Financial markets' imperfections and technology adoption

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Declaration

I hereby declare that the work presented in this thesis is my own.

Chapter 2 was undertaken as joint work with Evangelia Vourvachaki.

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Abstract

This thesis examines information imperfections in asset markets and its impact on economic performance through technology adoption and innovation.

In a rational setting, where equity market participants take into account common public information in addition to their private signals about fundamentals, equity prices are persistently biased towards the public signals. Chapter 2 investigates the real effect of such mis-pricing, when R&D producing firms rely on equity finance. Relating to the recent technology stocks boom, the model shows how market's optimism causes more innovations. Furthermore, such optimism can generate gains in aggregate consumption.

Chapter 3 analyzes equity markets' role in facilitating ownership transfer from entrepreneurs investing in adopting technology to managers running these firms once technology is adopted. Information imperfections in equity market affect entrepreneurs' willingness to invest in frontier technology in two ways. First, uncertainty about equity price or lack of market liquidity discourages technology adoption. This can explain slow technology adoption and limited venture capitalists' participation in under-developed equity markets. Second, imperfectly informed market participants take fast adoption as a positive signal. The resulting increase of expected market value encourages technology adoption. Probability of fast technology adoption is highest at an intermediate number of informed investors.

Chapter 4 looks more closely into the extent of asset mis-pricing by endoge-

nizing the variance of investors' private signals. Better quality of freely available public information reduces incentives to invest in private information and can magnify the extent of asset mis-pricing. Furthermore, in a dynamic setting, investors' react more slowly on changes of the fundamentals because incentives to invest in research are low in early trading periods. The chapter also shows that availability of longer price history might not bring asset prices closer to the fundamentals, as investors choose to free-ride on other investors' research efforts.

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Chapter 1

Introduction

More developed financial systems are expected to promote entrepreneurial and innovative activities and thereby long-term growth. This idea is supported by a wide literature on the impact of credit constraints on growth[1]. In addition to debt, the development of equity markets and related instruments such as venture capital or private equity, provide firms with increasing variety of funding mechanisms. As equity prices are subject to uncertainty and potential mis-pricing even in the most developed countries, the implications of this to aggregate economy is still a question of an academic and policy debate. This thesis aims to contribute to this discussion by addressing questions in two broad areas. First, how does imperfect private information among equity market participants affect investments in technology and aggregate economic growth? Second, how is the extent of these information imperfections determined in an environment where investors can choose how much private information to obtain?

Equity markets are likely to be of particular importance for innovation for the following set of reasons addressed in this thesis:

[1] See for example Aghion, Howitt, and Mayer-Foulkes (1993). Also a comprehensive overview of the literature addressing the relationship between financial development and growth can be found in Levine (2005)

- An innovation project tends to lack assets that can be collateralized. Also, the success such project is uncertain, while offering high returns in the event of success. These characteristics makes equity likely to be a more feasible and desirable source for raising external funds for such projects than debt (see e.g. Brealey and Myers 2003, Allen and Gale 1999). Also, empirical evidence shows that R&D intensive firms are more likely to raise equity funds than other types of firms. (e.g. Aghion, Bond, Klemm, and Marinescu 2004, Carlin and Mayer 2003).

- Entrepreneurs that establish firms that innovate or adopt new technologies are likely to have a particular talent in identifying and starting up good projects. If at least some of these entrepreneurs do not have a superior managerial talent, welfare gains arise if firms can be easily sold. Such benefits of ownership transfers have been analyzed in a perfect information setting by Holmes and Schmitz Jr. (1990). Developed equity markets provide a good mechanism for this. Furthermore, good exit opportunities are crucial for venture capitalists to be willing to provide funds for technology investments.

While these aspects make the existence of equity markets important for firms that can invest in technology, equity prices can deviate from their fundamentals. A wide empirical literature has found that equity prices react slowly on changes in variables that proxy the fundamentals (e.g. Cutler, Poterba, and Summers 1991, Jegadeesh and Titman 1993, Chan, Jegadeesh, and Lakonishok 1996), tend to become overpriced (underpriced) after long record of positive (negative) news (e.g. De Bondt and Thaler 1985, Chopra, Lakonishok, and Ritter 1992, La Porta 1996) and are affected by market sentiment (e.g. Lee, Shleifer, and Thaler 1991, Swaminathan 1991).

Behavioral approach attributes these patterns to some degree of irrationality among investors. For example, investors could be making persistent mistakes in their expectations due to psychological factors (e.g. Barberis, Shleifer, and Vishny 1998). Even when some investors are rational, they would not necessarily eliminate asset mis-pricing in the presence of irrational traders, as it would be rational to benefit from forecasting the beliefs of irrational traders (e.g. De Long, Shleifer, Summers, and Waldmann 1990). However, persistent mis-pricing for a similar reason can occur also in a fully rational setting as pointed out more recently by Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006). This thesis adopts a noisy rational expectations approach and similar information structure to these two papers when modelling equity prices.

Throughout this thesis, it is assumed that investors can obtain information about firms' fundamental value (or dividends) from three sources: private signals, public signals[2] that are common for all investors and price signals. As long as none of these signals is perfect for an average investor, it is rational for them to take all these signals into account when forming their expectations. As a result, the errors in the public signals will affect the average beliefs and equity prices. In general, asset prices are a function of the true fundamental value, public signals and supply shocks. This has several implications for equity prices that are important for the mechanisms analyzed in the different chapters of this thesis:

1. Equity is typically mis-priced and is tilted from the true fundamental value towards the public signals.

2. The extent of mis-pricing is lower if the quality of private information in the market is high.

3. Especially when investors have short horizons and care about future equity

[2]The public signal could reflect for example "market sentiment", announcements by policy makers or opinion leaders and news more generally.

prices, then the mis-pricing generated by the availability of public signals is persistent. Equity prices adjust slowly to changes in the fundamental value and fast to changes in the public signals.

4. From the point of view of entrepreneurs that aim to sell their firms in the equity market, uncertainty about the future public signals generates further uncertainty about the price they can sell their firms for.

Chapter 2 addresses the firms' ability to invest in technology. It incorporates equity markets with information imperfections, as described above, into a Romer (1990) style growth model and analyzes the impact of mis-pricing on aggregate output and consumption. In order to highlight the effect of the mechanism, it is assumed that equity is the only source of funding for R&D firms. The source of uncertainty is assumed to be the productivity of final goods' sector in using the new technology. This translates into uncertainty about the demand innovative firms face in the future and therefore uncertainty about their future profits and dividends.

During periods when equity markets are optimistic (in the sense that public signal is higher than the true productivity) firms can raise more funds and produce more R&D. This implies that output increases faster. At the same time, investors with imperfect information get losses in the equity market. The main question of this chapter is whether such mis-pricing can lead to permanent gains or losses in aggregate consumption. It also analyzes how such an economy reacts to changes in the actual productivity compared to the case of perfect information setting. Chapter 2 shows that if the R&D sector is not very congested, some degree of market optimism is likely to be beneficial for the economy.

In the light of the recent technology stocks boom in the United States and other developed countries, the mechanism suggested in this chapter contributes to understanding the real effect of these events. It also suggests that policy makers could have an incentive to encourage some degree of market optimism.

Chapter 3 investigates how imperfect information in equity markets can affect firms' willingness to invest in technology. It takes a radically opposite approach to Chapter 2 in assuming that entrepreneurs in a small open economy can always find resources to invest in profitable technology adoption (or innovation) projects. In reality, even if debt financing is not suitable for R&D investments, such investments could be funded from own funds or by engaging a venture capitalist. The crucial assumption in this chapter is that entrepreneurs, who establish firms that adopt technology will sell their firms after the initial phase of development has passed. Similarly to Chapter 2, uncertainty is assumed to arise from the demand side and the mechanisms are analyzed in an endogenous growth framework (in the spirit of Aghion and Howitt 1992).

The chapter further assumes that entrepreneurs (alone or joining with a venture capitalist) have superior information about the fundamental value of their firms. As technology adoption decisions and investments are made before the firms are sold in the equity market, there is uncertainty about the public signals (or market sentiment) that affects the market value of the firms. There is also uncertainty about the noise trading in the period that firms will be sold[3]. Such uncertainty can discourage entrepreneurs from engaging into technology adoption projects that would be profitable in perfectly informed equity markets. This negative force is called "fear of unstable markets". At the same time, some degree of imperfect information could also increase the speed of technology adoption due to a second the force that is called "adoption to signal". This force emerges because technology adoption decision becomes an additional signal for imperfectly

[3]This is likely to be less important in relatively liquid equity markets.

informed equity market participants. They would rationally infer that entrepreneur would not invest in an expensive technology, if the fundamental value of the firm is too low. As a result, investors are willing to pay more for firms that invest in expensive technology. This could lead initial owners to undertake technology projects that they would not find profitable in perfectly informed markets. Nevertheless, investing in better technology still leads to higher output and wages, in aggregate level.

The chapter also addresses implications that arise from the presence of such mechanisms. First, to analyze policy makers' incentives to pursue policies towards transparency, the degree of information asymmetry between entrepreneurs and equity market participants is endogenized by introducing information cost. The chapter shows that policy makers can lack incentives to eliminate these information asymmetries due to "adoption to signal", even if they would be able to set equity market participants' information costs to zero. Second, assuming that in a small open economy context the foreign investors are relatively less informed than the local ones, the chapter analyzes if restricting foreign portfolio investments could be beneficial because it reduces the magnitude of "fear of unstable markets" force. It is shown that while such policy may increase the probability of fast technology adoption under specific circumstances, there is a trade-off between higher uncertainty and lower liquidity. The model is consistent with empirical evidence that lack of liquidity is likely to lead to underpriced equity (see e.g. Sadka 2000, Pastor and Stambaugh 2003, Amihud 2002, Acharya and Pedersen 2005). The implied lower expected market value of the firms reduces entrepreneurs' incentives to invest in new technologies.

The mechanisms analyzed in this chapter could explain the current important differences in aggregate performance among transition countries despite their similar initial share of human capital, stage of development and institution at the beginning of 1990's. It can also provide insights for explaining the differences in venture capital investments across countries. Finally, the mechanisms analyzed could have also been relevant during the technology stocks boom in developed countries, where there might have been a role for "innovation to signal" force.

The mechanisms in Chapters 2 and 3 rely on the fact that equity market participants have imperfect private information. While Section 3.3 in Chapter 3 addresses the fact that the average quality of private information can be endogenous, it is done in a simple setting with one trading period for each risky asset and two types of investors: informed and uninformed[4]. **Chapter 4** looks at endogenous quality of private information in a multiple trading periods partial equilibrium setting similar to Allen, Morris, and Shin (2006). It assumes that short-lived investors' trading decisions are based on public, price and private signals about the liquidation value of a single risky asset. Investors are heterogeneous in the private signals they observe and they choose the precision of their private signals before observing the signal and trading. It is shown that if higher quality public signals are available, it reduces incentives to acquire better quality private signals. As a result, policies that aim to reduce the variance of public signals have an ambiguous effect on the extent of mis-pricing. While a public signal of better quality is expected have a smaller error, the impact of this error becomes magnifies through the higher weight investors put on this signal when forming their expectations.

Furthermore, short-lived investors who care about next period's asset price rather than the fundamental value of the assets not willing to put a lot of research effort in analyzing events that are further in the future. Therefore, the

[4]The technological progress in Chapter 2 is modelled as quality improvements in the spirit of Aghion and Howitt (1992) and Aghion, Comin, and Howitt (2006). In that setting firms can sustain their monopoly power only for one period before being driven out of the market. This implies that firms are traded in only one period.

slow-reaction on the news on the fundamentals is more pronounced than in Allen, Morris, and Shin (2006), where short-lived investors effectively have constant quality of private information over time.

Another interesting implication of endogenous information costs arises when comparing the extent of potential mis-pricing of assets that have different length of price history available. If the precision of private information is fixed, longer price history necessarily reduces the extent of mis-pricing. This is because of the additional information revealed in historical prices. In the case of endogenous information costs this might no longer be the case. Longer price history reduces the marginal benefit of improving the quality of private signals and encourages investors to free-ride on the research efforts of investors that traded in earlier periods. Depending on the assumptions about marginal research cost, this could even lead to assets with longer price history to be more mis-priced.

Some on these results are sensitive to the assumption of short-lived agents. To investigate the implications of this, Chapter 4 extends to consider long-lived agents who have preferences over consumption on the final liquidation date as in Brown and Jennings (1989). In such case[5], asset prices react on the changes in fundamentals much faster. Chapter 4 also shows that in such case investors prefer to obtain a higher quality private signal as early as possible and do not have incentives to delay their research efforts and wait for further information to be revealed in prices. This is because their private signal will remain useful in later trading periods and they can foresee their future demand for the risky asset. While the price adjustment dynamics change in the case of non-myopic investors, the direction of impact of other variables that affect incentives to acquire private information remain unchanged.

[5]As also shown by Allen, Morris, and Shin (2006) for a special case of infinite variance of noise trading.

The rest of the thesis is structured as follows. Chapter 2 analyzes the impact of equity mis-pricing on equity funded and R&D driven growth, when equity market participants are short-lived and have noisy private, public and price signals. Chapter 3 addresses the aggregate effects of information asymmetries between equity market participants and entrepreneurs adopting technology and aiming to sell their firms in equity markets. Chapter 4 proceeds by looking at investors' incentives to acquire costly private information about the fundamentals and the extent of asset mis-pricing that arises in a setting where public information is available. Finally, Chapter 5 highlights the links between the insights coming from the different chapters, presents the implications for economic policy and discusses weaknesses present in the analytical framework that give directions for future research.

Chapter 2

Equity mis-pricing and R&D growth

2.1 Introduction

The recent developments in the stock market prices of the United States' technology intensive firms poses the following question. If these stocks were overpriced, what would be the aggregate economic impact of this? Could it be that the United States experienced higher output following this boom? If firms that produce R&D rely on equity finance, overpricing in equity markets provides them with cheaper funds and enables them to produce more R&D. Hence, as long as R&D is the driver of economic growth, there can be gains in aggregate consumption due to optimism in equity markets. At the same time, optimism can have an opposite effect, because investors get losses when they invest in overpriced equity markets. This chapter presents a theoretical model that investigates how equity mis-pricing transmits to the real economy through R&D growth.

Figure 2.1 presents data on real price earnings ratio from the firms listed in S&P500 over the period 1970-2002, along with a proxy for R&D output, as given

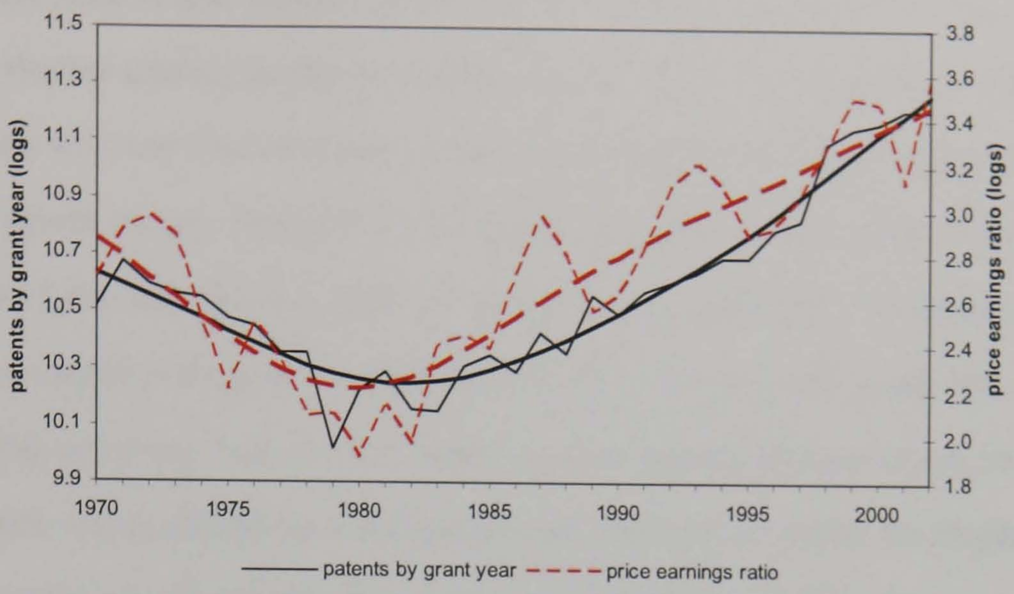

Figure 2.1: S&P500 price earning ratio and USPTO patents granted to non-government institutions

by number of patents granted by USPTO to the United States non-government institutions[1]. The two series co-move along time, reflecting the pattern of productivity growth of the United States over the same period. Most rational expectations models explain this correlation by the forward looking nature of the equity market. However, if the market price deviates from the fundamentals, then the correlation can also be driven by the funds available for R&D activities through equity issue. The existence of such feedback becomes of particular interest in relation to Information and Communication Technologies (ICT), for which the same correlation pattern is present. Importantly, the ICT producing sector is highly intensive in R&D and patenting activity (Carlin and Mayer 2003), while recent growth accounting identifies this sector as the driver of economic growth (Jorgenson, Ho, and Stiroh 2005).

In endogenous growth models like Romer (1990), Grossman and Helpman

[1] Figure 1 presents the series in log levels and their respective trends (Hodrick-Prescott filter with $\lambda = 100$ for annual data). Data on price-earnings ratios and patents is from Robert J. Schiller and Bronwyn H. Hall websites respectively.

(1991) and Aghion and Howitt (1992), R&D is the engine of growth. This chapter keeps a similar setting in the production side. There are competitive final good producers, who use capital varieties that are produced by the monopolistic intermediate goods sector. Similarly to Comin and Gertler (2004), intermediate goods sector produces also its own R&D, in a decentralized manner. The key difference to all aforementioned papers, is that this chapter considers the possibility of equity mis-pricing resulting from the information imperfections of consumers (investors). Consumers are modelled as overlapping generations, in order to emphasize on investors who care about the short-term movements in equity prices.

Accounting for the potential mis-pricing of equities is important. There is empirical support on equity prices under-reacting on the changes in fundamentals (see Chapter 1 for references). Also, most fund managers find it important to take into account their own perception of market view about a particular asset, in addition to their private fundamental research (e.g. Menkhoff 1998).

Several models in finance emphasize on irrationality (e.g. animal spirits) in the investors' behavior that result in the mis-pricing of asset (e.g. Barberis, Shleifer, and Vishny 1998). However, mis-pricing can occur in a purely rational setting, as shown by papers on higher order expectations by Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006). In these papers, the necessary component for mis-pricing to occur is the existence of heterogeneous, noisy private information together with common, noisy public information[2]. Such setting results in a rational expectation equilibrium, where all investors end up taking both signals into account and asset prices are affected by the public signal. This chapter takes a similar approach on the information structure and therefore the frictions considered result from rational behavior.

[2] "Public information" or a "public signal" is distinct from the price signal in this chapter.

The channel through which equity prices affect R&D growth in this chapter, is that R&D is financed by issuing equity. The reason why equity financing is likely to be more important for the R&D producing than other sectors in the economy, is that this sector is not appealing for debt contracts, while internal finance is may not provide sufficient funds. R&D production activity lacks collateral and carries agency problems, driven by the uncertainty about the success of innovations and the demand for them. As a result, debt financing may not be desirable or possible for the innovating firms (see discussion on bankruptcy costs in Brealey and Myers 2003). Empirical evidence by Aghion, Bond, Klemm, and Marinescu (2004) based on the United Kingdom data shows that firms that report R&D are more likely to raise equity than those that do not. Also, the probability of equity financing increases with R&D intensity. Carlin and Mayer (2003) investigate OECD countries' data and find support for Allen and Gale's (1999) hypothesis that equity market is more relevant for raising funds for the R&D intensive firms.

In order to highlight the main mechanism, the chapter abstracts from the choice of financing for the R&D producers. It assumes that their only available source of financing is equity. Furthermore, the success of innovation is assumed to be certain, as in endogenous growth models like Romer (1990), Evans, Honkapohja, and Romer (1998) and Comin and Gertler (2004). This assumptions are mainly for simplicity of argument to emphasize on the main questions of this chapter. The uncertainty about the level of demand for the innovations in the model, is introduced through a labour augmenting productivity shock in the final good sector.

The main question addressed in this chapter is the impact of pure optimism on aggregate economy. The chapter shows how market optimism reflected in a public signal above the true productivity can induce expansion of the economy. In particular, it analyzes whether such deviation of equity prices from the fundamental value coincides with higher consumption, despite resulting in losses in the equity market for the consumers.

Another question relates to the impact of an actual change in the fundamentals (transitory or not, anticipated or not). The performance of the model economy, where there are various sources of imperfect information, is compared to the economy, where the fundamentals are known to the financial market participants. In addition, the chapter addresses the impact of noise trading shocks. Finally, the chapter provides an insight into how changes in the quality of different sources of information and risk aversion would affect the response to the public signal and true productivity shocks.

The chapter is organized as follows. Section 2.2 sets up the production and consumption side of the economy for the general case of infinite horizon. Section 2.3 describes the information structure and presents the analytical solution and the comparative statics for the three-period horizon. In such setting there is only uncertainty about the liquidation value of intermediate goods firm in the second period. Section 2.4 presents numerical results for the infinite horizon model. It confirms the conclusions for the three-period model and discuss the additional insights provided by this framework. Section 2.5 summarizes the main findings and discusses the incentives of policy makers that can affect the public signal (e.g. central banks) to make truthful statements.

2.2 The general setup

2.2.1 Production side

Final good producers and intermediate goods production

Competitive final good producers use all labour (L) and capital varieties ($x_t(j)$) that are available in the economy in period t ($j \in [0, A_t]$; A_1 is given). The capital depreciates fully in a period. The sector rents capital varieties from the intermediate good's sector for a price $p_{x_t}(j)$ and pays wage w_t to consumers. The final output is the numeraire and its price is normalized to one. Therefore, the final goods' producers solve

$$\max_{L, x_t(j)} \left\{ Y_t - \int_0^{A_t} p_{x_t}(j) x_t(j) dj - w_t L, \text{ s.t. } Y_t = (\phi_t L)^{1-\alpha} \int_0^{A_t} x_t^{\alpha}(j) dj \right\}, \tag{2.1}$$

where $\phi_t \sim \mathcal{N}(\overline{\phi}, 1/\beta_\phi)$ is a labour augmenting productivity shock[3]. This is also the publicly known prior distribution of productivity. At the beginning of period t, ϕ_t is known, but there is uncertainty for all future ones.

Monopolistic intermediate goods producers use η units of final good in order to produce one unit of capital good. They maximize profits:

$$\max_{p_{x_t}(j), x_t(j)} \left\{ \pi_t(j) = p_{x_t}(j) x_t(j) - \eta x_t(j), \text{ s.t. } p_{x_t}(j) = \frac{\partial Y_t}{\partial x_t(j)} \right\}.$$

Given the symmetry among the intermediate goods firms, the demand for

[3]The normality assumption, while being unrealistic by allowing negative output, greatly simplifies the solution. It is also widely used assumption in the finance literature about the liquidation value of assets. By reasonable assumptions about the parameters the probability of negative output or asset prices is negligible. The main mechanism would remain valid with different distributional assumptions. The normality assumption is maintained in the other chapters of this thesis for the same reason.

intermediate goods is

$$x_t = (\frac{\alpha^2}{\eta})^{\frac{1}{1-\alpha}} L\phi_t. \tag{2.2}$$

Since the demand is linear in ϕ_t, the intermediate goods firms face uncertain future demand. Profits are also linear in ϕ_t and uncertain in the future,

$$\pi_t = \Gamma\phi_t, \text{ where } \Gamma = \eta(\frac{1-\alpha}{\alpha})(\frac{\alpha^2}{\eta})^{\frac{1}{1-\alpha}} L. \tag{2.3}$$

Since all firms of the sector get exactly the same profits, consumers treat all equity in the economy as one asset. The total supply of equity coincides with the number of capital producing firms.

The aggregate capital stock can be expressed as

$$K_t = \eta A_t x_t. \tag{2.4}$$

R&D production

The R&D production is modelled as in Comin and Gertler (2004). The firms conduct R&D to invent a new capital variety, one period in advance of producing and using this variety for the final good production. The input to R&D is final goods, which implies that R&D expenditures become procyclical. A firm that intends to enter the intermediate goods producing sector in period $t+1$ needs to undertake R&D during period t. It raises its R&D expenditures by issuing equity worth of $I_t(j)$. The R&D production function from the point of view of every investor is

$$A_{t+1}(j) - A_t(j) = \overline{\lambda}_t I_t(j), \tag{2.5}$$

where $\overline{\lambda}_t$ is taken as given and exogenous in every period. At the aggregate level the R&D productivity is given by

$$\overline{\lambda}_t = \lambda \left(\frac{I_t}{K_t}\right)^{\rho-1} K_t^{-1} A_t. \tag{2.6}$$

This allows for congestion in R&D, implied by the aggregate I_t entering negatively this expression, for $0 < \rho < 1$. The parameter ρ measures the extent of congestion, with higher value implying lower congestion and higher productivity for R&D (ρ is the elasticity of R&D output with respect to R&D intensity). The existence of knowledge spillover over time, is captured by A_t. The current value of capital stock (K_t) acts as a proxy for the embodied knowledge stock. The parameter λ is a pure scaling factor.

Since the firms conducting R&D do not have alternative sources of funding, they invest in R&D the funds they can raise in the equity market. Free entry to R&D production level ensures that R&D activity gives zero profits for every firm, $P_t(A_{t+1}(j) - A_t(j)) - I_t(j) = 0$, and the entire sector:

$$P_t(A_{t+1} - A_t) = I_t. \tag{2.7}$$

Despite the fact that the equity price depends on future productivities, R&D producing firms take this price as given, while the outcome of R&D production is known with certainty by all agents. R&D producers do not have any superior information about their future profits and are forced to paying these all out as dividends. This chapter abstracts from any agency problems between R&D produces and investors, in order to focus on the impact of investors' information imperfections.

2.2.2 Consumption side

The consumption side consists of overlapping generations of rational and non-rational consumers, who work and invest in assets in the first period of their lives, and retire and consume in the second period. The short-lived agents assumption emphasizes the behavior of investors, who care about the short-term price movements in addition to the fundamental value of firms.

There is a continuum of short-lived rational consumers normalized in the interval [0,1], who make their asset allocation decisions when young. A rational (indexed as R) consumer i, born in period t , invests his labour income (w_tL) in stocks of the intermediate goods firms and the risk-free technology ($\tilde{Y}_t$). The latter offers a certain gross return $R \geq 1$ on funds invested in it during the previous period (M_{t-1})[4]:

$$\tilde{Y}_t = RM_{t-1}. \tag{2.8}$$

There are no short-selling or borrowing constraints.

Rational consumers maximize the CARA utility

$$U_t = -e^{-\tau c_{r,t+1}(i)},$$

where risk aversion is measured by τ. Using the consumer's budget constraint, the consumption of a rational consumer i can be expressed as

$$c_{r,t+1}(i) = (P_{t+1} + \pi_{t+1})h_{r,t}(i) + RM_{r,t}(i) = (P_{t+1} + \pi_{t+1} - RP_t)h_{r,t}(i) - Rw_tL,$$

where $h_{r,t}(i)$ and $M_{r,t}(i)$ represent respectively consumer i's equity and risk-free asset demand. Using (2.3), consumer i's optimal demand for stocks can be express

[4]The risk-free asset could be another final good technology, storage or foreign assets.

as

$$h_{r,t}(i) = \frac{E[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t(i)] - RP_t}{\tau \operatorname{Var}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t(i)]}, \tag{2.9}$$

where $\Omega_t(i)$ is the information set available for consumer i in period t and defined explicitly in Sections 2.3.1 and 2.4.1.

The aggregate demand of rational consumers for stocks and the risk-free asset are $H_{r,t}$ and $M_{r,t}$ respectively. The latter is given by $M_{r,t} = w_t L - P_t H_{r,t}$. The aggregate consumption of rational consumers in period t is equal to $C_{r,t} = (P_t + \pi_t)H_{r,t-1} + RM_{r,t-1}$.

The non-rational (indexed as n) consumers, born in period t, differ from the rational consumers only in two respects: they are not endowed with labour and they demand a random quantity of stocks[5]. The existence of non-rational consumers with random equity demand is necessary to make the equity prices not fully revealing (a paradox first addressed by Grossman and Stiglitz 1976)[6]. The equity demand of non-rational investors in period t is

$$\begin{aligned} H_{n,t} &= A_{t+1} - s_t, \\ s_t &\sim \mathcal{N}\left(0, \frac{1}{\beta_s}\right), \end{aligned}$$

where s_t is the noise trading shock. The mean of the non-rational consumers' equity demand is equal to aggregate supply of assets in period t (A_{t+1}), in order to ensure that the financial market do not have excess or shortage of liquidity on average. Equity market clearing implies $A_{t+1} = H_{n,t} + H_{r,t} \Longrightarrow s_t = H_{r,t}$.

As the budget constraint for the non-rational consumers is similar to that of the rational consumers, their investment in the risk-free technology in period t

[5]The wage income does not affect the demand for stocks with CARA utility maximization under no short-selling or borrowing constraints. Therefore, the split of wage income between rational and non-rational consumers does not affect the aggregate results.

[6]See also Chapter 4.

is $M_{n,t} = -P_t H_{n,t}$. The non-rational consumers born in $t-1$ consume $C_{n,t} = (P_t + \pi_t)H_{n,t-1} + RM_{n,.t-1}$ in period t.

The aggregate investment in the alternative technology and aggregate consumption are respectively

$$M_t = M_{r,t} + M_{n,t} = w_t L - P_t A_{t+1}, \tag{2.10}$$

$$C_t = C_{r,t} + C_{n,t} = (P_t + \pi_t)A_t + RM_{t-1}. \tag{2.11}$$

2.2.3 Goods market clearing

The goods market clearing condition in period t is

$$\tilde{Y}_t + Y_t = C_t + K_t + I_t + M_t.$$

Using equations (2.1), (2.2), (2.4), (2.8), (2.10), (2.11) and $w_t L = (1-\alpha)Y_t$, one can simplify this to

$$(\Gamma\phi_t - \pi_t)A_t = I_t - P_t(A_{t+1} - A_t).$$

The LHS of this equals zero, given (2.3) and the RHS equals zero, because all the funds raised from the equity market are used for R&D investment, (2.7). Therefore, the market clears out in all interim periods. The details on the market clearing conditions in the initial and terminal period of the three-period economy are shown in Appendix A.1.

2.3 Three-period model

2.3.1 Information structure and equilibrium equity prices

The production of final good (described by (2.1)) takes place only in periods 1 and 2. The productivity in the first period (ϕ_1) the number of intermediate firms (A_1) and the initial investment in alternative technology (M_0) are given. The uncertainty concerns the productivity in the second period (ϕ_2). Since the last period that intermediate goods producers operate is period 2, the dividend paid during this period is the liquidation value of the firm.

The R&D production (described by (2.5),(2.6)and (2.7), where $t = 1$), occurs only in period 1, since there is no any demand for capital varieties developed in period 2. Hence, there is no equity market in period 2 ($P_2 = 0$). Consumers born in period 2 receive labour income (w_2L), and invest it all into the alternative technology, because it is the only available asset. This implies, $C_3 = Rw_2L$.

The information structure is similar to that of Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006). In period 1, every rational consumer i born in that period, receives a private signal $\nu(i) = \phi_2 + \varepsilon_\nu(i)$, where $\varepsilon_\nu(i) \sim \mathcal{N}(0, 1/\beta_\nu)$. The trading decisions will be based on three sources of information: this private signal, the public signal $\phi_2 \sim \mathcal{N}(\bar{\phi}, 1/\beta_\phi)$, and the equity price signal. Therefore, the information set is $\Omega_1(i) = \{\nu(i), \overline{\phi}_2, P_1\}$. The variance of the price signal depends on the variance of the net supply $(1/\beta_s)$. The private signals, the public signal and the noise trading shock are assumed to be uncorrelated with each other and across consumers.

By aggregating the demand of all rational consumers (2.9) and equating this demand with net supply of stocks to rational consumers gives

$$P_1 = \frac{\Gamma}{R}\left(\overline{E}[\phi_2|\Omega_1] - \tau\Gamma\,\mathrm{Var}\left(\phi_2|\Omega_1\right)s_1\right), \tag{2.12}$$

where average expectations of all rational consumers in period are denoted as $\overline{E}[..|\Omega_t] = \int_0^1 E[..|\Omega_t(i)]di$ for period t. The quality of information will be the same for all of these consumers, therefore $\mathrm{Var}\left[\phi_2|\Omega_1(i)\right] = \mathrm{Var}\left(\phi_2|\Omega_1\right)$, for all i.

The equilibrium equity price is found by assuming the price function and then solving for the coefficients with the method of undetermined coefficients. The equilibrium equity price equation is

$$P_1 = \frac{\Gamma}{R}\phi_2 + \frac{\Gamma}{R}z_1\left(\overline{\phi}_2 - \phi_2\right) - \frac{\Gamma}{R}z_{s,1}s_1, \tag{2.13}$$

where

$$\begin{aligned} z_1 &= \frac{\beta_\phi}{\beta_\phi + (\frac{\beta_\nu}{\tau\Gamma})^2\beta_s + \beta_\nu} \\ z_{s,1} &= \frac{\tau\Gamma + (\frac{\beta_\nu}{\tau\Gamma})^2\beta_s}{\beta_\phi + (\frac{\beta_\nu}{\tau\Gamma})^2\beta_s + \beta_\nu}. \end{aligned} \tag{2.14}$$

See Appendix A.2 for details of the derivation.[7] The term $\frac{\Gamma}{R}z_1\left(\overline{\phi}_2 - \phi_2\right)$ in (2.13) captures the extent of mis-pricing. This is due the presence of the common and noisy public signal. If none of the signals is perfect, it is rational for each consumer to take all of them into account, when forming their expectations about productivity in period 2. While the noise in private signals averages to the true productivity (ϕ_2), when aggregating over all rational consumers, the mean of the public signal ($\overline{\phi}_2$) does not average out to the true productivity.

Market optimism is defined as $\overline{\phi}_2 > \phi_2$, which means that equity is overpriced,

[7] See also Chapter 4 about solving a similar problem..

even if the noise trading is at its mean ($s_1 = 0$).

2.3.2 Benchmark price equation

If private information was perfect ($1/\beta_\nu \to 0$), or there was no public information ($1/\beta_\phi \to \infty$), then the term $(\overline{\phi}_2 - \phi_2)$ in (2.13) would disappear. Define the economy with perfect private signals as the "Benchmark economy". In this case, consumers do not take into account the public signals.

By using equations (2.13) and (2.14), the equilibrium price in the Benchmark economy is

$$P_1^B \equiv \lim_{1/\beta_\nu \to 0} (P_1) = \frac{\Gamma}{R}\phi_2 - \frac{\Gamma}{R}s_1.$$

In the analysis that follows, equity is considered to be "overpriced" ("underpriced"), if the equilibrium price in the setting described in Section 2.3.1: the "Model economy" exceeds (is below) in the one in the Benchmark economy.

2.3.3 Results

R&D and output growth

Aggregating R&D production (2.5) and combining it with (2.6) and (2.7) for $t = 1$, R&D growth can be express as

$$g_A \equiv \frac{A_2 - A_1}{A_1} = \left(\lambda^{\frac{1}{\rho}} \frac{1-\alpha}{\alpha}\right)^{\frac{\rho}{1-\rho}} \left(\frac{P_1}{\pi_1}\right)^{\frac{\rho}{1-\rho}}. \tag{2.15}$$

Therefore, R&D growth during period 1 depends positively on the equity prices, given the profits paid as dividends in the same period. As it was established in Section 2.3.1, equilibrium equity price is affected by the public signal. Given (2.13), equation (2.15) shows that a pure improvement in the market sentiment increases R&D growth.

The magnitude of the equity mis-pricing depends on the weight on the public signal (z_1) in the equity pricing equation. An increase of this weight amplifies the positive impact of market optimism (negative impact in the case of pessimism) on equity prices and R&D growth. As shown in Appendix A.3, this weight increases in the variance of private signal and decreases in the variance of the public signal. These variances reflect the relative quality of the different sources of information. The weight on the public signal also increases in the variance of noise trading and risk aversion. Both of these worsen the quality of the price signal. The former is a direct impact of more volatile noise trading. The latter is due to the lower demand and participation of rational-consumers in the equity market. Finally, the prices can be affected by the noise trading in period 1. This is true also in the case that the rational consumers have prefect information.

Output growth that comes from the productive activity of the agents can be expressed as

$$g_Y \equiv \frac{Y_2 - Y_1}{Y_1} = (1 + g_A)\frac{\phi_2}{\phi_1} - 1. \tag{2.16}$$

Output growth has two parts. One is coming from the exogenous productivity growth and the other from R&D growth. Output growth always reacts stronger on R&D growth, when it is accompanied with actual productivity growth. As in the case of R&D growth, output growth is affected by all determinants of the equity prices.

Comparison with the Benchmark economy

This section compares the reactions of the Model and the Benchmark economy to changes in the true productivity, market perception and noise trading. The initial scenario is set to be such that there is no productivity growth ($\phi_1 = \phi_2$), public signal is correct ($\bar{\phi}_2 = \phi_2$) and noise trading shock is at its mean ($s_1 = 0$). Details of all derivations are in Appendix A.3 and A.4.

1. Non-justified improvement of market sentiment (increase of $\bar{\phi}_2$).

Public signal does not affect equity prices, nor any other variable in the Benchmark economy. In the Model economy, equity will be overpriced. Higher prices allow more funds for R&D investment and imply increase in R&D growth. Through R&D growth output growth increases as well. Hence, in terms of output, the impact of an optimistic market is clearly positive compared to the Benchmark economy.

The impact on consumption is different across generations. Consumption of the first generation (C_1) will be higher. This is because these agents sell their equity holdings at a higher price and retire before the lower liquidation value of firms is realized. Consumption of the generation who traded on the market (C_2) will decrease. They get losses from the equity market, given that the realized profits of the intermediate goods firms are lower than they expected on average. Note that this generation does not gain from the expansion of the economy and higher wages, because their wages were determined from 1st period's output. Due to their mis-perception about the fundamentals, they invested too much in the equity market and too little in the risk-free technology. The consumption of the next generation (C_3), will be higher, because of the expanded output.

The overall impact on the welfare[8] is likely to be positive. Welfare gains will be present if R&D growth rises enough to compensate for the capital losses from the equity market. This requires relatively low congestion (high ρ) in the innovating sector. If congestion is too high, further R&D activity is not very productive

[8] We define the measure of welfare as the discounted sum of the aggregate consumption levels of different generations. The discount factor is based on return from the alternative technology (R). See Appendix A.4 for derivations. The results on welfare should be seen as purely indicative as there is no well-established convention for analyzing welfare in over-lapping generations setting.

and the extra funds invested in it, induce little gains in terms of output. In such case the gains of the last generation would be small, while the second generation gets remarkable capital losses. Also, for a given level on congestion, there is an upper bound in how much the public signal can exceed the true productivity for welfare gains to be sustained. Overall, for reasonable range of parameters, the gains are realized for the Model economy. All these effects would be stronger, if the mis-pricing is larger.

2. Above mean demand from the noise traders (decrease of s_1).

Equity prices increase in both economies. Which economy reacts stronger depends on the variances of noise trading and signals. However, unless the precision of public and private signals is very low (when $\beta_\phi + \beta_\nu < \tau\Gamma$), then prices the Benchmark economy will react more on the noise trading shock. This is because in the Model economy the agents use the equity prices also as signals of the underlying productivity. Therefore, the agents facing imperfect information, consider the chance of an actual productivity increase and take a less aggressive position against the non-rational traders. This is assumed to be the case in the conclusions that follow.

The R&D and output growth increase more in the Benchmark economy. First period consumption increases because of the higher equity prices. Negative noise trading shock means that the noise traders cause excess demand of assets by borrowing from storage to invest in the equity market. The rational traders react to guarantee lower losses for themselves (in the Benchmark they guarantee no losses). However, there are large losses in the equity market at the aggregate level. As a result, the second period consumption falls and does so more in the Benchmark economy. Consumption in period 3 increases, because of the higher R&D growth and output. This increase is be larger in the Benchmark economy.

Provided that R&D congestion is relatively low and the noise trading shock is not too big, there are welfare gains compared to the initial scenario. The welfare gains in the Benchmark economy are higher.

A discussion about the differences in the impact of the noise trading and the public signal changes is postponed to Section 2.4.2, since these differences are better understood in the infinite horizon setting.

3. A productivity shock that is not accompanied with a change in the public signal (increase of ϕ_2).

Prices increase in both cases, however the increase in prices is larger in the Benchmark economy. This is because the Model economy puts a positive weight on the unchanged public signal. As a result, the equity market will be underpriced. Therefore, R&D growth will also increase in both cases, but will increase more in the Benchmark economy. The same is true for the output growth. In both cases there is growth due to both the exogenous productivity growth (same in both cases) and R&D growth (higher in Benchmark economy).

Consumption in period 1 increases in both cases, because of higher stock prices. Since the stock price increases more in the Benchmark economy, the consumption will increase more as well. The wage income of the generation consuming in the second period is not affected. In the Benchmark economy, these consumers price equity correctly and will not get any excess gains from the equity market. Hence, in the Benchmark economy, the consumption in the second period is the same as in the initial scenario. In the Model economy, consumers get "surprise" excess gains from the equity market. Thus, their consumption will be higher than in the Benchmark economy. Consumption in period 3 increases in both cases, because the expansion of the economy increases their wages. Since the output growth is smaller in the Model economy compared to the Benchmark case, so is consumption in the 3rd period. Welfare will increase in both cases. The welfare in the Model economy will be lower than in the Benchmark economy, as long as congestion is not too high and the public signal is not too low compared to the actual productivity. Again, the magnitude of the difference between the Model and the Benchmark depends on the variances of different signals.

4. A productivity shock that is accompanied with equivalent change in the public signal (increase of both, $\bar{\phi}_2 = \phi_2$).

The stock prices increase by the same amount in both cases. Therefore, the R&D and output growth rates will increase to the same extent in both cases. Consumption will react in the same way as well, in either economy. That is C_1 and C_3 are increasing (higher prices and output respectively), while C_2 stays the same, as there are not excess losses or gains from the asset market. Welfare will be higher and the same in both economies.

