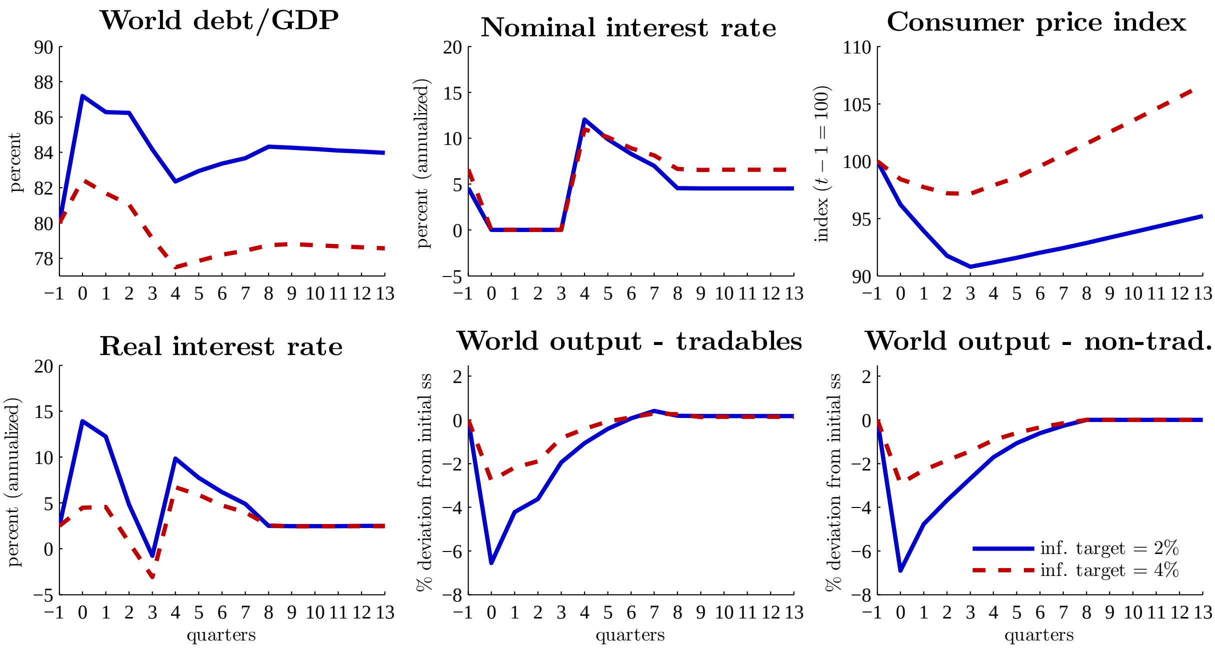


Figure 1.10: Response to deleveraging shock - higher inflation target.
Note: Consumer price index denotes the monetary union aggregate consumer price index. The real interest rate is the nominal interest rate deflated by inflation in the tradable sector.

with nominally rigid wages keeps supply subdued, until wage flexibility is restored. Instead, after four quarters of deleveraging the debt overhang is reduced and aggregate demand recovers. The combination of low supply and high demand puts upward pressure on prices, so the central bank has to raise nominal rates in order to dampen the rise in consumption demand and to prevent inflation from exceeding the 2 percent inflation target. This explains the slow recovery that takes two years to complete.

1.6.2 Raising the inflation target

One policy that can mitigate the recession during debt deleveraging consists in adopting a higher inflation target. Figure 2.6 compares two monetary unions with different steady state inflation targets.[45] The solid lines refer to the baseline economy, in which the inflation target is 2 percent a year, while the dashed lines refer to an economy with a higher inflation target, of 4 percent a year.

Even with a 4 percent inflation target the deleveraging shock pushes the monetary union into a liquidity trap that lasts four quarters. However, the adjustment is much less traumatic

[45] This section looks at two economies whose steady state inflation target is different. An alternative would be to consider a change in the inflation target in response to the tightening of the borrowing limit. However credibility issues are likely to prevent a central bank from changing the inflation target in the middle of a deleveraging episode. This point is discussed by Eggertsson (2008), who considers credibility issues faced by the FED during the Great Depression.

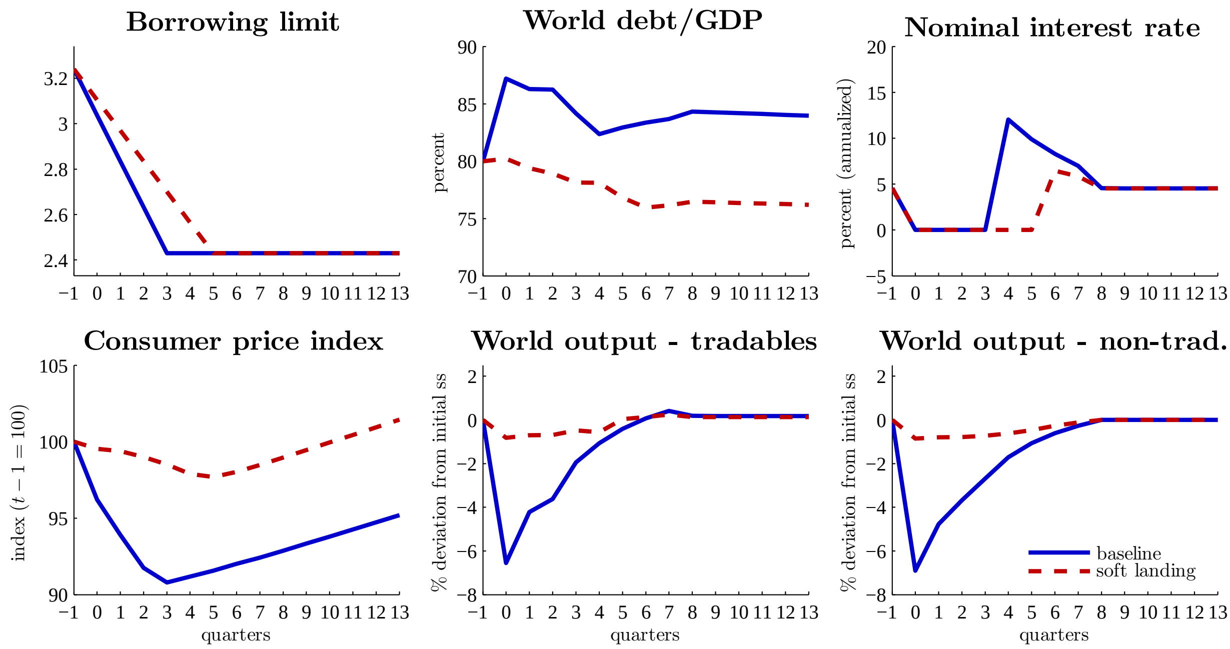


Figure 1.11: Response to deleveraging shock - soft landing.

in the economy with higher inflation target. In fact, a higher inflation target guarantees a smaller drop in output, as well as less deflation and lower real rates throughout the liquidity trap. The reason is the following. In the last period of the liquidity trap, period $t = 3$, the real interest rate, defined as the nominal interest rate deflated by inflation in the tradable sector, is equal to the inverse of the inflation target. This happens because the nominal interest rate is equal to zero, so the real rate is equal to the inverse of the inflation rate. Since the central bank hits the inflation target once the liquidity trap is over, the expected inflation in period $t = 3$ is equal to the inflation target. This means that the real interest rate in the last period of the liquidity trap is lower the higher the inflation target.

A lower real rate stimulates demand for consumption in the last period of the trap, limiting the fall in prices and the contraction in output. Moreover, the lower real rate in the last period of the trap has also a positive effect on demand during the previous periods, since aggregate demand depends on the path of all the future interest rates. It follows that during the previous periods too deflation is lower and the drop in output is smaller. Indeed, raising the inflation target from 2 to 4 percent halves the fall in output during the liquidity trap. This experiment suggests that a higher inflation target may be helpful in limiting the recession during a deleveraging episode in a monetary union.

1.6.3 A "soft landing" scenario

In the first phase of the 2008/2009 recession, public flows passing via the ECB played a major role in cushioning the fall in foreign credit in the countries at the Eurozone periphery, as shown by Lane and Milesi-Ferretti (2012). In this section I consider a simple experiment to evaluate the effectiveness of policies that slow down the adjustment in debtor countries, inducing a "soft landing" type of adjustment. More precisely, I compare the baseline scenario in which the borrowing limit takes four quarters to reach its new steady state value, to an economy in which the borrowing limit takes six quarters to reach its new steady state value. The results are shown in figure 2.8. The solid lines refer to the baseline economy, while the dashed lines refer to the soft landing scenario.

Figure 2.8 makes clear that the intervention aiming at slowing the adjustment to the new credit conditions significantly reduces deflation and the output contraction. This happens because a gradual tightening of the borrowing limit prevents abrupt cuts in consumption and reduces the fall in the interest rate needed to reach market clearing. So, although now the liquidity trap lasts six quarters, two quarters more than in the baseline scenario, the adjustment is smoother and the recession is milder. Moreover, in the soft landing scenario deleveraging, as captured by the reduction in the world debt-to-GDP ratio, is faster. This happens because the slower adjustment in the borrowing limit prevents the sharp fall in GDP that causes the initial rise in the world debt-to-GDP ratio in the baseline economy.

This experiment suggests that interventions that limit the surprise effect of a deleveraging shock can play a role in mitigating the recession associated with an episode of debt deleveraging.

1.7 Conclusion

I propose a multi-country model for understanding deleveraging among a group of financially integrated countries. The model highlights a novel economic mechanism that makes episodes of debt deleveraging particularly painful for monetary unions. Deleveraging leads to a drop in the world interest rate, both because high-debt countries are forced to save more in order to reduce their debt and because the rest of the world experiences an increase in the desire to accumulate precautionary savings. In the absence of nominal rigidities, deleveraging also triggers a rise in production in high-debt countries. If wages are nominally rigid but nominal exchange rates are allowed to float, the rise in production involves a nominal depreciation in high-debt countries. In a monetary union, the combination of nominal wage rigidities and fixed exchange rates prevents any increase in production in indebted countries. This amplifies the fall in the world consumption demand and the drop in the world interest rate. Hence, monetary unions are particularly prone to enter a liquidity trap during an episode of deleveraging. In a liquidity trap deleveraging generates a deflationary union-wide recession, hitting high-debt countries especially hard.

The analysis presented in this paper can be extended in a number of directions. First, the model could be used to understand the role of fiscal policy in a monetary union undergoing a process of deleveraging. In particular, the recent experience of the Eurozone has sparked a lively debate on the role of fiscal transfers and mutual insurance inside monetary unions. The model has the potential to shed light on this key policy issue, and I plan to tackle it in future research. In addition, it would be interesting to consider collateral constraints in which asset prices, for instance house prices, play a role in determining access to credit. Mendoza (2010) uses a small open economy model to show how economies in which borrowing depends on the price of capital can endogenously enter deleveraging episodes. An open research question concerns the interactions between these types of constraints and the zero lower bound in a model of the world economy.[46]

[46] See Fornaro (2012a) for a small open economy model in which nominal wage rigidities and exchange rate policies interact with occasionally binding collateral constraints.

Chapter 2

Debt Deleveraging, Debt Relief and Liquidity Traps

2.1 Introduction

Since the onset of the recent global crisis several countries have embarked in a process of private debt deleveraging (figure 2.1).[1] The path toward lower debt has been characterized by a severe global recession, taking place in a low interest rate environment limiting the scope for conventional monetary policy stimulus. Against this background, some commentators have argued that debt relief policies, that is policies that reduce the debt burden of indebted households, could play a key role in easing the recovery.[2] However, we still lack a clear understanding of the macroeconomic channels through which debt relief policies might affect the economy and of their implications for welfare.

I tackle these issues using an analytical framework suitable to study the positive and normative implications of debt relief policies during episodes of debt deleveraging. I derive two key results. First, I show that a program of debt relief leads to an expansion in employment and output if deleveraging pushes the economy in a liquidity trap, that is a case in which the nominal interest rate hits the zero lower bound. Second, I show that debt relief during a liquidity trap may benefit both debtors and creditors and generate a Pareto improvement in welfare.

I reach these results studying a tractable model of debt deleveraging. The model is simple

[1]McKinsey (2010, 2012) and Koo (2011) describe the process of international deleveraging that began with the 2007-2008 financial crisis.

[2]For example, this view is maintained by Geanakoplos and Koniak (2009) and Sufi (2012). In fact, debt relief is not just a theoretical possibility. Iceland has been implementing debt relief programs for financially distressed households since the end of 2008. Ireland is now in the process of implementing similar policies.

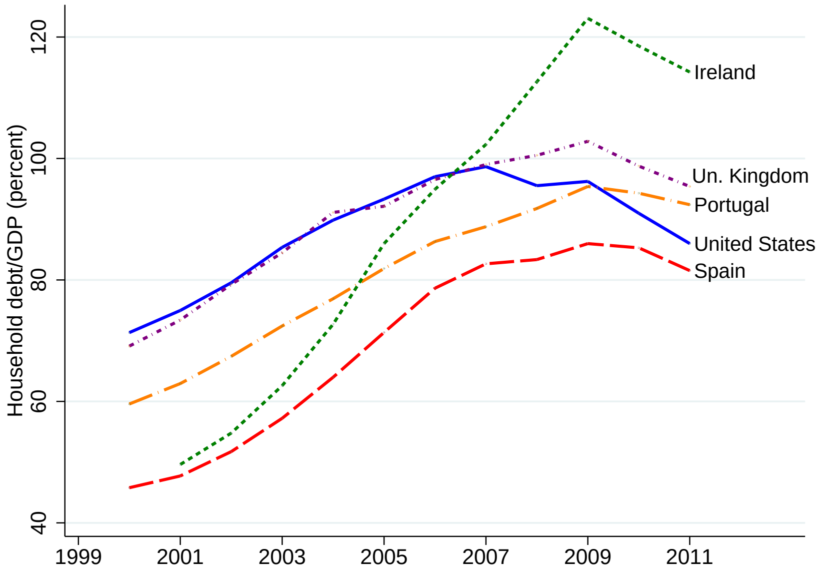


Figure 2.1: Household debt-to-GDP ratio, 2000 – 2011. Notes: data are from the OECD.

enough so that its properties can be derived analytically, without resorting to local approximations. This is important since local approximations might perform poorly when employed to study liquidity traps.[3] Despite its simplicity the model captures salient features of debt deleveraging episodes. There are two groups of households, debtors and creditors. Deleveraging is triggered by a shock that forces debtors to reduce their debt. The process of debt reduction generates a fall in aggregate demand and in the interest rate. If the shock is large enough the economy falls in a liquidity trap characterized by low inflation, leading to involuntary unemployment due to the presence of downward nominal wage rigidities. In this context I study the impact of a policy of debt relief, which I capture with a transfer of wealth from creditors to debtors.

Debt relief leads to an increase in aggregate demand, because borrowing-constrained debtors have a higher propensity to consume out of income than creditors. If the economy is in a liquidity trap the increase in demand generates an increase in output, since in a liquidity trap there is involuntary unemployment precisely because aggregate demand is weak. Through this channel a program of debt relief has an expansionary impact on employment and output.

Debt relief can also give rise to a Pareto improvement in welfare. While it is not surprising that debtors should gain from a policy of debt relief, it is not obvious that creditors could benefit too. In fact, a Pareto improvement in welfare is possible only if debt relief generates an expansion in output large enough to compensate creditors for the loss in wealth due to the transfer to debtors. I show that this is more likely to be the case the more the central

[3]Braun et al. (2012) show that local approximations can lead to qualitatively, as well as quantitatively, inaccurate results when employed to study economies experiencing a liquidity trap.

bank is concerned with stabilizing inflation. To understand this result, consider that during the recovery from a liquidity trap real wages have to fall to a level consistent with full employment. Since nominal wages are downwardly rigid, higher inflation speeds up the process of wage adjustment and leads to a faster recovery, while low inflation during the recovery is associated with persistent unemployment.[4] Because of this effect, a policy of debt relief that limits the rise in unemployment during the liquidity trap has a larger positive impact on employment, output and welfare the more the central bank is concerned with keeping inflation low during the recovery. Moreover, I show that targeting inflation during a liquidity trap can open the door to multiple equilibria. In this case, an appropriate transfer scheme can eliminate undesirable equilibria.

The rest of the paper is structured as follows. I start with a discussion of the related literature. Section 2 introduces the model. Section 3 shows how an episode of deleveraging can generate a liquidity trap. Section 4 studies the normative and positive impact of debt relief. Section 5 discusses several extensions, including the case of a monetary union. Section 6 concludes.

Related literature. This paper is related to several strands of the literature. First, the paper is about debt deleveraging and liquidity traps. Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) study the impact of deleveraging shocks on the interest rate in closed economies, while Benigno and Romei (2012) and Fornaro (2012b) consider deleveraging in open economies. I contribute to this literature by studying the impact of debt relief policies in economies undergoing a period of debt deleveraging.

The paper is also related to the literature on fiscal policy and liquidity traps. A non-exhaustive list of papers studying fiscal policy during liquidity traps is Eggertsson and Woodford (2006), Christiano et al. (2011), Mertens and Ravn (2010), Correia et al. (2011), Mankiw and Weinzierl (2011), Werning (2011), Bilbiie et al. (2012), Braun et al. (2012), Carlstrom et al. (2012), Farhi and Werning (2012) and Rendahl (2012). While these contributions focus on government expenditure or public debt, this paper considers the role of pure transfers from creditors to debtors.

The focus on transfers connects this paper to Werning and Farhi (2012), who study transfers among members of a monetary union. My model describes a closed economy, but most of its insights can be extended to the case of a monetary union as I discuss in section 2.5.3. While the rationale for transfers in Werning and Farhi (2012) arises because of the presence of idiosyncratic shocks and nominal rigidities, in this paper transfers are welfare improving

[4]This feature of the model is consistent with the empirical findings of Calvo et al. (2012), who show that recoveries from financial crises are characterized by a trade-off between inflation and unemployment.

because of the interaction between an aggregate deleveraging shock and the zero lower bound on the nominal interest rate. The role of transfers in stabilizing economic fluctuations is also studied in McKay and Reis (2012). While McKay and Reis (2012) consider the impact of automatic stabilizers on business cycle fluctuations, this paper analyzes the role of debt relief, a discretionary and exceptional form of policy intervention, during sharp recessions. Bianchi (2012) studies bailout policies in the form of transfers from households to firms. Bianchi (2012) focuses on a real economy in which monetary policy is neutral, while in this paper the interaction between debt relief and monetary policy is crucial.

A key feature of the model is the presence of nominal wage rigidities. There is extensive evidence in support of the existence of downward nominal wage rigidities. Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2010) and Fabiani et al. (2010) document the existence of downward wage rigidities using micro data. From a macro perspective Olivei and Tenreyro (2007, 2010) and Christiano et al. (2005) highlight the key role of nominal wage rigidities as a transmission channel for monetary policy. There is also evidence suggesting that nominal wages fail to adjust downward during deep recessions. Eichengreen and Sachs (1985), Bernanke and Carey (1996) and Bordo et al. (2000) discuss the role of wage rigidities during the Great Depression. Schmitt-Grohé and Uribe (2011) document the importance of downward nominal wage rigidities in the context of the 2001 Argentine crisis and of the Great Recession in countries at the Eurozone periphery. This paper also relates to models of downward nominal wage rigidities, such as Akerlof et al. (1996), Benigno and Ricci (2011), Schmitt-Grohé and Uribe (2011, 2012) and Daly and Hobijn (2013).

2.2 Model

Consider a closed economy inhabited by households and firms. There is also a central bank that conducts monetary policy. Time is discrete and indexed by t and there is perfect foresight.

2.2.1 Households

There is a continuum of households of measure one. The lifetime utility of a generic household i is

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t^i\right). \tag{2.1}$$

In this expression, C_t^i denotes consumption, β is the subjective discount factor and the period utility function $U(\cdot)$ is specified as

$$U\left(C_t^i\right) = \frac{C_t^{i\,1-\gamma} - 1}{1-\gamma},$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In every period each household is endowed with $\bar{L}$ hours of labor. Households supply inelastically their labor endowment to the labor market, but, due to the presence of nominal wage rigidities, a household may be able to work only $L_t^i < \bar{L}$ hours.[5] Hence, when $L_t^i = \bar{L}$ for every household i the economy is operating at full employment, otherwise there is involuntary unemployment.

Households trade in one period riskless bonds. Bonds are denominated in units of consumption good and pay the real interest rate r_t.[6] The budget constraint of the household is

$$P_t C_t^i + \frac{P_t B_{t+1}^i}{1+r_t} = W_t L_t^i + P_t B_t^i + \Pi_t^i + T_t^i. \tag{2.2}$$

The left-hand side of this expression represents the household's expenditure. P_t is the nominal price level in period t, hence $P_t C_t^i$ is the expenditure of the household in consumption expressed in units of money. B_{t+1}^i denotes the purchase of bonds made by the household at time t at price $P_t/(1+r_t)$. If $B_{t+1}^i < 0$ the household is a borrower.

The right-hand side captures the household's income. W_t denotes the nominal wage, so $W_t L_t^i$ is the household's labor income. Labor is homogeneous across households and every household receives the same wage W_t. $P_t B_t^i$ is the gross return on investment in bonds made at time $t-1$ expressed in units of money. Π_t^i are the nominal profits that the household receives from firms. Firms are wholly owned by households and equity holdings within these firms are evenly divided among them. Finally, T_t^i is a lump sum transfer taken as given by the household.

There are frictions in the financial markets and households are subject to a borrowing limit. In particular, each period debt repayment cannot exceed the exogenous limit κ_t, so that the bond position has to satisfy

$$B_{t+1}^i \geq -\kappa_t. \tag{2.3}$$

This constraint captures in a simple form a case in which a household cannot credibly commit in period t to repay more than κ_t units of the consumption good to its creditors in period $t+1$.

Each period the household chooses B_{t+1}^i to maximize the present discounted value of utility

[5] In section 2.5.2 I discuss the case of elastic labor supply.

[6] I focus on bonds denominated in real terms to simplify the analysis. Considering nominal bonds should not alter the key results of the paper.

(4.1), subject to the budget constraint (2.2) and the borrowing limit (4.9). The household's optimal choice of bonds satisfies

$$U'\left(C_t^i\right) = \beta\left(1 + r_t\right) U'\left(C_{t+1}^i\right) + \mu_t^i \tag{2.4}$$

$$\mu_t^i\left(B_{t+1}^i + \kappa_t\right) = 0, \text{ with } \mu_t^i \geq 0, \tag{2.5}$$

where $U'(\cdot)$ is the first derivative of the period utility function and μ_t^i is the Lagrange multiplier on the borrowing limit, normalized by the gross real interest rate $1 + r_t$. Expression (2.4) is the standard Euler equation for bonds, which guarantees optimal consumption smoothing over time. Expression (2.5) is the complementary slackness condition on constraint (4.9), which ensures that the borrowing limit is not violated.

2.2.2 Firms

There is a large number of firms that use labor as the only factor of production. Each period a firm that employs L_t units of labor produces L_t^{α} units of the consumption good, where $0 < \alpha < 1$.[7] The nominal profits of the representative firm are

$$\Pi_t = P_t L_t^{\alpha} - W_t L_t. \tag{2.6}$$

Each firm chooses employment L_t to maximize profits, taking the price of the consumption good and the wage as given. Profit maximization implies

$$\alpha L_t^{\alpha-1} = \frac{W_t}{P_t}. \tag{2.7}$$

At the optimum firms equate the marginal product of labor, the left-hand side of the expression, to the real marginal cost, the right-hand side.

2.2.3 Downward nominal wage rigidities

Nominal wages are downwardly rigid, and wage dynamics must satisfy

$$W_{t+1} \geq \phi\left(u_t\right) W_t,$$

[7] To introduce constant returns-to-scale in production one could assume that a firm that employs L_t units of labor produces $L_t^{\alpha} K^{1-\alpha}$ units of the consumption good, where K is a fixed production factor owned by the firm, for example physical or organizational capital. The production function in the main text corresponds to the normalization $K = 1$.

where $u_t = 1 - L_t/\bar{L}$ is the unemployment rate and the function $\phi(\cdot)$ satisfies $\phi'(\cdot) \leq 0$. The term $\phi(u_t)$ introduces a feedback from the unemployment rate to wage dynamics. Specifically, when $\phi'(\cdot) < 0$ a higher unemployment rate is associated with more downward wage flexibility.

Given this constraint on wage dynamics, employment satisfies the complementary slackness condition

$$\left(\bar{L} - L_t\right)\left(W_{t+1} - \phi(u_t) W_t\right) = 0,$$

which says that unemployment arises only if wages cannot fall enough for the labor market to clear.

2.2.4 Central bank

The central bank uses the nominal interest rate i_t as its policy instrument.[8] The nominal interest rate is related to the real interest rate by the Fisher equation

$$1 + i_t = (1 + r_t)\,\pi_{t+1}, \tag{2.8}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate between period t and period $t+1$.

I focus on central banks that follow targeting rules.[9] First, I consider a central bank that targets an inflation rate $\bar{\pi}$.[10] Second, I consider a central bank whose main objective is to guarantee full employment and that, conditional on having reached full employment, also targets inflation $\bar{\pi}$.[11] However, it might not always be possible for the central bank to attain its desired target because of the zero bound on the nominal interest rate $i_t \geq 0$.

[8]More formally, assume that there are government bonds paying the nominal interest rate i_t. Also assume that households cannot take a negative position in government bonds. The central bank can set the nominal interest rate through open market operations in government bonds, and in equilibrium government bonds are in zero net supply.

[9]Another possibility would be to assume a benevolent central bank that maximizes households' welfare. However, since households are heterogeneous modeling an optimizing central bank involves taking a stance on how the central bank values the utility of different individuals. I prefer not to follow this approach and I consider central banks that target aggregate variables, because in reality the mandate of most central banks is specified in terms of inflation and employment targets.

[10]In this paper I employ a notion of inflation targeting that is perhaps more restrictive than the one commonly understood in the literature on monetary policy. In fact, in general adhering to a policy of inflation targeting does not prevent the central bank from changing its inflation target in response to changes in the economy. Instead, the inflation targeting policy that I consider in this paper does not allow for changes in the target. In practice central banks in advanced economies are extremely reluctant to change their inflation target, even following major shocks such as the 2007-2008 financial crisis and the following recession.

[11]Later, in section 2.5.1, I study the case of a central bank that sets monetary policy according to an interest rate rule.

2.2.5 Market clearing and equilibrium

I consider equilibria in which every household works the same number of hours. Hence, equilibrium on the labor market is attained when

$$L_t^i = L_t. \tag{2.9}$$

Moreover, I focus on equilibria in which transfers are balanced every period across households so that

$$\int_0^1 T_t^i \,\mathrm{d}i = 0. \tag{2.10}$$

Market clearing for the consumption good is reached when aggregate consumption is equal to aggregate output

$$\int_0^1 C_t^i \,\mathrm{d}i = L_t^{\alpha}. \tag{2.11}$$

These conditions imply that bonds are in zero net supply in every period, $\int_0^1 B_{t+1}^i \,\mathrm{d}i = 0$. We are now ready to define an equilibrium.

Definition 2 *An equilibrium is a set of processes* $\{C_t^i, L_t^i, B_{t+1}^i, \mu_t^i, L_t, r_t, P_t, W_t\}_{t=0}^{\infty}$ *and a sequence of distributions for bond holdings* $\Psi_t(B)$*, such that given an exogenous process for* $\{\kappa_t\}_{t=0}^{\infty}$*, a sequence of interest rates and transfers* $\{i_t, T_t^i\}_{t=0}^{\infty}$ *and the initial distribution* $\Psi_0(B)$*, in every period* t

- *The households' decisions are optimal given prices* $\{r_t, W_t, P_t\}_{t=0}^{\infty}$*, that is for every household* i *they satisfy*

$$U'\left(C_t^i\right) = \beta\left(1 + r_t\right) U'\left(C_{t+1}^i\right) + \mu_t^i$$

$$B_{t+1}^i \geq -\kappa, \text{ with equality if } \mu_t^i > 0.$$

- *Firms' maximize profits given prices* $\{r_t, W_t, P_t\}_{t=0}^{\infty}$

$$\alpha L_t^{\alpha-1} = \frac{W_t}{P_t}.$$

- *The complementary slackness for the wage setting condition holds*

$$\left(\bar{L} - L_t\right)\left(W_{t+1} - \phi\left(u_t\right) W_t\right) = 0.$$

- *Transfers are balanced every period*

$$\int_0^1 T_t^i \,\mathrm{d}i = 0.$$

- $\Psi_t(B)$ *is consistent with the decision rules*

- *Markets for bonds and labor clear*

$$L_t^i = L_t \leq \bar{L}.$$

$$\int_0^1 B_{t+1}^i \,\mathrm{d}i = 0.$$

2.2.6 Steady state

I focus on an economy that features a deterministic steady state in which there is no conflict between targeting inflation or employment. This requires the following assumptions. The parameters $\bar{\pi}$ and β are such that

$$\bar{\pi} \geq \beta.$$

The function $\phi(\cdot)$ is such that

$$\phi(0) \leq \bar{\pi}.$$

Hence, in steady state inflation is equal to its target $\bar{\pi}$ and the economy is at full employment. I also limit the analysis to steady states in which transfers are equal to zero for every household, that is $T^i = 0$ for every i.

In steady state each household features a constant consumption stream. Combining this condition with the Euler equation (2.4) gives the steady state real interest rate

$$r = \frac{1}{\beta} - 1,$$

where the absence of a time subscript denotes variables referring to the steady state. The steady state consumption of a generic household i is

$$C^i = \bar{L}^\alpha + \frac{rB^i}{1+r}, \tag{2.12}$$

where B^i is the stock of bonds owned by the household in steady state. This expression implies that the only source of heterogeneity across households in steady state consumption is due to differences in wealth. In particular, households that have a higher wealth consume more in steady state.

2.3 Debt deleveraging and liquidity traps

In this section I consider an episode of deleveraging and show how deleveraging can push the economy into a liquidity trap characterized by low inflation and involuntary unemployment. I start by considering economies without transfers and set $T_t^i = 0$ for every i and t.

Assume that at the start of period 0 some households are debtors and some are creditors. In particular, a fraction n of the households are debtors and each debtor starts with initial assets $-B_0 < 0$. The remaining fraction of households $1-n$ are creditors and each creditor starts with assets $n/(1-n)B_0 > 0$.[12] In what follows, I will denote debtors with the superscript d and creditors with the superscript c. This simple form of initial heterogeneity in bond holdings makes the analysis particularly tractable, while preserving the fundamental insights that could be derived from a model featuring a more realistic initial wealth distribution.

In period 0 the economy is hit by an unexpected *deleveraging shock*, that is a sudden tightening of credit conditions that forces debtors to reduce their debt positions. I capture the deleveraging shock with an unexpected fall in the borrowing limit κ, so that $\kappa_0 = \bar{\kappa}$ where $0 < \bar{\kappa} < B_0$. The tightening of the borrowing constraint forces debtors to reduce their debt by the amount $B_0 - \bar{\kappa}$ and triggers a process of deleveraging. To simplify the analysis, I assume that the shock to κ is permanent, so that $\kappa_t = \bar{\kappa}$ in every period $t \geq 0$.

Irrespective of whether the central bank targets inflation or employment, the central bank responds to the deleveraging shock by decreasing the nominal interest rate. To see this point it is useful to start by considering a case in which the central bank is not constrained by the zero lower bound on the nominal interest rate, and hence in which inflation is always equal to the target and the economy always operates at full employment. In this case, creditors' consumption in period 0 is given by

$$C_0^c = \bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \frac{\bar{\kappa}}{1+r_0}\right).$$

From period 1 on the economy enters a steady state in which creditors' consumption is constant and equal to

$$C^c = \bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}.$$

Let us now consider the implications for the interest rate. Suppose that the real interest rate

[12]The existence of initial heterogeneity in bond holdings can be due to past idiosyncratic shocks, as in Guerrieri and Lorenzoni (2011) and Fornaro (2012b).

does not respond to the deleveraging shock and so $r_0 = r$. In this case $C_0^c > C^c$ and creditors experience a decrease in consumption between period 0 and period 1. But if $r_0 = r$ the Euler equation implies that creditors' consumption must be constant between periods 0 and 1, a contradiction. Hence r_0 must respond to the deleveraging shock.

In fact, it is possible to show that during period 0 the real interest rate falls below its steady state value.[13] Intuitively, the deleveraging shock forces debtors to increase their savings so as to reduce their debt positions. At full employment the interest rate must fall so that creditors, which are not borrowing constrained, become willing to absorb the forced savings coming from debtors.[14]

By the Fisher equation (2.8), the fall in the real interest rate exerts a depressive impact on the nominal interest rate. Hence, a deleveraging shock exposes the economy to the risk of experiencing a liquidity trap, that is a case in which the nominal interest rate hits the zero lower bound.[15] Indeed, for a sufficiently large shock, that is if $B_0 - \bar{\kappa}$ is sufficiently large, the economy enters a liquidity trap for sure.

