The London School of Economics and Political Science

Financial Contracts, Bankruptcy and Product Market Competition

Thesis submitted by

Paul Emil Marie Povel

to the

University of London

for the degree of

Doctor of Philosophy

May 1998

Abstract

This thesis consists of three self–contained game–theoretic analyses of the contractual relationship between borrowers and lenders. A key element of this relationship is asymmetric information: the players tend to have superior information concerning their strategic variables than their opponents. Optimal contracts for different environments are derived and studied. They include 'bankruptcy' games, which are designed to structure the parties' bargaining under certain circumstances.

The first chapter questions the idea that being a unique lender to a firm is better than sharing the lender's role. Even borrowers with poor prospects will apply for loans, if their main goal is to be financed, and re–financed if necessary. With one lender, refinancing is always provided once former loans are 'sunk'. With two lenders, the situation may be different: inefficient negotiations have to determine how the overall loss is allocated. Some borrowers may therefore not be refinanced, and this may keep borrowers with poor prospects from applying for loans.

The second chapter extends this model by adding a timing dimension: a borrower finds out about poor prospects earlier than his lender. He can ask for refinancing, or simply 'wait and pray'. Either 'soft' contracts or 'tough' contracts may be optimal contracts: 'soft' contracts treat the borrower well if he asks for refinancing, while 'tough' contracts don't (and the lender will not have the option of refinancing). 'Hybrid' contracts are strictly worse than the two 'pure' types. From this we draw conclusions for the design of bankruptcy laws, and for empirical work on bankruptcy.

The third chapter analyses the interdependence of financial and production decisions. Debt contracts are frequently thought to lead to excessive risk taking — in a Cournot setup this means excessive production. At the same time, debt is a costly type of financing, which should reduce production. This conflict is analysed in a setting which allows to endogenise 'debt' contracts. The main result is that there is no excessive production, and financial constraints reduce output. However, for large levels of 'inherited' debt, it may be that output increases in the level of debt.

Contents

List of Figures

Acknowledgements

During my postgraduate studies I had the opportunity to meet many fellow economists, both at the L.S.E. and during my travels and research stays outside the L.S.E. From all of them I learnt interesting and important lessons about economics, and about being an economist. Patrick Bolton and Martin Hellwig were undoubtedly the most influential of them all. Both have greatly supported me over the past years, patiently waiting for their advice and hints to bear fruit in my work.

Patrick was a wonderful thesis supervisor, more patient than one could possibly imagine, always full of energy and ideas, always interested, no matter how pressing other duties were. He has read many versions of all parts of this thesis, provided comments and corrections, and pointed out many difficulties. I have always admired his ability, not only to quickly grasp jumbled ideas, but to expand them immediately, and to share his immense creativity with whoever he was speaking or listening to. My most enduring impression of him is the 'human touch' that he brought into our 'cooperation': always more a friend than a teacher, and always more a motivator than a supervisor; he is and endowed with an enthusiasm, that is simply contagious.

Martin Hellwig has always been a source of wisdom and of sharp criticism. His careful and precise advice and comments have supported me since I left Basel, and I am grateful for them. He encouraged me to learn economic theory 'properly', by going to Bonn and joining the European Doctoral Program in Quantitative Economics (E.D.P.). He also had the tedious task of writing many references for me, most of them for scholarships, and I can't imagine how far I would have got without his help. Finally, his support was accentuated during the last months of my PhD, and strongly influenced the last chapter of this thesis. He invited me to come to Mannheim during this time, and by asking important and difficult questions he helped to greatly improve both my understanding of the model and its exposition.

Many thanks are due to everybody at the institutes which welcomed me during these years: (in chronological order) the University of Bonn, the L.S.E., the Financial Markets Group at the L.S.E., E.C.A.R.E., D.U.L.B.E.A. and C.E.M.E.

in Brussels, and the University of Mannheim.

Urs Schweizer from the *Wirtschaftspolitsche Abteilung* in Bonn was doubly helpful, firstly as my supervisor during the first two years of studies, and secondly as coordinator of the E.D.P. He has the difficult job of integrating new students into the doctoral programme in Bonn, and I admire him for the energy he puts into this task. He made sure that I was admitted to the programme, and I am thankful for the trust he had in me.

At the L.S.E., John Moore took over as supervisor when Patrick fell ill. My discussions with him have always been much more helpful than he will ever realise. He also helped my research in a very concrete way, with the offer to accompany him, Philippe Aghion and Oliver Hart, when they were visiting insolvency practitioners in London during a week, to discuss both insolvency practice and their own proposal for a new insolvency procedure. As he said himself, this was a unique opportunity to be in contact with both theory and practice, which was very inspiring.

At the end of my first year at the L.S.E., David Webb welcomed me at the Financial Markets Group, and I could enjoy not only its excellent (and constantly improving) infrastructure, but also the team–spirit that he had managed to build up. David's sense of humour was as important in achieving this success as his determination to do a good job, both as a teacher and as head of the F.M.G.

The third chapter of this thesis is joint work with Michael Raith, and grew out of a larger research project that I started with him. Most of the work was carried out at the European Center for Advanced Research in Economics (E.C.A.R.E.) at the Université Libre de Bruxelles. I am grateful to everybody at E.C.A.R.E. and at the Department of Economics of the U.L.B., for making this research stay successful, in particular Mathias Dewatripont.

Although the research project has turned out to be rather complex, I have benefited from cooperation with Michael. I learnt much about Industrial Organization during this time, and Michael also volunteered to work through earlier versions of the first two chapters of this thesis, raising numerous valid and very helpful objections.

The final work on the thesis was carried out at the Department of Economics of the University of Mannheim. I am grateful to Martin Hellwig both for offering me a place in his institute, and for very substantial criticisms on earlier versions of the third chapter, that helped considerably to clarify my ideas and to structure the analysis of the model.

Thanks are also due to Philippe Aghion, Avner Shaked and Josef Zechner, for their interest in my work, and for inspiring and very motivating discussions.

The work on this thesis would have been impossible without the generous financial assistance that I received during these years from (in chronological order) the *Stipendienkommission Basel–Landschaft*, the Swiss National Science Foundation, the *Stiftung zur Förderung der jungen WissenschaftlerInnen an der Universität Basel*, the *Stiftung der Basler Kantonalbank*, the *Freiwillige Akademische Gesellschaft Basel* (all in Switzerland), the L.S.E. and the European Union (through both its Human Capital and Mobility Program and the European Doctoral Program in Quantitative Economics). I have also greatly benefited from financial assistance from the University of Bonn, the Financial Markets Group at the L.S.E., and the University of Mannheim. Furthermore, the University of Bonn, the European Doctoral Program in Quantitative Economics, the Jerusalem Summer School in Economic Theory, the International Summer School in Wallerfangen/Saar, the Department of Economics at the L.S.E., the Financial Markets Group at the L.S.E., *Studienzentrum Gerzensee* and the Center for Economic Policy Research (C.E.P.R.) facilitated my participation at several workshops and conferences.

Introduction

The purpose of this thesis is to explore *strategic* aspects of bankruptcy: problems of bargaining power outside of and inside bankruptcy, informational problems, and interdependencies of financial decisions or situations with the investment or output market decisions of firms. A key element for the analysis is asymmetric information: it limits the efficiency of using contracts to determine what actions economic agents agree to take under what circumstances. Bankruptcy is analysed as part of a 'wider' contract, which is not 'complete' (i.e. some relevant clauses that would improve all parties' payoffs cannot be added to the contract), and problems of bargaining power in possible renegotiations are found to have a strong impact on what types of contract are optimal in equilibrium.

Bankruptcy is an important element of the business world, not only because of the large number of bankruptcies that happen every year[1], but also because of its sometimes drastic consequences. For a small firm, bankruptcy will typically mean "liquidation": a buyer is found for the assets of the firm, jobs will be lost and creditors will be repaid less than they are owed. Large firms may do better: being "too big to fail", their existence will rarely be endangered. Nevertheless, bankruptcy will imply that financial claims are not repaid, and that jobs are lost, frequently including those of the firm's managers.

Bankruptcies can get quite some media coverage. One factor that determines the degree of attention is the size of the firms, or the number of jobs that may be lost. The interest may also be motivated by the causes of a bankruptcy, e.g.

[1] The *Statistisches Bundesamt* reports more than 19'000 business insolvencies in 1997 in the area of the former F.R.G., with aggregate liabilities of \$15,5bn (press release, 5. March 1998). For the U.S., the Financial Times (3.3.1998) reports more than 83'000 businesses failing, with liabilities over \$37bn.

serious fraud (as in the case of Barings Bank), or major management mistakes (as in the case of Herstatt Bank in Germany). Additionally, the U.S. have experienced some spectacular cases: the U.S. reorganisation procedure,[2] *Chapter 11*, protects firms (and their management teams) from their creditors to an extent unseen in other countries[3]. In some cases Chapter 11 may have been abused to achieve other goals than dealing with inadequate balance sheets (see Delaney (1992)): Texaco reduced damages awarded to a competitor by a court; Manville dealt with product liability charges; Continental Airlines 'renegotiated' union contracts.[4]

Bankruptcy plays an important role in several branches of economic theory. At a very general level, it is an essential ingredient to the idea that 'market forces' ensure that the assets of an economy are brought to their most productive use. In such a survival–of–the–fittest view, firms have to compete for the assets they want to use, as well as for the customers that buy their products. Those which are inefficient users of some assets will be driven into bankruptcy, while the best users of an economy's assets survive. A failure by the bankruptcy mechanism to sort out viable from nonviable companies will lead to a suboptimal use of an economy's assets. The investments that are the easiest to finance may not be the most efficient ones, and assets that are locked in failed investments may not be freed for more efficient users.

From a contract–theoretic (as well as legal) perspective, bankruptcy laws are important because they affect the respective rights and bargaining positions of contracting parties. For example, it is very hard to contract around bankruptcy laws: they are not default options, which become rules if contracts do not specify alternatives, but instead are mandatory rules, which cannot be 'contracted away'. Similarly, decisions are made by voting, and dissenting minorities may be forced

[2] U.S. bankruptcy laws are divided in "Chapters". Roman figures (e.g. Chapter X, Chapter XI) refer to the 'Chandler Act' of 1938, which was a major revision of the 'Bankruptcy Act' of 1898. Arabic figures (e.g. Chapter 7, Chapter 11) refer to the 'Bankruptcy Code' of 1978.

[3] For a concise survey of the U.S. Bankruptcy Code see Section 2 in Senbet and Seward (1995).

[4] Additionally, Chapter 11 cases have attracted attention because of their costliness. It is not clear, however, whether these costs are really significantly different from those of other major restructurings, e.g. liquidations, mergers or going publics.

to back down.

Furthermore, the bargaining positions of contracting parties may be different inside bankruptcy than outside of it. In Chapter 11, for instance, a bankrupt debtor receives not only protection from all claimants, but also the possibility to continue running the firm, using new super–priority "debtor–in–possession" financing. This may be different from the bargaining position of a debtor who tries to renegotiate his obligations privately. Not surprisingly, this has lead to complaints about Chapter 11 in the business press.[5]

Besides influencing the bargaining positions, formal bankruptcy procedures can also affect the objectives of the contracting parties. This happens because decisions are made by majority–voting, and minorities (even absent or unknown) may be bound by such an agreement. Some classes may even have an incentive to collude, to transfer wealth to themselves from other parties[6]. While in a transaction–cost free world there cannot be disagreement about any decision, in bankruptcy each party has an incentive to care for its own payoff, only. For instance, secured creditors may favour the liquidation of a firm over its (risky) continuation, while unsecured creditors (and equityholders) might have opposite preferences.

Finally, bankruptcy laws have strong effects on firms' investment behaviour. This is now well understood, and referred to as either "risk–shifting" or "debt overhang" problems. Both describe agency problems that may arise if a firm has issued both debt and equity. The key idea is uncertainty: the use of debt financing has the result that the payoff to equityholders is a convex function of income, and therefore a firm may undertake nonprofitable variance–increasing investments, and forgo profitable variance–reducing ones. An example for this agency problem is the bankruptcy case of Eastern Airlines. Weiss and Wruck

[5] See e.g. "Management by Bankruptcy" (Fortune 31.10.1983), "Bankruptcy as a Corporate Management Tool" (A.B.A. Journal 1.1.1987), "The Uses and Abuses of Chapter 11" (Economist, 18.3.1989) and "The Kindness of Chapter 11" (Economist 25.5.1991).

[6] This problem is very concrete in Chapter 11 cases, where the debtor has the right to classify claims into classes. It can happen that small dissenting groups are joined with larger groups, with completely different types of claims, but whose interests are aligned with those of the debtor's management.

(1998) describe how the airline was kept operating (a "high–variance negative present value project"), even though it was forced to gradually sell off all assets to finance its current losses.

Eastern Airlines also points at another important effect that bankruptcy laws have on nonfinancial decisions: it may affect the competitive position of a firm, as well as that of its rivals. Eastern Airlines is an extreme example for this effect. Chapter 11 did not only protect it from its creditors, but it also shielded it from its competitors. A 'cooperative' judge allowed it to reduce its prices below costs, in order to gain back market share, and to start its economic recovery.[7] This recovery never happened, and after almost two years Eastern had to be shut down, but undoubtedly Eastern's prices had a major impact on its competitors' price setting and earnings. On the other hand, its financially sounder competitors could not answer by using 'predatory strategies' themselves, e.g. by cutting their prices to drive Eastern into insolvency and therewith out of the market, as Eastern was bankrupt, already.

Bankruptcy procedures are supposed (both in theory and practice) to achieve many goals at the same time, many of them being conflicting goals. Not surprisingly, this makes bankruptcies very complex episodes. This is most striking if one regards the 'firm' as a nexus of contracts: many parties meet repeatedly, at different points of time, to negotiate their contributions to the firm's production process. These parties may have different preferences and objectives. In a second–best world, in which some contingencies are noncontractible, it can happen that the parties have to renegotiate their contracts: some unexpected or undescribable event may occur, which limits the firm's capability of fulfilling its promises, and nowhere it is specified which party should suffer how much. This can lead to several types of inefficiency, which court–supervised mandatory bankruptcy procedures are supposed to mitigate.

Firstly, each party will have an incentive to renegotiate its contract with the firm secretly, to improve its own position relative to that of other contracting parties. For instance, a supplier may require pre–payment of deliveries, which

---

[7] cf. Borenstein and Rose (1995) and Weiss and Wruck (1998). At one stage, Eastern Airlines charged \$12 for the connection New York – Washington (Economist, 18.3.1998).

strains the firm's liquidity; creditors may ask for immediate repayment of their loans, if necessary by forcing the firm to sell some of its assets; a bank may reduce the firm's overdraft facility. This is a zero–sum game, in which the parties spend resources to grab a larger share of a pie that is too small. Apart from wasting resources, this 'run on the assets' may also endanger a recovery of the firm. For this reason, reorganisation procedures put a 'stay' on all claims, i.e. no debt–collection efforts may be undertaken. Free from harassment from its creditors, the firm has time to consider its alternatives and negotiate with its claimants (the advantage for the lenders is that they need not worry about possible 'runs' on the firm's assets anymore).

Secondly, clearly defined rights and obligations in bankruptcy avoid a costly duplication of debt–collection efforts from the side of the firm's creditors. It is sufficient if these efforts are delegated to one party, for instance a trustee appointed by the court, as in Germany, or a (privately appointed) Receiver, as in the U.K. Furthermore, a court will have better possibilities than private claimants to investigate the reasons of the bankruptcy, as well as the whereabouts of the debtor's assets (this is the interpretation of "bankruptcy" in Gale and Hellwig (1985): the lender investigates the borrower's income).

Thirdly, under court supervision, the parties can reach agreements by voting, which bind minorities. As a consequence, no party has veto power which would allow it to hold up the other negotiating parties. This is a relevant advantage of bankruptcy procedures, compared with privately negotiated reorganisations. In the U.S., for instance, private renegotiations of public debt (under the Trust Indenture Act of 1939) require that *all* holders of that debt agree with a change of the terms of the contract (this difficult starting point for negotiations with possibly atomistic debtholders is analysed in Gertner and Scharfstein (1991)). This was made worse by a decision in the LTV bankruptcy case, where some creditors had agreed to reschedule or cancel some of their claims in private negotiations, and afterwards LTV filed for bankruptcy. The court decided that the generous creditors' claims were the rescheduled ones, not the original claims, which has added additional disadvantages to private workouts ever since.

Additionally, court supervision allows to bind minorities which are absent during the negotiations, or even unknown (the 'Scheme of Arrangement' in the U.K. requires large majorities for this case). This can be relevant in connection with cases of product liability or environmental damage.

These collective action or coordination problems have not generated much interest from the side of economic theorists recently. One reason is that they may be interesting from a theorist's point of view, but they are not specific to problems around bankruptcy. Another reason is that contract theorists cannot really provide consistent theories that explain why *mandatory* procedures are necessary, or even why contracts are *incomplete,* i.e. why parties rely on laws and courts to decide for them, instead of making their contracts more detailed.

Apart from the above coordination problems, bankruptcy procedures are supposed to solve several *strategic* problems.

The function that is reflected in most economic writing about bankruptcy is its role as a *bonding device:* if poor performance is followed by punishment, e.g. by the loss of one's job, or of future incomes, this should improve the *ex ante* incentives of the managers or shareholders to ensure that the performance is good enough. The use of bankruptcy as a bonding device is helpful to provide incentives for more efficient behaviour *outside of bankruptcy.* Examples of papers that use this bonding function of bankruptcy include Hart and Moore (1995) and Bolton and Scharfstein (1996), whose models are based on a possibility to punish by liquidating a project; in Diamond (1984), a debt contract explicitly specifies the degree of punishment as a function of the debtor's default; in Gale and Hellwig (1985), the punishment is introduced in the form of 'inspection costs'.

Next, bankruptcy is supposed to shape the parties' incentives *inside bankruptcy.* An important function of bankruptcy is to separate viable from nonviable firms, such that the former can be refinanced, and the latter sold or liquidated. This is not an easy task, as normally it is difficult to tell whether the financial distress that some firm is experiencing is temporary or fundamental, i.e. whether the maturity structure of its financial obligations is poorly matched with its cash flows, or whether the firm's operations are systematically loss–making. Whoever decides on the fate of the firm in the end will have to rely on the information and advice that the negotiators contribute.

Unfortunately, under most countries' bankruptcy laws, the payoffs from different alternatives that each party can expect are only imperfectly linked with the total payoff. A party may therefore favour a suboptimal alternative, because it gives it a higher payoff. One reason for this could be different preferences over different outcomes. Another reason lies in the priority structure of different claims, and 'absolute priority rules', which require that certain obligations are to be fulfilled before others can be addressed.

For instance, secured lenders may prefer liquidation to reorganisation, in particular if the liquidation value is sufficient to repay their outstanding loans[8] (banks must also be concerned with the effect of a liquidation on their image both as lenders and as deposit takers). Unsecured lenders may prefer reorganisations, because they participate in the upside (their claims may be repaid if the reorganisation is successful, and as trade creditors they may profit additionally if the firm's health is restored), but not in the downside[9] (if their share of the liquidation value is small). The shareholders of an insolvent firm will always favour its refinancing, as long as they can keep a share in the reorganised entity. The employees of the firm will be interested in in keeping their jobs.[10] The managers will additionally worry about their future position in the job market (e.g. about their reputation).

The influence of certain parties on the outcome of a bankruptcy can be strong. In Germany, a rescue is practically impossible without the cooperation of a firm's banks. Similarly, in the U.K. the holder of a 'Floating Charge' must be convinced to cooperate. On the other hand, bankruptcy judges sometimes have strong preferences for rescuing firms. This is most explicit in France, where creditors have little say in bankruptcies. In the U.S., the influence seems to depend on the bankruptcy courts, or the judges. For instance, the Bankruptcy Court of Miami seems to have taken a tough line on bankrupt firms in the past two decades, while

---

[8] For them, 'continuation' is a risk–shifting attempt from the side of claimants with lower priority.

[9] The unsecured creditors will argue that the firm is exposed to a debt overhang, which causes inefficient underinvestment.

[10] Trade unions, however, may sometimes care more about their industry–wide bargaining power, cf. Weiss and Wruck (1998).

the Southern District of New York has built up a reputation for being excessively management–friendly (see e.g. Delaney (1992) or Baird (1995)).

While the bonding role of bankruptcy is well understood, the incentive problems *inside* bankruptcy have not been studied in great detail, yet. Given that there seem to be many types of incentive problem connected with bankruptcy negotiations, which can have a strong impact on the final outcome, as well as on incentives outside of bankruptcy, more research in this area is necessary. This is the purpose of the present thesis.

The complexity and multitude of incentive problems makes it necessary to concentrate on a subset of problems, while abstracting from others. For instance, negotiation problems between several lenders are analysed in chapter 1, but ignored in chapters 2 and 3; the firm's income is modeled in reduced forms ("high" or "low") in chapters 1 and 2, while in chapter 3 we introduce a production process, as well as an oligopolistic market for the firm's output; in chapters 1 and 2, financing is necessary to finance some fixed setup investment, while in chapter 3 financing is needed to cover production costs, and is therefore scalable; in chapters 1 and 2, a 'failed' firm can be 'rescued', which is not an issue in chapter 3.

In its modeling technique, this thesis follows a traditional approach in corporate finance: fix some elements of the legal environment (e.g. the types of financial contracts that exist, or the bankruptcy laws), assume some market (or contracting) imperfections, and then explain existing institutions (e.g. "debt contracts", "leverage", etc.) as a second–best response to these imperfections. Depending on the interests of the researcher, more or less of the legal environment is assumed to be given exogenously. A large strand of the literature analyses the use of 'leverage', i.e. the debt–equity ratio, as a second–best tool in solving incentive problems. Another strand (the 'security design' literature) analyses in what sense the financial contracts that we observe can be considered as optimal responses to some informational or agency problems (for a survey of both literatures see Harris and Raviv (1992)). The two fields are joined in a series of models that endogenises both the structure of financial contracts (debt, equity), and the maturity and priority structure of *several* classes of claimants (see e.g. Berglöf and von Thadden (1994), Dewatripont and Tirole (1994), Hart and Moore (1995), Rajan and Winton (1995) and Repullo and Suarez (1998)).

The approach taken in this thesis is that of *security design.* This is most explicit in Chapter 3, where the financial contract that the parties sign (a debt contract) is derived from primitives as an optimal contract. In the other two chapters, the structure of the repayment functions is not sufficiently detailed to be able to speak of 'debt' or 'equity', say. The primary interest with those chapters was in endogenising other aspects of financial relationships: the *number* of contracting parties in chapter 1, and the respective bargaining positions in renegotiations in chapter 2. Chapter 2 represents "security design" in a wider sense, as the goal was to explain the *structure* of optimal bankruptcy laws, as a function of the economic environment.

Chapter 1 of this thesis addresses a mechanism which is frequently used in agency models: threaten to make ex post inefficient decisions to improve some other party's ex ante incentives. The use of bankruptcy as a bonding device is an example for this type of mechanism: the creditor threatens to liquidate the firm if a promised repayment is not made, and this threat induces the owner–manager of the firm to increase his (unobservable) effort, say, or to pay out all (unobservable) earnings.

Renegotiation is a threat to the efficacy of this type of contracts: once the parties have reached a stage at which the contract tells them to make some inefficient decision, both can improve their payoff by rescinding the old contract, and avoid the inefficiency. Chapter 11 offers a forum for this type of renegotiation: a bankrupt firm can renegotiate with its creditor, and if there is a gain from not liquidating, the chances are high that it will not happen. Not surprisingly, many authors explicitly or implicitly assume that there is no Chapter 11 in their models (see e.g. Hart and Moore (1995, footnote 8) or Bolton and Scharfstein (1996, p. 20)).

Chapter 1 explores these renegotiation problems, and shows how they can be addressed in a very natural way. It explicitly models the inefficiencies that can arise if several parties have to negotiate their contributions to some 'public good' (in this case the rescue of a firm), and it offers an explanation for features of rescue negotiations that may sometimes seem to be irrational to an outside observer (e.g. delays caused by negotiations about details, when a bankrupt firm is about to collapse). In the model, the possibility to renegotiate leads to an extreme reversal of bargaining positions: while a unique lender would normally enjoy a strong bargaining position, the possibility to renegotiate robs her of a possibility to threaten to be 'tough' (to use bankruptcy as a *bonding device*). At first sight, the introduction of a second lender should be considered as weakening a first lender's bargaining position. However, given the inefficiencies of multi–party bargaining, the introduction of a second lender introduces a commitment possibility which a single lender does not have, and its bargaining power is improved at a stage where the lender needed it most.

Chapter 1 also offers an analysis of a renegotiation setup that lies between the two most commonly used extremes. Some models work with one lender, only. Others assume that the firm issued public debt to atomistic bondholders, and that therefore renegotiation is impossible. With *two* lenders, elements from both setups are combined: few, nonatomistic lenders take the effects of their negotiating on the final outcome into consideration, and therefore are more likely to allow for ex post efficient decisions than atomistic lenders; on the other hand, the higher the number of negotiating parties, the more valuable it is to 'hold out', and even nonatomistic negotiators will therefore sometimes achieve inefficient outcomes.

Like chapter 1, chapter 2 analyses conflicting incentive schemes in dynamic agency problems: bankruptcy serves as a bonding device to solve some agency problem (information revelation in chapter 1, effort provision in chapter 2), but once the proper incentives have been provided, the parties would gain from renegotiating the contract to become softer, i.e. more efficient ex post. In chapter 1 the main actors at this stage are the lenders. In chapter 2, the distribution of bargaining positions is more intricate. Loosely speaking, the only bargaining power that the firm has is *not* to start bargaining — once the creditor knows that there is scope for renegotiations, the firm has nothing in its hand to increase its payoff above the status quo.

The key idea behind chapter 2 is that the bonding role of bankruptcy requires a harsh treatment of a firm's manager, say, but that this treatment should not be *too* harsh — if it were, his cooperation in revealing a need for renegotiation could not be ensured. This captures a very serious problems in countries with 'tough' bankruptcy regimes: if failure is followed by strong punishment, no debtor will voluntarily admit that there is trouble ahead — as long as there is hope, and the trouble is not publicly observable, it may be better to hope that things turn to the good, and do nothing.

This does not only address an important problem with insolvencies in Germany or the U.K., say, where insolvency practitioners frequently complain that they get involved in failed companies much too late. It also points at the fact that bankruptcy is not just a random event, outside of the control of a firm's management. Quite to the contrary, a firm may find it easy to change its balance sheet such that a formal bankruptcy is postponed (see also Delaney (1992)). Thus, the distinction that is normally made in economic theory between incentives *inside* or *outside* of bankruptcy is not precise: as long as the manager of a distressed firm is able to shift the start of bankruptcy in time, and may have incentives to do so, the different incentive problems can only be analysed together.

A central role in chapter 2 is played by *Absolute Priority Rules* (APR). These rules require that in bankruptcy, claims with high priority are repaid in full before claims with lower priority are repaid. This makes it difficult — in theory — to 'reward' a manager–owner for revealing financial difficulties as early as possible. Interpreted literally, APRs require that shareholders receive nothing if their firm is bankrupt, and therefore they will never allow information about financial difficulties to be revealed before they would become visible, anyway. In other words, a strict interpretation of APRs puts a lot of weight on the bonding role of bankruptcy, and ignores possibilities to improve ex post decisions.

APRs are a generally accepted element of the corporate finance literature, because this literature typically uses bankruptcy as a bonding device in its models.[11] In reality, the implementation of APRs is less strict. There is a large empirical literature which reports major systematic violations of APRs in U.S. business bankruptcies (started by Franks and Torous (1989)). Two historical

[11] Even in Aghion, Hart and Moore (1992) the need to adhere to APRs is not questioned.

examples are less well–known. Firstly, the predecessor of Chapter 11, Chapter XI of the 1938 Chandler Act, had no APR provisions. This procedure was intended for small business reorganisations, and the conflict modeled in chapter 2 may have been relevant when the Act was drafted (cf. N.B.R.C. (1997a, Working Group Proposal # 1) and (1997b, p. 549)). Secondly, in 1705 Queen Anne of England signed a statute called "An act to prevent frauds frequently committed by bankrupts", which

> [...] provided that those debtors who cooperated fully would be discharged and would take five percent of whatever assets were gathered. Debtors who did not meet with their creditors, who lied to them, or who refused to reveal the whereabouts of all their assets would be hanged.
>
> Baird (1987, p. 174)

Chapter 2 criticises the requirement of APRs in 'soft' bankruptcy procedures like Chapter 11: 'soft' procedures are designed to lead to early revelation of refinancing needs, and APRs are a tool to make bankruptcy procedures 'tough'. This conflict is not obviously necessary, and chapter 2, unlike much of the literature, is very precise about what is good about Chapter 11, and what is bad about it.

The main goal of chapter 3 is to expand the nonfinancial side of the firm in models of corporate finance. The typical approach to modeling would be the one of chapters 1 and 2: a firm is called a 'project', which can be set up by investing some fixed amount of money, and which will be liquidated at the end of n periods. The income that the project generates is typically stochastic, and frequently the support is reduced to "high" and "low".

While this is helpful in simplifying the analysis, it is important to know whether the results of the reduced–form models are changed if real investment, output and marketing decisions are added to the model. In particular because the interdependence of financial contracts and investment behaviour has been the centre of attraction for the corporate finance literature during the past two decades: problems of overinvestment (e.g. in the form of excessive risk taking) or underinvestment (e.g. due to debt overhang) are driving forces in many models.

In the model of chapter 3, the firm's financial needs are determined by its production plans, and are therefore scalable; it faces competitors on the output market, which affects its earnings; and it does not sign some exogenously given financial contract, but designs an optimal contract. The analysis shows that standard simplifications come at a price, as they can generate misleading results.

Chapter 1

Multiple Banking as a Commitment Not to Rescue

Multiple Banking as a Commitment Not to Rescue

Paul POVEL

Financial Markets Group, London School of Economics, and ECARE

London School of Economics, Financial Markets Group, London WC2A 2AE, United Kingdom. E–Mail: povel@lse.ac.uk.

I would like to thank Patrick Bolton and John Moore for their invaluable advice and encouragement. I am especially grateful to Patrick Bolton, who provided helpful support by carefully commmenting on earlier versions of this paper. I am also indebted to Philippe Aghion, Beatriz Armendariz, Ed Green, Denis Gromb, Michael Raith, David Webb, and seminar participants at the London School of Economics. Financial support from the Swiss National Science Foundation, Stipendienkommission Basel–Landschaft, Stiftung zur Förderung der jungen WissenschaftlerInnen an der Universität Basel, Stiftung der Basler Kantonalbank, the European Union (Human Capital and Mobility Program), and the European Doctoral Program in Quantitative Economics is gratefully acknowledged. I have benefited from the hospitality of the University of Bonn, Wirtschaftspolitische Abteilung, where part of this research was done.

Abstract

We present a model in which nonatomistic investors finance projects and endogenously choose not to be unique lenders. That is, multiplicity of lenders is not assumed but arises endogenously. We assume that due to informational asymmetries investors cannot tell apart entrepreneurs with good projects from those who will have to be refinanced. While refinancing is sequentially rational, it is not profitable to start such a project. A unique lender cannot commit not to refinance a project. Two creditors, however, will have to agree on their respective degrees of debt forgiveness. Due to inefficiencies in their negotiations they can credibly commit not always to refinance a distressed project. This is harmful ex post, once the need to refinance has arisen, but it is helpful ex ante, as it can keep entrepreneurs with nonpromising projects from demanding to be financed.

Journal of Economic Literature Classification Numbers: C72, D92, G21, G33, G34, G38

1.1 Introduction

This paper analyses a bank's incentives to forgive debt and refinance a distressed firm. We compare the decision of a unique lender with that of two banks, which have jointly provided a loan to the firm. We show that banks may prefer such co–financing, even if they enjoy a strong bargaining position relative to the firm. The main difference between single and multiple banking lies in the negotiations that are necessary, if the firm cannot repay its debt but it could profitably be refinanced.

Suppose that refinancing is profitable, once an initial investment is sunk, but that ex ante it is not. Some firms will need refinancing, others not, and the creditors would like to finance the latter, only. The entrepreneurs of the respective firms, however, who are informed about their prospective financial needs, are only interested in receiving a loan, irrespective of whether it will be performing well or badly. If the creditors could commit not to refinance a firm, the entrepreneurs with ex ante unprofitable firms would prefer to be inactive, instead of being forced to liquidate their firm prematurely. A single lender cannot credibly commit to being tough, as it is always sequentially rational to refinance a distressed firm, once the initial loan is sunk. We argue that introducing multiplicity on the side of the lenders can make such a commitment possible. Even if they agree on the need to rescue the firm, two lenders will have to bargain about the distribution of the overall loss. Asymmetric information between the banks is the cause of inefficiencies in the rescue decision: with positive probability the firm is not refinanced, and it is liquidated, instead.

There is a large literature now, which analyses the effects of single or multiple lending on the decisions of a firm. One strand of the literature analyses the effects that the structure of the creditors' claims has on the possibilities to reorganise a distressed firm. Gertner and Scharfstein (1992) and Detragiache (1994) for instance assume that bonds are held by atomistic investors and therefore cannot be renegotiated. They analyse the effects of different bankruptcy regimes on the possibilities to reorganise a distressed firm.

These effects can be used strategically by a firm, i.e. different financial structures can be used to achieve different goals. Several papers have asked the question why a firm may prefer to have one or many creditors. The difference between the market–based financial system in the US and the bank–based system in Germany and Japan are striking, and an analysis of the relative advantages of the two systems is an important research program.

A frequently stated advantage of the 'main bank' financial system in Germany and Japan is that distressed firms are rescued more frequently (See e.g. Hoshi et al. (1990) for the case of Japan, and Edwards and Fischer (1994) for the case of Germany). Some theoretical papers have analysed the conditions under which 'main bank' finance is more efficient than a system with multiple lenders (See e.g. Fischer(1990), von Thadden (1995), and Dewatripont and Maskin (1995)). As Edwards and Fischer (1990) conclude, however, these models are not compatible with the empirical evidence for the German case. While in the models at most one 'main bank' can emerge, in reality a German firm has more than one 'Hausbank'. The question to analyse is thus why we may observe more than one nonatomistic lender. Several answers are possible.

First, one could argue that banks are risk averse and want to spread out their risk exposure by sharing risks with their competitors. This is certainly true, but not a very satisfying explanation from a theoretical point of view. Banks are usually thought of as 'large', compared with the size of the average firm. They should therefore be able to diversify away most of their risks, as was modeled in Diamond (1984). This makes them de facto risk neutral, and they should not suffer from risk exposure. After all, it is the banks' business to deal with risks and to allocate them optimally, and not to avoid risks. Additionally, it would be interesting to know whether there is more behind multiple banking than mere risk–sharing.

Second, a bank may lack the funds to finance a project. Dewatripont and Maskin (1995) suggested that such smallness could be a solution to the Soft Budget Constraint problem in centralised economies. Inability to finance a project exclusively may be a real problem when firms are very large. However, even in cases when the firms are very small, compared with their banks, we find multiplicity. As before, there is a need for additional explanations.

Third, firms may want to have many banks because this protects them from being exploited by too strong a partner, as was suggested in von Thadden (1992). This third rationale for multiple banking implies that neither the banks nor the firms enjoy exceptionally strong bargaining positions in their relationship. Normally it is however conjectured that the banks are in the strong position. Many situations can occur in which a firm has to rely on its bank or banks and in which the bank can cheaply 'punish' earlier unfriendly behaviour.

Finally, a recent literature analyses the use of multiple claimants, holding different types of securities, in solving agency problems: the investors may have poor incentives either to really monitor their debtor, or to make proper use of their information (e.g. to liquidate a firm). See e.g. Diamond (1993), Berglöf and von Thadden (1994), Dewatripont and Tirole (1994), Rajan and Winton (1995), and Repullo and Suarez (1995).

The present paper offers a rationale for multiplicity, which complements the explanations above. We argue that multiplicity is requested by the banks, who use it as a commitment device for eventual renegotiations of the lending contracts. The inefficiencies that arise in rescue negotiations (the banks have to determine their respective degrees of debt forgiveness) are a threat for entrepreneurs with bad projects. If the inefficiencies are sufficiently strong, this allows the banks to deter nonprofitable projects, and to finance high quality ones, only.

The idea that multiplicity can serve as a commitment device was first stated in Hellwig (1991). Dewatripont and Maskin (1995) analyse the role of 'multiple lending' in hardening the 'soft budget constraint' of a firm. In their model, however, multiplicity is a credible commitment not to rescue only because of the assumption that lenders are 'small', and cannot provide both an initial and a refinancing loan. Bolton and Scharfstein (1996) analyse a renegotiation problem that is similar in spirit to ours. In their model, too, multiplicity is used as a commitment to be inefficient in renegotiations, with the result that high quality firms borrow from two creditors, while low quality firms prefer to borrow from a single creditor. Our model differs from theirs in different aspects. First, we work in a complete contracting environment. There is no variable in this model, which is 'observable but not verifiable.' In Bolton and Scharfstein (1996), the entrepreneur can hide the returns of the project, and claim that the returns had been low. An optimal contract 'punishes' him by threatening to liquidate the assets that are still valuable to him. In our model, the banks want to keep away nonprofitable projects, i.e. projects with a low probability of being successful. Second, we model the renegotiation process explicitly, and base it on observations from a financial system with 'main banks'. Bolton and Scharfstein (1996) use the Nash Bargaining Solution and the Shapley Value, instead, to model bargaining outcomes.