2.4 Infinite horizon model

2.4.1 Information structure and equilibrium equity prices

The production and the consumption side of the model is described in Section 2.2. There is uncertainty about all future labour augmenting productivity shocks. Their prior distribution of these shocks $\phi_t \sim \mathcal{N}(\overline{\phi}, 1/\beta_\phi)$. Every rational consumer i trading in period t receives a private signal about the productivity T periods ahead $\nu_t(i) = \phi_{t+T} + \varepsilon_{\nu,t}(i)$, where $\varepsilon_{\nu,t}(i) \sim \mathcal{N}(0, 1/\beta_\nu)$. He also inherits the private signals from his ancestors (i.e. he gets a signal from a rational consumer i born in $t-1$ about ϕ_{t+T-1}, from one born in $t-2$ about ϕ_{t+T-2} etc.). The earliest private signal that remains still useful in period t is $\nu_{t-T+1}(i)$.

Notice that $\overline{\phi}$ is a public signal that coincides with the long-term productivity in the economy. In order to allow for the possibility of more frequent release of public signals and temporary optimism or pessimism in the market, assume that investors also receive new public signals every period. Similarly to private signals, investors trading in period t receive a public signal about productivity T periods ahead, $\widetilde{\phi}_t = \phi_{t+T} + \varepsilon_{\widetilde{\phi},t}$, where $\varepsilon_{\widetilde{\phi},t} \sim \mathcal{N}(0, 1/\beta_{\widetilde{\phi}})$. The earliest public signal that will be still useful in period t is $\widetilde{\phi}_{t-T+1}$.

Finally, these consumers obtain information about the future productivities from current and historical prices. The earliest price that is useful in predicting price future dividends is P_{t-T+1}. Therefore, the information set available for a rational consumer i in t is $\Omega_t(i) = \{\nu_t(i), ..., \nu_{t-T+1}(i), P_t, ..., P_{t+T+1}, \widetilde{\phi}_t, ...\widetilde{\phi}_{t-T+1}, \overline{\phi}\}$. All private and public signals, as well as the noise trading are assumed to be uncorrelated over time and with each other. Private signals are also uncorrelated across consumers.

Aggregating individual consumers' demand for equity and equating aggregate demand with aggregate supply, equity price becomes

$$P_t = \frac{1}{R}\overline{E}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t] - \tau \operatorname{Var}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t]s_t. \tag{2.17}$$

The conditional variance of the sum of next period's price and dividend is the same for all investors and over time.

The equilibrium price equation solved by the method of undetermined coefficients is

$$P_t = \Gamma Z_1'\Phi_t + \Gamma\widehat{Z}'\widehat{\Phi}_t + \Gamma\overline{z}\overline{\phi}, \tag{2.18}$$

where $\Phi_t = (\phi_{t+1}, .., \phi_{t+T}, \frac{s_{t-T+1}}{\Gamma}, .., \frac{s_t}{\Gamma})'$ is a vector of unknowns that includes future productivities and unknown net noise supplies. Note that s_{t-T} is perfectly revealed by the prices in $t-T$. $\widehat{\Phi}_t = (\widehat{\phi}_{t-T+1}, ..., \widehat{\phi}_t)'$ is the vector of updated public signals. It excludes the information in private signals and prices. Thus, $\phi_{t+T}|\widetilde{\phi}_t, \overline{\phi} \sim \mathcal{N}(\widehat{\phi}_t, 1/\beta_{\widehat{\phi}})$, where $\beta_{\widehat{\phi}} = \beta_{\phi} + \beta_{\widetilde{\phi}}$ and $\widehat{\phi}_t = \frac{\beta_\phi}{\beta_{\widehat{\phi}}}\overline{\phi} + \frac{\beta_{\widetilde{\phi}}}{\beta_{\widehat{\phi}}}\widetilde{\phi}_t$. Finally, $Z_1 = (z_1, .., z_T, -z_{s,1}, .., -z_{s,T})'$ and $\widehat{Z} = (\widehat{z}_1, .., \widehat{z}_T)'$ are vectors of coefficients that can be solved for numerically. The solution method for the equity price equation is presented in Appendix A.5. It is noteworthy that in this setting equity prices depend not only on the true productivities and current noise trading shock, but also on the historical noise trading and the public signals.

Solving the infinite horizon model is done in two stages: first, numerically solving for the equilibrium price equation and second, using this equation in order to solve for the remaining endogenous variables of the model.

The Benchmark price equation is found under the assumption of perfect private signals and is presented in Appendix A.6.

2.4.2 Results

As in the three-period case, R&D and output growth depend on the equity price, which is itself affected by public signals:

$$
\begin{aligned}
g_{A,t} &= \left(\lambda^{\frac{1}{\rho}}\frac{1-\alpha}{\alpha}\right)^{\frac{\rho}{1-\rho}}\left(\frac{P_t}{\pi_t}\right)^{\frac{\rho}{1-\rho}} \\
g_{Y,t} &\equiv \frac{Y_{t+1}-Y_t}{Y_t} = (1+g_{A,t})\frac{\phi_{t+1}}{\phi_t} - 1
\end{aligned}
\tag{2.19}
$$

Details on the parameters used to obtain the results below, are listed in Appendix A.7.[9] Similarly to the three-period model, in the "initial scenario" noise trading at its mean, the productivity is constant and all public signals are correct, (i.e. $s_{t+k} = 0$, $\phi_{t+k} = 1$, $\overline{\phi} = 1$, $\widehat{\phi}_{t-T+k} = 1$, $\widetilde{\phi}_{t-T+k} = 1$ for all $k \in (-\infty, \infty)$).

[9]The numerical outcome of the model should be treated with caution. Given that due complications of identifying good proxies for the variables relevant for measuring the quality of information in equity market, the figures presented here are provided purely for the purpose of illustrating the direction of impact and the economy's dynamics.

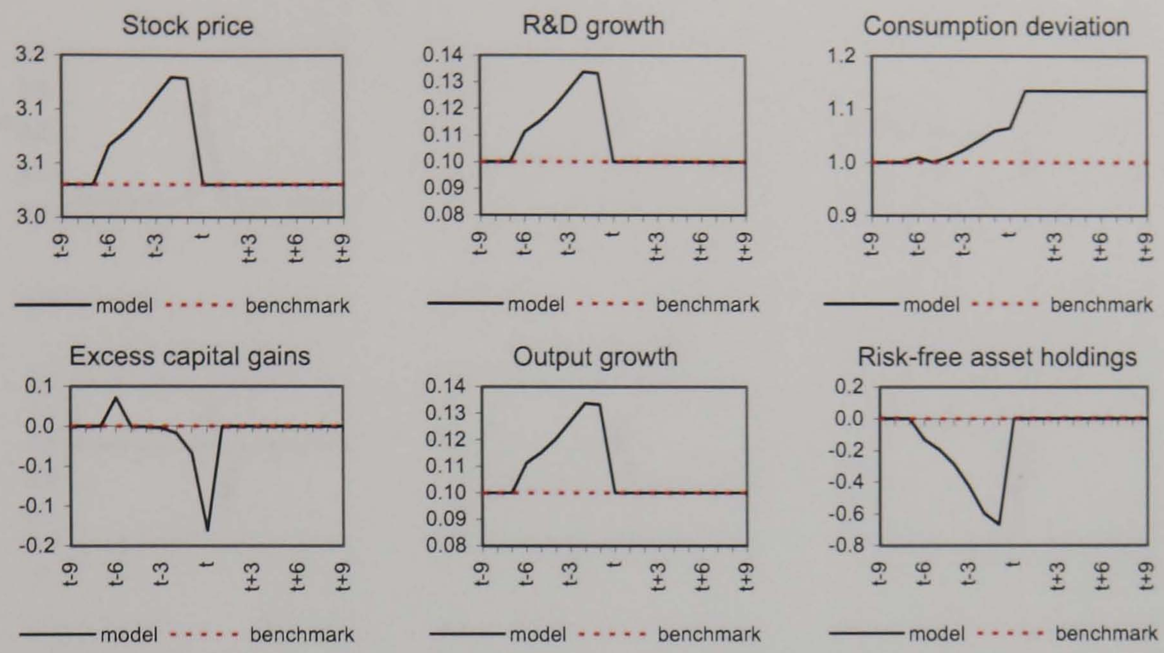

Figure 2.2: Impact of an unjustified improvement of the market sentiment in t.

In this state, the Model and Benchmark economy behave exactly the same. Consumers do not obtain any excess gains or losses from the equity market, and R&D, output and consumption grow at the same rate.

1. A non-justified improvement of market perception about productivity in t (increase in $\bar{\phi}_t$).

This implies an increase in $\widetilde{\phi}_{t-T}$ and therefore in $\widehat{\phi}_{t-T}$. The Benchmark economy is not affected and behaves identically to the initial scenario.

Figure 2.2 depicts the impact of this on stock prices (P_t), excess capital gains ($P_t + \pi_t - RP_{t-1}$), R&D growth ($g_{A,t}$), output growth ($g_{Y,t}$), alternative asset holdings (M_t) and consumption. The latter is measured as deviation from the initial scenario (C_t/C_t^{intl} for the Model and C_t^B/C_t^{intl} for the Benchmark economy, where C_t^{intl} is the consumption level corresponding to the initial scenario).

As in the three-period model, market optimism increases stock prices, R&D growth and output growth for all periods from $t-T$ to $t-1$.

All the generations consuming between periods $t-T+1$ and t get excess losses in

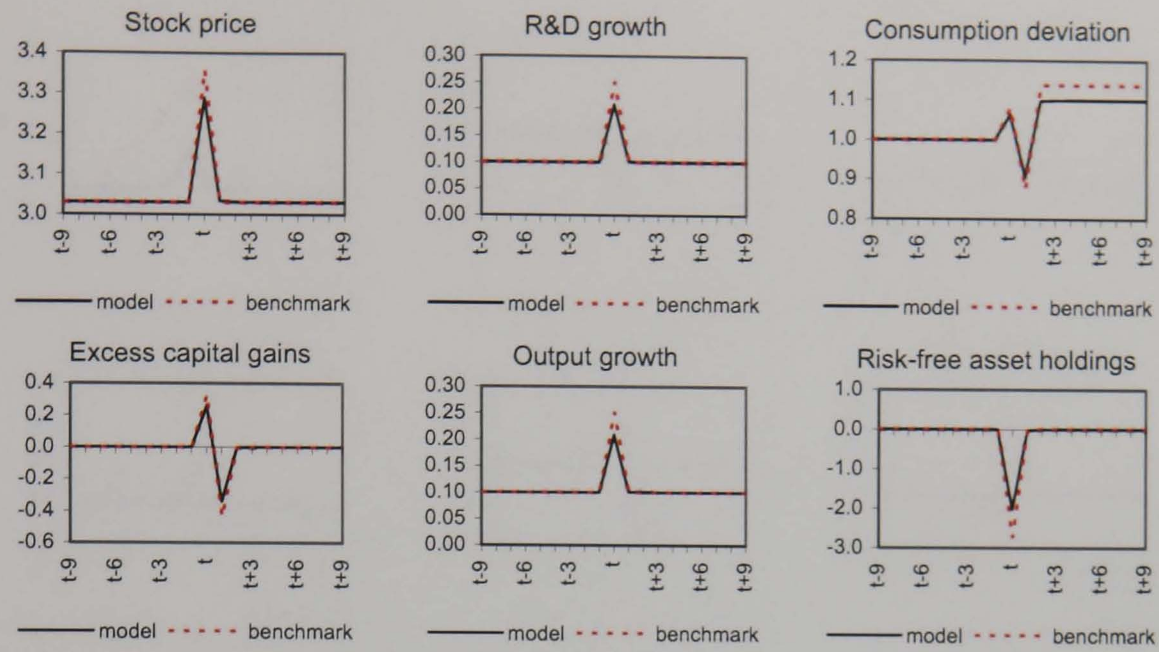

Figure 2.3: Impact of a positive noise trading shock in t.

the equity market that increase over time. As opposed to the three-period model, generations that get excess losses in equity market, can still have higher consumption compared to the initial scenario ($C_{t-T+k} > C^{intl}_{t-T+k} = C^{B}_{t-T+k}$ for $k \in [2, T]$), due to the faster increase of wages. An overall loss always present only for the first generation, that observes a higher public signal ($C_{t-T+1} < C^{intl}_{t-T+1} = C^{B}_{t-T+1}$), because they do not benefit from the expansion of the economy. Generations from t onwards will gain in terms of consumption like the last generation in the three-period model. The potential for welfare gains from pure market optimism is even stronger in the infinite horizon model.

2. Above mean demand from the noise traders (decrease of s_t)

For the parameters chosen, a positive noise trading shock ($s_t > 0$) implies higher prices in both economies in t. As one can see in Figure 2.3, prices increase more in the Benchmark economy. In the Model economy, the prices remain higher until $t + T - 1$, because the noise trading shock will not be fully revealed to these investors until then. However, this persistence is small compared to the impact

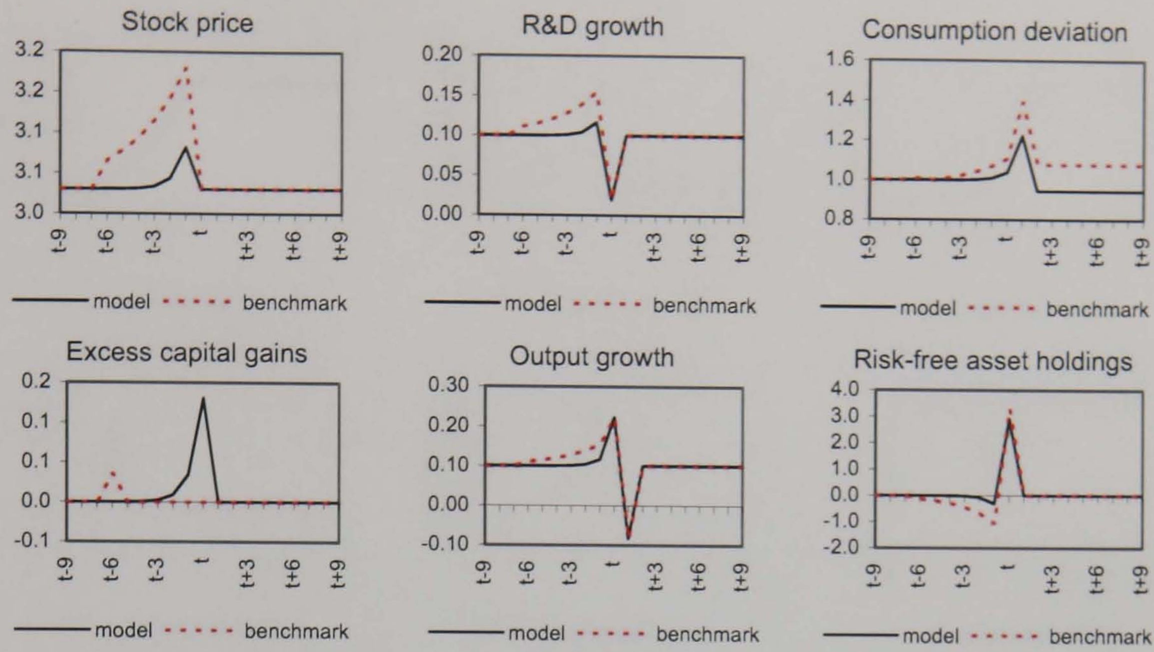

Figure 2.4: Impact of a temporary productivity shock in t, not anticipated from the public signals.

in t, when noise trading shock has a direct impact on equity prices through lower demand.

Unlike the case of an optimistic public signal, the positive impact of higher equity prices will be short-lived. This is because contemporaneous noise trading shock affects the equity prices much more than historical noise trading, since it has a direct impact on equity prices. Thus, equity prices return to their pre-shock level fast (immediately in the Benchmark case). Also, in the Model economy the historical noise trading affects only through the informativeness of historical price signals. The latter are very noisy signals of future productivity and thus have no remarkable persistent impact on the equity prices. Therefore, the increase in the future consumption levels will be smaller compared to the impact of the market optimism.

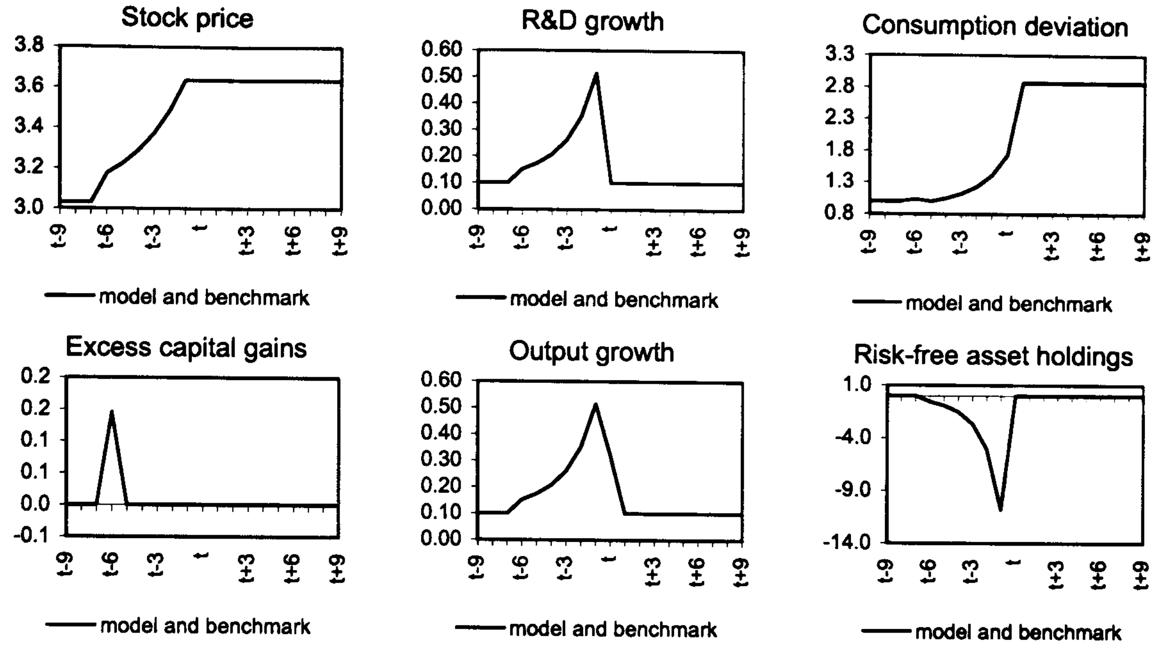

Figure 2.5: Impact of a permanent and fully anticipated increase of productivity in t

3. A temporary productivity shock that is not accompanied with a change in the public signal (increase of ϕ_t).

An implication of temporary increase of productivity (Figure 2.4), that could not be analyzed in the three-period economy is the fact that in period t the price will return to the initial level, while the profits of intermediate firms will be higher. Hence, R&D growth will fall below the initial scenario for one period. While the acceleration of R&D growth will be lower in the Model economy during periods $t - T$ to $t - 1$, the fall in t is exactly as high compared as in the Benchmark economy. The economy with information imperfections and effectively pessimistic public signal, will not take the full advantage of the positive productivity shock and this can even result in lower consumption levels compared to the initial scenario for the Model economy.

In contrast to a temporary productivity shock, a permanent improvement of productivity would not be accompanied with a drop in R&D as shown in Figure

2.5. It is clear by now that any optimism (pessimism) in public signals would imply gains (losses) in later generations consumption. Therefore, the figure only shows the impact of permanent productivity shock that is fully anticipated in public signals.

2.5 Concluding remarks

This chapter analyzed the effect of information imperfection on aggregate economy through equity funded R&D. Comparing the aggregate performance of such Model economy with the perfect private information Benchmark one, leads to the following conclusions.

First, the Model economy tends to perform worse that the Benchmark economy in the event of true productivity shocks and noise trading shocks that increase the equity prices. In the first case, it is a question of market pessimism and equity underpricing, if the market sentiment does not match these changes. In the second case, the rational investors do not take a sufficiently aggressive position against the noise traders.

Second, the Model tends to perform better than the Benchmark economy in the incidence of market optimism. It leads to gains in consumption of all future generations at the expense of possible losses of the earliest generation(s). This is because overpricing of assets results in more R&D being produced. Even if the demand in the future does not justify the initial equity prices, more R&D will have a positive impact on future generations consumption levels through higher output. The positive effect of some optimism is present, when the R&D production market is not already highly congested. The Model also suggests that the lower is the congestion, the greater will be the extent of market optimism that delivers gains over time.

Regarding the improvements in ICT sector, too much congestion was unlikely to be the case in 1990s. For example, during the "dot-coms boom" in the United States, there could have been scope for market optimism. That would have had a positive impact by creating more firms that helped improving the ICT faster, even though the market value of many of these firms fell substantially later on. These firms were typically created with venture capital funding and involved fast exit of founders through an initial public offering. Considering the stage from development to exit as one period[10], technology investments can be interpreted as having been effectively equity funded.

These dynamics pose a dilemma from a policy perspective. It is clear that if the public signal would coincide with the true productivity, the negative side effects of equity overpricing would disappear. Some policy making institutions are likely to have the ability to affect the public signal (e.g. the central bank's comments about economic conditions and outlook). However, given the possible consumption gains (losses) from market optimism (pessimism), there are incentives to preserve an asymmetric behavior from such policy maker. This is because, unless the mean of public signal (e.g. market sentiment) is extremely high compared to the fundamentals, it is unlikely that issuing a low public signal is welfare improving (in terms of aggregate consumption over time). Yet, in a pessimistic market the policy makers should clearly try to inject optimism to the market. The downside of this is that such asymmetric policies, are likely to reduce the credibility of policy maker's optimistic statements among the market participants.

Another implication of the model is that the extent of equity market mis-pricing depends on the quality of information. The better is the private information and the more informative are the price signals compared to the public ones, the

[10] Chapter 3 looks explicitly at the issues that arise between initial development and exit stage.

smaller will be the bias caused by mis-pricing.[11] In order to reduce the quality of information in the public signal, the policy makers could avoid making statements about economic outlook and asset prices. However, the dilemma and trade-off of welfare among different generations remains. In addition to the quality of information, a decrease in the risk aversion, would also reduce the magnitude of the mis-pricing, by improving the price signals.

The discussion about the likely implications of relaxing some restrictive assumptions present in this chapter and directions for future research that arise from this is postponed until Chapter 5.

[11] This is purely due to the fact that investors assign lower weight on public signal in their expectation. See discussion in Chapter 4.4.2 about the quality of public signal and the extent of mis-pricing that considers additional effects arising from endogenous quality of private information and the fact that lower quality public signal is likely to have a larger error.

Chapter 3

The speed of technology adoption with imperfect information in equity markets

3.1 Introduction

There is a growing interest in the connections between financial institutions and economic growth in the literature[1]. This chapter suggests a new mechanism how the development of equity markets and related institutions can determine the speed of technology adoption.[2]

Equity markets have an important role in transferring ownership rights from entrepreneurs, who establish firms to managers running these firms. This chapter analyzes the technology adoption or innovation decisions that are made before the initial public offering. If equity market participants have imperfect information

[1] See Levine (2005) for a comprehensive review of existing theoretical and empirical literature on this topic.

[2] Empirical studies by Beck and Levine (2004) and Rousseau and Wachtel (2000) show that more developed equity markets have a positive impact on economic growth.

about the value of a firm, an entrepreneur's incentives to invest in adopting the newest and most expensive technologies are affected. High uncertainty and low expected market value of the firm can discourage investment in the most advanced technologies - the "fear of unstable markets" force. At the same time, provided that the market accepts that an entrepreneur has better information about the value of his firm than the average equity market participant, his decision to invest in the newest accessible technology issues a positive signal to the market - the "adoption to signal" force. The number of informed investors determines which of these two forces dominates.

When the number of informed investors is small, entrepreneurs become discouraged and choose to adopt technology slowly. This can also be a reason that leads a country to persistently slow technology adoption. Fast technology adoption is most likely with an intermediate number of informed investors. In this case, entrepreneurs have the highest expected gains from investing in the newest technology, in order to issue a positive signal to the uninformed participants in the market. In countries with very developed financial markets and a large number of informed investors, both the discouraging "fear of unstable markets" and encouraging "adopting to signal" force disappear. This implies a non-monotonic relationship between the level of equity market development and the speed of technology adoption.

There are exogenous and endogenous factors that can affect the number of informed investors. For example, some countries could rely on a higher number of informed investors because of cultural links that allow some foreign investors to be informed for a lower cost (e.g. Scandinavian investors in the Baltic States or Austrian investors in Hungary). Furthermore, the number of informed investors is likely to be lower in countries with weak institutions for facilitating access to information (e.g. accounting standards and laws), and therefore less developed equity markets[3].

In order to analyze the link between policies that facilitate access to information and the speed of technology adoption, the basic model is extended to allow for any uninformed investor to become informed at a fixed cost. Within this context, the policies that affect the information cost also affect the speed of technology adoption. The chapter demonstrates that the probability of fast technology adoption is maximized when this information cost is above zero. Because faster technology adoption implies faster growth in local wages and output, a local policy maker would choose zero information costs (i.e. full transparency). Setting the information cost to zero eliminates the possibility that entrepreneurs would issue a positive signal by adopting technology fast.

Another topic addressed in this chapter is the impact of participation of different types of foreign agents. If foreign agents (a part of foreign direct investment) have access to new technology at lower costs, their participation increases the probability of fast technology adoption. Nevertheless, the same two forces remain in action; if the number of informed equity market participants is low, these foreigners might not participate and projects that would be profitable on perfectly informed equity markets are not undertaken.

Participation of foreign portfolio equity investors increases the liquidity of the firms they trade. Higher liquidity has a positive impact on market prices of firms, and increases incentives to invest in fast technology adoption. At the same time, if portfolio investors are largely uninformed, their participation can increase uncertainty and discourage fast technology adoption. The chapter shows the conditions under which forbidding foreign portfolio equity investments encourages fast tech-

[3]La Porta, de Silanes, Shleifer, and Vishny (2005) show that laws mandating disclosure benefit stock markets.

Figure 3.1: Impediments for venture capital investor (US and European respondents)

nology adoption.

The setup of the model relies on two crucial assumptions. First, an entrepreneur has to sell his firm before it generates profits. The need to exit would emerge endogenously if some agents have a comparative advantage to be entrepreneurs rather than managers, as in Holmes and Schmitz Jr. (1990). Also, venture capitalists can be seen as agents who are skilled in judging whether or not a particular technology adoption is worth investing in. They are generally not constrained in credit market and prefer to exit fast (Jovanovic and Szentes 2007). Venture capitalist involvement in running the firms is likely to limit the extent of potential agency problems between the venture capitalist and the entrepreneur. Lack of good exit opportunities is a major concern for these agents while assessing investments to developing countries (Lerner and Pacanins 1997). Figure 3.1 shows that venture capitalists perceive the concerns about successful exit to be a bigger impediment than lack of skilled workers or weak intellectual property laws[4]. Among the less developed countries, Asia is often considered as one of the most attractive locations for venture capital (Aylward 1998, Survey by Deliotte Touche Tochmatsu and EVCA 2006). While this region does not have more skilled labor than competing regions, it has more developed equity markets. Furthermore, Asia has better legal and regulatory environment than Latin America and transition countries that have not entered European Union by 2006 (Appendix B.1). Good exit opportunities facilitate long-term investments and allow for efficient use of entrepreneurial skills.

The second crucial assumption is that the rational uninformed investors' trading decisions are based on noisy information from asset prices, the technology adoption decision, and a noisy public signal. The last captures the impact of market sentiment as in Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006) . Crises in emerging markets and transition countries at the end of 1990s suggest that shifts in market sentiment is an important factor in these countries. Empirical studies show that portfolio capital flows to these countries are largely unrelated to the fundamentals in these countries (e.g. Garibaldi, Mora, Sahay, and Zettelmeyer 2002, Kose, Prasad, Rogoff, and Wei 2004, Prasad, Rajan, and Subramanian 2006).

Appendix B.2 looks at the relationship between GDP growth, R&D expenditures and the level of development of the equity market. Figures B.1 and B.2 show that in transitions countries[5] where securities markets developed faster, R&D ex-

[4]Data source is for Figure 3.1 is Survey by Deliotte Touche Tochmatsu and EVCA (2006). The venture capitalists surveyed are not necessarily investing in these regions. Important impediments that are excluded from the Figure 3.1 are "Lack of quality deals that fit investment profile" and "Lack of knowledge and expertise of business environment" that are likely to be specific to a particular venture capitalist.

[5]This group of countries provides a good comparison group. In addition to the high and similar share of educated labor, transition countries were similar in terms of GDP per capita and institutions in 1991 after USSR dissolved.

penditures have been higher and income per capita has grown faster from 1991 to 2004. At the same time, the relationship appears non-monotonic, which is consistent with the predictions of this chapter. Figures B.3 shows similar patterns for R&D expenditures in high and upper-middle income countries.[6]

The model predicts that openness to capital flows does not guarantee fast technology adoption, unless there are institutions that encourage enough investors to be informed. This is consistent with empirical findings on the effect of openness to capital flows on growth. This effect is found to be positive only if it coincides with more developed institutions, while it is ambiguous otherwise (e.g. Klein and Olivei 1999, Edwards 2001, Edison, Levine, Ricci, and Sløk 2002, Prasad, Rajan, and Subramanian 2006).

The chapter relates to the existing theoretical literature on the determinants of the speed of technology adoption. Differences in the speed of adoption could arise from the lack of skilled labor that make the frontier technologies inappropriate for countries with lack of skilled labor (e.g. Acemoglu 2002). While this argument is likely to be crucial in countries with the lowest shares of educated labor force, it is harder to explain the differences among countries where the share of educated labor force is similar to that of developed countries (e.g. transition countries). In this chapter, the productivity of the labor force in using technology adopted is uncertain. The speed of technology adoption depends on the interaction of this productivity and number of informed investors. If the productivity of labor force is low, fast technology adoption is less likely for a given number of informed

[6]The mechanisms analyzed in this chapter can be argued to apply for investments more generally. The chapter focuses on investments in technology for the following three reasons: 1) Technology sector is likely to be more dependent on entrepreneurial talent, which implies more potential benefits from good exit opportunities 2) This sector is an engine of aggregate economic growth. 3) Venture capitalists that are addressed in this chapter are largely specialised in technology intensive industries. Figure B.4 is Appendix B.2 also shows that the correlation of a proxy for equity market development with more general investments is similar, but weaker that in the specific case of investments in technology.

investors. However, the speed of technology adoption can differ in countries where this productivity is not significantly different.

Obstacles for technology adoption can also be commitment problems and credit constraints (e.g. Gertler and Rogoff 1990, Aghion, Bacchetta, and Banerjee 2004, Aghion, Comin, and Howitt 2006). In order to emphasize the role of the equity market in providing exit opportunities, rather than access to funding, the chapter abstracts from credit constraints. Credit constraints of local agents are unlikely to explain, for example, why foreign venture capitalists do not invest more in less developed countries with relatively skilled and inexpensive labor.[7] Furthermore, the large private capital flows to some developed countries observed in the 1990s (see e.g. Kose, Prasad, Rogoff, and Wei 2004) could have reduced the importance of pure credit constraints in these countries.

Closer to the this chapter are Bencivenga, Smith, and Starr (1995) and Levine (1991) that analyze the impact of liquidity of equity markets and the need for exit in a closed economy. Like in this chapter lack of liquidity reduces incentives to invest in technology adoption. However, information imperfections analyzed in the current chapter add further mechanisms. These papers also do not address the link between technology adoption and institutions facilitating the access to information.

The arguments presented in this chapter are also closely related to the literature on institutions (e.g. Parente and Prescott 1994), which assumes that weaker institutions increase the cost of technology adoption. Therefore, worse institutions should imply slower technology adoption. Marimon and Quadrini (2006) model the start-up cost in an environment with limited contract enforceability that creates incentives for new entries to innovation sector. As long as there are

[7] In fact credit constraints could also arise endogenously (e.g. venture capitalists are unwilling to provide funding for good projects in countries with bad exit opportunities)

new entries accumulation of knowledge is decreasing in start-up costs. While new entries is assumed to be the case in this chapter focus on the need for exit, weak institutions that increase the cost of technology adoption (e.g. property rights, taxation, or other obstacles in establishing or running a firm) could be incorporated in the model. The two main forces found would still remain in action. An innovative result in this chapter is a non-monotonic relationship between fast technology adoption and equity market development, because of technology adoption decisions becomes a signal to the market.

The remainder of this chapter is organized as follows. Section 3.2 presents the model with a fixed number of informed agents. Section 3.3 endogenizes the number of informed investors, and discusses the incentives for a policy maker to choose policies enhancing transparency. Section 3.4 provides a brief discussion on the possibility of gains from forbidding foreign portfolio equity investment in the local asset market. Section 3.5 concludes.

3.2 The model

The model is a small open economy general equilibrium model with rational expectations. It builds on the endogenous growth literature with quality improvements of technology (e.g. Aghion and Howitt 1992, Aghion, Comin, and Howitt 2006) and rational expectations literature (e.g. Grossman 1976, Allen, Morris, and Shin 2006, Kodres and Pritsker 2002, Yuan 2006)

The local economy is populated with overlapping generations of agents endowed with one unit of raw labor each period. These agents work and invest in asset market, in the first period of their lives and consume only in the second period of their lives. The measure of local rational agents is μ. These agents, "investors" can be informed (type $i = I$) or uninformed (type $i = U$). There are similar overlapping generations of foreign agents endowed with exogenous wealth W_t^* in each period investing in the asset market. In the world economy, there are $\hat{\mu}_t^I = \mu_t^I + \mu_t^{*I}$ informed and $\hat{\mu}_t^U = (\mu - \mu_t^I) + \mu_t^{*U}$ uninformed investors.[8]

Some rational local agents have also special skills to be entrepreneurs, who establish local monopolistic firms engaging in technology adoption. Each local entrepreneur can adopt technology alone or in a joint venture with one foreign agent. The firm is called to be established by an "initial owner" where the exact ownership structure is not important.

All rational agents have mean-variance preferences

$$U_t = E[c_{t+1}|\Omega_t] - \frac{\tau}{2}\operatorname{Var}(c_{t+1}|\Omega_t), \tag{3.1}$$

where c_{t+1} is consumption, Ω_t is the available information set in t and τ is a measure of risk aversion.

None of the agents is borrowing or short-sales constrained. The assets traded are local equity (risky asset) and a foreign risk-free bond with a gross return $R \geq 1$ available with infinitely elastic supply. Equity market consists of the shares of local monopolistic firms that engage in technology adoption.

In addition to rational investors, there are noise traders who demand a stochastic quantity (s_t) of risky asset portfolio. All noise traders are assumed to be local unless specified otherwise and they do not receive wage income[9]. The existence of noise traders is necessary for risky asset prices not to be fully revealing (the

[8] μ_t^I and μ_t^{*I} are the numbers of local and foreign informed investors and μ_t^{*U} is the number of foreign uninformed investors

[9] Their location has no impact on conclusions apart from those in Section 3.4. With mean-variance utility, the split of wage income between noise traders and local rational agents does not affect aggregate conditions and conclusions in the model.

Grossman and Stiglitz (1976) paradox). The equity market clearing condition is

$$\hat{\mu}_t^I \hat{h}_t^I + \hat{\mu}_t^U \hat{h}_t^U + s_t = S_t, \tag{3.2}$$

where S_t is the supply of risky asset and $\hat{h}_t^I$ is the demand of risky asset by every informed investor and $\hat{h}_t^U$ is the demand by every uninformed investor.

The production side of the economy consists of a competitive final good production sector and a monopolistic intermediate goods sector.

The price of the final good is normalized to one. The final good producers use raw local labor, L, and i distinct intermediate goods that are produced by local monopolists. Each of these intermediate goods, $x_t(j)$, has quality $A_t(j)$ ($j \in [0, 1]$). For example, the intermediate good could be a computer designed to perform a particular task in the production line ($x_t(j)$) and the vintage of the computer ($A_t(j)$) would determine how fast it will perform the task. The production function is

$$Y_t = (\phi_t L)^{1-\alpha} \int_0^1 A_t^{1-\alpha}(j) x_t^{\alpha}(j) dj, \tag{3.3}$$

where ϕ_t measures the productivity of local labor force in using the technology.

This productivity is uncertain before the period when actual production takes place (i.e. uncertainty about ϕ_t resolves in period t) and can be decomposed into two parts

$$\phi_t = \theta_t + u_t, \tag{3.4}$$

where θ_t is the explainable part of productivity that is uncorrelated across the time and with any other shocks, and u_t is a residual; $u_t \sim \mathcal{N}(0, 1/\beta_u)$ that is also uncorrelated across time with any other shocks.[10] The explainable component

[10]The reasons behind and implication of normality assumption are the same as in Chapter 2.

measures factors like education, training, working culture, management practices, etc. The unexplainable component could be affected by factors like the health of the workers, natural disasters, etc.

Final good producer buys each intermediate good, $x_t(j)$, from a local monopolists in sector j for a price $p_{x,t}(j)$. Intermediate good producers in each sector j use one unit of final good to produce one unit of intermediate good. All intermediate goods depreciate fully in one period.

Section 3.2.1 shows how the uncertainty about the productivity of labor force in using technology (ϕ_t) translates in uncertainty about the future demand for intermediate goods and the profits of local monopolists.

Initial owners establish firms two periods before these firms produce. They can adopt the frontier technology (A_t^*)[11] that grows at an exogenous rate,

$$g^* \equiv \frac{A_{t+1}^* - A_t^*}{A_t^*} \text{ for any } t. \tag{3.5}$$

For each intermediate good j there is only one initial owner, whose effort is needed for technology adoption in each period. In addition to this effort, technology adoption requires an investment in final goods.

Initial owner in sector j born in t, decides whether to invest in fast $(A_{t+2}(j) = A_{t+2}^*)$ or slow $(A_{t+2}(j) = A_{t+1}^*)$ technology adoption. Growth of the world frontier technology (3.5) allows for new firms to produce with higher quality of technology each period $(A_{t+2}(j) \geq A_{t+1}(j))$. New monopolies drive old monopolies out of the market and monopolistic profits can be sustained only for one period.[12]

(see footnote 3 there). The mechanisms would remain valid with other distributional assumptions.

[11]The frontier can also be interpreted as the newest technology that can be accessible and useful for a particular country, instead of the newest available technology worldwide.

[12]The strict inequality, $A_{t+2}(j) > A_{t+1}(j)$, holds if technology is adopted fast in period t, because $A_{t+2}(j) = A_{t+2}^*$ and $A_{t+1}(j) \in \{A_{t+1}^*, A_t^*\}$. The same is true if slow adoption is chosen in consecutive periods, i.e. $A_{t+2}(j) = A_{t+1}^*$ and $A_{t+1}(j) = A_t^*$. $A_{t+2}(j) = A_{t+1}(j)$ only if in

Adopting the newest technology is more expensive than adopting an older one. The fixed cost of establishing a fast adopting firm is

$$I_t = \left(A^*_{t+2} - A^*_{t+1}\right) \hat{\zeta}(\cdot). \tag{3.6}$$

The cost of fast technology adoption is assumed to be proportional to the gain in technology from fast adoption in the period the firm will be active. The cost of adoption per technology gain for an initial owner is

$$\hat{\zeta}(\cdot) = \min[\zeta(\cdot), \zeta^*],$$

where $\zeta(\cdot) > 0$ is the cost for each local entrepreneur alone and $\zeta^* > 0$ for each potential foreign agent participating. The cost $\zeta(\cdot)$ can be constant or an increasing function of the distance from the frontier. The latter would capture the assumption that fast technology adoption may be harder for local agents, who are less familiar with the frontier technology.

Without loss of generality, the required investment to establish a firm that adopts technology slowly is zero. The technology adoption decision is denoted with

$$\tilde{1}_{I_t}(j) = \begin{cases} 1, \text{ if fast adoption is chosen in } t \text{ in sector } j \\ 0, \text{ if slow adoption is chosen in } t \text{ in sector } j \end{cases} \tag{3.7}$$

Initial owner born in t knows the explainable component of the productivity (θ_{t+2}). Given that the initial owner has to retire before his firm produces profits, he sells his firm in the equity market. This assumption about the timing captures

t initial owner chooses slow technology adoption $A_{t+2}(j) = A^*_{t+1}$, while initial owner born in $t-1$ adopted fast $A_{t+1}(j) = A^*_{t+1}$. It is assumed that in this case In new monopoly will still drive the old incumbent out of the market. Implicit assumption behind this is that an intermediate good firm cannot sustain exactly the same quality for more than one period.

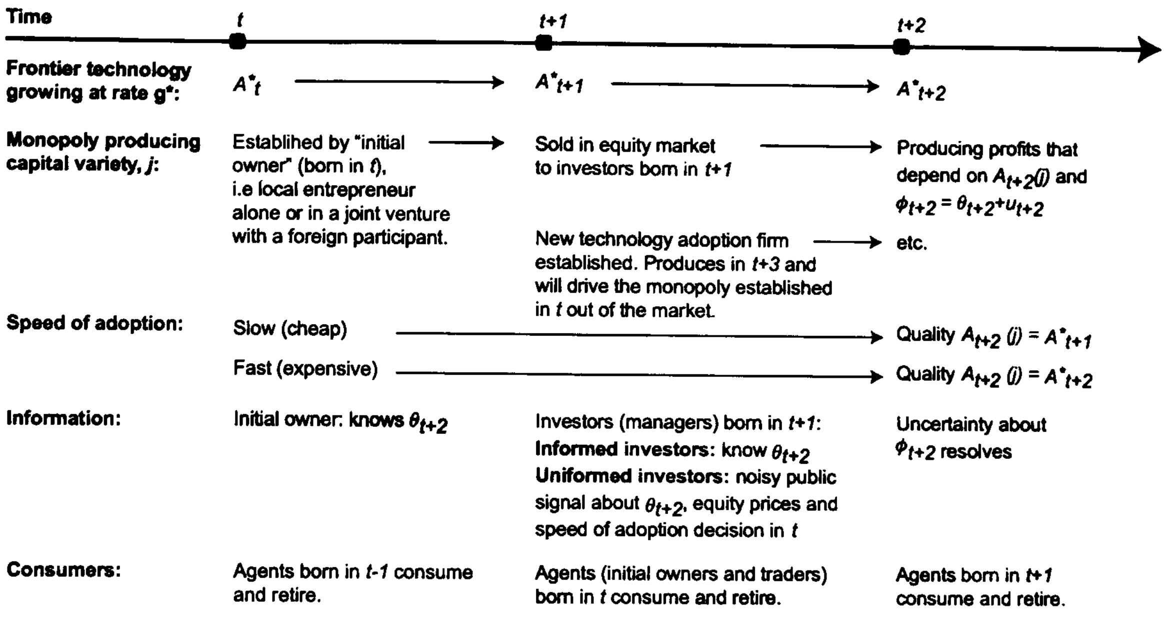

Figure 3.2: Timeline of events

the need for exit and ownership transfers.

