London School of Economics and Political Science

An analysis of leverage ratios and default probabilities

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I am greatly indebted to Wouter. Working under his supervision has truly been an honour. The thesis benefited from the support of relatives and friends, who will be thanked one by one in person.

Declaration

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Abstract

The thesis consists of three independent chapters.

In *Chapter 1 (page 7) - Counter-cyclical defaults in "costly state verification" models* - I argue that a pro-cyclical risk-free rate can solve the problem of pro-cyclical defaults in "costly state verification" models. Using a partial equilibrium framework, I compute numerically the coefficient of a Taylor rule that delivers pro-cyclical output, pro-cyclical capital and counter-cyclical defaults. This parametrization is consistent with the empirical evidence on Taylor rules.

In *Chapter 2 (page 67) - Monetary Policy, Leverage, and Default* - I use the Bernanke, Gertler and Gilchrist (1999) model to study the effect of monetary policy on the probability that firms default on loans. I argue that a monetary expansion affects defaults through two opposing partial equilibrium effects. It increases defaults because it leads firms to take on more debt and leverage up net worth, and it decreases defaults because the cost of borrowing decreases and aggregate demand shifts out, increasing firms' profits and net worth. I argue that the leverage effect could explain the empirical partial equilibrium finding by Jimenez *et al.* (2008) that defaults on new loans increase after a monetary expansion. I then argue that this effect does not hold in general equilibrium due to an increase in firms' profits. In the full model, defaults decrease after a monetary expansion, although the effect equals only few basis points.

In *Chapter 3 (page 131) - Monetary Policy and Defaults in the US* - I study empirically the effect of an unexpected monetary expansion on the delinquency rate on US business loans, residential mortgages and consumer credit. I consider several identification strategies and use Granger-causality tests to assess the exogeneity of the time series of monetary shocks to the Fed's forecast of defaults on loans. I then compute impulse responses using a variant of the Local Projection method by

Jorda (2005). I find that the delinquency rates do not respond to monetary shocks in a statistically significant way. The paper suggests that the risk-taking incentives analyzed in partial equilibrium by several existing contributions might not be strong enough to prevail and increase aggregate defaults, although they could explain why defaults do not significantly decrease.

CHAPTER 1

Counter-cyclical defaults in "costly state verification" models

Abstract

This chapter argues that a pro-cyclical risk-free rate can solve the problem of pro-cyclical defaults in "costly state verification" models. Using a partial equilibrium framework, the paper computes numerically the coefficient of a Taylor rule that delivers pro-cyclical output, pro-cyclical capital and counter-cyclical defaults. This parametrization is consistent with the empirical evidence on Taylor rules.

JEL Classification: E44, E52

Keywords: Financial accelerator, costly state verification, pro-cyclical default

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1 Introduction

Financial frictions are a key ingredient of several macroeconomic models. One of the approaches used in the literature to model financial frictions follows the "costly state verification" model by Townsend (1979) (as for instance in Carlstrom and Fuerst (1997), Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999)). In the "costly state verification" model (CSV hereafter) credit markets are affected by asymmetric information regarding the idiosyncratic shock that hits the borrowers' revenues. Borrowers observe the realization of the shock at no cost, while lenders observe it only if they incur an auditing cost. In this framework, agents could potentially sign a contract in which the borrowing rate is a function of the realization of the shock. Such a contract, though, would generate moral hazard, since the borrower has the incentive to under-report the shock in order to pay a lower interest rate. The lender anticipates this incentive and designs a contract that specifies a non-contingent borrowing rate which is paid by the borrower as long as his ex-post revenues exceed the repayment obligation. If the idiosyncratic shock is such that the repayment obligation cannot be met, the borrower defaults and the lender recovers the entire revenues, net of auditing costs.

While the CSV approach has the interesting feature that endogenous dynamics in firms' net worth generate hump-shaped impulse responses for output, it has the inconvenience that it delivers pro-cyclical defaults (Gomes, Yaron and Zhang (2003), Covas and Den Haan (2012) and Quadrini (2011)). As explained by Covas and Den Haan (2012), this is due to a "substitution effect" on the determination of the default rate. In equilibrium, the borrower weights the benefit of borrowing more to invest more against the cost of paying to the lender a higher share of expected revenues. This higher share is necessary to convince the lender to issue the additional credit, but it increases the borrower's default probability because it increases his debt burden. In the model, when aggregate productivity increases, the borrower has the incentive to shift from low investment and low default into high repayment share and high default in order to enjoy the increase in productivity. While the data display pro-cyclical investment and output, they do not display pro-cyclical defaults (Vassalou and Xing (2004)).

The literature has followed two approaches to reconcile the CSV model with the empirical evidence on default rates. The first approach identifies elements *within* the CSV model which break down the substitution effect. Accordingly, Dorofeenko, Lee and Salyer (2008) and Medina (2006) show respectively that either a decrease in the uncertainty on the idiosyncratic shock or an increase in its expected value generates pro-cyclical output and counter-cyclical defaults. The business cycle is hence interpreted in terms of time-varying idiosyncratic uncertainty (in Dorofeenko, Lee and Salyer (2008)) and in terms of time-varying expected idiosyncratic productivity (in Medina (2006)), rather than in terms of standard aggregate productivity shocks. The second approach, instead, seeks a solution *outside* the CSV model by nesting the standard model in a more comprehensive environment. Accordingly, Covas and Den Haan (2012) generate both pro-cyclical output and counter-cyclical defaults by adding an equity contract to the CSV model, as long as the cost of issuing equity is sufficiently counter-cyclical.

This paper suggests an alternative solution. It argues that, although pro-cyclical defaults are an intrinsic property of the partial equilibrium CSV setting, they could disappear once the model is nested in a general equilibrium framework where the opportunity cost of lending is an increasing function of aggregate productivity. The intuition is simple. An increase in the risk-free rate generates an opposite substitution effect if compared to the one triggered by a positive productivity shock. This is because the higher cost of borrowing *decreases* the incentive to invest more and leads the borrower to substitute towards lower repayment share and lower default. It follows that one could calibrate a positive response of the real interest rate to productivity in order to simultaneously generate counter-cyclical defaults and pro-cyclical output. This approach seems encouraging, given that Taylor rules imply a closely-related positive relationship between interest rates and the real activity.

Specifically, the paper computes numerically the simplified Taylor rule that breaks down the positive correlation between output and defaults in the CSV model, following an aggregate productivity shock. The analysis is unconventional if compared to standard monetary models. For instance, the model is in real terms and does not feature inflation. Additionally, given the absence of nominal rigidities, the central bank directly controls the real interest rate, while the output gap is proxied by the level of aggregate productivity. These shortcomings should be weighed against the purpose of the numerical exercise proposed, which is to suggest a new direction for solving the problem of pro-cyclical defaults in CSV models. A heuristic assessment of the model shows that the computed parametrization of the Taylor rule is not rejected at conventional type-one errors by the empirical estimates on Taylor rules by Clarida, Galí and Gertler (1998 and 2000) and Orphanides (2001).

The model generates pro-cyclical output and counter-cyclical defaults at the apparent cost of generating counter-cyclical capital. As it will be shown, a very small pro-cyclicality in the borrower's net worth is enough to generate the correct cyclicality also in capital. The increase in net worth is exogenous in the model, but it can be realistically reconciled in terms of an evaluation effect that works through asset prices. This effect would be in place in a proper general equilibrium environment, which is left for future research.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the modeling environment. Section 4 illustrates the determinants of the equilibrium and shows graphically how a pro-cyclical real interest rate simultaneously generates pro-cyclical output and counter-cyclical defaults. Section 5 calibrates the non-linear version of the model and computes numerically the Taylor rule that delivers the desired outcome. Section 6 compares it to the empirical evidence.

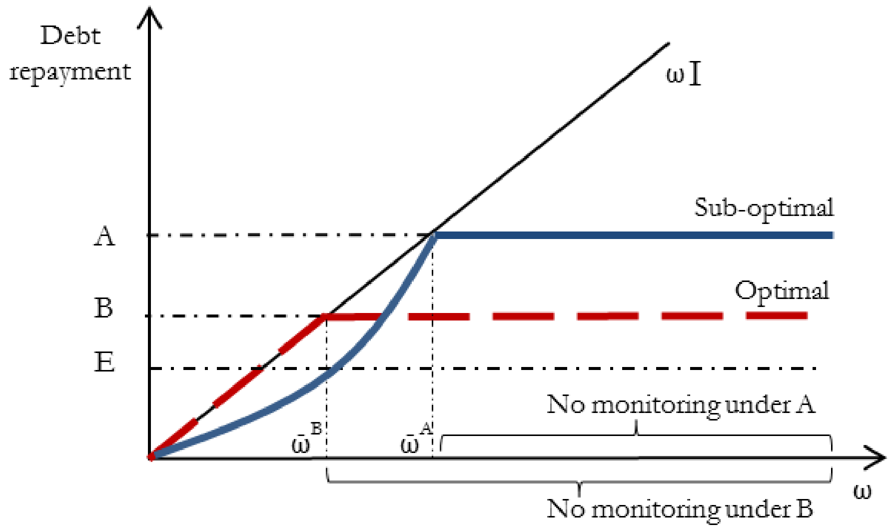
2 Related literature

The costly state verification approach was developed by Townsend (1979) to rationalize the existence of financial contracts in which the repayment rate is fixed, i.e. not contingent on the realization of shocks. The key elements of the model are the ex-post asymmetric information between the borrower and the lender concerning the revenues from the borrower's project and the presence of a dead-weight monitoring (or auditing) cost for the lender to observe the realization of the borrower's idiosyncratic shock. The model assumes risk neutrality of both the borrower and the lender.

The asymmetric information concerning the idiosyncratic shock that hits the borrower's revenues implies that the debt repayment cannot be state-contingent on the states of the world where monitoring does not occur. As mentioned in the introduction, if this was the case, the borrower would have the incentive to exploit his informational advantage by under-reporting the realization of the shock in order to pay a lower interest rate. Townsend departs from the fact that feasibility requires a non-contingent borrowing rate. He then shows that, when monitoring occurs, optimality prescribes that the lender recovers the entire return on revenues net of monitoring cost and leaves nothing to the borrower. This is what the literature refers to as a simple debt contract.

Figure 1 helps develop the intuition of the result by Townsend (1979). Assume that the borrower's revenues are given by ωI, where ω is the privately observed idiosyncratic shock and I is the level of investment. Suppose that the borrower borrows the entire amount I at the fixed interest rate r_b. The figure shows the re-

Figure 1: Optimality of a simple debt contract



lationship between the realization of the shock ω (horizontal axis) and the debt repayment (vertical axis), interpreted as the interest rate and principle on the amount borrowed ($(1 + r_b)I$). The intuition builds on the fact that a) monitoring implies a dead-weight loss, b) when monitoring occurs the lender could either keep the entire revenues or leave part of it to the borrower, and c) the higher the repayment in the states of no monitoring, the more likely it is that the borrower defaults and forces the lender to pay the monitoring cost. Knowing this, the lender can potentially obtain a certain expected return E on lending by either demanding a high repayment $A = (1 + r_b^A)I$ in the non-monitoring states and leaving some revenues for the borrower in case monitoring occurs (i.e. if $\omega < \bar{\omega}^A$), or by demanding a low repayment $B = (1 + r_b^B)I$ in case no monitoring occurs and keeping the entire revenues in case monitoring occurs (i.e. if $\omega < \bar{\omega}^B$). While yielding by construction the same expected return E to the lender, these combinations are not equivalent for the entrepreneur. In fact, the lower default probability attached to the repayment scheme B implies that expected monitoring costs are lower under B, which leaves more revenues to the borrower in expected value, if compared to repayment scheme

A. As displayed by the dotted red line in figure 1, the optimal contract takes the form of a simple debt contract because, by minimizing monitoring costs, it yields a higher utility to the borrower and leaves the lender indifferent. Having established that the optimal contract takes the form of a simple debt contract, it only remains to compute the optimal level of borrowing I and the optimal non-monitoring repayment B (or equivalently, the borrowing rate r_b^B).

The costly state verification model has found extensive application in the macroeconomic literature. Carlstrom and Fuerst (1997) used it in a general equilibrium framework to study the role of net worth in the dynamics of output. The authors build a model in which entrepreneurs borrow from capital mutual funds to transform consumption goods into capital goods using a technology that is subject to idiosyncratic uncertainty. Carlstrom and Fuerst (1997) show that this setting generates hump-shaped response functions for output, because a positive productivity shock increases asset prices and pushes up revenues for several quarters ahead. This increases net worth both directly (because the entrepreneur has a higher wealth) and indirectly (because he optimally reduces consumption). The generated hump-shaped response of net worth leads entrepreneurs to delay part of the investments to the peak of the hump when the higher net worth reduces agency costs. Such a behaviour in investment generates a positive autocorrelation in output, a result that is consistent with the data.

While offering a tractable microfoundation of a debt contract, the costly state verification model has the inconvenience that defaults are pro-cyclical, because they are increasing in the exogenous return of the borrower's investment. This is due to a substitution effect that leads the borrower to respond to an increase in productivity by paying a higher share of expected revenues to the lender in order to convince him to issue more credit. The model by Carlstrom and Fuerst (1997) inherits this property from the baseline costly state verification contract used.

Dorofeenko, Lee and Salyer (2008) fix this problem in the Carlstrom and Fuerst (1997) model by introducing time-varying uncertainty in the idiosyncratic shock and by generating business cycles through a variation in this uncertainty, instead of through a variation in the productivity parameter. Their paper shows that a decrease in the idiosyncratic uncertainty increases output and reduces the default rate, delivering counter-cyclical default. Medina (2006) takes a similar approach and introduces time-varying expected idiosyncratic productivity. He shows that an increase in the expected idiosyncratic shock increases capital supply more than in the case in which a similar increase in productivity is generated by an aggregate shock. This stronger increase in credit supply reduces the price of capital, the default rate and the external finance premium.[1]

Covas and Den Haan (2012) do not depart from Carlstrom and Fuerst (1997). They take a different approach and introduce an equity contract and firms' heterogeneity into the partial equilibrium model by Townsend (1979). They show that the pro-cyclicality of default is reduced if both equity and debt contracts are in place. They then show that if the cost of issuing equity is sufficiently counter-cyclical a) the pro-cyclicality of defaults disappears for both small and large firms, b) equity is pro-cyclical for the bottom 25% firms, in accordance with the empirical evidence on all firms except the top 1% firms, and c) shocks are magnified instead of dampened as in the standard the debt contract, at least with respect to small firms.

[1]Faia and Monacelli (2007) solve the closely-related problem of a pro-cyclical external finance premium of the Carlstrom and Fuerst (1997) by letting the expected idiosyncratic productivity of investment increase following a positive aggregate productivity shock. Their model still features pro-cyclical default, but generates a counter-cyclical external premium because the endogenous increase in the average idiosyncratic productivity increases the expected return to the lender, and this leads the lender to accept a lower borrowing rate. The authors note that a pro-cyclical expected idiosyncratic productivity magnifies the effect of a productivity shock if compared to the case of constant average idiosyncratic productivity, although not if compared to the frictionless case. This feature of the model is also discussed by Covas and Den Haan (2012).

3 CSV model with a linear production function

The previous section has contextualized the costly state verification model in the macroeconomic literature. In this section I lay down the standard model and comment on its key properties.

3.1 Environment

The standard CSV environment features a risk neutral entrepreneur who has access to a production function but has limited net worth to buy capital. The entrepreneur borrows external finance from a risk neutral lender who faces an opportunity cost of lending equal to an exogenous risk-free interest rate r.[2] I follow Carlstrom and Fuerst (1997) and assume that the loan market is competitive, in the sense that lenders compete to provide loans to borrowers and design the debt contract to maximize the expected utility of the borrower.[3]

The borrower has access to a production function that features idiosyncratic uncertainty. The realization of the idiosyncratic shock is costlessly observed by the entrepreneur, but it can be observed by the lender only if he pays an observation cost. This information asymmetry makes state-contingent contracts subject to moral hazard because the borrower has the incentive to under-report the realization of the shock in order to pay a lower interest rate. The lender anticipates the entrepreneur's incentive to misreport revenues and sets a non-contingent borrowing rate that ensures truth-telling by the entrepreneur.

[2]Carlstrom and Fuerst (1997), Dorofeenko, Lee and Salyer (2008), Medina (2006) and Faia and Monacelli (2007) normalize the opportunity cost of lending to 1. As in Bernanke, Gertler and Gilchrist (1999), I assume that monetary policy affects debt contracts by influencing the opportunity cost of lending.

[3]Carlstrom and Fuerst (1997) defend this assumption by considering it plausible that entry into lending is more likely than entry into entrepreneurial activity. Note that, in this setting, competition does not have the standard meaning of price taking behaviour because the borrowing rate is set in the debt contract, together with the amount borrowed.

Assume the following linear production function:

$$\omega\theta k + (1-\delta)k.$$

k stands for the level of capital,[4] θ is an aggregate productivity parameter, ω is the non-negative idiosyncratic shock with known cumulative density function $\Phi(\omega)$ and $E(\omega) = 1$ and $\delta < 1$ is the depreciation rate.[5] The linearity assumption helps develop the intuition and is abandoned in Section 5.

Assume that the cumulative distribution function of ω implies a hazard rate that increases in ω, which holds true for conventional distribution functions. Formally, assume that [6]

$$\frac{d}{d\omega}\frac{\Phi'(\omega)}{1-\Phi(\omega)} > 0. \qquad \text{(Assumption 1)}$$

The contract lasts one period. At the beginning of the period both agents observe θ. The entrepreneur, who is endowed with net worth n, borrows $k-n$ from the lender at the non-contingent interest rate r_b and invests k. At the end of the period ω is realized. If $\omega\theta k+(1-\delta)k \geq (1+r_b)(k-n)$ the entrepreneur pays back $(1+r_b)(k-n)$ and keeps the remaining as profits. Instead, if $\omega\theta k + (1-\delta)k < (1+r_b)(k-n)$ the entrepreneur defaults on the loan and the lender recovers $\omega\theta k + (1-\delta)k$ net of

[4] Since the model is static, capital and investment coincide. The difference becomes important only in Section 5, where the shock to net worth is computed so that the volatility of capital (instead of investment) relative to output matches the data. This is because in this model the variation in k is always smaller than the variation in $E(y)$ following a productivity shock, making it more convenient to interpret k as capital rather than investment.

[5] Part of the literature simplifies the maximization problem of the debt contract by assuming that depreciation does not affect the production function of the borrower, but enters the production function of an intermediate good producer who rents capital from borrowers on a competitive market (Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) and Faia and Monacelli (2007)). This approach has the inconvenient feature that the idiosyncratic shock is not a structural shock anymore. I comment on this point further in note 17.

[6] Equivalently, Bernanke, Gertler and Gilchrist (1999) assume $\frac{d}{d\omega}\frac{\omega\Phi'(\omega)}{1-\Phi(\omega)} > 0$. In their paper the auditing cost is assumed proportional to the ex-post revenues rather than the expected revenues, as instead is the case in Faia and Monacelli (2007), Covas and Den Haan (2012) and here.

the observation cost $\mu\theta k$ (with $\mu < 1$). The fact that the idiosyncratic shock ω can take values from 0 to ∞ implies the existence of a threshold value $\bar{\omega}$ for ω below which the debt repayment cannot be met and the entrepreneur defaults.[7] Agents are assumed not to be allowed to renegotiate the contract after ω is realized.[8]

The assumption that θ is observed before the contact is signed strengthens the pro-cyclicality of defaults featured by the contract because agents know that the end-of-period productivity will be high and they want to benefit from it by investing more at the beginning of the period. Bernanke, Gertler and Gilchrist (1999) dampen this effect by assuming that the debt contract is signed before aggregate productivity is known, which implies that agents only behave in expectation of θ. In this paper I assume that θ is known at the beginning of the period in order to strengthen pro-cyclical defaults, since this is the problem addressed in the paper.

3.2 Maximization problem and equilibrium

The contract maximizes the expected utility of the borrower under the indifference condition of the lender. The maximization problem is

$$\max_{\{k,\bar{\omega},r_b\}} \int_{\bar{\omega}}^{\infty} \big[\omega\theta k + (1-\delta)k - (1+r_b)(k-n)\big] d\Phi(\omega),$$

subject to

$$\int_0^{\bar{\omega}} \big[\omega\theta k + (1-\delta)k - \mu\theta k\big] d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \big[(1+r_b)(k-n)\big] d\Phi(\omega) \geq (1+r)(k-n), \quad (1)$$

$$\bar{\omega}\theta k + (1-\delta)k = (1+r_b)(k-n). \quad (2)$$

[7] Covas and Den Haan (2012) note that the contract by Townsend (1987) is optimal only if μ is interpreted as an observation cost rather than as a bankruptcy cost (i.e. a cost that includes indirect effects like liquidation costs, lost sales, financial distress etc.). This point is articulated further in the calibration.

[8] Default would not occur in equilibrium if one allows for ex-post renegotiation of the loan, as for instance in Kiyotaki and Moore (1997).

Equation (1) guarantees that the lender covers the opportunity cost in expectation. Equation (2) defines the threshold value $\bar{\omega}$ as a function of the borrowing rate r_b and capital k.

Define $F(\bar{\omega})$ and $G(\bar{\omega})$ the shares of expected revenues θk that go respectively to the borrower and to the lender. These functions can be computed by substituting equation (2) into the objective function and the lender's participation constraint, obtaining

$$F(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega d\Phi(\omega) - \big[1 - \Phi(\bar{\omega})\big]\bar{\omega} \quad ; \quad G(\bar{\omega}) = 1 - F(\bar{\omega}) - \mu\Phi(\bar{\omega}). \tag{3}$$

Simple algebra shows that $F(\bar{\omega})$ is a strictly decreasing function in the entire support of $\bar{\omega}$. This happens because an increase in $\bar{\omega}$ coincides with an increase in the default probability, and because it implies that the entrepreneur pays a higher borrowing rate if he does not default (as follows from equation (2)). $G(\bar{\omega})$, instead, is not monotone in $\bar{\omega}$. In fact, an increase in $\bar{\omega}$ increases the borrowing rate, but it also decreases the probability that the lender receives it. Assumption 1 ensures that, in the relevant part of the support of ω, the function $G(\bar{\omega})$ is increasing. If this was not the case a reduction in $\bar{\omega}$ would increase the expected utility of both agents. The positive relationship between $G(\bar{\omega})$ and $\bar{\omega}$ plays a crucial role in the model and follows from the fact that promising a higher share of expected revenues to the lender implies that it will be harder for the borrower to actually pay it.[9]

Simplify the maximization problem using constraint (2) to substitute r^b out of the objective function and out of constraint (1). The maximization problem then becomes

[9] Assumption 1 ensures that the equilibrium level $\bar{\omega}^*$ satisfies $G'(\bar{\omega}^*) \geq 0$. See Covas and Den Haan (2012), Appendix C and Bernanke, Gertler and Gilchrist (1999), Appendix A.

$$\max_{\{k,\bar{\omega}\}} F(\bar{\omega})\theta k,$$

$$\text{subject to} \quad G(\bar{\omega})\theta k + (1-\delta)k \geq (1+r)(k-n). \tag{4}$$

The objective function is increasing in k and decreasing in $\bar{\omega}$. Intuitively, the borrower would like to borrow an infinite amount and promise a zero share or expected revenues to the lender. This, of course, violates the constraint.[10]

To see what pins down the equilibrium, rewrite equation (4) as

$$k \leq \frac{1+r}{\delta + r - G(\bar{\omega})\theta} n \equiv k_{max}(\underset{+}{n}, \underset{+}{\theta}, \underset{+}{G(\bar{\omega})}, \underset{-}{r}). \tag{5}$$

Equation (5) shows that the debt amount $k_{max} - n$ that the lender is willing to supply to the borrower is an increasing function of n, θ and $G(\bar{\omega})$ and a decreasing function of r. Other things equal, the lender is more willing to lend if the entrepreneurial net worth is higher, if the loan is backed by a higher aggregate productivity, if the borrower promises a higher share of expected revenues or if the opportunity cost of lending is lower.

Reconsider the maximization problem in light of equation (5). We saw that the entrepreneur would like to borrow an infinite amount and promise a zero share of expected revenues. Constraint (5) shows that in order to invest more the borrower must promise to the lender a higher share of expected revenues to convince him to supply more credit. A trade-off emerges for the borrower: he either promises a low share of expected revenues and attracts limited capital, or accepts to promise a higher share of expected revenues, invests more and runs a higher default probability.

Assuming an interior solution, the equilibrium level of $\bar{\omega}$ is the one that ensures

[10]From equation (2), when $\bar{\omega} = 0$, $1 + r_b = (1-\delta)\frac{k}{k-n}$, implying $\lim_{k\to\infty} 1 + r_b = 1 - \delta < 1 + r$.

that the marginal cost of a higher promised share of expected revenues equals the marginal benefit of running a bigger-scale investment. Substitute equation (5) in the objective function and derive the optimality condition (6):

$$-F'(\bar{\omega}) = F(\bar{\omega})\frac{G'(\bar{\omega})}{\frac{\delta+r}{\theta} - G(\bar{\omega})}. \tag{6}$$

Equation (6) uniquely pins down the optimal default rate $\bar{\omega}^*$ as an increasing function of $\frac{\theta}{\delta+r}$ (see Covas and Den Haan (2012), Appendix C). Substitute $\bar{\omega}^*$ into the constraint and get the equilibrium capital k^*. To close the model, solve for r_b^* using equation (2).

4 Generating counter-cyclical defaults

In this section I explain how a pro-cyclical real interest rate eliminates the pro-cyclicality of defaults implicit in the CSV contract. I first give the key intuition using the three equilibrium equations of the model. I then develop the argument by studying the role of parameters θ and r in isolation and then together. In this section the analysis is carried out graphically using the linear production function. In the next section the analysis is replicated numerically using a non-linear production function.

4.1 Key intuition of the exercise

The key intuition of the exercise can be gained in three steps after rewriting equilibrium $\bar{\omega}$, k and y as [11]

[11] By assuming sufficiently many agents we can interpret y as aggregate output instead of *ex post* output. This is convenient in order to disregard the specific *ex post* realization of the idiosyncratic shock from considerations on the cyclical behaviour of y.

$$\bar{\omega}^* = f\left(\frac{\theta}{\delta + r}\right)\ ,\ \ f'(.) > 0, \tag{7}$$

$$k^* = \frac{1 + \frac{1-\delta}{\delta+r}}{1 - G(\bar{\omega}^*)\frac{\theta}{\delta+r}} n, \tag{8}$$

$$y^* = \theta k^* = \frac{1 + r}{\frac{\delta+r}{\theta} - G(\bar{\omega}^*)} n. \tag{9}$$

1. An increase in θ increases the default rate, capital and output;

2. If r increases in response to an increase in θ to keep the ratio $\frac{\theta}{\delta+r}$ constant, defaults remain constant, capital decreases but output still increases;

3. By continuity, if r responds to an increase in θ by marginally *decreasing* $\frac{\theta}{\delta+r}$, the default rate decreases, capital decreases and output still increases.

In other words, if the real interest rate remains unchanged, a productivity shock delivers pro-cyclical default. To reconcile the model with the data, it suffices to move the real interest rate pro-cyclically, since an appropriate variation of r delivers counter-cyclical defaults and pro-cyclical output.[12] From equation (8) we see that the right cyclicality of default and output comes at the apparent cost of generating counter-cyclical capital. This problem disappears as soon as one also gives a very small pro-cyclical shock to net worth n (more on this later).

4.2 The effect of an increase in productivity θ

The initial equilibrium is shown in figure 2, point A. The optimal combination of $\{k, \bar{\omega}\}$ is shown in the left panel, the corresponding expected output $E(y) = \theta k$ in the right panel. In the left panel, the dash-dotted lines show iso-profit curves, the

[12] The effect generated by an increase in r is qualitatively identical to the effect generated by a negative productivity shock, but quantitatively different due the the depreciation parameter δ. Appendix A discusses this point and shows that the result of the analysis does not rely on the assumption of $\delta < 1$.

upward-sloping line shows constraint (4) and the vertical line gives the optimality condition (6).[13]

Figure 2: Initial equilibrium

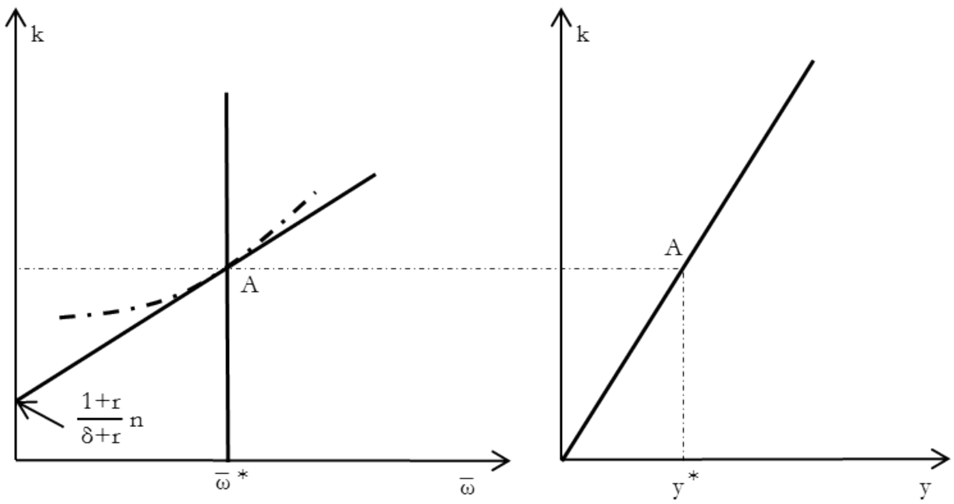
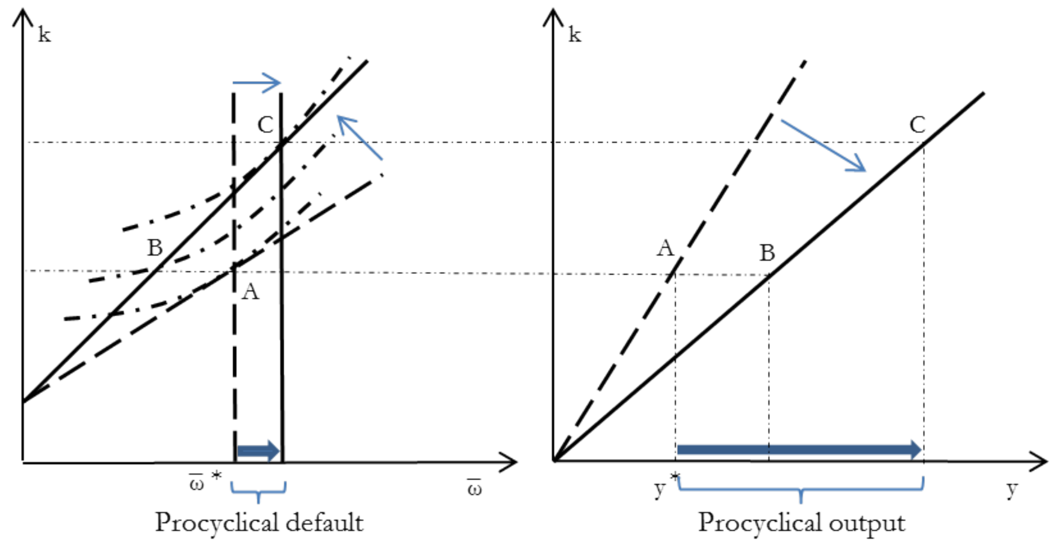


Figure 3 shows the effect of a positive productivity shock. When θ increases, the constraint slacks and rotates upwards. If investment was artificially constant (say, due to a fixed investment technology) the entrepreneur would reduce the share of expected revenues $G(\bar{\omega})$ promised to the lender and the default probability would decrease (point B). Since k is not fixed, the entrepreneur increases the level of investment in order to benefit from the higher productivity. As implied by equation (6), it is optimal for the entrepreneur to increase the promised share of expected revenues in order to convince the lender to issue more credit up to point C. The increase in $\bar{\omega}$ increases the probability that the entrepreneur goes bankrupt, generating pro-cyclical default. This effect occurs because the increase in θ moves the entrepreneur from preferring relatively small levels of capital and default probability to accepting a higher default probability in order to invest more. This is the substitution effect mentioned in the introduction, which is captured graphically by the change in the slope of the constraint.

[13]For simplicity, the constraint is graphed linearly in the space $\{k, \bar{\omega}\}$, although the true relation from equation (4) is mildly convex.

Figure 3: Positive productivity shock: $\theta \uparrow$



Proposition 1: the CSV model predicts pro-cyclical output, pro-cyclical capital and pro-cyclical defaults:

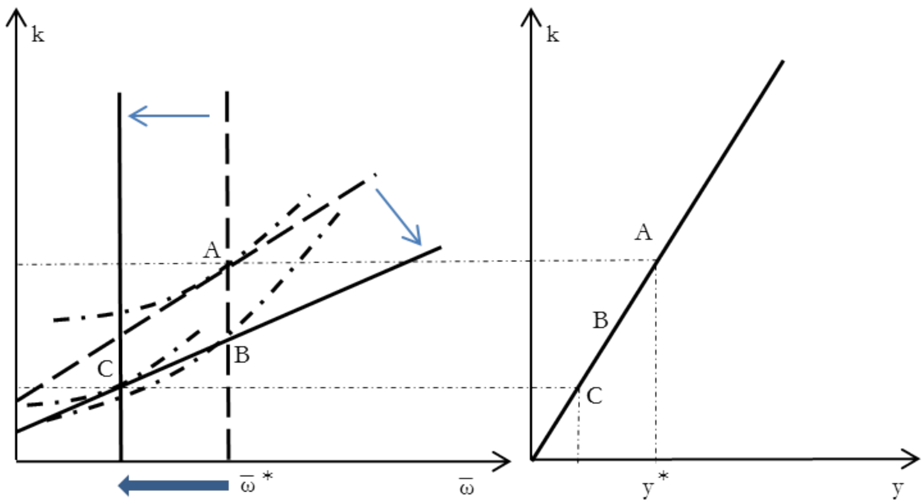
$$\rho_{\bar{\omega},\theta} > 0 \;\; ; \;\; \rho_{y,\theta} > 0 \;\; ; \;\; \rho_{k,\theta} > 0 \;.$$ [14]

4.3 The effect of an increase in the risk-free interest rate r

When r increases, the constraint shifts downwards and becomes flatter. We saw from equation (5) that, other things equal, credit supply decreases in r. If this was the only effect in place, the equilibrium in figure 4 would move from point A to point B. As implied by equation (6), when r decreases it is optimal for the entrepreneur to reduce investment even further by decreasing the share of expected revenues promised to the lender (point C). In fact, a higher risk-free rate decreases the discounted productivity and generates an incentive for the entrepreneur to shift from a high level of capital and a high default probability to a low level of capital and a low default probability.

[14] ρ stands for correlation: $\rho_{Y,Z} = Corr(Y, Z)$.

Figure 4: Increase in the real interest rate: $r \uparrow$



Proposition 2: the CSV model predicts that output, capital and default decrease in the exogenous risk-free interest rate:

$$\rho_{\bar{\omega},r} < 0 \quad ; \quad \rho_{y,r} < 0 \quad ; \quad \rho_{k,r} < 0$$

4.4 Combining θ and r to generate counter-cyclical default

Assume now that the risk-free rate responds to the productivity shock. Formally, define $\tilde{r}$ and $\tilde{\theta}$ the initial values of r and θ and assume the following simplified Taylor rule :

$$r = \tilde{r} + \gamma \frac{\theta - \tilde{\theta}}{\tilde{\theta}}. \tag{10}$$

The parameter γ measures the sensitivity of the interest rate to the productivity gap. A 1% deviation of aggregate productivity from its equilibrium level increases r by $\gamma * 0.01$, i.e. by $\gamma * 100$ basis points.

The Taylor rule in equation (10) is very simple and replaces the responsiveness of the interest rate to real activity with the responsiveness of the interest rate to aggregate productivity. Additionally, it does not feature inflation. These shortcoming are considered unimportant for the numerical exercise proposed. Real activity is arguably positively correlated with the productivity parameter, and the model does not feature inflation, so cannot account for it in the Taylor rule.[15]

Consider the initial equilibrium in figure 5 (point A, top and bottom graph). Following a positive productivity shock the constraint rotates upwards, leading the entrepreneur to substitute towards a higher investment (point B, top and bottom graph, equivalent to point C in figure 3). Under the Taylor rule from equation (10), the increase in θ prescribes an increase in the risk-free rate, which rotates the constraint backwards and shifts it downwards. The magnitude of this rotation depends on the parameter γ.

As argued in Appendix A, there exists a level of $\gamma = \gamma_L$ such that the interest rate increases by just enough to restore the previous default rate and still increase output (point C in figure 5, top graphs). If the risk-free rate increases even more by a maximum level $\gamma = \gamma_H$, defaults decrease and output shrinks to the initial equilibrium level (point D in figure 5, bottom graphs). This means that, by continuity, any level of γ strictly between γ_L and γ_H delivers a reduction in defaults and an expansion in output, i.e. pro-cyclical output and counter-cyclical default. This result is synthesized in Proposition 3, which gives the key intuition of the paper:

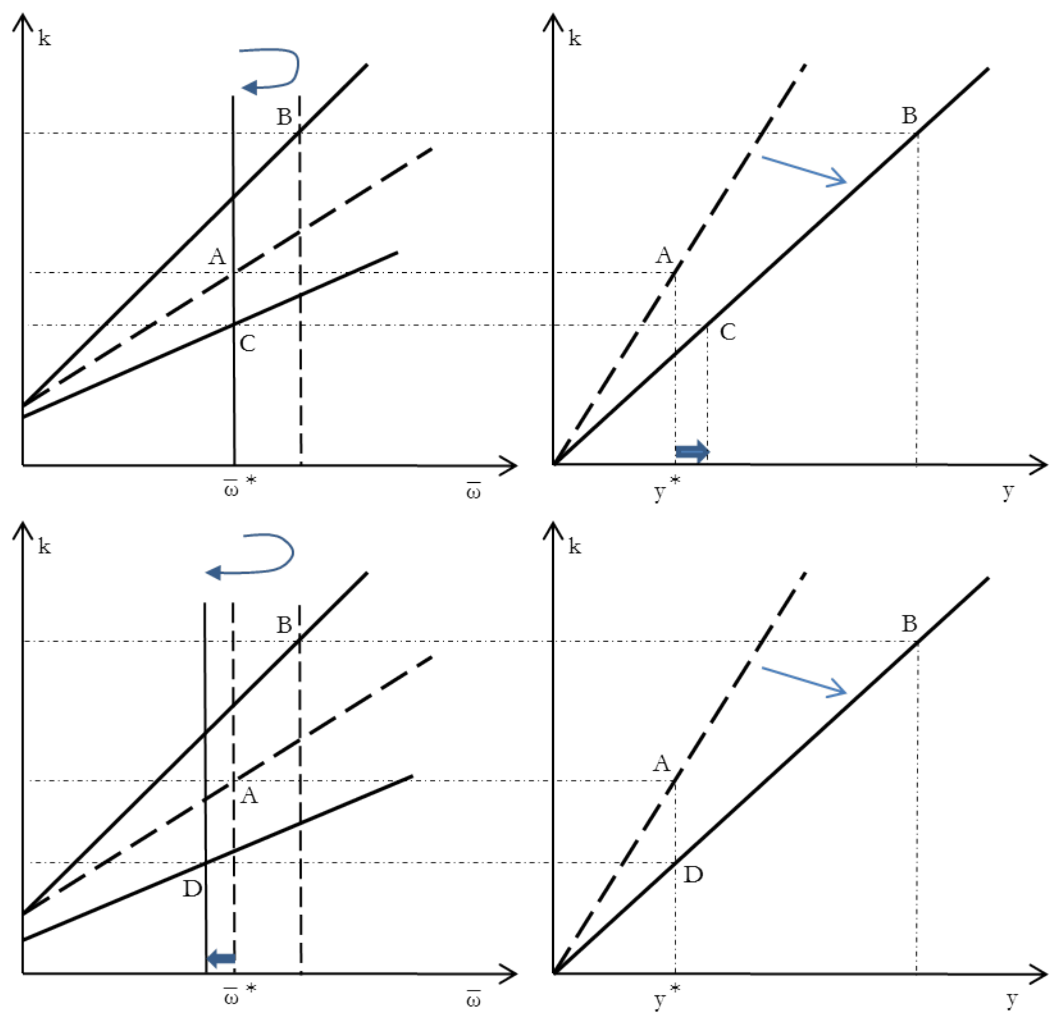
> <u>Proposition 3</u>: Given the stylized Taylor rule as in equation (10), there exists a set $\Omega_\gamma \equiv (\gamma_L, \gamma_H)$ of the parameter γ such that output is pro-cyclical, the default rate is counter-cyclical and capital counter-cyclical.

$$\rho_{\bar{\omega},\theta} < 0 \;\; ; \;\; \rho_{y,\theta} > 0 \;\; ; \;\; \rho_{k,\theta} < 0 \; .$$

The inconvenience of the result in Proposition 3 is that capital becomes counter-

[15] Faia and Monacelli (2007) and Schmitt-Grohé and Uribe (2007) use the level of output, which is argued to improve the implementability of the rule by the central bank.

Figure 5: Generating counter-cyclical default



cyclical. This problem is of first-order importance, considered that the goal of the exercise is to improve the cyclical behaviour of the key variables of the costly state verification model. I solve this problem numerically in the next section by assuming that net worth marginally increases following a positive productivity shock. This assumption is quite realistic, because in general equilibrium asset prices would arguably increase following an increase in investment, and this would generate a positive evaluation effect on net worth. The exercise shows that a very small increase in n is enough to fix the cyclicality of capital while keeping the correct cyclicality of output and defaults.

In the next section I calibrate the model and compute numerically the set Ω_γ from Proposition 3. Before doing so I introduce the version of the model with a non-linear production function.