Condition 3 *The parameters satisfy*

$$\frac{\beta}{\bar{\pi}} \frac{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right)} > 1.$$

Proposition 2 *If condition* 3 *holds and* $T_t^i = 0$ *for every* i *and* t *the economy is in a liquidity trap in period 0,* $i_0 = 0$. *Then there is unexpected undershooting of the inflation target,* $\pi_0 < \bar{\pi}$, *and involuntary unemployment,* $L_0 < \bar{L}$. *Moreover, the economy exits the liquidity trap in period 1, i.e.* $i_t > 0$ *and* $\pi_t \geq \bar{\pi}$ *for* $t > 0$.

Proof. See appendix. ■

In a liquidity trap the nominal interest rate hits the zero lower bound and the real interest rate is equal to the inverse of expected inflation. There is unemployment because consumption demand is too weak to sustain full employment. Intuitively, the interest rate cannot fall enough to induce creditors to absorb the forced savings of debtors at full employment. Hence, firms react to the excess supply of consumption good by cutting prices, and so inflation is lower

[13]See the proof of proposition 2.

[14]See Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) for more discussion on the link between deleveraging and low interest rates.

[15]As emphasized by Krugman (1998), the central bank could avoid hitting the zero lower bound constraint by increasing expected inflation, that is by setting π_1 high enough so that $(1 + r_0)\pi_1 > 0$. However, this strategy conflicts with our assumptions about the objectives of the central bank. In fact, in the absence of a liquidity trap in period 0, once period 1 comes the central bank will want to set $\pi_1 = \bar{\pi}$. Hence, any announcement of a higher π_1 is not credible. Eggertsson and Woodford (2003) discuss these credibility issues in the context of a standard New-Keynesian model.

than the target. Low inflation coupled with nominal wage stickiness leads to high real wages, which discourage firms' labor demand and employment. This adjustment process goes on until output has fallen enough so as to eliminate the excess supply on the goods' market.

Though, as stated by proposition 2, the liquidity trap lasts only one period the impact on inflation and employment can be more persistent. The persistence arises because real wages increase during the liquidity trap, and so during the recovery real wages have to fall to restore full employment. Due to the presence of downward nominal wage rigidities, inflation may affect the speed at which real wages fall during the recovery. In particular, if inflation is too low nominal wages may not fall fast enough to immediately restore full employment once the liquidity trap is over. Hence, during the recovery a trade-off between inflation and employment may arise.[16] Indeed, we can distinguish two regimes. For sufficiently mild recessions the recovery is immediate and involves no trade-off between inflation and employment. I will refer to this case as mild recessions. Instead, for large recessions the central bank faces a trade-off between inflation and employment during the recovery.

More formally, the economy is in a mild recession if the following condition holds.

Condition 4 L_0 *satisfies*

$$U'\left(L_0^{\alpha} + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right) = \frac{\beta}{\bar{\pi}}U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)$$

$$\left(\frac{L_0}{\bar{L}}\right)^{1-\alpha}\bar{\pi} \geq \phi\left(0\right)$$

$$L_0 < \bar{L}.$$

Proposition 3 *If condition* 4 *holds and* $T_t^i = 0$ *for every* i *and* t *the economy is in a liquidity trap in period 0. Moreover, the economy is at full employment,* $L_t = \bar{L}$*, and inflation is equal to its target,* $\pi_t = \bar{\pi}$*, for all* $t > 0$*.*

Proof. See appendix. ■

2.4 Debt relief and liquidity traps

We are now ready to consider the impact of debt relief policies. I model debt relief as a lump-sum transfer from creditors to debtors occurring when the deleveraging shock hits the economy, that is in period 0. Specifically, in period 0 each debtor receives T units of the

[16] The existence of a trade-off between inflation and employment during the recovery is consistent with the empirical evidence provided by Calvo et al. (2012).

consumption good, financed with a tax $n/(1-n)T$ levied on each creditor. Formally, $T_0^d = T$, $T_0^c = -n/(1+n)T$ and $T_t^i = 0$ for every i and for $t > 0$. The period 0 budget constraints respectively of debtors and creditors now become

$$P_0 C_0^d + \frac{P_0 B_1^d}{1+r_0} = W_0 L_0 - P_0 B_0 + T + \Pi_0 \tag{2.13}$$

$$P_0 C_0^c + \frac{P_0 B_1^c}{1+r_0} = W_0 L_0 + \frac{n}{1-n}\left(P_0 B_0 - T\right) + \Pi_0. \tag{2.14}$$

This transfer scheme captures a variety of policies aiming at transferring wealth from creditors to debtors: a program of debt relief, fiscal transfers from creditors to debtors or even defaults. I am interested in inspecting the impact of a transfer from creditors to debtors on employment and output and in deriving conditions under which such a transfer is Pareto improving in welfare terms. I start to analyze the impact of transfers during mild recessions, which represent a particularly tractable case useful to build up intuition. I then move to the more complex case of large recessions.

2.4.1 A simple case: debt relief during mild recessions

In this section I focus on debt relief during mild recessions characterized by immediate recoveries, and I will thus assume that condition 4 holds. Let us start by considering the impact of a marginal transfer.

Proposition 4 *If conditions 4 holds, that is if in the absence of transfers the economy is in a liquidity trap characterized by a mild recession, a marginal transfer from creditors to debtors leads to an increase in employment and to a Pareto improvement in welfare. Moreover, a liquidity trap is a necessary condition to obtain a Pareto improvement in welfare from a marginal transfer from creditors to debtors.*

Proof. See appendix. ■

Proposition 4 states that a marginal transfer from creditors to debtors is Pareto improving if the economy is in a liquidity trap characterized by a mild recession. To grasp the intuition behind this result, consider that when condition 4 holds the economy reaches the steady state in period 1, right after the liquidity trap is over. Inspecting equation (2.12) one can see that a transfer in period 0 cannot affect steady state consumption, and so to trace the impact of a marginal transfer on welfare we just have to take into account the impact on consumption in period 0.

During the recovery from a mild recession inflation is equal to its target and so the real interest rate during the liquidity trap is equal to the inverse of the inflation target. We can then write creditors' Euler equation as

$$U'\left(C_0^c\right) = \frac{\beta}{\bar{\pi}} U'\left(C^c\right).$$

Differentiating this expression with respect to T and using the fact that $\partial C^c/\partial T = 0$ gives $\partial C_0^c/\partial T = 0$, so the transfer does not affect creditors' consumption in period 0. Hence, the transfer is Pareto improving if it leads to an increase in debtors' consumption in period 0.

To derive the impact of the transfer on C_0^d, first differentiate creditors' budget constraint (2.14) in period 0 with respect to T

$$\frac{\partial C_0^c}{\partial T} = -\frac{n}{1-n} + \alpha L_0^{\alpha-1} \frac{\partial L_0}{\partial T}.$$

Since by the Euler equation $\partial C_0^c/\partial T = 0$, we have

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n} \frac{L_0^{1-\alpha}}{\alpha} > 0,$$

so that the transfer leads to an increase in employment and output. In fact the expansion in output must be just enough to compensate creditors' for the loss in consumption due to the transfer, so as to leave period 0 creditors' consumption unchanged. Finally, differentiating debtors' budget constraint (2.13) with respect to T gives

$$\frac{\partial C_0^d}{\partial T} = 1 + \alpha L_0^{\alpha-1} \frac{\partial L_0}{\partial T} > 0.$$

From this expression it is clear that the transfer has a positive impact on debtors' consumption, both because of its direct effect and because of its positive impact on employment, and hence it is Pareto improving.

A transfer from creditors to debtors is expansionary because it stimulates aggregate demand. On the one hand, debtors' consumption demand rises one for one with income, because debtors are borrowing constrained. Hence, the transfer positively affects debtors' consumption demand. On the other hand, creditors' consumption demand is determined by the real interest rate and by expected consumption, which are not affected by the transfer if the economy is in a mild recession. Consequently, the transfer does not affect creditors' demand for consumption. The result is that the transfer generates an increase in aggregate demand which leads to an increase in inflation and production.

To understand why the transfer leads to a Pareto improvement in welfare, consider that the zero lower bound constraint on the interest rate negatively affects welfare both for creditors and debtors. The increase in aggregate demand due to the transfer relaxes the zero lower bound constraint, because it generates an increase in the interest rate that would clear the market for consumption. The relaxation of this constraint allows for a Pareto improvement in welfare.

The second part of proposition 4 states that a transfer cannot lead to a Pareto improvement in welfare if the deleveraging shock does not push the economy in a liquidity trap. To understand this result, consider that if the zero lower bound constraint never binds the economy always operates at full employment, which means that a transfer cannot induce an expansion in output. But without an increase in output creditors cannot be compensated for the loss due to the transfer. Hence, a transfer cannot be Pareto improving if the economy never enters a liquidity trap.

Having characterized the impact on welfare of a marginal transfer, I now turn to the Pareto optimal policy. I define a Pareto optimal transfer as the transfer that maximizes debtors' welfare, leaving creditors at least as well off as in the equilibrium without transfer.[17]

Definition 5 *The Pareto optimal transfer maximizes debtors' welfare, leaving creditors at least as well off as in the equilibrium without transfer.*

Proposition 5 *If condition* 4 *holds, the Pareto optimal transfer restores full employment. The optimal transfer* T^* *satisfies*

$$U'\left(\bar{L}^{\alpha}+\frac{n}{1-n}\left(B_0-\bar{\pi}\bar{\kappa}-T^*\right)\right)=\frac{\beta}{\bar{\pi}}U'\left(\bar{L}^{\alpha}+\frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

Proof. In the appendix. ■

Proposition 5 says that the Pareto optimal transfer during a mild recession restores full employment. To visually illustrate the impact of the Pareto optimal transfer during a mild recession I use a numerical example. Though the model is too simple to lend itself to a calibration exercise, I choose the parameters to target salient features of the US, so as to give a feeling of the magnitude of the effects implied by the model.

Every period corresponds to one year. I set the discount factor to $\beta = 0.9756$, so that in steady state the real interest rate is equal to 2.5 percent. This is close to the real interest in the US in 2007, at the onset of the financial crisis. The coefficient of relative risk aversion is

[17] Alternatively, one could define a Pareto optimal transfer as the transfer that maximizes creditors' welfare, leaving debtors at least as well off as in the equilibrium without transfer. In the case of a mild recession this definition would lead to an indeterminate transfer, because there is a range of transfers that leave creditors' utility unchanged, while having a positive impact on debtors' welfare.

Table 1: Parameters

	Value	Source/Target
Discount factor	$\beta = 0.9756$	$r = 0.025$
Risk aversion	$\gamma = 2$	Standard value
Labor share	$\alpha = 0.65$	Standard value
Fraction of debtors	$n = 0.6044$	Share of constrained consumption = 58%
Labor endowment	$\bar{L} = 1$	Normalization
Initial debt per debtor	$B_0 = 1.6546$	Debt/GDP in initial steady state = 100%
Inflation target	$\bar{\pi} = 1.02$	Fed inflation target
Wage rigidities	$\phi_0 = 1$	At full employment wages cannot fall
	$\phi_1 = 0.3939$	Wages fall by 2 percent per year at 5 percent unemployment

Table 2.1: Parameters

set to $\gamma = 2$, a standard value in the real business cycle literature. The labor share is set to $\alpha = 0.65$, consistent with US data.

The fraction of debtors is set to $n = 0.6044$ to target a share of constrained consumption in the initial steady state of 58 percent, which is the same target used by Hall (2011). Moreover, I normalize the labor endowment to one $\bar{L} = 1$ and set the initial debt per borrower to $B_0 = 1.6546$, so as to target a debt-to-GDP ratio in the initial steady state of 100 percent. This is the household debt-to-GDP ratio in the US in 2007. The inflation target is set to $\bar{\pi} = 1.02$, in line with the Fed's definition of price stability.

To model wage rigidities I adopt the same functional form proposed by Schmitt-Grohé and Uribe (2012) and assume

$$\phi(u) = \phi_0 (1 - u_t)^{\phi_1} .$$

I set $\phi_0 = 1$, so that in absence of involuntary unemployment nominal wages cannot fall. Following Schmitt-Grohé and Uribe (2012), I set ϕ_1 so that at an unemployment rate of 5 percent nominal wages can fall frictionlessly by 2 percent per year. This target implies $\phi_1 = 0.3939$.

Figure 4.2 displays the transitional dynamics following a deleveraging shock calibrated so that the debt-to-GDP ratio in the final steady state is equal to 94 percent. The solid lines refer to an economy without transfers. The drop in the borrowing limit forces debtors to deleverage and so the debt-to-GDP ratio falls. The central bank responds to the deleveraging shock by lowering the nominal interest rate and the economy falls into a liquidity trap that lasts one period. Inflation undershoots its target, and, due to the presence of nominal wage rigidities, real wages rise generating involuntary unemployment. Aggregate consumption falls, and the fall in consumption is particularly sharp for debtors.[18] Since we are considering a mild

[18] In fact, in this example creditors' consumption rises during the liquidity trap. However, there are cases in which the fall in output during the liquidity trap is so severe that also creditors' consumption falls.

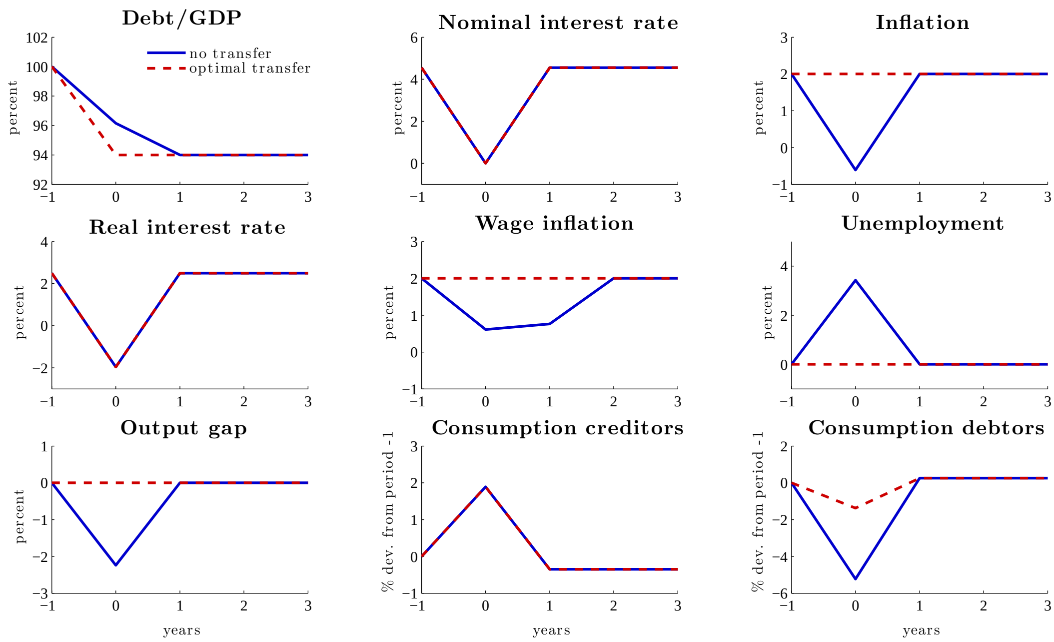


Figure 2.2: Deleveraging and Pareto optimal transfer during a mild recession.

recession, inflation goes back to target and the economy is at full employment starting from period 1.

The dashed lines in figure 4.2 illustrate the impact of the Pareto optimal transfer. The transfer stimulates debtors' consumption and this has a positive impact on aggregate demand. The increase in aggregate demand brings the economy to full employment, thus closing the output gap and leaving inflation equal to the target. Creditors' consumption is not affected by the transfer because the transfer has no impact on creditors' expected consumption and on the real interest rate during the liquidity trap. Finally, though the optimal transfer restores full employment it does not lift the economy out of the liquidity trap, and the nominal interest rate hits the zero lower bound during period 0.

2.4.2 Debt relief during large recessions

I now turn to the more complex case of large recessions. Large recessions generate a trade-off between inflation and employment during the recovery and so the strategy followed by the central bank has an impact on how the economy behaves once the liquidity trap is over. Moreover, since households are forward looking monetary policy decisions that affect the recovery also have an impact on the behavior of the economy during the liquidity trap. Because of this, the impact of transfers on employment and welfare during large recessions crucially depends on whether the central bank targets inflation or employment. To illustrate this point I start by analyzing the case of a central bank that targets employment, before turning to a central bank

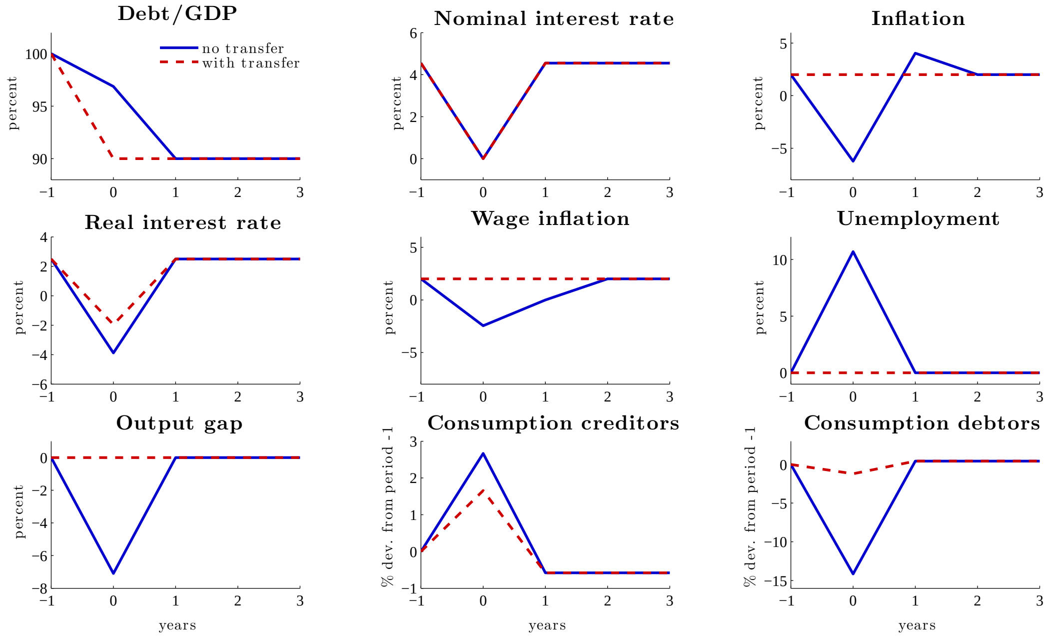


Figure 2.3: Deleveraging and impact of transfer during a large recession with employment targeting.

that targets inflation.

Employment targeting

Suppose that the economy is in a large recession and that the central bank targets employment. Then during the recovery the central bank overshoots its inflation target, so as to make real wages fall to a level consistent with full employment. In turn, the inflation burst that the economy experiences during the recovery leads to a lower real interest rate during the liquidity trap, thus mitigating the impact of the binding zero lower bound constraint on the economy. These dynamics are illustrated by the solid lines in figure 4.3, which show the response of the economy to a deleveraging shock sufficiently large so as to violate condition 4.[19]

The following proposition summarizes the impact of a transfer on an economy undergoing a large recession with a central bank targeting employment.

Proposition 6 *Assume that condition* 3 *holds, that condition* 4 *is violated and that the central bank targets employment. Then a marginal transfer has an expansionary impact on employment. Moreover, a marginal transfer cannot lead to a Pareto improvement in welfare.*

Proof. See appendix. ■

As in the case of mild recessions, a transfer is expansionary because it transfers wealth to debtors, who have a higher propensity to consume out of wealth than creditors. Thus the

[19]The shock is calibrated so that the final steady state features a debt-to-GDP ratio of 90 percent. The other parameters are kept as in section 2.4.1.

transfer stimulates aggregate demand, relaxes the zero lower bound constraint on the nominal interest rate and raises inflation and employment.

However, a transfer unambiguously reduces creditors' consumption during the liquidity trap and so it cannot lead to a Pareto improvement in welfare. The intuition is as follows. A transfer stimulates employment during the trap, thus leading to an increase in L_0. Combining firms' optimality conditions in periods 0 and 1 with the wage setting equation in period 1 and the condition $L_1 = \bar{L}$ gives a relationship between period 1 inflation π_1 and employment during the liquidity trap L_0

$$\pi_1 = \phi(0)\left(\frac{\bar{L}}{L_0}\right)^{1-\alpha}.$$

This expression implies that an increase in L_0 leads to a reduction in expected inflation π_1. This happens because a rise in L_0 limits the fall in prices during the trap, thus limiting the rise in future inflation needed to reduce real wages to a level consistent with full employment. In turn, the fall in expected inflation leads to a rise in the real interest rate during the liquidity trap, which induces creditors to cut their consumption. Hence, if the central bank targets employment a transfer during a large recession generates a fall in creditors' consumption and it cannot be Pareto improving.

This point is illustrated by the dashed lines in figure 4.3, which display the impact of a transfer that restores full employment. The transfer leads to an increase in employment, but it also causes a fall in expected inflation that induces creditors to reduce their consumption. The result is that the Pareto optimal transfer in the case of a large recession with employment targeting is equal to zero.[20]

Inflation targeting

Under inflation targeting deleveraging tends to generate deeper recessions than under employment targeting. This happens because during a liquidity trap expected inflation is lower, and thus the real interest rate is higher, if the central bank follows a policy of inflation targeting. Through this channel, targeting inflation deepens the shortage of aggregate demand and the fall in output during a liquidity trap compared to a policy of employment targeting.

Perhaps more worryingly, targeting future inflation during a liquidity trap opens the door to multiple equilibria. To grasp the intuition behind this result it is useful to express the behavior of the economy during a liquidity trap under inflation targeting in terms of aggregate

[20] Of course, this does not necessarily mean that a transfer is not desirable on welfare terms. In fact, the transfer generates an increase in debtors' consumption and welfare. Depending on the weights that society attaches to the welfare of debtors and creditors a transfer might have a positive impact on aggregate welfare.

supply and demand schedules. To derive an aggregate supply (AS) schedule combine firms' optimality conditions in periods 0 and 1 with the wage setting equation in period 1 and the condition $\pi_1 = \bar{\pi}$ to obtain

$$Y_0 = Y_1 \left(\frac{\phi(u_1)}{\bar{\pi}} \right)^{\frac{\alpha}{1-\alpha}},$$

where $Y_t = L_t^\alpha$ denotes aggregate output. The AS curve implies a positive relationship between current and future output during a liquidity trap. Intuitively, lower production during the trap is associated with lower inflation and higher real wages. Since the inflation rate in period 1 is given by the inflation target and the adjustment in wages is constrained by the downward rigidities, also real wages in period 1 are increasing in Y_0. Hence lower output in period 0 is associated with higher real wages and lower output in period 1, creating a positive relationship between Y_0 and Y_1.

The aggregate demand (AD) schedule can be derived rearranging the Euler equation for creditors and imposing $r_0 = 1/\bar{\pi} - 1$

$$Y_0 = U'^{-1} \left(\frac{\beta}{\bar{\pi}} U' \left(Y_1 + \frac{n}{1-n} \frac{r_1 \bar{\kappa}}{1+r_1} \right) - \frac{n}{1-n} \left(B_0 - \bar{\pi}\bar{\kappa} - T \right) \right).$$

Also the AD curve describes a positive relationship between Y_0 and Y_1.[21] Intuitively, if creditors expect income to be higher in period 1, i.e. a higher Y_1, they also anticipate that period 1 consumption will be higher and so their demand for consumption in period 0 increases. This in turn stimulates aggregate demand during the liquidity trap, leading to a higher production, i.e. a higher Y_0.

Combining the AS and AD curves together can generate multiple equilibria during a liquidity trap. Suppose that agents expect future output to be high. Then they will want to consume more during the trap and also output during the trap will be high. In turn a high output during the trap validates expectations of a high future output because it implies lower real wages during the liquidity trap, which lead to lower future real wages and higher future production.

Figure 2.4 illustrates two possible shapes of the AS and AD curves. The solid lines refer to the AS curve, while the dashed lines refer to the AD curve in the absence of transfers.[22] The left panel is obtained using the same parameters as in figure 4.3. In this case the curves intersect only once and so the equilibrium is unique. The right panel captures the possibility of multiple equilibria. In this example all the parameters are kept as in the example on the

[21] This is because C_1^c is increasing in Y_1, despite the fact that r_1 is decreasing in Y_1. See the proof of proposition 2.

[22] The AD curves are truncated because for some values of Y_1 the model does not have a solution.

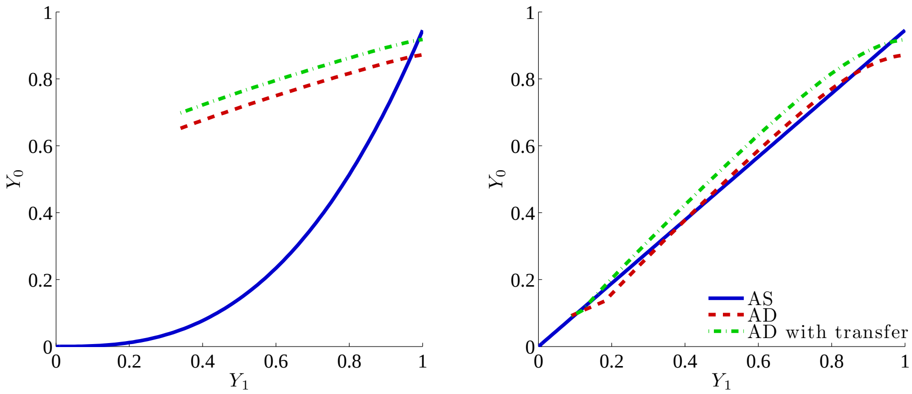


Figure 2.4: Multiple equilibria and transfers under inflation targeting.

left panel, except that $\phi_1 = 0$ so that wages do not respond to unemployment. In this case the curves intersect three times and so there are three possible equilibria.

The impact of a marginal transfer on employment is potentially ambiguous. This is illustrated by the dash-dotted lines in figure 2.4. Graphically, a transfer makes the AD curve shift up. In the case depicted by the left panel the transfer unambiguously leads to an expansion in output and employment during the liquidity trap. However, in the case depicted in the right panel the impact of the transfer on output and employment is a priori ambiguous. In fact, even with the transfer there are three possible equilibria, and the impact of the transfer on employment depends on the starting equilibrium and on how the transfer affects expectations. This suggests that implementing a policy of debt relief during a liquidity trap might have a perverse impact on employment if the central bank follows a policy of inflation targeting.

One implication of this result is that implementing a transfer such as the one described in proposition 5 might not restore full employment, because other equilibria might be consistent with that transfer in addition to the full employment one. Luckily, it is possible to design transfer schemes that eliminate multiple equilibria and lead to full employment.

Proposition 7 *Suppose that condition* 3 *holds and that the central bank targets inflation. Then a transfer scheme defined as*

$$T = \tilde{T} + \chi \left(L_0^\alpha - \bar{L}^\alpha \right)$$

where $\chi > (1-n)/n$ *and* $\tilde{T}$ *solves*

$$U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - \tilde{T}\right)\right) = \frac{\beta}{\bar{\pi}} U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

restores full employment and leads to a Pareto improvement in welfare.

Proof. In the appendix. ∎

Proposition 7 describes the transfer scheme involving the smallest transfer from creditors to debtors consistent with full employment. The transfer described in proposition 7 is decreasing in output. Intuitively, multiple equilibria arise because expectations of low future output translate into weak aggregate demand by creditors leading to low output during the trap. The transfer reduces the response of creditors' demand to changes in expected future output, ruling out multiple equilibria.

The proposition also states that an appropriately designed transfer leads to a Pareto improvement in welfare.[23] The key to this result is the fact that the transfer produces an increase in output during the trap, which during a large recession generates an increase in future output and future consumption. The expectation of higher future consumption, and the fact that the interest rate is given by $r_0 = 1/\bar{\pi} - 1$ and not affected by the transfer, stimulates creditors' consumption during the trap. Hence creditors' consumption stream increases following the transfer. Debtors experience an even larger increase in their consumption stream, because this indirect effect is complemented by the direct increase in income due to the transfer. Hence, the transfer makes both creditors and debtors better off.

These effects are illustrated by figure 2.5.[24] Without transfers deleveraging generates a deep and persistent recession. Unemployment is persistent because with inflation equal to the target it takes a few periods for wages to fall back to a level consistent with full employment. Instead, the transfer described in proposition 7 restores full employment. Moreover, the transfer has a positive impact both on creditors' and debtors' consumption.

Deriving the Pareto optimal transfer, defined as the transfer that maximizes debtors' welfare leaving creditors at least as well off as in the initial equilibrium, in the case of large recessions with inflation targeting can be cumbersome and I will leave it to future research.[25] Here I just notice that the Pareto optimal transfer is larger than $\tilde{T}$, the smallest transfer that restores full employment. As shown in the proof to proposition 7, under that transfer both creditors and debtors are better off compared to an equilibrium with large recession and no transfer. Hence, a marginal increase in the transfer generates a rise in debtors' utility, while leaving creditors still better off compared to the equilibrium without transfer. This also implies that

[23] Notice that by invoking condition 3 proposition 7 does not refer to sunspot liquidity traps, that is cases in which a liquidity trap equilibrium coexist with an equilibrium in which the zero lower bound constraint is not binding. I do not address this case because, despite an extensive search, I could not find a parameter configuration that leads to sunspot liquidity traps. In the case of sunspot liquidity traps the proposition should be qualified by acknowledging that the transfer scheme proposed implies a Pareto improvement in welfare with respect to equilibria in which the zero lower bound constraint is binding.

[24] The parameters are the same as in figure 4.3. Under this parameter configuration the equilibrium without transfer is unique.

[25] The main difficulty comes from the fact that under the Pareto optimal transfer the borrowing limit may not be binding for debtors, which complicates significantly the analysis.

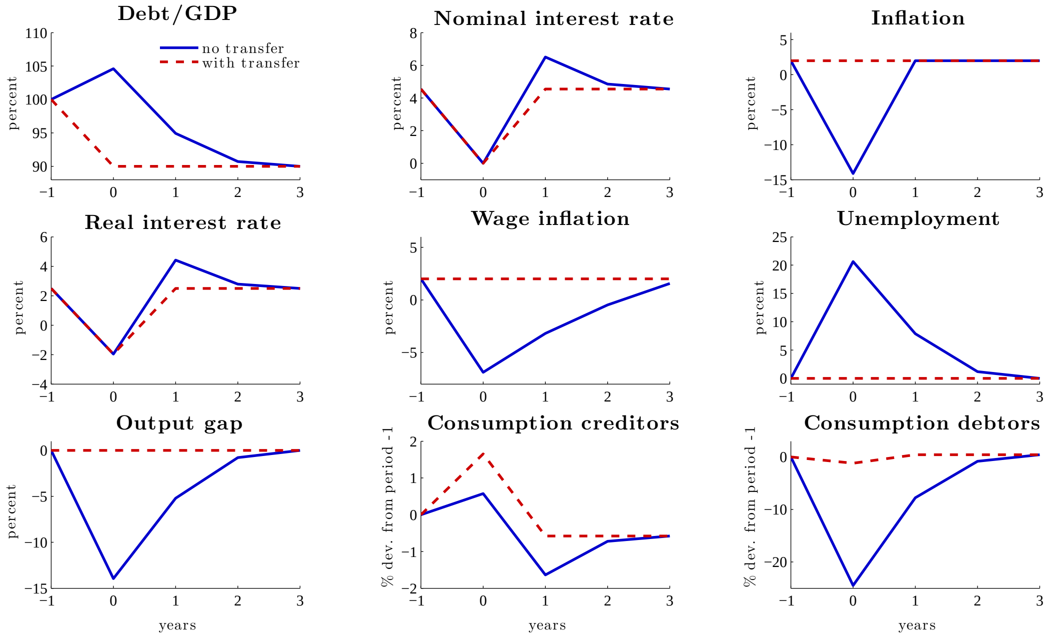


Figure 2.5: Deleveraging and impact of transfer during a large recession with inflation targeting.

the Pareto optimal transfer lifts the economy out of the liquidity trap, because for creditors' Euler equation to hold the interest rate must satisfy $r_0 > 1/\bar{\pi} - 1$ if $T > \tilde{T}$.[26]

Summarizing, the case for debt relief policies during a liquidity trap is particularly strong if the central bank follows a policy of inflation targeting. Not only a transfer can lead to an increase in welfare both for creditors and debtors, but an appropriately designed transfer scheme can also eliminate the possibility of multiple equilibria.

2.5 Extensions

I now consider a few extensions to the baseline model. I start by analyzing the case of a central bank that conducts monetary policy according to an interest rate rule. I then consider the role of disutility from working. I conclude this section with a discussion of the similarities and differences between debt relief in a closed economy and in a monetary union.