The firm established in t is bought by investors (local and foreign) trading in period $t+1$ equity market. Informed investors have the same information as the initial owner; the information set that is relevant for their trading decision is $\Omega^I_{t+1} = \{\theta_{t+2}\}$. Rational uninformed investors obtain information from prices of firms traded, $P_{t+1}(j)$, and the technology adoption decision made one period earlier, $\tilde{1}_{I_t}(j)$. They also receive a noisy public signal at the beginning of period $t+1$,

$$\tilde{\theta}_{t+2} = \theta_{t+2} + \epsilon_{\tilde{\theta},t+2}, \text{ where } \epsilon_{\tilde{\theta},t+2} \sim \mathcal{N}(0, 1/\beta_{\tilde{\theta}}). \tag{3.8}$$

The public signal would capture the "market sentiment". Information set of uninformed investors is $\Omega^U_{t+1} = \{\tilde{\theta}_{t+2}, P_{t+1}(0), .., P_{t+1}(1), \tilde{1}_{I_t}(0), ..., \tilde{1}_{I_t}(1)\}$.

An initial owner in sector j, who is an investor of type $i \in \{U, I\}$ born in $t+1$ has information set $\Omega^{i,e(j)}_t = \{\theta_{t+2}, \Omega^i_t\}$. Figure 3.2 summarizes the main mechanism and timing.

The final goods are used in the local market for aggregate consumption (C_{t+1}), capital ($\int_0^1 x_t(j)dj$) and investments to technology adoption ($\int_0^1 \tilde{1}_{I_{t+1}}(j)I_{t+1}(j)dj$). These expenditures have to equal to aggregate production Y_{t+1} and net inflow of goods from abroad (F_{t+1}). The local goods market clearing condition is

$$C_{t+1} + \int_0^1 x_t(j)dj + \int_0^1 \tilde{1}_{I_{t+1}}(j)I_{t+1}(j)dj = F_{t+1} + Y_{t+1}. \tag{3.9}$$

Final good production sector employs all local labor force. Hence, the labor market clearing condition is

$$L = \mu. \tag{3.10}$$

Solving this model for period t involves first deriving the profits of local monopolies in period $t+2$ in Section 3.2.1. Using this, the equilibrium in period $t+1$ equity market and the market value of the monopolistic firms in period $t+1$ will be derived in Section 3.2.3. After that, the technology adoption decision will be derived for period t in Section 3.2.4 and local goods market clearing decision will be proven to hold in any period in Section 3.2.5.

3.2.1 Production decisions

In period $t+2$, a final good producer takes prices of the intermediate goods ($p_{x,t+1}(j)$) and wages (w_{t+2}) as given and solves

$$\max_{L, x_{t+2}(j)} Y_{t+2} - w_{t+2}L - \int_0^1 p_{x,t+2}(j)x_{t+2}(j)dj,$$

where Y_{t+2} is given by (3.3) and L is the raw labor and $x_{t+2}(j)$ is an intermediate good j.

Intermediate good firm in sector j solves

$$\max_{p_{x,t+2}(j), x_{t+2}(j)} \pi_{t+2}(j) = p_{x,t+2}(j)x_{t+2}(j) - x_{t+2}(j) \text{ st. } p_{x,t+2}(j) = \frac{\partial Y_{t+2}}{\partial x_{t+2}(j)}.$$

Optimal solution implies a demand function for an intermediate good that is linear in the labor productivity and the quality of technology,

$$x_{t+2}(j) = \left(\alpha^2\right)^{\frac{1}{1-\alpha}} \phi_{t+2} L A_{t+2}(j). \tag{3.11}$$

The equilibrium profit in sector j is

$$\pi_{t+2}(j) = \Gamma A_{t+2}(j)\phi_{t+2}, \tag{3.12}$$

where

$$\Gamma \equiv \frac{1-\alpha}{\alpha} \left(\alpha^2\right)^{\frac{1}{1-\alpha}}.$$

Replacing the labor market clearing condition (3.10) and demand for intermediate capital goods, (3.11), in the production function (3.3) the aggregate final good production also becomes linear in the level of technology and productivity of labor force

$$Y_{t+2} = \left(\alpha^2\right)^{\frac{\alpha}{1-\alpha}} \mu A_{t+2}\phi_{t+2}, \tag{3.13}$$

where $A_{t+2} = \int_0^1 A_{t+2}(j)dj$ is the average quality of technology. The equilibrium wages are proportional to the aggregate final good production:

$$w_{t+2} = (1-\alpha)\frac{Y_{t+2}}{\mu}. \tag{3.14}$$

3.2.2 Identical technology adoption decisions

Initial owners are not borrowing constrained and can always finance their investment in technology adoption. From (3.12) the only difference between the firms in different sectors is $A_{t+2}(j)$. The productivity of labor force and information about this productivity (θ_{t+2}), the cost of technology adoption and the frontier technology is the same in each sector j. This implies that all initial owners make identical choices and all intermediate capital goods are produced with the same quality of technology, i.e. for any j

$$\begin{aligned} \tilde{1}_{I_t}(j) &= \tilde{1}_{I_t} \\ A_{t+2}(j) &= A_{t+2}. \end{aligned} \tag{3.15}$$

As a result, there is a continuum of monopolistic firms whose profits are perfectly correlated. Modelling all these firms and their owners is equivalent to modelling one risky asset and one initial owner for all monopolists in the country. The price of all firms will be the same

$$P_{t+1}(j) = P_{t+1}. \tag{3.16}$$

3.2.3 Equity market

Using results from Section 3.2.2, (3.4) and (3.12), the profits of local monopolists can be expressed as

$$\pi_{t+2} = \Gamma(\theta_{t+2} + u_{t+2})A_{t+2}. \tag{3.17}$$

Each monopolistic firm has one divisible share, so the supply of risky asset in

$t+1$

$$S_{t+1} = \int_0^1 1dj = 1. \tag{3.18}$$

The demand of noise traders is

$$s_{t+1} \sim \mathcal{N}(0, 1/\Gamma^2 A_{t+2}^2 \beta_s), \tag{3.19}$$

and s_{t+1} is uncorrelated across time and with any other shocks. The assumption that the variance of noise trading is proportional to the inverse of $\Gamma^2 A_{t+2}^2$ guarantees that the variance of the price signals of uninformed investors does not increase over time. As it will be pointed out later in the chapter, relaxing this assumption would strengthen the results. While noise traders do not receive wage income, they still invest in asset market and their consumption is

$$c_{t+2}^N = (\pi_{t+2} - RP_{t+1})s_{t+1},$$

where P_{t+1} is the equilibrium price of the risky asset.

From (3.1) the trading decision for type $i \in \{I, U\}$ can be expressed as

$$\begin{aligned} \max U_{t+1}^i &= E[\hat{c}_{t+2}^i|\Omega_{t+1}^i] - \frac{\tau}{2}\operatorname{Var}(\hat{c}_{t+2}^i|\Omega_{t+1}^i) \\ \text{st. } \hat{c}_{t+2}^i &= (\Gamma(\theta_{t+2} + u_{t+2})A_{t+2} - RP_{t+1})\hat{h}_{t+1}^i + R\hat{W}_{t+1}, \end{aligned} \tag{3.20}$$

where $\hat{c}_{t+2}^i$ is the consumption of investor of type i in period $t+2$ and τ is a measure of risk aversion. $\hat{W}_{t+1}$ is the wealth or wage income that can be invested on asset markets for agent i. If the agent is local $\hat{W}_{t+1} = W_{t+1} = w_{t+1}$ is wage income given by (3.14), if he is foreign $\hat{W}_{t+1} = W_{t+1}^*$ is exogenous wealth. It should be pointed out that initial owners of monopolists established in $t+1$ trade in the asset market as well. However, under CARA type utility, with no borrowing or short-sales constraints and information structure assumed, the trading and adoption decisions are independent and can be solved for separately[13].

As is well known, type i investor's, who can be local or foreign, demand for risky asset is

$$\hat{h}^i_{t+1} = \frac{E\left[\pi_{t+2}|\Omega^i_{t+1}\right] - RP_{t+1}}{\tau \operatorname{Var}(\pi_{t+2}|\Omega^i_{t+1})}. \tag{3.21}$$

As described at the beginning of Section 3.2, the information set that is relevant for informed investors is $\Omega^I_{t+1} = \{\theta_{t+2}\}$. Therefore, if investor is informed

$$\begin{aligned} E\left[\pi_{t+2}|\Omega^I_{t+1}\right] &= \Gamma\theta_{t+2}A_{t+2}, \\ \operatorname{Var}(\pi_{t+2}|\Omega^I_{t+1}) &= \Gamma^2 A^2_{t+2}\frac{1}{\beta_u}. \end{aligned} \tag{3.22}$$

Uninformed investors obtain information from asset prices, public signal (3.8) and technology adoption decision (3.7) and (3.15). All firms traded in $t+1$ have the same price (3.16). Replacing the optimal demand of informed investors ((3.21) and (3.22)) and supply of risky asset (3.18) in the asset market clearing condition (3.2) the information uninformed investors obtain from asset prices: the price signal is[14]

$$\tilde{P}_{t+1} = \theta_{t+2} + \frac{\tau\Gamma A_{t+2}}{\hat{\mu}^I_{t+1}\beta_u} s_{t+1}. \tag{3.23}$$

The information set to uninformed investors can be expressed as $\Omega^U_{t+1} = \{\tilde{\theta}_{t+2}, \tilde{P}_{t+1}, \tilde{1}_{I_t}\}$. Given that initial owners have superior information (know θ_{t+2}), we can conjecture that their decision to invest in fast; $\tilde{1}_{I_t} = 1$ (slow; $\tilde{1}_{I_t} = 0$) adoption implies that $\theta_{t+2} \geq \bar{\theta}_{t+2}$ ($\theta_{t+2} < \bar{\theta}_{t+2}$). This conjecture is verified in Section 3.2.4. Section 3.2.4 also shows that the threshold ($\bar{\theta}_{t+2}$) is known to uninformed investors trading in $t+1$.

[13] Independence is first assumed to be the case. Later, Appendix B.5 will prove it formally.
[14] See Appendix B.3 for further details.

Expected profits and variance for an uninformed investor are

$$
\begin{aligned}
& E\left[\pi_{t+2}|\Omega^U_{t+1}\right] = && (3.24)\\
= \ & \Gamma A_{t+2}\left(z_{t+1}\tilde{\theta}_{t+2} + (1-z_{t+1})\tilde{P}_{t+1} + \sqrt{z_{v,t+1}}\lambda_{\tilde{1}_{I_t}}(b_{t+1})\right), \\
& \operatorname{Var}(\pi_{t+2}|\Omega^U_{t+1}) = \\
= \ & \Gamma^2 A^2_{t+2} z_{v,t+1}\left(1-\lambda^2_{\tilde{1}_{I_t}}(b_{t+1}) + b_{t+1}\lambda_{\tilde{1}_{I_t}}(b_{t+1})\right) + \Gamma^2 A^2_{t+2}\frac{1}{\beta_u},
\end{aligned}
$$

where

$$
\begin{aligned}
b_{t+1} &\equiv \frac{1}{\sqrt{z_{v,t+1}}}\left(\bar{\theta}_{t+2} - z_{t+1}\tilde{\theta}_{t+2} - (1-z_{t+1})\tilde{P}_{t+1}\right), && (3.25)\\
z_{v,t+1} &\equiv \frac{1}{\beta_{\bar{\theta}} + \left(\frac{\hat{\mu}^I_{t+1}\beta_u}{\tau}\right)^2\beta_s}, \qquad z_{t+1} \equiv \beta_{\bar{\theta}} z_{v,t+1}
\end{aligned}
$$

and $\lambda_{\tilde{1}_{I_t}}(b_{t+1})$ is the inverse Mills ratio[15]. The derivation of these these expressions is presented in Appendix B.3.

For the intuition behind the conditional expected value for an uninformed investor, assume for a moment that all investors are uninformed. In such case, they get information only from the public signal $\tilde{\theta}_{t+1}$ (3.8) and the technology adoption decision (3.7) and (3.15), because asset prices do not reveal any extra information. Fast (slow) technology adoption implies $\theta_{t+2} \geq \bar{\theta}_{t+2}$ ($\theta_{t+2} < \bar{\theta}_{t+2}$) and the

conditional distribution of θ_{t+2} becomes a truncated normal. This implies $E\left[\pi_{t+2}|\tilde{\theta}_{t+2}, \tilde{1}_{I_t}\right] = \Gamma A_{t+2}\left(\tilde{\theta}_{t+2} + \sqrt{1/\beta_{\bar{\theta}}}\lambda_{\tilde{1}_{I_t}}(\sqrt{\beta_{\bar{\theta}}}(\bar{\theta}_{t+2} - \tilde{\theta}_{t+2})\right)$. The expectations differ from the perfect information (3.22) in two respect. First, there is noise in the public signal, $\tilde{\theta}_{t+2}$, that can increase or reduce expected value of the firm.

[15]If $\tilde{1}_{I_t} = 1$, $\lambda_{\tilde{1}_{I_t}=1}(b_{t+1}) = \frac{\phi(b_{t+1})}{1-\Phi(b_{t+1})}$. If $\tilde{1}_{I_t} = 0$, $\lambda_{\tilde{1}_{I_t}=0}(b_{t+1}) = -\frac{\phi(b_{t+1})}{\Phi(b_{t+1})}$,. where $\phi(.)$ and $\Phi(.)$ are standard normal p.d.f. and c.d.f respectively.

This could reflect the "market sentiment". Second, fast (slow) technology adoption implies a positive (negative) Mills ratio and expectations about the fundamental are higher (lower) even if the public signal is correct.

Including informed investors in the model $\theta_{t+2}|\tilde{\theta}_{t+2}\tilde{P}_{t+1} \sim \mathcal{N}(z_{t+1}\tilde{\theta}_{t+2} + (1 - z_{t+1})\tilde{P}_{t+1}, z_{v,t+1})$. Incorporating the information revealed by technology adoption decision again results the labor productivity having a truncated normal distribution from the perspective of an uninformed investor. Expected value is closer to the fundamental and technology adoption decision has less effect on the expected value if $z_{v,t+1}$ is smaller. This is the case when other signals have lower variance (e.g. $\beta_{\tilde{\theta}}$, β_s, $\hat{\mu}^I_{t+1}$ are higher).

The equilibrium price can be derived by replacing (3.18), (3.21), (3.22) and (3.24) into the market clearing condition (3.2). The equilibrium risky asset price is a function of the expectations of informed investors, the expectations of uninformed investors, the liquidity premium and the risk premium. As the expression is lengthy, and only the relevant limiting cases are analyzed, full details are left to Appendix B.3.

If the number of informed investors approaches infinity (or the variance of public information is zero), then the equilibrium asset prices equal the discounted expected profits by informed investors:

$$P^{PI}_{t+1} = \frac{\Gamma A_{t+2}}{R}\theta_{t+2}. \tag{3.26}$$

In such case, the equilibrium asset prices will be fully revealing, investors' asset holdings approach zero, and the risk premium and liquidity premium are pushed to zero. The implications of imperfect information in financial markets can be compared with this benchmark.

In a more realistic environment, the number of informed investors is limited.

Given that this chapter analyzes a small open economy, it is reasonable to assume that the number of uninformed foreign investors who can invest in the local risky assets is infinite compared to the size of the local market. If the number of uninformed investors approaches infinity, the excess returns of uninformed investors approach to zero. Using (3.23) and (3.24), equilibrium asset prices can be expressed as

$$\begin{aligned} P_{t+1} =& \frac{\Gamma A_{t+2}}{R}\left(z_{t+1}\tilde{\theta}_{t+2} + (1-z_{t+1})\theta_{t+2} + \sqrt{z_{v,t+1}}\lambda_{\bar{1}_{I_t}}(b_{t+1})\right) \\ &+ \frac{z_{s,t+1}\Gamma^2 A_{t+2}^2}{R} s_{t+1}, \end{aligned} \tag{3.27}$$

where

$$z_{s,t+1} \equiv (1-z_{t+1})\frac{\tau}{\hat{\mu}_{t+1}^{I}\beta_u}. \tag{3.28}$$

In this case, asset prices are affected by the public signal, noise trading, and an extra term that captures the impact of the signal from the adoption decision. Looking at the expressions for z_{t+1}, $z_{s,t+1}$ and $z_{v,t+1}$ ((3.25) and (3.28)), it is clear that the larger the number of informed investors $(\hat{\mu}_{t+1}^{I})$, the closer the asset price will be to the perfect financial markets benchmark $(\hat{\mu}_{t+1}^{I} \longrightarrow \infty \Longrightarrow z_{t+1}, z_{v,t+1} \longrightarrow 0)$. Both the public signal $\tilde{\theta}_{t+2}$ and noise trading s_{t+1} create uncertainty in asset prices. The latter affects asset prices through the information revealed to uninformed investors by price signals.

Without infinitely many investors (whether uninformed or informed), asset prices would be lower *ceteris paribus,* because the local asset market would not be liquid enough. Noise trader demand would have a direct impact on asset prices, in addition to its impact on the uninformed investors' price signals. This question will be revisited in Section 3.4, when analyzing the impact of forbidding foreigners to invest in local asset market. Until then, the number of foreign uninformed investors is assumed to be infinite.

3.2.4 Adoption decision

Initial owners' technology adoption decision in period t is based on their knowledge of the explainable part of productivity, θ_{t+2}. There is uncertainty about the asset price in period $t+1$, because these agents do not know the next period market perception (signal $\tilde{\theta}_{t+2}$) and noise trading (s_{t+1}).

From (3.1) and the independence of trading and technology adoption decision, investment in fast technology adoption is optimal if $U_t(\tilde{1}_{I_t}=1) \geq U_t(\tilde{1}_{I_t}=0)+RI_t$, where the utility from fast adoption

$$U_t(\tilde{1}_{I_t}=1) = E\left[P_{t+1}|\theta_{t+2},\tilde{1}_{I_t}=1\right] - \frac{\tau}{2}\operatorname{Var}(P_{t+1}|\theta_{t+2},\tilde{1}_{I_t}=1), \tag{3.29}$$

while the utility from slow adoption

$$U_t(\tilde{1}_{I_t}=0) = E\left[P_{t+1}|\theta_{t+2},\tilde{1}_{I_t}=0\right] - \frac{\tau}{2}\operatorname{Var}(P_{t+1}|\theta_{t+2},\tilde{1}_{I_t}=0). \tag{3.30}$$

It can be seen from (3.27) that the selling price of firms that adopt technology fast is always higher. This is because asset prices are proportional to A_{t+2} and from (3.5) $A^*_{t+2} > A^*_{t+1}$.

Explicit derivation of $E\left[P_{t+2}|\theta_{t+2},\tilde{1}_{I_t}\right]$ and $\operatorname{Var}(P_{t+2}|\theta_{t+2},\tilde{1}_{I_t})$ is complicated by the fact that asset prices (3.27) include the inverse Mills ratio $(\lambda_{\tilde{1}_{I_t}}(b_{t+1}))$. While b_{t+1} is an observable constant for investors trading in $t+1$, it depends on $\tilde{\theta}_{t+2}$ and $\tilde{P}_{t+1}$, which are unknown in period t. As a result, b_{t+1} has a normal distribution from the point of view of initial owner who is deciding on speed of technology adoption. The moments of Mills ratio with normally distributed b_{t+1} are, to the best of my knowledge, impossible to derive in closed form. However, the

Mills ratio can be approximated with a linear or polynomial function. The results presented in this chapter employ the linear approximation for simplicity. This is sufficient because the most interesting cases for analysis occur in the neighborhood of $\lambda_{\bar{1}_{I_t}}(0)$, where initial owners are close to being indifferent between fast and slow technology adoption.[16].

Two forces affecting the technology adoption decision

Proposition 1 *Initial owners choose to adopt the technology fast ($A_{t+2} = A^*_{t+2}$) if the observable component of productivity satisfies $\theta_{t+2} \geq \bar{\theta}_{t+2}$, where*

$$\begin{aligned} \bar{\theta}_{t+2} &= \frac{R^2}{\Gamma}\hat{\zeta}(\cdot) - \frac{2+g^*}{g^*}\sqrt{z_{v,t+1}}\eta_1 \\ &\quad + \frac{\tau}{2}\frac{\Gamma(2+g^*)}{R}A^*_{t+1}(1-\eta_2)^2 z_{v,t+1}, \end{aligned} \tag{3.31}$$

and η_1 and η_2 are constants from the linear approximation of the inverse Mills ratio satisfying $\eta_1,\eta_2 > 0$ and $\eta_2 < 1$.

Proof. Presented in Appendix B.4. ■

It can be seen from (3.31) that the threshold depends on the variables and constants that are observable by all agents. Therefore, uninformed investors trading in period $t+1$ know the value of $\bar{\theta}_{t+2}$.

Corollary 2 *In perfect financial markets (i.e. if all investors are informed), the threshold simplifies to*

$$\bar{\theta}^{PI}_{t+2} = \frac{R^2}{\Gamma}\hat{\zeta}(\cdot). \tag{3.32}$$

Replacing $\lim\limits_{\hat{\mu}^I_{t+1}\to\infty} z_{v,t+1} = \lim\limits_{\mu_I\to\infty}[\beta_{\bar{\theta}} + \left(\hat{\mu}^I_{t+1}\beta_u/\tau\right)^2\beta_s]^{-1} = 0$ in (3.31) yields (3.32).

[16]From (3.24) $E[b_{t+1}|\theta_{t+2}] = \frac{1}{\sqrt{z_{v,t+1}}}(\bar{\theta}_{t+2} - \theta_{t+2})$, $\bar{\theta}_{t+2}$ is the threshold above which fast technology adoption will be undertaken.

As long as some investors are uninformed, there are two opposite forces that affect the adoption decision: **"fear of unstable markets"** and **"adoption to signal"**.

The "fear of unstable markets" force is captured by the term $\frac{\tau}{2}\frac{\Gamma(2+g^*)}{R}A^*_{t+1}(1-\eta_2)^2 z_{v,t+1}$ in (3.31). Uncertainty about the price on exit can discourage risk averse agents from adopting the frontier technology, which they would find profitable in perfect asset markets (3.32).

The "adoption to signal" term is captured by $\frac{2+g^*}{g^*}\sqrt{z_{v,t+1}}\eta_1$ in (3.31). Investors who establish local monopolies have superior information compared to the average investor who determines the market value of their firm. They know that these investors will take fast adoption as an indication of higher profitability. As a result, initial owners might invest in fast adoption to gain from uninformed investors, even if they would not do so in perfect financial markets (3.32). The possibility of these gains remains despite of the fact that uninformed investors are rational and aware of the force.

Both of these forces decrease with the number of informed investors: $\frac{\partial z_{v,t+1}}{\partial \bar{\mu}^I_{t+1}} < 0$.

Corollary 3 *If productivity of labor is such that initial owners are indifferent between fast or slow adoption in perfect financial markets ($\theta_{t+2} = \bar{\theta}^{PI}_{t+2}$), they will be discouraged from adopting due to the "fear of unstable markets" in imperfectly informed financial markets if*

$$2R < \tau\Gamma g^* A^*_{t+1}\frac{(1-\eta_2)^2}{\eta_1\sqrt{\beta_{\bar{\theta}} + \left(\frac{\bar{\mu}^I_{t+1}\beta_u}{\tau}\right)^2\beta_s}}. \tag{3.33}$$

Indifference between fast or slow adoption in perfect markets implies that $\theta_{t+2} = \frac{R^2}{\Gamma}\hat{\zeta}(\cdot)$ from (3.32). Applying this fact to (3.31), and using constants from

(3.24) and (3.27) shows (3.33). Corollary 3 has some interesting implications.

A lower number of informed investors ($\hat{\mu}^I_{t+1}$) magnifies the "fear of unstable markets". We can think of the number of informed investors as a measure of the size or development of local financial markets. Therefore, the model suggests that countries with underdeveloped financial markets are more likely to adopt frontier technology slowly, even if the productivity is high enough to justify fast technology adoption in perfect financial markets.

An increase of the number of informed investors encourages "adopting to signal", but causes the resulting gains to decrease. This implies that this force is likely to be most important at an intermediate number of informed investors. If the number of informed investors is very high, potential gains are negligible. Figure 3.3 illustrates how the threshold for fast technology adoption (3.31) depends on the number of informed investors. It plots the relationship between productivity (θ_{t+2}), number of informed investors ($\hat{\mu}^I_{t+1}$) and speed of technology adoption. In perfect financial markets fast (slow) adoption would occur in areas A and B (C and D). In imperfect markets fast (slow) adoption occurs areas A and C (B and D). In B slow technology adoption is due the "fear of unstable markets" force and in C fast technology adoption is due to the "adoption to signal" force.

Higher risk aversion (τ) pushes initial owners towards the "fear of unstable markets". One reason for this is the direct impact of higher risk aversion, making initial owners care more about uncertainty in the following period. There is also a secondary effect, since higher risk aversion reduces the quality of price information through lower demand for the risky asset from informed investors. A higher variance for the unexplainable component of productivity ($1/\beta_u$) has similar effect on the quality of price signal. With an infinite number of traders, the unexplainable component of productivity affects initial owners only through its' impact on price

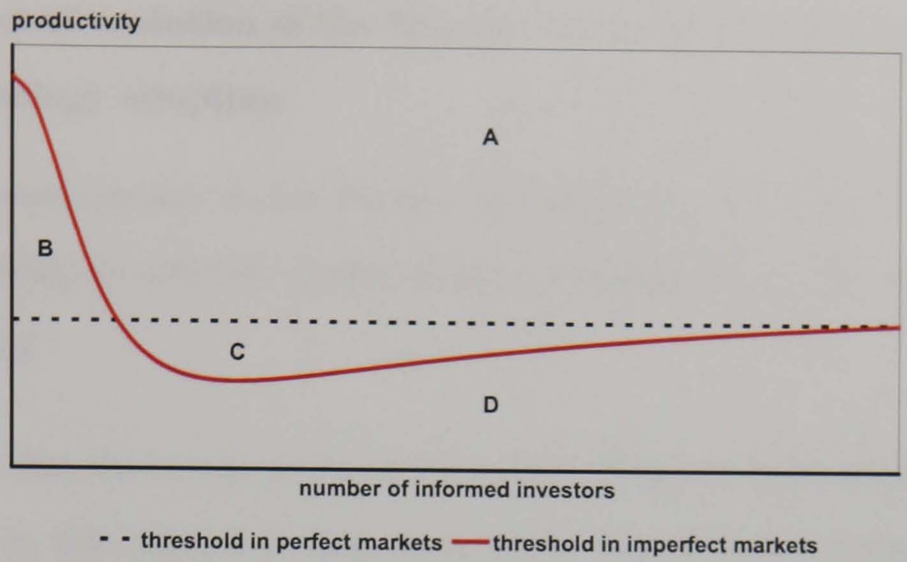

Figure 3.3: Threshold productivity for fast technology adoption

signals.

Similarly, higher variance of the public signal $(1/\beta_{\bar{\theta}})$ and noise trading $(1/\beta_s)$ increase the uncertainty investors are facing, increasing the "fear of unstable markets". In these cases, there is another secondary effect at play, as these move the equilibrium equity price closer to fundamentals (see (3.27)). However, this force is not strong enough to eliminate the negative direct impact from higher uncertainty.

An increase of the risk-free rate has a dual effect on incentives to invest in fast technology adoption. First, there is a direct effect, by which the threshold productivity has to be higher to make investment in fast adoption worthwhile (3.31). This effect is present also in perfect financial markets. Second, (3.33) implies that a higher risk-free rate reduces the impact of "fear of unstable markets force", because it implies a lower variance of equity prices. This suggests that an increase of risk-free rate (R) reduces the probability of fast technology adoption less in imperfect equity markets.

The impact of evolution of the frontier - tendency towards persistently slow technology adoption

Claim 4 *Improvements in the frontier technology have a negative impact on a country's ability to adopt the world frontier technology (A^*_{t+2}), due to information imperfections.*

Assume that the cost of adoption for a given change in technology is constant, $\hat{\zeta}(\cdot) = \hat{\zeta}$. In this case, it is clear from (3.32) that if productivity would stay constant at some level $\bar{\theta} \geq \frac{R^2}{\Gamma}\hat{\zeta}$, a country can always keep up with adopting the newest technology under perfect financial markets.

In imperfect financial markets, the impact of "fear of unstable markets" will increase with the level of technology. Keeping up with the adoption of newest technology with imperfectly informed investors, has to imply an increase in the number of informed investors (or other variables that would lower the threshold or an increase of productivity). Furthermore, a higher growth rate of frontier technology reduces the gains from "adopting to signal" while increasing the negative impact of "fear of unstable markets". If, for example, pure "fear of unstable markets" discourages initial owners from adopting fast in period t, the next generations will also not adopt fast, *ceteris paribus*.

The intuition for this is the following. By (3.17), monopolistic profits increase with the evolution of frontier technology. Uninformed investors do not know how well local labor is able to use any technology, and therefore uncertainty about profits is higher at higher technology levels. This result is driven by the assumption that uncertainty regarding the productivity of using any level of technology is the same.[17]

[17]If, this uncertainty is higher for the more advanced technology adopted, evolution of the frontier technology makes it even harder to sustain fast technology adoption.

If in addition, we assume that the cost of adoption is an increasing function of the distance to the frontier (for example, $\hat{\zeta}(\frac{A^{*}_{t+2}}{A_t}) = \hat{\zeta}(\frac{1+g}{(A_t/A^{*}_{t+1})}$, $\hat{\zeta}'(\cdot) > 0)$ and similarly to Aghion, Comin, and Howitt (2006), the improvements in the frontier would be even more discouraging. Failing to adopt fast in some period would in such case make it also more costly to adopt fast in the following period and the threshold (3.31) increases[18].

Assuming that the variance of price signal has constant quality over time (3.19) eliminated another mechanism that would imply further impact of "fear of unstable markets" with the growth of technology. If the variance of noise trading would not fall with $\Gamma^2 A^2_{t+2}$, the price signals would become worse over time, because a limited number of informed investors holds a relatively smaller proportion of firms. In such case the tendency towards persistently slow technology adoption would also be stronger.

Countries that have big and well developed financial markets (the number of either local or foreign informed investors is large) are less affected by both forces analyzed. This is consistent with developed countries having less volatile capital markets, and high technology level. The model suggests that this outcome does not require developed countries to have either more skilled labor force (higher θ_t) or lower technology adoption costs.

Impact of the participation of a foreign investor

It can be seen from Proposition 1 that, even with foreign initial owners capable of cheaper adoption technology ($\hat{\zeta} = \zeta^* < \zeta(\cdot)$), the impact of the two forces analyzed would be also present and the dominating force does not depend on the adoption cost (Corollary 3). Nevertheless, the threshold $\bar{\theta}_{t+2}$ is lower than the threshold if

[18] This argument is more relevant if firms are established by local entrepreneurs alone.

the local entrepreneurs operates alone:

$$\begin{aligned}\bar{\theta}^{loc}_{t+2} &\equiv \frac{R^2}{\Gamma}\zeta(\cdot) - \frac{2+g^*}{g^*}\sqrt{z_{v,t+1}}\eta_1 \\ &+\frac{\tau}{2}\frac{\Gamma\left(2+g^*\right)}{R}A^*_{t+1}\left(1-\eta_2\right)^2 z_{v,t+1}\end{aligned}$$

It is clear that if the fast technology adoption is more costly for a foreigner ($\zeta^* > \zeta(\cdot)$), he would never participate. This is due to the assumption that the adoption of any technology requires effort by a local entrepreneur, who is the only agent with the relevant skills to adopt in local intermediate goods' sector j. With a similar argument, there is no foreign participation, if slow technology adoption would be optimal for the possible joint venture with the local entrepreneur and foreign investor that can adopt technology fast for a cost $\hat{\zeta} = \zeta^* < \zeta(\cdot)$. Therefore, the relevant cases to analyze are when $\theta_{t+2} \geq \bar{\theta}_{t+2}$, $\bar{\theta}_{t+2} < \bar{\theta}^{loc}_{t+2}$ and $\zeta^* < \zeta(\cdot)$. It is assumed that in a joint venture, foreigner has all the bargaining power.

First, the local entrepreneur alone might choose slow technology adoption, while fast adoption would be undertaken in a joint venture ($\bar{\theta}_{t+2} \leq \theta_{t+2} < \bar{\theta}^{loc}_{t+2}$). If the local entrepreneur's reward in the joint venture (received in period $t+1$) is $q_{sl,t+1}$, his participation constraint is $q_{sl,t+1} \leq U_t(\tilde{1}_{I_t} = 0)$. With the foreigner having the bargaining power, this holds with equality. The foreigner will bear all costs of fast adoption $\zeta^*(A^*_{t+2} - A^*_{t+1})$ and receives the gains from higher firm value $q^*_{sl,t+1} = U_t(\tilde{1}_{I_t} = 1) - U_t(\tilde{1}_{I_t} = 0)$ in $t+1$. The foreign agent can be seen acting as a venture capitalist, by providing funding and receiving a risky return.

Second, the local entrepreneur might be able to adopt fast technology alone, but it is cheaper in the joint venture ($\theta_{t+2} \geq \bar{\theta}^{loc}_{t+2}$). In such case the local entrepreneur's utility from the joint venture equals to his opportunity cost $q_{fa,t+1} = U_t(\tilde{1}_{I_t} = 0) - R\zeta(\cdot)(A^*_{t+2} - A^*_{t+1})$ and foreigner pays for adoption cost $\zeta^*(A^*_{t+2} -$

A^*_{t+1}) and extracts $q^*_{fa,t+1} = R\zeta(\cdot)(A^*_{t+2} - A^*_{t+1})$ from local entrepreneur. This essentially means that the local entrepreneur will hire the services of the foreigner to reduce his costs. It requires highly productive labor, little negative impact from uncertainty related to imperfect information and low cost of fast technology adoption for locals. This case is less realistic in developing countries.

3.2.5 Local goods market clearing

The local goods market condition is given by (3.9). Appendix B.6 proves that it holds. The net inflow of goods from abroad (F_{t+1}) is determined as follows. In period $t+1$, there is an inflow of returns from risk-free asset (the investment local investors/consumers made in period t) and of foreigners investment to the local risky asset (monopolistic firms established in t). There is an outflow of period $t+1$ investment in the risk-free asset by locals, and period $t+1$ profits claimed by foreign investors. If in addition to local entrepreneur, foreign investors participate in technology adoption, there are additional capital inflows and outflows from their investment to the technology and exit.

The predictions from goods market clearing are standard. If domestic investment is higher, because fast technology adoption in undertaken, net foreign asset position will be lower. In such case agents need to borrow more or invest less in the risk-free asset.

Given that the foreign partner always compensates the opportunity cost to the local entrepreneur (Section 3.2.4), foreign participation in technology adoption projects does not affect aggregate consumption of the generation that forms a joint venture with foreign agents (see Appendix B.6)[19]. If foreign investors are

[19] Relaxing the assumption that technology adoption requires the unique skills of a local entrepreneur could allow for welfare losses from foreign investors' participation in technology adoption. Especially if optimal speed of adoption is slow.

capable of adopting fast technology that locals are not, consumption of future generation is higher because of higher wages ((3.14) and (3.13)).

3.3 Endogenous number of informed investors and incentives for transparency

3.3.1 Equilibrium number of informed investors

Section 3.2.4 highlighted the fact that the number of informed investors ($\hat{\mu}^I_{t+1}$) is one of the crucial determinants for the speed of adoption in a small open economy. So far, this number is taken as exogenous. This section assumes that uninformed investors can become informed for a fixed cost (D_{t+1}) during the trading period. When an uninformed investor decides whether to become informed, he does not know what value of θ_{t+2} he will observe after paying the information cost. He will compare his expected utility as an informed investor with his expected utility from staying uninformed, conditional on his available information set, Ω^U_{t+1}. He will decide to become informed if

$$E\left[U^I_{t+1}|\Omega^U_{t+1}\right] - RD_{t+1} \geq E\left[U^U_{t+1}|\Omega^U_{t+1}\right].$$

The information cost function is assumed to be given by a known time specific constant in $t+1$, $D_{t+1} = \delta_{t+1}\vartheta_{t+1}$, where δ_{t+1} is a constant that measures how expensive becoming informed is at any level of technology, and ϑ_{t+1} is a constant that allows uninformed investors to discover more easily if technology adoption decision issues a false signal.[20]

[20] The cost being proportional to $\vartheta_{t+1} \equiv 1 - \lambda^{\cdot 2}_{\bar{1}_{I_t}}(b_{t+1}) - b_{t+1}\lambda_{\bar{1}_{I_t}}(b_{t+1})$ affects the $\mathrm{Var}[\theta_{t+2}|\Omega^U_{t+1}]$. It assumes that information cost is lower if according to other signals, uninformed investors would expect the productivity to be low (high), and the country nevertheless

The number of informed investors cannot be negative, $\hat{\mu}_{t+1}^{I} \geq 0$ and $\mu_{t+1}^{I} \geq 0$. Assuming the existence of some local investors who become informed at zero cost, i.e. $\mu_{t+1}^{I} > 0$, could be justified since at least some local investors are likely to be able to understand local information better. They could also have more direct contact with managers of firms, superior knowledge of the local labor force and business environment and better access to "inside information".

Proposition 5 *An investor will choose to become informed if $\delta_{t+1} \leq \bar{\delta}_{t+1}$. In equilibrium, the cost of information will equal to the gains from becoming informed and the equilibrium number of informed investors*

$$\hat{\mu}_{t+1}^{I} = \begin{cases} \mu_{t+1}^{I}, \text{ if } \delta_{t+1} > \bar{\delta}_{t+1} = \frac{\beta_u}{R2\tau} \frac{1}{\beta_{\tilde{\theta}} + \frac{\mu^2 \beta_u^2}{\tau^2} \beta_s} \\ \sqrt{\frac{\tau}{\beta_u \beta_s} \left(\frac{1}{R2\delta_{t+1}} - \frac{\beta_{\tilde{\theta}} \tau}{\beta_u} \right)}, \text{ otherwise.} \end{cases} \tag{3.34}$$

Proof. See Appendix B.7. ■

Intuitively, becoming informed is profitable as long as the cost is not too high compared to the freely available information. As investors do not know what signal they get, the gain from information is the opportunity to reduce the variance of their returns. The more investors become informed, the more informative asset prices will be. More informative prices limit the gains from better private information. If any uninformed investor finds it profitable to become informed, the equilibrium number of informed investors equalizes the gains of better information with its costs. If information cost is high and no uninformed investor finds it profitable to become informed, the equilibrium number of informed investors is given by the number of investors, who are informed for zero cost.

As can be seen from (3.34), the number of informed investors is higher if

adopted fast (slow). Therefore, this assumption works against the distortions analyzed by limiting the initial owner's potential gains from "adopting to signal". It simplifies the analysis, because ϑ_{t+1} is unknown in period t.

either the risk-free return (R) or information costs (δ_{t+1}) are low. If the public signal is more informative (high $\beta_{\bar{\theta}}$), less investors decide to become informed. Similarly, lower variance of noise trading ($1/\beta_s$) reduces the number of uninformed investors who find it profitable to become informed, because price signals are more informative. Higher risk aversion (τ) and ($1/\beta_u$) affect the incentives to acquire costly information in two opposite ways. First, they reduce the willingness of the investors to invest in the risky asset and pay the information costs. Second, they increase the incentives to bear information costs, because lower participation of informed investors reduces the informativeness of price signals. The second effect dominates as long as it is optimal for any investor to pay the information cost ($\delta_{t+1} \leq \bar{\delta}_{t+1}$)[21].

It can be seen from (3.34) that the equilibrium number of informed investors does not depend on the level or growth rate of the technology. Even though technology improvements imply higher profits, the adoption decision is made before the trading period and is known to all participants of financial markets. The risky asset price adjusts to take this improvement into account for any number of informed investors.[22]

3.3.2 Adoption with endogenous number of informed investors and incentives for transparency

This section assumes that $\delta_{t+1} \leq \bar{\delta}_{t+1}$, which implies that at least some uninformed investors will decide to become informed. Replacing (3.34) in (3.31) and

[21] These results are consistent with findings in Chapter 4.

[22] This result relies on the assumption that variance on noise trading decreases over time (3.19). If this is not the case, less informative asset prices at higher technology level would give more incentives to paying the information costs. However, this would only offset the extra negative impact from "fear of unstable markets" that is eliminated in the current setup.

simplifying the threshold gives

$$\bar{\theta}_{t+2} = \frac{R^2}{\Gamma}\zeta(\cdot) - \frac{2+g^*}{g^*}\sqrt{\frac{R2\tau}{\beta_u}}\sqrt{\delta_{t+1}}\eta_1 + \\ +\tau^2\Gamma\left(2+g^*\right)A^*_{t+1}\left(1-\eta_2\right)^2\frac{1}{\beta_u}\delta_{t+1} \tag{3.35}$$

The forces of "fear of unstable markets" and "adopting to signal," and the factors influencing these, are still present with an endogenous number of informed investors. The technology adoption decision becomes a function of the cost of information δ_{t+1}. Policies towards transparency by local policy makers could affect this cost. This creates a link between technology adoption and institutions that affect financial markets' development.

In order to investigate the policy maker's incentives for transparency, consider an extreme case where it has full control over δ_{t+1}. Suppose the policy maker's objective is to maximize the chances for the country to adopt fast. This objective can be justified, because it allows for output and wages ((3.13) and (3.14)) to increase earlier and therefore increases the consumption of agents benefiting from this. Maximizing the probability of fast technology adoption is equivalent to minimizing the threshold, i.e.

$$\delta^{opt}_{t+1} = \arg\min_{\delta_{t+1}}\left(\bar{\theta}_{t+2}\right),$$

where $\bar{\theta}_{t+2}$ is given by (3.35).[23]

Proposition 6 *If a policy maker has a full control over the cost of information,*

[23] Appendix B.9 shows that the results are similar, if the local policy maker chooses the precision of the public signal, for the same policy objective.

he will set the cost to be

$$\delta_{t+1}^{opt} = \left(\frac{\eta_1 \sqrt{R\beta_u}}{g^* \Gamma A_{t+1}^* (1-\eta_2)^2 \sqrt{2\tau^3}} \right)^2 > 0.$$

Proof. See Appendix B.8. ■

This proposition suggests that the local policy maker does not choose full transparency ($\delta_{t+1}^{opt} = 0$). The reason comes from the "adopting to signal" force. As long as some investors are uninformed, initial owners would find it optimal to adopt fast at a lower level of productivity than would be possible in perfectly informed equity markets. It is important to point out that the counter-intuitive policy encouraging "too fast" technology adoption is justified because the policy maker is local. The extra opportunities of fast technology come at the expense of losses of foreign uninformed investors. Given that the local market is limited in size, asset holdings of local uninformed investors are marginal. In equilibrium $RP_{t+1} = E\left[\pi_{t+2} | \Omega_{t+1}^U\right]$, and from (3.21) each of the local uninformed investors holds in equilibrium $h_{t+1}^U = \hat{h}_{t+1}^U = 0$. At the same time, local informed investors are expected to get excess gains from asset market as long as there are not infinitely many informed investors.[24]

Both the higher level and growth rate of frontier technology imply more incentives towards transparency. As discussed in the Section 3.2.4, evolution of frontier technology implies higher uncertainty. Therefore, countries that try to keep up with improvements in the frontier technology are expected to aim to become more

[24] Policy makers' objective could also be maximizing the utility of local agents. This is more cumbersome mainly because it is hard to identify what is the reasonable information set the local policy maker has. However, it would not alter the optimal information cost being above zero in this setup. Agents affected by the choice of δ_{t+1} are 1) local entrepreneurs born in t, 2) local investors born in $t+1$ and 3) workers born in $t+2$. Higher probability of fast technology adoption is beneficial for agents 1) and 3). Lower transparency is beneficial for an average local investor. Therefore, the local policy with such objective function is likely to set information cost higher and not lower compared to the one analyzed.

transparent over time,

$$\frac{\delta_{t+2}^{opt}}{\delta_{t+1}^{opt}} = \left(\frac{A_{t+1}^*}{A_{t+2}^*}\right)^2 = \left(\frac{1}{1+g^*}\right)^2$$

Other variables that increase the optimal transparency are higher risk aversion (τ), variance of unexplainable component of productivity ($1/\beta_u$) and lower risk-free interest rate (R). As it can be seen from (3.35), these changes tilt towards the dominance of "fear of unstable markets" force. Section 3.3.1 showed that for a given information cost, the same variables give incentives to more uninformed investors to become informed. However, this is not sufficient and policy maker would give further incentives to acquire costly information through higher transparency.