5 Numerical computation of Ω_γ

The intuition of the exercise has been laid down using a linear version of the production function of the borrower. Covas and Den Haan (2012) show that this assumption does have important implications on defaults because it makes the default rate independent on the borrower's net worth. In this section I move to the version of the model with a non-linear production function before doing the numerical exercise on the Taylor rule suggested above. The non-linearity assumption improves considerably the ability of the model to replicate empirical moments with realistic parameter values, while still modeling ω as a structural shock of the model.[16]

5.1 The CSV model with a non-linear production function

Assume that capital k yields revenues $\omega\theta k^\alpha$, with $\alpha < 1$. To improve further the ability of the model to match the data, I follow Covas and Den Haan (2012) and add two features to the model. First, I assume that the production function is subject to a fixed cost η. Second I assume that depreciation is stochastic and depends negatively on the ex-post realization of the idiosyncratic shock. These two assumptions do not affect the result qualitatively, and help the model avoid

[16] Bernanke, Gertler and Gilchrist (1999) reconcile the linearity assumption of the production function of the borrower with an aggregate standard Cobb-Doublas production function by assuming that borrowers obtain returns on capital linearly by renting capital on competitive markets to intermediate good producers. This assumption implies that the idiosyncratic shock that affects the debt contract is no longer a structural shock of the model, but enters in a reduced form way by affecting the market return to capital. While an idiosyncratic shock smaller than 1 can be thought of as some unexpected operational cost incurred, it is hard to rationalize an idiosyncratic shock above unity, since it would imply that the entrepreneur receives more than the agreed state-contingent rental rate on the market of capital. I follow Covas and Den Haan (2012) and keep the non-linearity in the production function of the borrower, which is then dealt with numerically.

featuring unrealistically high values of the depreciation rate or unrealistically low values of α.[17] Under these assumptions the production function of the borrower is

$$\omega\theta k^{\alpha} + (1 - \delta_0 e^{\delta_1 \omega})k - \eta, \tag{11}$$

with $\delta_0 > 0, \delta_1 < 0$ and $\eta > 0$. δ_0 captures the depreciation rate when the ex-post realization of the idiosyncratic shock is exactly zero and δ_1 detects how fast depreciation falls in the realization of the shock. η is the fixed production costs.

Appendix C shows that with this production function the maximization problem can be rewritten as

$$\max_{\{k,\bar{\omega}\}} F(\bar{\omega})\theta k^{\alpha} - H(\bar{\omega})\delta_0 k,$$

$$\text{subject to} \quad G(\bar{\omega})\theta k^{\alpha} + (1 - T(\bar{\omega})\delta_0)k - \eta \geq (1 + r)(k - n). \tag{12}$$

Like in Section 3, $F(\bar{\omega})$ and $G(\bar{\omega})$ represent the share of expected revenues θk^{α} respectively to the borrower and to the lender.[18] $H(\bar{\omega})$ and $T(\bar{\omega})$, instead, represent the share of expected depreciation $\delta_0 \int_0^{\infty} e^{\delta_1 \omega} d\Phi(\omega)$ that is subtracted respectively to the borrower's and to the lender's expected return. These functions are computed in Appendix C.

The above maximization yields the following optimality condition:

$$\alpha\theta k^{\alpha-1} F(\bar{\omega}) - H(\bar{\omega})\delta_0 = -\frac{F'(\bar{\omega})\theta k^{\alpha} - H(\bar{\omega})\delta_0 k}{G'(\bar{\omega})\theta k^{\alpha} - T(\bar{\omega})\delta_0 k}\left[r + T(\bar{\omega})\delta_0 - \alpha\theta k^{\alpha-1} G(\bar{\omega})\right]. \tag{13}$$

As in the optimality condition (6) from Section 3, the left-hand side detects the borrower's marginal cost of running a higher default probability while the right-

[17] Ambler and Paquet (1994) show that stochastic depreciation improves the performance of a standard real business cycle model by breaking down the counter-factual high correlation of the model between labour productivity and hours worked, without deteriorating the predictions of the model concerning other comovements. Without stochastic depreciation and fixed production cost the model requires a depreciation close to 1 to match the desired moments.

[18] $F(\bar{\omega})$ and $G(\bar{\omega})$ represent the share of gross expected revenues θk^{α} rather than of net expected revenues $\theta k^{\alpha} - \eta$.

hand side gives the marginal benefit of doing so, with $\frac{F'(\bar{\omega})\theta k^{\alpha}-H(\bar{\omega})\delta_0 k}{G'(\bar{\omega})\theta k^{\alpha}-T(\bar{\omega})\delta_0 k}$ the Lagrange multiplier attached to constraint (12). Equations (12) and (13) nest equations (4) and (6) as a special case for $\delta_1 = 0$ and $\eta = 0$. Propositions 1-3 discussed in Section 3 still hold in the case considered in this section, as will be shown numerically below.

The key difference between the linear and the non-linear model is that in the non-linear version of the model the equilibrium threshold value of defaults $\bar{\omega}$ is not uniquely pinned down by parameter values, but becomes a function of k. This implies a system of 2 equations in two unknowns (equations (12) and (13) in $\bar{\omega}$ and k) and makes equilibrium default a function of the borrower's net worth.

5.2 Calibration

The time period of the model is a year. Parameters are calibrated to match the empirical moments for the US of the default rate $\Phi(\bar{\omega})$, the borrowing rate r_b and the leverage ratio k/n. I will compare the calibration used in this paper to the calibration in Covas and Den Haan (2010 and 2012) (hereafter CDH), Bernanke, Gertler and Gilchrist (1999) (hereafter BGG) and Carlstrom and Fuerst (1997) (hereafter CF). The calibration is shown in table 1. The idiosyncratic shock ω is assumed to follow a log normal distribution with unitary expected value, like in the above and other papers. The model pins down the endogenous variables $\{k, \bar{\omega}, r_b\}$ as a function of parameters $\{\alpha, \theta, \eta, \delta_0, \delta_1, r, n, \gamma, \tilde{\theta}\}$. The equilibrium equations of the model are rewritten below for convenience.

$$\alpha\theta k^{\alpha-1}F(\bar{\omega}) - H(\bar{\omega})\delta_0 = -\frac{F'(\bar{\omega})\theta k^{\alpha} - H(\bar{\omega})\delta_0 k}{G'(\bar{\omega})\theta k^{\alpha} - T(\bar{\omega})\delta_0 k}\left[r + T(\bar{\omega})\delta_0 - \alpha\theta k^{\alpha-1}G(\bar{\omega})\right], \tag{Optimality}$$

$$G(\bar{\omega})\theta k^{\alpha} + (1 - T(\bar{\omega}\delta_0)k - \eta \geq (1+r)(k-n), \tag{Constraint}$$

$$1 + r_b = \frac{\bar{\omega}\theta k^{\alpha} + (1 - \delta_0 e^{\delta_1 \bar{\omega}})k - \eta}{k - n}, \qquad \text{(Borrowing rate)}$$

$$r = \tilde{r} + \gamma \frac{\theta - \tilde{\theta}}{\tilde{\theta}}, \qquad \text{(Taylor rule)}$$

The model presents strong non linearities. For this reason, I calibrate it using a sequential approach instead of using an algorithm that jointly searches for the combination of parameters that meets the restrictions of the model. I do so because I found that the initial guess for the parameter values plays a strong role in determining whether the algorithm ultimately delivers the desired equilibrium, or if it fails to meet the moment restrictions imposed to the calibration. Additionally, the approach used in the paper facilitates the understanding of the trade-offs implicit in the calibration of the model, and facilitates the understanding of which parameters are more helpful to meet which moment conditions. The sequential approach used is discussed extensively in Appendix D.

Table 1: Calibration

	Parameter	**Value**	**Calibration**	**Source**
Opportunity cost	r	0.0204	literature	FED
Marginal returns to k	α	0.30	literature	
Borrower's net worth	n	0.4762	literature	
Bankruptcy cost	μ	0.54	data	
Variance of e^{ω}	σ	1.1944	data	
Depreciation parameters	δ_0	0.3995	data	
	δ_1	-3.0180	data	
Fixed cost of production	η	0.1406	data	
Aggregate productivity	θ	0.8730	data	
	Moment	**Model**	**Data**	**Source**
Default rate	$\Phi(\bar{\omega})$	0.0227	0.0220	CDH (2010)
Leverage ratio	k/n	2.1057	2.10	CDH (2010)
Borrowing rate	r_b	0.0473	0.0469	FED
Expected depreciation	$\delta_0 \int_0^{\infty} e^{\delta_1 \omega} d\Phi(\omega)$	0.10	0.10	

Parameters calibrated from the literature

The exogenous interest rate r is set equal to the sample average of the real federal funds rate. Being an interest rate on interbank lending, the fed funds rate is a valid measure of the opportunity cost of lending. Information on the average default rate will be taken from Covas and Den Haan (2010), who report it for the period between 1986 and 2004. I use the same period to compute the sample average of the fed funds rate, which is $r = 0.0204$ (see Appendix B). The value is relatively small if compared to the literature. BGG, CF and other papers use an annual rate of 0.0404. CDH use $r = 0.022$.

The parameter α is set at 0.30, which is slightly below the conventional value of 0.33 from the literature. Covas and Den Haan (2012) argue that models without labour (as the model used here) should use values as high as 0.7, in accordance with the empirical evidence by Guffrey and Moore (1991). I do not follow CDH, given that in this model a value of α above 0.30 implies a value of μ that is unrealistically high. The trade-off between α and μ is investigated numerically in Appendix D.

The parameter n is normalized so that the equilibrium level of capital equals approximately 1 when the target on the leverage ratio is met.

Parameters calibrated from the data

The remaining parameters are set to match the following three empirical moments. The first moment is a default rate $\Phi(\bar{\omega})$ equal to 0.022, which I take from Covas and Den Haan (2010). The second moment is on the borrowing rate r_b between the borrower and the lender. Being a rate on a risky loan, I match its initial value with the average real interest rate charged by US banks on loans, which equals 0.0469 in the period between 1986 and 2004 (see Appendix B). The third moment is the initial leverage ratio k/n of 2.10, in line with the literature. The value is taken from CDH (2010). BGG use 2.

The parameter σ stands for the second moment of the normal random variable e^{ω} and implies a variance of the idiosyncratic shock ω equal to 3.1651. On average the depreciation rate equals the conventional value of 0.10 and it increases up to almost $\delta_0 = 0.40$ when the realization of ω is 0. The aggregate productivity parameter θ equals 0.8730, implying an equilibrium average productivity $\frac{\theta k^{\alpha}-\eta}{k}$ of $0.7311 = 73.11\%$. This value is relatively high but still realistic.

A consensus is still missing in the literature regarding the estimate of the parameter μ. The disagreement hinges on whether μ should only include direct bankruptcy costs or also indirect costs like foregone revenues and financial disruption.[19] Existing contributions use values that range from 0.12 to 0.36. I calibrate μ to the relatively high value of 0.54 to avoid the parameter α falling below the standard value of 0.33 (see Appendix D). 0.54 could be potentially reconciled empirically only if one interprets auditing costs by also including indirect costs, although this would make the debt contract suboptimal (see note 8). CDH and BGG use values of μ of respectively 0.15 and 0.12, which they obtain by calibrating the model to match the moment on the default rate. CF use 0.25.

Table 2: Initial equilibrium

	Value	**Variable**
k^*	1.0027	capital
$k^* - n$	0.5265	borrowing
$\bar{\omega}^*$	0.0449	default threshold
r_b^*	0.0473	borrowing rate
k^*/n	2.1057	leverage ratio
$\Phi(\bar{\omega}^*)$	0.0227	default probability
$(1+r)(k^* - n)$	0.5373	outside option of the lender
$(1+r_b)(k^* - n)$	0.5514	debt repayment
$\theta k^{*^{\alpha}} - \eta$	0.7331	expected output
$\theta k^{*^{\alpha}}$	0.8737	expected revenues
$\bar{\omega}\theta k^{*^{\alpha}}$	0.0392	threshold value of revenues

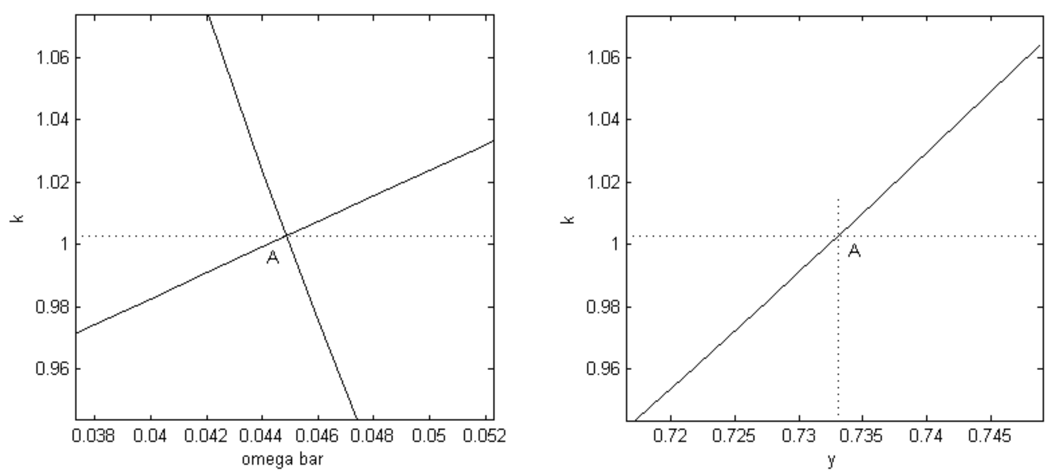
The initial equilibrium is shown in table 2. The borrower borrows 0.5265 at an

[19] See Carlstrom and Fuerst (1997) for a discussion on this issue.

interest rate of 4.73% and buys capital for 1.0027. The opportunity cost $(1+r)(k-n)$ for the lender equals 0.5373. The expected value of output $E(y) = \theta k^{\alpha} - \eta$ equals 0.7331. The borrower defaults as soon as the realization of the shock leads revenues below 0.0392, an event that happens with 2.27% probability.

The initial equilibrium is shown graphically in figure 6, which is the numerical equivalent of figure 2 from Section 3. The left panel reads the equilibrium level of k and $\bar{\omega}$ as the intersection of the optimality condition and the budget constraint (downward sloping and upward sloping lines, respectively). The right panel reads the equilibrium level of output as a function of equilibrium investment and is interpreted from the vertical axis to the horizontal axis.

Figure 6: Initial equilibrium



Having described the calibration and the initial equilibrium, I now replicate numerically the analysis from Section 3 by studying separately the effect of a productivity shock and of a variation in the risk-free interest rate. I then combine the two effects and compute numerically the set Ω_{γ} from Proposition 3. Tables are provided with equilibrium values and percentage or basis point variations corresponding to figures 3 to 5 from Section 3. Figures 9 to 11 in the appendix show the numerical equivalent of figures 3 to 5.

5.3 The effect of an increase in productivity θ

Consider a 1 percent increase in aggregate productivity that leads θ from 0.8730 to 0.8817 (table 3). If r remains constant, capital and output increase by respectively 0.25% and 1.28% while defaults increase by 5 basis points (point C). Had capital remained constant (point B), output would have increased by only 1.19% and default would have decreased by 1 basis point. As shown in Section 3, it is optimal for the entrepreneur to react to an increase in θ by expanding investment, increasing borrowing and accepting an increase in the default probability of 5 basis points (point C).

Table 3: Increase in productivity θ

(levels)	θ	$\bar{\omega}$	k	$E(y)$	$\Phi(\bar{\omega})$	r_b
point A	0.8730	0.0449	1.0027	0.7331	0.0227	0.0473
point B	0.8817	0.0447	1.0027	0.7418	0.0225	0.0476
point C	0.8817	0.0453	1.0052	0.7425	0.0231	0.0480
(variations)		$\frac{\dot{\theta}}{\theta}$	$\frac{\dot{k}}{k}$	$\frac{\dot{E(y)}}{E(y)}$	$\dot{\Phi(\bar{\omega})}$	
from A to B		1%	0	1.19%	-0.0001	
from A to C		1%	0.25%	1.28%	0.0005	

5.4 The effect of an increase in the risk-free interest rate r

Consider now the case of an interest rate shock. Keep θ at the initial value and assume that the real interest rate r increases by 1 percentage points from 0.0204 to 0.0304. As shown in table 4, capital and output decrease respectively by 2.78% and 1% while defaults decrease by 33 basis points (point C). Had the promised share of revenues $G(\bar{\omega})$ remained constant (point B), output would have dropped by only 0.51% and defaults would have remained unchanged. As shown in Section 3, it is optimal for the borrower to react to an increase in r by promising a lower share of expected revenues to the lender in order to further decrease the level of capital, and this decreases the default probability by 33 basis points.

Table 4: Increase in the exogenous interest rate r

(levels)	r	$\bar{\omega}$	k	$E(y)$	$\Phi(\bar{\omega})$	r_b
point A	0.0204	0.0449	1.0027	0.7331	0.0227	0.0473
point B	0.0304	0.0449	0.9884	0.7293	0.0227	0.0581
point C	0.0304	0.0415	0.9748	0.7257	0.0193	0.0559
(variations)		$\dot{r}$	$\frac{\dot{k}}{k}$	$\frac{\dot{E(y)}}{E(y)}$	$\dot{\Phi(\bar{\omega})}$	
from A to B		0.01	-1.43%	-0.51%	0	
from A to C		0.01	-2.78%	-1%	-0.0033	

5.5 Combining θ and r to generate counter-cyclical default

Consider again the productivity shock from table 3. It can be computed that to avoid the 5 basis points increase in defaults that follows from a 1% increase in θ it suffices for the interest rate r to increase by 14 basis points from 0.0204 to 0.0218 (table 5, top part). This implies a weaker increase in output, which now increases by 1.13% . If the interest rate increases not only up to 0.0218 but up to 0.0331, output goes back to the initial level but the default rate drops by 37 basis points (table 5, bottom part). This means that, starting from a real interest rate of 0.0204, the 1% increase in productivity does not increase defaults as long as the interest rate increases by between 14 and 127 basis points, i.e. up to between 0.0218 and 0.0331. The implied parameter values for γ are $\Omega_\gamma \equiv (\gamma_L, \gamma_H) = (0.1368, 1.2728)$ (bold values in table 5). This range delivers pro-cyclical output, counter-cyclical default and counter-cyclical capital, in accordance with Proposition 3 from Section 3. Depending on the specific parameter value of γ in the range Ω_γ output increases up to 1.13%, defaults decrease by a maximum of 37 basis points and capital decreases by between 0.15% and 3.26%

5.6 Restoring the pro-cyclicality of capital

Table 5 shows that the pro-cyclicality of the real interest rate r fixes the cyclicality of output and default at the apparent cost of losing the pro-cyclicality in capital.

Table 5: Generating counter-cyclical defaults after $\frac{\dot{\theta}}{\theta} = 0.01$

TOP PART: Weak response of r to θ ($\gamma = \gamma_L$)

	r	$\bar{\omega}$	k	$E(y)$	$\Phi(\bar{\omega})$	r_b
point A	0.0204	0.0449	1.0027	0.7331	0.0227	0.0473
point B	0.0204	0.0453	1.0052	0.7425	0.0231	0.0480
point C	0.0218	0.0449	1.0012	0.7413	0.0227	0.0482
	γ	$\dot{r}$	$\frac{\dot{k}}{k}$	$\frac{\dot{E(y)}}{E(y)}$	$\dot{\Phi(\bar{\omega})}$	
from A to B		1%	0.25%	1.28%	0.0005	
from A to C	**0.1368**	0.0014	-0.15%	1.13%	0	

BOTTOM PART: Strong response of r to θ ($\gamma = \gamma_H$)

	r	$\bar{\omega}$	k	$E(y)$	$\Phi(\bar{\omega})$	r_b
point A	0.0204	0.0449	1.0027	0.7331	0.0227	0.0473
point B	0.0204	0.0453	1.0052	0.7418	0.0225	0.0476
point D	0.0331	0.0411	0.9700	0.7331	0.0188	0.0583
	γ	$\dot{r}$	$\frac{\dot{k}}{k}$	$\frac{\dot{E(y)}}{E(y)}$	$\dot{\Phi(\bar{\omega})}$	
from A to B		1%	0.25%	1.28%	0.0005	
from A to D	**1.2728**	0.0127	-3.26%	0%	-0.0037	

It can be computed that this problem disappears if net worth n increases by as little as 0.096637%. Of course a variation in net worth is purely exogenous in this model and it is simply proposed as a numerical exercise to reconcile the dynamics of capital with the rest of the analysis. Nevertheless, a pro-cyclical variation in net worth does not seem unrealistic because an evaluation effect on n would be in place in a general equilibrium extension of the model, following an increase in θ.

Table 6 computes new estimates of γ when aggregate productivity increases by 1% and when net worth n is hit by a positive shock (second column). We saw in Section 5.5 that, given a constant net worth, defaults decrease as long as the interest rate r increases by between 14 and 127 basis points (panel a in table 6). Assume now that, following the same productivity shock, net worth increases by $0.003 = 0.3\%$ (panel b). It can be shown that as long as r responds to the productivity

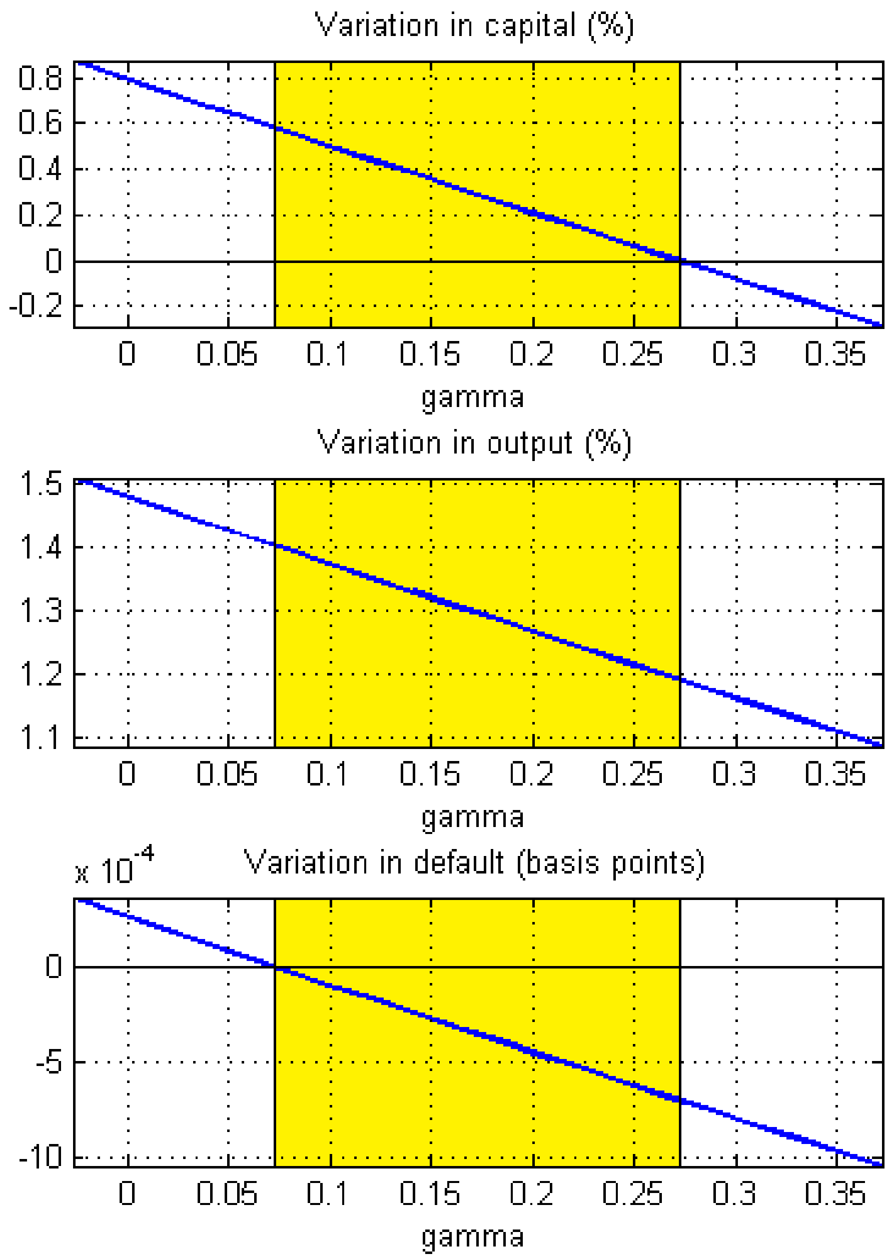
Table 6: Restoring the pro-cyclicality of capital after $\frac{\dot{\theta}}{\theta} = 0.01$

	$\frac{\dot{n}}{n}$	new r	γ	$\frac{\dot{k}}{k}$	$\frac{\dot{E(y)}}{E(y)}$	$\dot{\Phi(\bar{\omega})}$	$\frac{\dot{k}/k}{\dot{E(y)}/E(y)}$
Panel a	0	0.0204	0	0.2521	1.2828	0.0005	
		0.0213	0.0854	0	1.1918	0.0002	
		0.0218	0.1368	-0.1503	1.1375	0	
		0.0331	1.2728	-3.2624	0	-0.0037	
Panel b	0.003	0.0204	0	0.6	1.4081	0.0003	
		0.0214	0.0958	0.3167	1.3061	0	
		0.0224	0.2042	0	1.1918	-0.0004	
Panel c	0.005	0.0204	0	0.8319	1.4914	0.0002	
		0.0211	0.0691	0.6269	1.4177	0	
		0.0232	0.2840	0	1.1918	-0.0008	
Panel d	0.008	0.0204	0	0.1797	1.6161	0.0001	
		0.0207	0.0289	1.0935	1.5852	0	
		0.0244	0.4044	0	1.1918	-0.0013	
Panel e	0.0047	0.0204	0	0.7971	1.4789	0.0003	
		0.0211	0.0731	0.5802	1.4009	0	0.4142
		0.0221	0.1726	0.2885	1.2959	-0.0004	0.2226
		0.0231	0.2720	0	1.1918	-0.0007	0
		$\Omega_\gamma = (\mathbf{0.0731}, \mathbf{0.2720})$					

shock by between 9 and 20 basis points, the model delivers pro-cyclical output, pro-cyclical capital and counter-cyclical default, which is the right cyclicality on all three variables. The result is driven by the fact that an increase in net worth increases capital without affecting defaults, as can be seen from equations (7) and (8). To help develop the intuition, the same effect is computed for shocks to net worth of different size (0.5% in panel c, 0.8% in panel d). We see that the higher the pro-cyclical shock to n and the wider the range of r that deliver the right cyclicality of k, $E(y)$ and $\Phi(\bar{\omega})$.

As made clear by table 6, the computation of the range of parameter values for γ depends on the pro-cyclical shock given to n (Ω_γ equals (0.0958, 0.2042) for $\dot{n}/n = 0.003$, (0.0691, 0.2840) for $\dot{n}/n = 0.005$ and (0.0289, 0.4044) for $\dot{n}/n = 0.008$). Given the need to compute one set of estimates for Ω_γ to be compared with the empirical estimates available in the literature, I choose the size of the shock to

Figure 7: Comparing different values of γ under $\frac{\dot{\theta}}{\theta} = 0.01$



net worth so that it implies an average value of γ for which k is 0.2222 times as volatile as output. This relative volatility between k and $E(y)$ is taken from the real business cycle literature.[20] When this criterion is applied, a 1% productivity shock requires a 0.47% increase in n and an increase in r between 7 and 27 basis points. This implies a set of acceptable values of γ of Ω_γ =(**0.0731, 0.2720**) (panel e from table 6), which are the values of Ω_γ used in the rest of the paper.

The intuition from panel e of table 6 is represented graphically in figure 7, which shows, for different values of γ, the variation in capital, expected output and defaults following a 1% productivity shock and a 0.47% increase in net worth. The vertical lines in all three graphs represent $\gamma_L = 0.0731$ and $\gamma_H = 0.2720$, i.e. the threshold values computed above. We see that in the shaded yellow area the pro-cyclical real interest rate r delivers pro-cyclical capital, pro-cyclical output and counter-cyclical default.

5.7 Robustness checks on the computation of Ω_γ

The above section found that parameter values of γ in the range of (0.0731, 0.2720) for the Taylor rule (10) in a standard costly state verification model deliver pro-cyclical capital, pro-cyclical output and counter-cyclical default. This section shows that the computed range is robust to alternative values of the productivity shock and of the auditing cost. Overall, the model performs well if γ does not exceed approximately 0.25.

Table 9 computes Ω_γ for different values of the productivity shock (first column). As explained above, net worth is increased pro-cyclically in order to deliver a median value of γ for which the volatility of capital relative to output matches the value found empirically by the real business cycle literature. Stronger productivity shocks slightly increase the range of γ within which capital, output and defaults behave

[20] Prescott (1986) and King and Rebelo (1999) find that capital is around one forth as volatile as output, while investment is around three times as volatile as output. The model presented here is static, implying that k is best interpreted as the capital stock rather than its variation from a previous period.

with the correct cyclical patterns.

Table 10 computes Ω_γ when the auditing cost μ takes different values. It was explained that the relatively high value of μ was required in order to match the empirical moments, while still using a realistic value of α (see Section 5.2 and Appendix D). The set Ω_γ was computed again assuming a higher value of μ, recalibrating the model and assuming an appropriately pro-cyclical net worth, as explained above. The values of γ that give the right cyclicality for capital, output and default remain in the range between approximately 0.07 and 0.25.

6 Comparing Ω_γ with the empirical evidence

The exercise carried so far has shown that, in a standard costly state verification model, if the risk free rate increases between approximately 7 and 27 basis points following a 1% positive productivity shock, the right cyclicality of capital, output and default is generated, as long as net worth marginally increases.

Comparing these estimates to the existing empirical evidence on Taylor rules is not immediate. The proposed model, in fact, does not feature inflation and hence can only host half of a standard Taylor rule. Additionally, by disregarding nominal frictions that draw a wedge between actual and potential output, it can only approximate the output gap with a percentage difference of aggregate productivity from its calibrated equilibrium level. Nevertheless, a heuristic analysis can help to understand if the model is at least moving in the right direction. A proper empirical assessment of the performance of the model would require a general equilibrium extension, which is left for future research.

The empirical literature debates on the identification approach to the estimation of Taylor rules along three dimensions. First, if the policy rule should be estimated considering lagged, contemporaneous or forward levels of output and inflation. Sec-

Table 7: Empirical evidence on Taylor rules

ρ	β_π	β_y	p-value on $H_0 : \beta_y = 0.2720$	at 5% one-sided	at 1% one-sided
			PART A: Clarida, Galí, Gertler (1998)		
			1979:m3 - 1993:m12 Germany		
0.91	1.31	0.25	0.2912	Fail to reject	Fail to reject
0.91	1.10	0.28	0.4207	Fail to reject	Fail to reject
0.91	1.29	0.28	0.3949	Fail to reject	Fail to reject
0.91	1.23	0.25	0.2912	Fail to reject	Fail to reject
0.91	1.37	0.35	0.0256	Reject	Fail to reject
			1979:m10 - 1994:m12 USA		
0.92	1.79	0.007	0.0000	Reject	Reject
0.94	2.20	0.14	0.0934	Fail to reject	Fail to reject
0.90	1.05	0.44	0.0082	Reject	Reject
			1982:m10 - 1994:m12 USA		
0.97	1.83	0.56	0.0359	Reject	Fail to reject
0.97	1.99	0.75	0.0279	Reject	Fail to reject
0.96	1.26	0.52	0.0000	Reject	Reject
			PART B: Clarida, Galí, Gertler (2000)		
			1979:q3 - 1996:q4 USA		
0.93	2.15	0.93	0.0586	Fail to reject	Fail to reject
0.76	1.97	0.55	0.1770	Fail to reject	Fail to reject
0.73	2.01	0.56	0.2412	Fail to reject	Fail to reject
0.78	2.62	0.83	0.0231	Reject	Fail to reject
0.78	2.73	0.92	0.0183	Reject	Fail to reject
			PART C: Orphanides (2001)		
			1987:q1 - 1993:q4 USA		
0.77	0.57	1.25	0.0300	Reject	Fail to reject
0.77	0.62	1.33	0.0081	Reject	Reject
0.72	1.18	1.15	0.0049	Reject	Reject
0.66	1.64	0.97	0.0000	Reject	Reject

ond, if the estimation should allow for a smooth variation of the interest rate or not. Third, if the analysis should use revised or real-time data. With regard to the first two points, I restrict the attention to empirical estimates that consider forward looking feedback rules and which add a smoothing parameter. As for the third point, I use estimates on both revised and real-time data.

Table 7 displays the estimates computed by Clarida, Galí and Gertler (1998, 2000) and Orphanides (2001) for different time horizons, countries and identification strategies. The table reports point estimates for the smoothing parameter, the coefficient on inflation and the coefficient on the variable capturing economic activity.[21] Overall, the parameters range within the commonly accepted interval, i.e. 0.9 for the autoregressive component, above 1 for the coefficient on inflation and between 0.5 and 1 for the coefficient on output (Rudebush (2001)). Note that these estimates differ slightly from the values assumed by Taylor (1993), who imposes zero persistence in the policy rate and uses 1.5 and 0.5 for inflation and output, respectively.

I use the estimates from table 7 together with standard errors (unreported in table 7) to test if the data reject the hypothesis that the true coefficient on output differs from 0.2720. This is the highest value computed in the previous section which reconciles the costly state verification model with the cyclicality of capital, output and defaults. Failing to reject the null hypothesis suggests that the model yields a

[21] Formally, the model estimated takes the following form:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\beta_\pi E_t(\pi_{t+k}) + \beta_y E_t(y_{t+k})\right] + v_t,$$

where i_t is the nominal interest rate and $E_t(\pi_{t+k})$ and $E_t(y_{t+k})$ represent the expectation of k-period-ahead inflation and economic activity. Due to the persistence in the nominal interest rate, the coefficients β_π and β_y represent the long run effect on the nominal interest rate of a permanent increase in inflation and output and not the contemporaneous effect. Estimates from Clarida, Galí and Gertler (1998) measure output as contemporaneous output gap and 1 year-ahead inflation. Lines 2, 7 and 10 add 12-month-lagged inflation as regressor, lines 3, 8 and 11 add money supply, line 4 adds the federal funds rate and like 5 the real exchange rate. Estimates from Clarida, Galí and Gertler (2000) measure output as output gap, excepted the 2^nd and 3^rd estimations reported where detrended output and the unemployment rate are used, respectively. The first 3 estimates use 1 quarter-ahead inflation and 1 quarter-ahead output, the 4^ht and 5^th estimation use 1 year-ahead inflation and, respectively, 1 and 2 quarter-ahead of output. Orphanides (2001) uses real time data and, in order, 1, 2, 3 and 4 quarter-ahead inflation and output gap. Orphanides estimates the equation using least squares, Clarida, Galí and Gertler use the Generalized Method of Moments under the assumption that passed values of output and inflation matter for the policy decision only to the extent that they contribute to form the expectation of future output and inflation. In part A, the estimation for the USA uses 2 lagged values of the nominal interest rate instead of 1. The parameter value reported is the sum of the 2 coefficients.

numerical value for the Taylor rule that is consistent with the empirical evidence from table 7. The p-value of the test is reported in the fourth column. Out of the 20 estimates considered, the data reject the hypothesis that the true value of β_y is 0.2720 12 times at a 5% type-one error with a one-sided hypothesis and 6 times at a 1% type-one error with a one-sided hypothesis. Overall the data do not seem to reject the value of 0.2720 computed numerically in the exercise of Section 5 as the true value of the Taylor rule. Nevertheless, the result of the test on the empirical performance of the model is mixed and depends on the estimates considered.

7 Conclusions

This paper argues that the counterfactual pro-cyclicality of default rates in "costly state verification" models disappears if the risk-free rate moves pro-cyclically. As explained, the success of such an approach lies in the fact that upward movements in the real interest rate trigger a substitution effect of opposite sign, if compared to the one generated by an increase in productivity. Since Taylor rules prescribe a positive response of the interest rate to output, exploiting a Taylor rule in this direction could prove successful in reconciling the "costly state verification" model with the cyclicality of output, capital and defaults.

The paper computes numerically the coefficient of a simplified Taylor rule that prescribes a positive response of the risk-free rate to a productivity shock. It is shown that the model generates pro-cyclical output and counter-cyclical defaults. To generate pro-cyclical capital it suffices to add an infinitesimal pro-cyclical shock to net worth which mimics an evaluation effect on asset prices.

Comparing the coefficients on the Taylor rule computed numerically in the paper with the empirical estimates available in the literature is not immediate, since the specification of the model differs from empirical estimations along important dimensions. Nevertheless, an approximate comparison suggests that the coefficient computed numerically in the exercise (slightly below 0.2720) lies close to the coefficient value estimated in the literature (between 0.25 and 0.9). In particular, the data does not systematically reject the hypothesis that the true coefficient in the Taylor rule equals the value, computed numerically in the paper, which reconciles the "costly state verification" with the cyclicality of output, capital and defaults. Caution should be paid, though, since the final assessment of the idea developed in this paper should not ultimately rely on the partial equilibrium analysis presented here. A proper general equilibrium extension of the model is left for future research.

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8 Appendix A: The role of depreciation

This appendix explains the role of the depreciation parameter in the linear version of the model and shows that the result from Proposition 3 does not rely on the assumption of $\delta < 1$.

In the special case of $\delta = 1$, equations (7), (8) and (9) become

$$\bar{\omega}^* = f\left(\frac{\theta}{1+r}\right) , \quad f'(.) > 0, \tag{14}$$

$$k^* = \frac{n}{1 - G(\bar{\omega}^*)\frac{\theta}{1+r}}, \tag{15}$$

$$y^* = \theta k^* = \frac{n}{\frac{1}{\theta} - \frac{G(\bar{\omega}^*)}{1+r}}. \tag{16}$$

It is clearly evident that an increase in θ and a decrease in r increase equilibrium investment by the same amount as long as they have the same impact on $\frac{\theta}{1+r}$. The effect on output is stronger in the former case. This means that if r increases in response to an increase in θ by just enough to keep k constant, output still increases due to the productivity effect. Hence, by continuity, if r responds by just enough to marginally *decrease* $\frac{\theta}{1+r}$, defaults and capital decrease and output increases.

If $\delta < 1$, then equations (7), (8) and (9) equal

$$\bar{\omega}^* = f\left(\frac{\theta}{\delta+r}\right) , \quad f'(.) > 0, \tag{17}$$

$$k^* = \frac{1 + \frac{1-\delta}{\delta+r}}{1 - G(\bar{\omega}^*)\frac{\theta}{\delta+r}} n, \tag{18}$$

$$y^* = \theta k^* = \frac{1+r}{\frac{\delta+r}{\theta} - G(\bar{\omega}^*)} n. \tag{19}$$

An increase in θ and a decrease in r now have different effects on k even if the ratio $\frac{\theta}{\delta+r}$ is affected identically. For a given impact on $\frac{\theta}{\delta+r}$ the increase in k is stronger if generated by a decrease in r. y increases in both cases, but by more in case the variation in $\frac{\theta}{\delta+r}$ is triggered by a decrease in θ. The validity of Proposition 3 can be seen by noting that, if r increases in response to an increase in θ by just enough to keep $\frac{\theta}{\delta+r}$ constant, defaults are unchanged, capital decreases and output increases. By continuity, an increase in r that marginally decreases $\frac{\theta}{\delta+r}$ decreases defaults, but still increase output, delivering the result.

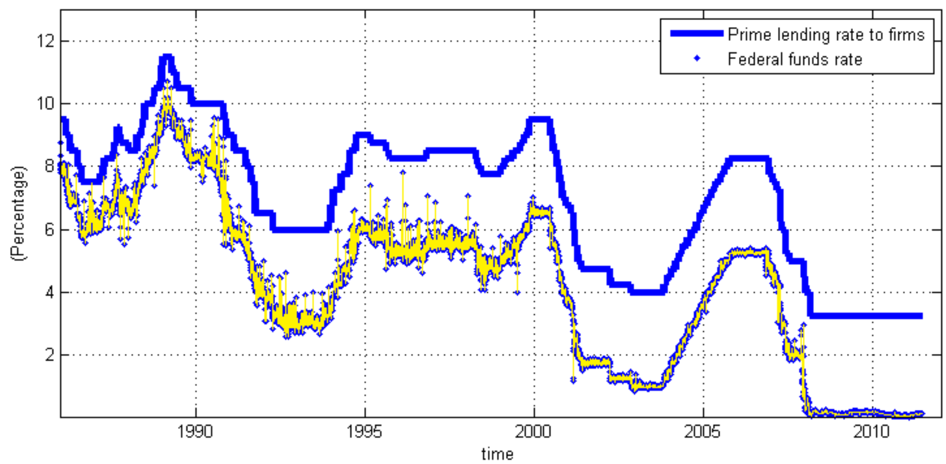
9 Appendix B: Data used to calibrate r and r^b

I extracted daily data from January 1986 to May 2012 from the Federal Reserve Board statistical release. Two variables were considered, the federal funds rate (H15/RIFSPFF_N.WW) and the average prime lending rate on bank loans to firms (H15/RIFSPBLP_N.WW). The evolution of these variables is shown in figure 8. As explained in Section 4.1, the fed funds is used to calibrate the exogenous interest rate r and the prime borrowing rate is used to calibrate the endogenous borrowing rate r_b. For comparability, the data used to measure the interest rate sample averages cover the period 1986 - 2004, as in Covas and Den Haan (2010). Summary statistics are also provided for the entire time period spanned by the dataset. Data on inflation rates were taken from the CPI index available from the Bureau of Labor Statistics (series CUUR0000SA0).

