LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

# ESSAYS IN FINANCIAL CONTRACT THEORY

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Thesis submitted to the Department of Finance of the London School of Economics for the degree in Doctor of Philosophy
London, May 2014

# Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).



I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I confirm that Chapter 3 is jointly co-authored with Giorgia Piacentino and Chapter 4 is jointly co-authored with Giorgia Piacentino and Anjan Thakor.

# Abstract

This thesis contains three theory essays on the role of contracting in financial markets. The first essay, called Procyclical Promises, shows that in the presence of two contracting frictions—capital diversion and renegotiation—increasing the cyclicality of an entrepreneur's output can increase his debt capacity with potentially important implications for the macroeconomy and government policy. The second essay, called The Downside of Public Information in Contracting, studies a principal-agent problem with a verifiable public signal. It demonstrates that when agents are competitive, decreasing the precision of the public signal can be Pareto improving in a wide class of environments. We apply the framework to a problem of delegated portfolio management and argue that our results suggest that regulators should insist that credit ratings agencies coarsen their ratings categories. The third essay, called Credit Market Competition and Corporate Investment, uses a general equilibrium framework to study the effect of the price and supply of credit on firms' project choices. It shows that for only intermediate levels of credit market competition do firms choose efficient projects.

# Acknowledgements

I would like to thank my advisors Jean-Pierre Zigrand and Balazs Szentes. Eight years ago I took a summer course with JP and he inspired me to become an economist. He has been an exemplary primary advisor throughout my PhD. He forced me to be independent and he demanded I develop my own opinions. But he also thought seriously about my ideas when I described them to him and he responded with insightful (and sometimes severe) criticism.

Balazs made spirited, direct demands of my work and I learnt a lot trying to meet them. He would ask impossible questions and remain unsatisfied until I found an answer. Defending my work to Balazs was always a pleasure. His aggressive yet fair probing made me understand my own work better.

The essays in my thesis benefited from conversations with many people, notably Ron Anderson, Ulf Axelson, the late Sudipto Bhattacharya, Bruno Biais, Max Bruche, Mike Burkart, Jon Danielsson, Amil Dasgupta, Philip Dybvig, Erik Eyster, Jack Favilukis, Daniel Ferreira, Stéphane Guibaud, Radha Gopalan, Denis Gromb, Christian Julliard, Ohad Kadan, John Kuong, Kai Li, Dong Lou, Igor Makarov, Ian Martin, Marc Martos-Vila, Adrien Matray, John Moore, Philippe Mueller, Francesco Nava, Bob Nobay, Clemens Otto, Daniel Paravisini, Paul Pfleiderer, Christopher Polk, Ronny Razin, Antoinette Schoar, Alan Schwatz, Joel Shapiro, Rob Shimer, Dimitri Vayanos, David Webb, Wei Xiong, Kathy Yuan, Kostas Zachariadis, and Stefan Zeume. Anjan Thakor is a co-author on the last chapter and I have learnt a lot from working with him. I would also like to thank my friend Colm Friel for hundreds of hours of discussion about economics, especially for patiently explaining to me how financial markets and central banks really work.

Thanks to my parents for continuous support. My dad, at eight-six-years-old, still reads everything I write with a scrupulous eye. His understanding remains the best test of whether I am making good sense.

I owe the most to Giorgia Piacentino. When she and I were first-year PhD classmates she scribbled a note to me during class that redefined my approach to research. I had asked the professor a question about quadratic variation (which I thought was fairly smart-sounding) only to have Gio write "Think as an economist!" on the paper in front of me. Later we revised together, and I learnt to think like an economist while I relearnt corporate finance from her. In the meantime, she became my co-author and my girlfriend. She extracts the best from me at every level of scholarship. It is a continuing joy to work with her.

# Contents

# Chapter 1

# Introduction

The three essays in this thesis study the connection between individual contracting frictions and the larger economy. The first essay shows that limits on borrowers' commitment create risk premia for procyclical capital goods and generate fluctuations in capital prices and output. The second essay shows that competition among privately informed agents can prevent them from providing valuable insurance to their clients. The third essay shows that some competition among creditors can mitigate the incentive distortions that debt creates for borrowers, but that high competition can lead to new inefficiencies.

In the first essay, called Procyclical Promises, I construct a model of endogenous borrowing constraints based on limited repayment enforcement. It shows that entrepreneurs' output procyclicality increases their debt capacity, causing fluctuations in capital prices and expected aggregate output. Because project liquidation values are high when capital is expensive, creditors are more willing to finance projects that pay off in booms. Hence, procyclical entrepreneurs stretch their endowments further with leverage, allocating more capital to productive projects and driving up the price of capital in the market. Procyclical assets are good collateral and trade at a premium in equilibrium. Even though the worst recessions occur after credit booms in which procyclical firms are highly levered, borrowing is inefficiently low from the second-best perspective.

In the second essay, called The Downside of Public Information in Contracting, Giorgia Piacentino and I propose a model of delegated investment with a public signal that suggests that (i) contracts do not have to refer to the public signal in order to overcome incentive problems; (ii) contracts include references to the public signal not to address incentive problems, but rather to help agents compete; and, in contrast to the contracting literature, (iii) decreasing the precision of the public signal leads to Pareto improvements. We apply this framework to a problem of delegated portfolio choice in which contracts make references to credit ratings. Our model suggests that wider rating categories make everyone better off.

The third essay, called Credit Market Competition and Corporate Investment, is joint with Giorgia Piacentino and Anjan Thakor. It develops a general equilibrium model to examine how interbank competition influences the types of projects borrowing firms invest in. There are two main results. First, at low levels of interbank competition, firms invest excessively in (riskier) specialized projects, whereas at high levels of interbank competition, firms invest excessively in (safer) standardized projects. Efficient project choices arise in equilibrium for only intermediate levels of competition. Second, the emergence of relationship lending eliminates the inefficiency for low levels of competition, but not the inefficiency for high levels of competition.

# Chapter 2

# Procyclical Promises

## 2.1 Introduction

Does a firm's dependence on the macroeconomy affect its ability to borrow? Does the price of an asset depend on the cyclicality of its output? Can regulating debt levels increase social welfare? The literature suggests that countercyclical output loosens firms' borrowing constraints; that procyclical assets trade at a discount; and that limiting leverage can increase welfare. (See, e.g., Shleifer and Vishny (1992), Sharpe (1964), and Stein (2012).) This paper shows that in the presence of two contractual frictions—capital diversion and renegotiation—these established conclusions are invalid. In the model below, the more procyclical is a borrower's output, the higher is his debt capacity; capital is more expensive when invested in procyclical projects; and taxing countercyclical industries to subsidize procyclical industries increases utilitarian welfare.

The setting is an infinite-horizon economy with two goods, capital and a consumption good called fruit. Capital plays a dual role: it produces fruit and secures loans. Capital is the only collateral. At each date, productive entrepreneurs pledge capital to borrow fruit from less-productive investors. Each entrepreneur uses this credit to buy more capital and invest in a risky project. Before his project bears fruit, the entrepreneur learns whether his project has succeeded. At this point he may divert capital. Diversion entails early project liquidation and comes at the expense of future fruit revenues. Next, the entrepreneur can renegotiate his debt, making his creditor a take-it-or-leave-it offer. If the creditor rejects the offer, he seizes the capital behind the loan and liquidates at the market price. An entrepreneur considering diversion faces a trade-off: to divert and forgo future revenues or to continue and make debt repayments.

In equilibrium, the entrepreneur always diverts capital when he learns his project has failed, since he gains nothing from continuing. Repayment is nil regardless of the price of capital. When an entrepreneur learns that his project has succeeded, he has the incentive to keep his capital in productive use because it will bear fruit in the future. Thus he continues his project and proceeds to renegotiate repayments with his creditor.

A project is called procyclical if expected aggregate output is high when it succeeds and a project is called countercyclical otherwise. In equilibrium, capital prices move one-for-one with expected aggregate output. Therefore, a procyclical entrepreneur puts his creditor in a strong bargaining position in the event of renegotiation, since valuable collateral backs his promise when he succeeds and has the incentive not to divert capital. Creditors can extract repayment effectively from procyclical entrepreneurs and therefore lend relatively freely to them.

To see why procyclicality increases debt capacity, suppose first that an entrepreneur has a procyclical technology that fails in recessions, when capital is cheap, and succeeds in booms, when capital is dear. In a recession, he fails, so he diverts capital and makes no repayment. In a boom, he succeeds and has the incentive to continue. Now capital prices are high and creditors can threaten to seize valuable collateral to extract a high repayment from the entrepreneur.

In contrast, consider an entrepreneur with a countercyclical technology that succeeds in recessions and fails in booms. In a recession, he succeeds so he continues his project and makes a repayment. However, because capital is cheap, his creditor assumes a weak bargaining position and the repayment is low. In a boom, he fails, so he diverts capital and leaves his creditor empty-handed. He diverts capital when it is most valuable. Such a countercyclical entrepreneur repays only in recessions, when capital prices are low.

The comovement between project success and capital prices determines the value of an entrepreneur's repayment promise. If two entrepreneurs differ only in their projects' cyclicality, then the more procyclical entrepreneur's expected repayment is higher than that of the more countercyclical entrepreneur. The reason is that procyclical entrepreneurs continue their projects and make repayments exactly when their creditors have high capital liquidation values and thus extract more from renegotiation. Creditors are thus more willing to lend to procyclical entrepreneurs ex ante.

What are the aggregate consequences of such capital diversion and renegotiation? To answer this question, I build a dynamic economy that embeds the bilateral relationships between investors and entrepreneurs described above. To isolate the effects of the enforcement frictions in connection with entrepreneurs' cyclicality, I allow only the distribution fruit endowments to change from date to date. The types of players and their technologies are time invariant. Two types of entrepreneurs are born every period: procyclical entrepreneurs and countercyclical entrepreneurs. Investors are infinitely lived with a deterministic, decreasing-returns-to-scale technology that produces fruit from durable capital. Entrepreneurs live for just two periods. When they are young, they borrow and buy capital to invest in risky, constant-returns-to-scale technologies. When they are old, they either divert capital or produce, then they renegotiate their debts, sell their capital, and consume. The exogenous variation in the model comes from only the random allocation of entrepreneurs' endowments. At each date, one of three states realizes: either no entrepreneurs have endowments, only procyclical entrepreneurs have endowments, or only countercyclical entrepreneurs have endowments.

In equilibrium, an entrepreneur with zero endowment cannot borrow and therefore does not produce. The reason is that even though entrepreneurs can make more productive use of capital than investors, entrepreneurs never repay more than the market value of their capital, given that they make the renegotiation offer. Therefore, only entrepreneurs with positive endowments hold capital; hence, capital is invested in procyclical projects whenever procyclical entrepreneurs have endowments and, likewise, capital is invested in countercyclical projects whenever countercyclical entrepreneurs have endowments.

The second main result is that capital is more expensive when it is invested in procyclical projects than when it is invested in countercyclical projects. The mechanism is as follows: because procyclical entrepreneurs can borrow more than countercyclical entrepreneurs, they buy more capital to scale up their projects. The residual capital supply held by investors is thus lower when procyclical entrepreneurs invest than when countercyclical entrepreneurs invest. Decreasing the quantity of capital left for investors drives up its price since investors' technology has decreasing returns. This price premium for procyclical capital is a collateral premium. Entrepreneurs borrow more against procyclical capital, buying more capital on margin and driving up its price.

The difference between capital prices at dates when procyclical entrepreneurs have endowments and when countercyclical entrepreneurs have endowments results from only the interaction between capital diversion and renegotiation. Only the combination of the two frictions makes high capital prices a valuable threat for creditors when entrepreneurs succeed but not when they fail. Consequently, such price fluctuations are absent in the four natural benchmark models—namely in the same economy but with perfect contractual enforcement, with no borrowing whatsoever, with renegotiation but without capital diversion, and with capital diversion but without renegotiation. Macroeconomic fluctuations are endogenous in the sense that they appear only as a result of the two limits to contractual enforcement together.

I proceed to study welfare and to suggest a policy intervention. The main result of this analysis is that a social planner aiming to maximize output would wish to transfer countercyclical entrepreneurs' endowments to procyclical entrepreneurs. The reason is that procyclical entrepreneurs stretch their endowments further than countercyclical entrepreneurs do. Thus transferring wealth to procyclical entre-preneurs ex ante yields a superior allocation of capital in aggregate. Such a tax-subsidy scheme causes higher leverage and more investment in risky projects, since it has the effect of moving capital away from investors who have safe (but unproductive) technologies. Even though realized output is lowest when heavily levered entrepreneurs' projects fail, the intervention induces higher leverage ex ante only to deepen such output troughs. Thus the prescription casts doubt on unqualified macro-prudential regulatory policies advocating capping leverage in booms to smooth output: while leverage may lead to crises, it may still be inefficiently low due to private enforcement constraints. Note further that a social planner who can levy ex post taxes on procyclical entrepreneurs can implement an ex ante Pareto improvement by transferring wealth back to countercyclical entrepreneurs after the procyclical entrepreneurs have used their endowments to borrow and produce.

While my results contrast with some established conclusions in the literature, the frictions I study are ubiquitous in the real world and my main predictions are consistent with empirical findings. In most finance models, such as the CAPM, procyclical assets trade at a discount because risk-averse investors cannot diversify away the systematic risk they add to portfolios. To focus on enforcement frictions, I assume that agents are risk-neutral and I thereby shut down the effect of risk-sharing on prices. I discover and analyze a positive side of procyclicality: procyclicality mitigates enforcement frictions. Thus, procyclical assets' collateral premium in my model contrasts with countercyclical assets' insurance premium in classical models. This benefit of procyclicality may account for part of the CAPM's failure to explain observed returns (Fama and French (2004)).

Both of the frictions I focus on are of first-order importance for real world firms. Mironov (2008) documents the importance of flagrant diversion, calculating that Russian companies syphoned off upward of ten percent of GDP in both 2003 and 2004. Moreover, such "looting" is not restricted to developing countries and is indeed common in the US, as Akerlof and Romer (1993) details. In addition to managers' explicit theft, so-called self-dealing, tunnelling, and asset substitution correspond to diversion in my model—these agency frictions are all well-documented afflictions even in countries with strong legal systems (see, e.g., Shleifer and Vishny's 1997 corporate governance survey). Renegotiation is equally pervasive. For example, Roberts and Sufi (2009) finds that more than ninety percent of private loans to public firms are renegotiated before maturity.

The model makes a number of empirical predictions. In the cross-section, it suggests that firms with procyclical cash flows take on relatively high leverage, consistent with evidence in Campbell, Polk and Vuolteenaho (2010) and Maia (2010). Aggregating to study the time-series, I find that entrepreneurs' leverage is procyclical, which, since all borrowers in my model exhaust their debt capacity, agrees with Korajczyk and Levy (2003)'s empirical finding that constrained firms' debt-equity ratios are procyclical. Finally, my model speaks to the "essential feature of business cycles" (Basu and Fernald (2001) p. 225) that productivity is procyclical. In the model, productive entrepreneurs' debt capacity increases in booms, allowing them to acquire more capital and resulting in increased aggregate productivity. Further, the model provides several novel empirical predictions to test. Notably, it suggests that procyclical firms' investment and leverage are more sensitive to endowment shocks than are countercyclical firms' investment and leverage.

The remainder of the introduction describes the paper's context in the literature and its incremental contribution relative to several papers (subsection 2.1.1). Section 2.2 sets up the formal model and section 2.3 solves it. Section 2.4 describes the four benchmarks to contextualize the results. Section 2.5 describes the welfare-improving policy intervention. Section 2.6 states real world analogues of model variables (subsection 2.6.1) and enumerates predictions about the signs of coefficients of linear regressions from correlations in the model (subsection 2.6.2) and from a natural experiment viewed as a shock to the model (subsection 2.6.3).

### 2.1.1 Related Literature

As in Shleifer and Vishny (1992), in my model general equilibrium asset liquidation values determine debt capacity. In both their model and mine, asset buyers' funding constraints and the wedge between the value of the assets in first- and second-best use, which they term "asset illiquidity", pin down liquidation values. They assume that debtors cannot reschedule their loans, so liquidation values do not matter when borrowers succeed and repay, but only matter when they fail and default, when creditors seize collateral and sell it to the highest bidder. They model two firms in an industry with correlated projects; they emphasize that when one is forced to liquidate the other is likely to be cash-strapped, its financial constraints preventing it from acquiring its competitor's old assets, leaving them to be redeployed inefficiently by an industry outsider. In my model, in contrast, liquidation values are most important when projects succeed because they determine outsiders' threat points in renegotiation, while, when entrepreneurs fail, they have incentives to divert capital, decreasing the quantity of liquidatable assets. Shleifer and Vishny conclude that, because cyclical assets are illiquid in downturns, “cyclical and growth assets are poor candidates for debt finance” (p. 1359); in my model, the interaction between renegotiation and capital diversion flips the result.

Hart and Moore (1998) also focuses on the interaction between these frictions. As in my model, creditors’ right to foreclose on capital is the essential enforcement mechanism. In their three-date model, an entrepreneur requires a fixed capital investment to start a project comprising risky returns and asset liquidation values at the middle and final dates. The main results say, roughly, that when only the interim payoffs are risky, optimal debt contracts maximize financial slack, whereas when only the terminal return is risky, optimal debt contracts constitute entrepreneurs’ “maximum equity participation”. Depending on his project’s specific risks, an entrepreneur either borrows to capacity to maintain a cushion of working capital or puts up all of his own money to take on as little debt as possible in order to minimize liquidation when the surplus lost from foreclosure is greatest. They do not analyze the comovement of liquidation and continuation values, the variable of primary interest for me. More specifically, after the entrepreneur gets his enterprise off the ground, he renegotiates his debts and scales up his project at the interim date, when he also potentially diverts cash flows but not assets in place. Liquidation—tantamount to withdrawal of the entrepreneur’s specific capital—preempts the project’s bearing fruit at the final date, when the entrepreneur will never repay anything. I weaken the assumption that assets in place cannot be diverted, supposing instead that a market exists where the entrepreneur can liquidate by himself. In my model the creditor faces a further constraint to repayment: the debtor diverts unless his project’s terminal cash flows less repayments exceed his revenues from diverting capital early. Because projects are scaleable, entrepreneurs always borrow to capacity—or write the "fastest" debt contract in Hart and Moore's language—not because they wish to maintain financial slack, but, rather, because they want to buy more capital. My innovations with respect to this paper are, firstly, to show that the comovement between liquidation values and inside returns—procyclicality—is a valuable resource for financially constrained entrepreneurs and, secondly, to endogenize liquidation values in a dynamic general equilibrium framework.

Kiyotaki and Moore (1997) studies price and output fluctuations when a small, unanticipated technological shock hits the steady state of an infinite-horizon economy in which entrepreneurs must post assets to secure their loans—capital famously plays a dual role, it yields output and serves as collateral. The resulting price change is the same order of magnitude as the productivity change. Because prices represent the entire future productivity of assets, in the Arrow–Debreu world a momentary change in productivity leads "the price to experience a tiny blip" (p. 214). But, in Kiyotaki and Moore's model, since increased productivity loosens borrowing constraints allowing further asset purchases which, in turn, increase productivity and loosen borrowing constraints (repeat), the interplay between the two functions of capital converts the blip into a wallop. The feedback loop between slackened budget constraints and increased borrowing capacity works both within and between periods, effects which Kiyotaki and Moore refer to as the static and intertemporal multipliers. The collateral multiplier (section 2.3.4) in my model relies on the same spiralling back-and-forth, but, since the constrained agents—the entrepreneurs—are short-lived, the long-term consequences of immediate constraints are absent. But, because my model is stochastic and repayments depend on both entrepreneurs' success and the aggregate state tomorrow, the one-period-ahead effects are subtler; my analysis separates the changes in the price today from changes in the price tomorrow. Capital demand curves can slope upward in Kiyotaki and Moore (1997), because more expensive capital means more valuable collateral, which comes with increased borrowing capacity, output, and profits. My overlapping generations set-up renders cumulative wealth unimportant: prices are only forward-looking—the only state variable is the aggregate state. Demand curves have the vanilla downward slope in today's price, but they slope upward in the expected price when repayment occurs, namely in the event that entrepreneurs succeed tomorrow. Price changes, not price levels, matter in my model; specifically, the ratio of the expected price given success to the price today—entrepreneurs' cyclicality—determines demand today. Assuming that entrepreneurs live for only two dates allows me to respond to the challenge that Kiyotaki and Moore pose in their concluding remarks, "The pressing next step in the research is to construct a fully fledged stochastic model, in which a shock is not a zero probability event and is rationally anticipated" (p. 243), but my main contribution is to demonstrate that price and output fluctuations result endogenously from the collateral frictions alone, even absent exogenous variation in productivity. In my model economic fluctuations arise even when no blip at all shows up in the Arrow–Debreu archetype.

In a 2003 paper, Krishnamurthy builds a stripped-down version of Kiyotaki and Moore's model to analyze the hypothesis that state-contingent hedging contracts prevent the economy from amplifying shocks. He shows that, even if insurers require collateral to force entrepreneurs to repay, permitting hedging kills amplification. When limited enforcement is two-sided, however, and entrepreneurs also demand collateral from insurers to secure their hedges, the supply of collateral fails to stretch far enough to insure all risk and the amplification mechanism reemerges. Since in my model entrepreneurs can divert capital as well as cash flows, the optimal state-contingent contract yields the same transfers as standard debt after renegotiation or capital diversion (cf. the discussion in section 2.2.6).

Lorenzoni (2008) uses the Krishnamurthy (2003) structure to assess the welfare consequences of leverage. In a three-date model, entrepreneurs first borrow from investors via state-contingent contracts and then, at the interim date, they receive perfectly correlated payoffs and invest in deterministic constant-returns projects. As in my model, deep-pocketed investors have a decreasing-returns technology; they are marginal since entrepreneurs are constrained. In both models, entrepreneurs' borrowing constraints lead to variations in the residual capital supply held by investors—and thus to changes in the marginal productivity of capital—that drive price fluctuations. In Lorenzoni's model, entrepreneurs sell poorly performing assets to pay their creditors. As they liquidate more capital, investors hold more, lowering marginal productivity and, therefore, prices, thereby forcing entrepreneurs to sell more capital to meet their debts. The more they borrow the more they must promise to repay and the more they must liquidate when returns are low. Because agents are price-takers, they fail to internalize the negative impact of heavy leverage on prices. The main result is that, because of this pecuniary externality, the competitive equilibrium is constrained suboptimal: because highly levered entrepreneurs must liquidate assets to repay their debts at the middle date, the unproductive investors hold too much capital in expectation. Capping borrowing leads to a Pareto improvement. Lorenzoni warns against over-borrowing. In my model, entrepreneurs do not borrow enough. The inefficiency is more direct: repayment constraints prevent investors from lending to the agents who use it most productively and, as in Lorenzoni's low-return states, the economy never achieves the first-best allocation. Transferring wealth to productive agents with more balance sheet capacity, namely to procyclical entrepreneurs, increases welfare because they gear up to invest more. While my prescription is orthogonal to Lorenzoni's, it requires the caveat that price effects like those he focuses on must not be too big: as procyclical agents receive subsidies they drive up the capital price, reducing their ability to stretch their endowments and damping the benefits of the transfers (that entrepreneurs's initial wealth is not too large suffices for the result to hold at the margin, cf. Proposition 25).

The theoretical framework of general equilibrium with endogenous contracts and collateral constraints that Geanakoplos built in his 1997 article "Promises, Promises" has lead him to argue, like Lorenzoni, for the regulation of excessive leverage, citing increased volatility and severe crashes as features of an economy in which the leverage cycle is left unmanaged. In their 2004 paper, he and Kubler use the equilibrium concept to demonstrate that a maturity mismatch can arise endogenously, causing inefficient liquidation when collateral prices fall. As in Lorenzoni's model, small borrowers do not take the collateral price effects into account when they borrow, leading to excessive leverage which a regulator should cap for a Pareto improvement. The work is most important for my model in its conceptual underpinnings. Geanakoplos's definition of a contract as a promise-collateral pair determined in equilibrium motivates my definition of loan contracts (compare with the discussion in section 2.2.6); I also borrow the notion that some goods function as collateral (my capital) while others do not (my fruit)—property rights are effectively enforced only over capital goods.

Many models study the role of limited enforcement in dynamic financial contracting or in macroeconomics. Cooley, Marimon and Quadrini (2004) does both. They use a general equilibrium framework to endogenize entrepreneurs' outside options from repudiation in an infinite-horizon model of capital diversion. They show that limited enforcement amplifies productivity shocks: when new projects are highly productive, investors must give them strong incentives not to abandon their commitments and search for new opportunities, loosening incentive constraints and allowing more efficient capital allocation. Like Kiyotaki and Moore (1997), they consider steady state equilibria and simulate their responses to exogenous shocks. Analogously, in my model, high capital prices increase entrepreneurs' incentives to abscond, but since they are short-lived with bang-bang technologies, capital price fluctuations do not determine equilibrium repudiation—they quit only when their own projects fail—but affect only renegotiated repayments. Contractual constraints make economies that rely on debt to allocate capital sensitive to productivity shocks. Kiyotaki and Moore (1997) and Cooley et al. (2004) each demonstrates a channel by which limited enforcement amplifies shocks, based on constraints to check renegotiation and capital diversion, respectively. My model shows that the two mechanisms do more when they interact: beyond aggravating extrinsic output changes, they generate fluctuations endogenously.

## 2.2 Model

### 2.2.1 Background Environment

The background structure is the probability space $(\mathfrak{S}, \mathsf{F}, \mathbb{P})$ with the set of states $\mathfrak{S} = \big\{(\omega_t)_{t\in\mathbb{Z}}\ ;\ \omega_t \in \{a, b, 0\}\big\}$, the natural filtration $\mathsf{F}$ of $\omega_t$ (viewed as a random process), and the probability $\mathbb{P}$ with $\mathbb{P}\{\omega_s \,|\, \mathcal{F}_t\} = 1/3$ for each $\mathcal{F}_t \in \mathsf{F}$ and any $\omega_s$ with $s > t$.