Finally, the policies towards transparency could alternatively be modelled as the policy maker choosing the variance of public signal. Appendix B.9 shows that the implications of this are the same as in the case of fixed information costs.

3.4 Closing the local asset market to foreign portfolio investors

One of the reasons why countries restrict the foreign portfolio investments is the potential instability of these flows. This section analyzes if preventing foreigners to trade in the local equity market can make fast technology adoption more likely. Since the justification for capital restrictions implies that foreign capital is less informed than local, assume that all potential foreign investors are uninformed and all local investors are informed but limited in number ($\mu = \mu_{t+1}^I$ is finite). Assume that the restrictions of foreign investors imply that none of the foreign investors can invest in the country ($\mu^{*U}_{t+1} = \mu^{*I}_{t+1} = 0$). This section analyzes two cases in this framework, where the location of noise traders is different.

In the first case, assume that all noise traders are local. Using (3.18), the optimal demand (3.21), (3.22), and the equity market clearing condition (3.2), the equilibrium price can be expressed as

$$P^R_{t+1} = \frac{\Gamma A_{t+2}\theta_{t+2}}{R} - \frac{\tau\Gamma^2 A^2_{t+2}}{\mu\beta_u R} + \frac{\tau\Gamma^2 A^2_{t+2}}{\mu\beta_u R} s_{t+1}. \tag{3.36}$$

Because the size of local market is limited, equity prices contain a liquidity premium and a risk premium. Both premiums are decreasing in the number of local informed investors (μ). A liquidity premium is introduced because the limited number of local investors cause excess supply of risky asset. As a result, asset prices will be lower and excess gains of local rational investors higher. This has a new discouraging impact on the incentives to adopt fast. There is still uncertainty in the local market from noise traders and the "fear of unstable markets" has an impact. Absence of uninformed investors, who take fast adoption as a signal of high productivity, eliminates any potential gains from "adopting to signal". If all noise traders are local, there is more uncertainty regarding the asset price despite less uncertainty from the impact of sentiment in international markets. In such case it is never optimal for a country to forbid uninformed, but rational foreigners from investing in the country, if the goal of the local policy maker is to encourage fast technology adoption. Appendix B.10 proves it formally.

In the second case, it is assumed that all noise traders are foreign. Then

$$P^R_{t+1} = \frac{\Gamma A_{t+2}\theta_{t+2}}{R} - \frac{\tau\Gamma^2 A^2_{t+2}}{\mu\beta_u R}.$$

On one hand, initial owners deciding the speed of technology adoption, face no uncertainty and the "fear of unstable markets" force disappears. On the other hand, absence of "adoption to signal" force and liquidity premium reduce the incentives to invest in fast adoption. In this case there exists a possibility that restricting foreign portfolio investments can encourage faster technology adoption (see Appendix B.10). However this possibility exists only under specific conditions. First, the number of local informed investors has to be low enough such the "fear on unstable markets" force would dominate if the country was open to foreign portfolio investments (Section 3.2.4). Otherwise, participation of uninformed foreign investors would eliminate the liquidity premium and allow "adopting to signal". Second, the variance of foreign noise trading or unexplainable component of labor productivity has to be high. This implies that price signals are not sufficiently informative. Third, the number of local informed investors cannot be very low, i.e. the local market is very small. In such case, the need for liquidity is pressing.

Hence, the model suggests that countries that could benefit from fast technology adoption by restricting foreign uninformed capital are those with small, but not the smallest local equity markets. In this case, potential benefits would arise only if the uncertainty in the purely domestic market is very low compared to uncertainty associated with the behavior of foreign investors.

3.5 Concluding remarks

This chapter presented an alternative answer to the question, why is the speed of technology adoption different across countries?. It argues that if ownership transfers of firms that engage in technology adoption have to be made in imperfectly informed equity markets, two opposite forces arise: a negative "fear of unstable markets" force and a positive "adopting to signal" force. These forces affect the incentives for developers to adopt the accessible frontier technology.

The relative importance of these forces depends on the size of financial markets. "Adopting to signal" is likely to be most influential in countries where equity markets are at an intermediate level of development, while "fear of unstable markets" should dominate in underdeveloped markets. The less precise are the signals uninformed traders base their decisions on, the stronger these forces are. The importance of both forces falls with the number of informed investors; it follows that countries with well informed (developed and large) financial markets are less affected. Nevertheless, if the recent developments in the United States' and other developed countries' technology sector assets were a bubble, it suggests that there would be room for "adoption to signal" (in this case it should be seen as "innovation to signal") even in developed countries.

Fast technology adoption tends to be more difficult to sustain because of the participation of uninformed traders. Provided that the number of informed investors and cost of technology adoption does not change, the evolution of the frontier technology implies an increasing importance of the "fear of unstable markets". This is because uncertainty about the ability of labor in using any technology creates higher uncertainty about profits if the technology is more advanced and therefore profits are higher.

The mechanisms analyzed in this chapter affect both local agents and foreign investors (such as venture capitalists) intending to invest in establishing new firms. Lack of informed investors in the equity market, can discourage foreign investors from participating in projects where they could reduce the costs associated with adopting the frontier technology. The limited presence of venture capitalists in most developing countries is likely to be affected by the weakness and instability of local asset markets.

When the number of informed investors is made endogenous, by letting the local policy maker to determine the magnitude of information costs, it is shown that countries would not choose to be completely transparent. This situation arises from the "adopting to signal" force. Nevertheless, a policy maker has incentives to enhance more transparency over time to keep up with adopting the frontier technology.

The model considered two extremes cases generating information asymmetries: the number of informed investors being exogenous, and the local policy maker having full control over information costs. In the more realistic case, where the local policy maker has some, but not full, control over the information costs, both policies and exogenous factors will determine the number of informed investors.

The better performance of transition countries that joined the European Union in 2004, when compared with those that did not, could be explained by their ability to attract informed investors from neighboring developed countries more easily. Estonia is a stark example of a country that has been very active in adopting Internet and Communication Technologies in 1990s, and attracted venture capital funded Skype, arguably due to the impact of the "adopting to signal" force. At the same time, Romania or Ukraine, which have similar shares of educated labor, have lower rates of technology adoption, and may have been more affected by the "fear of unstable markets" force.

The model assumed that openness to international portfolio capital flows guarantees sufficient liquidity in the local equity market. In reality, less developed equity markets can also lack liquidity, even if they are open, because the number of foreign investors who are interested in investing in these countries is low. The liquidity premium has a further negative impact on the incentives to adopt costly frontier technology in less developed equity markets, and forbidding foreign portfolio equity flows would increase it. In countries where the local equity markets are smallest, the need for attracting foreign portfolio equity flows to generate liquidity is pressing, and entry of foreign traders is likely to encourage investments in fast technology adoption. Gains from preventing foreign portfolio equity flows are only possible under very specific conditions in this setup. First, local equity markets have to be small, such that the "fear of unstable markets" would dominate in open equity capital markets. Second, policies should enable only local investors to be well informed, while the foreign investors are largely uninformed and their behavior is highly uncertain. Only in such case, the benefits from lower uncertainty could potentially offset the losses due to a higher liquidity premium. In countries with intermediate and big equity markets, the "fear of unstable markets" has little negative effect and even a small additional liquidity provided by the participation of foreign traders would justify openness to foreign portfolio equity investments.

The model does not specify whether firms are listed in the local or foreign stock market. Listing in a well established stock exchange (e.g. NASDAQ) can allow a firm to access a larger number of informed potential buyers. Also, the regulations of well developed stock exchange should reduce information costs. However, for most of the firms from developing countries, fixed costs associated with an initial public offering in NASDAQ are likely too high and they have to rely on the local equity market. Therefore, this possibility is available only for the most successful and innovative firms. Moreover, the most successful and innovative local firms can be more easily sold to a strategic foreign owner. As long as the price the strategic owner pays for a firm reflects its market value, the mechanism suggested in this chapter remains valid. If the local equity market is very underdeveloped and most firms are transferred directly between local agents, both potentially the low number of informed buyers and lack of liquidity are likely to discourage fast technology adoption.

Chapter 4

Optimal research in financial markets with heterogeneous private information: a rational expectations model

4.1 Introduction

Empirical evidence shows that asset prices adjust slowly to changes in the fundamentals (see Chapter 1 for references) and investors appear to put relatively low weight on their private signals when making their trading decisions[1]. Furthermore, when investment decisions are made by financial institutions, rather than individual agents, they have to hire researchers in order to obtain private signals.

[1]For example, Menkhoff (1998) presents results of a survey among participants of foreign exchange market in Germany, who were asked to evaluate the relative importance of fundamentals, technical analysis and monitoring order flows in their trading decisions. The importance of fundamental research was evaluated to be only around 45 per cent. The survey also shows that investors consider that psychological factors and opinion leaders' viewpoints are relevant to the market.

They can choose both, the size of their research department and the research effort that is put into a particular asset.[2]

The aim of this chapter is to analyze the extent of asset mis-pricing and the dynamics of asset prices in a setting where the choice of private information is explicitly modelled. Research cost is modelled as the real resources that investors commit to pay in order to obtain a private signal of a particular precision. In order to address to extent of asset mis-pricing in a rational setting, research costs are incorporated into the setting of Allen, Morris, and Shin (2006), where the asset prices dynamics are affected by the existence of common public information.

In their setting, the availability of public information generates slow adjustment to changes in the fundamentals because of higher order beliefs. Rational and short-lived investors take the public signal into account not only because it reveals information about the fundamental, but also because it helps to predict future asset prices. The further away is the liquidation date, the higher is the importance of public signal in predicting future asset prices.[3]

The derived dynamics in Allen, Morris, and Shin (2006) rely on constant precision of private signals over time. There are two opposite forces that affect the dynamics of asset prices when research effort is made endogenous. First, over time the fundamental affects asset prices more, and therefore private signals become better predictors of next period's price. This gives investors the incentives to acquire more precise private signals over time. Second, because investors obtain more information from historical prices over time, they become less willing to

[2] Although not directly compatible with the modelling strategy in this chapter, research costs can also arise from the opportunity cost of fundamental research in terms of the speed of reaction fast on the developments in the market.

[3] It is worth pointing out that the liquidation date should not be interpreted only as the maturity of a bond or the liquidation of a firm. It could be any future event that affects the value of an asset (e.g. the success of an particular investment project that affects the future value of equity prices).

spend resources on better private information in later periods, because the benefit of better private information are lower, when more information from other sources is available.

This chapter shows that with endogenous precision of private information, the first force dominates in determining the time path of research costs and slow reaction to changes in the fundamentals becomes even more pronounced compared to Allen, Morris, and Shin (2006).

The second force is important when comparing two assets that have the same number of trading periods left until the liquidation date, but different length of price history. Under specific assumptions about the research costs, the asset with longer price history becomes even more mis-priced. This is in contrast with a setting that does not incorporate endogenous research decision, where the extent of mis-pricing would be always lower for an asset with longer price history.

The chapter also addresses the dependence of optimal precision on the other parameters of model (e.g. precision of public information, risk aversion, risk-free rates etc.) and the importance of the assumption that investors are myopic. The latter is addressed by incorporating research costs and public signal in a two-period model with long-lived agents in the spirit of Brown and Jennings (1989).

Finally, the chapter discusses the effectiveness of policy makers' efforts to reduce the extent of mis-pricing by increasing the precision of public information (e.g. through making truthful statements about the developments in the asset markets or encouraging development of independent research institutes that provide publicly available information). This is also important in understanding the impact of developments in technologies that facilitates access to information.

Information costs and their impact on asset prices has been addressed by Calvo and Mendoza (1999), who focus on portfolio choice and herding behavior due to information costs. Closer to this chapter are rational expectations models with information costs starting from Grossman and Stiglitz (1976) and more recently Veldkamp (2006), Nieuwerburgh and Veldkamp (2006), Peress (2006) and Dow, Goldstein, and Guembel (2007). All these papers focus on impact of information costs in one trading period setting.

The chapter also relates to Bacchetta and van Wincoop (2006) that similarly to Allen, Morris, and Shin (2006) emphasize on the importance of higher order expectation and to rational expectations literature more generally (e.g. Grossman and Stiglitz 1976, Hellwig 1980, Diamond and Verrecchia 1981, Kyle 1985, Singleton 1986, Brown and Jennings 1989, Grundy and McNichols 1989, He and Wang 1995).

The rest of the chapter is structured as follows. Section 4.2 sets up the model and shows how research and trading decisions are determined. Section 4.3 discusses the results. It solves the model in a simple one-period setting and proceeds with analyzing the multi-period setting. Section 4.4 presents extensions of the baseline model. It analyzes its dynamics, when investors are long-lived and discusses the impact of higher precision of public information on asset prices. Section 4.5 summarizes the results.

4.2 The model

4.2.1 Assumptions

Assume that there are overlapping generations of short-lived investors, who can invest in a risky asset and an alternative asset providing a known risk-free return $r_t \geq 1$, on each trading period t. The risky asset has liquidation value Θ at date $T+1$, and is traded in periods 1 to T. Investors do not know the liquidation value, but they can obtain information about it from public signals, private signals and prices. The characteristics of these signals will be described later in this section.

Every period, there is a continuum of rational investors normalized in the interval $[0, 1]$, endowed with W units of funds. They buy assets at date t and consume at date $t+1$. A risk averse investor i, who is trading at date t, has the mean-variance utility function

$$U_{i,t} = E\left[c_{i,t+1}|\Omega_{i,t}\right] - \frac{\tau}{2}\operatorname{Var}(c_{i,t+1}|\Omega_{i,t}), \tag{4.1}$$

where the risk aversion is measured by the constant τ and $\Omega_{i,t}$ the information set available for investor i in period t, which will be specified shortly.

The budget constraint for investor i trading in period t is

$$c_{i,t+1} = h_{i,t}(P_{t+1} - r_t P_t) + r_t W - r_t \kappa(\beta_{i,t}),$$

where $c_{i,t+1}$ is his consumption in $t+1$, $h_{i,t}$ is his demand for the risky asset, W is the endowment and $\kappa(\beta_{i,t})$ is the research cost function, which will be specified shortly. The price of the risky asset at time t is denoted by P_t. Notice also that $P_{T+1} = \Theta$.

In addition to rational traders, there are noise traders in each period, modelled as a noisy net supply of the risky asset $s_t \sim \mathcal{N}(0, 1/\delta)$. Supply shocks are assumed to be uncorrelated along time and across both public and private signals of investors in every time period. The market clearing condition is $H_t = s_t$ for every period t, where H_t is the aggregate demand.

There is some initial public information about the risky assets. The liquidation value is drawn from the prior distribution that is the public signal $\Theta \sim \mathcal{N}(y, 1/\alpha)$.

Investors trading in period t can obtain a costly and noisy private signal about the fundamental value of the asset, $\nu_{i,t} = \Theta + \varepsilon_{i,t}$, $\varepsilon_{i,t} \sim \mathcal{N}\left(0, 1/\beta_{i,t}\right)$. It is assumed that $\varepsilon_{i,t}$ is uncorrelated across investors, time, supply shocks and the noise in public signal.

Investors can choose the variance of their private signal $(1/\beta_{i,t})$, if they spend $\kappa(\beta_{i,t})$ on research. The research cost function should capture the idea that having a more precise view about the fundamental value must be more expensive, and knowing the true value with certainty is too expensive. This implies the following conditions: $\kappa'(\beta) > 0$, $\kappa''(\beta) \geq 0$, $\kappa(\beta) \rightarrow \infty$ as $\beta \rightarrow \infty$ and $\kappa(0) = 0$. Further analysis of the importance of the functional form of cost function is presented in the Section 4.3.2.

The sequence of events is the following:

1) The fundamental value Θ is drawn by nature at date 0.

2) The first generation of investors simultaneously choose their research cost for the risky asset, before observing private signals and the price. The information set about fundamentals available to each of them is the same: $\Omega_{-1} = \{y\}$.

3) These investors trade and period 1 market clears. The information available for their trading decision also contains private signals and the price signal. So $\Omega_{i,1} = \{y, \nu_{i,1}, P_1\}$. In the following period, they will receive income corresponding to the price of each asset, consume and retire without revealing their private signals to the following generations.[4]

4) The generations 2 to T will make their research and trading decisions in a similar manner. The only difference is that they will obtain information about the fundamental also from historical prices. So the information set available for their research decision is $\Omega_{-t} = \{y, P_1, \ldots, P_{t-1}\}$. Their trading decisions will be based on information set $\Omega_{i,t} = \{\Omega_{-t}, \nu_{i,t}, P_t\}$. Apart from the generation trading

[4] Relaxing this assumption and allowing investors to inherit the private signals from previous generations, would make investors heterogeneous in their research. See Section 4.5 for a discussion about the likely implications of this.

at T, the consumption of investors will depend on prices in $t+1$ rather than the fundamental value Θ.

5) At $T+1$ the risky asset will be liquidated and generation T will obtain Θ.

This setting implies that investors are heterogeneous in their trading decisions, while being homogeneous in their research decision.

4.2.2 Solution method

Investors of each generation face a two stage decision problem. The model is solved by first deriving investors' demand and equilibrium prices, taking the research decisions as given. Given the solution from that stage, the optimal research costs can be derived conditional on the available information.

From the utility function (4.1), we can find the demand for the risky asset of investor i,

$$h_{i,t} = \frac{E[P_{t+1}|\Omega_{i,t}] - r_t P_t}{\tau \operatorname{Var}(P_{t+1}|\Omega_{i,t})}. \tag{4.2}$$

Equilibrium prices

Assume that the asset price follows a linear rule on each period, $P_t = \eta_t(\lambda_t y_t + \mu_t\Theta - s_t)$. The term y_t is a public signal about asset in period t. It includes the initial public signal y and information from prices in trading periods 1 to $t-1$. The coefficients η_t, λ_t, μ_t can be found by the method of undetermined coefficients from the market clearing condition. (See Appendix C.1 for the derivation.)

Rearranging the pricing equation, investors can observe a price signal $\tilde{P}_t$ in period t, such that $\tilde{P}_t \equiv (P_t/\eta_t - \lambda_t y_t)/\mu_t = \Theta - s_t/\mu_t$, and $\tilde{P}_t|\Theta \sim \mathcal{N}\left(\Theta, \mu_t^{-2}\delta^{-1}\right)$. The updated distribution of Θ conditional on prices will also be normal. It will be the public signal for the investors trading in period $t+1$: $\Theta|\tilde{P}_t \sim \mathcal{N}\left(y_{t+1}, \alpha_{t+1}^{-1}\right)$.

The public signal evolves over time as

$$y_{t+1} = \frac{\alpha_t y_t + \mu_t^2 \delta \tilde{P}_t}{\alpha_{t+1}}, \tag{4.3}$$

$$\alpha_{t+1} = \alpha_t + \mu_t^2 \delta. \tag{4.4}$$

The first generation investors cannot observe historical prices, so

$$\alpha_1 = \alpha \text{ and } y_1 = y. \tag{4.5}$$

As investors trading in period t will also obtain a private signal $\nu_{i,t}$, they will believe that the fundamental to be drawn from

$$\begin{aligned} \Theta|\Omega_{i,t} &\sim \mathcal{N}\left((y_t\alpha_t + \mu_t^2\delta\tilde{P}_t + \beta_{i,t}\nu_{i,t})V_{i,\Theta,t}, V_{i,\Theta,t}\right), \\ \text{where } V_{i,\Theta,t} &\equiv \frac{1}{\alpha_t + \mu_t^2\delta + \beta_{i,t}} \end{aligned}$$

What follows focuses on a symmetric equilibrium. By assumption, investors are identical in their preferences and have the same information Ω_{-t} in the research decision stage. Furthermore, all investors face the same continuous research cost technology. These assumptions ensure the existence of a symmetric equilibrium, where all investors choose the same optimal research cost; $\beta_{i,t} = \beta_t$ for all i. This implies also the same variance $V_{\Theta,t} \equiv (\alpha_t + \mu_t^2\delta + \beta_t)^{-1}$.

Aggregating the demand of all investors averages out the noise in the private signals, however it does not average out the noise in the public signal.

Lemma 7 *The prices in period t will be determined by coefficients z_t and $z_{s,t}$, so that*

$$P_t = \frac{(1 - z_t)y_t + z_t\Theta - z_{s,t}s_t}{R_t}, \tag{4.6}$$

where

$$z_t = z_{t+1}(\mu_t^2\delta + \beta_t)V_{\Theta,t} + (1 - z_{t+1})\frac{\mu_t^2\delta}{(\alpha_t + \mu_t^2\delta)}, \tag{4.7}$$

$$z_{s,t} = (1 - z_t)\frac{\mu_t\delta}{\alpha_t} + \frac{\tau}{R_{t+1}}\left(z_{t+1}^2 V_{\Theta,t} + z_{s,t+1}^2\frac{1}{\delta}\right). \tag{4.8}$$

$$R_t \equiv \prod_{k=t}^{T+1} r_k$$

Proof. See Appendix C.1. ■

Using (4.3), (4.4), (4.5) and $\tilde{P}_t \equiv \Theta - s_t/\mu_t$ an alternative way to represent the equilibrium asset price is in term of the initial public signal y and historical noise trading as

$$P_t = \frac{w_{y,t}y + (1 - w_{y,t})\Theta - z_{s,t}s_t - (1 - z_t)\frac{\mu_{t-1}\delta}{\alpha_t}s_{t-1} - \ldots - (1 - z_t)\frac{\mu_1\delta}{\alpha_t}s_1}{R_t}, \tag{4.9}$$

where the weight on initial public signal

$$w_{y,t} \equiv (1 - z_t)\frac{\alpha}{\alpha_t} \tag{4.10}$$

This is similar to Proposition 2 in Allen, Morris, and Shin (2006). Integrating out the noisy supply, asset prices will be a weighted average of the true value and the initial public signal public signal (y). It is clear that assets will be closer to the fundamentals if z_t is closer to 1 (equivalently $w_{y,t}$ is closer to zero).

In period $T + 1$ the risky asset will be liquidated at the fundamental value. Also, there will be no more opportunities to invest in the risk-free asset. So, the terminal conditions will be

$$z_{T+1} = 1,\ z_{s,T+1} = 0 \text{ and } r_{T+1} = 1. \tag{4.11}$$

Finally, comparing the pricing equation (4.6) with the linear pricing rule assumed initially, it is clear that

$$\mu_t = \frac{z_t}{z_{s,t}}. \tag{4.12}$$

The dynamic system (4.3)-(4.5) and (4.7)-(4.12) solves for the values of z_t and $z_{s,t}$, given the values of $\beta_1$to β_T.

The research decision

The optimal research cost decision is equivalent to choosing $\beta_{i,t}$ taking other investor's $\beta_{j,t}$, $j \neq i$ as given. Replacing the optimal demand into the utility function, simplifying and taking expectations implies that the optimal research decision solves

$$\max_{\beta_{i,t}} \frac{E[(E[P_{t+1}|\Omega_{i,t}] - r_t P_t)^2 | \Omega_{-t}]}{2\tau \operatorname{Var}(P_{t+1}|\Omega_{i,t})} - r_t \kappa(\beta_{i,t}). \tag{4.13}$$

By doing research, investors can gain a reduction in the variance of their estimated next period price. They can also gain a better view about risky return opportunities. Given that investors are not short-sales constrained, they can gain equally from either an increase or a decrease in the next period price.

From the pricing equation (4.6) the variance of the following period price is

$$\begin{aligned} \operatorname{Var}(P_{t+1}|\Omega_{i,t}) &= \frac{1}{R_{t+1}^2} V_{i,P,t}, \\ \text{where } V_{i,P,t} &\equiv z_{t+1}^2 V_{i,\Theta,t} + \frac{z_{s,t+1}^2}{\delta}. \end{aligned} \tag{4.14}$$

It is clear from the equation above that the conditional variance of expected price should be lower if investor i decides to research a lot ($\beta_{i,t}$ is high and $V_{i,\Theta,t}$ is low) and if the weight on the fundamental value (z_{t+1}) in the next period pricing equation is high.

Investors face additional sources of uncertainty about the expected return of the risky asset at the research decision stage, compared to the trading stage. They do not know the realization of their private signal, as well as the price signal they can observe in t. By doing research, they can influence the variance of the private signal. The only information they have about the fundamental value is the public signal y_t. Noting that $E[\nu_{i,t}|\Omega_{-t}] = E[\tilde{P}_t|\Omega_{-t}] = E[\Theta|\Omega_{-t}] = y_t$, it is easy to show that $E[E[P_{t+1}|\Omega_{i,t}] - r_t P_t|\Omega_{-t}] = 0$. Thus, the expected squared return from the risky asset equals its variance (see Appendix C.2 for derivation)

$$\begin{aligned}&\mathrm{Var}[E[P_{t+1}|\Omega_{i,t}] - r_t P_t|\Omega_{-t}] = \\ &\tfrac{1}{R_{t+1}^2}\left[(z_{i,t} - z_t)^2 \frac{1}{\alpha_t} + (\tilde{z}_{i,s,t} - z_{s,t})^2 \frac{1}{\delta} + z_{t+1}^2 \beta_{i,t} V_{i,\Theta,t}^2\right],\end{aligned} \tag{4.15}$$

where $z_{i,t}$ and $\tilde{z}_{i,s,t}$ are the weights an individual investor puts on true value and current noise trading respectively.[5]

The benefits of research for an individual investor come from the reduction of the conditional variance of prices ($\mathrm{Var}(P_{t+1}|\Omega_{i,t})$), the difference in the weight he puts on true value in his expectations compared to market price ($z_{i,t} - z_t$), difference in the weight on noise trading ($\tilde{z}_{i,s,t} - z_{s,t}$) and a better private signal. The latter is captured by $z_{t+1}^2 \beta_{i,t} V_{i,\Theta,t}^2$ and has a bigger impact when expected next period prices are closer to the true value of the fundamental (z_{t+1} is high).

It should be pointed out that in symmetric equilibrium the benefits of different weight on true value disappear ($z_{i,t} = z_t$), while different weight on noise trading does not ($\tilde{z}_{i,s,t} \neq z_{s,t}$). The latter is because noise trading shock affects individual investors' expectations only through price signals, while equilibrium prices are in addition directly affected by the noise traders' demand.

[5] $z_{i,t} \equiv (1 - z_{t+1})\frac{\mu_t^2 \delta}{\alpha_{t+1}} + z_{t+1}(\beta_{i,t} + \mu_t^2 \delta) V_{i,\theta,t}$ and $\tilde{z}_{i,s,t} \equiv \frac{\mu_t \delta}{\alpha_{t+1}} - z_{t+1} \frac{\mu_t \delta}{\alpha_{t+1}} \beta_{i,t} V_{i,\theta,t}$.

Proposition 8 *In a symmetric equilibrium, all investors equate the marginal cost of research with marginal benefit of it (MB_t) and $\{\beta_1, \ldots, \beta_T, \mu_1, \ldots, \mu_T\}$ solve*

$$\begin{aligned} r_t \frac{\partial \kappa(\beta_{i,t})}{\partial \beta_{i,t}}\bigg|_{\beta_t} &= MB_t, \qquad (4.16) \\ \text{where } MB_t &\equiv \frac{z_{t+1}^2 V_{\Theta,t}^2}{2\tau}\left(\frac{\tau^2}{\delta R_{t+1}^2} + z_{t+1}^2 \beta_t \frac{V_{\Theta,t}^2}{V_{P,t}^2} + \frac{1}{V_{P,t}}\right) \end{aligned}$$

simultaneously with (4.12),(4.4), (4.5), (4.7),(4.8) and (4.11)).[6]

Proof. See Appendix C.3. ∎

Equation (4.16) shows that the marginal benefit of research is zero if $V_{\Theta,t} \to 0$. Therefore, investors have no incentives to invest in private research if either public signal is perfect ($\alpha \to \infty$ or $\alpha_t \to \infty$) or variance of noise trading is zero ($\delta \to \infty$) making price signals fully informative. The latter is consistent with Grossman and Stiglitz (1976), that shows that in markets with no noise trading no investor has incentive to acquire costly information and the asset prices cannot reveal information about the fundamental.

4.3 Results

4.3.1 One trading period example

As in Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006) a multi-period pricing equation with higher order beliefs can only be solved numerically. Therefore, to establish some basic relationships, consider at first a one trading period example.

For $T = 1$ the solution of the model simplifies as $z_2 = 1$, $z_{s,2} = 0$. The value

[6] The term $V_{P,t} \equiv z_{t+1}^2 V_{\theta,t} + \frac{z_{s,t+1}^2}{\delta}$.

of μ_1 can be found from (4.7), (4.8) and (4.12) to be $\mu_1 = \beta_1/\tau$; the variance of the private signals is higher, prices will be more informative, for a given level of noise in supply. Prices will also be more informative if risk aversion (τ) is lower, since rational investors will invest more in risky assets. Finally, because the asset will be liquidated in period 2, $V_{P,1} = V_{\Theta,1} = \left(\alpha + \beta_1 + \frac{\beta_1^2}{\tau^2}\delta\right)^{-1}$ and there is no historical prices implying $\alpha_2 = \alpha_1 + \mu_1^2\delta$, with $\alpha_1 = \alpha$.

For the analysis of the impact of availability of price history, let us drop time indexes and denote the variables in a one trading period model with "$*$". Simplifying (4.16) the marginal benefit of research MB^* can be now expressed as

$$MB^* = \frac{\tau}{2\delta}V_{\Theta}^{*2} + \frac{\beta^*}{2\tau}V_{\Theta}^{*2} + \frac{1}{2\tau}V_{\Theta}^{*}. \tag{4.17}$$

Appendix C.4 shows that marginal benefit of research is decreasing in β^* ($\partial MB^*/\partial\beta^* < 0$)[7].

The same appendix also reports the sensitivity of optimal precision to parameters of the model. It is straightforward from (4.17) that the lower the variance of the fundamental (V_{Θ}^*) investors perceive, the lower is their marginal benefit from obtaining better private signals. Therefore, exogenous parameters, such as existence of public signal that is freely available and of higher quality, reduces incentives to acquire better private information. The same applies to the effect of lower variance in noise trading that increases the quality of price signals.

The dependence of the research decision on risk aversion is ambiguous. There are three forces affecting it in different directions. First, a higher risk aversion makes the investors to care more about the possibility of reduction of variance that can be achieved by investing in better private signals. Second, more risk

[7]This together with constant or increasing marginal cost of research guarantees the uniqueness of the equilibrium.

averse investors are less willing to invest in risky assets in the first place, and therefore have less incentives to research instead of investing in the risk- free asset. Finally, higher risk aversion makes the demand of other rational investors in the risky assets lower, making prices less informative, which again increases the incentives to research. If the level of risk aversion is very high, the optimal research cost is increasing in risk aversion. In low and intermediate values of risk aversion, optimal research cost could be increasing or decreasing in risk aversion. (See Appendix C.4. and Figure C.1 in the same appendix)

A higher risk-free return reduces the optimal research cost by increasing the opportunity cost of research as compared to investing in the risk-free asset. This implies that the perceived variance of risky asset increases and the demand for risky asset falls more than the direct reallocation effect suggests.

The weight on the true value in the pricing equation, z_1, simplifies to

$$z^* = \frac{\frac{\beta^{*2}}{\tau^2}\delta + \beta^*}{\alpha + \frac{\beta^{*2}}{\tau^2}\delta + \beta^*}, \tag{4.18}$$

which is clearly increasing in β^*. This means that prices are expected to be closer to the common public signal, if less research is chosen. Hence, if investors choose to research less, a "good" risky asset is likely to be under- priced and a "bad" asset overpriced. An asset being "good" ("bad") in this context is an asset with true value above (below) y.

Finally, it is worth emphasizing that if there is no public signal ($\alpha \to 0$), investors incentives to acquire private information still depend on the other parameters of the model (e.g. variance of noise trading, risk aversion), however this would only affect the variance of asset prices, but not the expected dynamics due mis-pricing, as $z^* = 0$.

4.3.2 Multi-period model

Development of research costs over time.

In a multi-period setting, investors can choose their research effort in every period. Different generations of investors differ in the information that is available to them and the number of trading periods left until the fundamental is realized. For example, if there are T trading periods, investors that trade in period T base their research decisions of information set $\{y, P_1, .., P_{T-1}\}$ compared to investors that trade in period 1 and observe only $\{y\}$. From (4.13), it is also clear that their objectives differ. Considering the same example, their returns from investing in the risky asset depend on $\Theta - r_T P_T$ and $P_2 - r_1 P_1$ respectively. As a result, different generations of investors have different incentives to invest in private information.

Lemma 9 *Incentives to invest in private signals increase monotonically over time, i.e.*

$$\beta_t > \beta_{t-1} \text{ for any } t.$$

Proof. See Appendix C.5. ∎

This result is driven by two forces that reduce earlier periods' investors marginal benefit from research. First, the weight on the true value (z_t) in the pricing equation (4.6) increases over time. As fundamental research can only reduce the uncertainty regarding the true value of the asset, the marginal benefit of research is lower in earlier periods. Second, earlier periods' investors face higher uncertainty due to noise trading, which they cannot reduce by conducting fundamental research. These two forces reduce earlier periods investors' marginal benefit from research.

There is also a third force that has an opposite impact in terms of generating incentives to invest in private information; the availability of price history. As

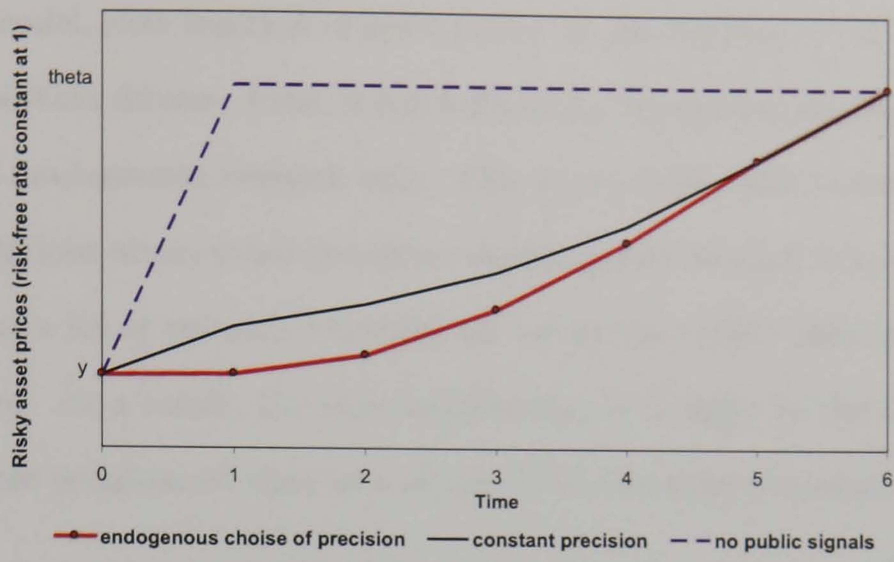

Figure 4.1: Adjustment to changes in the fundamental

long as at least some research is done in earlier periods, the variance of the public signals (evolving according to (4.4) and (4.5)) decreases over time. As in the one-period model, the lower is the variance of freely available signals, the lower is investors' willingness to research. Therefore, this force lowers investors' incentives to research over time. Nevertheless, this force is not strong enough to offset the lower marginal benefit of research in earlier trading periods.

Figure 4.1 illustrates Proposition 9 and compares risky expected asset prices (integrating out the supply shocks) in three settings:

- The model in this chapter with endogenous precision of private signal;
- Allen, Morris, and Shin (2006) setting where precision of private signals are constant over time and set equal to the endogenous precision in the last trading period;
- a model without public signal.[8]

[8]The expected price adjustment would be exactly the same with or without endogenous choice of private signals.

In this model, slow reaction of asset prices on the changes of the fundamental has two important drivers. First, is not rational to "go against the market" following internal fundamental research only. This would imply that investors overlook their expectations about other investors' expectations. Second, it is also not optimal to spend a lot of research resources on events that affect asset prices further in the future. As a result, the slow adjustment to changes in the fundamentals becomes more pronounced than in a setting with constant precision.[9]

Asset prices, price history and marginal research cost.

In a setting where the quality of private information is fixed longer price history necessarily implies less mis-pricing, because historical prices reveal additional information about the fundamentals and reduce the weight on the initial public signal. However, if precision of private signals is endogenous, longer price history offers additional freely available information that reduces incentives to pay information costs.

This section analyzes these effects and compares the prices of two assets in the last trading period. One of the assets is a one-period asset (as analyzed in Section 4.3.1 and denoted with "$*$") and the other is a T-period asset. Taking expectations with respect the supply shock, the extent of mis-pricing of these two assets will be the same if the weight on initial public signal y is the same for both assets, i.e. from (4.9)

$$w_{y,T} = w_y^*$$

[9] An alternative and equivalent way to interpret Lemma 9 would be that adjustment is faster because investors have incentives to acquire better private signals over time or that adjustment is slower at first and faster when the liquidation date is closer. The interpretation chosen in this chapter puts emphasis on the factors that potentially increase mis-pricing. Notice also that in the absence of public signal, investors would not acquire perfect private signals, but their incentives would not be affected by the expected low weight on public signal in future asset prices. The precision that would be chosen in such setting is closer to the one in last trading period in current setup.

Using then (4.7) and (4.10), the same extent of mis-pricing would imply that the variance of the risky asset in the last trading period is the same for both assets

$$V_{\Theta,T} = V_{\Theta}^{*}$$

For this condition to be satisfied, it must hold that $\beta_T + \frac{\beta_T^2}{\tau^2}\delta + \mu_{T-1}^2\delta + ... + \mu_1^2\delta = \beta^* + \frac{\beta^{*2}}{\tau^2}\delta$. Given that the precision of information revealed in historical prices $\mu_{T-1}^2\delta + ... + \mu_1^2\delta > 0$, the marginal benefit of research and optimal research effort must be lower for the asset with longer price history (i.e. $\beta_T < \beta^*$).

Proposition 10 *If research cost function takes the form $\kappa(\beta_{i,t}) = K_1\beta_{i,t}^2 + K_2\beta_{i,t}$, there exist a relationship between parameters $\bar{K}_1$ and $\bar{K}_2$*

$$\bar{K}_2 = 2\frac{\tau^2}{\delta}\bar{K}_1 + \sqrt{\frac{\bar{K}_1}{\tau r_T}}, \tag{4.19}$$

such that the extent of expected asset mis-pricing on one-period asset and multi-period asset during the last trading period is exactly the same ($V_{\Theta,T} = V_{\Theta}^{*}$ and $w_{y,T} = w_y^*$).

$$\begin{aligned} \text{If } K_1 &> \bar{K}_1 \text{ and } K_2 \leq \bar{K}_2, \text{ then } w_{y,T} < w_y^*. \\ \text{If } K_1 &\leq \bar{K}_1 \text{ and } K_2 > \bar{K}_2, \text{ then } w_{y,T} > w_y^*. \end{aligned}$$

Proof. See Appendix C.6. ∎

Figure 4.2 illustrates Proposition 10. The intuition behind the proof of Proposition 10 is the following. For a given variance of the fundamental, marginal benefit increases with the precision of private information. This arises from the fact that if investors have more precise private signal, they expect to gain more from it in terms of predicting the returns from trading. If marginal research cost in is

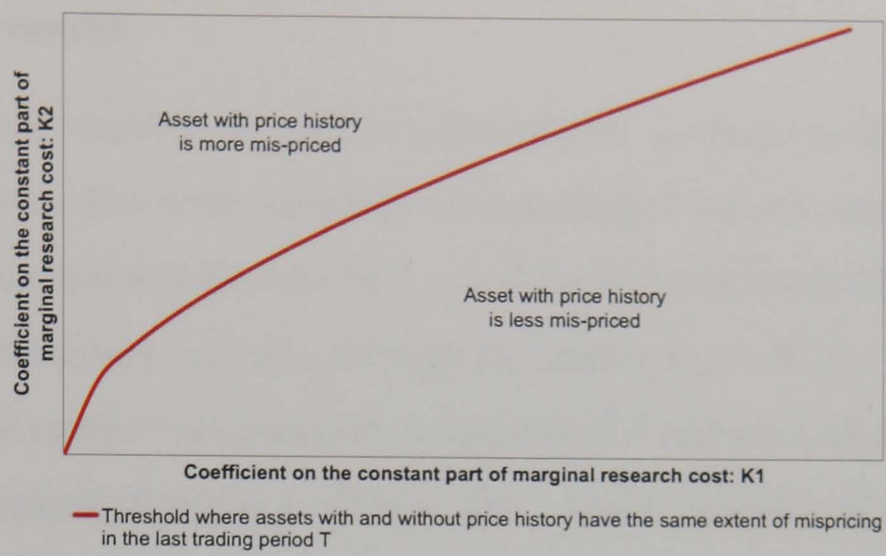

Figure 4.2: Marginal research cost and impact of price history

constant, investment in assets with different price history must give exactly the same marginal benefit. At the same time, the marginal benefit from obtaining a better private signal is always higher for assets about which less information is freely available and therefore $\beta^* > \beta_T$. Hence, it must hold that $V_\Theta^* < V_{\Theta,T}$ and there is less mis-pricing if no price history is available.