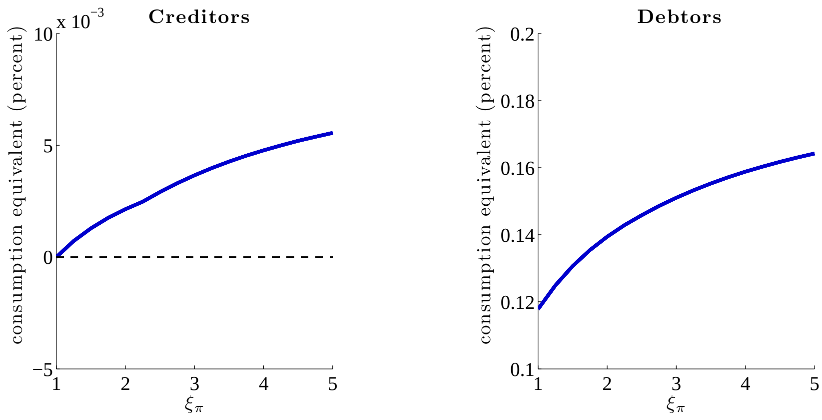


Figure 2.6: Welfare gains from transfer with interest rate rule.

2.5.1 Interest rate rule

One popular way of modeling monetary policy is through interest rate rules. In this section I consider a central bank that sets the policy rate according to the simple rule

$$1 + i_t = \max\left(1, (1+i)\left(\frac{\pi_t}{\bar{\pi}}\right)^{\xi_\pi}\right), \tag{2.15}$$

where $i = (1+r)\bar{\pi} - 1$ is the steady state nominal interest rate and $\xi_\pi \geq 1$ is a parameter determining how aggressively the central bank responds to deviations of inflation from the target. A higher value of ξ_π is associated with a stronger aversion to inflation variability, and when $\xi_\pi \to \infty$ the central bank is effectively implementing a policy of inflation targeting. Notice that the rule takes into account the fact that monetary policy is constrained by the zero lower bound on the nominal interest rate.[27]

Based on the analysis of section 2.4.2, one could conjecture that a transfer is more likely to lead to a Pareto improvement in welfare the more aggressively the central bank responds to deviations of inflation from the target, i.e. the higher ξ_π. In this section I show that this conjecture is correct.

Considering a central bank that conducts monetary policy according to rule (2.15) makes it difficult to derive analytical results, hence I will resort to numerical simulations. To investigate whether a transfer is more likely to be Pareto improving the more aggressively the central bank responds to inflation, I compute the welfare gains for creditors and debtors from a transfer

[26] Instead, characterizing the transfer that maximizes creditors' welfare leaving debtors at least as well off as in the initial equilibrium is easier. Indeed, this happens when the transfer described in proposition 7 is implemented.

[27] Benhabib et al. (2001) show that an interest rate rule such as the one described in expression (2.15) can give rise to expectation-driven liquidity traps. I consider a central bank that is able to avoid expectation-driven liquidity traps, for instance by implementing the exit strategy proposed by Schmitt-Grohé and Uribe (2012).

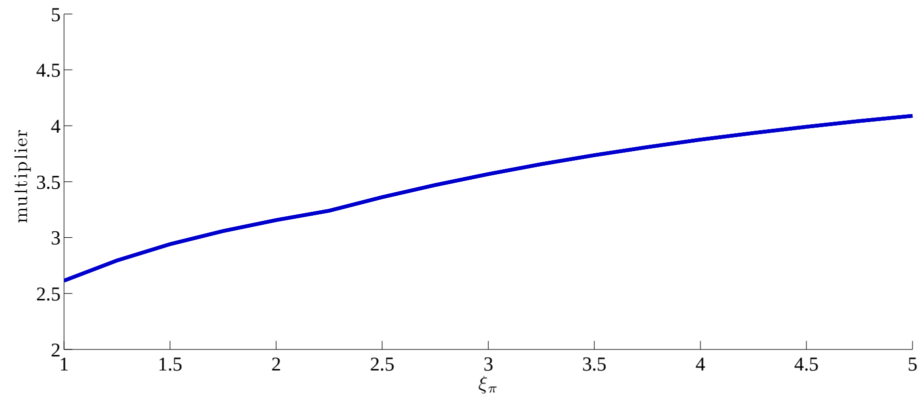


Figure 2.7: Impact on output of transfer with interest rate rule.

equal to 1 percent of full employment GDP, that is $T = 0.01\bar{L}^\alpha/n$, for a range of values of ξ_π given a shock that pushes the economy into a large recession.[28] I compute the welfare gains from implementing a transfer as the proportional increase in the consumption stream that a household living in the economy with no transfer must receive in order to be indifferent between remaining in the no-transfer economy and switching to an economy with the transfer.[29]

Figure 2.6 shows the results. The transfer leads to a Pareto improvement in welfare for any value of $\xi_\pi > 1$.[30] Moreover the welfare gains of both debtors and creditors are increasing in ξ_π, confirming the conjecture that a transfer is more likely to lead to a Pareto improvement in welfare the more aggressively the central bank responds to deviations of inflation from the target.

This result is due to the fact that the impact of a transfer on output is larger the more the central bank is concerned with stabilizing inflation. This happens because the speed of the recovery from a large recession is increasing in inflation. To illustrate this point I computed the transfer multiplier, defined as

$$\frac{\mathbf{Y}^T - \mathbf{Y}^{NT}}{nT},$$

where

$$\mathbf{Y} = \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t},$$

is the present value of output and the superscripts T and NT denote respectively allocations

[28]The other parameters are the same as in figure 4.3.

[29]Formally, the welfare gain η^i for $i = c, d$ is defined as

$$\sum_{t=0}^{\infty} \beta^t U\left(\left(1+\eta^i\right) C_t^{i,NT}\right) = \sum_{t=0}^{\infty} \beta^t U\left(C_t^{i,T}\right),$$

where the superscripts NT and T denote allocations respectively in the economy without and with transfer.

[30]If $\xi_\pi = 1$ the transfer makes debtors better off, but it leads to a small welfare loss for creditors.

with and without transfer.[31] Figure 2.7 shows that the multiplier increases with ξ_π, so that the impact of the transfer on output is larger the more aggressively the central bank responds to deviations of inflation from the target.

2.5.2 Disutility from working

In the baseline model households do not experience disutility from working, a typical assumption in the literature on involuntary unemployment.[32] However, the literature on monetary policy commonly assumes that households experience disutility from working and that the labor supply is elastic.[33]

The presence of disutility from working makes it less likely for a debt relief policy to produce a Pareto improvement in welfare. This happens because creditors need to be compensated not only for the loss in wealth due to the transfer, but also for the disutility due to the increase in labor effort. However, the presence of disutility from working does not eliminate the possibility of Pareto improving transfers.

To make this point I use a numerical example. Suppose that households experience disutility from working during period 0. Specifically assume that the lifetime utility of a household is given by

$$\frac{C_0^{i\,1-\gamma}-1}{1-\gamma}-\psi\frac{L_0^{i\,1+\theta}}{1+\theta}+\sum_{t=1}^{\infty}\beta^t U\left(C_t^i\right),$$

where $\psi > 0$ is a parameter determining the disutility from working and $\theta \geq 0$ determines the elasticity of labor supply. Notice that to simplify the analysis I assume that labor disutility arises only during period 0, while from period 1 on the model is exactly identical to the baseline.

The solid lines in figure 2.8 illustrate the impact on welfare of a transfer equal to 1 percent of full employment GDP as a function of the elasticity of labor supply θ. The economy is hit by a shock large enough to generate a large recession and the central bank targets inflation. For each value of θ I calibrated ψ so that given the pattern of consumption in the initial steady state aggregate labor is exactly equal to $\bar{L}$.[34] For comparison, the dashed lines show the welfare gains from the transfer in the baseline model without disutility from working.

Figure 2.8 shows that introducing disutility from labor does not eliminate the possibility of Pareto improving transfers. The figure also shows that, perhaps unsurprisingly, a Pareto

[31] I discount output with the steady state real interest rate because I want to abstract from the impact of the transfer on the interest rate.

[32] See Pissarides (2000).

[33] An example of a monetary model with involuntary unemployment and disutility from labor effort is Erceg et al. (2000).

[34] The other parameters are the same as in figure 2.

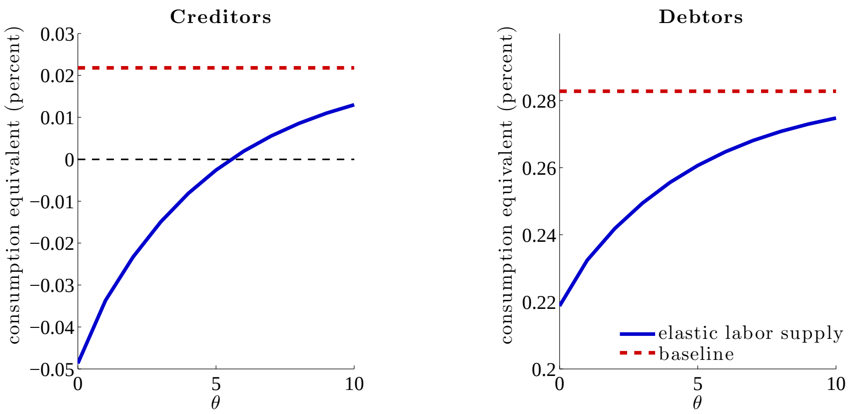


Figure 2.8: Welfare gains from transfer with disutility from working.

improvement in welfare is more likely to materialize the more inelastic the labor supply, i.e. the higher θ. Indeed, for this particular numerical example the transfer is Pareto improving for values of θ greater than 5.

It is important to stress that the model with elastic labor supply is likely to bias downward the gains from a policy of debt relief. The reason is that the preferences considered in this section threat equally voluntary and involuntary leisure. Instead, the empirical evidence suggests that involuntary leisure has a negative impact on welfare.[35]

2.5.3 Debt relief policies in monetary unions

Though the model describes a closed economy, its fundamental insights apply to the case of a monetary union undergoing an episode of deleveraging, as long as countries are heterogeneous in their debt positions.[36] In particular, a transfer from creditor to debtor countries should lead to an economic expansion and possibly to a Pareto improvement in welfare, especially if the central bank of the union is mainly concerned with targeting inflation.[37]

However, there is an important difference between the case of a closed economy and a monetary union. In fact, in a closed economy a benevolent government will implement a policy

[35]See Winkelmann and Winkelmann (1998). Moreover, there is empirical evidence suggesting that, everything else held constant, people living in countries with a lower unemployment rate are happier, as documented by Di Tella et al. (2001).

[36]Indeed, from a modeling perspective the only difference would be that in a monetary union in which labor is immobile across countries differences in wages could arise. See Benigno and Romei (2012) and Fornaro (2012b) for models of deleveraging in monetary unions.

[37]This seems to fit the case of the Eurozone well. In the Eurozone a group of countries, the periphery, is characterized by high foreign debt and is undergoing a period of private debt deleveraging, while the rest of the union, the core, has low foreign debt, or even a positive stock of foreign assets, and is not experiencing a contraction in credit. Moreover, the mandate of the European Central Bank is to maintain price stability. The analysis above suggests that in this case a transfer from the core to the periphery should lead to an expansion in output and potentially to a Pareto improvement in welfare.

of debt relief if this leads to a Pareto improvement in welfare. This might not be the case in a monetary union. To see this point, imagine a monetary union composed of a continuum of countries, each one of them being infinitesimally small. In this world, a creditor country does not have an incentive to unilaterally forgive its debtors. In fact, being infinitesimally small a single country does not take into account the impact of its actions on aggregate demand and output. Hence, in a monetary union the implementation of a Pareto improving debt relief policy requires coordination across member countries. I am exploring these coordination issues in ongoing research.

2.6 Conclusion

Debt deleveraging can push the economy into a liquidity trap characterized by involuntary unemployment and low inflation. During these episodes, debt relief policies lead to an expansion in employment and output and can benefit both creditors and debtors.

One natural direction in which the analysis could be extended is to consider the impact of debt relief on moral hazard. In fact, the anticipation of a future debt relief might give an incentive to borrowers to increase their debt during times in which access to finance is plentiful. Moral hazard could thus partly counteract the positive impact of debt relief on welfare, and the interactions between the two represents a fruitful area for future research.[38]

[38] See Bianchi (2012) for some recent work on the interaction between bailouts and moral hazard.

Chapter 3

Financial Crises and Exchange Rate Policy

3.1 Introduction

Since the financial liberalization wave of the 1980s, several countries have experienced financial crises characterized by sudden arrests of international capital inflows and sharp drops in output, consumption and asset prices.[1] These episodes, known as sudden stops, have sparked great interest in the design of monetary and exchange rate policies in financially fragile economies. Should these economies let their exchange rate float or rather anchor it to a foreign currency? Should monetary policy be concerned only with its traditional objective of granting price stability or should it also care about financial stability?

In this paper, I address these questions focusing on a pecuniary externality originating from frictions on the international credit markets. I present a theoretical framework that shows how the combination of financial frictions and nominal rigidities gives rise to a trade-off between financial and price stability. My main result is that a narrow focus on price stability can lead to a sub-optimal monetary policy in sudden stop-prone economies.

I study a small open economy with imperfect access to the international financial markets. Domestic agents borrow from foreign investors against collateral. Collateral consists in a physical asset used in production, land, valued at market price. When the collateral constraint binds a financial accelerator mechanism akin to Fisher's debt deflation arises: aggregate demand for land falls, the price of land drops and collateral declines. Since domestic agents are atomistic, they do not take into account the general equilibrium effect of their actions on the price of

[1]Diaz-Alejandro (1985) is the classic reference on the link between financial liberalization and financial crises in emerging economies. Calvo et al. (2004) provide an overview of the facts characterizing sudden stop events.

land and on the value of their collateral. This is the pecuniary externality that creates scope for policy interventions in the financial markets.

Wages are nominally rigid.[2] During a financial crisis nominal wages fail to adjust downward, potentially worsening the impact of financial turmoil on the real economy. The central bank can mitigate the downturn associated with a financial crisis by engineering an exchange rate depreciation that increases the competitiveness of the economy. Importantly, the stimulus provided by an exchange rate depreciation has a positive effect on the aggregate demand for land and on the value of collateral. Through this channel, exchange rate policy affects domestic agents' access to the international credit markets during crisis events.

Many narratives of financial crisis episodes have given a central role to the interaction between capital flows, asset prices and wage rigidities. Consider the recent events in the Eurozone periphery. Prior to 2008, several European countries underwent a period characterized by fast build-up of foreign debt. Rising real estate prices likely contributed to the credit boom, since housing represents an important source of collateral. Conversely, the crisis that followed has been characterized by a vicious cycle of falling capital inflows and plummeting asset prices.[3] In addition, many commentators have argued that the combination of rigidities in wage setting and fixed exchange rates has exacerbated the severity of the crisis.[4] This is the kind of episodes that the model is meant to capture.

I use the model to compare the performance of three alternative monetary rules: a fixed exchange rate rule and two types of floating exchange rate regimes. The first type of float considered is a policy of strict wage inflation targeting. This rule eliminates all the distortions arising from nominal wage stickiness and corresponds to the price stability rule of closed-economy sticky price models. The second type of float is a policy of flexible exchange rate targeting in which the central bank intervenes to smooth out deviations of the exchange rate from a target. This rule parallels flexible price level targeting rules in closed-economy models

[2]A growing body of evidence emphasizes how nominal wage rigidities represent a key transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Christiano et al. (2005) using an estimated medium-scale DSGE model of the US economy. Moreover, Olivei and Tenreyro (2007) show that monetary policy shocks in the US have a bigger impact on output if they occur during the first or second quarter of the year. They argue that this finding can be explained with the fact that most US firms adjust wages during the fourth quarter, and hence wages tend to be more rigid during the first half of the year. There is also evidence describing the role of nominal wage rigidities in exacerbating the downturn during financial crises, especially if coupled with fixed exchange rates. This point is made by Eichengreen and Sachs (1985) and Bernanke and Carey (1996) in the context of the Great Depression, while Schmitt-Grohé and Uribe (2011) document the importance of wage rigidities for the 2001 Argentine crisis and for the 2008-2009 recession in the Eurozone periphery. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2010) and Fabiani et al. (2010).

[3]McKinsey (2010) and Merler and Pisani-Ferry (2012) describe the accumulation of debt, especially foreign debt, in countries at the Eurozone periphery during the run up to the 2008 financial crisis and the subsequent sudden stop in capital inflows, giving rise to deleveraging by the private sector.

[4]This point is forcefully made by Feldstein (2010) and Krugman (2010).

and represents a simple alternative to wage inflation targeting. In addition, this rule is interesting because it implies a more expansionary monetary policy stance during crisis events compared to the strict wage inflation targeting rule.

The main result of the paper concerns the role of financial frictions in determining the welfare ranking between strict wage inflation targeting and the flexible exchange rate targeting rule. I show that in a version of the model in which the collateral constraint is replaced by a fixed borrowing limit, and hence in which Fisher's debt deflation channel is shut down and financial crises are not present, the strict wage inflation targeting rule delivers higher welfare gains than the flexible exchange rate targeting rule for any initial state of the world. This finding is in line with the well known result that, in models in which the only distortions come from monopolistic competition and from nominal rigidities, a policy that corrects for nominal rigidities approximates well the optimal policy.[5]

I then show that the pecuniary externality implied by the Fisherian deflation mechanism has the potential to change the welfare ranking among the policy rules considered. In fact, once the Fisherian deflation mechanism is introduced the initial stock of foreign assets owned by domestic households becomes a key determinant of the welfare ranking. For high levels of net foreign assets the probability of a future crisis is small and a policy of targeting wage inflation is preferred, due to its good performance in managing normal business cycle fluctuations. For low levels of net foreign assets the risk of a crisis is high and flexible exchange rate targeting becomes the preferred regime, since it does a better job in mitigating the fall in the price of land and in capital inflows during crisis events compared to the wage inflation targeting rule. In contrast, the peg is always welfare dominated by the other two rules. This happens because during tranquil times the peg does not remove the distortions due to wage stickiness, while during crisis times pegging the exchange rate amplifies the fall in the price of land and in capital inflows compared to the other two regimes.

A second set of results concerns the impact of the monetary regime on precautionary savings and crisis probability. The currency peg is the regime that stimulates more the accumulation of precautionary savings, followed by the policy of targeting wage inflation and by the flexible exchange rate targeting rule. The intuition is simple: the more crises disrupt economic activity, the more agents accumulate precautionary savings to reduce the risk of experiencing a sudden stop. Since the peg is the regime under which crises have the strongest impact on output and consumption, the peg is also the regime under which the accumulation of precautionary savings

[5]Kollmann (2002) and Schmitt-Grohé and Uribe (2007) derive this result using models with monopolistic competition in the product market and nominal price rigidities. However, a similar logic should apply to models with monopolistic competition in the labor market and in which the presence of sticky wages is the only source of nominal rigidities.

is stronger. Moreover, since crises are milder when the central bank adopts a flexible exchange rate targeting rule, agents accumulate less precautionary savings under flexible exchange rate targeting than under a policy of strict wage inflation targeting. The outcome is that the currency peg is the regime featuring the lowest crisis probability, while the probability of experiencing a sudden stop is highest under a policy of flexible exchange rate targeting.

This paper is related to two strands of the literature. The first one focuses on the design of monetary policy in financially fragile small open economies. Cespedes et al. (2004), Moron and Winkelried (2005) and Devereux et al. (2006) compare the performance of different monetary regimes in small open economies featuring financial market imperfections. Contrary to this paper, their models focus on business cycle fluctuations and are not suited to study economies occasionally subject to financial crises. Christiano et al. (2004), Cook (2004), Gertler et al. (2007), Braggion et al. (2007) and Curdia (2007) all use quantitative models to analyze the impact of monetary policy interventions during crisis times. In their frameworks crises are unexpected one-shot events, while this paper presents a model in which crises alternate with tranquil times and crisis probabilities are rationally anticipated by agents. This allows the analysis of the impact of monetary policy on the probability of entering a crisis, an issue on which the existing literature is silent. Moreover, this literature typically finds that the presence of financial frictions does not alter the welfare ranking among monetary policy rules, while the key insight of this paper is that financial frictions are a key determinant of which policy rule delivers higher welfare. Aghion et al. (2004), Caballero and Krishnamurthy (2003), Bordo and Jeanne (2002) and Benigno et al. (2011b) consider monetary economies featuring both tranquil periods and crises. However their focus is on static models, while the dynamics of debt accumulation play a key role in the model presented in this paper.[6] Finally, this paper shares with Schmitt-Grohé and Uribe (2011) the focus on the performance of different exchange rate regimes in economies subject to the risk of experiencing a deep recession. However, in their model recessions are exogenous events and there is no financial amplification, while in this model the probability of entering a crisis is endogenous and the interaction between the exchange rate regime and Fisher's debt deflation is key.

The second strand of related literature employs dynamic real business cycle models featuring occasionally binding credit constraints and financial accelerator mechanisms to describe economies prone to sudden stops and to draw implications about policy conduct in small open economies. Examples are Mendoza (2010), Bianchi (2011), Benigno et al. (2011a), Jeanne and Korinek (2010) and Bianchi and Mendoza (2010). The novelty of this paper with respect to

[6] I refer to these frameworks as static because they consider economies that last two or three periods, in which the stock of external debt at the onset of a crisis is essentially taken as an exogenous variable.

this literature resides in the focus on monetary policy and on the interplay between Fisher's debt deflation and nominal wage rigidities.

The rest of the paper is structured as follows. Section 2 describes the analytical framework. Section 3 presents the results using numerical simulations. Section 4 concludes.

3.2 Model

Consider an infinite-horizon small open economy. Time is discrete and indexed by t. The economy is populated by a continuum of mass 1 of households that consume a single tradable good and engage in financial transactions with foreign investors. There is also a large number of competitive firms that produce the consumption good using factors of production supplied by the households and a central bank that uses the interest rate on domestic bonds as its policy instrument.

3.2.1 Firms and production

Firms are owned by the households. They are competitive, take all prices as given and produce the tradable consumption good according to the production function

$$Y_t = z_t F(L_t, M_t, K_t), \tag{3.1}$$

where Y_t denotes output, $F(\cdot)$ is a decreasing-returns-to-scale production function and z_t is a total factor productivity (TFP) shock.[7] The productivity shock follows a finite-state, stationary Markov process and represents the only source of uncertainty in the model. Firms produce using labor L_t, an intermediate input M_t and land K_t. All the factors of production are purchased or rented from domestic households.

As in Obstfeld and Rogoff (2000), each household supplies a differentiated labor input. L_t is a CES aggregate of the differentiated labor services

$$L_t = \left[\int_0^1 L_t^{i\,\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where L_t^i denotes the labor input purchased from household i and $\sigma > 1$.

Purchasing power parity holds so $P_t = S_t P_t^*$. P_t and P_t^* are respectively the domestic and foreign currency price of the consumption good. S_t denotes the nominal exchange rate, defined

[7]Decreasing returns to scale in production can derive from the assumption that production also requires the input of managerial capital, of which each firm has a fixed supply normalized to 1.

as the units of domestic currency needed to buy one unit of the foreign currency. For simplicity, I assume that P_t^* is constant and normalize it to 1. Hence, the domestic currency price of the consumption good is equal to the nominal exchange rate $P_t = S_t$.

In every period, the representative firm maximizes profits

$$\Pi_t = S_t Y_t - \int_0^1 W_t^i L_t^i di - R_t^M M_t - R_t^K K_t, \tag{3.2}$$

where W_t^i is the wage rate of household i, R_t^M is the price of the intermediate input and R_t^K is the rental rate of land, all expressed in units of the domestic currency.

The minimum cost of a unit of aggregate labor L_t is given by

$$W_t = \left[\int_0^1 W_t^{i1-\sigma} di\right]^{\frac{1}{1-\sigma}},$$

which can be taken as the aggregate wage. Using this definition, profit maximization implies equality between factor prices and marginal productivities:

$$W_t = S_t z_t F_L(L_t, M_t, K_t) \tag{3.3}$$

$$R_t^M = S_t z_t F_M(L_t, M_t, K_t) \tag{3.4}$$

$$R_t^K = S_t z_t F_K(L_t, M_t, K_t), \tag{3.5}$$

where F_L, F_M and F_K are the derivatives of the production function respectively in L_t, M_t and K_t. Finally, cost minimization gives the demand for household's i labor

$$L_t^i = \left(\frac{W_t}{W_t^i}\right)^\sigma L_t. \tag{3.6}$$

3.2.2 Households

Households are the main actors in the economy. Each household derives utility from consumption C_t^i and experiences disutility from labor effort L_t^i. The lifetime utility of a generic household i is given by

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U\left(C_t^i, L_t^i\right)\right]. \tag{3.7}$$

In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time t and β is the subjective discount factor. The period utility function $U(\cdot)$ is assumed to be increasing in the first argument, decreasing in the second argument, strictly concave and twice continuously differentiable.

Each household can trade in one period, non-state contingent foreign and domestic bonds. The foreign bond is traded with foreign investors, it is denominated in units of the foreign currency and pays a fixed gross interest rate R^*, determined exogenously in the world market. The domestic bond is denominated in units of the domestic currency, pays the gross interest rate R_t and is traded only among domestic agents.[8] Moreover, households can purchase and sell units of land.

The budget constraint of household i in terms of the domestic currency can be written as

$$\begin{aligned} S_t C_t^i + S_t B_{t+1}^{*i} + B_{t+1}^i + Q_t(K_{t+1}^i - K_t^i) =& W_t^i L_t^i + R_t^K K_t^i + S_t R^* B_t^{*i} + R_{t-1} B_t^i + \\ & \Pi_t + \left(R_t^M - S_t P^M\right) M_t^i. \end{aligned} \tag{3.8}$$

The left-hand side of this expression represents the household's expenditure. This is given by the sum of consumption expenditure $S_t C_t^i$, investment in foreign bonds $S_t B_{t+1}^{*i}$, investment in domestic bonds B_{t+1}^i and net purchases of land $Q_t(K_{t+1}^i - K_t^i)$. Q_t is the price of land at time t in units of the domestic currency, while K_t^i denotes the household's holdings of land at the beginning of period t.

The right-hand side captures the household's income. $W_t^i L_t^i$ is the household's labor income, $R_t^K K_t^i$ is the income derived from renting land to firms, while $S_t R^* B_t^{*i}$ and $R_{t-1} B_t^i$ denote respectively the gross return on investment in foreign and domestic bonds made at time $t-1$. Π_t are the profits received from firms. Finally, the household imports from foreigners the intermediate input M_t^i and sells it to domestic firms. The world price of the intermediate input expressed in the foreign currency is constant and denoted by P^M. Hence, $R_t^M - S_t P^M$ is the return in units of the domestic currency that the household receives from purchasing one unit of the imported input from foreign producers and selling it to domestic firms.

A fraction ϕ of the intermediate input has to be paid at the start of the period and requires working capital financing. To finance the purchase of the imported input the household obtains a working capital loan from foreign investors at the start of the period and repays it at the end of the same period. I assume that the interest rate on these intra-period loans is zero.[9]

Foreign investors restrict loans so that total foreign debt, including both inter-temporal debt in one-period bonds and intra-period loans, does not exceed a fraction κ of the foreign

[8]This assumption is meant to capture the fact that in small open economies loans from foreign investors are most often denominated in a foreign currency.

[9]One could assume that intra-period loans pay an interest rate equal to R^*. This alternative formulation would not change in any way the key results of the paper.

currency value of the household's end of period land holdings

$$\phi P^M M_t^i - B_{t+1}^{*i} \leq \kappa \frac{Q_t}{S_t} K_{t+1}^i. \tag{3.9}$$

This constraint ensures that the loan-to-value ratio of domestic households does not exceed the limit κ.[10] This international collateral constraint is meant to capture in reduced form an environment in which informational and institutional frictions affect the credit relationship between domestic and foreign agents. A constraint of this form arises if land can be used as collateral to mitigate the frictions on the international credit markets. Domestic bonds are not subject to the collateral constraint since they are not traded by foreign investors.[11]

I introduce nominal rigidities by assuming that each household has to set its nominal wage W_t^i at the very start of the period, before the realization of the productivity shock z_t is known.[12] Each household acts as a monopolistic supplier of its labor input and sets its wage to maximize the expected present discounted value of utility (3.7), subject to the budget constraint (3.8) and firms' demand for its labor (3.6). The optimal wage satisfies

$$-E_{t-1}\left[U_L(C_t^i, L_t^i)L_t^i\right] = \frac{\sigma - 1}{\sigma} W_t^i E_{t-1}\left[\frac{U_C(C_t^i, L_t^i)}{S_t} L_t^i\right], \tag{3.10}$$

where $U_C(\cdot)$ and $U_L(\cdot)$ denote the derivative of the period utility function with respect to consumption and labor. At the margin, the expected disutility from an increase in labor effort, the left-hand side, is equal to the expected utility from higher revenue, the right-hand side.

Once wages are set, households are willing to satisfy firms' labor demand as long as the real wage, that is the wage expressed in units of the foreign currency, does not fall below the marginal rate of substitution between consumption and leisure

$$\frac{W_t^i}{S_t} \geq -\frac{U_L(C_t^i, L_t^i)}{U_C(C_t^i, L_t^i)}. \tag{3.11}$$

Given the pre-set wage and the realization of the productivity shock, each period the household chooses C_t^i, B_{t+1}^{*i}, B_{t+1}^i, K_{t+1}^i and M_t^i to maximize the expected present discounted value of utility (3.7), subject to the budget constraint (3.8) and the collateral constraint (3.9).

[10] Similar collateral constraints are widely used in the literature on sudden stops. Mendoza (2010) shows that models featuring this form of financing constraints can reproduce quantitatively well both business cycles and sudden stop episodes in emerging economies.

[11] The implications of segmented international and domestic financial markets is also explored, for example, in Caballero and Krishnamurthy (2001). For simplicity, here I abstract from frictions in the domestic credit market.

[12] The assumption that wages are set at the start of the period, rather than one period in advance, reduces significantly the computational costs involved by the global solution method used to solve the model numerically.

The optimality condition for B^i_{t+1} can be written as

$$\frac{U_C(C^i_t, L^i_t)}{S_t} = \beta R_t E_t \left[\frac{U_C(C^i_{t+1}, L^i_{t+1})}{S_{t+1}} \right]. \tag{3.12}$$

The optimal investment in domestic bonds is such that the marginal utility from spending one unit of domestic currency in period t consumption is equal to the expected marginal utility from investing one unit of domestic currency in domestic bonds and consuming the return in period $t+1$.

The optimal choice for B^{*i}_{t+1} is given by

$$U_C(C^i_t, L^i_t) = \beta R^* E_t \left[U_C(C^i_{t+1}, L^i_{t+1}) \right] + \mu^i_t, \tag{3.13}$$

where μ^i_t is the Lagrange multiplier on the collateral constraint, and by the complementary slackness condition

$$\mu^i_t \left(\kappa \frac{Q_t}{S_t} K^i_{t+1} - \phi P^M M^i_t + B^{*i}_{t+1} \right) = 0. \tag{3.14}$$

The left-hand side of expression (3.13) is the marginal utility from spending one unit of foreign currency in period t consumption. If the collateral constraint does not bind ($\mu^i_t = 0$) this is equated to the expected utility from investing one unit of foreign currency in foreign bonds and consuming the return in period $t+1$. When the collateral constraint binds ($\mu^i_t > 0$), B^{*i}_{t+1} is determined by the collateral that the household can offer to foreign investors, as stated by condition (3.14). In this case, the household is not free to borrow as much as it would like from foreign investors and the marginal utility of period t consumption is bigger than the expected marginal utility cost of borrowing on the international credit market.

Combining equations (3.12) and (3.13) gives

$$\beta R_t E_t \left[U_C(C^i_{t+1}, L^i_{t+1}) \frac{S_t}{S_{t+1}} \right] = \beta R^* E_t \left[U_C(C^i_{t+1}, L^i_{t+1}) \right] + \mu^i_t. \tag{3.15}$$

When the collateral constraint is not binding this equation is just the usual uncovered interest parity condition, which rules out arbitrage opportunities between domestic and foreign bonds. However, when $\mu^i_t > 0$ the uncovered interest parity condition breaks down and the expected return in terms of utility from investing in domestic bonds is greater than the expected utility from investing in foreign bonds. The presence of a spread between the cost of borrowing on the domestic market and the world interest rate in states in which the collateral constraint binds is due to the assumption that only foreign loans enter the collateral constraint.[13] Whether

[13]Intuitively, when the collateral constraint binds the household cannot borrow as much as it would like on

the spread materializes through an increase in the domestic interest rate, a movement of the exchange rate or a combination of both depends on the actions of the monetary authority.