Finally, a recent strand of literature analyses the strategic use of single or multiple lending as a commitment device with respect to nonfinancial decisions. More precisely, these papers (see e.g. Yosha (1995) or Bhattacharya and Chiesa (1995)) study the relative advantages of public or bank lending, if the two regimes have different effects on how sensitive information can leak to a firm's competitors. They thus provide more and richer explanations for multilateral lending, which add new aspects to the purely financial models.

The paper also adds a new variant of a bargaining model to the game theoretic literature. We model the negotiations between the banks as a *war of attrition.* As soon as the banks have been informed that the firm must be refinanced, negotiations start. In these negotiations, each of the two banks tries to convince its opponent to write down the larger fraction of its claims. A rescue is only possible if one of the banks gives in: it frees the way to a rescue of the firm by accepting its opponent's rescue plan. The reason why the banks eventually give in is that a rescue may become impossible, and the firm has to be liquidated. Each bank has a privately known valuation for the business relationship with the firm, which it loses if the latter is liquidated. The impossibility to rescue can arise at any time, as soon as the parties have started to bargain, and the longer the rescue is delayed, the more likely it becomes that the banks are forced to liquidate the firm. If a bank has a high valuation at risk, it has strong incentives to accept its opponent's plan, only to ensure that the firm is rescued. As the opponent could have an even higher valuation, however, it also has an incentive to hold out for a while. This tradeoff determines the banks' strategies in the war of attrition.

Admati and Perry (1991), Fernandez and Glazer (1991), and Abreu and Gul

(1994) are other papers, in which two parties must come to an agreement in time consuming negotiations. We could have used variants of these models, instead of the war of attrition, to capture the inefficiencies of the renegotiation process. The models in the three papers, however, are somewhat technical, too, and do not generate more elegant results than our model. We believe, therefore, that the war of attrition is a good compromise between the requirements for the analysis and the tractability of the results.

The rest of the paper is structured as follows: In Section 1.2, the projects and the entrepreneurs are introduced, and the difficulties of a single bank are discussed. The model is extended in Section 1.3, where two banks finance a firm, and renegotiate if it must be refinanced. These renegotiations are modeled as a war of attrition. Section 1.4 solves this model to find the equilibrium of the renegotiation stage, as well as that of the whole game. Section 1.5 presents some empirical evidence, and discusses implications and extensions of the model. Section 1.6 concludes. Proofs are in the Appendix.

1.2 The Model With One Bank

There is a large number of entrepreneurs who can start one project each. Each entrepreneur privately knows the type of project that he can start, either 'good' or 'bad'. The proportion of entrepreneurs with 'good' projects, γ, is common knowledge.

The timing of a project is the following. In the first period, an investment I must be sunk. In period 2 the project types become publicly observable. Payoffs are earned in the third (the last) period. A 'good' project earns $R > I$, while a 'bad' project earns zero. Both project types can be liquidated, which earns r, where $0 \leq r < R$. A 'bad' project can be 'rescued' in period 2: if an additional amount J is invested, a payoff $\bar{R}$ is earned, instead of zero.

Assumption 1.1 *It is profitable to rescue a 'bad' project in period 2, as* $\bar{R} - J > r$*. However, it is not profitable to finance a 'bad' project ex ante:* $\bar{R} - J - I < 0$*. Neither should a random sample of projects be financed:* $\gamma(R - I) + (1 - \gamma)(\bar{R} - J - I) < 0$.

The entrepreneurs' payoffs depend on whether their projects were started and completed. If a project was not started, the entrepreneur earns zero utility. If the project was started, and either completed successfully (if 'good') or rescued (if 'bad'), his utility is $M > 0$. If a project was started and then liquidated, this causes harm to the entrepreneur, and his payoff is $-m$ (where $m > 0$).

The entrepreneurs have no wealth of their own, and need outside finance to start their projects. We assume that a project cannot be separated from its entrepreneur. 'Good' projects cannot be continued without him, and 'bad' projects cannot be rescued — both types would have to be liquidated. The entrepreneurs are protected by limited liability. No punishment can be used legally to influence the entrepreneurs' decisions, except for the liquidation of the project, which gives them negative utility.

As we assume that it is not profitable to finance a cross section of projects, an investor must find a way to separate the 'good' from the 'bad' projects. Ideally, only the former would be financed. A bank could propose a contract which specifies that 'bad' projects are liquidated in period 2. It would like to commit never to refinance, as this would prevent the entrepreneurs with 'bad' projects from applying for initial loans I. Unfortunately, as one can easily verify, such a threat is not credible. Entrepreneurs with both 'good' and 'bad' projects will apply for I, as those with 'good' projects have nothing to fear, and those with 'bad' projects know that there will be a rescue. As a result, the single bank faces a random sample of projects, and it has to reject all loan requests. Due to a lack of commitment no project is undertaken, even though there would be valuable investment opportunities.

1.3 The Model With Two Banks

The lack of a commitment possibility in the case of a single bank can be overcome (at least partially) by having more than one creditor for each project. If each of two banks provides, say, half of the initial loan, both have some rights over the returns of the firm at $t = 3$. If the entrepreneur asks for the additional loan J, a part of the total investment will have to be written off. The banks will bargain over how much each should forgive. If this bargaining is sufficiently inefficient, and the consequences of this inefficiency cause harm to the entrepreneur, the underinvestment problem can be solved.

It will be shown below, that two banks can commit to rescue with a probability which is strictly smaller than one. There is a critical value for this probability, which we denote by $\bar{q}$. It is determined by the entrepreneurs' utility functions:

$$\bar{q}M - (1 - \bar{q})m = 0. \tag{1.1}$$

If an entrepreneur's 'bad' project is rescued with probability $\bar{q}$ and liquidated with probability $(1 - \bar{q})$, his expected payoff is exactly zero. He is thus indifferent between applying for a loan, and being inactive (which earns a sure payoff zero). If the rescue probability is strictly below $\bar{q}$, he prefers not to apply for the loan. In this case, only the entrepreneurs with 'good' projects apply for funding. Therefore, if the banks can credibly commit not to refinance with a probability larger than $(1 - \bar{q})$, multiple banking strictly dominates bilateral lending relationships.

The model with two banks incorporates some observations about private workouts and bankruptcy negotiations that are reported in the business press, in empirical and descriptive papers (e.g. Fischer (1990) and Edwards and Fischer (1994)), in studies on the banking system and insolvency procedures in Germany, and in the large literature on the reform of the bankruptcy laws in Germany. These observations, or 'stylised facts', are:

1) Banks seem to have a strong bargaining position.
2) The parties involved try to keep the negotiations secret.
3) The banks want to terminate the negotiations quickly.
4) It is likely that customers and suppliers are lost if they hear that there are rescue negotiations.
5) Whether to rescue or not is rarely subject to dispute.
6) The parties rather bargain about who is to sacrifice how much.

We have used these observations to construct a model of debt renegotiations, such that it captures important elements of an existing financial system, and it generates results which can again be compared with reality. To do so, we must expand the model with a single bank, by adding some assumptions. Two comments will be helpful before this is done. First, all additional assumptions could have been added to the model with a single bank, without changing any of the results. This has not been done, as it would have complicated the exposition unnecessarily. Second, we will make assumptions that are much more restrictive than is necessary to generate the results. Again, this is done to simplify the notation. Where assumptions are 'extreme', we mention this fact, and offer weaker alternatives to the reader.

We model the renegotiation process between the two banks of a firm as a *War of Attrition.* Each of the two banks tries to convince its opponent to carry the burden of refinancing. An outside observer of the negotiations will find that no progress is being made for a while: the banks fail to come to an agreement on how to split the overall loss $\bar{R} - I - J$, if there should be a rescue. The negotiations can end in two different ways. Either one of the banks gives in, i.e. it accepts the rescue plan of its opponent. Or fate turns against the firm: a rescue becomes impossible for exogenous reasons, and it must be liquidated. In the latter case, each bank incurs a loss (additional to the financial loss). The size of this loss is privately known by the respective bank. In equilibrium, the higher it is, the more a bank fears liquidation, and the less it is willing to reject its opponent's rescue plan.

We now introduce the extensions of the single banking model, proceeding along the observations listed above. The equilibrium of the war of attrition will be analysed in Section 1.4.

The first observation states that banks are the main players in rescue negotiations. This is captured by assuming that they are the only bargaining parties, and by assuming that the courts strictly enforce *Absolute Priority Rules.* These rules specify that no party may receive any of the returns of the firm, if the banks have neither been repaid in full, nor have agreed to such a payment.

Observation 2 describes how the banks want to keep the negotiations secret. It is helpful in achieving this goal to conclude an agreement as quickly as possible (See Observation 3). The reason for this wish for secrecy lies in the bankruptcy laws, which in most countries favour the banks (France is a notable exception). The assets of the firm usually are used as collateral for the loans from the banks, and absolute priority rules enforce the need to repay these claims first. The customers and suppliers are the parties who typically do badly in bankruptcy. Similar to a bank run, they have every incentive to request what they are owed, as soon as they discover the firm's problems, and not to engage in any new trades (except possibly on a cash–only basis). We model this sensitivity of a rescue to the cooperation of these parties as a heavily reduced form of observation 4.

Assumption 1.2 *At any time during the rescue negotiations, the public can discover that there are such negotiations going on. This happens by the time t with probability $F(t)$. If the negotiations have been discovered, a rescue becomes immediately impossible, and the project must be liquidated.*

Assumption 1.2 is much more extreme than is necessary for the results. Nevertheless, it is not unrealistic. Firms whose assets consist almost exclusively of human capital are an example. If the competitors of an advertising company find out that it is in difficulties, they will try to hire its best employees on the spot. Robbed of its most valuable 'assets', the distressed company is not worth rescuing anymore, and must be liquidated. For this reason, a formal insolvency in this industry can end after a couple of hours. Furthermore, there is anecdotal evidence from the UK, which indicates that secrecy may be a crucial requirement for a successful rescue. The Bank of England assists in the rescue of distressed large companies, by coordinating the parties' efforts as soon as possible. It is not uncommon that in the negotiation meetings the parties have to use coded names to identify the distressed firms, even if everybody is informed about the real ones. Secrecy may also be relevant if without it potential customers are lost. Imagine that a customer in a travel agency hears that an airline is financially distressed. He has strong incentives, in this case, to book his ticket with another airline, as the airline may be bankrupt before he has completed his journey.

We have to make some technical assumptions, in order to make the model tractable:

Assumption 1.3 *The 'discovery technology' F of the public has a mass point with measure $\pi > 0$ at $t = 2$, and a density f with support $(2, \tau]$, where $\tau < \infty$.*

The mass point at $t = 2$ is necessary for the uniqueness of the equilibrium strategies.[1] These are determined by two differential equations, the solution of which is not unique without a socalled boundary condition. The mass point leads to a static lottery over rescue and liquidation at $t = 2$, which gives us such a boundary condition. This lottery is a logic extension of the dynamic war of attrition game to a discrete pre–stage, and is therefore used in the model: as will be shown below, the dynamic war of attrition is the limiting case of a discrete time game, if the length of a time unit becomes infinitesimal.

Assumption 1.3 further restricts the support of f to a finite interval. The reason for this is that the results would be difficult to interpret if $\tau = \infty$ (it would be possible that the banks bargain endlessly). It is by no means a necessary assumption. Furthermore, one can easily imagine why the firm's distress should be discovered in finite time. For example, there may be legal obligations to make the distress public, if certain contingencies arise.

Observation 5 states that the negotiating parties normally agree that the firm should be rescued (if they start to negotiate). This is captured by the complete information about the costs and returns of a rescue, and by the assumption that a rescue is profitable (Assumption 1.1). Not everything is common knowledge between the negotiating parties, however.

Assumption 1.4 *After signing the initial loan contract, each bank B_i develops a privately known valuation ℓ_i for the business relations with the firm. ℓ_i is a loss that the bank incurs if the firm is liquidated. The valuations are independently and identically distributed, with a common probability density function g. g is strictly positive on its support $\mathbb{R}_+$, continuous and differentiable. Denote the cumulative distribution function with G.*

There are many possible interpretations for the loss of ℓ_i if the firm is liquidated. For instance, it may be an estimate of future profits from dealing with the firm. Alternatively, the bank may incur costs or lose profits because the liquidation of its debtor damages its public image or leads to tighter supervision by the banking regulator. Finally, ℓ_i may parametrise agency problems within the bank. A bank

[1] A simple alternative to the mass point assumption will be discussed below, see Lemma 1.1.

manager's career prospects may be worsened, if 'his' firm must be liquidated. Similarly, the bank manager and the entrepreneur may have become good friends. In both cases, the decision making unit in the bank would lose something if the firm is liquidated, and would prefer to rescue it.

The banks' willingness to assist a distressed debtor is frequently underlined in studies of the German financial system (see e.g. Schneider–Lenné (1992)). It is questioned in Fischer (1990). His evidence, however, is based on interviews with insolvency practitioners, and can therefore be assumed to be biased to the banks' disadvantage. In their analysis of private workouts in the US, Gilson et al. (1990) conclude that restructuring is the more likely, the more debt is owed to banks. This may be caused by the banks' superior skills and capabilities in attempting to rescue a firm, but it may also signal that banks are more willing to rescue a firm than other creditors. In the model this willingness to rescue is captured by the valuation ℓ_i.

Assumption 1.4 and the next assumption jointly capture observation 6, that the banks bargain about who has to bear how much of the loss. The set of outcomes that the banks can achieve is restricted to simplify the analysis, that is how the net surplus s (the returns $\bar{R}$ minus the cost J and the opportunity cost r) from rescuing can be split (it is positive because of assumption 1.1.)

Assumption 1.5 *The banks fight for the whole surplus $s := (\bar{R} - r - J)$. No offer to share the surplus is made or accepted. If one bank gives in it receives its share r_i in the liquidation value r of the firm from the other bank, where r_1 and r_2 are specified in the initial contract. The winning bank is committed to rescue the firm immediately, but may keep the returns for itself.*

As before (see Assumption 1.2), the formulation of Assumption 1.5 is much stronger than necessary. A sharing rule saying that the gross surplus $\bar{R} - J$ can only be shared in proportions α and $(1 - \alpha)$, where $\alpha \neq \frac{1}{2}$, would be sufficient. This would lead to significant complications of the analysis, however, which are not rewarded by the additional insight that one gets.

This completes the introduction of the model with two banks. As one can easily see, the assumptions that have been added in this section could also have been introduced in the single bank model, without changing anything. The loss of a valuation ℓ_i if the firm is liquidated would make a single lender even more willing to refinance a 'bad' project. This rescue happens already without the valuation, however.

In the model with one bank, a strategy for the bank consisted of a financing and a refinancing decision. In the case with two banks it is slightly more complicated. We first consider the part of the strategy which is used in the rescue negotiations. If a firm needs refinancing, the sequence of events is the following. First, the banks decide whether they want to give in immediately. If none of the banks has given in, the negotiations are discovered with probability π, and the firm must be liquidated. With probability $(1 - \pi)$ the continuous time war of attrition starts. We assume that if both banks give in simultaneously, each 'wins' with probability $\frac{1}{2}$.

A strategy is a function $T_i : \mathbb{R}_+ \longrightarrow [2, \tau]$, which determines for each moment of time whether a bank B_i with valuation ℓ_i should give in or not. It will be shown in Section 1.4, that if the equilibrium strategy tells this bank to stop at time $T_i(\ell_i)$, it will stop at every later time, as well. Thus, we will define T_i as determining the first time at which a bank plans to stop. This includes the static lottery which is played because of the mass point in F at $t = 2$.

One may wonder why the banks cannot renegotiate the lending contract, after it has been signed. Both are fully aware of the inefficiency that will arise, if the contract is renegotiated using the war of attrition. Why cannot one bank (or a third bank) take over all debt for a flat price? Suppose B_1 would make such an offer to B_2. B_2 would claim to have a valuation $\ell_2 = 0$ and not to fear the war of attrition, in order to increase the takeover price. B_1 would claim to have the same valuation, to decrease the price. None of the two has any incentive to admit having a positive valuation, until a rescue is really needed. In this case, however, the war of attrition will start. The time that passes by is the only credible information about one's valuation, as talk is 'cheap', and neither before nor during the war of attrition the parties can renegotiate more efficiently. Even a bank with valuation $\ell_i = \infty$ would wait until a rescue is necessary, as it might be that the opponent gives in. Nothing is lost by waiting until $t = 2$, at which time both banks can prevent a liquidation with probability one by giving in.

1.4 Equilibrium Strategies

The first step in solving the renegotiation game is to determine which types would want to start the war of attrition, and which types would prefer to give in immediately, in order to secure the rescue of the firm. If no bank gives in immediately, the negotiations are discovered with probability π (the mass point in F), and the firm is liquidated. With probability $(1-\pi)$ the continuous time war of attrition starts.

A bank with a very high valuation at stake will not want to gamble for the surplus s, and stop immediately. We must determine which is the lowest valuation, for which this is still true. Denote this cut–off value of bank B_i with λ_i. If its valuation is $\ell_i > \lambda_i$, it should strictly prefer to give in immediately, while if it is $\ell_i < \lambda_i$, it should want to start the war of attrition, and plan to stop later than $t = 2$.

We define λ_i as the valuation with which a bank B_i is indifferent between giving in immediately, and starting the war of attrition, if it is sure that the opponent will either give in immediately (with probability $1 - G(\lambda_2)$), or will start the war of attrition without giving in (with probability $G(\lambda_2)$).

Consider the bank with valuation $\ell_i = \lambda_i - \varepsilon$, where $\varepsilon > 0$. Given the definition of λ_i, there must be a $\delta > 0$, such that it will strictly prefer to start the negotiations, if the probability that the opponent gives in immediately, as soon as the negotiations have started, is δ. Thus, a bank with a valuation below λ_i has an incentive to hold out for a strictly positive amount of time. A bank with a valuation higher than λ_i, however, strictly prefers to give in immediately.

Lemma 1.1 *The cut–off values λ_1 and λ_2 are defined implicitly by*

$$\lambda_1 = \frac{s}{2\pi}\left(\frac{1-G(\lambda_2)}{G(\lambda_2)}\right) \qquad \text{and} \qquad \lambda_2 = \frac{s}{2\pi}\left(\frac{1-G(\lambda_1)}{G(\lambda_1)}\right). \tag{1.2}$$

Since λ_i is continuous and monotonic in λ_j, a symmetric solution exists. It can happen that there are multiple solutions, since the two equations in Lemma 1.1

must be solved simultaneously. We assume that the banks play the symmetric solution in this case[12], and denote the common cut–off value with λ.

As was mentioned before, the war of attrition is only one of many possible ways to model negotiations with inefficient delays. The model could have been slightly simplified by assuming that the support of G is bounded (see Assumption 1.4). Suppose it was common knowledge that the highest value ℓ_i that a bank can attribute to its business relationship with a firm is $A < \infty$, because a bank's line manager cannot 'bet the ranch'. The results would be qualitatively the same, except that we would have $\lambda = A$. Our formulation allows for banks that give in immediately with a certain probability, depending on the parameters of the model.

If both banks decided to stay in the game, the war of attrition starts. A strategy T_i in this war of attrition specifies the earliest instant at which a bank wishes to stop, given the realisation of its potential loss, ℓ_i. Lemma 1.2 derives some characteristics that equilibrium strategies must have. In Proposition 1.1 we will show that these necessary conditions are also sufficient conditions for the existence of a unique equilibrium, together with the boundary conditions that are determined in Lemma 1.1.

Lemma 1.2 *Let T_1 and T_2 be equilibrium strategies of the game defined above. Then the strategy T_i is (i) strictly decreasing in the liquidation loss ℓ_i, (ii) continuous, and (iii) differentiable. (iv) Bank B_i stops at τ if and only if $\ell_i = 0$.*

In equilibrium it will never be the case that the bank with the higher loss level will decide to stay in longer than its opponent. The threat of the public's discovery must have strictly more weight in a bank's reasoning the higher ℓ_i is, while the gain from winning, the surplus s, is constant. Only a bank with zero liquidation loss will wait until τ, and it will not want to stop earlier than τ. A bank with

[2] A sufficient condition for uniqueness can be found by inverting $\lambda_2(\lambda_1)$, and requiring that the slope of this inverse is never equal to the slope of $\lambda_1(\lambda_2)$. It is, however, difficult to interpret:

$$\frac{g(\ell)}{[G(\ell)]^2} \cdot \frac{g\left(G^{-1}\left[\frac{s}{2\pi\ell+s}\right]\right)}{\left[G\left(G^{-1}\left[\frac{s}{2\pi\ell+s}\right]\right)\right]^2} \neq \left(\frac{2\pi}{s}\right)^2 \qquad \forall \ell \in \mathbb{R}_+.$$

strictly positive loss level will either stop immediately at $t = 2$ (if it has costs $\ell_i \geq \lambda$) or at some moment after $t = 2$ but earlier than τ.

At every moment, both players update their beliefs about the opponent's valuation. Since T_i is continuous and strictly decreasing, each ℓ_i is mapped one–to–one with a stopping time $T_i(\ell_i)$. T_i can be inverted to yield a function $L_i : [2, \tau] \rightarrow \mathbb{R}_+$. At each instant t_i there is a valuation $L_i(t_i)$ with which a bank would plan to stop. As time passes by, a player's expectation about the maximal valuation that his opponent could possibly have decreases. Bygones are not 'bygones' in this game: every second that passes by signals information about a bank's valuation, and is relevant for the present and future decisions of the opponent.

As was mentioned above, the finiteness of τ is not a necessary condition for the tractability of the model. If the function f had an infinite support, then Lemma 1.2(*iv*) would state that banks with zero liquidation loss never stop, and banks with strictly positive loss levels plan to stop at some finite time.

L_i, the inverse of the strategy function T_i, is the lowest cost level that would make bank B_i want to stop at time t. It will be helpful for characterising the equilibrium strategies in the following. These are determined by finding for each moment t_1 a valuation $L_1(t_1)$, such that B_1 is exactly indifferent between stopping at t_1, and waiting for a small amount of time Δ, and giving in then (The derivation is similar to that of the cut–off values λ_i).

If the bank gives in at time t_1, its payoff is r_1 for sure. We require this payoff to be equal to the expected payoff, if it decides to wait until $t_1 + \Delta$:

$$r_1 = \tfrac{G(L_2(t_1+\Delta))}{G(L_2(t_1))} \left[\tfrac{(F(t_1+\Delta)-F(t_1))}{1-F(t_1)} (r_1 - L_1(t_1)) + \tfrac{(1-F(t_1+\Delta))}{1-F(t_1)} r_1 \right] \qquad (1.3)$$
$$+ \tfrac{G(L_2(t_1))-G(L_2(t_1+\Delta))}{G(L_2(t_1))} \left[\tfrac{F(t_1+\Delta)-F(t_1)}{1-F(t_1)} (r_1 - L_1(t_1)) + \tfrac{1-F(t_1+\Delta)}{1-F(t_1)} (\bar{R} - r_2 - J) \right].$$

The second expected payoff (on the right hand side of (1.3)) has four components. The opponent may have a low valuation, and plan to give in later than $t_1 + \Delta$. By this time, the negotiations may have been discovered, and the firm must be liquidated. The bank receives its share r_1 of the liquidation value r, but loses $L_1(t_1)$. If the negotiations are not discovered, it will give in at time $t_1 + \Delta$, which earns r_1. On the other hand, the opponent may plan to give in between t_1 and $t_1 + \Delta$. As before, the negotiations may be discovered, or they may not. In the latter case, the firm is rescued. The bank pockets the surplus $\bar{R} - J$, and pays

r_2 to the opponent. We abstract from the possibility that both may give in at $t_1 + \Delta$ simultaneously, as the probability that this happens is negligible.

(1.3) can be simplified by rearranging, subtracting r_1 on both sides, and by substituting s for $(\bar{R} - r_1 - r_2 - J)$. A division of both sides by Δ leads to

$$\frac{G(L_2(t_1)) - G(L_2(t_1+\Delta))}{G(L_2(t_1))\Delta}\left(\frac{1-F(t_1+\Delta)}{1-F(t_1)}\right)s = -\left(\frac{F(t_1+\Delta)-F(t_1)}{(1-F(t_1))\Delta}\right)L_1(t_1). \tag{1.4}$$

Since the strategies are differentiable everywhere it is possible to take the limit as Δ goes to zero. The same procedure can be repeated for the second bank, and we get a system of two differential equations:

$$L_2'(t_1) = -\left(\frac{G(L_2(t_1))}{g(L_2(t_1))}\right)\left(\frac{f(t_1)}{1-F(t_1)}\right)\frac{L_1(t_1)}{s}, \tag{1.5}$$

$$L_1'(t_1) = -\left(\frac{G(L_1(t_2))}{g(L_1(t_2))}\right)\left(\frac{f(t_2)}{1-F(t_2)}\right)\frac{L_2(t_2)}{s}. \tag{1.6}$$

Given the strategy of the opponent, (1.5) determines the optimal response of bank B_1, if it has loss level $L_1(t_1) = \ell_1$ (the two are equivalent, if the equilibrium strategy tells bank B_i with cost level ℓ_i to stop at time t_i) and bank B_2 plays strategy $L_2(\cdot)$. If (1.5) were an inequality, B_1 would either want to wait longer than t_1 (if $<$), or it would want to have stopped earlier (if $>$).

Since by Assumption 1.2 the probability density function g is strictly positive on $\mathbb{R}_+$, G has an inverse function $G^{-1} : [0,1] \to \mathbb{R}_+$. (1.5) and (1.6) can be integrated, and this leads to the following reaction function for bank B_i:

$$L_j(t_i) = G^{-1}\left(G(\lambda)\cdot\exp\left\{-\int_2^{t_i}\left(\frac{f(t)}{1-F(t)}\right)\frac{L_i(t)}{s}\,dt\right\}\right). \tag{1.7}$$

(1.7) implicitly describes the strategy of bank B_j that makes bank B_i exactly indifferent between stopping at t_i and stopping at $t_i + \Delta$ (where Δ is a small amount of time), given its cost level $L_i(t_i)$. The analogous can be done to derive the strategy of the other bank. The solution to these two equations will give us the equilibrium strategies for the banks. We will continue with the differential equations (1.5) and (1.6), and show that there is a unique equilibrium. The reaction functions will be helpful in Section 1.5, where we present some comparative statics.

With the help of the differential equations and the boundary conditions it is now possible to describe the equilibrium strategies of the players for the whole renegotiation game.

Proposition 1.1 *The renegotiation game has a unique symmetric Bayesian equilibrium, which is implicitly described by the system of differential equations (1.5) and (1.6), and the boundary conditions* $T_1(\lambda) = T_2(\lambda) = 0$. *The equilibrium strategy for bank* B_i *is:*

$$\textit{Type } \ell_i \textit{ stops at } t \textit{ if and only if } \ell_i \geq L_i(t),$$

where $L_i(t)$ *is determined in (1.7).*

We can now find the equilibrium strategies for the whole game with two banks, including the financing decision. Whether an entrepreneur with a bad project applies for a loan in the first period depends on the probability with which his project is rescued in the second period. In (1.1) we determined an upper bound $\bar{q}$ to this probability, such that 'bad' projects are not financed.

Proposition 1.2 *If the probability of non–rescue due to bargaining delays is high enough,*

$$\int_0^\lambda 2\,F\Big(T_1(\ell_1)\Big)G(\ell_1)g(\ell_1)\,d\ell_1 \geq \tfrac{M}{M+m}, \tag{1.8}$$

the entrepreneurs will apply for the initial loan if and only if the project is of the 'good' type.

Proposition 1.2 is the main result of the paper. There are cases in which a financial system with multiple banking performs strictly better than one with single bank lending. If the condition in (1.8) is met, the banks prefer to require co–financing by a second bank to being a single lender.

1.5 Empirical Implications

The main result of the paper is that banks might want to syndicate a loan to a firm, if they fear to find themselves in a harmfully weak bargaining position if the firm has to be refinanced. The loan is shared for strategic reasons, and the banks propose to share even if they have all bargaining power. There can be other reasons for why loans are syndicated, however, like (see the Introduction) risk aversion, the sheer size of the loan, or because the strong competition on the lenders' side. These reasons complement each other, and it is not clear which one was the most important if a loan has been shared.

There is some empirical work on this question for the US and for Germany. For the US, Gilson et al. (1990) have analysed the performance of private workouts. One of their results is that debt restructurings are more likely if the number of lenders is small, which could support the result above. For the case of Germany, Fischer (1990) and Edwards and Fischer (1994) report that all but the very small firms have several 'main banks', which could be interpreted as supporting the conclusions in this paper.

Interesting evidence is reported in Armendariz (1994). She analyses the performance of several development banks, i.e. the default rates of their loans. Some of these banks require that projects are co–financed by commercial banks, while others usually are the unique providers of capital. The former enjoy considerably less arrears in the repayment of their loans. Her interpretation of these facts is that the requirement of co–financing hardens the Soft Budget Constraint of development projects, exactly what the results above suggest.

A similar observation can be made if firms grow: suppose that for a small firm $\bar{R}_s - J_s > I_s$, while for a larger firm $\bar{R}_l - J_l < I_l$. Then a 'main bank' could require that a growing firm finds a second main lender, for instance by committing to finance only a fraction of a major investment. Similarly, a bank could require co–financing if fixed costs of rescuing a firm are higher than the net surplus s for small firms, but lower for larger firms.

We now analyze other implications of the model. The equilibrium strategies of all parties are unique, and therefore we can analyse the effects of varying some of the parameters of the model.

Proposition 1.3 *A higher expected value of the firm $\bar{R}$, a lower liquidation value r and a lower additional loan J lead to later concessions. This in turn implies that the liquidation of a 'bad' firm becomes more likely.*

The intuition behind Proposition 1.3 is clear: if the prize is increased, and the expected costs of fighting remain unchanged, the banks have an incentive to fight longer. The implications for rescue negotiations are surprising, however. Of two otherwise identical candidates for a rescue, the one with a higher post–rescue return $\bar{R}$, i.e. the more profitable, is more likely to be liquidated. Similarly, the one with a lower liquidation value is more likely to be liquidated. This seems to be counterintuitive, as usually we would expect a valuable rescue to be undertaken. The result follows from two modeling assumptions. First, the negotiations are inefficient, as the 'cake' that is to be split can disappear at any time. Second, the banks' valuations for the surplus from a rescue and for the rescue itself are independent. Suppose that s depends on the number of employees of the firm, and that the banks' public relations suffer if they cause unemployment by not assisting a distressed debtor (they lose ℓ_i). In this case we would expect a bank to be more willing to rescue if the firm is larger.

A 'valuable' firm could therefore be rescued for different reasons, either because a rescue is profitable (large s), or because failing to rescue would cause indirect costs (large ℓ_i). The second reason is an incentive problem that is similar to the one underlying our assumption: once a project has been financed, its investors have too strong incentives to refinance it (see Mitchell (1993) or Aghion et al. (1996) on the problems that this can cause for banking regulation).

The result should hold, however, in situations in which the valuations ℓ_i are small, compared with the surplus from a rescue, s. One could analyse the refinancing decisions of foreign banks, that care less about their public image outside their home country. Similarly, one could analyse these decisions in sectors, regions, or during time periods, in which unemployment and bankruptcies are not considered as being major problems.

A further implication of Proposition 1.3 concerns the allocation of the assets of a distressed firm. Many bankruptcy procedures are court–led, and contain rules that are meant to protect the interests of all parties. This may make it difficult to use the assets in the most efficient way, as for instance their quick sale to the highest bidder. The liquidation value of a firm is therefore lower than necessary if a formal procedure is started, with the consequence that a rescue becomes less likely.

The variables R and I (the return of a 'good' project and the initial investment) have no effect on the strategies, because of the simplified structure of the model. As was suggested above, we could allow a bank's valuation ℓ_i to depend on the size of its stake in the firm. The larger the loan, the more the bank is exposed to public scrutiny, and the more it will therefore be willing to cover up 'mistakes' by rescuing the firm.

Similarly, the relative shares r_i in the liquidation value r play no role. The reason for this is that the bank receives a payment of at least r_i whatever the outcome of the negotiations. We could easily change the sharing rule such that r_i plays a role in the banks' renegotiation strategies. For instance, a sharing rule could require that the bank that gives in receives a share $\alpha < \frac{1}{2}$ of the surplus.

Next, consider a variation in the public discovery technology, the density function f. Suppose that π remains constant, and that f is changed to f_1 such that the hazard rate is higher (the term $f/(1-F)$ on the RHS of (1.7)). Assume that this makes the second discovery technology is superior, i.e. it becomes more skewed to the left. The RHS of (1.7) becomes more negative, and in order to restore the equilibrium L_2 must become steeper and L_1 must decrease.

Proposition 1.4 *Assume that early discovery becomes more likely, such that the hazard rate of the discovery technology $f/(1-F)$ increases. Then the banks tend to give in earlier.*

Rescue negotiations can become more difficult to hide, if the disclosure requirements for banks or firms are tightened. The introduction of a new business paper in a region can have a similar effect. The effect of a change in the discovery technology by varying π is similar: an increase in π leads to a reduced stopping time for all types (see Lemma (1.1)). Unfortunately the effect on the likelihood of liquidation is not easy to specify for the general case, as two effects are opposed: the banks stop earlier but discovery becomes more likely. This would be interesting, as one could derive implications for disclosure rules of stock markets, or for the benefits of having a more transparent economy. Consider the following change, however:

Proposition 1.5 *Suppose that the support of f is rescheduled such that $f_1(t) = f(\alpha \cdot t)$, where $\alpha < 1$. Then the banks tend to stop earlier, but the probability of liquidation is unchanged.*

Suppose that the speed of all information channels is increased symmetrically. In this case the moment of sure discovery τ has an effect on the stopping time of a bank with cost $\ell_i = 0$, but not on the *relative* stopping times of the other types (as it does not appear in the derivations). In this case, the improvement of the discovery technology had no material effect. Thus, stricter disclosure requirements can be neutral, and therefore (depending on the parameters) welfare reducing or improving.

Similarly, we can analyse changes in the distribution of types. Here the 'hazard rate' is somewhat complicated, as the types are revealed 'backwards', i.e. the first types that reveal themselves by stopping are those with high costs ℓ_i. The 'reversed' hazard rate is thus $g(\ell)/G(\ell)$. We encounter the same difficulties as in Proposition 1.4, as we can determine (using the equilibrium conditions (1.5) and (1.6)) the effect on the banks' strategies, but not the effect on the probability of liquidation.

Proposition 1.6 *Assume that the probability of ℓ being low is higher, such that the 'reverse' hazard rate of the type distribution g/G increases. Then the banks tend to give in earlier.*

This seems to be a surprising result, as one would expect 'tougher' banks to hold out longer. However, the result states that a bank with type ℓ will stop earlier. This is intuitive, as it must be more pessimistic about its strength relative to other types. The overall effect cannot be determined without making assumptions on the functional forms of f and g.

Negotiation costs can easily introduced to the model. They have been omitted for simplicity, but can be expected to have an effect on rescue negotiations. Examples for such costs are the need to set up a management team which analyses the firm's state and the rescue plans (i.e. the opportunity costs of sending bank managers to attend negotiations), legal costs (the costs of hiring lawyers), or the material costs of planning and negotiating (expenses for business consultants and industry experts, travel expenses).

Proposition 1.7 *Assume that each bank incurs a continuous cost c per unit of time dt, while the negotiations take place. Then the banks tend to stop earlier than in the case of no costs, and rescues are more likely.*

Even though this type of bargaining costs reduces the net surplus from a rescue, this material loss has no effect on the banks' decisions. At each instant, the past costs are sunk, and 'bygones are bygones'. However, c has an effect on the decision whether to wait another infinitesimal amount of time. It decreases the expected payoff from waiting, and therefore the banks stop earlier with higher costs. Thus, while the already incurred costs have no effect, the costs that have to be incurred if the negotiations continue are relevant for the decision to stop.

Finally, the entrepreneurs' utility functions are relevant. As m, the utility loss that an entrepreneur incurs if his project is liquidated, increases, funds become available for more parameter settings. Thus, there is a use in this model for the stigma that is attached to a business failure. While we do not want to suggest that this is a good way of solving incentive problems, we can conclude from the model that the financing patterns of two regions or industries should be different if bankruptcy is 'not a big deal' in one of them, while it has strong negative connotations in the other.

1.6 Conclusions

This paper studies the difference between single and multiple banking. It concentrates on renegotiation problems, which are shown to be solved better in the case of multiple banking. We assume that entrepreneurs ask banks for loans, such that they can start projects. These may be of a 'good' or 'bad' type, where the type of a project can be observed by the respective entrepreneur, only. 'Bad' projects need refinancing at an intermediate stage, which makes them nonprofitable from an ex ante perspective. However, once the initial loan is lost, refinancing is better than the only alternative, liquidation.

A single bank cannot commit not to refinance a bad project, which would keep entrepreneurs with 'bad' projects from applying for a loan. Two banks, however, can commit not to refinance with some probability. The reason for this are inefficiencies in the negotiations between the banks, when they have to agree on their respective degree of debt forgiveness. If the probability of liquidation is sufficiently high, entrepreneurs with 'bad' projects do not ask for a loan at all.

We model the negotiations as a war of attrition. Each of the two banks incurs a privately known loss, if the firm is liquidated, and therefore would like to have it refinanced. Additionally, refinancing is profitable, once the initial loan is sunk. The banks have to agree on how to split the costs and revenues, if they refinance the firm. These negotiations take time, and the longer they last, the more likely it becomes that a rescue becomes impossible (for exogenous reasons). In order to prevent this, the banks plan to 'give in' after a while, i.e. to let the opponent pocket the gain from rescuing, only to make sure that the firm is refinanced. There is a unique equilibrium in this game: the higher the potential loss, the earlier a bank decides to give in. The negotiations can last for a while, if both banks' potential losses are low, and therefore the firm is liquidated with positive probability.