When marginal research cost depends also on the precision of private signals, $\beta^* > \beta_T$ also implies that marginal cost of research is higher for an asset with not price history. When there is no fixed component in the marginal cost of research ($K_2 = 0$), increasing the precision up to the point where $V_\Theta^* = V_{\Theta,T}$ is always too expensive and assets with longer price history are less mis-priced.

Finally, while intuitively there is no reason why K_2 cannot be positive, it should be pointed out that $K_2 > 0$ can lead to a corner solution. In the case of one trading period model, if $K_2 \geq (\tau/2\delta\alpha^2 + 1/2\tau\alpha)$ then $\beta^* = 0$.

Numerical results

Given that the sensitivity of optimal private signals' precision to changes in the exogenous parameters of the model are too complicated to derive analytically, this section presents numerical results for $T = 5$. In order to the possibility of a corner solutions, assume that research cost takes the form $\kappa(\beta_{i,t}) = K_1\beta_{i,t}^2$.

Numerical results[10] are presented in Appendix C.7 and are consistent with the sensitivity analysis discussed in Section 4.3.1. Higher precision of public signal and noise trading reduce the optimal precision in all periods (Figure C.2). The pattern indicates also that in the case of less variable noise trading, the incentives to acquire better private signals could be lower in later trading periods, where a lot of information is already revealed by the history of asset prices. At the same time, lower variance of public signal could be more influential in reducing incentives to research in earlier periods, when public signal is a good predictor of future asset prices. Figure C.3 shows the impact of this for the dynamics of asset prices, when integrating of the supply shocks.[11]

The negative effect of higher research costs is straightforward (Figure C.4) and the effect to expected prices is similar to the impact of lower noise trading. The impact of risk aversion is not reported, as it is ambiguous for the reasons already discussed in Section 4.3.1. Nevertheless, it should be pointed out that in dynamic setting higher risk aversion has an additional positive effect on the incentives to invest in private signals, since it reduces the quality of historical price signals.

Permanent increase of interest rates in period 3 reduces the incentives to research in all periods. The incentives to research are lower also in trading periods

[10] Numerical results not reported in this thesis suggest that the direction of the effects described does not depend on specific assumptions about the parameters. The choice of parameters is arbitrary and aims to show the direction of effects rather than their magnitude.

[11] In the case of higher precision of public signal the asset price dynamics shown does not take into account the fact that more precise signal also implies higher probability that y is close to θ. This issue is explicitly addressed in Section 4.4.2.

1 and 2, because investors can forecast future interest rates. Thy know that this implies lower weight on the fundamentals in the future asset prices[12]. In the case of a temporary increase of the interest rate, incentives to obtain better private signals increase after the shock is over. While not very obvious from the graph, the optimal precision is slightly higher immediately after interest risk-free rates have decreased back (period 4), because lower demand for risky asset in earlier periods implies worse historical price signals. This creates incentives to research more in order to compensate this (Figure C.5).

4.4 Extensions

4.4.1 Long-lived agents

This section relaxes the assumption that the agents are short-lived and investigates the research cost decision and resulting asset prices in a two trading period setting in the spirit of Brown and Jennings (1989). For simplicity, assume also that the return on alternative asset is $r_1 = r_2 = 1$, and research cost is given by $\kappa(\beta_{i,t}) = K_1\beta_{i,t}^2$. Long-lived agents have preferences over their last period consumption that is given by

$$c_{i,3} = h_{i,2}(\Theta - P_2) + h_{i,1}(P_2 - P_1) + w - \kappa(\beta_{i,1}) - \kappa(\beta_{i,2}).$$

This section assumes that investors can choose to receive i.i.d. private signals in both trading periods and select the precision of these signals ($\beta_{i,1}$ and $\beta_{i,2}$) before the first trading period. Therefore, the information set in the research decision stage is $\Omega_{-1} = \{y\}$, in the first trading period $\Omega_{i,1} = \{y, \nu_{i,1}, P_1\}$ and in

[12]This is due to the perfectly forecastable risk-free rates. If the change would be truly unexpected, incentives to research would only decrease on the period that the shock happens.

the second trading period $\Omega_{i,2} = \{y, \nu_{i,2}, \nu_{i,1}, P_1, P_2\}$. The rest of the assumptions and notation remains unchanged.

Following the method of Brown and Jennings (1989)[13] the demand in the second trading period is

$$h_{i,2} = \frac{E[\Theta|\Omega_{i,2}] - P_2}{\tau V_{i,\Theta,2}}$$

and in the first period

$$h_{i,1} = \frac{E[P_2|\Omega_{i,1}] - P_1}{\tau} G_i + \frac{E\left[E[\Theta|\Omega_{i,2}] - P_2|\Omega_{i,1}\right]}{\tau} Q_i, \tag{4.20}$$

where G_i and Q_i depend on $\mathrm{Var}(P_2|\Omega_{i,1})$, $\mathrm{Var}(E[\Theta|\Omega_{i,2}]|\Omega_{i,1})$ and $\mathrm{Cov}(P_2, E[\Theta|\Omega_{i,2}]|\Omega_{i,1})$, which are defined in Appendix C.8.

Equilibrium asset prices are given by

$$\begin{aligned} P_1 &= (1-\acute{z}_1)y + \acute{z}_1\Theta - \acute{z}_{s,1}s_1 \\ P_2 &= (1-\acute{z}_2)y_2 + \acute{z}_2\Theta - \acute{z}_{s,2}s_2, \end{aligned} \tag{4.21}$$

where

$$\begin{aligned} \acute{z}_1 &= \frac{G-Q}{G}\left[(1-\acute{z}_2)\frac{\mu_1^2\delta}{\alpha+\mu_1^2\delta} + \acute{z}_2(\beta_1+\mu_1^2\delta)V_{\Theta,1}\right] + \frac{Q}{G}(\beta_1+\mu_1^2\delta)V_{\Theta,1} \\ \acute{z}_{s,1} &= (1-\acute{z}_1)\frac{\mu_1\delta}{\alpha} + \frac{\tau}{G}, \\ \acute{z}_2 &= (\mu_2^2\delta + \beta_1 + \beta_2)V_{\Theta,2}, \\ \acute{z}_{s,2} &= (\tau + \mu_2^2\delta)V_{\Theta,2}. \end{aligned} \tag{4.22}$$

Given that in equilibrium $\beta_{i,1} = \beta_1$ and $\beta_{i,2} = \beta_2$, then it follows also that $G_i = G$

[13] Also in Brunnermeier (2001), pp. 107-110

and $Q_i = Q$ for every i. Similar to the case of short-lived agents'; μ_1 and μ_2 solve

$$\mu_1 = \frac{\acute{z}_1}{\acute{z}_{s,1}},\ \mu_2 = \frac{\acute{z}_2}{\acute{z}_{s,2}}.$$

See Appendix C.8 for the derivations.

In the case that $\beta_2 = 0$, this setting can be compared with the two-period model with myopic investors and identical and constant quality private signal. In this case, the weight on the fundamental in the second trading period is the same in both settings ($z_2 = \acute{z}_2$) and closer to the higher in the first trading period[14]. Long-lived investors foresee their expected demand in the second trading period. If the risk from the second period prices is high (Q is relatively high compared to G), they demand higher quantity already in the first period. This implies that there is less scope for higher-order expectations; investors are less influenced by the value of public signal in predicting next period's prices. As a result, the weight on the fundamental in the equilibrium asset price equation is higher.

Research cost decision follows the same logic as in the short-lived agents case and gives the following proposition.

Proposition 11 *Provided that long-lived investors commit to pay research costs before the first trading period, they will have incentives to choose to research only in the first trading period.*

$$\beta_2 = 0 \text{ and } \beta_1 > 0$$

Proof. See Appendix C.9. ■

The intuition behind Proposition 11 is that before first trading period higher private signal in the second period brings no additional benefits, because investors

[14]With $\beta_2 = 0$, from (4.7) and (4.22) $\acute{z}_1 = \frac{G-Q}{G} z_1 + \frac{Q}{G}(\beta_1 + \mu_1^2\delta)V_{\theta,1} = z_1 + \frac{Q}{G}\left((\beta_1 + \mu_1^2\delta)V_{\theta,1} - z_1\right) = z_1 + \frac{Q}{G}(1 - z_2)\left(\frac{\beta_1\alpha}{(\alpha+\mu_1^2\delta+\beta_1)(\alpha+\mu_1^2\delta)}\right) \geq z_1$

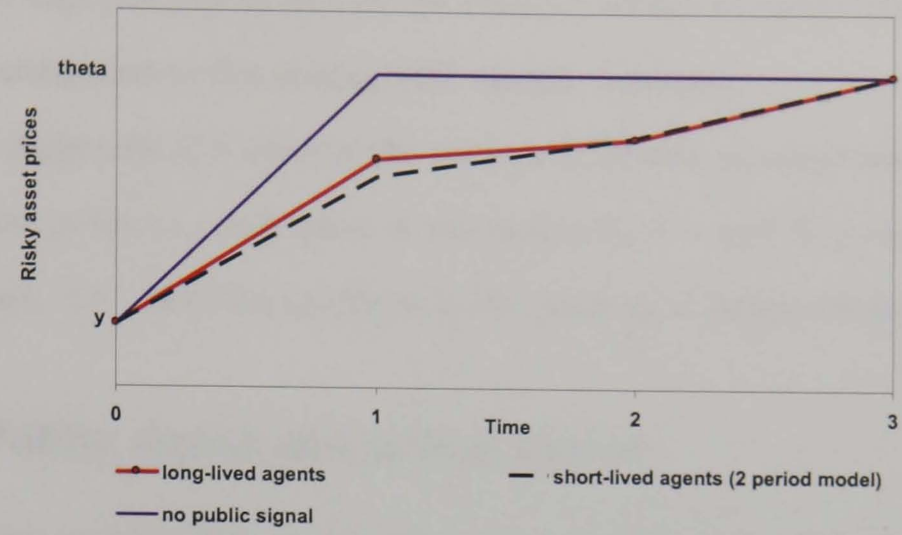

Figure 4.3: Adjustment of asset prices on a change in the fundamentals; long-lived vs. short-lived investors

will remember their private signal in the second trading period. Also, given that second period private signal is realized late, before trading period 1, the conditional variance of the returns between different trading periods and liquidation date is independent of β_2.

Figure 4.3 compares the reaction of asset prices to the change in the fundamental in the model with long-lived investors to the one with short-lived investors. Both models assume two trading periods and the same research costs. These two settings are comparable, because investors in both settings have one private signal in every trading period.

The reason why expected prices change between period 1 and 2 in long-lived agent's setting is that period 1 price signal increases the information available in period 2, if variance in noise trading is not too high. This creates incentives for long-lived investors not to demand the full amount of risky asset they want already in the first trading period.

Asset prices are tilted towards the public signal also when investors are long-lived, because it reveals information about the fundamental. However, the effect of higher order expectations as well as low research efforts in earlier trading periods diminishes compared to the setting with myopic investors.

Finally, Appendix C.9 reports the numerical results on sensitivity of optimal precision and prices to parameters of the model (α, δ, τ and K_1), when investors are long-lived. The direction of effects is the same as in other settings analyzed.

4.4.2 Public signal and policy maker

Policy makers make public statements that could have an effect on the public signal. When the precision of public signal increases, it also imply a average smaller error in the public signal. It is clear from (4.6) and (4.21) that mis-pricing would disappear in the public signal was perfect (i.e. $y = \Theta$). Asset prices in such case would equal to the present discounted value of the fundamental and noise trading, even though no investors has incentives to acquire private information.

While, better quality public signal (higher α) reduces the error in the public signal, it also increases the weight on public signal in investors' expectations and reduces the incentives to acquire private information as shown in the previous sections. As long as the policy maker is unable to issue perfect signals, the effect of improving the quality of public information can have an ambiguous effect. This section gives a simple example of this in one trading period setting.[15]

First, the extent of mis-pricing (χ_{mp}) can be specified as the absolute value of the difference between the asset price (integrating out the supply shocks) and true value of asset, that is

$$\left|\chi_{mp}\right| = \left|(1 - z^{*})y + z^{*}\Theta - \Theta\right|.$$

The public signal can be written as $y = \Theta + \varepsilon_y$, where error in the public signal is

[15]The same mechanism is valid in other settings.

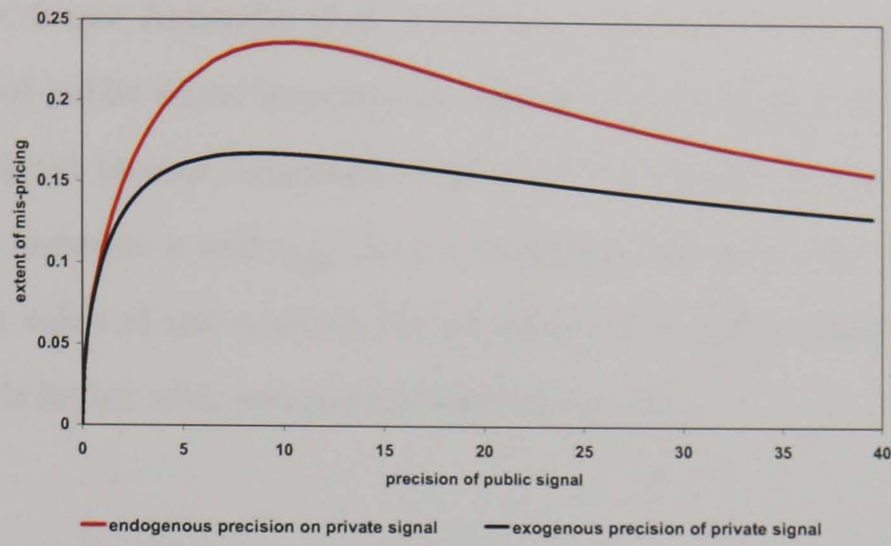

Figure 4.4: Precision of public signal and the extent of mis-pricing

$\varepsilon_y \sim \mathcal{N}(0, 1/\alpha)$.

Consider the case that the error in the public signal is one standard deviation and above its mean; $\varepsilon_y = 1/\sqrt{\alpha}$. Using (4.18), V_Θ^*, the extent of mis-pricing becomes

$$\chi_{mp} = V_\Theta^* \sqrt{\alpha}.$$

If the precision of private information does not change (i.e. β^* is exogenous)

$$\frac{\partial \chi_{mp}}{\partial \alpha} = V_\Theta^* \frac{1}{2\sqrt{\alpha}} - V_\Theta^{*2} \sqrt{\alpha} = V_\Theta^{*2} \frac{\left(\beta^* + \frac{\beta^{*2}}{\tau^2}\delta - \alpha\right)}{2\sqrt{\alpha}}.$$

The extent of mis-pricing is ambiguous due higher weight investors put on the public signal when forming their expectations. Therefore, unless α is high, it can increase the extent of mis-pricing.

If β^* is endogenous,

$$\frac{\partial \chi_{mp}}{\partial \alpha} = V_\Theta^* \frac{1}{2\sqrt{\alpha}} - V_\Theta^{*2} \sqrt{\alpha} + V_\Theta^{*2} \sqrt{\alpha} \left(1 + 2\frac{\beta^*}{\tau^2}\delta\right) \frac{\partial \beta^*}{\partial \alpha}.$$

As $\frac{\partial \beta^*}{\partial \alpha} < 0$ (see Appendix C.4), it becomes even more likely that improving the quality of public signal increases the extent of mis-pricing, because errors in the public signal become magnified. Figure 4.4[16] illustrates this by plotting the relationship between α and χ_{mp} for one standard deviation error in the public signal. The value of the α where higher precision of public signal implies less mis-pricing is higher with endogenous information costs.

4.5 Concluding remarks

This chapter presented a model where investors choose the precision of their private information and make their trading decisions according to both public and private information and . This leads to following the conclusions.

When the quality of public signal is higher, the incentives to invest in private information are lower. Public signal affects asset prices more if its precision is higher, because of two reasons. First, all investors assign higher weight on the common public signal when forming their expectations about the fundamental or next period asset prices. Second, as incentives to acquire lower variance private signals are lower they, assign lower weight on their private signal. This implies that policies that increase the quality on public information can have an ambiguous effect on potential mis-pricing. On the one hand, higher quality public signals are on average closer to the fundamental and therefore reduce potential mis-pricing. On the other hand, as private information becomes endogenously more noisy, the error in public signal becomes magnified.

The model is also consistent with the findings of Grossman and Stiglitz (1976), that lower variance in noise trading reduces incentives to acquire better private

[16]The precision of public signal is chosen equal to the optimal precision in the case when there is no public signal; $\alpha = 0$. Other parameters are the same as in Appendix C.7.

signals. This also increases the potential mis-pricing of assets as long as public signal is available.

As already shown by Allen, Morris, and Shin (2006) in a similar setting with short-lived investors, the existence of a public signal makes asset prices biased towards it. Furthermore, this bias is larger if investors trade assets that have more time left to maturity. Allowing investors to choose the precision of their private signal magnifies this bias in earlier trading periods. Given that investors care more about the price that they will get for their assets in the market when exiting, rather than the underlying fundamentals, their incentive to research is lower the earlier they trade compared to the maturity date of their assets.

Furthermore, for a fixed time until maturity, a longer price history reduces incentives to invest in private signal, as investors have an incentive to free-ride on the research effort made by the earlier trading periods' investors. This implies that longer price history does not necessarily imply less mis-pricing and depends on the curvature of research cost function. If marginal research cost is constant, assets with longer price history are even more mis-priced. This implies that mis-pricing is potentially important also in more mature assets markets. Clearly, this effect would be smaller or even reversed when in more mature asset are associated with lower research costs.

The model also predicts that higher current and future risk-free rates reduce the incentives to obtain information about risky assets. Therefore, the demand for the risky asset falls not only because of the substitution effect, but also because investors endogenously face higher variance of risky asset. If the increase of risk-free rate is temporary, then later periods' investors would be more willing to research, in order to compensate for the worse signals coming from historical asset prices.

Finally, the predictions of the model are sensitive to the assumption that short-lived investors. If investors are long-lived, they have incentives to obtain higher quality private information as early as possible. Also, with long-lived investors, the reaction to changes in the fundamentals is faster than with short-lived investors. It would probably be more realistic to assume that both myopic and long-lived investors participate in the market. In this case the adjustment dynamics to shocks in fundamentals are likely to be in between the these two cases. The existence of myopic investors can be justified by agents' that foresee their future liquidity shocks. Furthermore, when the agent is an asset management firm, the short horizon can be rationalized, because the performance of these firms and their staff tends to be judged by the short term performance of their portfolio.

The assumption in this chapter is that the short-lived investors do not inherit historical private signals and the long-lived investors decide research effort in all periods before the first trading period is rather restrictive. Both cases overlook the dynamics that would arise if investors have heterogeneous information when deciding about research costs. It is likely that investors who inherit private signal that is further from the signal emerging from other channels, have higher incentives to research more. This is likely to make the degree of mis-pricing smaller, but would not eliminate the main dynamics.

The assumptions about the information structure could be extended as well. The chapter treats private and public signals as distinct and uncorrelated. More realistic assumption might be that some private agents can also have an impact of public information (e.g. financial institutions making their research available). Analyzing the incentives of these agents to issue truthful signals and the market for information in a dynamic setting.

Chapter 5

Conclusion

In chapters 2-4, this thesis addressed the impact of imperfect information in equity markets and endogenous choice of the quality of private information on technology adoption and thereby aggregate economic performance. There are two important channels through which equity market affects innovation and technology adoption. First, equity is an important source of funding for technology sector and affects its ability to invest. Second, well developed equity markets facilitate talented entrepreneurs to sell their firms and affect their willingness to invest. At the same time, equity prices are subject to uncertainty and potential mis-pricing arising from the commonly available public information and investors' lack of willingness to invest in private information.

Chapter 2 analyzed a setting where R&D is financed by issuing equity in an environment where equity market participants are imperfectly informed. This chapter showed that imperfect information implies under-reaction to the changes of fundamentals compared to the environment where equity market participants are perfectly informed. This would be the case, because an improvement in the fundamentals is not necessarily reflected in the public signal and equity market is pessimistic. However, when equity market is optimistic (i.e. public signal about the fundamental is higher than the fundamental), there are potential gains in long-term consumption, because higher equity prices imply more investments in R&D and therefore higher productivity and output, despite the fact that in the short run investors get losses in the equity market.

Chapter 3 addressed the importance of equity markets in facilitating the exit of initial owners. It assumed that the initial owner of a technology adopting firm has superior information about the fundamental value of his firm and sells his firm in equity market. If an average potential buyer of the firm is less informed, two forces emerge that affect initial owners' willingness to invest in technology. First, there is a negative "fear of unstable markets" force that arises from the uncertainty about the future market value of the technology adopting firm. Second, there is a positive "adoption to signal" force that arises from the fact that fast technology adoption decision becomes a positive signal that increases the market value of the technology adopting firm. Other imperfections in the equity market, such as lack of liquidity, further reduce incentives to invest in new technologies.

Finally, Chapter 4 analyzed the endogenous choice of precision of private information in an environment where public information is available. It showed that the slow reaction to changes in fundamentals becomes magnified as long as investors have short-horizons. This is due to investors' reluctance to spend resources in order to analyze events that are far in the future and expected to have only a limited effect on next periods' asset prices. Also, existence of longer price history does not guarantee that the potential mis-pricing of assets is smaller. This arises from investors incentives to free-ride on research done by investors that traded on earlier periods.

The policy implications that are drawn from the above listed finding are the following:

- The existence of liquid equity markets is beneficial for aggregate economy. Furthermore, policy actions that are likely to reduce liquidity (such as forbidding foreign portfolio investments) can be harmful, even if the equity prices involve high uncertainty.

- The presence of imperfect information in equity markets has an impact on aggregate economic performance. If these imperfections become an obstacle for growth (e.g. information about the firms in a country is too hard to obtain), policy makers should aim to reduce information imperfections, by improving the relevant institutions.

- If policy makers would have the ability to eliminate information imperfections in the asset market, they may lack incentives to do so. Market optimism is beneficial for growth because it allows the technology sector to raise more equity funds and make larger investments. Some degree of uncertainty is beneficial, because it allows firms to signal their high profitability by investing in new technologies.As higher level of technology implies higher future output and wages, policy makers' incentives would depend on how much they value these gains compared to the losses that the current investors receive in the equity market. Furthermore, the findings in Chapter 2 indicate that policy makers could have an incentive to issue only positive signals, which can make such signals not credible.

- Policies that improve the quality of publicly available information have an ambiguous effect on the extent of mis-pricing in the asset market. While, better quality public information implies a smaller error in public signal, this error has a larger effect on asset prices. This is driven by the fact that improvements in public signal make investors' expectations more biased towards the public signal and reduce their incentives to acquire better quality private information.

As several policy dilemmas arise from these findings, optimal and feasible policy to address the presence of imperfect is information in equity markets is one interesting direction of future research.

The analytical frameworks developed in the main chapters bears some important simplifying assumptions and their mechanisms could be analyzed further in more general settings. The following discussion aims to identify the importance of these assumptions and potential links among the different mechanisms developed in this thesis. Through this analysis, it points out additional directions of future research.

Both Chapter 2 and Chapter 3 are stylized in their assumptions regarding the source of funding available for the innovative firms. There is a lack of endogenous choice of funding among the different sources available: equity, venture capital and own funds. While this assumption is unlikely to be important for the validity of main predictions of the models, it could affect the magnitude of different forces and introduce new mechanisms. It would also have important implications for policy decisions regarding, for example, the regulation of venture capital.

Chapters 2 and 3 also lack the choice of timing the terms of exit and/or issue of new equity to fund investment in technology. It is likely that owners of the firms would have incentives to delay (rush into) doing so when equity markets are pessimistic (optimistic). Given that in this thesis there is no heterogeneity among firms that invest in technology, the choice of timing is unlikely to add further insights. However, when there is heterogeneity in entrepreneurial talent or quality of innovation project, the choice of timing could generate important additional effects. For example, it would give further insights about the characteristics and quality of firms that decide to go for an initial public offering or raise equity funds in "hot markets", such as during the "dot-coms" boom.

Furthermore, introducing a degree of optimism in the setting of Chapter 3 would affect the magnitude of the present mechanisms. First, if entrepreneurs expect the equity market to be optimistic in the future due to a public signal that is expected to fade out slowly, their incentives to invest in technology would be higher, and not only ability as in Chapter 2. This would introduce new forces in the setting of Chapter 3. Second, if entrepreneurs would be optimistic (over-confident) about their firm, the magnitude of "adoption to signal" force would be smaller, because technology adoption decision would become a less informative signal.

The uncertainty about the profits in technology sector in Chapters 2 and 3 arises purely from the demand side. A more realistic assumption would be to incorporate uncertainty regarding the success of a technology project. This is likely to reduce the magnitude of the gains from market optimism addressed in Chapter 2. However, the gains would not be eliminated. The main mechanisms in Chapter 3 would not be altered as well, since fast technology adoption would remain a positive signal about a particular firm and uncertainty about the market value of the firm would remain a discouraging force.

In addition to information imperfections, equity markets can also be imperfect because they lack liquidity. Lower liquidity necessarily reduces equilibrium equity prices, because there is too little demand for equity. The negative impact arising from the lack of liquidity when firms are sold in equity markets is addressed in Section 3.4 (see also Bencivenga, Smith, and Starr 1995). In Chapter 2, it is straightforward to see that allowing for the equity markets to lack of liquidity would also have a negative effect on purely equity financed R&D investment. All this relies on the assumption that information imperfections and liquidity can be addressed separately. The potential interactions between equity market liquidity and the ease of obtaining information gives rise to further scope for research. One can conjecture that in a multiple country/asset setting, investors would be reluctant to invest in countries where information about listed firms is hard to obtain and this can make the equity market endogenously less liquid. This can result in less liquid equity markets and therefore smaller benefits arising from the presence of imperfect information in equity markets.

Chapter 4 showed that allowing for endogenous choice of the precision of private signals and different investment horizon is important for the degree of asset mis-pricing. While modelling equity markets in Chapter 2 and 4 is not directly comparable, these finding suggest the following. Given that short-lived investors lack incentives to invest in information about events that are far in the future, the magnitude of mechanisms analyzed in Chapter 2 are likely to be higher with endogenous private information acquisition. However, when allowing for at least some investors to have long investment horizons in Chapter 2, the benefits of market optimism would be smaller, because public signal shocks would become less persistent.

To summarize, this thesis argued that well functioning and liquid equity markets are likely to be important for growth. It showed that endogenous precision of private signals is likely to magnify asset mis-pricing. It highlighted the importance of funding and exit as channels through which uncertainty and mis-pricing in equity markets can have an important, potentially positive impact on aggregate economy. The robustness of these findings to alternative set-ups, and the empirical quantification of these mechanisms is left for future research.

Appendix A

Appendix for Chapter 2

A.1 Market clearing in three-period model

In the three-period economy, the final goods' production takes places only in periods 1 and 2. Therefore R&D production occurs only in period 1. Market clearing conditions for different periods are as follows.

$$
\begin{aligned}
\tilde{Y}_1 + Y_1 &= C_1 + K_1 + I_1 + M_1 \\
\tilde{Y}_2 + Y_2 &= C_2 + K_2 + M_2 \\
\tilde{Y}_3 &= C_3
\end{aligned}
$$

The market clearing in the first period is identical to Section 2.2.3. Intermediate firms will be liquidated in period 2. Therefore, the investors who invested in it will receive just the dividend and $P_2 = 0$. This means that $C_2 = \pi_2 A_2 + RM_1$. Using (2.1), (2.2), (2.4), (2.8), (2.10), the market clearing condition can be simplified to $\pi_2 = \Gamma\phi_2$, which holds true by (2.3).

There is no other productive activity is taking place in period 3 apart from the risk-free technology. Given that there is no equity market in period 2, consumers born in period 2 can only invest in the risk-free asset. Hence, $C_3 = \tilde{Y}_3 = RM_2$.

A.2 Equilibrium equity price for three-period model

The starting point is the pricing equation (2.12). To find the average conditional expectations and variance of ϕ_2, assume that prices follow a linear rule

$$P_1 = \mu_1(\mu_2\overline{\phi}_2 + \mu_3\phi_2 - s_1) \tag{A.1}$$

Given that prices reveal information, we can rewrite this and define the price signal as

$$\tilde{P}_1 \equiv \frac{P_1/\mu_1 - \mu_2\overline{\phi}_2}{\mu_3} = \phi_2 - \frac{s_1}{\mu_3}$$

To summarize, the signals that rational consumer i has are:

- Public signal $\overline{\phi}_2 \sim \mathcal{N}(\phi_2, 1/\beta_\phi)$
- Private signal $\nu(i)|\phi_2 \sim \mathcal{N}(\phi_2, 1/\beta_\nu)$
- Price signal $\tilde{P}_1|\phi_2 \sim \mathcal{N}(\phi_2, 1/\mu_3^2\beta_s)$

Using Bayesian updating, the distribution of ϕ_2 based on all the information investor i has is

$$\phi_2|\Omega_1(i) \sim \mathcal{N}\left(\frac{\beta_\phi\overline{\phi}_2 + \mu_3^2\beta_s B_1 + \beta_\nu\nu(i)}{\beta_\phi + \mu_3^2\beta_s + \beta_\nu}, \frac{1}{\beta_\phi + \mu_3^2\beta_s + \beta_\nu}\right).$$

This implies that that the variance is the same from the point of view of every investor. The average conditional expectation is

$$\overline{E}[\phi_2|\Omega_1] = \frac{\beta_\phi \overline{\phi}_2 + \mu_3^2 \beta_s (\phi_2 - s_1/\mu_3) + \beta_\nu \phi_2}{\beta_\phi + \mu_3^2 \beta_s + \beta_\nu}$$

Replacing this into (2.12), one can solve for coefficients μ_1, μ_2 and μ_3, by equating coefficients in (2.12) and (A.1), as

$$\begin{aligned}
\mu_1 &= \frac{\tau\Gamma^2 + \frac{\beta_s \beta_\nu}{\tau}}{R(\beta_\phi + (\frac{\beta_\nu}{\tau\Gamma})^2 \beta_s + \beta_\nu)} \\
\mu_2 &= \frac{\beta_\phi}{\tau\Gamma + \frac{\beta_s \beta_\nu}{\tau\Gamma}} \\
\mu_3 &= \frac{\beta_\nu}{\tau\Gamma}
\end{aligned}$$

This gives the equilibrium equity price equation as in (2.13).

A.3 Comparative statics for three-period model

Price Rule

From the price rule ((2.13) and (2.14)), we can find the dependence of price on the underlying parameters:

$$\begin{aligned}
\frac{\partial P_1}{\partial \phi_2} &= \frac{\Gamma}{R}(1 - z_1) > 0 \\
\frac{\partial P_1}{\partial \overline{\phi}_2} &= \frac{\Gamma}{R} z_1 > 0 \\
\frac{\partial P_1}{\partial R} &= -\frac{1}{R} P_1 < 0 \\
\frac{\partial P_1}{\partial s_1} &= -\frac{\Gamma}{R} z_{s,1} < 0
\end{aligned}$$

$$\frac{\partial P_1}{\partial z_1} = \frac{\Gamma}{R}\left(\bar{\phi}_2 - \phi_2\right) > 0 \text{ iff } \bar{\phi}_2 - \phi_2 > 0$$
$$\frac{\partial P_1}{\partial z_{s,1}} = -\frac{\Gamma}{R} s_1 < 0 \text{ iff } s_1 > 0$$

Provided that $z_1 = z_1(\beta_\nu, \beta_\phi, \beta_s, \tau; \Gamma)$ and $z_{s,1} = z_{s,1}(\beta_\nu, \beta_\phi, \beta_s, \tau; \Gamma)$, then by applying the chain rule one may recover the effect of the precision parameters and risk aversion. For example: $\frac{\partial P_1}{\partial \tau} = \frac{\partial P_1}{\partial z_1}\frac{\partial z_1}{\partial \tau} + \frac{\partial P_1}{\partial z_{1,s}}\frac{\partial z_{1,s}}{\partial \tau}$. When the noise trading shock is set equal to its unconditional mean ($s_1 = 0$), the second effect becomes zero. Given that the for weight on public signal (z_1) it is true that:

$$\frac{\partial z_1}{\partial \beta_\phi} = \frac{\beta_\nu + \left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_s}{\left[\beta_\phi + \left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_s + \beta_\nu\right]^2} > 0$$

$$\frac{\partial z_1}{\partial \beta_s} = -\frac{\left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_\phi}{\left[\beta_\phi + \left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_s + \beta_\nu\right]^2} < 0$$

$$\frac{\partial z_1}{\partial \beta_\nu} = -\frac{\beta_\phi\left[\frac{2\beta_\nu\beta_s}{(\tau\Gamma)^2} + 1\right]}{\left[\beta_\phi + \left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_s + \beta_\nu\right]^2} < 0$$

$$\frac{\partial z_1}{\partial \tau} = \frac{2}{\tau^3}\frac{\beta_s\beta_\phi\left(\frac{\beta_\nu}{\tau\Gamma}\right)^2}{\left[\beta_\phi + \left(\frac{\beta_\nu}{\tau\Gamma}\right)^2 \beta_s + \beta_\nu\right]^2} > 0$$

It then follows that: $\frac{\partial P_1}{\partial \tau} > 0$ iff $\bar{\phi}_2 - \phi_2 > 0$. In all cases that that consider the effect of risk aversion below, the noise trading shock is kept at zero; $s_1 = 0$.

R&D and output growth

R&D growth is descried by (2.15). It depends positively on the equity price:

$\frac{\partial g_A}{\partial P_1} = \frac{\rho}{1-\rho}\frac{g_A}{P_1} > 0$. Therefore:

$$\begin{aligned}
\frac{\partial g_A}{\partial \phi_2} &= \frac{\partial g_A}{\partial P_1}\frac{\partial P_1}{\partial \phi_2} > 0 \\
\frac{\partial g_A}{\partial \bar{\phi}_2} &= \frac{\partial g_A}{\partial P_1}\frac{\partial P_1}{\partial \bar{\phi}_2} > 0 \\
\frac{\partial g_A}{\partial R} &= \frac{\partial g_A}{\partial P_1}\frac{\partial P_1}{\partial R} < 0 \\
\frac{\partial g_A}{\partial \tau} &= \frac{\partial g_A}{\partial P_1}\frac{\partial P_1}{\partial \tau} > 0 \text{ iff } \bar{\phi}_2 - \phi_2 > 0
\end{aligned}$$

Moreover, from (2.16) and given that the growth in R&D has a positive effect on output growth: $\frac{\partial g_Y}{\partial g_A} = \frac{\phi_2}{\phi_1} > 0$, it follows:

$$\begin{aligned}
\frac{\partial g_Y}{\partial \phi_2} &= \frac{\partial g_Y}{\partial \phi_2} + \frac{\partial g_Y}{\partial g_A}\frac{\partial g_A}{\partial \phi_2} > 0 \\
\frac{\partial g_Y}{\partial \bar{\phi}_2} &= \frac{\partial g_Y}{\partial g_A}\frac{\partial g_A}{\partial \bar{\phi}_2} > 0 \\
\frac{\partial g_Y}{\partial R} &= \frac{\partial g_Y}{\partial g_A}\frac{\partial g_A}{\partial R} < 0 \\
\frac{\partial g_Y}{\partial \tau} &= \frac{\partial g_Y}{\partial g_A}\frac{\partial g_A}{\partial \tau} > 0 \text{ iff } \bar{\phi}_2 - \phi_2 > 0
\end{aligned}$$

Note that when there is actual productivity growth $\phi_2 > \phi_1$the effect of different parameters on g_Y increases in magnitude.

Benchmark Model

Regarding the pricing equation in the Benchmark economy:

$$\begin{aligned}
\frac{\partial P_1^B}{\partial \phi_2} &= \frac{\Gamma}{R} > 0 \\
\frac{\partial P_1^B}{\partial \bar{\phi}_2} &= \frac{\partial P_1^B}{\partial \tau} = 0 \\
\frac{\partial P_1^B}{\partial R} &= -\frac{P_1^B}{R} < 0.
\end{aligned}$$

Moreover, $\frac{\partial g_A^B}{\partial P_1^B} = \frac{\rho}{1-\rho}\frac{g_A^B}{P_1^B} > 0$. Comparing the implied R&D growth under the

Benchmark and the Model economy:

$$\frac{g_A^B}{g_A} = \left(\frac{P_1^B}{P_1}\right)^{\frac{\rho}{1-\rho}} = \frac{(\phi_2 - s_1)^{\frac{\rho}{1-\rho}}}{\left[z_1(\bar{\phi}_2 - \phi_2) + \phi_2 - z_{1,s}s_1\right]^{\frac{\rho}{1-\rho}}}$$

For $s_1 = 0$ and given the congestion parameter ρ, it follows that $\frac{g_A^B}{g_A} \geq 1$ iff $\phi_2 \geq \bar{\phi}_2$. i.e., when the public signal is pessimistic. This goes through also when comparing g_Y^B with g_Y.

Consumption Levels

The consumption in the last period: $C_3 = Rw_2L = R\frac{A_1}{\alpha}\pi_2(1+g_A)$, depends positively on the second-period profits, $\frac{\partial C_3}{\partial \pi_2} = \frac{C_3}{\pi_2} > 0$, and R&D growth, $\frac{\partial C_3}{\partial g_A} = R\frac{A_1}{\alpha}\pi_2 > 0$. Also, there is a positive direct effect of the alternative asset return, $\frac{\partial C_3}{\partial R} = \frac{C_3}{R} > 0$. Therefore:

$$\begin{aligned}
\frac{\partial C_3}{\partial \phi_2} &= \frac{\partial C_3}{\partial \pi_2}\frac{\partial \pi_2}{\partial \phi_2} + \frac{\partial C_3}{\partial g_A}\frac{\partial g_A}{\partial \phi_2} > 0 \\
\frac{\partial C_3}{\partial \bar{\phi}_2} &= \frac{\partial C_3}{\partial g_A}\frac{\partial g_A}{\partial \bar{\phi}_2} > 0 \\
\frac{\partial C_3}{\partial R} &= \frac{\partial C_3}{\partial R} + \frac{\partial C_3}{\partial g_A}\frac{\partial g_A}{\partial R} \text{ (ambiguous)} \\
\frac{\partial C_3}{\partial \tau} &= \frac{\partial C_3}{\partial g_A}\frac{\partial g_A}{\partial \tau} > 0 \text{ iff } \bar{\phi}_2 - \phi_2 > 0
\end{aligned}$$

Comparing with the Benchmark model's last-period consumption:

$$C_3^B - C_3 = R\frac{A_1}{\alpha}\pi_2\left(g_A^B - g_A\right) > 0 \text{ iff } \bar{\phi}_2 - \phi_2 \leq 0$$

Regarding the second-period consumption $C_2 = RA_1\pi_1\left[\left(\frac{\pi_2 - RP_1}{R\pi_1}\right)(1+g_A) + \frac{1}{\alpha}\right]$, it depends positively on profits, $\frac{\partial C_2}{\partial \pi_1} = \frac{RA_1}{\alpha} > 0$ and $\frac{\partial C_2}{\partial \pi_2} = A_1(1+g_A) > 0$, negatively on equity price, $\frac{\partial C_2}{\partial P_1} = -A_1R(1+g_A) < 0$, and the effect of the R&D growth depends on whether there are realized excess gains or losses in the equity market, $\frac{\partial C_2}{\partial g_A} = A_1 (\pi_2 - RP_1) > 0$ iff $\pi_2 - RP_1 > 0$. The direction of the effect from risk-free asset return is ambiguous, $\frac{\partial C_2}{\partial R} = \frac{A_1 \pi_1}{\alpha} - A_1 P_1 (1 + g_A)$. In effect, all of the following effects become ambiguous:

$$\begin{aligned}
\frac{\partial C_2}{\partial \phi_2} &= \frac{\partial C_2}{\partial \pi_2}\frac{\partial \pi_2}{\partial \phi_2} + \frac{\partial C_2}{\partial P_1}\frac{\partial P_1}{\partial \phi_2} + \frac{\partial C_2}{\partial g_A}\frac{\partial g_A}{\partial \phi_2} \\
\frac{\partial C_2}{\partial \bar{\phi}_2} &= \frac{\partial C_2}{\partial P_1}\frac{\partial P_1}{\partial \bar{\phi}_2} + \frac{\partial C_2}{\partial g_A}\frac{\partial g_A}{\partial \bar{\phi}_2} \\
\frac{\partial C_2}{\partial R} &= \frac{\partial C_2}{\partial R} + \frac{\partial C_2}{\partial P_1}\frac{\partial P_1}{\partial R} + \frac{\partial C_2}{\partial g_A}\frac{\partial g_A}{\partial R} \\
\frac{\partial C_2}{\partial \tau} &= \frac{\partial C_2}{\partial P_1}\frac{\partial P_1}{\partial \tau} + \frac{\partial C_2}{\partial g_A}\frac{\partial g_A}{\partial \tau}
\end{aligned}$$

Comparing with the Benchmark model second-period consumption:

$$C_2^B - C_2 = A_1[(\pi_2 - RP_1^B)\left(1 + g_A^B\right) - (\pi_2 - RP_1)(1 + g_A)]$$

It is noteworthy that when noise trading is at its mean ($s_1 = 0$), the second period consumers never get excess gains or losses in the asset market ($\pi_2 = RP_1^B$). Therefore, it this case $C_2^B - C_2 = -A_1[(\pi_2 - RP_1)(1 + g_A)]$. This implies that consumers in the model economy get losses (gains), if the market is optimistic (pessimistic), i.e. when $\bar{\phi}_2 - \phi_2 > 0$ (< 0) and $C_2^B > C_2$ ($C_2^B < C_2$).