Table 8: Federal funds rate and prime lending rate to firm

Time period	**Variable**	**mean**	**std.dev.**	**p1**	**p99**
01/01/1986 - 31/03/2012	Fed funds rate	0.0422	2.57	0.08	9.69
	Prime rate	0.0697	2.24	3.25	11.5
	Inflation	2.9%	1.30	-1.3%	6.3%
01/01/1986 - 31/12/2004	Fed funds rate	0.0504	2.24	0.98	9.79
	Prime rate	0.0769	1.90	4	11.5
	Inflation	3%	1.13	1.1%	6.17%

Figure 8: Federal funds rate and lending rate to firm



10 Appendix C: The full model with non-linear production, stochastic depreciation and fixed production cost

The production function is given by

$$\omega\theta k^{\alpha}+(1-\delta_0 e^{\delta_1\omega})k-\eta \quad , \quad \delta_1<0 \; , \; \eta>0.$$

This implies that the debt contract solves the following maximization problem:

$$\max_{\{k,\bar{\omega},r_b\}}\int_{\bar{\omega}}^{\infty}\big[\omega\theta k^{\alpha}-\eta+(1-\delta_0 e^{\delta_1\omega})k-(1+r_b)(k-n)\big]d\Phi(\omega),$$

subject to

$$\int_0^{\bar{\omega}}\big[\omega\theta k^{\alpha}+(1-\delta_0 e^{\delta_1\omega})k-\mu\theta k^{\alpha}-\eta\big]d\Phi(\omega)+\int_{\bar{\omega}}^{\infty}\big[(1+r_b)(k-n)\big]d\Phi(\omega),\geq(1+r)(k-n) \tag{20}$$

$$\bar{\omega}\theta k^{\alpha}-\eta+(1-\delta_0 e^{\delta_1\bar{\omega}})k=(1+r_b)(k-n). \tag{21}$$

Substitute now the second constraint in the objective function:

$$\int_{\bar{\omega}}^{\infty} \left[\omega\theta k^{\alpha} + (1-\delta_0 e^{\delta_1\omega})k - \bar{\omega}\theta k^{\alpha} + (1-\delta_0 e^{\delta_1\bar{\omega}})k\right] d\Phi(\omega),$$

$$\int_{\bar{\omega}}^{\infty} \left[(\omega-\bar{\omega})\theta k^{\alpha}\right] d\Phi(\omega) - \delta_0 k\Big[\int_{\bar{\omega}}^{\infty} e^{\delta_1\omega} d\Phi(\omega) - [1-\Phi(\bar{\omega})]e^{\delta_1\bar{\omega}}\Big],$$

$$F(\bar{\omega})\theta k^{\alpha} - \underbrace{\Big[\int_{\bar{\omega}}^{\infty} e^{\delta_1\omega} d\Phi(\omega) - [1-\Phi(\bar{\omega})]e^{\delta_1\bar{\omega}}\Big]}_{H(\bar{\omega})} \delta_0 k,$$

$$F(\bar{\omega})\theta k^{\alpha} - H(\bar{\omega})\delta_0 k,$$

with $H(\bar{\omega}) = 0$ if $\delta_1 = 0$, and with

$$F'(\bar{\omega}) = -[1-\Phi(\bar{\omega})] < 0,$$

$$F''(\bar{\omega}) = \Phi'(\bar{\omega}) > 0,$$

$$H'(\bar{\omega}) = -[1-\Phi(\bar{\omega})]\delta_1 e^{\delta_1\bar{\omega}} = F'(\bar{\omega})\delta_1 e^{\delta_1\bar{\omega}} > 0,$$

$$H''(\bar{\omega}) = -[1-\Phi(\bar{\omega})]\delta_1^2 e^{\delta_1\bar{\omega}} + \Phi'(\bar{\omega})\delta_1 e^{\delta_1\bar{\omega}} < 0.$$

Substitute the second constraint in the first constraint:

$$\int_0^{\bar{\omega}} \left[\omega\theta k^{\alpha} - \eta + (1-\delta_0 e^{\delta_1\omega})k - \mu\theta k^{\alpha}\right] d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left[\bar{\omega}\theta k^{\alpha} - \eta + (1-\delta_0 e^{\delta_1\bar{\omega}})k\right] d\Phi(\omega) \geq (1+r)(k-n),$$

$$G(\bar{\omega})\theta k^{\alpha} - \eta + \Big(1 - \delta_0\Big[\int_0^{\bar{\omega}} e^{\delta_1\omega} d\Phi(\omega) + [1-\Phi(\bar{\omega})]e^{\delta_1\bar{\omega}}\Big]\Big)k \geq (1+r)(k-n),$$

$$G(\bar{\omega})\theta k^{\alpha} - \eta + \Big(1 - \delta_0\Big[\int_0^{\infty} e^{\delta_1\omega} d\Phi(\omega) - \int_{\bar{\omega}}^{\infty} e^{\delta_1\omega} d\Phi(\omega) + [1-\Phi(\bar{\omega})]e^{\delta_1\bar{\omega}}\Big]\Big)k \geq (1+r)(k-n),$$

$$G(\bar{\omega})\theta k^{\alpha} - \eta + k - \underbrace{\Big(\int_0^{\infty} e^{\delta_1\omega} d\Phi(\omega) - \int_{\bar{\omega}}^{\infty} e^{\delta_1\omega} d\Phi(\omega) + [1-\Phi(\bar{\omega})]e^{\delta_1\bar{\omega}}\Big)}_{T(\bar{\omega})} \delta_0 k \geq (1+r)(k-n),$$

$$G(\bar{\omega})\theta k^{\alpha} - \eta + (1 - T(\bar{\omega})\delta_0)k \geq (1 + r)(k - n),$$

with $T(\bar{\omega}) = \int_0^{\infty} e^{\delta_1 \omega} d\Phi(\omega) - H(\bar{\omega}) = 1$ if $\delta_1 = 0$ and with [22]

$$G'(\bar{\omega}) = -F'(\bar{\omega}) - \mu\Phi'(\bar{\omega}) > 0,$$

$$G''(\bar{\omega}) = -F''(\bar{\omega}) - \mu\Phi''(\bar{\omega}) < 0,$$

$$T'(\bar{\omega}) = -H'(\bar{\omega}) < 0,$$

$$T''(\bar{\omega}) = -H''(\bar{\omega}) > 0.$$

The maximization problem then rewrites as

$$\max_{\{k,\bar{\omega}\}} F(\bar{\omega})\theta k^{\alpha} - H(\bar{\omega})\delta_0 k,$$

$$\text{subject to } \; G(\bar{\omega})\theta k^{\alpha} + (1 - T(\bar{\omega})\delta_0)k \geq (1 + r)(k - n). \tag{22}$$

Note that the sum of what agents obtain in expected value (right-hand side of equation (23)) equals expected revenues plus expected non-depreciated capital net of expected auditing costs (left-hand side of equation (23)):

$$\begin{aligned} \theta k^{\alpha} - \eta + \Big(1 - \delta_0\big[\int_0^{\infty} e^{\delta_1 \omega} d\Phi(\omega)\big]\Big)k - \mu\Phi(\bar{\omega})k = \\ = \big(F(\bar{\omega}) + G(\bar{\omega})\big)\theta k^{\alpha} - \eta + \big(1 - T(\bar{\omega})\delta_0 - H(\bar{\omega})\delta_0\big)k. \end{aligned} \tag{23}$$

11 Appendix D: Calibration

The model derived in Section 5 consists of the following 5 equations:

[22] The first inequality holds only in the lower part of the support of ω. See also note 10.

$$CON(\bar{\omega}, \mu, \sigma, \theta, k, \alpha, \delta_0, \delta_1, \eta, r, n) = 0 \tag{24}$$

$$OPT(\bar{\omega}, \mu, \sigma, \theta, k, \alpha, \delta_0, \delta_1, r, n) = 0 \tag{25}$$

$$1 + r_b = BR(\bar{\omega}, \theta, k, \alpha, \delta_0, \delta_1, r, n, \eta) \tag{26}$$

$$def = DFT(\bar{\omega}, \sigma) \tag{27}$$

$$E(deprec) = DPR(\sigma, \delta_0, \delta_1) \tag{28}$$

Equation (24) represents the indifference condition of the lender, equation (25) the optimality condition pinning down the default threshold value, equation (26) the borrowing rate implied by equation (2), equation (27) the default probability implied by the default threshold value and equation (28) the expected depreciation rate. The set of parameters involved is displayed in each equation. The model is calibrated in $\{\mu, \sigma, \theta, \alpha, \delta_0, \delta_1, \eta, r, n\}$ in order to deliver an expected depreciation $E(deprec)$ of 0.10 and in order to guarantee that in equilibrium the endogenous variables $\bar{\omega}$ and k deliver the following three moments:

$$def = 0.0220$$

$$r_b = 0.0469$$

$$\frac{k}{n} = 2.10$$

The initial guess of the set of parameters is crucial to determine whether the numerical algorithm matches the desired empirical moments or not. This is due to the strong non-linearities present in the model. Instead of searching numerically for the initial guess that brings the calibration sufficiently close to the desired equilibrium, I follow an alternative, relatively simpler and less time-consuming approach, which consists of 8 steps:

1. guess a parameter value for σ and use equation (27) to compute the corresponding equilibrium value of $\bar{\omega}$ so that the target value on the default rate is met;

2. guess a parameter value for δ_0 and use equation (28) to compute the corresponding value of δ_1 so that, given σ, the target value on the expected depreciation is met;

3. set $r = 0.00204$, the average real fed funds rate;

4. set $\alpha = 0.30$, a value in line with the literature;

5. restrict the attention to an equilibrium in which k lays in the neighbourhood of 1. From the target value of the leverage ratio, n must equal $1/2.10 = 0.4762$;

6. given $r, \alpha, n, k, \sigma, \bar{\omega}, \delta_0, \delta_1$ and the target value of r_b, guess a relatively high parameter value of μ and compute the value of η that implies that equations (24) and (26) yield the same θ;

7. move back to step 1 and 2 and search numerically for the parameter values of σ and δ_0 that satisfy equation (25). If such a combination of σ and δ_0 can be found it means that the entire set of parameters delivers the target values for $\frac{k}{n}$, $\Phi(\bar{\omega})$ and r_b and by construction meets the target value of the expected depreciation rate;

8. repeat the exercise by decreasing μ to the lowest value that still allows the calibration over σ and δ_0 to deliver the desired target values for the leverage ratio, the default rate and the borrowing rate.

This approach is convenient because, by calibrating one parameter at the time, it gives much more flexibility in using visual inspection to identify the most appropriate initial guess. This would not be the case, however, if one contemporaneously calibrates the model on the entire set of parameters and lets the algorithm search for the combination of values that delivers the desired outcome. In fact, call Λ the set of values for $\{\mu, \sigma, \theta, \alpha, \delta_0, \delta_1, \eta, r, n\}$ found using the suggested step-by-step procedure and displayed in table 1 from Section 5.2. The non-linear algorithm *fsolve* in Matlab delivers approximately the same solution if the initial guess of the parameters lies in the neighborhood of Λ, but it fails to do so if the initial guess is far away from Λ. It was found, in fact, that the algorithm delivers the desired equilibrium as long as the initial guess is up to 50% away from Λ, where each parameter was allowed to randomly deviate either upwards or downwards. The algorithm fails to find the desired equilibrium as soon as the initial guess diverges from Λ by slightly more than 50%.

A second advantage of the step-by-step procedure explained above lies in the fact that it helps to develop an intuition of the trade-offs that the model implies for the parameters. Ideally, one would like to calibrate the model with a value of α equal to 0.70 and a relatively low value of the auditing cost (see Section 5.2). Table 11 shows the search for the lowest parameter value for μ compatible with $\alpha = 0.30$, as explained in step 8. The line marked with the right-ward arrow indicates the set of parameters chosen in the final calibration. The table shows that, having fixed α to 0.30, values of μ above 0.55 allow the model to match the desired empirical moments perfectly, but as soon as μ decreases below 0.54 the model performs poorly very rapidly. An additional heuristic assessment of the trade-off between α and μ is shown in table 12, which displays the calibration of the model for values of α slightly above 0.30 and adjusts μ according to step 8. Again, the line marked with the right-ward arrow indicates the final calibration. An increase in α of 0.01 requires an increase in the auditing cost μ of approximately 0.015 to let the model still perform well. This implies that the model cannot possibly generate the right moments using high values of the marginal return to capital and low values of the auditing cost simultaneously. Note also that no trade-off appears to be in place between α and the parameters other than μ, since the computed values of $\sigma, \delta_o, \delta_1, \eta, \theta$ are fairly in line with the parameter values found under $\alpha = 0.30$. A decrease in the auditing cost can instead be reconciled with $\alpha = 0.30$ if the risk-free interest rate increases above 0.0204. Table 13 calibrates the model considering values of r 25 basis points below and 25 and 50 basis points above the value used in the calibration. The model is found to be compatible with lower values of μ only if r increases above 0.0204, which was the value chosen on US data, as explained in Section 5.2 and in Appendix B.

Figure 9: Increase in productivity

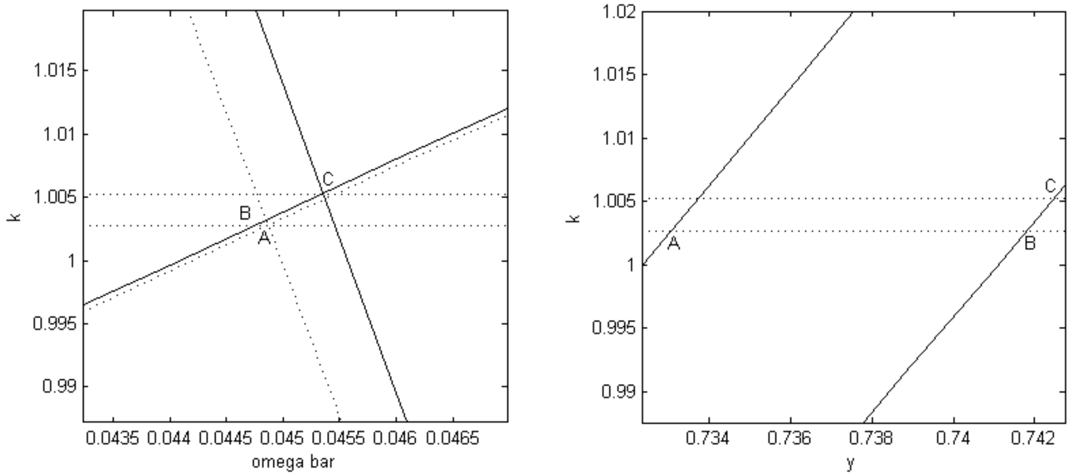


Figure 10: Increase in the exogenous interest rate

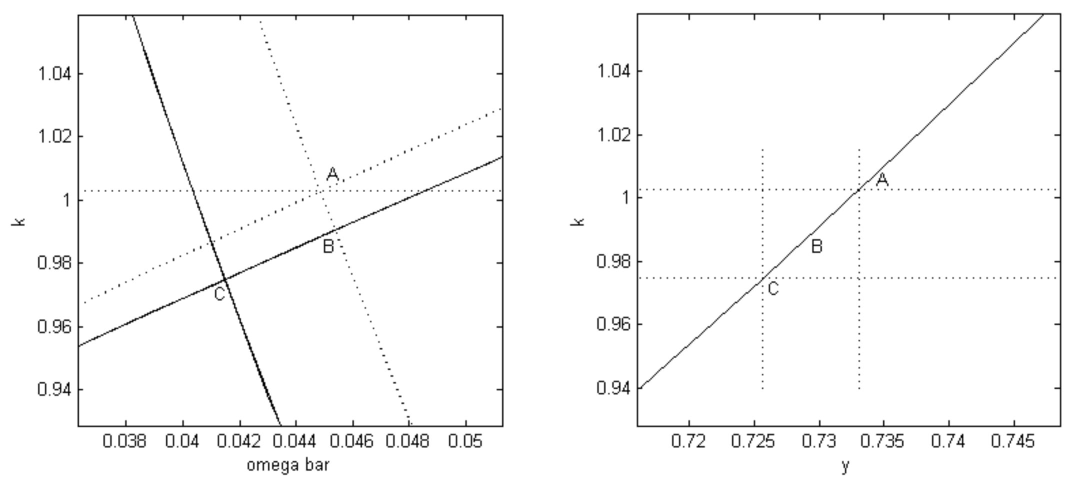


Figure 11: Weak (top) vs. strong (bottom) response of r to θ

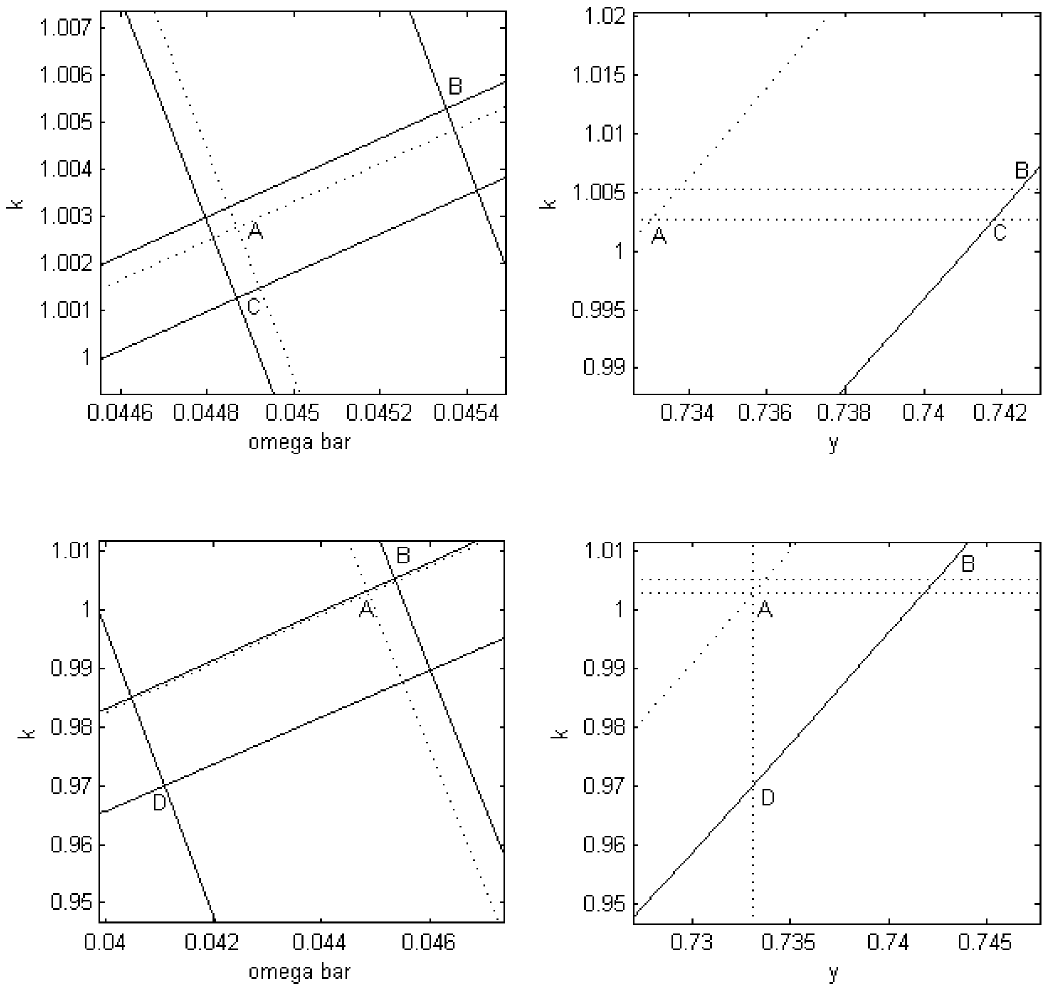


Table 9: With different productivity shocks

$\frac{\dot{\theta}}{\theta}$	$\frac{\dot{n}}{n}$	new r	γ	$\frac{\dot{k}}{k}$	$\frac{\dot{y}}{y}$	$\dot{\Phi(\bar{\omega})}$
0.005	0.0027	0.0208	0.0732	0.2899	0.70	0
		0.0218	0.2709	0	0.5959	-0.0004
		$\Omega_\gamma = (0.0731, 0.2709)$				
0.01	0.0047	0.0211	0.0731	0.5802	1.4009	0
		0.0231	0.2720	0	1.1918	-0.0007
		$\Omega_\gamma = (\mathbf{0.0732}, \mathbf{0.2720})$				
0.05	0.0238	0.0240	0.0714	2.9443	7.0533	0
		0.0344	0.2804	0	5.9591	-0.0034
		$\Omega_\gamma = (0.0714, 0.2804)$				
0.10	0.0485	0.0273	0.0691	6.0076	14.2330	0
		0.0496	0.2917	0	11.9183	-0.0064
		$\Omega_\gamma = (0.0691, 0.2917)$				
0.20	0.1029	0.0330	0.0632	12.8655	29.1247	0
		0.0848	0.3219	0	23.8365	-0.0118
		$\Omega_\gamma = (0.0632, 0.3219)$				

Table 10: With different auditing costs

μ	$\frac{\dot{n}}{n}$	new r	γ	$\frac{\dot{k}}{k}$	$\frac{\dot{y}}{y}$	$\dot{\Phi(\bar{\omega})}$
0.60	0.0048	0.0211	0.0705	0.5857	1.4088	0
		0.0230	0.2570	0	1.1969	-0.0007
		$\Omega_\gamma = (0.0705, 0.2570)$				
0.59	0.0048	0.0211	0.0707	0.5873	1.4129	0
		0.0230	0.2605	0	1.1998	-0.0007
		$\Omega_\gamma = (0.0707, 0.2605)$				
0.58	0.0047	0.0211	0.0709	0.5893	1.4177	0
		0.0230	0.2640	0	1.2033	-0.0007
		$\Omega_\gamma = (0.0709, 0.2640)$				
0.57	0.0047	0.0211	0.0713	0.5891	1.4170	0
		0.0231	0.2678	0	1.2028	-0.0007
		$\Omega_\gamma = (0.0713, 0.2678)$				
0.56	0.0047	0.0211	0.0708	0.5911	1.4219	0
		0.0229	0.2528	0	1.2063	-0.0008
		$\Omega_\gamma = (0.0708, 0.2528)$				
0.55	0.0046	0.0211	0.0707	0.5902	1.4195	0
		0.0228	0.2391	0	1.2046	-0.0008
		$\Omega_\gamma = (0.0707, 0.2391)$				
0.54	0.0047	0.0211	0.0732	0.5802	1.4009	0
		0.0231	0.2720	0	1.1918	-0.0007
		$\Omega_\gamma = (\mathbf{0.0731}, \mathbf{0.2720})$				

Table 11: Calibration with different values of μ given $\alpha = 0.30$

	Parameters						Equilibrium		
	μ	σ	δ_0	δ_1	η	θ	$\Phi(\bar{\omega}^*)$	r_b^*	$\frac{k^*}{n}$
	0.60	1.1418	0.4291	-3.2376	0.1309	0.7961	0.0220	0.0469	2.1000
	0.59	1.1510	0.4244	-3.2056	0.1320	0.8076	0.0220	0.0469	2.1000
	0.58	1.1604	0.4194	-3.1711	0.1333	0.8196	0.0220	0.0469	2.1000
	0.57	1.1698	0.4143	-3.1340	0.1348	0.8321	0.0220	0.0469	2.1000
	0.56	1.1789	0.4088	-3.0916	0.1367	0.8450	0.0220	0.0469	2.1000
	0.55	1.1883	0.4031	-3.0476	0.1389	0.8584	0.0220	0.0469	2.1000
$\Rightarrow$	0.54	1.1944	0.3995	-3.0180	0.1406	0.8730	0.0227	0.0473	2.1057
	0.53	1.1922	0.3999	-3.0214	0.1416	0.8887	0.0251	0.0489	2.1266
	0.52	1.1922	0.3999	-3.0214	0.1423	0.9053	0.0273	0.0504	2.1460
	0.51	1.1902	0.4003	-3.0243	0.1433	0.9222	0.0303	0.0523	2.1713
	0.50	1.1897	0.4004	-3.0250	0.1442	0.9399	0.0333	0.0542	2.1967

Table 12: Calibration with different values of α

	Parameters							Equilibrium		
	α	μ	σ	δ_0	δ_1	η	θ	$\Phi(\bar{\omega}^*)$	r_b^*	$\frac{k^*}{n}$
$\Rightarrow$	0.30	0.54	1.1944	0.3995	-3.0180	0.1406	0.8730	0.0227	0.0473	2.1057
	0.31	0.55	1.1918	0.4001	-3.0224	0.1402	0.8573	0.0238	0.0481	2.1157
	0.32	0.57	1.1923	0.3999	-3.0212	0.1389	0.8281	0.0225	0.0473	2.1046
	0.33	0.58	1.1914	0.4001	-3.0225	0.1384	0.8142	0.0233	0.0478	2.1112
	0.34	0.60	1.1911	0.4001	-3.0229	0.1372	0.7878	0.0223	0.0471	2.1022
	0.35	0.61	1.190	0.4005	-3.0255	0.1367	0.7752	0.0230	0.0470	2.1085

Table 13: Calibration with different values of r given $\alpha = 0.30$

	Parameters							Equilibrium		
	r	μ	σ	δ_0	δ_1	η	θ	$\Phi(\bar{\omega}^*)$	r_b^*	$\frac{k^*}{n}$
	0.0179	0.55	1.1987	0.3987	-3.0123	0.1417	0.9113	0.0241	0.0483	2.1181
$\Rightarrow$	0.0204	0.54	1.1944	0.3995	-3.0180	0.1406	0.8730	0.0227	0.0473	2.1057
	0.0229	0.52	1.1854	0.4013	-3.0312	0.1406	0.8633	0.0236	0.0479	2.1136
	0.0254	0.50	1.1770	0.4030	-3.0432	0.1400	0.8681	0.0235	0.0479	2.1136

CHAPTER 2

Monetary Policy, Leverage, and Default

Abstract

I use the Bernanke, Gertler and Gilchrist (1999) model to study the effect of monetary policy on the probability that firms default on loans. I argue that a monetary expansion affects defaults through two opposing partial equilibrium effects. It increases defaults because it leads firms to take on more debt and leverage up net worth, and it decreases defaults because the cost of borrowing decreases and aggregate demand shifts out, increasing firms' profits and net worth. I argue that the leverage effect could explain the empirical partial equilibrium finding by Jimenez *et al.* (2008) that defaults on new loans increase after a monetary expansion. I then argue that this effect does not hold in general equilibrium due to an increase in firms' profits. In the full model, defaults decrease after a monetary expansion, although the effect equals only few basis points.

JEL Classification: E44, E52

Keywords: Monetary policy, risk taking channel, costly state verification, default probability

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1 Introduction

This paper uses the "costly state verification" contract by Townsend (1979) to study the effect of monetary policy on the probability that firms default on loans. A recent empirical literature has argued that a monetary expansion decreases firms' default probability on existing loans and increases their default probability on new loans. In particular, Jimenez, Ongena, Peydró and Saurina (2008), Ioannidou, Ongena and Peydró (2008) and Lopez, Tenjo and Zarate (2011) argue that this asymmetric effect on new versus existing loans holds for Spain, Bolivia and Columbia, respectively. The analysis of these papers is carried out in partial equilibrium. In this paper I first use the Townsend (1979) model to argue that a leverage effect could potentially contribute to explain the partial equilibrium result that defaults on new loans are higher after a monetary expansion. I then argue that, according to the Bernanke, Gertler and Gilchrist (1999) extension of the debt contract by Townsend (1979), the increase in defaults on new loans does not hold in general equilibrium. In the general equilibrium model, defaults on both new and existing loans decrease, although by only a few basis points. The results that defaults effectively remain constant in general equilibrium is consistent with the empirical analysis by Buch, Eickmeier and Prieto (2011) and by Piffer (2014), who do not find a statistically significant response to a monetary expansion of the issuance of risky loans by banks and on the delinquency rate on business loans, respectively.

The model isolates two intuitions. The first intuition relates to the increase in defaults in partial equilibrium. The debt contract by Townsend has only one period, hence it features only new loans. A decrease in the risk free rate exerts two opposing effects on defaults, a revenue effect and a leverage effect. The revenue effect predicts that defaults decrease because a monetary expansion increases firms' revenues by decreasing the cost of borrowing or by increasing aggregate demand. The leverage effect, instead, predicts that defaults increase because firms react to the lower cost of borrowing by leveraging up their net worth. In the partial equilibrium model, the leverage effect dominates for any calibration of the parameters. The second intuition relates to the general equilibrium effect. In general equilibrium, firms' return on capital is not held constant. In the model, the increase in investment following the monetary expansion increases the price of capital and hence the return on capital, pushing firms' profits up. The increase in profits dominates for all calibrations considered, ultimately leading to a decrease in defaults for both new and existing loans. Leverage decreases in general equilibrium because of the accumulation of net worth by firms, following the increase in profits. The effect on leverage and defaults is quantitatively very small and only equal to a few basis points in response to a monetary expansion that decreases the policy rate by 100 basis points.

The "costly state verification" contract by Townsend (1979) has found many applications in the macroeconomic literature. Nevertheless, to the best of my knowledge, it has rarely been used explicitly to study the effect of monetary policy on default probabilities.[1] One exception is Hafstead and Smith (2012), who also find that a monetary expansion decreases the default rate by few basis points. The literature has discussed the fact that the general equilibrium application of the Townsend model proposed by Bernanke, Gertler and Gilchrist (1999) and used in this paper is suboptimal, because the borrowing rate is not made state contingent on the aggregate shock. As discussed in Carlstrom, Fuerst and Paustian (2011), an optimal contract would imply a smaller variation in firms' net worth in response to shocks, since part of the effect is absorbed by a state-contingent borrowing rate. The results of this paper hold also using an optimal contract, as implied in the

[1] Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) use the "costly state verification" model to generate hump-shaped impulse response functions for output and to model a financial accelerator, respectively. As discussed for example by Gomes, Yaron and Zhang (2003) and Quadrini (2011), the "costly state verification" model generates procyclical defaults when business cycles are generated by productivity shocks. Dorofeenko, Lee and Salyer (2008), Medina (2006) and Covas and Den Haan (2012) propose possible modifications of the model in order to reconcile it with countercyclical defaults, as observed in the data.

analysis by Dmitriev and Hoddenbagh (2013).

The paper relates to the literature on the "risk taking channel of monetary policy". This literature was sparked by Rajan (2006) and Borio and Zhu (2008), who argued that a monetary expansion leads lenders to "search for yield" and worsens their perception of risk. Their contributions are part of a broader literature on the effects of monetary policy on financial stability. Eichengreen *et al.* (2011), Adrian and Shin (2008) and Blanchard, Dell'Ariccia and Mauro (2010) offer an overview of the debate.

The model contributes to the literature by stressing the role of ex-ante homogeneous borrowers who have access to the same production function and draw idiosyncratic productivity shocks from a common distribution. Borrowing characteristics depend endogenously on the leverage ratio of the borrower and determine the level of the idiosyncratic shock below which the borrower defaults on debt. An alternative approach would be to stress the role of the quality of the project financed rather than the leverage ratio of the borrower. In Stiglitz and Weiss (1981) borrowers either differ ex ante in terms of riskyness of projects, or they can ex post divert funds towards more risky projects. The size of the investment does not play a role in either of these two cases. A similar approach is followed by Drees, Eckwert and Vardy (2012), where agents have one unit to invest and choose between a risky and a safe project. In contrast, several papers abandon the assumption of a fixed unit of investment, but they do not use a debt contract or they focus on mechanisms that differ from the one studied in this paper. For example, Riccetti, Russo and Gallegati (2011) use a model with "rule-of-thumb" agents who interact through non-Walrasian prices. Acharya and Naqvi (2012) and Angeloni, Faia and Lo Duca (2011) study liquidity shocks and endogenous bank runs. Challe, Mojon and Ragot (2012), Dubecq, Mojon and Ragot (2009), Agur and Demertzis (2012) and Cociuba, Shukayev and Ueberfeldt (2012) develop models in which the choice of the leverage ratio interacts with the ex-ante heterogeneity of borrowers. Bruno and Shin (2012) use a model in which leverage ratios interact with the real value of the currency. Dell'Ariccia, Laeven and Marquez (2010) analyze a setting in which the lender controls the default probability of the borrower through monitoring. Valencia (2011) considers the special case of a monopolistic bank. Fahri and Tirole (2009) consider projects that might need to be refinanced before maturity. Stein (2012) analyzes the role of the debt maturity structure.

The rest of the paper is organized in three Sections. Section 2 uses the partial equilibrium "costly state verification" model to develop the intuition behind the revenue and leverage effects. Section 3 uses the Bernanke, Gertler and Gilchrist (1999) model to extend the partial equilibrium analysis to general equilibrium. The last Section concludes and suggests possible extensions for future research.

2 A simple model with defaults

This section uses the "costly state verification" model in partial equilibrium to highlight the leverage effect. Sections 2.1 describes the model, Section 2.2 analyses the optimality condition.

2.1 The model

A risk-neutral entrepreneur has limited net worth N. At the beginning of the period he borrows $QK - N$ from a risk-neutral lender, where K stands for the level of investment and Q for the price of investment.[2] The entrepreneur has access to the following production function:

[2] Q is not strictly necessary for the purpose of the partial equilibrium model of this section, but it will play an important role in Section 3. Risk neutrality is assumed to simplify the exposition of the model, but it is not required to derive the optimality of the debt contract, as shown in Townsend (1979).

$$y = \omega R_K K.$$

The shock ω represents an idiosyncratic productivity shock with support $[0, \infty)$, expected value of 1 and cumulative distribution function $\Phi(\omega)$. R_k stands for the deterministic aggregate return on the risky technology and is known by both agents at the beginning of the period. No aggregate shock enters the partial equilibrium model. Capital fully depreciates and cannot be consumed at the end of the production.[3]

Information is symmetric ex-ante but asymmetric ex-post. The contract is signed at the beginning of the period. The shock ω is realized at the end of the period. If the shock was costlessly observed by both parties at the end of the period, then state-contingent contracts would allow the borrowing rate of the contract to be some optimal function of the ex-post entrepreneur's revenues. Assume, instead, that ω is costlessly observed by the entrepreneur, but it is not observed by the lender unless he pays a fraction $\mu < 1$ of ex-post revenues $\omega R_k K$. This observation cost captures the idea that borrowers typically have a richer information set regarding their project relative to lenders. Given asymmetric information, the repayment scheme cannot be contingent on the shock, because it would lead the entrepreneur to opportunistically under-report ω in order to pay a lower interest rate.[4]

Monetary policy enters the model through the opportunity cost of lending, as in Woodford (2003) and Curdia and Woodford (2009). Let $R < R_k$ represent

[3]The main results of the model do not depend on the linearity assumption in the production function nor on the assumption of full depreciation, which only make the intuition clearer. See Covas and Den Haan (2012) for a study of non-linearities in production and capital depreciation in "costly state verification" models. The general equilibrium extension of the model used in Section 3 allows for capital depreciation.

[4]Verification is assumed to be deterministic, although Townsend (1979) shows that it is dominated by stochastic verification. The assumption that the verification cost is proportional to ex-post revenues is without loss of generality. Townsend (1979) shows that the optimality of the debt contract holds irrespectively on whether this cost is constant or not in the realization of the revenues of the borrower.

this opportunity cost of lending. In the model, the central bank controls R either directly, as in the partial equilibrium version of this section, or through nominal price rigidities, as in the general equilibrium version of Section 3.

Entrepreneurs borrow from lenders in competitive markets. Perfect competition takes the form of lenders competing among each other to provide loans to entrepreneurs. Under the assumptions of perfect competition among lenders and risk neutrality of all agents, the contract maximizes the expected profits of the entrepreneur under the condition that the expected return on lending equals the opportunity cost of lending.[5]

Townsend (1979) shows that in this setting the optimal contract is a simple debt contract, that is, the borrower either repays a fixed amount independently on the realization of the shock or defaults.[6] Let R_b stand for the borrowing rate. When the borrower and the lender agree on $K - N$ and R_b they indirectly agree on an endogenous threshold value $\bar{\omega}$ for ω below which the entrepreneur's revenues are insufficient to cover the debt repayment obligation. This threshold value is pinned down by $\bar{\omega} R_k K = R_b(QK - N)$. If $\omega > \bar{\omega}$, then the entrepreneur pays back $R_b(QK - N)$ and keeps profits $\omega R_k K - R_b(QK - N)$. If $\omega < \bar{\omega}$, then the entrepreneur defaults and the lender recovers $(1 - \mu)\omega R_k K$.

The contract maximizes the expected profit of the entrepreneur subject to the

[5]This form of perfect competition is taken from the original paper by Townsend (1979) and from several papers that draw from it, for example Covas and Den Haan (2012) and Faia and Monacelli (2006).

[6]The key intuition behind the optimality of the debt contract comes from the fact that it is optimal to limit the probability that the dead-weight observation cost will be incurred. To do so, the lender leaves no revenue to the entrepreneur in case of default in order to reduce the borrowing rate to be paid in case of no default. The debt contract is optimal if μ is an observation cost, i.e. a cost that the lender must pay to discover that the entrepreneur cannot afford to repay the debt. It is not optimal, however, if μ is only a bankruptcy cost, i.e. a cost that the lender must pay to seize the revenues of the entrepreneur. In fact, if it was not costly to observe the revenue but only to organize, say, the dissembling of the firm and the sale of the entrepreneurs' assets, then in the event of a default it would be optimal ex-post to renegotiate the loan and avoid incurring the cost. This is not the case for an observation cost, because the lender needs to observe that the ex-post return on revenues is insufficient to cover the entrepreneur's repayment obligation before agreeing to renegotiate, otherwise entrepreneurs would demand to renegotiate irrespectively of the realization of ω. The above remark is discussed, for instance, in Covas and Den Haan (2012).

participation constraint of the lender. The maximization problem is solved in $\bar{\omega}, R_b, K$ and is written as

$$\max_{\{\bar{\omega}, R_b, K\}} \int_{\bar{\omega}}^{\infty} \omega R_k K - R_b(QK - N) d\Phi(\omega),$$

subject to

$$\bar{\omega} R_k K = R_b(QK - N), \tag{1}$$

$$[1 - \Phi(\bar{\omega})]R_b + \Phi(\bar{\omega})(1 - \mu)\frac{E(\omega \mid \omega < \bar{\omega})R_k K}{QK - N} \geq R. \tag{2}$$

Equation (1) defines the threshold value $\bar{\omega}$ as a function of R_b and K. Equation (2) gives the participation constraint of the lender. This constraint imposes that the expected return on lending (left-hand side) is not lower than the opportunity cost of lending (right-hand side).[7]

It is convenient to simplify the maximization problem by substituting out the endogenous variable R_b from equation (1) into equation (2) and into the objective function. This substitution reduces the maximization problem to

$$\max_{\bar{\omega}, K} \; F(\bar{\omega}) R_k K,$$

$$\text{subject to} \; \frac{G(\bar{\omega}) R_k K}{QK - N} \geq R, \tag{3}$$

where $F(\bar{\omega})$ and $G(\bar{\omega})$ are equal to

[7] An equivalent way of writing down the participation constraint of the lender is

$$(1 - \mu) \int_0^{\bar{\omega}} \omega R_k K d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} R_b(QK - N) d\Phi(\omega) \geq R(QK - N).$$

$$F(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega d\Phi(\omega) - [1 - \Phi(\bar{\omega})]\bar{\omega},$$
$$G(\bar{\omega}) = 1 - F(\bar{\omega}) - \mu \int_{0}^{\bar{\omega}} \omega d\Phi(\omega).$$

To develop intuition behind the notation, use the expressions for $F(\bar{\omega})$ and $G(\bar{\omega})$ and the condition $E(\omega) = 1$ to derive the following equality:

$$\underbrace{\Big[1 - \mu \int_{0}^{\bar{\omega}} \omega d\Phi(\omega)\Big] R_k K}_{\text{net expected output}} = R_k K \Big[\underbrace{F(\bar{\omega})}_{\substack{\text{share to the} \\ \text{entrepreneur}}} + \underbrace{G(\bar{\omega})}_{\substack{\text{share to} \\ \text{the lender}}} \Big]. \tag{4}$$

Equation (4) shows that $F(\bar{\omega})$ and $G(\bar{\omega})$ determine the shares of expected output $R_k K$ net of expected monitoring costs $\mu \int_0^{\bar{\omega}} \omega R_k K d\Phi(\omega)$ allocated to respectively the entrepreneur and the lender. These shares are implicitly pinned down by the debt contract, once the borrower and the lender agree on $K - N$ and R_b.

Since the expected monitoring cost $\mu \int_0^{\bar{\omega}} \omega d\Phi(\omega)$ for each unit of $R_k K$ is relatively small for any realistic calibration of μ, $F(\bar{\omega})$ and $G(\bar{\omega})$ approximately add up to one.[8] Borrowing conditions that indirectly imply a higher share of expected revenues to the borrower $F(\bar{\omega})$ imply a lower share of expected revenues to the lender $G(\bar{\omega})$ and vice-versa. The leverage effect is driven by the positive first derivative of $G(\bar{\omega})$ with respect to the default threshold $\bar{\omega}$.[9] An increase in the share of expected revenues

[8] $F(\bar{\omega})$ and $G(\bar{\omega})$ add up to 0.9983 in the partial equilibrium exercise of Section 2.2 and add up to 0.9983 in the general equilibrium analysis of Section 3.

[9] More precisely, $G(\bar{\omega})$ is increasing in $\bar{\omega}$ only in the lower support of $\bar{\omega}$. Since a level of $\bar{\omega}^*$ where $G(\bar{\omega})$ decreases would be suboptimal (both parties would benefit from a reduction in $\bar{\omega}$) we can disregard the decreasing part of $G(\bar{\omega})$ from the analysis. $\bar{\omega}^*$ will be pinned down by equation (5). Bernanke, Gertler and Gilchrist (1999) show that condition $G'(\bar{\omega}^*) > 0$ holds if $\Phi(\omega)$ is such that

$$\frac{d}{d\omega} \omega \frac{d\Phi(\omega)}{1 - \Phi(\omega)} > 0.$$

This condition is satisfied for standard distributions, including the log normal distribution used promised to the lender $G(\bar{\omega})$ is associated with an increase in the default threshold $\bar{\omega}$ because it is harder for the entrepreneur to meet the higher repayment obligation to the lender.

$$F(\bar{\omega}) + G(\bar{\omega}) \approx 1 \quad \text{with} \quad F'(\bar{\omega}) < 0 \text{ and } G'(\bar{\omega}) > 0.$$

Return now to the simplified maximization problem. To understand the intuition of the model, note that the expected return on lending $\frac{G(\bar{\omega})R_k K}{QK-N}$ in equation (3) is affected by $G(\bar{\omega})$ and K in opposite directions. It increases in the share $G(\bar{\omega})$ to the lender because, given K, the lender receives a higher share of expected revenues. It decreases in K because, given $G(\bar{\omega})$, an increase in investment pushes up the entrepreneur's leverage and decreases the relative buffer provided by his net worth.

To solve the maximization problem, substitute for K using constraint (3) in the objective function and derive the optimality condition with respect to $\bar{\omega}$:

$$-F'(\bar{\omega}^*) = F(\bar{\omega}^*)\frac{G'(\bar{\omega}^*)}{Q \cdot \left(\frac{R_k}{R}\right)^{-1} - G(\bar{\omega}^*)}. \tag{5}$$

Equation (5) pins down the equilibrium default threshold $\bar{\omega}^*$ and the default probability $\Phi(\bar{\omega}^*)$.[10] The economic intuition behind equation (5) is explained in the next section in detail. To close the model, substitute $\bar{\omega}^*$ in constraint (3) and compute the equilibrium level of investment K^*. Last, use $\bar{\omega}^*$ and K^* to compute the equilibrium borrowing rate R_b from equation (1).

later.

[10] As shown in Covas and Den Haan (2012), the equilibrium default threshold is independent on the entrepreneur's net worth due to the linearity in the production function.

2.2 Interpreting the optimality condition

The threshold value $\bar{\omega}^*$ pinned down by equation (5) is a decreasing function of R for any parametrization of the model (see Covas and Den Haan (2012), appendix C).[11] This means that a decrease in the opportunity cost of lending *increases* the equilibrium default probability of the entrepreneur.

To develop the economic intuition behind this result, it is convenient to decompose the equilibrium effect into two partial effects. When R decreases,

> the default probability *decreases* because, for each level of borrowing, perfect competition pushes down the borrowing rate, making debt cheaper (**revenue effect**);
>
> the default probability *increases* because the reduction in the opportunity cost of lending increases the discounted profits, pushing up the optimal level of investment and leading the entrepreneur to leverage up his net worth (**leverage effect**).