The optimality condition for land K_{t+1}^i is

$$\frac{Q_t}{S_t}U_C(C_t^i, L_t^i) = \beta E_t\left[U_C(C_{t+1}^i, L_{t+1}^i)\frac{R_{t+1}^K + Q_{t+1}}{S_{t+1}}\right] + \frac{Q_t}{S_t}\kappa\mu_t^i. \tag{3.16}$$

The left-hand side is the marginal cost in terms of utility of an extra unit of land investment. The right-hand side captures the marginal benefit from increasing the household's land holdings. The first term is the marginal return in terms of utility of renting a unit of land to firms in period $t+1$ and selling it at the end of the period. The second term is the value that the household gets from relaxing the collateral constraint by increasing its stock of land.

The last first order condition gives the optimal choice of M_t^i:

$$R_t^M = S_t P^M\left(1 + \frac{\mu_t^i}{U_C\left(C_t^i, L_t^i\right)}\right). \tag{3.17}$$

When the collateral constraint does not bind the price at which the intermediate input is sold to domestic firms is equated to its world price expressed in units of the domestic currency. If the collateral constraint binds the amount of intermediate input that the household can import is limited by the value of its collateral. This shows up in the first order condition as an increase in the price of the imported input.[14]

3.2.3 Equilibrium

The solution is symmetric across households and in equilibrium individual and aggregate per capita variables are identical. For example aggregate consumption per capita C_t is given by

$$C_t = \int_0^1 C_t^i di = C_t^i, \tag{3.18}$$

where the last equality comes from the fact that each household makes the same choices in equilibrium. Similarly, in equilibrium the aggregate net foreign asset position of the economy B_t^* is such that

$$B_t^* = B_t^{*i}, \tag{3.19}$$

the international credit market. This induces the household to stand ready to pay a higher rate on domestic loans, because they are not subject to the collateral constraint.

[14]Through this channel an episode of binding collateral constraint is associated with disruptions in trade credit and inefficient use of imported inputs.

and the individual and aggregate wage coincide

$$W_t = W_t^i. \tag{3.20}$$

To derive the resource constraint of the economy, notice that since the domestic bond is traded only among domestic households its net supply must be equal to zero, i.e. equilibrium on the domestic bond market requires $B_t^i = 0$ for every t. The aggregate stock of land is assumed constant and equal to K, so that in equilibrium the households' net purchases of land must be zero. Using these equilibrium conditions, the expression for firms' profits (3.2) and the household's budget constraint (3.8) gives the aggregate resource constraint of the economy

$$C_t + B_{t+1}^* = Y_t - P^M M_t + R^* B_t^*. \tag{3.21}$$

This expression says that the aggregate expenditure of the economy, the sum of consumption plus investment in foreign bonds, must be equal to aggregate income, which is given by the sum of the gross domestic product $(Y_t - P^M M_t)$ plus the gross return on foreign bonds purchased during the previous period.

Finally, market clearing for the factors of production requires:

$$L_t = L_t^i \tag{3.22}$$

$$M_t = M_t^i \tag{3.23}$$

$$K_t = K_t^i = K. \tag{3.24}$$

We are now ready to define a rational expectations equilibrium as a set of stochastic processes $\left\{C_t^i, C_t, B_{t+1}^{*i}, B_{t+1}^*, L_t^i, L_t, M_t^i, M_t, K_{t+1}^i, K_{t+1}, Y_t, W_t^i, W_t, R_t^M, R_t^K, Q_t, \mu_t^i, S_t\right\}_{t=0}^{\infty}$ satisfying (3.1), (3.3)-(3.5), (4.11)-(3.14) and (3.16)-(3.24), given the exogenous process $\{z_t\}_{t=0}^{\infty}$, the central bank's policy $\{R_t\}_{t=0}^{\infty}$ and initial conditions B_0^* and z_{-1}.[15]

3.2.4 Central bank and exchange rate policy

The central bank uses the interest rate on domestic loans as the monetary policy instrument. I focus the analysis on the case in which the central bank credibly commits to a policy rule at the start of period 0, before period 0 wages are set, and then sticks to that policy forever. The

[15] z_{-1} has to be included among the initial conditions because at the beginning of each period t households use the value of productivity in $t-1$ to form expectations in the wage setting equation (4.11).

general form of the interest rate rule can be written as

$$R_t = R^* \left(\frac{W_t}{W_{t-1}}\right)^{\xi_W} \left(\frac{S_t}{\bar{S}}\right)^{\xi_S}. \tag{3.25}$$

The parameter ξ_W allows the central bank to control the wage inflation rate. The parameter ξ_S controls the response of the interest rate to movements of the exchange rate around a target level $\bar{S}$.

I consider three policy rules. First, I consider a policy of strict wage inflation targeting in which $\xi_W \to \infty$. Under this rule the central bank credibly commits to a policy of zero nominal wage inflation. To achieve this goal the central bank acts so as to replicate the flexible wage equilibrium in any date and state. In this way, households lack an incentive to change the nominal wage and keep their wages constant in every period. This rule offsets all the distortions coming from nominal rigidities and captures the traditional price stability objective of central banks.

Second, I consider a policy of flexible exchange rate targeting in which $\xi_S > 0$ and $\xi_W = 0$. By implementing this policy the central bank provides a nominal anchor to the economy, while allowing some flexibility in the exchange rate. This rule corresponds to a policy of flexible price level targeting in closed-economy models and it represents a simple alternative to targeting wage inflation.[16]

The third regime considered is a perfectly credible currency peg in which $\xi_S \to \infty$. This policy is interesting because it captures the case of dollarized countries or of countries belonging to a monetary union. Moreover it will be used to calibrate the model using data from Eurozone peripheral countries.

3.2.5 The Fisherian deflation mechanism

Before proceeding to the numerical results, it is useful to build some intuition about the financial amplification mechanism at the heart of the model. To this end, in this section I present a brief partial equilibrium analysis that provides insights about the ability of the model to generate crisis events.

Let's start by combining equations (3.16) and (3.13) to write the equilibrium real price of land as

$$\frac{Q_t}{S_t} = \frac{\beta E_t \left[U_C(C_{t+1}, L_{t+1}) \frac{R^K_{t+1} + Q_{t+1}}{S_{t+1}}\right]}{(1-\kappa)\, U_C(C_t, L_t) + \kappa \beta R^* E_t \left[U_C(C_{t+1}, L_{t+1})\right]}.$$

[16] The results would be similar if I assumed that the central bank was targeting a depreciation rate, rather than a level for the exchange rate.

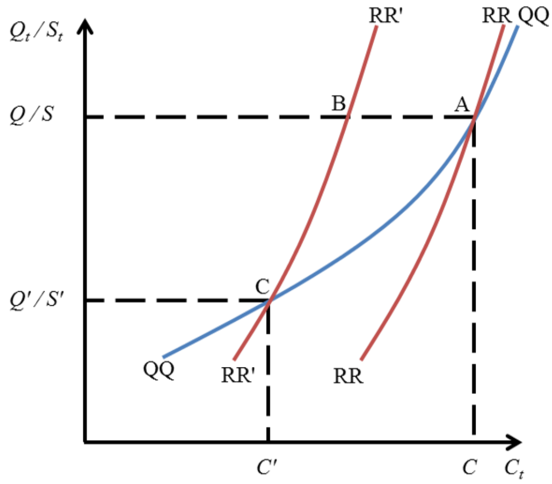


Figure 3.1: Equilibrium with Fisherian Deflation

Since $U_C(C_t, L_t)$ is decreasing in C_t, this equation gives a positive relationship between the real price of land and current consumption. This is due to the households' desire to smooth consumption over time, which implies that the rate at which future returns from land holdings are discounted is decreasing in current consumption. I will refer to this relationship as the QQ curve.

In states in which the collateral constraint binds another positive relationship between Q_t/S_t and C_t arises in equilibrium. To see this combine the resource constraint (3.21) and the binding collateral constraint (3.9) to obtain

$$C_t = z_t F(L_t, M_t, K_t) - (1 + \phi) P^M M_t + R^* B_t + \kappa \frac{Q_t}{S_t} K.$$

To gain intuition about this equation, consider that an increase in the price of land corresponds to an increase in the value of collateral that domestic households can offer to foreign investors. If households are borrowing constrained they will respond to the increase in the value of their collateral by borrowing more to finance current consumption. Hence the positive relationship between Q_t/S_t and C_t. I will call this relationship the RR curve.

Figure 3.1 shows how these two relationships give rise to a financial amplification mechanism based on Fisher's debt deflation. The figure depicts the effects of a negative TFP shock, that is a fall in z_t, in states in which the collateral constraint binds. The initial equilibrium is at point A. The negative TFP shock makes the RR curve shift left to RR'. In absence of financial amplification households would be forced to reduce their consumption, but this would not affect the value of their collateral and the new equilibrium would correspond to point B.

However, the reduction in consumption generates a fall in the demand for land and in its price which tightens the collateral constraint. Households are then forced to decrease their foreign borrowing and further cut their consumption. This gives rise to a vicious cycle of falls in consumption, land price and capital inflows that amplifies the impact of the initial shock. The result is that the Fisherian deflation mechanism moves the economy to the equilibrium depicted by point C, featuring depressed values of consumption and land price.

This simple partial equilibrium analysis shows how the presence of the collateral constraint can be a powerful source of nonlinearity in the response of the economy to exogenous shocks. The numerical results presented in the next section illustrate how the occasionally binding collateral constraint allows the model to reproduce salient features of crisis events in open economies and how it affects the outcome of monetary policy decisions.

3.3 Parameterization and results

The model cannot be solved analytically and I analyze its properties using numerical simulations. A period in the model corresponds to one year, in accordance with the empirical evidence suggesting that wage contracts are set on average once a year.[17] The values of the parameters are chosen using annual data from five small open economies belonging to the Eurozone periphery: Greece, Ireland, Italy, Portugal and Spain. For each country the period considered starts with the year of adoption of the Euro and ends in 2010.[18] I focus on this sample because it features a homogeneous exchange rate policy and because these countries are currently experiencing a period of financial turmoil. The calibration strategy consists in choosing values for the parameters so that the model with monetary policy characterized by a currency peg matches some key aspects of the countries in the sample.

3.3.1 Functional forms and parameterization

The functional forms for preferences and technology are:

$$U(C,L) = \frac{\left(C - \frac{L^{\omega}}{\omega}\right)^{1-\gamma} - 1}{1-\gamma},$$

$$F(L,M,K) = L^{\alpha_L} M^{\alpha_M} K^{\alpha_K},$$

with $\omega \geq 1$, $\gamma \geq 1$, $\alpha_L \geq 0$, $\alpha_M \geq 0$, $\alpha_K \geq 0$ and $\alpha_L + \alpha_M + \alpha_K < 1$. The period utility function takes the form introduced by Greenwood et al. (1988). This type of preferences eliminates the

[17]See Olivei and Tenreyro (2010).

[18]For Ireland, Italy, Portugal and Spain the period considered is 1999-2010, while for Greece it is 2001-2010. Unless otherwise stated, the data come from Eurostat and from the World Development Indicators.

wealth effect on labor supply and are widely used in the quantitative literature on small open economies as they are able to reproduce small open economies' business cycles better than separable preferences.[19] The production function is the standard Cobb-Douglas aggregator.

The risk aversion parameter is set at $\gamma = 2$, a standard value in the real business cycle literature. The Frisch elasticity of labor supply $1/(\omega - 1)$ is set equal to 1, in line with evidence by Kimball and Shapiro (2008). The parameter σ is set to 3 as in Smets and Wouters (2003). The world real interest rate is set to $R^* = 1.03$, a reasonable value for the interest rate charged to small open economies during tranquil times. The stock of land K and the price of the intermediate input P^M are both normalized to one without loss of generality.

The measure of gross output (Y) in the data consistent with the one in the model is the sum of GDP plus imported inputs. The average share of imported inputs in gross output in the sample considered is 0.127, hence $\alpha_M = 0.127$. I assume a labor share in GDP of 0.64 and so $\alpha_L = 0.64(1 - \alpha_M) = 0.558$. I set $\alpha_K = 0.044$ following Bianchi and Mendoza (2010). The discount factor β is set to 0.958 to match an average net foreign assets-to-GDP ratio in the model with a currency peg of -0.41.[20] This is the average net foreign assets-to-GDP ratio across the five sample countries during the period since Euro adoption up to 2007, computed using data from Lane and Milesi-Ferretti (2007).

The productivity shock z_t follows a log-normal AR(1) process $log(z_t) = \rho log(z_{t-1}) + \eta_t$. This process is approximated with the quadrature procedure of Tauchen and Hussey (1991) using 5 nodes. The first order autocorrelation ρ and the standard deviation of the productivity shock σ_z are set so that the model economy under a peg reproduces the average across the five sample countries of the corresponding moments for the cyclical component of GDP per capita (which are respectively 3.1 percent and 0.65).[21] This procedure yields $\rho = 0.9$ and $\sigma_z = 0.0155$.

The parameter κ is set so that the unconditional probability of experiencing a crisis in the currency peg version of the model economy is 5.5 percent, in line with the observed frequency of sudden stops in the cross-country data set of Eichengreen et al. (2006). To be consistent with their definition, a crisis in the model occurs when the credit constraint binds and this leads to an improvement in the current account that exceeds one standard deviation. This calibration results in a value of κ equal to 0.38. The fraction of imported inputs that has to be

[19] Mendoza (1991) is an early example of a small open economy model using GHH preferences. Correia et al. (1995) compare different utility functions in a small open economy model and show that GHH preferences provide the best fit with the data.

[20] In the model the net foreign assets-to-GDP ratio is $B^*_{t+1}/(Y_t - P^M M_t)$.

[21] More precisely, for the five countries in the sample I computed the logarithm of per capita GDP during the period 1960-2010 and removed a smooth trend using the Hodrick-Prescott filter with a smoothing parameter of 100. I then computed for each country the standard deviation and the first order autocorrelation of the detrended series, restricting the sample to the years since the adoption of the Euro. The average standard deviation across the countries in the sample is 3.1 percent, while the average first order autocorrelation is 0.65.

Table 1: Parameters

	Value	Source/Target
Risk aversion	$\gamma = 2$	Standard DSGE value
Frisch elasticity of labor supply	$1/(\omega - 1) = 1$	Kimball and Shapiro (2008)
Elasticity of demand for labor	$\sigma = 3$	Smets and Wouters (2003)
World interest rate	$R^* = 1.03$	Standard DSGE value
Stock of land	$K = 1$	Normalization
World price of imported input	$P^M = 1$	Normalization
Imported input share in output	$\alpha_M = 0.127$	Sample average
Labor share in output	$\alpha_L = 0.558$	Labor share in GDP = 64%
Land share in output	$\alpha_K = 0.044$	Bianchi and Mendoza (2010)
Discount factor	$\beta = 0.958$	NFA/GDP = −41%
TFP process	$\sigma_z = 0.0155$, $\rho = 0.9$	Std. dev. and autoc. of GDP
Credit coefficient	$\kappa = 0.38$	Frequency of crises = 5.5%
Working capital coefficient	$\phi = 0.42$	Working capital/GDP = 6%
Coefficient on interest rate rule	$\xi_S = 1.5$	Standard value
Exchange rate target	$\bar{S} = 1$	Normalization

Table 3.1: Parameters

paid in advance ϕ is set to 0.42 to match an average working capital-to-GDP ratio of 6 percent. This is the same target as in Mendoza and Yue (2011).

The exponent on the exchange rate in the flexible exchange rate targeting rule ξ_S is set to 1.5, a value commonly used in closed-economy sticky price models to capture the response of policy rates to inflation or to deviations of the price level from its target. I later show how the main results of the paper hold true for a variety of values for this coefficient. Finally, the exchange rate target $\bar{S}$ is normalized to 1.

3.3.2 Debt dynamics

The solution is approximated numerically by applying the time iteration method proposed by Coleman (1990) over a discretized state space. This global solution method preserves the nonlinearities induced by the occasionally binding collateral constraint. The state of the economy in period $t \geq 0$ is given by the triplet $\{B_t^*, z_t, z_{t-1}\}$. The previous period productivity shock z_{t-1} must be included among the state variables because it is used by households at the start of the period to form the expectations needed to set their wages. The endogenous state B_t^* is discretized using 700 equally spaced points.

To understand how the model is able to generate both tranquil and crisis times, it is instructive to look at the households' foreign borrowing decision rules. Figure 3.2 shows the optimal choice of next period foreign bonds as a function of the current holdings of foreign

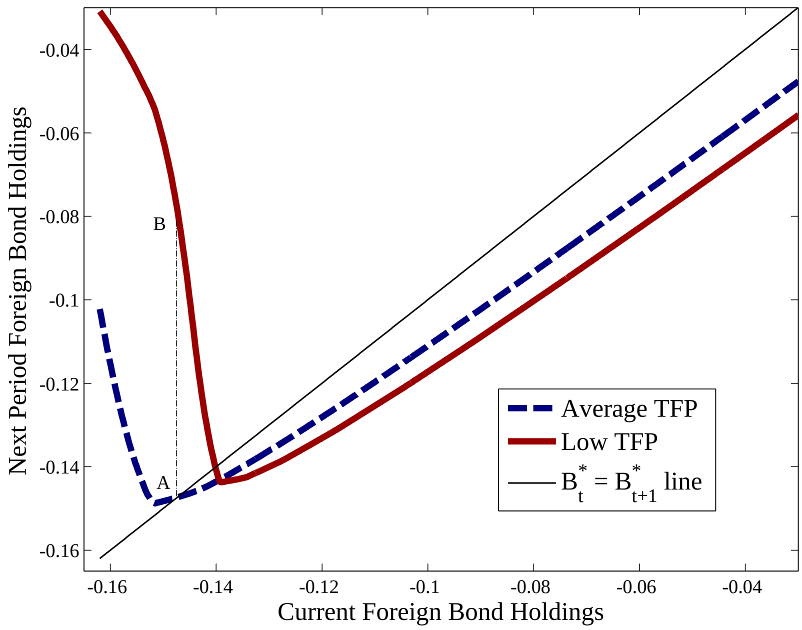


Figure 3.2: Foreign Debt Dynamics

bonds for two different realizations of the TFP shock.[22]

The Fisherian deflation mechanism generates non-monotonic policy functions. The point at which the bond decision rules switch slope corresponds to the value of current foreign bond holdings for which the collateral constraint is satisfied with equality but does not bind. To the right of this point the collateral constraint is not binding and the policy function is upward sloped. When the collateral constraint is not binding domestic agents' investment in foreign bonds is increasing in the value of their wealth at the start of the period, as it is standard in models in which the current account is used to smooth consumption over time. To the left of the kink the collateral constraint binds and the policy function becomes downward sloped. This happens because, for a given choice of next period foreign bonds, both consumption and the price of land are increasing in the stock of foreign bonds held by the households at the start of the period. Hence, a decrease in the start-of-period holdings of foreign bonds is associated with a fall in the value of collateral and, if agents are borrowing constrained, with a decline in foreign debt. This gives rise to a negative relationship between current and future bond holdings in states in which the collateral constraint is binding.

Figure 3.2 also illustrates the process through which the economy enters a crisis. Point A corresponds to a steady state in which the TFP shock is equal to its mean value. At this point, the stock of foreign debt accumulated by domestic agents is big enough to expose the

[22]The decision rule depicted by the solid line is conditional on z_{t-1} being equal to the mean value of TFP and z_t being two standard deviations below mean, while the decision rule represented by the dashed line is conditional on z_{t-1} and z_t being both equal to the mean. Both decision rules refer to agents living under a currency peg. The decision rules for the other two regimes exhibit similar shapes.

economy to the risk of a sudden stop in the event of a negative TFP shock. Facing a negative TFP shock households try to smooth the impact on consumption by increasing their foreign borrowing. This makes the collateral constraint bind and triggers the Fisherian deflation mechanism which generates a drop in the price of land and in the value of collateral pledgeable to foreign investors. Domestic agents are then forced to cut their foreign borrowing and the economy experiences a sudden stop, that is a drastic decrease in capital inflows. For instance, a negative two-standard-deviations TFP shock causes a fall in foreign borrowing which moves the economy to the equilibrium depicted by point B. After the crisis, domestic agents resume their process of debt accumulation until the economy becomes again vulnerable to the risk of a sudden stop.

Another important feature of the model that can be inferred from the figure is that whether a negative shock makes the collateral constraint bind depends on the stock of foreign assets owned by domestic households at the start of the period. The figure shows that for sufficiently high levels of foreign assets, corresponding to the region to the right of the kink in the 'low TFP' line, a negative two-standard-deviations TFP shock does not make the collateral constraint bind. Conversely, for sufficiently low levels of foreign assets, the region to the left of the kink in the 'low TFP' line, a negative two-standard-deviations TFP shock causes the collateral constraint to bind and triggers the financial amplification mechanism.

3.3.3 Crisis event analysis

This section describes how the exchange rate regime affects the behavior of the economy during crises. To compare the response of economies with different exchange rate regimes to a typical crisis event I use the following procedure. I simulate the model economy under a currency peg for 100000 periods, drop the first 1000 periods and then collect all the crisis events, that is periods in which the collateral constraint binds and the current account-to-GDP ratio exceeds one standard deviation. Then I construct five year windows centered around each crisis episode and calculate the median productivity shock across all of these event windows in each year $t-2$ to $t+2$, the median holdings of foreign bonds at $t-2$ and the median productivity shock at $t-3$. Finally, I feed this sequence of shocks and initial values for the state variables to the decision rules of each model economy and compute the corresponding endogenous variables.

The results are shown in figure 3.3. All the variables are in percentage deviations from their ergodic mean except for the current account-to-GDP ratio, the exchange rate and the policy rate. The policy rate corresponds to the interest rate on domestic bonds, deflated by the expected exchange rate depreciation.

Let us start by describing the crisis dynamics under a currency peg, which correspond to the solid lines in figure 3.3. Initially the economy is on a steady state in which the productivity shock is equal to its mean value, the collateral constraint is not binding, the policy rate is equal to the world interest rate and the net foreign assets are constant. In period t the economy is hit by a negative TFP shock, the collateral constraint becomes binding and the economy enters a crisis.

During the crisis GDP drops by more than 5 percentage points below its ergodic mean. This happens because of three effects. First, the negative TFP shock induces a fall in output for a given amount of factors of production employed. Second, there is an inefficient fall in the imports of the intermediate input because households' access to working capital loans is limited by the collateral constraint. Third, the combination of nominal wage rigidities and fixed exchange rate prevents real wages from adjusting downward to accommodate the fall in firms' labor demand caused by the two previous effects. Because of this, employment falls by nearly 6 percentage points below its ergodic mean.

Consumption falls by more than GDP to almost 8 percentage points below trend. This is due to the fact that the binding collateral constraint prevents households from using the current account to smooth the impact on consumption of the fall in GDP. Indeed, the economy experiences a decrease in capital inflows which translates into a sharp rise in the current account-to-GDP ratio. Moreover, the central bank is forced to raise the policy rate above the world interest rate in order to defend the peg. Finally, the Fisherian deflation mechanism generates a fall in the foreign currency price of land of more than 8 percentage points.

During the fourth period productivity remains below trend, but output and consumption recover because of two effects. First the sudden stop causes a sharp decrease in foreign debt, which relaxes the collateral constraint so that it is no longer binding. This allows households to increase their imports of the intermediate input and of the consumption good, thus having a positive effect on output and aggregate consumption. Second, since the TFP shock is persistent after the first period of productivity below trend households revise downward their expectations of future labor demand and lower their wages accordingly. The drop in wages helps the recovery with his positive impact on employment and GDP.

The dashed lines in figure 3.3 illustrate the behavior of the economy when the central bank implements a policy of strict wage inflation targeting. The economy with wage inflation targeting and the currency peg exhibit similar dynamics in the two years before the crisis. However, when in period t the crisis hits the behavior of the two economies diverges.

Under wage inflation targeting the central bank lets the exchange rate depreciate during the

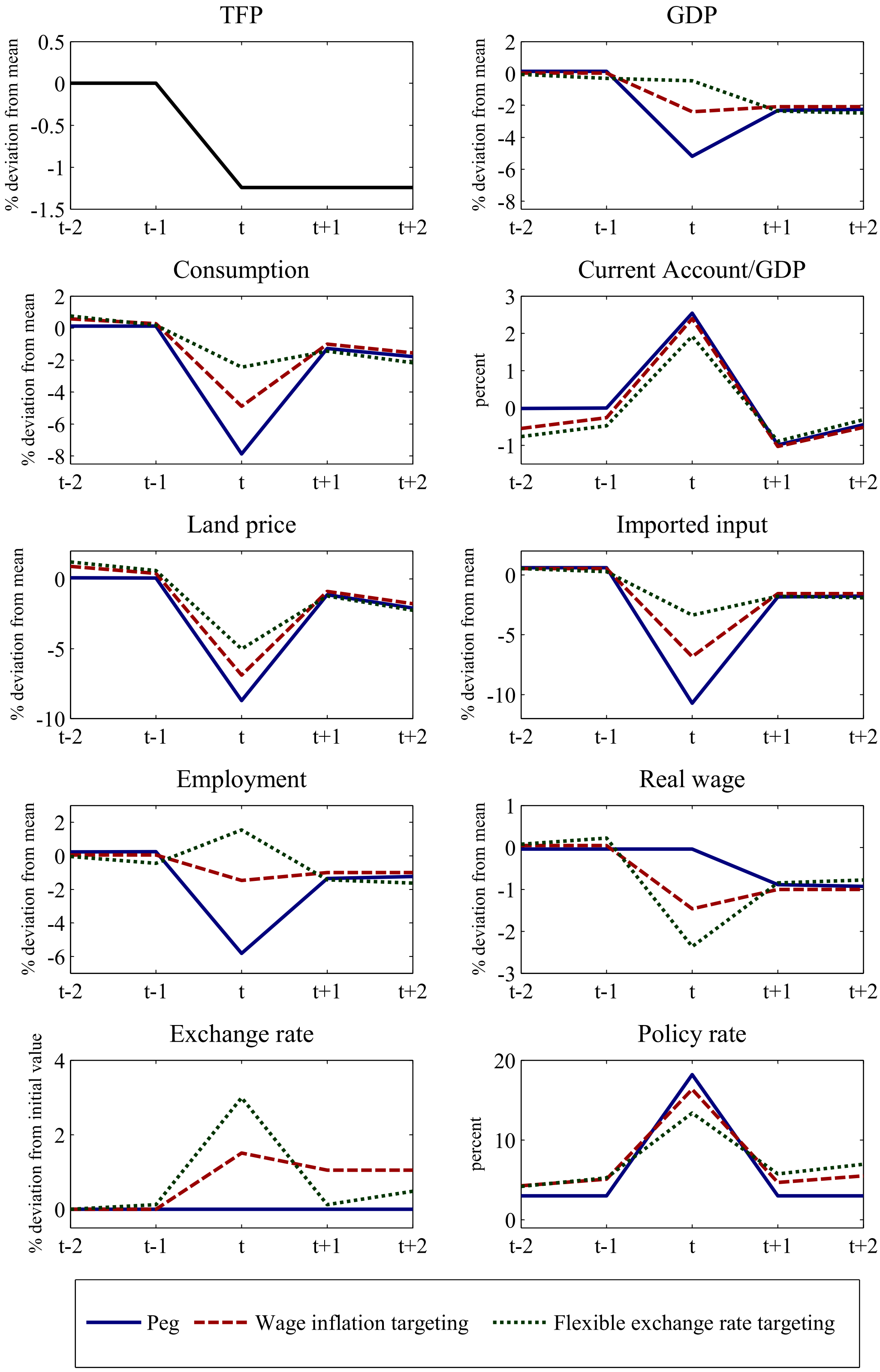


Figure 3.3: Crisis event analysis

sudden stop, in order to reduce real wages in response to the fall in firms' demand for labor. This affects the economy through several channels. First, the decrease in the cost of labor pushes firms to increase employment and this has a positive impact on output. Moreover, the increase in output allows households to consume more. This in turn sustains the demand for land and its price and relaxes households' collateral constraints. Finally, due to the assumption of liability dollarization the depreciation reduces the value for foreign investors of a unit of domestic currency and this tightens domestic agents' borrowing limit. In equilibrium the positive impact on the price of land prevails and the depreciation increases the value of the collateral pledgeable to foreign investors. Indeed, the depreciation interacts with the financial amplification mechanism and produces a virtuous cycle of increases in consumption, land price and capital inflows.

The outcome is that under wage inflation targeting the impact of the sudden stop on output, consumption and land price is milder than under the currency peg. GDP falls by only 2 percent below its ergodic mean, consumption falls by 5 percent below its mean and the price of land falls by 7 percentage points below its mean. The policy rate spikes up during the crisis, but the increase is smaller than in the case of the currency peg.

The dotted lines show the behavior of the economy when the monetary authority follows a policy of flexible exchange rate targeting. Under this regime the exchange rate depreciates during the sudden stop by more than under wage inflation targeting, while the policy rate increases by less.[23]

The reduction in the cost of labor is sufficiently big so that employment rises above trend during the crisis and output barely falls below its ergodic mean. Also, flexible exchange rate targeting exhibits the smallest drops in consumption, which falls by just 2 percent below trend, and land price, which falls by nearly 5 percent below its ergodic mean, compared to the other two regimes.

The event analysis suggests that the flexible exchange rate targeting rule fares better than the other two rules in stabilizing output, consumption and the price of land during sudden

[23] To understand why the exchange rate depreciates under a policy of flexible inflation targeting it is useful to write equation (3.15) as

$$\beta E_t \left[U_C(C_{t+1}, L_{t+1}) \left(R_t \frac{S_t}{S_{t+1}} - R^* \right) \right] = \mu_t^i.$$

Now suppose that a shock makes the collateral constraint bind and so $\mu_t^i > 0$. Also suppose that the shock does not influence the 'long-run' value of the exchange rate, S_{t+1}, or the future marginal utility from consumption. Then a binding collateral constraint translates either in an increase in the domestic nominal interest rate R_t or in an increase in S_t, that is a nominal exchange rate depreciation. Under a policy of flexible exchange rate targeting the adjustment passes through both margins and so an episode of binding collateral constraint is associated with a nominal depreciation and a rise in the domestic interest rate. Moreover, a weaker response of monetary policy to deviations of the exchange rate from its target, that is a lower value of ξ_s, leads to a larger depreciation.

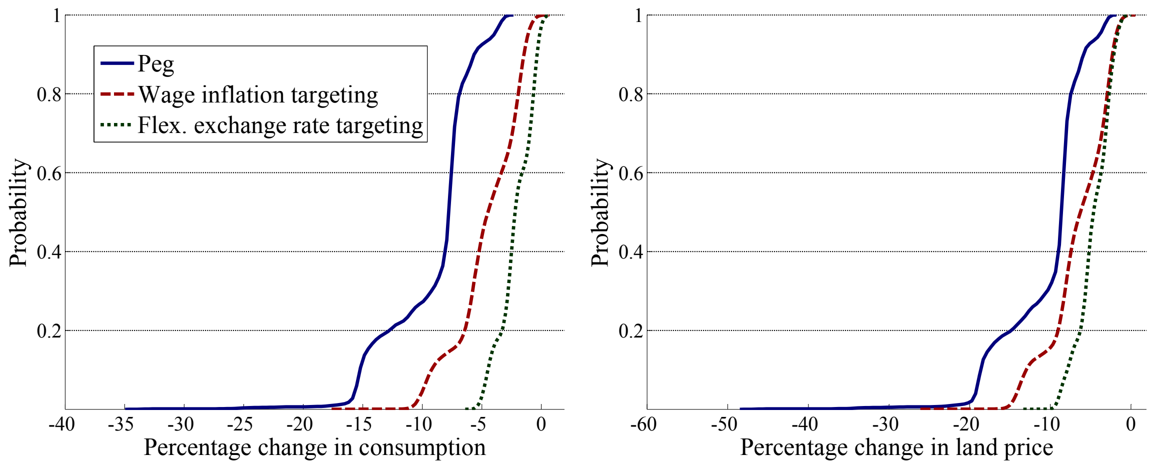


Figure 3.4: Cumulative distribution of impact effect of crises on consumption (left panel) and land price (right panel)

stops. Figure 3.4 further illustrates this point by showing the ergodic cumulative probability distribution of the response of consumption and land price to sudden stops under the three exchange rate policies, expressed as percentage deviations from their ergodic means.[24] The figure shows that both the economy with wage inflation targeting and the currency peg assign non-trivial probabilities, respectively 20 percent and 90 percent, to consumption drops of more than 6 percent, the maximum fall in consumption experienced by the economy with flexible exchange rate targeting. Similarly, the economy with flexible exchange rate targeting assigns a negligible probability to falls in land price below 10 percent, while this happens with almost a 20 percent probability under wage inflation targeting and with more than a 30 percent probability under a peg.

3.3.4 Debt accumulation and precautionary savings

The exchange rate regime not only affects the economy during sudden stops, but it also has an impact on debt accumulation during tranquil times and on the probability that the economy slides into a crisis.