The model is designed to isolate the advantage of multiplicity for the lenders. We thus abstract from many aspects which are relevant for the choice between bilateral and multilateral finance, as well as for reorganisation procedures. One of these is the tradeoff between single and multiple banking. Bolton and Scharfstein (1996) analyse a case where either single or multiple lending may be optimal, and also derive results for voting rules, as well as for the optimal use of assets as collateral. Similarly, the effects of different bankruptcy laws need further analysis. In the model the two banks decide to share the highest priority rank. It would be interesting to analyse a model in which their claims have different ranks. A further topic for future analysis is whether and how a distressed firm is rescued, if the banks do not enjoy the highest priority rank.

Appendix: Proofs

Proof of Lemma 1.1

(1.9) compares the respective payoffs for bank B_1 with valuation λ_1, given λ_2:

$$G(\lambda_2)r_1 + (1-G(\lambda_2))\left(\frac{\bar{R}-J-r}{2}+r_1\right) \tag{1.9}$$

$$= (1-G(\lambda_2))(\bar{R}-r_2-J)+G(\lambda_2)\left[(1-\pi)(\bar{R}-r_2-J)+\pi(r_1-\lambda_1)\right]$$

The left hand side of (1.9) is the expected payoff if bank B_1 gives in immediately. With probability $G(\lambda_2)$ the opponent has a low valuation and does not give in. The firm is rescued, and the bank receives r_1. With probability $1 - G(\lambda_2)$ the opponent gives in, as well, and the net surplus is shared (in expected terms).

The right hand side of (1.9) is the payoff if the bank gives in as soon as the war of attrition has started. With probability $1-G(\lambda_2)$ the opponent has a high valuation and will give in immediately. The bank rescues, pockets the surplus $\bar{R} - J$, and pays r_2 to the opponent. With probability $G(\lambda_2)$ the war of attrition starts. It is discovered with probability π, and the firm is liquidated. With probability $(1-\pi)$, the game could continue, but by definition the bank plans to stop, which earns r_1.

Some simplifications of (1.9) and of an analogous equation for bank B_2 lead to the two equations in Lemma 1.1. There is always an interior solution for the cut–off levels: If λ_i goes to zero, the $\lambda_j(\lambda_i)$ goes to infinity, while if λ_i goes to infinity it goes to zero. □

Proof of Lemma 1.2

(i) We first show that T_i is nonincreasing, and then that it is strictly decreasing. By utility–maximisation it must be the case that

$$\mathcal{V}_1(t_1, T_2(\cdot), \ell_1) \geq \mathcal{V}_1(t_1', T_2(\cdot), \ell_1) \quad \forall t_1', \;\; \forall t_1 = T_1(\ell_1) \tag{1.10}$$

and

$$\mathcal{V}_1(t_1', T_2(\cdot), \ell_1') \geq \mathcal{V}_1(t_1, T_2(\cdot), \ell_1') \quad \forall t_1, \;\; \forall t_1' = T_1(\ell_1'), \tag{1.11}$$

where $\mathcal{V}_i(t_i, T_j(\cdot), \ell_i)$ is the expected payoff of bank B_i with cost level ℓ_i, if it stops at t_i, and bank B_j plays strategy $T_j(\cdot)$:

$$\mathcal{V}_i(t_i, T_j(\cdot), \ell_i) = Pr\{T_j(\ell_j) \geq t_i\}\Big(F(t_i)(r_i - \ell_i) + (1 - F(t_i))r_i\Big) \tag{1.12}$$
$$+ \int_{\{\ell_j | T_j(\ell_j) < t_i\}} \Big[F(T_j(\ell_j))\Big(r_i - \ell_i - (\bar{R} - r_j - J)\Big) + (\bar{R} - r_j - J)\Big] g(\ell_j)\, d\ell_j.$$

The payoff of a bank depends on the chosen stopping time t_i, the opponent's strategy T_j and the (privately known) loss ℓ_i of losing the firm. With probability $Pr\{T_j(\ell_j) \geq t_i\}$ the opponent plans to stop later than t_i. If the public discovered the negotiations (This happens with probability $F(t_i)$), the payoff is $(r_i - \ell_i)$. If the secret was kept well, the bank receives r_i from bank B_j who rescues the firm. The second term of (1.12) is the equivalent if the opponent plans to stop earlier. Here the bank receives $(\bar{R} - J)$ if the firm can be rescued and pays r_j to the opponent.

We can rewrite these two inequalities (1.10) and (1.11) using (1.12). Subtracting the RHS of (1.11) from the LHS of (1.10), and the LHS of (1.11) from the RHS of (1.10), we get

$$Pr\{T_2(\ell_2) \geq t_1\}F(t_1)(\ell_1' - \ell_1) + \int_{\{\ell_2 | T_2(\ell_2) < t_1\}} F(T_2(\ell_2))(\ell_1' - \ell_1) g(\ell_2)\, d\ell_2$$
$$\geq Pr\{T_2(\ell_2) \geq t_1'\}F(t_1')(\ell_1' - \ell_1) + \int_{\{\ell_2 | T_2(\ell_2) < t_1'\}} F(T_2(\ell_2))(\ell_1' - \ell_1) g(\ell_2)\, d\ell_2$$

or, rearranging,

$$\Big[\Big(1 - Pr\{T_2(\ell_2) < t_1\}\Big)(F(t_1) - F(t_1'))\Big](\ell_1' - \ell_1)$$
$$\geq \left[\int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} F(t_1') g(\ell_2)\, d\ell_2 - \int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} F(T_2(\ell_2)) g(\ell_2)\, d\ell_2\right] (\ell_1' - \ell_1).$$

If $t_1 > t_1'$, the following holds:

$$\Big(1 - Pr\{T_2(\ell_2) < t_1\}\Big)(F(t_1) - F(t_1')) \geq 0,$$
$$0 \geq \int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} [F(t_1') - F(T_2(\ell_2))]\, g(\ell_2)\, d\ell_2,$$

and it must be the case that $\ell_1' \geq \ell_1$. On the other hand, if $t_1' > t_1$:

$$\Big(1-Pr\{T_2(\ell_2) < t_1\}\Big)(F(t_1')-F(t_1))$$
$$\geq Pr\{t_1 < T_2(\ell_2) < t_1'\}(F(t_1') - F(t_1))$$
$$\geq \int_{\{\ell_2 | t_1 < T_2(\ell_2) < t_1'\}} [F(t_1') - F(T_2(\ell_2))]\, g(\ell_2)\, d\ell_2,$$

and it must be the case that $\ell_1 \geq \ell_1'$. Thus for all t_1, t_1', in equilibrium $(\ell_1' - \ell_1) \cdot (t_1' - t_1) \leq 0$, i.e. the strategies are nonincreasing in the liquidation loss.

Assume that T_1 is not strictly decreasing, i.e. there are $\ell_a, \ell_b > \ell_a$, such that for all $\ell \in [\ell_a, \ell_b]$, $T_1(\ell) = \theta$. Then there is an $\varepsilon > 0$ such that all types ℓ_2 with $T_2(\ell_2) \in (\theta - \varepsilon, \theta]$ prefer to wait until θ and stop then, if the opponent did not stop. Then the types $\ell \in [\ell_a, \ell_b]$ could gain by stopping at $\theta - \varepsilon$ instead of θ: The probability of winning is not affected, but the risk of losing ℓ is diminished.

(ii) Assume that T_1 is discontinuous at ℓ. Then there are $t_a, t_b > t_a$ such that a type ℓ never stops at any $t \in (t_1, t_2)$. A type ℓ_2 with $T_2(\ell_2) \in [t_a, t_b)$ would thus wait only until t_a, and stop if the opponent did not stop. This implies that no one stops at any $t \in (t_a, t_b)$. But then there are types $\hat{\ell_1}$ and an $\varepsilon > 0$ such that $T_1(\hat{\ell_1}) \in [t_b, t_b + \varepsilon]$, who prefer stopping at some $t \in (t_a, t_b)$.

(iii) Assume that T_i is not differentiable at ℓ.

(a) Let the lefthand derivative be higher than the righthand derivative (T_i is flatter to the left of ℓ). Then there is an $\varepsilon > 0$ such that no type ℓ_j stops at any $t \in (T_i(\ell) - \varepsilon, T_i(\ell)]$. It pays to wait longer since after the point of discontinuity it becomes relatively likely that the opponent stops. This holds since both f and g are continuous and differentiable.

(b) Let the righthand derivative be higher. Then there is an $\varepsilon > 0$ such that no type ℓ_j stops at $t \in [T_i(\ell), T_i(\ell) + \varepsilon)$. It pays to stop earlier since it becomes more likely that B_i stops immediately after $T_i(\ell)$.

In both cases T_j is not continuous, contradicting *(ii)* above.

(iv) If type $\ell_1 = 0$ stops at $\theta < \tau$, all types with higher loss level stop earlier because of Lemma 1.2*(i)*. Then in equilibrium no type of the other bank should stop later than θ. There is an $\varepsilon > 0$ such that types $\ell_1 \in (0, \varepsilon]$ find it profitable to wait until θ and wait for the opponent to stop. □

Proof of Proposition 1.1

The differential equations are Lipschitz–continuous on $[2, \tau]$ which implies that a solution exists and is unique (See e.g. Birckhoff and Rota (1978, Ch. 6)). At each $t \in [2, \tau]$, the (expected) payoffs from stopping or non–stopping can be compared, as was done in deriving (1.5). Since the strategies are strictly decreasing, at $t < T_1(\ell_1)$, i.e. if $L_1(t) > \ell_1$, the payoff to bank B_i with loss level ℓ_1 will be higher if it waits. The opposite holds for $L_1(t) < \ell_1$. For all $t \geq t_1$, type $L_1(t_1)$ can only decrease his payoff by waiting, and will stop whenever possible.

The players constantly update their beliefs using Bayes' Rule. If a player stops at the wrong time (this is the only deviation that is possible) the opponent will have no difficulties in updating his beliefs: If a player stops too early, the game is over and beliefs are not relevant anymore. If a player waits too long, the strategy tells him to stop immediately: Type ℓ_i stops at any time t if $t > T_i(\ell_i)$. Again, the opponent can update his beliefs without problems. □

Proof of Proposition 1.2

Follows directly from the Assumptions and Proposition 1.1. □

Proof of Proposition 1.3

The reaction curves L_i (see (1.7)) are shifted outward, if s is increased. The indirect effect via the cut–off value λ goes in the same direction: $\lambda_i(\lambda_j)$ is shifted outward, as well (see Lemma 1.1). □

Proof of Propositions 1.4, 1.5, and 1.6

As Proposition 1.3: analyse the equilibrium conditions (1.5) and (1.6), and the indirect effect via the cutoff value λ in Lemma 1.1. □

Proof of Proposition 1.7

(1.7) is changed to

$$L_2(t_1) = G^{-1}\left(G(\lambda) \cdot \exp\left\{-\int_2^{t_1}\left[\left(\frac{f(t)}{1-F(t)}\right)\frac{L_1(t)}{s} + \frac{c}{s}\right] dt\right\}\right). \qquad (1.13)$$

A comparison of (1.13) with (1.7) shows that all types will want to stop earlier, including zero–cost types. □

Chapter 2

Optimal ‘Soft’ or ‘Tough’ Bankruptcy Procedures

Optimal 'Soft' or 'Tough' Bankruptcy Procedures

Paul POVEL

Financial Markets Group, London School of Economics, and
E.C.A.R.E. and D.U.L.B.E.A., Université Libre de Bruxelles

May 1996

London School of Economics, Financial Markets Group, London WC2A 2AE, United Kingdom. E–Mail: povel@lse.ac.uk. Correspondence: E.C.A.R.E., Université Libre de Bruxelles, Av. F.D. Roosevelt 39, B–1050 Bruxelles, Belgium. E–Mail: povel@ulb.ac.be.
I would like to thank Patrick Bolton for assistance, support, and many comments on earlier drafts of this paper. I am grateful to John Moore for encouraging me to work on this topic, and for subsequent invaluable discussions. I am also indebted to Philippe Aghion, Mathias Dewatripont, Oliver Hart, Klaus Schmidt, Urs Schweizer, Jaime Zender, and seminar participants in Bonn, at the London School of Economics, and at ECARE for helpful comments on an earlier version of this paper. Financial support from the Freiwillige Akademische Gesellschaft (Basel) and the European Union (Human Capital and Mobility Program) is gratefully acknowledged. I have benefited from the hospitality of the University of Bonn, Wirtschaftspolitische Abteilung, where part of this paper was written.

Abstract

This paper studies optimal bankruptcy laws in a framework with asymmetric information. The key idea is that the financial distress of a firm is not observed by its lenders for quite a while. As early rescues are much cheaper than late rescues, it may pay if the creditors are forgiving in bankruptcy, thereby inducing the revelation of difficulties as early as possible. Either 'tough' or 'soft' bankruptcy laws can be optimal. 'Hybrid' procedures are found to be redundant, and possibly harmful. Absolute Priority Rules may be helpful as a part of pure liquidation procedures, but their introduction is (partly) inconsistent with the design of 'soft' procedures like Chapter 11. The paper also reviews evidence on the performance of Chapter 11, questioning many negative results.

JEL No G32, G33, G38

Keywords: Bankruptcy, Reorganisation, Chapter 11,
Absolute Priority Rule, New Value Exception

2.1 Introduction

Even though the *causes* of a bankruptcy may be exogenous, the timing, i.e. the *start* of a formal procedure is highly endogenous. 'Bankruptcy' does not hit a firm like a flash. There are plenty of methods for a firm to hide or even cover up financial difficulties. Cash flows can be freed to finance current losses, e.g. by cutting R&D or replacement investments, or by reducing the quality of the firm's products. Changes in the accounting practices can achieve the same goal. Artificial reductions in the valuations of obligations and increases in those of assets can generate additional 'income'. Typically these methods are easy to implement, and it is difficult to observe or even prove that a business decision was not based on sound principles.

One advantage of delaying formal bankruptcy by hiding the financial difficulties is that this also delays (or possibly even prevents) its usually unpleasant consequences for the managers and owners of the firm. Another advantage is that the difficulties could be of a temporary nature, and that the 'breakthrough' (or 'turnaround') will come later than expected. The manager/owner of the firm could simply 'wait and pray', and if he is lucky nobody else can tell that there had ever been difficulties. The costs of doing so are borne by the creditors: Early rescues of a firm are typically cheaper than late rescues, and more likely to be successful. Delays cause opportunity costs because the assets of the firm cannot be brought to their most efficient use. Further costs arise if the delay is achieved by cutting investments in the future of the firm (R&D, plants and machinery, reputation, etc.). Not only does this make it more costly to rescue the firm (as more wrong decisions must be corrected), but a rescue may even become impossible.

In this paper we analyse the tradeoff between two conflicting goals of a firm's creditors. On the one hand, they want a bankruptcy procedure to be tough on the borrower, as a harsh punishment may increase his incentive to generate sufficient earnings to repay. On the other hand, the creditors want to prevent the waste of resources that takes place if a rescue is necessary but not undertaken in time. Clearly, if bankruptcy is a strong punishment, a borrower keeps the unpleasant information for himself and prefers to 'wait and pray'. An obvious method to obtain the necessary information is to reward its revelation. However, this implies that the borrower is rewarded for poor outcomes. This works against the 'effort' incentives: it limits the extent to which the borrower can be punished, and if 'effort' is relevant, its provision must be ensured by raising the entrepreneur's payoff after good outcomes. Thus, the creditors have to trade off a waste of resources if a rescue is possible for higher costs of effort provision. It is not clear a priori whether one of the incentive problems is more relevant, or if both can be solved at the same time.

More concretely, we model an entrepreneur who can start a project by investing both effort (either a high or a low level), and a fixed amount of capital. The outcome can be good or poor. Effort has the disadvantage that it causes disutility, but it also increases the probability of realising a good outcome. The entrepreneur does not have the funds to invest, and must therefore borrow from an investor. Writing a financial contract is made difficult by two types of asymmetric information. Firstly, the entrepreneur's effort choice is unobservable. The contract can only be contingent on the final outcome, and a wedge between the entrepreneur's respective payoffs after good and poor outcomes is necessary to provide an incentive to invest enough effort. We assume that it is essential that the entrepreneur chooses the high effort level, if the project is to be profitable. Given that Limited Liability prevents very low payoffs for the entrepreneur, the wedge must be created by offering a sufficiently high payoff if a good outcome is realised.

Secondly, at an intermediate stage the entrepreneur receives a signal about the prospects of his project. This signal is not observable by anyone except himself. The creditor would be interested in this information, however, as it could be possible and profitable to invest more money in a bad project. If she wants to realise such an efficiency gain (or reduce her expected loss), she has to 'buy' the information from the entrepreneur. She cannot rely on him to just inform her that he needs more money, as he could also choose a 'wait and pray' strategy: If no additional money is invested, a bad project may nevertheless become a good project with some probability. The revelation of bad news is costly in terms of effort incentives: a reward for telling the truth is paid when poor outcomes are likely. This drives up the payoff that the entrepreneur must receive if a good outcome is realised, to make sure that he invests the high effort level.

This tradeoff between being soft and tough when the prospects are bad (in 'bankruptcy') is exacerbated by another (realistic) assumption: if a borrower demands more funds for a rescue, the creditor cannot tell whether a rescue is really worth undertaking, or whether the borrower is simply going to use up those funds to keep the firm afloat for a while (the latter is a complaint that can be heard in many Chapter 11 cases). Thus, a misuse of the creditor's softness may be quite expensive, and the equilibrium contract must prevent such waste.

While it is obvious that a borrower would prefer to be treated well in bankruptcy, we would not a priori expect a creditor to share this wish. We therefore isolate the creditor's willingness to be soft and forgive debt by assuming that the investor has all bargaining power. That is, she designs the contract, and the entrepreneur can only accept or reject it. In equilibrium the investor will propose a standard debt contract, to which she has added a 'bankruptcy clause': this clause determines the actions and events if the entrepreneur announces that he will probably be unable to repay his debt. The 'bankruptcy clause' can be either 'soft' on the manager, inducing an early revelation of information, or 'tough', i.e. treating him as badly as possible if poor outcomes are realised.

Our work has implications for the design of bankruptcy laws. 'Optimal bankruptcy laws' are identified as those 'bankruptcy clauses' that the parties would have added to their contract, if there were no transaction costs in relation to writing such a 'complete' contract. A bankruptcy law is thus 'optimal', if it replicates the optimal contract in a transaction cost free environment. One result is that either tough procedures or soft procedures may be optimal, but mixed procedures, which contain elements from both types of procedure, may be much worse than the two pure procedures. This result is relevant for the bankruptcy laws in many countries. In the UK and in Germany, for instance, there have been attempts to introduce 'softer' bankruptcy laws. At the same time the drafters of the new legislation tried to preserve the 'punishing role' of bankruptcy. Our paper shows that this can backfire. In the UK, the procedure called Administration is rarely used, as a creditors who holds a Floating Charge can opt out and start a much tougher procedure (Administrative Receivership); Similar results should be expected in the daily practice of the new *Insolvenzordnung* in Germany, where the bargaining position of the manager or owner has not been significantly improved, compared with the current law.

A second result concerns Absolute Priority Rules. These rules establish a creditor's right to object to payoffs that holders of lower ranked claims (for instance equityholders) receive, if this creditor's claim has not been repaid. Our result is that a soft procedure must violate Absolute Priority Rules to some degree. The entrepreneur must be rewarded if he cooperates in a rescue by revealing information early, whether the debtors have been repaid or not. In the extreme, he should be rewarded even if the firm must be liquidated, and no debt is repaid at all. This extreme result highlights the difficulties that a consistently designed 'soft' procedure might encounter in practice, in particular if confronted with much more appealing (and traditional) arguments in favour of 'tough' procedures. Note, however, that this violation of Absolute Priority Rules concerns only the ranking of debt and equity. Nothing is implied about the use of Absolute Priority Rules between different classes of debt.

There are striking similarities between the soft bankruptcy procedure in this paper, and Chapter 11, the procedure that is used in the US for reorganisations. One of the major differences is that there are no 'direct rewards' in Chapter 11. Quite the contrary, there are Absolute Priority Rules which are supposed to be strictly enforced (if the parties disagree). We argue that indirect reward systems are being used, instead. Equity can earn a reward in Chapter 11 cases because it is endowed with a strong bargaining position. We argue that this need to reward indirectly is the main source of many inefficiencies that are blamed on the procedure, and that much of the bad press of Chapter 11 needs to be revised or targeted more precisely. It is also an inefficient way of rewarding, as a lender's bargaining position determines the payoff. If he knows that he is perfectly substitutable, the incentive to file, and thereby reveal the bad news, are small.

Our work has implications for other aspects of the US Bankruptcy Code, and for the revision which is currently under way. The 'New Value Exception' can be interpreted as a cheap method to 'reward' shareholders for filing early: manager–

owners of distressed firms are commonly assumed to have superior information about the value of the assets and their best use. By allowing them to cash in on this superior information the reward that a soft procedure requires is much less costly to the lenders than a direct reward.

This paper adds to the literature on bankruptcy (see e.g. Baird (1995) or White (1996) for a survey) by showing how the softness of a bankruptcy procedure may improve the timing of restructuring decisions. For the purpose of clarifying the analysis only, we are ignoring other important aspects of bankruptcy. In particular, we assume that there are no collective action problems on the side of the creditors. These problems have been discussed in earlier contributions, and possible solutions have been suggested (see e.g. the mechanisms suggested by Bebchuk (1988) and Aghion et al (1992 and 1994)). While these papers concentrate on *ex post* bargaining problems, we in this paper concentrate on *ex ante* incentive problems, i.e. decisions of firms outside of bankruptcy.

Several papers have analysed the effects that the US procedures Chapter 7 and Chapter 11 have on a firm's incentive to cooperate either outside or in bankruptcy (or both). These include Bebchuk (1991), Bebchuk and Chang (1992), Mooradian (1994) and White (1994). Our paper differs from these in two dimensions. Firstly, we do not start with exogenously given bankruptcy procedures, but derive 'optimal procedures' from first principles. Secondly, we analyse two types of incentive problems that arise 'outside of bankruptcy'. On the one hand, a procedure has effects on both the efficiency of the bargaining outcome and on ex ante effort and investment decisions; these have been thoroughly studied in the earlier literature. On the other hand, a procedure will also need to secure the entrepreneur's cooperation in *starting* a rescue, which is a relatively new topic.[1]

Related papers include Gromb (1994), Heinkel and Zechner (1993), Giammarino and Nosal (1995) and Berkovitch, Israel, and Zender (1995). Gromb (1994) analyses a creditor's willingness to forgive debt in a multiperiod lending situation, extending the model in Bolton and Scharfstein (1996) to many periods. As in our

[1]The conjecture that soft procedures may be useful to induce early bankruptcy filings can be found in Jackson (1986, ch. 8), Baird (1991, 1993, and 1995), White (1989 and 1996), and Aghion et al. (1992 and 1994).

model, the investor has all bargaining power, and the entrepreneur is protected by limited liability. Nevertheless, the investor has strong incentive to forgive debt, because threats to terminate a lending relationship may not be credible.

Heinkel and Zechner (1993) and Giammarino and Nosal (1995) analyse the effects of different contractual and legal regimes on creditors' debt forgiveness, and on the entrepreneur's renegotiation timing decision. One of the results in Heinkel and Zechner (1993) is that there can be gains from allowing for deviations from Absolute Priority Rules.

Berkovitch, Israel, and Zender (1995) analyse boundaries to using bankruptcy as a bonding device. If an entrepreneur is treated too badly in bankruptcy, this has a negative impact on his ex ante choice between investing in marketable nonspecific human capital, or in more productive firm–specific human capital. As a solution, the authors suggest to introduce bankruptcy laws that give the entrepreneur additional bargaining power if things go badly. One possibility is to give him the exclusive right to make the first offer in a Rubinstein bargaining game, and to artificially delay any following counteroffers (as in Chapter 11).

Other related problems are studied in Fudenberg and Tirole (1995), Boot and Thakor (1993), Dewatripont and Tirole (1994), and Aghion et al. (1996). Fudenberg and Tirole (1995) study the incentive of a manager to smoothen the reported income and the dividends of a firm over time. The manager corrects low values upwards and high values downwards because he wants to protect the private benefits that he derives from controlling the firm. Boot and Thakor (1993) and Dewatripont and Tirole (1994) analyse the incentive of a bank regulator to close down undercapitalised banks, if bank closures may cast doubt on his past monitoring effort or ability. Aghion et al. (1996) study the problem of restoring the banking systems of Eastern Europe to health. A government that wants to recapitalise distressed banks must be careful how it treats the managers of these banks, as it needs their information on their real financial needs. A treatment that is too soft leads to excessive demands for refinancing, while if the treatment is too tough, bad loans are hidden and simply rolled over.

A potential further application of our model is Golden Parachutes. Knoeber (1986) analyses their effect on a manager's incentive to invest effort, if an optimal contract requires deferred payments, i.e. when the effects of the manager's effort become visible. The manager has a weak bargaining position if the firm changes owners in a 'hostile takeover', as the new owners could simply fire him and keep the deferred promised payments for themselves. His model could be extended to include the manager's fight against a takeover, or even his encouragement of a takeover, by using our model.

Finally, in independent research, Levitt and Snyder (1996) have analysed a similar agency problem, and derive similar results.[2] A principal employs an agent, who must invest unobservable effort to start a project. The final outcome can be either 'good' or 'bad', where the agent's effort increases the probability of the former. At an intermediate stage, the agent privately observes the probability of the 'good' outcome. If this probability is low, the principal would like to share this information, as he could costlessly terminate the project prematurely (the likely bad outcome is a loss). Early termination, however, makes initial effort more costly, as it decreases the effect that effort has on whether the good or the bad outcome will be observed in the end.

The papers differ completely in the extent to which the results are applied. Our paper was motivated by the observation that there are important conflicting goals that reorganisation procedures are expected to achieve. Our model has important implications for both the design of bankruptcy laws, and for the large literature which analyses the performance of Chapter 11 in the US.

While the models are different, the type of results that are generated are quite similar. In our model, the optimal procedure may be either 'tough' or 'soft', a result which cannot be generated in a model which is simpler than the one presented here (see Section 2.4). In Levitt and Snyder (1996) there is a corresponding result, which states that the *extent* of revelation that is induced by their optimal incentive schemes will depend on the parameters of the model. However, this result only holds if their contract space is constrained to non–randomized decisions. With randomizing, Levitt and Snyder (1996) obtain an unrealistic optimal contract which is similar to our 'contract for many types' in Section 2.4. In our paper, we additionally address renegotiation issues, i.e. the

[2]We thank John Moore for drawing our attention to their work.

problem of the lack of credibility of a 'tough' procedure, which would be difficult in the model by Levitt and Snyder (1996).

The rest of the paper is structured as follows. In Section 2.2 the model is presented. In Section 2.3 we derive the equilibrium contracts, and some implications for bankruptcy laws. In Section 2.4 we discuss simplifications and extensions of the model, and justify the unorthodox use of *three* project types. In Section 2.5 we introduce a renegotiation possibility, which restricts the use of tough bankruptcy procedures. Section 2.6 compares the 'soft' procedure with Chapter 11, and reviews and reinterprets evidence of its performance. Section 2.7 concludes. All proofs are in the Appendix.

2.2 The Model

There is an entrepreneur, who can start a project but has no wealth of his own, and an investor. She may offer to finance the entrepreneur's project, if they can find a contract under which both parties break even.

The project extends over up to five periods. In period 1, an amount K must be invested, and the entrepreneur must invest effort $\tilde{e} \in \{0, e\}$, which the investor cannot observe (this can easily be generalised to a continuous choice, see Section 2.4).

In period 2, the type of the project is realised. It is g ('good') with probability $\tilde{e}$, b ('bad') with probability $a(1-\tilde{e})$, and ϕ ('failure') with probability $(1-a)(1-\tilde{e})$. Thus, if the entrepreneur did not invest effort in period 1, the type must be either b or ϕ. The type can be observed by the entrepreneur, only.

In period 3, the project can be refinanced by investing an amount J (a 'rescue'). This money could be necessary to install a new organisational structure, say, or to start a price war. If the project type is g, the payoffs are unchanged, as a firm that is doing well presumably does not have to change its organisation, or market strategy. If the type is b, the project becomes a 'good' project (type g). Here a reorganisation of the firm's policies might be helpful, and for simplicity we assume that it is guaranteed to be successful. Finally, with the 'failure' type we model a firm that should be liquidated, but this fact is not common knowledge.

We assume that it would starve publicly in period 4, but if it is refinanced, the additional funds J allow the project to continue until $t = 5$.

In period 4, a 'failure' type that was not refinanced in period 3 'starves' publicly, and must be liquidated. A 'bad' type that was not refinanced becomes a 'good' type with probability b. The idea is that the entrepreneur has a 'wait and pray' strategy, i.e. he can hope that there will be a 'breakthrough' or 'turnaround' of his business (if he is lucky in the end, the investor is not able to tell that there had been difficulties at an intermediate stage).

Finally, in period 5, the (verifiable) payoffs are earned. A 'good' project earns Y, a 'bad' project $y < Y$. A 'failure' earns nothing, whether it has been refinanced or not. If the project was not terminated earlier, the entrepreneur additionally earns a private benefit r. That is, if the project type was ϕ, and it was not refinanced, the entrepreneur does not earn the private benefit r, while in all other cases he does.

While the entrepreneur is perfectly informed about every variable as the project progresses, the investor cannot observe the effort decision, nor can she observe the type of the project in period 2. This captures the idea that while the *causes* of financial distress may be exogenous, the *start* of 'bankruptcy' is not an exogenous event, but can in most cases be delayed or triggered early by a distressed firm. As was outlined in the Introduction, a firm can change its accounting practices, and value assets and obligations differently. Similarly, it can economise on its investments, in order to be able to finance current losses.

The entrepreneur must make two decisions. First, he must decide how much effort to invest in period 1. Second, he must decide whether to reveal his information in period 3. If the type of his project is 'bad', he can make its rescue possible by revealing the type. He can also claim that the type is 'good', and hope for a breakthrough in period 4. If he is lucky with this 'wait and pray' strategy, the investor will not be able to tell whether the type was really 'good' or not.

We observed in the Introduction that late rescues are more costly and less likely to succeed than early ones. We model this by assuming that 'early' rescues (in period 3) are profitable,

$$Y - J > bY + (1 - b)y, \tag{2.1}$$

while later rescues are impossible. While the rescue of a 'bad' type is profitable ex post (in period 3), ex ante it is not:

$$Y - J < K. \tag{2.2}$$

We also assume that a 'failure' type project should not be refinanced:

$$J > r. \tag{2.3}$$

There is no monetary profit from refinancing a 'failure' project, and the entrepreneur's private benefit from completing the project is smaller than the cost of rescuing.

The players have the following utility functions:

$$\text{Investor:} \qquad V = X_I, \tag{2.4}$$

$$\text{Entrepreneur:} \qquad U = X_E - c\tilde{e} + \tilde{r}, \tag{2.5}$$

where X_I and X_E are monetary incomes, c is a positive constant, $\tilde{e}$ is the effort that the entrepreneur invests at $t = 1$, and $\tilde{r}$ is zero, if the project was terminated prematurely, and r otherwise (r may be the satisfaction of having completed a project, for instance).

We assume that the investor has all bargaining power from the start. She designs the contract initially, and once the project has started, the entrepreneur can be replaced at any instant, without affecting the payoffs. We make this assumption to *isolate* the investor's willingness to forgive debt, in order to allow for an early rescue if this is necessary. It is quite intuitive that the entrepreneur should prefer a 'soft' over a 'tough' contract, but this is less straightforward if one considers the position of the investor.

Note that the investor's bargaining power is not unrestricted, however: the entrepreneur is protected by *limited liability.* He cannot receive negative transfers, and the worst punishment that can be inflicted on him is a loss of control over the project and of any income.

We make some assumptions on the parameters of the model. First, the private benefit r is not high enough to induce the entrepreneur to invest effort without any monetary incentives. We can write this condition as

$$c > (1 - a)r. \tag{2.6}$$

Second, the efficient choice for the entrepreneur is to invest the high effort level e, and the project is viable if he does. A sufficient condition for this to hold, even in the case with asymmetric information, is that the inequality

$$(1-a)Y + aJ > c - (1-a)r \tag{2.7}$$

is sufficiently strict.

2.3 Optimal Contracts

In this section we derive and compare the contracts that the investor may want to propose in equilibrium. In a first best environment, the entrepreneur would invest the high effort level, and the investor would have the necessary information for an efficient rescue decision. With asymmetric information, this is not necessarily the case. When searching for the contract that maximises her payoff, the investor must take into account several constraints: the entrepreneur should accept the contract, he should invest effort, he should reveal the project type if the investor needs to know it, and the limited liability constraint must be met. We use the Revelation Principle to find the optimal contract. As we follow a standard procedure, the maximisation program and the solution are in the Appendix.

We find two types of contract, of which one will be the equilibrium contract, depending on the parameters of the model. These are described in Propositions 2.1 and 2.2.

Proposition 2.1 *The following* ***Tough Contract*** $(\mathcal{C}^T)$ *may be an optimal contract for the investor:*

§1 At $t = 1$, the entrepreneur receives K from the investor and invests it in his project.

§2 If at $t = 3$ the entrepreneur reveals that the project is of either a 'bad' or a 'failure' type, the entrepreneur is fired immediately, and all eventual earnings belong to the Investor. She may refinance or liquidate the project, and keep the returns for herself.

§3 If §2 does not apply, and the return is Y, the entrepreneur makes a payment

$Y - \frac{c}{1-ab} + \frac{1-a}{1-ab}\, r$ to the Investor. If the entrepreneur cannot make this payment, the returns of the project belong to the Investor.

The tough contract has much similarity with a standard debt contract. §1 and §3 specify the use of the loan K and the repayment if the project was successful (the investor receives $Y - \frac{c}{1-ab} + \frac{1-a}{1-ab}\, r$). §2 makes provisions for the case in which the entrepreneur has told the investor that he may not be able to repay the amount specified in §3. §2 can be interpreted as a 'bankruptcy clause', which could also be omitted if there were an equivalent bankruptcy law. This bankruptcy law would be a pure liquidation procedure: if the investor discovers that the project type is either b or ϕ, she may take over control of the assets and of all returns. Obviously, the entrepreneur has no incentive to reveal a b or ϕ type under a tough bankruptcy regime, as the payoff from remaining silent is strictly higher for type b and unchanged for type ϕ.

Thus, by proposing a tough contract, the investor willingly ignores the possibility to rescue the project early. Before we discuss the possible reasons for this, we present the alternative contract.

Proposition 2.2 *The following **Soft Contract** ($\mathcal{C}^S$) may be an optimal contract for the investor:*

§1 At $t = 1$, the entrepreneur receives K from the investor and invests it in his project.

§2.a If at $t = 3$ the entrepreneur reveals that the project is of a 'bad' type, the investor refinances the project, and if at $t = 5$ the return after rescuing is Y, the entrepreneur pays $Y - b \cdot \frac{c}{1-ab}$ to the Investor.

§2.b If at $t = 3$ the entrepreneur reveals that the project is a 'failure', the project is liquidated, and the entrepreneur receives a payment r from the Investor.

§3 If §2 does not apply, and the return is Y, the entrepreneur makes a payment $Y - \frac{c}{1-ab}$ to the Investor. If the entrepreneur cannot make this payment, the returns of the project belong to the Investor.

Like the tough contract, $\mathcal{C}^S$ contains elements from a standard debt contract (§1 and §3), and bankruptcy clauses (§2.a and §2.b). The debt contract part differs only in the size of the repayment in the last period: it is higher with a tough contract, i.e. the entrepreneur's reward for successful outcomes is higher if $\mathcal{C}^S$ is signed. The bankruptcy elements of $\mathcal{C}^T$ and $\mathcal{C}^S$ differ considerably. Under a soft regime, the entrepreneur decides (in equilibrium) to reveal his type in the third period. If the type is b, he receives a payoff which is the same as his expected payoff if he decided to 'wait and pray', i.e. if he pretended that the project was a g type. Similarly, the entrepreneur has no incentive to lie if the type of his project is ϕ. Revealing this information earns him a reward r, while the best lie, pretending to have a 'bad' project, would earn him exactly the same payoff (in the form of a private benefit, however).

Proposition 2.3 *One of the two contracts $\mathcal{C}^S$ and $\mathcal{C}^T$ achieves the highest payoff for the investor. That is, no other contract achieves a strictly higher payoff.*

Propositions 2.1 to 2.3 are proved jointly, using the Revelation Principle (see the Appendix). The intuition behind the results is that the contract should first of all provide an incentive to put in effort, as otherwise the project is not profitable. The next question is how 'expensive' it is to get the entrepreneur to reveal bad news *truthfully*, making sure for instance that if the project is a 'failure' the entrepreneur does not claim that it is 'bad', only to earn the private benefit r. This revelation is 'expensive' in the sense that it makes the provision of effort incentives more difficult. A reward for not lying about bad outcomes limits the 'punishment' for bad outcomes. The entrepreneur invests effort only if the wedge between the payoffs for good and poor outcomes (bad or failure) is sufficiently large. Thus, the only choice that the investor has is to *increase* the payoff for good outcomes. Depending on the parameters this might be quite expensive, compared with the gain that can be made by rescuing a 'bad' project. In this case, the investor prefers to ignore this possibility, and the ('tough') contract aims only at providing effort incentives.