In the rest of this section I analyze the revenue and leverage effects, graphically, mathematically, and with a simple numerical exercise. Section 3 studies how general equilibrium effects impact on the revenue and leverage effects and on their interaction.

Graphical representation

Figure 1, panel a, shows combinations of K and $G(\bar{\omega})$. As can be seen from the simplified maximization problem, these combinations differ for the implied expected profit of the entrepreneur and for whether they satisfy the participation constraint of the lender.

[11] Covas and Den Haan (2012) study the relationship between $\bar{\omega}^*$ and R_k. Their proof extends to the relationship between $\bar{\omega}^*$ and R, after accounting for how R_k and R affect the discounted return to capital R_k/R in the optimality condition (5).

The red dash-dotted lines represent iso-profit curves of the entrepreneur. Expected profits increase towards the top left part of the graph, where the entrepreneur invests more with a contract that implies a lower share of expected revenues to the lender. For the entrepreneur, it would be best to borrow an infinite amount and promise a zero share of expected revenues to the lender. Of course, this violates the participation constraint of the lender.

The participation constraint of the lender is shown in the solid, convex line. All combinations of $(K, G(\bar{\omega}))$ below the solid line satisfy the participation constraint of the lender, while all combinations above the line violate the participation constraint of the lender. The initial equilibrium is shown in point A, which is the point along the participation constraint which reaches the highest possible iso-profit curve of the entrepreneur.

The participation constraint of the lender is upward sloping in the space $(K, G(\bar{\omega}))$. This positive slope reflects the *leverage effect* that drives the result of the model. To see why, rewrite constraint (3) in terms of an upward limit to the entrepreneur's leverage ratio:

$$\frac{K}{N} \leq \frac{1}{Q - \frac{R_k}{R}G(\bar{\omega})} = k(\bar{\omega}). \tag{6}$$

Equation (6) shows that the maximum entrepreneurial leverage compatible with the participation constraint of the lender is an increasing function of $G(\bar{\omega})$. Given net worth N, and assuming that the constraint is initially binding, the entrepreneur can invest more only by borrowing more. For given $G(\bar{\omega})$, the increase in borrowing violates the participation constraint of the lender. In fact, it increases the entrepreneur's leverage ratio, which reduces the relative buffer that the constant net worth provides to the risky loan. To compensate the lender for this leverage effect, the entrepreneur must pay a leverage premium that takes the form of a higher $G(\bar{\omega})$, i.e. a higher share of expected revenues to the lender. The increase in $G(\bar{\omega})$ has a negative effect on the expected profit of the entrepreneur because it decreases $F(\bar{\omega})$ due to the higher default probability. Nevertheless, it is necessary to move along the participation constraint and convince the lender to issue more credit.

Consider now what happens when the opportunity cost of lending decreases. A decrease in R rotates the constraint upwards and expands the set of combinations compatible with the participation constraint of the lender. This is because perfect competition pushes down the return on lending, requiring now a lower borrowing rate for any given level of lending. The crucial issue is, which point along the new participation constraint reaches the highest iso-profit curve of the entrepreneur.

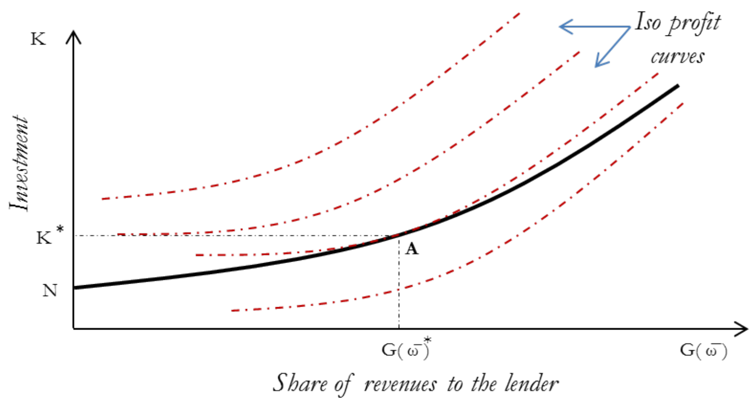
Consider first the special case in which the level of investment K is forced to remain at the initial value. This case is shown in figure 1, panel *b*. If K remains constant, the upward rotation of the constraint moves the equilibrium from point A to point B. In B, the entrepreneur borrows the same amount and pays a lower borrowing rate due to perfect competition. This is the *revenue effect*. The lower cost of debt implies a lower share of revenues to the lender and hence a lower default probability of the entrepreneur. If this was the only effect in place, the reduction in R would decrease the equilibrium default probability.

In this debt contract, though, point B is not optimal because the entrepreneur has an incentive to move along the new participation constraint of the lender. In fact, the productivity R_k of each unit of investment has remained the same, but the *discounted* return to capital R_k/R has increased, making each unit of investment more productive in discounted terms. From point B, investing more requires a higher expected share of revenues $G(\bar{\omega})$ promised to the lender due to the leverage effect discussed above. This means that any point on the new participation constraint on the right of point B implies an increase in the default probability relative to point B, although not necessarily relative to point A.

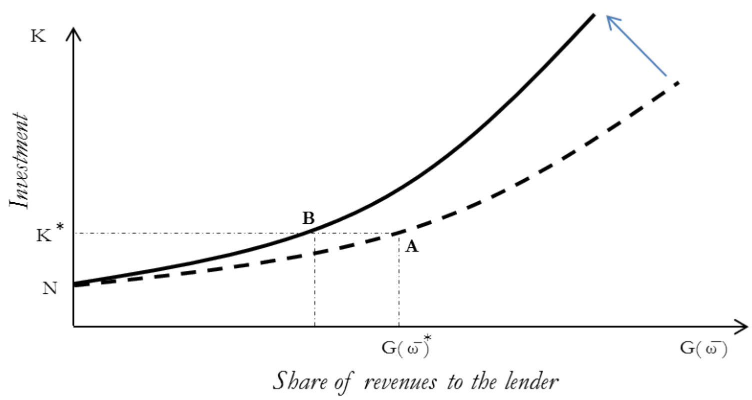
In principle, it could be that the optimal increase in the leverage ratio is not

Figure 1: Effects of a decrease in the opportunity cost of lending

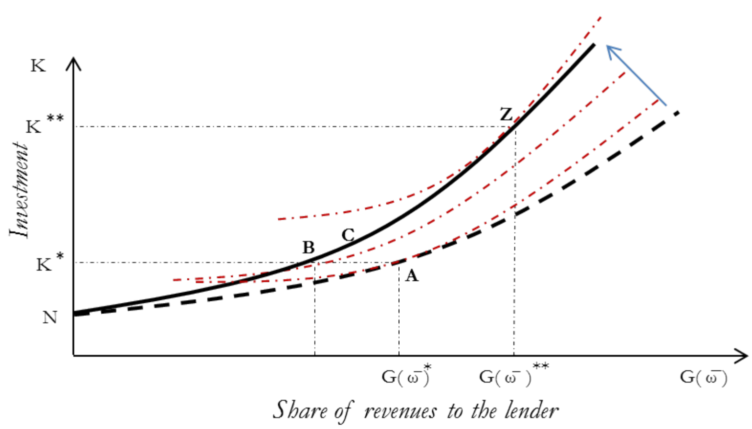
panel a) Initial Equilibrium



panel b) Revenue effect only



panel c) Both revenue and leverage effects



Notes: A decrease in the opportunity cost of lending relaxes the participation constraint of the lender and reduces the cost of borrowing. The entrepreneur reacts to the lower cost of borrowing by leveraging up its net worth in order to invest more and reach point Z. The equilibrium default probability is higher in point Z relative to point A due to the leverage premium demanded by the lender.

strong enough to offset the revenue effect, leading to a new equilibrium with a lower default probability relative to the initial equilibrium of point A. This is the case of point C of figure 1, panel c. In this contract, instead, the leverage effect always dominates on the revenue effect, taking the new equilibrium to the *right* of point A (point Z). This is implied mathematically by the optimality condition (5), which determines a negative relationship between $\bar{\omega}^*$ and R. The new equilibrium features a higher default probability because the entrepreneur finds it so profitable to invest more that he accepts to pay a higher share of expected revenues to the lender.

Mathematical derivation

An alternative way of seeing the intuition above is to derive the intermediate steps behind the optimality condition (5). Substitute the participation constraint (3) in the objective function and rewrite the maximization problem of the debt contract only in the endogenous variable $\bar{\omega}$:

$$\max_{\bar{\omega}} \quad F(\bar{\omega})k(\bar{\omega})NR_k.$$

$k(\bar{\omega})$ is defined by equation (6) and is increasing in $\bar{\omega}$ due to the leverage effect commented on above. Taking the first order condition in $\bar{\omega}$ gives

$$-\frac{dF(\bar{\omega})}{d\bar{\omega}}k(\bar{\omega}) = F(\bar{\omega})\frac{dk(\bar{\omega})}{d\bar{\omega}}. \tag{7}$$

Equation (7) states that, in an interior solution, the entrepreneur accepts a debt contract only if it implies that the marginal benefit and the marginal cost of a higher default threshold $\bar{\omega}$ are equal. The marginal cost of a higher $\bar{\omega}$ equals the foregone expected revenue implicit when accepting a higher share of revenues to the lender $(-\frac{dF(\bar{\omega})}{d\bar{\omega}}k(\bar{\omega}))$. The marginal benefit of a higher $\bar{\omega}$ equals the benefit associated with the corresponding marginal increase in the leverage ratio $(F(\bar{\omega})\frac{dk(\bar{\omega})}{d\bar{\omega}})$.

Define now the leverage premium as the increase in $G(\bar{\omega})$ associated with a 1% increase in the leverage ratio of the entrepreneur. The implicit function theorem gives

$$\frac{dG(\bar{\omega})}{\frac{dk(\bar{\omega})}{k(\bar{\omega})}} = \underbrace{Q \cdot \left(\frac{R_k}{R}\right)^{-1} - G(\bar{\omega})}_{\text{Leverage Premium}} > 0.$$

The leverage premium is strictly positive under the assumption that the maximization problem has an interior solution for K. This assumption is satisfied if, in the optimum, $\bar{\omega}$ satisfies $R_k < Q\frac{R}{G(\bar{\omega})}$.[12]. Substituting out $\frac{dk(\bar{\omega})}{k(\bar{\omega})}$ in equation (7) and rearranging the terms gives

$$-F'(\bar{\omega}) = F(\bar{\omega})\frac{G'(\bar{\omega})}{\text{Leverage Premium}},$$

which is the optimality condition of equation (5). It can be shown mathematically that equation (5) pins down a negative relationship between R and $\bar{\omega}^*$. This means that, in response to a decrease in R, the marginal benefit of expanding the size of investment outweighs the marginal cost of running a higher default probability associated with the increase in leverage.

Numerical exercise

The graphical and mathematical representations developed the intuition behind the leverage effect, but they are not suitable for a discussion of the borrowing rate. In fact, it is possible to show numerically that the sign of the variation in R_b is not uniquely pinned down. To analyze the behaviour of R_b and of the spread on the risk free interest rate I use a simple numerical example. In both the numerical example below and the general equilibrium analysis in the next section the borrowing rate

[12] If this was not the case, the aggregate return R_k would be high enough to make the asymmetric information irrelevant because the lender would supply an infinite amount of credit for any level of $\bar{\omega}$.

decreases and the spread increases following a monetary expansion.

The calibration of the exercise is chosen to simplify the intuition as far as possible, leaving a full calibration of the model to Section 3. Net worth is normalized to 1, as well as the price of capital. The initial net opportunity cost of lending is set at 2% and the net return to capital at 4%, implying a net discounted return to capital of 1.96%. The variance of the idiosyncratic shock and the observation cost μ are calibrated to match an equilibrium leverage ratio of 2 and a default probability of 3%, obtaining $Var(\omega) = 0.3434$ and $\mu = 0.1655$. Note that the normalization of net worth to 1 implies that investment and leverage coincide.

Table 1: A decrease in the opportunity cost of lending from 2 % to 1 %

	Opp. cost of lending R	Borrower's leverage $K \equiv K/N$	Borrowing rate R_b	Spread (in bps) $R_b - R$	Default probability $\Phi(\bar{\omega}) \cdot 100$	Borrower's E(revenues) $F(\bar{\omega})R_k K$
Case A: *Initial equilibrium*	1.02	2	1.0278	78	3%	1.0556
Case B: *Constant leverage*	1.01	2 (**0**)	1.0172 (**-106**)	72 (-6)	2.80% (**-20**)	1.0659
Case C: *Intermediate case*	1.01	2.01 (**+50**)	1.0175 (**-103**)	75 (-3)	2.90% (**-10**)	1.0660
Case D: *Constant default*	1.01	2.02 (+100)	1.0178 (-100)	78 (0)	3% (0)	1.0661
Case Z: *Final equilibrium*	1.01	2.2057 (**+1034**)	1.0242 (**-35**)	142 (+64)	5.14% (**+214**)	1.0671

Notes: In the initial equilibrium the entrepreneur borrows 1 at 2.78%, he invests 2 and runs a 3% probability of default. When the opportunity cost of lending decreases from 2% to 1%, the entrepreneur could invest the same amount and enjoy a default probability of 2.80% as a consequence of the lower cost of borrowing (revenue effect). Instead, the entrepreneur increases his leverage from 2 to 2.2057 to invest more (leverage effect). This increase in leverage pushes up his default probability from 3% to 5.14% despite the overall decrease in the borrowing rate.

Given these parameter values, the entrepreneur and the lender sign a contract in which the entrepreneur borrows 1 unit from the lender at an interest rate of 2.78%, equivalent to a 78 basis point spread on the risk free rate of 2%. The entrepreneur invests 2 units. At the end of the period, if the realization of the shock is such that revenues exceed the gross repayment obligation of 1.0278, then the entrepreneur pays back his debt and keeps what is left as profits. Otherwise, the entrepreneur defaults and the lender obtains revenues, net of monitoring costs.

Table 1 compares optimal vs. suboptimal combinations of $K, \bar{\omega}, R_b$ when the opportunity cost of lending decreases from 2% to 1%, keeping all the other parameters fixed. To simplify the comparison, I report in parenthesis the basis point variations from the initial equilibrium.

Start from the initial equilibrium and decrease the opportunity cost of lending. Following the decrease in R, if the entrepreneur invests the same amount, then perfect competition reduces the cost of the loan by 106 basis points (case B). The lower cost of debt pushes down the default probability by 20 basis points, which is in fact reflected in the decrease in the spread on the risk-free rate from 78 to 72 basis points. This is the revenue effect.

A constant level of investment, though, is not optimal, because the reduction in the risk-free rate pushes up the net discounted return to capital from 1.96% to 2.97% (unreported in the table), making each unit of investment more productive in discounted expected terms. If the entrepreneur increases investment by 0.01, as in case C, then the increase in leverage is relatively weak and the default probability still decreases. Of course, the entrepreneur is not constrained to borrow only 0.01 more. If he borrows 0.02 more, case D, then his leverage ratio increases by just enough to offset the revenue effect, keeping the default probability unchanged. In the new equilibrium of case Z, instead, the default probability *increases* despite the ultimate decrease in the borrowing rate because the increase in leverage offsets the revenue effect. In the case considered in this numerical example, the reduction in the opportunity cost of lending from 2% to 1% leads the entrepreneur to increase his leverage from 2 to approximately 2.20, which pushes his default probability from 3% to 5.14% despite the decrease in the borrowing rate from 2.78% to 2.42%. The lender is indifferent to the new default probability because he prices the loan accordingly by demanding an increase in the spread on the risk-free rate from 78 to 142 basis points. The entrepreneur, instead, is better off by accepting the increase in defaults in exchange for an increase in investment, as shown by the increase in his expected profits (last column of table 1).

3 A New Keynesian extension of the model

The analysis of the previous section showed that, in the "costly state verification" model, the default probability of the borrower can be implicitly rewritten as a function of the discounted return to capital and the entrepreneur's leverage (equation (8) below).[13] A decrease in R reduces the borrowing cost (revenue effect), but since the entrepreneur reacts by increasing his leverage, the default probability ultimately increases (leverage effect).

$$\text{Prob(default)} = f\Big(\frac{R_k}{R}, \frac{K}{N}\Big), \quad \text{with} \quad \underbrace{f_1' < 0}_{\text{Revenue effect}} \quad \text{and} \quad \underbrace{f_2' > 0}_{\text{Leverage effect}} \quad . \tag{8}$$

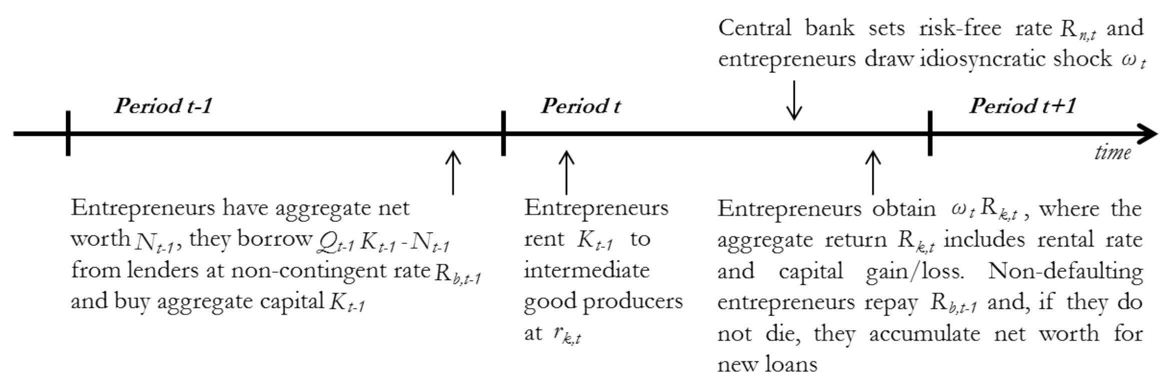
From equation (8), we see that whether the default probability still increases in general equilibrium depends on the behaviour of R_k and N and on their feedback effects on K. In this Section I address these issues using a New Keynesian model. Section 3.1 describes the full model, Section 3.2 shows the calibration, Section 3.3 discusses the results and the robustness of the results.

[13]Non linearities prevent from deriving equation (8) analytically.

3.1 The full model

The New Keynesian model used in this section draws from the model developed by Bernanke, Gertler and Gilchrist (1999) and from several papers related to their contribution. Since most of the features of the model are standard in the literature, I discuss them only briefly, unless they are key in driving the result. Christiano, Trabandt and Walentin (2011) provide an excellent description of New Keynesian models. An assessment of the costly state verification model in a general equilibrium framework can also be found in Christiano, Motto and Rostagno (2008), Hafstead and Smith (2012) and Dmitriev and Hoddenbagh (2013).

Figure 2: Timing of the full model



The model is populated by seven representative agents: households, lenders, capital producers, entrepreneurs, intermediate good producers, retailers and a central bank. The timing of their interaction is shown in figure 2. The basic structure is as follows. At the end of period $t-1$ entrepreneurs use net worth and loans to buy capital for the next period. At the beginning of period t, capital is rented out to intermediate good producers. Then, each entrepreneur draws an idiosyncratic shock for the return to capital and the central bank sets the risk-free nominal interest rate. At the end of period t, entrepreneurs either pay back their debt or default, depending on the realization of the idiosyncratic shock. A constant fraction of non-defaulting entrepreneurs retires. The entrepreneurs that do not default nor retire accumulate aggregate revenues into end-of-period net worth, take new loans and proceed to period $t+1$. The debt contract is written in real terms.

Households

Households are risk averse and derive utility from a basket of imperfectly substitutable consumption goods and from leisure. The instantaneous utility function is $\log(C_t) - \chi \frac{H_t^{1+1/\eta}}{1+1/\eta}$. Households are infinitely lived, they discount future utility by $\beta < 1$ and postpone consumption through the deposit services of lenders. These services take the form of deposits and pay the real risk-free rate R_{t-1} at period t.

Lenders

At time $t-1$, lenders collect deposits from households and provide loans to entrepreneurs at the non-contingent real borrowing rate $R_{b,t-1}$. Lenders remunerate deposits at the risk free rate R_{t-1}, as implied by four assumptions. First, that there is perfect competition among lenders in providing deposit services to households. Second, that lenders perfectly diversify the idiosyncratic shocks of entrepreneurs by providing loans to an infinite number of entrepreneurs. Third, that entrepreneurs are risk neutral and hence are willing to bear aggregate uncertainty without requiring compensation. Fourth, that households have the outside option of postponing consumption through the risk-free bond instead of through deposits.[14] Lenders make zero profits and do not consume.

Capital producers

Each period, capital producers buy all non-depreciated capital from entrepreneurs, invest in new units of capital and sell the new stock to entrepreneurs. The technology

[14] Since risk free bonds and deposits yield the same return, only the latter are modeled explicitly in the budget constraint of the households.

that produces new units of investment goods is subject to investment adjustment costs. These costs capture disruption costs, replacement of installed capital and costly learning. The adjustment cost function is taken from Christiano, Eichenbaum and Evans (2005), Christiano, Motto and Rostagno (2008) and Smets and Wouters (2003):

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{9}$$

The adjustment costs are zero in steady state and have constant second derivative, i.e. $S(1) = S(1)' = 0$, $S(1)'' = \nu > 0$. These costs equal $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\nu}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$.[15]

Entrepreneurs

Entrepreneurs are risk neutral. At time $t-1$ they own net worth accumulated from retained earnings and borrow from lenders to buy capital from capital producers. Capital is then rent out at time t to intermediate good producers in competitive markets.

The ex-post realization of the aggregate real return to capital $R_{k,t}$ is a function of the real rental rate on capital $r_{k,t}$ plus the capital gain or loss from non-depreciated capital:

$$R_t^K = \frac{r_t^k + (1-\delta)Q_t}{Q_{t-1}}. \tag{10}$$

As explained below in more detail, $R_{k,t}$ is affected by aggregate uncertainty because an unanticipated monetary policy shock to the nominal risk-free rate affects the investment decisions at time t and unexpectedly moves price Q_t. The ex-post return to capital that the entrepreneur actually receives is $\omega_t R_{k,t}$, i.e. the aggregate return

[15] The assumption of convex adjustment costs is justified by convenience rather than realism. Existing empirical evidence documents that investment decisions on micro data display lumpiness, periods of inactions and spikes that are inconsistent with the smooth behaviour of investment implied by convex adjustment costs. It appears, though, that the severity of these non-linearities is dampened by the aggregation, as argued by Caballero (1999) and Cooper and Haltiwanger (2006), and by general equilibrium effects, as argued by Veracierto (2002).

to capital $R_{k,t}$ after it has been hit by the idiosyncratic shock ω_t.[16]

Non-defaulting entrepreneurs obtain aggregate revenues V_t given by

$$V_t = F(\bar{\omega}_t) R_t^K Q_{t-1} K_{t-1}, \tag{11}$$

where, by the law of large numbers, $F(\bar{\omega})$ detects now the ex-post share of aggregate revenues to non-defaulting entrepreneurs.[17]

To avoid an accumulation of net worth that allows the entrepreneur to fully self-finance investment with retained earnings, an exogenous fraction $1-\gamma$ of non-defaulting entrepreneurs die and the same mass of entrepreneurs is born. This assumption is standard in the literature. Being risk-neutral, the entrepreneurs who remain in business allocate their revenues into net worth. Reborn entrepreneurs enter their first debt contract with a positive net worth that can be made arbitrarily small, as in De Fiore, Teles and Tristani (2011) and in Carlstrom, Fuerst and Paustian (2012). Entrepreneurial net worth at the end of period t is given by

$$N_t = \gamma V_t. \tag{12}$$

Intermediate good producers

Entrepreneurs rent capital to intermediate good producers. Intermediate good producers use capital and labor in the following Cobb-Douglas production function to produce intermediate goods.

$$Y_t = K_{t-1}^{\alpha} H_t^{1-\alpha}. \tag{13}$$

[16] Contrary to the partial equilibrium model, ω_t is not a structural shock to production, but enters the model as a shock to the market return to capital $R_{k,t}$. Christiano, Motto and Rostagno (2008) avoid this shortcoming of the Bernanke, Gertler and Gilchrist (1999) model by assuming that the idiosyncratic shock affects the share of the stock of capital that can be effectively used in production.

[17] Q enters equation (11) because equation (10) defines R_k as a ratio.

Retailers

Retailers buy intermediate goods, diversify them costlessly and sell them to consumers as final consumption goods. Retailers enjoy some price-setting power because households view final goods as imperfect substitutes. Under the assumption of Calvo price setting, prices are set as a mark-up over the weighted average of marginal costs over time. This nominal rigidity gives monetary policy real effects in the short term.

Central bank

The key driver of the model is the monetary policy shock in the Taylor rule. The monetary shock enters at time t by changing the risk-free nominal interest rate R_t^n between t and $t+1$. In doing so, it affects the opportunity cost of lending and the investment decisions starting from period t onwards. The central bank sets the policy rule according to the following Taylor rule:

$$\frac{R_t^n}{R_{ss}^n} = \Big(\frac{R_{n,t-1}}{R_{ss}^n}\Big)^{\rho}\Big[\Big(\frac{\Pi_t}{\Pi_{ss}}\Big)^{\gamma_\pi}\Big(\frac{Y_t}{Y_{ss}}\Big)^{\gamma_y}\Big]^{1-\rho} e^{\epsilon_t}. \tag{14}$$

Equation (14) is similar to the Taylor rule used by Faia and Monacelli (2007) and Schmitt-Grohé and Uribe (2007), as it features inflation and output in deviations from their value in steady state.[18] Bernanke, Gertler and Gilchrist (1999) use a similar Taylor rule, except that γ_y is set equal to zero.

An important remark is due before continuing. Debt contracts are signed each period based on the expectation of the return to capital in the following period. Agents know that the ex-post realization of the return to capital can differ from

[18]It differs from the Taylor rule in Faia and Monacelli (2007) because it does not include the price of assets, and it differs from the Taylor rule in Schmitt-Grohé and Uribe (2007) because it considers the response of the policy rate to contemporaneous instead of future levels of inflation and output.

its expected value because of the effect that policy shocks have on the capital gain on non-depreciated capital. Entrepreneurs are willing to bear this aggregate uncertainty due to their risk neutrality, implying that the borrowing rate $R_{b,t-1}$ is not a function of unexpected changes to $R_{k,t}$. The non-contingency of $R_{b,t-1}$ on either $R_{k,t}$ or ω_t implies that the default threshold $\bar{\omega}_t$ is a function of the aggregate shock, otherwise equation (2) would be violated for some values of $R_{k,t}$.[19]

This link between $\bar{\omega}_t$ and the ex-post realization of $R_{k,t}$ is crucial in the Bernanke, Gertler and Gilchrist (1999). Contracts are signed based on the expectation of the default probability. Whether the actual rate of default is below or above the expected default probability depends on whether the ex-post return to capital is respectively above or below its expected value. This effect is realistic and detects the fact that an unexpected increase in firms' revenues reduces their default rate because it increases the cash flow that can be used to meet the repayment obligation. As will be explained, an unexpected monetary expansion takes R_k above expectation and pushes up revenues, which on impact mitigate the default rate of outstanding loans.[20]

[19] The participation constraint of the lender is rewritten below for convenience:

$$[1-\Phi(\bar{\omega}_t)]R_{b,t-1} + \Phi(\bar{\omega}_t)(1-\mu)\frac{E_{t-1}(\omega_t \mid \omega_t < \bar{\omega}_t)R_{k,t}K_{t-1}}{Q_{t-1}K_{t-1} - N_{t-1}} \geq R_{n,t-1}.$$

In the partial equilibrium model, $R_{k,t}$ is known before the contract is signed and the lender obtains $R_{n,t-1}$ in expected value, but not ex-post, unless he diversifies. In the general equilibrium model, $R_{k,t}$ is stochastic and not known when the contract is signed. If $\bar{\omega}_t$ was not a function of the realization of $R_{k,t}$, an unexpected increase in $R_{k,t}$ would increase the return on lending above $R_{n,t-1}$, because it would increase the return on defaulting entrepreneurs (term $(1-\mu)\frac{E_{t-1}(\omega_t|\omega_t<\bar{\omega}_t)R_{k,t}K_{t-1}}{Q_{t-1}K_{t-1}-N_{t-1}}$ in the above equation), making the participation constraint not binding ex-post. In this model, it is assumed that the zero profit condition holds in any state of the world. $\bar{\omega}_t$ adjusts to $R_{k,t}$ to ensure that the ex-post return on lending equals the opportunity cost of lending. While the debt contract used in Section 2 is optimal, the general equilibrium extension of the contract used in this Section is not, as for instance discussed by Carlstrom, Fuerst and Paustian (2012). In Carlstrom, Fuerst and Paustian (2012) lenders write down a contract in which the borrowing rate $R_{b,t}$ is a function of the realization of $R_{k,t}$.

[20] The revenue effect in the general equilibrium model is partially different from the one in the partial equilibrium analysis from Section 2. While in both cases revenues increase, in partial equilibrium they increase due to the lower cost of borrowing, while in general equilibrium they increase due to the increase in aggregate demand. In both cases, it is the higher level of revenues that pushes down the default rate.

3.2 Calibration

I calibrate the model using observations on Spain, the country for which Jimenez *et al.* (2008) estimate duration models on defaults. The calibration is shown in table 2. The model is calibrated quarterly. For convenience, table 2 reports the equilibrium interest rate and the default probability in annualized terms. A wider range of parameter values is considered in Section 3.4 to assess the robustness of the results.

Table 2: Calibration of the full model

Description		**Parameter**	**Value**	**Target / Source**
Discount factor		β	0.9979	Interbank rate
Variance of idiosyncratic component e^{ω}		σ^2	0.0256	Spanish leverage and defaults
Observation cost for the lender		μ	0.0925	Spanish leverage and defaults
Probability that entrepreneur retires		$1-\gamma$	0.0139	Bernanke *et al.* (1999)
Weight on disutility of labour		χ	18.70	Bernanke *et al.* (1999)
Policy rate persistence		ρ	0.87	Sauer and Sturm (2007)
Weight on inflation in Taylor rule		γ_π	1.85	Sauer and Sturm (2007)
Weight on output in Taylor rule		γ_y	0.59	Sauer and Sturm (2007)
Investment adjustment cost		ν	3.60	Christiano *et al.* (2005)
Elasticity of substitution across varieties		ϵ	10	Chari *et al.* (2000)
Marginal product of capital		α	0.35	Bernanke *et al.* (1999)
Depreciation rate		δ	0.025	Bernanke *et al.* (1999)
Frisch elasticity of labour		η	3	Bernanke *et al.* (1999)
Probability of Calvo price optimization		$1-\psi$	0.25	Bernanke *et al.* (1999)
Description	**Moment**	**Model**	**Data**	**Source**
Annualized default rate	$\Phi(\bar{\omega})$	0.0292	0.0291	Jimenez *et al.* (2008)
Leverage ratio	K/N	3.03	3.0785	Kalemli-Ozcan *et al.* (2012)
Annualized risk free rate	R^n	0.0085	0.0085	Interbank rate
Discounted return to K	R_K/R	1.0049	1.0049	Bernanke *et al.* (1999)

Notes: the model is calibrated to match the leverage ratio and the default probability of Spanish firms, following the empirical results by Jimenez et al. *(2008). The rest of the parameters are calibrated from the literature. Alternative values of the calibration are considered in Section 3.3.*

Parameters calibrated from the data

The discount factor β is calibrated to ensure that the steady state risk-free rate R^n equals the quarterly average of the policy rate used by Jimenez *et al.* (2008). This rate is the German interbank rate between 1986 and 1998 and the European Overnight Index Average (EONIA) between 1999 and 2006. In the model, the nominal and the real interest rates coincide in the steady state, because in the steady state inflation is zero, as for instance in Curdia and Woodford (2009). To be consistent with this assumption, I calibrate β to match the observed real rather than the nominal value of the average interbank rate. This value is 0.85% annually, which equals 4.60% nominal interest rate net of 3.75% inflation. The implied quarterly discount factor is 0.9979. The relatively high value of this parameter reflects the very low level of the real interest rate in Spain. Section 3.3 shows that this non conventional value of β is not crucial for the result.

The idiosyncratic shock ω_t is assumed to be log normally distributed. The variance of ω_t and the observation cost μ are calibrated to match the default probability and the leverage ratio of Spanish firms. Jimenez *et al.* (2008) estimate an average default probability of 0.6% quarterly, or 2.4% annually, for the median loan one quarter after loan origination. I adjust this measure to 0.7291% quarterly, or 2.92% annually, to partially control for differences between the debt contract in the model and the median loan in Jimenez *et al.* (2008) (see Appendix A for the details).[21] A measure of the leverage ratio, instead, is not available from their paper. The leverage ratio of Spanish firms is taken from Kalemli-Ozcan, Sorensen and Yesiltas (2012), who find a median leverage ratio of listed and non-listed non-financial Spanish firms of 3.0785.[22]

[21] The model features non-collateralized, one-quarter-ahead loans issued by an intermediary at a fixed interest rate. In the dataset used by Jimenez *et al.* (2008), almost 85% of the loans have no collateral, the average maturity is 5 quarters, around 90% are issued by a commercial or saving bank and most have non adjustable rates. See also Jimenez and Saurina (2004) for a description of the data in the Spanish credit registry.

[22] The dataset assembled by Kalemli-Ozcan, Sorensen and Yesiltas (2012) covers the period between 2000 and 2009, which falls short of the period for which I calibrate the default probability of firms. Alternative empirical estimates are less suitable for the calibration because they cover a shorter period of time and focus on only one particular type of firms (Reverte (2009) studies firms listed in the IBEX35 index in years 2005 and 2006, Inchausti (1997) concentrates on firms listed on the Valencia Stock Exchange, Garcia-Teruel and Martinez-Solano (2008) focus on small and medium enterprises in the manufacturing sector, Ferreira and Vilela (2004) study publicly traded firms between 1987 and 2000). I use the median instead of the mean value because the latter (9.0443) is heavily biased upwards due to relatively few outliers. Spanish firms tend to have a higher leverage than European firms. In the dataset by Kalemli-Ozcan, Sorensen and Yesiltas (2012), 90% of the observations of the median leverage ratio of non-financial firms in the

The exogenous probability γ that entrepreneurs retire and the relative disutility of labour χ in the instantaneous utility function follow from the calibration to 200 basis points of the annual spread between the aggregate return to capital R_k and the risk free rate R. This measure is taken from Bernanke, Gertler and Gilchrist (1999).

The last parameters calibrated from the data regard the Taylor rule. The identification strategy by Jimenez *et al.* (2008) relies on the exogeneity of the policy rate. The authors defend this assumption by arguing that the policy rate is set by the European Central Bank with limited attention to Spanish-specific considerations. It is not possible to fully account for this assumption in a New Keynesian model, because an exogenous risk-free rate would leave inflation expectations undetermined, giving room for sunspot equilibria. The divergence between the model and the empirical estimation by Jimenez *et al.* (2008) is reduced using the following approach. I start from the empirical estimation of the Taylor rule of the European Central Bank computed by Sauer and Strum (2007). These estimates are 0.87 for the persistence parameter ρ, 1.85 for the coefficient γ_π attached to inflation and 0.59 for the coefficient γ_y attached to output. Then, to mimic the relative attention that the European Central Bank presumably gives to Spanish-specific business cycle considerations, I multiply the coefficients on inflation and output by a parameter $\lambda \in [0, 1]$, which reflects the relative role played by the Spanish business cycle in the determination of the common monetary policy by the European Central Bank. A high (low) value of λ implies a relatively endogenous (exogenous) response of the policy rate to the Spanish business cycle. It then remains to calibrate the parameter λ.

Calibrating λ requires an assessment of how close the Spanish business cycle is to the business cycle of the monetary union. The literature on the synchroniza-

European Monetary Union range between 2.29 and 3.34, with the distribution skewed to the left (the skewness equals 1.1535). I am grateful to the authors for having shared these statistics.

tion of business cycles does not reach a consensus on this point, since the results strongly depend on the sample, the methodology, the data and the data transformation (see for instance the discussion by De Haan, Inklaar and Jong-A-Pin (2008) and by Giannone, Lenza and Reichlin (2008)). In this paper I calibrate λ using the analysis developed by Camacho, Perez-Quiros and Saiz (2008), who move beyond the assessment of business cycle syncronization across countries and compare the duration, amplitude and shape of the business cycles of European and non European countries. Their analysis shows that the Spanish business cycle is not significantly dissimilar from the business cycle of the monetary union, as commented on extensively in Appendix B. For this reason I calibrate λ to 0.9, which implies a baseline calibration of (γ_π, γ_y) is 0.9*(1.85,0.59) = (1.665, 0.531). This approach is of course an approximation, but it serves the purpose of the analysis of the paper.[23] Alternative calibrations of the Taylor rule will be considered in the analysis of the results.

Parameters calibrated from the literature

The remaining parameters are set according to the literature.

The parameter ν stands for the second derivative of the adjustment cost function of investment. ν and β pin down the elasticity of investment to a temporary and a permanent increase in the price of installed capital, which equal $\frac{1}{\nu}$ and $\frac{1}{\nu\cdot(1-\beta)}$, respectively (see Christiano, Eichenbaum and Evans (2005)). By construction, ν does not affect the steady state of the model, although it does play a role in the dynamics outside the steady state. The value of ν is taken from Christiano, Eichenbaum and Evans (2005) and implies an elasticity of 0.28 to a temporary increase in the price of installed capital. I use this value as a lower bound in the calibration, because

[23] An equally approximate approach has been used in the literature to estimate the Taylor rules for the European central bank in its early years of operation or to estimate the hypothetical Taylor rule of the European Central Bank before it started its operations in 1999. See the discussion on this point in Sauer and Sturm (2007).

the high value of β in this paper implies an elasticity of investment to a permanent increase in the price of installed capital of 131%. Robustness checks consider values of ν up to the level required to generate an elasticity of 38%, which is the value implicit in the calibration by Christiano, Eichenbaum and Evans (2005).

The elasticity of substitution across varieties of consumption goods ϵ is set equal to 10. This value is based on Basu (1996) and Basu and Kimball (1997) and is also used by Chari, Kehoe and McGrattan (2000). The literature tends to use lower values. Faia and Monacelli (2006) use 8, Schmitt-Grohé and Uribe (2007) use 5. I consider all these values for robustness.

The Frisch elasticity of labour supply η is set equal to 3, as in Bernanke, Gertler and Gilchrist (1999). This value is consistent with Cho and Cooley (1994) and King and Rebelo (1999), who report Frisch elasticities that range from 2.6 to 4. This is the range of values that I also consider in the robustness checks.[24]

As in Bernanke, Gertler and Gilchrist (1999) and Christiano, Trabandt and Walentin (2011), the probability $1-\psi$ that the retailer optimizes his price is set equal to 0.25, implying an average period of price adjustment of 4 quarters. I will also consider values up to 0.5, implying an average price duration of half a year.

The parameter α for the marginal productivity of capital is set equal to 0.35. There is a general agreement on the plausible set of values for this parameter, at least in models with both capital and labour in the production (see Covas and Den Haan (2012), note 29).

The depreciation rate δ is set to 0.025, corresponding to a 10 % annual rate. There is a general agreement on the value of δ, at least within this class of models.

[24] Empirical estimates of the Frisch elasticity on macroeconomic data tend to exceed estimates based on microeconomic data, which are usually below 1. See Christiano, Trabandt and Walentin (2011) for an explanation of this divergence and Reichling and Whalen (2012) for a survey of different empirical estimates of the elasticity of labour supply.

3.3 Results

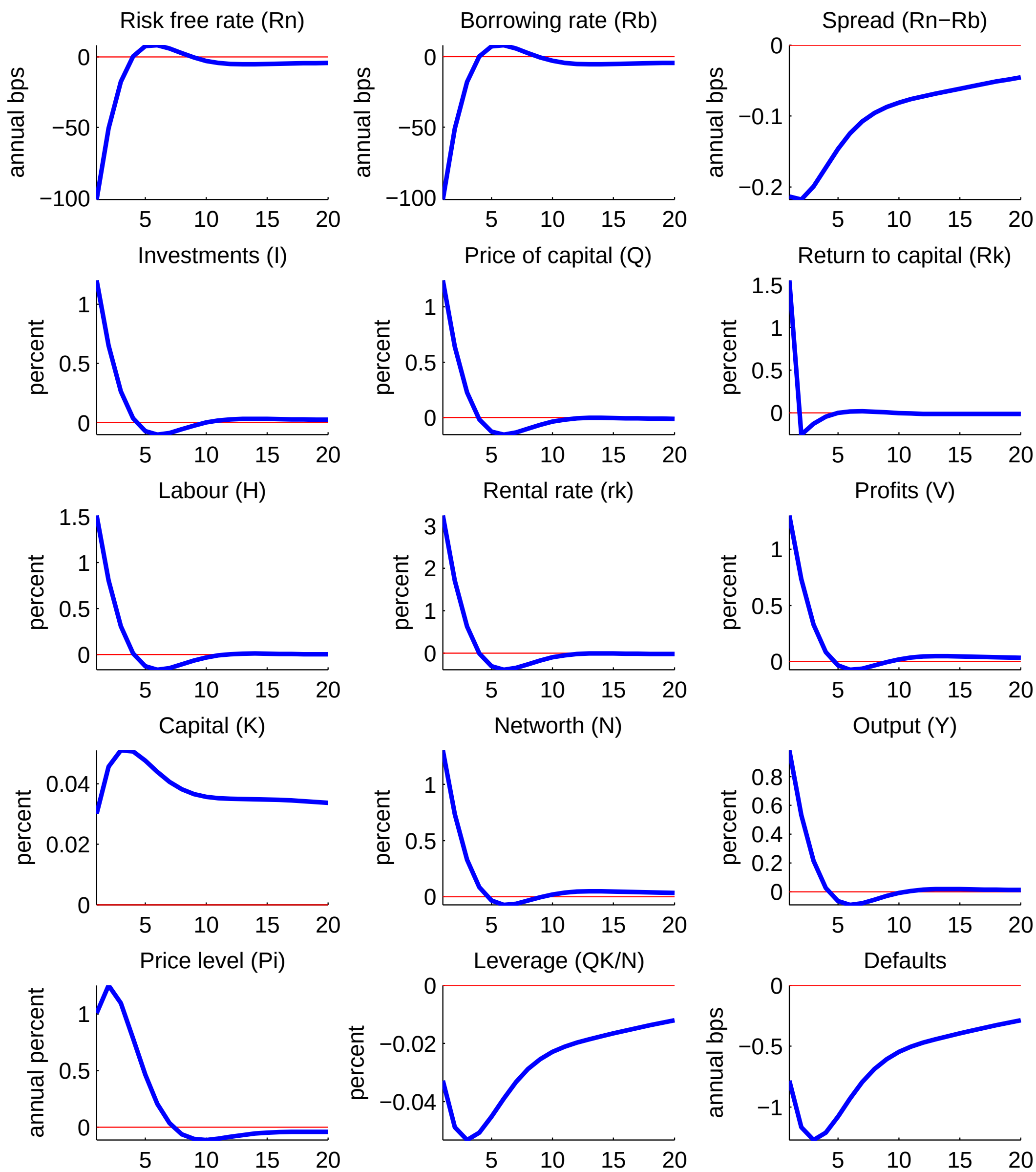
Figure 3 shows the effects of a 100 annual basis point decrease in the nominal interest rate R_t^n generated by a policy shock ϵ_t in the Taylor rule. The horizontal axis represents time, which is expressed in quarters. Variables are displayed in either basis point or percentage point deviations from the steady state, as specified for each variable. Interest rates, inflation and the default rate are reported in annualized terms.