Figure 3.5 displays the ergodic cumulative probability distribution of foreign bond holdings for the three policy rules considered. Both the economy with wage inflation targeting and the one with flexible exchange rate targeting tend to reach higher levels of foreign debt than the peg. For instance, the probability of experiencing levels of foreign debt higher than the

[24]To construct this figure I performed for each model economy a 100000-period long simulation, dropped the first 1000 periods and collected all the crisis events. The figure plots for each economy the cumulative probability distribution function of the percentage deviations of consumption and land price from their ergodic means conditional on the economy being in a crisis.

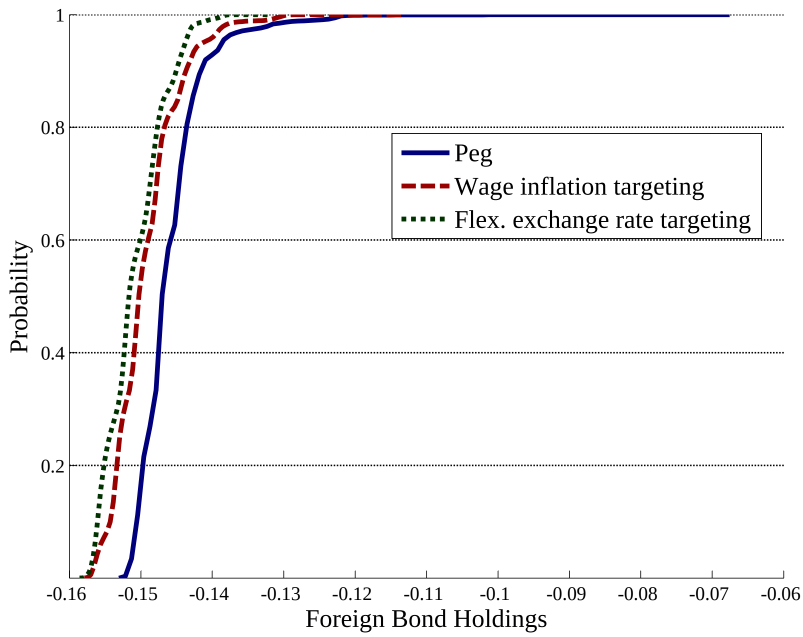


Figure 3.5: Ergodic cumulative probability distribution of foreign bond holdings

maximum attained by the currency peg is around 30 percent both for the economy with wage inflation targeting and for the one with flexible exchange rate targeting.

The reluctance of agents living under a currency peg to reach high levels of foreign debt can be explained with the fact that a higher level of foreign debt increases the chances that a negative shock makes the collateral constraint bind. Since episodes of binding collateral constraint are more disruptive under a currency peg than under the two other monetary regimes, households living under a peg take smaller levels of foreign debt to reduce the risk of entering a crisis. Consistent with this, the economy with flexible exchange rate targeting, which is the regime under which crises have the mildest effects, reaches very high levels of foreign debt more often than the economy with wage inflation targeting.

This can also be seen by looking at precautionary savings, defined as the difference between the borrowing limit and foreign debt.[25] Table 2 shows that the peg has the highest average precautionary savings-to-GDP ratio (1.4 percent), followed by the economy with wage inflation targeting (0.7 percent) and by the economy with the flexible exchange rate targeting rule (0.5 percent). This indicates the existence of a positive relationship between the severity of crises and the amount of precautionary savings that agents accumulate.

By accumulating precautionary savings households influence the probability that the economy enters a crisis. Table 2 shows that the unconditional probability of entering a crisis is 5.5 percent for the economy with a fixed exchange rate, while the crisis probability is 8.3 percent

[25] Formally, precautionary savings at time t are defined as $\kappa K Q_t / S_t + B^*_{t+1} - \phi P^M M_t$.

Table 2: Precautionary savings and crisis probability

	Wage inflation targeting	Flexible exchange rate targeting	Currency peg
Precautionary savings/GDP	0.7	0.5	1.4
Crisis probability	8.3	9.3	5.5

Note: Precautionary savings are defined as the difference between the collateral value of land and total foreign debt, $\kappa K Q_t/S_t + B^*_{t+1} - \phi P^M M_t$. A crisis event is defined as a period in which the collateral constraint binds and the current account-to-GDP ratio exceeds one standard deviation.

Table 3.2: Precautionary savings and crisis probability

for the economy with wage inflation targeting and 9.3 for the economy with flexible exchange rate targeting.

3.3.5 Long run moments

This section documents how the monetary policy regime affects the business cycle moments of the economy. Table 3 displays the long-run business cycle moments for the three policies considered, computed using each economy's ergodic distribution. The economy with the currency peg exhibits the highest business cycle variability in GDP, labor and consumption, signaling the role of shock absorber that flexible exchange rates perform in the model.[26] The economy with wage inflation targeting is characterized by lower volatility in GDP and labor than the economy with flexible exchange rate targeting, but by higher volatility in consumption. This can be explained with the fact that the flexible exchange rate targeting rule does a better job in insulating consumption from the effect of crises.

The model produces a higher variability in GDP than in consumption, a typical feature of emerging markets subject to the risk of financial crises highlighted by Neumeyer and Perri (2005). This is due to the fact that the Fisherian deflation mechanism interferes with households' desire to smooth consumption over time. This can be seen by looking at the cyclicality of the trade balance-to-GDP ratio. In absence of frictions in the credit market the trade balance would be procyclical, because households would smooth the impact of productivity shocks on consumption by decreasing net exports during periods of low productivity. Instead, the binding collateral constraint forces agents to reduce their foreign borrowing, and hence to increase their net exports, when productivity is low generating a countercyclical trade balance-to-GDP ratio. By looking at the cyclicality of the trade balance we can see that consumption smoothing works worst under the peg, which has the highest negative cyclicality of the trade-balance-to-

[26] For empirical evidence on the shock-absorbing role of flexible exchange rates see Broda (2004).

Table 3: Long Run Moments

	Standard deviation			Correlation with GDP			Autocorrelation		
	WIT	FERT	PEG	WIT	FERT	PEG	WIT	FERT	PEG
GDP	2.47	2.51	3.14	1.00	1.00	1.00	0.89	0.85	0.65
Consumption	2.80	2.60	3.60	0.94	0.95	0.96	0.64	0.79	0.48
Trade balance/GDP	0.78	0.60	0.89	−0.15	0.04	−0.30	−0.22	−0.27	−0.19
Employment	1.28	1.61	2.62	1.00	0.89	0.94	0.86	0.52	0.32
Leverage	1.48	1.32	2.29	−0.37	−0.56	−0.59	0.30	0.43	0.43
Land price	3.62	3.29	4.11	0.91	0.87	0.94	0.56	0.69	0.51
Exchange rate	1.28	0.96	0.00	−1.00	−0.36	−	0.86	−0.05	−
Policy rate	3.94	2.54	5.36	−0.37	−0.22	−0.49	−0.09	−0.16	−0.04

Note: WIT stands for the economy with strict wage inflation targeting, FERT stands for the flexible exchange rate targeting rule and PEG stands for the currency peg. Autocorrelation refers to the first-order autocorrelation. Leverage is defined as $(-B^*_{t+1} + \phi P^M M_t)S_t/KQ_t$. The policy rate is the domestic nominal interest rate R_t, deflated by the expected exchange rate depreciation.

Table 3.3: Long run moments

GDP ratio, while the flexible exchange rate targeting rule is the regime that guarantees better consumption smoothing, since its trade balance-to-GDP ratio is mildly procyclical.

The Fisherian deflation mechanism also affects the business cycle moments of land price and leverage.[27] Land price is much more volatile than GDP and strongly procyclical under the three regimes. The flexible exchange rate targeting is the regime with the lowest land price volatility, while the peg exhibits the highest volatility in land price. Also leverage is most volatile under the peg, while the lowest volatility is attained under the flexible exchange rate targeting rule. Leverage is countercyclical under the three policy regimes, due to the fact that when the collateral constraint binds, and thus when leverage has reached its maximum κ, GDP tends to fall.

The exchange rate is more volatile under the wage inflation targeting regime, compared to the economy with flexible exchange rate targeting. Both regimes exhibit small volatilities in the exchange rate compared to data from small open economies, in accordance with the well known difficulty of DSGE models in accounting for the volatility of nominal exchange rates (see for example Kollmann (2002) and Gertler et al. (2007)). In both regimes the exchange rate is countercyclical due to the fact that it tends to depreciate following bad productivity shocks. While the first-order autocorrelation of the exchange rate is strongly positive under a policy of wage inflation targeting, it becomes mildly negative when the central bank follows a policy of flexible exchange rate targeting.

The policy rate tends to be more volatile than GDP, and the highest policy rate volatility is attained under the peg. Interestingly, the model generates countercyclical policy rates,

[27] The leverage ratio is given by $(-B^*_{t+1} + \phi P^M M_t)S_t/KQ_t$.

because crises are associated with spikes in the domestic interest rate. The flexible exchange rate targeting rule is the regime that guarantees the lowest countercyclicality of the policy rate.

3.3.6 Welfare

This section compares the welfare performance of the three monetary regimes considered. I compute the welfare gains of moving from the policy regime r to regime s as the proportional increase in consumption for all possible future histories that households living under regime r must receive in order to be indifferent between remaining in regime r and switching to regime s. Formally, the welfare gain η at a state (B_0, z_{-1}) is defined as

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(C_t^r\left(1+\eta(B_0,z_{-1})\right), L_t^r\right)\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(C_t^s, L_t^s\right)\right],$$

where the superscripts r and s denote allocations in the economy with the corresponding policy regime. Since the central bank commits to a regime at the start of period 0, before wages are set and the TFP shock is known, I compute the welfare compensation η contingent on the initial stock of foreign bonds B_0 and on the past TFP shock z_{-1}. Importantly, this welfare measure takes into account the impact on welfare of the transition to the steady state implied by the new policy.

I start by showing how the presence of the Fisherian deflation mechanism affects the welfare ranking between the strict wage inflation targeting and the flexible exchange rate targeting rule. To this end, I compute the welfare gains of moving from a policy of wage inflation targeting to the flexible exchange rate targeting rule both for the benchmark model, in which the Fisherian deflation channel is present, and for a version of the model in which the collateral constraint (3.9) is replaced by

$$\phi P^M M_t^i - B_{t+1}^{*i} \leq \kappa \bar{Q} K_{t+1}^i,$$

where $\bar{Q}$ is a constant.[28] In this case households are subject to a fixed borrowing limit, there is no financial amplification and the economy never experiences a financial crisis.

Figure 3.6 plots the welfare gains of moving from wage inflation targeting to the flexible exchange rate targeting rule as a function of B_0^*, conditional on z_{-1} being equal to $E(z)$.[29] The dashed line refers to the economy with a fixed borrowing limit. Absent the Fisherian deflation channel, a policy of wage inflation targeting delivers higher welfare than targeting the exchange

[28] In the numerical simulations $\bar{Q}$ is set equal to the average price of land in the benchmark model with a currency peg.

[29] The welfare gains for other values of z_{-1} have similar shapes.

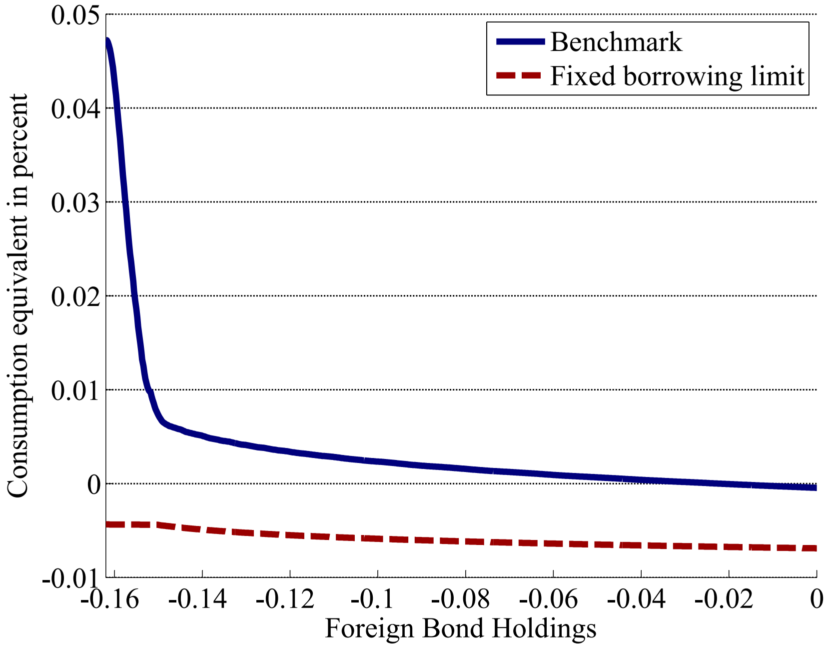


Figure 3.6: Welfare gains of switching from wage inflation targeting to flexible exchange rate targeting

rate for any initial state of the world. This happens because with a fixed borrowing limit there are only two sources of inefficiency. First, on average production is inefficiently low due to the presence of monopolistic competition in the labor market. Second, the assumption of nominal wage stickiness may lead to inefficient wedges between the wage rate and the marginal rate of substitution between consumption and leisure. These two sources of inefficiency are standard in monetary economics and we know that a policy that corrects for nominal rigidities and replicates the equilibrium with flexible wages is close to the optimal policy in this setting.

The Fisherian deflation mechanism introduces another source of inefficiency, based on a pecuniary externality. Atomistic households do not internalize the effect of their actions on the price of land and thus on the value of their collateral. A benevolent social planner that internalizes the impact of its decisions on prices has an incentive to sustain the price of land in states in which the collateral constraint binds, in order to increase the value of the collateral pledgeable to foreign investors. This creates an incentive for the central bank to deviate from its traditional objective of pursuing price stability and to adopt policies that sustain households' access to the international credit markets during crisis times, by mitigating the fall in the price of land in states in which the collateral constraint binds.

The relevance of this source of inefficiency is highlighted by the solid line in figure 3.6, which displays the welfare gains of moving from wage inflation targeting to the flexible exchange rate targeting rule for the benchmark economy. The figure shows that once the Fisherian deflation mechanism is introduced the welfare ranking between a policy of wage inflation targeting and a policy of flexible exchange rate targeting crucially depends on the initial stock of foreign assets owned by domestic households. For high levels of initial foreign assets, corresponding to low initial foreign debt, the wage inflation targeting rule delivers higher welfare gains. This happens because a policy of targeting wage inflation does a good job at managing normal business cycle fluctuations. If the economy starts with a high stock of net foreign assets the probability of a future crisis is small and so welfare is mostly affected by business cycle fluctuations.

As the initial stock of net foreign assets decreases the welfare gains of sticking to the wage inflation targeting rule diminish until adopting the flexible exchange rate targeting rule becomes the preferred option. This happens because a policy of flexible exchange rate targeting does a better job at mitigating the fall in the price of land and at granting access to international credit during crisis events, compared to a policy of targeting wage inflation. Lower foreign assets are associated with higher probability of entering a crisis in the future and so households living in an economy with a low stock of net foreign assets attach more value to the good crisis management properties of the flexible exchange rate targeting rule. The gains from adopting the flexible exchange rate targeting rule become significantly higher for very low levels of initial net foreign assets, because these are the states of the world in which a negative TFP shock triggers a financial crisis.

In the stochastic steady state, the average welfare gains of moving from a policy of strict wage inflation targeting to the flexible exchange rate targeting rule are positive but small, about 0.007 percentage points of permanent consumption. On the one hand, this can be explained with the fact that the gains of adopting a policy of flexible exchange rate targeting are concentrated in states of the world in which the collateral constraint binds. Due to the accumulation of precautionary savings this happens with a small probability in steady state. On the other hand, this finding is in line with Lucas' result on the small welfare costs of business cycle fluctuations in models with CRRA utility, trend-stationary income and no idiosyncratic uncertainty.[30]

Figure 3.7 shows the welfare gains of moving from wage inflation targeting to a currency peg, again as a function of B_0^* and conditional on z_{-1} being equal to $E(z)$. The figure indicates that a policy of strict wage inflation targeting is preferred to a currency peg for both versions of the model and for any value of initial net foreign assets. This suggests that the peg does a poor job in managing both normal business cycle fluctuations and crisis events. In the benchmark

[30] Adding an endogenous growth process as in Barlevy (2004) or allowing for heterogenous agents as in Krusell et al. (2009) is likely to increase the welfare differences between the two regimes.

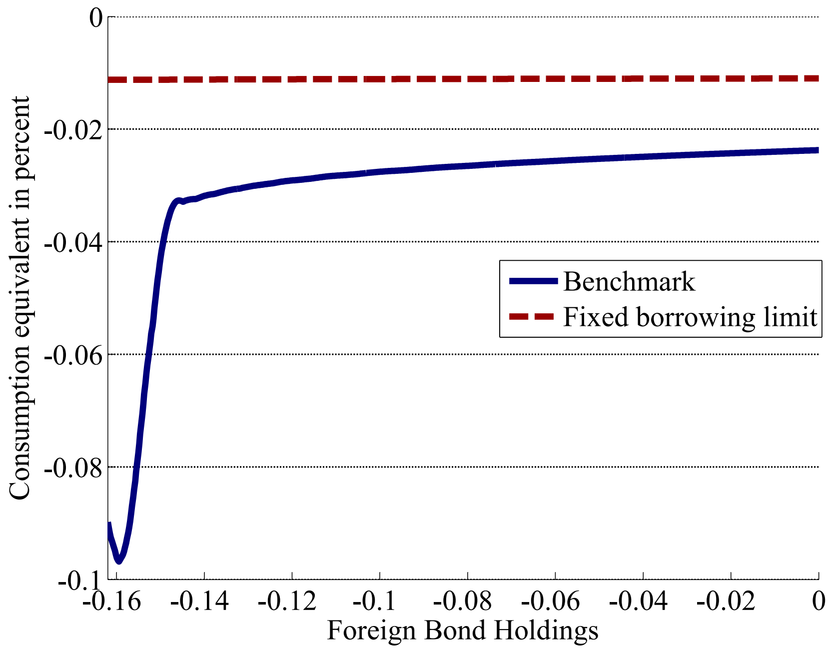


Figure 3.7: Welfare gains of switching from wage inflation targeting to currency peg

version of the model, lower initial net foreign assets are associated with higher welfare costs from pegging the exchange rate. This is due to the fact that the currency peg amplifies the fall in the price of land and worsens households access to international credit during crises.[31]

Considering the stochastic steady state, the average welfare losses from moving from a policy of strict wage inflation targeting to a peg are small, around 0.041 percentage points of permanent consumption. However, compared to the welfare gains of moving to a policy of flexible exchange rate targeting the welfare costs of adopting a currency peg are significantly larger.

3.3.7 Robustness checks

This section examines the robustness of the main results of the paper to changes in some key parameters. I start by investigating whether the result that the flexible exchange rate targeting rule welfare dominates the wage inflation targeting rule in the benchmark version of the model is robust to changes in ξ_S, the parameter that governs the response of the central bank to deviations of the exchange rate from its target. To this end, I computed the average welfare

[31] For very high levels of debt the welfare losses of moving from wage inflation targeting to a peg become decreasing in the initial stock of debt. Presumably this happens because very high initial levels of debt are associated with a high probability of a severe crisis generating a sharp deleveraging that reduces significantly the probability of entering a crisis again in the future.

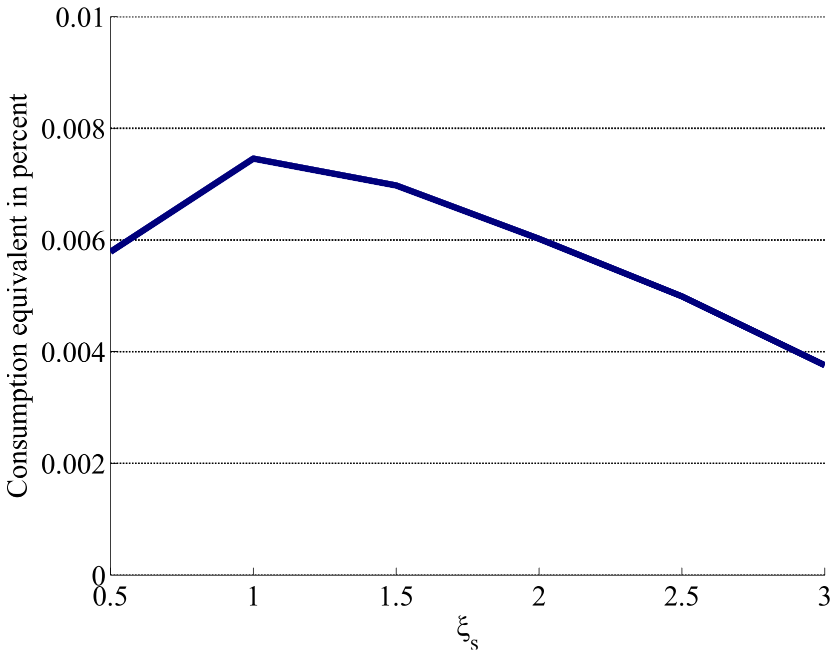


Figure 3.8: Average welfare gains of switching from wage inflation targeting to flexible exchange rate targeting

gains that agents living in the stochastic steady state of the economy with the wage inflation targeting regime would experience from switching to a flexible exchange rate targeting rule for a variety of values of ξ_S. The results, displayed by figure 3.8, indicate that the flexible exchange rate targeting rule is preferred to a policy of targeting wage inflation over a whole range of values for ξ_S. Among the values of ξ_S considered, setting ξ_S equal to 1 guarantees the highest average welfare gains from adopting a flexible exchange rate targeting rule.

Table 4 presents the sensitivity of the main results of the paper with respect to several parameters. The qualitative results seems not to be affected by changes in the key parameters of the model. In particular, strict wage inflation targeting is always welfare dominated by the flexible exchange rate targeting rule, and the currency peg is always the regime characterized by the worst performance in terms of welfare. Moreover, the flexible exchange rate targeting rule is always the regime under which crises have the mildest impact on the economy, while the currency peg always features the lowest crisis probability.

However, some parameters have a significant effect on the quantitative results. Indeed, the differences in the welfare performance of the three regimes increase significantly if the coefficient of relative risk aversion rises or if the fraction of land holdings that can be offered as collateral increases. This suggests that different calibrations of the model may yield higher welfare gains from adopting an appropriate monetary policy regime.

Table 4: Robustness Checks

	Welfare gains from WIT to		Crisis probability			Mean impact effect of financial crises								
						GDP			Consumption			Land price		
	FERT	PEG	WIT	FERT	PEG	WIT	FERT	PEG	WIT	FERT	PEG	WIT	FERT	PEG
benchmark	0.007	−0.041	8.3	9.3	5.5	−2.0	0.2	−5.9	−4.8	−2.3	−9.5	−3.0	−2.2	−4.7
$\gamma = 1.8$	0.003	−0.031	10.1	10.1	7.4	−1.5	−0.2	−5.0	−3.7	−2.1	−7.9	−2.1	−1.7	−3.5
$\gamma = 2.2$	0.015	−0.090	7.5	9.0	4.5	−2.2	0.2	−6.6	−5.2	−2.0	−10.7	−3.6	−2.4	−5.7
$1/(\omega - 1) = 0.9$	0.005	−0.042	8.5	9.2	5.4	−1.9	−0.2	−6.0	−4.5	−2.2	−9.6	−3.0	−2.2	−4.9
$1/(\omega - 1) = 1.1$	0.009	−0.042	8.3	9.0	5.3	−2.1	0.7	−6.1	−4.9	−2.1	−9.8	−3.0	−2.0	−4.7
$\sigma_z = 0.013$	0.007	−0.034	9.3	10.9	5.4	−1.3	0.8	−5.1	−3.7	−1.6	−8.7	−2.4	−1.6	−4.4
$\sigma_z = 0.018$	0.006	−0.049	7.9	8.7	4.9	−2.4	−0.3	−7.1	−5.3	−2.7	−11.1	−3.4	−2.4	−5.4
$\kappa = 0.36$	0.005	−0.036	8.6	9.1	5.7	−1.9	−0.2	−5.9	−4.5	−2.2	−9.3	−2.9	−2.1	−4.7
$\kappa = 0.4$	0.010	−0.060	7.9	9.0	4.9	−2.4	−0.8	−6.3	−5.1	−2.1	−10.4	−4.2	−2.1	−5.1
$\phi = 0.32$	0.010	−0.038	8.0	9.0	5.3	−2.0	0.1	−5.7	−5.0	−2.0	−9.6	−3.2	−2.1	−4.8
$\phi = 0.52$	0.005	−0.043	8.6	9.0	5.7	−2.0	−0.3	−6.2	−4.5	−2.3	−9.5	−2.8	−2.1	−4.6

Note: WIT stands for the economy with strict wage inflation targeting, FERT stands for the flexible exchange rate targeting rule and PEG stands for the currency peg.

Robustness checks

3.4 Conclusion

This paper has examined the performance of alternative monetary policy rules in a small open economy model with an occasionally binding collateral constraint that limits access to foreign credit and with nominal wage rigidities. The main finding is that the presence of pecuniary externalities in the credit markets introduces a trade-off between price and financial stability. For low levels of external debt the probability of a future crisis is small and a policy that eliminates the distortions coming from nominal rigidities by targeting wage inflation is the best rule. For high levels of external debt the probability of a future crisis is high and targeting wage inflation is dominated by a flexible exchange rate targeting rule, because the latter policy is better at mitigating the fall in output, consumption and capital inflows during crisis events. In contrast, pegging the exchange rate is always welfare dominated by the wage inflation targeting rule. A second key finding is that the exchange rate regime affects both the behavior of the economy during crisis events and the probability that the economy enters a crisis, through its impact on debt accumulation during tranquil times. The more the monetary policy regime mitigates the impact of crises on output and consumption, the more private agents engage in foreign debt accumulation during tranquil times and the higher is the probability of experiencing a crisis.

The paper represents a first step in the analysis of monetary policy in dynamic general equilibrium models featuring tranquil and crisis times. The model is kept voluntarily simple to reduce the computational complexities involved by the derivation of a global numerical solution. An interesting area for future research would be to extend the model in order to make it more suitable to deliver quantitative results. Two possible extensions are the inclusion of endogenous capital accumulation and of a dynamic wage-setting process.

Chapter 4

Reserve Accumulation, Growth and Financial Crises

4.1 Introduction[1]

One of the most spectacular recent trends in the international monetary system is the considerable built up of foreign exchange reserves by emerging countries, in particular East Asian economies and China.[2] As shown by figure 4.1a, the average reserves-to-GDP ratio in developing countries more than doubled between 1980 and 2010, increasing from 9.5 to 23.3 percent. The increase has been particularly marked in East Asia, where the average reserves-to-GDP ratio passed from 15.5 percent in 1980 to 55.3 percent in 2010.[3]

The large accumulation of foreign reserves is not just interesting in itself, but it also represents a key element for understanding the direction and allocation of international capital flows among developing economies. As noticed by Gourinchas and Jeanne (2011), while the neoclassical growth model would suggest that capital should be directed towards those economies that experience faster productivity growth, in the data we observe that faster growing economies are associated with lower net capital inflows (figure 4.1b). Moreover, Alfaro et al. (2011) show that the positive correlation between current account surpluses and growth is purely driven by public flows, while private flows conform with the predictions of the neoclassical growth model. In fact, they find that the current account surpluses of fast growing economies are due to their policy of fast accumulation of international reserves (figure 4.1c), while current account deficits

[1]This chapter is coautored with Gianluca Benigno.

[2]See Ghosh et al. (2012) for a discussion of the accumulation of reserves by developing countries in the last three decades.

[3]Developing countries refer to a sample of 66 developing economies. East Asia refers to the unweighted average of China, Hong Kong, Indonesia, South Korea, Malaysia, Philippines, Singapore and Thailand. All the data are from the World Bank Development Indicators.

in countries that experienced dismal growth performances are driven by inflows of foreign aid.

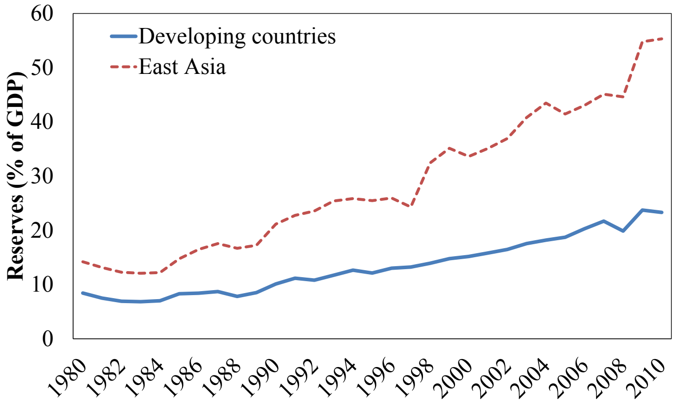
Our main objective in this paper is to provide a framework that explains the joint behavior of private and public capital flows in fast growing emerging economies. We study a two-sector, tradable and non-tradable, small open economy. There are two key elements. First, firms in the tradable sector absorb foreign knowledge by importing intermediate inputs. This mechanism provides the source of growth in our economy, but its benefits are not internalized by individual firms since knowledge can be used freely by all the firms in the economy. Second, private agents have limited access to international financial markets and the economy is exposed to the risk of sudden stops in capital inflows.

The combination of growth externalities and financial frictions provides a powerful incentive for the government to accumulate reserves. First, we show that during tranquil times the government can use reserve accumulation to exploit the knowledge spillovers in the tradable sector. In fact, an increase in foreign exchange reserves leads to a real currency depreciation and to a reallocation of production toward the tradable sector. This stimulates the use of imported inputs, the absorption of foreign knowledge and productivity growth.

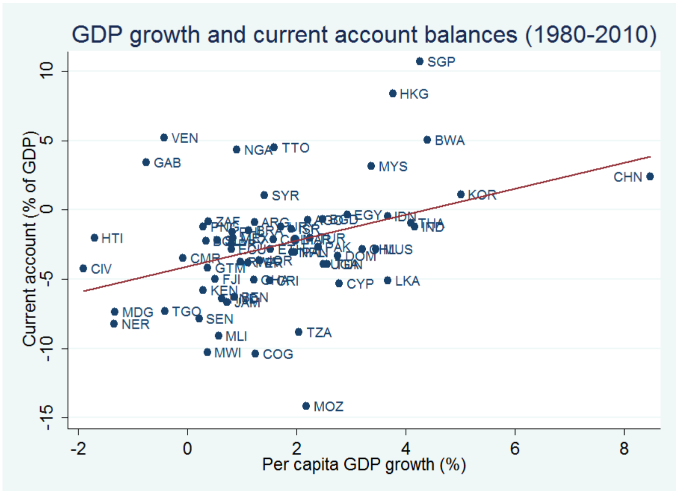
This mechanism is effective as long as there is imperfect substitutability between private and public flows. Indeed, in the neoclassical growth model the accumulation of international reserves would be offset by private capital inflows. Instead, in our framework the offsetting effect is not complete because the risk of a sudden stop limits the willingness of private agents to accumulate debt in response to an increase in the stock of reserves by the government. Hence, while the economy as a whole runs a current account surplus and gathers foreign reserves, the private sector accumulates foreign liabilities, consistent with the empirical findings of Alfaro et al. (2011).

Second, we show that the presence of knowledge externalities provides an incentive for the government to use reserves during financial crises, in order to counteract the loss of access to private credit by firms in the tradable sector. Indeed, our framework reproduces the pattern of gross capital flows observed by Broner et al. (2011) in emerging markets. During financial crises both gross inflows, in the form of private credit, and gross outflows, in the form of reserve accumulation, decrease, since the government uses its stock of reserves to provide loans to firms that have lost access to foreign financing. Through this channel, reserve management positively affects growth by cushioning the impact of financial crises on output and productivity growth.

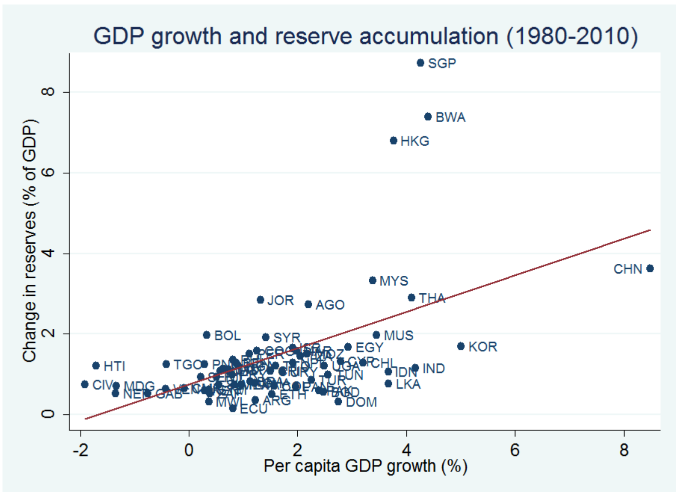
We then examine the normative implications of reserve accumulation. We first show that a social planner that is unconstrained in terms of policy tools would choose not to accumulate reserves but to rely on sectoral subsidies. We argue, similarly to what Korinek and Servén



(a) Reserves in percent of GDP



(b) Average per capita GDP growth and average current account balances between 1980 and 2010



(c) Average per capita GDP growth and average reserve accumulation between 1980 and 2010

Figure 4.1: Motivating facts

(2010) suggest, that in practice sectoral subsidies may conflict with WTO rules or other trade agreements. In this case, a policy of reserve accumulation can be used to circumvent these restrictions. We compute within a class of simple rules the optimal reserve policy and we find that, despite being a second-best policy tool, the welfare gains from optimal reserve management can be significant. As an example, we find that the gains from public intervention in capital flows for a country that is opening itself to international capital markets are in the order of a 1 percent permanent increase in consumption. Moreover, we find that the bulk of these welfare gains come from the use of reserves during financial crises.