Overlaid is an extensive form game in which the refinement from $\mathcal{F}_{t-1}$ to $\mathcal{F}_t$ is nature acting at date $t$, termed "the realization of $\omega_t$". The histories that include the realization of $\omega_t$ but not of $\omega_{t+1}$ constitute period $t$.

"Today" and "tomorrow" refer to $\mathcal{F}_t$- and $\mathcal{F}_{t+1}$-measurable variables from the point of view of period $t$.

### 2.2.2 Goods, Players, and Technologies

The numeraire is a perishable consumption good called fruit and measured in pounds. Capital is in supply $K$ and produces fruit according to players' technologies; it does not depreciate. $p_t$ denotes the price of capital at date $t$. A player with technology $\tau \in \{\alpha, \beta, \gamma\}$ and capital $k$ produces $\tau(k)(\omega)$ of fruit if $\omega$ realizes tomorrow.

A unit continuum of long-lived players called investors have deterministic technology $\tau = \gamma$, where $\gamma' > 0$, $\gamma'' < 0$, and $\gamma'(0) = A$. They are deep-pocketed in fruit. At each date they act to maximize the expected value of future consumption discounted at gross rate $R$,

$$U_t(c) = \mathbb{E}_t\left[\sum_{s=t}^{\infty} \frac{c_s}{R^{s-t}}\right],$$

over feasible consumption profiles $\{c_s\}_{s\geq t}$ (given beliefs about other players' action profiles).

Entrepreneurs are short-lived players with risky technologies. At each date a unit of $\alpha$-entrepreneurs and a unit of $\beta$-entrepreneurs are born, where $\alpha$-entrepreneurs have $w$-pound endowments if $\omega_t = a$ and nothing otherwise and, likewise, $\beta$-entrepreneurs have $w$-pound endowments if $\omega_t = b$ and nothing otherwise. An entrepreneur born at date $t$ with technology $\tau$ is called a $t$- or $\tau$-entrepreneur, depending on the context; at date $t$ he is called young and at date $t+1$ (when he will eventually die) he is called old.

$\alpha$-entrepreneurs have technology

$$\alpha(k)(\omega_{t+1}) = \begin{cases} 3Ak & \text{if } \omega_{t+1} = a, \\ 0 & \text{otherwise} \end{cases}$$

and $\beta$-entrepreneurs have technology

$$\beta(k)(\omega_{t+1}) = \begin{cases} 3Ak & \text{if } \omega_{t+1} = 0, \\ 0 & \text{otherwise.} \end{cases}$$

(See figure 2.1 for a pictorial representation of the entrepreneurs' technologies.) An entrepreneur born at date $t$ acts to maximize his expected consumption at $t+1$.

A project is an entrepreneur's technology given his capital investment. Liquidation is the extraction of capital from a project before it bears fruit. A project is successful if $\tau(k)(\omega_{t+1}) \neq 0$.

$e$ and $i$ denote typical entrepreneurs and investors; $\alpha$, $\beta$ refer to types of entrepreneurs. Below, $k_t^\tau$ denotes the capital $\tau$-entrepreneurs hold and and $k_t^e$ denotes the capital entrepreneurs hold cumulatively, $k_t^e = k_t^\alpha + k_t^\beta$.

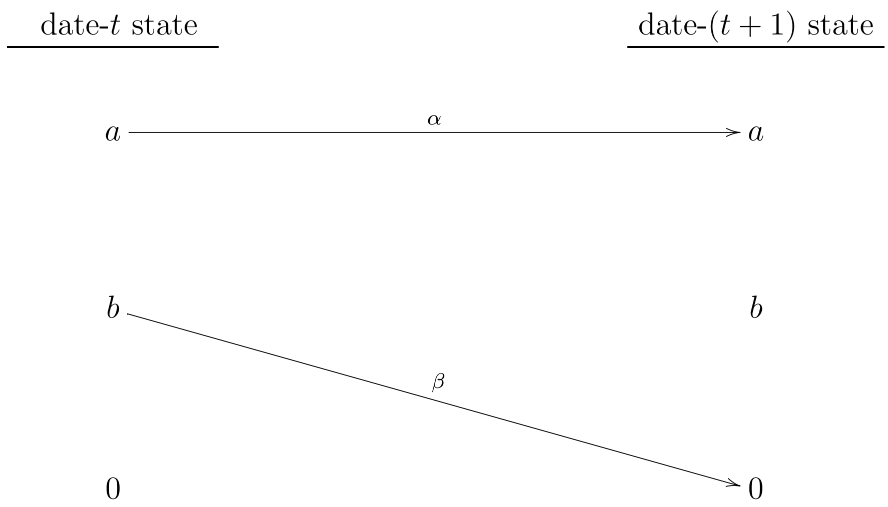


Figure 2.1: The heads of the arrows represent the states in which the respective entrepreneurs' technologies pay off. The tails of the arrows represent the states in which these entrepreneurs have positive endowments. In equilibrium, only entrepreneurs with positive endowments invest, so the arrows represent all the risky production in the economy.

### 2.2.3 Contracts

A contract $\mathsf{c} = (F, \ell)$ is a promise to repay $F$ pounds tomorrow in exchange for $\ell$ pounds today. Contracts are bilateral between a creditor and a debtor (but see the comment in section 2.2.6). The debtor is long the contract and the creditor is short it. If the debtor fails to repay $F$, the creditor has the right to seize the debtor's capital; seizure destroys the successful project's fruit. With all contracts comes the risk that the debtor will divert capital, denoted $\zeta = \mathrm{d}$, or, if he does not, $\zeta = \neg\mathrm{d}$, the risk that he will renegotiate to a repayment $F' < F$. Subsection 2.2.4 below describes the timing of the stage game that players play in each period, including the renegotiation protocol, which follows Hart and Moore (1998) and ascribes bargaining power to entrepreneurs. Denote a $\tau$-entrepreneur's actual repayment at $t+1$ associated with contract $\mathsf{c}$ by the random variable $T_{t+1}(F, k; \tau) = T(F, k)$, for short—viz. the contract $\mathsf{c}$ written at date $t$ induces equilibrium transfer $T(F,k)(\omega_{t+1})$ when $\omega_{t+1}$ realizes at date $t+1$. The value of the promise to repay $F$ from a $\tau$-entrepreneur with capital $k$ is $\mathbb{E}_t\left[T(F,k)\right]/R$ to an investor.

### 2.2.4 Stage Game

In each period $t \in \mathbb{Z}$, first the state realizes, revealing the payoffs of old entrepreneurs. Then, young entrepreneurs are born, determining the date-$t$ price of capital and thus the liquidation values of old entrepreneurs' collateral. Old entrepreneurs either divert and liquidate their projects or wait for them to bear fruit, only to renegotiate their debts; then they sell their capital in the market before they consume and die. Meanwhile, young entrepreneurs borrow to fund their projects and buy capital in the market, determining the liquidation values for investors and the previous generation of entrepreneurs.

The following sequence of moves describes the extensive form of the stage game.

1. $\omega_t$ realizes. $t$-entrepreneurs are born.
2. Each old entrepreneur $e$ either diverts capital $\zeta = \mathrm{d}$ or does not $\zeta = \neg\mathrm{d}$.
    - If $\zeta = \mathrm{d}$, $e$ sells his capital $k_{t-1}$ in the market (viz. he submits an order $-k_{t-1}$ that returns $p_t k_{t-1}$ when the market clears in round 6 below); he makes no transfer to his creditor.
3. If $\zeta = \neg\mathrm{d}$, $e$'s project pays off and he offers a repayment $F'$ to his creditor.
    - If the creditor accepts the offer, $\xi = \mathrm{a}$, or if $F' \geq F_{t-1}$, then $e$ makes him transfer $F'$; if the creditor rejects the offer, $\xi = \neg\mathrm{a}$, then the creditor seizes $e$'s capital $k_{t-1}$, to obtain $p_t k_{t-1}$.
4. Each $t$-entrepreneur an (arbitrary) investor a contract $\mathsf{c}_t = (F_t, \ell_t)$.
    - Each investor accepts or rejects the offer.
5. Each young entrepreneur and each investor submits a demand for capital $k_t(p_t)$ (subject his budget constraint).
6. The price $p_t$ clears the capital market.
7. Old entrepreneurs and investors consume; young entrepreneurs and investors invest.

### 2.2.5 Solution Concept

The solution concept is Markov equilibrium.

Since, therefore, $p_t$ depends on only $\omega_t$, henceforth use the following notation.

**Notation 1.** *Write*

$$p^{\omega_t} := p_t$$

*and*

$$\bar{p} := \mathbb{E}\left[p_{t+1}\right] = \frac{p^a + p^b + p^0}{3}.$$

And note that $\bar{p} \equiv \mathbb{E}_t\left[p_{t+1}\right]$.

### 2.2.6 Assumptions

The assumption below that investors are relatively impatient ensures that prices are never so high (cf. Lemma 5) that entrepreneurs prefer to divert capital and liquidate than to consume the fruit of a successful project tomorrow (Lemma 8).

**Assumption 2.2.1.**

$$R > 4/3. \tag{2.1}$$

The assumption suffices to streamline proofs and ensure uniqueness of the equilibrium action profile and price system (equations (2.11)-(2.13)) by providing a uniform bound on prices.

To ensure that entrepreneurs' borrowing constraints bind—that they do not hold all capital (corollary 14)—assume further that the entrepreneurs' endowment is small relative to the supply of capital. Specifically, assume that entrepreneurs' endowments are always less than the present value of the economy's maximum expected output, i.e. the expected output obtained if entrepreneurs were to invest all capital.

**Assumption 2.2.2.**

$$Rw \leq AK. \tag{2.2}$$

Note that since entrepreneurs' technologies return nil given failure, scrapping unsuccessful projects is efficient; no inefficient liquidation will occur in equilibrium. As a result, nothing is lost in assuming that debt is non-contingent, i.e. that the repayment promise does not depend on the state, $F(\omega_t) = F$ for all $\omega_t$. Equilibrium transfers remain unchanged if the aggregate state is contractible and contracts are optimal because failing entrepreneurs will always divert their capital (cf. Lemma 8). Further, the assumption that contracts are bilateral serves only to simplify the analysis. A richer set-up in which entrepreneurs borrow from multiple creditors via covered debt contracts $\mathsf{c} = (F, \ell, \bar{k})$, where $F$ is the face value, $\ell$ is pounds borrowed, and $\bar{k}$ is the capital securing the specific loan delivers the same results.

### 2.2.7 Notations

The outcome of their projects will determine old entrepreneurs' behaviour. The following definition gives a notation for the state in which projects succeed.

**Notation 2.** *$\sigma(\tau) = \omega_t$ if the project $\tau(k)$ succeeds in state $\omega_t$, i.e. $\sigma(\alpha) = a$ and $\sigma(\beta) = 0$.*

The price of capital given success will determine young entrepreneurs' borrowing capacity. Since projects succeed in only one state, the expected capital price given success is just the price in that state,

$$\mathbb{E}\left[\, p_{t+1} \,\middle|\, \omega_{t+1} = \sigma(\tau) \,\right] = p^{\sigma(\tau)}.$$

A special notation for this price is convenient.

**Notation 3.**

$$P^{\tau} := p^{\sigma(\tau)}.$$

This notation facilitates the notion of project cyclicality as the ratio of the value of capital given success to the value of capital today.

**Definition 2.2.1.** *The cyclicality $\chi$ of a project is*

$$\chi_t^{\tau} := \frac{P^{\tau}}{p_t}.$$

A project is called procyclical if it succeeds when prices are increasing or $\chi \geq 1$ and called countercycical if it succeeds when prices are decreasing or $\chi < 1$. In equilibrium an increasing bijection will pair prices and expected output, so procyclicality will coincide with success when expected output increases.

# 2.3 Results

## 2.3.1 Investors' Indifference Condition and Price Bounds

An investor $i$ who holds capital $k_t^i > 0$ at date $t$ must be indifferent between consuming and buying capital. The condition that $\gamma'(0) = A$ ensures the identity holds even in the corner in which investors hold no capital, $k_t^i = 0$. Since investors are deep-pocketed and risk neutral and $\gamma$ is concave, the pricing identity follows immediately from the first-order condition

$$\frac{\partial}{\partial k}\bigg|_{k=k_t^i}\left(-p_t k + \frac{1}{R}\Big(\gamma(k) + \mathbb{E}_t\left[p_{t+1}k\right]\Big)\right) = 0.$$

Lemma 4 below summarizes.

**Lemma 4.**

$$p_t = \frac{\gamma'\big(k_t^i\big) + \mathbb{E}_t\left[\,p_{t+1}\right]}{R} \tag{2.3}$$

*where $k_t^i$ is the capital held by any investor $i$ at date $t$.*

This expression implies that the price of capital is bounded above by that of a perpetuity that pays $A$ at each date when the gross interest rate is $R$.

**Lemma 5.**

$$p_t \leq \frac{A}{R-1}.$$

*Proof.* The result follows from $\gamma'' < 0$:

$$\begin{aligned}
p_t &= \frac{1}{R}\big(\gamma'(k^i) + \mathbb{E}_t\left[p_{t+1}\right]\big) \\
&\leq \frac{1}{R}\big(\gamma'(0) + \mathbb{E}_t\left[p_{t+1}\right]\big) \\
&\leq \frac{\gamma'(0)}{R} + \frac{\gamma'(0)}{R^2} + \frac{\mathbb{E}_t\left[p_{t+2}\right]}{R} \\
&\leq \gamma'(0)\left(\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \cdots\right) \\
&= \frac{\gamma'(0)}{R-1} \\
&= \frac{A}{R-1}.
\end{aligned}$$

□

The investors' indifference condition and the restriction to Markov equilibria provide a lower bound on prices.

**Lemma 6.** *For any $\omega \in \{a, b, 0\}$,*

$$3Rp_t > p^{\omega_t}. \tag{2.4}$$

*Proof.* Immediately from equation (2.3),

$$\begin{aligned} 3Rp_t &= 3\gamma'(k_t^i) + 3\mathbb{E}_t\left[\, p^{\omega_{t+1}} \right] \\ &= 3\gamma'(k_t^i) + p^a + p^b + p^0 \\ &> \max\left\{p^a, p^b, p^0\right\} \\ &\geq p^{\omega_t}. \end{aligned}$$

□

Lemma 6 constitutes a bound on cyclicality:

$$\begin{aligned} \chi_t^\tau &= \frac{P^\tau}{p_t} \\ &< \frac{\max\left\{p^a, p^b, p^0\right\}}{p_t} \\ &\leq 3R. \end{aligned} \tag{2.5}$$

### 2.3.2 Renegotiation

Lemma 5 and Assumption 2.2.1 (that $R > 4/3$) suffice to solve the stage game by backward induction. First: because the entrepreneur has the bargaining power, he repays at most his creditor's seizure value.

**Lemma 7.** *If $\zeta = \neg \mathrm{d}$, an entrepreneur with capital $k$ who is long a contract with face value $F$ repays*

$$T(F, k) = \min\left\{F\,,\; p_{t+1}k\,\right\}.$$

*Proof.* See appendix 2.8.2 for the standard argument. □

### 2.3.3 Capital Diversion

A failing entrepreneur may divert capital and liquidate it, obtaining the value of his assets in place, forgoing his project's fruit but avoiding paying his debts. A successful entrepreneur repays as long as his payoff from continued production is sufficiently high relative to his anticipated repayment. Now, Lemma 8 demonstrates that Assumption 2.2.1 ensures that successful entrepreneurs continue their projects and thus make transfers to their creditors. The result emphasizes the importance of dynamic borrowing relationships; debtors repay their debts only because they anticipate future cash flows and must avoid early liquidation.

**Lemma 8.** *A $\tau$-entrepreneur plays $\zeta = \neg \mathrm{d}$ if and only if $\omega_t = \sigma(\tau)$.*

*Proof.* The proof is in appendix 2.8.3. Sufficiency follows from noting that if $F > 0$ an entrepreneur with no cash flow always diverts because otherwise he would forfeit $F$. Necessity results from bounding prices relative to cash flows using Lemma 5 and Assumption 2.2.1.

□

### 2.3.4 Collateral Multiplier

When an entrepreneur purchases investment capital, his stock of collateral expands, thus allowing him to borrow to acquire still more capital. This dual role of capital creates a multiplier effect whereby an increase in capital leads to a further increase in capital.

An investor accepts a $\tau$-entrepreneur's offer to borrow $\ell$ against the promise to repay $F = \infty$ whenever present value of the expected transfer—the probability of success times the value of collateral given success divided by the investors' discount rate—exceeds the value of the loan or the borrowing constraint

$$\ell \leq \mathbb{E}_t\left[T(F,k)\right] = \frac{P^\tau k}{3R} \tag{2.6}$$

is satisfied, where $k$ is the capital held by the entrepreneur. With fruit $w$ he can buy $w/p_t$ units of capital which he can pledge to borrow $P^\tau w/(3Rp_t)$ pounds, with which he will buy an additional $P^\tau_{t+1} w/(3Rp_t^2)$ units of capital, which, in turn, he can pledge to borrow. The entrepreneur may repeat this buy-pledge-borrow sequence ad infinitum. Viz., with fruit endowment $w$ an entrepreneur can acquire capital up to

$$\begin{aligned} k^\tau &= \frac{w}{p_t} + \left(\frac{P^\tau}{3Rp_t}\right)\frac{w}{p_t} + \left(\frac{P^\tau}{3R}\right)^2\frac{w}{p_t} + \cdots \\ &= \frac{w}{p_t}\sum_{n=0}^{\infty}\left(\frac{P^\tau}{3Rp_t}\right)^n \\ &= \frac{3w}{3p_t - P^\tau/R}. \end{aligned} \tag{2.7}$$

Proposition 9 below demonstrates that entrepreneurs' balance sheets stretch by a multiplier that depends only on their cyclicality; the proof arrives at the same formula as the series above as a solution of the linear system of binding budget and borrowing constraints.

**Proposition 9.** *A $\tau$-entrepreneur with endowment $w$ can hold assets worth up to $S^{\chi_t^\tau} w$ where*

$$S^\chi := \frac{3R}{3R-\chi}.$$

*Proof.* The maximum liability $\ell$ a $\tau$-entrepreneur can secure with capital $k$ is given by his binding borrowing constraint (inequality (2.6) holding with equality)

$$\ell = \frac{P^\tau k}{3R} \tag{2.8}$$

and the maximum capital an entrepreneur can obtain comes from his binding budget constraint given this loan,

$$p_t k = w + \ell. \tag{2.9}$$

Substituting $k$ from equation (2.9) into equation (2.8) gives

$$\begin{aligned}\ell &= \frac{P^\tau}{3Rp_t}\big(w + \ell\big)\\ &= \frac{\chi_t^\tau\big(w + \ell\big)}{3R}.\end{aligned}$$

Rearranging gives

$$\ell = \frac{\chi_t^\tau w}{3R - \chi_t^\tau} \tag{2.10}$$

and the asset value is

$$w + \mathrm{DC}^{\chi_t^\tau} = \frac{3Rw}{3R - \chi_t^\tau}.$$

□

The constant of proportionality $S^\chi$, called the collateral multiplier, describes the gross maximum feasible leverage of an entrepreneur with cyclicality $\chi$—his ability to lever up does not depend on his equity endowment $w$—

$$\frac{\text{assets}}{\text{equity}} \leq \frac{S^\chi w}{w} = \frac{3R}{3R - \chi}.$$

Figure 2.2 illustrates the maximal balance sheet expansion.

The expression for the maximum size of an entrepreneur's balance sheet immediately gives an expression for his maximal liability, or debt capacity $\mathrm{DC}^\chi$, which is likewise proportional to his endowment by a multiplier which depends on only cyclicality, as now stated in corollary 10.

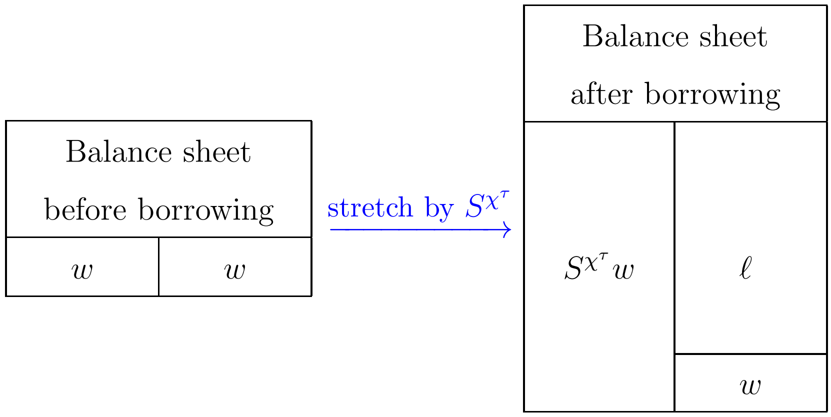


Figure 2.2: Entrepreneurs' balance sheets expand by up to the collateral multiplier $S^\chi$.

**Corollary 10.** *An entrepreneur with endowment $w$ has debt capacity*

$$\mathrm{DC}^\chi(w) = \frac{\chi\, w}{3R - \chi}.$$

The formula for the collateral multiplier reveals that cyclicality is valuable to entrepreneurs, granting them commitment power: the procyclical entrepreneurs can borrow more and invest more, as corollary 11 now states.

**Corollary 11.** *The multiplier $S^\chi$ and the debt capacity $\mathrm{DC}^\chi$ are increasing in entrepreneurs' cyclicality $\chi$.*

*Proof.* Immediate from differentiation of $S^\chi$ and $\mathrm{DC}^\chi$.

□

A procyclical borrower can not only borrow more than a countercyclical borrower initially, but he can also buy more capital with his loan and thus reuse his initial liquidity to lever up even further. Thus, the sensitivity of debt capacity to cyclicality increases in cyclicality, as stated formally in corollary 12. The observation offers an insight tangential to the main results: more levered firms are more sensitive to cyclicality $\chi_t^\tau = P^\tau / p_t$, and therefore must adjust their balance sheets more in response to fluctuations in the price $p_t$.

**Corollary 12.** *The multiplier $S^\chi$ and the debt capacity $\mathrm{DC}^\chi$ are convex in entrepreneurs' cyclicality $\chi$.*

*Proof.* Immediate from second differentiation of $S^\chi$ and $\mathrm{DC}^\chi$ and the bound $\chi < 3R$ from inequality (2.5).

□

Now, corollary 13 states the immediate result that, since debt capacity is proportional to equity, penniless entrepreneurs have no way to raise funds.

**Corollary 13.** *Entrepreneurs with endowment zero do not invest, i.e. $k_t^\beta = 0$ if $\omega_t \in \{a, 0\}$ and $k_t^\alpha = 0$ if $\omega_t \in \{b, 0\}$.*

Finally, the upper bound on entrepreneurs' ability to borrow combines with Assumption 2.2.2 (which says that entrepreneurs' endowments are not too large) to imply that entrepreneurs never hold all of the capital, ensuring an interior solution.

**Corollary 14.** *Entrepreneurs never hold all of the capital, $k_t^e < K$.*

*Proof.* The proof is in appendix 2.8.4. It supposes that entrepreneurs do hold all the capital in one state and uses the Markov assumption to tighten the lower bound on the price. It then combines the upper bound on balance sheet size (Proposition 9) with Assumption 2.2.2 for a contradiction. □

### 2.3.5 Entrepreneurs Borrow to Capacity

Entrepreneurs will always borrow to capacity. Since they consume only when they are old, they borrow as much as they can so long as expected repayments are not prohibitively high relative to capital prices today. To prefer strictly to borrow, entrepreneurs must be infra-marginal; that they never hold all of the capital (corollary 14) will suffice.

Any investor to whom an entrepreneur with capital $k$ offers $\mathsf{c} = (F, \ell)$ accepts if and only if

$$\frac{\min\{F\ ,\ P^{\tau}k\}}{3R} \geq \ell,$$

since the debtor repays only one-third of the time, when he succeeds. Each $t$-entrepreneur thus determines $k, F$, and $\ell$ to solve the programme of maximizing

$$\mathbb{E}_t\left[p_{t+1}k\right] + \frac{1}{3}\Big(3Ak - \min\{F\ ,\ P^{\tau}k\}\Big)$$

subject to

$$p_t k \leq w + \ell,$$
$$\ell \leq \frac{\min\{F\ ,\ P^{\tau}k\}}{3R}$$

(having omitted the time subscripts and player superscripts on the choice variables). The expectation in the objective embeds the value of liquidation in the state when the project succeeds as well as in both states when it fails.

**Lemma 15.** *$F \geq P^{\tau}k$.*

*Proof.* Since his objective is increasing in $k$, the entrepreneur's programme reduces to determining $k$ and $F$ to maximize

$$\bar{p}\,k + \frac{1}{3}\Big(3Ak - \min\{F\,,\;P^{\tau}k\}\Big)$$

subject to the borrowing constraint

$$p_t k \le w + \frac{\min\{F\,,\;P^{\tau}k\}}{3R}.$$

Now suppose (in anticipation of a contradiction) $F < P^{\tau}k$. The objective is increasing in $k$ and decreasing in $F$ so the constraint

$$p_t k \le w + \frac{F}{3R}$$

binds. The unconstrained objective is

$$\left(\frac{w}{p_t} + \frac{F}{3Rp_t}\right)\bar{p} + \left(\frac{w}{p_t} + \frac{F}{3Rp_t}\right)A - \frac{F}{3} = \frac{1}{p_t}\left[\left(\frac{A+\bar{p}}{R} - p_t\right)\frac{F}{3} + (A+\bar{p})w\right].$$

Equation (2.3) and the assumption that $\gamma' < A$ imply

$$p_t = \frac{\gamma'(k_t^i) + \bar{p}}{R} \le \frac{A+\bar{p}}{R}.$$

If the inequality is strict, then the objective is strictly increasing in $F$ so the solution contradicts the assumption $F < P^{\tau}k$.