The 100 basis point decrease in R^n pushes down the annualized borrowing rate by just as much, effectively leaving the spread unchanged (the decrease is smaller than one basis point).[25] The decrease in the borrowing rate increases end-of-period investment by 1.2% and pushes up the price of capital by 1.24%. This is because capital producers anticipate that entrepreneurs will demand more capital starting from the end of the period. The unanticipated increase in the price of capital unexpectedly increases the return to capital R_k above steady state by 1.5%. This increase is mainly driven by the capital gain on non-depreciated capital, but it also reflects a 3.25% increase in the rental rate of capital and a 1.4% increase in labour. Capital starts to increase, although the increase is small.

The unexpected increase in the return to capital pushes up entrepreneurs' revenues and net worth by 1.3%. This decreases on impact the annualized default probability, although the effect is very small and equals less than a basis point. On impact, the market value of leverage QK/N decreases because the unexpected increase in net worth dominates the increase in the price of capital. Output increases by almost 1% and inflationary pressures arise from the first period onwards.

[25] In the model by Curdia and Woodford (2009) the spread increases, reflecting the cost of intermediation. The increase in the spread in their model reflects the assumption of convex intermediation costs and is lost when this cost is assumed proportional. The empirical evidence on the behaviour of defaults is mixed. Eickmeier and Hofmann (2012) find that a monetary policy expansion decreases the spread on commercial paper and on commercial and industrial loans, increases the spread on consumer loans and does not affect significantly the Moody's corporate bond spread, i.e. the spread of BAA corporate yield over AAA corporate yield.

Figure 3: Expansionary monetary policy shock of 100 basis points annually



Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

From the second period the nominal interest rate reverts to the mean. The accumulation of capital would push the leverage ratio up, but the higher level of net worth dominates, leading to a further decrease in the leverage ratio. The default rate follows the leverage ratio, although the effect on defaults remains quantitatively small, being equal to just a basis point 3 quarters from the shock. As the leverage ratio reverts to the mean, the default rate reverts to steady state. Note also that the ex-post return to capital decreases below steady state after the first period due to the capital loss on non-depreciated capital when Q_t decreases towards steady state. Capital remains above steady state for much longer than output, a fact consistent with the very small increase in capital, the low value of α in the production function and the temporary decrease in labour 5 quarters after the shock.

Figure 3 ultimately shows that the leverage effect discussed in Section 2 does not hold in the general equilibrium model used in the paper. The reason being that, in general equilibrium, the decrease in the cost of borrowing pushes up profits, which push the default rate down. Put it differently, it is true that firms react to the lower cost of borrowing by borrowing more and leveraging up net worth, but since profits increase in general equilibrium due to the increase in the price of capital, leverage ultimately decreases, pushing down defaults. In the rest of this section I argue that the result is robust to alternative calibrations of the model. For simplicity, the figures corresponding to robustness checks are reported at the end of the paper. In all cases considered I give a monetary shock such that the policy rate decreases by 100 basis points. I find that defaults always decrease, i.e. it was not possible to find a parametrization of the model that replicates qualitatively in general equilibrium the partial equilibrium result from Section 2. The order of magnitude of the decrease in defaults was always very small and equal to only a few basis points. This implies that defaults were effectively constant.

The model was first recalibrated using alternative values of the structural parameters $\{\rho, \lambda, \nu, \epsilon, \alpha, \delta, \eta, \psi\}$, keeping constant the calibration necessary to match the empirical moments explained in Section 3.2. Figure 6 shows that the higher the reduction in the parameters on inflation and output in the Taylor rule, the higher the decrease in the borrowing rate, hence the higher the increase in profits, the decrease in leverage and ultimately the decrease in defaults. Figure 8 shows that the more frequently retailers reoptimize prices, the sooner the nominal risk free rate reverts to the mean and the less profits increase, implying a smaller decrease in leverage and defaults. No particular change in the results was found when considering alternative values of the other parameters.

The model was then recalibrated around higher values of the steady state risk-free rate, which was remarkably low in the baseline calibration due to the relatively high value of inflation in Spain (see Section 3.2). The new value considered is 4% annually, implying a value of β of 0.99 (figure 9). Similarly, the model was recalibrated with a default probability of 1% and 4% annually (figure 11) and a value of the leverage ratio of 2 and of 4 (figure 10). Overall, I found that defaults decrease more the higher the risk-free rate, the higher the steady state level of defaults, and the lower the steady state leverage ratio. The deviations from steady state remain nevertheless very small.

Last, I studied if the persistence in the policy rate affects the variation in defaults. To do so, I generated a series of subsequent monetary shocks that replicate the policy rate generated by a single and persistent monetary shock. The exercise is designed to study the role of the expectation that entrepreneurs have regarding future variations in the policy rate. In the case of no persistence, agents are continually surprised by monetary shocks, while in the case with persistence they know the time path of the risk free rate from the second period onwards. In general, with a positive persistence parameter in the Taylor rule, agents anticipate that the policy rate remains low for more than one period. Since the Euler equation prescribes an increase in consumption for many periods to come, entrepreneurs increase investment in order to satisfy the high demand from intermediate good producers. On the contrary, when the persistence in the policy rate is zero, agents do not anticipate a strong increase in consumption and investment increases by less. The results of the exercise are shown in figure 12. Compared to the baseline model of $\rho = 0.87$, a zero persistence effectively decreases the spread by less, hence increases investment by less. Since this leads to a milder increase in the price of capital, the increase in profits is lower, implying a smaller decrease in defaults.

4 Conclusions

This paper argues that the default probability on loans by firms effectively remains unchanged after an expansionary monetary shock. Jimenez *et al.* (2008) show that a monetary expansion leads to a lower default probability on existing loans and to a higher default probability on new loans. Their result holds in partial equilibrium. I have argued that a leverage effect on the side of firms might contribute to explain their result on new loans. I have then used the Bernanke, Gertler and Gilchrist (1999) model to argue that this increase might not hold in general equilibrium. In the model, this happens because the monetary expansion increases firms' profits, which in turn push up their net worth, decrease leverage and ultimately decrease the default rate. The effect on defaults equals only a few basis points, implying that defaults effectively remain constant. The result is consistent with the general equilibrium analysis in Piffer (2014), where I argue that one cannot reject the hypothesis that delinquency rates on business loans in the US respond to a monetary shock.

The model used in this paper features ex ante homogeneous borrowers. Whether defaults increase or decrease after a monetary expansion depends on how the leverage ratio of firms changes. An interesting extension of the model would be to allow borrowers to draw idiosyncratic shocks from distributions with different variance and allow for monetary policy to affect the allocation of credit across risky versus safe borrowers. The mechanism would be complementary to the leverage effect discussed in the paper, and could potentially yield interesting results in both partial and general equilibrium. The analysis is left for future research.

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Appendix A: Calibrating the default probability using duration models

This appendix gives a short introduction to duration models and explains how the empirical results by Jimenez *et al.* (2008) are used in the calibration of the model. The interested reader is referred to Kleinbaum and Klein (2005) and Keifer (1988) for a more comprehensive analysis of duration analysis.

Duration analysis on default rates use ex-post information on the actual time to default to infer the stochastic process generating the observed switching from non-default to default.[26] Define $f(t)$ the probability density function determining the time to default, where t represents the time after loan origination. From $f(t)$ and from the corresponding cumulative distribution function $F(t)$, compute the surviving function $S(t) = 1-F(t)$, which gives the probability that the loan survives until time t. Having a dataset on loans until time T, assume the functional form of $f(t)$ up to a parameter set and construct the log likelihood as the combination of densities and cumulative densities, respectively for observations of loans defaulted before T and for observations of loans still outstanding at time T or that have been repaid. Last, compute the parameters that maximize the likelihood that the default patter is the observed one.

It is convenient to synthesize the information content of $f(t)$ into the hazard function $h(t)$, which detects the instantaneous rate of default at time t given that the loan has not defaulted yet (alternatively, the conditional rate of default per time unit in an infinitesimally small interval). Formally, $h(t)$ is defined as

$$h(t_0) = \lim_{\epsilon \to 0} \frac{P(t_0 < t < t_0 + \epsilon \mid t \geq t_0)}{\epsilon}.$$

[26] The switching from one state to another is studied extensively also in other branches of the Social Sciences, as for instance Medicine and Epidemiology.

$h(t_0)$ detects approximately $P(t_0 < t < t_0+\epsilon \mid t \geq t_0)$, i.e. the probability of default between time t_0 and $t_0+\epsilon$ conditioning on the fact that the default has not occurred yet, with ϵ being the approximating factor:[27]

$$P(t_0 < t < t_0 + \epsilon \mid t \geq t_0) \approx h(t_0)\epsilon.$$

Duration analysis uses heuristic data inspection to infer a realistic hazard rate (usually the exponential or the Weibull distribution), computes the corresponding $f(t)$, constructs the likelihood function and solves the problem parametrically.[28]

It is common in the literature to study the role of other variables (or covariates) by assuming that they affect the hazard rate in the following convenient form:

$$h(t_0) = h_0(t_0) \cdot e^{X_t \beta},$$

where $h_0(t_0)$ stands for the baseline hazard function. A positive (negative) estimate of β_j implies that an increase in the covariate shifts the entire hazard function proportionally up (down).

Following the above strategy, Jimenez *et al* (2008) estimate the hazard function for loans in Spain from 1985 to 2006, obtaining [29]

[27] One can rewrite $h(t_0)$ as the ratio between the probability density function and the surviving function, or equivalently, as the rate of decrease of the surviving function:

$$h(t_0) = \lim_{\epsilon \to 0} \frac{F(t_0+\epsilon) - F(t_0)}{\epsilon} \frac{1}{1-F(t_0)} = \frac{f(t)}{S(t)} = -\frac{dlogS(t)}{dt}.$$

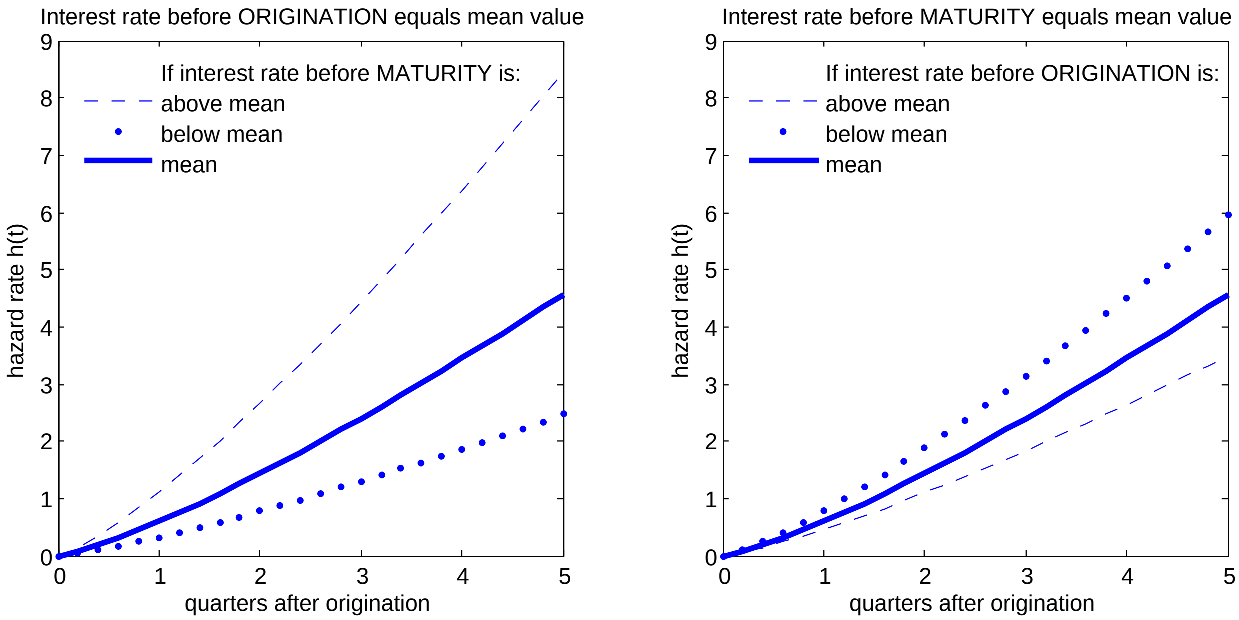
From this it follows that $P(t_0 < t < t_0 + \epsilon \mid t \geq t_0)$ can be rewritten as

$$\int_{t_0}^{t_0+\epsilon} \frac{f(t)}{1-F(t_0)} dt = \frac{F(t_0+\epsilon) - F(t_0)}{1-F(t_0)} \approx \frac{F(t_0) - F(t_0) + F'(t_0)\epsilon}{1-F(t_0)} = \frac{f(t_0)}{1-F(t_0)}\epsilon = h(t_0)\epsilon.$$

[28] Given $h(s)$, the corresponding cumulative distribution function is $F(s) = 1 - e^{-\int_0^t h(s)ds}$. Cox (1972) proposes a partial likelihood estimation approach that exploits the *ordering* in which the switch from a state to another, not the exact time. The result by Jimenez *et al.* (2008) is robust to this alternative estimation strategy.

[29] The computation of equation (15) from Jimenez *et al.* (2008) exploits the information from their paper that the hazard rate equals 0.6 for average interest rate before and after the origination, given that some estimates included in the regression are not reported in their paper.

Figure 4: Estimated hazards rate in Spain, 1985 and 2006



Notes: The default probability of Spanish firms estimated by Jimenez et al. *(2008) increases as the loan approaches maturity, it increases in the policy rate before maturity and decreases in the policy rate before origination.*

$$h(t) = 2.2614 \cdot t^{2.2614-1} e^{-0.127 \cdot i_{before} + 0.293 \cdot i_{after} - 2.0074}. \quad (15)$$

i_{before} and i_{after} are the key variables of the regression and represent the monetary policy interest rate on the quarter before the loan is originated and the monetary policy interest rate after origination and before the loan comes to maturity, respectively. The term -2.0074 includes the estimated effect of the covariates evaluated at the median value. The regression results suggest that, other things equal, a 1 percentage point increase in the interest rate before the loan was issued decreases the hazard rate by 0.127 percent along the entire life-time of the loan. Instead, a 1 percentage point increase in the interest rate after the loan was issued and before it comes to maturity increases the hazard rate by 0.293 percent along the entire life-time of the loan.[30]

[30] To argue that this shift in risk profiles is intentionally taken by banks, Jimenez *et al.* (2008)

The result is robust to alternative estimation strategies and is shown graphically in figure 4.[31] Both graphs represent the instantaneous rate of default for the median loan with 5 quarters maturity for each point in time between origination and maturity. The continuous lines show the case in which the interest rates before and after origination are equal to the mean value of 4.1%. The left graphs shows the case of loans originated when the policy rate is at mean value and that reached maturity when it is at its sample maximum (dashed line) or sample minimum (dotted line). Conversely, the right graph shows the case of loans for which the policy interest rate is equal to its mean value before maturity and that were originated when the policy rate was at its sample maximum (dashed line) or at its sample minimum (dotted line). We see from figure 4 that a decrease in the policy rate after the loan is originated decreases the probability of default, while a decrease in the policy rate before the loan is originated increases the probability of default.

Equation (15) includes the entire dataset and hence detects the hazard rate of the median loan. As emphasized in Section 3.3, loans in the model differ from the median loan from the Spanish credit registry for being non-collateralized and for having one quarter maturity. To control for this difference, I adjust the corresponding dummy variables in the term -2.0074 in equation (15). The corresponding hazard function is

$$h(t) = 2.2614 \cdot t^{2.2614-1} e^{-0.127 \cdot i_{before} + 0.293 \cdot i_{after} - 1.8125}. \qquad (16)$$

Under this alternative parametrization of the hazard function, the default rate of

take the estimated probability density function of defaults as a proxy of the expectation that loan officers attach to defaults of different loan applications. In this paper I only interpret their result as suggestive evidence of an equilibrium phenomenon rather than as an effect intentionally driven by banks' behaviour.

[31] All other estimations considered by the authors deliver a negative sign for the coefficient on i_{before} and a positive sign for the coefficient on i_{after}. The magnitude of the estimates for the first coefficient are between -0.0052 and 0.127 and for the second coefficient between 0.044 and 0.350, always remaining significantly different from zero at 1% significance.

a loan with median interest rate of 4.1% both before origination and before maturity goes from 0.60 to 0.7291 quarterly, or from 2.4% to 2.92% annually. This is the value used in the calibration

Equation (16) can be used to compare the effect on the default rate predicted by the model and found empirically by Jimenez *et al.* (2008). It has been said that the default rate of an adjusted median loan when the policy rate is constant at the median interest rate of 4.1% is 2.92%. Equation (16) implies that when the interest rate decreases temporarily by 100 basis points, then the default rate of existing loans decreases to 2.18% and the default rate of new loans increases to 3.31%.[32]

A similar analysis is carried out by Ioannidou *et al.* (2009) for Bolivia and by Lopez *et al.* (2011) for Colombia. In Bolivia, when the policy rate (i.e. the federal funds rate) equals its mean before origination and before maturity (4.25%) the default rate of loans is 1.84%. A 1% decrease in the policy rate decreases the default rate on existing loans approximately to 1.51% and increases the default rate on new loans approximately to 2.14%. In Colombia, when the policy rate (i.e. the interbank rate) equals its mean (2.18%) the default rate of loans is 1.69%. A 1% decrease in the policy rate decreases the default rate on existing loans approximately to 1.47% and increases the default rate on new loans approximately to 1.71%. This means that the result by Jimenez *et al.* (2008) is robust when using data on Bolivia, but is bigger than what is found for Columbia.

[32]Without the adjustment for non collateralized, one quarter maturity loans the annual default rate of 2.4% decreases to 1.8% for existing loans and increases to 2.72% for new loans, making the model marginally more successful to match both variations.

Appendix B: Calibrating the Taylor rule

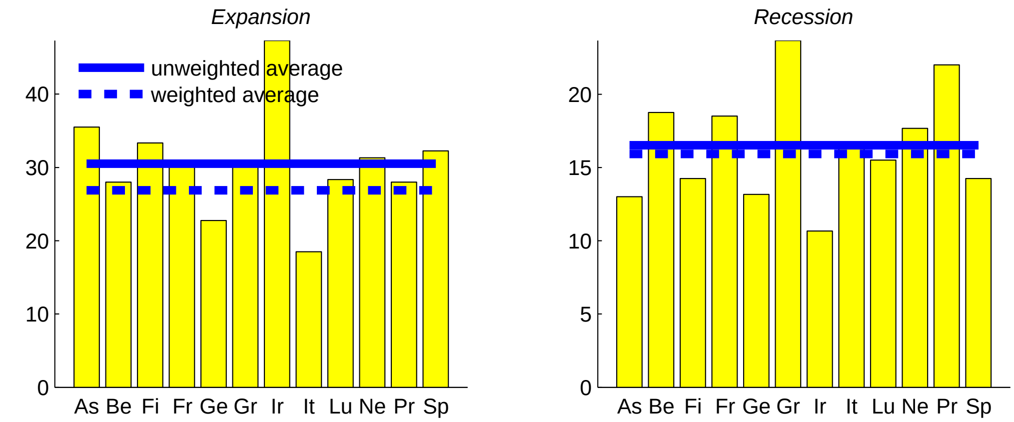
The calibration of the Taylor rule requires the assessment of how similar the Spanish business cycle is from the business cycle of the Eurozone as a whole. This assessment then leads to the calibration of the parameter λ in Section 3.2. To do so I use the analysis by Camacho, Perez-Quiros and Saiz (2008). They analyze different properties of the business cycle of several countries. Since the default rate is calibrated on data up to 2006, I only compare the Spanish business cycle to the one of the countries that joined the Eurozone before 2006. The business cycle is studied using the level of the seasonally adjusted monthly observations of the industrial production index.

Three features of business cycles are studied by Camacho *et al.* (2008): the duration, the amplitude and the shape. The duration is defined as the average number of months between two consecutive turning points of the cycle. The amplitude is defined as the average percentage variation of industrial production between two consecutive turning points of the cycle. The shape is defined as the divergence of the average business cycle from a linear trajectory between two consecutive turning points of the cycle. A positive value of the shape of a business cycle is associated with a convex cycle, i.e. a cycle that begins slowly and then accelerates, if expansionary, or that begins rapidly and then decelerates, if recessionary. Similarly, a negative value of the shape of a business cycle is associated with a concave cycle, i.e. a cycle that begins rapidly and then decelerates, if expansionary, or that begins slowly and then accelerates, if recessionary.

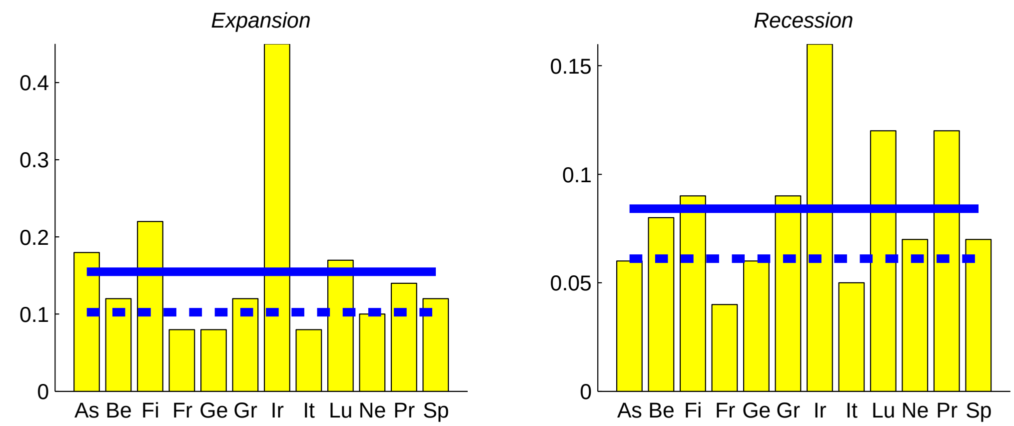
Figure 5 reports the results by Camacho *et al.* (2008), table 3, for the countries of interest for the calibration in this paper. The horizontal solid and dashed lines show the unweighted and weighted average, respectively. The weights used are computed from the relative size of real GDP in each country, relative to the real GDP of the Eurozone. Overall, we see that, relative to the weighted Eurozone, the

Figure 5: Comparison of the business cycle in the euro-zone, 1990M1-2004M3

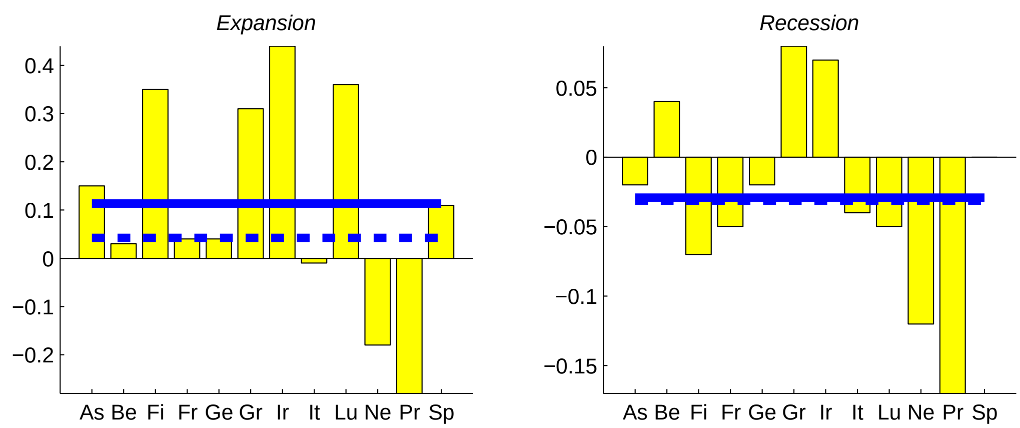
a) Duration (months)



b) Amplitude (percent)



c) Shape



Notes: the figure replicates table 3 in Camancho et al. *(2008). The horizontal line shows the unweighted average for the euro zone. The duration is computed as the average number of months between two consecutive turning points in the business cycle. The amplitude is computed as the average percentage difference between two consecutive turning points in the business cycle. The shape is computed as a normalized measure of how non linear the average path between two consecutive turning points in the business cycle:* > 0*: convex,* < 0*: concave,* 0*: linear.*

Spanish business cycle tends to have longer expansions and shorter recessions, wider cycles, more convex expansions and more linear recessions.

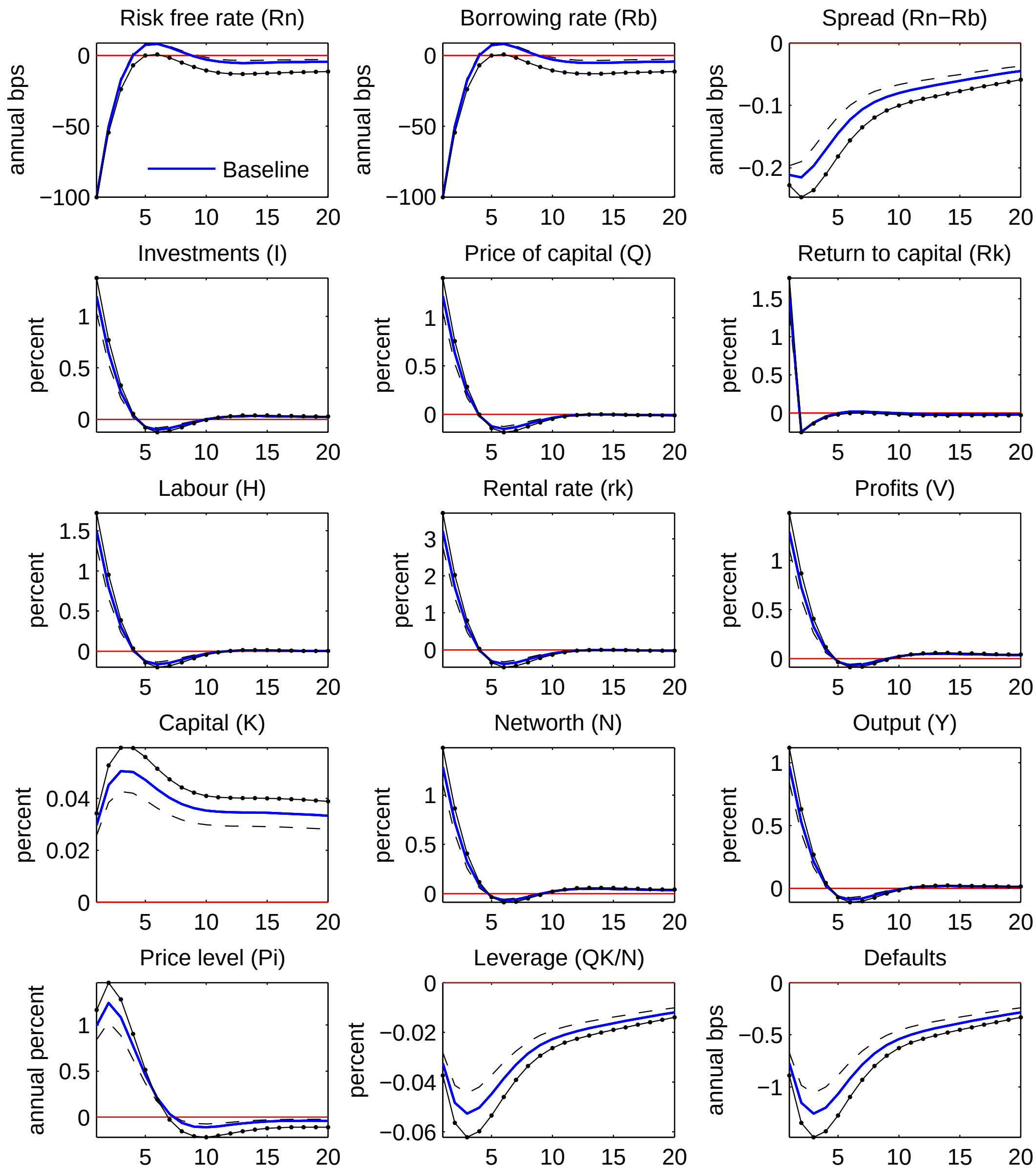
Table 3: Difference from Eurozone business cycle (1 = most similar)

	Unweighted average			**Weighted average**		
country	duration	amplitude	shape	duration	amplitude	shape
Austria	7	3	2	10	8	6
Belgium	5	1	6	2	5	4
Finland	6	8	8	7	11	9
France	2	11	4	4	6	2
Germany	11	9	3	6	2	1
Greece	10	2	9	11	7	10
Ireland	12	12	11	12	12	12
Italy	8	10	5	8	4	3
Luxemburg	3	5	7	1	10	8
Netherlands	1	7	10	3	1	7
Portugal	9	6	12	9	9	11
Spain	4	4	1	5	3	5

Notes: the table lists the countries in terms of how different they are from the weighted and unweighted Eurozone average, where weights refer to real GDP of the different countries. Countries that are closer to the Eurozone average are ranked first. The table builds from the results in Table 3 of Camacho et al. *(2008).*

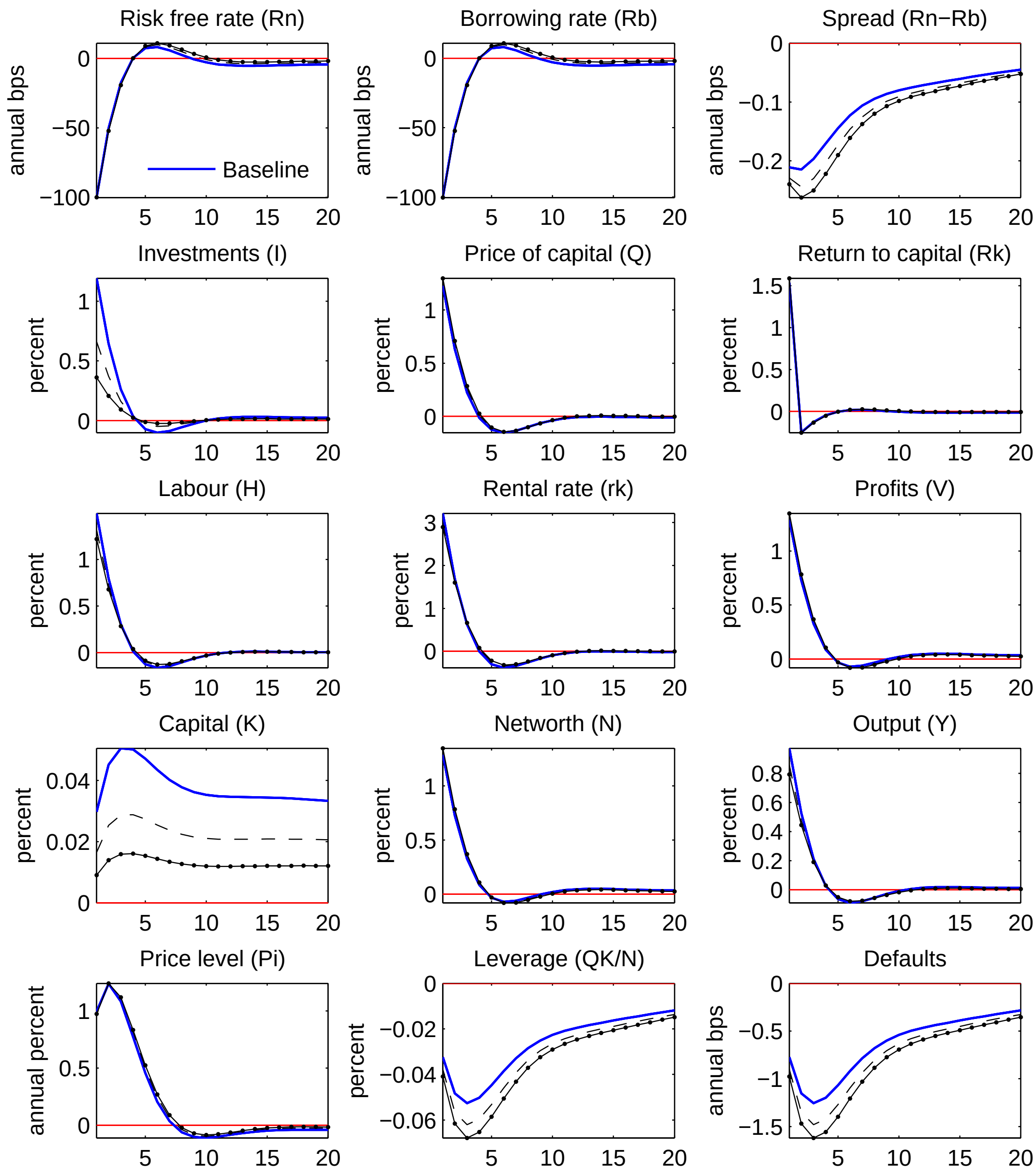
It is not clear *a priori* how one should judge the relative similarity of the typical Spanish business cycle to the Eurozone business cycle. A quantitative assessment would require the assessment of relative weights to be attached to the different features of the business cycle. In this paper I follow a qualitative approach, which is less affected by the judgment of the researcher. Table 3 builds on the data from figure 5 by ordering the countries in the Eurozone according to how different they are from the Eurozone average. A value of 1 (12) is associated with a country that is the most similar (the most dissimilar) to the Eurozone average. The country that has an overall average duration of the business cycle closest to the unweighted Eurozone average are the Netherlands, followed by France and Luxembourg. The country that has average amplitude of the business cycle closest to the unweighted Eurozone average is Belgium, followed by Greece and Austria. Overall, we see that Spain ranks only fourth according to the duration and amplitude and first according to shape, when considering the unweighted Eurozone average. Similarly, Spain ranks fifth according to duration and shape, and third according to amplitude, when considering the weighted Eurozone average. Overall, the analysis implies that the Spanish business cycle is not particularly dissimilar from the business cycle of the Eurozone as a whole. For this reason I calibrate the parameter λ in Section 3.2 to an arbitrary 0.90, i.e. I decrease the parameter values estimated by Sauer and Strum (2007) by 10%. Alternative values are used to assess the robustness of the results.

Figure 6: Robustness checks (solid is $\lambda = 0.8$, dashed is $\lambda = 1$, solid-dot is $\lambda = 0.6$)



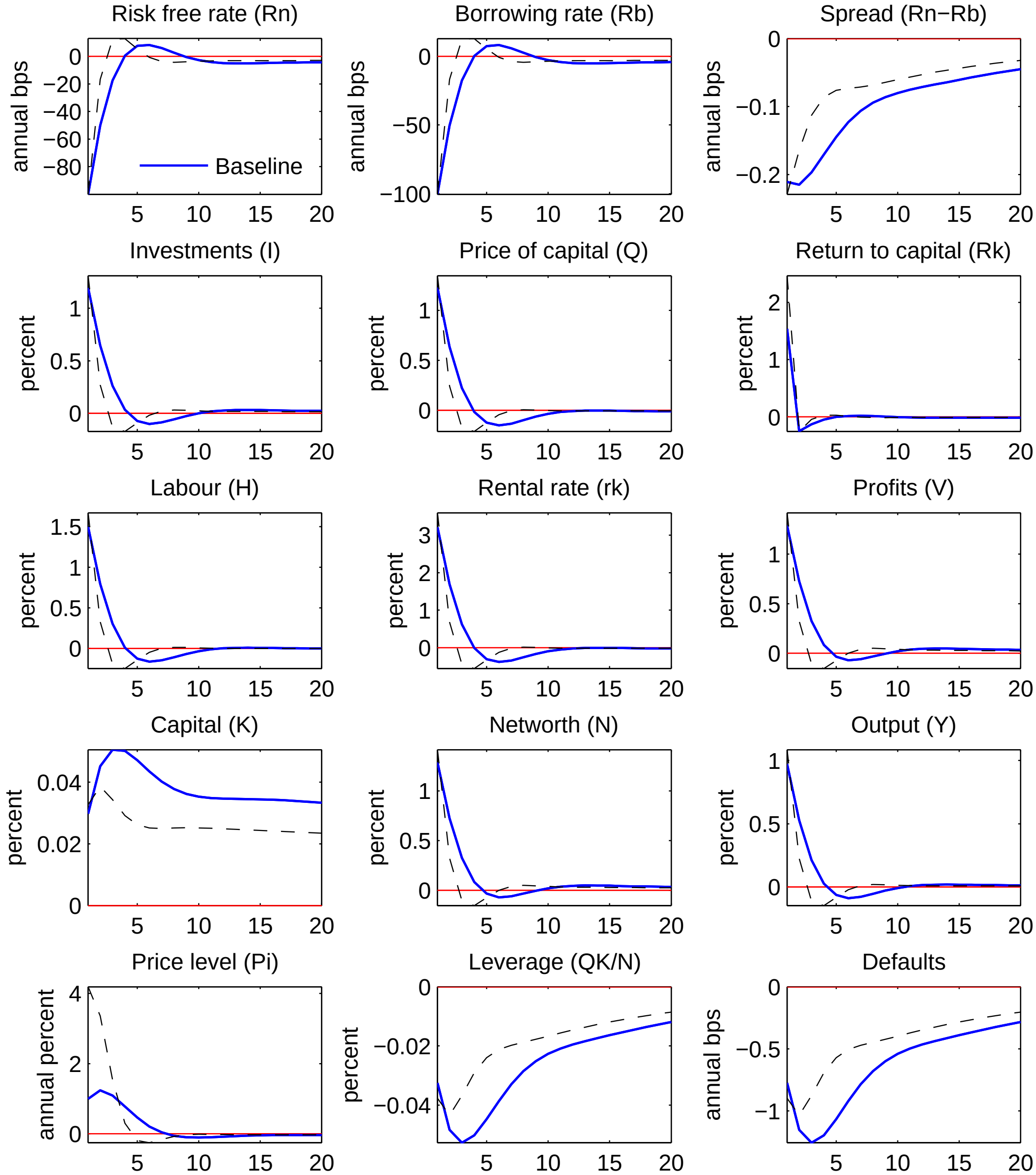
Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 7: Robustness checks (solid is ν =?, dashed is ν =?, solid-dot is ν =?)