Finally, we show that our model also rationalizes the negative relationship between inflows of foreign aid and growth observed in low income countries. We model foreign aid as public loans provided to the government by foreign institutions. We show that inflows of foreign aid lead to an appreciated real exchange rate, less productive resources in the tradable sector and slower accumulation of knowledge and growth, in the spirit of the resource curse literature.[4]

The rest of the paper is structured as follows. We start by discussing our key assumptions and the related literature. Then, in section 2 we introduce the framework. Section 3 presents the social planning allocation and discusses the political barriers that may prevent a government from implementing the first best through sectoral subsidies. Section 4 provides intuition about the effect of reserve management. Section 5 presents the results of our policy experiment on financial liberalization and provides estimates of the welfare gains from implementing the optimal reserve policy. Section 7 considers the impact of inflows of foreign aid. Section 7 concludes.

Discussion of key elements. Our theory rests on two key elements: the existence of knowledge spillovers in the tradable sector and the limited and intermittent access to international credit markets. Here we discuss the empirical evidence that underpins these assumptions.

We study an economy that grows by absorbing foreign knowledge. The existence of international knowledge spillovers is well established in the literature on global growth. The foundations for the theoretical study of cross-country knowledge flows were laid down by Grossman and Helpman (1991), while Klenow and Rodriguez-Clare (2005) stress how a model of the world economy has to feature international knowledge spillovers in order to be consistent with the growth patterns observed in the data.

There is also a sizable literature emphasizing the role of trade in facilitating the transmission

[4]The data can also be explained by the fact that donors may want to allocate more aid to the countries with lower growth. However, the empirical evidence provided by Rajan and Subramanian (2011) is consistent with the mechanism described by our model.

of knowledge across borders. The idea is that in order to have access to the international pool of knowledge a country has to import foreign products or export to foreign markets. We choose to focus on the transmission of knowledge through the imports of intermediate inputs because we feel that this is the channel for which more empirical evidence is available. Our starting point is the empirical analysis of Coe et al. (1997). They find that imports of capital goods and materials represent a key channel through which discoveries made in developed countries spill over to developing economies. Subsequent research, surveyed by Keller (2004), has confirmed the significant role of imports in the process of international knowledge diffusion. More recently, plant-level evidence on the positive impact of imports of intermediate goods on productivity has emerged. For instance, Amiti and Konings (2007) using Indonesian plant-level data find a positive effect on productivity from a decrease in tariffs on intermediate inputs.

Another line of research has tried to identify a positive effect on productivity from exporting. This may happen, for example, if exporting allows firms to become familiar with foreign technologies that increase their productivity, the so called learning-by-exporting effect. Isolating this effect is hard, because the most productive firms tend to self-select themselves into the export sector. Despite this difficulty, some firm-level evidence in support of learning-by-exporting effects has been find by Blalock and Gertler (2004), using Indonesian data, and by Park et al. (2010), who use data from Chinese firms. Importantly, our qualitative results would carry through in a model in which firms absorb foreign technology by exporting, rather than by importing intermediate inputs.[5]

In our model productivity growth through the absorption of foreign knowledge is present only in the tradable sector. We make this stark assumption to simplify the exposition, but our qualitative results would remain in a setting in which knowledge spillovers are stronger in the tradable sectors compared to the non-tradable ones. Rodrik (2008) provides some indirect evidence consistent with this assumption. He finds that real exchange rate depreciations stimulate growth in developing countries and that this effect is increasing in the size of the tradable sector. In addition, Rodrik (2012) considers cross-country convergence in productivity at the industry level and finds that this is restricted to the manufacturing sectors. This finding is consistent with the idea that international knowledge spillovers are confined to, or at least more intense in, the manufacturing sectors. Since manufacturing represents the bulk of the sectors

[5]There is a long-standing tradition in the growth literature that emphasizes the role of learning-by-doing effects. This literature, that dates back to Arrow (1962), sees the accumulation of knowledge as a by product of the production process. Krugman (1987) and Young (1991) are early studies of learning-by-doing effects in open economy models. Our qualitative results would hold in a model in which learning-by-doing is the engine of growth, as long as learning-by-doing effects are stronger in the tradable sector and not fully internalized by firms.

producing tradable goods, Rodrik's finding lends support to our assumption that knowledge spillovers are more important in the tradable sectors.

Finally, in our model knowledge is a non-excludable good, and hence it can be used freely by any firm in the economy. We still lack a good empirical understanding of the extent to which knowledge can be appropriated by individual firms. However, it seems reasonable to assume that, at least partly, the knowledge accumulated inside a firm can spill over to other firms. For example, this may happen trough imitation or through the hiring of workers that embody the technical knowledge developed in a rival firm. Indeed, the assumption that knowledge is only partially excludable is a feature of the most influential endogenous growth frameworks, such as the models developed by Romer (1986), Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). It is important to stress that, while we assume that knowledge is a completely non-excludable good, the mechanism that we describe would still hold in a framework in which knowledge is partially excludable.

We now turn to our assumptions about financial markets. We consider an economy that periodically sees its access to international credit markets curtailed. This assumption is meant to capture the sudden stop episodes, that is periods in which capital inflows are severely reduced, experienced by many emerging countries. These episodes are often associated with banking crises and deep recessions. In our model, sudden stops have a negative impact on production because they interfere with firms' ability to secure trade credit and hence to satisfy their demand for imported inputs. Mendoza (2010) shows that a model with this feature is able to capture the behavior of measured TFP around sudden stop episodes. Moreover, Mendoza and Yue (2011) provide empirical evidence on the fall in the use of imported inputs around crisis episodes culminating in a sovereign default. Our specification of financial frictions also allows us to capture the negative long run impact of crises on growth highlighted by the empirical analysis of Cerra and Saxena (2008).

During financial crises the government can use its stock of foreign exchange reserves to provide trade credit to firms, so as to help firms to overcome the loss of access to foreign financing. Central banks in emerging countries often use reserves to provide dollar loans to banks to avoid disruptions in trade credit during sudden stops. For instance, this was the case in Korea and Indonesia during the 1997 Asian Crisis and in Brazil in 2002-2003.[6] More recently,

[6]Ronci and Wang (2006) describe central banks' interventions to finance trade credit during these episodes. In 1997, the Bank of Korea used 2.3 billion dollars from its stock of reserves to provide loans to banks to finance imports of raw materials and purchase export bills of exchange from exporters. In Indonesia the central bank deposited 1 billion dollars of its international reserves in 12 foreign banks as a guarantee to letters of credit issued by Indonesian banks for the financing of imports by export-oriented firms. Finally, in Brazil the central bank provided 1.8 billion dollars between August 2002 and early 2003 to banks to meet demand for export finance.

several emerging countries used reserves to contain disruptions in trade credit following the 2008 financial crisis.[7] More broadly, our model captures the positive impact of active reserve management on output during financial crises. Dominguez et al. (2012) show how emerging countries used their stock of reserves to mitigate the fall in output in the aftermath of the 2008 financial crisis.

Related literature. This paper is related to several strands of the literature. Our framework provides a plausible explanation for the negative correlation between productivity growth and capital inflows in developing countries observed by Prasad et al. (2007), Gourinchas and Jeanne (2011) and Alfaro et al. (2011). Gourinchas and Jeanne (2011) and Alfaro et al. (2011) find that the current account surpluses observed in fast growing developing economies is driven by their policy of reserve accumulation and this motivates our focus on foreign exchange reserves. The central role of government intervention in shaping capital flows to developing countries relates our paper to the so-called "Bretton Woods 2" perspective on the international monetary system of Dooley et al. (2003), according to which the large accumulation of international reserves by the public sector in emerging economies is part of an export-led growth strategy. Our paper is also related to Rodrik (2008), who provides empirical evidence in favor of a causal link from real exchange rate undervaluation to growth.

From a theoretical perspective, our paper is connected to the growing literature providing formal models that reproduce the negative correlation between growth and capital inflows characterizing developing countries. Examples include Aghion et al. (2006), Angeletos and Panousi (2011), Broner and Ventura (2010) and Sandri (2010). These papers all focus on private capital flows, while in our model the negative correlation between growth and capital inflows is driven by reserve accumulation by the public sector. Aguiar and Amador (2011) provide a model in which public flows may generate a negative correlation between growth and capital inflows, but the mechanism that they emphasize is different from ours. In fact, in their model the government decreases its stock of foreign debt in order to credibly restrain from expropriating the return from private investment, thus stimulating investment and growth. In contrast, in our framework reserve accumulation by the public sector shifts productive resources toward the tradable sector in order to exploit the knowledge spillovers coming from the imports of foreign capital goods.

Our paper is also related to the literature examining the determinants of reserve accumulation in emerging markets. Aizenman and Lee (2007) and Korinek and Servén (2010) emphasize the link between reserve accumulation and growth externalities, while Durdu et al. (2009) and

[7] See Chauffour and Farole (2009).

Jeanne and Rancière (2011) focus on the precautionary motive of holding international reserves. In Bianchi et al. (2012) reserves are used as a buffer against rollover risk. Bacchetta et al. (2011) suggest that the accumulation of foreign reserves can be used to supply saving instruments to domestic agents when domestic financial markets are imperfect and private agents have limited access to foreign credit. Our framework encompasses the first two approaches and differs critically from the existing literature in the modeling of public versus private capital flows.

4.2 Model

We consider an infinite-horizon small open economy. Time is discrete and indexed by t. The economy is populated by a continuum of mass 1 of households and by a large number of firms. Firms are owned by the households and produce tradable and non-tradable consumption goods. Moreover, firms producing the tradable good engage in financial transactions with foreign investors. There is also a government that manages foreign exchange reserves.

4.2.1 Households

The representative household derives utility from consumption and supplies inelastically one unit of labor each period. The household's lifetime expected utility is given by

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right]. \tag{4.1}$$

In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time t , $\beta < 1$ is the subjective discount factor, $\gamma > 0$ is the coefficient of relative risk aversion and C_t denotes a consumption composite good. C_t is defined as a Cobb-Douglas aggregator of tradable C_t^T and non-tradable C_t^N consumption goods[8]

$$C_t = \left(C_t^T\right)^{\omega} \left(C_t^N\right)^{1-\omega}, \tag{4.2}$$

where $0 < \omega < 1$ denotes the share of expenditure in consumption that the household allocates to the tradable good.

[8]A Cobb-Douglas consumption aggregator is needed to ensure the existence of a balanced growth path.

Each period the household faces the following flow budget constraint

$$C_t^T + P_t^N C_t^N = W_t + \Pi_t^T + \Pi_t^N. \tag{4.3}$$

The budget constraint is expressed in units of the tradable good. The left-hand side represents the household's expenditure. We define P_t^N as the relative price of the non-tradable good in terms of the tradable good, so $C_t^T + P_t^N C_t^N$ is the household's consumption expenditure expressed in units of the tradable good. The right-hand side represents the income of the household. W_t denotes the household's labor income. Π_t^T and Π_t^N are the dividends that the household receives from firms operating respectively in the tradable and in the non-tradable sector. For simplicity, we have assumed that domestic households do not trade directly with foreign investors. As we will see below, households can access international financial markets indirectly through their ownership of firms.

Each period the representative household chooses C_t^T and C_t^N to maximize expected utility (4.1) subject to the budget constraint (4.3). The first order conditions are

$$\frac{\omega C_t^{1-\gamma}}{C_t^T} = \lambda_t \tag{4.4}$$

$$\frac{(1-\omega) C_t^{1-\gamma}}{C_t^N} = \lambda_t P_t^N, \tag{4.5}$$

where λ_t denotes the Lagrange multiplier on the budget constraint, or the household's marginal utility of wealth. By combining (4.4) and (4.5), we obtain the standard intratemporal equilibrium condition that links the relative price of non-tradable goods to the marginal rate of substitution between tradable and non-tradable goods

$$P_t^N = \frac{1-\omega}{\omega} \frac{C_t^T}{C_t^N}. \tag{4.6}$$

According to this expression, P_t^N is increasing in C_t^T and decreasing in C_t^N. In what follows we will use P_t^N as a proxy for the real exchange rate.

4.2.2 Firms in the tradable sector

The tradable sector is meant to capture a modern sector characterized by dynamic productivity gains and open to financial transactions with foreign investors. Firms in the tradable sector produce using labor L_t^T, an imported intermediate input M_t and the stock of accumulated knowledge X_t, according to the production function

$$Y_t^T = \left(X_t L_t^T\right)^{\alpha_T} M_t^{1-\alpha_T}, \tag{4.7}$$

where Y_t^T is the amount of tradable goods produced in period t and $0 < \alpha_T < 1$ is the labor share in gross output in the tradable sector. Knowledge is non-rival and can be freely used by firms producing tradable goods.

Firms in the tradable sector have access to international credit markets. First, they can trade in a non-contingent one period bond denominated in units of tradable goods that pays a fixed gross interest rate R. At the end of the period the representative firm distributes to the households the dividends

$$\Pi_t^T = Y_t^T - W_t L_t^T - P^M M_t - B_{t+1} + RB_t - T_t. \tag{4.8}$$

In this expression B_t denotes the firm's holding of foreign bonds at the start of period t. When $B_t < 0$ the firm is a borrower. W_t is the wage paid to workers in the tradable sector, P^M is the price of the imported input and T_t are lump-sum taxes paid to the government.[9]

Second, firms in the tradable sector are subject to a working capital constraint. A fraction ϕ of the intermediate inputs has to be paid at the beginning of the period and requires working capital financing. To finance their working capital, firms have access to intraperiod loan contracts. Under these contracts, the funds borrowed by firms at the start of the period have to be repaid at the end of the same period. We assume that the interest rate charged on intraperiod loans is equal to zero. The domestic government provides an amount D_t of working capital loans. The remaining part $\phi P^M M_t - D_t$ has to be covered using intraperiod loans from foreign investors.

In addition, we introduce financial frictions by assuming that at the end of the period each firm can choose to default on its debts toward international investors. In case of default international investors are able to collect an amount of tradable goods equal to $\kappa_t X_t$.[10] To prevent defaults, international investors impose on domestic firms the borrowing constraint

$$\phi P^M M_t - D_t - RB_t \leq \kappa_t X_t, \tag{4.9}$$

[9] The assumption that taxes are paid by firms in the tradable sector, rather than by households, is made to simplify the exposition and it does not affect our results.

[10] The presence of the term X_t in the borrowing constraint ensures the existence of a balanced growth path. Alternatively, we could assume that investors can recover a fraction of the output produced by the firm. However, this alternative formulation would complicate the derivation of a numerical solution, without adding significant insights to our analysis.

where κ_t measures the tightness of the borrowing constraint. On the left-hand side, we have the net liabilities of the firm at the beginning of period t. Notice that both the intertemporal loans and the loans used to finance the working capital expenses enter the constraint. We introduce credit shocks in the model by assuming that the parameter κ_t is stochastic. In what follows we refer to a financial crisis as a period in which the borrowing constraint (4.9) holds with equality.

Each period the representative firm chooses L_t^T, M_t and B_{t+1} to maximize its expected stream of dividends discounted by the households' marginal utility of wealth

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\lambda_t\Pi_t^T\right], \tag{4.10}$$

subject to the borrowing constraint (4.9). The optimality conditions are given by

$$\alpha_T Y_t^T = W_t L_t^T \tag{4.11}$$

$$(1-\alpha_T)Y_t^T = P^M M_t\left(1+\phi\frac{\mu_t}{\lambda_t}\right) \tag{4.12}$$

$$\lambda_t = \beta R E_t\left[\lambda_{t+1}+\mu_{t+1}\right] \tag{4.13}$$

$$\mu_t\left(\phi P^M M_t - D_t - RB_t - \kappa_t X_t\right) = 0, \quad \mu_t \geq 0, \tag{4.14}$$

where μ_t denotes the multiplier on the borrowing constraint. Equation (4.11) represents the optimal demand for labor, which implies equality between the marginal product of labor and the wage. The optimal demand for imported inputs is given by equation (4.12). When the borrowing constraint is not binding ($\mu_t = 0$), the marginal product of the imported input is equated to its price. When the borrowing constraint is binding ($\mu_t > 0$), firms are unable to purchase the desired amount of imported inputs. This shows up in the equation as an increase in the marginal cost of purchasing one unit of the imported input. Equation (4.13) is the modified Euler equation for the case in which international borrowing might be constrained. The expectation of a future binding borrowing constraint has an effect similar to an increase in the cost of intertemporal debt that induces agents to decrease their borrowing. Finally, equation (4.14) is the complementary slackness condition for the borrowing constraint.

4.2.3 Knowledge accumulation

The stock of knowledge available to firms in the tradable sector evolves according to

$$X_{t+1} = \psi X_t + M_t^{\xi} X_t^{1-\xi}, \tag{4.15}$$

where $\psi \geq 0$ and $0 \leq \xi \leq 1$. This formulation captures the idea that imports of foreign capital goods represent an important transmission channel through which discoveries made in developed economies spill over to developing countries. As mentioned above, we assume that knowledge is a non-rival and non-excludable good. This, combined with the assumption of a large number of firms in the tradable sector, implies that firms do not internalize the impact of their actions on the evolution of the economy's stock of knowledge.

4.2.4 Firms in the non-tradable sector

The non-tradable sector represents a traditional sector with stagnant productivity, closed to financial transactions with foreign investors. The non-tradable good is produced using labor, according to the production function $Y_t^N = \left(L_t^N\right)^{\alpha_N}$. Y_t^N is the output of the non-tradable good, L_t^N is the amount of labor employed and $0 < \alpha_N < 1$ is the labor share in gross output in the non-tradable sector.[11]

The dividends distributed by firms in the non-tradable sector can be written as

$$\Pi_t^N = P_t^N Y_t^N - W_t L_t^N. \tag{4.16}$$

In this expression we have used the fact that in equilibrium firms in both sectors produce and that this requires equalization between the wages offered in the two sectors. Profit maximization implies

$$\alpha_N P_t^N L_t^{N\alpha_N - 1} = W_t. \tag{4.17}$$

This equation represents the optimal demand for labor from firms in the non-tradable sector. Similar to firms in the tradable sector, firms in the non-tradable sector equate the marginal product of labor to the wage rate.

[11] To ensure constant returns to scale in the production of non-tradable goods, we can assume that production is carried out using labor and land according to a constant-returns-to-scale Cobb-Douglas aggregator. The production function in the main text obtains if the supply of land is fixed and normalized to one.

4.2.5 Credit shocks

The only source of uncertainty in the model concerns κ_t, the parameter that governs the sum that foreign lenders can recover in case of default. Our aim is to model an economy in which tranquil times alternate with crises. The simplest way to capture this is to assume that κ_t can take two values, κ_H and κ_L with $\kappa_H > \kappa_L$. We will choose values for κ_H such that when $\kappa_t = \kappa_H$ the borrowing constraint (4.9) does not bind, while the value for κ_L will be such that when $\kappa_t = \kappa_L$ the borrowing constraint may bind, depending on B_t and on the actions of the government. As mentioned above, we refer to a period in which the borrowing constraint binds as a financial crisis. Moreover, denoting by ρ_i for $i = H, L$ the probability that $\kappa_t = \kappa_i$ knowing that $\kappa_{t-1} = \kappa_i$, we will set $\rho_H > 0.5$ so that crises are rare events and $\rho_L > 1 - \rho_H$ so that crisis events have some persistence.

4.2.6 Government

The government collects taxes from firms in the tradable sector T_t, provides working capital loans D_t to firms and trades in foreign exchange reserves FX_t.[12] In the spirit of Gertler and Karadi (2011), we assume that lending from the government entails some efficiency losses. Specifically, we assume that in order to lend to firms a sum equal to D_t, the government has to employ an amount of tradable goods equal to $D_t/(1-\theta)$, with $0 \leq \theta \leq 1$. Of this amount, D_t is repaid by firms to the government at the end of the period, while $D_t\theta/(1-\theta)$ is lost during the intervention. Hence, the higher θ is, the less efficient is the government in providing liquidity to firms.

We can then write the government budget constraint expressed in units of tradable goods as

$$FX_{t+1} = R^{FX} FX_t + T_t - D_t \frac{\theta}{1-\theta}, \tag{4.18}$$

where R^{FX} is the gross interest rate paid on reserves. To capture some defining features of foreign exchange reserves, we assume that the interest rate paid on reserves is not greater than the interest rate charged on private loans ($R^{FX} \leq R$) and that the government cannot hold negative amounts of foreign reserves

$$FX_t \geq 0. \tag{4.19}$$

Moreover, the resources employed to provide working capital loans to firms at the start of the

[12] In our framework the accumulation of reserves is financed through lump-sum taxes. In practice, central banks finance the accumulation of foreign reserves by issuing domestic currency, i.e. through seignorage. We leave for future research a study of the distortions induced by the financing of reserve through seignorage.

period cannot exceed the start of period holdings of foreign reserves

$$\frac{D_t}{1-\theta} \leq R^{FX} FX_t. \tag{4.20}$$

To simplify the analysis, we restrict our attention to simple forms of intervention. In particular, we assume that to finance reserve accumulation the government levies a tax equal to a fraction χ of the output of tradable goods during tranquil times, while following a bad credit shock the government sets the tax to zero, that is

$$T_t = \begin{cases} \chi Y_t^T & \text{if } \kappa_t = \kappa_H \\ 0 & \text{if } \kappa_t = \kappa_L \end{cases} \tag{4.21}$$

where $0 \leq \chi \leq 1$. In addition, we assume that during crises the government provides loans to firms until their borrowing constraint stops binding or until the size of the intervention exceeds a fraction χ^{WK} of the start-of-period stock of reserves. Formally, we assume that

$$D_t = Min\left(\phi P^M M_t^{unc} - RB_t - \kappa_t X_t,\ \ \chi^{WK}(1-\theta)R^{FX} FX_t\right), \tag{4.22}$$

where $0 \leq \chi^{WK} \leq 1$ and M_t^{unc} is the amount of intermediate inputs that firms would choose in absence of financial frictions, that is if $\phi = 0$.

4.2.7 Market clearing and competitive equilibrium

Market clearing for the non-tradable good requires that the amount consumed is equal to the amount produced

$$C_t^N = \left(L_t^N\right)^{\alpha_N}. \tag{4.23}$$

Combining (4.23), with the households' budget constraint (4.3), the definitions of firms' profits in the tradable and non-tradable sectors (4.8) and (4.16), and the government budget constraint (4.18), we obtain the market clearing condition for the tradable good

$$C_t^T = Y_t^T - P^M M_t - B_{t+1} + RB_t - FX_{t+1} + R^{FX} FX_t - \frac{\theta}{1-\theta} D_t. \tag{4.24}$$

Finally, equating the demand and supply of labor gives

$$L_t^T + L_t^N = 1. \tag{4.25}$$

We are now ready to define a rational expectation equilibrium as a set of stochastic processes $\{C_t, C_t^T, C_t^N, P_t^N, \lambda_t, Y_t^T, L_t^T, L_t^N, M_t, B_{t+1}, \mu_t, W_t, X_{t+1}, FX_{t+1},\ T_t, D_t\}_{t=0}^{\infty}$ satisfying (4.2), (4.4)-(4.7), (4.11)-(4.14), (4.17)-(4.18) and (4.21)-(4.25), given the exogenous process $\{\kappa_t\}_{t=0}^{\infty}$, the government policy $\{\chi, \chi^{WK}\}$ and initial conditions B_0, FX_0 and X_0.

The model has a balanced growth path in which $C_t^T, Y_t^T, M_t, P_t^N, B_{t+1}$ and W_t all grow at the same rate as X_t. The real exchange rate grows at a positive rate in the balanced growth path because productivity in the tradable sector exhibits positive trend growth, while productivity in the non-tradable sector is fixed. This is the classic Balassa-Samuelson effect. Since also $GDP_t = Y_t^T - P^M M_t + P_t^N Y_t^N$ grows at the same rate as X_t, we will refer to the growth rate of the stock of knowledge as the growth rate of the economy.

4.2.8 Discussion: public and private capital flows

A novel feature of our framework is the distinction between public capital flows in the form of foreign reserves FX_t and private capital flows B_t. Before we move forward in the analysis, we want to emphasize the roots of the imperfect substituability between the internationally traded private bond and foreign reserves.

The first difference is related to the fact that in our framework domestic agents have an imperfect access to international private capital markets. In fact, domestic agents are subject to an occasionally binding borrowing constraint that limits their access to foreign credit. Crucially, the possibility of the constraint being binding in the future affects agents' behavior also when they are not constrained. In particular, a positive probability of hitting the constraint in the future limits the accumulation of private debt during periods in which access to foreign credit is plentiful. We also assume that foreign reserves provide a lower return compared to private bonds ($R^{FX} \leq R$). Moreover, similarly to what is also assumed in a first-generation currency crises model, reserves are subject to a lower bound ($FX_t \geq 0$) so that they can only be accumulated.

These features make the two assets imperfect substitutes. We note here that imperfect substitutability between B_t and FX_t would hold even if $R^{FX} = R$ as long as there is a possibility that the borrowing constraint that private agents face might be binding. This feature of the model creates the key difference with respect to the neoclassical growth model in which the accumulation of foreign reserves would be exactly offset one-for-one by private capital inflows. It also differs from the tradition in international finance as in Kouri (1981) and Branson and Henderson (1985) in which imperfect substitutability is exogenously assumed rather than arising endogenously. From our reading of the literature the distinction between the private and public nature of capital flows is novel and differs from existing contributions that identify the international reserves accumulated by the government with the economy's stock of net foreign assets.

Reassuringly, our model is consistent with the cyclical pattern of gross capital flows characterizing developing countries as described by Broner et al. (2011). In our framework tranquil times are periods of positive capital inflows, in the form of increases in private debt, as well as positive capital outflows, in the form of accumulation of official reserves. Conversely, during crises there is a retrenchment in gross capital flows. Capital inflows diminish as firms cut their stock of foreign debt, while capital outflows fall because the government employs its stock of reserves to mitigate the impact of the crisis. Because of these effects, in our model gross capital flows are procyclical, consistent with the findings of Broner et al. (2011).[13]

4.3 Social planner

Before considering the foreign reserve policy, we first characterize the social planner allocation. This is useful to build intuition about the source of inefficiency in the competitive equilibrium that creates scope for policy intervention.

The planner maximizes domestic households' utility (4.1), subject to the economy-wide resource constraints (4.23), (4.24) and (4.25), the borrowing constraint (4.9) and the two constraints on reserve management (4.19) and (4.20). Importantly, the social planner takes into account the effect that imported inputs have on the accumulation of knowledge, and so also the equation describing the evolution of the stock of knowledge (4.15) enters as a constraint in the planner's problem.

Appendix C.1 provides a formal characterization of the social planning allocation. Here we notice that, as long as $R^{FX} < R$, the social planner chooses not to hold reserves, that is she sets $FX_{t+1} = 0$ for every t.[14] Intuitively, the social planner chooses not to hold reserves because they represent an inefficient saving vehicle compared to foreign bonds, as they pay a lower interest rate. This happens notwithstanding the fact that reserves can be used to provide liquidity during crises. To understand this result, notice that the working capital constraint is affected by the private net foreign asset position at the beginning of period t. Due to the lower interest rate paid on reserves compared to private bonds, the most efficient way from the social planner perspective to relax the constraint in period t is by reducing the net debt position in period $t-1$ (i.e. increasing B_t), rather than accumulating reserves and using them

[13]Moreover, Broner et al. (2011) find that developing countries reduce their stock of official reserves during crises.

[14]If $R^{FX} = R$ the planner may hold foreign reserves, but imposing $FX_{t+1} = 0$ for every t on her allocation does not prevent the planner from reaching the first best. See the appendix for the details.

in the event of a crisis.

As showed in appendix C.1, the social planner allocation is characterized by the same equations as the competitive equilibrium in which $FX_{t+1} = D_t = 0$ is imposed in every period.[15] The only difference is given by equation (4.12), the optimality condition that determines the choice of imported inputs. In fact, in the social planner allocation equation (4.12) is replaced by

$$P^M\left(1+\phi\frac{\mu_t^{SP}}{\lambda_t^{SP}}\right) = (1-\alpha_T)\frac{Y_t^T}{M_t} + \underbrace{\beta\xi\left(\frac{X_t}{M_t}\right)^{1-\xi} E_t\left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}}\left(\alpha_T\frac{Y_{t+1}^T}{X_{t+1}} + \kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right)}_{\text{growth externality}},$$

where μ_t^{SP} is the Lagrange multiplier on the borrowing constraint (4.9) and λ_t^{SP} is the Lagrange multiplier on the resource constraint for tradable goods (4.24). The left-hand side of this expression represents the marginal cost of increasing the use of imported inputs, taking into account the impact of the borrowing constraint, captured by the term μ_t^{SP}. The first term on the right-hand side is the benefit from the increase in the output of tradable goods generated by an increase in the use of imported inputs. These two terms are equivalent to the ones that would arise in the competitive equilibrium allocation (4.12). The second term on the right-hand side is specific to the social planner problem and captures the benefits derived from the increase in the stock of knowledge implied by an increase in the use of imported inputs. Increasing the stock of knowledge is beneficial for two reasons. First, the social planner internalizes the fact that a higher usage of imported inputs today leads to higher knowledge and higher productivity tomorrow and thus to a higher amount of tradable goods produced in the future. Second, the social planner internalizes the fact that an increase in productivity tomorrow relaxes the borrowing constraint by increasing the sum that foreign investors can recover in case of default. These two effects imply that in every period the amount of imported inputs used is higher in the social planner allocation than in the competitive equilibrium without policy intervention. Because of this, the economy grows at a faster rate under the social planner allocation compared to the competitive equilibrium with no policy intervention.

It is possible to decentralize the social planner allocation in the competitive equilibrium by subsidizing the purchase of imported inputs at rate

$$\tau_t = \frac{\beta\xi}{P^M}\left(\frac{X_t}{M_t}\right)^{1-\xi} E_t\left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}}\left(\alpha_T\frac{Y_{t+1}^T}{X_{t+1}} + \kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right),$$

[15] To be precise, if the economy starts with a positive amount of reserves ($FX_0 > 0$) and it is hit by a bad credit shock during the first period ($\kappa_0 = \kappa_L$) the planner may use the initial stock of reserves to finance working capital and D_0 may be positive. Even in this case, $FX_{t+1} = 0$ for any t and so $D_t = 0$ for any $t > 0$.

while financing the subsidy using lump-sum taxes. This subsidy scheme is able to restore the first best, but in practice this form of intervention might be politically hard to implement. For instance, a government might not be able to openly subsidize firms in the export sector due to the existence of trade agreements such as the WTO rules. In the next section we show how an appropriate management of foreign exchange reserves can serve as a second best policy to internalize the growth externalities in the tradable sector, without breaking the rules dictated by free trade agreements.

4.4 Reserve policy and growth

In this section we discuss the mechanisms through which a policy of reserve accumulation during tranquil times and liquidity provision during crisis times works. In particular we are interested in providing intuition on how foreign reserves can be used as a second best policy tool aimed at internalizing the growth externalities in the tradable sector.