Finally, consumption in the first period is $C_1 = (\pi_1 + P_1) A_1 + RM_0$. It depends positively on first-period profits, $\frac{\partial C_1}{\partial \pi_1} = A_1 > 0$, and equity price, $\frac{\partial C_1}{\partial P_1} = A_1 > 0$. Therefore:

$$\begin{aligned}
\frac{\partial C_1}{\partial \phi_2} &= \frac{\partial C_1}{\partial P_1}\frac{\partial P_1}{\partial \phi_2} > 0 \\
\frac{\partial C_1}{\partial \bar{\phi}_2} &= \frac{\partial C_1}{\partial P_1}\frac{\partial P_1}{\partial \bar{\phi}_2} > 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial C_1}{\partial R} &= \frac{\partial C_1}{\partial P_1}\frac{\partial P_1}{\partial R} < 0 \\
\frac{\partial C_1}{\partial \tau} &= \frac{\partial C_1}{\partial P_1}\frac{\partial P_1}{\partial \tau} > 0 \text{ iff } \bar{\phi}_2 - \phi_2 > 0
\end{aligned}
$$

Comparing the Model with the Benchmark model for the first-period consumption:

$$
C_1^B - C_1 = A_1\left(P_1^B - P_1\right) \geq 0 \text{ iff } \bar{\phi}_2 - \phi_2 \leq 0
$$

A.4 Welfare

Welfare in the three-period model is defined as $W \equiv C_1 + \frac{C_2}{R} + \frac{C_3}{R^2}$. W^{intl} is the welfare corresponding to the initial scenario of both productivity and noise trading shock always equal to their unconditional means ($\phi_1 = \phi_2 = \overline{\phi}_2$, $s_1 = 0$). Note that in this case there are no excess asset market gains or losses ($\pi_2^{intl} = \pi_1 = RP_1^{intl}$) and that the initial scenario coincides for the Benchmark and Model economy:

$$
\begin{aligned}
& W - W^{intl} = \\
= \; & \left(C_1 - C_1^{intl}\right) + \left(\tfrac{C_2 - C_2^{intl}}{R}\right) + \left(\tfrac{C_3 - C_3^{intl}}{R^2}\right) = \\
= \; & \left[A_1\left(P_1 - P_1^{intl}\right)\right] + \left[\tfrac{1}{R}A_1\left(\pi_2 - RP_1\right)\left(1 + g_A\right)\right] + \\
& + \left[\tfrac{1}{R}\tfrac{A_1}{\alpha}\pi_2\left(g_A - g_A^{intl}\right) + \tfrac{1}{R}\tfrac{A_1}{\alpha}\left(\pi_2 - \pi_1\right)\left(1 + g_A^{intl}\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
& W^B - W^{intl} \\
= & \left(C_1^B - C_1^{intl}\right) + \left(\tfrac{C_2^B - C_2^{intl}}{R}\right) + \left(\tfrac{C_3^B - C_3^{intl}}{R^2}\right) = \\
= & \left[A_1\left(P_1^B - P_1^{intl}\right)\right] + \left[\tfrac{1}{R}A_1\left(\pi_2 - RP_1^B\right)\left(1 + g_A^B\right)\right] \\
& + \left[\tfrac{1}{R}\tfrac{A_1}{\alpha}\pi_2\left(g_A^B - g_A^{intl}\right) + \tfrac{1}{R}\tfrac{A_1}{\alpha}\left(\pi_2 - \pi_1\right)\left(1 + g_A^{intl}\right)\right]
\end{aligned}
$$

Welfare implications for the shocks considered in Section 2.3.3 are the following.

Increase of $\bar{\phi}_2$.

The Benchmark economy model is not affected by a pure public signal shock. Therefore, $W^B - W^{intl} = 0$, while:

$$
\begin{aligned}
& W - W^{intl} = W - W^B = \\
= & A_1\tfrac{\Gamma}{R}\left(z_1\left(\bar{\phi}_2 - \phi_2\right)\right) - A_1 z_1 \tfrac{\Gamma}{R}\left(\bar{\phi}_2 - \phi_2\right)\left(1 + g_A\right) + \tfrac{1}{R}\tfrac{A_1}{\alpha}\pi_2\left(g_A - g_A^{intl}\right) = \\
= & -g_A A_1 \frac{\Gamma}{R}\left(z_1\left(\bar{\phi}_2 - \phi_2\right)\right) + \frac{\Gamma}{R}\frac{A_1}{\alpha}\phi_2\left(g_A - g_A^{intl}\right) = \\
= & A_1 \frac{\Gamma}{\alpha R}\phi_2\left[g_A\left(1 - \alpha z_1 \frac{\bar{\phi}_2}{\phi_2} + \alpha z_1\right) - g_A^{intl}\right].
\end{aligned}
$$

There are welfare gains from a false yet positive public signal, if the technology growth caused by it is fast enough, so that the following condition is met:

$$
\frac{g_A^{intl}}{g_A} = \left(\frac{1}{1 + z_1\frac{\bar{\phi}_2}{\phi_2} - z_1}\right)^{\frac{\rho}{1-\rho}} < 1 - \alpha z_1 \frac{\bar{\phi}_2}{\phi_2} + \alpha z_1.
$$

Since the congestion parameter $\rho \in (0,1)$ and given that $z_1 \in (0,1)$ and $\frac{\bar{\phi}_2}{\phi_2} > 1$, it follows that the LHS of the inequality is decreasing in ρ. Hence, this condition is more likely to hold, the higher is ρ. The nonlinearity of the inequality does not allow for a simple condition on the parameter values. This can be demonstrated in the simple case of $\rho = \frac{1}{2}$ (numerical exercises in Section 2.4.2 use $\rho = 0.9$). In this case the inequality simplifies to:

$$z_1 \left(\frac{\bar{\phi}_2}{\phi_2} - 1\right) \left[1 - \alpha \left(1 - z_1 + z_1 \frac{\bar{\phi}_2}{\phi_2}\right)\right] > 0 \text{ or } \frac{\bar{\phi}_2}{\phi_2} < 1 - \frac{1}{z_1} + \frac{1}{\alpha z_1}$$

Therefore, we get an "upper bound" on the percentage deviation of the public signal from the actual productivity, for the Model economy to deliver gains compared to the initial scenario/Benchmark economy: $\frac{\bar{\phi}_2 - \phi_2}{\phi_2} < \frac{1-\alpha}{\alpha} \frac{1}{z_1}$. For a value of α around 0.3 and the extreme case of $z_1 = 1$, there would be welfare gains, even if the public signal exceeds the true productivity by a factor of 3.3 (that however seems unlikely given the underlying distribution). Lower weight on the public signal increases that upper bound. Therefore, as long as the public signal is not too high and the congestion parameter is not too low, the optimistic market sentiment improves the welfare.

Increase of ϕ_2.

The consumption will be higher compared to the initial scenario for all generations in both the Model and the Benchmark, delivering higher welfare ($W > W^{intl}$, $W^B > W^{intl}$). The difference in welfare gains between the Model and the Benchmark simplifies to:

$$W - W^B = A_1 \frac{\Gamma}{\alpha R} \phi_2 [g_A (1 - \alpha z_1 \frac{\bar{\phi}_2}{\phi_2} + \alpha z_1) - g_A^B]$$

and there will be welfare gains in the Model economy compared to the Benchmark one, if:

$$\frac{g_A^B}{g_A} = \left(\frac{1}{1 + z_1 \frac{\bar{\phi}_2}{\phi_2} - z_1}\right)^{\frac{\rho}{1-\rho}} < 1 - \alpha z_1 \frac{\bar{\phi}_2}{\phi_2} + \alpha z_1$$

Given that now $\bar{\phi}_2 < \phi_2$, the LHS fraction is greater than 1 and thus it is now increasing in ρ. Therefore, the Benchmark model will result in the higher welfare, as long as the congestion is not too low and the public signal is not very extremely pessimistic.

Increase of both, $\bar{\phi}_2 = \phi_2$

The welfare will be higher and the same in both economies compared to the initial scenario ($W = W^B > W^{intl}$).

Decrease of s_1

Both economies can be compared to the initial scenario as follows:

$$\begin{aligned} W - W^{intl} &= A_1 \frac{\Gamma}{R} \left(z_{1,s} s_1 g_A + \frac{1}{\alpha} \phi_2 \left(g_A - g_A^{intl} \right) \right), \\ W^B - W^{intl} &= A_1 \frac{\Gamma}{R} \left(s_1 g_A^B + \frac{1}{\alpha} \phi_2 \left(g_A^B - g_A^{intl} \right) \right). \end{aligned}$$

There are welfare gains in the Model economy and Benchmark economy if

$$\begin{aligned} \frac{g_A^{intl}}{g_A} &= \left(\frac{\phi_2}{\phi_2 - z_{1,s} s_1} \right)^{\frac{\rho}{1-\rho}} < \frac{\alpha}{\phi_2} z_{1,s} s_1 + 1 \text{ and} \\ \frac{g_A^{intl}}{g_A^B} &= \left(\frac{\phi_2}{\phi_2 - s_1} \right)^{\frac{\rho}{1-\rho}} < \frac{\alpha}{\phi_2} s_1 + 1 \end{aligned}$$

respectively. What follows, assumes that $\beta_\phi + \beta_\nu > \tau\Gamma$, in which case $z_{1,s} < 1$. In either case of the above inequalities, the fraction in the LHS expression is less than one, and thus decreasing in ρ. Therefore, if ρ is high and $|s_1|$ is not too high compared to ϕ_2, there will be welfare gains in the model economy and benchmark economy compared to the initial scenario. Comparison between the Model and Benchmark economy, gives the following condition:

$$\frac{g_A^B}{g_A} = \left(\frac{\phi_2 - s_1}{\phi_2 - z_{1,s} s_1} \right)^{\frac{\rho}{1-\rho}} < \frac{\frac{\alpha}{\phi_2} z_{1,s} s_1 + 1}{\frac{\alpha}{\phi_2} s_1 + 1}$$

and the Benchmark model will give higher utility for higher values of ρ provided the noise trading shock is not too large.

A.5 Equilibrium equity prices in infinite horizon model.

Similarly to the three-period model, the solution method starts from (2.17) and assumes that the price equation will be in the form of (2.18).

Conditional on the public signals *only*, the "prior" distribution of Φ_t ($2T \times 1$) is

$$
\begin{aligned}
\Phi_t &\sim \mathcal{N}(\widehat{\Phi}_{0,t}, \Sigma_\Phi), \text{ where} \\
\widehat{\Phi}_{0,t} &= (\widehat{\phi}_{t-T+1}, .., \widehat{\phi}_t, 0, .., 0)' \\
\Sigma_\Phi &= \begin{pmatrix} \frac{1}{\beta_{\widehat{\phi}}} & 0 & & & \cdots & 0 \\ 0 & \ddots & & & & \vdots \\ & & \frac{1}{\beta_{\widehat{\phi}}} & & & \\ & & & \frac{1}{\Gamma^2\beta_s} & & \\ \vdots & & & & \ddots & \vdots \\ 0 & & & & \cdots & \frac{1}{\Gamma^2\beta_s} \end{pmatrix}
\end{aligned}
$$

For the price signals, the adjusted prices (that can be interpreted as price signals) are defined as

$$
\begin{aligned}
\tilde{P}_{t-k} &= P_{t-k} - \Gamma\overline{z}\overline{\phi} - \Gamma\widehat{Z}'\widehat{\Phi}_t - \Gamma\sum_{l=1}^{k}\left(z_l\phi_{t-k+l} - z_{s,l}\frac{s_{t-T-k+l}}{\Gamma}\right), \\
\text{for } k &= [0, T-1]
\end{aligned}
$$

The vector of observables for investor i trading in period t is defined as $\Lambda_t(i) = (\widetilde{P}_t, ..., \widetilde{P}_{t-T+1}, v_{t-T+1}(i), ..., v_t(i))'$, $\Lambda_t(i)$ is $(2T \times 1)$. Then

$$\Lambda_t(i) = \Gamma Z \Phi_t + \varepsilon_t,$$

where $\quad \varepsilon_t = (0, ..., 0, \varepsilon_t(i), ..., \varepsilon_{t-T+1}(i))'$

$$Z = \begin{pmatrix} z_1 & \cdots & z_{T-1} & z_T & -z_{s,1} & \cdots & -z_{s,T-1} & -z_{s,T} \\ z_2 & \cdots & z_T & 0 & -z_{s,2} & \cdots & -z_{s,T} & 0 \\ \vdots & \ddots & 0 & 0 & \vdots & \ddots & 0 & 0 \\ z_T & \cdots & 0 & 0 & -z_{s,T} & \cdots & 0 & 0 \\ 1/\Gamma & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & 0 & 0 & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 1/\Gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\Gamma & 0 & 0 & 0 & 0 \end{pmatrix}$$

This implies:

$$\begin{aligned} \Lambda_t(i)|\Phi_t \quad &\sim \quad \mathcal{N}(\Gamma Z \Phi_t, \Sigma_\Lambda), \text{ where} \\ \Sigma_\Lambda \quad &= \quad \begin{pmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \frac{1}{\beta_v} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & \frac{1}{\beta_v} \end{pmatrix} \end{aligned}$$

Using the projection theorem, one can find the updated distribution of the unobservables, conditional on the observables for each of the consumer i

$$\begin{aligned} E[\Phi_t|\Lambda_t(i)] &= \widehat{\Phi}_{0,t} + \Gamma\Sigma_\Phi Z'(\Gamma^2 Z\Sigma_\Phi Z' + \Sigma_\Lambda)^{-1}(\Lambda_t(i) - \Gamma Z\widehat{\Phi}_{0,t}) \\ \text{Var}[\Phi_t|\Lambda_t(i)] &= \Sigma_\Phi - \Gamma^2\Sigma_\Phi Z'(\Gamma^2 Z\Sigma_\Phi Z' + \Sigma_\Lambda)^{-1} Z\Sigma_\Phi \equiv V_\Phi \end{aligned}$$

Notation V_Φ indicates that the conditional variance of unobservables is constant over time and the same from the point of view of every consumer. Aggregating over all rational investors, we get the average expectations of the unobservables

$$\overline{E}[\Phi_t|\Lambda_t] = (I - QZ)\widehat{\Phi}_{0,t} + QZ\Phi_t,$$

where $Q \equiv \Gamma^2\Sigma_\Phi Z'(\Gamma^2 Z\Sigma_\Phi Z' + \Sigma_\Lambda)^{-1}$ and I is the $(2T \times 2T)$ identity matrix. Using this, derive

$$\overline{E}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t] = \Gamma((z_T + \widehat{z}_T + \overline{z})\overline{\phi} + (Z_2'(I - QZ) + \widehat{Z}_2')\widehat{\Phi}_{0,t} + Z_2'QZ\Phi_t),$$

where $Z_2' = (1, z_1, .., z_{T-1}, 0, -z_{s,1}, ..., -z_{s,T-1})$ and $\widehat{Z}_2' = (0, \widehat{z}_1, .., \widehat{z}_{T-1}, 0, 0, ..., 0)$. Also

$$\text{Var}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t] = \Gamma^2\left[Z_2'V_\Phi Z_2 + z_T^2\frac{1}{\beta_\phi} + \widehat{z}_T^2\frac{\beta_{\hat{\phi}} - \beta_\phi}{\beta_{\hat{\phi}}\beta_\phi} + \frac{z_{s,T}^2}{\Gamma^2}\frac{1}{\beta_s}\right].$$

The variance is verified to be constant over time and depends on the coefficients and precision of shocks and $\overline{E}[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t]$ is linear in future productivities, historical noise trading and public signals, while s_t enters into price equation from (2.17) directly. Therefore, the prices take the form of the conjectured price equation and we can find Z_1 and $\widehat{Z}$ numerically by replacing the above results into (2.17) and equating coefficients with (2.18).

A.6 Benchmark price equation in infinite horizon model

For the infinite horizon Benchmark model, assume that investors get a perfect private signal about ϕ_{t+T} in t and are aware also of the private signals about ϕ_{t+1} to ϕ_{t+T-1}. This means that the public signal $(\overline{\phi})$ will be useful only from period $t+T+1$ onward, because there is no agents with better about this than the information than the public signal (i.e. the prior distribution $\phi_{t+T+k} \sim \mathcal{N}(\overline{\phi}, 1/\beta_{\phi})$, for $k > 1$). Thus, the information set available in t is $\Omega_t^B = \Omega_t^B(i) = \{\phi_{t+1}, ..., \phi_{t+T}\}$. The uncertainty in prices also comes from the noise trading in every period, $s_{t+k} \sim \mathcal{N}(0, 1/\beta_s)$.

Note that all rational consumers have the same expectations $(E[..|\Omega_t^B(i)] = \overline{E}[..|\Omega_t^B] = E[..|\Omega_t^B]$ and $\text{Var}\left[..|\Omega_t^B(i)\right] = \text{Var}\left[..|\Omega_t^B\right]$ and the law of iterated expectations holds in this case. Therefore,

$$E[P_{t+1} + \Gamma\phi_{t+1}|\Omega_t^B] = \Gamma\sum_{k=1}^{T}\frac{1}{R^{k-1}}\phi_{t+k} + \Gamma\frac{1}{R^{T-1}(R-1)}\overline{\phi},$$

which is the present discounted value of the expected future profits. The variance:

$$\text{Var}\left(P_{t+1} + \Gamma\phi_{t+1}|\Omega_t^B\right) = \text{Var}\left[\frac{1}{R^{T-1}}\Gamma\phi_{t+T+1} + s_{t+1}\tau\,\text{Var}\left(P_{t+2} + \Gamma\phi_{t+2}|\Omega_t^B\right)\right].$$

There are two points to notice here. First, prices in $t+1$ are a function of ϕ_{t+T+1}, which is not known in t. Therefore, we have to consider its variance as well (investors know that the price will move due to additional perfect signal being issued). Second, the quality of information is the same over time. Hence,

$\mathrm{Var}\left(P_{t+1}+\Gamma\phi_{t+1}|\Omega_t^B\right)=\mathrm{Var}\left(P_{t+2}+\Gamma\phi_{t+2}|\Omega_t^B\right)\equiv V^B$ and

$$V^B=\frac{1+\sqrt{1-4\frac{\Gamma^2\tau^2}{R^{2(T-1)}\beta_\phi\beta_s}}}{2\frac{\tau^2}{\beta_s}}.$$

The Benchmark pricing equation is :

$$P_t^B=\sum_{k=1}^{T}\frac{1}{R^k}\Gamma\phi_{t+k}+\frac{1}{R^T(R-1)}\Gamma\overline{\phi}-\frac{1}{R}\tau V^B s_t$$

Only the period t noise trading affects the equity price. The term $\frac{1}{R^T(R-1)}\Gamma\overline{\phi}$ reflects the lack of knowledge about the long term productivity.

A.7 Parameters used in numerical solution for infinite horizon model

parameters	T	α	ρ	η	L	R	λ	τ	β_ϕ	β_ν	$\beta_{\tilde{\phi}}$	β_s
values	6	0.3	0.9	1	13.46	1.33	0.14	2	2.5	2.5	2.5	2.5

In the initial scenario $s_{t+k}=0$, $\phi_{t+k}=1$, $\overline{\phi}=1$, $\widehat{\phi}_{t-T+k}=1$, $\widetilde{\phi}_{t-T+k}=1$ for all $k\in(-\infty,\infty)$.

The results are for $T=6$[1]. The choice for the capital share (α) is standard. The congestion parameter (ρ) is from Comin and Gertler (2004). The gross interest rate (R) corresponds to yearly interest rate of approximately 6.6 per cent and is chosen such that $M_t=0$ (for all t) for the initial scenario. The labour force (L) is chosen to normalize $\Gamma=1$. The productivity or R&D (λ) is chosen to give R&D growth ($g_{A,t}$) of 0.1 (corresponds to ≈ 2 per cent yearly growth rate). The coefficient of risk aversion (τ) is the same as in Bacchetta and van Wincoop (2006).

[1]The length of one time period can be viewed to be 5 years. Thus, given the setting, consumers receive public and private signals about the productivity around 30 years ahead

The precisions β_ϕ, β_ν, $\beta_{\tilde{\phi}}$, and β_s are chosen to be equal. This that around 30% of the weight in the investors' expectations is put on the public signal.

Appendix B

Appendix for Chapter 3

B.1 Labor, stock market and institutions.

Data for 1996-2004, medians

	Labor with sec. educ.*	Stock mrk. cap.	Turnover ratio	Rule of law	Regul. qual.
Asia	28.2	44.5	80.3	0.30	0.35
Latin America	33.3	24.5	15.3	-0.29	0.26
Transition (EU)	62.9	13.4	38.4	0.60	0.74
Other transition	56.6	10.4	8.9	-0.07	0.28
EU (excl. new)	45.0	66.8	72.8	1.74	1.39
United States	na	133.9	141.4	1.73	1.46

* no data available after 2001

Table B.1: Labor force, stock markets and institutions

Indicators:

Labor with sec. educ. - Percentage of labor force with at least secondary education out of total labor force. Source: World Bank Development Indicators

Stock mrk. cap. - Ratio of stock market capitalization to GDP. Source: Financial Sector Development Indicators. World Bank

Turnover ratio - Stock market turnover ratio equals to stocks traded divided by stock market capitalization. Source: Financial Sector Development Indicators.

World Bank

Rule of law. - Index is in scale -2.5 to 2.5 and measures the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, the police, and the courts, as well as the likelihood of crime and violence. Source: Kaufmann, Kraay, and Mastruzzi (2006)

Regul. qual. - Regulatory quality index. Index is in scale -2.5 to 2.5 and measures the ability of the government to formulate and implement sound policies and regulations that permit and promote private sector development. Source: Kaufmann, Kraay, and Mastruzzi (2006)

Country groups:

Asia - China, Hong Kong, India, Indonesia, Korea, Malaysia, Philippines, Singapore, Thailand

Latin America - Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela

Transition (EU) - transition countries that joined European Union in 2004, i.e. Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovak Republic, Slovenia

Other transition - other transition countries that have initial PPP adjusted GDP per capita above 3.0 thousand USD in 1991, i.e. Belarus, Bulgaria, Croatia, Georgia, Kazakhstan, FYR Macedonia, Romania, Russian Federation, Ukraine.

European Union (excl. new) - European Union members excluding "Transition (EU)" and Luxembourg, i.e. Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Portugal, Spain, Sweden, United Kingdom

B.2 Income per capita and R&D expenditure

Figures B.1 and B.2 present data on transition countries that are comparable in various characteristics. These countries are similar in high share of educated labour force, institutions and lack of securities markets at the time when Soviet Union dissolved and initial level of GDP. The figures exclude five transition countries that had substantially lower initial PPP adjusted GDP per capita (below 3.0 thousand USD) in 1991. The remaining countries have mean 6.6 thousand USD and standard deviation 2.0 thousand USD.

Figure B.1 shows the relationship between development of equity markets and PPP adjusted GDP per capita in US dollars (World Bank Development Indicators). Figure B.2 shows similar relationship with R&D expenditures per capita in PPP adjusted US dollars (World Bank Development Indicators). The measure of equity markets' development is "Securities market & non-bank financial institutions index" (European Bank for Reconstruction and Development). The index evaluates countries on a scale 1-4.5, where 1: little progress; 2: Formation of security exchanges, market-makers and brokers, some trading in government paper and/or securities; rudimentary legal and regulatory framework for the issuance and trading securities; 3: substantial issuance of securities by private enterprises, secure clearance and settlement procedures, and some protection of minority shareholders, emergence of non-bank financial institutions and associated regulatory environment; 4: securities laws and regulation approaching the IOSCO standards, substantial market liquidity and capitalization, well functioning non-bank financial institutions and effective regulation; 4.5 standards and performance norms of advanced industrial economies, full coverage or securities laws and regulations with the IOSCO standards, fully developed non-bank intermediation. In 1989 all transition countries had index "1". The value of index for the EU average and the United States is taken to be equal to 4.5 (the maximum index value). which is consistent with the definition.

Figure B.3 shows similar patterns for R&D expenditures in high and upper-middle income countries as classified by World Bank. The measure of equity markets development used is the "Equity Size Index" (Financial Sector Development Indicators, World Bank). The index is an average of scaled market capitalization to GDP, value traded to GDP and turnover ratio: value traded to market capitalization. Scaling is done according to the median and standard deviation of the variables such that most scaled values are in the interval [2.5, 7.5].

Figure B.4. shows that the patterns for more general investments and the "Equity Size Index" in high and upper-middle income countries is similar than for R&D expenditures, but weaker.

Figures show a concave and possibly non-monotonic relationship that would be consistent with the predictions of the model in Chapter 3. The patterns are similar if using other available measures of equity market development (e.g. number of IOSCO principles implemented, realized equity return volatility) or technology adoption (e.g. number of personal computers or internet users per 1000 people and GDP growth).

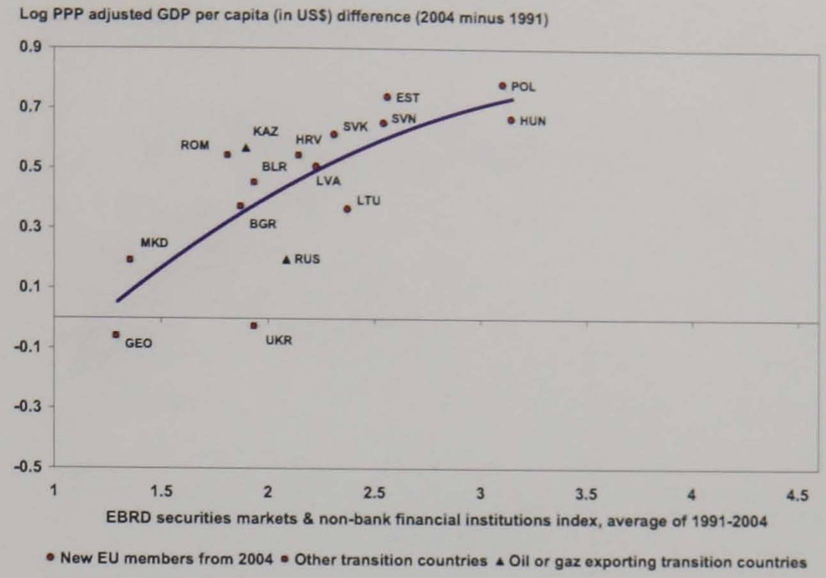

Figure B.1: Securities market and GDP growth in transition countries

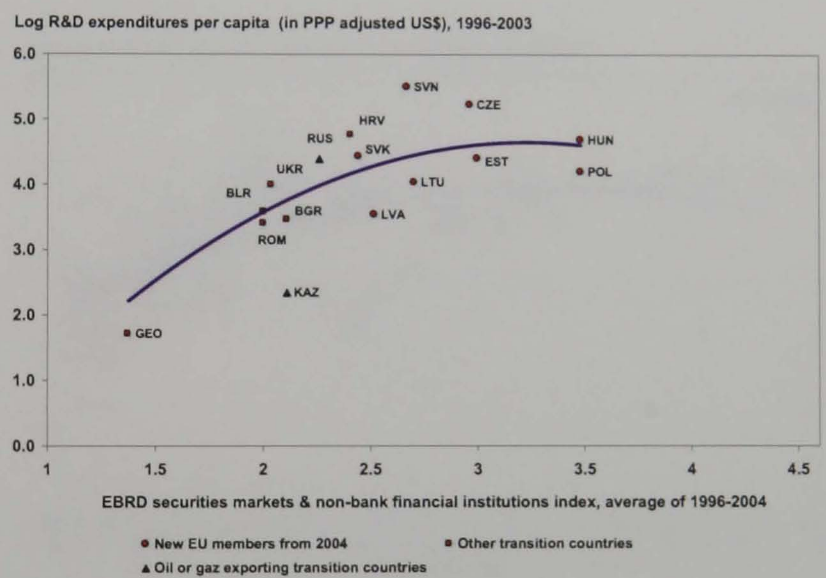

Figure B.2: Securities market and R&D expenditures in transition countries

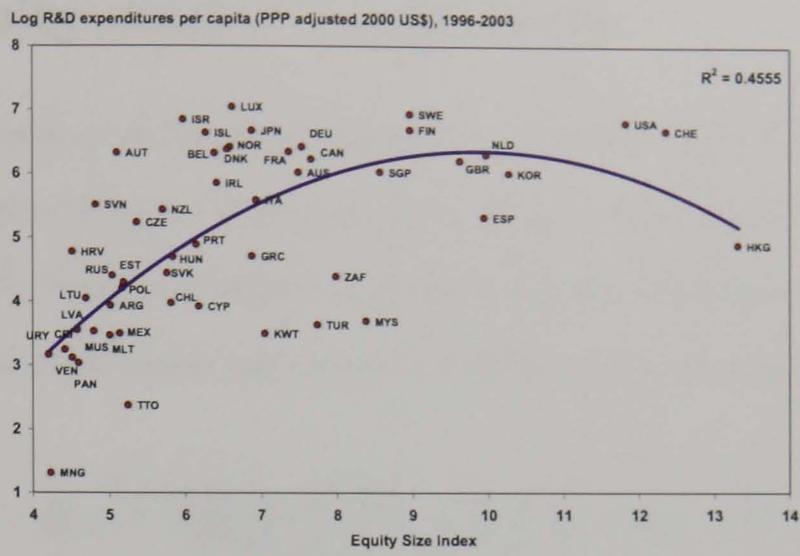

Figure B.3: Equity market and R&D expenditures in high and upper middle income countries

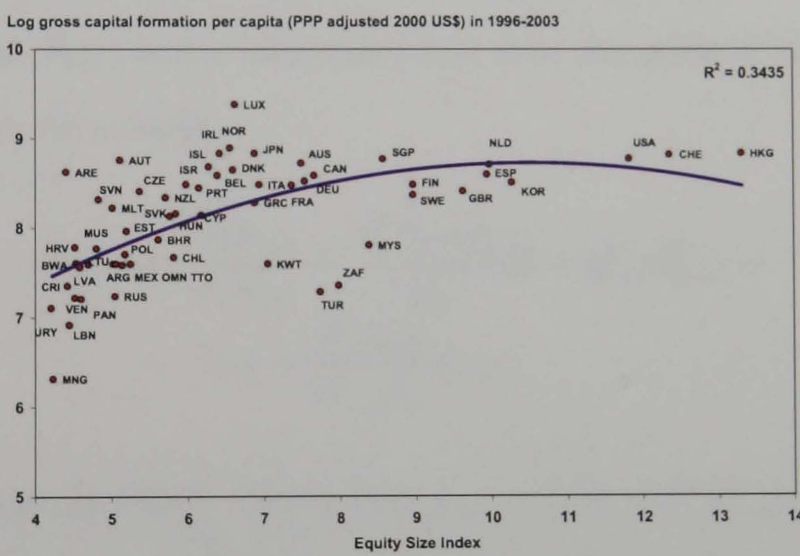

Figure B.4: Equity markets and investments in high and upper middle income countries

B.3 Equity market equilibrium

The optimal demand of informed traders is specified in (3.21) and (3.22). Uninformed investors obtain information from their public signal (3.8), adoption decision made by the initial owners in period t and the stock price. Replacing the optimal demand of informed agents into the asset market clearing condition (3.2). This implies,

$$\hat{\mu}^I_{t+1}\frac{\Gamma\theta_{t+2}A_{t+2}-RP_{t+1}}{\tau\Gamma^2A^2_{t+2}\frac{1}{\beta_u}}+\hat{\mu}^U_{t+1}\hat{h}^U_{t+1}+s_{t+1}=1. \tag{B.1}$$

Uniformed investors are all identical and they know their demand $(\hat{h}^U_{t+1})$. This means that they also know the demand of all other uninformed investors. Therefore, the price signal can be found from rearranging the equation into observable (the price signal, $\tilde{P}_{t+1}$) and unobservable part from the point of view of any uninformed investor. As a result

$$\begin{aligned}\tilde{P}_{t+1} &\equiv \frac{RP_{t+1}}{\Gamma A_{t+2}}-\frac{\tau\Gamma A_{t+2}\frac{1}{\beta_u}}{\mu_I}(1-\hat{\mu}^U_{t+1}\hat{h}^U_{t+1}) = \\ &= \theta_{t+2}+\frac{\tau\Gamma A_{t+2}}{\hat{\mu}^I_{t+1}\beta_u}s_{t+1},\end{aligned}$$

which is the same as (3.23). Given that $s_{t+1}\sim\mathcal{N}(0,1/\Gamma^2A^2_{t+2})$, the conditional distribution $\tilde{P}_{t+1}|\theta_{t+2}\sim\mathcal{N}\left(\theta_{t+2},\frac{1}{\beta_s\left(\hat{\mu}^I_{t+1}\beta_u\right)^2\tau^{-2}}\right)$. This implies that

$$\theta_{t+2}|\tilde{\theta}_{t+2},\tilde{P}_{t+1},\sim\mathcal{N}\left(\frac{\beta_{\tilde{\theta}}\tilde{\theta}_{t+2}+\beta_s\left(\hat{\mu}^I_{t+1}\beta_u\right)^2\tau^{-2}\tilde{P}_{t+1}}{\beta_{\tilde{\theta}}+\beta_s\left(\hat{\mu}^I_{t+1}\beta_u\right)^2\tau^{-2}},\frac{1}{\beta_{\tilde{\theta}}+\beta_s\left(\hat{\mu}^I_{t+1}\beta_u\right)^2\tau^{-2}}\right).$$

Defining coefficients z_{t+1} and $z_{v,t+1}$ as in (3.25)

$$\theta_{t+2}|\tilde{\theta}_{t+2},\tilde{P}_{t+1},\sim\mathcal{N}\left(z_{t+1}\tilde{\theta}_{t+2}+(1-z_{t+1})\tilde{P}_{t+1},z_{v,t+1}\right)$$

Uninformed investors also get information from knowing the adoption decision of local informed investors. We can conjecture that if in previous period the speed of technology adoption was chosen to be fast ($\tilde{1}_{I_t} = 1$), it implies $\theta_{t+2} \geq \bar{\theta}_{t+2}$. If it was slow ($\tilde{1}_{I_t} = 0$) then $\theta_{t+2} < \bar{\theta}_{t+2}$. This conjecture is verifies in Appendix B.4. Following Green (2000), pp. 899 the moments of truncated normal can be expressed as

$$
\begin{aligned}
E\left[\theta_{t+2}|\Omega^U_{t+1}\right] &= \left(z_{t+1}\tilde{\theta}_{t+2} + (1 - z_{t+1})\tilde{P}_{t+1} + \sqrt{z_{v,t+1}}\lambda_{\tilde{1}_{I_t}}(b_{t+1})\right) \qquad \text{(B.2)} \\
\mathrm{Var}(\theta_{t+2}|\Omega^U_{t+1}) &= z_{v,t+1}\left(1 - \lambda^2_{\tilde{1}_{I_t}}(b_{t+1}) + b_{t+1}\lambda_{\tilde{1}_{I_t}}(b_{t+1})\right),
\end{aligned}
$$

where $b_{t+1} \sim \mathcal{N}(0,1)$ defined as in (3.25) and $\lambda_{\tilde{1}_{I_t}}$ is inverse Mills ratio; $\lambda_{\tilde{1}_{I_t=1}}(b_{t+1}) = \frac{\phi(b_{t+1})}{1-\Phi(b_{t+1})}$ and $\lambda_{\tilde{1}_{I_t=0}}(b_{t+1}) = -\frac{\phi(b_{t+1})}{\Phi(b_{t+1})}$, where $\phi(.)$ and $\Phi(.)$ are standard normal p.d.f. and c.d.f respectively. Using the (3.4) and the independence of u_{t+1} from the public signal and noise trading shocks, the expectation of profits is (3.24) for uninformed investors.

Plugging the demand of uninformed investors (3.21) and (3.24) in (B.1) equilibrium equity price

$$
P_{t+1} = \frac{\hat{\mu}^I_{t+1}\dfrac{\Gamma\theta_{t+2}A_{t+2}}{\tau\Gamma^2A^2_{t+2}\frac{1}{\beta_u}} + \hat{\mu}^U_{t+1}\dfrac{E\left[\pi_{t+2}|\Omega^{*U}_{t+1}\right]}{\tau\,\mathrm{Var}\left(\pi_{t+2}|\Omega^{*U}_{t+1}\right)} - 1 + s_{t+1}}{R\left(\dfrac{\hat{\mu}^I_{t+1}}{\tau\Gamma^2A^2_{t+2}\frac{1}{\beta_u}} + \dfrac{\hat{\mu}^U_{t+1}}{\tau\,\mathrm{Var}\left(\pi_{t+2}|\Omega^{*U}_{t+1}\right)}\right)}.
$$

As the number of foreign uninformed investors $\mu^U_{t+1} \to \infty$, which also implies $\hat{\mu}^U_{t+1} \to \infty$, the price becomes equal to the discounted expected profits by uninformed investors.

$$
P_{t+1} = \frac{E\left[\pi_{t+2}|\Omega^U_{t+1}\right]}{R}
$$

Using then (3.24) and (3.23)

$$\begin{aligned}
P_{t+1} &= \frac{\Gamma A_{t+2}}{R}\left(z_{t+1}\tilde{\theta}_{t+2} + (1 - z_{t+1})\tilde{P}_{t+1} + \sqrt{z_{v,t+1}}\lambda_{\bar{1}_{I_t}}(b_{t+1})\right) = \\
&= \frac{\Gamma A_{t+2}}{R}\left(\begin{array}{l} z_{t+1}\tilde{\theta}_{t+2} + (1 - z_{t+1})\theta_{t+2} + \\ +(1 - z_{t+1})\frac{\tau\Gamma A_{t+2}}{\hat{\mu}^I_{t+1}\beta_u}s_{t+1} + \sqrt{z_{v,t+1}}\lambda_{\bar{1}_{I_t}}(b_{t+1}) \end{array}\right).
\end{aligned}$$

B.4 Proof of Proposition 1

Assume that there exists a threshold level of productivity $\bar{\theta}_{t+2}$ above which fast technology adoption is optimal. Assume also that this threshold is observable for uninformed investors trading in the next period.

The approximation of Mills ratio with a linear function around 0 is $\lambda_{1_{I_t}}(b_{t+1}) \approx \eta_2 b_{t+1} + \eta_1(-1)^{1-\bar{1}_{I_t}}$. Mills ratios for right and left truncation is a reflection from origin. Therefore, the absolute value of intercept is the same for right and left truncation. For example estimated in the range $[-1, 1]$ $\eta_2 = 0.6247$ and $\eta_1 = 0.8377$ or in the range $[-3, 3]$ $\eta_2 = 0.5701$, $\eta_1 = 1.1101$. For the left truncation the ratio is effectively 0 below -3 and close to linear above 3. In the linear area of Mills ratio function, the slope is below 1 and the function is convex in between. Therefore, in any symmetric range around 0 the slope must be below $\eta_2 < 1$. For left (right) truncation Mills ratio is an increasing and convex (concave) function above (below) 0, which implies $\eta_1, \eta_2 > 0$.

Using this, the equity prices can be expressed as

$$\begin{aligned}
&P_{t+1} \approx \\
&\frac{1}{R}\Gamma A_{t+2}\left(\begin{array}{l} z_{t+1}\tilde{\theta}_{t+2} + (1 - z_{t+1})\theta_{t+2} + z_{s,t+1}\Gamma A_{t+2}s_{t+1} + \\ +\sqrt{z_{v,t+1}}(\eta_2 b_{t+1} + (-1)^{1-\bar{1}_{I_t}}\eta_1) \end{array}\right).
\end{aligned}$$

Expanding the price by replacing in b_{t+1}, $\tilde{\theta}_{t+2}$ and $\tilde{P}_{t+2}$, the price becomes

$$P_{t+1} \approx \frac{1}{R}\Gamma A_{t+2}\left(\begin{array}{l} (1-\eta_2)\,\theta_{t+2} + (1-\eta_2)\,z_{t+1}\epsilon_{\bar{\theta},t+2} + \\ +(1-\eta_2)\,z_{s,t+1}\Gamma A_{t+2}s_{t+1} + \\ \eta_2\bar{\theta}_{t+2} + \sqrt{z_{v,t+1}}(-1)^{1-\tilde{1}_{I_t}}\eta_1) \end{array}\right). \tag{B.3}$$

From here we can find the conditional moments as

$$E\left[P_{t+1}|\theta_{t+2}, 1_{I_t}\right] = \frac{\Gamma A_{t+2}}{R}\left[(1-\eta_2)\,\theta_{t+2} + \eta_2\bar{\theta}_{t+2} + \sqrt{z_{v,t+1}}(-1)^{1-\tilde{1}_{I_t}}c_1)\right]$$

$$\mathrm{Var}\left(P_{t+1}|\theta_{t+2}, 1_{I_t}\right) = \frac{\Gamma^2 A_{t+2}^2}{R^2}\left(1-\eta_2\right)^2\left[\frac{z_{t+1}^2}{\beta_{\bar{\theta}}} + \frac{z_{s,t+1}^2}{\beta_s}\right] = \frac{\Gamma^2 A_{t+2}^2}{R^2}\left(1-\eta_2\right)^2 z_{v,t+1}$$

By definition, if $\tilde{1}_{I_t} = 1$, then $A_{t+2} = A^*_{t+2}$ and if $\tilde{1}_{I_t} = 0$, then $A_{t+2} = A^*_{t+1}$. Investors, will choose to adopt the technology fast if $U_t(\tilde{1}_{I_t} = 1) \geq U_t(\tilde{1}_{I_t} = 0) + RI_t$. Using the moments just derived and the adoption cost function (3.6), the condition for fast technology adoption is

$$\frac{\Gamma\left(A^*_{t+2} - A^*_{t+1}\right)}{R}\left((1-\eta_2)\,\theta_{t+2} + \eta_2\bar{\theta}_{t+2}\right) \geq R\left(A^*_{t+2} - A^*_{t+1}\right)\hat{\zeta}(\cdot) - \\ -\frac{\Gamma\left(A^*_{t+2} + A^*_{t+1}\right)}{R}\sqrt{z_{v,t+1}}\eta_1 + \frac{\tau}{2}\frac{\Gamma^2\left(A^{*2}_{t+2} - A^{*2}_{t+1}\right)}{R^2}\left(1-\eta_2\right)^2 z_{v,t+1}.$$

This can be simplified by expressing it in terms of growth rate of frontier $g^* =$

$\frac{A^*_{t+2}}{A^*_{t+1}} - 1$ as

$$(1-\eta_2)\,\theta_{t+2} + \eta_2\bar{\theta}_{t+2} \geq \frac{R^2}{\Gamma}\hat{\zeta}(\cdot) -$$
$$-\tfrac{(2+g^*)}{g^*}\sqrt{z_{v,t+1}}\eta_1 + \frac{\tau}{2}\frac{\Gamma\,(2+g^*)\,A^*_{t+1}}{R}\,(1-\eta_2)^2\, z_{v,t+1}$$

If the productivity is at the threshold $\theta_{t+2} = \bar{\theta}_{t+2}$, initial owner is indifferent between adopting fast or slow. This implies

$$\begin{aligned}\bar{\theta}_{t+2} &= \frac{R^2}{\Gamma}\hat{\zeta}(\cdot) - \frac{\Gamma\,(2+g^*)}{Rg^*}\sqrt{z_{v,t+1}}\eta_1 + \\ &\quad + \frac{\tau}{2}\frac{\Gamma^2\,(2+g^*)\,A^*_{t+1}}{R^2}\,(1-\eta_2)^2\, z_{v,t+1}\end{aligned}$$

Replacing $\bar{\theta}_{t+2}$ into the condition for adoption above and simplifying, the condition for fast adoption becomes

$$\begin{aligned}\theta_{t+2} &\geq \bar{\theta}_{t+2} = \frac{R^2}{\Gamma}\hat{\zeta}(\cdot) - \frac{(2+g^*)}{g^*}\sqrt{z_{v,t+1}}\eta_1 \\ &\quad + \frac{\tau}{2}\frac{\Gamma\,(2+g^*)\,A^*_{t+1}}{R}\,(1-\eta_2)^2\, z_{v,t+1}.\end{aligned}$$

$\bar{\theta}_{t+2}$ depends on R, $\hat{\zeta}(\cdot)$, Γ, g^*, $z_{v,t+1}$, $\beta_{\bar{\theta}}$ and β_s that are all known to uninformed investors in period $t+1$. This proves that there exists and observable threshold for fast technology adoption. This also confirms the guess in Chapter 3.2.3 (and Appendix B.3)

B.5 Independence of adoption and trading decisions

In period $t+1$ some investors trading in the financial markets are also initial owners of monopolistic firms that produce profits in period $t+3$ (investment $\tilde{1}_{I_{t+1}} I_{t+1}$ will produce profits $\pi_{t+3} = \Gamma A_{t+3}(\theta_{t+3} + u_{t+3})$). Assume that such agent is an investor of type $i \in \{I, U\}$ in his trading decision. The information set that is relevant for his trading decision is Ω^i_{t+1} (that is $\Omega^U_{t+1} = \{\tilde{\theta}_{t+2}, \tilde{P}_{t+1}, \tilde{1}_{I_t}\}$ or $\Omega^I_{t+1} = \{\theta_{t+2}\}$). The information that is relevant for his technology adoption decision is θ_{t+3}. He solves

$$\max_{\hat{h}^i_{t+1}, \tilde{1}_{I_{t+1}}} E[c^{e,i}_{t+2} | \Omega^i_{t+1}, \theta_{t+3}] - \tfrac{\tau}{2} \operatorname{Var}\left(c^{i,e}_{t+2} | \Omega^i_{t+1}, \theta_{t+3}\right),$$

$$\text{st. } c^{e,i}_{t+2} = c^i_{t+2} + c^e_{t+2}$$

$$c^i_{t+2} = (\Gamma(\theta_{t+2} + u_{t+2}) A_{t+2} - R P_{t+1}) \hat{h}^i_t + R \hat{W}^i_{t+1}$$

$$c^e_{t+2} = \tilde{1}_{I_{t+1}} P_{t+2}(\tilde{1}_{I_{t+1}} = 1) + (1 - \tilde{1}_{I_{t+1}}) P_{t+2}(\tilde{1}_{I_{t+1}} = 0) - \tilde{1}_{I_{t+1}} R I_{t+1},$$

where $c^{e,i}_{t+2}$ is total consumption, c^i_{t+2} is consumption from trading, c^e_{t+2} is consumption from the adoption decision, $\hat{W}^i_{t+1}$ is the wealth of such agent (equals to wage income if the agent is a local entrepreneur alone) and $\hat{h}^i_t$ his risky equity demand. Notation $P_{t+2}(\tilde{1}_{I_{t+1}} = 1)$ and $P_{t+2}(\tilde{1}_{I_{t+1}} = 0)$ is to point out that equity price will be different, depending on the adoption decision (as profit and equity price depends on the quality of technology, i.e. the profits depend on A^*_{t+3} if $\tilde{1}_{I_{t+1}} = 1$ or A^*_{t+2} if $\tilde{1}_{I_{t+1}} = 0$).