The inequality must bind:

$$\frac{\gamma'(k_t^i) + \bar{p}}{R} = \frac{A+\bar{p}}{R}$$

or $\gamma'(k_t^i) = A$, so $k_t^i = 0$ and $k_t^e = K$, which contradicts corollary 14.

□

Pictorial Representation of the Equilibrium

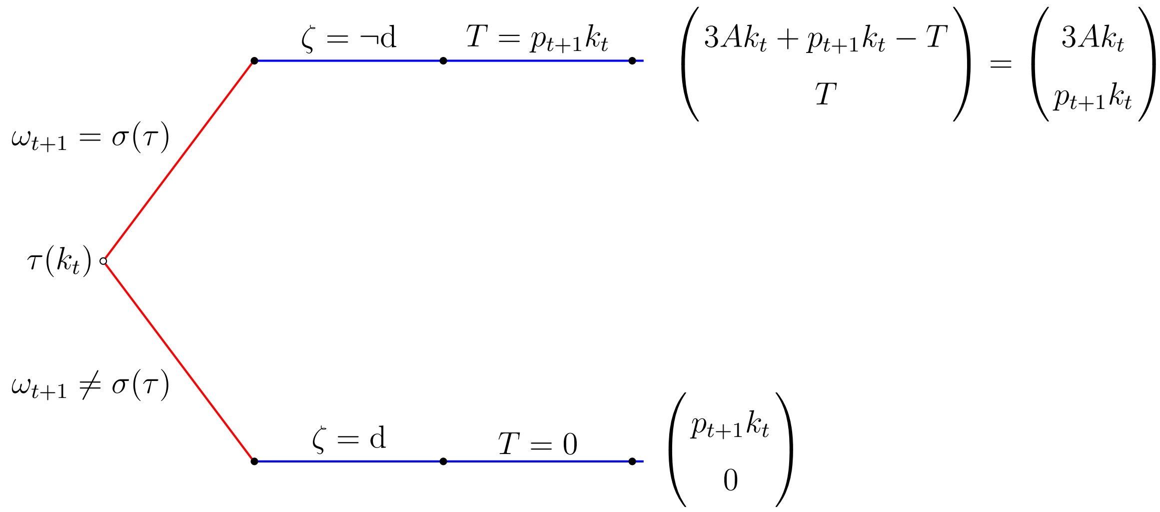


Figure 2.3: A reduced-form tree representation of the equilibrium of the stage game between an entrepreneur and his creditor. The tree incorporates the lemmata 8, 7, and 15. The entrepreneur's payoffs are above the creditor's in the payoff profiles.

### 2.3.6 Prices

Lemma 15 says entrepreneurs always borrow to capacity and equation (2.7) says entrepreneurs hold maximal capital, so, if $\omega_t = a$,

$$k_t^e = S^{\chi_t^\alpha} w / p_t = \frac{3w}{3p^a - P^\alpha / R},$$

and if $w_t = b$

$$k_t^e = S^{\chi_t^\beta} w / p_t = \frac{3w}{3p^b - P^\beta / R}.$$

Corollary 13 says that only $\alpha$ entrepreneurs invest in state $a$ and only $\beta$ entrepreneurs invest in state $b$, so if $\omega \in \{a, b\}$ then

$$k_t^i = K - S^\tau w / p_t,$$

and if $\omega = 0$ then $k_t^e = 0$ and $k_t^i = K$. The equilibrium price system now follows from equation (2.3), establishing Proposition 16 below.

**Proposition 16.** *In equilibrium, the prices solve*

$$Rp^a = \bar{p} + \gamma'\big(K - S^{\chi_t^\alpha} w/p^a\big), \tag{2.11}$$

$$Rp^b = \bar{p} + \gamma'\big(K - S^{\chi_t^\beta} w/p^b\big), \tag{2.12}$$

$$Rp^0 = \bar{p} + \gamma'(K). \tag{2.13}$$

**Proposition 17.** *The system (2.11)-(2.13) has a solution (a Markov equilibrium exists).*

*Proof.* The proof is in appendix 2.8.5. It recasts the system (2.11)-(2.13) as a fixed point problem in order to apply Brouwer's theorem after some massaging to ensure the image is compact despite the singularities in the denominator of $S^{\chi_t^\tau}$. □

Analysis of the system in Proposition 16 gives the next main result: when $\alpha$-entrepreneurs have positive endowments prices are higher than when $\beta$-entrepreneurs have positive endowments. Procyclicality, not insurance, is the valuable resource in this economy.

**Proposition 18.**

$$p^0 < p^b < p^a.$$

*Proof.* The proof is in two steps.

Step 1: Lemma 6 implies immediately that

$$\chi_t^\tau < 3R$$

so $S^{\chi} > 0$, giving that

$$p^0 < \min\left\{p^a, p^b\right\}$$

by $\gamma'' < 0$.

Step 2: Suppose (in anticipation of a contradiction) that $p^b \geq p^a$ so

$$Rp^b - Rp^a = \gamma'\left(K - \frac{3Rw}{3Rp^b - p^0}\right) - \gamma'\left(K - \frac{3Rw}{3Rp^a - p^a}\right) \geq 0,$$

having subtracted equation (2.11) from equation (2.12). Or, equivalently, by $\gamma'' < 0$,

$$K - \frac{3Rw}{3Rp^b - p^0} \leq K - \frac{3Rw}{3Rp^a - p^a}.$$

Since the denominators are positive by Lemma 6,

$$3Rp^a - p^a \geq 3Rp^b - p^0.$$

Rewrite to see that

$$3R(p^a - p^b) \geq p^a - p^0 > 0,$$

where the final inequality follows from step 1 and implies that $p^a > p^b$, a contradiction.

□

## 2.4 Benchmarks

### 2.4.1 Complete Markets/Perfect Enforcement

Since agents are risk-neutral, with no enforcement problems the most productive agents hold all of the capital. The marginal return on capital is $A$ in every state, because investors' technologies don't change. Proposition 19 now follows.

**Proposition 19.** *With perfect enforcement,*

$$p^a = p^b = p^0 = \frac{A}{R-1}.$$

There is no aggregate price risk in the economy.

### 2.4.2 No Borrowing

When agents cannot borrow at all, entrepreneurs spend their endowments and only their endowments on capital. In states $a$ and $b$ their (binding) budget constraints read

$$p^{\omega} k^{e,\omega} = w,$$

so $k^{e,a} = 1/p^a$, $k^{e,b} = 1/p^b$, and $k^{e,0} = 0$. The pricing equation (2.3) implies

$$Rp^a = \bar{p} + \gamma'\Big(K - w/p^a\Big), \tag{2.14}$$

$$Rp^b = \bar{p} + \gamma'\left(K - w/p^b\right), \tag{2.15}$$

$$Rp^0 = \bar{p} + \gamma'\left(K\right).$$

**Proposition 20.** *With no borrowing,* $p^a = p^b$*.*

*Proof.* Suppose (in anticipation of a contradiction) that $p^a > p^b$. Subtracting equation (2.15) from (2.14) implies

$$\gamma'\Big(K - w/p^a\Big) > \gamma'\left(K - w/p^b\right) > 0$$

and, since $\gamma'$ is decreasing, $p^b > p^a$, a contradiction. Thus $p^b \leq p^a$. Repeating the argument supposing $p^b > p^a$ gives the result. $\square$

### 2.4.3 Renegotiation without Capital Diversion

With renegotiation but not capital diversion borrowers repay the value of their capital in each state,

$$T = p_{t+1}k_t^e$$

so the binding borrowing constraint reads

$$\ell = \frac{\bar{p}k_t^e}{R}$$

and the budget constraint implies

$$p^{\omega}k^{e,\omega} = w + \ell = w + \frac{\bar{p}k^{e,\omega}}{R}$$

if $\omega \in \{a, b\}$ and $k^{e,0} = 0$. The price system is now

$$Rp^a = \bar{p} + \gamma'\left(K - \frac{Rw}{Rp^a - \bar{p}}\right), \tag{2.16}$$

$$Rp^b = \bar{p} + \gamma'\left(K - \frac{Rw}{Rp^b - \bar{p}}\right), \tag{2.17}$$

$$Rp^0 = \bar{p} + \gamma'(K).$$

**Proposition 21.** *Without capital diversion, $p^a = p^b$.*

*Proof.* The proof is in appendix 2.8.6. It is almost identical to the proof of Proposition 20. ☐

### 2.4.4 Capital Diversion without Renegotiation

If borrowers divert capital when it is profitable but never renegotiate their debts, they repay only when they succeed, with repayments capped by incentive constraints, when they play $\zeta = \neg\mathrm{d}$ whenever

$$3Ak_t + p_{t+1}k_t - T \geq p_{t+1}k_t$$

or $T \leq 3Ak_t$. The proof of Lemma 15, stating that entrepreneurs assume maximum leverage, implies here that entrepreneurs set the maximum face value that will induce repayment, or $F = 3Ak_t$. As in the full model, only entrepreneurs with positive endowments can borrow in equilibrium, but the result no longer follows from the formula (2.10) for entrepreneurs' debt capacity and requires a separate proof.

**Lemma 22.** *Without renegotiation, entrepreneurs with zero endowment do not borrow.*

*Proof.* The proof is in two steps. Step 1 demonstrates that if $t$ prices are low, entrepreneurs are never constrained. Step 2 shows without constraints prices are high, a contradiction.

Step 1: A $\tau$-entrepreneur with capital $k$ repays nil when he fails and at most $3Ak$ when he succeeds, so his binding borrowing constraint gives his maximal liability,

$$\ell = \frac{Ak}{R}.$$

If his endowment is nil, his budget constraint reads

$$p_t k \leq \frac{Ak}{R}.$$

If $p_t \leq A/R$ he is unconstrained and if $p_t > A/R$ he cannot borrow.

Step 2: Suppose (in anticipation of a contradiction) that $p_t \leq A/R$. Call the state $\omega$ so $p_t = p^\omega$. Then entrepreneurs are unconstrained and the pricing equation (2.3) gives

$$\begin{aligned} p_t = p^\omega &= \frac{A + \bar{p}}{R} \\ &= \frac{3A + p^a + p^b + p^0}{3R} \\ &\geq \frac{3A + p^\omega}{3R} \end{aligned}$$

which combines with the hypothesis to give

$$p^{\omega} \geq \frac{3A}{3R-1} > \frac{A}{R} \geq p^{\omega},$$

a contradiction.

Therefore $p_t < A/R$ and entrepreneurs without endowments cannot borrow.

□

The price system without renegotiation follows from entrepreneurs' borrowing to capacity: if $\omega \in \{a, b\}$ then

$$p^{\omega} k^{e,\omega} = w + \frac{Ak^{e,\omega}}{R}$$

or

$$k^{e,\omega} = \frac{Rw}{Rp^{\omega} - A}$$

and if $\omega = 0$ then $k^{e,0} = 0$.

$$Rp^a = \bar{p} + \gamma'\Big(K - \frac{Rw}{Rp^a - A}\Big), \tag{2.18}$$

$$Rp^b = \bar{p} + \gamma'\left(K - \frac{Rw}{Rp^b - A}\right), \tag{2.19}$$

$$Rp^0 = \bar{p} + \gamma'(K).$$

**Proposition 23.** *Without renegotiation, $p^a = p^b$.*

*Proof.* The proof is in appendix 2.8.7. It is almost identical to the proofs of Proposition 20 and Proposition 21.

□

## 2.5 Welfare and Policy

### 2.5.1 Welfare

The expectation of $t$-output (isomorphic to utilitarian welfare thanks to transferable utility) is

$$\begin{aligned} W_t :=& \mathbb{E}\left[\alpha\left(k_t^\alpha\right)+\beta\left(k_t^\beta\right)+\gamma\left(k_t^\gamma\right)\right] \\ =& \frac{1}{3}\left(\frac{3ARw}{3Rp^a-p^a}+\gamma\left(K-\frac{3Rw}{3Rp^a-p^a}\right)+\right. \\ &\left.+\frac{3ARw}{3Rp^b-p^0}+\gamma\left(K-\frac{3Rw}{3Rp^b-p^0}\right)+\gamma(K)\right). \end{aligned}$$

If a $t$-entrepreneur is equally likely to be type-$\alpha$ or type-$\beta$, increases in output are ex ante Pareto improvements—all unborn entrepreneurs are better off.

### 2.5.2 Taxes and Subsidies

Allocating more capital to entrepreneurs increases welfare because it allows the most productive agents to invest more. Reallocating wealth only among entrepreneurs may also lead to an ex ante Pareto improvement (in the sense just described in section 2.5.1 above). A social planner who must break even in expectation can levy a tax $\varepsilon$ on $\beta$-entrepreneurs in state $b$ and subsidize $\alpha$-entrepreneurs in state $a$, making welfare

$$\begin{aligned} W_t(\varepsilon) :=& \frac{1}{3}\left(\frac{3AR(w+\varepsilon)}{3Rp_\varepsilon^a-p_\varepsilon^a}+\gamma\left(K-\frac{3R(w+\varepsilon)}{3Rp_\varepsilon^a-p_\varepsilon^a}\right)+\right. \\ &\left.+\frac{3AR(w-\varepsilon)}{3Rp_\varepsilon^b-p_\varepsilon^0}+\gamma\left(K-\frac{3R(w-\varepsilon)}{3Rp_\varepsilon^b-p_\varepsilon^0}\right)+\gamma(K)\right). \end{aligned}$$

Subscripts now denote values of the transfer $\varepsilon$ (and no longer time). A dot above a variable denotes the rate of change with respect to the tax level, $\dot{x} := dx/d\varepsilon$. The shorthands

$$\gamma'_a := \gamma'\left(K - \frac{3Rw}{(3R-1)p_0^a}\right), \qquad \gamma''_a := \gamma''\left(K - \frac{3Rw}{(3R-1)p_0^a}\right),$$
$$\gamma'_b := \gamma'\left(K - \frac{3Rw}{3Rp_0^b - p_0^0}\right), \qquad \gamma''_b := \gamma'\left(K - \frac{3Rw}{3Rp_0^b - p_0^0}\right)$$

save space below.

The next result, Lemma 24, gives a necessary and sufficient condition for a transfer from $\beta$-entrepreneurs to $\alpha$-entrepreneurs to increase welfare.

**Lemma 24.** $\dot{W}_t(0) > 0$ *if and only if*

$$1 - w\,\frac{\dot{p}_0^a}{p_0^a} > -\frac{A - \gamma'_a}{A - \gamma'_b}\,\frac{(3R-1)p_0^a}{3Rp_0^b - p_0^0}\left(1 + w\,\frac{3R\dot{p}_0^b - \dot{p}_0^0}{3Rp^b - p^0}\right). \tag{2.20}$$

*Proof.* Differentiating $W_t$ gives

$$\frac{d}{d\varepsilon}\Big|_{\varepsilon=0}\frac{w+\varepsilon}{(3R-1)p_\varepsilon^a} > \frac{A - \gamma_b{}'}{A - \gamma_a{}'}\,\frac{d}{d\varepsilon}\Big|_{\varepsilon=0}\frac{w-\varepsilon}{3Rp_\varepsilon^b - p_\varepsilon^0}.$$

Applying the quotient rule and rearranging gives the result.

□

$\alpha$-entrepreneurs borrow more efficiently than $\beta$-entrepreneurs, so transferring a pound from a $\beta$-entrepreneur to an $\alpha$-entrepreneur increases efficient capital investment. This direct effect means that so long as the indirect price effects, which in turn determine changes in balance sheet capacity, are not too large, a social planner indeed wishes to transfer wealth to procyclical entrepreneurs in aggregate. A sufficient condition is that entrepreneurs' wealth is not too large, as stated in Proposition 25 presently.

**Proposition 25.** *If $w$ is small, a marginal transfer from $\beta$-entrepreneurs to $\alpha$-entrepreneurs increases welfare, i.e.* $\dot{W}_t(0) > 0$.

*Proof.* Since the coefficient on the right-hand side of inequality (2.20) is negative,

$$-\frac{A-\gamma'_a}{A-\gamma'_b}\frac{(3R-1)p_0^a}{3Rp_0^b-p_0^0}<0,$$

as long as the ratios

$$\frac{\dot{p}_0^a}{p_0^a} \quad \text{and} \quad \frac{3R\dot{p}_0^b-\dot{p}_0^0}{3p_0^b-p_0^0}$$

making $w$ small ensures the condition is satisfied. Since

$$\frac{\gamma'(K)}{R-1}\leq p^\omega\leq\frac{A}{R-1},$$

it suffices to show that $\dot{p}_0^w$ is finite. Perturbing the price system (2.11)-(2.13) and differentiating with respect to $\varepsilon$ about $\varepsilon=0$ reveals that $(\dot{p}_0^a,\dot{p}_0^b,\dot{p}_0^0)$ solves the linear system

$$\begin{pmatrix} 3R-1-\frac{9R\gamma''_a w}{(3R-1)(p_0^a)^2} & -1 & -1 \\ -1 & 3R-1-\frac{27R^2\gamma''_b w}{(3Rp_0^b-p_0^0)^2} & \frac{9R\gamma''_b w}{(3Rp_0^b-p_0^0)^2}-1 \\ -1 & -1 & 3R-1 \end{pmatrix}\begin{pmatrix}\dot{p}_0^a\\ \dot{p}_0^b\\ \dot{p}_0^0\end{pmatrix}=\begin{pmatrix}-\frac{9R\gamma''_a}{(3R-1)p_0^a}\\ \frac{9R\gamma''_b}{3Rp_0^b-p_0^0}\\ 0\end{pmatrix},$$

which is well-defined for any $(p_0^a,p_0^b,p_0^0)$ satisfying the bounds (2.5.2) and any $w$.

□

## 2.6 Predictions

### 2.6.1 Framework and Definitions

This section recasts the model in terms of (theoretically) measurable quantities to state some testable implications. As emphasized, the interaction between the two kinds of limited enforcement—the inability to commit not to renegotiate debt and not to divert capital—effects all of the main results; therefore, the predictions below apply when enforcement frictions are very important, for example in developing countries in which creditor rights are weak and enforcement is costly.

Define the return on capital as the price ratio,

$$r_{t+1} := \frac{p_{t+1}}{p_t}$$

and call its one-step-ahead expectation the expected return,

$$\bar{r}_t := \mathbb{E}_t \left[ r_{t+1} \right].$$

The "beta" of an entrepreneur's project $\tau$ is its linear projection on to capital returns,

$$\text{beta}_t^\tau := \frac{\text{Cov}_t \left[ r_{t+1}, \tau \right]}{\text{Var}_t \left[ r_{t+1} \right]}.$$

Since the equilibrium is Markov, the conditional variance of returns is constant. Define

$$\Sigma := \text{Var}_t \left[ r_{t+1} \right]$$

to write

$$\text{beta}_t^\tau := \frac{\text{Cov}_t \left[ r_{t+1}, \tau \right]}{\Sigma}$$

and compute the covariance:

$$\begin{aligned}
\text{Cov}_t \left[ r_{t+1}, \tau \right] &= \mathbb{E}_t \left[ r_{t+1} \tau \right] - \mathbb{E}_t \left[ r_{t+1} \right] \mathbb{E}_t \left[ \tau \right] \\
&= \mathbb{E}_t \left[ \frac{p_{t+1}}{p_t} \mathbb{1}_{\{\omega_{t+1} = \sigma(\tau)\}} \tau \right] - \mathbb{E}_t \left[ r_{t+1} \right] \mathbb{E}_t \left[ \tau \right] \\
&= \frac{1}{3} \frac{P^\tau}{p_t} 3A - A \bar{r}_t \\
&= A \big( \chi_t^\tau - \bar{r}_t \big)
\end{aligned}$$

(having made use of the success indicator notation 2). The next lemma summarizes the calculation and reveals that $\text{beta}_t^\tau$ proxies for cyclicality in the model.

**Lemma 26.**

$$\text{beta}_t^\tau = \frac{A(\chi_t^\tau - \bar{r}_t)}{\Sigma}.$$

Total expected output is

$$\mathbb{E}_t\left[\text{output}_{t+1}\right] = Ak_t^e + \gamma(K - k_t^e)$$

so productivity (normalized by $K$) is

$$\text{productivity}_t := A + \gamma'(K - k_t^e). \tag{2.21}$$

Since capital is the only durable asset in the economy and an increasing bijection maps prices to expected output, use capital prices to proxy for the state of the economy,

$$\text{market}_t := Kp_t.$$

Now since $p_t$ is high exactly when $k^e$ is high (because $\gamma' < A$), expected output is high exactly when the market is high. Call date $t$ a "boom" if $\text{market}_t$ is high.

The asset value or size of an entrepreneur's enterprise is the sum of his equity endowment $w$ and the present value of his debt $\ell_t^\tau$,

$$\text{size}_t^\tau := w + \ell_t^\tau.$$

A $\tau$-entrepreneur's (gross) leverage is his size divided by his equity,

$$\text{leverage}_t^\tau := \frac{\text{size}_t^\tau}{w}.$$

### 2.6.2 Correlations

Since debt capacity is increasing in cyclicality and entrepreneurs are always maximally levered (Lemma 15 and Corollary 11), size and leverage are increasing in beta.

**Prediction 2.6.2.1.** *Size is increasing in cyclicality.*

*In the cross-sectional regression*

$$\text{size}_t^\tau = \beta\,\text{beta}_t^\tau + \varepsilon_t,$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$.*

**Prediction 2.6.2.2.** *Leverage is increasing in cyclicality.*

*In the cross-sectional regression*

$$\text{leverage}_t^\tau = \beta\,\text{beta}_t^\tau + \varepsilon_t,$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$.*

Booms occur when procyclical agents can borrow, giving the analogous predictions in the time-series.

**Prediction 2.6.2.3.** *Average size is high in booms.*

*In the time-series regression*

$$\overline{\text{size}}_t = \beta\,\text{market}_t + \varepsilon_t,$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$.*

**Prediction 2.6.2.4.** *Average leverage is high in booms.*

*In the time-series regression*

$$\overline{\text{leverage}}_t = \beta\,\text{market}_t + \varepsilon_t,$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$.*

The procyclicality of constrained firms' leverage is well-documented empirically. See, for example, Korajczyk and Levy (2003).

From the proof of Proposition 18, entrepreneurs hold more capital in $a$-states than in $b$-states and more capital in $b$-states than in 0-states:

$$K > K - \frac{3Rw}{3Rp^b - p^0} > K - \frac{3Rw}{(3R-1)p^a}$$

or

$$k^{e,0} < k^{e,b} < k^{e,a}.$$

Thus, immediately from the definition (equation (2.21)), productivity is high in booms.

**Prediction 2.6.2.5.** *Productivity is high in booms.*

*In the time-series regression*

$$\text{productivity}_t = \beta \text{market}_t + \varepsilon_t,$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$.*

This prediction volunteers an explanation of the puzzle of procyclical productivity originating with Hall (1988). I think the explanation that productivity is higher because of loosening borrowing constraints improves capital allocation—that resources flow more efficiently to productive firms in booms than in recessions due to increased debt capacity—may be new.

### 2.6.3 A Natural Experiment

Since the collateral multiplier $S^\chi$ is increasing in cyclicality (corollary 11), procyclical entrepreneurs' balance sheets are more sensitive to their endowments than are countercyclical entrepreneurs'. A shock to endowments, resulting, for example, from foreign capital flowing into a newly opened economy, provides a natural experiment for difference-in-differences analysis of the model's predictions.

Specifically, suppose that a uniform, unanticipated positive shock to endowments occurs at date $t^*$ so that all endowments are $\overline{w}$ before or at $t^*$ and are $\underline{w} < \overline{w}$ after $t^*$. Proposition 9 and Lemma 15 say that

$$\text{size}_t^\tau = S^{\chi_t^\tau} w = \frac{3Rw}{3R - \chi_t^\tau},$$

so if $H$ and $L$ denote entrepreneurs with cyclicality $\chi^H$ and $\chi^L$ respectively

$$\begin{aligned}\left(\text{size}_{t+1}^H - \text{size}_{t+1}^L\right) - \left(\text{size}_t^H - \text{size}_t^L\right) &= \left(S^{\chi^H}\overline{w} - S^{\chi^L}\overline{w}\right) - \left(S^{\chi^H}\underline{w} - S^{\chi^L}\underline{w}\right) \\ &= \left(\frac{3R}{3R-\chi^H} - \frac{3R}{3R-\chi^L}\right)\left(\overline{w} - \underline{w}\right) > 0,\end{aligned}$$

and

$$\left(\text{debt}_{t^*+1}^H - \text{debt}_{t^*+1}^L\right) - \left(\text{debt}_{t^*}^H - \text{debt}_{t^*}^L\right) = \left(\frac{\chi^H}{3R-\chi^H} - \frac{\chi^L}{3R-\chi^L}\right)\left(\overline{w} - \underline{w}\right) > 0.$$

which immediately give the following predictions for the panel regressions of size and leverage against cyclicality.

**Prediction 2.6.3.1.** *Positive shocks to endowments increase the size of procyclical firms more than of countercyclical firms.*

*If a positive shock to capital occurs at time $t^*$, then in the panel regression*

$$\text{size}_t^\tau = \alpha + \beta \mathbb{1}_{\left\{\text{beta}_t^\tau \geq \text{beta}^*\right\}} \mathbb{1}_{\left\{t \geq t^*\right\}} + \gamma \mathbb{1}_{\left\{\text{beta}_t^\tau \geq \text{beta}^*\right\}} + \delta \mathbb{1}_{\left\{t \geq t^*\right\}},$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$, for any* beta$^*$.

**Prediction 2.6.3.2.** *Positive shocks to endowments increase the leverage of procyclical firms more than of countercyclical firms.*

*If a positive shock to capital occurs at time $t^*$, then in the panel regression*

$$\text{leverage}_t^\tau = \alpha + \beta \mathbb{1}_{\left\{\text{beta}_t^\tau \geq \text{beta}^*\right\}} \mathbb{1}_{\left\{t \geq t^*\right\}} + \gamma \mathbb{1}_{\left\{\text{beta}_t^\tau \geq \text{beta}^*\right\}} + \delta \mathbb{1}_{\left\{t \geq t^*\right\}},$$

*the estimate of the coefficient $\beta$ is positive, $\hat{\beta} > 0$, for any* $\text{beta}^*$.