We start by examining the impact of foreign reserve accumulation in states in which the borrowing constraint is not binding. Combining equations (4.11), (4.12) and (4.17) and using the fact that when the borrowing constraint does not bind $\mu_t = 0$, we obtain the demand for imported inputs, M_t, as a function of the real exchange rate, P_t^N

$$M_t = \left(\frac{1-\alpha_T}{P^M}\right)^{\frac{1}{\alpha_T}} X_t \left[1 - \left(\frac{\alpha_N}{\alpha_T}\frac{P_t^N}{X_t}\left(\frac{P^M}{1-\alpha_T}\right)^{\frac{1-\alpha_T}{\alpha_T}}\right)^{\frac{1}{1-\alpha_N}}\right].$$

When the real exchange rate appreciates (P_t^N rises) the demand for imported inputs decreases. Intuitively, an increase in P_t^N, the relative price of non-tradable goods, increases the marginal product of labor in the non-tradable sector. This causes a shift of labor out of the tradable sector that decreases the productivity of the imported intermediate inputs and induces firms to reduce M_t. This suggests that in order to increase the use of imported inputs and the growth rate of the economy above their competitive equilibrium values, the government can implement policies that reduce P_t^N, that is to engineer a real exchange rate undervaluation.[16]

To understand the link between reserve accumulation and real exchange rate determination in tranquil times, we combine equations (4.6), (4.18) and (4.24) and use the fact that during

[16]We refer to a policy-induced real exchange rate undervaluation when the real exchange rate, net of the Balassa-Samuelson effect, is undervalued in the competitive equilibrium allocation with policy intervention compared to its value in the laissez-faire equilibrium.

tranquil times $D_t = 0$ to obtain

$$P_t^N = \frac{1-\omega}{\omega} \frac{Y_t^T - P^M M_t - B_{t+1} + RB_t - FX_{t+1} + R^{FX} FX_t}{C_t^N}.$$

Holding everything else constant, this equation implies a negative relationship between P_t^N and FX_{t+1}. The intuition is simple: In order to accumulate foreign reserves the government needs to withdraw resources from the private sector. Since only tradable goods can be sold to foreigners in exchange for reserves, the government must appropriate tradable goods from the private sector.[17] Private agents are then forced to reduce their consumption of tradable goods. This leads to a real exchange rate depreciation which in turns stimulates production in the tradable sector and imports of the intermediate good. Through this channel, a policy of accumulating reserves during tranquil times has the potential to increase the growth rate of the economy and to internalize, at least partly, the growth externalities present in the tradable sector.

Clearly, in general equilibrium a change in FX_{t+1} affects all the other endogenous variables. In particular private agents tend to offset the impact of the increase in foreign reserves on consumption by borrowing from abroad. Indeed, in a model in which private borrowing and reserves are perfect substitutes, the accumulation of FX_{t+1} would be counterbalanced by a corresponding decline in B_{t+1}. In our framework the imperfect substitutability between the two assets prevents private agents from completely offsetting the actions of the government.

We now illustrate the general equilibrium implications of a policy of reserve accumulation during tranquil times by examining how the stochastic steady state of our economy varies when we change the value of χ, our proxy for the resources employed to accumulate reserves during tranquil times.[18]

The six panels of figure 4.2 show the long-run mean values of the following variables: the growth rate of GDP, the percentage deviations of the real exchange rate from its value in the equilibrium with no policy intervention, the trade balance-to-GDP ratio, the private net foreign assets-to-GDP ratio, consumption of tradable goods and aggregate consumption as a function of χ, the fraction of tradable output devoted to reserve accumulation during tranquil times. The real exchange rate is normalized by the stock of knowledge to control for the Balassa-Samuelson effect. The same normalization is applied to consumption of tradable goods and to

[17]In our model, we can think of tradable goods as a proxy for the international currency.

[18]More precisely, for each value of χ we solved the model numerically. Then we drew a 10000 periods-long simulation, discarded the first 100 periods, and computed the long run average values of the variables of interest. In all the simulations we set $\chi^{WK} = 0$, details on the value of the other parameters are provided in section 4.5.1.

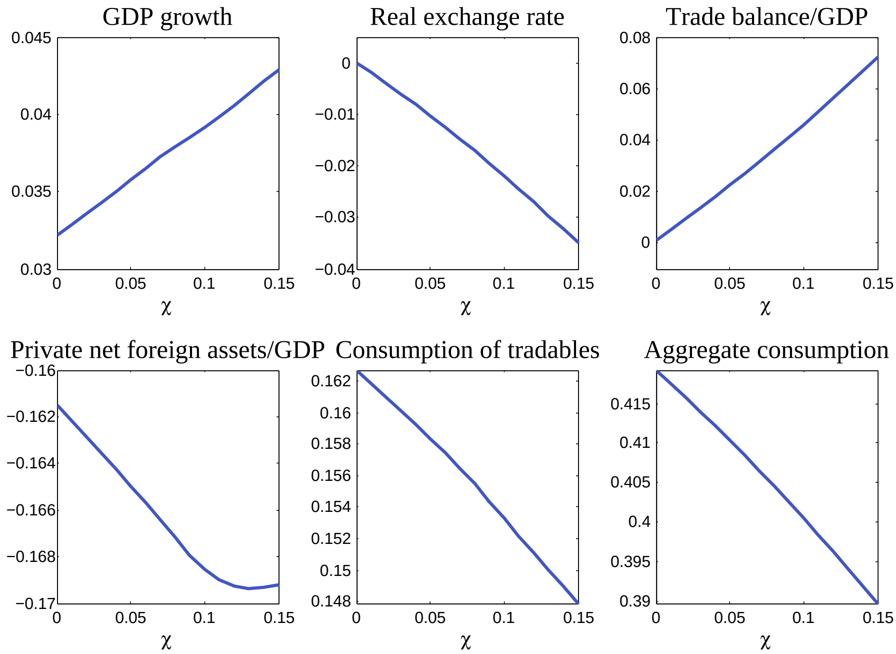


Figure 4.2: Impact of reserve accumulation

aggregate consumption.

As suggested by the partial equilibrium analysis, the growth rate of the economy is increasing in the amount of resources devoted to reserves accumulation during tranquil times. Stronger accumulation of foreign exchange reserves also produces a depreciation of the real exchange rate and an increase in the trade balance-to-GDP ratio. Both of these effects are driven by the fall in the consumption of tradable goods caused by the withdrawal of resources from private agents. The increase in the production of tradable goods implied by the real exchange rate depreciation also contributes to the improvement in the trade balance-to-GDP ratio.

Figure 4.2 shows that as the government increases the pace at which it accumulates foreign exchange reserves the private foreign debt-to-GDP ratio rises. As we mentioned above, this occurs as private agents partially offset the increase in public savings implied by faster reserve accumulation by decreasing private savings and hence by accumulating more foreign debt.[19]

De-trended consumption of tradable goods and aggregate consumption are both decreasing in the rate of reserve accumulation. This highlights a key trade-off that determines the impact on welfare of government intervention. On the one hand, faster reserve accumulation induces

[19]For very high rates of reserve accumulation the private foreign debt-to-GDP ratio decreases with the growth rate of the stock of reserves. This happens because the positive impact of reserve accumulation on production and hence on GDP outweighs the growth in the stock of private debt. However, the stock of foreign debt increases monotonically with the resources devoted to reserve accumulation.

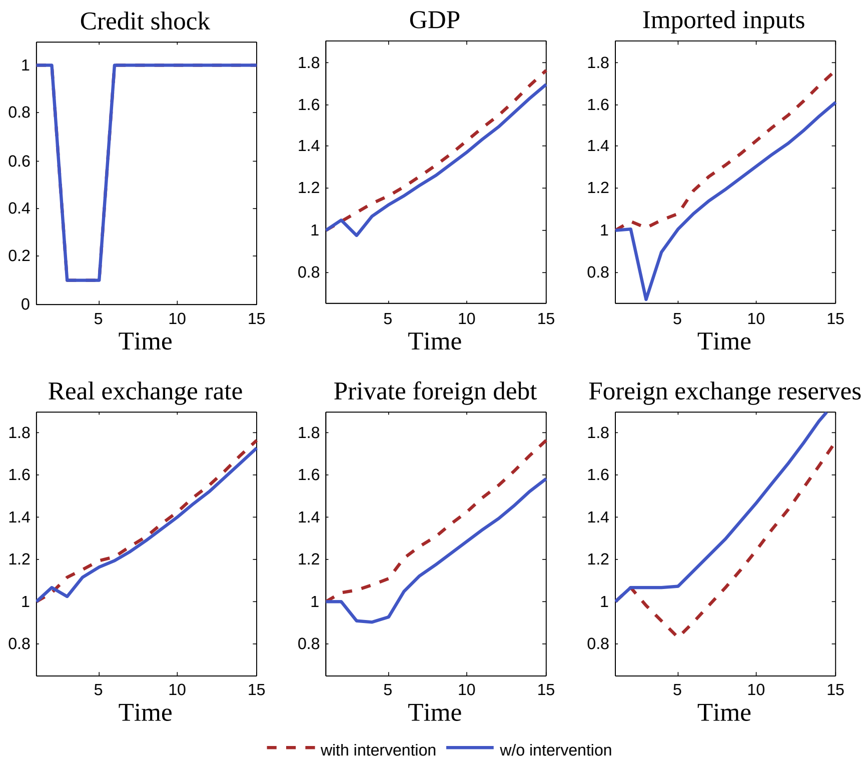


Figure 4.3: Intervention during crises

higher growth and this has a positive effect on welfare. On the other hand, in order to accumulate foreign exchange reserves the government has to subtract resources that would otherwise be consumed, and this affects welfare negatively. The balance between these two effects determines whether reserve accumulation during tranquil times has a positive or negative impact on welfare, as we will document later.

We now turn to the impact of crisis-times interventions. During crisis times, the borrowing constraint binds and the amount of imported inputs used in production is given by

$$M_t = \frac{X_t \kappa_L + RB_t + D_t}{\phi P^M}.$$

This equation makes clear that in order to increase the amount of imported inputs used by firms above its value in the equilibrium without intervention, the government has to provide working capital loans during crisis events (i.e. set $D_t > 0$). Hence, in the model the existence of growth externalities in the tradable sector, coupled with financial frictions, provides a justification for the use of reserves during crises.

Figure 4.3 compares the response to a negative credit shock for two different economies.[20]

[20]To construct this figure, we simulated the economy with $\chi = 0.09$ and $\chi^{WK} = 1$ for 10000 periods, discarded the first 100 periods and then collected all the periods with a negative credit shock ($\kappa_t = \kappa_L$). We then constructed windows around each period t with a bad credit shock going from $t-2$ years before the

The solid lines refer to an economy in which the government does not intervene during the crisis ($\chi^{WK} = 0$). When the bad credit shock hits the economy in period 3, firms become borrowing constrained, they are forced to cut their imports of intermediate inputs and this negatively affects production of tradable goods and GDP. The real exchange rate depreciates because households have to cut their consumption of tradable goods and because labor flows toward the non-tradable sector, thus increasing the supply of non-tradable goods. Moreover, since credit shocks are persistent, households decrease their stock of inter-temporal foreign debt in order to self-insure against the increased risk of a future bad credit shock.

The dashed lines refer to the case in which the government uses its stock of reserves to provide working capital loans to firms in the tradable sector ($\chi^{WK} > 0$). When the bad credit shock hits the economy, the government starts drawing down its stock of reserves to finance the purchase of imported inputs. This mutes the impact of the credit shock on GDP and on the real exchange rate. In addition, the bad credit shock generates a milder decrease in foreign debt compared to the case with no intervention, because households anticipate that the government will intervene in case of a future bad credit shock.

Notice that the crisis entails a permanent difference in the level of GDP between the two economies. This stems from the fact that in our model an economy hit by a crisis never fully recovers to its pre-crisis growth path.[21] Because of this reason, intervening during crises has a positive impact on the average growth rate of the economy.

One interesting feature of the model is that the relationship between growth and the real exchange rate depends on whether the economy is borrowing constrained or not. In fact the binding borrowing constraint reverses the negative relationship between growth and real exchange rate observed during tranquil times. This happens because to stimulate growth during crises the government has to provide loans to firms in the tradable sector. This shifts productive resources toward the tradable sector, allowing households to consume more tradable goods. At the same time, the production of non-tradable goods decreases and so the real exchange rate appreciates, creating a positive relationship between real exchange rate, use of imported inputs and growth.

shock to $t + 12$ years after. We then collected the median path for κ_t and the median initial values for the state variables B_{t-2} and FX_{t-2} across all the windows. Finally, we fed this path for the credit shock and these initial conditions to the model without intervention during crises ($\chi^{WK} = 0$) and to the model with intervention ($\chi^{WK} = 1$).

[21] Cerra and Saxena (2008) provide empirical evidence showing that countries that are hit by a crisis hardly get back to their pre-crisis growth path.

4.5 Financial liberalization and optimal management of foreign exchange reserves

In this section we use our framework to describe the impact of international reserve management on the transition from financial autarky to a regime in which foreign borrowing is allowed, but limited by the borrowing constraint (4.9). This experiment demonstrates the model's ability to reproduce the pattern of growth, capital flows and reserve accumulation observed in the data. Moreover, we use this exercise to evaluate the significance of the welfare gains that can be obtained through an appropriate management of foreign exchange reserves.

4.5.1 Parameters

The model cannot be solved analytically and so we must resort to numerical simulations. In order to preserve the non-linearities present in our framework we solve the model using a global solution method.[22] The model is too simple to lend itself to a careful calibration exercise, hence we choose reasonable values for the parameters in order to illustrate the model's properties.

Some parameters are standard in the literature. The risk aversion parameter is set at $\gamma = 2$. The interest rate at which domestic agents can borrow from foreign investors is assumed equal to $R = 1.04$, while the discount factor is set to $\beta = 1/R$. We choose identical labor shares in the two sectors $\alpha_T = \alpha_N = 0.65$. The share of tradable goods in consumption is set to $\omega = 0.341$ as in Durdu et al. (2009). The price of imported inputs P^M is normalized to 1 without loss of generality.

The parameters governing the financial frictions are set so that the version of the model without government intervention reproduces salient characteristics of developing countries. We set the borrowing limit κ_L equal to 0.1. This gives an average net foreign assets-to-GDP ratio of -16 percent, in the range of the values commonly observed in developing countries.[23] The probability of experiencing a bad credit shock is set to $1 - \rho_H = 0.1$ as in Jeanne and Rancière (2011), while the probability of exiting an episode of financial turbulence is set to $1 - \rho_L = 0.5$, following Alfaro and Kanczuk (2009). The fraction of imported inputs that has to be paid in advance ϕ is set to 0.33 to match an average working capital-to-GDP ratio of 6 percent. This is the same target as in Mendoza and Yue (2011).

To parameterize the process for the accumulation of knowledge we use the estimates pro-

[22] More precisely, we solve the model by iterating on the equilibrium conditions as proposed by Coleman (1990).

[23] The precise value of κ_H does not affect the simulations, as long as it is sufficiently high so that the borrowing constraint does not bind when $\kappa_t = \kappa_H$.

Table 1: Parameters

Parameter	Symbol	Value
Risk aversion	γ	2
Interest rate on private borrowing	R	1.04
Discount factor	β	$1/R$
Labor share in output in tradable sector	α_T	0.65
Labor share in output in non-tradable sector	α_N	0.65
Share of tradable goods in consumption	ω	0.341
Price of imported inputs	P^M	1
Borrowing limit	κ_L	0.1
Probability of bad credit shock	$1-\rho_H$	0.1
Probability of exiting bad credit shock	$1-\rho_L$	0.5
Working capital coefficient	ϕ	0.33
Elasticity of TFP w.r.t. imported inputs	ξ	0.15
Constant in knowledge accumulation process	ψ	0.34
Interest rate on reserves	R^{FX}	1
Efficiency of government intervention during crises	θ	0.5

Table 4.1: Parameters

vided by Coe et al. (1997). They find that the elasticity of TFP with respect to imports of machinery and equipment in developing countries is close to 0.3. They do not estimate which part of the effect can be attributed to spillovers that are not internalized by firms, so 0.3 is likely to be an upper bound for our parameter ξ. We take a pragmatic approach and set $\xi = 0.15$. The constant in the knowledge accumulation process ψ is set to 0.34, in order to match an average growth rate of 3 percent in the competitive equilibrium without government intervention.

The gross interest rate paid on reserves R^{FX} is equal to 1. This gives a spread between private borrowing cost and the interest rate paid on reserves of 4 percent, in the range of the values considered by Rodrik (2006). We could not find good estimates for θ, the parameter that determines the efficiency of government intervention during crises. Hence, we somehow arbitrarily set it to 0.5. Our intuition is that our main results would not be affected by changes in the value of this parameter.

4.5.2 Results

We start by exploring how the foreign reserve policy affects the adjustment process of an economy that opens up to international capital flows. To capture the opening to international credit markets, we look at economies that start with no foreign debt ($B_0 = 0$) and with no reserves ($FX_0 = 0$) and we follow them during the transition to a steady state in which foreign borrowing is allowed, but constrained by condition (4.9). We also assume that the economy starts in tranquil times ($\kappa_0 = \kappa_H$).

We compare two different economies. First, we look at an economy in which the government does not intervene, that is in which $\chi = \chi^{WK} = 0$. Second, we consider an economy in which the government optimally chooses the parameters governing the foreign reserve policy, χ and χ^{WK}. To compute the optimal policy we constructed grids for χ and χ^{WK} and then we searched for the combination of these two parameters that maximizes the expected lifetime utility of the representative household. Given our parametrization the optimal policy is characterized by $\chi = 0.09$, which implies that the government devotes 9 percent of the output of tradable goods to the accumulation of reserves during each tranquil period, and $\chi^{WK} = 1$, which means that the government is willing to use up to its whole stock of reserves to intervene during crises.

We derived forecast functions that describe the transition from financial autarky to the steady state with financial liberalization using the following procedure. For each model economy we performed 100000 stochastic simulations lasting for 15 periods each, taking as initial conditions $B_0 = FX_0 = 0$ and $\kappa_0 = \kappa_H$. For each period we then averaged across all the simulations to obtain our forecast functions. Figure 4.4 shows the results of the experiment. To facilitate comparison, GDP, consumption of tradable goods, consumption of non-tradable goods and the real exchange rate are all expressed in percentage deviations from their first-period value in the equilibrium without government intervention.

Start by considering the solid lines, which describe the economy without government intervention. Upon opening to the international credit markets, the economy embarks in a period of accumulation of foreign debt that lasts for around five years, when the private net foreign assets-to-GDP ratio reaches its steady state value of -16 percent. The accumulation of foreign debt is the result of two forces. On the one hand, households living in an economy that is growing faster than the rest of the world, as we are implicitly assuming, have the desire to frontload their consumption stream and this pushes domestic agents to accumulate foreign debt. On the other hand, a high stock of foreign debt increases the negative impact of a bad credit shock on production of tradable goods. Because of this, domestic agents accumulate precautionary savings to self-insure against the risk of a bad credit shock and this puts a brake to the buildup of foreign debt. The counterpart to the process of debt accumulation are the high initial current account deficits, that progressively decrease until the current account-to-GDP ratio reaches its steady state value of -1 percent.

The first years following financial liberalization also see a progressive increase in the growth rate of the economy. This happens because foreign borrowing props up the consumption of tradable goods for a given amount of tradable goods produced. This gives an incentive to shift

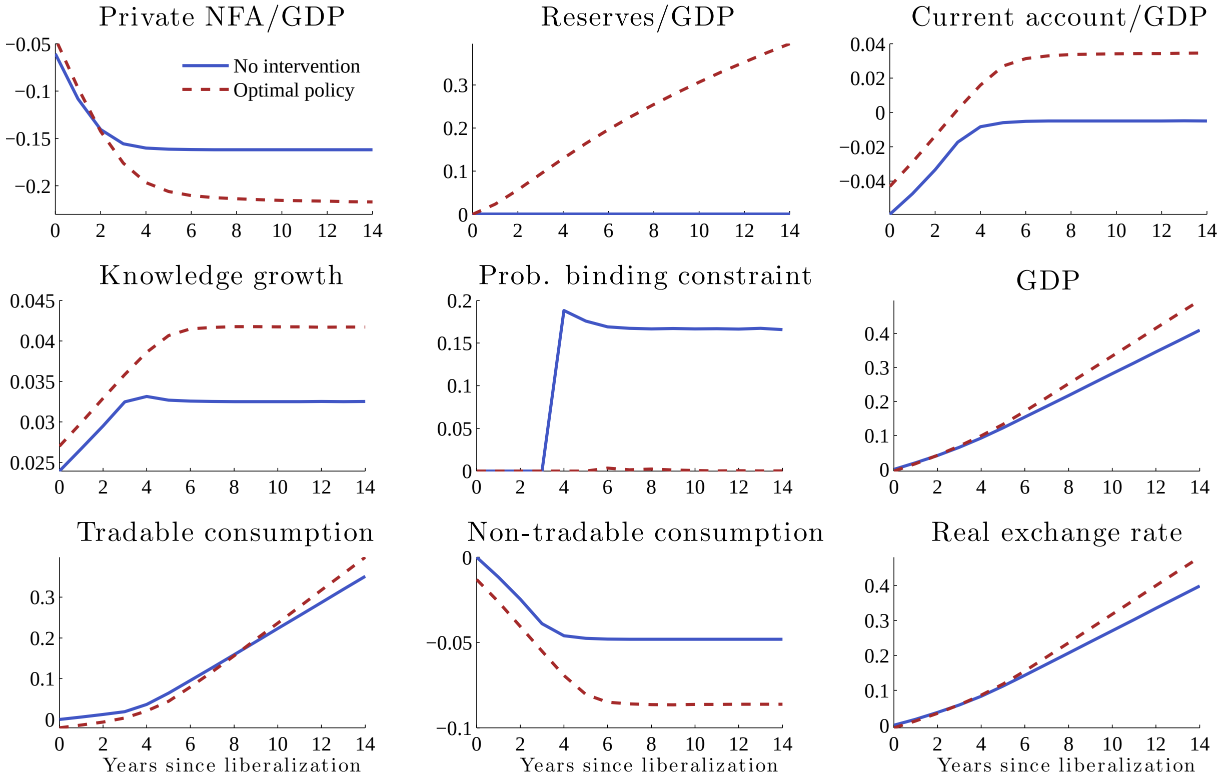


Figure 4.4: Impact of reserve policy

labor toward the production of non-tradable goods, which is higher during the first years after liberalization compared to its steady state value. As the economy approaches its steady state, progressively more labor is allocated to the production of tradable goods, more intermediate inputs are imported and the growth rate of the economy increases until it reaches its steady state value.

Finally, during the first years after the opening to international credit markets the probability of experiencing a binding borrowing constraint is zero, because of the low stock of initial debt. As the stock of foreign debt increases, so does the probability of entering a financial crisis.

The dashed lines refer to the economy in which the government implements the optimal policy. After the opening to the international credit markets the government starts to accumulate foreign reserves at a fast pace. In fact, in the first fifteen years after financial liberalization the reserves-to-GDP ratio passes from 0 to almost 40 percent. Afterward, the reserves-to-GDP ratio keeps growing until it reaches its steady state value of 84 percent. Because of this policy, net capital inflows are lower compared to the laissez-faire equilibrium. Indeed, in steady state the current account-to-GDP ratio in the economy with policy intervention is 5 percentage points higher than in the economy without intervention.

The economy with government intervention posts higher current account surpluses despite higher accumulation of foreign debt from the private sector. The large buildup of private debt is driven by two effects. First, as discussed in section 4.4, private agents take on foreign debt to partly offset the impact of reserve accumulation on consumption. Second, in the economy with government intervention the incentives for private agents to build a stock of precautionary savings are weaker, because firms in the tradable sector anticipate that the government will supply working capital financing during crisis events. The result is that in steady state the private net foreign assets-to-GDP ratio is 5 percentage points lower compared to the economy without policy intervention.

Despite the reaction of private agents and because of the imperfect substitutability between private and public capital flows, the government policy succeeds in engineering a real exchange rate undervaluation that shifts productive resources out of the non-tradable sector and into the production of tradable goods.[24] Moreover, the government intervention during crises reduces to almost zero the probability of facing a binding borrowing constraint. These two effects lead to a higher use of imported inputs and to a faster growth rate of the economy compared to the equilibrium with no policy intervention. In fact, in steady state the growth rate of the stock of knowledge is 1 percent higher than under laissez-faire.

The model is thus able to replicate the negative correlation between growth and capital inflows observed in the data. Moreover, consistent with empirical evidence, the correlation is driven by the accumulation of foreign reserves from the public sector.

Figure 4.4 can also be used to illustrate the intuition underlying the impact on welfare of government interventions. During the first years after financial liberalization, consumption of tradable goods is lower in the economy with policy intervention compared to the laissez-faire equilibrium. This happens because the government appropriates tradable goods from the private sector to finance the accumulation of reserves. However, the government policy also leads to faster growth and this explains why from year 9 on the consumption of tradable goods becomes higher in the equilibrium with policy intervention compared to the one without intervention. Hence, the government faces a trade-off between lower consumption of tradable goods in the present, in exchange for faster growth and thus higher consumption of tradable goods in the future.

[24]Notice that the undervaluation refers to the real exchange rate purged from the Balassa-Samuelson effect. In absolute terms, the real exchange rate in the economy with policy intervention is undervalued compared to the laissez-faire equilibrium only during the first years after liberalization. Due to faster productivity growth in the tradable sector induced by reserve accumulation, the real exchange rate in the economy with government intervention eventually becomes more appreciated than in the economy with no intervention.

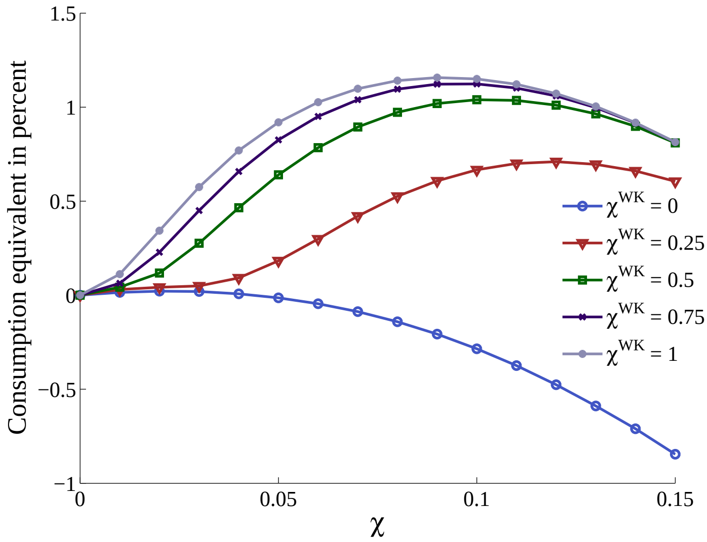


Figure 4.5: Welfare impact of policy interventions

To describe the impact on welfare of different reserve management policies, we report the welfare gains that can be obtained from government intervention for an economy that undergoes financial liberalization. We compute the welfare gains of moving from the equilibrium with no intervention to a generic policy regime i as the proportional increase in consumption for all possible future histories that households living in the economy with no policy intervention must receive in order to be indifferent between remaining the no-intervention economy and switching to policy regime i. Formally, the welfare gain η is defined as

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left((1+\eta)\, C_t^n\right)^{1-\gamma}}{1-\gamma} \right] = E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{i\,1-\gamma}}{1-\gamma} \right],$$

where the superscripts n and i denote allocations respectively in the economy with no policy intervention and under a generic policy regime i. Since we want to look at economies that start from financial autarky we set the initial states to $B_0 = 0$, $FX_0 = 0$ and $\kappa_0 = \kappa_H$.

Figure 4.5 presents the results of our welfare analysis by plotting the welfare gains as a function of the resources employed to accumulate reserves during tranquil times χ, for different intensities of the intervention during crises χ^{WK}.

The first thing to notice is that the welfare gains from policy intervention are quantitatively significant. For instance, the optimal policy delivers welfare gains above 1 percent of permanent consumption equivalent. Moreover, the bulk of the welfare gains seems to come from the ability to provide liquidity to firms during crises. This can be seen from the large welfare differences between the economy with no intervention during crises ($\chi^{WK} = 0$) and those in which the government does intervene to provide liquidity during periods of financial turbulence ($\chi^{WK} > 0$). In addition, under the welfare maximizing rule reserves are accumulated at a fast pace, since 9 percent of the output of tradable goods is devoted to the accumulation of reserves each tranquil period.

Interestingly, some welfare gains, albeit small, can be obtained through the accumulation of foreign exchange reserves also when they cannot be used to intervene during crises. This can be seen by looking at the $\chi^{WK} = 0$ line, which reaches its maximum corresponding to a consumption equivalent of 0.02 percent when $\chi = 0.02$. Thus reserve accumulation can be a welfare enhancing policy also when reserves cannot perform their traditional role of liquidity provider during financial crises.

4.6 Foreign aid

As noticed by Alfaro et al. (2011), the observed current account deficits in countries with dismal growth performances are due to inflows of foreign aid, in the form of development loans provided by institutions such as the World Bank or regional development banks. In this section we consider an economy that receives foreign aid and show that our model is consistent with the relationship between flows of foreign aid, current account balances and growth observed in the data.

To focus on the role of foreign aid, we abstract from reserve accumulation and so we set $FX_t = 0$ in every period. Instead, every period the government receives foreign aid in the form of a loan Z_{t+1}, which, for simplicity, carries a zero interest rate. The government uses the loan to repay its previous debts and transfers the difference to households via a lump-sum transfer H_t.[25] Hence, the government budget constraint is

$$Z_{t+1} = Z_t + H_t.$$

We parameterize the flow of foreign aid to target a steady state aid-to-GDP ratio, as measured by H_t/GDP_t, of 6 percent, following the estimate for Cote d'Ivoire reported by Arellano et al. (2009). Notice that since in every period the transfer to households is positive then $Z_{t+1} - Z_t = H_t > 0$ in every t, and the government accumulates foreign liabilities due to the inflows of foreign aid. This implies that, keeping everything else constant, foreign aid tends to increase the current account deficits of a country.

[25]The results would not change if the transfer was made to firms instead.

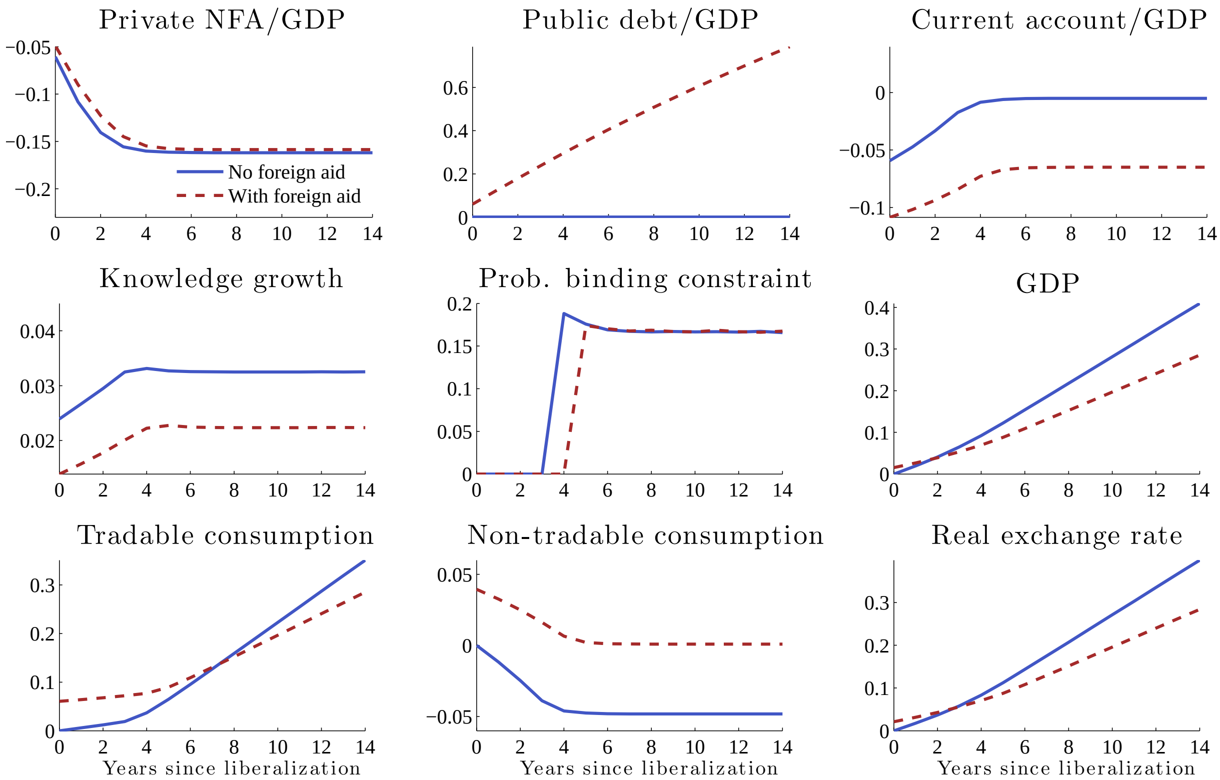


Figure 4.6: Impact of foreign aid

To consider the general equilibrium implications of foreign aid we perform an experiment analogous to the financial liberalization exercise in section 4.5. Specifically, we compare the transition after the opening of the capital account of two economies, with and without foreign aid.[26] Figure 4.6 shows the transition for the economy without aid, solid lines, and for the economy receiving foreign aid, dashed lines.[27]

The inflows of foreign aid lead to current account deficits, because of the accumulation of foreign liabilities by the government. Moreover, the economy that receives foreign aid features a lower growth rate compared to the economy without intervention. In fact, aid from abroad, which is essentially an exogenous transfer of tradable goods, induces households to increase consumption of tradables. In turn, the increase in consumption of tradables generates a real exchange rate appreciation, inducing a shift of productive resources toward the non-tradable sector. Hence, the inflows of foreign aid lead to lower production in the tradable sector, smaller use of imported inputs and slower growth of the stock of knowledge. Through this channel,

[26] As in the previous section, we derived forecast functions by performing 100000 stochastic simulations lasting for 15 periods each, taking as initial conditions $B_0 = Z_0 = 0$ and $\kappa_0 = \kappa_H$. We then average across all the simulations to obtain our forecast functions.