Proposition 2.3 has important implications for bankruptcy laws. The second paragraph in both $\mathcal{C}^S$ and $\mathcal{C}^T$ could be provided by a bankruptcy law, instead of being added to the contract. This may seem to be a redundant exercise in the present model, but in more realistic contracting environments a 'robust' principle may be a good substitute for elaborate individual complete contracts. Unfortunately, there is no convincing theory that shows both why contracts are chosen to be incomplete, and how publicly provided mandatory laws can improve upon complete contracts. When discussing the relevance of our results for bankruptcy laws, we follow the literature in assuming that there is a need for such laws, and ask how they should be designed. As in e.g. Baird (1995) or Aghion et al. (1994), we claim that optimal laws should replicate the clauses that the negotiating parties would have added to their contract in the absence of transaction costs.

In the following we will interpret the bankruptcy clauses in the two contracts as being 'optimal bankruptcy laws'. A tough bankruptcy law could for instance require that the entrepreneur immediately loses all control rights to a trustee. Furthermore, the creditor may have the right to decide on the use of the assets of the firm. Finally, the entrepreneur will receive a payment only if the debt has been fully repaid. One can easily identify this set of rules as a pure liquidation procedure, as we can find it in many countries (e.g. Chapter 7 in the US). With a soft bankruptcy law, the entrepreneur may lose control of the firm, or he may not. The relevant difference to the tough law is that he receives a payoff after filing, even if some debt is not repaid (even if no debt is repaid at all). Chapter 11 is the only procedure that has some characteristics in common with the soft bankruptcy law. We will investigate these similarities, as well as the differences, in more depth in Section 2.6.

We can reinterpret Propositions 2.1, 2.2, and 2.3 as stating that either soft or tough bankruptcy laws may be optimal, and that no other type of law is strictly better. In particular, mixed forms of law may be strictly worse than either of the pure forms. Consider the following example:

Hybrid Contract *($\mathcal{C}^H$):*

§1 At $t = 1$, the entrepreneur receives K from the investor and invests it in his project.

§2.a If at $t = 3$ the entrepreneur reveals that the project is of a 'bad' type, the investor refinances the project. The entrepreneur receives a payment only if all debt has been repaid.

§2.b If at $t = 3$ the entrepreneur reveals that the project is a 'failure', the project is liquidated. Payments as in §2.a.

§3 If §2 does not apply, and the return is Y, the entrepreneur makes a payment $Y - Z^H$ to the Investor. If the entrepreneur cannot make this payment, the returns of the project belong to the Investor.

As in the case of $\mathcal{C}^S$ and $\mathcal{C}^T$, the hybrid contract contains a standard debt contract part, and a bankruptcy clause. Under this contract, the entrepreneur will claim to have a g type if the real type is either g or b. With a b type his monetary payoff is zero after a rescue, while the 'wait and pray' strategy promises a positive expected monetary payoff. However, the ϕ type has an incentive to claim having a b type. The investor would then refinance the project, which is nonprofitable in monetary terms, but secures the entrepreneur's private benefit r. The hybrid bankruptcy law is strictly worse than both pure laws: the 'bad' type is not rescued, the 'failure' type is refinanced, and the transfer payment Z^H is as high as the success payment $\frac{c}{1-ab}$ in $\mathcal{C}^S$ (if effort is to be provided).

How are the transfer payments in $\mathcal{C}^T$ and $\mathcal{C}^S$ determined? In the case of the tough contract $\mathcal{C}^T$, the entrepreneur can keep some of the return only if the project earned Y (remind that no information is revealed in period 3). This monetary payoff must be high enough to ensure that effort is invested. The case of the soft contract $\mathcal{C}^S$ is more complicated. Additionally to the effort problem, the reward scheme must satisfy several truthtelling constraints. Some of these are depicted in Figure 1.

Figure 1 shows the set of soft contracts (including the optimal soft contract $\mathcal{C}^S$), parametrised by the monetary payoffs that the entrepreneur receives (keeps) after 'good' and 'bad' outcomes, Z_{Yg} and Z_b (Z_b is paid after a successful rescue).

Three incentive constraints for soft contracts are shown. First, the effort incentive constraint, (IC_e). If the 'bad' payoff is too high, compared with the 'good' one, the entrepreneur has no incentive to put in effort. This constraint is binding in equilibrium, and the entrepreneur is indifferent between investing effort and not investing. Second, (IC_{gb}) is the truthtelling constraint if the type is g. If the reward for admitting that the project is of a 'bad' type is too high, the entrepreneur with a 'good' project may pretend to have a 'bad' one. As

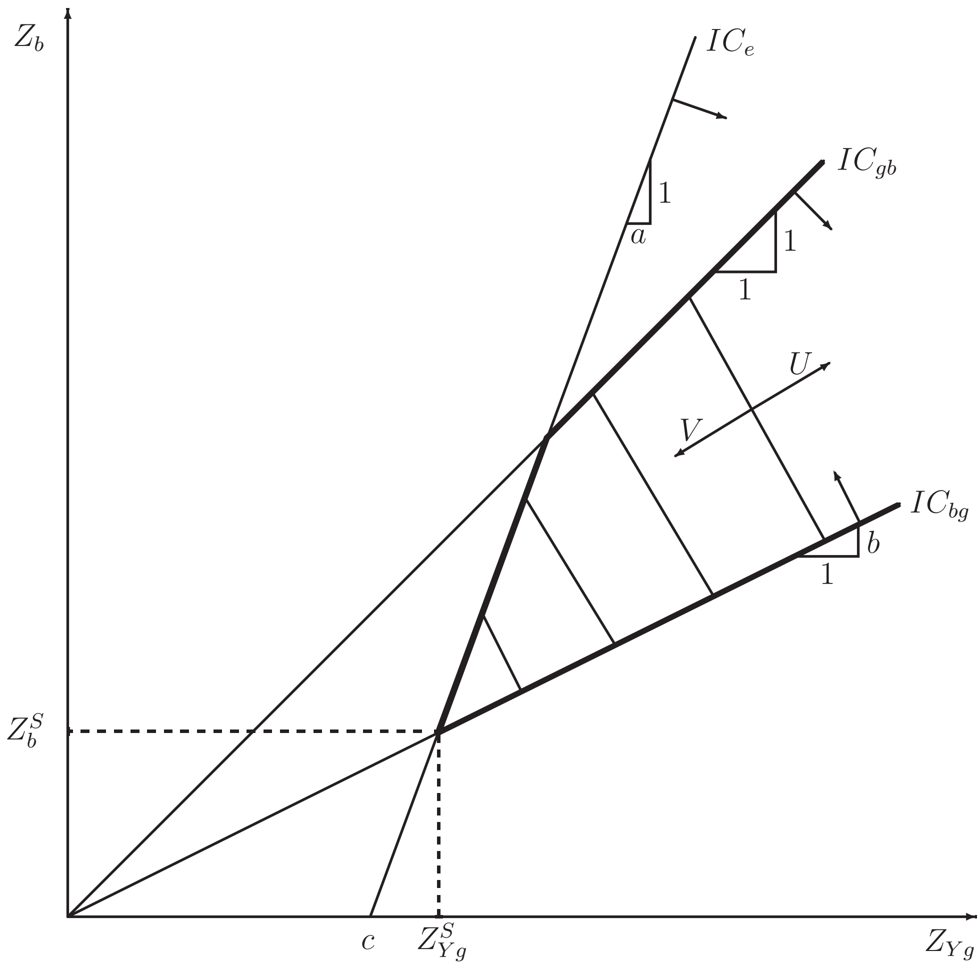


Figure 2.1: *Incentive constraints for the soft contract* $\mathcal{C}^S$

J has no effect on the final outcome Y, the investor would not be able to tell what the real type was. This constraint is not binding in equilibrium. Third, the truthtelling constraint for the 'bad' type is (IC_{bg}). If the monetary payoff for a 'bad' type is not high enough, the entrepreneur prefers to pretend that the type is g, and decides to 'wait and pray'. This constraint is binding in equilibrium, and the entrepreneur with a 'bad' type is indifferent between revealing this fact and pretending that the type is g. We have omitted the truthtelling constraints that refer to type ϕ (four constraints). The only of these that is binding is that an entrepreneur with type ϕ should not claim to have type b, only to be rescued.

The area enclosed by the thick lines is the set of incentive compatible soft contracts, in which the entrepreneur invests effort, and reveals the type truthfully. Inside the area some indifference curves are shown. The preferences of the two parties over soft contracts are exactly opposed, where the investor prefers contracts with lower monetary payoffs for the entrepreneur, while the latter's preferences are exactly opposed. As the investor proposes the contract, her most preferred incentive compatible soft contract (Z^S_{Yg}, Z^S_b) lies in the southwest corner of the area (marked by broken lines).

The figure is not easy to complete, as the preferences and incentive constraints are different contracts that are not 'soft', and the payoff functions are discontinuous at regime switches. The south east area for instance, where (IC_{bg}) is violated (the truthtelling constraint of the bad type), contains the set of tough contracts. With tough contracts, there is no revelation at all in equilibrium: Z_b is so low that the entrepreneur prefers to 'wait and pray' with a b type, and therefore the investor is not interested in rewarding the revelation of type ϕ, either. The players' preferences are again strictly opposed in the set of tough contracts, and their indifference curves are vertical. The equilibrium monetary pay that the entrepreneur can expect after a 'good' outcome Y lies between c and Z^S_{Yg} (not shown). This payoff is as low as possible, and the 'tough' (IC_e) (not shown either) is binding in equilibrium.

While the players' preferences over the two contracts cannot be shown graphically, this is possible algebraically. The entrepreneur exerts effort under both contracts, and the expected private benefit is the same under both contracts, too. Therefore his preferences are determined by the financial rewards, only. Under the soft contract the rewards are higher in all states, and we have proved

Proposition 2.4 *The entrepreneur strictly prefers the soft contract $\mathcal{C}^S$ over the tough contract $\mathcal{C}^T$.*

Within both sets of contracts, the players' preferences are strictly opposed. The preferences over contract *types* are not necessarily opposed, however, as the payoff funcions are not continuous at the incentive constraints.

Proposition 2.5 *The investor prefers the soft contract $\mathcal{C}^S$ over the tough contract $\mathcal{C}^T$ if and only if*

$$\Delta := (1-e)a\Big[(1-b)(Y-y) - J\Big] - \frac{1-a}{1-ab}\, r \;>\; 0. \tag{2.8}$$

Δ is the difference between the investor's payoff from offering $\mathcal{C}^S$ and from offering $\mathcal{C}^T$. We can analyse its elements in more detail, i.e. analyse the effect of different project parameters on the investor's preferences:

Proposition 2.6 *The investor's preference for $\mathcal{C}^S$ over $\mathcal{C}^T$ increases in the rescue gain. It decreases in the effect of effort on type, in the probability of the 'failure' type, in the size of the private benefit, and in the likelihood of a 'breakthrough' of a 'bad' type.*

The above proposition follows from differentiating Δ in (2.8) with respect to its variables. That $\mathcal{C}^S$ is the more attractive the higher the rescue gain $(1-b)(Y-y)-J$, is obvious. Δ decreases in the effect of effort on type, since with high values of e the rescue decision and gain become less likely. As e approaches 1, the expected gains from rescuing converge to zero. The cost of rescuing, however, remain strictly positive: the difference between the entrepreneur's expected monetary payoffs is $\frac{1-a}{1-ab}\,r$,, if e goes to 1. The reason for this is that he must receive a bribe r if the type is ϕ, which is strictly unproductive (its only benefit is to *prevent* a lie with wasteful consequences). This bribe decreases the wedge between the expected payoffs from investing and not investing effort in period 1. In order to satisfy the effort incentive constraint, the expected payoff for a g type must be increased, relative to that in the tough contract.

Next, we differentiate Δ with respect to a, the conditional probability that the type is not ϕ, if it is not g. This derivative is negative, and we can conclude that as the probability of a 'failure' type increases, the soft contract $\mathcal{C}^S$ becomes less attractive. There are two reasons for this. First, as a decreases, the possibility to rescue the project becomes less likely to arise. Second, the costs of revelation increase in a, as the 'failure' types must be bribed not to claim to have 'bad' types.

The effect of the entrepreneur's private benefit r on Δ is negative, as well. With either contract, r increases the investor's payoff, as it is a payoff to the entrepreneur which is linked to the outcomes g and b, but which is not earned if the type is ϕ. It is thus more likely to be earned if effort is invested in period 1. This costless (for the investor) incentive on the effort decision is aligned with those that the investor wants to provide, and serves as a substitute for the payments that she must make. In the soft contract, however, r is the bribe, that must be paid to the entrepreneur with project type ϕ. This second effect more than offsets the first effect on the investor's payoff: while her payoff with contract $\mathcal{C}^T$ increases in r, its payoff with $\mathcal{C}^S$ decreases in r.

Finally, b is the probability of a 'breakthrough' for a 'bad' type. That is, if the parties decide to 'wait and pray' with type b, the probability of being lucky is b. b increases revelation costs, even though with types b and ϕ the entrepreneur is always indifferent between lying and telling the truth. The reason for this is that the incentives to invest effort require a higher payoff after successful projects. The entrepreneur's monetary payoffs paid under $\mathcal{C}^S$ are higher than those under $\mathcal{C}^T$, and the difference (the revelation costs) is increasing, too. The second reason for the decreased attractiveness of $\mathcal{C}^S$ is simply that the opportunity costs of rescuing decrease in b: hoping for a breakthrough pays more, the higher b is.

The disutility of effort, ce, has a direct effect only via e. c appears in the payoffs to the entrepreneur, but it cancels out when we compare the investor's respective payoffs. The reason for this is that the model is simplified: the effort levels are fixed. In a more general setting (e.g. with continuous effort, see Section 2.4) it would play a role similar to e: the more weight the investor must give to the effort decision, relative to the truth revelation problem, the less attractive the soft contract becomes.

2.4 Simplifications and Extensions

The model in this paper is somewhat unusual in that it has three types of project ('good', 'bad', and 'failure'), instead of the usual two. The reason for this is that the results become unrealistic if the model is too simple. More precisely, if there were only a good and a bad type, the investor could obtain the entrepreneur's private information at no cost. She could make the entrepreneur with a bad type exactly indifferent between revealing his information, and claiming to have a good type, without affecting the incentive to invest effort. This is easy to see if we assume $a = 1$, or $r = 0$: Δ in Proposition 2.5 would always be strictly positive.

Obviously, it is not realistic that all potentially insolvent firms should be rescued. Similarly, it is realistic to assume that the managers of a firm who know that it should be liquidated would nevertheless try to have it refinanced, by claiming that a rescue is both possible and profitable. Finally, it is realistic to assume that it is hard for outsiders to tell whether a firm should be rescued, or whether the managers just claim that it should.

The failure type ϕ is a simple way to introduce these elements of realism to the model. Not that the relevant aspect that ϕ adds is not the enlarged type space. With many bad types, even with a continuum, the results remain degenerate (see below). The relevant aspect that ϕ adds is a new truthtelling constraint, which is binding. The failure type can claim to be a bad type, but must be prevented from doing so at a cost. There is no gain from separating the type from the other, but there is a loss if it is not separated, and the investor wants to rescue b types.

Adding additional types who could claim to be good types is not sufficient to achieve the result. Suppose there were two bad types, b and B, that can both be rescued. Suppose further that $B > b$, and that $r = 0$ (for simplicity). If under a soft contract the investor promises to rescue both bad types, a type b could claim to have type B. The truthtelling constraints require that both are at least indifferent between revealing their type, and claiming to have a good type. As $B > b$, type b should be expected to earn an information rent in a soft contract. If the project is rescued, the outcome is the same for both types, and the investor would not be able to tell whether it was b or B. The investor can propose the following contract, however:

Contract for many bad types *($\mathcal{C}^B$):*

§1 At $t = 1$, the entrepreneur receives K from the investor and invests it in his project.

§2.a If at $t = 3$ the entrepreneur reveals that the project is of type B, the investor refinances the project with probability $(1 - \varepsilon)$, where $\varepsilon > 0$. All returns belong to the Investor, whether the project is rescued or not. If there is no rescue, and the project returned Y, the entrepreneur receives a payment $\frac{1}{\varepsilon} \cdot \frac{c}{1-ab}$ from the Investor.

§2.b If at $t = 3$ the entrepreneur reveals that the project is of type b, the investor refinances the project with probability one. If the rescue was successful, the entrepreneur makes a payment $Y - b \cdot \frac{c}{1-ab}$ to the Investor.

§3 If §2 does not apply, and the return is Y, the entrepreneur makes a payment $Y - \frac{c}{1-ab}$ to the Investor. If the entrepreneur cannot make this payment, the returns of the project belong to the Investor.

One can easily verify that all types will be truthfully revealed. Furthermore, revelation costs go to zero in the limit, as ε goes to zero. That is, both the entrepreneur's information rent and the losses due to inefficient rescue decisions go to zero, and the soft contract dominates the tough contract. Exactly the same mechanism could be used to separate a higher number of bad types, or even a continuum. §2.a would have to be applied to all types, except for the lowest–b type, for which §2.b has to be applied.

Both this model and the contract $\mathcal{C}^B$ are unsatisfactory for several reasons. One is the fact that revelation is virtually costless and that the bad projects are (almost) always rescued. Next, it is not possible to characterise the optimal contract without further assumptions. The investor's payoff decreases in ε, so she would want to choose it as small as possible. It cannot be set to zero, however, and therefore we need e.g. a budget constraint for the investor to specify the optimal contract.[3] A further unattractive aspect of the model is that the tough contract is not completely tough. The investor can easily separate the two bad

[3] In a somewhat different model, Levitt and Snyder (1996) encountered similar difficulties. In their model, the principal can achieve a second best result in the limit, using a mechanism similar to $\mathcal{C}^B$. They solve the problem by assuming that the principal cannot randomize.

types by rescuing type b, only, and making this type exactly indifferent between revelation, and claiming to have type g (or B). As before, this type of revelation is costless. The preferences of the players over different contracts are as in our model above. For any $\varepsilon > 0$ we could define an equation as in Proposition 2.5, which determines the investor's preferences over the two contracts.

How robust are the results or our model to changes in other assumptions? Consider first the effort decision. Suppose that the entrepreneur has to choose an effort level e in the interval $[0, 1]$, which causes disutility $c\frac{e^2}{2}$. With some additional assumptions the problem is well behaved, and easy to solve. The investor's maximisation problem is unchanged, except for (IC_e): on the right hand side (cf. the Proof of Propositions 2.1 and 2.2 in the Appendix) we now have ce, instead of c, and this condition is binding by definition. The investor now maximises over all feasible e, i.e. she compares the optimal contracts for each e, and chooses the best effort level and the respective contract for her equilibrium offer.

Proposition 2.7 *Extend the model such that the entrepreneur can choose an effort level $e \in [0, 1]$, at cost $c\frac{e^2}{2}$. Then either a tough or a soft contract (which are similar to the contracts described in Propositions 2.1 and 2.2) may be optimal, depending on the parameters. The entrepreneur chooses a smaller effort level if the soft contract is offered.*

Thus, the result that *either* a soft *or* a tough contract may be optimal is *not* driven by the discrete effort choice assumption. Furthermore, as one would expect, the induced effort level is lower with the soft contract. Nevertheless, the investor may prefer the soft contract, because it involves a more efficient rescue decision. The condition on the parameters corresponding to the definition of Δ in Proposition 2.5 is changed to

$$\hat{\Delta} = \left(1 - \frac{e^S}{2} - \frac{e^T}{2}\right) a\left[(1-b)(Y-y) - J\right] - (1-a)\, r, \tag{2.9}$$

which can be positive or negative.

Other changes in the assumptions do not affect the qualitative results, as long as the changes are not too large. Introducing equity for instance, has small effects, as long as the entrepreneur's initial wealth is not sufficient to finance the project. The same is true if the project earns a lower income if it is separated from the entrepreneur. The 'outside option principle' applies in this case: up to a certain degree the additional bargaining power does not change the results. If the need for his presence endows him with a strong bargaining position in renegotiations, however, the entrepreneur need not fear bankruptcy anymore, and he may be willing to reveal a bad type even without a soft contract.

Unlimited liability affects the results considerably. It allows the investor to punish the entrepreneur for bad outcomes, and therefore he cannot capture any information rents. The investor can easily guarantee first best decisions, and if the punishments are transfer payments (say, the entrepreneur expects a high exogenous income in the last period), the investor can even achieve her first best payoff.

2.5 Optimal Contracts without Commitment

The contracts in Section 2.3 were optimal under the restriction that there is no renegotiation during the game. However, in reality it is not possible to exclude renegotiations of a contract. No court would try to enforce a contract that all parties want to rescind and substitute by another contract. Under a tough bankruptcy law, the parties can gain from negotiating around the official procedure, but this possibility causes more harm (in terms of effort) than it helps (in terms of more efficient rescue decisions).

Proposition 2.8 *If the parties signed contract $\mathcal{C}^T$, there is scope for renegotiations between period 1 and period 2 (at time 1.5, say). The renegotiated contract is a soft contract (a bad project is rescued), but with lower transfers to the entrepreneur than in $\mathcal{C}^S$, if the type is g or b. It gives the entrepreneur unchanged utility, while the investor's payoff is increased. Being able to predict this, the entrepreneur will choose the low effort level in period 1.*

As the investor has all bargaining power, she can propose the renegotiated contract. She can easily find the cheapest soft contract: instead of an effort incentive constraint, it must satisfy a participation constraint from the side of the entrepreneur. His expected payoff may not be lower, otherwise the renegotiated contract would not be accepted, and the initial contract would be valid. Revelation was costly under the optimal soft contract $\mathcal{C}^S$, but here the transfer r to type ϕ is cross subsidised by the other two types. If the transfer for good outcomes was Z_Y^T under the tough contract $\mathcal{C}^T$, the renegotiated good transfer will be

$$Z_{Yg}^R = Z_Y^T - \frac{(1-e)(1-a)}{e+(1-e)ab}\, r \tag{2.10}$$

(The transfer to a bad type will be $b \cdot Z_{Yg}^R$). While the payoff to a ϕ type is strictly higher under the renegotiated contract (r instead of zero), the payoff to the other two types is strictly lower. The entrepreneur can, of course, predict the possibility to renegotiate, and, expecting a smaller differential between good outcomes and bad outcomes, has no incentive anymore to put in effort.

Both in reality and in the model, renegotiation is a serious issue. The advantage of the tough contract was the use of bankruptcy as a bonding device: no prisoners are taken if there is no success. If renegotiations cannot be exluded, the ex post inefficiency which makes $\mathcal{C}^T$ tough is not credible anymore.

One possibility to restore an efficient choice of effort is to start with higher transfers in the original tough contract. However, as one can easily construct, the lowest such transfer will be renegotiated to the best soft contract $\mathcal{C}^S$, which could therefore have been written as initial contract.

Another possibility is to allow for one round of renegotiation at $t = 1.5$. Following Fudenberg and Tirole (1990) we can construct a renegotiation proof contract for period 1, which will be substituted by one of two contracts that the investor proposes at $t = 1.5$. In equilibrium, the entrepreneur chooses to invest effort with probability f, where $0 < f < 1$. The investor does not know whether effort was invested, and offers a menu of 2 contracts, from which the entrepreneur may choose one.

Proposition 2.9 *Assume that $abc > (1-a)r$, and that the parties expect renegotiations at $t = 1.5$. The investor's renegotiation offer will consist of one soft contract $\mathcal{C}^{S0}$, which the entrepreneur may accept or reject. In equilibrium, the entrepreneur invests effort with probability f, where $0 < f < 1$. If he did not* *invest effort, he will accept the new contract $\mathcal{C}^{S0}$, and the project is either rescued or liquidated in period 3. If he did invest effort, he rejects the new contract, and the tough contract $\mathcal{C}^{T}$ remains valid. Whatever the effort decision is, the entrepreneur's expected payoff is not changed by the renegotiations.*

Knowing which contracts he will choose at $t = 1.5$ in each case, the entrepreneur is indifferent between investing effort or not. He is thus willing to play a mixed strategy, i.e. investing effort with probability f. The investor, on the other hand, cannot profitably renegotiate this type of contract, given her belief that the entrepreneur has invested effort with probability f. The contract is renegotiation proof in the sense that there is no strict gain that can be realised by renegotiating before $t = 1.5$ (at $t = 1.25$, say). The tough contract with renegotiation is not unique, unfortunately: for any $f \in (0,1)$, the renegotiation offer is the same. The entrepreneur is indifferent between investing effort and not investing, and therefore there is a continuum of renegotiated tough contracts, which are all identical, except for the effort decision f.

Even if it is renegotiated, the contract $\mathcal{C}^{T}$ remains a tough contract. With probability $(1-f)$ the rescue decisions are efficient, but with probability f, the parties ignore the possibility to rescue early. As in the case of the full commitment contracts, we can analyse the players' preferences over the two contracts. While the entrepreneur's payoff is not affected by the renegotiation possibility, this is not true for the investor.

Proposition 2.10 *The investor prefers the soft contract $\mathcal{C}^{S}$ over the tough contract $\mathcal{C}^{T}$, which will possibly be renegotiated to $\mathcal{C}^{S0}$, if and only if*

$$f \cdot (1-e)a\Big[(1-b)(Y-y) - J\Big] - \frac{1-a}{1-ab}\, r \qquad (2.11)$$

$$+e\Big[(1-f)[Y-a(Y-J)] + (1+f)(1-a)r - (1-f)c\Big] \;>\; 0.$$

Thus, even if we allow for renegotiations after the effort decision, the parties can write a tough contract, which may be either better or worse than the soft contract (from the investor's point of view).

A third approach to renegotiation is to analyse how costly renegotiations are, and how likely they are to be successful. Aspects which are not modeled above can make renegotiations of contracts costly, maybe to such an extent, that renegotiations will not happen. Inefficient bargaining may be one reason for imperfect renegotiations. It is generally assumed that the more parties participate in negotiations, the more difficult it gets to achieve an efficient outcome. Many actions that are necessary for a renegotiation are costly, and their provision by one party is a public good for all parties of a contract. Information has to be gathered and distributed, a negotiation procedure must be specified, and negotiations must take place. As private renegotiations require unanimous consent, there is ample space for holdups from each side, if there is asymmetric information between the negotiating parties. We have not modeled this aspect of bankruptcy procedures due to lack of space. Two models with inefficiencies in renegotiations that are due to asymmetric information are analysed in Bolton and Scharfstein (1996) and in Povel (1995).

Inefficient renegotiations may make the tough contract $\mathcal{C}^T$ credible, in particular if the costs of renegotiation are fixed to a large extent. They are not a watertight means to prevent renegotiations, however. Additional obstacles to private renegotiations of debt contracts are provided by the sections of company and bankruptcy laws. Many bankruptcy laws have features that make formal bankruptcy more attractive than informal workouts. These advantages can induce the contracting parties to trigger a formal bankruptcy procedure, which on its turn may make renegotiation difficult or costly.

The obstacles to renegotiation in formal bankruptcy may be the following: minimal requirements on capital structure must be met for a reorganisation to be approved by the courts (In Germany, a 'Vergleich' is possible only if 35% of the unsecured debt can be repaid within a year); too many parties may enjoy highest priority, i.e. too many parties have a high threat point, and may find it hard to give in (claims by employees and the government usually make large fractions of total indebtedness; trade creditors in Germany can secretly secure their claims by using the retention titles clauses); secured creditors may dispose of 'their' assets (in Germany, secured creditors have to be repaid before a procedure can even be started; in the UK, a lender secured by a 'floating charge' may appoint her own 'administrative receiver'); finally, Absolute Priority Rules provide additional scope for holdups, as payments to low ranked claims can be vetoed by claimants with higher ranks.

The legal environment can also make both formal bankruptcy more attractive, and private renegotiations more difficult. 'Tax losses' are an important asset in bankrupt companies, and they are transferable to other companies (and therefore valuable) only in formal bankruptcy procedures. Similarly, in formal bankruptcy, it is often possible to evade regulation. Private renegotiation can be made difficult if refinancing parties can be accused of having been a 'Shadow Director', or of having caused additional losses by delaying an imminent insolvency. For example, a bank that participates too much in the informal reorganisation of a German company may face such accusations if the company does not recover, with the consequence that its claims lose priority ranks.

While the obstacles to renegotiation that come from the multiplicity of the creditors seem to be unavoidable, the claim that tough bankruptcy laws are designed to *prevent* renegotiations would probably go to far. The discussions around the recent reform of the German bankruptcy laws show up the conflict of interests: as in the UK, the main goal of bankruptcy law is to enforce the repayment of debts. Therefore, a bankruptcy procedure is expected to resemble the tough §2 in $\mathcal{C}^T$. After reviewing the insolvency practice in the UK, a parliamentary commission wrote:

> It is a basic objective of the law to support the maintenance of commercial morality and encourage the fulfilment of financial obligations. Insolvency must not be an easy solution for those who can bear with equanimity the stigma of their own failure or the responsibility for the failure of a company under their management. (Cork Report, 1982, Chapter 4, at 191)

(Similar statements can be found in German legal writings.) On the other hand, both countries have reshaped their 'reorganisation' procedures recently, with the intention of minimising the losses from unnecessary liquidations. Not only do the contracting parties try to realise all gains from bargaining (as one would expect from looking at bargaining models), but the state tries to overcome the legal obstacles, as well. These obstacles are a consequence of other inefficiencies, that may or may not be related to bankruptcy, which is far from saying that they have been or should be introduced as a commitment device.

To sum up, inefficient renegotiations may add credibility to a tough contract, but cannot guarantee it. While some sources of inefficiency arise naturally (the multiplicity and diversity of creditors), others are artificial. In either case, the contracting parties will try hard to negotiate around the obstacles, thereby watering down their commitment to be tough.

A final method to deal with renegotiation problems is to accomodate them, and to design bankruptcy laws such that the conflict cannot arise anymore. One way of doing this is to introduce a real soft procedure, which rewards cooperation in reorganisations, and nevertheless does not worsen the ex ante incentives of managers or owners too much. The problem with this solution is that if the creditor breaks even with a tough contract, but not with a soft one, a potentially profitable project will not be undertaken.

2.6 Is Chapter 11 the 'Soft' Procedure?

The similarity between the soft bankruptcy clause in Proposition 2.2 and Chapter 11, the US reorganisation procedure, is striking. Both promise some direct or indirect reward to the entrepreneur, if he files for bankruptcy. Indeed, it was the intention of the drafters of the procedure to induce firms to admit their difficulties as soon as possible (See House of Representatives 1977, p.233–4).

There are two major differences between the two procedures, however. First, the US courts systematically reject filings by firms with a single lender. The argument behind those rejections is that there cannot be any bargaining problems with only two parties (there is no 'Common Pool Problem', See Jackson (1986)). However, the reason for rewarding and protecting the entrepreneur under a soft contract was different: he is only willing to reveal his type if this does not make him too vulnerable. If his filing for a reward under $\mathcal{C}^S$ is rejected, he has revealed his type, and is at the mercy of the investor. With the US rejection rule, Chapter 11 is a tough reorganisation procedure for borrowers with exactly one lender, and a soft procedure if there are more lenders.

Second, Chapter 11 contains Absolute Priority Rules, which are supposed to be enforced by the bankruptcy courts. The recent discussions around the 'New Value Exception' (see e.g. Baird and Jackson (1988), Baird (1993, Chapters 3 and 10), or Westbrook (1993)) make clear that financial reward to the entrepreneur, as suggested by the model, are not feasible. Nevertheless, Chapter 11 has characteristics that makes it a soft procedure. There is a system of indirect rewards at use, which the entrepreneur can earn, because Chapter 11 endows him with considerable bargaining power. If he files for protection under Chapter 11, this puts a stay on all claims against the firm. No trustee is appointed, and the entrepreneur cannot easily be removed from his position. As a 'Debtor in Possession', he can take on new debt, which has higher priority than all earlier debt, to keep the firm running. Finally, and most importantly, he has the exclusive right to propose a reorganisation plan for at least a couple of months, possibly for years. There is ample evidence that this system of indirect rewards is effective and actually being used (See e.g. Franks and Torous (1989)).

Unfortunately, this indirect type of reward has proved costly. Rescues are delayed, unnecessary uncertainties are created, and the assets of a distressed firm are not used in the most efficient way. Furthermore, large legal and administrative costs are associated with Chapter 11 cases. There is a large literature now, which lists the different types of costs that can arise in Chapter 11 procedures. Below we list some important points that are raised, and then show that many of the conclusions that are made in this literature are less robust than one would imagine.

Frequently mentioned drawbacks of Chapter 11 in both the business press (see e.g. Fortune (1983) and Economist (1992)) or the bankruptcy literature (see e.g. Aghion et al. (1994)) include:

1. *The loss of stigma that used to be attached to bankruptcy;*

2. *Chapter 11 is soft on management: this weakens the bonding role of debt;*

3. *A powerful debtor means smaller payoffs for the creditors;*

4. *The procedure is time consuming, which can lead to considerable losses of value;*

5. *There are high legal and administrative costs.*

Chapter 11 is judged as being inefficient because the entrepreneur has too much bargaining power. Its 'softness' seems to be directly or indirectly responsible for the costliness of the procedure, which should therefore be made 'tougher'.

Bradley and Rosenzweig (1992) collected data on the relative performance of Chapter 11, and its predecessors, Chapters X and XI. Their paper is a particularly sharp criticism of Chapter 11, and it is frequently quoted in the literature. Some of their results are the following:[4]

Since the introduction of Chapter 11 in 1978,

1. *the bankruptcy decision has become more endogenous*

2. *the frequency of bankruptcy filings has "increased dramatically"*

3. *a smaller fraction of bankrupt firms are delisted from the major exchanges in the year before their filing*

4. *bankrupt firms are generally in better financial condition*

Bradley and Rosenzweig (1992) find that managers can file for protection from their creditors whenever it pleases them (result 1), and that this possibility is being made use of without real need (results 2 to 4). They conclude that Chapter 11 protects bad or lazy managers from the market for corporate control, both outside and in bankruptcy, and should therefore be amended.

Using the model, we will now show that many of the above arguments do not necessarily imply that Chapter 11 should be made tougher on management, or even amended. We look at the results in Bradley and Rosenzweig (1992), first. Their first finding is actually a *desired* result. It was the intention of the soft procedure that firms file before their distress must be publicly admitted. As the entrepreneur files voluntarily in the model, his filing must be 'endogenous'. The

[4] A series of papers has attacked the authors' methodology; See e.g. Bhandari and Weiss (1993), and the references cited in their footnote 4. See also Bradley (1997).

second finding follows necessarily from the fact that the firms are to file earlier. While under a tough bankruptcy procedure, a project is completed without interruptions and earns a high payoff with probability $e + ab(1 - e)$ (the probability of type g and a 'lucky' type b), the same probability with a soft procedure is e. In the latter case, the formerly 'lucky' bad types count as 'bankrupts' as well. Nothing about welfare losses can be derived from simply counting the number of filings, however: as was shown in the model, the higher number of filings may be preferred by the investor. The third and fourth findings of Bradley and Rosenzweig (1992) are again a consequence of the earlier filings. The gain from the earlier filing derives from the possibility to reorganise an acceptably healthy firm, which is much cheaper and easier than to reorganise a deeply insolvent firm. As with finding 2, nothing can be said about the aggregate effects.

Thus, the results in Bradley and Rosenzweig (1992), when reinterpreted, state that Chapter 11 may be doing quite well. Its introduction in 1978 seems to have had a positive effect on both the number of filings, and on the economic conditions of the filing firms, as was the intention of the creators of the law.

Similarly, the other arguments (points 1 to 5 above) can be reviewed. Consider first point 2, the idea that the bonding role of debt is weakened. We have seen that a procedure can be soft and *nevertheless* preserve the bonding role of debt. In the model, the effort incentive constraint is always binding, and the entrepreneur always invests the high effort level. The key is that the procedure should be soft, but not *too soft*. Chapter 11 is soft on the manager: Lo Pucki and Whitford (1993) found five in 43 studied cases, in which the CEO of a firm in Chapter 11 received considerable payments during the proceedings (These payments were agreed to as employment contracts). However, it is not too soft: even though the entrepreneur is treated well in Chapter 11, he is not invulnerable. Gilson (1990) and Gilson and Vetsuypens (1993) show that directors and CEOs of publicly traded firms that file for Chapter 11 frequently lose their jobs (about one in two) and that the remuneration of the remaining managers is significantly reduced.

Thus, in Chapter 11 cases, we observe that managers are treated badly, but not too badly. The lower bound to the reward is determined by the alternatives that the entrepreneur can choose. Typically, these consist of a later bankruptcy, that may be much more harmful than Chapter 11. Thus, there is evidence both for the sufferings of managers in Chapter 11 (Z_b^S is not too high in the language of the model) and for a reward for filing (Z_b^S is not too small), and the bonding role of debt may still be effective.

The criticism in point 1 points at the danger of putting too much emphasis on the bonding role of bankruptcy procedures when designing a soft procedure. As was shown above, this can lead to a 'hybrid' procedure which is strictly worse than both a soft or tough procedures.

Similarly, one has to be more precise when arguing that a soft procedure increases the cost of capital (point 3). It is true that the entrepreneur's payoffs are higher under the soft procedure (this is why he strictly prefers it to the tough one). However, when designing the contract, the investor also takes into consideration the gains from a more efficient rescue decision. If these are high, she prefers to offer the soft contract, which indicates (since she keeps all profits) that the *ex ante* costs of capital are *lower*.

Arguments 4 and 5 remain true, but, as was mentioned in the last section, these inefficiencies are required because the entrepreneur cannot, presently, *expect* a direct reward. As reorganisations in other countries and informal workouts show, there is no need for a reorganisation to be costly or time consuming. Thus, if one wants to make the procedure more efficient, the question should be whether direct rewards can be introduced, and whether some form of Absolute Priority Rule is necessary in Chapter 11.