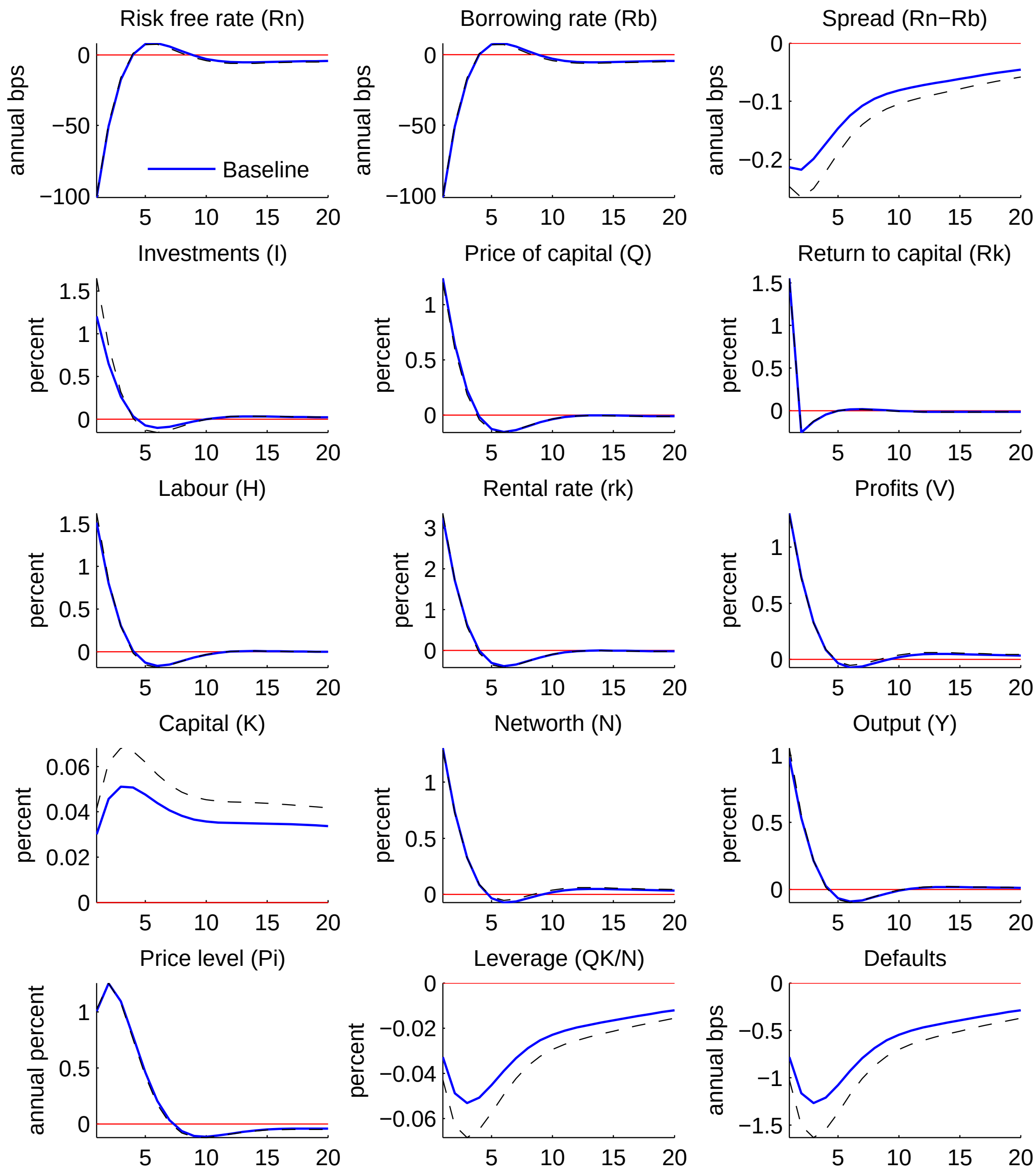
Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 8: Robustness checks (solid is $1-\psi = 0.25$, dashed is $1-\psi = 0.5$)



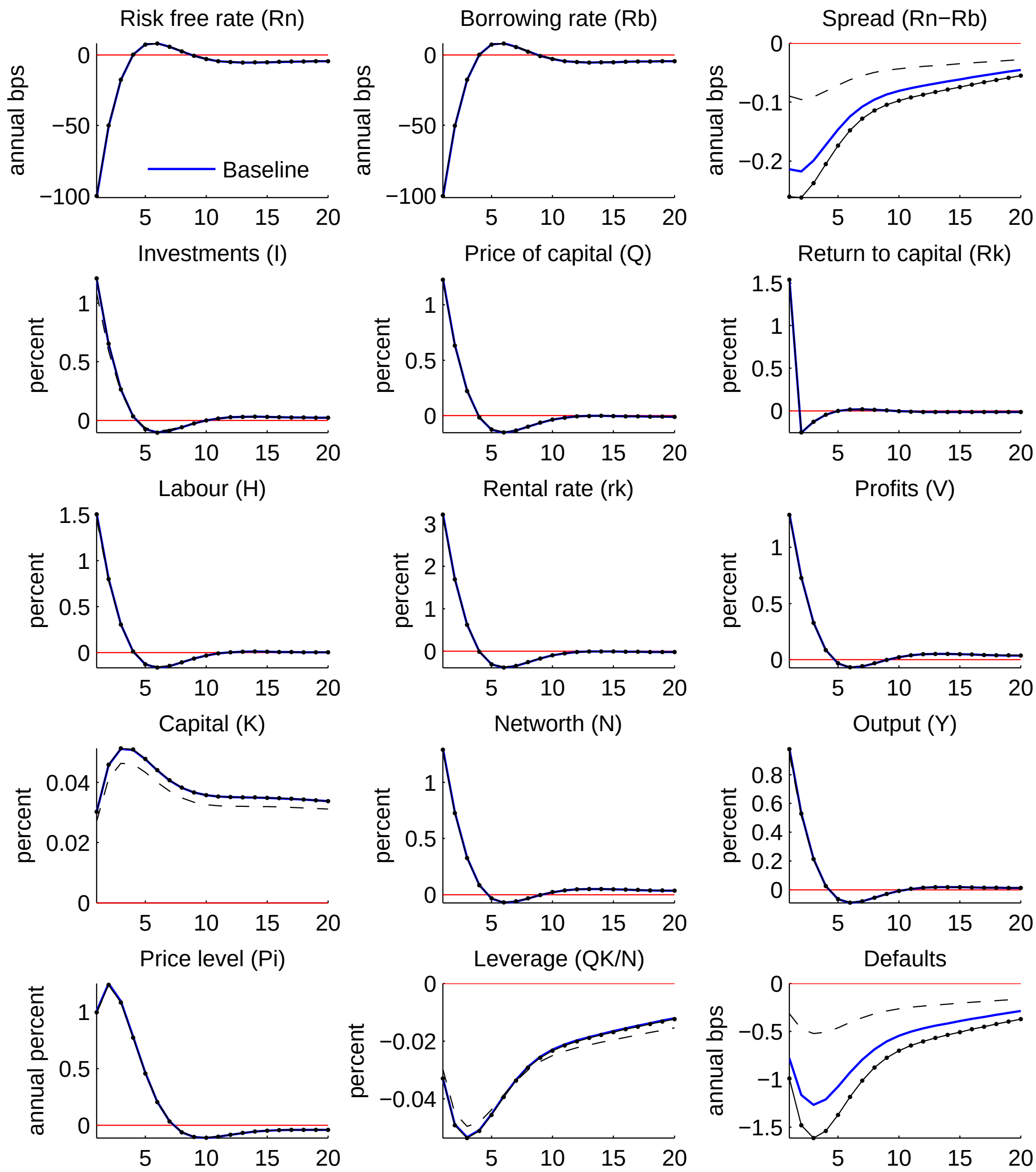
Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 9: Robustness checks (solid is $\beta = 0.9979$, dashed is $\beta = 0.99$



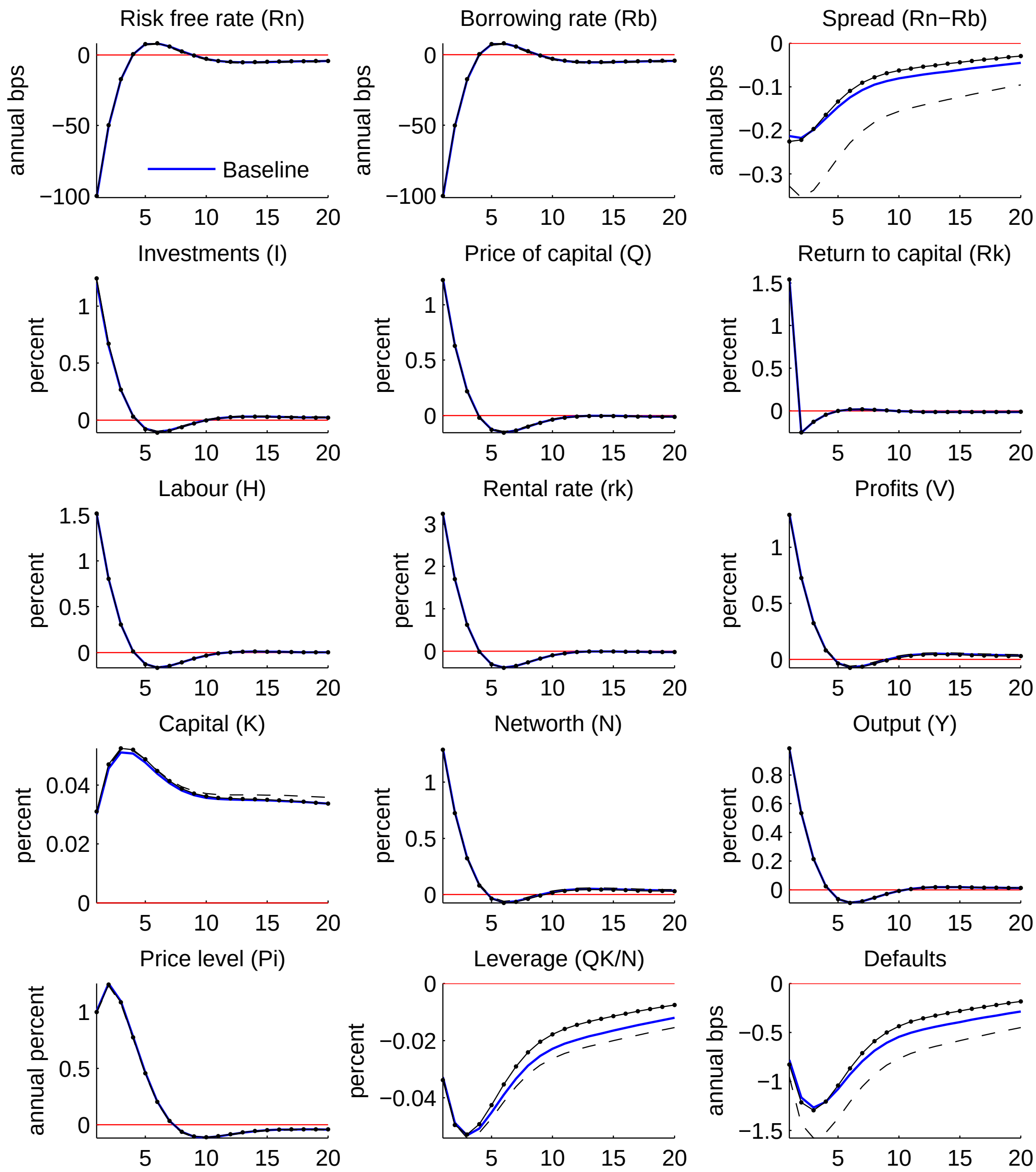
Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 10: Robustness checks (solid is $\Phi(\bar{\omega}) = 0.01$, dashed is $\Phi(\bar{\omega}) = 0.04$)



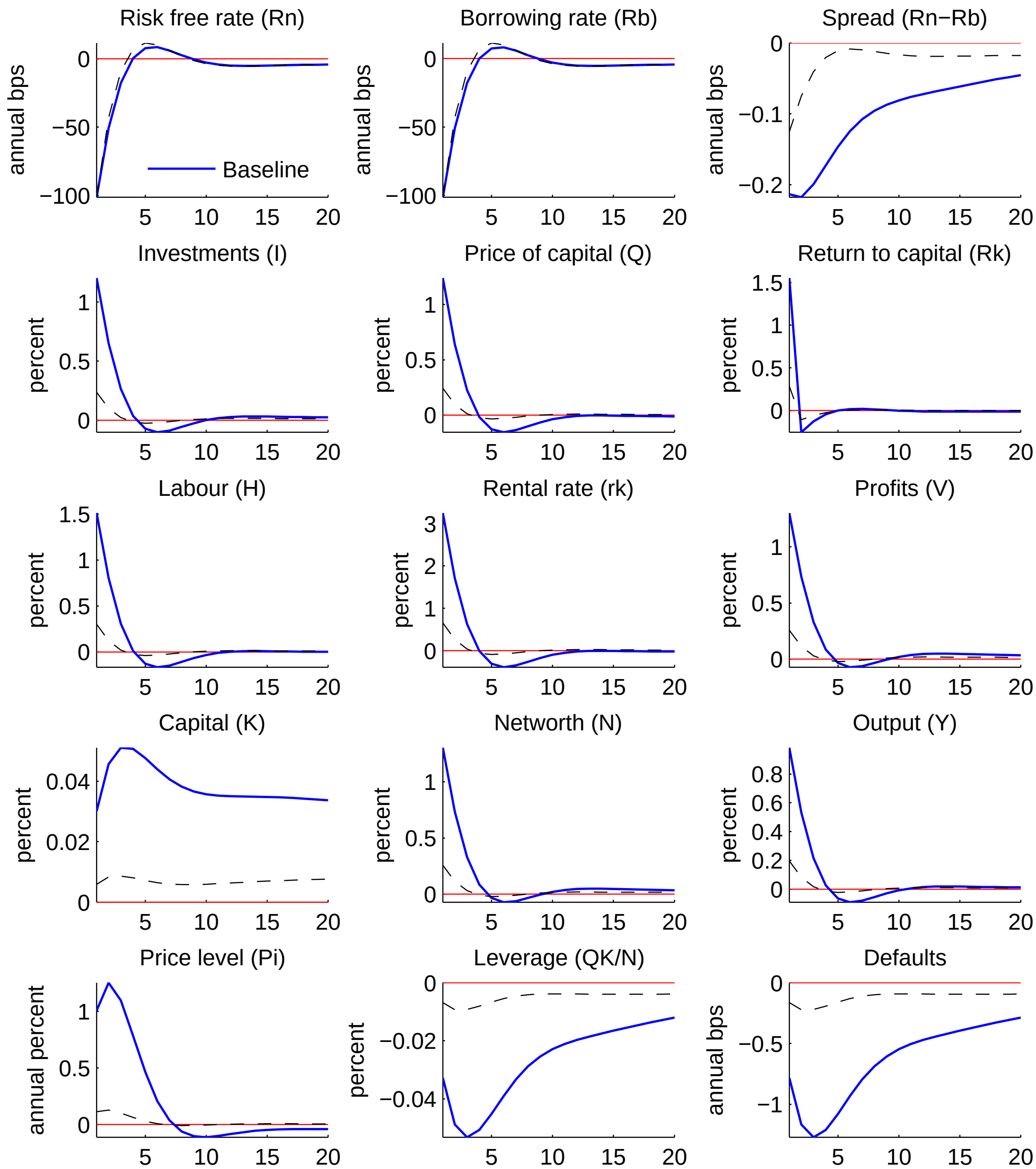
Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 11: Robustness checks (solid is $K/N = 2$, dashed is $K/N = 4$)



Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

Figure 12: Robustness checks (solid is $\rho = 0.87$ and one shock, dashed is $\rho = 0$ and many shocks)



Notes: All variables are shown in deviation from the steady state. The expansionary monetary shock is calibrated so to generate a 100 basis point decrease in the nominal interest rate.

CHAPTER 3

Monetary Policy and Defaults in the US

Abstract

I study empirically the effect of an unexpected monetary expansion on the delinquency rate on US business loans, residential mortgages and consumer credit. I consider several identification strategies and use Granger-causality tests to assess the exogeneity of the time series of monetary shocks to the Fed's forecast of defaults on loans. I then compute impulse responses using a variant of the Local Projection method by Jorda (2005). I find that the delinquency rates do not respond to monetary shocks in a statistically significant way. The paper suggests that the risk-taking incentives analyzed in partial equilibrium by several existing contributions might not be strong enough to prevail and increase aggregate defaults, although they could explain why defaults do not significantly decrease.

JEL Classification: E44, E52

Keywords: Monetary policy, risk taking channel, Granger causality, monetary shocks, default, Vector Autoregressive models

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1 Introduction

This paper studies empirically the effect of a monetary expansion on the probability that firms and households default on loans. On the one hand, one could expect that a monetary expansion reduces this default probability, because it boosts revenues and income by lowering the cost of borrowing and by increasing output. On the other hand, several recent empirical papers have used a partial equilibrium analysis to argue that banks take on more risk after a monetary expansion. The findings of these papers, discussed more extensively below, suggest the opposite prediction, i.e. that a monetary expansion could *increase* aggregate defaults in general equilibrium, as long as banks' risk-taking incentives are sufficiently strong. In this paper I show evidence against this hypothesis. Specifically, I argue that the response of aggregate defaults on US loans to an unexpected monetary expansion is not statistically different from zero when one accounts for general equilibrium effects, i.e. when one allows profits, GDP and other variables to respond to the monetary expansion. The results of the paper are in line with Buch, Eickmeier and Prieto (2011), which, to the best of my knowledge, is the only paper that addresses the general equilibrium impact of risk-taking in the US. The contribution of the paper differs from Buch *et al.* (2011) by not restricting the analysis to new loans, and by using a wider set of identification strategies.

In this paper, I measure the default probability on loans using the aggregate delinquency rate on three types of loans: business loans, residential mortgages and consumer credit. Aggregate delinquency rates are defined as the ratio of the value of loans whose repayment is overdue for more than a month, relative to the value of total loans. While the use of aggregate data helps to study defaults in general equilibrium, it also poses a key identification challenge. As is well known in the literature, the policy rate is endogenous to the state of the economy and tends to decrease in recessions. Since recessions are associated with higher defaults, a negative correlation emerges between the policy rate and the delinquency rate. The goal of the paper is to address whether, to some extent, this negative correlation is also driven by a reverse causality, i.e. by an increase in defaults *in response* to the decrease in the policy rate. To address this endogeneity problem I follow the literature and use monetary shocks, i.e. the variations in the policy rate not driven by the systematic intervention of the Fed to the current and future state of the economy.[1]

To analyze the effect of monetary policy on the default on loans one needs to extract variations in the policy rate that are orthogonal to the Fed's forecasts of defaults on loans (see for example the discussion by Cochrane (2004)). Suppose, in fact, that one finds that an unexpected monetary expansion is associated with a subsequent increase in defaults. If the monetary shocks are not orthogonal to the Fed's forecast of defaults, then the increase in defaults could still reflect the fact that the Fed decreased the policy rate in anticipation of the financial distress associated with a future downturn of the business cycle. The Fed's forecasts of defaults on loans is not available, but there are several approaches that one can follow to tackle the issue indirectly. For instance, Romer and Romer (2004) identify variations in the policy rate that are exogenous to the contemporaneous rate of unemployment and to the Fed's forecasts of output and inflation. If these variables are sufficient statistics for future defaults, then one can use the Romer and Romer shocks for the analysis of defaults. Alternatively, one can start from a reduced-form time series model and impose theoretical restrictions to identify monetary shocks. The estimated shocks are appropriate to study defaults to the extent that the variables

[1]While it is generally believed that the policy rate is always set by central banks in response to the state of the economy, the decision process within monetary policy committees is likely to be noisy. This noise is conveniently exploited in the literature for identification purposes. A more detailed discussion of monetary shocks is available, for instance, in Bernanke and Mihov (1998) and Romer and Romer (2004). An analysis of the decision making process in monetary policy committees is available for instance in Riboni and Ruge-Murcia (2010) and Hansen, McMahon and Velasco Rivera (2012).

included in the model span the information set used by the Fed to forecast future defaults. Another approach could be to use the unexpected variation of the federal funds rate implied by financial markets, as suggested by Kuttner (2001). Each of these methods has advantages and disadvantages in generating monetary shocks that are appropriate for the research question of the paper.

In this paper I use an approach that departs from the methodologies outlined above. I first collect five estimated time series of monetary shocks corresponding to five methodologies used in the literature. The approaches considered are a VAR in the spirit of Canova and De Nicoló (2002), a Medium scale Bayesian VAR by Banbura, Giannone and Reichlin (2010), a Factor Model by Forni and Gambetti (2010), the Romer and Romer shocks estimated by Coibion, Gorodnichenko, Kueng and Silvia (2012) and the shocks estimated by Barakchian and Crowe (2013) using Futures Contracts. Developing a theoretical argument as to which approach is more likely to deliver shocks that are exogenous to the Fed's forecasts of defaults is very subjective. Empirically, the first best approach would be to exploit the time series of the Fed's forecasts of the defaults, which is not available. In this paper I propose an indirect approach that relies on the following idea. While the Fed's forecasts on defaults are not known, they are realistically some function of variables that are likely to be informative of the future financial position of firms and households. I take these variables as proxies of the unobserved Fed's forecasts of defaults and dismiss candidate time series of monetary shocks that are Granger caused by any of these variables. The variables used are previous delinquency rates, the aggregate leverage ratios and the aggregate debt-to-GDP ratios of firms and households. This approach limits the analysis to the time series of monetary shocks from only one methodology, namely the Medium Bayesian VAR. The shocks are then used to compute impulse responses following a variant of the method by Jorda (2005), Romer and Romer (2004) and Chang and Sakata (2007).

The paper finds no statistically significant effect of a monetary shock on any of the three aggregate delinquency rates considered. The results are robust to alternative measures of monetary shocks used, as well as to alternative identification and specification strategies. Whether this evidence supports or not the findings on the risk-taking channel of monetary policy is subject to judgment. On the one hand one can argue that banks' risk taking should not be of particular concern for central banks, given that there is no statistically significant evidence that delinquency rates increase. For instance, this is the interpretation suggested by Buch, Eickmeier and Prieto (2011). On the other hand, though, one could conclude that risk taking is quantitatively quite significant. In fact, it could explain why there is no statistical evidence that delinquency rates *decrease*, as could be realistically expected. Using aggregate data from the Survey on Terms of Business Lending, Buch, Eickmeier and Prieto (2011) show that, while risk taking does not emerge for the banking sector as a whole, it does emerge when considering only small banks. Addressing whether this result is robust to the use of delinquency rates was not possible, given data availability. The question remains open for future research.

The paper relates to the literature on the risk-taking channel of monetary policy that follows Rajan (2006) and Borio and Zhu (2008). This literature explores the possibility that central banks face a trade-off between output and financial stability, in addition to the well-known trade-off between output and inflation. I take the probability of defaults on loans issued by banks as a proxy of the financial stability of banks. Using microeconomic data from the credit registry on Spain, Jimenez, Ongena, Peydró and Saurina (2010) find that a monetary expansion decreases the default probability on existing loans, but increases the default probability on new loans. Ioannidou, Ongena and Peydró (2009) and Lopez, Tenjo and Zarate (2011) show that the results by Jimenez, Ongena, Peydró and Saurina (2010) also hold for Bolivia and Colombia. An empirical analysis for the US using microeconomic data is provided by Maddaloni and Peydró (2011), Altunbas, Gambacorta and Marquez-Ibanez (2014), Paligorova and Santos (2012), De Nicoló, Dell'Ariccia, Laeven and Valencia (2010) and Dell'Ariccia, Laeven and Suarez (2013). These authors find that a monetary expansion leads banks to loosen credit standards, run a higher default probability, charge lower interest rate spreads to risky borrowers relative to safe borrowers, have a higher ratio of risk-weighted assets to total assets and grant loans classified by banks as more risky. Several theoretical contributions have modeled the possibility that a monetary expansion affects the allocation of credit across borrowers characterized by different *ex ante* risk profiles (see for example Challe, Mojon and Ragot (2013), Dubecq, Mojon and Ragot (2009), Agur and Demertzis (2012) and Fahri and Tirole (2009)). Related but different mechanisms are modeled in Drees, Eckwert and Vardy (2012), Acharya and Naqvi (2012), Dell'Ariccia, Laeven and Marquez (2010), Stein (2012), Valencia (2011), Bruno and Shin (2012), Angeloni and Faia (2013), Cociuba, Shukayev and Ueberfeldt (2012) and Piffer (2014).

Section 2 explains the key variables used in the analysis. Section 3 outlines the empirical strategy used. Section 4 reports the results. Section 5 concludes.

2 Dataset

To measure the probability of default on loans I use delinquency rates. The delinquency rate is the ratio of the value of delinquent loans over the value of outstanding loans, where a borrower is "delinquent" if the loan repayment (interests or principal) is overdue for more than 30 days. I consider three delinquency rates depending on the type of loan considered: business loans, residential mortgages and consumer credit. The Federal Reserve Bank collects these time series from the Call Report filed for all US-chartered commercial banks on a quarterly basis. Nevertheless, only certain aggregate time series are made publically available. In this paper I use delinquency rates on loans issued by all commercial banks.[2] The series start in 1987Q1 for the delinquency rates on business loans and consumer credit, and in 1991 for residential mortgages.[3] Aggregate data from the Flow of Funds reveal that the three types of loans considered account in total for around 40% of banks total assets. This ratio has been relatively constant since the 1970s, but hides a progressive increase of the volume of residential mortgages and a progressive decrease in the volume of business loans (and, to a lesser extent, of consumer credit). Figure 7 at the end of the paper provides further details.

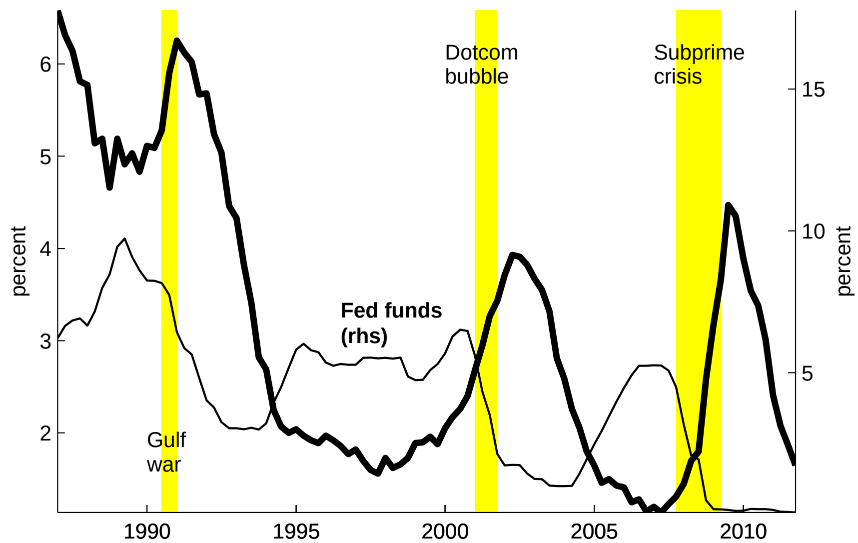
Figure 1 shows the evolution of the delinquency rates. The three time series are compared to the evolution of the federal funds rate (thin line, plotted on the right-hand side of the graphs). As explained in the introduction, the analysis of the paper is carried out using data for the period until 2007Q2. For completeness, the figure covers also the period after 2007Q2. The vertical shaded bands indicate the NBER recessions covered. These correspond to the recessions following the Gulf war, the dotcom bubble and the subprime crisis. Table 1 complements the analysis of figure 1 by reporting summary statistics on the mean, the 2.5th and 97.5th percentiles, the standard deviation and the contemporaneous correlation with real GDP. This correlation is computed after filtering out low and seasonal frequencies using an X-12 ARIMA algorithm from the US Census Bureau and the HP filter, respectively.

[2]The Federal Reserve also publishes the delinquency rate aggregated for the top 100 banks and for the non top 100 banks, according to assets. An aggregation for small banks only is not available. The results are robust to the use of the delinquency rates for top and non-top 100 banks, as discussed in Section 4.

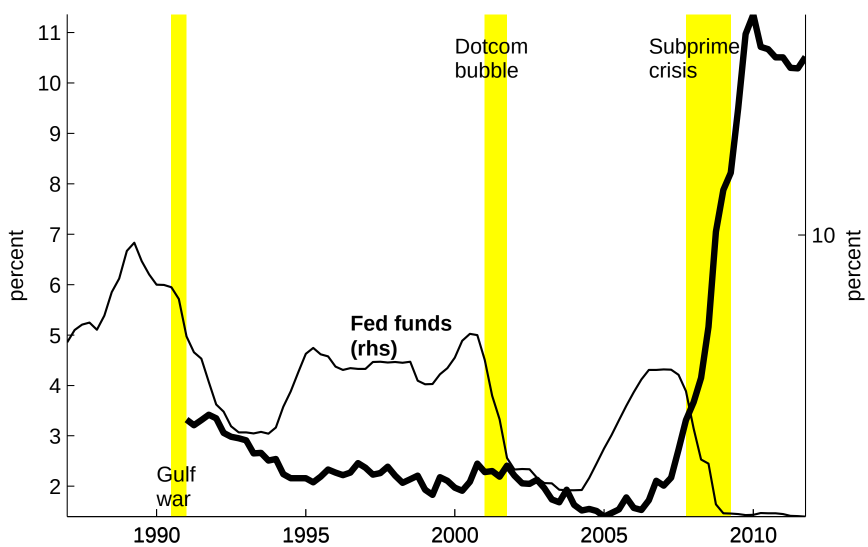
[3]When the borrower becomes severely delinquent (traditionally after six months from the first omitted payment) the lender charges off from his balance sheet the amount of the loan that is considered noncollectable. This allows the lender to book the corresponding loss in the income statement and benefit from a tax exemption on the loss. I use delinquency instead of charge-off rates since the latter are more noisy. The results are robust to the use of charge-off rates. Further details on delinquency and charge-off rates are available on http://www.federalreserve.gov/releases/chargeoff/about.htm. An alternative measure could be the rate of non-performing loans by commercial banks, which nevertheless combines business and non-business loans in a single time series.

Figure 1: Delinquency rates on three types of loans

a) business loans

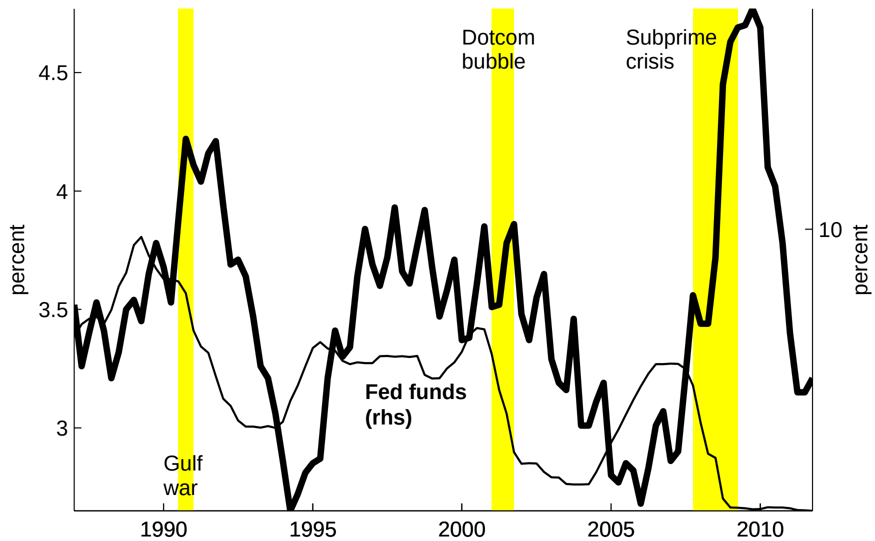


b) residential mortgages



(continues ...)

c) consumer credit



Notes: the shaded bands indicate NBER recessions, the thin lines show the federal funds rate, the thick lines show the delinquency rate (top graph) and the leverage ratio of firms (bottom graph). The period considered is 1987Q1-2011Q4. Only the period until 2007Q2 will be included in the analysis of Sections 3 and 4.

The smoothing parameter used in the HP filter equals 1600, which is standard for quarterly data.

Overall, figure 1 and table 1 suggest that:

1. the delinquency rates average between 3% and 3.5% for all types of loan considered, although the average delinquency rate on residential mortgages decreases to 2.21% when excluding the period after 2007Q2;

2. the delinquency rate on residential mortgages spiked during the subprime crisis. The delinquency rate on the other loans increased too, but less remarkably;

3. the delinquency rates on business loans and residential mortgages display a downward trend when one excludes the period after 2007Q2;

Table 1: Summary statistics on delinquency rates

	Mean %	Percentile (2.5)	Percentile (97.5)	Standard deviation	Correlation with real GDP
Until 2011Q4					
business loans	3.10	1.20	6.25	1.57	-0.76
residential mortgages	3.49	1.49	10.82	2.86	-0.75
credit loans	3.50	2.72	4.69	0.47	-0.63
Until 2007Q2					
business loans	3.21	1.17	6.27	1.65	-0.60
residential mortgages	2.21	1.47	3.34	0.48	-0.19
credit loans	3.41	2.70	4.18	0.38	-0.31

Notes: The mean and percentiles are computed on the level of the corresponding variable. The correlation with real GDP is computed after filtering out seasonal and low frequencies using an X-12 ARIMA filter and an HP filter, respectively. The smoothing parameter used in the HP filter equals 1600.

4. all delinquency rates are countercyclical, especially the delinquency rate on business loans;

5. when excluding the period after 2007Q2, the delinquency rate on business loans is largely more volatile than the delinquency rate on residential mortgages and consumer loans;

3 Empirical strategy

The empirical strategy used in this paper consists of three steps. In the first step I collect several candidate time series of monetary shocks. In the second step I use Granger-causality tests to identify the time series of shocks that are less likely to display endogeneity with respect to the Fed's forecasts of defaults on loans. In the third step I compute impulse responses using local projection methods.

Table 2: List of the candidate time series of monetary shocks considered

Time series approach	
1) VAR	
2) Medium Bayesian VAR	Banbura, Giannone and Reichlin (2010)
3) Factor Model	Forni and Gambetti (2010)
Romer and Romer approach	
4) Romer and Romer shocks	Coibion, Gorodnichenko, Kueng and Silvia (2012)
Financial data approach	
5) Futures contract data	Barakchian and Crowe (2013)

Notes: The table lists the five candidate time series of monetary shocks used in the paper.

3.1 Step 1: collect time series of monetary shocks

The literature has proposed several methodologies to estimate monetary shocks. I start by considering five time series of monetary shocks, which fall into three main approaches: the "Time Series approach", the "Romer and Romer approach" and the "Financial Data approach". The methodologies used are listed in table 2. In this section I discuss the ability of each of these methodologies to deliver estimates of monetary shocks that satisfy the key requirement for the application of this paper. Some of the time series of monetary shocks are available before 1987Q1. Only the subset of the time series after 1987Q1 will be used for the computation of impulse responses in Section 3.3, given that the delinquency rates are available only after 1987Q1 (1991Q1 for the delinquency rate on residential mortgages). When necessary, monthly time series are converted to quarterly time series using the unweighted sum within each quarter, in line with Cochrane and Piazzesi (2002) and Romer and Romer (2004).

The first candidate time series of monetary shocks is estimated using a standard VAR model, in the spirit of Bernanke and Blinder (1992), Christiano, Eichenbaum and Evans (1999) and Canova and De Nicoló (2002). One option could be to include the delinquency rates in the VAR. The limited availability of the time series for the delinquency rates discourages this approach, since it would imply a significant loss of degrees of freedom in the estimation of the VAR.[4] Alternatively, I use variables, available on a longer time period, that are likely to be informative of the future financial position of firms and households. The VAR includes the following five variables: the log of real GDP, the log of GDP deflator, the federal funds rate, the log of firms' leverage ratio and the log of households' leverage ratio. The VAR, which is estimated using OLS, includes 4 lags, a constant, a linear trend and seasonal dummies. The order in which variables enter the model does not matter, since I identify the model using sign restrictions. The sign restrictions imposed are that 1) real GDP does not increase during the first two quarters, 2) interest rates do not increase during the first two quarters, and 3) if a price puzzle is displayed, it does not last for more than three years.[5] For simplicity, I only use the estimates of monetary shocks corresponding to the median target. Appendix A discusses the estimation and identification of the VAR model in detail. I do not use a Cholesky identification for the VAR model given that the model is estimated on quarterly data.[6]

The VAR model outlined above includes only 5 variables. The literature has discussed extensively the fact that central banks use a wider information set to guide the course of the policy rate. The second and third candidate time series of monetary shocks address this issue. The second time series of monetary shocks is computed using the Bayesian estimation of large VARs proposed by Banbura, Giannone and Reichlin (2010). I use the medium specification of their model, which

[4] Section 4 shows that the results are consistent with the use of VAR models that include delinquency rates.

[5] While unconventional, the sign restriction on the behaviour of the GDP deflator is in line with the observation that the literature typically finds a price puzzle that lasts up to around three years For a discussion of the price puzzle see Boivin *et al.* (2011). For robustness, I also used a Cholesky identification, which did not change the results and which delivered monetary shocks that did not satisfy the Granger-causality restrictions explained in the next section.

[6] The recursive timing of monetary shocks is more likely to hold for monthly data, as discussed for instance in Bernanke, Boivin and Eliasz (2005).

includes 20 variables, because they show that the predictive power of such a model is superior to a larger VAR model. I will refer to this model as the Medium Bayesian VAR. The paper by Banbura *et al.* (2010) delivers monetary shocks for the period between 1962M1 and 2003M12. I replicate their analysis for the period between 1976M1 and 2007M6.[7] The third time series of monetary shocks is taken from the Factor Model by Forni and Gambetti (2010), which includes 112 variables. The period covered is 1973M4-2007M12. The identification is recursive in both papers, as suggested by the use of monthly data. The variables potentially informative of the repayment possibilities of firms and households are personal income, indexes of industrial production, the tightness of unemployment in different sectors, stock price indexes and a range of interest rates.[8]

It is hard to judge whether a VAR with a small number of variables is more or less successful in generating monetary shocks that are exogenous to the Fed's forecasts of default, compared to a larger model. In fact, the inclusion of variables involves costs and benefits. On the one hand, adding more variables allows one to capture a wider information set potentially used by the Fed to assess the future course of delinquencies. On the other hand, it requires more structure to be imposed on the model.[9] The remaining candidate time series of monetary shocks account for this consideration.

The fourth candidate time series of monetary shocks that I use follows the contri-

[7]Contrary to their specification, I do not add a prior on the sum of coefficients, given that the purpose of my analysis is not to improve the forecasting power of the model. The results were not affected by the exclusion of this prior.

[8]The variables included in the Medium Bayesian VAR are personal income, consumption, industrial production index, capital utilization, unemployment rate, number of employees on non farm sector, housing starts, producer price index, the price index of personal consumption expenditures, the CPI index, the trade-weighted US dollar index, hourly earnings of production, the federal funds rate, M1, M2, total reserves, non borrowed reserves, the industrial production index, the business loans and the loans on residential real estate. The list of the 112 variables in the Factor Model by Forni and Gambetti (2010) can be found in the on-line supplement material of the original paper.

[9]Banbura, Giannone and Reichlin (2010) do so by estimating the model with Bayesian techniques. Forni and Gambetti (2010) do so by assuming that the dynamics of the data are ultimately generated by four structural shocks that affect 16 unobservable factors.

bution by Romer and Romer (2004). In short, Romer and Romer identify an index of the intended variations of the federal funds rate. They then regress this index on the Fed's forecasts of real output growth, the GDP deflator and the unemployment rate. The residuals of such regressions capture the variations in the policy rate that are orthogonal to the Fed's expectations of the state of the economy captured by the forecasts. To some extent it is reasonable to expect that the Romer and Romer shocks are at least in part exogenous to the Fed's forecast of defaults, since future GDP is realistically informative of the future financial position of borrowers. The original Romer and Romer shocks cover the period between 1969M3 and 1996M12. I use the series by Coibion, Gorodnichenko, Kueng and Silvia (2012), who extend the analysis by Romer and Romer (2004) up to 2008M12. Using the original estimates of the Romer and Romer shocks did not change the results.

The fifth and last candidate time series of monetary shocks considered uses a methodology suggested by Kuttner (2001). This approach exploits market data from the prices on futures contracts on the federal funds rate and computes the variation in the policy rate that was not expected by the markets. The main advantage of this approach is that it does not rely on a specific structural representation of the data. The main disadvantage is that it also captures, as monetary shocks, variations of the federal funds rate that are potentially well intended by the Federal Reserve Bank as the response to the state of the economy, but that had either not been expected by the market, or that had been expected differently.[10] As a measure of monetary shocks, I use the time series by Barakchian and Crowe (2013), who build on Kuttner (2001) and generate estimates for the period between 1988M1 and 2008M12.[11,12]

[10]This risk is partly reduced by the fact that the time window used around the policy event lasts only 24 hours.

[11]Cochrane and Piazzesi (2002) use a similar approach and exploit the variations in the one-month eurodollar rate between two days before and one day after the variation in the target rate of the federal funds rate.

[12]I am grateful to the authors of the above-mentioned papers for having shared estimates and codes.

3.2 Step 2: select time series of monetary shocks

Table 3 provides summary statistics for the five candidate time series of monetary shocks discussed in the previous section. It shows the standard deviation of each time series, the contemporaneous correlations across series and the autocorrelation of each series at 4 and 8 lags. The table reports summary statistics both for the full sample period available and for the period after 1987Q1. I do so because, as explained above, the estimation of impulse responses uses only the time series after 1987Q1 or 1991Q1, as imposed by the availability of the delinquency rates.

Table 3: Summary statistics of the candidate time series of monetary shocks

	st.dev	cross correlations					autocorrelations (4 lags)	autocorrelations (8 lags)
Full sample								
1) VAR	0.93	1					.04	.02
2) Medium Bayesian VAR	0.57	.57	1				-.12*	-.01***
3) Factor Model	1.61	.34	.28	1			.03	.06
4) Romer and Romer shocks	0.58	.34	.55	.04	1		-.09*	.06*
5) Futures Contracts	0.92	.01	-.05	-.08	.10	1	-.04	-.24
After 1987Q1								
1) VAR	0.69	1					.01***	0*
2) Medium Bayesian VAR	0.34	.23	1				-.12**	-.16**
3) Factor Model	1.52	.15	.03	1			.07***	.07**
4) Romer and Romer shocks	0.26	.45	.05	-.27	1		.14***	-.03***
5) Futures Contracts	1.18	.02	-.01	-.10	.29	1	-.04	-.24

*Notes: The table shows the correlations across the five candidate time series of monetary shocks and the standard deviations. The tests on autocorrelations are carried out using the Box and Ljung statistic * ** significant at 1%, ** significant at 5%, * significant at 10%.*

According to table 3, the highest volatility is displayed by the monetary shocks from the Factor Model, while the lowest volatility is displayed by the Romer and Romer shocks. Correlations are overall small, especially after 1987Q1, and are negative one third of the time, although very close to zero. This indicates that the description of the non-systematic variation of the policy rate varies considerably among the five methodologies considered.[13] As for autocorrelations, the Box and Ljung test finds that autocorrelations differ from zero in a statistically significant way for several series, especially after 1987Q1. This suggests some caution in interpreting the candidate monetary shocks as structural shocks. It was explained in the previous section that 4 of the 5 time series used in the paper are quarterly transformations of monthly time series. I found that the Box and Ljung test tended to reject the hypothesis of no autocorrelations also for the originals series (see table 6 at the end of the paper).[14] Figure 2 complements table 3. It uses rolling windows of 48 quarters to study how the standard deviations and the cross correlations vary over time. The figure shows that the volatility of monetary shocks has steadily decreased after the 1980s for all the five measures of monetary shocks included, a finding consistent with Primiceri (2005). The decrease in standard deviations is quite considerable and equals up to around 50% for the Romer and Romer shocks and for the shocks from the VAR and the Medium Bayesian VAR. A similar downward trend is displayed by cross correlations. This suggests that, in general, the candidate estimates of monetary shocks imply a progressively less consistent pattern of the non-systematic response of the policy rate. The only exception is the positive correlation between the shocks from the VAR model and the Romer and Romer shocks, which increases over time.

A candidate time series of monetary shocks is appropriate for the analysis of this paper only if it does not reflect the endogenous response of the policy rate to the Fed's expectation of future defaults on loans. Put it differently, monetary shocks

[13] A relatively strong correlation is found between the Romer and Romer shocks and the shocks generated by the VAR. This fact is also pointed out in Coibion (2012).

[14] The results from Section 3.3 use the series from a Medium Bayesian VAR that included 12 months as lags. I found that the Box and Ljung test failed to reject the hypothesis of no autocorrelation only when including more than as much as 6 years of lags. i.e. 72 months. Computational constraints did not allow to draw from the distribution of so many parameters. For this reason I use the specification of the model with 12 lags.

should not include the Fed's anticipatory response to future defaults, a response realistically implied by the Fed's response to the business cycle. In principle, all the available monetary shocks might still be capturing to some extent this endogenous response. It is hard to judge whether this is the case using an *a priori* theoretical argument. In fact, all the methodologies discussed in Section 3.1 have strengths and weaknesses in capturing the Fed's response to defaults. A direct empirical assessment is even less viable, given that the time series of the Fed's forecast of defaults is not available. An indirect approach becomes necessary. In this paper, I propose one based on the following idea. While unobserved, the Fed's expectation at time t of future defaults must be some (unknown) function of variables up to time $t-1$ realistically informative of the future financial strength of borrowers. The variables used in this paper are:

1. the delinquency rate on business loans;
2. the delinquency rate on residential mortgages;
3. the delinquency rate on consumer credit;
4. the leverage ratio of firms;
5. the leverage ratio of households;
6. the debt-to-GDP ratio of firms;
7. the debt-to-GDP ratio of households;
8. the ratio of banks' reserves to total loans.

The first 7 variables measure the contemporaneous financial position of firms and households, while the last variable captures banks' expectations of future defaults of firms and households on loans. One can proxy the unobserved Fed's forecast of defaults with the above 8 variables and impose the restrictions that the candidate monetary shocks are not Granger-caused by any of these variables.