## 2.7 Conclusions

The contractual frictions of capital diversion and renegotiation interact so as to make procyclicality a valuable resource for entrepreneurs—it increases their power to commit to repay loans and therefore allows them to lever up. Borrowers' inability to commit not to renegotiate loans makes collateral valuable to creditors even when their debtors are not near bankruptcy, because it determines creditors' seizure value during renegotiation. Borrowers' incentive to divert capital in anticipation of default—to line their own pockets and avoid handing over good quality assets to their creditors—decreases the quantity and quality of assets that creditors can liquidate when they repossess a firm. The threats of renegotiation and capital diversion interact. They make collateral relatively more valuable to creditors when debtors' projects succeed than when they fail. Creditors, therefore, value the comovement between liquidation values and borrower success: debtor procyclicality is a valuable resource for creditors because it allows them to enforce repayment. Since creditors can enforce repayment of loans to procyclical borrowers most effectively ex post, they are willing to lend to them ex ante. Thus procyclicality is a valuable resource for borrowers: it grants them the power to commit and allows them to lever up.

The main new insight is that output procyclicality can loosen borrowing constraints. Further, the mechanism does not affect only individual firms, but has implications for the aggregate economy. When productive agents are procyclical, they are unconstrained and can borrow to buy capital, thereby effecting efficient capital allocation. Capital prices increase to reflect the productivity of capital in its best use. In contrast, when the productive agents are countercyclical, their ability to borrow is limited and they cannot acquire the capital that they require to produce. Hence, capital remains poorly allocated and, since the marginal buyer may be relatively unproductive, capital prices are low. This mechanism questions the literature's conclusion that procyclical assets necessarily trade at a discount since they add market risk to investors' portfolios. Procyclical assets can also mitigate enforcement frictions leading them to demand a collateral premium.

Financing frictions matter for the macroeconomy not only because they determine the allocation and price of capital but also because they generate endogenous fluctuations in productivity and expected output. When the two frictions of capital diversion and renegotiation are both present in the economy, capital prices and expected output fluctuate even when they do not in the benchmark economies with one or both of the fractions removed. Thus the interaction between capital diversion and renegotiation creates an endogenous component of the business cycle.

In accordance with many papers in the literature (e.g., Lorenzoni (2008), Stein (2012)), my welfare analysis suggests that policy makers must take firms' financing constraints into account. But, in contrast to these models, in my model leverage is inefficiently low because entrepreneurs' endogenous private borrowing limits stifle efficient capital allocation. To increase aggregate efficiency, the government must provide liquidity to firms that make the best use of their capital as collateral—those firms that can stretch their endowments and lever up. When contractual enforcement is limited by capital diversion and renegotiation, the government should subsidize procyclical firms. Since limits to lending result from borrowers' inability to commit to repay, subsidizing private lenders—injecting capital into the banking sector—may not increase lending or aid capital allocation. Direct subsidies to procyclical entrepreneurs are necessary.

## 2.8 Appendices

### 2.8.1 Proof of Lemma 5

The result follows from $\gamma'' < 0$:

$$\begin{aligned}
p_t &= \frac{1}{R}\big(\gamma'(k^i) + \mathbb{E}_t\left[p_{t+1}\right]\big) \\
&\leq \frac{1}{R}\big(\gamma'(0) + \mathbb{E}_t\left[p_{t+1}\right]\big) \\
&\leq \frac{\gamma'(0)}{R} + \frac{\gamma'(0)}{R^2} + \frac{\mathbb{E}_t\left[p_{t+2}\right]}{R} \\
&\leq \gamma'(0)\left(\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \cdots\right) \\
&= \frac{\gamma'(0)}{R-1} \\
&= \frac{A}{R-1}.
\end{aligned}$$

### 2.8.2 Proof of Lemma 7

An entrepreneur never plays $F' > F$ because $F' = F$ induces the same action ($\xi = \mathrm{a}$) and gives him a higher payoff. Suppose $F' < F$.

If $F' < p_{t+1}k$ the creditor plays $\xi = \neg\mathrm{a}$, leaving the entrepreneur with nil, so $F' \geq p_{t+1}k$. If $F' > p_{t+1}k$ then $\xi = \mathrm{a}$, but $F'' = (F' + p_{t+1}k)/2$ is superior for the debtor and $\xi = \mathrm{a}$ still, so $F' \leq p_{t+1}k$. Thus if $F' < F$ then $F' = p_{t+1}k$ and $F' = F$ otherwise, which is to say $F' = \min\{F\,,\,p_tk\}$.

### 2.8.3 Proof of Lemma 8

If $\omega_{t+1} \neq \sigma(\tau)$, then $\zeta = \neg\mathrm{d}$ yields $p_{t+1}k_t - \min\{F\,,\,p_{t+1}k_t\}$ and $\zeta = \mathrm{d}$ yields $p_{t+1}k_t$, but

$$p_{t+1}k_t - \min\{F\,,\,p_{t+1}k_t\} \geq p_{t+1}k_t$$

only if $F \leq 0$ (which, due to Lemma 7 above, implies he would have no debt), so a failing entrepreneur always plays $\zeta = \mathrm{d}$.

If $\omega_{t+1} = \sigma(\tau)$, $\zeta = \neg\mathrm{d}$ yields $3Ak_t + p_{t+1}k_t - \min\{F\,,\, p_{t+1}k_t\}$ and $\zeta = \mathrm{d}$ yields $p_{t+1}k_t$; rearranging implies the entrepreneur does not abscond so long as

$$\min\{F\,,\, p_{t+1}k_t\} < 3Ak_t$$

which holds since

$$\begin{aligned}\min\{F\,,\, p_{t+1}k_t\} &\leq p_{t+1}k_t \\ &\leq p_{t+1}k_t \\ &\leq \frac{Ak_t}{R-1} \\ &< \frac{Ak_t}{4/3-1} \\ &= 3Ak_t,\end{aligned}$$

where the last inequality follows from Assumption 2.2.1.

### 2.8.4 Proof of Lemma 14

Suppose (in anticipation of a contradiction) that at $\omega \in \{a, b, 0\}$ $k_t^e = K$ and consequently $k_t^i = 0$. Equation (2.3) gives the price

$$p^{\omega} = \frac{\gamma'(0) + \bar{p}}{R} = \frac{A + \bar{p}}{R}.$$

For $\omega' \neq \omega$,

$$p^{\omega'} = \frac{\gamma'(k_t^i) + \bar{p}}{R} \geq \frac{\bar{p}}{R}.$$

Since, from above,

$$
\begin{aligned}
3\bar{p} &= p^a + p^b + p^0 \\
&= \frac{A+\bar{p}}{R} + \frac{2\,\bar{p}}{R} \\
&= \frac{A+3\bar{p}}{R}, \\
\bar{p} &\geq \frac{A/3}{R-1}
\end{aligned}
$$

and

$$
p^{\omega'} \geq \frac{A/3}{R(R-1)}.
$$

Combine this inequality with equation (2.7) above to compute:

$$
\begin{aligned}
k_t^e &\leq \frac{3Rw}{3Rp^{\omega} - P^{\tau}} \\
&= \frac{3Rw}{3(A+\bar{p}) - P^{\tau}} \\
&= \frac{3Rw}{3A + p^a + p^b + p^0 - P^{\tau}} \\
&\leq \frac{3Rw}{3A + 2\min\{p^a, p^b, p^0\}} \\
&\leq \frac{3Rw}{3A + 2\frac{A/3}{R(R-1)}} \\
&< \frac{Rw}{A} \\
&\leq K,
\end{aligned}
$$

by Assumption 2.2.2, contradicting $k_t^e = K$.

### 2.8.5 Proof of Lemma 17

The proof recasts solutions of system (2.11)-(2.13) as fixed points of a continuous mapping from a closed ball to itself and applies Brouwer's theorem.

For simplicity, employ the convention that a real number divided by zero is infinity and that a real number minus infinity is minus infinity—$x/0 = \infty$ and $x - \infty = -\infty$.

First, extend $\gamma$ to the extended real line by defining the function $\bar{\gamma} : \mathbb{R} \cup \{-\infty, \infty\} \to \mathbb{R}$ via

$$\bar{\gamma}'(k) := \begin{cases} \gamma'(0) & \text{if } k < 0, \\ \gamma'(k) & \text{if } k \in [0, K], \\ \gamma'(K) & \text{if } k > K. \end{cases}$$

$\bar{\gamma}'$ inherits monotonicity from $\gamma$.

Now define the compact domain

$$\Omega := \left\{ \left(p^a, p^b, p^0\right) \in \mathbb{R}^3 \;\middle|\; 0 \le p^\omega \le \frac{A}{R-1},\ p^\omega \le 3Rp^{\omega'} \text{ for all } \omega, \omega' \right\}$$

and the function $\Gamma : \Omega \to \mathbb{R}^3$ by the action

$$\Gamma : \begin{pmatrix} p^a \\ p^b \\ p^0 \end{pmatrix} \mapsto \frac{1}{R} \begin{pmatrix} \dfrac{p^a + p^b + p^0}{3} + \bar{\gamma}'\left(K - \dfrac{3Rw}{3Rp^a - p^a}\right) \\ \dfrac{p^a + p^b + p^0}{3} + \bar{\gamma}'\left(K - \dfrac{3Rw}{3Rp^b - p^0}\right) \\ \dfrac{p^a + p^b + p^0}{3} + \bar{\gamma}'\left(K\right) \end{pmatrix}.$$

Away from the singular points of the argument of $\bar{\gamma}'$, continuity of $\Gamma$ is immediate. In their neighbourhoods, namely as $p^a \searrow 0$ or $p^0 \nearrow 3Rp^b$, $\bar{\gamma}'$ is flat since the argument is negative, $\bar{\gamma}' \equiv \gamma'(0)$, giving continuity.

Now observe that $\Gamma(\Omega) \subset \Omega$ because $\bar{\gamma}'$ is decreasing. Since $\bar{\gamma}' \le A$ and

$$\bar{p} \le \max\left\{p^a, p^b, p^c\right\} \le \frac{A}{R-1},$$

for any $\omega \in \{a, b, 0\}$,

$$\begin{aligned} p^\omega &\leq \frac{A/(R-1) + \gamma'(0)}{R} \\ &= \frac{A}{R}\left(\frac{1}{R-1} + 1\right) \\ &= \frac{A}{R-1} \end{aligned}$$

and

$$p^\omega \geq \frac{\gamma'(K)}{R} \geq 0,$$

or $0 \leq p^\omega \leq A/(R-1)$. Finally, since $\bar{\gamma}\,' > 0$,

$$3Rp^\omega \geq p^a + p^b + p^0 \geq \max\left\{p^a, p^b, p^0\right\},$$

thus $3Rp^\omega \geq p^{\omega'}$ for any $\omega$ and $\omega'$ and $\Gamma : \Omega \to \Omega$. $\Gamma$ has a fixed point by Bouwer's theorem. The point solves (2.11)–(2.13)—in which $\gamma\,'$ replaces $\bar{\gamma}\,'$—so long as $\gamma$ is well-defined there, namely if entrepreneurs' capital is indeed nonnegative and not greater than the total supply. Positivity is immediate from $S^\chi \geq 0$ and corollary 14 (the proof of which depends only on the bounds on $\gamma\,'$, which coincide with those on $\bar{\gamma}\,'$) implies $k^e < K$. A fixed point exists.

### 2.8.6 Proof of Proposition 21

Suppose (in anticipation of a contradiction) $p^a > p^b$. Subtracting equation (2.17) from equation (2.16) implies

$$\gamma\,'\left(K - \frac{Rw}{Rp^a - \bar{p}}\right) - \gamma\,'\left(K - \frac{Rw}{Rp^b - \bar{p}}\right) > 0$$

and, since $\gamma\,'$ is decreasing,

$$\frac{Rw}{Rp^a - \bar{p}} > \frac{Rw}{Rp^b - \bar{p}}$$

or $p^b > p^a$, a contradiction. Thus $p^b \leq p^a$. Repeating the argument supposing $p^b > p^a$ gives the result.

### 2.8.7 Proof of Proposition 23

Suppose (in anticipation of a contradiction) $p^a > p^b$. Subtracting equation (2.19) from equation (2.18) implies

$$\gamma'\left(K - \frac{Rw}{Rp^a - A}\right) - \gamma'\left(K - \frac{Rw}{Rp^b - A}\right) > 0$$

and, since $\gamma'$ is decreasing,

$$\frac{Rw}{Rp^a - A} > \frac{Rw}{Rp^b - A}$$

or $p^b > p^a$, a contradiction. Thus $p^b \leq p^a$. Repeating the argument supposing $p^b > p^a$ gives the result.

# Chapter 3

# The Downside of Public Information in Contracting

## 3.1 Introduction

Expert delegated asset managers invest on behalf of inexpert clients. They offer contracts to their clients which often make reference to credit ratings.[1] But why do they propose compensation schemes that depend on public information, such as credit ratings, even though clients employ them for their private information? The contracting literature suggests that contracting on a public signal can mitigate the incentive problem between a principal and his agent (Nalebuff and Scharfstein (1987), Cremer and McLean (1988), Riordan and Sappington (1988)). Do references to credit ratings mitigate delegated asset managers' incentive to shift risk?

We propose a model of delegated investment with a public signal that suggests (i) that contracts do not have to refer to the public signal in order to overcome the

[1] According to the Bank for International Settlements (2003), "it is common, for example, for fixed income investment mandates to restrict the manager's investment choices to investment grade credits"; that is to say that they restrict their portfolios to securities rated BBB- or higher by Standard & Poor's or Baa3 or higher by Moody's.

incentive problem; (ii) that contracts include references to the public signal not to address the incentive problem, but rather to help agents compete; and, in contrast to the contracting literature, (iii) that decreasing the precision of the public signal leads to Pareto improvements.

A clear regulatory prescription follows from this last result: broaden ratings categories, i.e. coarsen the contractible public information partition. Our suggestion is consistent with regulators' assertions that institutions should quit responding robotically to ratings. For example, in 2010 the Financial Stability Board told the G20 Finance Ministers that

> Investment managers and institutional investors must not mechanistically rely on CRA ratings for assessing the creditworthiness of assets. This principle applies across the full range of investment managers and of institutional investors, including money market funds, pension funds, collective investment schemes (such as mutual funds and investment companies), insurance companies and securities firms... [Investment managers should limit] the proportion of a portfolio that is CRA ratings-reliant.

We build a model with two key frictions: first, agents have private information and, second, the principal and the agents differ in their attitudes toward risk. The agents' private information creates the motive for delegation and the difference in risk attitudes creates the misalignment of incentives. Both the principal and the agents are risk averse, but we make no assumption as to who is more risk averse. Further, the difference between the risk aversion coefficients of the principal and the agent can be arbitrarily large. However, we require that the utility functions of the principal and the agents are in the same class of hyperbolic absolute risk-aversion—i.e. that the absolute risk tolerance of the agent is an affine transformation of the absolute risk tolerance of the principal.

The timing of the model is as follows: first, identical agents offer contracts competitively. Each agent's contract can depend on the final wealth, the agent's action and the realization of the public signal, but not on the agent's private information. The agents offer the contracts before the realization of the public signal and before they learn their private information. Second, the public signal realizes and the principal decides which agent to employ to invest on his behalf. Third, the agent learns his private information and takes an action. The agent's private information pertains to the conditional distribution of final wealth given each of his possible actions. Finally, wealth realizes and the principal and agent divide it according to the initial contract.

The first result is that the contract that depends on final wealth alone both solves the incentive problem and implements efficient risk sharing. The reason is that the contract that implements efficient risk sharing makes the principal and agent equally sensitive to the final payoff; since the only incentive problem comes from the difference in risk aversion, this optimal sharing rule aligns the agent's incentives with the principal's. Therefore the principal can delegate the decision to the agent knowing that the agent will act in their joint interest given the contract is the efficient sharing rule. Put differently, the first-best action is incentive compatible, thus there is no need to introduce the public signal into the contract. Note that this intuition is robust only if the principal's and agent's preferences belong to the same HARA class.[2]

[2] To understand why this intuition is not correct for other preferences, see Pratt (2000).

The second result is that the equilibrium contract does indeed depend on the public signal even though it does not mitigate the incentive the problem. To see why this is the case, suppose an equilibrium in which all agents offer contracts that do not depend on the public signal and observe that an agent has a profitable deviation. Because agents are competitive, in any equilibrium in which contracts do not depend on the public signal, agents must break even in expectation across all realizations of the public signal. Thus, for realizations of the public signal for which the surplus is high, the employed agent receives more than his reservation utility. But now a competing agent can undercut him in this high surplus state by offering a contract contingent on the public signal. Extending this argument implies that agents must break even not only in expectation, but also for every realization of the public signal. They achieve this by writing the public signal into their contracts.

The third main result is that decreasing the precision of the public signal is Pareto improving. Since, by the last result above, agents receive the same payoff (their reservation utility) for each realization of the public signal, they do not bear any risk over the realization of the public signal. Therefore, the principal bears all the risk associated with the public signal. That is to say that the agent's competition prevents them from providing insurance to the principal. But, decreasing the precision of the public signal attenuates the negative welfare effects that result from the failure of insurance. To see the advantage of a less precise public signal more clearly, consider the extreme case of a fully uninformative public signal. This is equivalent to the case of contracting without a public signal. In this case, by the first result above, the optimal contract implements both efficient risk sharing and solves the incentive problem. Therefore, the only effect of decreasing the precision of the public signal is to improve the insurance that the agent provides to the principal: decreasing the precision of the public signal makes everyone better-off.

Our model provides some useful insight into the role of credit ratings in the delegated asset management industry. One of the most important functions of ratings is their role in institutional asset management contracts. We apply our framework to a specific model of delegated portfolio choice, interpreting the public signal as the credit rating of a risky security. We make the model concrete by considering a two-asset world with a riskless bond and a risky security. The agent's private information is his knowledge of the distribution of the return of the risky security and his action is the allocation of the principal's wealth to the risky security. For this part of the paper we restrict attention to the case in which both the principal and agent have quadratic utility (but still differ in their aversion to risk). In this setting we can solve not only for the optimal contract but also for the equilibrium action/portfolio weight in closed-form. This allows us to establish the main results via explicit calculation. In particular, to show that decreasing the precision of the credit rating improves welfare, we write down the players' indirect utilities explicitly and compare them across different ratings partitions. Our application is more than an illustration of our theoretical analysis. It comes with a strong policy prescription: broaden ratings categories to improve risk sharing. Broadening ratings categories allows portfolio managers to provide insurance as well as expertise to their clients.

This example also allows us to demonstrate that at least two predictions of our model are consistent with stylized facts. First, the equilibrium contract is affine in wealth, as are most real-wold asset managers' contracts. Second, the equilibrium contract is higher powered in the event that ratings are good, which we interpret as an economic boom. The prediction is consistent with empirical evidence on fund flows: capital flows from money market funds to equity funds—i.e. from funds with low-powered composition to funds with high-powered compensation—as economic conditions improve (see, for example, Chalmers, Kaul and Blake (2010)).

Our result that improved public information decreases welfare is reminiscent of Hirshleifer (1971). He argued that more private information could inhibit trading to share risk in a market setting. For us, simply the ability to contract on information to be revealed later inhibits risk sharing. Further, in our model the public signal not only inhibits risk sharing but also is unnecessary to mitigate the incentive problem between the principal and agent. Several papers have found that public signals are unambiguously welfare-improving in principal-agent settings with adverse selection, notably Nalebuff and Scharfstein (1987), Cremer and McLean (1988), Riordan and Sappington (1988). In these papers the public signal is verifiable ex post. They rely on large punishments to implement the agent's truth-telling. Kessler, Lülfesmann and Schmitz (2005) question these findings by including limited liability with endogenous punishments; they find that public information can decrease efficiency in some cases. We alter the set-up in a different way—in our model players are risk-averse and public information is verifiable ex interim rather than ex post—and we find that better public information is always welfare-decreasing. In addition to the literature on contracting in the presence of a public signal, our paper relates to the literature on socially optimal group decision making (Amershi and Stoeckenius (1983), Pratt and Zeckhauser (1989), Pratt (2000), Wilson (1968)). This work typically does not study strategic behaviour. One exception is Wilson (1984), which studies a social planner who must induce agents to reveal private information. Our application to asset management is related to the literature on delegated portfolio choice (Bhattacharya and Pfleiderer (1985), Dybvig, Farnsworth and Carpenter (2010), Palomino and Prat (2003), Stoughton (1993)). None of these papers considers the role of public information, but Admati and Pfleiderer (1997) and He and Xiong (2013) do. Admati and Pfleiderer (1997) studies the role of performance benchmarks in a classical delegated investment setting and He and Xiong (2013) studies the role of penalties based on publicly observed market quantities (mainly based on tracking error) when the agent is a portfolio manager and the principal is a fund family. There is also an active theory literature studying credit rating agencies (Bolton, Freixas and Shapiro (2012), Bar-Isaac and Shapiro (2010), Donaldson and Piacentino (2012), Kurlat and Veldkamp (2011), Manso (2014), Mathis, McAndrews and Rochet (2009), Skreta and Veldkamp (2009)). Unlike these papers, we take ratings as exogenous and study the affect of their precision on private contracts.

## 3.2 Model

The model constitutes an extensive game of incomplete information in which agents first compete in contracts in the hope of being employed by a single investor and then invest his capital on his behalf.

**Players**

There is a single principal with a unit wealth and von Neumann–Morgenstern utility $u_\mathrm{P}$ and at least two competitive agents with von Neumann–Morgenstern utility $u_\mathrm{A}$ and outside option $\bar{u}$. The principal and the agents differ in their risk aversion. We make no assumption as to whether the principal or the agent is more risk averse, but, for the proof of our main result, we require that both utility functions are in the same class of hyperbolic absolute risk-aversion. Specifically, their absolute risk tolerances are affine with the same slope,

$$-\frac{u'_\mathrm{P}(w)}{u''_\mathrm{P}(w)} = a_\mathrm{P} + bw \tag{3.1}$$

and

$$-\frac{u'_\mathrm{A}(w)}{u''_\mathrm{A}(w)} = a_\mathrm{A} + bw \tag{3.2}$$

for $a_i > -bw$ for all $w$ and for $i \in \{\mathrm{P}, \mathrm{A}\}$.[3] Note that this assumption imposes no restriction on the magnitude of the difference between the principal's and agent's risk aversions. When we consider the application to delegated asset management (Section 3.4) we assume that players have quadratic utility; quadratic utility satisfies conditions (3.1) and (3.2) with $b = -1$.

Agents have private information, captured by their type $\sigma$. A public signal $\rho$ conveys information about $\sigma$. In the application to delegated asset management, $\sigma$ represents agents' expert knowledge about the risk of the market securities and $\rho$ represents the securities' credit ratings.

[3]For example, when $b = 0$ conditions (3.1) and (3.2) imply that the principal and the agents have exponential utility with constant coefficients of absolute risk aversion $a_\mathrm{P}^{-1}$ and $a_\mathrm{A}^{-1}$.

### Actions and Contracts

The principal wishes to delegate investment to an agent because he is better informed; however, he anticipates a misalignment of investment incentives since his risk aversion differs from the agents'.

Contracts attempt to align incentives to mitigate the downside of delegated asset management. Each agent $a$ offers contract $\Phi_a$ which may depend on the final wealth $w$, the public signal $\rho$, and his action $x$. The agent chooses $x$ after he has entered the contract. The action choice affects only the distribution of the final wealth $\tilde{w}(x)$. We assume that $\tilde{w}$ is a concave function of $x$ for every state of the world. In our portfolio management application in Section 3.4, we interpret $x$ as the proportion of wealth invested in an asset. Note that the agent's type $\sigma$ does not enter the contract because it is not verifiable; however, $\rho$ may enter the contract as a proxy.

### Timing

After agents announce their contracts, the principal observes $\rho$ and employs an agent who chooses $x$ after learning $\sigma$. Then, wealth realizes and players divide it according to the initial contract. Formally, the timing is as follows:

1. Agents simultaneously offer contracts $\Phi_a$.
2. $\sigma$ and $\rho$ realize.
3. The principal observes $\rho$ and the profile of contracts $\{\Phi_a\}_a$ and hires an agent $a^*$.
4. Agent $a^*$ chooses $x$.

5. Final wealth realizes and it is distributed such that agent $a^*$ is awarded $\Phi_{a^*}(w)$ and the investor keeps $w - \Phi_{a^*}(w)$.

Note that key to our timing is that players learn $\rho$ after agents offer contracts but before the principal employes an agent. In Section 3.4.2, we demonstrate that our results are robust to the inclusion of a second public signal that realizes after the agent has been employed. Nevertheless, the timing is sensitive to the agent's offering contracts before they learn $\sigma$. Our timing shuts down any signaling incentives.

### Note on Notation

We frequently omit the arguments of variables. The contract $\Phi$ always depends on wealth $w$, the agent's action $x$, and the public signal $\rho$, as well as the offering agent $a$, but we frequently write just $\Phi(w)$. The agent chooses the action given his type $\sigma$, but we usually write just $x$ for $x(\sigma)$. Later we will introduce a social planner's problem, in which the welfare function places weight $\mu_\rho$ on the agent given the realization $\rho$ of the public signal. We sometimes suppress this dependence and write $\mu$ for $\mu_\rho$. Finally, the social planner's sharing rule $\varphi$ depends on final wealth directly and on the public signal indirectly via the welfare weight. While we sometimes write formally $\varphi_{\mu_\rho}(w)$, we frequently abbreviate to $\varphi_\mu(w)$ or even just $\varphi(w)$.

## 3.3 Results

### Competition Is Rating-by-Rating

We first show that agents must break even for every realization of the public signal. This will allow us to transform our game into a family of principal-agent problems, one for each realization of the public signal. That is to say that for every realization of the public signal the agent must offer the contract that maximizes the principal's utility and assures him at least his reservation payoff.