To summarise, the evidence of the performance of Chapter 11 leads to some negative results for the design of the procedure, but it does not follow that a soft procedure does worse than a tough procedure. Early rescues may pay, and, as was shown in the model, even with limited liability, which makes the revelation of project types costly, the incentive to invest effort can be sufficiently strong. Undoubtedly, the entrepreneur does better under a soft procedure, as his expected payoff after bad outcomes is increased. However, his payoff for good outcomes is increased even more. The wedge between the two payoffs is large enough to provide him with incentives to invest effort, if this can reduce the probability of bad outcomes, and the investor is glad to finance these higher payoffs, as she gains from the more efficient use of the assets.

2.7 Conclusion

This paper studies both the decision of an entrepreneur, when to reveal to his lenders that his firm is in financial distress, in order to initiate a reorganisation. As early rescues are likely to be more successful and cheaper than delayed rescues, the creditors want to receive this information as early as possible. The entrepreneur must be convinced to reveal his information, as he could carry on, playing a 'wait and pray' strategy at the creditors' expense. It may pay for the latter to be 'forgiving', if the entrepreneur admits that he lost their money, even if they are in a much stronger bargaining position than the entrepreneur. This may also be too expensive in terms of ex ante incentives of the entrepreneur, however, and the creditors could prefer to ignore the possibility of more efficient rescue decisions in this case.

Assuming that 'optimal laws' should replicate the clauses of those 'optimal contracts' that the parties would write if the transaction costs of contracting were sufficiently low, we derive implications for the design of bankruptcy laws. Both soft and tough bankruptcy laws may be optimal in equilibrium, depending on the economic and legal environment. A procedure which contains both soft and tough elements, however, is never better than both of the other two, and may be strictly worse.

A soft bankruptcy law requires that a reward is paid to the entrepreneur, if he successfully cooperated in a rescue by starting it early. This reward must be paid even if some of the debt of the firm is not repaid. At the extreme, it must be paid even if the firm must be liquidated, and the returns are negligible (as with a failure type in the model). This may seem 'unfair' at first. It makes clear, however, how important it is to separate clearly between the different goals that a bankruptcy procedure is supposed to achieve, and how important it is do design a procedure consistently.

Chapter 11, the US reorganisation procedure, has much in common with the soft bankruptcy law, but there are some relevant differences. The most important of these is that the creditors can demand that the 'Absolute Priority Rule' be enforced, which prohibits payments to equity if claims with higher ranks have not been repaid. Nevertheless, Chapter 11 is a soft procedure. It provides the entrepreneur with indirect rewards: by endowing him with bargaining power (he can make the procedure costly and inefficient), he can extract a payoff from the creditors. This may be an inefficient reward scheme, but the main goal of a soft procedure, inducing early bankruptcy filings, may be achieved. This is one of the insights that the model provides, when we look at empirical evidence on Chapter 11. While there are undoubtedly severe inefficiencies in a Chapter 11 procedure, these are not necessary elements of a soft procedure. The arguments against Chapter 11 may be valid, but they do not imply that the 'softness' of the procedure should be reduced.

It would be interesting to analyse the possibility of choosing between *several* bankruptcy procedures. It could be that having e.g. two bankruptcy procedures (as is the case in the US) or even more (as in the UK) provides a more flexible legal environment and thereby allows for easier contracting (see Schwartz, 1997). One problem that must arise here is that of signalling through the choice of procedures. White (1994) analyses these problems at the stage where a distressed firm has to select a bankruptcy procedure. More severe signaling problems may arise, however, when two parties want to sign a lending contract and the borrower proposes to be treated well in the case of bankruptcy. This must make the lender rather pessimistic about the borrower's intentions.

The paper has important consequences for the review of the US Bankruptcy Code, which is currently in progress. Over the past years there has been a discussion about the New Value Exception, and whether it is or should be part of Chapter 11. It deals with the question under what circumstances the former shareholders should be allowed to participate in the reorganised firm, e.g. as 'new' shareholders. There is concern about the unfairness of the possibility to abuse this option, as e.g. a majority shareholder must be assumed to have superior information about the value of the assets and investment opportunities of the firm. However, exactly this represents a cheap way to 'reward' (as required by a soft procedure), as the lenders' loss is much smaller than the gain to the shareholder, if the latter gets a good deal.

In a similar spirit, the idea that shareholders get 'unfairly' good deals in workouts or prepackaged Chapter 11 filings should be reconsidered. As noted by e.g. Bebchuk and Chang (1992), Berkovitch, Israel and Zender (1995) and Franks and Torous (1989), shareholders have a *de facto* right to delay regular Chapter 11 procedures, and use it as a threat point in Chapter 11 negotiations to improve their payoff. Allowing the use of such threats in workout and 'Prepack' negotiations could greatly improve the efficiency of the US code. As anecdotal evidence shows (for instance from Germany), there is no need for rescues to be time consuming, or connected with high costs.

Appendix: Proofs

Proofs of Propositions 2.1 and 2.2

The direct mechanism consists of:

$$\begin{aligned} \text{A message space} \quad & M = \{g, b, \phi\} \\ \text{rescue decisions} \quad & P_g, P_b, P_\phi \in [0,1] \\ \text{a payoff function (for E)} \quad & Z : \{Y, y, 0, Y-J, -J\} \times M \to \mathbb{R}_+ \end{aligned}$$

The optimal direct mechanism is found by solving the following maximisation program:

$$\begin{aligned} \max \quad & e\Big(P_g(Y - J - Z_g) + (1-P_g)(Y - Z_{Yg})\Big) && (Max) \\ & + \ (1-e)a\Big(P_b(Y - J - Z_b) + (1-P_b)\Big[b(Y - Z_{Yb}) + (1-b)(y - Z_{yb})\Big]\Big) \\ & + \ (1-e)(1-a)\Big(P_\phi(-J - Z_\phi) + (1-P_\phi)(-Z_{0\phi})\Big) \end{aligned}$$

such that

$$\begin{aligned} P_g Z_g \ + \ (1-P_g)Z_{Yg} + r \ - \ a\Big(P_b Z_b + (1-P_b)\Big[bZ_{Yb} + (1-b)Z_{yb}\Big] + r\Big) && (IC_e) \\ - \ (1-a)\Big(P_\phi(Z_\phi + r) + (1-P_\phi)Z_{0\phi}\Big) \quad \geq \quad c \end{aligned}$$

$$P_g Z_g + (1-P_g)Z_{Yg} + r \ \geq \ P_b Z_b + (1-P_b)Z_{Yb} + r \qquad (IC_{gb})$$

$$P_g Z_g + (1-P_g)Z_{Yg} + r \ \geq \ P_\phi(Z_\phi + r) + (1-P_\phi)Z_{0\phi} \qquad (IC_{g\phi})$$

$$\begin{aligned} & P_b Z_b + (1-P_b)\Big[bZ_{Yb} + (1-b)Z_{yb}\Big] + r && (IC_{bg}) \\ & \qquad \geq \ P_g Z_g + (1-P_g)\Big[bZ_{Yg} + (1-b)Z_{yg}\Big] + r \end{aligned}$$

$$P_b Z_b + (1-P_b)\Big[bZ_{Yb} + (1-b)Z_{yb}\Big] + r \ \geq \ P_\phi(Z_\phi + r) + (1-P_\phi)Z_{0\phi} \qquad (IC_{b\phi})$$

$$P_\phi(Z_\phi + r) + (1-P_\phi)Z_{0\phi} \geq P_g r \qquad (IC_{\phi g})$$

$$P_\phi(Z_\phi + r) + (1-P_\phi)Z_{0\phi} \geq P_b r \qquad (IC_{\phi b})$$

are satisfied, as well as the limited liability conditions (no transfer Z_{ij} may be negative). The first constraint (IC_e) is the (simplified) effort constraint. As we assume that the inequality in (2.7) is sufficiently strict, the investor will want the entrepreneur to invest the high effort level. The truthtelling constraints are (IC_{gb}) to $(IC_{\phi b})$, where the first index refers to the true type, and the second to the type the entrepreneur should not pretend to be.

The propositions are proved by simplifying the program, and by reducing it to case distinctions. For every case we will either derive the optimal transfer scheme, or show that it leads to a contradiction.

The first simplification is to set $Z_{yg} = 0$, which cannot violate any ICs, since it appears on the RHS of (IC_{bg}), only. Next, $P_\phi = 0$. Suppose it were strictly positive. Then the investor could decrease it by some $\varepsilon > 0$, and increase $Z_{0\phi}$ by $\delta_1 = \frac{Z_\phi - Z_{0\phi} + r}{1-P_\phi}\,\varepsilon$, without violating any of the ICs. If $\delta_1 < 0$, and $Z_{0\phi} = 0$, she can instead increase Z_ϕ by $\delta_2 = \frac{Z_\phi + r}{P_\phi}\,\varepsilon > 0$, again without violating any of the ICs. Her own payoff is thereby increased.

There is no need for the investor to set $Z_{0\phi} > r$. Otherwise, it could be reduced without violating any of the ICs, thereby improving the investor's payoff. With $Z_{0\phi} \leq r$, $(IC_{g\phi})$ and $(IC_{b\phi})$ are redundant, as they must be satisfied from the limited liability constraints.

P_g must be zero. Otherwise, the investor could profitably reduce it by $\varepsilon > 0$, thereby saving $J \cdot \varepsilon > 0$. By increasing Z_g by $\delta_1 = \frac{Z_g - Z_{Yg}}{P_g}\,\varepsilon$, no IC is violated (The effect on (IC_{bg}) is that the RHS is decreased by $(1-b)Z_{Yg}\varepsilon$). If $\delta_1 < 0$, and $Z_g = 0$, she can instead decrease Z_{Yg} by $\delta_2 = \frac{Z_{Yg}}{1-P_g}\,\varepsilon > 0$, which has the same effect on the ICs. Z_{Yg} must be strictly positive because of (IC_e).

With $P_g = 0$, $(IC_{\phi g})$ is always satisfied, and we can omit it. Therefore, $(IC_{\phi b})$ must be binding, as the investor could otherwise profitably decrease $Z_{0\phi}$.

Next, (IC_{bg}) must be binding in equilibrium. Suppose it is not. The investor could decrease either of Z_b, Z_{Yb}, or Z_{yb} (at least one must be strictly positive from (IC_{bg}) and (IC_e)), without violating any of the ICs.

From this follows that (IC_e) must be binding, as well. Suppose it is not. Then (IC_{gb}) must be binding, as the investor could otherwise reduce Z_{Yg}, without violating any of the ICs. Then, Z_{Yb} must be zero. Otherwise the investor could decrease it by $\varepsilon > 0$, and increase Z_{yb} by $\delta_1 = \frac{b}{1-b}\,\varepsilon > 0$, without violating any of the ICs. (IC_{gb}) would not bind anymore, and the investor could profitably reduce

Z_{Yg}. Similarly, we can conclude that $Z_b = 0$. Then, however, from (IC_{gb}) follows $Z_{Yg} = 0$, which contradicts (IC_e).

Consider first the case in which (IC_{gb}) is binding. From (IC_{gb}) and (IC_{bg}) follows that $P_b Z_b = 0$ and $(1 - P_b)Z_{yb} = 0$. We can calculate the following equilibrium transfers as a function of P_b:

$$Z_{Yg} = \frac{c - (1-a)(1-P_b)\,r}{1-ab}, \quad Z_{Yb} = \frac{c-(1-a)(1-P_b)\,r}{(1-P_b)(1-ab)}, \quad Z_{0\phi} = P_b r. \tag{2.12}$$

We can substitute these values into the investor's objective function (Max). Differentiating with respect to P_b gives the optimal contract for this case. This derivative is Δ, as defined in (2.8). If $\Delta < 0$, the investor wants to set $P_b = 0$, while if $\Delta > 0$, she wants to choose the highest value of P_b that is possible. The equilibrium transfers if $\Delta < 0$ are those of the tough contract $\mathcal{C}^T$ in Proposition 2.1. The optimal transfers if $\Delta > 0$ cannot be determined, as Z_{Yb} goes to infinity as P_b approaches one. The limits are

$$\lim_{P_b \uparrow 1} Z_{Yg} = \frac{c}{1-ab}, \qquad \lim_{P_b \uparrow 1}(1-P_b)Z_{Yb} = \frac{c}{1-ab}, \qquad \lim_{P_b \uparrow 1} Z_{0\phi} = r. \tag{2.13}$$

These values can be used to calculate an upper bound to the investor's payoff. This is not done here, since one can easily verify that the payoff from the soft contract (which is an optimal transfer scheme if $\Delta > 0$) is strictly higher.

Consider now the case where (IC_{gb}) is not binding. We can calculate the transfers to the entrepreneur, as a function of P_b, except for the 'bad' type. For the 'bad' type, we can calculate the utility transfer:

$$Z_{Yg} = \frac{c-(1-a)(1-P_b)r}{1-ab}, \tag{2.14}$$

$$P_b Z_b + (1-P_b)[bZ_{Yb} + (1-b)Z_{yb}] = bZ_{Yg}, \qquad Z_{0\phi} = P_b r.$$

We can now determine the investor's payoff, as a function of P_b, and, as before, take the derivative with respect to P_b. Again, this derivative is Δ. If $\Delta > 0$, $P_b = 1$ in the optimum, and the transfers are those of the soft contract $\mathcal{C}^S$ in Proposition 2.2. If $\Delta < 0$, the investor would prefer to set $P_b = 0$. This gives us a continuum of equilibria, which are all equivalent in their payoff with the transfer scheme in the tough contract, $\mathcal{C}^T$. The multiplicity derives from the fact that there is some scope to vary Z_{Yb} and Z_{yb} without any effect, as long as neither of them is negative, and $[bZ_{Yb} + (1-b)Z_{yb}] = bZ_{Yg}$. Since the payoff is unchanged, we have restricted our attention on the case where $Z_{yb} = 0$: this is the only transfer scheme which is easy to implement (in the form of $\mathcal{C}^T$). □

Proof of Proposition 2.3

As has been shown in the Proof of Propositions 2.1 and 2.2, contracts other than $\mathcal{C}^T$ and $\mathcal{C}^S$ are either not incentive compatible, or do not achieve a higher payoff than both $\mathcal{C}^T$ and $\mathcal{C}^S$. □

Proof of Proposition 2.5

If $\Delta \leq 0$, the investor's payoff is maximised by choosing the tough mechanism, while if $\Delta \geq 0$, she prefers the soft mechanism. This can easily be shown by calculating and comparing the four payoffs. □

Proof of Proposition 2.8

Suppose the tough transfer scheme is $Z^T_{Yg} = Z^T_{Yb} = Z^T$, and all other transfers are zero. The investor can offer the following soft contract at $t = 1.5$:

$$\hat{Z}_{Yg} = Z^T - \frac{(1-e)(1-a)}{e + (1-e)ab} r, \qquad \hat{Z}_b = b\hat{Z}_{Yg}, \qquad \hat{Z}_{0\phi} = r. \tag{2.15}$$

This leaves the entrepreneur's expected payoff unchanged, and leads to truthful revelation of types and the rescue of the 'bad' type. Because of the more efficient rescue decision, the investor's payoff must be strictly higher. Next, we show that, if the entrepreneur can predict such renegotiations, his effort constraint is violated. The LHS of (IC_e) under the renegotiated mechanism is changed to $(1-ab)\hat{Z}_{Yg}$, or

$$c - (1-a)r - (1-ab)\frac{(1-e)(1-a)}{e + (1-e)ab} r, \tag{2.16}$$

which is strictly smaller than c: (IC_e) is violated. □

Proof of Proposition 2.9

As in Propositions 2.1 and 2.2, we apply the Revelation Principle, using as 'types' not the project types (which are not known yet), but the effort level that the entrepreneur has invested, either e or 0. The investor can offer mixtures of soft and tough contracts, i.e. lotteries over the two that will be played in period 3, after the entrepreneur has observed the project type. The (slightly simplified) investor's program consists of an objective function (omitted here), two interim individual rationality constraints,

$$P_\ell(aZ_b^{S0}+r)+(1-P_\ell)(abZ_{Yb}^{T0}+ar)=\tfrac{abc}{1-ab}+\tfrac{a-ab}{1-ab}r+M, \qquad (IIR^0)$$

$$\begin{aligned} P_h[eZ_{Yg}^{Se}+(1-e)aZ_b^{Se}+r]+(1-P_h)[e(Z_{Yg}^{Te}+r)+(1-e)a(bZ_{Yb}^{Te}+r)] \quad (IIR^e) \\ = ec+\tfrac{abc}{1-ab}+\tfrac{a-ab}{1-ab}+M, \end{aligned}$$

and two interim incentive (truthtelling) constraints,

$$P_\ell(aZ_b^{S0}+r)+(1-P_\ell)(abZ_{Yb}^{T0}+ar)\geq P_h[aZ_b^{Se}+r]+(1-P_h)a[bZ_{Yb}^{Te}+r], \; (IIC^0)$$

$$\begin{aligned} P_h[eZ_{Yg}^{Se}+(1-e)aZ_b^{Se}+r]+(1-P_h)[e(Z_{Yg}^{Te}+r)+(1-e)a(bZ_{Yb}^{Te}+r)] \quad (IIC^e) \\ \geq P_\ell[(e+(1-e)a)Z_b^{S0}+r]+(1-P_\ell)[e(Z_{Yg}^{T0}+r)+(1-e)a(bZ_{Yb}^{T0}+r)]. \end{aligned}$$

P_ℓ and P_h are the probabilities with which a soft contract will be used in period 3 (P_h if effort was invested, P_ℓ if not). Z_{Yg}^{Te} etc. are transfers to the entrepreneur. The lower indices refer to the outcome (Y, and g was announced), and the upper indices to the type of contract (Te for the tough part of the contract for the e type). The program is formulated in a less general than that of Propositions 2.1 and 2.2, but the omitted additional variables would cancel out anyway. M is the information rent that the entrepreneur may earn by renegotiating. It must be identical for both types, as without renegotiation the entrepreneur is indifferent between investing effort and not, while with renegotiations he *should* be indifferent (because he is to play a mixed strategy).

We first show that $P_\ell = 1$. Suppose $P_\ell < 1$. The investor could increase it by $\varepsilon > 0$, and increase Z_{Yb}^{T0} by $\delta_1 = \frac{abZ_{Yb}^{T0}-aZ_b^{T0}-(1-a)r}{(1-P_\ell)ab}\cdot\varepsilon$. This leaves (IIR^0)

and (IIC^0) unchanged, decreases the right hand side of (IIC^e), and increases the investor's payoff. If $Z_{Yb}^{T0} = 0$, she can decrease Z_b^{S0} by $\delta_2 = \frac{aZ_b^{S0}+(1-a)r}{P_\ell a} \cdot \varepsilon$, which has the same effects. $Z_b^{Yb} = 0$ would lead to a contradiction with either $M \geq 0$ or the assumption that $abc > (1-a)r$, as we could rewrite (IIR^0) as $-(1-P_\ell)(1-a)r = \frac{abc-(1-a)r}{1-ab} + M$.

The simplified program can now be solved. It yields the following constraint,

$$P_h \leq \frac{(1-ab)M}{(1-a)r}, \tag{2.17}$$

which gives us an upper bound for P_h for every level of information rent. This allows us to reduce the investor's program to a maximisation with only M as a variable. The derivative of this objective function with respect to M is negative, and therefore the investor will set $P_h = M = 0$. The renegotiated contracts are thus

$$P_\ell = 1, \qquad Z_b^{S0} = \frac{1}{a}\left[\frac{abc}{1-ab} - \frac{1-a}{1-ab}r\right], \qquad Z_{0\phi}^{S0} = r,$$

(all other transfers zero) for the 0 type, and no new contract for the e type. $\square$

Proof of Proposition 2.10

As in Proposition 2.5. $\square$

Chapter 3

Liquidity Constraints, Production Costs and Output Decisions

Liquidity Constraints, Production Costs and Output Decisions *

Paul Povel

University of Mannheim and

London School of Economics

Michael Raith

Graduate School of Business

University of Chicago

May 1998

* This paper is based on research that was done while both authors were at ECARE, Brussels. We would like to thank Patrick Bolton and Martin Hellwig for their encouragement and guidance, and Judy Chevalier for helpful discussions. Financial support from the Stipendienkommission Basel–Landschaft (Povel), and the European Commission through its HCM Programme (Raith) is gratefully acknowledged.

Paul Povel, Dept. of Economics, University of Mannheim, D-68131 Mannheim, Germany; email: povel@pool.uni-mannheim.de; phone +49 (621) 292-5170; fax +49 (621) 292-1037

Michael Raith, Graduate School of Business, University of Chicago, 1101 E. 58th Street, Chicago, IL 60637; email: michael.raith@gsb.uchicago.edu; phone (773) 702-7324; fax (773) 702 0458

Abstract

This paper analyses the effects of liquidity constraints on a firm's output decisions by emphasizing the role of production costs. We present a simple duopoly model in which firms have to produce goods and incur production costsbefore they can offer their products in the market. A financially constrained firm may choose to obtain external funds by agreeing on a financial contract with a bank, which we derive *endogenously*. After signing the contract, the firm chooses its level of production. Finally, its revenue, and hence the ability to repay the loan, depends on the firms' output levels and the realization of a stochastic demand function.

We find that with endogenous debt contracts, existing debt has no effect on a firm's *desired* output level, as compared to a firm with a deep pocket. The requirement that production costs be debt-financed, however, places a constraint on the output level. As a result, in equilibrium, firms are forced to internalise the expected costs of possible bankruptcy, which leads firms to reduce output. This main result, and some other results we obtain, are consistent with empirical evidence.

Keywords: liquidity constraints, debt contracts, product market competition

JEL-Codes: G32, G33, L13

3.1 Introduction

There is a large literature which analyses the interdependence of financial and product market decisions of firms. The theoretical work is inconclusive: Most notably, Brander and Lewis (1986) have argued that leveraged firms compete more aggressively in the output market, because in the presence of uncertainty, debt should make borrowers more risk–loving, or more "aggressive".[1] Several other theories predict that for different reasons, debt or financial constraints leads firms to compete less aggressively.[2] Most empirical studies conclude that firms with high debt levels and/or in financial distress compete less aggressively, i.e. invest less and set higher prices.[3]

In this paper, we emphasise the role of debt–financed *production costs* for a firm's financial and output decisions. We analyse the interdependence of the two decisions in a model in which *debt* contracts arise as optimal contracts, designed as a solution to agency problems that are caused by asymmetric information. In so doing, we synthesise the analyses of Brander and Lewis (1986), Bolton and Scharfstein (1990) and Gale and Hellwig (1985), and extend several of the results obtained in these key papers. We find that financially constrained firms produce less than unconstrained firms. This and other results we obtain are consistent with the empirical evidence.

We focus on the case of Cournot duopolists, one of which is "financially constrained": it cannot produce the standard Cournot output using its internal funds, only (or it may even have inherited debt, which must be refinanced). Unlike most of the recent literature,[4] we study the influence of a firm's financial situation (inherited from the past) on its current decisions (financing, output), instead of the

---

[1] Maksimovic (1988) obtains a similar result. In Hendel's (1996) model, firms in distress generate cash by cutting price in order to dump their inventories on the market.

[2] cf. Gale and Hellwig (1985), Jensen (1986), Fudenberg and Tirole (1986), Bolton and Scharfstein (1990), Phillips (1995) and Chevalier and Scharfstein (1996).

[3] cf. Opler and Titman (1994), Chevalier (1995a,1995b), Phillips (1995), Kovenock and Phillips (1997). Zingales (1997), however, finds evidence of tougher pricing by overleveraged firms in the trucking industry.

[4] An exception is Gale and Hellwig (1985).

correlation of *current* debt and *current* output decisions.

The structure of our model is as follows. First, the financially constrained firm may design a financial contract, which it offers to competing investors. Second, each firm produces its output, where variable costs are financed by both retained earnings and borrowed money. Third, the firms' stochastic earnings are realised: revenue is determined by both the output choices and a stochastic demand intercept. These earnings are not observable. All along the way, the financially constrained firm fulfils its obligations as specified in the contract (e.g. it makes a repayment after obtaining its earnings). Finally, the firms earn a second period payoff. This payoff is contractable, and the financial contract will make use of a possibility to liquidate the firm, such that it cannot earn this second period payoff.

We obtain the following results:

1. Extending the work of Bolton and Scharfstein (1990), we derive an optimal, incentive–compatible, renegotiation–proof contract between the firm and an Investor, given the assumption that the firm's realised profit is not verifiable. The resulting contract has a simple structure which resembles a standard debt contract in that the firm is required to repay a fixed amount. If the firm fails to repay in full, however, it is not forced to go bankrupt for sure as with a standard debt contract. Rather, the probability of bankruptcy is a decreasing function of the amount repaid. This provides the incentive for the firm to repay as much as it can, possibly its entire earnings. Thus, instead of *assuming* that firms issue debt (as in Brander and Lewis (1986)), we analyse a setup in which debt arises naturally, as an institutional solution to some agency problem.

Extending Bolton and Scharfstein's analysis from their two–state model, in which there is no output decision for the firm to make, to a full oligopoly model is not straightforward. In our model, realised profit depends on a stochastic demand intercept, which is a continuous random variable, and on the respective output decisions of two firms. More importantly, however, the revelation game which leads to the optimal contract has a much more complicated structure. Firstly, the firm's output is unobservable, and must also be revealed to the lender (it is private information). The optimal contract must also make sure that the firm produces exactly the output level that the parties have agreed on. The contracting problem in Bolton and Scharfstein (1990) is simpler, because investment is a verifiable zero–one decision. Second, the firm's earnings depend on its output choice, and the earnings that it can report at the last stage depend on *both* the real output level, and the *reported* one.

With one output choice stage and (potentially) two revelation stages, the borrower has a much richer strategy set than if he simply had to reveal a state of the world once. It is therefore surprising, that the simple debt contract, which makes little use of revelation games, turns out to be an optimal contract.

2. If debt serves no direct purpose except possibly to finance *fixed costs* (as in Brander and Lewis (1986), and many related papers), *and* a firm and an Investor sign an optimal contract as described above, then debt has no effect on the choice of output at all. As a consequence, a firm cannot gain anything by issuing "strategic debt" — the result in Brander and Lewis (1986) is driven by the *assumption* that debt is used.

3. If variable costs must be financed, however, the (inherited) financial constraints of a firm *do* have an effect on its decisions (and those of its rivals): the constrained firm produces less than it would without its constraints. Two effects are at work here: Firstly, the amount of borrowing puts a limit on the output that can be produced, because we look at a case with positive marginal costs. Secondly, borrowing is *costly*, as the (endogenous) debt contract leads to dead-weight losses if promised repayments cannot be made. Both effects work against the "limited liability" effect modeled in Brander and Lewis (1986), and more than compensate it.

Even though we analyse sequential decisions (first borrowing, then output, then uncertain earnings and repayments), the model is relatively easy to handle. In equilibrium, the financially constrained firm commits to some output level at the financing stage, by borrowing an amount that is exactly sufficient to cover the production costs. Brander and Lewis' "limited liability effect" works in a model that has the same sequential structure, and in which the lenders can correctly predict all actions, and their result is that firms want to incur debt, because this commits them to be more aggressive quantity–setters. By fine–tuning their debt level, a firm can aim for the payoff of a Stackelberg–leader, and therefore increase its ex ante value. We get the opposite result: financially constrained firms produce smaller outputs than financially unconstrained firms, and a firm's value does not increase in its financial constraints.

Gale and Hellwig (1985) also derive an underinvestment result, but in a different model. Firstly, theirs is a costly–state–verification model, in which a lender can observe and verify the borrower's (stochastic) earnings, but only at a cost. In our model, this is not possible (the 'inspection costs' are infinitely high), and the contract has to use other means to ensure that the expected payments to the lender are sufficiently high (our contract uses the firm's survival, i.e. its possibility to earn additional payoffs, as a 'hostage'). Secondly, In Gale and Hellwig (1985) the borrower's investment is contractable, and the agreed on output level is part of the contract. This also means that the financial and output decisions are *simultaneous*. In our model, the output level is unobservable, and therefore cannot be element of a contract. We also study a sequence of decisions, where borrowing is followed by output choice.[5,6]

These differences are not reproduced in the results, as in both Gale and Hellwig (1985) and our model there is underinvestment by financially constrained firms. While one would have expected that the sequential structure leads to moral hazard problems at the output choice stage (as in Brander and Lewis (1986)), the opposite is the case: because the firm has to finance variable production costs, the amount it borrows puts a limit on the output it can produce, and the risk–shifting problem has no bite.

4. The degree of underinvestment is not monotonically increasing in a firm's financial constraints. It *is* monotonic for firms with *limited* retained earnings, but not for firms with negative levels of retained earnings, which we interpret

[5] An additional difference is the assumption in Gale and Hellwig (1985) that a firm is bankrupt with probability zero or one. See e.g. Mookherjee and Png (1989), for an extension of the costly–state–verification model (without investment decisions) to a continuous outcome space.

[6] Faure–Grimaud (1997) analyses a model with observable but nonverifiable output decisions, and also derives an underinvestment result. His analysis implicitly uses the assumption that financial and output decisions are simultaneous, as in Gale and Hellwig (1985).

as 'inherited debt' (obligations which have to be paid off before production can start). The effect of inherited debt is similar to that of fixed costs: the initial loan must pay both the initial obligations and the current production costs, and the higher the amount that has to be rolled over, the larger the share of the firm's earnings that has to go to the lender. This also increases the probability of liquidation, which follows insufficient repayments by the borrower.

With small financial constraints, an increase in the constraints will lead to reduced borrowing and output: the firm trades off current earnings and liquidation probability, and the effect on the latter is stronger. With high levels of inherited debt, however, the situation is different. Here, the firm may not be able to transfer sufficient amounts of money to the lender if it reduces its borrowing: increasing the fixed costs of production means that production has to become *more profitable*, if it is to take place at all. Thus, with high inherited debt, a firm finds itself with its back to the wall, and must become suicidal to survive.[7]

5. Perhaps surprisingly, constrained firms with relatively high retained earnings (i.e. not too far below the deep–pocket level) choose not to incur any debt, and rather finance production only out of retained earnings. The reason is that even if the level of debt is close to zero, a firm that chooses debt financing faces a *discrete* increase in its effective marginal cost of production. Therefore, a firm chooses debt financing for production only if it is *sufficiently* constrained. Thus, precisely because there is a discrete difference between the costs of internal and external funds, the absence of debt does not rule out the presence of financial constraints.

As mentioned above, our analysis can be viewed as combining the analyses of Brander and Lewis (1986), Bolton and Scharfstein (1990) and Gale and Hellwig (1985). From Brander–Lewis, we adopt the explicit oligopoly model and the two–

[7] Gale and Hellwig also obtain a nonmonotonicity result. See also Aghion, Dewatripont and Rey (1997), who analyse the effort incentives of the manager of a financially constrained firm. In a patent race model, in which R&D effort is not fully contractable, the manager's effort provision depends in a nonmonotonic way on the firm's financial situation.

stage structure of the game: When choosing its output level, an indebted firm takes the required repayment to the bank as given, but takes into account the effect of its output choice on the current–period profit and hence the ability to repay. In Brander–Lewis, the combination of this sequential structure with the *assumption* that standard debt contracts are used drives the effect that makes a leveraged firm more aggressive. Here, we show that with endogenously derived contracts, this effect vanishes.[8]

Note that the need to pre–finance production costs drives the underinvestment result in our model. With marginal costs of production equal to zero, debt obligations have no effect on a firm's output choice. The same holds for a firm's *desired* output choice in a model with positive marginal costs of production: this desired level is the same for any debt level, but the firm is not able to achieve it, because at the (earlier) contracting stage it decided not to borrow enough money. What drives down its output aspirations are the (increasing) marginal costs of borrowing, which are caused by the need to sometimes liquidate the firm.

The use of non–liquidation as a 'hostage' in the financial contract exactly wipes out the "limited liability effect" that is modeled in Brander and Lewis (1986). This reminds of the use of the "punishment function" in Diamond (1984). We want to stress that this effect arises as a *by–product* of the revelation mechanism: The incentive constraints for the truthful revelation of the firm's income generate the debt–like structure, and the incentive problem at the output choice stage does not add any structure, because it is not binding anymore, once the truthtelling problems have been solved. Solving the incentive problems 'backwards' is the right approach, because of the sequential structure of the game: no decision by the firm can be observed by the lender, and therefore all decisions at all stages can only be made contingent on earlier *revelations* by the firm.

The predictions of our model are consistent with most existing empirical evidence: debt weakens a firm's competitive position. An alternative theory to explain why debt may make firms less aggressive is offered by Chevalier and

[8] With small but positive marginal costs of production, the results of Brander and Lewis (1986) are unchanged: A firm that issues "strategic debt" will simply have to use up some of the borrowed funds to finance production.

Scharfstein (1996). They model a market with switching costs, in which the short–run profit–maximizing prices exceeds the prices that maximize profits in the long–run. The latter are relatively low because firms are concerned about attracting new customers in order to maintain their market share; in the short run, however, a firm may want to exploit its locked–in customers and raise its price, thereby increasing its current period profit, at the expense of its future market share. A highly indebted firm faces an existential threat (of bankruptcy), which gives it a double incentive to concentrate on short–run profit maximisation: firstly because this increases its survival chances, and secondly because it must discount future gains from investing in market share more heavily.

Other theories are based on 'classical' papers in corporate finance, which analyse overinvestment and underinvestment problems as a function of a firm's financial structure. Jensen and Meckling (1976) argue that issuing debt leads to poor investment incentives. They analyse a manager's *effort* incentives, which are also the central problem in the model of Aghion, Dewatripont and Rey (1997).

Myers and Majluf (1984) show that firms may want to ignore positive NPV investments if they cannot finance them internally, because external funds are more expensive than internal funds. Jensen (1986) and Hart and Moore (1995) extend this, by showing that debt is a valuable tool in that it forces a firm to pay out its earnings to investors. Jensen argues that managers of highly leveraged firms have less opportunities to waste 'free cash flows' on unprofitable investments, which increase their own private benefits. Chevalier (1995a, 1995b) speculates that this could have been the reason for 'unnecessary' price wars between supermarket chains in the U.S., and that improved governance (through takeovers) led to a reduction in these negative NPV 'investments in market share'. Phillips (1992) analyses a market model with two periods, in which there is not enough room for two firms in the second stage. Predation in the first stage is one (costly) way to lead to a rival's exit. His model uses the idea that debt finance is more costly than internal funds in a duopoly model, and a firm may want to incur debt to drain its future funds, and therewith signal to its rivals that there is no need for predation, as it will not be able to profitably finance its second period investment.

The rest of the paper is structured as follows. The model and the contracting problems are introduced in Section 3.2. We first analyse a monopolist's choice of financial contract and then output, solving for his equilibrium strategy by backwards induction (we set up the mechanism design problem in Appendix A). in Section 3.3 we show how problems of asymmetric information (after the firm's output choice) structure the financial contract. In Section 3.4 we analyse the firm's output choice (given the contract), and the preceeding contract choice decision. We show how our results extend to the case of a duopoly in Section 3.5. The empirical implications of our model are discussed in Section 3.6. Section 3.7 concludes.

3.2 The Model

There are two players, an entrepreneur (E) who can start a project, and an investor (I). Both are risk neutral. The timing is as follows:

1. E owns retained earnings k_0 (if $k_0 < 0$, we will also speak of 'inherited debt'). He can offer a financial contract to I to borrow k_1. I can accept or reject.

2. E produces output q at cost $c \cdot q \leq k_0 + k_1$. I cannot observe E's choice of q.

3. E offers his output q for sale in a market with stochastic demand. The realised price is $\max\{0, \theta - q\}$, where θ is a stochastic intercept uniformly distributed over $[a - \varepsilon, a + \varepsilon]$. Thus, E's earnings are $\max\{0, q(\theta - q)\}$. While the distribution of θ is common knowledge, only E, but not I, can observe θ and the earnings.

4. The project can be terminated or continued. If the project is continued, E earns an additional payoff π_2. If it is terminated, there is no additional payoff.

Notice that E produces some output and is then *committed* to selling every single unit in the market, at price zero if necessary. In reality E would want to destroy part of the output if he learns that demand is weak, to maximise his profit. For instance, the output could be perishable goods (e.g. fashion items, bestsellers, etc.) and once the output is inside the retail distribution network E cannot control or recall it anymore. Similarly, E cannot produce to order, or store some of the redundant output for later sales (as the diamond cartel seems to do). We analyse the extreme case in which all output that is produced is actually sold, even at a price of zero, because it greatly simplifies the analysis. We believe that this assumption does not affect our results qualitatively. We plan to use a richer model at a later stage, in which we also analyse issues of predation: a financially sound firm may want to dump large quantities into the market if its rival is highly indebted, because driving the rival into bankruptcy means being able to enjoy a monopoly in future periods.

If q and θ were observable and verifiable, E and I could agree to produce the profit–maximizing quantity

$$\begin{aligned}\bar{q} &= \arg\max_q \int_q^{a+\varepsilon} q(\theta - q)\frac{1}{2\varepsilon}\, d\theta - c \cdot q \qquad (3.1)\\ &= \frac{2}{3}(a+\varepsilon) - \frac{1}{3}\sqrt{(a+\varepsilon)^2 + 12c\varepsilon}.\end{aligned}$$

Assuming that it is profitable (in expected terms) to do so, E and I could also agree on a division of profits (depending on θ) such that both break even on average.