With the linear approximation of Mills ratio specified in Appendix B.4, the equilibrium equity price (B.3) depends of three uncertainty terms,

$$P_{t+2} = \frac{\Gamma A_{t+3}}{R}(1-\eta_2)\left(\theta_{t+3} + z_{t+2}\epsilon_{\bar{\theta},t+3} + z_{s,t+2}\Gamma A_{t+2}s_{t+2} + time\_specific\_cons\right)$$

This implies that

$$\mathrm{Cov}(c^e_{t+2}, c^i_{t+2}) \propto \mathrm{Cov}(\theta_{t+2} + u_{t+2}, \theta_{t+3} + z_{t+2}\epsilon_{\bar{\theta},t+3} + z_{s,t+2}A_{t+2}s_{t+2}) = 0,$$

because by assumption there is no correlation between the shocks and no serial correlation. Using this, the moments of $c^{i,e}_{t+1}$ can be expresses as

$$\begin{aligned} E[c^{e,i}_{t+1}|\Omega^i_{t+1}, \theta_{t+3}] &= E[c^i_{t+1}|\Omega^i_{t+1}] + E[c^e_{t+1}|\theta_{t+3}] \\ \mathrm{Var}\left(c^{e,i}_{t+1}|\Omega^i_{t+1}, \theta_{t+3}\right) &= \mathrm{Var}\left(c^i_{t+1}|\Omega^i_{t+1}\right) + \mathrm{Var}\left(c^e_{t+1}|\theta_{t+3}\right). \end{aligned}$$

The utility function used implies that optimal equity demand does not depend on wealth. Therefore utility from equity trading and developing can be solved separately as (3.20) and

$$\max_{\tilde{1}_{I_{t+1}}} E[c^e_{t+1}|\theta_{t+3}] - \tfrac{\tau}{2}\mathrm{Var}\left(c^e_{t+1}|\theta_{t+3}\right),$$

which is equivalent to (3.29) and (3.30) for $t+1$.

B.6 Local goods market clearing

Case 1. Consider the case when initial owners are local entrepreneurs alone. The aggregate budget constraint for all young local agents in period t.

$$\mu w_t = \tilde{1}_{I_{t-1}} H_t P_t(\tilde{1}_{I_{t-1}} = 1) + (1 - \tilde{1}_{I_{t-1}}) H_t P_t(\tilde{1}_{I_{t-1}} = 0) + M_t + \tilde{1}_{I_t} I_t,$$

where M_t is aggregate risk-free foreign asset demand by local agents and $H_t \equiv (\mu_t^I \hat{h}_t^I + (\mu - \mu_t^I)\hat{h}_t^U + s_t)$ and is the aggregate equity demand by local agents. Due to the lack of wealth effects with CARA utility, local and foreign informed and uninformed investors' equity demand is the same, i.e. $h_t = h_t^{*I} = \hat{h}_t^I$ and $h_t = h_t^{*I} = \hat{h}_t^I$. The aggregate consumption of these agents during next period will be

$$C_{t+1} = \mu c_{t+1} + c_{t+1}^N = \pi_{t+1} H_t + RM_t + \tilde{1}_{I_t} P_{t+1}(\tilde{1}_{I_t} = 1) + (1 - \tilde{1}_{I_t}) P_{t+1}(\tilde{1}_{I_t} = 0).$$

The asset market clearing condition (3.2) can be rewritten as $\mu_t^I \hat{h}_t^I + (\mu - \mu_t^I)\hat{h}_t^U + \mu_{t+1}^{*I} \hat{h}_t^I + \mu_{t+1}^{*U} \hat{h}_t^U + s_{t+1} = 1$ or $H_t + H_t^* = 1$, where total risky asset demand by foreigners is $H_t^* = \mu_{t+1}^{*I} \hat{h}_t^I + \mu_{t+1}^{*U} \hat{h}_t^U$. Replacing this in aggregate consumption, it becomes

$$C_{t+1} = \pi_{t+1}(1 - H_t^*) + RM_t + \tilde{1}_{I_t} P_{t+1}(\tilde{1}_{I_t} = 1) + (1 - \tilde{1}_{I_t}) P_{t+1}(\tilde{1}_{I_t} = 0).$$

From the first period of life budget constraint, the aggregate holdings of risk-free asset are

$$M_t = \mu w_t - (1 - H_t^*) \left(\tilde{1}_{I_{t-1}} P_t(\tilde{1}_{I_{t-1}} = 1) + (1 - \tilde{1}_{I_{t-1}}) P_t(\tilde{1}_{I_{t-1}} = 0) \right) - \tilde{1}_{I_t} I_t$$

It is clear that if a country invests in technology adoption in period t, it's foreign asset holdings will be smaller (or foreign debt higher).

The net foreign asset position of the country (F_{t+1}) has following components:

1. $\tilde{1}_{I_t} H_{t+1}^* P_{t+1}(\tilde{1}_{I_t} = 1) + (1 - \tilde{1}_{I_t}) H_{t+1}^* P_{t+1}(\tilde{1}_{I_t} = 0)$ inflow if foreign investment to local asset;

2. $H_t^* \pi_{t+1}$ outflow of profits from previous period investments;

3. M_{t+1} outflow of locals' investment to the world asset (or inflow of debt);

4. RM_t inflow of previous period world asset revenues (or outflow of debt repayment).

$$F_{t+1} = \tilde{1}_{I_t} H^*_{t+1} P_{t+1}(\tilde{1}_{I_t} = 1) + (1 - \tilde{1}_{I_t}) H^*_{t+1} P_{t+1}(\tilde{1}_{I_t} = 0) - H^*_t \pi_{t+1} - M_{t+1} + RM_t$$

Using that $\tilde{1}_{I_{t+1}}(j) = \tilde{1}_{I_{t+1}}$ and $I_{t+1}(j) = I_{t+1}$ from Section 3.2.2, domestic good's market clearing (3.9) becomes

$$F_{t+1} + Y_{t+1} = C_{t+1} + \int_0^1 x_t(j)dj + \tilde{1}_{I_{t+1}} I_{t+1}.$$

It can be shown to hold. Replacing F_{t+1}, C_{t+1} and M_{t+1}in the goods' market clearing condition and simplifying we obtain

$$-\mu w_{t+1} + Y_{t+1} = \pi_{t+1} + \int_0^1 x_t(j)dj$$

Using (3.11), (3.13) and (3.14) this simplifies to

$$\pi_{t+1} = \Gamma \phi_t A_{t+1}$$

and holds by (3.12). The goods' market clears.

Case 2. As analyzed in Section 2.4.3, if initial owners include a foreign investor, the speed of technology adoption must be fast, $\tilde{1}_{I_{t-1}} = 1$. If fast technology adoption is possible only with the participation of foreign investor

$$C_{t+1} = \pi_{t+1}(1 - H^*_t) + RM_t + P_{t+1}(\tilde{1}_{I_t} = 0).$$

$$M_t = \mu w_t - (1 - H_t^*) P_t(\tilde{1}_{I_{t-1}} = 1)$$

There will be an additional capital inflow, because the foreign participant will bear all the technology adoption cost $I_{t+1} = \zeta(A_{t+3}^* - A_{t+2}^*)$ and an additional outflow of foreigners' earnings from exiting $P_{t+1}(\tilde{1}_{I_t} = 1) - P_{t+1}(\tilde{1}_{I_t} = 0)$. The resulting net foreign asset position

$$\begin{aligned} F_{t+1} = & H_{t+1}^* P_{t+1}(\tilde{1}_{I_t} = 1) - H_t^* \pi_{t+1} - M_{t+1} + RM_t \\ & + I_{t+1} - P_{t+1}(\tilde{1}_{I_t} = 1) + P_{t+1}(\tilde{1}_{I_t} = 0). \end{aligned}$$

Replacing these in the goods' market clearing condition, simplifies to the same condition as Case 1.

If foreign investors participate for reducing the fast adoption cost, then capital additional capital inflow and outflow are $\zeta(A_{t+2}^* - A_{t+1}^*)$ and $R\zeta(\cdot)(A_{t+2}^* - A_{t+1}^*)$. Consumption, risk-free asset holdings and net foreign asset position are

$$C_{t+1} = \pi_{t+1}(1 - H_t^*) + RM_t + P_{t+1}(\tilde{1}_{I_t} = 1) - R\zeta(\cdot)(A_{t+2}^* - A_{t+1}^*)$$

$$M_t = \mu w_t - (1 - H_t^*) P_t(\tilde{1}_{I_{t-1}} = 1)$$

$$\begin{aligned} F_{t+1} = & H_{t+1}^* P_{t+1}(\tilde{1}_{I_t} = 1) - H_t^* \pi_{t+1} - M_{t+1} + RM_t \\ & + \hat{\zeta}(A_{t+3}^* - A_{t+2}^*) - R\zeta(\cdot)(A_{t+2}^* - A_{t+1}^*). \end{aligned}$$

Using $I_{t+1} = \hat{\zeta}(A_{t+3}^* - A_{t+2}^*)$ market clears similarly to the above. As the foreign participation always compensates the opportunity cost for local agents, forming a joint venture does not have an impact on aggregate consumption in $t + 1$. It can also be seen from the relations above that the net foreign asset position is always lower if the country adopts fast technology. Local good's market clears in period t if a foreigner participates in the project.

B.7 Proof of Proposition 5.

The demand of uninformed investors with wealth (or wage income), $\hat{W}^U_{t+1}$, is given by (3.21) and (3.24). They know this demand with certainty. The utility from staying uninformed is given by

$$\begin{aligned}
&E\left[U^U_{t+1}|\Omega^U_{t+1}\right] = \\
&\frac{E\left[\pi_{t+2}|\Omega^U_{t+1}\right] - RP_{t+1}}{\tau \operatorname{Var}\left(\pi_{t+2}|\Omega^U_{t+1}\right)} E\left[\pi_{t+2} - RP_{t+1})|\Omega^U_{t+1}\right] \\
&+ R\hat{W}^U_{t+1} - \frac{\left(E\left[\pi_{t+2}|\Omega^U_{t+1}\right] - RP_{t+1}\right)^2}{2\tau \operatorname{Var}\left(\pi_{t+2}|\Omega^U_{t+1}\right)^2} \operatorname{Var}(\pi_{t+2}|\Omega^U_{t+1})
\end{aligned}$$

This simplifies to

$$E\left[U^U_{t+1}|\Omega^U_{t+1}\right] = \frac{\left(E\left[\pi_{t+2}|\Omega^U_{t+1}\right] - RP_{t+1}\right)^2}{2\tau \operatorname{Var}\left(\pi_{t+2}|\Omega^U_{t+1}\right)} + R\hat{W}^U_{t+1}.$$

It they decide to become informed their demand is given by (3.21) and (3.22). However, they do not know what productivity they will observe after becoming informed and therefore their demand as informed. Replacing the demand as informed in the utility function, the utility can be expressed as

$$U^I_{t+1} = \frac{\left(\Gamma A_{t+2}\theta_{t+2} - RP_{t+1}\right)^2}{2\tau\Gamma^2 A^2_{t+2}\frac{1}{\beta_u}} + \frac{\Gamma A_{t+2}u_{t+2}\left(\Gamma A_{t+2}\theta_{t+2} - RP_{t+1}\right)}{\tau\Gamma^2 A^2_{t+2}\frac{1}{\beta_u}} + R\hat{W}^U_{t+1}.$$

Taking expectations of this conditional on the information the uninformed investor has

$$E\left[U_{t+1}^{I}|\Omega_{t+1}^{U}\right]=\frac{\Gamma^{2}A_{t+2}^{2}\operatorname{Var}\left(\theta_{t+2}|\Omega_{t+1}^{U}\right))+\left(E\left[\pi_{t+2}|\Omega_{t+1}^{U}\right]-RP_{t+1}\right)^{2}}{2\tau\Gamma^{2}A_{t+2}^{2}\frac{1}{\beta_{u}}}+R\hat{W}_{t+1}^{U}$$

Given that the number of uninformed investors is infinite, asset prices correspond to the expectations of uninformed investors. This means $E\left[\pi_{t+2}|\Omega_{t+1}^{U}\right]-RP_{t+1}=0$. An investor will decide to become informed if $E\left[U_{t+1}^{I}|\Omega_{t+1}^{I}\right]-RD_{t+1}\geq E\left[U_{t+1}^{U}|\Omega_{t+1}^{U}\right]$. This implies the following condition

$$\frac{\operatorname{Var}\left(\theta_{t+2}|\Omega_{t+1}^{U}\right)}{2\tau\frac{1}{\beta_{u}}}\geq RD_{t+1}$$

The conditional variance of the productivity has to be high enough, such that the cost of becoming more informed is compensated by better expected arbitrage opportunities as an informed investor. Using $D_{t+1}=\delta_{t+1}\vartheta_{t+1}$, $\operatorname{Var}\left(\theta_{t+2}|\Omega_{t+1}^{U}\right)=z_{v,t+1}\vartheta_{t+1}$ from (B.2) and $\vartheta_{t+1}\equiv\left(1-\lambda_{\bar{1}_{I_t}}^{2}(b_{t+1})+b_{t+1}\lambda_{\bar{1}_{I_t}}(b_{t+1})\right)$, this becomes

$$\delta_{t+1}\leq\frac{\beta_{u}z_{v,t+1}}{R2\tau}\equiv\bar{\delta}_{t+1}$$

Investors find it optimal to invest adoption as long as δ_{t+1} is small enough. However, $z_{v,t+1}=\frac{1}{\beta_{\bar{\theta}}+\left(\hat{\mu}_{t+1}^{I}\beta_{u}\right)^{2}\beta_{s}\tau^{-2}}$ is a decreasing function of the number of informed investors. This is because with the higher number of informed investors makes asset price more revealing, thereby reducing the gains from being informed. If the number of local investors is large enough, such that $\delta_{t+1}>\frac{\beta_{u}}{R2\tau}\frac{1}{\beta_{\bar{\theta}}+\left(\hat{\mu}_{t+1}^{I}\beta_{u}\right)^{2}\beta_{s}\tau^{-2}}$ holds, no uninformed investor will decide to become informed. If this is not the case, investors will become informed until the gains from becoming informed are driven to 0. This means that in equilibrium, the number of uninformed investors is

$$\hat{\mu}^I_{t+1} = \sqrt{\frac{\tau}{\beta_u \beta_s}\left(\frac{1}{R2\delta_{t+1}} - \frac{\beta_{\tilde{\theta}}\tau}{\beta_u}\right)}.$$

This root is always real, it being negative implies that $\delta_{t+1} > \frac{\beta_u}{\beta_{\tilde{\theta}}\tau R2}$, which is satisfied as long as there is at least one investor who decides to become informed in addition to those who are informed for a zero cost.

The dependence of equilibrium number of informed investors on R, δ_{t+1}, $\beta_{\tilde{\theta}}$ and β_s is straightforward. Sufficient condition for $\frac{\partial \hat{\mu}^I_{t+1}}{\partial \tau} > 0$ and $\frac{\partial \hat{\mu}^I_{t+1}}{\partial \beta_u} < 0$ is $\delta_{t+1} < \frac{\beta_u}{\beta_{\tilde{\theta}}\tau R2}$ (the condition for a real root).

B.8 Proof of Proposition 6

This is a simple optimization problem. Define constants $Q_1 \equiv \frac{R^2}{\Gamma}\hat{\zeta}(\cdot) > 0$; $Q_2 \equiv \frac{2+g^*}{g^*}\sqrt{\frac{R2\tau}{\beta_u}}\eta_1 > 0$; $Q_3 \equiv \frac{\tau}{2}\frac{\Gamma(2+g^*)}{R}A^*_{t+1}(1-\eta_2)^2\frac{R2\tau}{\beta_u} > 0$. Then $\delta^{opt}_{t+1} = \arg\min_{\delta_{t+1}}\left(Q_1 - Q_2\delta^{\frac{1}{2}}_{t+1} + Q_3\delta_{t+1}\right)$. First order condition of this gives $\delta^{opt}_{t+1} = \left(\frac{Q_2}{2Q_3}\right)^2$. Second order condition, $\frac{\partial^2\left(Q_1 - Q_2\delta^{\frac{1}{2}}_{t+1} + Q_3\delta_{t+1}\right)}{\partial^2\delta_{t+1}} = \frac{Q_2}{4\delta_{t+1}{}^{\frac{3}{2}}} > 0$ confirms it as minimum. Replacing the constants back in δ^{opt}_{t+1} gives the proposition.

B.9 Policy maker's choice of precision of public signal

Assume that instead of information cost, the local policy issues the public signal and chooses the precision of it. The policy maker chooses in period t how precise signal he would get about θ_{t+2} in period $t+1$ and commits to issuing his observed signal in $t+1$. For example, the local policy maker could establish an independent research department and choose the size of it.[1] The policy maker solves

$$\beta_{\bar{\theta},t+1}^{opt} = \arg\min_{\beta_{\bar{\theta}}} \left(\bar{\theta}_{t+2}\right),$$

where $\bar{\theta}_{t+2}$ is given by (3.31).As the chosen precision can chance over time, consider $\beta_{\bar{\theta},t+1}$ instead of $\beta_{\bar{\theta}}$, which means that $z_{v,t+1} = \frac{1}{\beta_{\bar{\theta},t+1}+\left(\frac{\hat{\mu}_{t+1}^{I}\beta_u}{\tau}\right)^2\beta_s}$. The solution of this problem is

$$\beta_{\bar{\theta},t+1}^{opt} = \begin{cases} 0, \text{ if } \hat{\mu}_{t+1}^{I} > \frac{g^*\tau\Gamma A_{t+1}^*(1-\eta_2)^2\tau}{R\eta_1\sqrt{\beta_s}\beta_u} \\ \left(\frac{g^*\tau\Gamma A_{t+1}^*(1-\eta_2)^2}{R\eta_1}\right)^2 - \left(\frac{\hat{\mu}_{t+1}^{I}\beta_u}{\tau}\right)^2\beta_s, \text{ otherwise} \end{cases}$$

It is clear that $\beta_{\bar{\theta},t+1}^{opt}$ is finite. Perfect public signal would require $\beta_{\bar{\theta},t+1}^{opt} \to \infty$. Therefore, similarly to Section 3.3.2, local policy maker does not choose full transparency. Here, if the number of informed investors is sufficiently high, the policy maker would issue no public signal $\beta_{\bar{\theta},t+1}^{opt} = 0$, in such case there is no reason to aim to offset the "fear of unstable markets" force and more precise public signal would only limit the gains from "adopting to signal".

B.10 Restrictions on foreigners portfolio equity investments

Case 1. All noise traders are local. Let as assume that there exits a threshold $\bar{\theta}_{t+2}^{R} < \bar{\theta}_{t+2}$, such that adoption is more likely in the case of restricting foreign capital.

[1] The approach with the choice of information cost is preferred, because local policy maker could have incentives to declare higher productivity to encourage faster technology adoption. This can make the public signal he issues not credible. The assumed independent research department that would avoid such problem may be less realistic than facilitating investors to access information directly.

Using the equilibrium price in the case of $\mu_{t+1}^{*U} = \mu_{t+1}^{*I} = 0$ and $\mu_{t+1}^{I} = \mu$

$$\begin{aligned} E[P_{t+1}^R|\theta_{t+2}] &= \tfrac{\Gamma A_{t+2}\theta_{t+2}}{R} - \left(\tfrac{\tau}{\mu\beta_u}\right)\frac{\Gamma^2}{R}A_{t+2}^2 \\ \mathrm{Var}\left(P_{t+1}^R|\theta_{t+2}\right) &= \left(\tfrac{\tau}{\mu\beta_u}\right)^2\tfrac{\Gamma^2 A_{t+2}^2}{R^2}\tfrac{1}{\beta_s} \end{aligned}$$

It is optimal to pursue fast adoption if $U_t(\tilde{1}_{I_t} = 1) \geq U_t(\tilde{1}_{I_t} = 0)$. At the threshold $\bar{\theta}_{t+2} = \bar{\theta}_{t+2}^R \implies U_t(\tilde{1}_{I_t} = 1) \geq U_t(\tilde{1}_{I_t} = 0)$. Using the moments above, the threshold can be expressed as

$$\bar{\theta}_{t+2}^R = \tfrac{R^2}{\Gamma}\hat{\zeta}(\cdot) + \left(\tfrac{\tau}{\mu\beta_u}\right)\tfrac{\Gamma}{R}(2+g)A_{t+1}^*\left(R + \tfrac{\tau}{2\beta_s}\left(\tfrac{\tau}{\mu\beta_u}\right)\right)$$

Using $z_{v,t+1} = \frac{1}{\beta_{\tilde{\theta}}+\left(\frac{\mu\beta_u}{\tau}\right)^2\beta_s}$, after simplification $\bar{\theta}_{t+2}^R < \bar{\theta}_{t+2}$ implies.

$$\frac{1}{g^*}\sqrt{z_{v,t+1}}\eta_1 < -\frac{\Gamma}{R}A_{t+1}^*\left(\frac{\frac{\tau}{2}(1-(1-\eta_2)^2)+\beta_{\tilde{\theta}}\left(\frac{\tau}{\mu\beta_u}\right)\left(R+\frac{\tau}{2\beta_s}\left(\frac{\tau}{\mu\beta_u}\right)\right)+\left(\frac{\mu\beta_u}{\tau}\right)\beta_s R}{\beta_{\tilde{\theta}}+\left(\frac{\mu\beta_u}{\tau}\right)^2\beta_s}\right) \quad \text{(B.4)}$$

As all variables and constants in this inequality are positive and $(1-\eta_2)^2 < 1$, this implies $LHS > 0$ and $RHS < 0$. This contradicts existence of threshold where adoption is more likely with restricting foreign portfolio equity investments.

Case 2. None of the noise traders are local. As before, assume that there exits a threshold $\bar{\theta}_{t+2}^R < \bar{\theta}_{t+2}$, such that adoption is more likely in the case of restricting foreign capital.

In the absence of noise traders $E[P_{t+1}^R|\theta_{t+2}]$ is as above and $\mathrm{Var}\left(P_{t+1}^R|\theta_{t+2}\right) = 0$. The threshold for fast adoption becomes

$$\bar{\theta}_{t+2}^R = \tfrac{R^2}{\Gamma}\hat{\zeta}(\cdot) + \left(\tfrac{\tau}{\mu\beta_u}\right)\Gamma(2+g)A_{t+1}^*$$

First, from (3.32) it is clear that $\bar{\theta}_{t+2}^R > \bar{\theta}_{t+2}^{PI}$. This implies that potential

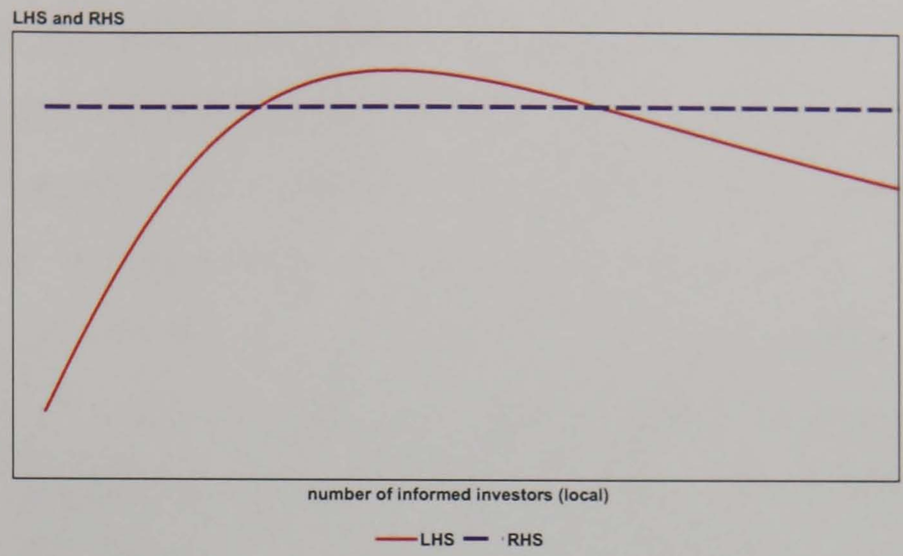

Figure B.5: Possibility of faster technology adoption when forbidding foreign portfolio investments

gains from closing the access to foreign uninformed investors could arise only if the number of local informed investors is sufficiently small. In such case the "fear of unstable markets" force is stronger than "adopting to signal" force (area B in Figure 3.3 in Section 3.2.4). Using 3.33, we can find the following condition, where "fear of unstable markets" force is stronger

$$\mu < \bar{\mu} \equiv \frac{\tau}{\beta_u}\sqrt{\frac{1}{\beta_s}\left(\tau\Gamma g^* A^*_{t+1}\frac{(1-\eta_2)^2}{\eta_1 2R}\right)^2 - \frac{\beta_{\bar{\theta}}}{\beta_s}}$$

Comparing the thresholds, with imperfect equity markets, $\bar{\theta}^R_{t+2} < \bar{\theta}_{t+2}$ implies that

$$RHS = R < \frac{\frac{\mu\beta_u}{\tau}}{\sqrt{\beta_{\bar{\theta}}+\frac{\mu^2\beta_u^2}{\tau^2}\beta_s}}\left(\frac{\tau}{2}(1-\eta_2)^2\frac{1}{\sqrt{\beta_{\bar{\theta}}+\frac{\mu^2\beta_u^2}{\tau^2}\beta_s}} - \frac{R}{\Gamma g^* A^*_{t+1}}\eta_1\right) = LHS \quad (B.5)$$

This condition cannot also be met in very small number of local informed investors ($\mu \to 0$ implies contradiction).

It can hold for some $\mu \in (0, \bar{\mu})$. This requires that the variance in foreign noise trading is high and/or unexplainable component of productivity, $(1/\beta_s)$ and/or $(1/\beta_u)$ is high. Furthermore, low risk-free interest rate and higher growth and level of technology make the condition to hold more easily. The graph below provides an illustration for this for values: $R = 1$, $\tau = 6$, $\beta_u = 2$, $\beta_s = 0.25$, $\beta_{\tilde{\theta}} = 0.25$, $\Gamma = 1$, $g^* = 0.1$, $A^*_{t+1} = 100$, $g^* = 0.1$ and $\eta_1 = 0.8377$, $\eta_2 = 0.6247$ (approximation of Mills ratio between -1 and 1). Closing the access to foreign investors implies lower threshold for fast adoption for the values of μ, where $LHS > RHS$ line in (B.5) on Figure B.5.

Case 3. Some noise traders are local. The results are in between Case 1 and Case 2

Appendix C

Appendix for Chapter 4

C.1 Equilibrium asset prices

Consider an investor of generation T. His demand for the risky asset is

$$h_{i,T} = \frac{(E[\Theta|\Omega_{i,T}] - r_t P_T)}{\tau V_{i,\Theta,T}}.$$

Replacing in the conditional expectations and variance, we can rewrite this as

$$h_{i,T} = \frac{1}{\tau}(y_T \alpha_T + \mu_T^2 \delta \tilde{P}_T + \beta_{i,T} \nu_{i,t} - r_t P_T(\alpha_t + \mu_t^2 \delta + \beta_{i,t})).$$

Aggregating this across investors, and noting that in the symmetric equilibrium $\beta_{i,T} = \beta_T$, the aggregate demand is

$$H_T = \frac{1}{\tau}(y_T \alpha_T + \mu_T^2 \delta \tilde{P}_T + \beta_T \Theta_t - r_t P_T(\alpha_t + \mu_t^2 \delta + \beta_t)).$$

Equating supply and demand, replacing in $\tilde{P}_T = \Theta - H_T/\mu_T$ and rearranging will make the equilibrium price

$$P_T = \frac{\tau + \mu_T\delta}{r_t(\alpha_t + \mu_t^2\delta + \beta_t)}\left(\frac{\alpha_T}{\tau + \mu_T\delta}y_T + \frac{\beta_T + \mu_T^2\delta}{\tau + \mu_T\delta}\Theta - s_t\right).$$

Equating the initially assumed coefficients with the results of market clearing, gives

$$\eta_T = \frac{\tau(\tau^2 + \delta\beta_t)}{r_t(\alpha_t + \frac{\beta_t^2}{\tau^2}\delta + \beta_t)}, \lambda_T = \frac{\alpha_t\tau}{\tau^2 + \delta\beta_t}, \mu_T = \frac{\beta_t}{\tau}.$$

The equilibrium price can also be expressed as

$$P_T = \frac{1 - z_T}{r_t}y_T + \frac{z_T}{r_t}\Theta - \frac{z_{s,T}}{r_t}s_t,$$

where

$$z_T = \frac{\beta_T + \mu_T^2\delta}{\alpha_t + \mu_t^2\delta + \beta_t}, z_{s,T} = \frac{\tau + \mu_T\delta}{\alpha_t + \mu_t^2\delta + \beta_t}.$$

Next, it can be shown that for any trading period t, where in the next trading period equilibrium price can be expressed as

$$P_{t+1} = \frac{1 - z_{t+1}}{R_{t+1}}y_{t+1} + \frac{z_{t+1}}{R_{t+1}}\Theta - \frac{z_{s,t+1}}{R_{t+1}}s_{t+1}$$

there are z_t and $z_{s,t}$ such that a similar pricing equation for period t can be formed.

Investors trading in t know the value of y_{t+1} as it incorporates the price signals for periods 1 to t. The expected value and variance of the next period price of asset are

$$
\begin{aligned}
E[P_{t+1}|\Omega_{i,t}] &= \frac{1-z_{t+1}}{R_{t+1}}y_{t+1} + \frac{z_{t+1}}{R_{t+1}}E[\Theta|\Omega_{i,t}], \\
\mathrm{Var}[P_{t+1}|\Omega_{i,t}] &= \frac{z_{t+1}^2}{R_{t+1}^2}V_{i,\Theta,t} + \frac{z_{s,t+1}^2/\delta}{R_{t+1}^2}.
\end{aligned}
$$

Replacing in the conditional expectations, variance and the public signal and aggregating it across investors, result in the aggregate demand equation

$$
\begin{aligned}
H_T &= \frac{1}{\tau}R_{t+1}\frac{(1-z_{t+1})\left(\alpha_t y_t + \mu_t^2\delta\Theta - \mu_t\delta s_t\right)\dfrac{1}{\alpha_t+\mu_t^2\delta}}{z_{t+1}^2 V_{i,\Theta,t} + z_{s,t+1}^2/\delta} + \\
&\quad + \frac{z_{t+1}\left(\alpha_t y_t + \beta_t\Theta + \mu_t^2\delta\Theta - \mu_t\delta s_t\right)V_{i,\Theta,t} - r_t P_t}{z_{t+1}^2 V_{i,\Theta,t} + z_{s,t+1}^2/\delta}.
\end{aligned}
$$

Equating this to supply and rearranging, there will be z_t and z_s (equations (4.7) and (4.8) respectively), such that the pricing equation (4.6) can be formed. The coefficients η_t, λ_t and μ_t solve

$$
\eta_{t+1} = \frac{z_{s,t+1}}{R_{t+1}}, \qquad \lambda_{t+1} = \frac{(1-z_{t+1})}{z_{s,t+1}}, \qquad \mu_{t+1} = \frac{z_{t+1}}{z_{s,t+1}}.
$$

Explicit expressions for η_t, λ_t and μ_t cannot be derived for $t < T-1$.

It is easy to see that replacing in the terminal conditions $z_{T+1} = 1$, $z_{s,T+1} = 0$ and $r_{T+1} = 1$, (4.7) and (4.8) also simplify to the expressions z_T and $z_{s,T}$ derived.

C.2 Derivation of the variance of excess returns from risky asset

Using the pricing equation (4.6) and conditional expectations of the true value, the expected return of the risky asset for investor i can be expressed as

$$
\begin{aligned}
&E[P_{t+1}|\Omega_{i,t}] - r_t P_t = \\
&\frac{1}{R_{t+1}}\left[(1-z_{t+1})\frac{(\alpha_t y_t + \mu_t^2 \delta \tilde{P}_t)}{\alpha_{t+1}} + z_{t+1}(\alpha_t y_t + \beta_{i,t}\nu_{i,t} + \mu_t^2 \delta \tilde{P}_t)V_{i,\Theta,t}\right] + \\
&-\frac{1}{R_{t+1}}\left[(1-z_t)y_t + z_t\Theta - z_t s_t\right].
\end{aligned}
$$

The public signal is $\tilde{P}_t = \Theta - s_t/\mu_t$ and the private signal $\nu_{i,t} = \Theta + \varepsilon_{i,t}$. Defining $z_{i,t} \equiv (1-z_{t+1})\frac{\mu_t^2\delta}{\alpha_{t+1}} + z_{t+1}(\beta_{i,t} + \mu_t^2\delta)V_{i,\Theta,t}$ and $z_{i,s,t} \equiv \frac{\mu_t\delta}{\alpha_{t+1}} - z_{t+1}\frac{\mu_t\delta}{\alpha_{t+1}}\beta_{i,t}V_{i,\Theta,t}$, $E[P_{t+1}|\Omega_{i,t}] - r_t P_t$ can be rearranged as

$$
\begin{aligned}
&\frac{1}{R_{t+1}}\left[(z_{i,t} - z_t)\,\Theta + (-\tilde{z}_{i,s,t} + z_{s,t})s_t + z_{t+1}\beta_{i,t}V_{i,\Theta,t}\varepsilon_{i,t}\right. \\
&\left. + z_{t+1}\beta_{i,t}V_{i,\Theta,t}\varepsilon_{i,t} + \text{a non stochastic term wrt. } \Omega_{-t}\right].
\end{aligned}
$$

Since $\mathrm{Var}[\varepsilon_{i,t}|\Omega_{-t}] = 1/\beta_{i,t}$, $\mathrm{Var}[s_t|\Omega_{-t}] = 1/\delta$ and $\mathrm{Var}[\Theta|\Omega_{-t}] = 1/\alpha_t$, the variance of the expected return, $\mathrm{Var}[E[P_{t+1}|\Omega_{i,t}] - r_t P_t|\Omega_{-t}]$ is

$$
\frac{1}{R_{t+1}^2}\left[(z_{i,t} - z_t)^2\frac{1}{\alpha_t} + (\tilde{z}_{i,s,t} - z_{s,t})^2\frac{1}{\delta} + z_{t+1}^2\beta_{i,t}V_{i,\Theta,t}^2\right].
$$

C.3 Proof of Proposition 8

Replacing (4.14) and (4.15) in expected utility at the research decision stage (4.13) gives

$$E[U|\Omega_{-t}] = \frac{\left[(z_{t,i}-z_t)^2\frac{1}{\alpha_t}+(-\tilde{z}_{s,i,t}+z_{s,t})^2\frac{1}{\delta}+z_{t+1}^2\beta_{i,t}V_{\Theta,i,t}^2\right]}{2\tau V_{P,i,t}} - \kappa(\beta_{i,t})r_t$$

An individual investor takes the market variables z_t and $z_{s,t}$ as given and the first order condition of his optimization problem is

$$\begin{aligned}
&2\tau r_t\frac{\partial\kappa(\beta_{i,t})}{\partial\beta_{i,t}} = \\
&-\frac{\left[(z_{t,i}-z_t)^2\dfrac{1}{\alpha_t}+(-\tilde{z}_{s,i,t}+z_{s,t})^2\dfrac{1}{\delta}+z_{t+1}^2\beta_{i,t}V_{\Theta,i,t}^2\right]}{V_{P,i,t}^2}\frac{\partial V_{P,i,t}}{\partial\beta_{i,t}}+\\
&+\frac{1}{V_{P,i,t}}\left[2\left(z_{t,i}-z_t\right)\frac{1}{\alpha_t}\frac{\partial z_{t,i}}{\partial\beta_{i,t}}-2\left(-\tilde{z}_{s,i,t}+z_{s,t}\right)\frac{1}{\delta}\frac{\partial\tilde{z}_{s,i,t}}{\partial\beta_{i,t}}\right]\\
&+\frac{1}{V_{P,i,t}}\left[+z_{t+1}^2V_{\Theta,i,t}^2+2z_{t+1}^2\beta_{i,t}V_{\Theta,i,t}\frac{\partial V_{\Theta,i,t}}{\partial\beta_{i,t}}\right],
\end{aligned}$$

where $\frac{\partial z_{t,i}}{\partial\beta_{i,t}} > 0$ and finite, $\frac{\partial V_{\Theta,i,t}}{\partial\beta_{i,t}} = -V_{\Theta,i,t}^2$, $\frac{\partial V_{P,i,t}}{\partial\beta_{i,t}} = z_{t+1}^2\frac{\partial V_{\Theta,i,t}}{\partial\beta_{i,t}} = -z_{t+1}^2V_{\Theta,i,t}^2$ and $\frac{\partial\tilde{z}_{s,i,t}}{\partial\beta_{i,t}} = -z_{t+1}\mu_t\delta V_{\Theta,i,t}^2$.

In a symmetric equilibrium all investors choose the same precision $\beta_{i,t} = \beta_t$. This implies that $z_{t,i} = z_t$, $V_{\Theta,i,t} = V_{\Theta,t}$, $V_{P,i,t} = V_{P,t}$ and $\tilde{z}_{s,i,t} = \tilde{z}_{s,t}$, where $\tilde{z}_{s,t} \equiv \frac{\mu_t\delta}{\alpha_{t+1}} - z_{t+1}\frac{\mu_t\delta}{\alpha_{t+1}}\beta_{i,t}V_{i,\Theta,t}$. Using (4.7) and (4.8) $\tilde{z}_{s,t} = z_{s,t} + \frac{\tau}{R_{t+1}}V_{P,t}$ and the first order condition becomes

$$\begin{aligned}
&r_t\left.\frac{\partial\kappa(\beta_{i,t})}{\partial\beta_{i,t}}\right|_{\beta_{i,t}=\beta_t} = \\
&\frac{z_{t+1}V_{\Theta,t}^2}{2\tau}\left(z_{t+1}\frac{\tau^2}{\delta R_{t+1}^2}+z_{t+1}^3\beta_t\frac{V_{\Theta,t}^2}{V_{P,t}^2}+2\frac{\tau\mu_t}{R_{t+1}}+z_{t+1}\frac{1}{V_{P,t}}-2z_{t+1}\beta_t\frac{V_{\Theta,t}}{V_{P,t}}\right)
\end{aligned} \quad (C.1)$$

This can be simplified further. Using (4.7), (4.8) and (4.12)

$$z_{s,t} = (1 - z_{t+1}\beta_t V_{\Theta,t})\frac{\mu_t \delta}{\alpha_t + \mu_t^2 \delta} + \frac{\tau}{R_{t+1}} V_{P,t} = \frac{z_t}{\mu_t}.$$

Inserting (4.7) into the above equation again an simplifying

$$2\frac{\tau \mu_t}{R_{t+1}} = 2\frac{z_{t+1}\beta_t V_{\Theta,t}}{V_{P,t}} \tag{C.2}$$

and (C.1) becomes (4.16). Given the precision in each period $\{\beta_1, \ldots, \beta_T\}$, coefficients $\{\mu_1, \ldots, \mu_T\}$ can be solved as specified in Section 4.2.2. Full symmetric equilibrium solves for $\{\beta_1, \ldots, \beta_T\}$ and $\{\mu_1, \ldots, \mu_T\}$ simultaneously.

Finally, it can be shown that there exists at least one equilibrium for positive and finite parameters $(\alpha, \tau, \delta, r_1, .., r_T)$ of the model and a cost function $\kappa(\beta_{i,t})$, where $\beta_t \geq 0$ for any t.

First, it is clear from (4.16), that $MB_t \geq 0$ for any feasible parameter values.