[27] The economy without foreign aid corresponds to the competitive equilibrium without policy intervention.

our model is able to reproduce the correlation between foreign aid, growth and current account deficits observed in the data.[28]

The impact of foreign aid on welfare is a priori ambiguous. On the one hand, foreign aid corresponds to an exogenous increase in the income of the household, which has a positive impact on welfare. On the other hand, foreign aid induces a shift of productive resources out of the tradable sector that exacerbates the inefficiencies due to the learning externality. The impact of this second effect on welfare is negative. Indeed, under our baseline parameterization foreign aid negatively affects welfare. In fact, η, the permanent increase in the consumption stream that an agent living in the economy without policy intervention has to receive to be indifferent with switching to the economy with foreign aid, is equal to -1.1 percent. This means that a permanent foreign aid transfer of around 6 percent of GDP has a significant negative impact on welfare.

4.7 Conclusion

This paper presents a framework that it is able to reproduce two facts characterizing the international monetary system: Fast growing emerging countries i) Run current account surpluses, ii) Accumulate international reserves and receive net private inflows. In our framework the government uses foreign exchange reserves to internalize the growth externalities present in the tradable sector and to provide liquidity to private agents during periods of financial stress. This creates a positive link between reserve accumulation, current account surpluses and growth. Importantly, in our framework official reserves and private debt are imperfect substitutes, so that the reserve policy of the government cannot be perfectly offset through borrowing by private agents.

We use the model to compare the laissez-faire equilibrium and the optimal reserve policy in an economy that is opening to international capital flows. We find that the optimal reserve management entails a fast rate of reserve accumulation, as well as higher growth and larger current account surpluses compared to the economy with no policy intervention. We also find that the welfare gains of reserve policy are significant, in the order of 1 percent of permanent consumption equivalent. Finally, we show that our model is consistent with the pattern of low growth and current account deficits driven by inflows of foreign aid observed in low income countries.

[28] It is possible that the negative correlation between foreign aid and growth is due to the fact that donors allocate more aid to those countries in which growth is weaker. However, the empirical analysis of Rajan and Subramanian (2011) lends support to the mechanism described by our model.

The simple framework that we propose can be extended in a number of directions to study several issues related to the international monetary system. For example, extending the model to a two country framework sheds light on the impact of reserve accumulation from developing countries on global interest rates and on the country issuing the reserve currency (Benigno and Fornaro (2012)). It would also be interesting to introduce into the model the possibility for the government to implement controls on private capital flows. We conjecture that the imposition of barriers to private borrowing would make the impact of reserve accumulation on growth more effective. In light of this, the model could provide an explanation for the practice of imposing tight controls on capital flows characterizing many developing economies. Another interesting avenue of research would be to consider alternative financing schemes for reserve accumulation: allowing for distortionary financing would entail a further cost of the reserve accumulation policy that might limit its effectivness and benefits. We are planning to address these topics in future research.

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Appendix

Appendix A

International Debt Deleveraging

A.1 Numerical solution method

To solve the model numerically I employ the method proposed by Guerrieri and Lorenzoni (2011).

Computing the steady state of the model involves finding the interest rate that clears the bond market at the world level. The first step consists in deriving the optimal policy functions $C^T\left(B, A^T\right)$, $C^N\left(B, A^T\right)$ and $L\left(B, A^T\right)$ for a given interest rate R. To compute the optimal policy functions I discretize the endogenous state variable B using a uniformly spaced grid with 600 points, and then iterate on the Euler equation and on the intratemporal optimality conditions using the endogenous gridpoints method of Carroll (2006). Using the optimal policies, it is possible to derive the inverse of the bond accumulation policy $g(B, A^T)$. This is used to update the conditional bond distribution $\Psi\left(B, A^T\right)$ according to the formula $\Psi_\tau\left(B, A^T\right) = \sum_{A^T} \Psi_{\tau-1}\left(g(B, \tilde{A}^T), \tilde{A}^T\right) P(A^T|\tilde{A}^T)$ for all $B \leq -\kappa$, where τ is the τ-th iteration and $P(A^T|\tilde{A}^T)$ is the probability that $A^T_{t+1} = A^T$ if $A^T_t = \tilde{A}^T$. The bond accumulation function is not invertible at $B = -\kappa$, but the formula above holds if $g(-\kappa, A^T)$ is defined as the largest B such that $B' = -\kappa$ is optimal. Once the bond distribution has converged to the stationary distribution, I check whether the market for bonds clears. If not, I update the guess for the interest rate.

To compute the transitional dynamics, I first derive the initial and final steady states. I then choose a T large enough so that the economy has approximately converged to the final steady state at $t = T$ (I use $T = 200$, increasing T does not affect the results reported). The next step consists in guessing a path for the interest rate, with $R_T = R'$. I then set the policy functions for consumption in period T equal to the ones in the final steady state and iterate backward on the Euler equation and on the intratemporal optimality conditions to find the sequence of optimal policies $\{C_t^T(B, A^T), C_t^N(B, A^T), L_t(B, A^T)\}$. Next, I use the optimal policies to compute the sequence of bond distributions $\Psi_t(B, A^T)$ going forward from $t = 0$ to $t = T$, starting with the distribution in the initial steady state. Finally, I compute the world demand for bonds in every period and update the path for the interest rate until the market clears in every period.

To compute the transitional dynamics with wage rigidities I follow a similar method. The only difference is that the equilibrium in period $t = 0$ has to be modified to take into account the presence of pre-set wages.

Also the numerical solution to the model with multi-period wage rigidities is derived using the method described above. However, the presence of another endogenous state variable, the period 0 nominal wage, for the periods in which wages are partly rigid increases significantly the time needed to obtain a numerical solution.

A.2 A model with interest rate spreads

In the model presented in the main text all the agents, and countries, are subject to the same interest rate. However, in many cases tight credit conditions manifest themselves with high interest rates. In fact, often countries whose access to the international credit markets is restricted are charged a spread over the interest rate paid by unconstrained countries. This appendix shows how it is possible to reconcile this fact with the model without changing any of the results. In particular, in this appendix I present a model in which the borrowing limit is enforced through interest rate spreads and show that this model is isomorphic to the framework studied in the main text. The discussion draws on Uribe (2006).

For simplicity I focus on the economy without nominal rigidities described in section 1.2, but the results can be extended to the case of a monetary union with nominal wage rigidities. Suppose that the representative household in country i is charged the country-specific interest rate $R_{i,t}$, potentially different from the world interest rate R_t. Suppose also that there is no limit to how much the household can borrow at the interest rate $R_{i,t}$. The Euler equation then writes

$$U_{C_{i,t}^T} = R_{i,t}\beta E_t\left[U_{C_{i,t+1}^T}\right].$$

In the model in the main text, in which the household is constrained by the borrowing limit

(4.9) and it is charged the world interest rate R_t, the Euler equation can instead be written as

$$U_{C^T_{i,t}} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{U_{C^T_{i,t}}}} \beta E_t \left[U_{C^T_{i,t+1}} \right],$$

where $\mu_{i,t}$ is the Lagrange multiplier on the borrowing constraint.

Notice that if the household is charged the interest rate

$$R_{i,t} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{U_{C^T_{i,t}}}},$$

the two Euler equations coincide. Moreover, we have that $R_{i,t} = R_t$ when $\mu_{i,t} = 0$, while $R_{i,t} > R_t$ when $\mu_{i,t} > 0$.[1] Intuitively, investors can make sure that an household respects the borrowing limit by charging a positive spread over the world interest rate anytime the household would violate the constraint if charged the world interest rate. In equilibrium, we would thus observe that high-debt constrained countries are charged a positive spread over the world interest rate.

To obtain a version of the model with interest rate spreads isomorphic to the model in the main text, we must make sure that the resource constraint of the household is not affected by the interest rate spreads. Indeed, whenever the constraint is binding there is a financial rent given by the difference between the cost of funds for the investor and the interest rate that the borrower would like to pay. In the model in the main text this rent accrues to the borrower, since constrained borrowers are charged the world interest rate, that is the cost of funds for investors. We must then ensure that financial rents go fully to the borrower also in the version of the model with interest rate spreads. Following Uribe (2006), this can be done by assuming the existence of domestic financial intermediaries that borrow at the world interest rate R_t and lend to households at the interest rate $R_{i,t}$. Assuming that the profits of the domestic financial intermediaries are fully rebated to households in a lump sum fashion, we obtain that the economy with the borrowing constraint described in the main text is isomorphic to the economy with spreads described in this appendix.

[1] Using the fact that $\mu_{i,t} \geq 0$ and rearranging the Euler equation in the main text, it is easy to check that $\mu_{i,t} R_t / U_{C^T_{i,t}} < 1$.

A.3 Proof of proposition 1

To prove that from the perspective of a single country the equilibrium with flexible wages attains the first best I characterize the solution to the social planner problem for a single country. Importantly, the social planner in a single country takes the world interest rate as given, since a single country is too small to influence the world interest rate.

The social planner in a generic country i chooses $C_{i,t}^T$, $C_{i,t}^N$, $L_{i,t}$, $L_{i,t}^T$, $L_{i,t}^N$ and $B_{i,t+1}$, taking the path for the interest rate $\{R_t\}_{t=0}^{\infty}$ and the initial bond position $B_{i,0}$ as given, to maximize expected utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{i,t}^T, C_{i,t}^N, L_{i,t}\right)\right],$$

subject to the resource constraints

$$C_{i,t}^T = A_{i,t}^T\left(L_{i,t}^T\right)^{\alpha_T} + B_{i,t} - \frac{B_{i,t+1}}{R_t} \tag{A.3.1}$$

$$C_{i,t}^N = A^N\left(L_{i,t}^N\right)^{\alpha_N} \tag{A.3.2}$$

$$L_{i,t}^T + L_{i,t}^N = L_{i,t}, \tag{A.3.3}$$

and the borrowing constraint

$$B_{i,t+1} \geq -\kappa. \tag{A.3.4}$$

The first order conditions are

$$U_{C_{i,t}^T} = \lambda_{i,t}^T$$

$$U_{C_{i,t}^N} = \lambda_{i,t}^N$$

$$-U_{L_{i,t}} = \lambda_{i,t}^L$$

$$\lambda_{i,t}^T\alpha_T A_{i,t}^T\left(L_{i,t}^T\right)^{\alpha_T-1} = \lambda_{i,t}^L$$

$$\lambda_{i,t}^N\alpha_N A^N\left(L_{i,t}^N\right)^{\alpha_N-1} = \lambda_{i,t}^L$$

$$\frac{U_{C_{i,t}^T}}{R_t} = \beta E_t\left[U_{C_{i,t+1}^T}\right] + \mu_{i,t}$$

$$B_{i,t+1} \geq -\kappa, \quad \text{with equality if } \mu_{i,t} > 0,$$

where U_x denotes the first derivative of the utility function with respect to x and λ_i^T, λ_i^N, λ_i^L and μ_i are the Lagrange multiplier associated respectively with constraint $(A.3.1)$, $(A.3.2)$, $(A.3.3)$ and $(A.3.4)$.

Defining

$$w_{i,t} = \frac{\lambda^L_{i,t}}{\lambda^T_{i,t}}$$

$$p^N_{i,t} = \frac{\lambda^N_{i,t}}{\lambda^T_{i,t}},$$

it is easy to verify that the social planner allocation coincide with the equilibrium conditions of the flexible wage version of the model presented in section 1.2. ■

Appendix B

Debt Deleveraging, Debt Relief and Liquidity Traps

B.1 Proofs

B.1.1 Proof of proposition 2

I first show that if condition 3 holds then the economy is in a liquidity trap in period 0 and $i_0 = 0$. Suppose that the zero lower bound constraint on the nominal interest rate does not bind. Then the central bank hits the inflation target, $\pi_0 = \bar{\pi}$ and the economy operates at full employment, $L_0 = \bar{L}$. Moreover, the real interest rate satisfies $r_0 \geq \bar{\pi}^{-1} - 1$. The Euler equation for creditors, equation (2.4), then implies

$$\beta\left(1+r_0\right)\frac{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \frac{\bar{\kappa}}{1+r_0}\right)\right)} = 1.$$

The left-hand side of this expression is increasing in r_0, hence we have that

$$\frac{\beta}{\bar{\pi}}\frac{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right)} \leq 1,$$

which contradicts condition 3. Hence, we must have $i_0 = 0$.

I now characterize the equilibrium in a liquidity trap. Since $i_0 = 0$ then $r_0 = \pi_1^{-1} - 1$. The Euler equation for creditors is then

$$U'\left(L_0^\alpha + \frac{n}{1-n}\left(B_0 - \pi_1\bar{\kappa}\right)\right) = \frac{\beta}{\pi_1}U'\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right). \tag{B.1.1}$$

Suppose that the economy is at full employment in period 0 ($L_0 = \bar{L}$). Then the central bank can hit both its inflation and employment targets in period 1 and so $L_1 = \bar{L}$, $\pi_1 = \bar{\pi}$ and $r_1 = r$. So the Euler equation for creditors writes

$$U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right) = \frac{\beta}{\bar{\pi}} U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right),$$

which contradicts condition 3. Hence, equilibrium labor is given by the $L_0 < \bar{L}$ that solves equation $(B.1.1)$ and there is involuntary unemployment.

Moreover, the fact that there is involuntary unemployment implies that the constraint on wage setting, equation (2.2.3), binds and so

$$\frac{W_0}{W_{-1}} = \phi\left(L_0\right) < \bar{\pi},$$

where the last inequality follows from assumption 2.2.6. Combining the optimality conditions for firms, equation (2.6) in periods -1 and 0, the fact that the economy starts in steady state and so $L_{-1} = \bar{L}$ and the wage setting equation gives

$$\left(\frac{\bar{L}}{L_0}\right)^{1-\alpha} \pi_0 = \frac{W_0}{W_{-1}} = \phi\left(L_0\right) < \bar{\pi}.$$

This condition implies that $\pi_0 < \bar{\pi}$.

I now show that the economy exits the liquidity trap in period 1 and so $i_t > 0$ and $\pi_t \geq \bar{\pi}$ for $t > 0$. There are two cases to consider. First, consider a central bank that targets employment. Assume that in periods $t > 0$ the zero lower bound constraint does not bind and so the employment target is hit, that is $L_t = \bar{L}$ for $t > 0$. Then the economy enters a steady state with $r_t = r = 1/\beta - 1$, which, by assumption 2.2.6, implies $i_t > 0$ for $t > 0$. This means that the economy exits the liquidity trap in period 1, validating our conjecture that the central bank hits the employment target in periods $t > 0$. Moreover, we can write the equation that determines the evolution of employment as

$$\pi_t = \left(\frac{L_t}{L_{t-1}}\right)^{1-\alpha} \frac{W_t}{W_{t-1}} \geq \left(\frac{L_t}{L_{t-1}}\right)^{1-\alpha} \phi\left(L_t\right). \tag{B.1.2}$$

Recall that $\phi(0) < \bar{\pi}$ and that $\phi'(\cdot) \leq 0$. For $t > 1$ we have $L_{t-1} = \bar{L}$ and so the central bank can satisfy $(B.1.2)$ by setting $\pi_t = \bar{\pi}$. In $t = 1$ $L_{t-1} = L_0 < \bar{L}$ and so $\pi_1 \geq \bar{\pi}$.

Let us now consider a central bank that targets inflation. Suppose that the inflation target is hit in periods $t > 0$, so $\pi_t = \bar{\pi}$ for $t > 0$. Then L_t evolves according to

$$L_t = \min\left(\bar{L}, \left(\frac{\bar{\pi}}{\phi\left(L_t\right)}\right)^{\frac{1}{1-\alpha}} L_{t-1}\right) \geq L_{t-1},$$

where the inequality follows from $\phi(0) < \bar{\pi}$ and $\phi'(\cdot) \leq 0$. Hence the growth rate of labor is nonnegative.

This implies that also creditors' consumption C_t^c grows over time and increases with L_t. Suppose this is not the case in $t = 1$. Then $r_1 < r$. Moreover

$$C_1^c > C_2^c \leftrightarrow L_1^\alpha + \frac{r_1}{1+r_1}\bar{\kappa} > L_2^\alpha + \frac{r_2}{1+r_2}\bar{\kappa}.$$

Since $L_1 \leq L_2$ this condition implies that $r_2 < r_1 < r$. Repeating this logic forward we see that the economy converges to the steady state only if creditors' consumption grows over time. This means that $r_t \geq r_{t+1} \geq r$ for all $t > 0$. By assumption 2.2.6 this also implies that $i_t > 0$ for all $t > 0$, so that the economy exits the liquidity trap in period 1 and inflation is never lower than the target in periods $t > 0$.

Throughout the proof I have assumed that debtors are against their borrowing limit in $t = 0$ and during the transition to the new steady state. To conclude the proof I show that this is indeed the case. Let us start with period $t = 0$. Suppose that debtors are not against their borrowing limit, then the Euler equation for debtors implies

$$U'\left(L_0^\alpha - B_0 + \pi_1\bar{\kappa}\right) \leq \frac{\beta}{\pi_1}U'\left(L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}\right) \leq U'\left(L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}\right),$$

where the last inequality follows from $\pi_1 \geq \bar{\pi} \geq \beta$. Then it must be that

$$L_0^\alpha - B_0 + \pi_1\bar{\kappa} \geq L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}. \tag{B.1.3}$$

The Euler equation for creditors implies

$$U'\left(L_0^\alpha + \frac{n}{1-n}\,B_0 - \pi_1\bar{\kappa})\right) = \frac{\beta}{\pi_1}U'\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right).$$

Since $L_0 \leq L_1$ this implies that

$$B_0 > \bar{\kappa}\left(\pi_1 + \frac{r_1}{1+r_1}\right),$$

which contradicts condition $B.1.3$, proving that debtors are against their borrowing limit during the liquidity trap.

Also in $t > 0$ debtors are against their borrowing limit, i.e. $B_t^d = -\bar{\kappa}$ for all t. In the case of a central bank that targets employment this follows from the fact that the economy enters in steady state in period 1, and in steady state the bond positions are continually rolled over.[1] In the case of a central bank that targets inflation, it is possible to show that in absence of the borrowing constraint B^d grows at rate $B_{t+1}^d / B_t^d = (1 + g_{t+1})^\gamma / \beta$, where $g_{t+1} = L_{t+1}/L_t - 1$. Since $g_t > 0$ during the transition, this means that debtors would like to increase their debt during the transition. This would violate the borrowing constraint and so $B_t^d = -\bar{\kappa}$ for all t.■

B.1.2 Proof of proposition 3

Since condition 4 implies condition 3, the proof of the first part of the proposition follows directly from the proof of proposition 2.

We have to prove that the economy reaches a steady state with full employment and inflation equal to target in period 1. Combining firms' optimality conditions in period 0 and 1 with the constraint on wage setting in period 1 gives

$$\left(\frac{L_0}{L_1}\right)^{1-\alpha} \pi_1 \geq \phi \left(1 - u_1\right).$$

This equation implies that if condition 4 holds, it is feasible for the central bank to hit both the inflation and the employment target in period 1 and to set $\pi_1 = \bar{\pi}$, $L_1 = \bar{L}$ and $u_1 = 0$. ■

B.1.3 Proof of proposition 4

I first show that a marginal transfer from creditors to debtors can lead to a Pareto improvement in welfare only if the economy is in a liquidity trap. Suppose that the economy is not in a liquidity trap in period 0. Then a marginal transfer from creditors to borrowers in period 0 has the following impact on creditors' consumption

$$\frac{\partial C_0^c}{\partial T} = \frac{n}{1-n} \left(\frac{B_1}{(1+r_0)^2} \frac{\partial r_0}{\partial T} - \frac{1}{1+r_0} \frac{\partial B_1}{\partial T} - 1 \right) \tag{B.1.4}$$

$$\frac{\partial C_t^c}{\partial T} = \frac{n}{1-n} \left(\frac{\partial B_1}{\partial T} (1-\beta) \right)$$

for all t> 0,where I have used the fact that out of a liquidity trap $L_t = \bar{L}$ and $\partial L_t / \partial T = 0$. Because of the borrowing constraint $\partial B_1 / \partial T \leq 0$ so $\partial C_t^c / \partial T \leq 0$ for all $t > 0$. Moreover,

[1]Strictly speaking, debtors are not constrained in steady state since their desired bond position is exactly $B^d = -\bar{\kappa}$.

differentiating the period 0 creditors' Euler equation with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = \frac{\beta}{U''(C_0^c)}\left(\frac{\partial r_0}{\partial T}U'(C_1^c) + (1+r_0)\frac{\partial C_t^c}{\partial T}U''(C_1^c)\right). \tag{B.1.5}$$

Equation $(B.1.4)$ implies that $\partial C_0^c/\partial T > 0$ only if $\partial r_0/\partial T > 0$. But equation $(B.1.5)$ implies that if $\partial r_0/\partial T > 0$ then $\partial C_0^c/\partial T < 0$, and hence $\partial C_0^c/\partial T < 0$. This implies that out of a liquidity trap a transfer from creditors to debtors unambiguously hurts creditors and hence cannot lead to a Pareto improvement in welfare.

I now show that a marginal transfer from creditors to debtors leads to a Pareto improvement in welfare if the deleveraging shock pushes the economy into a mild recession, that is if condition 4 is satisfied. Proposition 3 implies that the economy reaches a steady state with full employment in period 1 and so a marginal transfer does not affect C_t^c or C_t^d for $t > 0$. Consumption in period 0 are

$$C_0^d = L_0^\alpha - B_0 + \bar{\pi}\bar{\kappa} + T$$

$$C_0^c = L_0^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T\right).$$

Differentiating these expressions with respect to T gives

$$\frac{\partial C_0^d}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} + 1 \tag{B.1.6}$$

$$\frac{\partial C_0^c}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} - \frac{n}{1-n}. \tag{B.1.7}$$

These expressions imply that a transfer can be Pareto improving only if $\partial L_0/\partial T > 0$, otherwise the transfer will unambiguously hurt creditors. The Euler equation for creditors, equation $(B.1.1)$ can be written as

$$C_0^c = U'^{-1}\left(\frac{\beta}{\pi}U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)\right).$$

Differentiating this expression with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = 0.$$

Combining this expression with equations $(B.1.7)$ and $(B.1.6)$ yields

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n}\frac{1}{\alpha}L_0^{1-\alpha} > 0 \tag{B.1.8}$$

$$\frac{\partial C_0^d}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} + 1 > 0, \tag{B.1.9}$$

which completes the proof. ■

B.1.4 Proof of proposition 5

I start by showing that if 3 holds an appropriate transfer restores full employment. Suppose that the optimal choice of bonds by debtors is $B_1^d = -\bar{\kappa}$. Then, full employment is restored by a transfer T^*, implicitly defined by

$$U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T^*\right)\right) = \frac{\beta}{\bar{\pi}}U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

Notice that since $\bar{\pi} \geq \beta$, T^* satisfies

$$T^* \leq B_0 - \bar{\kappa}\left(\bar{\pi} + \frac{r}{1+r}\right). \tag{B.1.10}$$

This condition implies that the optimal transfer does not lift the economy out of the liquidity trap and so $i_0 = 0$. Finally, we must show that $T = T^*$ is consistent with $B_1^d = -\bar{\kappa}$. Suppose this is not the case then the Euler equation for debtors implies

$$U'\left(L_0^\alpha - B_0 + \bar{\pi}\bar{\kappa} + T^*\right) < \frac{\beta}{\bar{\pi}}U'\left(\bar{L}^\alpha - \frac{r}{1+r}\bar{\kappa}\right).$$

This expression implies

$$T^* > B_0 - \bar{\kappa}\left(\frac{r}{1+r} + \bar{\pi}\right),$$

which contradicts $(B.1.10)$, thus proving that it is optimal for borrowers to set $B_1^d = -\bar{\kappa}$.

We have proved that a transfer T^* restores full employment, I now show that T^* is the Pareto optimal policy. Expressions $(B.1.7)$, $(B.1.8)$ and $(B.1.9)$ imply that an increase in the transfer T leads to Pareto improvement in welfare if $\frac{\partial L_0}{\partial T} > 0$, which is the case if $L_0 \leq \bar{L}$. Hence the Pareto optimal policy sets $L_0 = \bar{L}$. ■

B.1.5 Proof of proposition 6

I start by showing that if the central bank targets employment and the economy is in a large recession a marginal transfer from creditors to debtors leads to an expansion in employment. Differentiating the euler equation for creditors and using the fact that if the central bank targets employment the transfer has no impact on C_t^c for $t > 0$ gives

$$\frac{\partial C_0^c}{\partial T} = -\frac{\beta}{\pi_1^2}\frac{U'\left(C_1^c\right)}{U''\left(C_0^c\right)}\frac{\partial \pi_1}{\partial T}. \tag{B.1.11}$$

Differentiating creditors' budget constraint in period 0 with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} - \frac{n}{1-n}\left(\bar{\kappa}\frac{\partial \pi_1}{\partial T} + 1\right). \tag{B.1.12}$$

Moreover, using the fact that the wage setting condition binds in period 1 gives

$$\pi_1 = \phi(0)\left(\frac{\bar{L}}{L_0}\right)^{1-\alpha}.$$

Differentiating this expression with respect to T gives

$$\frac{\partial \pi_1}{\partial T} = -(1-\alpha)\phi\left(0\right)\bar{L}^{1-\alpha}L_0^{\alpha-2}\frac{\partial L_0}{\partial T}. \tag{B.1.13}$$

Combining $(B.1.11)$, $(B.1.12)$ and $(B.1.13)$ yields

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n}\left(\alpha L_0^{\alpha-1} + (1-\alpha)\phi(0)\bar{L}^{1-\alpha}L_0^{\alpha-2}\left(\frac{n}{1-n}\bar{\kappa} - \frac{\beta}{\pi_1^2}\frac{U'\left(C_1^c\right)}{U''\left(C_0^c\right)}\right)\right).$$

This expression implies $\partial L_0/\partial T > 0$ so that a marginal transfer has an expansionary impact on employment.

To prove that a marginal transfer cannot be Pareto improving it is sufficient to notice that by equation $(B.1.13)$ if $\partial L_0/\partial T > 0$ then $\partial \pi_1/\partial T < 0$ and by equation $(B.1.11)$ $\partial \pi_1/\partial T < 0$ implies $\partial C_0^c/\partial T < 0$. ■

B.1.6 Proof of proposition 7

I start by showing that the transfer scheme proposed in proposition 7 leads to a unique equilibrium characterized by full employment. Assuming that the borrowing constraint always binds for debtors, the Euler equation for creditors can be written as

$$L_0^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T\right) = \left(\frac{\beta}{\bar{\pi}}\right)^{-\frac{1}{\gamma}}\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right).$$

Plugging $T = \tilde{T} + \chi\left(L_0^\alpha - \bar{L}^\alpha\right)$ in this expression gives

$$\left(L_0^\alpha - \bar{L}^\alpha\right)\left(1 - \chi\frac{n}{1-n}\right) = \left(\frac{\beta}{\bar{\pi}}\right)^{-\frac{1}{\gamma}}\left(L_1^\alpha - \bar{L}^\alpha + \frac{n}{1-n}\left(\frac{r_1}{1+r_1} - \frac{r}{1+r}\right)\bar{\kappa}\right).$$

The right-hand side of this expression is not greater than zero.[2] Since $\chi > (1-n)/n$ the only possible solution to this equation is $L_0 = \bar{L}$, which proves that the transfer scheme proposed gives rise to a unique equilibrium characterized by full employment.[3] The proof that $B_t^d = -\bar{\kappa}$ for all t follows along the lines of the proof of proposition 5.

To prove that the transfer leads to a Pareto improvement in welfare, first consider that if the central bank targets inflation C_t^d and C_t^c for $t > 0$ are weakly increasing in L_0. Moreover, the tax leads to an increase in period 0 debtors' consumption, both because of its direct effect and because of the increase in employment. Hence, to show that the transfer leads to a Pareto improvement in welfare we need to show that its impact on period 0 creditors' consumption is nonnegative. To see that this is the case, consider that the transfer leads to an increase in C_1^c, while leaving $r_0 = 1/\bar{\pi} - 1$ unchanged. By creditors' Euler equation this implies that the transfer also leads to an increase in C_0^c and thus leads to a Pareto improvement in welfare. ■

[2]Recall that C_1^c is increasing in L_1, as shown in the proof of proposition 1.

[3]Notice that when $L_0 = \bar{L}$ the economy enters the steady state in period 1 and so $L_1 = \bar{L}$ and $r_1 = r$.

Appendix C

Reserve Accumulation, Growth and Financial Crises

C.1 Social planner allocation

In this appendix we formally characterize the social planner allocation. The social planner chooses $\left\{C_t^N, C_t^T, L_t^T, L_t^N, M_t, B_{t+1}, FX_{t+1}, D_t\right\}_{t=0}^{\infty}$ to maximize households' expected utility (4.1), subject to the economy-wide resource constraints (4.23), (4.24) and (4.25), the borrowing constraint (4.9), the two constraints on reserve management (4.19) and (4.20) and the law of motion for the stock of knowledge (4.15). The first order conditions of the planner's problem can be written as

$$(1-\omega)\frac{C_t^{1-\gamma}}{C_t^N} = \lambda_t^N,$$

$$\omega\frac{C_t^{1-\gamma}}{C_t^T} = \lambda_t^{SP}$$

$$\alpha_N\left(1-L_t^T\right)^{\alpha_N-1}\lambda_t^N = \alpha_T A_t X_t^{\alpha_T} L_t^{\alpha_T-1} M_t^{1-\alpha_T}\lambda_t^{SP}$$

$$\begin{aligned} P^M\left(1+\phi\frac{\mu_t^{SP}}{\lambda_t^{SP}}\right) &= (1-\alpha_T)\frac{Y_t^T}{M_t} \\ &+ \beta\xi\left(\frac{X_t}{M_t}\right)^{1-\xi} E_t\left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}}\left(\alpha_T\frac{Y_{t+1}^T}{X_{t+1}} + \kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right) \end{aligned} \tag{C.1.1}$$

$$\lambda_t^{SP} = \beta R\left(\lambda_{t+1}^{SP} + \mu_{t+1}^{SP}\right) \tag{C.1.2}$$

$$\lambda_t^{SP} = \beta R^{FX}\left(\lambda_{t+1}^{SP} + \mu_{t+1}^{FX}\right) + \nu_t \tag{C.1.3}$$

$$\mu_t^{SP} = \frac{\mu_t^{FX}}{1-\theta} + \frac{\theta}{1-\theta}\lambda_t^{SP}, \tag{C.1.4}$$

plus the complementary slackness conditions for the inequality constraints. λ_t^N, λ_t^{SP}, μ_t^{SP}, ν_t and μ_t^{FX} are the Lagrange multipliers respectively on constraints (4.23), (4.24), (4.9), (4.19) and (4.20).

Combining equations $(C.1.2)$, $(C.1.3)$ and $(C.1.4)$ gives

$$\beta R\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{SP}\right)=\beta R^{FX}\left(1-\theta\right)\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{SP}\right)+\nu_t.$$

This expression has strong implications for the social planner's management of foreign reserves. Start by assuming that $R^{FX} < R$. Then the equation above implies that $FX_t = 0$ for each $t > 0$. This means that if the return on foreign reserves is less than the return on foreign bonds the social planner chooses to hold a zero amount of reserves during each period. If the social planner starts with a positive amount of reserves she may use them to finance the purchase of imported inputs during the initial period, but she will choose to hold no reserves from period 1 on.

Now consider the case $R^{FX} = R$, so that the return on the two assets is equalized. If $\theta = 0$, then it is easy to see that B_t and FX_t become perfect substitutes and that the planner cares only about the economy's net foreign asset position $B_t + FX_t$ and not about its composition between private bonds and reserves. If $\theta > 0$, that is if using reserves during crises is costly, the two assets cease to be perfect substitutes, but the planner is again indifferent about the composition of foreign assets as long as the foreign assets position allows her to set $D_t = 0$ for each $t > 0$. In any case, also when $R^{FX} = R$, setting $FX_{t+1} = 0$ in every period does not prevent the planner from reaching the first best allocation.[1]

Indeed, the social planner allocation is characterized by the same equations as the competitive equilibrium in which $FX_t = D_t = 0$ is imposed in every $t > 0$. The only exception concerns the optimality condition for imported inputs, which is replaced by equation $(C.1.1)$. This happens because the social planner internalizes the impact of imported inputs on the stock of knowledge, while atomistic agents don't.

[1] Again, the social planner might use reserves to provide working capital loans during period 0 if it starts with a positive amount of reserves.