**Lemma 27.** *The employed agent $a^*$ breaks even for each realization $\rho$ of the public signal, or*

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi_{a^*}(\tilde{w})\Big)\,\Big|\,\tilde{\rho}=\rho\right]=\bar{u}$$

*for all $\rho$.*

*Proof.* The proof is in Appendix 3.6.1. ☐

That agents receive their reservation utility in equilibrium is unsurprising because they are competitive. The takeaway from Lemma 27 above is that agents receive their reservation utility for *every realization of the public signal.* There cannot be an equilibrium in which agents break even in expectation over all possible realizations (unless they break even for every realization). In fact, if that is the case, then an agent who receives less than his reservation utility for some realization of the public signal must receive in excess of his reservation utility for another realization. But since the agent is getting strictly in excess of his reservation utility for this realization, another agent can undercut him by offering a contract that grants him more than his reservation utility and allocates more of the surplus to the principal.

The proof is by contraction. It is standard except for one subtlety. We first suppose that an agent receives strictly in excess of his reservation utility for some realization of the signal. This agent must therefore be employed given this realization. But then another agent, otherwise unemployed and receiving his reservation utility, would undercut the employed agent for this realization of the signal. Therefore, à la Bertrand competition, the agents must break-even given this realization. The only subtlety of the proof is that agents' contracts affect their incentives and hence their actions. Thus, when a deviant agent offers the principal a contract, the principal must take the effect of this contract on the agent's action into account. Our proof circumvents this issue by constructing a deviation that preserves the incentives of the originally employed agent while allocating more surplus to the principal. Specifically, if the supposed equilibrium contract is $\Phi$ the deviation $\Phi_\varepsilon(w) := u_{\mathrm{A}}^{-1}\big(u_{\mathrm{A}}(\Phi(w)-\varepsilon)\big)$ preserves the employed agent's incentives and allocates more of the surplus to the principal.

The argument in the proof of Lemma 27 also implies that the contract must maximize the principal's utility for every realization of the signal $\rho$ as is summarized in Corollary 28 below. The reason is that if the employed agent does not maximize the principal's utility, then another agent can deviate to a contract more favorable to the principal that also leaves him a small surplus above $\bar{u}$.

**Corollary 28.** *If $\Phi_{a^*}$ is the contract of the employed agent $a^*$ given rating $\hat{\rho}$ and there is another contract $\hat{\Phi}$ such that*

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \hat{\Phi}(\tilde{w})\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \Phi_{a^*}(\tilde{w})\Big) \mid \tilde{\rho} = \hat{\rho}\right],$$

*then it must be that*

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) \mid \tilde{\rho} = \hat{\rho}\right] < \bar{u}.$$

**Principal-Agent Formulation**

Lemma 27 and Corollary 28 taken together say that the principal chooses the contract that maximizes his expected utility subject to the constraint that the agent receives his reservation utility for every realization of the signal $\rho$. That is to say that the equilibrium contract solves the principal-agent problem for every $\rho$. The twist on a standard principal-agent problem is that the agent's participation constraint depends on the public signal.

**Proposition 29.** *For each realization $\rho$ of the public signal, the contract of the employed agent $a^*(\rho)$ solves the following principal-agent problem:*

$$\begin{cases} \textit{Maximize} \;\; \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x) - \Phi\big(\tilde{w}(x), x, \rho\big)\Big) \,\big|\, \tilde{\rho} = \rho\right] \\ \\ \textit{subject to} \;\; \mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(x), x, \rho\big)\Big) \,\big|\, \tilde{\rho} = \rho\right] = \bar{u} \quad \textit{and} \\ \\ x \in \arg\max \left\{\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big(\tilde{w}(\xi), \xi, \rho\big) \,\big|\, \tilde{\sigma} = \sigma\right] \;\; ; \;\; \xi \in \mathbb{R}\right\} \end{cases} \tag{3.3}$$

*over contract $\Phi$.*

### Equilibrium Contract as the Solution of a Social Planner's Problem

For each realization of the public signal $\rho$, we transform the principal-agent problem into a social planner's problem. The social planner will maximize social welfare subject to the agent's incentive compatibility constraint. Call the agent's welfare weight $\mu_\rho$ for a given $\rho$. This will coincide with the Lagrange multiplier on the agent's participation constraint in the principal-agent problem for a given $\rho$. This approach allows us eliminate the agent's participation constraints temporarily to focus on incentive compatibility.

Now use the method of Lagrange multipliers to eliminate the participation constraint and say that the problem is to maximize

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x) - \Phi\big(\tilde{w}(x), x, \rho\big)\Big) + \mu_\rho\Big[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(x), x, \rho\Big) - \bar{u}\Big] \,\Bigg|\, \tilde{\rho} = \rho\right]$$

subject to

$$x \in \arg\max \Big\{ \mathbb{E}\Big[ u_{\mathrm{A}}\Big( \Phi\big(\tilde{w}(\xi), \xi, \rho\big) \Big| \tilde{\sigma} = \sigma \Big] \;\; ; \;\; \xi \in \mathbb{R} \Big\}$$

over contract $\Phi$.

For any Lagrange multiplier $\mu_\rho$ the problem is equivalent to the social planner's problem with welfare weight $\mu_\rho$ associated with the agent. That is to say that, for given $\mu_\rho$, we can omit the agent's outside option $\bar{u}$ and solve the following social planner's problem for $\Phi$ and $x$.

$$\begin{cases} \text{Maximize } \;\mathbb{E}\left[ u_{\mathrm{P}}\Big( \tilde{w}(x) - \Phi\big(\tilde{w}(x), x, \rho\big) \Big) + \mu_\rho u_{\mathrm{A}}\Big( \Phi\big(\tilde{w}(x), x, \rho\big) \,\Bigg|\, \tilde{\rho} = \rho \right] \\ \\ \text{subject to } \; x \in \arg\max \Big\{ \mathbb{E}\Big[ u_{\mathrm{A}}\Big( \Phi\big(\tilde{w}(\xi), \xi, \rho\big) \Big| \tilde{\sigma} = \sigma \Big] \;\; ; \;\; \xi \in \mathbb{R} \Big\} . \end{cases} \tag{3.4}$$

Below we solve the problem for a generic Lagrange multiplier and only later we use the agent's binding participation constraint to solve for $\mu_\rho$ for each $\rho$. Transforming the game into a social planner's problem reveals that the task is to trade off efficient risk sharing with implementing efficient investment.

**The Efficient Sharing Rule Implements Efficient Investment**

We now find the contract that solves the social planner's problem. We do this by characterizing the first-best contract and action—i.e. those that the social planner would choose if he had perfect information. We then show that given the first-best contract the first-best action is incentive compatible, so the solution to the social planner's problem coincides with the first-best outcome. Thus, in fact, there is no tension between risk sharing and efficient investment in equilibrium.

**Proposition 30.** *If the contract is the efficient sharing rule, then the incentive compatible action is the social optimum.*

*Namely, if $\varphi$ maximizes*

$$u_{\mathrm{P}}(w-\varphi)+\mu_\rho u_{\mathrm{A}}(\varphi)$$

*then*

$$x \in \arg\max \left\{ \mathbb{E}\left[ u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(\xi)\big)\Big) \,\Big|\, \tilde{\sigma}=\sigma \right] \right\}$$

*implies*

$$x \in \arg\max \left\{ \mathbb{E}\left[ u_{\mathrm{P}}\Big(\tilde{w}(\xi)-\varphi\big(\tilde{w}(\xi)\big)\Big) + \mu_\rho u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(\xi)\big)\Big) \,\Big|\, \tilde{\sigma}=\sigma \right] \right\}.$$

*Proof.* The proof is in Appendix 3.6.2. □

The main takeaway of Proposition 30 is that for any $\rho$ the efficient contract implements the efficient action.

In the proof we first find the efficient $\varphi$. We then demonstrate that, given this $\varphi$, the agent would choose the social optimum. That is to say that the action that the agent chooses coincides with the action a social planner would choose if he had the agent's private information.

To understand the connection between incentive alignment and risk sharing, recall that a sharing rule $\varphi$ is efficient if it maximizes $u_{\mathrm{P}}(w-\varphi)+\mu_\rho u_{\mathrm{A}}(\varphi)$ for each realization of $w$ or

$$u'_{\mathrm{P}}(w-\varphi(w)) = \mu_\rho u'_{\mathrm{A}}(\varphi(w)). \tag{3.5}$$

On the other hand, the sharing rule $\varphi$ aligns the incentives of the principal and the agent globally if one's utility function is an affine transformation of the other's given the sharing rule $\varphi$, or

$$u_{\mathrm{P}}(w-\varphi(w)) = \alpha u_{\mathrm{A}}(\varphi(w)) + \beta$$

for some $\alpha > 0$ and $\beta \in \mathbb{R}$. Differentiating this condition with respect to $w$ gives

$$u'_{\mathrm{P}}(w-\varphi(w)) = \frac{\alpha\varphi'(w)}{1-\varphi'(w)} u'_{\mathrm{A}}(\varphi(w)).$$

This last condition coincides with the condition above of efficient risk sharing (condition (3.5)) exactly when $\mu_\rho = \dfrac{\alpha\varphi'(w)}{1-\varphi'(w)}$, which is possible if and only if $\varphi'$ is a constant or $\varphi$ is affine. The only remaining step in the argument is to show that the efficient sharing rule is affine for the preferences we consider, which we show in Lemma 33 in Appendix 3.6.2.

**Coarser Public Signals Are Pareto-Improving**

Proposition 30 shows that the optimal contract eliminates the incentive problem for every $\sigma$ and the risk sharing problem for every $\rho$. The problem remains to share risk over realizations of the public signal. The next result states that less precise public signals Pareto dominate more precise public signals. The reason is that the public signal does not mitigate the incentive problem but only hinders risk sharing.

From now on, since the optimal contract solves the incentive problem, we omit incentive constraints and focus directly on the social planner's problem (with complete information) as per Proposition 30.

**Proposition 31.** *Coarser public signals Pareto-dominate finer ones: for any signal $\tilde{\rho}_c$ and $\tilde{\rho}_f$ such that $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$, the ex ante equilibrium utility of all players is weakly higher given $\tilde{\rho}_c$ than $\tilde{\rho}_f$.*

*Proof.* Below we omit the dependence of $\varphi$ on $x$ because by Proposition 30 the efficient $x$ is chosen for every $\sigma$ independently of $\rho$. Below call $\varphi_{\mu_{\rho_f}}$ and $\varphi_{\mu_{\rho_c}}$ the efficient sharing rules associated with fine and coarse public signals respectively.

First, the agent's participation constraint given $\tilde{\rho}_f$ is

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right] = \bar{u}.$$

Now, since $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$, use the law of iterated expectations and the condition above to observe that

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right] = \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right]\,\middle|\,\tilde{\rho}_c\right] = \mathbb{E}\left[\bar{u}\,\middle|\,\tilde{\rho}_c\right] = \bar{u}.$$

This says that $\varphi_{\mu_{\rho_f}}$ satisfies the participation constraint given $\rho_c$. Since $\varphi_{\mu_{\rho_c}}$ solves the principal-agent problem given $\rho_c$—viz. it maximizes the principal's utility given the agent's participation constraint—

$$\mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right] \geq \mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right].$$

Now we use the inequality above and we apply the law of iterated expectations again to prove that the principal is better off given the coarser ratings, namely

$$\begin{aligned}\mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\right)\right] &= \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right]\right] \\ &\geq \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right]\right] = \mathbb{E}\left[u_{\mathrm{P}}\left(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\right)\right].\end{aligned}$$

Since agents always break-even and the principal is better off with coarser public signals $\tilde{\rho}_c$ Pareto dominates $\tilde{\rho}_f$. □

The main step of the proof is to show that a contract that is feasible given a fine signal structure is also feasible given a coarse signal structure. This follows directly from the law of iterated expectations. Since coarsening the signal structure expands the set of feasible contracts, it can only increase the principal's objective (recall that the incentive constraints are not binding by Proposition 30). Since the agent always breaks even, increasing the principal's profits constitutes a Pareto improvement.

The intuition behind this result comes from Lemma 27. Because competition makes agents break even state-by-state, there is one participation constraint for each realization of the public signal. Thus, with a finer signal structure there are more realizations of the public signal and, thus, more constraints on the principal's objective. Because we know from Proposition 30 that the efficient action is always taken, these constraints restrict only risk sharing between the principal and the agent. A finer signal structure shuts down risk sharing and reduces welfare.

## 3.4 An Example: Portfolio Choice with Quadratic Utility

### Setup

To fix ideas we consider the specific case of portfolio choice with quadratic utility. This example allows us to solve the model explicitly and thus it exposes the forces behind the more general proofs above.

The portfolio choice model has a risk-free bond with gross return $R_f$ and a risky asset with random gross return $\tilde{R}$. The agent's type $\sigma$ will be the standard deviation of $\tilde{R}$ and $\rho$ will be an imperfect public signal about this risk parameter. Call $\rho$ the credit rating of the risky security. The agent's action $x$ represents the proportion of wealth invested in the risky security; therefore,

$$\tilde{w}(x) = R_f + x\big(\tilde{R} - R_f\big).$$

We assume that all players have quadratic utility,

$$u_i(w) = -\frac{1}{2}\big(a_i - w\big)^2$$

for $i \in \{\mathrm{A}, \mathrm{P}\}$. The investor differs from the agents in his risk aversion. Note that the coefficient of absolute risk tolerance is $a_i - w$, so these utility functions are in the same class of hyperbolic absolute risk aversion as defined in equations (3.1) and (3.2).

We make some restrictions on the distribution of $\tilde{R}$ to simplify the belief updating. We assume that the mean return $\bar{R}$ of the risky asset is known and independent of the agent's type $\sigma$. In fact, since with quadratic utility players' expected utility depends on only the mean and variance of the distribution, all relevant asymmetric information is about the variance $\sigma^2$. Note that this assumption implies that the credit rating is informative only about the asset's risk and not about its expected return,

$$\mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\rho} = \rho\right] = \mathbb{E}\big[\tilde{R}\big].$$

With quadratic utility, players' marginal utility is decreasing when their wealth is large. We restrict parameters to ensure that players' wealth is not so large. In particular, it must be that the wealth of the principal and that of the agent are not too large, or, respectively,

$$w - \Phi(w) < a_{\mathrm{P}}$$

and

$$\Phi(w) < a_{\mathrm{A}}.$$

A sufficient condition for this is

$$\operatorname{supp} \tilde{w} \subset [0, a_{\mathrm{P}} + a_{\mathrm{A}}). \tag{3.6}$$

To guarantee this, make the technical assumption that

$$\left(\bar{R} - R_f\right)\left(R - \bar{R}\right) \leq \sigma^2 \tag{3.7}$$

for all pairs $(\sigma, R)$.[4]

### 3.4.1 Results

#### Competition Is Rating-by-Rating

Lemma 27 implies that agents must break even for each realization of the credit rating. Recall that the reason is that competition in contracts is Bertrand-like in the sense that the employed agent will receive his reservation utility conditional on any realization of the credit rating $\tilde{\rho}$; further agents act so as to maximize the investor's expected utility conditional on every $\rho$ subject to their participation constraints.

The proof of Lemma 27 is in Appendix 3.6.1, but re-iterating the main argument with these specific utility functions can clarify the proof. Recall that the proof is by contradiction. Supposing an equilibrium in which an agent receives in excess of his reservation utility for some realization of the public signal, a deviating agent can undercut him. However, we must be careful to take into account the effect of the new contract on the agent's incentives. We construct a deviation that does not distort incentives. With the current utility specification we can write it explicitly. In particular,

[4] Condition (3.7), sufficient for condition (3.6), comes from solving the game assuming that the agent's participation constraint binds, then writing a sufficient condition for it to bind in light of the equilibrium.

if the initial contract given a rating $\hat{\rho}$ is $\hat{\Phi}$, the contract for $\varepsilon > 0$ is

$$\hat{\Phi}_\varepsilon(w) := u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w}) - \varepsilon\Big)\Big) = a_{\mathrm{A}} - \sqrt{\big(a_{\mathrm{A}} - \hat{\Phi}(w)\big)^2 - 2\varepsilon}$$

gives the agent identical incentives to $\Phi$ and allocates more surplus to the principal.

**Principal-Agent Formulation and Social Planner's Problem**

Lemma 27 asserts that agents compete rating-by-rating, maximizing investor welfare subject to their participation constraints, that is to say that, for every realization $\rho$ of the credit rating, the contract of the employed agent and the corresponding portfolio weight solve the principal-agent problem of Proposition 29. Using the method of Lagrange multipliers we can transform the principal-agent problem into the social planner's problem summarized by the system (3.4). Now, unlike in the general case, we can compute simple expressions not only for the optimal contract but also for the agent's action $x$ and the Lagrange multiplier/welfare weight $\mu_\rho$.

**The Efficient Sharing Rule Implements Efficient Investment**

First, find the optimal sharing rule using the first-order condition in equation (3.5),

$$u'_{\mathrm{P}}\big(w - \varphi_\mu(w)\big) = \mu\, u'_{\mathrm{A}}\big(\varphi_\mu(w)\big),$$

or, for quadratic utility,

$$w - \varphi_\mu(w) - a_{\mathrm{P}} = \mu\big(\varphi_\mu(w) - a_{\mathrm{A}}\big)$$

for all $w$. Thus the efficient sharing rule is

$$\varphi_\mu(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{P}} - a_{\mathrm{A}}}{1 + \mu}. \tag{3.8}$$

Observe that the standard deviation $\sigma$ does not enter the expression, and thus that the social planner need not know the true variance to implement optimal risk sharing.

Given the optimal sharing rule, we now calculate the first-best investment in the risky security $x^*$ in the sense that $x^*$ is the investment that the social planner would make if he knew the standard deviation $\sigma$. The first-best will be useful in finding the solution to the second-best problem in which the social planner knows only $\rho$ and the agent chooses $x$. This $x$ in turn constitutes the equilibrium allocation of the model. The reason that it is useful to compute the first-best outcome is that we proceed to show that it is an attainable outcome of the second-best problem. Thus we solve the game by showing that the social optimum is attainable.

The social planner finds $x^*$ by computing the maximum of

$$\begin{aligned}&\mathbb{E}\left[u_{\mathrm{P}}\left(R_f + x\big(\tilde{R} - R_f\big) - \varphi_\mu\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right] \\ &\qquad + \mu\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_\mu\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right],\end{aligned} \tag{3.9}$$

over all $x$. Mechanical computations collected in Appendix 3.6.3 reveal that the optimal investment is

$$x^*(\sigma) = \frac{\big(\bar{R} - R_f\big)\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)}{\sigma^2 + \big(\bar{R} - R_f\big)^2}. \tag{3.10}$$

Note that the optimal investment does not depend on the welfare weight $\mu$, thus the social planner chooses the same $x^*$ for all $\mu$, even as $\mu \to \infty$. But, now, in the limit as $\mu \to \infty$, since in this case the social planner puts all the weight on the agent, his objective coincides with the agent's. Put differently, if the contract is the efficient sharing rule, the agent always takes the socially optimal action. This observation implies Proposition 30 in the context of this example.

### The Break-even Welfare Weight and Ex Ante Utility

Now we can characterize the employed agent's contract explicitly by finding the Lagrange multiplier $\mu_\rho$ for each $\rho$. For a given contract $\varphi_{\mu_\rho}$ the agent must break even, so we can determine $\mu_\rho$ directly from the agent's participation constraint:

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_\rho}\Big(R_f + x(\tilde{\sigma})\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\rho} = \rho\right] = \bar{u}. \tag{3.11}$$

This equation combined with the closed-form expressions for $\varphi_{\mu_\rho}$ and $x^*(\sigma)$ above allow us to compute $\mu_\rho$ in closed-form. A string of calculations employing the law of iterated expectations (cf. Appendix 3.6.4), says

$$\big(1 + \mu_\rho\big)^2 = \frac{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2}{2\,|\bar{u}|}\,\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2} \,\middle|\, \tilde{\rho} = \rho\right]. \tag{3.12}$$

This formula will be useful to express the ex ante utility of the principal and then to see constructively how changing the coarseness of the ratings partition affects investor welfare. In particular, within the framework of the example, we will be able to provide a less abstract proof of Proposition 31.

Before we proceed to the welfare analysis, we highlight one insight that the expression for the Lagrange multiplier offers. The mapping

$$\tilde{\sigma}^2 \mapsto \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\tilde{R} - R_f\big)^2}$$

under the expectation operator in equation (3.11) is concave, so that if the distribution of $\tilde{\sigma}^2$ spreads out (for example in the sense of second-order stochastic dominance), then $\mu_\rho$ decreases, suggesting that the more distribution risk the agent faces, the less the investor must compensate him despite his risk aversion, as captured by the social planner's lower welfare weight. This observation presents a puzzle: why would the agent, who is risk-averse, prefer a riskier distribution? The puzzle finds its resolution in the observation that higher dispersion of the variance comes with option value, and thus convexity, making him risk-loving over this kind of risk. The reason is that his investment decision comes after the realization of the variance, and thus the riskier decisions come with option value allowing him to adjust his investment decision to market conditions: when $\sigma^2$ is very low he will invest a lot in the risky asset, while when it is high he will invest relatively more in the riskless bond.

Now return to the main analysis. To analyze welfare we use the equilibrium welfare weight to find a formula for the investor's equilibrium expected utility given the rating $\rho$,

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\;\Big|\;\tilde{\rho}=\rho\right]=\bar{u}\,\mu_{\rho}^{2} \tag{3.13}$$

(see Appendix 3.6.5 for the short calculation). Thus his ex ante expected utility

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\right]=\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^{2}\right]. \tag{3.14}$$

**Coarser Credit Ratings Are Pareto-Improving**

Since competition means that agents always receive their reservation utilities, the main result that coarsening credit ratings makes everyone better-off follows from comparing the ex ante expected utility of the investor across ratings systems. To do this we use formula (3.14) above combined with the connection between convex functions, second-order stochastic dominance, and the law of iterated expectations.

Within the setting of the example, we can now provide a constructive proof for Proposition 31 above, which says that coarse credit ratings Pareto dominate finer ones.

Our proof has two main steps. We summarize these steps briefly before giving the full proof. The first step is to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right] = -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y \mid \tilde{\rho}\right]\Big)\right]$$

for (appropriately defined) $c > 0$, $f'' > 0$ and a random variable $Y$. The second step is to show that the expectation conditional on coarse ratings second-order stochastically dominates the expectation conditional on fine ratings,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right].$$

Whence utility is greater under coarse ratings because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated variable.

Step 1: Rewrite the investor's ex ante expected utility:

$$\begin{aligned}
\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right] &= \bar{u}\,\mathbb{E}\left[\left(\sqrt{\frac{(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f)^2}{2|\bar{u}|}\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \,\middle|\, \tilde{\rho}\right]} - 1\right)^2\right] \\
&= \frac{\bar{u}(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f)^2}{\sqrt{2|\bar{u}|}}\mathbb{E}\left[\left[\sqrt{\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \,\middle|\, \tilde{\rho}\right]} - 1\right]^2\right] \\
&= -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y|\tilde{\rho}\right]\Big)\right]
\end{aligned}$$

where

$$\begin{aligned}
c &:= \sqrt{|\bar{u}|/2}\,(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f)^2, \\
f(z) &:= \left(\sqrt{z} - 1\right)^2,
\end{aligned}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}.$$

Note that $c > 0$ and $f''(z) = z^{3/2}/2 > 0$.

Step 2: By definition,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right]$$

if there exists a random variable $\tilde{\varepsilon}$ such that

$$\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_c\right] + \tilde{\varepsilon}$$

and

$$\mathbb{E}\left[\tilde{\varepsilon} \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = 0.$$

For $\tilde{\varepsilon} = \mathbb{E}\left[Y \mid \tilde{\rho}_f\right] - \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]$ from the above, the condition is

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] - \mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = 0$$

or

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_c\right].$$

Given the assumption $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$ and since conditioning destroys information—$\sigma\big(\mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\big) \subset \sigma(\tilde{\rho}_c)$—apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] \,\middle|\, \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] \\
&= \mathbb{E}\left[Y \,\middle|\, \tilde{\rho}_c\right],
\end{aligned}$$

as desired.

### Asset Manager's Observed Contracts

The agent's equilibrium contract is

$$\varphi_{\mu_\rho}(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{P}} - a_{\mathrm{A}}}{1 + \mu_\rho} \tag{3.15}$$

where $\mu_\rho$ is defined in equation (3.12).

The compensation contract is affine in wealth, as are typical asset management contracts. For the next result (and the next result only), consider a simplified but realistic credit rating rule. Let $\tilde{\rho}$ define a partition of the realization of the variance $\sigma_0^2 < \sigma_1^2 < \cdots$, namely

$$\mathbb{P}\{\tilde{\sigma}^2 \in [\sigma_i^2, \sigma_{i+1}^2) \mid \rho_i\} = 1.$$

**Proposition 32.** *For $i < j$, $\mu_{\rho_i} < \mu_{\rho_j}$. Increases in the expected variance decrease the power of the contract, i.e. the slope in the wealth.*

*Proof.* Since

$$\frac{\sigma^2}{\sigma^2 + \left(\bar{R} - R_f\right)^2}$$

is increasing in $\sigma^2$,

$$\mathbb{E}\left[\left.\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right| \rho_i\right] < \frac{\sigma_{i+1}^2}{\sigma_{i+1}^2 + \left(\bar{R} - R_f\right)^2} < \mathbb{E}\left[\left.\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right| \rho_{i+1}\right].$$

This follows from definition (3.4.1), which implies that $\sigma_{i+1}$ is greater than the expectation of $\tilde{\sigma}$ conditional on $\rho_i$ but less than the expect ion of $\tilde{\sigma}$ conditional on $\rho_{i+1}$. Now, immediately from equation (3.12), $\mu_{\rho_i} < \mu_{\rho_{i+1}}$ and by induction $\mu_{\rho_i} < \mu_{\rho_j}$ whenever $i < j$. Combined with equation (3.15), the result implies that lower expected variances correspond to steeper wealth compensation for agents. $\square$

In the model, ratings describe the variance of the market portfolio. Define a "boom" a realization of $\tilde{\rho}$ implying low expected variance. With this interpretation, proposition 32 says that employed agents have higher powered contracts in booms than in busts. Since, almost uniformly, equity funds offer higher powered contracts than money market funds, the model predicts that the in-flows to equity funds relative to money market funds will be procyclical. Using a sample of US mutual fund data from 1991 to 2008, Chalmers et al. (2010) finds that investors direct funds away from money market funds towards equity funds when aggregate economic conditions improve, in line with our prediction.