However, q and θ are unobservable, and the contract cannot be made directly contingent on these values, nor on the firm's earnings. This severely constrains the set of financial contracts, under which both parties break even on average. Due to Limited Liability constraints, a contract cannot force E to pay out more than the project earned. The uncontingent repayment stipulated in the contract must therefore be no higher than the earnings that the project generates if the worst state of the world was realised. If E produces some output $q \geq a - \varepsilon$, the earnings are zero if $\theta \leq a - \varepsilon$, and so must be the repayment — borrowing with uncontingent contracts is only feasible if the contract induces an output choice $q < a - \varepsilon$.

While the realisations of q and θ may be unobservable, the parties can exchange *messages* about the realisations, and the latter *are* verifiable. Furthermore, a contract can specify transfer payments, and it can be made contingent on certain transfer payments. Finally, a contract can make use of randomising devices. The size of the space of possible contracts is very large, which makes the search for the optimal contract quite difficult, but we can simplify the task by making use of the Revelation Principle. With its use, we can determine an upper bound to the expected payoff that E might expect from any contract. As will be shown below, this highest payoff can be achieved by use of a very simple contract (see Proposition 3.2), which resembles a standard debt contract.

The standard mechanism design approach is to add 'message games' to a given game form: Whenever a player obtains new information, which is not shared by the other player, a mechanism ("contract") may make use of messages *about* this new information. Instead of being directly contingent on the information itself, the contract is contingent on messages about it. Obviously, a party with private information may try to misrepresent it, and the optimal contract has to take this possibility into account.

Nontrivial message games can be added to our game (as defined above) after two arrivals of private information: Firstly, E's quantity decision q is not observed by I. Secondly, only E can observe the realisation of θ. We solve for the optimal contract using backwards induction: The optimal contract is supposed to induce some output choice q and some repayment (as a function of θ), i.e. both parties *expect* these actions to be taken, and we require that at no time the players have an incentive to deviate from this equilibrium behaviour.

The design of an optimal contract, and the use of the Revelation Principle, are complicated by the fact that E has to choose two actions sequentially. It is well known that the application of the Revelation Principle is less straightforward with extensive form games, as compared with normal form games (see e.g. Myerson (1986)). Complications may arise as the extensive form allows for 'multistage' manipulations, and one must check that no player has an incentive to misrepresent his information at an early stage, only to (profitably) misrepresent other information at a later stage. On the other hand, some forms of misrepresentation may not always be available at later stages, which may limit the set of incentive constraints that an optimal contract has to take into consideration.

The optimal contract and the financial and output decisions are analysed in the two next Sections. In Section 3.3 we assume that the parties want to sign a contract which induces some output choice q, and determine a financial contract which firstly leads to this output choice, and which secondly is optimal: it maximises the payoff of the party who proposes the contract, while the other party breaks even in expected terms. In Section 3.4 we identify a simple contract which resembles a debt contract, and which has the same strategic properties as the optimal contract derived in Section 3.3 (it leads to the same payoffs and to the same revelation of information). We determine the set of feasible contracts, and we show that this is equivalent to determining the set of implementable output decisions. Given this set of feasible alternatives, we analyse E's contract and output choice.

3.3 Financial Contracts: The Monopoly Case

Let us start by explaining what complications one may expect. Given the information structure described in Section 3.2, E first decides on q, and then he can send two messages: one after his choice of q but before θ is realised, the other after θ has been realised. Depending on each message, the mechanism might also define payments that one party has to make to the other. It could therefore be the case that some messages are not feasible under certain circumstances, because the corresponding transfers cannot be made. For instance, assume that when deciding what quantity to produce, E owns some amount of money k, and that he produces output q at cost $c \cdot q$. The mechanism could require that after sending the first message E pays all his cash to I — he would be unable to claim (as part of his message) to have produced $\hat{q} < q$, as he would not be able to transfer $k - \hat{q}$.

As we will show below, these problems will not arise, i.e. the entrepreneur has no incentive at any stage to deviate from either the quantity choice that the contract is meant to induce, or from truthtelling. We proceed in two steps. First we analyse the requirements that the truthtelling constraints impose on an optimal contract (Lemma 3.1). Then we consider the entrepreneur's output choice problem, after having signed a financial contract which satisfies those requirements, and find that E's output choice is uniquely determined by the amount he borrows (Lemma 3.2). Proposition 3.1 then characterises the optimal financial contract that E will offer to I, if he intends to produce some output q.

In the Appendix, we show that the Revelation Principle can indeed be applied to our problem, and that the 'multistage' complications are not relevant. We define a game form for the direct mechanism, and solve for the best mechanism by backward induction, making sure that 'truthtelling' is indeed an equilibrium at every stage. Lemma 3.1 summarises the results of analysing the two revelation stages:

Lemma 3.1 (Structure of the Optimal *Direct* Mechanism) *Assume that E has produced an output $q \leq (k_0 + k_1)/c$ in stage I, and that the realised state of the world is θ. Consider a direct mechanism with announcements $\tilde{q}$ and $\hat{\theta}$. Denote by $r(\hat{\theta}|\tilde{q})$ the payment that E has to make if he announces first $\tilde{q}$, and then $\hat{\theta}$, and* *by* $\beta(\hat{\theta}|\tilde{q})$ *the corresponding probability that the project can be continued, i.e. that E can earn the additional payoff* π_2*. Finally, let* $\tilde{\delta} := k_0 + k_1 - c\tilde{q} \geq 0$ *be E's 'savings', i.e. the amount that was not spent if* $\tilde{q} < \frac{k_0+k_1}{c}$*.*

Then this direct mechanism is optimal, and it leads to truthful revelation of both q *and of the realised state of the world* θ*, if it has the following structure.*

$$r(\hat{\theta}|\tilde{q}) = \min\left\{D\ ,\ \tilde{\delta} + \tilde{q}(\hat{\theta} - \tilde{q})\right\}, \tag{3.2}$$

$$\beta(\hat{\theta}|\tilde{q}) = \min\left\{1\ ,\ 1 - \frac{D}{\pi_2} + \frac{\tilde{\delta} + \tilde{q}(\hat{\theta} - \tilde{q})}{\pi_2}\right\}, \tag{3.3}$$

where D *is a constant, i.e. independent of both* $\tilde{q}$ *and* $\hat{\theta}$*.*

Proof: See Appendix A. ■

The intuition behind Lemma 3.1 is that E tries to maximise his monetary repayment, in exchange for a higher probability of being rewarded (of earning π_2). The contract uses π_2 as a 'hostage', which E can buy back from I, if his funds are sufficiently high. E has the choice of repaying either with money, or with 'pain': the Investor does not derive any utility from liquidating, which therefore represents a deadweight loss. The optimal contract aims at minimising the probability that this happens, by requiring that after bad outcomes E transfers as much money as possible to I.

There is no scope for renegotiation of the threat to liquidate, if the repayment is insufficient, even though liquidating is wasteful. Our model could be reformulated in the spirit of Bolton and Scharfstein (1990), who analyse a twice repeated funding problem. Lending in the second period is unprofitable for the lender, since due to the unobservability of the borrower's earnings, the latter can always claim to have earned very little. However, borrowing in the second period is highly profitable for the borrower. The lender can commit to lending in the second period, if the repayment in the first period was sufficiently high. The second loan is used as a hostage, and renegotiation is made difficult by the fact that 'returning the hostage' (i.e. providing the second loan) is costly for the lender. In our model we analyse a limit case of this setup, by setting the lender's loss–from–lending in the second period equal to zero, while the profit of the borrower is a (positive) constant, π_2.

Given the structure that truthtelling imposes on the best direct mechanism, what is E's output choice? We can construct his expected payoff for any choice of q that he can finance (recall that $c \cdot q$ cannot be higher than $k_0 + k_1$). Given the mechanism as described in Lemma 3.1, the project is continued with probability one only if the earnings and savings are sufficient to repay D, i.e. if $q(\theta - q) + \delta \geq D$. In all other cases E hands over all money and the project is continued with probability β, as described in Lemma 3.1. E has an expected payoff of

$$\begin{aligned} \mathcal{E}U(q) &= \int_{a-\varepsilon}^{q} \left(1 - \frac{D}{\pi_2} + \frac{\delta}{\pi_2}\right) \pi_2 \frac{1}{2\varepsilon}\, d\theta \\ &+ \int_{q}^{\frac{D-\delta}{q}+q} \left(1 - \frac{D}{\pi_2} + \frac{q(\theta - q) + \delta}{\pi_2}\right) \pi_2 \frac{1}{2\varepsilon}\, d\theta \\ &+ \int_{\frac{D-\delta}{q}+q}^{a+\varepsilon} (\pi_2 - D + \delta + q(\theta - q)) \frac{1}{2\varepsilon}\, d\theta. \end{aligned} \tag{3.4}$$

The first integral contains payoffs for the cases in which $\theta \leq q$. In these cases the earnings are zero and the continuation probability is constant. The second integral contains payoffs if the earnings are positive but not sufficient to repay D:

$$\theta \in \left(q, \frac{D - \delta}{q} + q\right) \quad \Leftrightarrow \quad \delta \leq q(\theta - q) + \delta \leq D. \tag{3.5}$$

The third integral contains E's payoffs if the earnings are sufficient to repay D, and therefore $\beta(\theta) = 1$. We can simplify (3.4), and substitute δ:

$$\mathcal{E}U(q) = \pi_2 - D + \int_{q}^{a+\varepsilon} q(\theta - q) \frac{1}{2\varepsilon}\, d\theta + k_0 + k_1 - cq. \tag{3.6}$$

Note that the payoff function (3.6) is identical, up to a constant, with E's payoff function if his funds k_0 were unlimited:

$$\mathcal{E}U(q) = \int_{q}^{a+\varepsilon} q(\theta - q) \frac{1}{2\varepsilon}\, d\theta + k_0 - cq + \pi_2. \tag{3.7}$$

If we compare the first order conditions for the two maximisation problems, we see that they are identical, and we can conclude:

Lemma 3.2 *Assume that E has signed a financial contract with the structure described in Lemma 3.1. Then his output choice is uniquely determined by the amount k_1 that he borrows:*

if $k_0 + k_1 < c \cdot \bar{q}$, *E produces* $q = \frac{k_0 + k_1}{c} < \bar{q}$

while if $k_0 + k_1 \geq c \cdot \bar{q}$, *he produces exactly* $\bar{q}$.

Proof:

From the definition of $\bar{q}$ we know that E's first order condition at the quantity choice stage is positive if $q < \bar{q}$ and negative otherwise. Thus, he will produce $q = \bar{q}$, if this is feasible, and the highest feasible output otherwise. ■

We had noted above that once a financial contract has been signed, the debt level *per se* has no effect on the output decision. From Lemma 3.2, however, we can conclude that there is an *indirect* effect of debt on E's output choice. The latter is constrained by the size of the available funds, i.e. by the size of k_1, and this is of course correlated with D.

This result is important not only because of the implications discussed above (We will discuss this result in more detail in Section 3.6). It also means that when the borrowed amount is determined, the parties also determine the quantity that will be produced. While the two decisions (the offer of a financial contract, i.e. of a loan, and the output choice) are made sequentially, and should therefore be independent, the link via the availability of funds is so strong that — technically — the two decisions are made simultaneously.

At this point it is worth pointing out the following simplyfying result:

Lemma 3.3 *Without loss of generality, an optimal contract will provide for Maximum Equity Participation, i.e. the entrepreneur invests all his retained earnings* k_0.

Proof:

Suppose that E borrows k_1 and the intention of the contract is that he produces some output $q < (k_0+k_1)/c$, thus retaining $\delta := k_0+k_1-cq$ for later repayments. δ is not 'riskless debt': as E's output choice is not contractable, he could invest the whole borrowed amount and would choose to do so if $k_0 + k_1 \leq c\bar{q}$. If this would hurt the Investor, she would not accept the contract initially. While if the Investor gains, the Entrepreneur could design a different contract, under which the Investor's participation constraint is binding, and increase his own payoff. ■

Thus, if E wants to produce some output q, he can do so by designing a contract such that $k_0 + k_1 = cq \leq c\bar{q}$ (Lemmata 3.1, 3.2 and 3.3), and D is sufficiently high, such that the Investor breaks even. This is summarised in the following Proposition:

Proposition 3.1 *The best direct mechanism that E can design to implement an output $q \leq \bar{q}$ consists of β and r, as described in Lemma 3.1, a borrowed amount $k_1 = c\bar{q} - k_0$, and the smallest value of D (the promised repayment) such that*

(1) *$D \leq \pi_2$, and*

(2) *q, k_1, and D solve I's participation constraint with equality:*

$$\int_q^{\frac{D}{q}+q} q(\theta - q)\frac{1}{2\varepsilon}\, d\theta + \int_{\frac{D}{q}+q}^{a+\varepsilon} D\frac{1}{2\varepsilon}\, d\theta - k_1 = 0. \tag{3.8}$$

Proof:
Follows from Lemmata 3.1, 3.2, and 3.3, and the fact that E has all bargaining power when the contract is designed. D may not be larger than π_2 as otherwise E would always claim that the earnings were zero. ■

Note that complications may arise if for the optimal output q the promised repayment D is too large, compared with π_2 (part (2) of Proposition 3.1 is satisfied, but part (1) is violated). The last requirement in Proposition 3.1 constrains the set of feasible contracts but it does not prevent E from being financed. If D (as defined in (3.8)) is larger than π_2, both D and k_1 (and therefore q) have to be reduced, until $D = \pi_2$. Solving (3.8) with $D = \pi_2$ and $q = (k_0 + k_1)/c$ for k_1 yields the highest amount that E can borrow (in this case, $\beta(\theta) = 0$ for all $\theta \leq q$). Our results remain valid, however. The additional constraint does not affect the truthtelling constraints when θ has to be announced, because the structure in Lemma 3.1 is preserved. The same holds for the revelation of q, since D is constant. At the quantity choice stage nothing is changed since even without this additional constraint E's choice is a corner solution: the quantity that he can produce is determined by $(k_0 + k_1)$, and it is probably smaller than what he would like to produce ($\bar{q}$). From here onwards, we ignore the possibility that the optimal D may be too large, by assuming that π_2 is always sufficiently large.

3.4 A Monopolist's Contract and Output Choice

In Section 3.3 we have derived the structure of an optimal financial contract which induces an output choice q by the Entrepreneur, and makes sure that the Investor breaks even in expected terms. The former decision is not straightforward to implement, as E's output choice is unobservable. The latter result (the Investor breaks even) is similarly difficult to obtain, as E's earnings are unobservable, and he must be provided with an incentive to admit that there were some earnings, and to pay out some of it to the Investor.

The contract uses the future payoffs that E gains from the project as a 'hostage': if E claims that the earnings were low, and therefore repays little, his project is terminated with a probability that is increasing in the financial shortfall. In the optimum, E is indifferent between revealing the true earnings (and making the corresponding repayment) and claiming that the earnings were lower than they actually are (the repayment would be smaller, but so would be the continuation probability). With the optimum contract described in Lemma 3.1 and Proposition 3.1 E weakly prefers to reveal the realised state of the world (i.e. his earnings). This contract looks somewhat abstract, but we can construct a much simpler mechanism which leads to exactly the same incentives and outcomes:

Proposition 3.2 (Structure of an Optimal *General* Mechanism) *If E wants to produce some output q, he can achieve his highest payoff by offering the following contract:*

§*1* *E borrows k_1 from I, where $k_1 = cq - k_0$*

§*2* *E promises to repay D to I*

§*3* *If E repays less, his firm is liquidated with probability $(1 - \beta)$, where (if we denote with \$ the repayment)*

$$\beta(\$) = 1 - \frac{D}{\pi_2} + \frac{\$}{\pi_2}. \tag{3.9}$$

Proof:
The mechanism achieves the same outcomes as the one that was derived in Section 3.3, and the Entrepreneur's incentives are unchanged. ■

The contract described in Proposition 3.2 has much similarity with a standard debt contract. I is only interested in E's *cash holdings* after he has sold his products in the market. The contract need not specify an output or include any message games around the chosen output level. Similarly, the state of the world is only of indirect interest. The contract has an additional, more realistic feature: failing to repay 99% of a debt obligation is 'worse' than failing to repay 1%, as the probability of liquidation $(1-\beta)$ is increasing in the share of debt that is not repaid.

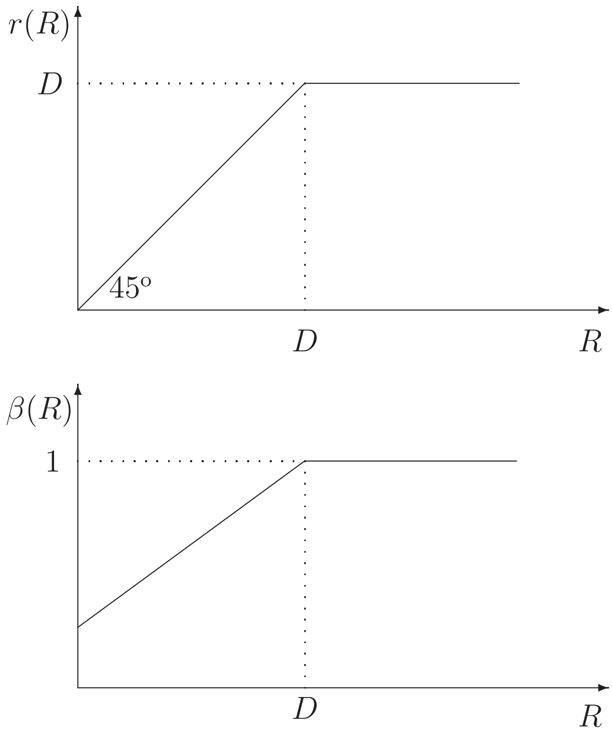


Figure 3.1: *Repayment and continuation probability as a function of earnings*

The repayment and continuation functions are depicted in Figure 3.1. E is 'bankrupt' if his earnings are below D, and in this case the continuation probability is less than one. The top graph shows E's payment to I, either D, or all earnings (if $R < D$). The bottom graph shows the continuation probability β, which is increasing in the repayment and exactly one if D is repaid. Note that even if the earnings and therefore also the repayment are zero, β may nevertheless be positive. This follows from the fact that E tries to maximise the expected probability of continuation (we would observe $\beta(0) = 0$ if π_2 is 'small', see the discussion after Proposition 3.1).

Proposition 3.2 almost concludes the application of the Revelation Principle to our model, which we discussed in Section 3.3. The aim had been to find the optimal contract, i.e. the best contract that E might possibly design. The solution to the 'related' problem of finding the optimal *direct* mechanism provided us with an upper bound to what E can achieve, and left us with the need to characterise a mechanism that actually implements this 'best' outcome. One example for such a 'more general' optimal mechanism is the contract above: For each output that E wants to produce, we can either determine a contract that allows him to do so, or we can be sure that this output is not feasible.

What remains to be determined is firstly the set of *implementable* outputs, i.e. which outputs q E can credibly commit to produce, if he proposes some optimal contract. This contract must make it feasible to produce that output (i.e. the borrowed amount must be sufficiently high), and E must have the incentive to produce that output, given his cash holdings (for instance, a promise to produce $q > \bar{q}$ is not credible, because E would prefer to produce exactly $\bar{q}$).

From here on, we will consider *both* positive and negative levels of retained earnings k_0. If $k_0 < 0$, we interpret it as 'inherited debt', which must be paid off if the project is to be continued.

Lemma 3.4 *A lower bound to the output levels that can be implemented by an optimal contract is determined by*

$$q \geq \begin{cases} \frac{k_0}{c} & \text{if} \quad k_0 \geq 0 \\ \underline{q}(k_0) & \text{if} \quad k_0 \leq 0 \end{cases} \tag{3.10}$$

where $\underline{q}(k_0) \in [0, \bar{q}]$ *is the smallest nonnegative solution (if it exists) to*

$$\int_{\underline{q}}^{a+\varepsilon} \underline{q}(\theta - \underline{q}) \frac{1}{2\varepsilon}\, d\theta - c\underline{q} \geq -k_0. \tag{3.11}$$

If $k_0 < 0$ *and there is no* $\underline{q} \in [0, \bar{q}]$ *which satisfies (3.11), then the project cannot be financed for any output level.*

Proof:

From Lemma 3.2 follows that E will produce $q = (k_0 + k_1)/c$ as long as this is less than $\bar{q}$. If $k_0 \geq 0$, the first part follows trivially. For $k_0 < 0$, we have to consider the project's net returns, and compare these with the cost of repaying (rolling over) the 'inherited debt'. Since the net returns that a project generates are increasing in q if $q \in [0, \bar{q}]$, the *smallest* implementable q is found by assuming that *all* earnings are handed over, and solving (3.11) for q with equality. If the smallest positive solution is higher than $\bar{q}$, financing is not feasible, since E cannot credibly commit to both invest some $q > \bar{q}$ *and* repaying. ■

(Note that none of the parties will *want* to implement some $q < a - \varepsilon$, as with $q \leq a - \varepsilon$ all borrowed funds constitute riskless debt. However, implementing some $q < a - \varepsilon$ is *possible*, by setting $k_0 + k_1 < c(a - \varepsilon)$.)

We denote the highest level of inherited debt with $\underline{k_0}$, i.e. $\underline{k_0}$ is the most negative level of retained earnings k_0 which can still be refinanced. If $k_0 = \underline{k_0}$, then all earnings are handed over to the Investor, whatever the state of the world (θ) is. Formally, $D = \bar{q}(a + \varepsilon - \bar{q})$, i.e. E promises to repay an amount that is equal to his earnings in the *best* state of the world. By definition, we must have $\theta_1 = a + \varepsilon$, and therefore the probability that E is *not* bankrupt (that he can repay his loan) is zero. Furthermore, it must be the case that the output is $\bar{q}$: smaller outputs reduce the expected earnings of the project, and therefore violate the Investor's participation constraint; higher outputs than $\bar{q}$ cannot be implemented because E would choose $\bar{q}$ at the output–choice stage.

Two further constraints (additionally to Lemma 3.4) have been derived in Section 3.3. We restate them here:

Lemma 3.5 *No output $q > \bar{q}$ can be implemented.*

Lemma 3.6 *Incentive compatibility requires that $D \leq \pi_2$.*

Both constraints in Lemmata 3.5 and 3.6 are imposed on E's maximisation program as boundary conditions, i.e. if the optimal choice of q for some k_0 lies outside the boundary, E will have to pick some corner solution (we have discussed the constraint $D \leq \pi_2$ in more detail after Proposition 3.1).

We noted after Lemma 3.2, that the debt level has an *indirect* effect on E's output choice, as the latter is constrained by the size of the available funds ($k_0 + k_1$). This implies that when E chooses the amount that he borrows, he simultaneously chooses the quantity that he will produce. For simplicity, we assume in the following that E chooses an output q that he wants to produce, and thereby determines the amount that he borrows by setting $k_1 = cq - k_0$.

Proposition 3.3 *If E is financially constrained, i.e. if $k_0 < c \cdot \bar{q}$, he chooses to borrow less than he would need to produce the deep pocket output $\bar{q}$, and therefore his output will be strictly smaller than $\bar{q}$.*

Proof:
See the Appendix. ■
(There is one exception to Proposition 3.3, to which we will come back when discussing Proposition 3.4 below).

The reason for this underinvestment is that external finance is strictly more costly than internal funds:

Corollary 3.1 *The marginal costs of expanding output are strictly higher if the funds for such an expansion are borrowed.*

Proof:
We want to show that $(\partial D/\partial q) > c$, as the former is the marginal cost of expanding output if funds are borrowed (see Appendix B). Using $D := q(\theta_1 - q)$,

$$\frac{\partial D}{\partial q} = -\frac{\int_q^{\theta_1}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta - c}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)}, \tag{3.12}$$

which is larger than c if

$$\int_q^{\theta_1}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta < 0. \tag{3.13}$$

From the Proof of Proposition 3.3 we know that this is the case in equilibrium.
■

The marginal costs of increasing output are higher with debt finance because of the deadweight loss that arises if the project has to be liquidated. The informational problems require that with the optimal contract the size of the 'payment' that E has to make is identical for all states of the world θ. If θ is low, some of this 'payment' will be made in liquidation probability: if E cannot repay D with his earnings, the contract fills in the shortfall with $(1 - \beta(\theta))\pi_2$, and E 'pays in pain units', instead of cash. Since the lender breaks even in expected terms with the optimal contract, the increase of the marginal costs is uniquely due to the deadweight loss.

Corollary 3.2 *The Entrepreneur (E) is not credit constrained. The Investor would be willing to provide more funds to increase his output, but E* chooses *to limit his borrowing. In particular, for intermediate levels of retained earnings, E chooses not borrow* at all: *There is a* $\widetilde{k_0} \in (0, c \cdot \bar{q})$ *such that for all* $k_0 \in [\widetilde{k_0}, c \cdot \bar{q}]$, *E produces* $q = (k_0/c)$.

Proof:
Follows from Lemmata 3.2 and 3.4 (which determine the set of feasible contracts), and the strictness of the result in Proposition 3.3. If there is a feasible contract with $q < \bar{q}$, then increasing q also increases the net total returns from the project. Therefore a higher output $\tilde{q} \in (q, \bar{q}]$ must be feasible, as well. ■

The reason for the lenders' willingness to provide funds is that E would really invest the additional funds (see Lemma 3.2), and would therefore generate sufficient earnings to repay a higher level of D. E does not take on more funds, because borrowing makes output expansions more expensive: it increases the marginal costs of production, as was shown in Corollary 3.2. This can be seen most strikingly if the retained earnings are 'almost' sufficient to produce the output $\bar{q}$. Here the firm is not *sufficiently constrained*, and it prefers not to borrow *at all*.

After having established this underinvestment result, we can study E's contract/output choice in more detail. In Proposition 3.4 we show that the output is not monotonic in E's financial constraints:

Proposition 3.4 *If E has small retained earnings or little inherited debt, his output is decreasing in his financial constraints, i.e. increasing in retained earnings and decreasing in inherited debt. If E has large retained debt, however, his output is increasing in the level of inherited debt. More precisely, there is a* $\widehat{k_0} \in (\underline{k_0}, 0)$ *such that for all* $k_0 \in (\widehat{k_0}, c \cdot \bar{q}]$, *output* q *is increasing in* k_0, *while for all* $k_0 \in [\underline{k_0}, \widehat{k_0})$, *output* q *is decreasing in* k_0.

Proof: See Appendix B. ■

The first part of this result is very intuitive. Financial constraints are a handicap for the firm, because of the increase of the costs of borrowing which was described in Corollary 3.1. (in terms of the payoff π_2 which is lost in the event of bankruptcy).

For strongly negative values of k_0, E is extremely constrained by the lender's participation constraint. He would like to borrow little (to decrease the probability of bankruptcy), but he must produce large outputs by borrowing more, as only by doing so he can repay enough to finance both the inherited debt *and* the production costs. This effect drives up E's quantity choice as k_0 becomes more and more negative: increasing output actually *decreases* the costs of capital, and the tradeoff between high current earnings vs. high continuation probability is not a tradeoff anymore: both goals are achieved by increasing q (to some extent). The same effect determines the lower bound $\underline{q}$ to the set of feasible outputs if k_0 is negative. The minimal output is increasing in inherited debt (i.e. decreasing in k_0 if $k_0 < 0$), as is described in Lemma 3.4.

As before, Proposition 3.1 applies, and the analysis can become more difficult is π_2 is small. Suppose that k_0 is strongly negative, i.e. close to $\underline{k_0}$. The lender's participation constraint determines the minimal output that must be produced, such that a lending contract is feasible at all. It also determines the promised repayment D. Now, with $D > \pi_2$, a financial contract is not viable. The highest level of initial debt that can be refinanced is thus bounded not only by the lender's participation constraint, but also by the requirement that it be satisfied if $D = \pi_2$.

We can summarise Lemmata 3.4 and 3.5 and Propositions 3.3 and 3.4 in Figure 3.2. The area inside the dotted lines is the set of outputs that can be induced by some optimal contract. For positive levels of k_0 it is constrained by the loan being *needed*, i.e. by the requirement that $c \cdot q > k_0$, as otherwise E could rely on his retained earnings, only. If $k_0 < 0$, the smallest output that can be sustained is $\underline{q}(k_0)$, as defined in Lemma 3.4. In any case, no output $q > \bar{q}$ can be induced, as was shown in Lemma 3.5, as at the output choice stage E would never choose an output larger than $\bar{q}$.

For high values of k_0, the firm is not financially constrained, and it chooses

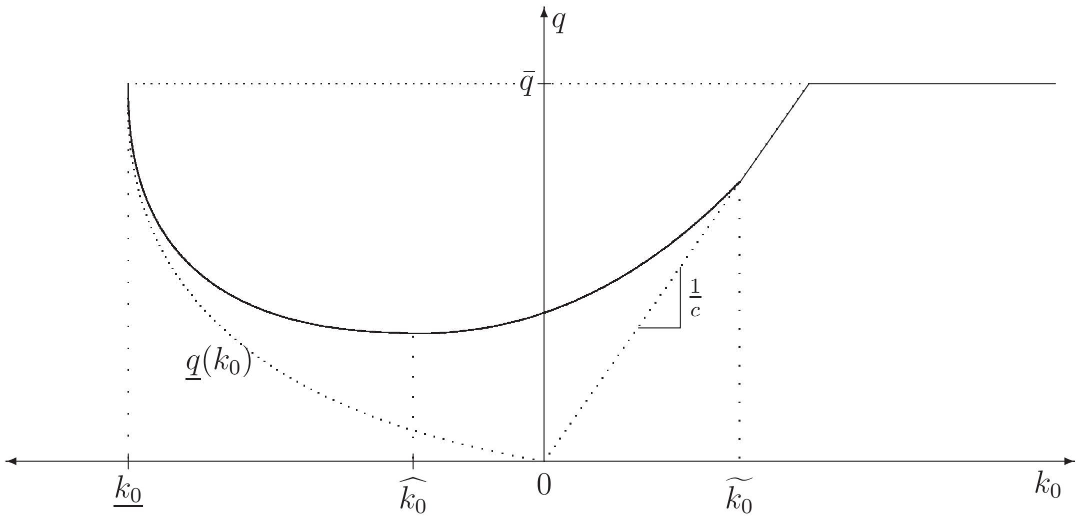


Figure 3.2: *Output as a function of retained earnings / inherited debt*

to produce $\bar{q}$. For lower retained earnings, the firm is financially constrained, but nevertheless does not want to borrow to increase its output. Instead, it relies on its retained earnings to produce some output (k_0/c). A firm with small retained earnings finds it advantageous to borrow, as in the tradeoff between survival and current earnings the current earnings become more attractive. Caution, however, leads these firms to limit the extent of their borrowing. In particular, the produced quantities decrease, as k_0 decreases. The same holds if k_0 is *negative* — we interpret this as 'inherited debt', which must be paid off if the firm is to be continued. Increasing levels of initial debt should make the firm more and more conservative, i.e. it wants to reduce its output. At the same time, however, the lender's participation constraint must be satisfied: the quantity that the borrower would want to produce cannot generate sufficient earnings to cover the costs of *both* rolling over the initial debt *and* the production costs. In this range, a continuation of the firm is only feasible if it borrows *more* than it would like, i.e. firm must become 'suicidal' to survive.

3.5 The Duopoly Case

Assume now that there are two firms 1 and 2 which produce homogeneous goods at output levels q^1 and q^2, respectively. Assume that the (stochastic) demand function is now

$$p(q^1, q^2) = \max\{0, \theta - q^1 - q^2\}. \tag{3.14}$$

Let us first consider the case in which both firms have unbounded internal funds. For notational ease we consider Firm 2's maximisation problem:

$$\max_{q^2} \int_{q^1+q^2}^{a+\varepsilon} q^2(\theta - q^1 - q^2)\frac{1}{2\varepsilon}\, d\theta - c \cdot q^2 + k_0^2 + \pi_2. \tag{3.15}$$

Its first order condition is

$$\int_{q^1+q^2}^{a+\varepsilon} (\theta - q^1 - 2q^2)\frac{1}{2\varepsilon}\, d\theta - c \ =!\ 0. \tag{3.16}$$

We can derive the reaction curve,

$$q^2(q^1) = \frac{2}{3}(a + \varepsilon - q^1) - \frac{1}{3}\sqrt{(a + \varepsilon - q^1)^2 + 12\varepsilon c} \tag{3.17}$$

and find that the problem is well behaved: the reaction curves are negatively sloped, and they are 'flat': $|\partial q^2/\partial q^1| < 1$, which can be found differentiating either (3.17) or the integral of (3.16), and noting that $a + \varepsilon - q^1 - q^2 > 0$:

$$\frac{\partial q^2}{\partial q^1} = -\frac{2}{3} + \frac{a + \varepsilon - q^1}{3\sqrt{(a + \varepsilon - q^1)^2 + 12\varepsilon c}} = -\frac{a + \varepsilon - q^1 - 2q^2}{2a + 2\varepsilon - 2q^1 - 3q^2} \tag{3.18}$$

(the slope must therefore lie between $-\frac{2}{3}$ and $-\frac{1}{3}$). The symmetric equilibrium output is

$$q^* := \frac{3}{8}(a + \varepsilon) - \frac{1}{8}\sqrt{(a + \varepsilon)^2 + 32c\varepsilon}. \tag{3.19}$$

By looking at the reaction curves we could repeat the analysis of Sections 3.3 and 3.4, as the rival's output is taken as constant in the above analysis. We assume that Firm 1 is financially constrained, while Firm 2 is not. Thus, k_0 and k_1 without superscripts refer to Firm 1 from here on.

As in the case of a monopolistic firm, we solve the game by backwards induction, starting with the last stage, in which the Firm 1 has to reveal its earnings, back to the first stage, in which it offers a financial contract to an Investor.

One can easily verify that the following holds:

Proposition 3.5 *With one financially constrained and one unconstrained firm, the equilibrium financial contract has the structure described in Section 3.3 and Proposition 3.2.*

The contract is unchanged because its structure was derived from incentive constraints *after* some output has been chosen. This led to the structure described in Proposition 3.2, and other incentive problems may, but need not, add structure to the contract.

As in the monopoly case, one can easily show that the amount of borrowing only determines the output that Firm 1 *can* choose, but not which output it may *want* to choose (cf. Lemma 3.2):

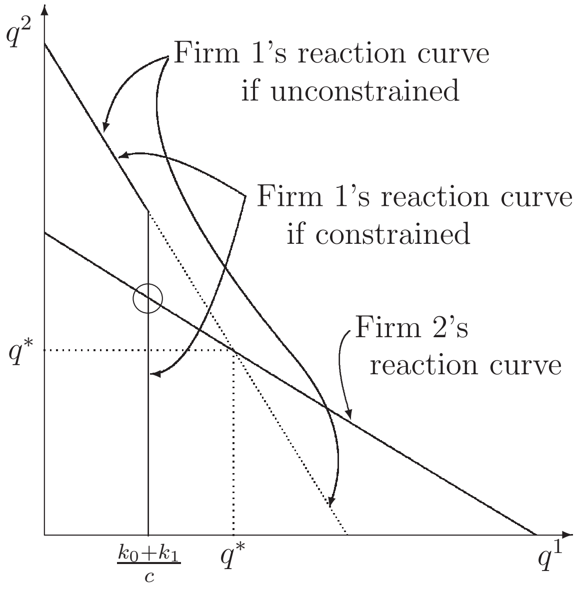
Proposition 3.6 *Assume that Firm 1 has signed a financial contract as described in Proposition 3.2. Then its output choice is uniquely determined by the amount k_1 that it borrows:*

$$\text{if} \quad k_0 + k_1 < c \cdot q^*, \quad \text{Firm 1 produces } q^1 = \tfrac{k_0+k_1}{c} < q^*$$

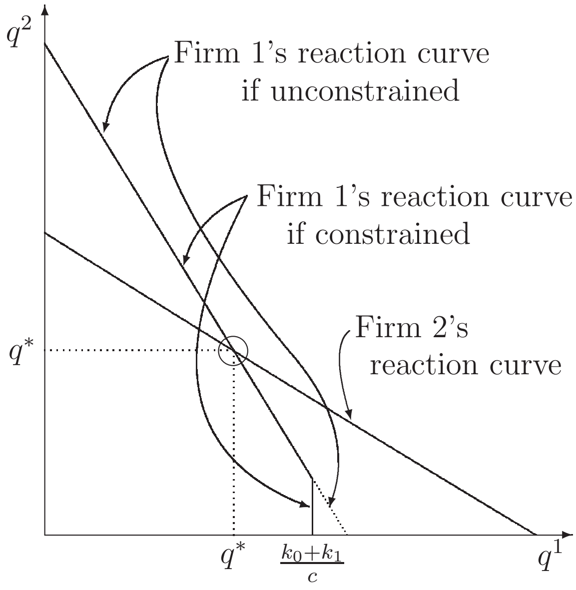
$$\text{while if} \quad k_0 + k_1 \geq c \cdot q^*, \quad \text{it produces exactly } q^*.$$

In other words, a financially constrained Firm 1 that borrows 'too much' is not constrained at the output choice stage, and an output higher than q^* (the Cournot equilibrium output) cannot be a Nash equilibrium for this subgame. On the other hand, if it remains constrained, it is committed to produce as much as possible. Formally, Firm 1's reaction curve at the output choice stage is kinked downwards (to become vertical) at $q^1 = (k_0+k_1)/c$, while Firm 2's reaction curve is unchanged (cf. Figure 3.3). If the kink lies to the left of $q^1 = q^*$, Firm 1 is committed to spend everything on production, and the equilibrium output choice (for this subgame) is the intersection of the (kinked for Firm 1) reaction curves. If the kink lies to the right of $q^1 = q^*$, however, Firm 1 cannot commit to spend everything, and the equilibrium output choices (for this subgame) will be $q^1 = q^2 = q^*$.