More precisely, I run Granger-causality tests using the following regression:

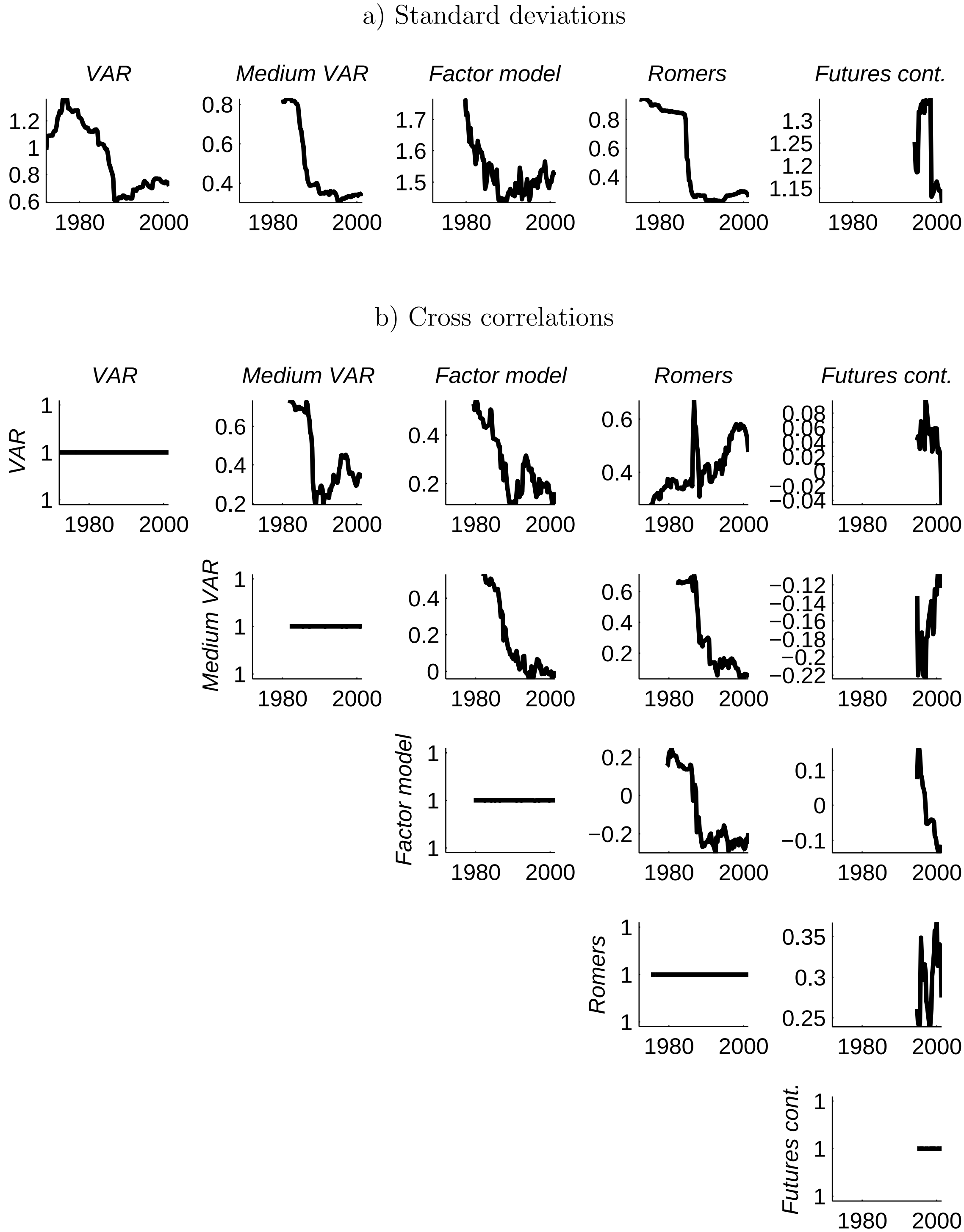
$$s_t = \gamma_0 + \gamma_1 v_{t-1} + \gamma_2 v_{t-2} + ... + \gamma_m v_{t-m} + \epsilon_{t+s}. \tag{1}$$

The scalar s_t indicates the candidate time series of monetary shocks evaluated at time t, the scalar v_{t-i} indicates the variable used in the test and evaluated at time $t-i$. I consider the eight variables above one at the time in order to retain degrees of freedom in the estimation of equation (1). The White test rejects the null hypothesis of heteroskedastic disturbances in equation (1) for virtually all regressions estimated.[15] I add lags of variable v_t from 1 up to $m = 2, ..., 12$. I then use an F-statistic to test the hypothesis that the parameters on all regressors, except the constant, are jointly equal to zero, i.e. $\gamma_1 = \gamma_2 = ... = \gamma_m = 0$. I rule out shocks that are Granger-caused at a type-one error of 5% by any of the seven variables when adding any number of lags from 1 up to 12. Each of the seven variables considered implies $12 \times 5 = 60$ tests, for a total of 420 tests. For robustness, I run additional tests using a quadratic and a cubic specification of equation (1), and using a Lagrange Multiplier test rather than an F test.

Table 4 shows the results of the F tests with a linear specification of equation (1). This table lists the "problematic" lags, i.e. the total number of lags from 1 until m including which Granger-causality is not rejected at 5% type-one error. To guide the reading of the table, consider first the case of the VAR shocks. Table 4 shows that Granger-causality is not rejected when including up to 8 lags of the delinquency rate on consumer credit. This suggests that, according to the logic of

[15]Part of the rejection of the null hypothesis of heteroskedasticity is likely to be due to the fact that the tests only use the part of the monetary shocks that overlap with the delinquency rates. The variation in time of the volatility of the variance of monetary shocks shown in figure 2 suggests that the null hypothesis would be probably rejected, at least for some estimates of monetary shocks, if one had a longer time series of the delinquency rate.

Figure 2: Time-varying summary statistics (rolling window = 48 quarters)



Notes: Summary statistics at period t are computed over a window of 48 quarters.

Table 4: Granger-causality restrictions on candidate time series of monetary shocks

	1) VAR	2) Medium Bayesian VAR	3) Factor model	4) Romer and Romer	5) Futures contracts
Delinquency rates					
Business loans			9-12	3-12	
Residential mortgages			1	6-12	5-6
Consumer credit	1-8		1-12	1-12	
Leverage ratios					
Firms			1-4		
Households			4-12	3-9	
Debt/GDP ratios					
Firms				10-12	
Households					
Reserves/Loans ratio					
Banks			6, 8	3-12	2, 11

Notes: The table shows the results of Granger-causality tests on the candidate time series of monetary shocks from Section 3.1. The table reports the number of lags m *in equation* (1) *including which the hypothesis of no Granger causality is rejected at 5% type-one error. The restrictions imposed are that none of the variables considered (delinquency rate, spread, leverage and dept-to-GDP ratios) Granger cause the candidate shocks.*

the test, VAR shocks might still be affected by endogeneity with respect to expected future defaults on consumer loans. Alternatively, consider the Romer and Romer shocks. The tests find that these shocks are Granger-caused by several variables, including all delinquency rates. Remember that the Romer and Romer shocks are computed from the regression of the intended variation of the policy rate on the unemployment rate and on the forecasts of output growth and inflation. Indeed, unreported tests find that output, the GDP deflator and the unemployment rate do not Granger-cause the Romer and Romer shocks, nor any of the other monetary shocks considered. Table 4 suggests that controlling for these variables might not be enough to detect the response of the policy rate to future defaults. The analysis delivers only one candidate time series of monetary shocks that meet the Granger-causality restrictions. These are the shocks corresponding to the Medium Bayesian

VAR model. The results of the tests are the same using a quadratic and cubic specification of equation (1) and using the Lagrange Multiplier test.[16]

The results from table 4 should be viewed with caution. In fact, by construction the test mistakenly rejects the null hypothesis of no Granger-causality with 5% probability. This implies that the four time series rejected by the tests (VAR, Factor Model, Romer and Romer and Futures Contracts) could well be just as exogenous to future defaults as the time series of shocks satisfying the restrictions (i.e., from the Medium Bayesian VAR), at least according to the logic of the test. Additionally, there is an (unknown) probability that the tests fail to reject the absence of Granger causality where this causality is actually present in the data. This implies that the time series of shocks satisfying the restrictions could have been selected as the appropriate candidate time series only by mistake. For this reason I will also discuss the impulse responses obtained from monetary shocks that do not satisfy the Granger-causality restrictions.

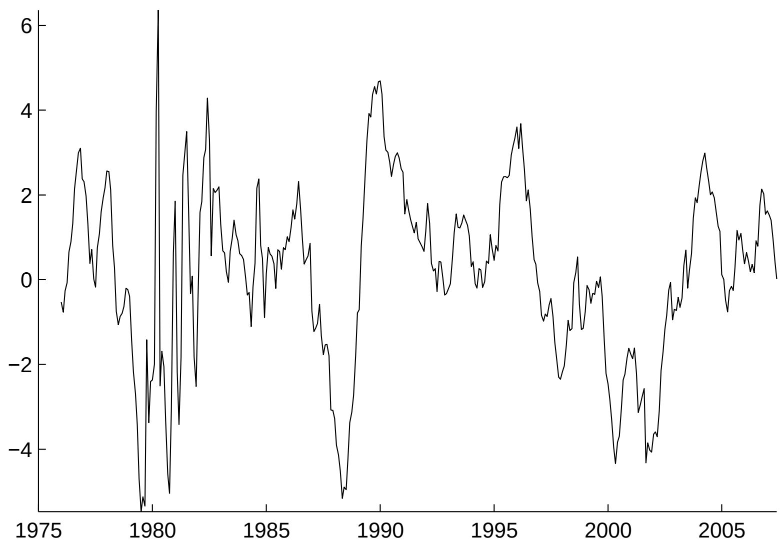
Figure 3 shows some of the features of the monetary shocks estimated from the Medium Bayesian VAR and used in the next section. By construction, these shocks are very noisy. I follow Coibion (2012) and show in panel a the cumulative monetary shocks, i.e. the sum at time t of all monetary shocks until time t.[17] An increase in the cumulative series of monetary shocks is associated with a positive (contractionary) marginal monetary shock. The graph in panel a shows some features consistent with Coibion (2012), including a period of expansionary monetary shocks in the late 1970s and a subsequent series of contractionary shocks during the Volcker period. Panel a only shows the dynamics of the random component of the

[16] With both a quadratic and a cubic specification of equation (1) the delinquency rate on business loans does not Granger-cause anymore the shocks by the Factor Model, and it Granger-causes the shocks estimated from Futures Contracts when adding 8 to 12 lags. The results from the Lagrange Multiplier test were always in line with the F test.

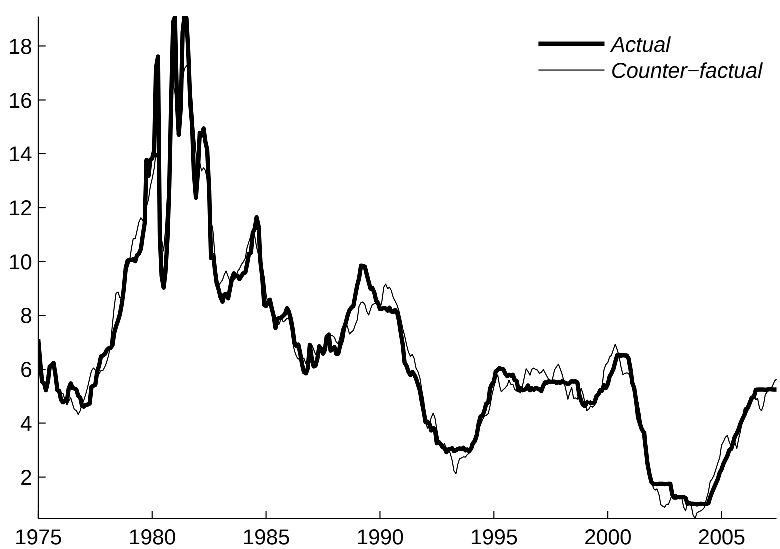
[17] The cumulative series of the monetary shocks corresponding to all 5 methodologies considered in Section 3.1 is shown in figure 6 of Appendix C. The figure also shows the cumulative shocks from the VAR with time-varying parameters by Primiceri (2005), which is used in Section 4 to discuss the robustness of the results.

Figure 3: Analysis of the monetary shocks from the Medium Bayesian VAR

a) Cumulative shocks (increase = contraction)



b) Fed funds rate (actual > counter. = contraction)



Notes: Panel a shows the cumulative monetary shocks between 1976M1 and 2007M6, computed for each time t as the sum of the series of monetary shocks from $t = 1$ until t. Panel b shows the level of the federal funds rate and the counter-factual level that, according to the Medium Bayesian VAR, would have occurred, had no monetary shock been given. The counter-factual time series is computed using the original data for the first $p = 12$ months, and then iterating recursively on the variables of the model. The computation uses the posterior means of the parameters estimated.

federal funds rate. To compare it to the systematic component of the federal funds rate I used the estimated Medium Bayesian VAR to generate pseudo data recursively by feeding into the model all structural shocks except the monetary shocks.[18] The results for the federal funds rate are shown in panel *b*. A level of the actual federal funds rate above the level that would have occurred, had monetary shocks not been given, is associated with a contractionary shock. The graph shows that, by construction, the systematic and the random component of the federal funds rate do not systematically point in the same direction. Other patterns emerge from figure 3. For instance, the overall contractionary monetary policy generated with the systematic response of the federal funds rate in the period between 1975 and 1980 has been met with a mix of contractionary and expansionary monetary shocks. This is also visible from panel *a*. The graph of panel *b* also suggests that, for instance, the expansionary monetary policy following the dotcom bubble had been initially combined with contractionary monetary shocks.

3.3 Step 3: compute impulse response functions

To compute the impulse responses of the delinquency rates I use variants of the local projection method by Jorda (2005). Local projections capture the relationship between the dependent variable and the structural shocks of interest, without relying on a specific, inverted autoregressive representation of the data, as for instance in VAR models. In addition, they allow to estimate impulse responses of variables not included in the initial model, as in this paper.

There are several ways of specifying single-equation regressions to compute impulse response functions. In this paper I use the one by Romer and Romer (2004). Define x_t the variable of interest and s_t the monetary shock. The baseline regression

[18] The first 12 observations of the pseudo data, lost due to the inclusion of lagged variables, are set equal to the data.

is

$$x_t = \alpha + \sum_{i=0}^{p_s} \beta_i s_{t-i} + \sum_{i=1}^{p_x} \gamma_i x_{t-i} + \epsilon_t. \tag{2}$$

The response of x to s is computed as $IRF_\tau = \bar{s} \cdot g_\tau$, where $\bar{s}$ is a parameter controlling for the sign and magnitude of the shock given and g_τ is defined as $g_0 = \beta_0,\ g_1 = \beta_1 + \gamma_1 g_0,\ g_2 = \beta_2 + \gamma_1 g_1 + \gamma_2 g_0,\ ...$. Contrary to Romer and Romer (2004), I add the contemporaneous level of the monetary shock in order to allow monetary shocks to affect the delinquency rates within the same period. In the rest of the paper I use 4 lags of the shock and 4 lags of the dependent variable, a parameter choice that was found not to affect the results particularly. The inclusion of the lagged values of the dependent value led to reject, at a 5% type-one error, the hypothesis that the errors are autocorrelated.[19]

To compute confidence intervals for the impulse response estimated from equation (2), one needs to take into account the generated regressor problem.[20] In fact, uncertainty enters the estimation of the impulse response from equation (2) both from the error term and from the estimation of the monetary shocks used as regressors. One could follow a frequentist approach and estimate equation (2) using ordinary least squares. Accordingly, one could then compute confidence intervals with a system of bootstraps that generates pseudo regressors s_t and pseudo errors ϵ_t. In this paper I follow a different approach and estimate equation (2) with Bayesian techniques, given that the monetary shocks used come from a Bayesian VAR. Write the Medium Bayesian VAR model from Section 3.1 as $Y = X \cdot B + U$, where the matrix $Y = [y_1, ..., y_n]$ includes the variables used in the model, the matrix $X = [x_1, ..., x_T]'$ with $x_t = [y'_{t-1}, ..., y'_{t-p}, 1]$ includes the lagged values of all variables as well as a constant and the matrix $U = [u_1, ..., u_n]$ includes the reduced-

[19]Basu, Fernald and Kimball (2006) use a similar specification, except that they do not include lagged values of the dependent variable. Similar regressions are used by Cochrane (2004), Cochrane and Piazzesi (2002), Chang and Sakata (2007) and Mésonnier and Stevanovic (2012).

[20]See for instance the discussion in Murphy and Topel (1985).

form shocks.[21] Call Ψ the variance-covariance matrix of the reduced-form shocks u_t. The Normal-inverse Wishart priors for B and Ψ used in Banbura *et al.* (2010) imply the following posterior distributions

$$\Psi \sim iW(\Sigma_{post}, d_{post}) \quad , \quad vec(B)|\Psi \sim N\big(vec(B_{post}, \Psi \otimes (X_*' X_*)^{-1}\big),$$

where the posterior hyper-parameters and the matrix X_* are defined extensively in their paper. For the estimation of equation (2), instead, I use a non-informative Normal-Gamma prior, implying a multivariate posterior t distribution for the parameters $\psi = [\alpha, \beta_0, ..., \beta_{p_s}, \gamma_1, ..., \gamma_{p_x}]'$. This distribution is centered around the OLS estimates of ψ, as shown for example by Koop (2013). Given this, coverage intervals are computed running the following steps 500 times:

1. draw Ψ and $vec(B)$ from their respective distributions;
2. compute the corresponding time series of monetary shocks, under the recursiveness assumption;
3. use these shocks to compute the least square estimates of ψ;
4. use ψ_{ols} to extract ψ from the multivariate t;
5. compute the corresponding impulse responses.

4 Results

Figure 4 shows the impulse responses of the delinquency rate on business loans, residential mortgages and consumer credit. The impulse given is the expansionary

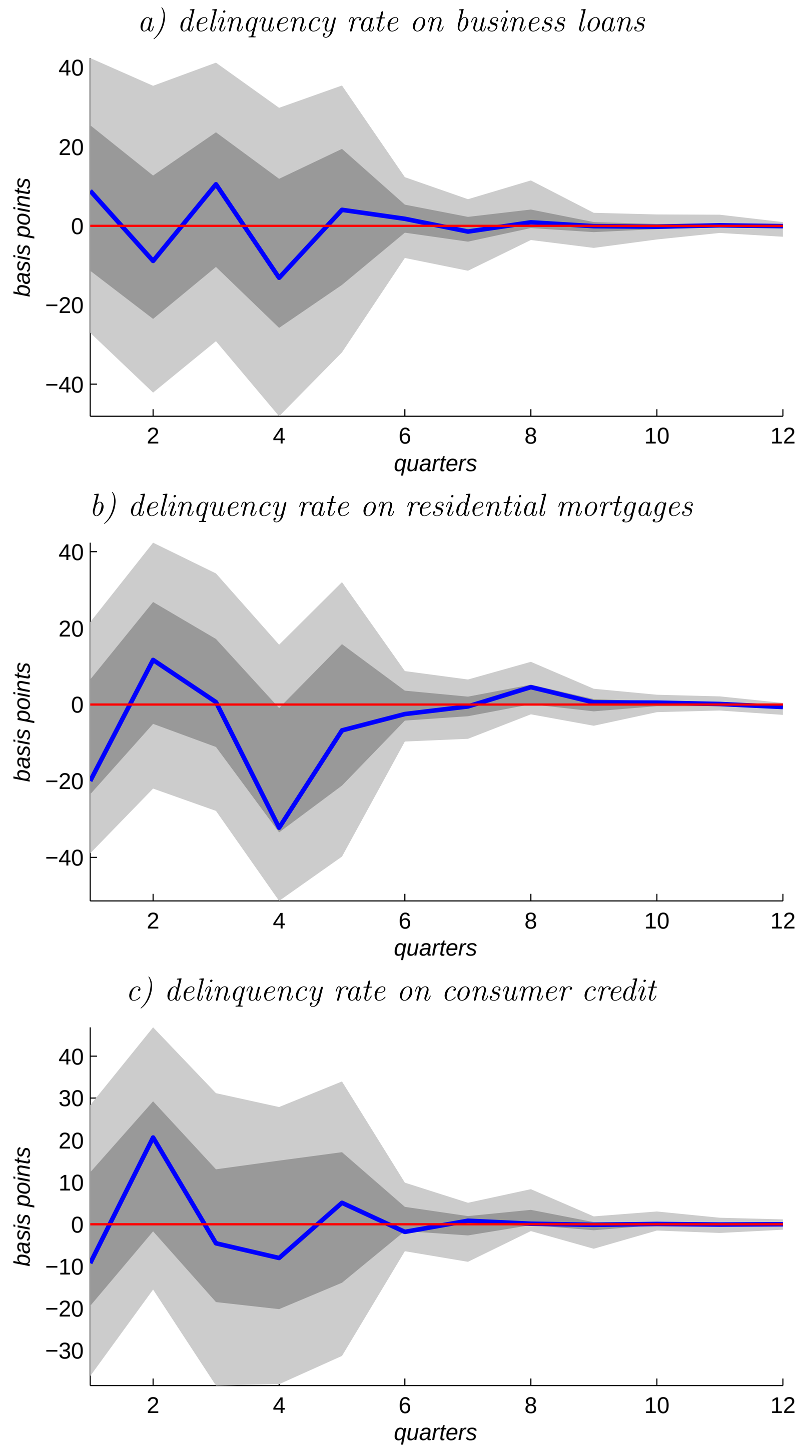
[21] Call n the number of variables in the VAR model, p the number of lags included and $T + p$ the number of available observations. Under this notation, the matrices Y and U have dimension $T \times n$, the matrix X has dimensions $T \times np + 1$, the matrix B has dimensions $np + 1 \times n$, the vector x_t has dimensions $np + 1 \times 1$ and the matrix Ψ has dimensions $n \times n$.

monetary shock that generates a 1% decrease in the federal funds rate in the Medium Bayesian VAR. The vertical axis indicates basis points, the horizontal axis indicates quarters. I report coverage intervals at 68% and 95%, in accordance with the literature. The figure unequivocally shows that there is no statistically significant variation of the aggregate delinquency rates, either at one standard deviation or at two standard deviations. Failing to reject the null hypothesis using a two-sided test does not imply that we would fail to reject the null hypothesis using a one-sided test. Figures 9 and 10 at the end of the paper show that the impulse responses of the delinquency rates remain statistically insignificant with one-sided tests.

The robustness of the results from figure 4 has been assessed along the following dimensions: the use of monetary shocks that do not satisfy the Granger-causality tests from Section 3.2, the use of HP residuals of the delinquency rates instead of the level, the use of seasonally adjusted levels and HP residuals of the delinquency rate, the use of monetary shocks estimated with a time-varying VAR model, the use of single VAR models that include the delinquency rates and the use of delinquency rates disaggregated at the level of top 100 and non top 100 banks in the US. The results are consistent with figure 4 and briefly discussed in the rest of this section. For convenience, the corresponding figures are reported at the end of the paper.

Table 5 lists the number of quarters in correspondence of which there is a statistically significant response at either one standard deviation or two standard deviations. The analysis reports the results corresponding to the five candidate times series of monetary shocks commented on in Section 3.1. The impulse responses corresponding to table 5 are reported in panels a to e of figure 11 at the end of the paper. Overall, we see that the hypothesis of no response of the delinquency rate to a monetary shock is not rejected in some cases when considering one standard deviation, but we almost always fail to reject it when considering two standard deviations. This supports the conclusion drawn from figure 4, which is that delinquency

Figure 4: Impulse response functions (expansionary monetary shock)



Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Table 5: Number of quarters displaying a statistically significant response or delinquency rates

	Levels			*HP residuals*		
	business loans	residential mortgages	consumer credit	business loans	residential mortgages	consumer credit
a) One standard deviation						
VAR	5	0	3	4	0	3
Medium B. VAR	0	1	0	0	2	0
Factor M.	1	3	7	3	2	5
Romer Romer	9	5	1	9	2	1
Futures cont.	3	4	2	2	2	2
b) Two standard deviations						
VAR	1	0	0	1	0	0
Medium B. VAR	0	0	0	0	0	0
Factor M.	0	0	2	1	0	2
Romer Romer	2	0	0	1	0	0
Futures cont.	0	0	0	0	0	0

Notes: the table reports the number of quarters at which the impulse responses of the delinquency rates were found to differ from zero with a statistical significance given by one and two standard deviations.

rates do not respond to a monetary expansion. The analysis was also carried out for the HP residuals of the delinquency rates instead of on their levels. The smoothing parameter used to filter delinquency rates is 1600, but the result was found to hold also using other values. The analysis was replicated for other values of the parameters p_x and p_s in equation (2), with no significant change in the results. When using seasonally adjusted data, VAR shocks implied a statistically significant increase at 5% type-one error in the HP residuals of all delinquency rates four quarters from the shock. The impulse responses are shown in panel f of figure 11 at the end of the paper. This is the only evidence supporting the hypothesis that defaults increase in general equilibrium following an expansionary monetary shock. The response corresponding to the other cases, instead, remained statistically insignificant.

The impulse responses from figure 4 were computed using estimates of monetary shocks that assumed a time-invariant model of the economy. The literature has discussed the fact that the transmission mechanism of monetary policy has changed over time. Primiceri (2005) addresses this issue using a VAR model with time varying parameters and a time-varying variance-covariance matrix of the structural shocks. He shows that the volatility of monetary shocks does change considerably across time, although the impulse responses do not change significantly depending on the exact point in time in which the structural shock is given. The VAR model in his paper includes three variables (the inflation rate, the unemployment rate and the 3-month treasury bill), is estimated for the period between 1963Q3 and 2001Q3 and is identified using the Cholesky structure. Figure 12 at the end of the paper shows the impulse responses computed using the time series of monetary shocks by Primiceri (2005).[22] The result that delinquency rates do not respond in a statistically significant way to a monetary shock are robust to the use of the monetary shocks by Primiceri.[23]

It has been said that the delinquency rates are available only after 1987Q1 (1991Q1 for the delinquency rate on residential mortgages). Four of the five methodologies considered so far delivered monetary shocks estimated in a time period that starts before 1987Q1. The robustness of the results from figure 4 was analyzed also using VAR models that include the delinquency rates. The advantage of this approach is that it avoids the generated regressor problem discussed in Section 3.3. The disadvantage is that it uses a shorter time period for the estimation. I include the three delinquency rates in the VAR one at a time in order to retain degrees

[22] The shocks are obtained from the updated MCMC algorithm by Del Negro and Primiceri (2014). I am grateful to the author for having shared his estimates.

[23] The impulse responses are computed using equation (2) estimated with Bayesian methods, as for figure 4. For simplicity, the coverage intervals reported in figure 12 only reflect the uncertainty in the parameters of equation (2) and not the uncertainty related to different draws of the time series for the structural shocks (see the discussion in Section 3.3). A full assessment of the coverage intervals would deliver wider standard errors, hence strengthening the result of the paper.

of freedom. In all three VAR models estimated I include, in addition to the delinquency rate, the log of real GDP, the log of the GDP deflator and the federal funds. The model is identified with sign restrictions, as the VAR model from Section 3.1 (see Appendix A). Figure 13 at the end of the paper shows the impulse responses of the delinquency rates to a monetary shock that decreases the federal funds rate by 1%. The error bands show 68% and 95% confidence intervals computed with a bootstrap on the reduced form shocks in the VAR. The results are in line with figure 4, and show that one cannot reject the hypothesis that delinquency rates do not respond to a monetary shock.

Last, the analysis was replicated using a different aggregation of the loans corresponding to the delinquency rates used. The analysis so far has used data on loans issued by all commercial banks. The Federal Reserve Bank computes these time series from microeconomic data, which remain confidential. To the best of my knowledge, the only disaggregated time series that is made publicly available regards the delinquency rate on loans issued by the top 100 banks according to assets, and on loans issued by all the other banks. Figures 14 and 15 show that the results remain unchanged using this disaggregation.

5 Conclusions

This paper has addressed the question of whether the risk taking incentives, analyzed extensively by empirical papers including Jimenez *et al.* (2011) and Maddaloni and Peydró (2012) in a partial equilibrium analysis, hold also in general equilibrium. To do so, I used time series analysis on the aggregate delinquency rates on loans issued by all commercial banks in the US. I argued that it is not possible to reject the hypothesis that a monetary shock does not affect delinquency rates. While risk taking incentives might not be strong enough to lead to an aggregate increase in defaults, as suggested in this paper and in Buch *et al.* (2011), they could explain why we fail to reject the hypothesis that defaults increase in response to a monetary expansion. One possible interpretation of the result of the paper is that, even in the event of an increase in risk taking by individual banks, the monetary expansion generates general equilibrium effects that increase firms' profits and households' income, hence pushing up their cash flow.

The analysis of the paper is consistent with the results by Buch *et al.* (2011). They use a FAVAR model augmented with data on the Survey of Terms of Business Lending and argue that the banking sector as a whole does not significantly take on new, more risky loans after a monetary expansion, but small banks do. It would be interesting to know if this result is also consistent with delinquency rates, since delinquency rates include all loans and do not limit the analysis to new loans. The availability of delinquency rates does not shed light on this point, leaving it an open research question.

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6 Appendix A: Specification and identification of the VAR model

The VAR model includes the log of real GDP, the log of the GDP deflator, the federal funds rate, the log of firms' leverage ratio and the log of households' leverage ratio. The inclusion of real GDP, the GDP deflator and the federal funds rate is standard in the empirical literature on monetary policy.[24] The remaining variables capture information on the financial situation of firms and households. The model is :

$$Y_t = \alpha_t + A(L)Y_{t-1} + R_t. \tag{3}$$

The 5×1 vector α_t includes a constant, a linear trend with no time breaks and seasonal dummies. The 5×5 matrix $A(L)$ includes four lags.[25] I estimate the model using least square estimation with data in the period between 1965Q1 and 2007Q2. The choice of 1965Q1 as the first observation of the sample is relatively standard in the literature and is taken from Christiano, Eichenbaum and Evans (1999) and Uhlig (2005).[26] The period after 2007Q2 is excluded from the analysis because it corresponds to a largely exceptional period, compared to the rest of the sample. While the transmission mechanism of monetary policy is likely to have undergone structural breaks and variations during the sample period considered, these variations were found by Primiceri (2005) to have a limited effect on impulse response functions.

The contemporaneous relationships among the variables included in Y_t are cap-

[24] One exception is Canova and De Nicoló (2002), who use the slope of term structure instead of the interbank overnight interest rate.

[25] The inclusion of four lags in the right-hand side is standard, as for instance in Den Haan and Sterk (2011). The Akaike, the Bayesian and the Hannan-Quinn information criteria suggested the use of respectively six, three and three lags. The results remain unchanged. The results from the Granger-causality tests in table 4 remain unchanged when adding up to as many as 12 lags in the VAR.

[26] The sample period included in Olivei and Tenreyro (2007), Coibion (2012) and Canova and De Nicoló (2002) starts in 1966Q1 1970M1 and 1973M1, respectively.

tured by the residuals. This implies that a shock to $R_{t,i}$ does not bear any structural interpretation. I follow the literature and assume that the reduced-form shocks are a linear combination of as many structural shocks S_t:[27]

$$R_t = P \cdot S_t. \tag{4}$$

Since the model is partially identified, the assumption that S_t has dimensions 5×1 is without loss of generality, as the 4 non-monetary shocks in S_t can be viewed as a combination of all non-monetary shocks driving Y_t.

Call Σ the variance-covariance matrix of R_t. The only condition imposed by the data on the estimation of P is

$$\Sigma = P \cdot P'. \tag{5}$$

There are infinite candidate matrices P that satisfy this condition. The literature solves this problem by introducing a limited set of theoretically-motivated assumptions. I follow Canova and De Nicoló (2002) and identify each model using sign restrictions. This approach allows for contemporaneous effects among the variables included in the analysis, which is consistent with Bernanke and Mihov (1998), Canova and De Nicoló (2002) and Canova and Pina (1999).[28] In particular, I gen-

[27] See Cooley and Roy (1985) for a discussion on this point.

[28] Bernanke and Mihov (1998) claim that one cannot rule out contemporaneous effects with quarterly data (as in this paper), and Canova and De Nicoló (2002) and Canova and Pina (1999) reject the hypothesis of no contemporaneous effects as inconsistent with a wide range of theoretical models. While part of the literature still considers it plausible to assume that a monetary shock affects real GDP only after one quarter (see for instance Olivei and Tenreyro (2007)), it is not clear whether a recursive ordering of the specific variables used in this paper would be appropriate. Including firms' balance sheet variables after the federal funds rate would realistically allow the monetary shock to affect firms' financing decisions within the same quarter of the shock, but it would unrealistically impose that the Federal Reserve Bank responds to shocks to the financing condition of firms with at least a lag of 3 months. Including, instead, balance sheet variables before the federal funds rate would realistically allow the Federal Reserve Bank to respond contemporaneously to shocks to the financial situation of firms, but would unrealistically impose that the monetary shock has no effect on firms' balance sheet variables within the first 3 months after the shock. In unreported robustness checks I found that the results do not change when considering a partially recursive structure in which the federal funds rate does not affect real GDP and the GDP deflator contemporaneously, but affects and responds to the other variables included. This partially recursive structure is used for instance by Eickmeier and Hofmann (2012).

erate several orthogonal representations of the data using the following algorithm:

1. generate $L = 1000$ orthogonal 5×5 matrices Q through a combination of reflection and rotation methods and through QR decompositions. Then generate L candidate matrices P_{cand} defined as $P_{cand} = P_{ee} \cdot Q$; P_{ee} stands for the eigenvalue-eigenvector decomposition of Σ (see Appendix B for the details);

2. use $\{P_{cand,l}\}_{l=1}^{L}$ to compute $5 \cdot L$ sets of impulse responses, i.e. L responses to a structural shock to each of the 5 elements in Y_t, one at the time;[29]

3. identify an expansionary monetary shock by ruling out models that display either an increase in the interest rate(s) during the first two quarters, a decrease in output during the first two quarters or a price puzzle that lasts more than three years. This set of restrictions is very general and broadly in line with the existing literature;[30] The algorithm generates 165 sign-restricted representations of the data. For simplicity, I will only use the median target

[29]I do not impose that the monetary shock enters as the last element of S_t in the identification through sign restrictions. As argued by Canova and De Nicoló (2002), what makes an impulse response a plausible candidate for the true impulse response to a monetary shock is not that the shock is given to the equation of the policy rate, but that it generates responses consistent with what economic theory associates to a monetary shock. This distinction is relevant because, in this application, generating models by only giving a shock to the third element of S_t would significantly reduce the set of impulse responses, despite the wide range of orthogonal matrices Q considered. The median target representation happens to correspond to a shock to the third element of Y_t, which is the equation of the federal funds rate. In other specifications of the model the median target did not correspond to a shock to the third element of R_t.

[30]The restrictions on real GDP and the GDP deflator reduce the risk of mistakenly capturing aggregate supply shocks as monetary shocks, while the restriction on the federal funds rate reduces the risk of capturing real demand shocks and ensures that the monetary shock is expansionary. In principle, the identified reduced-form shock could be driven by a negative money demand shock instead of an expansionary monetary shock. Unreported robustness checks found that the results hold when including the monetary aggregate M1, restricted to decrease for the first two quarters. Compared to the literature, the set of restrictions imposed is very general. Uhlig (2005) uses monthly data and identifies a monetary contraction by restricting the impact effect and the effect on the first 5 lags to display a decrease in prices, a decrease in non-borrowed reserves and an increase in the federal funds rate. Eickmeier and Hofmann (2012) identifies a monetary contraction by restricting both the impact effect and the effect on the first lag to display an increase in the federal funds rate, a decrease in real GDP, a decrease in the GDP deflator and a decrease in the M1 monetary aggregate. Canova and De Nicoló (2002) base their restrictions on pairwise correlations across variables instead of the sign of the impulse response, and impose restrictions up to the lag that ensures the identification of only one structural representation of the data.

sign-restricted representation á la Fry and Pagan (2011).[31]

7 Appendix B: algorithm to generate orthogonal matrices

The matrix P_{ee} used to generate 5000 matrices P_{cand} in Appendix A is computed by first decomposing the matrix Σ into a matrix A of its eigenvectors and a matrix B listing its eigenvalues along the diagonal. The matrix P_{ee} is then defined as $A * B^{0.5}$ and it satisfies the condition $P_{ee}P'_{ee} = \Sigma$ by construction. Given P_{ee}, the computation of L candidate matrices P_{cand} that satisfy condition 5 from Section 3.1 requires the generation of L 5×5 matrices Q satisfying condition $QQ' = I$, where 5 is the number of variables included in Y_t of equation (3). Since there are infinite possible representations of the data, the richer the set of Qs considered, the more likely it is that the true matrix P^* that has generated the data is actually replicated in the model.

I start by generating $3 \cdot M$ 2×2 orthogonal matrices. Rubio-Ramirez, Waggoner and Zha (2005) generate orthogonal matrices using the QR decomposition, while Canova and De Nicoló (2002) use Givens rotation.[32] In this paper I use both approaches. QR orthogonal matrices R_{qr} are generated using the orthogonal-triangular decomposition of a matrix. This operation decomposes a matrix $\tilde{A}$ into an orthogonal $\tilde{q}$ matrix and an upper triangular r matrix such that $\tilde{A} = \tilde{q}\tilde{r}$. To generate M QR orthogonal matrices I extract M 2×2 random matrices from a standard normal distribution. To each of them I apply the QR decomposition and

[31] The main difference between the median model and the median target model is that the former potentially switches from one representation to another. To avoid this, the median target is defined as the single representation that is closest to the median, after accounting for the appropriate normalization. See Fry and Pagan (2011) for a discussion.

[32] Fry and Pagan (2011) give an introduction to orthogonal matrices in sign-restricted models. See Paustian (2007) and Castelnuovo (2012) for an application of sign restrictions to New Keynesian models in a Monte Carlo experiment.

save the generated orthogonal matrices $\tilde{q}$ in the set of R_{qr}. Rotation matrices R_{rot} are instead defined as

$$R_{rot}(\theta) = \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix},$$

with $\theta \in [0, 2\pi]$.[33] The orthogonality of R_{rot} follows from the fact that $cos(\theta)^2 + sin(\theta)^2 = 1$ for $\forall\theta \in [0, 2\pi]$. To generate M rotation matrices I construct a grid of M points for the parameter θ in the space $[0, 2\pi]$ and then compute $R_{rot}(\theta)$ for each grid point. I extend Canova and De Nicoló (2002) and compute also M reflection matrices, which are defined as

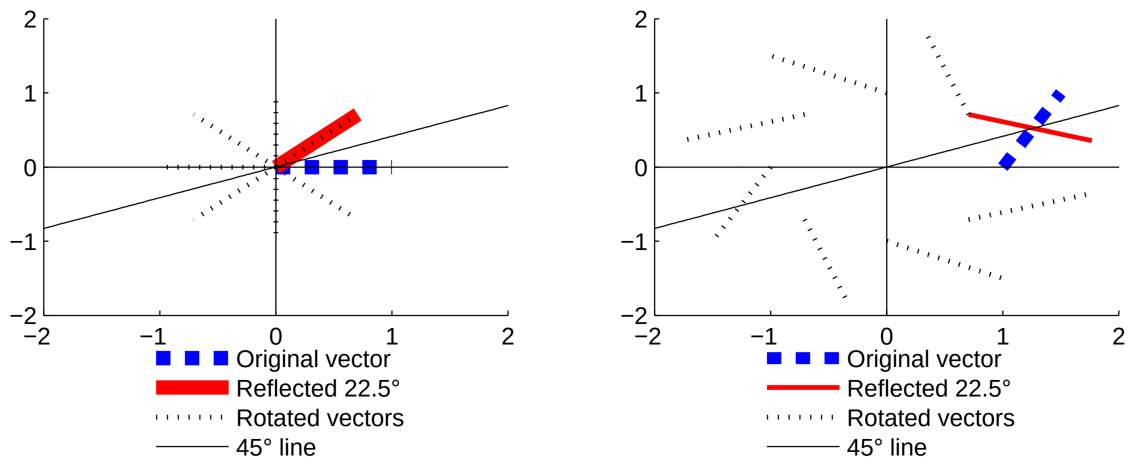
$$R_{refl}(\theta) = \begin{pmatrix} cos(\theta) & sin(\theta) \\ sin(\theta) & -cos(\theta) \end{pmatrix}.$$

Rotation matrices generate a vector $y = x \cdot R_{rot}$ such that the vector x is rotated clockwise (or a vector $y = R_{rot} \cdot x$ such that the vector x is rotated counter-clockwise) by an angle of θ about the origin of the Cartesian coordinate system. Reflection matrices, instead, generate a vector $y = x \cdot R_{refl}$ such that the vector x is reflected around the line that goes through the origin of the Cartesian coordinate system with slope $\theta/2$. While for some special cases the vectors coincide, the set of vectors generated through rotations does not fully overlap with the set of vectors generated by reflection matrices, as illustrated intuitively with the exercise in figure 5. It hence improves the analysis to account for both rotations and reflections in order to extend the set of structural representations replicated by the model.

Last, the $3 \cdot M$ 2×2 $R = \{R_{qr}, R_{rot}, R_{refl}\}$ orthogonal matrices generated through rotations, reflections and QR decompositions need to be combined into 6×6 orthogonal matrices. This is achieved using a multiplication similar in spirit

[33]Canova and De Nicoló (2002) construct the grid on $\theta \in (0, \pi/2)$.