### 3.4.2 Extensions

#### Imperfect Private Information

Suppose that the agent receives an imperfect signal about the variance. Namely, he observes the realization of a random variable $\tilde{s}$ that is not independent of $\tilde{\sigma}$. Then, equation (3.10) for the portfolio allocation becomes

$$x(\rho, s) = \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\right)}{\mathrm{Var}\left[\tilde{R} \,|\, \rho, s\right] + \left(\bar{R} - R_f\right)^2}.$$

The optimal contract is $\varphi_\mu(R_f + x(\rho, s)(R - R_f))$ (where an equation analogue to (3.12) determines $\mu$).

Clearly, whenever $\sigma(\tilde{\rho}) \subset \sigma(\tilde{s})$, then $x(\rho, s)$ does not depend on $\rho$ and our main result remains unchanged. If, instead, $\sigma(\tilde{\rho}) \not\subset \sigma(\tilde{s})$ then a trade-off arises: finer credit ratings still shut down risk sharing, but they increase allocational efficiency, i.e. the portfolio weight is closer to first best. The net welfare effect is then ambiguous.

Our model and policy prescriptions therefore apply to markets in which delegated portfolio managers are better informed than credit rating agencies.

**Additional Ratings' Changes**

In our model, ratings realize once, after agents offer contracts but before the investor employs an agent. In reality, ratings upgrades and downgrades are frequent and investors and agents have long-term relationships. In the model, if ratings change after the investor has employed an agent, the optimal contract above still induces the agent to invest efficiently. The new rating influences the portfolio allocation only insofar as it improves the agent's information (cf. the preceding discussion of imperfect signals). Ratings changes after the investor and agent commit to a relationship never decrease efficiency and can be beneficial if they improve information. Our model therefore suggests that making reference to ratings matters only because funds are looking to attract new investors or because their current investors may withdraw their funds. The idea finds support in the observation that hedge funds, who raise money infrequently via long-term contracts do not make reference to ratings in their contracts.

## 3.5 Conclusion

Motivated by delegated asset managers' frequent references to credit ratings in the contracts they offer their clients, we study a delegation problem with adverse selection in the presence of a public signal. We characterize the optimal contract between a risk-averse principal and a risk-averse agent and show that while it does indeed depend on the public signal, contracting on the public signal does not mitigate the incentive problem. In fact, in contrast to previous literature, we find that decreasing the precision of the public signal is Pareto improving. The reason is that contracting on final wealth implements efficient investment, so contracting on the public signal serves only to inhibit risk sharing. Agents include the public signal in their contracts only to help them compete.

We apply the model to a classical delegated portfolio management setting in which delegated asset managers' make a portfolio choice decision on behalf of their clients. In this setting, we interpret the public signal as a credit rating. Our main policy prescription is that credit rating agencies should provide information in forms prohibitive to their inclusion in rigid contracts. This helps asset managers to provide insurance to their clients in addition to expertise. Our recommendation is consistent with the popular suggestion that markets should eliminate the mechanistic reliance on ratings. Our model also suggests investment mandates may contribute to the cyclicality of mutual fund flows, providing further motivation for their abolition.

## 3.6 Appendices

### 3.6.1 Proof of Lemma 27

Suppose, in anticipation of a contradiction, an equilibrium in which the employed agent offers contract $\hat{\Phi}$ given signal $\hat{\rho}$ and receives strictly in excess of his reservation utility,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right] > \bar{u}. \tag{3.16}$$

We now show that another agent $\hat{\mathrm{A}}$ has a profitable deviation. In order for a contract $\hat{\Phi}_\varepsilon$ to be a profitable deviation for $\hat{\mathrm{A}}$ it must (i) make the principal employ him given $\hat{\rho}$ and (ii) give him expected utility greater than his reservation utility $\bar{u}$ given $\hat{\rho}$. The subtlety in this proof is that $\hat{\mathrm{A}}$'s contract determines not only the allocation of surplus, but also his action $x$. To circumvent the effect of changing actions on payoffs, we construct $\hat{\Phi}_\varepsilon$ to induce the agent to choose the same action that he would have chosen under $\hat{\Phi}$, but still to change the division of surplus. To summarize, $\hat{\Phi}_\varepsilon$ is a profitable deviation if given $\hat{\rho}$ (i) it gives the principal higher utility than does $\hat{\Phi}$,

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}-\hat{\Phi}_\varepsilon(\tilde{w})\Big)\mid\tilde{\rho}=\hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}-\hat{\Phi}(\tilde{w})\Big)\mid\tilde{\rho}=\hat{\rho}\right],$$

(ii) it gives the agent utility in excess of $\bar{u}$,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon(\tilde{w})\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right] > \bar{u},$$

and (iii) the set of incentive compatible actions under $\hat{\Phi}$ and $\hat{\Phi}_\varepsilon$ coincide,

$$\arg\max_x\left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon(\tilde{w})\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]\right\} = \arg\max_x\left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]\right\}.$$

One example of contract that satisfies these three conditions is

$$\hat{\Phi}_\varepsilon(\tilde{w}) := u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})-\varepsilon\Big)\Big) \tag{3.17}$$

given $\hat{\rho}$, so that

$$u_{\mathrm{A}}\big(\hat{\Phi}_{\varepsilon}\big) = u_{\mathrm{A}}\big(\hat{\Phi}\big) - \varepsilon. \tag{3.18}$$

Since $u'_{\mathrm{P}} > 0$, a sufficient condition for $\hat{\Phi}_{\varepsilon}$ to satisfy condition (i) is that

$$\tilde{w} - \hat{\Phi}_{\varepsilon}\big(\tilde{w}\big) > \tilde{w} - \hat{\Phi}\big(\tilde{w}\big),$$

or, substituting from equation (3.17),

$$\hat{\Phi}\big(\tilde{w}\big) > u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big) - \varepsilon\Big)\Big),$$

which is satisfied for $\varepsilon > 0$ by the inverse function theorem since $u'_{\mathrm{A}} > 0$.

Condition (ii) holds for $\epsilon > 0$ and sufficiently small. This follows from equation (3.18) and inequality (3.16) with the continuity of $u_{\mathrm{A}}$.

Finally, condition (iii) is immediate from equation (3.18) since affine transformations of utility do not affect choices.

Thus the investor will employ agent $\hat{\mathrm{A}}$ who will receive, given $\hat{\rho}$, utility greater than the utility that he would have received in the supposed equilibrium (in the supposed equilibrium he was unemployed and he was obtaining $\bar{u}$). Thus $\hat{\Phi}_{\varepsilon}$ is a profitable deviation for $\hat{\mathrm{A}}$ and $\Phi$ cannot be the contract of an agent employed at equilibrium given $\hat{\rho}$.

We have shown that the agent's expected utility given any $\rho$ cannot exceed $\bar{u}$. To conclude the proof, note that his utility can never be strictly less than $\bar{u}$ because then his expected utility would be less than his reservation utility.

### 3.6.2 Proof of Proposition 30

The proof relies on the following lemma.

**Lemma 33.** *The efficient sharing rule $\varphi$ is affine in wealth $w$.*

*Proof.* Assumptions (3.1) and (3.2) imply that

$$u_{\mathrm{P}}(w) = \frac{1}{b-1}\left(\frac{w}{b} + \frac{a_{\mathrm{P}}}{b^2}\right)^{\frac{b-1}{b}}$$

and

$$u_{\mathrm{A}}(w) = \frac{1}{b-1}\left(\frac{w}{b} + \frac{a_{\mathrm{A}}}{b^2}\right)^{\frac{b-1}{b}}.$$

The contract that implements efficient risk sharing is that which maximizes social surplus (for appropriate welfare weight $\mu$) for every realization of wealth. Now compute the efficient sharing rule directly via the first order approach:

$$\frac{\partial}{\partial \varphi}\Big(u_{\mathrm{P}}(w-\varphi) + \mu_\rho u_{\mathrm{A}}(\varphi)\Big) = 0$$

so

$$\left(\frac{w-\varphi}{b} + \frac{a_{\mathrm{P}}}{b^2}\right)^{-\frac{1}{b}} = \mu_\rho \left(\frac{\varphi}{b} + \frac{a_{\mathrm{A}}}{b^2}\right)^{-\frac{1}{b}}.$$

This implies

$$\varphi(w) = \frac{a_{\mathrm{P}} - \mu_\rho^{-b} a_{\mathrm{A}} + bw}{b\left(1+\mu_\rho^{-b}\right)},$$

which is affine in $w$. ☐

We begin the proof of Proposition 30 with the agent's incentive problem given the contract $\varphi$ and show through a series of manipulations that his incentives are aligned with those of the social planner. We rely on the fact that $u'_{\mathrm{P}}(w-\varphi) = \mu_\rho u'_{\mathrm{A}}(\varphi)$, from the definition of efficient risk sharing.

Incentive compatibility implies the first-order condition

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big) \,\Big|\, \tilde{\sigma} = \sigma\right] = 0$$

or

$$\mathbb{E}\left[u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

By Lemma 33 $\varphi'$ is a constant, thus we can pass it under the expectation operator. Further, since the right-hand side above is zero, we can remove $\varphi'$ from the equation entirely to get

$$\mathbb{E}\left[u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

And, since $u_{\mathrm{P}}'(w-\varphi)-\mu_\rho u_{\mathrm{A}}'(\varphi)=0$, the equation above re-writes as

$$\mathbb{E}\left[u_{\mathrm{P}}'\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0. \tag{3.19}$$

Now, in order to back out the social planner's objective from (3.19) observe that it suffices to subtract

$$\mathbb{E}\left[\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\left[u_{\mathrm{P}}'\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)-\mu_\rho u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\right]\,\Big|\,\tilde{\sigma}=\sigma\right], \tag{3.20}$$

which is zero since $u_{\mathrm{P}}'(w-\varphi)-\mu_\rho u_{\mathrm{A}}'(\varphi)=0$. Now subtracting expression (4.4) to equation (3.19) gives

$$\mathbb{E}\left[\Big(\tilde{w}'(x)-\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\Big)u_{\mathrm{P}}'\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]+ \\ +\mu_\rho\mathbb{E}\left[\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0$$

or

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)+\mu_\rho u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

This is the first-order condition of the social planner's problem if he knows $\sigma$. Since $u_{\mathrm{P}}$ and $u_{\mathrm{A}}$ are concave and $\tilde{w}$ is concave in $x$, the first order condition implies a global maximum, viz. the incentive compatible $x$ is a social optimum.

### 3.6.3 Computation of Optimal Investment

The problem stated in line (3.9) is to find the optimal investment $x^*$ given the optimal sharing rule stated in equation (3.8), namely

$$\varphi_\mu(w) = A + Bw,$$

where

$$A = \frac{\mu a_{\mathrm{A}} - a_{\mathrm{P}}}{1+\mu} \quad \text{and} \quad B = \frac{1}{1+\mu}. \tag{3.21}$$

That is, $x^*$ must maximize the expectation

$$-\frac{1}{2}\,\mathbb{E}\Bigg[\Big(R_f + x(\tilde{R}-R_f) - A - B\Big(R_f + x(\tilde{R}-R_f)\Big) - a_{\mathrm{P}}\Big)^2 \\ + \mu\Big(\Big(A + B\Big(R_f + x(\tilde{R}-R_f)\Big) - a_{\mathrm{A}}\Big)^2\Big) \,\Big|\, \tilde{\sigma} = \sigma\Bigg]$$

over all $x$. Thus the first-order condition says that for optimum $x^*$

$$\mathbb{E}\Bigg[(1-B)(\tilde{R}-R_f)\Big(R_f + x^*(\tilde{R}-R_f) - A - B\Big(R_f + x^*(\tilde{R}-R_f)\Big) - a_{\mathrm{P}}\Big) \\ + \mu\, B(\tilde{R}-R_f)\Big(A + B\Big(R_f + x^*(\tilde{R}-R_f)\Big) - a_{\mathrm{A}}\Big) \,\Big|\, \tilde{\sigma} = \sigma\Bigg] = 0,$$

thus

$$x^* = \frac{(\bar{R}-R_f)}{\mathbb{E}\big[(\tilde{R}-R_f)^2 \,\big|\, \tilde{\sigma}=\sigma\big]}\left(\frac{(1-B)(A+a_{\mathrm{P}}) - \mu\, B(A-a_{\mathrm{A}})}{(1-B)^2 + B^2\mu} - R_f\right).$$

Substituting in for $A$ and $B$ from equation (3.21) in the numerator gives

$$(1-B)(A+a_{\mathrm{P}}) - \mu B(A-a_{\mathrm{A}}) = \frac{\mu\,(a_{\mathrm{A}}+a_{\mathrm{P}})}{1+\mu}$$

and substituting in for $A$ and $B$ from equation (3.21) in the denominator gives

$$(1-B)^2 + B^2\mu = \frac{\mu}{1+\mu}.$$

Therefore

$$
\begin{aligned}
x &= \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)}{\mathbb{E}\big[\big(\tilde{R}-R_f\big)^2 \,\big|\, \tilde{\sigma}=\sigma\big]} \\
&= \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)}{\sigma^2+\big(\bar{R}-R_f\big)^2}.
\end{aligned}
$$

### 3.6.4 Computation of the Social Planner's Weight

Immediately from plugging in the expressions for $u_{\mathrm{A}}$, $\varphi_{\mu_\rho}$, and $x^*$ into equation (3.11), observe that

$$
\begin{aligned}
2|\bar{u}|\big(1+\mu_\rho\big)^2 &= \mathbb{E}\left[\left(R_f+\frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\Big(\tilde{R}-R_f\Big)-a_{\mathrm{P}}-a_{\mathrm{A}}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)^2\,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}-1\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)^2\left\{1-2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right]+\right. \\
&\qquad \left. +\,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right]\right\}.
\end{aligned}
\tag{3.22}
$$

Applying the law of iterated expectations gives

$$
\begin{aligned}
&1-\frac{2|\bar{\mu}|\big(1+\mu_\rho\big)^2}{\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)^2} \\
&= 2\mathbb{E}\left[\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right]-\mathbb{E}\left[\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right] \\
&= 2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\mathbb{E}\left[\big(\tilde{R}-R_f\big) \,\middle|\, \tilde{\sigma}\right]}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right]+\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right]}{\Big(\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2\Big)^2} \,\middle|\, \tilde{\rho}=\rho\right]
\end{aligned}
$$

and since

$$\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2\,\Big|\,\tilde{\sigma}\right]=\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2$$

we have

$$\begin{aligned}
&1-\frac{2|\bar{\mu}|\big(1+\mu_\rho\big)^2}{\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)^2}\\
&=\big(\bar{R}-R_f\big)^2\left\{\mathbb{E}\left[\frac{2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right]-\mathbb{E}\left[\frac{1}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right]\right\}\\
&=\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right].
\end{aligned}$$

Finally, solve for $(1+\mu_\rho)^2$ and cross multiply to recover equation (3.12).

### 3.6.5 Computation of Expected Utility Given $\rho$

Plug in to equation (3.13) and compute, maintaining at first the shorthand

$$\tilde{w}:=\tilde{w}\big(x^*(\tilde{\sigma})\big)=R_f+x^*(\tilde{\sigma})\big(R-R_f\big),$$

that is:

$$
\begin{aligned}
&\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}\big(x^*(\tilde{\sigma})\big) - \varphi_{\mu_\rho}\big(\tilde{w}\big(x^*(\tilde{\sigma})\big)\big)\Big) \;\Big|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\mathbb{E}\left[\Big(a_{\mathrm{P}} - \tilde{w} + \varphi_{\mu_\rho}\big(\tilde{w}\big)\Big)^2 \;\Big|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\mathbb{E}\left[a_{\mathrm{P}} - \tilde{w} + a_{\mathrm{A}} + \frac{\tilde{w} - a_{\mathrm{P}} - a_{\mathrm{A}}}{1+\mu_\rho} \;\middle|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\mathbb{E}\left[a_{\mathrm{P}} - \tilde{w} + a_{\mathrm{A}} + \frac{\tilde{w} - a_{\mathrm{P}} - a_{\mathrm{A}}}{1+\mu_\rho} \;\middle|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\big(a_{\mathrm{P}} + a_{\mathrm{A}} - \tilde{w}\big)^2 \;\Big|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\Big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f - x^*(\tilde{\sigma})\big(\tilde{R} - R_f\big)\Big)^2 \;\Big|\; \tilde{\rho} = \rho\right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\left(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f - \big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)\frac{\big(\bar{R} - R_f\big)\big(\tilde{R} - R_f\big)}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\right)^2 \;\middle|\; \tilde{\rho} = \rho\right] \\
&= -\frac{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\left(1 - \frac{\big(\bar{R} - R_f\big)\big(\tilde{R} - R_f\big)}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\right)^2 \;\middle|\; \tilde{\rho} = \rho\right].
\end{aligned}
$$

Now, from equation (3.22) above,

$$
\mathbb{E}\left[\left(1 - \frac{\big(\bar{R} - R_f\big)\big(\tilde{R} - R_f\big)}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\right)^2 \;\middle|\; \tilde{\rho} = \rho\right] = 2|\bar{u}|\left(\frac{1+\mu_\rho}{a_{\mathrm{P}} + a_{\mathrm{A}} - R_f}\right)^2,
$$

so, finally,

$$
\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}\big(x^*(\tilde{\sigma})\big) - \varphi\big(\tilde{w}\big(x^*(\tilde{\sigma})\big), \tilde{\sigma}, \rho\big)\Big) \;\Big|\; \tilde{\rho} = \rho\right] = \bar{u}\,\mu_\rho^2.
$$

# Chapter 4

# Credit Market Competition and Corporate Investment

## 4.1 Introduction

This paper develops a theoretical model to address the question of how interbank competition affects the *nature* of corporate investment, and how this effect is mitigated by the emergences of "relationship banking" (see, for example, Boot and Thakor (2000) and Petersen and Rajan (1995)). We conduct this analysis in the context of a general equilibrium model in which search fractions—akin to those in the labour market—and borrower-lender matching play an important role in determining equilibrium outcomes. The analysis sheds light on some existing evidence on relationship lending and produces new predictions.

The importance of the question we study is underscored by the fact that, in every country, banks play a dominant role in the allocation of credit, with significant consequences for corporate investment. It is widely recognised that interbank competition impinges on *how* banks allocate credit and *how much* of it they allocate (see, for example, Cetorelli (2001), and Ratnovski (2013)). This makes bank competition a central aspect of bank regulatory policy. Not surprisingly, it is a topic that has received substantial research attention.

At first blush, the question of whether higher bank competition is good for the economy seems to be hardly worth debating. One's economic intuition would say that higher competition should increase welfare, since banks would pay higher deposit rates and charge lower loans rates, facilitating higher deposit inflow and more lending to consumers and firms, with a consequent increase in economic growth. While some theoretical papers have confirmed this insight in banking models, others have pointed out that this standard intuition is off-the-mark because it overlooks special features that distinguish banks from other firms and also the potentially subtle effects of credit market frictions.

Pagano (1993) develops a model in which lower interbank competition leads to lower equilibrium lending and lower economic growth, a conclusion verified by Guzman (2000) in a general equilibrium model of capital accumulation. However, arrayed against this conclusion are numerous other papers with different results. Shaffer (1997) shows that a more competitive banking system may end up funding a lower-quality borrower pool. Cao and Shi (2000) argue that higher competition would increase loan rates and reduce loan supply. Dell'Ariccia (2000) develops a credit screening model to show that higher competition among banks may dilute their incentives to screen borrowers. Similarly, Manove, Padilla and Pagano (1998) show that, compared to a competitive banking system, banks in a monopolistic system will screen borrowers more and accept less collateral. There are also numerous theoretical papers that have examined the effect of interbank competition on risk-taking. Numerous papers have formalized the intuition that, by diminishing the charter values of banks, increased interbank competition generates incentives for banks to take higher risk (e.g. Hellmann, Murdock and Stiglitz (2000) and Repullo (2004)). However, some others have argued the opposite—by lowering the interest rates that banks charge their borrowers, increased interbank competition can induce *borrowers* to take less risk, thereby diminishing the default risk banks face. Martinez-Miera and Repullo (2010) extend this logic to show an inverse U-shaped relationship between bank competition and stability.

Our approach to the question of how interbank competition affects the credit market is different. We are concerned first with how competition impacts the *types* of projects firms invest in, and second with how this impact is influenced by the (endogenous) emergence of relationship banking. This is important because, in an environment in which firms have the discretion to choose whether to invest in say well-established routine technologies or in specialized innovative technologies, it would be useful to know the impact of interbank competition on this choice, since this would determine the kinds of firms and technologies that emerge and thrive in the economy. In this sense, our paper is in the tradition of Petersen and Rajan (1995) who show that young firms receive more credit when the banking system is less competitive, and of Boot and Thakor (2000) who develop a model in which greater competition among banks causes banks to make *more* relationship loans, but each loan has lower added value for the borrower.

The model we develop for our analysis is a general equilibrium model in which each firm can choose between a "standardized" (low-risk) project and a "specialized" ("differentiated", higher-risk) project. The bank can choose between relationship lending and arm's-length finance (or transaction lending, as in Boot and Thakor (2000)). In some states of nature the standardized project has higher social value and in others the specialized project has higher value. The bank commits to the type of financing before the firm chooses its project, and the firm's repayment obligation to the bank is determined by bargaining to divide the net surplus. Competition among banks affects the bank's choice of relationship lending versus arm's-length finance, and hence also affects the type of project the firm chooses.

Our main result is that the equilibrium involves the firm making the efficient project choice only for intermediate levels of interbank competition. When competition is sufficiently low, the specialized project is chosen excessively, relative to the first-best. When competition is sufficiently high, the standardized project is chosen excessively, relative to the first-best. Only for intermediate levels of competition does efficient project choice arise in equilibrium.

Our second main result is that the emergence of a relationship lender attenuates some of the inefficiencies in project choice. If a relationship bank extends credit, the inefficiency of excessive investment in the specialized project for low levels of competition disappears, but the inefficiency of excessive investment in the standardized project for high levels of competition remains.

To explain the core intuition behind these results, we first develop a "toy version" of the model. This model strips away some of the richness of the actual model in order to get to the intuition directly. After this the actual model is presented in order to more fully explore the various forces at work.

In contrast to theories in which monopolistic banking systems are the safest (e.g. Hellmann et al. (2000), Matutes and Vives (2000), and Repullo (2004)), in our analysis such banking systems are excessively risky since specialized projects are riskier than standardized projects. Our model predicts that the most competitive banking systems will involve the least risk, which sheds light on the country-level empirical evidence presented by Schaeck, Cihak and Wolfe (2009) that more competitive banking systems are less prone to systemic crises. However, the caveat suggested by our theory is that this attainment of safety comes at the cost of over-investment in (safe) standardized projects. Our result is also consistent with the empirical finding that firms invest less in R&D-intensive projects when credit competition is high (e.g. Hombert and Matray (2013)).

Another interpretation of our main result is that bank loan portfolios will become most liquid when interbank competition is the highest. This is because a standardized loan is more easily transferable and hence more liquid than a specialized loan. This seems to accord well with casual observation, but we are not aware of existing empirical evidence on this implication.

In addition to the papers discussed above, our work is related to the vast literature on relationship banking (e.g. Berlin and Mester (1992), Boot and Thakor (2000), Inderst and Mueller (2004), Petersen and Rajan (1995), Rajan (1992), and Sharpe (1990)); see Boot (2000) for a review. It is also related to how search frictions in the credit market (e.g. Diamond (1990)) affect credit outcomes, as in Wasmer and Weil (2004)).

The rest of the paper is organized in four remaining sections. Section 4.2 introduces the "toy model." Section 4.3 develops the actual model. Section 4.4 contains the analysis. Section 4.5 concludes. All formal proofs are in the Appendix.

## 4.2 Toy Model

In this section we introduce a simplified version of the model that pins down the intuition of the main results. At the core of the model is the project choice of an entrepreneur who needs outside capital to fund his investment. The entrepreneur chooses between two projects, called *standardized* and *differentiated* (or "specialized"). Both projects cost $I$ to implement. The standardized project is positive NPV with a deterministic cash flow, in particular it generates $V_{\mathrm{s}} > I$ for sure. The differentiated project, in contrast, is both information-sensitive and risky. To capture information sensitivity, we allow the differentiated project to be one of two types, which we refer to as high and low. Both the high-type and the low-type differentiated projects have binary risky cash flows. They pay off $V_{\mathrm{d}}$ when they succeed and zero otherwise. The difference between the types is the success probability. The high-type project yields $V_{\mathrm{d}}$ with probability $p_h$ whereas the low-type project yields $V_{\mathrm{d}}$ with probability $p_\ell < p_h$. We assume that the high-type differentiated project has the highest NPV but the low-type differentiated project has the lowest NPV, namely

$$p_\ell V_{\mathrm{d}} < V_{\mathrm{s}} < p_h V_{\mathrm{d}}.$$

This assumption implies that *efficient investment requires adapting to circumstances*, viz. an entrepreneur with a high-type differentiated project should undertake it, while an entrepreneur with a low-type differentiated project should undertake a standardized project.