The analysis of a firm's borrowing and output decisions is more complicated in the duopoly setup, because the timing of the model gives it a possibility to (sometimes) *precommit* to some output q^1. From the analysis above we know that if Firm 1 borrows less than it would need to produce the deep–pocket output, at the output–setting stage it will spend all cash holdings on producing output. We have implicitly assumed that financial contracts are observable. Firm 2 therefore can only react to Firm 1's output choice, which was made *earlier*, at the contracting stage.



(a) *Firm 1 borrowed 'little'*



(b) *Firm 1 borrowed 'much'*

Figure 3.3: *Reaction curves at the output choice stage (equilibrium circled).*

Thus, as long as its total funds at the output choice stage do not exceed $c \cdot q^*$, Firm 1 can choose any output combination (q^1, q^2) on Firm 2's reaction curve, by borrowing just enough to produce q^1. Firm 1 has a 'first–mover advantage' in this game, as it can commit to some output level before Firm 2 can decide.

However, this 'first–mover advantage' is of limited value for Firm 1. Where it would be interesting, the precommitment possibility does not exist: Firm 1 cannot commit to produce some output that is larger than the Cournot equilibrium output, and enjoy the benefits of being a Stackelberg–leader: if $k_0 + k_1 > c \cdot q^*$, the kink in the reaction curve of Firm 1 has no effect on the *intersection* of the two reaction curves at the quantity–setting stage (cf. Figure 3.3(b)).

Firm 1's (limited) precommitment possibility complicates the analysis of its borrowing decision, as it has to take into account a feedback over Firm 2's reaction curve. Technically speaking, Firm 2 must substitute Firm 1's reaction curve (3.17) for q^2 in its maximisation program,

$$\max_{q^1} \int_{q^1+q^2(q^1)}^{a+\varepsilon} q^1(\theta - q^1 - q^2(q^1)) \frac{1}{2\varepsilon}\, d\theta - q^1\Big(\theta_1(q^1, q^2(q^1)) - q^1 - q^2(q^1)\Big) + \pi_2 \quad (3.20)$$

s.th.

$$\int_{q^1+q^2(q^1)}^{\theta_1(q^1,q^2(q^1))} q^1(\theta - q^1 - q^2(q^1)) \frac{1}{2\varepsilon}\, d\theta \quad (3.21)$$

$$+\int_{\theta_1(q^1,q^2(q^1))}^{a+\varepsilon} q^1\Big(\theta_1(q^1,q^2(q^1)) - q^1 - q^2(q^1)\Big)\frac{1}{2\varepsilon}\,d\theta - cq^1 + k_0^1 = 0,$$

which complicates its first order condition at the contract choice stage.

Lemma 3.7 *The (limited) precommitment possibility of Firm 1 increases its incentives to borrow, and therefore its output.*

Proof:
This can easily be shown by differentiating (3.20), and collecting the terms with $\partial q^2/\partial q^1$. The sum of the terms with $\partial q^2/\partial q^1$ is negative, so is $\partial q^2/\partial q^1$, and therefore the first order condition becomes more positive. ■

However, the model still allows for clear results. A major simplification of the algebra is obtained by observing that Firm 1 is monopolist over the 'residual' demand function $p(q^1, q^2(q^1), \theta)$ (as long as $k_0 + k_1 \leq q^*$):

$$p(q^1, q^2(q^1), \theta) = \max\left\{ 0 \;,\; \theta - \frac{q^1}{3} - \frac{2(a+\varepsilon)}{3} + \frac{\sqrt{(a+\varepsilon-q^1)^2 + 12\varepsilon c}}{3} \right\}. \tag{3.22}$$

This 'new' demand function is differentiable, downward sloping and convex. Most results from the monopoly case can actually be derived for 'general' return functions $R(q, \theta)$, which we will use in this Section. The only change of notation that is necessary is the introduction of a value of θ, for which the price is zero given output q^1:

$$\underline{\theta}(q^1) \;:=\; q^1 + q^2(q^1) \;=\; \frac{q^1}{3} + \frac{a+\varepsilon}{3} - \frac{\sqrt{(a+\varepsilon-q^1)^2 + 12\varepsilon c}}{3} \tag{3.23}$$

(note that $\underline{\theta}(q^*) = q^* + q^*$). We also make an assumption that brings some 'Cournot' elements into the earnings function $R(\cdot)$:

$$R_{q^1\theta}(q^1, \theta) \;:=\; \frac{\partial^2 R(q^1, \theta)}{\partial q^1 \partial \theta} \;>\; 0, \tag{3.24}$$

where the lower indices refer to partial derivatives. As in the case of a monopolistic firm, we can show that the set of feasible contracts can be characterised by two parameters, the retained earnings (or inherited debt) k_0, and the output of Firm 1, q^1. The set of feasible contracts is constrained by a smallest level of output $\underline{q^1}$ if $k_0 < 0$ (defined as in Lemma 3.4), $q^1 \geq (k_0/c)$ and $q^1 \leq q^*$ (cf.

Lemma 3.4 and Proposition 3.6, and Figure 3.2). We can also define the highest level of inherited debt ($\underline{k_0} < 0$ that can be refinanced, which leads to the output q^* and promised repayment $q^*(a + \varepsilon - 2q^*)$ (Firm 1 will partially default with probability 1). Finally, the financial contract must satisfy $D \leq \pi_2$.

We can derive an underinvestment result similar to Proposition 3.3:

Proposition 3.7 *If Firm 1 is financially constrained, i.e. if $k_0 < c \cdot q^*$, it chooses to borrow less than it would need to produce the deep pocket output q^*, and therefore its output will be strictly smaller than q^*.*

Proof: See Appendix C. ■

The reason for this underinvestment is that the costs of expanding output are strictly higher with debt–financed production (cf. Corollary 3.1): while a deep–pocket firm's marginal costs of expanding output are c, for the financially constrained firm they are

$$\frac{\partial D(q^1, k_0)}{\partial q^1} = -\frac{\int_{\underline{\theta}}^{\theta_1} R_{q^1}(q^1, \theta)\frac{1}{2\varepsilon}\, d\theta - c}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)}, \tag{3.25}$$

which is strictly larger than c. As in the case of a monopolistic firm (cf. Corollary 3.2), a 'slightly' financially constrained firm will not borrow, and instead produce using its retained earnings, only:

Corollary 3.3 *Firm 1 is not credit constrained. The lender would be willing to provide more funds to increase its output, but Firm 1* chooses *to limit its borrowing. In particular, for intermediate levels of retained earnings, it chooses not borrow* at all: *There is a $\widetilde{k_0} \in (0, c \cdot q^*)$ such that for all $k_0 \in [\widetilde{k_0}, c \cdot q^*]$, Firm 1 produces $q^1 = (k_0/c)$.*

This result points at the need to be precise when talking about credit constraints. Obviously, Firm 1 is financially constrained, as it produces less than it would with larger retained earnings. At the same time, it does not borrow as much as it could: for any $k_0 \geq \underline{k_0}$, a lender would be willing to lend enough to both roll over k_0 and produce q^*.

The only result that cannot be easily reproduced in the duopoly setting is the u–shaped nonmonotonicity of output as a function of retained earnings (cf.

figure 3.2 and Proposition 3.4). However, the slope of $q^1(k_0)$ can be determined at the extremes of the nonmonotonicity. At one extreme, Firm 1 decides to default (partially) with probability 1 (at $k_0 = \underline{k_0}$, with $q^1 = q^*$ and $\theta_1 = a + \varepsilon$) because of its high inherited debt. Here the output is increasing in inherited debt (decreasing in k_0), because in order to be able to generate sufficient income to repay both the production costs and the rolled over debt, output must be high. At the other extreme, Firm 1 is just indifferent between borrowing an infinitesimal amount and not borrowing: at $\widetilde{k_0}$, Firm 1's production costs equal its retained earnings, i.e. $c \cdot q^1 = \widetilde{k_0}$, and the slope of $q^1(k_0)$ is positive. Together with the underinvestment result (Proposition 3.7) this allows to conclude:

Proposition 3.8 *A firm with intermediate financial constraints produces less than a firm with extremely small or large financial constraints.*

Proof: See Appendix C. ■

3.6 Implications

It is important to note that the underinvestment result is driven by the assumption of *costly expansion*, i.e. that production costs have to be paid before the products can be sold. Assume instead that that the variable production costs c are zero, and that instead the firms have to finance some fixed costs K. We can apply our analysis as before. The optimal contract will again be a contract as described in Proposition 3.2, consisting of a fixed promised repayment D and a continuation probability function β. The difference is that at the output choice stage Firm 1 is not constrained by the funds that it borrowed, as the expansion costs are zero. Therefore it will always choose the output that a financially unconstrained firm would choose. One can easily show:

Proposition 3.9 *Suppose that $c = 0$, and that instead Firm 1 must borrow to finance fixed costs K. Then current period debt has no effect on the firm's output decision. Furthermore, financial constraints have only a zero–one effect on the firm's financing decision: financing is either available or not, but it does not affect the firm's output decision otherwise.*

Thus, as long as $K - k_0$ is not too large, Firm 1 can find a lender who is willing to provide a loan. No matter how it obtains the money, the firm will always produce the same output that it would choose if it was not financially constrained.[9] The upper bound to K is

$$\overline{K} = \int_{q^*_{(c=0)}}^{a+\varepsilon} q^*_{(c=0)}(\theta - 2q^*_{(c=0)})\frac{1}{2\varepsilon}\,d\theta + k_0. \tag{3.26}$$

Note that the underinvestment result with costly expansions is not driven by the lender's worries over risk shifting. Even though she can constrain Firm 1's output choice by reducing the size of the loan, she does not want to do so. She would be willing to lend a larger amount than she does in equilibrium (cf. Corollary 3.3), and it is Firm 1 that decides to borrow less (cf. Proposition 3.7).

How do our results compare to those in Brander and Lewis (1986)? From Proposition 3.6 we can derive:

Corollary 3.4 *Firm 1's* desired *output does not depend on its financial situation.*

However, with positive variable costs its *actual* output choice is limited by the available funds, $k_0 + k_1$. This led to the underinvestment result: while at the quantity setting stage Firm 1 does not take into consideration the effects of his decisions on I's payoff, when offering a contract it has to internalise these effects. On the other hand, with zero variable costs Firm 1 can choose the desired output level. This implies that the *overinvestment* result in Brander and Lewis (1986) is driven by the exogeneity of the contractual structure. In their model debt changes the borrower's incentives at the quantity choice stage, which does not happen here. Furthermore, if one would introduce production costs which have to be paid before output can be sold, one should expect that if these costs are sufficiently high, the overinvestment result vanishes. In model with debt as an *optimal* contract and zero variable costs the overinvestment effect is exactly offset, while if the variable costs are positive, it is more than offset.

[9] This result contrasts with that in Faure–Grimaud (1997), who analyses a similar model, with the difference that output is observable but not verifiable. He derives an underinvestment result, which seems to be driven by his implicit assumption that the financial and output decisions are simultaneous (when choosing an output level, his firms take the lenders' participation constraints into consideration).

Brander and Lewis (1986, 1988) also have results on the *slope* of output as a function of debt, which, if positive, could be seen as a sign of a firm's increased "agressiveness". We obtained this type of result in Proposition 3.8: if a firm's *initial* debt is sufficiently high, then its output is increasing in this debt level.

In Brander and Lewis (1986), output is increasing in current–period debt, not in *inherited* debt, which we regard as a better measure of financial constraints. In Brander and Lewis (1988), bankruptcy costs are added to the 1986 model[10]. The result was that with exogenously given debt obligations, a firm's output choice is increasing in the debt level, and it is *smaller* than the no–debt output if the debt level is small, and *larger* otherwise.

This result is not the same as ours, however. Our result is driven by the need to pre–finance the output costs *and* possibly some inherited debt ($k_0 < 0$). If the latter is high, the project is only viable if a high output level is chosen, as low outputs cannot generate sufficiently high expected earnings to let the lender break even. The "limited-liability effect" in Brander and Lewis (1986,1988) is a risk shifting result: the borrower's payoff as a function of the state of the world becomes a convex function, which gives him an incentive to choose a more risky strategy — in a Cournot context this would mean a more *aggressive* strategy. Under a standard debt contract, a firm is only interested in the payoff it receives in the survival states. These are the high–demand states, which are associated with high marginal profits related to output. Hence, as debt increases, output increases. Under an optimal debt contract, however, a firm cares not only about the states in which it survives for sure (i.e. where $\beta = 1$), but also about the probability of continuation when it cannot repay its loan in full. It turns out that the gain from increasing quantity in the survival states identified by Brander and Lewis is exactly offset by the loss due to a decrease in the probability of continuation in the bankruptcy states, and without expansion costs the output decision is unchanged.

[10] The model is different, but can be reformulated to be more similar to the 1986 model: their notion of maximizing the value of the firm is equivalent to maximising the value of equity while making sure that the debtholders break even.

If we combine the definition of q^* (cf. (3.19)) with the underinvestment result (Proposition 3.7), we can conclude:

Corollary 3.5 *There is no scope for* strategic *debt, as opposed to debt which is incurred to finance some existing costs. A firm will not borrow and pay out the loan to its shareholders, as it cannot derive a benefit from doing so.*

In Brander and Lewis (1986) the advantage of debt is that it introduces a commitment to be agressive. While this has no advantage in the case of a monopolistic firm, in a duopolistic Cournot setup such a commitment is valuable, as it may force the opponent to reduce its output to that of a Stackelberg–follower, thereby increasing the indebted firm's profit.

In our model, however, this commitment is not available in this form. Debt has no effect on a firm's desired output level, as we had shown in Corollary 3.4. Additionally, it may even be connected with reduced 'agressiveness', if the firm decided to borrow to finance a costly output expansion. The first–mover advantage modeled in Brander and Lewis (1986) exists, but only where Firm 1 is handicapped and forced to produce *less* than a financially unconstrained firm.

We can analyse the effect of Firm 1's retained earnings or inherited debt on the value of the firms. We consider the market capitalisation, i.e. the sum of debt and share value. Since the value of debt is exactly zero in our model, this is equivalent to looking at the share value, only.

Proposition 3.10 *The value of Firm 1 increases in its retained earnings ($k_0 > 0$) and decreases in its inherited debt ($k_0 < 0$). The effect on the value of its financially unconstrained rival Firm 2 is unclear: it may either increase or decrease in k_0.*

Proof: See Appendix C. ■

This result explains the result in Corollary 3.5, that a firm will not issue 'strategic debt'. If its value were *decreasing* in k_0 over some interval $[k_0^a, k_0^b]$, it would pay for Firm 1 to issue some strategic debt, if it had retained earnings k_0^b. It could borrow $k_0^b - k_0^a$ and distribute it to its shareholders, and only then apply for a loan to refinance k_0^a and cover its production costs $c \cdot q^1(k_0^a)$.

The nonmonotonicity of q^1 in k_0 explains the second part of the result: the value of Firm 2 must be decreasing in Firm 1's output, and therefore it varies with q^1. Therefore, it will also be difficult to derive results on the aggregate market capitalisation of an industry as a function of the firms' financial constraints.

The result that output is nonmonotonic in k_0, and that this function may be u–shaped, could have several interesting implications, which we plan to analyse at a later stage.

For instance, if two firms have to pay off significant levels of initial debt, before they can produce, then we might observe that the more financially constrained firm is more aggressive than its competitor. This is a situation in which a financially constrained firm would like to be cautious and produce little, but its financial situation forces it to become aggressive. This could be a factor working against debt–financed predation. Suppose that two firms are financially constrained to different degrees: the 'stronger' firm may forgo a possibility to 'hurt' its competitor, as an attempt to do so could easily backfire (a similar effect has been modeled by Glazer (1994)).

On the other hand, it could be that a more general model could explain predatory behaviour. Consider two firms, with Firm 1 being "almost" financially constrained, while Firm 2 is unconstrained. Then it may pay for Firm 2 to increase its output in that period, such that the price is low (as in Bolton and Scharfstein (1990)). With minimal retained earnings in the next period, Firm 1 is forced to borrow, and therefore reduce its output below q^*. Firm 1's financial constraints may last for several periods, not only due to bad luck, but also because a high probability of bankruptcy in the future decreases the value of future production, which we had modeled as π_2. A small value of π_2 reduces D, the amount that Firm 1 can promise to repay, and therefore Firm 1's borrowing capability. Thus, by hurting an opponent, a financially unconstrained firm may force it to remain a financially constrained *small* firm for a while, shifting the output structure towards something that to an outsider looks like a Stackelberg–leader/follower situation.[11]

[11] This would be the continuous analogy to 'predation' in the discrete Bolton and Scharfstein (1990) model.

The result that output can be increasing in inherited debt (if the latter is sufficiently high) might explain some 'outliers' which are reported in the empirical literature. While most other empirical studies conclude that debt weakens a firm's competitive position, Borenstein and Rose (1995) find that almost bankrupt (i.e. highly leveraged) U.S. airlines became aggressive, compared with their less leveraged competitors, after the industry was deregulated. Our model contains elements of both: financial constraints make a firm weaker, but if the constraints become strong, they force a firm to become aggressive.

Another implication concerns the use of financial models in macroeconomics. Gale (1983) and Bernanke and Gertler (1989) have shown that capital market imperfections can amplify business cycles if a borrower's credit line depends positively on his own funds (cf. also Gale and Hellwig (1985)). These models rely on output being monotonically decreasing in a firm's financial constraints. While we obtain this result for firms with positive retained earnings and small inherited debt, we obtain the contrary in the case of large inherited debt.

We can derive some additional empirical implications of the model.

Proposition 3.11 *If one firm is financially constrained, and the other is not, the total industry output is (almost always) strictly less than what financially unconstrained firms would produce.*

Proof: This is an immediate consequence of the underinvestment result (see Proposition 3.7) and of the 'flatness' of the financially uncostrained rival's reaction curve (cf. (3.18)). For any output q_2, the best response of a financially constrained Firm 1 is smaller than if it were unconstrained, and while as a consequence Firm 2 produces more than q^* in equilibrium, this increase is smaller than the decrease of Firm 1's output. The qualifier "almost always" in Proposition 3.11 refers to the case where both firms have the highest possible levels of inherited debt, i.e. $k_0 = \underline{k_0}$. ■

The result in Proposition 3.11 should also hold if both firms are financially constrained. While it is difficult to determine precisely the shape of the reaction curves, we can conclude that the aggregate output is smaller than what a financially unconstrained duopoly would produce from the observation that a constrained firm's best response must lie *below* its unconstrained best response.

Coming back to the model with one constrained and one unconstrained firm, we can conclude the following:

Corollary 3.6 *Suppose that Firm 1 is financially constrained, while Firm 2 is unconstrained. If a parameter change leads to a change in* q^1*, the total output* $Q := q^1 + q^2$ *changes in the same direction as* q^1*.*

This follows from the flatness of the unconstrained firm's reaction curve: any change in q^1 is less than offset by a change of q^2. We can also immediately follow

Corollary 3.7 *Suppose that Firm 1 is financially constrained, while Firm 2 is unconstrained. If a parameter change leads to a change in* q^1*, the equilibrium price will change in the opposite direction.*

3.7 Conclusions

While recent empirical research by and large has found that financial constraints make firms "softer" in the product market, theoretical reasoning is driven by two apparently contradictory considerations: One the one hand, higher costs of external funds are expected to make a financially constrained firm softer. On the other hand, with existing debt, maximization of the equity value of a firm can lead to more risk-taking and aggressive behavior. This contrast cannot be resolved by pointing out the difference between ex-ante and ex-post incentives. In fact, the striking result of Brander and Lewis (1986) is that even if lenders fully anticipate a firm's ex-post behavior, debt can lead to an increase in the value of the firm because of strategic interaction in the output market. In this context, the contribution of our paper is threefold:

First, the optimal financial contract which we derive here differs from a standard debt contract in that a firm that cannot fully repay the required amount is not liquidated for sure. Rather, the probability of continuation depends on the amount it can repay. As a result, Brander and Lewis' limited-liability effect vanishes completely; i.e. debt has no strategic effect in the output market.

Second, in contrast to most of the current theoretical literature, we point out that both *fixed and variable costs* of production must be financed by the sum of available *internal and external funds*. This implies that if production must be financed before demand for the products is known, then, because of the neutrality result above, a firm effectively chooses its output level at the time of obtaining external funds. Hence, a firm is forced to internalise the costs of possible bankruptcy, and the resulting higher costs of debt-financing lead to a reduction of output, consistent with the empirical evidence.

Third, instead of distinguishing between "equity-financed" and "debt-financed" firms, we describe a firm's financial situation by its level of retained earnings. This seems more appropriate considering the fact that short-run debt is endogenously chosen. In particular, we show that even in the presence of financing constraints a firm may prefer not to use external financing. In addition, we can analyze how the level of short-run debt and output choice vary with the firm's retained earnings or existing debt, and find that this relationship is not monotonic.

Appendix A:

The Optimal Direct Revelation Mechanism

The Application of the Revelation Mechanism

The only strategic player in this enlarged game is E (I's only action is to accept or reject E's contract). E's strategy consists of firstly choosing an output $q \in \left[0, \frac{k_0+k_1}{c}\right]$. Secondly, E sends a message $m_I \in M_I(q)$ after his choice of q, where the set of feasible messages might depend on his choice of q. Thirdly, E sends a message $m_{II} \in M_{II}(q, m_I, \theta)$ after having observed θ. As before, the set of feasible messages might depend on past events. A strategy $s = (q, s_I(\cdot), s_{II}(\cdot))$ determines an action for the three decision, for each node that might be reached.

Denote with $\Gamma = (S, g)$ a mechanism that E could choose. It consists of a strategy space S for E and of an outcome function g, which determines both players' payoffs (U for E, V for I) as a function of the chosen actions and realised random variables. For example, g can determine monetary payoffs for both players and the probability of premature termination of the project, both depending on m_I and m_{II}. E's payoff depends on q and θ, as well, while I's payoff can depend on the messages, only.

Let $\Gamma^* = (S, g)$ be a mechanism that implements the outcome (s^*, U^*, V^*) as a subgame perfect equilibrium. Denote with s^* E's optimal strategy if he plays the mechanism Γ^*:

$$s^* = (q^*, s_I^*(q^*), s_{II}^*(q^*, s_I^*(q^*), \theta)). \tag{3.27}$$

As s^* is a subgame perfect equilibrium, the following inequalities must hold:

$$\mathcal{E}U(q^*, s_I^*(q^*), s_{II}^*(q^*, s_I^*(q^*), \theta), \theta) \geq \mathcal{E}U(q', m_I', m_{II}'(\theta), \theta) \quad \forall \textit{ feasible } q', m_I', m_{II}', \tag{3.28}$$

$$\mathcal{E}U(q^*, s_I^*(q^*), s_{II}^*(q^*, s_I^*(q^*), \theta), \theta) \geq \mathcal{E}U(q^*, m_I'', m_{II}''(\theta), \theta) \quad \forall \textit{ feasible } m_I'', m_{II}'', \tag{3.29}$$

$$U(q^*, s_I^*(q^*), s_{II}^*(q^*, s_I^*(q^*), \theta), \theta) \geq U(q^*, s_I^*(q^*), m_{II}'''(\theta), \theta) \quad \forall \textit{ feasible } m_{II}''', \ \forall \theta \in [a-\varepsilon, a+\varepsilon], \tag{3.30}$$

where $\mathcal{E}$ is the expectation operator with respect to θ. These inequalities must hold in particular if the messages $m'_I, m'_{II}, m''_I, m''_{II}$ and m'''_{II} are equilibrium strategies for off equilibrium histories, i.e. histories other than $(q^*, s^*_I(q^*), s^*_{II}(q^*, s^*_I(q^*), \theta))$:

$$\mathcal{E}U(q^*, s^*_I(q^*), s^*_{II}(q^*, s^*_I(q^*), \theta), \theta) \geq \mathcal{E}U(q', s^*_I(q'), s^*_{II}(q', s^*_I(q'), \theta), \theta) \tag{3.31}$$

$$\forall \textit{ feasible } q' \textit{ such that } s^*_I(q') \textit{ and } s^*_{II}(q', s^*_I(q'), \theta) \textit{ are feasible},$$

$$\mathcal{E}U(q^*, s^*_I(q^*), s^*_{II}(q^*, s^*_I(q^*), \theta), \theta) \geq \mathcal{E}U(q^*, s^*_I(q''), s^*_{II}(q^*, s^*_I(q''), \theta), \theta) \tag{3.32}$$

$$\forall \textit{ feasible } s^*_I(q'') \textit{ such that } s^*_{II}(q^*, s^*_I(q''), \theta) \textit{ is feasible},$$

$$U(q^*, s^*_I(q^*), s^*_{II}(q^*, s^*_I(q^*), \theta), \theta) \geq U(q^*, s^*_I(q^*), s^*_{II}(q^*, s^*_I(q^*), \theta'), \theta) \tag{3.33}$$

$$\forall \textit{ feasible } s^*_{II}(q^*, s^*_I(q^*), \theta'), \quad \forall \theta \in [a - \varepsilon, a + \varepsilon].$$

To apply the revelation principle, we have to show that there is a direct mechanism Γ^d, which implements the same outcome and payoffs as Γ^* as a truthtelling equilibrium. That is, in each stage E is asked to reveal his new private information and to choose certain actions. E should have no incentive to manipulate the mechanism by misrepresenting his private information, including manipulations of the mechanism that extend over several stages.

Define functions f_I and f_{II} to reproduce E's optimal strategy under the mechanism Γ^*, given the new information $\hat{q}$ and $\hat{\theta}$ that he reveals:

$$f_I(\hat{q}) = s^*_I(\hat{q}), \tag{3.34}$$

$$f_{II}(\hat{\theta}|\hat{q}) = s^*_{II}(\hat{q}, s^*_I(\hat{q}), \hat{\theta}). \tag{3.35}$$

Thus, instead of playing mechanism Γ^*, E could use a direct mechanism and a mediator, who offers to use the functions f_I and f_{II} to determine the payoffs. We can rewrite (3.31), (3.32) and (3.33):

$$\mathcal{E}U(q^*, f_I(q^*), f_{II}(\theta|q^*), \theta) \geq \mathcal{E}U(q', f_I(q'), f_{II}(\theta'|q'), \theta) \tag{3.36}$$

$$\forall \textit{ feasible } q' \textit{ such that } f_I(q') \textit{ and } f_{II}(\theta'|q') \textit{ are feasible},$$

$$\mathcal{E}U(q^*, f_I(q^*), f_{II}(\theta|q^*), \theta) \geq \mathcal{E}U(q^*, f_I(q''), f_{II}(\theta''|q''), \theta) \tag{3.37}$$
$$\forall \textit{ feasible } f_I^*(q'') \textit{ such that } f_{II}(\theta''|q'') \textit{ is feasible},$$

$$U(q^*, f_I(q^*), f_{II}(\theta|q^*), \theta) \geq U(q^*, f_I(q^*), f_{II}(\theta'|q^*), \theta) \tag{3.38}$$
$$\forall \textit{ feasible } f_{II}(\theta'|q^*), \ \ \forall \theta \in [a - \varepsilon, a + \varepsilon].$$

We can conclude from (3.36), (3.37) and (3.38) that truthtelling is an undominated strategy at all stages, and therefore that any payoff that can be achieved with some mechanism can also be achieved with the help of a mediator and a direct mechanism.

We have assumed in (3.37) that if E would decide to lie about his choice of q, there is always a feasible lie when he has to announce θ, such that

$$f_{II}(\theta''|q'') = s_{II}^*(q^*, s_I^*(q''), \theta) \tag{3.39}$$

where q and θ are the real values and q'' and θ'' the announcements. We will show below that this simplifying assumption has no side effects, as the revelation of q does not add any binding restrictions to the contract (the truthtelling constraints for the revelation of θ also lead to the truthful revelation of q).

If this were not true, the direct mechanism could be more restrictive than the corresponding indirect mechanism: only announcements of the type $s_{II}(q_1, s_I(q_2), \theta)$ with $q_1 = q_2$ are possible when E has to send the second message. In other words, the direct mechanism could offer less scope for manipulation than its corresponding indirect mechanism, and the set of implementable outcomes may seem larger than it actually is.

We now proceed to the construction of the optimal contract by backwards induction. We analyse the incentive constraints at each stage of the direct mechanism, starting with the last revelation stage, and assuming truthtelling/equilibrium behaviour at the earlier stages.

Proof: State–of–the–World Truthtelling Constraints

Lemma 3.8 *Assume that E has produced $\tilde{q} \leq (k_0+k_1)/c$ in stage I, and revealed it in stage II. Denote with θ the realised state of the world, and with $\hat{\theta}$ its* *announced value. Let* $\tilde{\delta} := k_0 + k_1 - c\tilde{q} \geq 0$ *be E's 'savings', i.e. the amount that was not spent if* $\tilde{q} < \frac{k_0+k_1}{c}$*. Then a direct mechanism* Γ^d *is optimal and provides incentives to announce the real* θ *if*

$$r(\hat{\theta}|\tilde{q}) = \min\left\{D(\tilde{q})\ ,\ \tilde{\delta} + \tilde{q}(\hat{\theta} - \tilde{q})\right\} \tag{3.40}$$

$$\beta(\hat{\theta}|\tilde{q}) = \min\left\{1\ ,\ 1 - \frac{D(\tilde{q})}{\pi_2} + \frac{\tilde{\delta} + \tilde{q}(\hat{\theta} - \tilde{q})}{\pi_2}\right\} \tag{3.41}$$

where β *is the probability that the firm can be continued (and E earns the private benefit* π_2*), and* r *is the payment that E has to make to I.* $D(\hat{q})$ *is independent of* $\hat{\theta}$*.*

Proof:
We simplify the notation by writing $r(\theta)$ and $\beta(\theta)$ instead of $r(\theta|\tilde{q})$ and $\beta(\theta|\tilde{q})$. Define $R(\theta|\tilde{q}) := \tilde{q}(\theta - \tilde{q}) + \tilde{\delta}$. As with r and β, we write $R(\theta)$ for simplicity.

Using k_0, k_1 and the (truthfully) announced $\tilde{q}$, the mediator knows the size of $\tilde{\delta}$. Thus, if E announces $\hat{\theta}$, the mediator believes that E's total cash holdings are $\tilde{q}(\hat{\theta} - \tilde{q}) + \tilde{\delta}$.

The truthtelling constraint for the announcement of θ requires that

$$\tilde{q}(\theta - \tilde{q}) + \tilde{\delta} - r(\theta) + \beta(\theta)\pi_2 \geq \tilde{q}(\theta - \tilde{q}) + \tilde{\delta} - r(\hat{\theta}) + \beta(\hat{\theta})\pi_2 \quad \forall \hat{\theta}, \theta, \tag{3.42}$$

i.e. that

$$-r(\theta) + \beta(\theta)\pi_2 \geq -r(\hat{\theta}) + \beta(\hat{\theta})\pi_2 \quad \forall \hat{\theta}, \theta. \tag{3.43}$$

The required payment $r(\theta)$ must be feasible, i.e.

$$r(\theta) \leq \tilde{q}(\theta - \tilde{q}) + \tilde{\delta} \quad \forall \theta \tag{3.44}$$

(this may constrain the firm's ability to misrepresent the state 'upwards', i.e. to announce a state $\hat{\theta} > \theta$, if θ is the real state).
Step 1 *For any* $\theta \in [a - \varepsilon, a + \varepsilon]$,

$$\beta(\theta) < 1 \quad \Rightarrow \quad r(\theta) = R(\theta), \tag{3.45}$$

$$r(\theta) < R(\theta) \quad \Rightarrow \quad \beta(\theta) = 1. \tag{3.46}$$

Suppose there is a θ such that both $\beta(\theta) < 1$ and $r(\theta) < R(\theta)$. Then E could increase both β and r such that his payoff is unchanged. I's expected payoff must have increased, and E has scope to improve his payoff (for instance, he could borrow a larger amount k_1).

Step 2 *For any two $\theta, \theta' \in [a-\varepsilon, a+\varepsilon]$,*

$$\beta(\theta) = \beta(\theta') \Leftrightarrow r(\theta) = r(\theta') \tag{3.47}$$

If this were not true, the truthtelling constraint (3.43) would be violated for either θ or θ'.

Step 3 *For any two $\theta, \theta' \in [a-\varepsilon, a+\varepsilon]$ such that $\beta(\theta) < 1$ and $\beta(\theta') < 1$,*

$$\theta < \theta' \Leftrightarrow \beta(\theta) < \beta(\theta'). \tag{3.48}$$

Notice first that $r(\theta) = R(\theta)$ and $r(\theta') = R(\theta')$ (this follows from Step 1). Suppose that $\beta(\theta) < \beta(\theta')$ but $\theta > \theta'$. Then $r(\theta) = R(\theta) > r(\theta') = R(\theta')$. But then E's truthtelling constraint (3.43) is violated if the state is θ. Next, consider $\theta < \theta'$. From this follows that $R(\theta) < R(\theta')$, and therefore that $r(\theta) < r(\theta')$. But then it must be the case that $\beta(\theta') > \beta(\theta)$, a s otherwise E's truthtelling constraint (3.43) is violated if the state is θ'.

Step 4 *For any two $\theta, \theta' \in [a-\varepsilon, a+\varepsilon]$ such that $\beta(\theta) < 1$ and $\beta(\theta') < 1$,*

$$(\beta(\theta) - \beta(\theta'))\pi_2 = r(\theta) - r(\theta'). \tag{3.49}$$

Assume (w.l.o.g.) that $\theta < \theta'$. From Steps 1 and 3 follows that $r(\theta) = R(\theta)$ and $r(\theta') = R(\theta')$ and that $\beta(\theta) < \beta(\theta')$. From the truthtelling constraint for any θ' with respect to any lower θ follows that

$$(\beta(\theta') - \beta(\theta))\pi_2 \geq r(\theta') - r(\theta). \tag{3.50}$$

If we divide both sides by $(\theta' - \theta)$ and take limits as $\theta \uparrow \theta'$, we derive

$$\frac{\partial \beta(\theta)}{\partial \theta} \cdot \pi_2 \geq \frac{\partial r(\theta)}{\partial \theta} \quad \forall \theta \text{ s. th. } \beta(\theta) < 1. \tag{3.51}$$

Suppose that in the optimal mechanism the inequality in (3.51) can be strict. Denote with $\tilde{\theta}$ the highest value of θ such that $\beta(\theta) < 1$ and the slope of β is strictly higher than (3.51). Define a function $\psi(\theta)$ such that

$$\psi(\tilde{\theta}) = \beta(\tilde{\theta}) \qquad \textit{and} \qquad \frac{\partial \psi(\theta)}{\partial \theta} \cdot \pi_2 = \frac{\partial r(\theta)}{\partial \theta} \quad \forall \theta. \tag{3.52}$$

Because of (3.50) and (3.51), β must lie below ψ. Then E could increase β for all $\theta < \tilde{\theta}$ up to ψ, without affecting any truthtelling constraint. This increases his payoff without changing I's payoff, and the original mechanism cannot have been optimal.

Step 5 *For any two* $\theta, \theta' \in [a - \varepsilon, a + \varepsilon]$, $\theta \neq \theta'$,

$$\beta(\theta) = \beta(\theta') \Rightarrow \beta(\theta) = \beta(\theta') = 1. \tag{3.53}$$

Suppose that $\beta(\theta) = \beta(\theta') < 1$. Then from Step 2, $r(\theta) = r(\theta')$, and from Step 1 $r(\theta) = R(\theta)$ and $r(\theta') = R(\theta')$, and therefore $R(\theta) = R(\theta')$, a contradiction.

From Steps 1–5 follows that the shapes of β and r are as described in Lemma 3.1: There is a θ_1 such that for all $\theta \in [\theta_1, a + \varepsilon]$, the repayment and the rescue probability are constant. For all $\theta \in [a - \varepsilon, \theta_1]$, the repayment includes the total cash holdings, and the rescue probability is increasing in the repayment. Denote with $D(\tilde{q})$ the amount that has to be repaid if $\theta > \theta_1$:

$$\beta(\theta_1) = 1 \qquad \textit{and} \qquad r(\theta_1) = D(\tilde{q}) := \tilde{q}(\theta_1 - \tilde{q}) + \tilde{\delta}. \tag{3.54}$$

Then we can calculate $\beta(\theta)$ for all $\theta < \theta_1$ (remind that (3.51) holds with equality):

$$\int_{\theta}^{\theta_1} \frac{\partial \beta(\tilde{\theta})}{\partial \tilde{\theta}} \cdot \pi_2 \, d\tilde{\theta} = \int_{\theta}^{\theta_1} \frac{\partial r(\tilde{\theta})}{\partial \tilde{\theta}} \, d\tilde{\theta}, \tag{3.55}$$

or,

$$(\beta(\theta_1) - \beta(\theta))\pi_2 = r(\theta_1) - r(\theta). \tag{3.56}$$

If we substitute $\beta(\theta_1) = 1$ and $r(\theta_1) = D(\tilde{q})$ using (3.54) and $r(\theta) = \tilde{q}(\theta - \tilde{q}) + \tilde{\delta}$, we can derive β as defined in Lemma 3.1. ■

Proof: Output–Choice Truthtelling Constraints

The truthtelling constraints for the last revelation stage require that the mechanism has the simple structure described in Lemma 3.8. We now analyse E's truthtelling constraints when he has to reveal his output choice, knowing that his announcement $\tilde{q}$ will determine the promised repayment $D(\tilde{q})$, i.e. the transfer $r(\theta|\tilde{q})$ and continuation probability $\beta(\theta|\tilde{q})$, as defined in Lemma 3.8.