Second, using (C.2) marginal benefit (4.16) can be written as

$$MB_t = \frac{\tau z_{t+1}^2 V_{\Theta,t}^2}{2\delta R_{t+1}^2} + z_{t+1}^2 \frac{V_{\Theta,t}^2 \tau \mu_t^2}{2R_{t+1}^2} + \frac{z_{t+1} V_{\Theta,t} \tau \mu_t}{2\tau R_{t+1}\beta_t}$$

Notice that $V_{P,t} = z_{t+1}^2 V_{\Theta,t} + z_{s,t+1}^2/\delta = z_{t+1}^2 \left(V_{\Theta,t} + \mu_{t+1}^2/\delta\right)$.[1] Using this in (4.16) can be written as

$$MB_t = \left(\frac{z_{t+1}^2 V_{\Theta,t}^2 \tau^2}{2\tau\delta R_{t+1}^2} + \beta_t \frac{V_{\Theta,t}^4}{2\tau\left(V_{\Theta,t} + \mu_{t+1}^2/\delta\right)} + \frac{V_{\Theta,t}^2}{\left(V_{\Theta,t} + \mu_{t+1}^2/\delta\right)2\tau}\right).$$

From here

$$\lim_{\beta_t \to 0} MB_t = \lim_{\beta_t \to 0}\left(\frac{z_{t+1}^2 V_{\Theta,t}^2 \tau^2}{2\tau\delta R_{t+1}^2} + \frac{V_{\Theta,t}^2}{\left(V_{\Theta,t} + \mu_{t+1}^2/\delta\right)2\tau}\right) > 0,$$

[1]For this replacement to be valid also in period T, define $\mu_{T+1} \equiv 0$

because $\lim_{\beta_t \to 0} V_{\Theta,t} = \lim_{\beta_t \to 0} \frac{1}{\alpha+\mu_1^2\delta+\ldots+\mu_t^2\delta+\beta_t} > 0$ and finite (at least $\alpha > 0$), and $z_{t+1} \in [0,1]$. At the same time

$$\lim_{\beta_t \to \infty} MB_t =$$

$$\lim_{\beta_t \to \infty} \left(\frac{z_{t+1}^2 V_{\Theta,t}^2 \tau^2}{2\tau\delta R_{t+1}^2} + \frac{V_{\Theta,t}^3}{2\tau\left(V_{\Theta,t}+\mu_{t+1}^2/\delta\right)\left(\frac{\alpha_t+\mu_t^2\delta}{\beta_t}+1\right)} + \frac{V_{\Theta,t}^2}{\left(V_{\Theta,t}+\mu_{t+1}^2/\delta\right)2\tau} \right) = 0,$$

because $\lim_{\beta_t \to \infty} V_{\Theta,t} = \lim_{\beta_t \to \infty} \frac{1}{\alpha+\mu_1^2\delta+\ldots+\mu_t^2\delta+\beta_t} = 0$.

These limits together with marginal cost and marginal benefit being continuous functions and the fact that marginal cost is either constant or increasing in $\beta_{i,t}$ implies that there exists a solution, where $\beta_t \geq 0$ for any t. If marginal cost is increasing in $\beta_{i,t}$ and does not have a fixed component, there exists an interior solution. If marginal cost has a constant component, there may be a corner solution; $\beta_t = 0$.

C.4 Sensitivity of optimal precision to parameters in one trading period model.

Marginal benefit of research in a one trading period model is given by (4.17). Noticing that $\frac{\partial V_\Theta^*}{\partial \beta^*} = -V_\Theta^{*2}(1+2\frac{\beta^*}{\tau^2}\delta)$, the derivative of marginal benefit of research (MB^*) with respect to precision β^* is

$$\begin{aligned}
\frac{\partial MB^*}{\partial \beta^*} &= \left(\frac{\tau}{\delta}V_\Theta^* + \frac{\beta^*}{\tau}V_\Theta^* + \frac{1}{2\tau}\right)\frac{\partial V_\Theta^*}{\partial \beta^*} + \frac{V_\Theta^{*2}}{2\tau} = \\
&= -V_\Theta^{*2}(1+2\tfrac{\beta^*}{\tau^2}\delta)\left(\frac{\tau}{\delta}V_\Theta^* + \frac{\beta^*}{\tau}V_\Theta^* + \frac{1}{2\tau}\right) - \frac{\beta^* V_\Theta^{*2}\delta}{\tau^3} < 0.
\end{aligned}$$

The marginal benefit of research is decreasing in β^*. Also by assumption marginal cost of research is either increasing in β^* or constant $(\partial^2\kappa(\beta^*)/(\partial\beta^*)^2 \geq 0)$ and $\frac{r^*\partial^2\kappa(\beta^*)}{(\partial\beta^*)^2} - \frac{\partial MB^*}{\partial\beta^*} > 0$.

Taking total derivatives of (4.16), when $MB_t = MB^*$ (4.17) and noticing that

$$\begin{aligned}
\frac{\partial MB^*}{d\alpha} &= -\left(\frac{\tau}{\delta}V_\Theta^* + \frac{\beta^*}{\tau}V_\Theta^* + \frac{1}{2\tau}\right)V_\Theta^{*2} < 0 \\
\frac{\partial MB^*}{d\delta} &= -\left(\frac{\tau}{\delta}V_\Theta^* + \frac{\beta^*}{\tau}V_\Theta^* + \frac{1}{2\tau}\right)\frac{\beta^{*2}}{\tau^2}V_\Theta^{*2} - \frac{\tau V_\Theta^{*2}}{2\delta^2} < 0 \\
\frac{\partial\kappa(\beta^*)}{\partial\beta^*} &\geq 0 \text{ by definition,}
\end{aligned}$$

we can express the sensitivity of optimal precision β^* to parameters of the model as follows:

$$\begin{aligned}
\frac{d\beta^*}{d\alpha} &= \frac{\frac{\partial MB^*}{d\alpha}}{\frac{r^*\partial^2\kappa(\beta^*)}{(\partial\beta^*)^2} - \frac{\partial MB^*}{\partial\beta^*}} < 0 \\
\frac{d\beta^*}{d\delta} &= \frac{\frac{\partial MB^*}{d\delta}}{\frac{r^*\partial^2\kappa(\beta^*)}{(\partial\beta^*)^2} - \frac{\partial MB^*}{\partial\beta^*}} < 0 \\
\frac{d\beta^*}{dr^*} &= -\frac{\frac{\partial\kappa(\beta^*)}{\partial\beta^*}}{\frac{r^*\partial^2\kappa(\beta^*)}{(\partial\beta^*)^2} - \frac{\partial MB^*}{\partial\beta^*}} < 0 \\
\frac{d\beta^*}{d\tau} &= -\frac{\frac{\partial MB^*}{d\tau}}{\frac{r^*\partial^2\kappa(\beta^*)}{(\partial\beta^*)^2} - \frac{\partial MB^*}{\partial\beta^*}} \text{ ambiguous,}
\end{aligned}$$

The direction of impact of risk aversion on marginal benefit is given determined by the sign of

$$\frac{\partial MB^*}{d\tau} = \left(\frac{\tau}{\delta}V_\Theta^* + \frac{\beta^*}{\tau}V_\Theta^* + \frac{1}{2\tau}\right)\frac{\beta^{*2}\delta}{\tau^3}V_\Theta^{*2} + \frac{V_\Theta^{*2}}{2\delta} - \frac{V_\Theta^*}{2\tau^2}$$

that is affected by three forces described in Section 4.3.1. Replacing in V_Θ^*, and

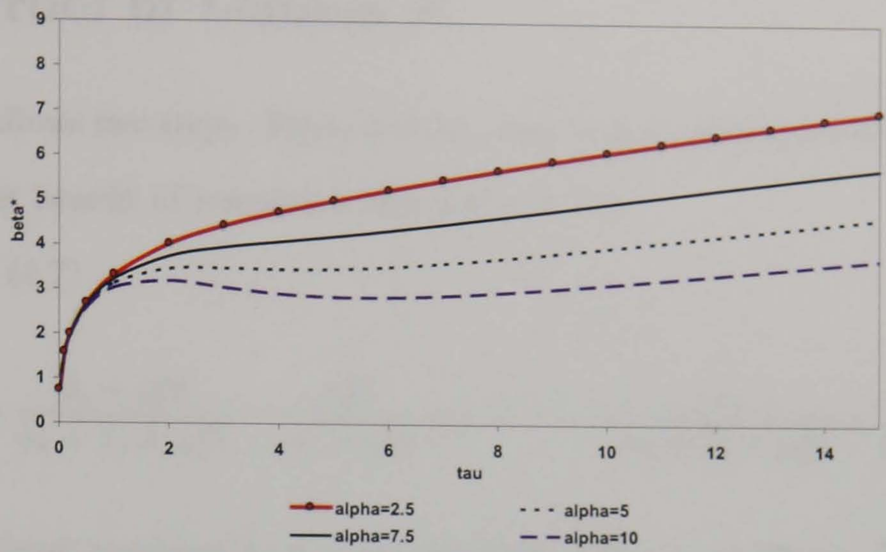

Figure C.1: Dependance of optimal precision on risk aversion at different values of public signal precision

simplifying

$$sgn\left(\frac{\partial MB^*}{d\tau}\right) = sgn\left(q_1\tau^4 + q_2\tau^2 + q_3\right),$$

where $q_1 = \alpha + \beta > 0$ and $q_2 = \delta(3\beta^* - (\alpha+\beta)^2)$, $q_3 = \beta^{*2}\delta^2(\beta^* - \alpha)$ can be positive or negative. As $\tau > 0$, $q_1\tau^4 + q_2\tau^2 + q_3 \geq 0$ implies that there are at most 3 possibilities given the other parameters of the model and the sign restrictions on all parameters:

- β^* is increasing in τ
- β^* is increasing in τ for high values of τ and decreasing otherwise.
- β^* is increasing in τ for high values of τ, decreasing in intermediate values of τ and increasing again for low values of τ.

Figure C.1 illustrates how the direction of impact of τ on β^* can depend on the value of α. (the assumptions about other parameters are the same as in Appendix C.7)

C.5 Proof of Lemma 9

The proof follows two steps. First, it shows that z_t is increasing over time. Second, it shows that benefit of research is increasing in z_{t+1}.

1) From (4.7)

$$\frac{\partial z_t}{\partial z_{t+1}} = \frac{\beta_t + \mu_t^2\delta}{\alpha_t + \beta_t + \mu_t^2\delta} - \frac{\mu_t^2\delta}{\alpha_t + \mu_t^2\delta} + 1 - 1 = -\frac{\alpha_t}{\alpha_t + \beta_t + \mu_t^2\delta} + \frac{\alpha_t}{\alpha_t + \mu_t^2\delta}.$$

Positive optimal precision $\beta_t > 0$ implies that $\frac{\alpha_t}{\alpha_t+\mu_t^2\delta} > \frac{\alpha_t}{\alpha_t+\beta_t+\mu_t^2\delta}$. Furthermore, as $0 < \frac{\alpha_t}{\alpha_t+\beta_t+\mu_t^2\delta} < \frac{\alpha_t}{\alpha_t+\mu_t^2\delta} < 1$, it is clear that

$$0 < \frac{\partial z_t}{\partial z_{t+1}} < 1.$$

Together with the terminal condition $z_{T+1} = 1$, this implies that $z_t < z_{t+1}$ for any t and z_t is increasing over time.

2) The only future variable in (4.16) is z_{t+1} and marginal benefit of research is increasing in z_{t+1} as

$$\frac{\partial MB_t}{\partial z_{t+1}} = z_{t+1}\frac{\tau V_{\Theta,t}^2}{\delta R_{t+1}^2} + 2z_{t+1}^3\beta_t\frac{V_{\Theta,t}^4}{\tau V_{P,t}^2} + z_{t+1}\frac{V_{\Theta,t}^2}{\tau V_{P,t}} > 0.$$

In equilibrium marginal cost equals to marginal benefit. Therefore, if z_{t+1} is higher, investors choose higher precision of private signal. Given that z_{t+1} is increasing over time, β_t is increasing over time as well.

C.6 Proof of Proposition 10

From (4.16) and (4.17) the research cost of one-period asset must satisfy

$$r_T \frac{\partial \kappa(\beta_i^*)}{\partial \beta_i^*}\bigg|_{\beta^*} = \frac{\tau}{2\delta} V_\Theta^{*2} + \frac{\beta^*}{2\tau} V_\Theta^{*2} + \frac{1}{2\tau} V_\Theta^*. \tag{C.3}$$

Similarly from (4.16), the research cost of T trading period asset must satisfy

$$r_T \frac{\partial \kappa(\beta_{i,T})}{\partial \beta_{i,T}}\bigg|_{\beta_T} = \frac{\tau}{2\delta} V_{\Theta,T}^2 + \frac{\beta_T}{2\tau} V_{\Theta,T}^2 + \frac{1}{2\tau} V_{\Theta,T} \tag{C.4}$$

If the extent of mis-pricing is the same, i.e. $\bar{V}_\Theta \equiv V_{\Theta,T} = V_\Theta^*$, the marginal cost of multi-period asset problem can be written as

$$r_T \frac{\partial \kappa(\beta_{i,T})}{\partial \beta_{i,T}}\bigg|_{\beta_T} = \frac{\beta_T}{2\tau} V_\Theta^{*2} + r_T \frac{\partial \kappa(\beta_i^*)}{\partial \beta_i^*}\bigg|_{\beta^*} - \frac{\beta^*}{2\tau} V_\Theta^{*2}. \tag{C.5}$$

This suggest a marginal cost function that is linear in β_T. Assuming a general cost function $\kappa(\beta_{i,t}) = K_1 \beta_{i,t}^2 + K_2 \beta_{i,t}$, a relationship between threshold values $\bar{K}_1$ and $\bar{K}_2$ that imply the same extent of mis-pricing can be found. From (C.5) and assumed cost function at $K_1 = \bar{K}_1$ and $K_2 = \bar{K}_2$

$$\begin{aligned} 2 r_T \bar{K}_1 (\beta_T - \beta^*) &= (\beta_T - \beta^*) \frac{1}{2\tau} V_\Theta^{*2} \Rightarrow \\ V_\Theta^* &= \sqrt{4 r_T \bar{K}_1 \tau}. \end{aligned}$$

Replacing this in (C.3) implies (4.19)

For deviation of K_2 and K_1 from threshold values, consider first a small deviation of $K_1 = \bar{K}_1 + \Delta K_1$ when $K_2 = \bar{K}_2$ and $\Delta K_1 > 0$. From (C.3), (C.4), (4.19) and $\bar{K}_1 = \frac{\bar{V}_\Theta^2}{4\tau r_T}$ we obtain

$$\begin{aligned}
2r_T\Delta K_1 &= \frac{\tau(V_{\Theta,T}^2 - \bar{V}_\Theta^2)}{2\delta\beta_T} + \frac{(V_{\Theta,T}^2 - \bar{V}_\Theta^2)}{2\tau} + \frac{(V_{\Theta,T} - \bar{V}_\Theta)}{2\tau\beta_T} = \\
&= \frac{\tau(V_\Theta^{*2} - \bar{V}_\Theta^2)}{2\delta\beta^*} + \frac{(V_\Theta^{*2} - \bar{V}_\Theta^2)}{2\tau} + \frac{(V_\Theta^* - \bar{V}_\Theta)}{2\tau\beta^*}.
\end{aligned}$$

As better quality public information (higher α_t by definition) implies less incentives to research, $\beta^* > \beta_T$. Given this, the above equality holds if $V_\Theta^* - \bar{V}_\Theta > V_{\Theta,T} - \bar{V}_\Theta \Rightarrow V_\Theta^* > V_{\Theta,T}$ and the extent of mis-pricing is higher for the one-period asset.

Next, consider $K_1 = \bar{K}_1$ when $K_2 = \bar{K}_2 + \Delta K_2$ and $\Delta K_2 > 0$. Similarly from (C.3), (C.4), (4.19) and $\bar{K}_1 = \frac{\bar{V}_\Theta^2}{4\tau r_T}$ we obtain

$$\begin{aligned}
r_T\Delta K_2 &= \frac{\tau}{2\delta}(V_{\Theta,T}^2 - \bar{V}_\Theta^2) + \frac{\beta_T}{2\tau}(V_{\Theta,T}^2 - \bar{V}_\Theta^2) + \frac{1}{2\tau}(V_{\Theta,T} - \bar{V}_\Theta) = \\
&= \frac{\tau}{2\delta}(V_\Theta^{*2} - \bar{V}_\Theta^2) + \frac{\beta^*}{2\tau}(V_\Theta^{*2} - \bar{V}_\Theta^2) + \frac{1}{2\tau}(V_\Theta^* - \bar{V}_\Theta)
\end{aligned}$$

Given that $\beta^* > \beta_T$, it must hold that $V_\Theta^* - \bar{V}_\Theta < V_{\Theta,T} - \bar{V}_\Theta \Rightarrow V_\Theta^* < V_{\Theta,T}$ and the extent of mis-pricing is higher for the one-period asset.

C.7 Dependence of optimal precision on the exogenous parameters of the model in multi-period setting.

The assumptions of the baseline case are the following: $\alpha = 2.5$, $\delta = 2.5$, $\tau = 6$, $K_1 = 0.00225$ and $r_t = 1$ for any t. The value of fundamental $\Theta = 1.2$ and mean of public signal $y = 1$. In the graph about asset prices, all noise trading shocks are zero.

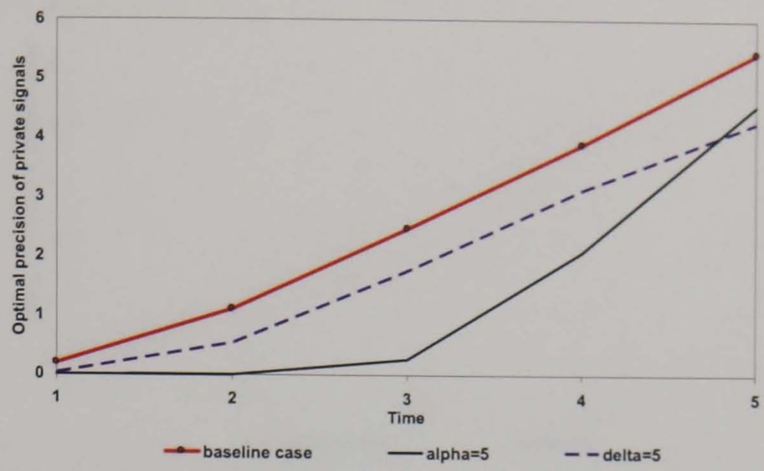

Figure C.2: Sensitivity of optimal precision of private signal to changes in precision of public signal and noise trading.

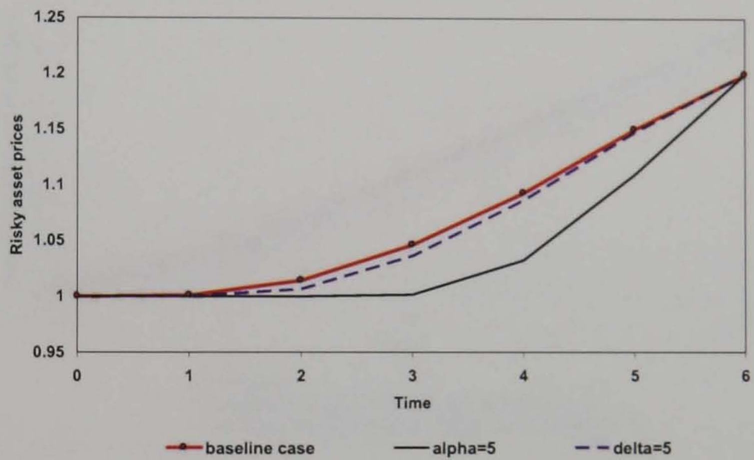

Figure C.3: Adjustment of prices with different precision of public signal and noise trading

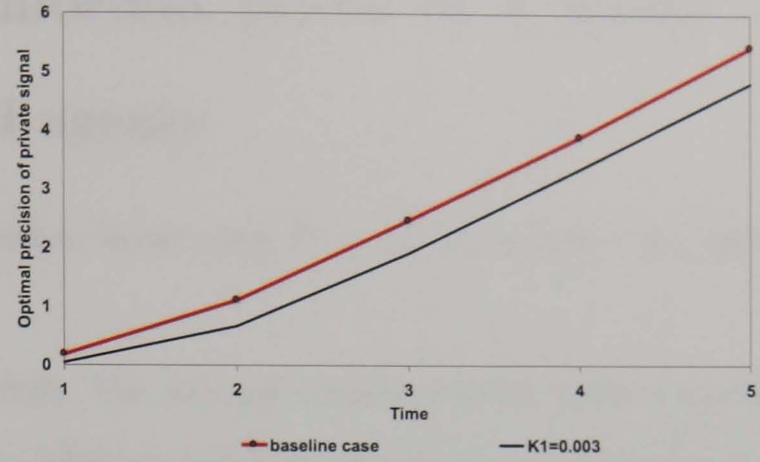

Figure C.4: Sensitivity of optimal precision of private signal tochanges in information cost..

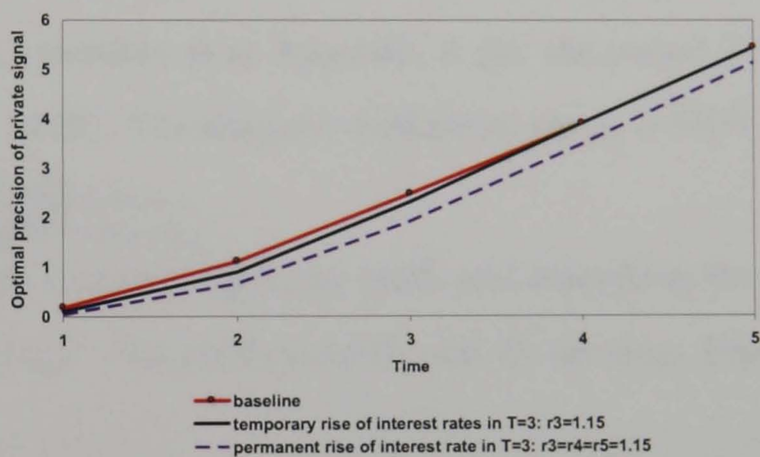

Figure C.5: Sensitivity of optimal precision of private signal to changes in risk-free interest rate.

C.8 Equilibrium prices in a model with long-lived agents

Assume prices follow linear rules $P_1 = \eta_1(\lambda_1 y + \mu_1 \Theta - s_1)$ and $P_2 = \eta_2(\lambda_2 y_2 + \mu_2 \Theta - s_2)$.

Second period. The solution remains similar to the short-lived agents problem with the only difference being the fact that investors have an additional private signal from the first trading period. Therefore, $V_{i,\Theta,2} = (\alpha_2 + \mu_2^2\delta + \beta_{i,2} + \beta_{i,1})$ and $E[\Theta|\Omega_{i,2}] = (\alpha_2 y_2 + \mu_2^2 \delta \tilde{P}_2 + \beta_{i,2}\nu_{i,2} + \beta_{i,1}\nu_{i,1})V_{i,\Theta,2}$. As before, the in research decision stage all investors are identical: $\beta_{i,1} = \beta_1$ and $\beta_{i,2} = \beta_2$ for every i.

The solution procedure as in Appendix A (for the period T) gives P_2, $\grave{z}_2$, $\grave{z}_{s,2}$ as in (4.21) and (4.22). The assumed coefficients are $\mu_2 = \frac{\beta_1+\beta_2}{\tau}$, $\lambda_2 = \frac{\alpha_2\tau}{\tau^2+\delta(\beta_1+\beta_2)}$ and $\eta_2 = \frac{\tau(\tau^2+\delta(\beta_1+\beta_2))}{\alpha_2+\frac{(\beta_1+\beta_2)^2}{\tau^2}\delta+\beta_{i,2}+\beta_{i,1}}$.

First period. Demand is given by (4.20) and depends on the joint distribution of P_2 and $E[\Theta|\Omega_{i,2}]$. The coefficients G_i and Q_i are from Brown and Jennings (1989).

$$
\begin{aligned}
G_i &= \frac{1 + V_{\Theta,i,2}^{-1} L_i}{M_i V_{\Theta,i,2}^{-1} + \mathrm{Var}(P_2|\Omega_{i,1})} \\
Q_i &= \frac{\mathrm{Var}(P_2|\Omega_{i,1}) - \mathrm{Cov}(P_2, E[\Theta|\Omega_{i,2}]|\Omega_{i,1})}{\left(M_i V_{\Theta,i,2}^{-1} + \mathrm{Var}(P_2|\Omega_{i,1})\right) V_{\Theta,i,2}} \\
M_i &\equiv \mathrm{Var}(P_2|\Omega_{i,1})\,\mathrm{Var}(E[\Theta|\Omega_{i,2}]|\Omega_{i,1}) - [\mathrm{Cov}(P_2, E[\Theta|\Omega_{i,2}]|\Omega_{i,1})]^2 \\
L_i &\equiv (\mathrm{Var}(P_2|\Omega_{i,1}) + \mathrm{Var}(E[\Theta|\Omega_{i,2}]|\Omega_{i,1}) - 2\,\mathrm{Cov}(P_2, E[\Theta|\Omega_{i,2}]|\Omega_{i,1}))
\end{aligned}
$$

In first trading period $V_{i,\Theta,1} = \left(\alpha + \mu_1^2\delta + \beta_{i,1}\right)^{-1}$ and $E[\Theta|\Omega_{i,1}] = \left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \beta_{i,1}\nu_{i,1}\right) V_{i,\Theta,1}$. Using this, expressions for P_2, $E[\Theta|\Omega_{i,2}]$ and $\grave{z}_{s,2} = \frac{\grave{z}_2}{\mu_2}$,we obtain the following expressions for the variance and covariance structure of P_2 and $E\left[\Theta|\Omega_{i,2}\right]$

$$\begin{aligned}
&\mathrm{Var}(P_2|\Omega_{i,1}) = \grave{z}_2^2\left(V_{i,\Theta,1} + \frac{1}{\mu_2^2\delta}\right)\\
&\mathrm{Var}(E\left[\Theta|\Omega_{i,2}\right]|\Omega_{i,1}) = V_{i,\Theta,2}V_{i,\Theta,1}\left(\mu_2^2\delta + \beta_{i,2}\right)\\
&\mathrm{Cov}(P_2, E\left[\Theta|\Omega_{i,2}\right]|\Omega_{i,1}) = \grave{z}_2 V_{i,\Theta,1}
\end{aligned}$$

From this $M_i = \grave{z}_2^2\frac{\beta_{i,2}}{\mu_2^2\delta}V_{i,\Theta,2}V_{i,\Theta,1}$. It implies that if no research is done in period 2 ($\beta_{i,2} = 0$), P_2 and $E\left[\Theta|\Omega_{i,2}\right]$ are perfectly correlated conditional on investor's information in period 1. Otherwise, they are positively correlated. Using this, we can simplify

$$\begin{aligned}
G_i &= V_{\Theta,i,1}^{-1} + \frac{\mu_2^2\delta(1-\acute{z}_2)^2}{\acute{z}_2^2}\\
Q_i &= V_{\Theta,i,1}^{-1} - \frac{\mu_2^2\delta\left(1-\acute{z}_2\right)}{\acute{z}_2}\\
L_i &= V_{i,\Theta,1}\left(1-\grave{z}_2\right)^2 + \grave{z}_2^2\frac{1}{\mu_2^2\delta} - V_{i,\Theta,2}
\end{aligned} \tag{C.6}$$

With symmetric equilibrium conditions ($\beta_{i,2} = \beta_2$ and $\beta_{i,2} = \beta_2$), it must hold that $V_{i,\Theta,1} = V_{\Theta,1}$ and the subscript "$_i$" can be dropped from coefficients G and Q. To derive the equilibrium prices, we also need investor i's expected value of P_2

$$\begin{aligned}
E\left[P_2|\Omega_{i,1}\right] &= E\left[(1-\acute{z}_2)y_2 + \acute{z}_2\Theta - \acute{z}_{s,2}s_2|\Omega_{i,1}\right] = (1-\acute{z}_2)y_2 + \acute{z}_2 E[\Theta|\Omega_{i,1}] =\\
&= (1-\acute{z}_2)\frac{\alpha y + \mu_1^2\delta\tilde{P}_1}{\alpha+\mu_1^2\delta} + \acute{z}_2\left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \beta_1\nu_{i,1}\right)V_{\Theta,1} =\\
&= (1-\acute{z}_2)\frac{\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1}{\alpha+\mu_1^2\delta} + \acute{z}_2\left(\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1 + \beta_1\nu_{i,1}\right)V_{\Theta,1}
\end{aligned}$$

and expected value of $E\left[\Theta|\Omega_{i,2}\right]$

$$\begin{aligned}
& E\left[E\left[\Theta|\Omega_{i,2}\right]|\Omega_{i,1}\right] = \\
= \; & E\left[(\alpha_2 y_2 + \mu_2^2\delta\tilde{P}_2 + \beta_2\nu_{i,2} + \beta_1\nu_{i,1})V_{\Theta,2}|\Omega_{i,1}\right] = \\
= \; & \left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \left(\mu_2^2\delta + \beta_2\right) E[\Theta|\Omega_{i,1}] + \beta_1\nu_{i,1}\right) V_{\Theta,2} \\
= \; & \left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \left(\mu_2^2\delta + \beta_2\right)\left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \beta_1\nu_{i,1}\right) V_{\Theta,1} + \beta_1\nu_{i,1}\right) V_{\Theta,2} = \\
= \; & (\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1 + \beta_1\nu_{i,1})V_{\Theta,1}
\end{aligned}$$

Using these, aggregating, equating demand with supply and simplifying, we obtain

$$\begin{aligned}
P_1 \; = \; & y\left(\tfrac{G-Q}{G}\left((1-\acute{z}_2)\tfrac{\alpha}{\alpha+\mu_1^2\delta} + \acute{z}_2\alpha V_{\Theta,1}\right) + \tfrac{Q}{G}\alpha V_{\Theta,1}\right) + \\
+ \; & \Theta\left(\tfrac{G-Q}{G}\left((1-\acute{z}_2)\tfrac{\mu_1^2\delta}{\alpha+\mu_1^2\delta} + \acute{z}_2\left(\mu_1^2\delta + \beta_1\right) V_{\Theta,1}\right) + \tfrac{Q}{G}(\mu_1^2\delta + \beta_1\Theta)V_{\Theta,1}\right) + \\
- \; & s_1\left(\tfrac{G-Q}{G}\left((1-\acute{z}_2)\tfrac{\mu_1\delta}{\alpha+\mu_1^2\delta} + \acute{z}_2\mu_1\delta V_{\Theta,1}\right) + \tfrac{Q}{G}\mu_1\delta V_{\Theta,1} + \tfrac{\tau}{G}\right).
\end{aligned}$$

From here, we can $\acute{z}_1$ and $\acute{z}_{s,1}$ can be presented as functions of $\acute{z}_2$ and $\acute{z}_{s,2}$ as in (4.22). Coefficients solve $\eta_1 = \acute{z}_{s,1}$, $\lambda_1 = \frac{(1-\acute{z}_1)}{\acute{z}_{s,1}}$, $\mu_1 = \frac{\acute{z}_1}{\acute{z}_{s,1}}$.

C.9 Proof of Proposition 11

In the spirit of Brown and Jennings (1989), the utility in period 1 can be represented as

$$\begin{aligned}
U_{1,i} \; = \; & h_{i,1}(E[P_2|\Omega_{i,1}] - P_1) + h_{i,1}\frac{Q_i}{G_i}E\left[E[\Theta|\Omega_{i,2}] - P_2|\Omega_{i,1}\right] - \frac{\tau}{2}h_{i,1}^2\frac{1}{G_i} + \\
& + \frac{\left(E\left[E[\Theta|\Omega_{i,2}]|\Omega_{i,1}\right] - E[P_2|\Omega_{i,1}]\right)^2}{2\tau(L_i + V_{i,\Theta,2})} - rw - \kappa(\beta_{i,1}) - \kappa(\beta_{i,2}),
\end{aligned}$$

where G_i, Q_i and L_i are given by (C.6). Using L_i from that, replacing in the demand in period 1 (4.20) and taking expectations, the research cost decision problem becomes

$$\max_{\beta_{i,1},\beta_{i,2}} \quad E[U_{1,i}|\Omega_{-1}] = \frac{E[(E[P_2|\Omega_{i,1}] - P_1)^2 \, |\Omega_{-1}]}{2\tau} G_i + \\ + \frac{E[(E[P_2|\Omega_{i,1}] - P_1)\,(E[E[\Theta|\Omega_{i,2}]] - E[P_2|\Omega_{i,1}])|\Omega_{-1}]}{\tau} Q_i + \\ + \frac{E[(E[E[\Theta|\Omega_{i,2}]] - E[P_2|\Omega_{i,1}])^2|\Omega_{-1}]}{2\tau} \left(\frac{Q_i^2}{G_i} + \frac{1}{(V_{i,\Theta,1}\,(1-\grave{z}_2)^2 + \grave{z}_2^2 \frac{1}{\mu_2^2\delta})} \right),$$

First, it is easy to show that as in short-lived agents' case

$E[(E[P_2|\Omega_{i,1}] - P_1)\,|\Omega_{-1}] = 0$ and $E[(E[E[\Theta|\Omega_{i,2}]] - E[P_2|\Omega_{i,1}])|\Omega_{-1}] = 0$ and therefore the benefit from research depends on the variance and covariance of expected returns between trading periods and liquidation date.

Expected value of period 2 prices in period 1 is $E[P_2|\Omega_{i,1}] = (1-\acute{z}_2)\frac{\alpha y + \mu_1^2\delta\tilde{P}_1}{\alpha+\mu_1^2\delta} + \acute{z}_2\left(\alpha y + \mu_1^2\delta\tilde{P}_1 + \beta_{i,1}\nu_{i,1}\right)V_{i,\Theta,1} =$
$= (1-\acute{z}_2)\frac{\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1}{\alpha+\mu_1^2\delta} + \acute{z}_2\left(\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1 + \beta_{i,1}\Theta + \beta_{i,1}\varepsilon_{i,1}\right)V_{i,\Theta,1}$ and expected value in period 1 of expected fundamental value is

$E\left[E\left[\Theta|\Omega_{i,2}\right]|\Omega_{i,1}\right] = (\alpha y + \mu_1^2\delta\tilde{P}_1 + \beta_{i,1}\nu_{i,1})V_{i,\Theta,1} =$
$= (\alpha y + \mu_1^2\delta\Theta - \mu_1\delta s_1 + \beta_{i,1}\Theta + \beta_{i,1}\varepsilon_{i,1})V_{i,\Theta,1}$. Using these and P_1 from (4.21).

After calculating these variances and covariance and plugging it back to the utility function and simplifying, we obtain

$$E[U_{1,i}|\Omega_{-1}] = \\ = \frac{1}{2\tau}\left[\begin{array}{l} G_i\left(\acute{z}_{i,1} - \acute{z}_1\right)^2\frac{1}{\alpha} + \beta_{i,1}V_{i,\Theta,1}^2\frac{\left(\acute{z}_2 G_i + (1-\acute{z}_2)Q_i\right)^2}{G_i} + \\ + G_i\left(\left(1-\acute{z}_{i,1}\right)\frac{\mu_1\delta}{\alpha} - \acute{z}_{s,1}\right)^2\frac{1}{\delta} + \\ + \frac{(1-\acute{z}_2)^2 V_{i,\Theta,1}\beta_{i,1}}{\left(V_{i,\Theta,1}(1-\grave{z}_2)^2 + \grave{z}_2^2\frac{1}{\mu_2^2\delta}\right)\left(\alpha+\mu_1^2\delta\right)} \end{array} \right] +$$

Where $\acute{z}_{i,1} \equiv \frac{G_i - Q_i}{G_i}\left[(1-\acute{z}_2)\frac{\mu_1^2\delta}{\alpha+\mu_1^2\delta} + \acute{z}_2(\beta_{i,1}+\mu_1^2\delta)V_{i,\Theta,1}\right] + \frac{Q_i}{G_i}(\beta_{i,1}+\mu_1^2\delta)V_{i,\Theta,1}$.

Research in period 2. Given that $V_{i,\Theta,1}$ does not depend on $\beta_{i,2}$, from (C.6), it is clear that also G_i, Q_i and $\acute{z}_{i,1}$ do not depend on $\beta_{i,2}$. Therefore marginal benefit on research (∂) in period 2 is zero for a long-lived investor before first trading period. The first order condition

$$\frac{\partial\kappa(\beta_{i,2})}{\partial(\beta_{i,2})} = 0$$

implies that optimal $\beta_{i,2} = 0$ as long as marginal cost $\frac{\partial\kappa(\beta_{i,2})}{\partial(\beta_{i,2})}$ is a function of $\beta_{i,2}$.

Research in period 1. From (C.6) $\frac{\partial G_i}{\partial\beta_{i,1}} = \frac{\partial Q_i}{\partial\beta_{i,1}} = 1$. Given this and $\frac{\partial V_{i,\Theta,1}}{\partial\beta_{i,1}}$Investor i's marginal benefit of research in period 1 is

$$MB_1(\beta_{i,1}) = \frac{1}{2\tau}\left(\begin{array}{l} \frac{(\acute{z}_{i,1}-\acute{z}_1)^2}{\alpha} + \frac{2G_i(\acute{z}_{i,1}-\acute{z}_1)^2}{\alpha}\frac{\partial\acute{z}_{i,1}}{\partial\beta_{i,1}} + V_{i,\Theta,1}^2\frac{(\acute{z}_2G_i+(1-\acute{z}_2)Q_i)^2}{G_i} + \\ -2\beta_{i,1}V_{i,\Theta,1}^3\frac{(\acute{z}_2G_i+(1-\acute{z}_2)Q_i)^2}{G_i} + 2\beta_{i,1}V_{i,\Theta,1}^2\frac{(\acute{z}_2G_i+(1-\acute{z}_2)Q_i)}{G_i} + \\ -\beta_{i,1}V_{i,\Theta,1}^2\frac{(\acute{z}_2G_i+(1-\acute{z}_2)Q_i)^2}{G_i^2} + \left((1-\acute{z}_{i,1})\frac{\mu_1\delta}{\alpha} - \acute{z}_{s,1}\right)^2\frac{1}{\delta} + \\ -\frac{2G_i\left((1-\acute{z}_{i,1})\frac{\mu_1\delta}{\alpha}-\acute{z}_{s,1}\right)}{\delta}\frac{\mu_1\delta}{\alpha}\frac{\partial\acute{z}_{i,1}}{\partial\beta_{i,1}} + \frac{(1-\acute{z}_2)^2V_{i,\Theta,1}^2}{\left(V_{i,\Theta,1}(1-\grave{z}_2)^2+\grave{z}_2^2\frac{1}{\mu_2^2\delta}\right)} + \\ +\frac{(1-\acute{z}_2)^4V_{i,\Theta,1}^3\beta_{i,1}}{\left(V_{i,\Theta,1}(1-\grave{z}_2)^2+\grave{z}_2^2\frac{1}{\mu_2^2\delta}\right)^2(\alpha+\mu_1^2\delta)} \end{array}\right),$$

where

$$\begin{aligned} \frac{\partial\acute{z}_{i,1}}{\partial\beta_{i,1}} &= (1-\acute{z}_2)\left((\beta_{i,1}+\mu_1^2\delta)V_{i,\Theta,1} - \frac{\mu_1^2\delta}{\alpha+\mu_1^2\delta}\right)\frac{G_i-Q_i}{G_i^2} \\ &\quad + \left(\acute{z}_2 + (1-\acute{z}_2)\frac{Q_i}{G_i}\right)\alpha V_{i,\Theta,1}\left(1-\mu_1^2\delta V_{i,\Theta,1}\right). \end{aligned}$$

In symmetric equilibrium $\beta_{i,1} = \beta_1$ for every i. This implies $V_{i,\Theta,1} = V_{\Theta,1}$, $G_i = G$, $Q_i = Q$, $\acute{z}_{i,1} = \acute{z}_1$ and $(1-\acute{z}_{i,1})\frac{\mu_1\delta}{\alpha} - \acute{z}_{s,1} = -\frac{\tau}{G}$. Also, From (C.6)

$G\acute{z}_2 + (1-\acute{z}_2)Q = V_{\Theta,1}^{-1}$, and $G - Q = \mu_2^2 \delta \frac{(1-\acute{z}_2)}{\acute{z}_2^2}$. We can simplify the equilibrium marginal benefit.

$$
MB_1 = \frac{1}{2\tau}\left(\begin{array}{l}
\frac{1}{G} - \beta_1 \frac{1}{G^2} + \frac{\tau^2}{G^2\delta} + 2\tau\mu_1 \frac{(1-\acute{z}_2)^2}{\acute{z}_2^2} \frac{\beta_1 \mu_2^2 \delta}{\alpha+\mu_1^2\delta} V_{\Theta,1} \frac{1}{G^2} \\
+\frac{2\tau\mu_1}{G} V_{\Theta,1}(\alpha+\beta_1) + \frac{V_{\Theta,1}^2}{\left(V_{\Theta,1} + \frac{\acute{z}_2^2}{(1-\acute{z}_2)^2}\frac{1}{\mu_2^2\delta}\right)} + \\
+\frac{V_{\Theta,1}^3 \beta_1}{\left(V_{\Theta,1} + \frac{\acute{z}_2^2}{(1-\acute{z}_2)^2}\frac{1}{\mu_2^2\delta}\right)^2 \left(\alpha+\mu_1^2\delta\right)}
\end{array}\right).
$$

Given that using (C.6) $\frac{1}{G} - \beta_1 \frac{1}{G^2} = \frac{\alpha + \mu_1^2\delta + \frac{\mu_2^2 \delta (1-\acute{z}_2)^2}{\acute{z}_2^2}}{G^2} > 0$ and all other variables in the expression for MB_1 are positive $MB_1 > 0$. If marginal research cost is an increasing function of $\beta_{i,1}$, this necessarily implies that optimal $\beta_{i,1} = \beta_1 > 0$, if $\kappa\left(\beta_{i,t}\right) = K_1 \beta_{i,t}^2$ as assumed.

C.10 Responsiveness of optimal precision and asset prices to parameters of the long-lived agents extension

The assumptions about baseline parameters of the model are the same as in Appendix C.7. Prices are reported assuming supply shocks at their zero mean in both periods. As before the fundamental $\Theta = 1.2$ and public signal $y = 1$

	β	P_1	P_2
baseline	5.33	1.132	1.149
$\alpha = 5$	4.07	1.080	1.102
$\delta = 5$	4.27	1.120	1.146
$\tau = 10$	6.25	1.141	1.149
$K_1 = 0.003$	4.69	1.126	1.143

Table C.1: Impact of parameters on the model in long-lived agents' setup

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