The entrepreneur borrows from the creditor via a debt contract with face value $F$. Credit competition affects project choice via its affect on the terms of debt, i.e. on this face value $F$. For simplicity, we assume here that an entrepreneur and a creditor divide the net surplus of their relationship fifty-fifty, where the net surplus is the total value created by the funded project less the outside options of the two parties. Thus, the outside options will be essential in determining the terms of debt $F$. We make the following three assumptions about the players' outside options: (1) The creditor's outside option is the value $I$ of his capital. (2) If the entrepreneur chooses the differentiated project, his outside option is zero, written $\pi_e^{\mathrm{d}} = 0$. The motivation is that the differentiated project is information-sensitive, and adverse selection in the credit market will make it impossible for the entrepreneur to find funding for his project elsewhere. Thus, if he chooses the differentiated project he is captive to his creditor. (3) If an entrepreneur chooses the standardized project, his outside option, labelled $\pi_e^{\mathrm{s}}$, depends on the competitiveness of the credit market. The motivation is as follows: since the project is information-insensitive, the entrepreneur will not suffer from the effects of adverse selection and other creditors will be able to fund his project. When the credit market is competitive, it will be easy to find another creditor to fund the project and the entrepreneur's outside option will be high; when the credit market is uncompetitive, it will be hard to find another creditor to fund the project and the entrepreneur's outside option will be low. Here we take $\pi_e^{\mathrm{s}}$ to represent credit competition directly. (In the full model, we model credit market competition explicitly within a search framework and demonstrate this connection with $\pi_e^{\mathrm{s}}$ explicitly.)

The stage is now set to present the main result: for only intermediate values of credit competition will the entrepreneur choose the efficient project. When credit competition is too low, the entrepreneur over-differentiates, choosing the differentiated project even when it is the low-type. When competition is too high, in contrast, the entrepreneur over-standardizes, choosing the standardized project even when he has a high-type differentiated project.

Before deriving the result explicitly in the toy model, we outline the intuition for each of these inefficiencies. First consider the entrepreneur with the low-type differentiated project. He wishes to standardize to increase the net surplus, but knows that by differentiating he will have an informational advantage, and may receive a cross-subsidy from the high-type. When competition is high the entrepreneur's strong bargaining position from standardization gives him the incentive not to choose the low-NPV differentiated project. For low levels of competition, however, the entrepreneur has no extra incentive to standardize and prefers to differentiate inefficiently. Now turn to the entrepreneur with the high-type differentiated project. He wishes to differentiate to increase the surplus, but knows that by doing so he will lower his outside option and thus worsen his borrowing terms. For high levels of credit competition the entrepreneur values his lower cost of borrowing from standardizing so much that he never differentiates. To summarize, credit competition mitigates the over-differentiation problem by increasing the value of safe projects to the entrepreneur. However, for high levels of competition this same effect leads the entrepreneur not to differentiate enough.

We now derive the result in the toy model. Suppose an efficient equilibrium and demonstrate how too much or too little credit competition leads the entrepreneur to deviate. Recall that $\pi_e^{\mathrm{s}}$ proxies for competition. The first step is to calculate the face values of debt $F_{\mathrm{s}}$ and $F_{\mathrm{d}}$ that the entrepreneur must promise to repay in order to fund the standardized and differentiated projects respectively. Recall that they divide the net surplus fifty-fifty. The net surplus from standardization is $V_{\mathrm{s}} - I - \pi_e^{\mathrm{s}}$, so $F_{\mathrm{s}}$ returns the creditor's investment $I$ to him in addition to half the net surplus,

$$F_{\mathrm{s}} = I + \frac{V_{\mathrm{s}} - I - \pi_e^{\mathrm{s}}}{2}.$$

The efficient outcome under consideration is separating, so only high-type entrepreneurs choose the differentiated project. Thus the net surplus given the differentiated project is $p_h V_{\mathrm{d}} - I - \pi_e^{\mathrm{d}} = p_h V_{\mathrm{d}} - I$ since $\pi_e^{\mathrm{d}} = 0$ by assumption. Now the creditor must receive his investment $I$ plus half the net surplus in expectation, so

$$p_h F_{\mathrm{d}} = I + \frac{p_h V_{\mathrm{d}} - I}{2}.$$

Now, when is it incentive compatible for the entrepreneur with the high-type project to differentiate? He chooses the differentiated project as long as

$$p_h(V_{\mathrm{d}} - F_{\mathrm{d}}) \geq V_{\mathrm{s}} - F_{\mathrm{s}}$$

which rewrites as

$$p_h V_{\mathrm{d}} - I \geq V_{\mathrm{s}} - I + \pi_e^{\mathrm{s}}$$

or, writing $\Delta V := p_h V_{\mathrm{d}} - V_{\mathrm{s}}$,

$$\pi_e^{\mathrm{s}} \leq \Delta V$$

which says exactly that entrepreneurs with high-type projects differentiate only if credit competition does not exceed the surplus gains from differentiation. When competition is too high entrepreneurs standardize inefficiently.

Next, when is it incentive compatible for the entrepreneur with the low-type project to standardize? He chooses the standardized project as long as

$$V_{\mathrm{s}} - F_{\mathrm{s}} \geq p_\ell (V_{\mathrm{d}} - F_{\mathrm{d}})$$

which rewrites as

$$V_{\mathrm{s}} - I + \pi_e^{\mathrm{s}} \geq \frac{p_\ell}{p_h}(p_h V_{\mathrm{d}} - I).$$

Here the right-hand side is the payoff of an entrepreneur with a low-type project from borrowing at the terms of the entrepreneur with a high-type project. When competition $\pi_e^{\mathrm{s}}$ is high, the inequality is satisfied because the gained bargaining position from standardization curbs the entrepreneur's incentive to over-differentiate. When competition is low, however, this mechanism is not in place; the inequality is violated and risk-shifting occurs.

Thus we see two-sides of credit market competition. Because higher competition encourages entrepreneurs with both high-type and low-type differentiated projects to standardize, increasing competition simultaneously curbs the incentive for entrepreneurs to over-differentiate when they have low-type projects while exacerbating the incentive for them to over-standardize when they have high-type projects. In particular the equations above imply immediately that entrepreneurs with both types of projects invest efficiently for only intermediate levels of credit competition, i.e. only if

$$\frac{p_\ell}{p_h}(p_h V_{\mathrm{d}} - I) - (V_{\mathrm{s}} - I) \leq \pi_e^{\mathrm{s}} \leq \Delta V. \tag{4.1}$$

Both these inefficiencies stem from the entrepreneur's choosing inefficiently to benefit from better funding terms. So far we have assumed that the creditor cannot observe the type of the differentiated project. Can a creditor with the expertise to assess the quality of the entrepreneur's project mitigate the inefficiencies that result from too much or too little credit competition? To address this question we introduce a *relationship lender* who can observe the type of the entrepreneur's differentiated project. Our next main result is that the entrepreneur with the low-type differentiated project no longer over-differentiates when competition is low if he obtains funding from a relationship lender. The reason is that over-differentiation is driven by the entrepreneur's incentive to access cheap funding by choosing the differentiated project even though it is low-type. However, this funding advantage disappears when a creditor can observe the project type.

The argument above shows that relationship lending can mitigate the over-differentiation problem and thus increase efficiency when there is little credit competition. For high competition, in contrast, the entrepreneur with the high-type differentiated project continues to over-standardize when competition is high, even if he obtains funding from a relationship lender. The reason is that the creditor's ability to observe the type of the entrepreneur's differentiated project does not make the entrepreneur less captive to his creditor when he chooses the differentiated project. Thus the entrepreneur with the high-type differentiated project still over-standardizes to avoid the relatively poor terms of debt he obtains when he differentiates. To summarize, relationship banking is valuable in relatively uncompetitive credit markets but fails to reduce the inefficient standardization that high competition causes.

While the toy model shows the main mechanism behind the main results, it relies on a number of shortcuts. Notably, we made the following assumptions to keep the analysis simple: (1) the creditor and entrepreneur split the surplus fifty-fifty, (2) the outside option of the entrepreneur with the differentiated project is zero, (3) the outside option of the entrepreneur with the standardized project proxies for credit market competition, and (4) relationship lending is costless. In the full model we micro-found or relax each of these assumptions using a dynamic search and matching framework. The complete model confirms the robustness of the intuition above as well as provides additional results. The most important result that the toy model does not deliver is that for very low levels of competition a creditor may be unwilling to make a positive NPV investment in a relationship lending technology due to a hold-up problem.

# 4.3 Model

## 4.3.1 Agents and Projects

There are two kinds of players, creditors and entrepreneurs. All players are risk-neutral and discount the future at net rate $r$ equal to the return on the money market account. A creditor $c$ provides start-up capital $I$ to penniless entrepreneur $e$ to fund a project $\delta$. $e$ has a choice between two projects, called *standardized*, $\delta = \mathrm{s}$, and *differentiated*, $\delta = \mathrm{d}$. A standardized project is information-insensitive and riskless. It pays off $V_{\mathrm{s}}$ for sure. In contrast, a differentiated project is information-sensitive—because its cash flow distribution is $e$'s private information—and risky—because its cash flows are random. Specifically, a differentiated project is one of two types, $\tilde{\tau} \in \{h, \ell\}$, which $e$ will observe before choosing $\delta \in \{\mathrm{d}, \mathrm{s}\}$. The $h$-type differentiated project pays off $V_{\mathrm{d}}$ with probability $p_h$ and zero otherwise, whereas the $\ell$-type differentiated project pays off $V_{\mathrm{d}}$ with probability $p_\ell < p_h$ and zero otherwise. The probability that the project is type $h$ is $\alpha$. All random variables are independent.

Before granting a loan, a creditor chooses the type of credit $\eta$ to provide. He either offers *relationship lending*, $\eta = \mathrm{r}$, or *arms-length finance*, $\eta = \mathrm{a}$. The difference between a relationship lender and an arm's-length lender is that the relationship lender can observe the differentiated entrepreneur's type, whereas an arm's length lender cannot. The creditor can always offer arm's-length finance at no cost, but to perform relationship lending he must pay a cost $k$. Finally, the cost $k$ is entrepreneur-specific, in other words it allows him to learn the type of only one entrepreneur's project.

An $h$-type differentiated project has the highest present value, but the standardized project has a higher present value than the $\ell$-type differentiated project, which has negative NPV, or

$$p_h V_{\mathrm{d}} - I > V_{\mathrm{s}} - I > 0 > p_\ell V_{\mathrm{d}} - I. \tag{4.2}$$

Further, a standardized project has higher present value than the average differentiated project, or

$$V_{\mathrm{s}} > \alpha p_h V_{\mathrm{d}} + (1 - \alpha) p_\ell V_{\mathrm{d}}. \tag{4.3}$$

Thus, it is efficient for the $h$-type entrepreneur to invest in the differentiated project and for the $\ell$-type entrepreneur to invest in the standardized project. We also assume that the efficiency gains from the efficient project choice exceed the cost of relationship lending, i.e.

$$\alpha p_h V_{\mathrm{d}} + (1 - \alpha) V_{\mathrm{s}} - k > V_{\mathrm{s}}.$$

or

$$k < \alpha \Delta V \tag{4.4}$$

where $\Delta V := p_h V_{\mathrm{d}} - V_{\mathrm{s}}$. The motivation for this assumption is that relationship lenders' ability to observe the project type gives them the flexibility to adapt to the different circumstances represented by the type of entrepreneurs' differentiated projects. The benefit of creditors' flexibility is the ability to ensure that entrepreneurs invest in the most efficient project in every circumstance. This is a benefit that creditors who extend arm's length finance do not enjoy. The assumption above says that the benefit of flexibility outweighs the cost $k$ of the relationship.

**Search and Matching**

Creditors and entrepreneurs find each other by searching in a decentralized market.

At time $t \in \{..., -1, 0, 1, ...\}$ a set $E_t$ of searching entrepreneurs matches with a set $C_t$ of creditors with intensity $m(|E_t|, |C_t|)$. Define $\theta_t := |C_t|/|E_t|$, the credit market competition. Assume the probability that a creditor finds an entrepreneur at time $t$,

$$q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|C_t|}, \tag{4.5}$$

and the probability an entrepreneur finds a creditor at time $t$,

$$Q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|E_t|}, \tag{4.6}$$

depend only on $\theta_t$ (for which, for example, $m$ being homogenous of degree one suffices).

Assume $m$ is such that $q$ and $Q$ are differentiable with $q' < 0$, $Q' > 0$, $q(0) = 1$, $Q(0) = 0$, and with $q(\theta) \to 0$ and $Q(\theta) \to 1$ as $\theta \to \infty$. As credit competition increases the likelihood that a creditor finds an entrepreneur decreases and that an entrepreneur finds a creditor increases.

To make the model stationary, assume that each player that leaves the market is replaced by a player of the same type.

**Stage Game Extensive Form**

When an entrepreneur $e$ is born he learns his type. When $e \in E_t$ matches with a creditor $c \in C_t$ for the first time, they play the extensive game defined by the timing below:

1. $c$ chooses between relationship lending and arm's-length finance, $\eta \in \{\mathrm{r}, \mathrm{a}\}$. r costs $k$ and enables the creditor to observe the type of the differentiated project.

2. If $c$ has played $\eta = \mathrm{r}$, he observes the type $\tilde{\tau} \in \{h, \ell\}$ of the differentiated project.

3. $e$ chooses between a differentiated project and a standardized project, $\delta \in \{\mathrm{d}, \mathrm{s}\}$. The choice is irreversible.

4. The face value of debt $F$ is determined as follows

   - With probability $\beta$, $c$ makes $e$ a take-it-or-leave-it offer $F_\delta^c$; $e$ accepts or rejects.

   - With probability $1 - \beta$, $e$ makes $c$ a take-it-or-leave-it offer $F_\delta^e$; $c$ accepts or rejects.

5. The project pays off $V_\delta \in \{V_\mathrm{s}, V_\mathrm{d}\}$. $e$ and $c$ divide the surplus according to the agreed contract.

If the relationship between $c$ and $e$ breaks down, they search again in the market. Since $e$'s choice $\delta$ is irreversible, it remains to specify the game played between an entrepreneur committed to a project $\delta$ and his new potential creditor $c'$.

1. $c'$ observes $\delta$.

2. $c'$ chooses between relationship lending and arm's-length finance, $\eta \in \{\mathrm{r}, \mathrm{a}\}$. r costs $k$ and enables the creditor to observe the type of the differentiated project.

3. The face value of debt $F$ is determined as follows

   - With probability $\beta$, $c$ makes $e$ a take-it-or-leave-it offer $F_\delta^{c'}$; $e$ accepts or rejects.

   - With probability $1-\beta$, $e$ makes $c'$ a take-it-or-leave-it offer $F_\delta^{e}$; $c'$ accepts or rejects.

If $c$ and $e$ are searching but not matched at date $t$ they search again.

Throughout the solution concept is Perfect Bayesian Equilibrium.

## 4.4 Results

This section establishes the main results (1) that the first-best is attainable for only intermediate levels of credit market competitiveness and (2) that relationship lending can mitigate the inefficiency but only for low levels of credit competition.

### 4.4.1 Background Mechanism

We establish the results first in partial equilibrium taking the players' continuation values as given. This analysis resembles that of the toy model in section 4.2. We then endogenize the continuation values and prove the results in terms of the ratio $\theta$ of creditors to entrepreneurs, which we call the credit market competition.

### Out of Equilibrium Beliefs

Since this is a dynamic game of asymmetric information, off-the-equilibrium path beliefs about the type $\tau$ of an entrepreneur's project will play a role in the analysis. Throughout we focus on equilibria supported by the following beliefs: if a creditor matches with an entrepreneur who has already differentiated his project (necessarily because he failed to obtain funding from the creditor he was initially matched with), then the creditor believes the project is type $\ell$. We summarize this with the following assumption.

**Assumption 4.4.1.** *If a creditor encounters an entrepreneur with an already differentiated project, he believes the project is type $\ell$, written $\mu(\tilde{\tau} = \ell \mid \delta = \mathrm{d}) = 1$.*

We emphasize that this is not an assumption on primitives, but just a statement about which Perfect Bayesian Equilibria we focus on. We find these equilibria most intuitive because the differentiated project is information-sensitive and we wish to capture the idea that it may be difficult to find funding for an information-sensitive project due to adverse selection. If he chooses a standardized project, an entrepreneur avoids the risk of adverse selection in the future. He obtains the advantage of a higher outside option and better terms of debt from his creditor.

Some readers may find it unappealing that we restrict the out of equilibrium beliefs. In this case they can replace the restriction with a more direct assumption on the nature of the differentiated project. For example, if it is also time-sensitive, because the rents from specialisation are available only if an entrepreneur implements his project before a competitor, then his outside option will be affected analogously.

Further, we add that for other out-of-equilibrium beliefs the model is still solvable and that for relatively pessimistic creditors or low average NPV differentiated projects the qualitative results still hold. Considering more optimistic beliefs of the creditor will indeed attenuate our results for high levels of credit competition, making it less likely that entrepreneurs will over-standardize, but it will exacerbate the over-differentiation inefficiency that we find for low levels of credit competition.

### Continuation Values

When $c$ and $e$ are matched, their decisions as to whether to engage in relationship lending and whether to undertake a differentiated project depend on the proportion of the total surplus from the match that they anticipate earning. For each player, a higher outside option leads to a greater share of net surplus.

At the time at which the face value of debt is determined, these outside options are equal to the players' continuation values from searching again in the market. Denote the continuation value of the creditor by $\pi_c$ and the continuation value of the entrepreneur with project $\delta$ by $\pi_e^\delta$. We emphasize that because his project choice is irreversible, the continuation value of the entrepreneur depends on his project choice. Note that, in general, these values could depend on time, but we suppress this possibility since we will focus on stationary equilibria. Further, the continuation value of the entrepreneur with the differentiated project could also depend on the type $\tau$ of his project. However, since the entrepreneur with the differentiated project will have his credit completely rationed in the future, this will also not be the case, as the following lemma implies.

**Lemma 34.** *The continuation value of the already differentiated entrepreneur is zero,* *i.e.* $\pi_e^d = 0$.

*Proof.* The short proof is in Appendix 4.6.1. □

### 4.4.2 First-Best

The efficient outcome of this model involves the entrepreneur choosing the efficient project, namely $\delta = \mathrm{d}$ when $\tau = h$ and $\delta = \mathrm{s}$ when $\tau = \ell$ and the creditor playing $\eta = \mathrm{a}$, avoiding the cost $k$ of relationship lending. Our question is when is this outcome ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$) implementable? Specifically, for which values of credit competition $\theta$ can it emerge in equilibrium? We now proceed to find conditions for it to be a stationary Perfect Bayesian Equilibrium of the model given the out-of-equilibrium beliefs $\mu(\ell \mid \mathrm{d}) = 1$.

For this to be an equilibrium, $e$ must self-select the efficient project without the discipline of his creditor—$c$ cannot observe the type of the differentiated project since $\eta = \mathrm{a}$. The main results come from finding conditions for $e$'s incentive constraints to be satisfied.

Before finding these conditions, we emphasize the equilibrium beliefs for clarity. Since beliefs must be consistent in equilibrium, creditors who observe entrepreneurs choose d must believe they have $h$-type projects.

#### Face Values

The face value $F$ of debt depends on which player is proposing the contract in round 4 of the stage game (subsection 4.3.1).

The proposer always offers the face value that makes his opponent indifferent between accepting and rejecting. Thus there are four cases: (1) when $e$ proposes and he has a standardized project, (2) when $c$ proposes when $e$ has a standardized project, (3) when $e$ proposes and he has a differentiated project, and (4) when $c$ proposes and $e$ has a differentiated project.

We now compute each of these face values under the equilibrium beliefs. Note that when $e$ chooses s, there is no asymmetric information so the face value just serves as a means to divide surplus—it will not enter substantively in the analysis. When the project is differentiated the face value of the debt contract will matter. Here the subscripts on the face values denote the project choice and the superscripts denote the proposer. When $e$ proposes the face value with a standardized project is

$$F_{\mathrm{s}}^{e} = \pi_c$$

and when $c$ proposes and $e$ has chosen a standardized project the face value is given by

$$V_{\mathrm{s}} - F_{\mathrm{s}}^{c} = \pi_e^{\mathrm{s}}.$$

When $e$ proposes with a differentiated project $c$ gets repaid with probability $p_h$ so the face value is given by

$$p_h F_{\mathrm{d}}^{e} = \pi_c$$

and when $c$ proposes and $e$ has chosen a differentiated project the face value is given by

$$p_h (V_{\mathrm{d}} - F_{\mathrm{d}}^{c}) = \pi_e^{\mathrm{d}}.$$

**Incentive Constraints**

If $c$ believes that everyone is playing according to the profile ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$), he never has incentive to deviate to $\eta = \mathrm{r}$ since it comes with a cost $k$ and no informational benefit. Thus to ask when this outcome is an equilibrium we can focus entirely on $e$'s incentive constraints. We find conditions first for $\delta_\ell = \mathrm{s}$ and then for $\delta_h = \mathrm{d}$.

In order for $\delta_\ell = \mathrm{s}$, the entrepreneur must prefer to standardize when his project is type $\ell$. Recall that in round 4 of the stage game the creditor proposes with probability $\beta$. Thus with probability $\beta$ $e$ receives his outside option and with probability $1-\beta$ $e$ pushes $c$ to his outside option. Hence for $e$ to choose to standardize with the $\ell$-type differentiated project it must be that

$$\beta\pi_e^{\mathrm{s}} + (1-\beta)(V_{\mathrm{s}} - F_{\mathrm{s}}^e) \geq \beta\pi_e^{\mathrm{d}} + (1-\beta)p_l(V_{\mathrm{d}} - F_{\mathrm{d}}^e)$$

or, plugging in,

$$(1-\beta)\big(V_{\mathrm{s}} - \pi_c\big) + \beta\pi_e^{\mathrm{s}} \geq (1-\beta)\frac{p_\ell}{p_h}\left(p_h V_{\mathrm{d}} - \pi_c\right). \tag{IC$_\ell$}$$

For $e$ to choose to differentiate with the $h$-type differentiated project it must be that

$$\beta\pi_e^{\mathrm{d}} + (1-\beta)p_h\left(V_{\mathrm{d}} - F_{\mathrm{d}}^e\right) \geq \beta\pi_e^{\mathrm{s}} + (1-\beta)\left(V_{\mathrm{s}} - F_{\mathrm{s}}^e\right)$$

or, plugging in,

$$(1-\beta)p_h V_{\mathrm{d}} \geq (1-\beta)V_{\mathrm{s}} + \beta\pi_e^{\mathrm{s}}. \tag{IC$_h$}$$

Combining the incentive constraints (IC$_\ell$) and (IC$_h$) implies that the first best can be attained in equilibrium if and only if

$$p_\ell V_{\mathrm{d}} - V_{\mathrm{s}} + \left(1 - \frac{p_\ell}{p_h}\right)\pi_c \;\leq\; \frac{\beta}{1-\beta}\,\pi_e^{\mathrm{s}} \;\leq\; \Delta V, \tag{4.7}$$

recalling that $\Delta V := p_h V_{\mathrm{d}} - V_{\mathrm{s}}$. These inequalities are the analogue of the inequalities (4.1) in the toy model described in Section 4.2 above. Next, we find the values of $\pi_c$ and $\pi_e^{\mathrm{s}}$ in terms of credit competition $\theta$.

### Value Functions

To demonstrate that if credit competition is either too intense or too weak real investment is choked off, compute the continuation values given that players believe $(\eta = \text{a}, \delta_h = \text{d}, \delta_\ell = \text{s})$ is the stationary action profile of the stage game.

We look for stationary equilibria of the model. The players' value functions from continuing to search determine the offers their opponents make in round 4 of the stage game. These offers determine the division of surplus. The creditor has a single continuation value $\pi_c$, whereas the entrepreneur's continuation value depends on his project choice, $\delta \in \{\text{s}, \text{d}\}$. Lemma 34 fixes $\pi_e^{\text{d}} = 0$. It remains to compute $\pi_e^{\text{s}}$ and $\pi_c$. $\pi_c$ and $\pi_e^{\text{s}}$ will be interdependent: a higher $\pi_c$ lowers $\pi_e^{\text{s}}$ because $e$ anticipates having to give a larger share of the net surplus to $c$ and vice versa.

On the $(\eta = \text{a}, \delta_h = \text{d}, \delta_\ell = \text{s})$ equilibrium path, creditors always fund entrepreneurs the first time that they are matched. But it must be incentive compatible for the creditor to agree to fund a standardized entrepreneur rather than search again to wait for a differentiated entrepreneur. As long as players are sufficiently impatient this will always be the case. Specifically, to ensure that standardized projects are funded, we make the following assumption on parameters.

**Assumption 4.4.2.**

$$r > \frac{\alpha\beta\Delta V}{V_{\text{s}} - I}.$$

Note that we view the time between dates as relatively long since it is the time taken to develop a lending relationship and implement a project. Thus we do not consider the assumption that $r$ is large to be overly restrictive.

We now proceed to compute the value functions maintaining the assumption that standardized entrepreneurs will find funding when they are matched. After computing the equilibrium value functions, observe that Assumption 4.4.2 suffices for this to be the case.

The players' value functions are their expected utilities today from continuing the game. Consider first the standardized entrepreneur. He will be matched tomorrow with probability $Q = Q(\theta)$ and will be unmatched with probability $1 - Q$. If he is unmatched he searches again. Since the equilibrium is stationary, he obtains $\pi_e^{\mathrm{s}}$ in this case. If he is matched the creditor is the proposer with probability $1 - \beta$. In this case again the entrepreneur obtains $\pi_e^{\mathrm{s}}$. With probability $\beta$, however, the entrepreneur is the proposer and in this case he obtains $(1 - \beta)(V_{\mathrm{s}} - \pi_c)$ (see Section 4.7 above). This description summarizes all the possibilities and allows us to write a formula for $\pi_e^{\mathrm{s}}$:

$$\pi_e^{\mathrm{s}} = \frac{Q\big(\beta\pi_e^{\mathrm{s}} + (1-\beta)(V_{\mathrm{s}} - \pi_c)\big) + (1-Q)\pi_e^{\mathrm{s}}}{1+r},$$

or

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)Q}{r + (1-\beta)Q}\big(V_{\mathrm{s}} - \pi_c\big). \tag{4.8}$$

The explanation for the expression for the creditor's value function $\pi_c$ resembles that of the entrepreneur who has a standardized project in the previous paragraph. It is two terms more complex, however. The first extra term arises because the creditor can be matched with two types of entrepreneurs: one with a high-type differentiated project (who plays $\delta = \mathrm{d}$) and one with a low-type differentiated project (who plays $\delta = \mathrm{s}$). The second extra term arises because the creditor earns interest on the capital $I$ he has not invested. We now describe the terms that we must take into account to write down the creditor's value function. He is matched with probability $q = q(\theta)$.