Lemma 3.9 *Assume that E has produced output q, and denote his announcement with $\tilde{q}$. Assume also that the mechanism has the structure described in Lemma 3.8. Then E has no incentive to lie about his choice of q if $D(\tilde{q})$ is constant for all $\tilde{q}$.*

Proof:
Consider first the case in which E announces the true value q. Given the mechanism described in Lemma 3.1, the project is continued with probability one only if the earnings and savings are sufficient to repay $D(q)$, i.e. if $q(\theta - q) + \delta \geq D(q)$. In all other cases E hands over all money and the project is continued with probability β, as described in Lemma 3.1. E has an expected payoff of

$$\begin{aligned} \mathcal{E}U(\textit{announce } q|q) &= \int_{a-\varepsilon}^{q} \left(1 - \frac{D(q)}{\pi_2} + \frac{\delta}{\pi_2}\right) \pi_2 \frac{1}{2\varepsilon}\, d\theta \\ &+ \int_{q}^{\frac{D(q)-\delta}{q}+q} \left(1 - \frac{D(q)}{\pi_2} + \frac{q(\theta - q) + \delta}{\pi_2}\right) \pi_2 \frac{1}{2\varepsilon}\, d\theta \\ &+ \int_{\frac{D(q)-\delta}{q}+q}^{a+\varepsilon} \left(\pi_2 - D(q) + \delta + q(\theta - q)\right) \frac{1}{2\varepsilon}\, d\theta \end{aligned} \tag{3.57}$$

if he reveals the true value of q. The first integral contains payoffs for the cases in which $\theta \leq q$. In these cases the earnings are zero and the continuation probability is constant. The second integral contains payoffs if the earnings are positive but not sufficient to repay $D(q)$:

$$\theta \in \left(q, \frac{D(q) - \delta}{q} + q\right) \quad \Leftrightarrow \quad \delta \leq q(\theta - q) + \delta \leq D(q). \tag{3.58}$$

The third integral contains E's payoffs if the earnings are sufficient to repay $D(q)$, and therefore $\beta(\theta) = 1$. We can simplify (3.57):

$$\mathcal{E}U(\textit{announce } q|q) \;=\; \pi_2 - D(q) + \delta + \int_{q}^{a+\varepsilon} q(\theta - q)\frac{1}{2\varepsilon}\, d\theta. \tag{3.59}$$

Suppose now that E lies and announces $\tilde{q} \neq q$, instead. For low values of θ, E will have to lie and announce $\tilde{\theta} \neq \theta$,

$$\tilde{\theta} = \frac{1}{\tilde{q}}\left(q(\theta - q) + \delta - \tilde{\delta}\right) + \tilde{q} \quad \Leftrightarrow \quad q(\theta - q) + \delta = \tilde{q}(\theta - \tilde{q}) + \tilde{\delta}, \tag{3.60}$$

as his announcements will have to lead to the same repayment that he should make if he announced the correct values of q and θ. We ignore complications that arise if $\tilde{q} < q$ and $\theta < q$: in these cases E does not have enough cash to pay the transfer that follows any announcement (as $\delta < \tilde{\delta}$). Denote with \$ the repayment that E can make. We assume that if $\$ < \tilde{\delta}$,

$$\beta(\$) = 1 - \frac{D(\tilde{q})}{\pi_2} + \frac{\$}{\pi_2}, \tag{3.61}$$

as with this formulation the continuation probability is maximised, and no new incentive problems are introduced. Alternatively, we could also assume that E simply keeps his cash and the project is terminated with probability one. E's payoff would be strictly lower, however.

If $\theta \leq q$, E's earnings are zero. E will announce a wrong value $\tilde{\theta}$ and repay δ. The continuation probability is

$$\beta(\tilde{\theta}) = 1 - \frac{D(\tilde{q})}{\pi_2} + \frac{\delta}{\pi_2}, \tag{3.62}$$

and the part of his expected payoff for these cases is

$$\int_{a-\varepsilon}^{q} \left(1 - \frac{D(\tilde{q})}{\pi_2} + \frac{\delta}{\pi_2}\right) \pi_2 \, \frac{1}{2\varepsilon} \, d\theta. \tag{3.63}$$

If the earnings are positive but insufficient to repay $D(\tilde{q})$, E will pay all money to I, in order to increase the continuation probability β:

$$\beta(\tilde{\theta}) = 1 - \frac{D(\tilde{q})}{\pi_2} + \frac{q(\theta - q) + \delta}{\pi_2}, \tag{3.64}$$

and the part of his expected payoff for these cases is

$$\int_{q}^{\frac{D(\tilde{q})-\delta}{q}+q} \left(1 - \frac{D(\tilde{q})}{\pi_2} + \frac{q(\theta - q) + \delta}{\pi_2}\right) \pi_2 \, \frac{1}{2\varepsilon} \, d\theta. \tag{3.65}$$

If θ is sufficiently high, E can repay $D(\tilde{q})$, and the part of his expected payoff for these cases is

$$\int_{\frac{D(\tilde{q})-\delta}{q}+q}^{a+\varepsilon} \left(\pi_2 - D(\tilde{q}) + \delta + q(\theta - q)\right) \frac{1}{2\varepsilon} \, d\theta. \tag{3.66}$$

The sum of (3.63), (3.65) and (3.66) is

$$\mathcal{E}U(\textit{announce } \tilde{q}|q) \;=\; \pi_2 - D(\tilde{q}) + \delta + \int_{q}^{a+\varepsilon} q(\theta - q) \frac{1}{2\varepsilon} \, d\theta. \tag{3.67}$$

Obviously, there are no incentive problems if (3.67) is exactly equal to (3.59), i.e. if $D(\tilde{q}) = D(q) \; \forall \tilde{q}, q$. ∎

Appendix B: The Monopoly Case

Proof: The Underinvestment Result

The firm maximises

$$\int_q^{a+\varepsilon} q(\theta - q)\frac{1}{2\varepsilon}\,d\theta - q(\theta_1 - q) + \pi_2. \tag{3.68}$$

There is no "$-cq$" term, as the costs of production are paid by using up the loan k_1; either the loan is insufficient to produce $\bar{q}$, and the firm spends everything, or it is more than sufficient, and the firm has taken on 'riskless debt'. We assume that in the latter case the firm reduces its borrowing.

When designing the debt contract, the firm must take the lender's participation constraint into consideration:

$$\int_q^{\theta_1} q(\theta - q)\frac{1}{2\varepsilon}\,d\theta + \int_{\theta_1}^{a+\varepsilon} q(\theta_1 - q)\frac{1}{2\varepsilon}\,d\theta + k_0 - cq = 0. \tag{3.69}$$

The first order condition at the contract choice stage is

$$-(\theta_1 - 2q) - q\frac{\partial\theta_1}{\partial q} + \int_q^{a+\varepsilon}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta. \tag{3.70}$$

The partial derivative of θ_1 with respect to q is obtained by implicit differentiation of the lender's participation constraint (3.69):

$$\frac{\partial\theta_1}{\partial q} = -\frac{\int_q^{\theta_1}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta + \int_{\theta_1}^{a+\varepsilon}(\theta_1 - 2q)\frac{1}{2\varepsilon}\,d\theta - c}{q\frac{a+\varepsilon-\theta_1}{2\varepsilon}}. \tag{3.71}$$

Rewrite the first order condition as

$$-(\theta_1 - 2q)\frac{\frac{a+\varepsilon-\theta_1}{2\varepsilon}}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}} + \frac{\int_q^{\theta_1}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta + (\theta_1 - 2q)\frac{a+\varepsilon-\theta_1}{2\varepsilon} - c}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}} + \int_q^{a+\varepsilon}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta. \tag{3.72}$$

Simplify, and 'split' "$-c$" in the fraction into $\int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta$ and $\int_{\theta_1}^{a+\varepsilon} c\frac{1}{2\varepsilon}\,d\theta$. The first order condition can then be rewritten as

$$\left(\int_q^{a+\varepsilon}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta - c\right) + \frac{\int_q^{\theta_1}(\theta - 2q)\frac{1}{2\varepsilon}\,d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}}. \tag{3.73}$$

The first term is exactly zero, if the 'deep pocket' quantity $\bar{q}$ is chosen, and the second must be negative. Therefore, the first order condition is negative, and the optimal output is smaller than $\bar{q}$. ∎

Proof: Concavity of the Objective Function

Differentiate the first order condition (3.73) with respect to q,

$$\int_q^{a+\varepsilon}(-2)\frac{1}{2\varepsilon}\,d\theta-(q-2q)\frac{1}{2\varepsilon} \tag{3.74}$$

$$+\quad\frac{\left(\int_q^{\theta_1}(-2)\frac{1}{2\varepsilon}\,d\theta-(q-2q)\frac{1}{2\varepsilon}+(\theta_1-2q)\frac{1}{2\varepsilon}\frac{\partial\theta_1}{\partial q}-c\frac{1}{2\varepsilon}\frac{\partial\theta_1}{\partial q}\right)\frac{a+\varepsilon-\theta_1}{2\varepsilon}}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^2}$$
$$-\quad\frac{\left(\int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta-\int_{a-\varepsilon}^{\theta_1}c\frac{1}{2\varepsilon}\,d\theta\right)\left(-\frac{1}{2\varepsilon}\frac{\partial\theta_1}{\partial q}\right)}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^2}.$$

Rearrange,

$$-\frac{2(a+\varepsilon-q)}{2\varepsilon}+\frac{q}{2\varepsilon}+\frac{-\frac{2(\theta_1-q)}{2\varepsilon}+\frac{q}{2\varepsilon}}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}} \tag{3.75}$$

$$+\frac{1}{2\varepsilon}\left(\frac{\int_{\theta_1}^{a+\varepsilon}(\theta_1-2q)\frac{1}{2\varepsilon}\,d\theta-\int_{\theta_1}^{a+\varepsilon}c\frac{1}{2\varepsilon}\,d\theta}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^2}\right)\frac{\partial\theta_1}{\partial q}+\frac{1}{2\varepsilon}\left(\frac{\int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta-\int_{a-\varepsilon}^{\theta_1}c\frac{1}{2\varepsilon}\,d\theta}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^2}\right)\frac{\partial\theta_1}{\partial q},$$

and use (3.71) to obtain

$$-\frac{2(a+\varepsilon)-3q}{2\varepsilon}-\frac{\frac{2\theta_1-3q}{2\varepsilon}}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}}-\frac{\left(\int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta+\int_{\theta_1}^{a+\varepsilon}(\theta_1-2q)\frac{1}{2\varepsilon}\,d\theta-c\right)^2}{2\varepsilon q\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^3}. \tag{3.76}$$

We can show that this is negative 'over the relevant range', i.e. if $q<(a+\varepsilon)/3$. Since $\bar q<(a+\varepsilon)/3$ and the firm will never choose a $q>\bar q$ at the quantity setting stage, higher values of q need not be taken into consideration when analysing the concavity at the contracting stage.

We use only the two first terms of (3.76). This sum is negative if

$$-2(a+\varepsilon)^2+2(a+\varepsilon)\theta_1+3q(a+\varepsilon)-3q\theta_1-2\theta_1\cdot 2\varepsilon+3q\cdot 2\varepsilon<0. \tag{3.77}$$

As $q<\bar q<(a+\varepsilon)/3$, it is sufficient to show that

$$-2(a+\varepsilon)^2+2(a+\varepsilon)\theta_1+(a+\varepsilon)(a+\varepsilon)-3q\theta_1-2\theta_1\cdot 2\varepsilon+(a+\varepsilon)\cdot 2\varepsilon<0. \tag{3.78}$$

This can be simplified to obtain

$$-(a+\varepsilon)(a-\varepsilon)-2\theta_1\Big(q-(a-\varepsilon)\Big)-q\theta_1, \tag{3.79}$$

which is indeed negative. ■

Proof: Slope of $q(k_0)$

The derivative of q with respect to k_0 is found by implicit differentiation of the first order condition (3.73):

$$\frac{\partial q}{\partial k_0} = -\frac{\frac{\theta_1-2q-c}{2\varepsilon}\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)\frac{\partial\theta_1}{\partial k_0} - \left(\int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta\right)\left(-\frac{1}{2\varepsilon}\right)\frac{\partial\theta_1}{\partial k_0}}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)^2 SOC}, \tag{3.80}$$

where SOC stands for the second order condition (3.76). As the latter is negative in equilibrium, the slope of $q(k_0)$ is the same as that of the Numerator of (3.80). Cancel constant terms from the Numerator, and rearrange,

$$\left(\int_{\theta_1}^{a+\varepsilon}(\theta_1-2q)\frac{1}{2\varepsilon}\,d\theta - \int_{\theta_1}^{a+\varepsilon} c\frac{1}{2\varepsilon}\,d\theta + \int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta\right)\frac{\partial\theta_1}{\partial k_0}. \tag{3.81}$$

The second term can be found by implicit differentiation of the lender's participation constraint (3.69):

$$\frac{\partial\theta_1}{\partial k_0} = -\frac{1}{q\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)}, \tag{3.82}$$

which is negative. Thus, the slope of $q(k_0)$ has the same sign as

$$-\frac{\int_q^{\theta_1}(\theta-2q)\frac{1}{2\varepsilon}\,d\theta + \int_{\theta_1}^{a+\varepsilon}(\theta_1-2q)\frac{1}{2\varepsilon}\,d\theta - c}{q\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)}, \tag{3.83}$$

which is the derivative of θ_1 with respect to q. We analyse the Numerator: Integrate,

$$\frac{\theta_1^2 - q^2 - 4q(\theta_1-q)}{4\varepsilon} + \frac{(a+\varepsilon-\theta_1)(2\theta_1-4q)}{4\varepsilon} - \frac{4\varepsilon c}{4\varepsilon}, \tag{3.84}$$

and substitute θ_1, which can be found by solving the lender's participation constraint (3.69) with equality:

$$\theta_1^* = a+\varepsilon - \sqrt{(a+\varepsilon-q)^2 - \frac{4\varepsilon}{q}(c\cdot q - k_0)}. \tag{3.85}$$

This yields

$$-q(a+\varepsilon-q) - \frac{2\varepsilon}{q}k_0, \tag{3.86}$$

Which is negative if k_0 is positive or not too negative.

If $k_0 \approx \underline{k}_0$, $q \approx \bar{q}$, $D \approx q(a+\varepsilon-q)$, and $\theta_1 \approx (a+\varepsilon)$. Then (3.80) reads

$$\left.\frac{\partial q}{\partial k_0}\right|_{\substack{q\uparrow\bar{q}\\ \theta_1\uparrow a+\varepsilon}} = \frac{\int_{\bar{q}}^{a+\varepsilon}(\theta-2\bar{q})\frac{1}{2\varepsilon}\,d\theta - c}{-\frac{2(a+\varepsilon)-3\bar{q}}{2\varepsilon}\bar{q}\left(\frac{a+\varepsilon-(a+\varepsilon)}{2\varepsilon}\right)^3 - \frac{2(a+\varepsilon)-3\bar{q}}{2\varepsilon}\bar{q}\left(\frac{a+\varepsilon-(a+\varepsilon)}{2\varepsilon}\right)^2 - \frac{(\int_{\bar{q}}^{a+\varepsilon}(\theta-2\bar{q})\frac{1}{2\varepsilon}\,d\theta-c)^2}{2\varepsilon}}, \tag{3.87}$$

which tends to minus infinity. Thus, there must be at least one minimum of $q(k_0)$, which lies strictly to the right of $\underline{k_0}$. This minimum is unique, which we show by contradiction. Suppose there is more than one. Pick the maximum between two minima, and denote the corresponding value of k_0 with $\hat{k}_0$, and the output with $\hat{q}$. In $\hat{k}_0$, we must have

$$\frac{\hat{q}^2(a+\varepsilon-\hat{q})}{2\varepsilon} + \hat{k}_0 = 0, \tag{3.88}$$

while for some value $k_0 < \hat{k}_0$ we must have $q < \hat{q}$ (because $q(k_0)$ has a maximum at $\hat{k}_0$), and

$$\frac{q^2(a+\varepsilon-q)}{2\varepsilon} + k_0 > 0. \tag{3.89}$$

Combining these two equations with $\hat{k}_0 > k_0$ leads to

$$q^2(a+\varepsilon-q) > \hat{q}^2(a+\varepsilon-\hat{q}), \tag{3.90}$$

which is only possible if $q > \bar{q}$, as $q < \hat{q} < \frac{a+\varepsilon}{3}$. ■

Appendix C: The Duopoly Case

Firm 1's maximisation program is

$$\max_{q^1} \int_{\underline{\theta}(q^1)}^{a+\varepsilon} q^1(\theta - q^1 - q^2(q^1))\frac{1}{2\varepsilon}\, d\theta - q^1\Big(\theta_1(q^1, q^2(q^1)) - q^1 - q^2(q^1)\Big) + \pi_2 \quad (3.91)$$

s.th.

$$\int_{\underline{\theta}(q^1)}^{\theta_1(q^1,q^2(q^1))} R(q^1, q^2(q^1), \theta)\frac{1}{2\varepsilon}\, d\theta \quad (3.92)$$

$$+\int_{\theta_1(q^1,q^2(q^1))}^{a+\varepsilon} R\Big(q^1, q^2(q^1), \theta_1(q^1, q^2(q^1))\Big)\frac{1}{2\varepsilon}\, d\theta - cq^1 + k_0^1 = 0$$

(the lender's zero–profit condition). We omit $q^2(q^1)$ in the following. The derivative of θ_1 with respect to q^1 is

$$\frac{\partial\theta_1}{\partial q^1} = -\frac{\int_{\underline{\theta}(q^1)}^{\theta_1} R_{q^1}(q^1,\theta)\frac{1}{2\varepsilon}\, d\theta + \int_{\theta_1}^{a+\varepsilon} R_{q^1}(q^1,\theta_1)\frac{1}{2\varepsilon}\, d\theta - c}{R_\theta(q^1,\theta_1)\frac{a+\varepsilon-\theta_1}{2\varepsilon}}. \quad (3.93)$$

We can derive the first order condition and rearrange it as in the proof of Proposition 3.3:

$$\left(\int_{q^1}^{a+\varepsilon} R_{q^1}(q^1,\theta)\frac{1}{2\varepsilon}\, d\theta - c\right) + \frac{\int_{q^1}^{\theta_1} R_{q^1}(q^1,\theta)\frac{1}{2\varepsilon}\, d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\, d\theta}{\frac{a+\varepsilon-\theta_1}{2\varepsilon}}. \quad (3.94)$$

Given the definition of q^*, this must be negative if $q^1 = q^*$, and the underinvestment result follows.

The concavity of the maximisation problem for a financially unconstrained firm is easy to show: the second derivative of the objective function (3.15) for Firm 2 is

$$-\frac{2(a+\varepsilon) - 2q^1 - 3q^2}{2\varepsilon}, \quad (3.95)$$

which is negative as $\max\{q^1, q^2\} < \frac{a+\varepsilon}{3}$ and $q^1 + q^2 < \frac{a+\varepsilon}{2}$ from the definition of the reaction curves.

For the case of the 'residual demand monopolist', we assume that the maximisation problem without financial constraints is sufficiently concave:

$$\int_{\underline{\theta}}^{a+\varepsilon} R_{q^1q^1}(q^1,\theta)\frac{1}{2\varepsilon}\, d\theta - \frac{R_{q^1}(q^1,\underline{\theta})}{2\varepsilon}\frac{\partial\underline{\theta}}{\partial q^1} \quad << \quad 0. \quad (3.96)$$

We can derive a second order condition with a structure similar to (3.76) for the case of a financially constrained Firm 1. If (3.96) is sufficiently negative, Firm 1's maximisation program will be concave over the relevant range of q^1.

The slope of $q(k_0)$ can be derived from the first order condition (3.94), and shown to have the same sign as (3.93). While it is not possible to determine the sign for general values of (k_0, q^1), this is possible for the extremes $\underline{k_0}$ (where $q^1(\underline{k_0}) = q^*$) and $\widetilde{k_0}$ (where $q^1(\widetilde{k_0}) = \widetilde{k_0}/c$). Not surprisingly (given the underinvestment result) the slope is negative in the first case and positive in the second.

The value of the financially constrained Firm 1 is described in (3.20). An infinitesimal change of k_0 has the following effect on this value:

$$\left(\int_{\underline{\theta}}^{a+\varepsilon} R_{q^1}(q^1,\theta)\frac{1}{2\varepsilon}\,d\theta - c + \frac{\int_{\underline{\theta}}^{\theta_1} R_{q^1}(q^1,\theta)\frac{1}{2\varepsilon}\,d\theta - \int_{a-\varepsilon}^{\theta_1} c\frac{1}{2\varepsilon}\,d\theta}{\left(\frac{a+\varepsilon-\theta_1}{2\varepsilon}\right)}\right)\frac{\partial q^1}{\partial k_0} - R_\theta(q^1,\theta_1)\frac{\partial \theta_1}{\partial k_0}. \tag{3.97}$$

The term in brackets in the first term is the first order condition and must be zero in equilibrium. The second term is negative, and therefore the whole expression is positive: the value of Firm 1 is increasing in k_0, i.e. increasing in its retained earnings, and decreasing in its retained debt.

References

Abreu, Dilip and Faruk Gul (1994): Bargaining and Reputation. Mimeo, Princeton University and Northwestern University.

Admati, Anat and Motty Perry (1991): Joint Projects without Commitment. *Review of Economic Studies* **58**, 259–276.

Aghion, Philippe, Patrick Bolton and Steven Fries (1996): Financial Restructuring in Transition Economies. Mimeo, London and Brussels.

Aghion, Philippe, Mathias Dewatripont, and Patrick Rey (1997): Agency Costs, Firm Behavior and the Nature of Competition. Mimeo, ECARE, Université Libre de Bruxelles.

Aghion, Philippe, Oliver Hart and John Moore (1992): The Economics of Bankruptcy Reform. *Journal of Law, Economics, & Organization,* **8**(3), 523–546.

Aghion, Philippe, Oliver Hart and John Moore (1994): Improving Bankruptcy Procedure. *Washington University Law Quarterly* **72**(3), 849–877.

Armendariz, Beatriz (1994): Development Banking. Mimeo, London School of Economics.

Baird, Douglas (1987): A World Without Bankruptcy. *Law and Contemporary Problems,* **50**(2), 173–193.

Baird, Douglas (1991): The Initiation Problem in Bankruptcy. *International Review of Law and Economics,* **11**, 223–232.

Baird, Douglas (1993): The Elements of Bankruptcy. Westbury, New York: The Foundation Press.

Baird, Douglas (1995): The Hidden Virtues of Chapter 11: An Overview of the Law and Economics of Financially Distressed Firms. Mimeo, University of Chicago Law School.

Baird, Douglas, and Thomas Jackson (1988): Bargaining After the Fall and the Contours of the Absolute Priority Rule. *The University of Chicago Law Review,* **55**, 738–789.

Bebchuk, Arye (1988): A New Approach to Corporate Reorganizations. *Harvard Law Review,* **101**, 775–804.

Bebchuk, Arye (1991): The Effects of CHapter 11 and Debt Renegotiation on ex ante Corporate Decisions. Discussioon Paper No. 104, Program in Law and Economics, Harvard Law School.

Bebchuk, Arye, and Howard F. Chang (1992): Bargaining and the Division of Value in Corporate Reorganization. *Journal of Law, Economics, & Organization,* **8**(2), 253–279.

Berglöf, Erik and Ernst–Ludwig von Thadden (1994): Short–Term versus Long–Term Interests: Capital Structure With Multiple Investors. *Quarterly Journal of Economics,* **109**, 1055–1084.

Berkovitch, Elazar, Ronen Israel and Jaime Zender (1995): The Design of Bankruptcy Law: A Case for Management Bias in Bankruptcy Reorganizations. Discussion Paper, University of Michigan.

Bernanke, Ben, and Mark Gertler (1989): Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review,* **79**(1), 14–31.

Bhandari, Jagdeep, and Lawrence Weiss (1993): The Untenable Case for Chapter 11: A Review of the Evidence. *American Bankruptcy Law Journal,* **67**, 131–150.

Bhattacharya, Sudipto and Gabriella Chiesa (1995): Proprietary Information, Financial Intermediation, and Research Incentives. *Journal of Financial Intermediation,* **4**, 328–357.

Birckhoff, Garrett and Gian–Carlo Rota (1978): *Ordinary Differential Equations.* Wiley, New York.

Bolton, Patrick, and David Scharfstein (1996): Optimal Debt Structure With Multiple Creditors. *Journal of Political Economy,* **104**(1), 1–25.

Bolton, Patrick, and David S. Scharfstein (1990): A Theory of Predation Based on Agency Problems in Financial Contracting. *American Economic Review,* **80**(1), 93–106.

Boot, Arnoud and Anjan Thakor (1993): Self–interested Bank Regulation. *American Economic Review,* **83**(2), 206–212.

Borenstein, Severin, and Nancy L. Rose (1995): Bankruptcy and Pricing Behavior in U.S. Airline Markets. *American Economic Review,* **85**(2), 397–402.

Bradley, Michael and Michael Rosenzweig (1992): The Untenable Case For Chapter 11. *Yale Law Journal,* **101**, 1043–1095.

Bradley, Michael (1997): A Re–Examination of the Effect of the 1978 Bankruptcy Reform Act on the Wealth of Corporate Stockholders. Mimeo, Fuqua School of Business, Duke University.

Brander, James and Tracy Lewis (1986): Oligopoly and Financial Structure: The Limited Liability Effect. *American Economic Review,* **76**(5), 956–970.

Chevalier, Judith A. (1995a): Capital Structure and Product–Market Competition: Empirical Evidence from the Supermarket Industry. *American Economic Review,* **85**(3), 415–435.

Chevalier, Judith A. (1995b): Do LBO Supermarkets Charge More? An Empirical Analysis of the Effects of LBOs on Supermarket Pricing. *Journal of Finance,* **50**(4), 1095–1112.

Chevalier, Judith A., and David S. Scharfstein (1996): Capital Market Imperfections and Countercyclical Markups: Theory and Evidence. *American Economic Review,* **86**(4), 703–725.

Cork Report (1982): Insolvency Law and Practice. Report of the Review Committee. London, HMSO.

Delaney, Kevin (1992): Strategic Bankruptcy. How Corporations and Creditors Use Chapter 11 to Their Advantage. University of California Press, Berkeley, Los Angeles, Oxford.

Dewatripont, Mathias and Eric Maskin (1995): Credit and Efficiency in Centralized and Decentralized Economies. *Review of Economic Studies,* **62**(4), 541–555.

Dewatripont, Mathias and Jean Tirole (1994): A Theory of Debt and Equity: Diversity of Securities and Manager–Shareholder Congruence. *Quarterly Journal of Economics,* **109**, 1028–1054.

Dewatripont, Mathias and Jean Tirole (1994): The Prudential Regulation of Banks. Cambridge, MIT Press.

Diamond, Douglas (1984): Financial Intermediation and Delegated Monitoring. *Review of Economic Studies,* **51**, 393–414.

Diamond, Douglas (1993): Seniority and Maturity of Debt Contracts. *Journal of Financial Economics,* **33**, 341–368.

The Economist (1992): When Firms Go Bust. 1st August 1992, 69–71.

Edwards, Jeremy and Klaus Fischer (1994): *Banks, Finance and Investments in Germany.* Cambridge University Press, Cambridge.

Faure–Grimaud, Antoine (1997): Product Market Competition and Optimal Debt Contracts: The Limited Liability Effect Revisited. Financial Markets Group Discussion Paper No. 261, London School of Economics.

Fernandez, Raquel and Jacob Glazer (1991): Striking for a Bargain Between Two Completely Informed Agents. *American Economic Review* **81**(1), 240–252.

Fischer, Klaus (1990): *Hausbankbeziehungen als Instrument der Bindung zwischen Banken und Unternehmen.* Unpublished doctoral dissertation, University of Bonn.

Fisher, Franklin (1989): Games Economists Play: a Noncooperative View. *Rand Journal of Economics,* **20**(1), 113–124.

Fortune (1983): Management by Bankruptcy. 31st October 1983, 69–72.

Franks, Julian and Walter Torous (1989): An Empirical Investigation of U.S. Firms in Reorganization. *Journal of Finance,* **44**(3), 747–769.

Fudenberg, Drew and Jean Tirole (1995): A Theory of Income and Dividend Smoothing Based on Incumbency Rents. *Journal of Political Economy,* **103**(1), 75–93.

Fudenberg, Drew and Jean Tirole (1990): Moral Hazard and Renegotiation in Agency Contracts. *Econometrica,* **58**(6), 1279–1319.

Fudenberg, Drew and Jean Tirole (1986): A Theory of Exit in Duopoly. *Econometrica* **54**(4), 943–960.

Gale, Douglas (1983): Competitive Models with Keynesian Features. *Economic Journal,* **93**(1), 17–33.

Gale, Douglas, and Martin Hellwig (1985): Incentive–Compatible Debt Contracts: The One–Period Problem. *Review of Economic Studies,* **52**, 647–663.

Gertner, Robert and David Scharfstein (1991): A Theory of Workouts and the Effect of Reorganization Law. *Journal of Finance,* **46**(4), 1189–1222.

Giammarino, Ronald, and Ed Nosal (1995): The Efficiency of Judicial Discretion in Bankruptcy Law. Mimeo, University of British Columbia.

Gilson, Stuart (1990): Bankruptcy Boards, Banks, and Blockholders. *Journal of Financial Economics,* **27**, 355–387.

Gilson, Stuart, Kose John and Larry Lang (1990): Troubled Debt Restructurings. An Empirical Study of Private Reorganisation of Firms in Default. *Journal of Financial Economics,* **27**, 315–353.

Gilson, Stuart, and Michael Vetsuypens (1993): CEO Compensation in Financially Distressed Firms: An Empirical Analysis. *Journal of Finance,* **48**(2), 425–458.

Glazer, Jacob (1994): The Strategic Effects of Long–Term Debt in Imperfect Competition. *Journal of Economic Theory,* **62**, 428–443.

Gromb, Denis (1994): Renegotiation in Debt Contracts. Mimeo, London School of Economics and Ecole Polytechnique, Paris.

Harris, Milton and Artur Raviv (1992): Financial Contracting Theory. Ch. 2 in Jean–Jacques Laffont (ed.), Advances in economic theory Sixth World Congress, Vol. II. Cambridge University Press.

Hart, Oliver and John Moore (1995): Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management. *American Economic Review,* **85**(3), 567–585.

Heinkel, Robert, and Josef Zechner (1993): Financial Distress and Optimal Capital Structure Adjustments. *Journal of Economics & Management Strategy* **2**(4), 531–565.

Hellwig, Martin (1997): Unternehmensfinanzierung, Unternehmenskontrolle und Ressourcenallokation: Was leistet das Finanzsystem? Discussion Paper No. 97/02, University of Mannheim, SFB 504.

Hellwig, Martin (1991): Banking, Financial Intermediation and Corporate Finance. In: A. Giovannini, C. Mayer (eds.), *European Financial Integration.* Cambridge University Press, Cambridge, pp. 35–63.

Hendel, Igal (1996): Competition Under Financial Distress, *Journal of Industrial Economics* **44**, 309-324

Hoshi, Takeo, Anil Kashyap and David Scharfstein (1990): The Role of Banks in Reducing the Costs of Financial Distress in Japan. NBER Working Paper No 3435.

House of Representatives Report No H.R.95–595, 95th Congress, 1st session: Bankruptcy Law Revision. Washington D.C. 1977.

Jackson, Thomas (1986): The Logic and Limits of Bankruptcy Law. Cambridge and London: Harvard University Press.

Jensen, Michael (1986): Agency Costs of Free Cash Flow, Corporate Finance and Takeovers. *American Economic Review,* **76**, 323–339.

Jensen, Michael and William Meckling (1976): Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. *Journal of Financial Economics,* **3**, 305–360.

Kaplan, Steven (1994): Campeau's acquisition of Federated: Post-bankruptcy results *Journal of Financial Economics,* **35**(1), 123–137.

Knoeber, Charles R. (1986): Golden Parachutes, Shark Repellents, and Hostile Tender Offers. *American Economic Review* **76**(1), 155–167.

Kovenock, Dan, and Gordon M. Philips (1995): Capital Structure and Product–Market Rivalry: How Do We Reconcile Theory and Evidence? *American Economic Review,* **85**(2), 403–408.

Kovenock, Dan, and Gordon M. Philips (1997): Capital Structure and Product Market Behavior: An Examination of Plant Exit and Investment Decisions. *Review of Financial Studies,* **10**(3), 767–803.

Levitt, Steven and Christopher Snyder (1996): Is No News Bad News? Information Transmission and the Role of "Early Warning" in the Principal–Agent Model. Mimeo, Harvard Society of Fellows and George Washington University.

LoPucki, Lynn and William Whitford (1993): Corporate Governance in the Bankruptcy Reorganization of Large, Publicly Held Companies. *University of Pennsylvania Law Review,* **141**(3), 669–800.

Maksimovic, Vojislav (1988): Capital Structure in Repeated Oligopolies. *Rand Journal of Economics,* **19**(3), 389–407.

Mitchell, Janet (1993): Creditor Passivity and Bankruptcy: Implications for Economic Reform. In: Colin Mayer and Xavier Vives (eds.), *Capital Markets and Financial Intermediation.* Cambridge University Press, Cambridge, pp. 197–224.

Mookherjee, Dilip and Ivan Png (1989): Optimal Auditing, Insurance, and Redistribution. *Quarterly Journal of Economics,* **104**(2), 399–415.

Myers, Stuart and Nicholas Majluf (1984): Corporate Financing and Ivestment Decisions When Firms Have Information That Investors Do Not Have. *Journal of Financial Economics,* **13**, 187–221.

Myerson, Roger (1986): Multistage Games With Communication. *Econometrica,* **54**(2), 323–358.

National Bankruptcy Review Commission (1997a): Proposals Initially Adopted By the National Bankruptcy Review Commission. Mimeo, Washington, 2. September 1997.

National Bankruptcy Review Commission (1997b): Bankruptcy: The Next Twenty Years. National Bankruptcy Review Commission Final Report. Mimeo, Washington, 20. October 1997 (http://www.nbrc.org).

Opler, Tim C., and Sheridan Titman (1994): Financial Distress and Corporate Performance. *Journal of Finance,* **49**(3), 1015–1040.

Philips, Gordon M. (1992): Financing Investment and Product Market Competition. Mimeo, Krannert School of Management, Purdue University.

Philips, Gordon M. (1995): Increased Debt and Industry Product Markets: An Empirical Analysis. *Journal of Financial Economics,* **37**, 189–238.

Povel, Paul (1995): Multiple Banking as a Commitment Not to Rescue. Discussion Paper, London School of Economics, Financial Markets Group.

Rajan, Raghuram and Andrew Winton (1995): Covenants and Collateral as Incentives to Monitor. *Journal of Finance,* **50**(4), 1113–1146.

Repullo, Rafael and Javier Suarez (1995): Monitoring, Liquidation, and Security Design. Mimeo, CEMFI (Madrid) and London School of Economics.

Repullo, Rafael and Javier Suarez (1998): Monitoring, Liquidation, and Security Design. *Review of Financial Studies,* **11**(1), 163–187.

Scharfstein, David and Jeremy Stein (1990): Herd Behavior and Investment. *American Economic Review,* **80**(3), 465–479.

Schneider–Lenné, E. (1992): Corporate Control in Germany. *Oxford Review of Economic Policy* **8**(3), 11–23.

Schwartz, Alan (1997): Contracting About Bankruptcy. *Journal of Law, Economics, & and Organization,* **13**(1), 127–146.

Senbet, Lemma and James Seward (1995): Financial Distress, Bankruptcy and Reorganization. Chapter 28 in Robert Jarrow, William Ziemba and Vojislav Maksimovic (eds.), Handbooks in Operations Research and Management Strategy, Vol. 9, Finance. Elsevier North Holland, Amsterdam.

von Thadden, Ernst–Ludwig (1995): Long–Term Contracts, Short–Term Investment and Monitoring. *Review of Economic Studies,* **62**(4), 557–575.

von Thadden, Ernst–Ludwig (1992): The Commitment of Finance, Duplicated Monitoring, and the Investment Horizon. Working Paper, University of Basel.

Weiss, Lawrence and Karen Wruck (1998): Information Problems, Conflicts of Interest, and Asset Stripping: Chapter 11's Failure in the Case of Eastern Airlines. *Journal of Financial Economics, forthcoming.*

Westbrook, Jay (1993): Chapter 11 Reorganisations in the United States. In: Harry Rajak (ed.), Insolvency Law: Theory and Practice. London, Sweet and Maxwell.

White, Michelle (1989): The Corporate Bankruptcy Decision. *Journal of Economic Perspectives,* **3**(2), 129–151.

White, Michelle (1994): Corporate Bankruptcy as a Filtering Device: Chapter 11 Reorganizations and Out–of–Court Debt Restructurings. *Journal of Law, Economics & Organization,* **10**(2), 268–295.

White, Michelle (1996): The Costs of Corporate Bankruptcy: A U.S.–European Comparison. In Jagdeep Bhandari (ed.), Corporate Bankruptcy: Legal and Economic Perspectives. Cambridge, Cambridge University Press.

Yosha, Oved (1995): Information Disclosure Costs and the Choice of Financing Source. *Journal of Financial Intermediation,* **4**, 3–20.

Zingales, Luigi (1997): Survival of the Fittest or Fattest? Exit and Financing in the Trucking Industry. *Journal of Finance*, forthcoming.