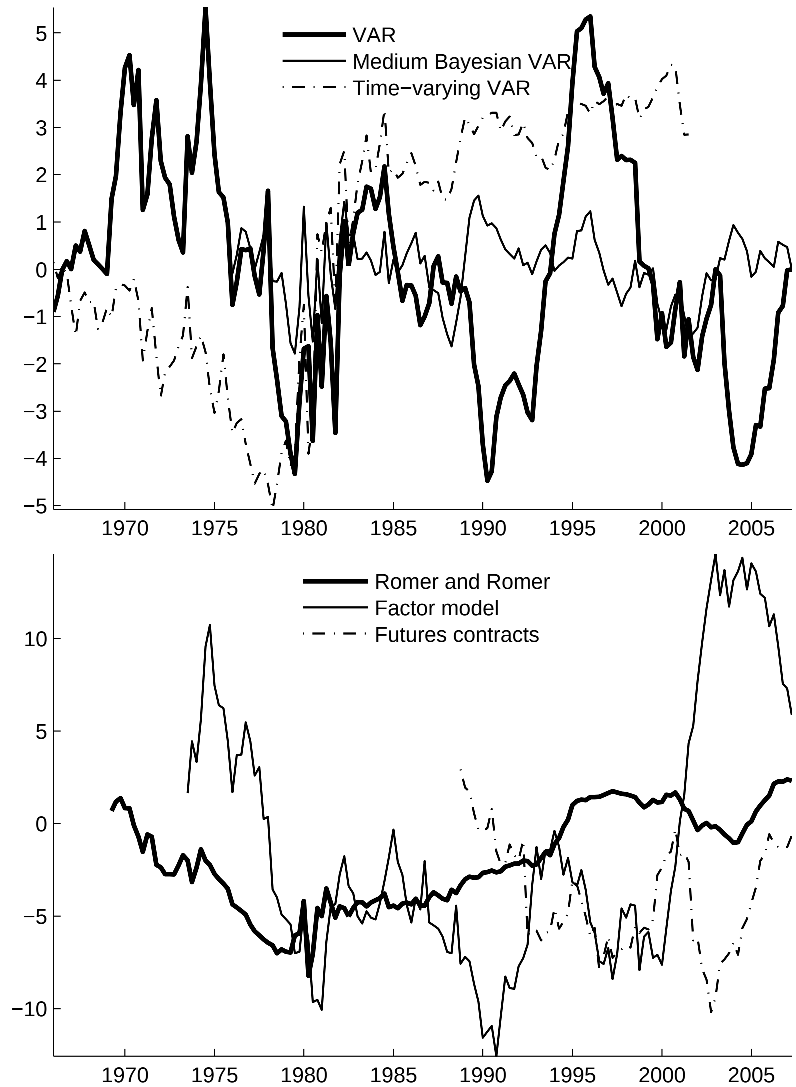
Figure 5: Rotation vs. reflection matrices



Notes: The left graph shows that the reflection around the 22.5 degree line of the vector between point (0,0) and point (1,0) (blue dashed line) gives the vector between point (0,0) and (0.7071, 0.7071) (red solid line). The same vector, though, is generated by a counter-clockwise rotation of the original vector of 45 degrees, or a clockwise rotation of the original vector of 315 degrees (black dotted line). The right graph, instead, shows the same case for the vector between (1,0) and (1.5,1). Reflection around the 22.5 degree line gives the vector between point (0.7071, 0.7071) and point (1.7678, 0.3536). No rotation can generate the same transformation achieved by the reflection.

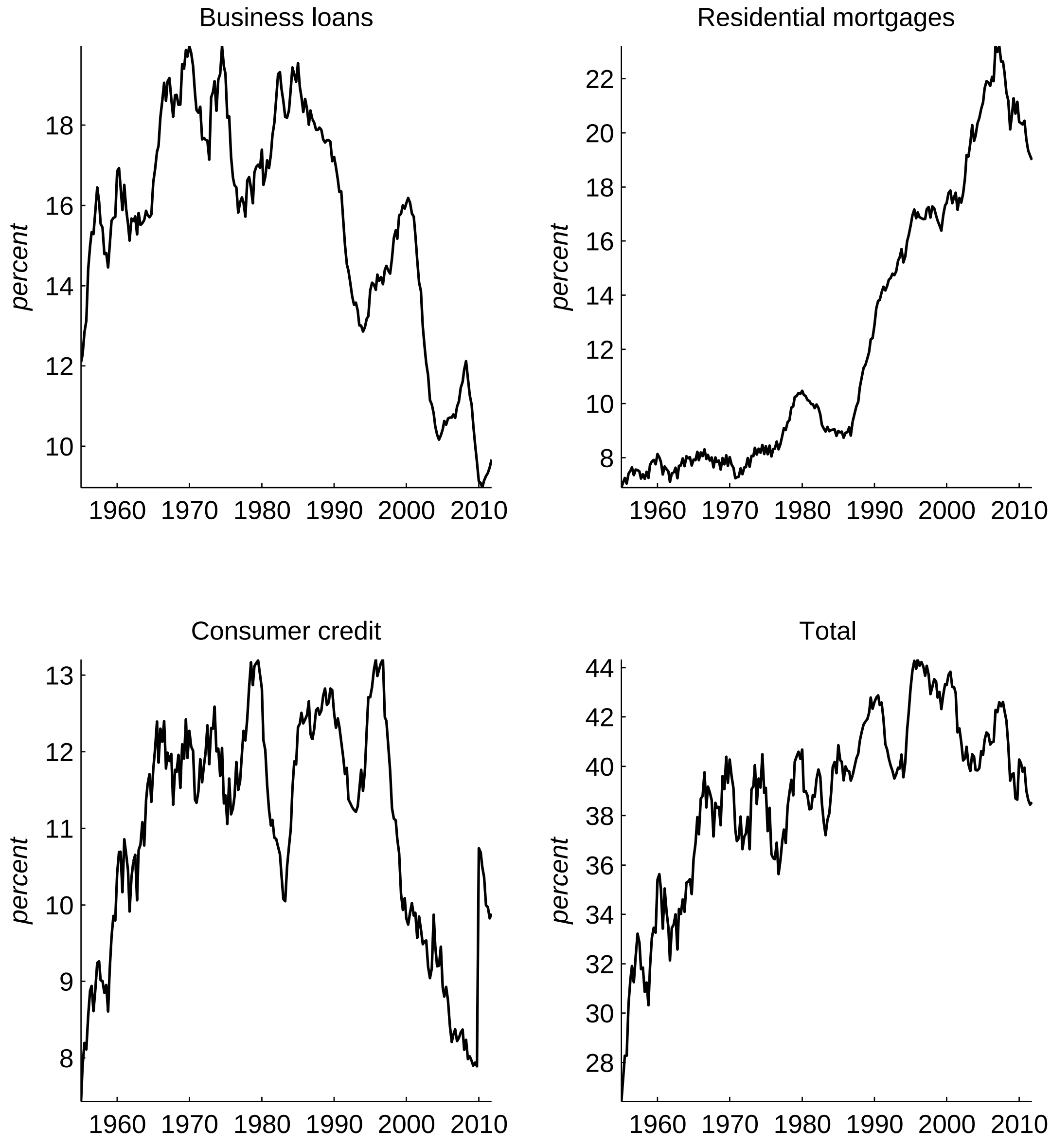
to Canova and De Nicoló (2002). For each of the L 5×5 orthogonal matrices $Q(i)$ that needs to be generated, extract 5 2×2 matrices R. Generate then a 5×5 identity matrix E. Select two of its columns a and b randomly and rewrite 4 of the elements of E as $E(a,a) = R(1,1)$, $E(a,b) = R(1,2)$, $E(b,a) = R(2,1)$ and $E(b,b) = R(2,2)$. The matrix Q is obtained by multiplying the 5 matrices E obtained. By construction Q is an orthogonal matrix and can be used to generate alternative structural representations of the data. Unreported robustness checks found that the results are not affected by the exact value of M as long as it is not too small. In the paper, M is set equal to 500.

Figure 6: Cumulative monetary shocks



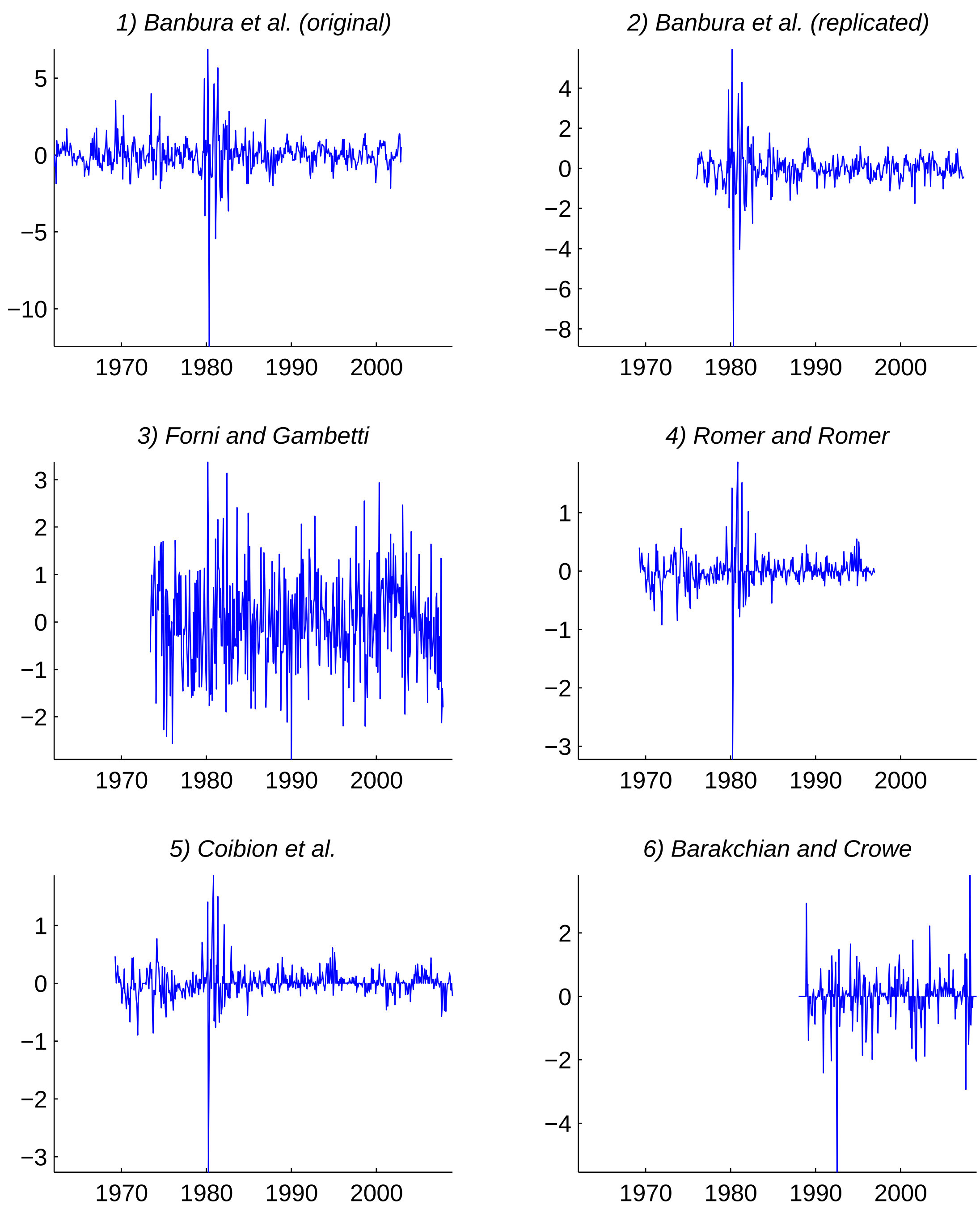
Notes: The figure shows the cumulative monetary shocks for the five methodologies discussed in Section 3.1, as well as for the methodology by Primiceri (2005) commented on in Section 4.

Figure 7: Ratio of loans relative to banks' total assets



Notes: The figure shows the ratio of the three types of loans considered relative to the size of banks' assets. The variables are computed using data from the Flow of Funds.

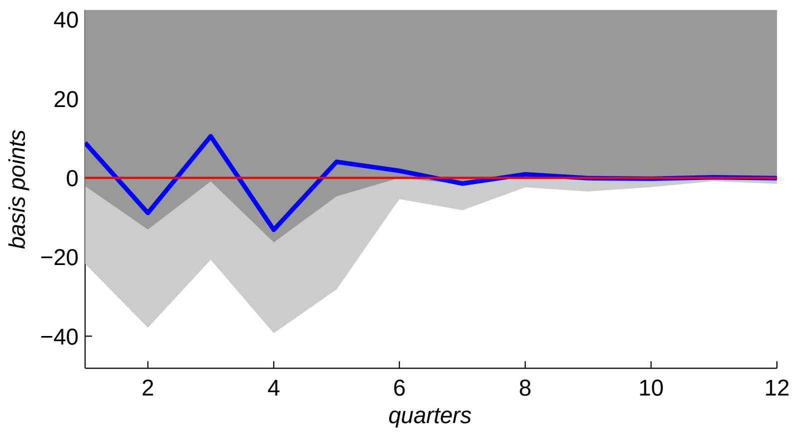
Figure 8: Monthly monetary shocks corresponding to table 6



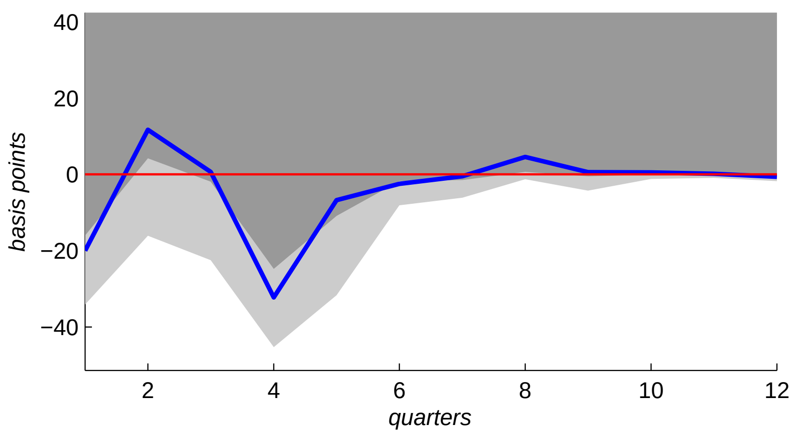
Notes: The table shows the 6 time series of the monthly monetary shocks whose autocorrelations are analyzed in table 6.

Figure 9: Same as figure 4 but one sided coverage interval

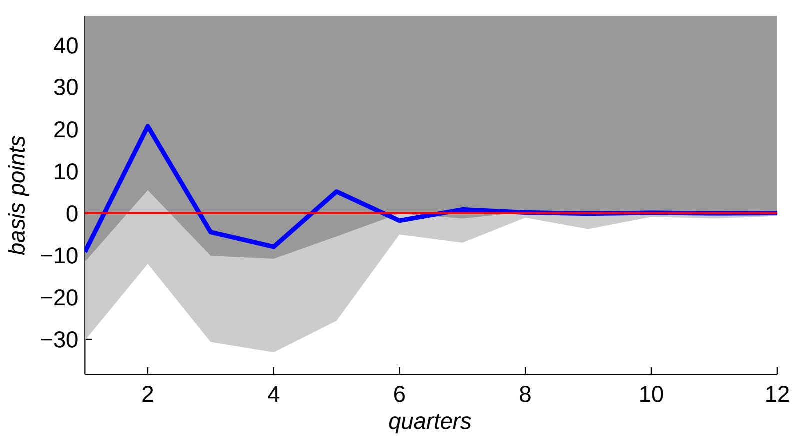
a) delinquency rate on business loans



b) delinquency rate on residential mortgages



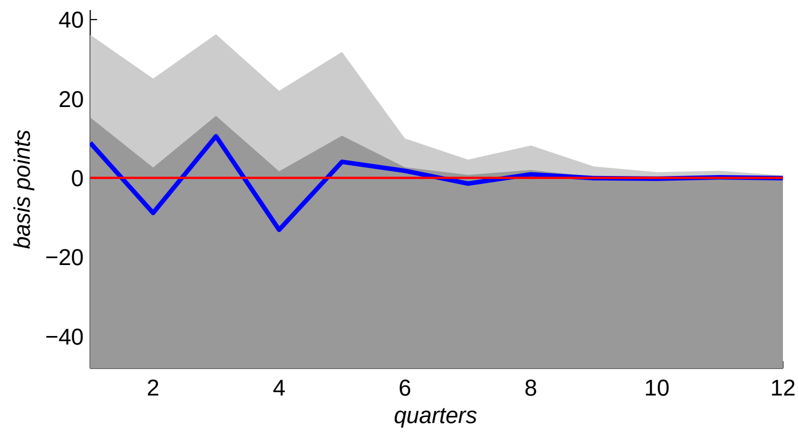
c) delinquency rate on consumer credit



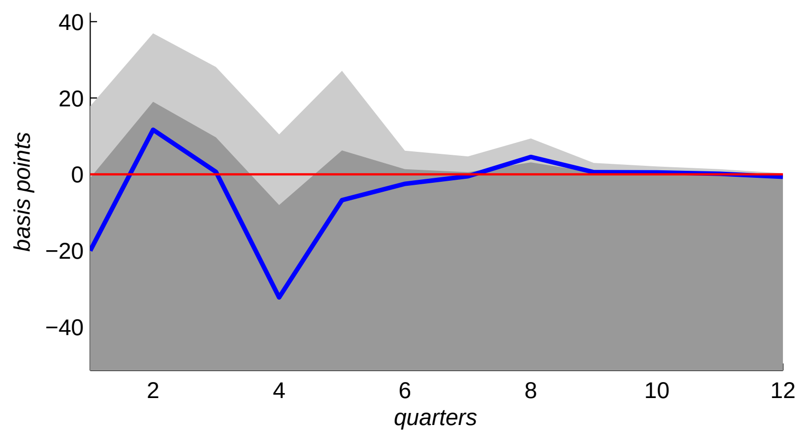
Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Figure 10: Same as figure 4 but one sided coverage interval

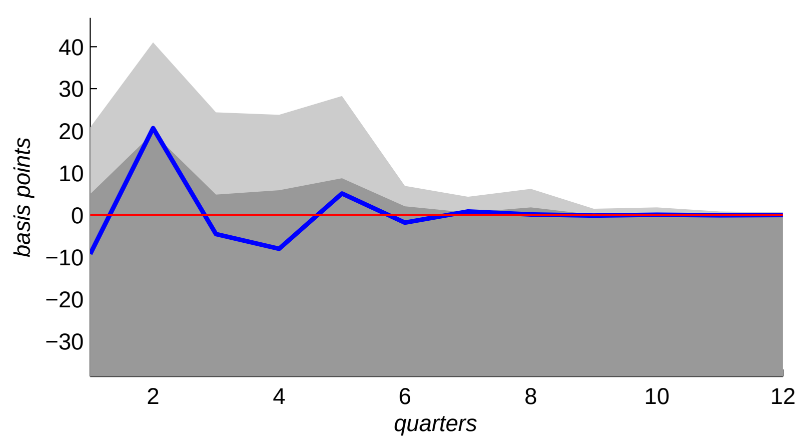
a) delinquency rate on business loans



b) delinquency rate on residential mortgages



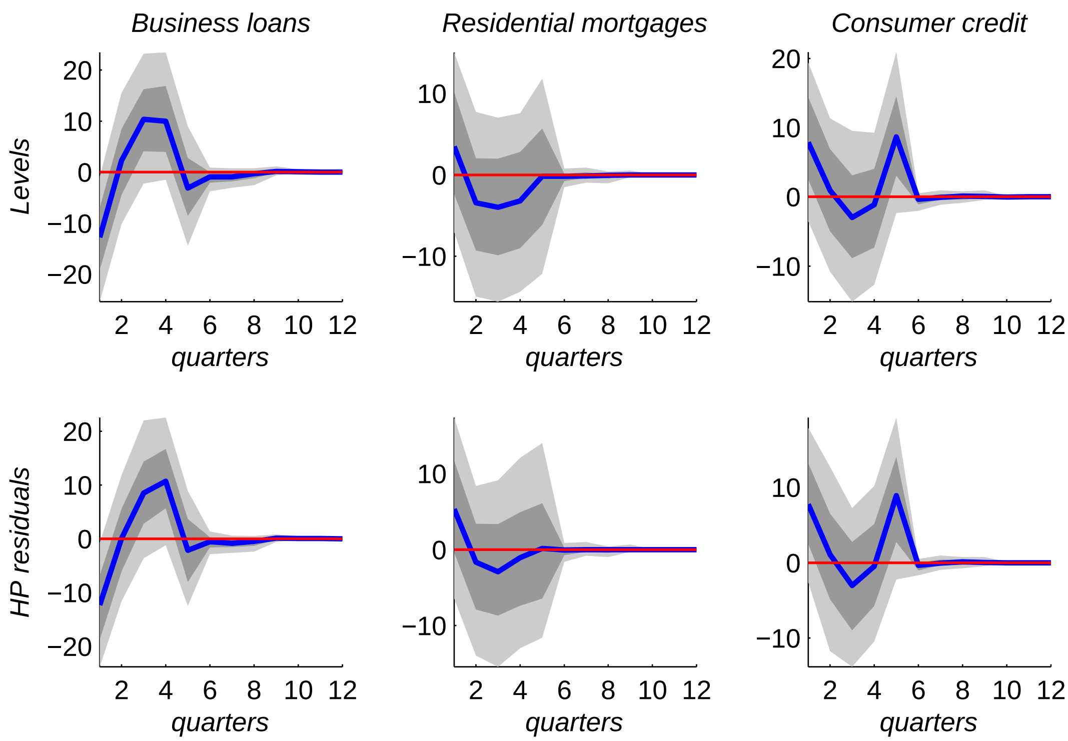
c) delinquency rate on consumer credit



Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Figure 11: Impulse responses corresponding to table 5

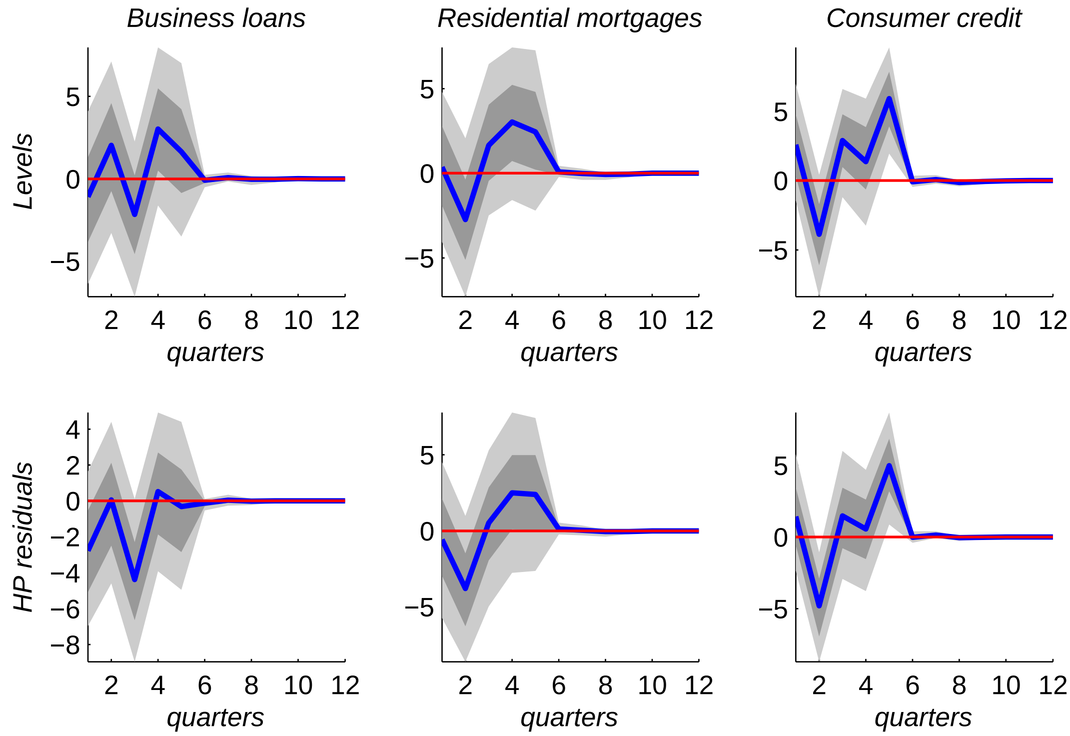
a) on VAR shocks



b) on Medium Bayesian VAR shocks

(continues)

c) on shocks from Factor Model



d) on Romer and Romer shocks

(continues)

e) on shocks from Futures Contracts

f) same as panel a) but with seasonally adjusted data

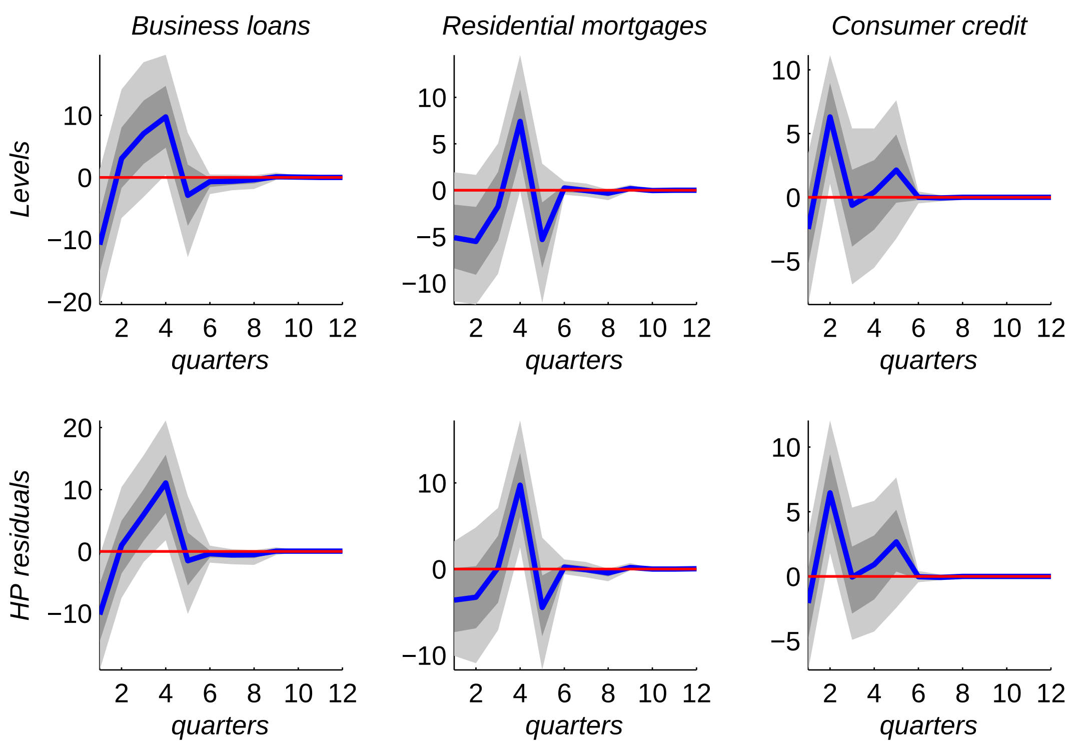
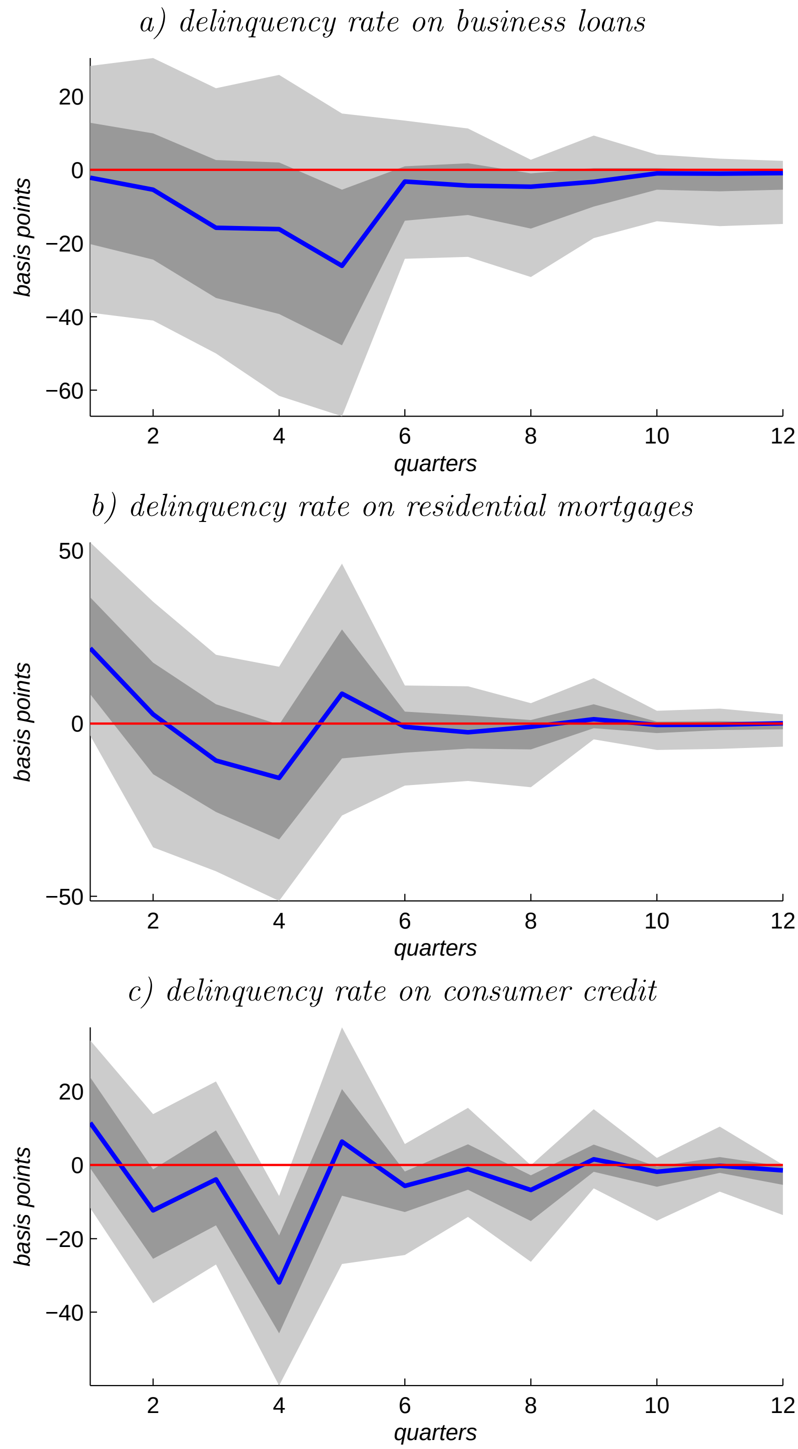
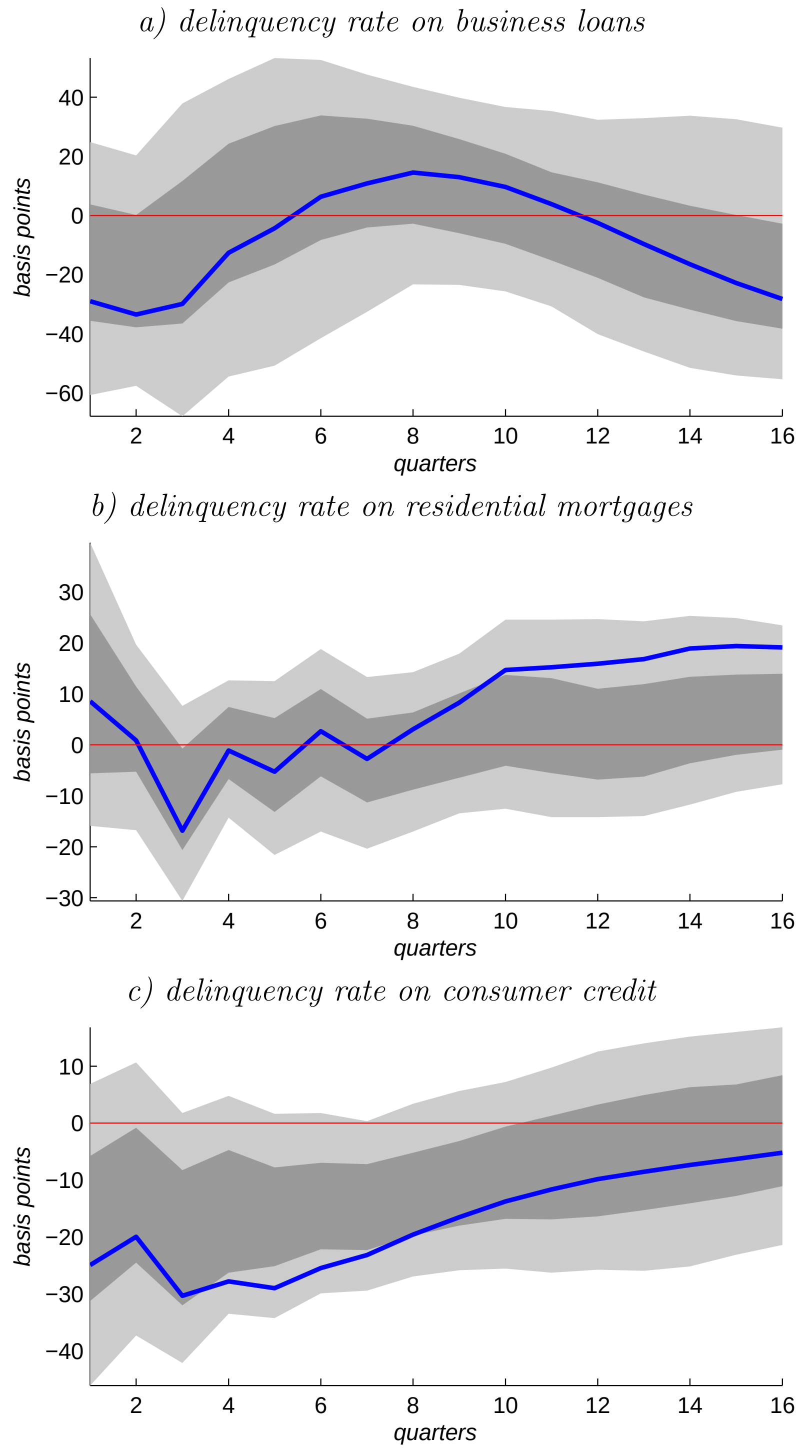


Figure 12: Impulse responses using monetary shocks from Primiceri (2005)



Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

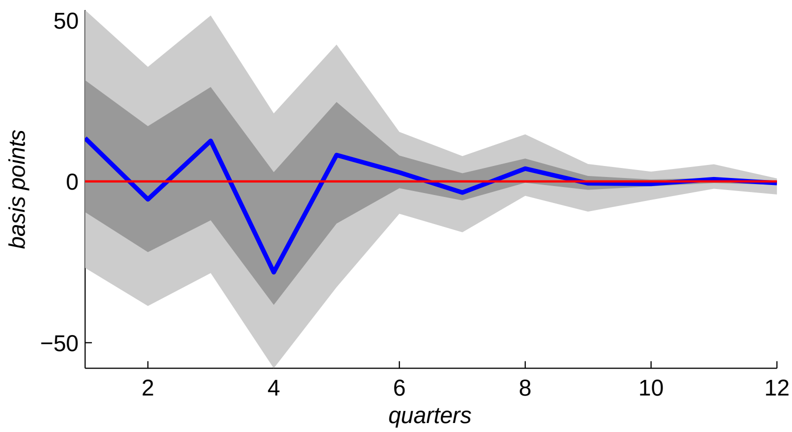
Figure 13: Impulse responses from small VARs with delinquency rates



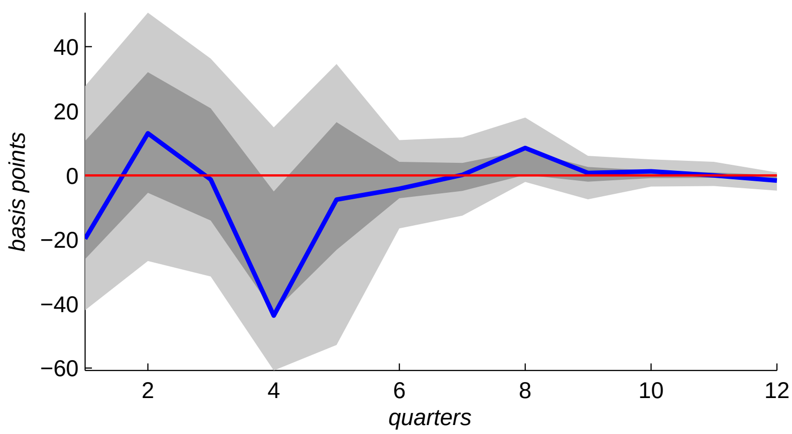
Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Figure 14: Impulse responses of delinquency on loans by top 100 banks

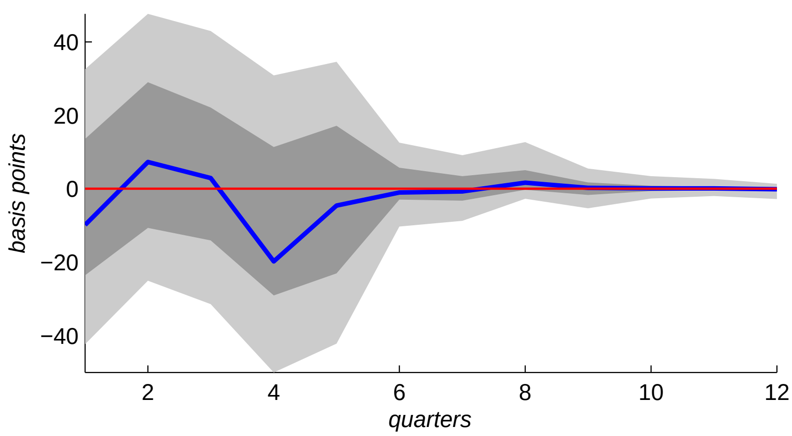
a) delinquency rate on business loans



b) delinquency rate on residential mortgages



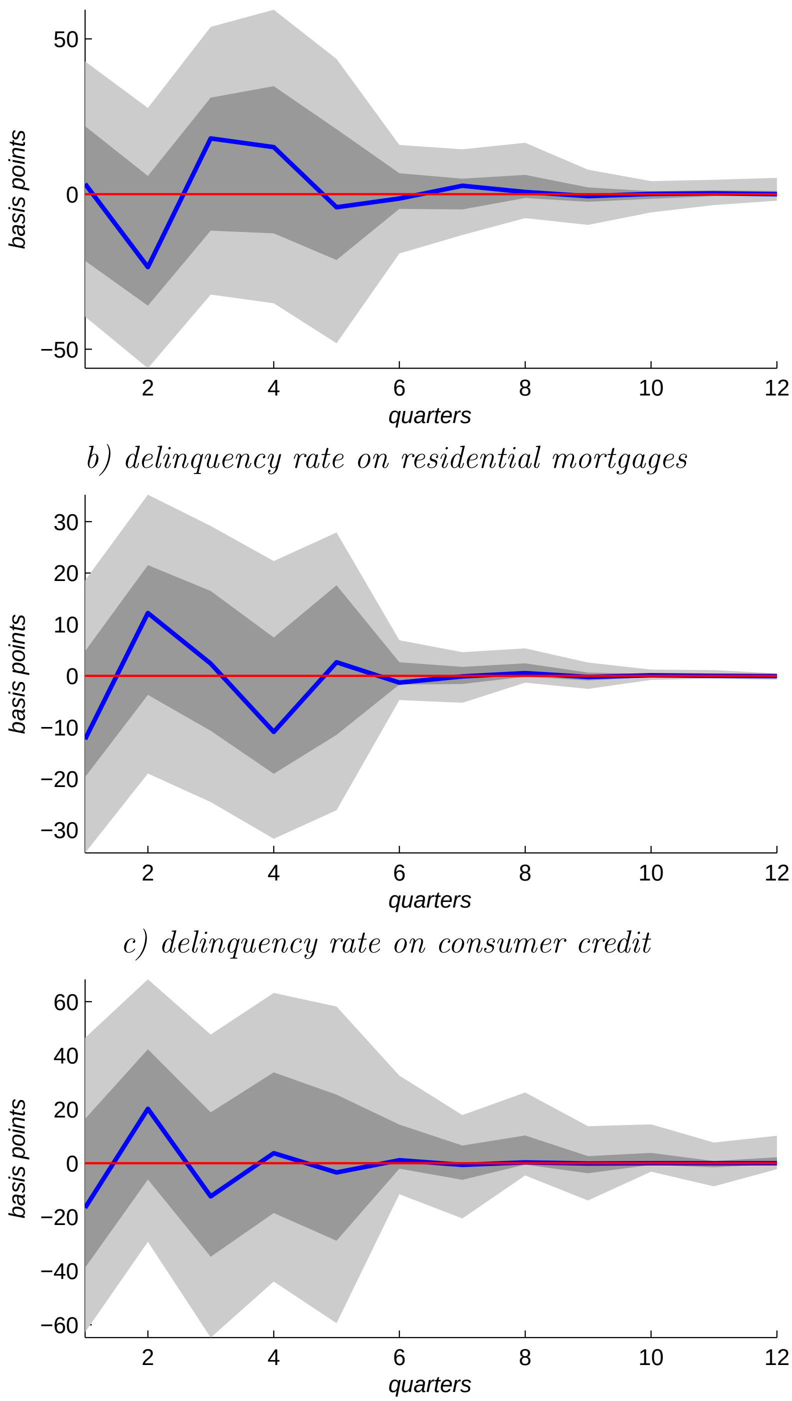
c) delinquency rate on consumer credit



Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Figure 15: Impulse responses of delinquency on loans by non top 100 banks

a) delinquency rate on business loans



Notes: The figure shows the impulse response functions of the delinquency rates in response to a monetary shock that decreases the federal funds rate by 1%.

Table 6: Analysis of autocorrelations

	Full sample				*After 1987*			
	monthly		quarterly		monthly		quarterly	
	$k = 4$	$k = 8$	$k = 4$	$k = 8$	$k = 4$	$k = 8$	$k = 4$	$k = 8$
1) Banbura, Giannone and Reichlin (2010), 1962M2-2002M12								
$\rho(k)$	-0.082	0.001	-0.109	0.047	0.185	-0.063	-0.313	0.127
pvalue	0.007	0.000	0.084	0.019	0.000	0.000	0.000	0.000
2) Banbura, Giannone and Reichlin (2010), replicated in this paper, 1976M1-2007M6								
$\rho(k)$	-0.062	-0.018	-0.124	-0.009	0.138	-0.069	-0.123	-0.164
pvalue	0.001	0.000	0.068	0.000	0.000	0.000	0.016	0.013
3) Forni and Gambetti (2010), 1973M6-2007M11								
$\rho(k)$	0.028	0.043	0.029	0.058	0.071	0.040	0.073	0.072
pvalue	0.743	0.641	0.128	0.384	0.601	0.352	0.009	0.047
4) Romer and Romer (2004), 1969M4-1996M12								
$\rho(k)$	0.007	0.117	-0.130	0.055	-0.054	0.009	0.212	-0.0372
pvalue	0.373	0.005	0.046	0.093	0.199	0.420	0.516	0.3520
5) Romer and Romer, extended by Coibion et al. *(2012), 1969M4-2008M12*								
$\rho(k)$	0.014	0.118	-0.090	0.055	0.087	0.070	0.149	-0.026
pvalue	0.130	0.000	0.082	0.078	0.000	0.000	0.000	0.000
6) Barakchian and Crowe (2013), 1988M1-2008M12								
$\rho(k)$	0.027	0.080	-0.037	-0.245	0.027	0.080	-0.037	-0.2453
pvalue	0.245	0.407	0.654	0.107	0.245	0.407	0.654	0.1075

Notes: the autocorrelation $\rho(k)$ is reported for 4 lags and for 8 lags. The pvalue reported corresponds to the Box and Ljung test of no autocorrelation. The statistics are reported both for the original monthly series and for the transformation into quarterly data as from Section 3.1. The monthly monetary shocks corresponding to this table are shown in figure 8.