With probability $1-q$ he is unmatched and searches again to receive $\pi_c$, by stationarity. If he is matched, with probability $\alpha$ he is matched with an entrepreneur who has an $h$-type differentiated project and who chooses to do this differentiated project. In this case $e$ proposes with probability $1-\beta$, leaving $c$ with $\pi_c$. With probability $\beta$ $c$ proposes and his utility is $p_h V_{\mathrm{d}}$ since $\pi_e^{\mathrm{d}} = 0$. Finally consider the case in which $c$ is matched with an entrepreneur who has an $\ell$-type differentiated project. $e$ then chooses to do the standardized project. Again, with probability $1-\beta$ $e$ proposes and $c$ gets $\pi_c$. With probability $\beta$ $c$ proposes and gets $V_{\mathrm{s}} - \pi_e^{\mathrm{s}}$. The final term is the interest $rI$ that the creditor earns from holding his capital in the money-market account while searching. This description summarizes all the possibilities and allows us to write a formula for $\pi_c$:

$$\pi_c = \frac{q\Big[\alpha\Big(\beta p_{\mathrm{h}} V_{\mathrm{d}} + (1-\beta)\pi_c\Big) + (1-\alpha)\Big(\beta\big(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}\big) + (1-\beta)\pi_c\Big)\Big] + (1-q)\pi_c + rI}{1+r}, \tag{4.9}$$

or

$$\pi_c = \frac{q\Big(\alpha\beta p_{\mathrm{H}} V_{\mathrm{d}} + (1-\alpha)\beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}})\Big) + rI}{r + \beta q}. \tag{4.10}$$

We can now solve for the equilibrium value functions, which are the solution of the system of equations (4.8) and (4.10).

**Lemma 35.** *In a stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$*, the* *value functions are given by*

$$
\begin{aligned}
\pi_e^{\mathrm{d}} &= 0,\\
\pi_e^{s} &= \frac{(1-\beta)Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\left(r\big(V_{\mathrm{s}}-I\big)-\alpha\beta q\Delta V\right),\\
\pi_c &= I+\frac{\beta q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\left(r\Big[\alpha\Delta V+V_{\mathrm{s}}-I\Big]+\alpha(1-\beta)Q\big(p_h V_{\mathrm{d}}-I\big)\right).
\end{aligned}
$$

One of the key short cuts we took in the toy model of Section 4.2 was that $\pi_e^{\mathrm{s}}$ proxied for competition. The next lemma says that $\pi_e^{\mathrm{s}}$ is strictly increasing in competition $\theta$, so the shortcut is now micro-founded.

**Lemma 36.** *In a stationary equilibrium with action profile* $(\eta=\mathrm{a},\delta_h=\mathrm{d},\delta_\ell=\mathrm{s})$, $\pi_e^{\mathrm{s}}$ *is increasing in* $\theta$*.*

*Proof.* The proof is in Appendix 4.6.2. It simply applies the quotient rule to $\pi_e^{\mathrm{s}}$ keeping in mind the assumptions on the matching probabilities, namely $q'(\theta)<0$ and $Q'(\theta)>0$. □

**The Two Sides of Credit Market Competition**

The inequalities (4.7) show the efficient outcome can be supported in equilibrium for only intermediate values of $\pi_e^{\mathrm{s}}$. Then, Lemma 36 shows that $\pi_e^{\mathrm{s}}$ indeed proxies for competition $\theta$. This section shows that the intuition established in the toy model of Section 4.2 is robust: when competition $\theta$ is too high or too low the efficient outcome is not an equilibrium.

**Proposition 37.** *Suppose that*

$$\left(1-\frac{p_\ell}{p_h}\right)\left(I+\frac{\beta}{\beta+r}\right)\left(\alpha\Delta V+V_{\mathrm{s}}-I\right)>V_{\mathrm{s}}-p_\ell V_{\mathrm{d}}$$

*and*

$$\frac{\beta}{1-\beta+r}\Big(V_{\mathrm{s}}-I\Big) > \Delta V.$$

*There is a stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *only if credit competition* $\theta$ *is neither too large nor too small.*

*Proof.* The proof is in appendix 4.6.3. It follows from considering the limits of $\pi_e^{\mathrm{s}}$ and $\pi_c$ as $\theta \to 0$ and $\theta \to \infty$ and comparing them to the bounds in the incentive constraints ($\mathrm{IC}_h$) and ($\mathrm{IC}_\ell$).

□

### 4.4.3 Second Best

We now ask whether the creditors' option to develop a relationship with the entrepreneur can restore efficient project choice when credit market competition is too high or too low for entrepreneurs to self-select the efficient project (as stated in Proposition 37).

Since relationship lending entails an expense $k$ for a creditor, efficiency is not restored fully, however recall that the assumptions (4.2), (4.3), and (4.4), say that

$$\big(\alpha p_h + (1-\alpha)p_\ell\big)V_{\mathrm{d}} < V_{\mathrm{s}} < \alpha V_{\mathrm{d}} + (1-\alpha)V_{\mathrm{s}} - k,$$

which imply that the surplus gains from efficient project choice outweigh the cost of relationship lending. Thus the second-best outcome is $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$. When does it constitute a stationary equilibrium?

### Off Equilibrium Path Behaviour

To find the conditions for this action profile to be part of an equilibrium, we must specify the behaviour off the equilibrium path. In particular, if $c$ plays $\eta = \mathrm{a}$ and $e$ plays $\delta = \mathrm{d}$, what does $c$ believe about the type of $e$'s project? We will focus on equilibria in which $e$ plays s following $c$ playing a. This is the unique off-path behaviour if the entrepreneur with the $h$-type differentiated project always prefers to play s than to pool with the entrepreneur with the $\ell$-type project. (If self-selected separation were possible we could implement first best anyway.) A sufficient condition for uniqueness is that the average NPV of differentiated projects is low, which we make precise with the next assumption.

**Assumption 4.4.3.**

$$\Delta V < \left( \frac{1}{\alpha p_h + (1-\alpha) p_\ell} - 1 \right) I.$$

Further we maintain the focus on only equilibria in which a creditor believes that if he encounters an entrepreneur with a differentiated project off-equilibrium he believes the project is type $\ell$, $\mu(\tau = \ell \mid \delta = \mathrm{d}) = 0$.

### Face Values

As in Subsection 4.4.2 above, the face value of debt depends on who is the proposer in round 4 of the stage game and on the project choice. Since we are looking for an equilibrium in which $\eta = \mathrm{r}$, the creditor also observes the type of the differentiated project. Note that the entrepreneur with the $\ell$-type differentiated project will not obtain funding for it—it is negative NPV and the creditor observes the type $\tau$. Thus, following $\eta = \mathrm{r}$, $e$ always plays $\delta = \mathrm{s}$ if he has an $\ell$-type differentiated project. Thus again there are four face values to compute: (1) when $e$ proposes and he has a standardized project, (2) when $c$ proposes and $e$ has a standardized project, (3) when $e$ proposes and he has an $h$-type differentiated project, and (4) when $c$ proposes and $e$ has an $h$-type differentiated project. The expressions for the face values are identical to those in Subsection 4.4.2. The key difference is that the continuation values $\pi_e^{\mathrm{s}}$ and $\pi_c$ are different. Also, keep in mind that, even though the notation is unchanged, when the project is differentiated the creditor observes whether $\tau = \ell$ or $\tau = h$ whereas before he did not. To summarize, the face values are

$$F_{\mathrm{s}}^e = \pi_c,$$

$$F_{\mathrm{s}}^c = V_{\mathrm{s}} - \pi_e^{\mathrm{s}},$$

$$F_{\mathrm{d}}^e = \pi_c / p_h,$$

$$F_{\mathrm{d}}^c = V_{\mathrm{d}}.$$

**Entrepreneurs' Incentive Constraints**

As already mentioned in the previous section, if $c$ has played $\eta = r$, then whenever $e$ has an $\ell$-type differentiated project he plays $\delta = \mathrm{s}$. Thus to check whether there is a stationary equilibrium with action profile $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$, we need to check the incentive constraint for only the entrepreneur with the $h$-type project. As in the first-best, he plays $\delta = \mathrm{d}$ whenever

$$(1-\beta) p_h V_{\mathrm{d}} \geq (1-\beta) V_{\mathrm{s}} + \beta \pi_e^{\mathrm{s}}$$

or

$$\pi_e^{\mathrm{s}} \leq \frac{1-\beta}{\beta} \Delta V.$$

This is the incentive constraint ($\mathrm{IC}_h$) already written down above, but bear in mind that $\pi_e^{\mathrm{s}}$ depends on the equilibrium.

**Creditors' Incentive Constraint**

Unlike in the first-best case, a creditor's incentive constraint now has bite. If he deviates to $\eta = \mathrm{a}$ he saves the cost $k$ of relationship lending, but forgoes the increased rents he gains when $e$ has an $h$-type differentiated project. If he chooses $\eta = \mathrm{a}$ he anticipates that the entrepreneur will standardize (see Assumption 4.4.3 and the discussion in Subsection 4.4.3). Therefore, his expected payoff is $\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c$. If he chooses $\eta = \mathrm{r}$ the expression for his payoff is more complicated for two reasons: (1) he must take into account the possibility that he is matched with an entrepreneur who has an $\ell$-type differentiated project as well as the possibility that he is matched with an entrepreneur who has an $h$-type differentiated project; and (2) he must pay the cost $k$ of relationship lending. If he is matched with an entrepreneur with an $\ell$-type differentiated project, $e$ standardizes and $c$'s payoff is again $\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c$, whereas if $c$ is matched with an entrepreneur with an $h$-type differentiated project, $e$ differentiates and $c$'s payoff is $\beta p_h F_{\mathrm{d}}^c + (1-\beta)\pi_c$. Thus $c$ plays $\eta = \mathrm{r}$ whenever

$$\alpha\left[\beta p_h F_{\mathrm{d}}^c + (1-\beta)\pi_c\right] + (1-\alpha)\left[\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c\right] - k \geq \beta F_{\mathrm{s}}^c + (1-\beta)\pi_c.$$

This simplifies to

$$p_h F_{\mathrm{d}}^c - \frac{k}{\alpha\beta} \geq F_{\mathrm{s}}^c,$$

or

$$\pi_e^{\mathrm{s}} \geq \frac{k}{\alpha\beta} - \Delta V. \tag{$\mathrm{IC}_c$}$$

Note that the incentive constraints ($\text{IC}_h$) and ($\text{IC}_c$) combine to say that the second-best action profile ($\eta = \text{r}, \delta_h = \text{d}, \delta_\ell = \text{s}$) is attainable if only if

$$\frac{k}{\alpha\beta} - \Delta V \ \leq \ \pi_e^{\text{s}} \ \leq \ \frac{1-\beta}{\beta}\,\Delta V.$$

Thus, taking $\pi_e^{\text{s}}$ again as a proxy for competition, we see that even with relationship lending efficient project choice is possible only for intermediate levels of credit competition. The inequalities above already convey the essence of our main results about relationship lending. First, relationship lending can mitigate the inefficiencies in entrepreneurs' project choice for only low levels of competition (the upper bound $(1-\beta)\Delta V/\beta$ coincides with the upper bound in the inequalities (4.7)). Second, only if the cost $k$ is small enough ($k \leq \alpha\beta\Delta V$), does relationship lending fully prevent the over-differentiation problem that was present for low levels of credit competition. If $k$ is larger, the creditor's incentive constraint will be violated and he will prefer to perform arm's-length lending and save the cost $k$: there is a hold-up problem because even though the total surplus created $\alpha\Delta V$ exceeds the cost $k$ (assumption (4.4)), the proportion of the increased surplus that $c$ receives may not exceed it.

**Value Functions**

To express the range of competition for which ($\eta = \text{r}, \delta_h = \text{d}, \delta_\ell = \text{s}$) is the stationary action profile in equilibrium, compute the players' continuation values as value functions. This is analogous and very similar to the calculations in Subsection 4.4.2. In fact, the expression for $\pi_e^{\text{s}}$ is identical:

$$\pi_e^{\text{s}} = \frac{Q\big(\beta\pi_e^{\text{s}} + (1-\beta)(V_{\text{s}} - \pi_c)\big) + (1-Q)\pi_e^{\text{s}}}{1+r},$$

or

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)Q}{r+(1-\beta)Q}\big(V_{\mathrm{s}} - \pi_c\big). \tag{4.11}$$

The only difference between $\pi_c$ in the efficient equilibrium (that with stationary action profile ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$) above) and $\pi_c$ in the equilibrium under consideration (with stationary action profile ($\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$)) is that when $c$ is matched with an entrepreneur he first pays $k$ to invest in a relationship with $e$. Thus his value function has an extra $-k$ term with probability $Q$ relative to equation (4.9) in Section 4.4.2:

$$\pi_c = \frac{q\Big[\alpha\Big(\beta\big(p_{\mathrm{h}}V_{\mathrm{d}}\big) + (1-\beta)\pi_c\Big) + (1-\alpha)\Big(\beta\big(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}\big) + (1-\beta)\pi_c\Big) - k\Big] + (1-q)\pi_c + rI}{1+r},$$

or

$$\pi_c = \frac{q\Big[\alpha\beta p_{\mathrm{H}}V_{\mathrm{d}} + (1-\alpha)\beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}) - k\Big] + rI}{r+\beta q}. \tag{4.12}$$

We can now solve for the equilibrium value functions, which are the solutions of the system of equations (4.11) and (4.12). The next lemma summarises them.

**Lemma 38.** *In stationary equilibrium with action profile* ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$)*, the value functions are given by*

$$\begin{aligned}
\pi_e^{\mathrm{d}} &= 0,\\
\pi_e^{s} &= \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}\Bigg(r\big(V_{\mathrm{s}} - I\big) + q\big(k - \alpha\beta\Delta V\big)\Bigg),\\
\pi_c &= I + \frac{q\Big(\beta\Big[r\Big(\alpha\Delta V + V_{\mathrm{s}} - I\Big) + \alpha(1-\beta)Q\big(p_h V_{\mathrm{d}} - I\big)\Big] - \big[r + (1-\beta)Q\big]k\Big)}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}.
\end{aligned}$$

### When Can Relationship Lending Restore Efficiency?

This subsection presents our two main results about relationship lending. First, relationship lending completely solves the over-differentiation inefficiency that arises for low levels of competition as long as $k \leq \alpha\beta\Delta V$. However, when $k > \alpha\beta\Delta V$ the inefficiency persists for sufficiently low levels of competition. Second, relationship lending does not mitigate the over-standardization problem that arises for high levels of competition. The next lemma states the first of these results.

**Proposition 39.** *Relationship lending restores efficiency for low levels of credit competition if and only if $k \leq \alpha\beta\Delta V$. I.e. for low $\theta$, there is a stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *if and only if $k \leq \alpha\beta\Delta V$*

*Proof.* The proof is in Appendix 4.6.4. It simply demonstrates when the creditor's incentive constraint ($\mathrm{IC}_c$) is violated.

□

As we already touched on above, the result depends on two forces. First, a creditor will never fund an $\ell$-type differentiated project when he observes its type because it has negative NPV by assumption (4.2). Thus, the entrepreneur with the $\ell$-type differentiated project knows that if he plays $\delta = \mathrm{d}$ he will receive payoff zero and prefers to standardize. However, the creditor must pay the entire cost $k$ to perform relationship lending. Even though the total surplus gains are positive by assumption (4.4), the proportion of those surplus gains allocated to the creditor may not suffice to justify bearing them *privately.* Due to this hold-up problem there may still be inefficient project choice for low levels of competition. Note that this intuition is already present from the creditor's incentive constraint ($\mathrm{IC}_c$) taking $\pi_e^{\mathrm{s}}$ as a proxy for competition. The proposition above combined with Lemma 38 simply confirms the intuition in general equilibrium.

The next main result relies more heavily on the general equilibrium framework provided by the search model. It says that the entrepreneur's incentive to over-standardize is stronger in the equilibrium with action profile $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$.

**Proposition 40.** *Relationship lending never restores efficiency for high levels of credit competition. I.e. if there is no stationary equilibrium with action profile* $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *for* $\theta > \bar{\theta}$*, then there is no stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *for* $\theta > \bar{\theta}$*.*

*Proof.* The proof is in Appendix 4.6.5.

□

Put more simply, if, for any level of competition $\theta$, the entrepreneur with the $h$-type differentiated project plays $\delta = \mathrm{s}$ in the equilibrium with action profile $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ then the entrepreneur with the $h$-type differentiated project plays $\delta = \mathrm{s}$ in the equilibrium with action profile $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$. That is to say that the incentive to standardize is even stronger when the creditor chooses relationship lending than when the creditor chooses arm's-length finance. The reason is that the creditor is in a weaker bargaining position looking forward when he is doing relationship lending. Because $c$ bears the cost $k$ his outside option is relatively low. $e$ takes advantage of $c$'s low outside option to negotiate better loan terms and capture more of the surplus. This means that standardization is even more attractive for $e$ when $c$ is playing $\eta = \mathrm{r}$ than when $c$ is playing $\eta = \mathrm{a}$, so relationship lending can only exacerbate the over-standardization problem.

## 4.5 Conclusions

This paper develops a general equilibrium model of competition in the banking market to investigate the question of how banking competition affects corporate investment. In the model entrepreneurs have two projects, a (safe) "standardized" project and a (risky) "differentiated" (or "specialized") project. Which efficient project to undertake depends on the state of nature, so efficient investment requires adapting to circumstances. The paper has two main results. The first main result is that project choice is inefficient when banking competition is both too high and too low. Specifically, when competition in the banking market is too low, entrepreneurs over-differentiate, inefficiently forgoing the standardized project. When competition is too high, in contrast, entrepreneurs over-standardize, inefficiently foregoing the differentiated project. The key force is that entrepreneurs with standardized projects can find funding more easily in the future, thus they can obtain better terms of debt from their creditors. This pushes entrepreneurs toward the standardized project, and the effect is strongest in competitive credit markets. The second main result is that relationship banking can mitigate the over-differentiation inefficiency that emerges for low levels of credit competition but not the over-standardization inefficiency that emerges for high levels of competition. The reason is that relationship banking allows banks to make more informed lending decisions, mitigating inefficiencies that arise from the asymmetric information that is associated with a specialized project. However, relationships with borrowers do not affect banks' ability to fund standardized projects; relationship banking cannot force entrepreneurs not to standardize. Our results emphasize that the composition of the banking market is intimately connected with the nature of real investment. While some papers (e.g. Hellmann et al. (2000) and Repullo (2004)) suggest that monopolistic banking systems are safe, we find a new channel by which uncompetitive banking systems induce firms to choose excessively risky projects. While these papers suggest that credit competition induces banks to take on too much risk, we show a countervailing force by which high competition leads entrepreneurs to choose safer projects. Our channel would cause the loans in banks' portfolios to be safer when competition is high, thus suggesting that, in order to study systemic risk, future research must jointly consider the effects of credit competition on entrepreneurs' project choice and on banks' risk-taking.

## 4.6 Appendices

### 4.6.1 Proof of Lemma 34

Given the assumption 4.4.1 on out-of-equilibrium beliefs, the creditor believes that he knows the quality of the project so will not pay $k$ to gain information via relationship lending. Further, the assumption 4.2 that the $\ell$-type differentiated project has negative NPV, there is no face value that will deliver a positive expected payoff to the creditor. Since the creditor's outside option is positive, in fact $\pi_c \geq I$, no lending can take place.

### 4.6.2 Proof of Lemma 36

First recall the expression for $\pi_e^{\mathrm{s}}$ from lemma 35:

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\left(r\big(V_{\mathrm{s}}-I\big)-\alpha\beta q\Delta V\right).$$

First observe that the first term in the product,

$$f(\theta) := \frac{Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)},$$

is increasing in $\theta$: compute the derivate with the quotient rule and group terms:

$$\begin{aligned}
f'(\theta) &= \frac{\partial}{\partial\theta}\left(\frac{Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\right)\\
&= \frac{Q'\big(r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)\big)-Q\big(r\beta q'+(1-\beta)Q'(r+\alpha\beta q)+(1-\beta)\alpha\beta Q q'\big)}{[r(r+\beta q)+(1-\beta)(r+\alpha\beta q)Q)]^2}\\
&= \frac{Q'\big(r(r+\beta q)+(1-Q)(1-\beta)Q(r+\alpha\beta q)\big)-Qq'\big(r\beta+\alpha\beta(1-\beta)Q}{[r(r+\beta q)+(1-\beta)(r+\alpha\beta q)Q)]^2}.
\end{aligned}$$

To see that this expression is positive, recall the assumptions on the matching function from subsection 4.3.1. Namely $q' < 0$ and $Q' > 0$. Thus $-q'$ is positive and so are all other terms, so $f' > 0$.

Now

$$\pi_e^{\mathrm{s}} = f(\theta)\Big(r\big(V_{\mathrm{s}} - I\big) - \alpha\beta q \Delta V\Big)$$

so

$$\frac{\partial \pi_e^{\mathrm{s}}}{\partial \theta} = f'(\theta)\Big(r\big(V_{\mathrm{s}} - I\big) - \alpha\beta q \Delta V\Big) - \alpha\beta q' f(\theta).$$

Assumption 4.4.2 and the result above that $f' > 0$ say that the first term is positive. $f$ is positive because all terms are positive. And $-q' > 0$ as above. Thus $\pi_e^{\mathrm{s}}$ is increasing in competition $\theta$.

### 4.6.3 Proof of Proposition 37

First note that $\pi_e^{\mathrm{s}}$ and $\pi_c$ as written in lemma 35 are continuous in $\theta$ since $q$ and $Q$ are continuous in $\theta$ and the denominators are always positive. Thus we must just show that an entrepreneur who has an $\ell$-type differentiated project chooses $\delta = \mathrm{d}$ when $\theta \to 0$ and that an entrepreneur who has an $h$-type differentiated project chooses $\delta = \mathrm{s}$ when $\theta \to \infty$. That is to say that inequality ($\mathrm{IC}_\ell$) is violated for low $\theta$ and inequality ($\mathrm{IC}_h$) is violated for high $\theta$.

Before checking $e$'s incentive constraints, note the limits of the value functions from lemma 35:

$$\begin{aligned}
&\lim_{\theta \to 0} \pi_c = I + \frac{\beta}{\beta + r}\Big(\alpha \Delta V + V_{\mathrm{s}} - I\Big), \\
&\lim_{\theta \to 0} \pi_e^{\mathrm{s}} = 0, \\
&\lim_{\theta \to \infty} \pi_e^{\mathrm{s}} = \frac{1-\beta}{1-\beta+r}\Big(V_{\mathrm{s}} - I\Big).
\end{aligned}$$

Consider first the incentive constraint of the entrepreneur with an $\ell$-type differentiated project. The constraint reads

$$p_\ell V_\mathrm{d} - V_\mathrm{s}\left(1 - \frac{p_\ell}{p_h}\right)\pi_c \leq \frac{\beta}{1-\beta}\pi_e^\mathrm{s}$$

or, as $\theta \to 0$,

$$p_\ell V_\mathrm{d} - V_\mathrm{s}\left(1 - \frac{p_\ell}{p_h}\right)\left(I + \frac{\beta}{\beta + r}\Big(\alpha\Delta V + V_\mathrm{s} - I\Big)\right) \geq 0.$$

This is violated by the first condition in the statement of the proposition. Therefore there is no efficient equilibrium when $\theta$ is small.

Now consider the incentive constraint of the entrepreneur with an $h$-type differentiated project. The constraint reads

$$\frac{\beta}{1-\beta}\pi_e^\mathrm{s} \ \leq\ \Delta V$$

or, for $\theta \to \infty$,

$$\frac{\beta}{1-\beta+r}\Big(V_\mathrm{s} - I\Big) \ \leq\ \Delta V.$$

This is violated by the second condition in the statement of the proposition. Therefore there is no efficient equilibrium when $\theta$ is large.

### 4.6.4 Proof of Proposition 39

From Lemma 38 observe that $\pi_e^\mathrm{s} \to 0$ as $\theta \to 0$. Recall from the creditor's incentive constraint ($\mathrm{IC}_c$) that he plays $\eta = \mathrm{r}$ if and only if

$$\pi_e^\mathrm{s} \geq \frac{k}{\alpha\beta} - \Delta V.$$

Thus this holds as $\theta \to 0$ if and only if

$$0 \geq \frac{k}{\alpha\beta} - \Delta V.$$

That is to say that relationship lending restores efficiency for low $\theta$—it eliminates the over-differentiation inefficiency—whenever $k \leq \alpha\beta\Delta V$.

### 4.6.5 Proof of Proposition 40

This proof involves comparing the incentive constraint ($\mathrm{IC}_h$) of the entrepreneur with the high-type differentiated project across the equilibria described in Lemma 35 and Lemma 38. This is the incentive constraint that says the entrepreneur who has an $h$-type differentiated project chooses to play $\delta = \mathrm{d}$. It reads always

$$\pi_e^{\mathrm{s}} \leq \frac{1-\beta}{\beta}\Delta V,$$

but $\pi_e^{\mathrm{s}}$ depends on the equilibrium. Write $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{a}}$ for the entrepreneur's value function as written in Lemma 35 and $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{r}}$ as written in Lemma 38.

Now, immediately from the expressions written in the lemmata,

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{r}} - \pi_e^{\mathrm{s}}|_{\eta=\mathrm{a}} = \frac{(1-\beta)qQk}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)} > 0.$$

So immediately, $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{r}} > \pi_e^{\mathrm{s}}|_{\eta=\mathrm{a}}$ which means that if

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{a}} > \frac{1-\beta}{\beta}\Delta V$$

then

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{r}} > \frac{1-\beta}{\beta}\Delta V.$$

That is to say if $e$ over-standardizes given the $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ equilibrium then he would also over-standardize given the $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ equilibrium.

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